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A model for real-time failure prognosis based on hidden Markov model and belief rule base

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ABSTRACT

As one of most important aspects of condition-based maintenance (CBM), failure prognosis has attracted an increasing attention with the growing demand for higher operational efficiency and safety in industrial systems. Currently there are no effective methods which can predict a hidden failure of a system real-time when there exist influences from the changes of environmental factors and there is no such an accurate mathematical model for the system prognosis due to its intrinsic complexity and operating in potentially uncertain environment. Therefore, this paper focuses on developing a new hidden Markov model (HMM) based method which can deal with the problem. Although an accurate model between environmental factors and a failure process is difficult to obtain, some expert knowledge can be collected and represented by a belief rule base (BRB) which is an expert system in fact. As such, combining the HMM with the BRB, a new prognosis model is proposed to predict the hidden failure real-time even when there are influences from the changes of environmental factors. In the proposed model, the HMM is used to capture the relationships between the hidden failure and monitored observations of a system. The BRB is used to model the relationships between the environmental factors and the transition probabilities among the hidden states of the system including the hidden failure, which is the main contribution of this paper. Moreover, a recursive algorithm for online updating the prognosis model is developed. An experimental case study is examined to demonstrate the implementation and potential applications of the proposed real-time failure prognosis method.

1. Introduction

With a growing demand for higher operational efficiency and safety in industry, failure prognosis, as one of the most important aspects in condition-based maintenance (CBM), has attracted considerable attention world-wide in the past three decades (Lu and Saeks, 1979; Jardin et al., 2006; Wang and Christer, 2000; Wang, 2002; Wang and Zhang, 2005; Dong and He, 2007).

The failure to be studied in this paper is often not observable directly (hidden), but it can be indirectly observed by some observable variables. For example, wear is a generic term for describing the deterioration of maintained plant. In most plant systems, wear is not directly observable, and can only be assessed via other measured condition information data such as metal concentrations or oil debris (Wang, 2007a). In this paper, the definition of failure is: if the probability of failure exceeds a limit, the system is considered to be in a failed state.

To predict the hidden failure, several failure prognosis methods such as filter based, qualitative knowledge based and hidden Markov model (HMM) based methods have been proposed (Bunks and McCarthy, 2000; Chen and Trivedi, 2005; Chen et al., 2005; Zhou et al., 2006; Wang, 2007a,b).

If the mathematical model or the statistical model of a system is known, the filter based methods including Kalman predictor (Yang and Liu, 1999; Yang, 2003), strong tracking predictor (Chen and Zhou, 2001), fuzzy Kalman predictor (Zhou et al., 2008) and particle predictor (Chen et al., 2005), can predict the hidden failure by estimating system states or parameters. If any qualitative knowledge about a system is known, the qualitative knowledge based methods, such as an expert system based method (Angeli and Chatzinikolaou, 2002) and Petri net based method (Yang and Liu, 1998), can also predict the failure. If the monitored observations are available, the HMM based methods can

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be used to predict the hidden failure by estimating the parameters of the HMM (Baruah and Chinnam, 2003; Zhang et al., 2005; Dong and He, 2007; Wang, 2007a,b).

There are at least two reasons why a HMM can predict failures. Firstly, a HMM is capable of characterizing a doubly embedded stochastic process with an underlying unobservable (hidden) stochastic process that can be linked to another set of stochastic processes. Thus, the failure can be treated as a hidden process and can be observed through system outputs. Secondly, there have been many publications on how to estimate the parameters of a HMM, which provide the required theoretical foundation (Rabiner, 1989; Ying et al., 2000; Lee et al., 2004; Zhang et al., 2005; Li et al., 2007).

But there are some shortcomings inherent to the above failure prognosis methods. The filter based methods can estimate the hidden failure represented by the system states or parameters. Unfortunately, they are not applicable to the cases where the mathematical models or the statistical models of complex systems are difficult to obtain. Although the qualitative knowledge based methods do not need the mathematical models, they may lead to combinatorial explosion and inaccurate prediction if systems are complex.

In addition to the above, there exist at least two problems in the current offline trained HMM based methods. Firstly, the failure prognosis model is trained offline (Zhang et al., 2005; Dong and He, 2007). This means that once the model is trained using historical data (Dong and He, 2007), the model parameters are fixed. However, it is possible in practice that new failure processes with different characteristics from the historical ones may occur and the trained model cannot reflect the new failure processes accurately. Therefore, the offline trained HMM are not applicable in those cases. Secondly, the offline training processes are often time consuming (Zhang et al., 2005; Dong and He, 2007), which is disadvantageous when real-time prognosis is required.

In order to solve the above two problems and meet the urgent need for developing approaches for fast and precise prognosis required by the next generation of diagnostic and prognostic systems (Jardin et al., 2006), online updating methods of HMM for real-time failure prognosis have been studied (Lin and Makis, 2002; Wang and Christer, 2000; Wang, 2007a,b). However, environmental factors are not considered in the above HMM based offline and online algorithms.

Environmental factors may play an important role in a failure process. For example, in a power system, a failure process may be affected by windy and stormy weather (Tanrioven et al., 2004). Some systems may fail to operate properly when noise, dusts, and vibration progressively develop. In order to reflect the relationships between environmental factors and a failure, a certain and accurate model is used by Wang and Hussin (2009). However, due to possible uncertain and nonlinear relationships between environmental factors and a failure process, an accurate model could be difficult to establish. What is available to us normally is expert knowledge usually in qualitative form and partial historical information about these relationships. Then the question is how the qualitative knowledge and the quantitative information can be used to improve the accuracy of failure prognosis. We propose to adopt the belief rule base (BRB) methodology developed recently by Yang et al. (2006, 2007) for the following reason.

The BRB methodology is described as being capable of capturing the relationships between system inputs and outputs that could be discrete or continuous, complete or incomplete, linear or nonlinear, non-smooth, or their mixture (Yang et al., 2006, 2007). It can also process incomplete or vague information. Some offline optimization models and recursive algorithms for training the BRB parameters have been proposed (Yang et al., 2007; Xu et al., 2007; Zhou et al., 2009). Moreover, a sequential learning algorithm for updating the BRB structure and parameters at the same time has also been developed (Zhou et al., 2010).

By combining the capabilities of the HMM and the BRB methodology, a new prognosis model, named as HMM–BRB based model, is proposed here to predict the hidden failure real-time, even under the influences from changes of environmental factors. The HMM is used to capture the relationships between the hidden failure and the monitored observations, and the BRB is to represent the relationships between the environmental factors and the transition probabilities among the hidden states of the system including the hidden failure. Based on expert knowledge, the use of the BRB to model the relationships between the environmental factors and failure process is the main contribution of this paper.

This paper is organized as follows. In Section 2, the problem of real-time failure prognosis is formulated and defined. In Section 3, a HMM–BRB based algorithm is proposed to predict the hidden failure real-time under the influences from changes of environmental factors. An experimental case study is presented to verify the proposed algorithm in Section 4. The paper is concluded in Section 5.

2. Problem formulation

In this section, the notations that will be used in this paper are given firstly. Then the problem formulation of real-time failure prognosis is presented. Finally, a new model composed of hidden Markov model (HMM) and belief rule base (BRB), named as HMM–BRB based model, is constructed to represent a real world system under the influences from changes of environmental factors.

2.1. Notations

The notations that will be used in this paper are listed as follows:

S	key parameter that can reflect the running condition of the system;
t	discrete-time index;
N	number of hidden state in the HMM;
$s_1, \ldots, s_N$	N hidden states of the HMM;
$\mathbf{y}(t)$	monitored observation of the system at time instant t;
$\mathrm{Prob}(\cdot\|\cdot)$	conditional probability;
$\boldsymbol{\pi}(t) = [\pi_1(t), \ldots, \pi_N(t)]^T$	probability vector of the hidden states at time instant t;
$\mathbf{Q} = [Q_1, \ldots, Q_W]^T$	parameter vector of the HMM–BRB based model;
$\boldsymbol{\lambda}(t) = [\lambda_{i,j}(t)]_{N\times N}$	transition probability matrix between the N hidden states at time instant t;

$b_i(t)$	probability of observing $\mathbf{y}(t)$ when $S(t) = s_i$;
$\mathbf{b}(t) = [b_1(t), \ldots, b_N(t)]^T$	vector composed of $b_1(t), \ldots, b_N(t)$;
μ_i	expectation of the hidden state s_i $(i = 1, \ldots, N)$;
σ	variance of the hidden state s_i $(i = 1, \ldots, N)$;
Δt	forecasting step;
$\mathbf{\Omega}(t)$	all the available information about the system up to time instant t;
P_{th}	pre-set threshold of failure;
$u_1, \ldots, u_M$	antecedent attributes of the BRB_i, i.e., environmental factors;
$\mathbf{u}(t) = [u_1(t), \ldots, u_M(t)]^T$	vector composed of $u_1(t), \ldots, u_M(t)$ at time instant t;
A_m^k	referential value of the mth antecedent attribute in the kth rule;
$\mathbf{A}_m = \{A_{m,j_m}, j_m = 1, \ldots, J_m\}$	a set of referential values for the mth antecedent attribute;
θ_k^i	relative weight of the kth rule in the BRB_i;
$\bar{\delta}_1^i, \bar{\delta}_2^i, \ldots, \bar{\delta}_M^i$	relative weights of M antecedent attributes used in the kth rule;
$D_{i,j}$	action which represents transition from hidden state s_i to hidden state s_j;
$\beta_{j,k}^i$	belief degree assessed to the jth consequent $D_{i,j}$ in the kth rule of the BRB_i;
L	rule number of the BRB_i;
$\mathbf{V}_i$	parameter vector of the BRB_i;
$f(\cdot\|\cdot)$	conditional probability density;
W	dimension of the parameter vector $\mathbf{Q}$;
$\mathbf{H}$	constraint set composed of constraints that the vector $\mathbf{Q}$ should satisfy;
$\prod_{\mathbf{H}}\{\cdot\}$	projection onto constraint set $\mathbf{H}$;
$\{\varepsilon(t)\}$	a sequence of step sizes at time instant t;
$\Gamma(\mathbf{Q}(t))$	derivative with respect to the parameter vector $\mathbf{Q}$ at time instant t;
$\mathbf{z}_d(t)$	partial derivative of $\boldsymbol{\pi}(t)$ with respect to $Q_d(t)$;
$E(\cdot)$	expectation of a random variable;
$U(s_i)$	utility of the evaluation grade s_i.

2.2. Problem formulation of real-time failure prognosis

Problem formulation of real-time failure prognosis is described as follows:

(1) Assume that there is a key parameter S that can reflect the running condition of a system. Furthermore, it is assumed that the system will progressively develop into an actual failure after it deviates from the normal running state, i.e., the system is in a defective stage but still working during this stage (Wang, 2007a). If S exceeds a pre-set threshold, the system is failed. Because the system failure may be caused by the fact that a component of the system is failed or some components are failed at the same time, the change process of S from normal to failure state can be considered as the system failure process, or simply system failure. It is noted that in some special cases, S denotes some parameters such as wear and drift, and can reflect the failure process directly. In some other cases, it can be the parameters that can reflect the failure process indirectly. For example, the temperature of a reactor can reflect the running condition of the reactor indirectly.

(2) Assume that the state S follows a finite-state, discrete-time, first-order Markov chain. Consequently, $S(t)$ is one of a finite number N of states $s_1, \ldots, s_N$ at time instant t, i.e., $S(t) \in \{s_1, \ldots, s_N\}$. Moreover, in many cases it is impossible to observe $S(t)$ such as the case of engine wear unless stripped down, but it can be indirectly estimated by the monitored observation $\mathbf{y}(t)$.

(3) Let $\boldsymbol{\pi}(t) = [\pi_1(t), \ldots, \pi_N(t)]^T$ be the probability vector of the hidden states at time instant t and $\text{Prob}(\cdot|\cdot)$ denote the conditional probability. $\pi_i(t)$ $(i = 1, \ldots, N)$ is written as:

$$\pi_i(t) = \text{Prob}(S(t) = s_i|\mathbf{y}(0), \ldots, \mathbf{y}(t-1), \mathbf{Q}) \quad \text{and} \quad \sum_{i=1}^{N} \pi_i(t) = 1 \tag{1}$$

where $\mathbf{Q}$ is the unknown parameter vector of the HMM–BRB based model.

(4) Let $\boldsymbol{\lambda}(t) = [\lambda_{i,j}(t)]_{N \times N}$ denote the transition probability matrix between the states of the Markov chain at time instant t, that is

$$\lambda_{i,j}(t) = \text{Prob}(S(t+1) = s_j|S(t) = s_i, \mathbf{Q}) \quad \text{and} \quad \sum_{j=1}^{N} \lambda_{i,j}(t) = 1, \qquad i = 1, \ldots, N \tag{2}$$

Eq. (2) can be considered as the system equation in the dynamic system.

(5) Assume $\mathbf{b}(t) = [b_1(t), \ldots, b_N(t)]^T$ and it follows that

$$b_i(t) = \text{Prob}(\mathbf{y}(t)|S(t) = s_i, \mathbf{Q}), \quad i = 1, \ldots, N \tag{3}$$

where $b_i(t)$ denotes the probability of observing $\mathbf{y}(t)$ when $S(t) = s_i$.

Eq. (3) can be considered as the observation equation in the dynamic system.

In this paper, the monitored observation $\mathbf{y}$ which is obtained by a sensor is continuous and random, so it is advantageous to use the HMM with continuous observation densities (Dong and He, 2007). It is assumed that $\mathbf{y}$ obeys the following Gaussian distribution:

$$b_i(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(\mathbf{y}(t)-\mu_i)^2}{\sigma}\right\} \tag{4}$$

where μ_i and σ are expectation and variance in the state s_i $(i = 1,\ldots,N)$, respectively. These two parameters, which are included in the parameter vector $\mathbf{Q}$, are unknown and need to be further estimated. It is noted that the normal distribution is chosen as an example in this paper and surely other distributions can also be used, but the final judgment will be based on the goodness of fit testing which is beyond the scope of this paper.

(6) Suppose a hidden state s_i $(i \in \{1,\ldots,N\})$ denotes '*Failure*' state of a system. Thus, the objective of real-time failure prognosis is to determine whether the following Eq. (5) is satisfied.

$$\text{if Prob}\{S(t+\Delta t) \in s_i|\mathbf{\Omega}(t)\} \geqslant P_{\text{th}}, \text{ then system is } \textit{Failure} \tag{5}$$

where $\text{Prob}\{S(t+\Delta t) \in s_i|\mathbf{\Omega}(t)\}$ is the probability to '*Failure*' after Δt steps. $\mathbf{\Omega}(t)$ denotes all the available information about the system up to time instant t. P_{th} is a pre-set threshold and $0 \leqslant P_{\text{th}} \leqslant 1$.

One of the key elements of this paper is to calculate $\text{Prob}\{S(t+\Delta t) \in s_i|\mathbf{\Omega}(t)\}$ using the available information, which provides a basis for real-time failure prognosis. Here the information mainly refers to the newly monitored observation.

As mentioned in Section 1, the failure process may be influenced by the environmental factors in engineering. In this paper, a new HMM–BRB based model will be presented to capture the relationships among the environmental factors, the failure process and the monitored observation.

2.3. The new HMM–BRB based model

Assume that the transition probability from one hidden state to itself or another, as shown in Eq. (2), may vary with the changes of environmental factors. In other words, the transition probability may be influenced by the environmental factors. In the example of a continuous stirred tank reactor as discussed in Section 4, the change of the temperature may lead to an increase or decrease of the transition probability from normal state to abnormal one, i.e., the failure process may be accelerated or decelerated. It is assumed that due to the complexity of a system, the relationships between the environmental factors and the failure process cannot be established accurately. It is noted, however, some rules can be extracted from human expert to reflect the qualitative relationships between these factors and the transition probabilities according to historic information and the analysis of the running patterns of the system. Moreover, these rules can be extended to belief rules as proposed by Yang et al. (2006). A collection of belief rules constructs a belief rule base (BRB) which is an expert system in fact (Yang et al., 2006). Compared with the traditional IF–THEN rule, a BRB can capture the dynamics of a system (Yang et al., 2006; Xu et al., 2007). Moreover, the offline and online learning algorithms for training the parameters of the BRB were developed to improve the forecasting ability of the BRB systems (Yang et al., 2007; Xu et al., 2007; Zhou et al., 2009).

There are N hidden states in the Markov chain and a BRB is used to model the relationships between the environmental factors and the transition probabilities from a hidden state s_i $(i = 1,\ldots,N)$ to all hidden states $s_1,\ldots,s_N$, so a total of N belief rule bases will be used. The kth belief rule in the ith BRB (named as BRB_*i*) can be constructed as follows (Yang et al., 2006):

$$\begin{aligned} R_{i,k}:\ & \text{If } u_1 \text{ is } A_1^k \wedge u_2 \text{ is } A_2^k \cdots \wedge u_M \text{ is } A_M^k, \text{ Then } \left\{\left(D_{i,1}, \beta_{1,k}^i\right),\ldots,(D_{i,N}, \beta_{N,k}^i)\right\} \\ & \quad \text{With a rule weight } \theta_k^i \text{ and attribute weight } \delta_{1,k}^i, \delta_{2,k}^i,\ldots,\delta_{M,k}^i \end{aligned} \tag{6}$$

where $u_1, u_2,\ldots,u_M$ denote the antecedent attributes in the kth rule of the BRB_*i* and can be seen as the environmental factors. $A_m^k (m = 1,\ldots,M,\ k = 1,\ldots,L)$ is the referential value of the mth antecedent attribute in the kth rule of the BRB_*i* and $A_m^k \in \mathbf{A}_m$. $\mathbf{A}_m = \{A_{m,j_m},\ j_m = 1,\ldots,J_m\}$ is a set of referential values for the mth antecedent attribute and J_m is the number of the referential values. θ_k^i $(\in R^+,\ k = 1,\ldots,L)$ is the relative weight of the kth rule of the BRB_*i*, and $\delta_{1,k}^i, \delta_{2,k}^i,\ldots,\delta_{M,k}^i$ are the relative weights of Mantecedent attributes used in the kth rule of the BRB_*i*. $\beta_{j,k}^i$ $(i = 1,\ldots,N, j = 1,\ldots,N)$ is the belief degree in the kth rule assessed to $D_{i,j}$ which denotes the jth consequent of the BRB_*i*. Note that "$\wedge$" is a logical connective to represent the "AND" relationship. In addition, assume $\delta_m^i = \delta_{m,k}^i$ and $\bar{\delta}_m^i = \delta_m^i / \max_{m=1,\ldots,M}\{\delta_m^i\}$.

The parameters of the *BRB_i* $(i = 1,\ldots,N)$ should satisfy the following constraints:

$$0 \leqslant \theta_k^i \leqslant 1,\ 0 \leqslant \bar{\delta}_m^i \leqslant 1,\ 0 \leqslant \beta_{j,k}^i \leqslant 1,\ \sum_{j=1}^{N} \beta_{j,k}^i = 1, \quad k = 1,\ldots,L,\ m = 1,\ldots,M,\ j = 1,\ldots,N \tag{7}$$

Let $\mathbf{V}_i$ denote a vector composed of the above parameters in the BRB_*i*. $\mathbf{V}_i$ is written as

$$\mathbf{V}_i = \left[\theta_1^i,\ldots,\theta_L^i, \bar{\delta}_1^i,\ldots,\bar{\delta}_M^i, \beta_{1,1}^i,\ldots,\beta_{N,L}^i\right]^T \tag{8}$$

In the BRB_*i*, $D_{i,j} (i = 1,\ldots,N,\ j = 1,\ldots,N)$ is an action (Yang et al., 2006) which represents transition from hidden state s_i to hidden state s_j. Due to the fact that a belief distribution is a generalized probability (Yang et al., 2006), $\beta_{j,k}^i$ which is the belief degree to $D_{i,j}$ can be treated as the transition probability from s_i to s_j. The BRB_*i* is used to capture the relationships between the environmental factors $u_1(t),\ldots,u_M(t)$ and the transition probabilities $\lambda_{i,1}(t),\ldots,\lambda_{i,N}(t)$ from s_i $(i = 1,\ldots,N)$ to $s_1,\ldots,s_N$. The mapping function reflected by the BRB_*i* is represented as:

$$O_i(\mathbf{u}(t)) = \left\{\left(D_{i,j}, \lambda_{i,j}(t)\right),\ i = 1,\ldots,N;\ j = 1,\ldots,N\right\} \tag{9}$$

where $O_i(\mathbf{u}(t))$ denotes the output of the BRB_*i* and $\mathbf{u}(t) = [u_1(t),\ldots,u_M(t)]^T$. The function $O_i(\cdot)$ denotes the evidential reasoning (ER) approach which is used as the inference tool of the BRB. The details of the algorithm to calculate $\lambda_{i,j}(t)$ will be given in Section 3.1 and Appendix A.

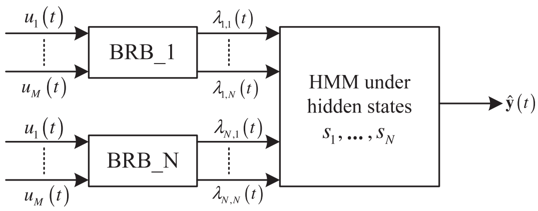

Fig. 1. The HMM–BRB based model.

(7) As a result of above description, the new HMM–BRB based model is shown in Fig. 1, where $u_1(t), \ldots, u_M(t)$ denote the inputs and $\hat{\mathbf{y}}(t)$ represents the output generated by the HMM–BRB based model. Let $\mathbf{Q}$ denote the parameter vector of the HMM–BRB based model and can be written as:

$$\mathbf{Q} = \left[\mathbf{V}_1^T, \ldots, \mathbf{V}_N^T, \boldsymbol{\mu}^T, \sigma\right]^T \tag{10}$$

where $\mathbf{V}_i$ is given in Eq. (8) and denotes the parameter vector of the BRB_*i*. $\boldsymbol{\mu} = [\mu_1, \ldots, \mu_N]^T$, and the expectation μ_i $(i = 1, \ldots, N)$ and the variance σ are given in Eq. (4).

Due to the capability of the BRB methodology (Yang et al., 2006), the inputs of the HMM–BRB based model, i.e., the environmental factors, $u_1(t), \ldots, u_M(t)$, could be either quantitative or qualitative information with various types of uncertainties, such as vagueness and incompleteness. This ensures that different factors affecting the failure prognosis results can be considered.

The initial values of parameters in the HMM–BRB based model may not be accurate. Therefore, it is necessary to update them using available information.

3. The HMM–BRB based real-time failure prognosis

In this section, a new HMM–BRB based real-time failure prognosis algorithm will be developed. The proposed recursive algorithm consists of two aspects: (i) recursive algorithm for online updating the HMM–BRB based model, and (ii) real-time failure prognosis.

3.1. Recursive algorithm for online updating the HMM–BRB based model

In this subsection, according to the maximum likelihood (ML) algorithm, a recursive algorithm is proposed to online update the HMM–BRB based model using the available input and output information.

Let $f(\mathbf{y}(n)|\mathbf{Q})$ denote the conditional probability density function (pdf) of the monitored observation $\mathbf{y}$ at time instant n. Suppose that $\mathbf{y}(1), \ldots, \mathbf{y}(t)$ are independent, so there is

$$f(\mathbf{y}(1), \ldots, \mathbf{y}(t)|\mathbf{Q}) = \prod_{n=1}^{t} f(\mathbf{y}(n)|\mathbf{y}(1), \ldots, \mathbf{y}(n-1), \mathbf{Q}) \tag{11}$$

where $\mathbf{Q}$ as given in Eq. (10) is the unknown parameter vector of the HMM–BRB based model.

Then the log-likelihood function in Eq. (11) can be constructed as follows:

$$\mathrm{L}_{t+1}(\mathbf{Q}) = \frac{1}{t+1} \sum_{n=1}^{t} \log f(\mathbf{y}(n)|\mathbf{y}(1), \ldots, \mathbf{y}(n-1), \mathbf{Q}) \tag{12}$$

According to the recursive algorithm for updating the HMM parameters (LeGland and Mevel, 1997), the following recursive algorithm for estimating the parameter vector $\mathbf{Q}$ is obtained when the log-likelihood function is maximized.

$$\mathbf{Q}(t+1) = \prod_{\mathbf{H}} \{\mathbf{Q}(t) + \varepsilon(t)\Gamma(\mathbf{Q}(t))\} \tag{13}$$

where $\mathbf{H}$ is a constraint set composed of the equality and inequality constraints that $\mathbf{Q}$ should satisfy, and $\prod_{\mathbf{H}}\{\cdot\}$ denotes the projection onto $\mathbf{H}$. $\{\varepsilon(t)\}$ is a sequence of step sizes and satisfies $\varepsilon(t) \geqslant 0$, $\lim_{t\to\infty}\varepsilon(t) = 0$ and $\sum_{t=1}^{\infty}\varepsilon(t) = \infty$. In addition, $\Gamma(\mathbf{Q}(t))$ denotes the derivative with respect to the vector $\mathbf{Q}$ at time instant t and is written as:

$$\Gamma(\mathbf{Q}(t)) = \left.\frac{\partial \log f(\mathbf{y}(t)|\mathbf{y}(1), \ldots, \mathbf{y}(t-1), \mathbf{Q})}{\partial \mathbf{Q}}\right|_{\mathbf{Q}=\mathbf{Q}(t)} \tag{14}$$

In the recursive algorithm as given in Eq. (13), the main task is to determine the derivative $\Gamma(\mathbf{Q}(t))$. The calculation methods of $\Gamma(\mathbf{Q}(t))$ will be given in detail in the following contents and Appendix A.

According to Eqs. (1) and (3) and Bayes theorem, in Eq. (14) there is

$$f(\mathbf{y}(t)|\mathbf{Q}) = \mathbf{b}(t)^T\boldsymbol{\pi}(t) \tag{15}$$

Let $\mathbf{Q} = [Q_1, \ldots, Q_W]^T$ and W denote the dimension of $\mathbf{Q}$. According to Eq. (10), there are $W = Z + N + 1$ and $Z = N \times (L + M + L \times N)$. Thus, the left side of Eq. (14) can be written as:

$$\Gamma(\mathbf{Q}(t)) = [\Gamma(Q_1(t)), \ldots, \Gamma(Q_W(t))]^T \tag{16}$$

In Eq. (16), $\Gamma(Q_d(t))$ $(d = 1, \ldots, W)$ needs to be determined. Substituting Eq. (15) into Eq. (14), $\Gamma(Q_d(t))$ can be calculated by

$$\Gamma(Q_d(t)) = \frac{\mathbf{b}(t)^T \mathbf{z}_d(t)}{\mathbf{b}(t)^T \boldsymbol{\pi}(t)} + \frac{\partial \mathbf{b}(t)^T}{\partial Q_d(t)} \frac{\boldsymbol{\pi}(t)}{\mathbf{b}(t)^T \boldsymbol{\pi}(t)} \tag{17}$$

where $\mathbf{z}_d(t)$ denotes the partial derivative of $\boldsymbol{\pi}(t)$ with respect to $Q_d(t)$ and it is given by

$$\mathbf{z}_d(t) = \frac{\partial \boldsymbol{\pi}(t)}{\partial Q_d(t)} \tag{18}$$

In Eqs. (17) and (18), the probability vector of the hidden states $\boldsymbol{\pi}(t)$, the partial derivatives $\partial \mathbf{b}(t)^T / \partial Q_d(t)$ and $\mathbf{z}_d(t)$ are needed. Their calculations are given in Appendix A.

The recursive algorithm in Eq. (13) is indeed a projection algorithm. That is, when the newly estimated parameter vector $\mathbf{Q}(t+1)$ is obtained at time instant $(t+1)$, a projection is operated to ensure $\mathbf{Q}(t+1)$ to be located in the constraint set $\mathbf{H}$. In this paper, $\mathbf{H}$ is composed of the constraints as given in Eq. (7). The projection algorithm for dealing with the constraints has been proposed by Zhou et al. (2009).

Compared with other methods such as probability based methods (Cagno et al., 2000; Davis et al., 2007), the BRB has an important characteristic to allow the direct expert intervention (Yang et al., 2007). In other words, experts can use the judgmental information to extrapolate the machine learnt rules to cover ranges which are not covered in historical data, so that the constructed BRB may capture all possible running patterns which can reflect the relationships between the environmental factors and the transition probabilities among the hidden states. This is helpful to improve the learning ability of the proposed recursive algorithm. In addition, as the probability transition matrix is constructed using the BRB, the task of parameter estimation is significantly reduced compared with using the ML algorithm for the estimation in a conventional HMM (Rabiner, 1989; Wang, 2007b).

3.2. Real-time failure prognosis

After the parameter vector, $\mathbf{Q}(t)$, is estimated at time instant t, the probability vector of the hidden states $\boldsymbol{\pi}(t)$, the expectation $\boldsymbol{\mu}(t)$ and the transition probability matrix $\lambda(t)$ can be determined. Let Δt denote the forecasting step. Thus, according to the characteristics of the Markov chain, the predicted expectation value of the hidden variable S after Δt steps can be calculated by

$$E(S(t+\Delta t)) = \left[\lambda(t)^{\Delta t} \boldsymbol{\pi}(t)\right]^T \boldsymbol{\mu}(t) \tag{19}$$

where $E(\cdot)$ denotes the expectation.

In our study, suppose $N = 2$ in the HMM–BRB based model, i.e., $S(t) \in \{s_1, s_2\}$, where the hidden states s_1 and s_2 denote '*Normal*' and '*Failure*', respectively. Moreover, s_1 and s_2 can be treated as two evaluation grades which can be related to the utilities according to the rules provided by the decisions maker (Yang, 2001). Let $U(s_i)$ $(i = 1, 2)$ be the utility of the evaluation grade s_i. The predicted probability to '*Failure*' can be determined by

$$\operatorname{Prob}\{S(t+\Delta t) \in s_2 | \boldsymbol{\Omega}(t)\} = \begin{cases} \frac{E(S(t+\Delta t)) - U(s_1)}{U(s_2) - U(s_1)}, & \text{if } U(s_1) < E(S(t+\Delta t)) < U(s_2) \\ \frac{U(s_1) - E(S(t+\Delta t))}{U(s_1) - U(s_2)}, & \text{if } U(s_2) < E(S(t+\Delta t)) < U(s_1) \end{cases} \tag{20}$$

Once the probability in Eq. (20) is calculated and the threshold P_{th} is given, whether the system is running at a normal or failure state after Δt steps can be determined by using Eq. (5).

Remark 1. According to Eqs. (19) and (20), if Δt is not pre-set, then the time to reach the failure state, i.e., the remaining useful life (RUL), can be calculated when let $\operatorname{Prob}\{S(t+\Delta t) \in s_2 | \boldsymbol{\Omega}(t)\} = 1$.

3.3. A procedure for real-time failure prognosis

As a result of the discussion in the previous subsection, the procedure of the new HMM–BRB based recursive algorithm for real-time failure prognosis may be summarized as follows:

Step 1. *Let $t = 0$. Assign initial values to the parameter vectors of the N belief rule bases $\mathbf{V}_1(t), \ldots, \mathbf{V}_N(t)$ and the parameters of the HMM $\boldsymbol{\pi}(t)$, $\mathbf{z}_d(t)$, $\boldsymbol{\mu}(t)$, $\sigma(t)$. The parameters in $\mathbf{V}_i(t)$ $(i = 1, \ldots, N)$ satisfy the constraints as given in Eq. (7). According to Eqs. (1) and (18), there are $\sum_{i=1}^{N} \pi_i(t) = 1$ and $\mathbf{z}_d(t) = \mathbf{0}_N$, where $\mathbf{0}_N$ denotes a column vector whose elements are all 0 and the dimension of $\mathbf{0}_N$ is N. From Eq. (10), $\mathbf{Q}(t)$ is obtained.*

Step 2. *Since the input $\mathbf{u}(t)$ and the monitored observation $\mathbf{y}(t)$ are available, Eq. (4) and Eq. (A.2) of Appendix A are used to determine the probability $\mathbf{b}(t)$ and $\partial \mathbf{b}(t)^T / \partial \mathbf{Q}(t)$, respectively. Then $\Gamma_d(\mathbf{Q}(t))$ $(d = 1, \ldots, W)$ can be calculated by substituting $\mathbf{b}(t)$, $\boldsymbol{\pi}(t)$, $\mathbf{z}_d(t)$ and $\partial \mathbf{b}(t)^T / \partial \mathbf{Q}(t)$ into Eq. (17). According to Eq. (16), $\Gamma(\mathbf{Q}(t))$ is obtained. Finally, $\mathbf{Q}(t+1)$ can be determined using Eq. (13) and $\Gamma(\mathbf{Q}(t))$.*

Step 3. *According to $\mathbf{V}_1(t), \ldots, \mathbf{V}_N(t)$, the transition matrix of the HMM $\boldsymbol{\lambda}(t)$ can be calculated by the ER approach using Eqs. A.7, A.8, A.9. Moreover, $\partial \mathbf{B}(t)/\partial \mathbf{Q}(t)$ and $\partial \lambda(t)/\partial \mathbf{Q}(t)$ are determined by Eqs. (A.6) and (A.10), respectively. Then $\mathbf{z}_d(t+1)$ can be obtained by substituting $\lambda(t)$, $\partial \mathbf{B}(t)/\partial \mathbf{Q}(t)$ and $\partial \lambda(t)/\partial \mathbf{Q}(t)$ into Eqs. A.3, A.4, A.5. $\boldsymbol{\pi}(t+1)$ is estimated using Eq. (A.1).*

Step 4. *After the input $\mathbf{u}(t+1)$ and the monitored observation $\mathbf{y}(t+1)$ are available, let $t = t+1$ and go to Step 2. Otherwise, go to Step 5.*

Step 5. *After the forecasting step Δt and the threshold P_{th} are given, Eqs. (19) and (20) can be used to predict the probability to the state 'Failure'. Then Eq. (5) is used to determine whether the system is failure after Δt steps.*

Remark 2. In the proposed recursive algorithm as given in Eq. (13), the observations are assumed to be independent. However, in engineering, they may not be independent and the other likelihood function should be adopted (Wang and Christer, 2000). Therefore, it is necessary to study a more general recursive algorithm for estimating the parameters of the HMM–BRB based model in future.

Remark 3. The nature of the proposed recursive algorithm is: The model parameters within a HMM–BRB based model are first updated, and then update the prediction using the updated HMM–BRB based model, which may produce over-fitting. In order to solve this problem, it is necessary to choose the appropriately initial parameters of the HMM–BRB based model. On the other hand, more conditioned observations are needed to train the HMM–BRB based model. Thus, the model parameters can converge to an optimal point and the updated model can give the accurate prediction.

4. An experimental case study

In this section, a continuous stirred tank reactor (CSTR) (Zhou and Frank, 1998) is used to demonstrate the implementation and validity of the proposed HMM–BRB based real-time failure prognosis method.

In our experimental case study, the CSTR model is used to generate simulated data. Then assume that the CSTR model is unknown and only the simulated data are used to predict the hidden failure of the CSTR by the proposed HMM–BRB based prognosis algorithm. The CSTR model is presented firstly.

4.1. Simulation model of continuous stirred tank reactor

A discrete-time CSTR model with one input and two outputs is described as

$$\begin{cases} \mathbf{x}(t+1) = \mathbf{x}(t) + dt \cdot g(\mathbf{x}(t), \upsilon(t)) + \mathbf{w}(t) \\ \mathbf{y}(t+1) = \mathrm{diag}\{1, 1.01\}\mathbf{x}(t+1) + \mathbf{v}(t+1) \end{cases} \tag{21}$$

where $t \geqslant 0$ is the discrete-time index. $\mathrm{diag}\{\cdot, \ldots, \cdot\}$ denotes a diagonal matrix. $\mathbf{x}$, υ and $\mathbf{y}$ are the state, the input and the output with appropriate dimensions, respectively. $\mathbf{w}$ and $\mathbf{v}$ are the process noise and the measurement noise, respectively. The nonlinear system function g is written as:

$$g(\mathbf{x}(t), \upsilon(t)) = \begin{bmatrix} \frac{q}{V}(C_{Af} - x_1(t)) - k_0\vartheta(t)x_1(t) \\ \frac{q}{V}(T_f - x_2(t)) + \frac{-\Delta H}{\rho C_p} k_0 \vartheta(t) x_1(t) + \frac{UA}{V\rho C_p}(\upsilon(t) - x_2(t)) \end{bmatrix} \tag{22}$$

where we have $\vartheta(t) = \exp(-E/(Rx_2(t)))$, $\mathbf{x} = [x_1, x_2] = [C_A, T_r]^T$, $\upsilon(t) = T_c$ and $\mathbf{y} = [y_1, y_2]^T$. The states x_1 and x_2 denote the reactant concentration and the reactor temperature, respectively. The input υ denotes the temperature of the cooling water in the jacket. The parameters are shown in Table 1.

In Eqs. (21) and (22), we choose the sampling interval as $dt = 0.2$ min. $\mathbf{w}(t)$ is Gaussian noise with covariance matrix $\mathrm{diag}\{0.005^2, 0.5^2\}$ and $\mathbf{v}(t+1)$ is Gaussian noise with covariance matrix $\mathrm{diag}\{0.004^2, 0.4^2\}$. The initial states are chosen as $x_1(0) = 0.22$ mol/L and $x_2(0) = 447$ K.

Let x_1^d denote the setting point of the reactant concentration and suppose $x_1^d = 0.2$ in this paper. Then the control objective of the CSTR system is to track x_1^d. For simplicity, we use a PID controller here. Let $K_p = 100$, $T_i = 0.4$, $T_d = 0.1$ and $\upsilon(0) = 419$. The controller is in a recursive form as

$$\upsilon(t) = \upsilon(t-1) + \tau_0\varepsilon(t) - \tau_1\varepsilon(t-1) + \tau_2\varepsilon(t-2) \tag{23}$$

$$\varepsilon(t) = \hat{x}_1(t|t) - x_1^d \tag{24}$$

$$\tau_0 = K_p\left(1 + \frac{dt}{T_i} + \frac{T_d}{dt}\right), \tau_1 = K_p\left(1 + \frac{2T_d}{dt}\right), \tau_2 = \frac{K_pT_d}{dt} \tag{25}$$

Under normal operations, the reactant concentration C_A is automatically controlled at the setting point and the reactor temperature T_r is kept in the given range. In this case, the energy released by reaction is partially absorbed by the jacket where the temperature of cooling water T_c is adjusted by the proposed PID controller. Assume that if T_r exceeds the pre-set threshold, a failure occurs in the CSTR.

Assume that due to contamination of the reactant or the leak of the reactor, the reactor volume V will decrease. Furthermore, assume that V changes as follows:

$$V(t) = 100 - 0.05^* t \tag{26}$$

It is assumed that as more production for the CSTR is needed, the fed-in flow rate of the reactant q should be increased. Moreover, assume that due to increase of the ambient temperature, the initial temperature of the reactant T_f also increases. The need of production and ambient temperature are chosen as the environmental factors in this experimental study. It can be observed that the changes of environmental factors can be reflected by q and T_f. Furthermore, it is assumed that before the reactant is fed to the reactor, q and T_f change as follows:

$$q(t) = \begin{cases} 100, & t < 200 \\ 100 + (t-200)^* 0.2, & t \geqslant 200 \end{cases}, \quad \text{and} \quad T_f(t) = \begin{cases} 400, & t < 250 \\ 400 + (t-250)^* 0.1, & t \geqslant 250 \end{cases} \tag{27}$$

Fig. 2 shows the change process of q and T_f. According to Eqs. (21)–(27), Fig. 3 gives the simulated observations of the reactor temperature, where the solid line denotes the simulated observation when only the failure as given in Eq. (26) exists, and the dotted line denotes the sim-

Table 1
The parameters of CSTR.

$q = 100$ L/min	$E/R = 5360$ K	$C_{Af} = 1$ mol/L	$UA = 11{,}950$ J/(min K)	$\rho = 1$ kg/L	$k_0 = \exp(13.4)$/min
$T_f = 400$ K	$-\Delta H = 17835.821$ J/mol	$V = 100$ L	$C_p = 0.239$ J/(g K)	$T_c = 419$ K	

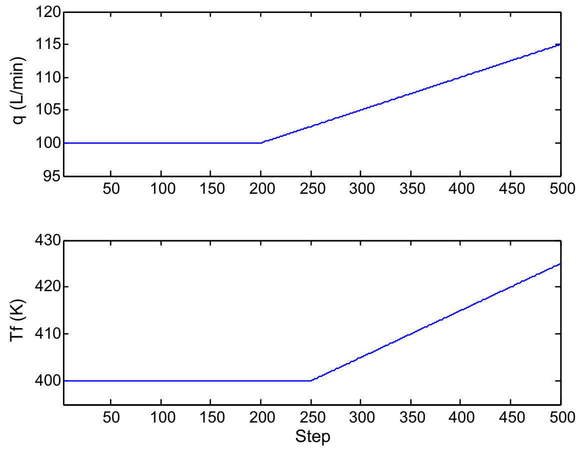

Fig. 2. The change processes of environmental factors.

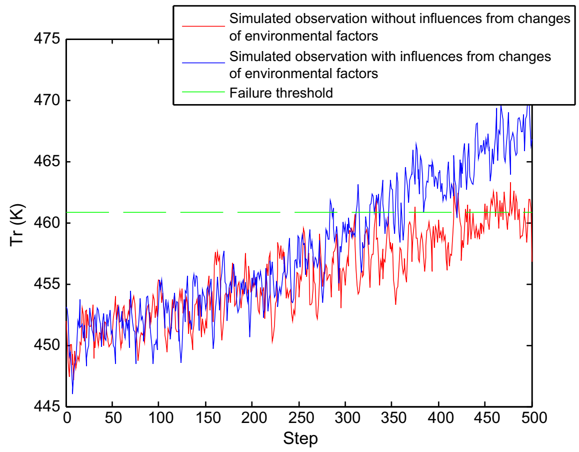

Fig. 3. The simulated observations with and without the influences from changes of environmental factors.

ulated observation when the failure and influences from the changes of environmental factors exist at the same time. From Fig. 3, it can be obviously seen that the simulated observation influenced by the changes of environmental factors exceeds the failure threshold more quickly than that when the environmental factors do not change. Therefore, it is necessary to consider the environmental factors when the failure is predicted.

Using the simulation model of CSTR, a set of 500 data are generated. Based on the data as given in Figs. 2 and 3, the objective of this experimental case study is to use the environmental factors as the input and the simulated observation of T_r as the output, i.e., the monitored observation, to verify the proposed HMM–BRB based model and real-time failure prognosis algorithm.

4.2. Construction of the HMM–BRB based model for the CSTR system

In our simulation study, it is assumed that the mathematical models such as Eqs. (21)–(27) to describe the physical behavior of the CSTR are unknown, so we cannot use them to predict the hidden failure. On the other hand, the qualitative knowledge based methods cannot be used either to predict the failure accurately. In order to use the HMM–BRB based model to simulate the CSTR system, the environmental factors, the hidden failure process and the monitored observation in the HMM–BRB based model may be chosen as follows:

(1) According to our knowledge and analysis, the reactor temperature T_r can reflect the running condition of the CSTR if the reactant concentration needs to be controlled at the setting point when the reactor volume V decreases. Besides from the failure as described in Eq. (26), the other failures, such as failure of the cooling jacket, failure of the PID controller and so on, all will lead to increase or decrease of T_r. Moreover, these failures may occur simultaneously. So T_r can be chosen as the key parameter to reflect the running condition of the CSTR. However, due to noise contamination and sensor drift as shown in Eq. (21), T_r cannot be observed directly, but can be indirectly reflected by y_2 that denotes the simulated observation with influences from changes of environmental factors. Therefore, we choose T_r and y_2 as the hidden failure process and the monitored observation, respectively.
(2) As mentioned in Section 4.1, the fed-in flow rate q and the initial temperature T_f of the reactant are treated as two characteristic parameters to reflect the environmental factors.

In the proposed model, it is important to determine the initial belief rules of the BRB system. The detailed method to determine the initial belief rules will be given in the next subsection.

4.3. Determination of the initial belief rules in the BRB system

When a BRB is constructed, two steps are usually included. Firstly, a basic structure of the BRB is determined. In other words, the antecedent attributes and consequent and their referential values should be chosen. Secondly, based on the analysis of running patterns and some available information such as expert knowledge and historical data, the initial parameters of the BRB need be determined.

4.3.1. Determination of the basic structure of the BRB

Since q and T_f are the inputs of HMM–BRB based model, they are chosen as the antecedent attributes of the BRB. T_r is chosen as the hidden variable of the HMM. The transition probabilities between the hidden states are treated as the consequents of the BRB.

In order to construct the BRB, some referential points should be assigned to the above variables. For q, three referential points are used and they are small (S), medium (M) and large (L). For T_f, two referential points are used and they are low (L) and high (H). In other words, there are

$$A_1^k \in \{\mathrm{S},\mathrm{M},\mathrm{L}\}, \quad \text{and} \quad A_2^k \in \{\mathrm{L},\mathrm{H}\} \tag{28}$$

For the hidden variable T_r, two hidden states s_1 and s_2 are used to denote the evaluation degrade '*Normal*' and '*Failure*', respectively. Therefore, for the consequent of $i$thBRB, two referential points are used: transition action from s_i to s_j (denoted as $D_{i,j}$), where $i = 1,2$ and $j = 1,2$. Let $\mathbf{D}_i = [D_{i,1}, D_{i,2}]^T$, where $\mathbf{D}_i$ $(i = 1,2)$ denotes the consequent vector of the $i$thBRB.

The referential points of q and T_f are in linguistic terms and need to be quantified in order to use the data as shown in Figs. 2 and 3. The quantified results as listed in Tables 2 and 3 need roughly cover the corresponding attribute value range. It is noted that the referential point $D_{i,j}(i = 1,2; j = 1,2)$ denotes an action and need not be quantified here.

From the referential points of the variables, two belief rule bases, named as BRB_1 and BRB_2, are constructed using the belief rule concept. The kth belief rule in the BRB_i (i = 1,2) is written as:

$$\begin{aligned} R_{i,k}:\ & \text{If } q \text{ is } A_1^k \wedge T_f \text{ is } A_2^k, \text{ Then } \left\{\left(D_{i,1}, \beta_{1,k}^i\right), \left(D_{i,2}, \beta_{2,k}^i\right)\right\} \\ & \text{With a rule weight } \theta_k^i \text{ and attribute weight } \delta_{1,k}^i, \delta_{2,k}^i \end{aligned} \tag{29}$$

where A_1^k and $A_2^k (k = 1,\ldots,6)$ are the referential values as defined in Tables 2 and 3. The BRB_i is used to capture the relationships between antecedent attributes and transition actions $D_{i,1}$ and $D_{i,2}$.

4.3.2. Qualitative Analysis of running patterns of the CSTR

In this subsection, according to expert knowledge and some basic principles such as mass balance, energy balance and thermodynamic law, the running patterns of the CSTR, i.e., the qualitative relationships among the fed-in flow rate q, the initial temperature of reactant T_f and the reactor temperature T_r will be analyzed, which provides a basis to determine the initial belief rules.

First of all, when the control is not considered in the CSTR, according to expert knowledge and the basic principles, it has been concluded that (1) if the reactant concentration is high, this means that not much reaction has occurred, so little energy has been released by reaction and T_r will not be much different than the feed and jacket temperatures; (2) if the reactant concentration is low, more energy will have been released and T_r will be higher (Rensselaer Polytechnic Institute, 1999).

Similar to the above, the following four cases can be obtained when the PID controller is used.

Case 1. If q is large, then the reactant in the reactor will increase, which leads to the increase of the energy released by reaction. Thus, T_r will be high. Otherwise, T_r will be low.
Case 2. If T_f is high, then T_r will be high due to the fact that the reactor temperature T_r is the same as the reactant temperature. Otherwise, T_r will be low.
Case 3. If q is small and T_f is low, then T_r will be low according to *Case 1* and *Case 2*. If q is large and T_f is high, then T_r will be high.
Case 4. If q is small and T_f is high, or q is large and T_f is low, then it is difficult to decide whether T_r is low or high. But it can be determined by experts or examining the historical data.

In this paper, it is assumed that if T_r exceeds the normal range, the failure process starts. Moreover, if T_r increases and is more than the pre-set threshold, a failure occurs in the CSTR. Thus, if T_r is high, the transition probability from hidden state '*Normal*' to '*Failure*' is large. Otherwise, it is small.

Table 2
The referential points of q.

Linguistic terms	S	M	L
Numerical values (L/min)	95	110	120

Table 3
The referential points of T_f.

Linguistic terms	L	H
Numerical values (K)	395	430

4.3.3. Determination of the initial belief rules

The initial belief rules can be established in the four ways (Xu et al., 2007): (1) Extracting belief rules from expert knowledge; (2) extracting belief rules by examining historical data; (3) using the previous rule bases for failure prognosis of the CSTR if available; and (4) random rules without any pre-knowledge.

For a complex system, prior knowledge may be not perfect, which leads to the construction of an incomplete or even inappropriate initial BRB structure. Also, too many rules in an initial BRB may lead to over-fitting, whilst too few rules may result in under-fitting. In order to solve this problem, a realistic method was proposed to adjust the structure and parameters of a BRB (Zhou et al., 2010). If the initial BRB is complete, the parameters in the BRB can be trained using the proposed offline and online learning algorithms (Yang et al., 2007; Zhou et al., 2009).

In our case, there are no previous belief rule bases to start with. Belief rules are extracted by examining the data and using the above qualitative analysis of the running patterns of the CSTR, and are used as the starting point for the proposed learning algorithm.

The initial belief degrees of BRB_1 and BRB_2 as listed in Tables 4 and 5 are given by an expert. For example, if q is L and T_f is H, the expert judges that the transition action from the state '*Normal*' to '*Failure*' must occur, so the expert assess that the belief degree to $D_{1,1}$ is 0 and the belief degree to $D_{1,2}$ is 1 according to the running patterns as described in *Case 3* of sub Section 4.3.2, where $D_{1,1}$ and $D_{1,2}$ denote the transition action from '*Normal*' to '*Normal*' and the transition from '*Normal*' to '*Failure*', respectively. Thus a belief rule is obtained in the second row of Table 4. Similarly, if q is S and T_f is L, according to the running patterns as described in *Case 3*, the expert assess that the belief degree to $D_{1,1}$ is 1 and the belief degree to $D_{1,2}$ is 0, which shows that the system is in the normal state. Thus a belief rule is obtained in the last row of Table 4. For the other belief rules in Table 4, the initial belief degrees cannot be determined according to the running patterns as described in *Case 4* of sub Section 4.3.2, but, in terms of the referential values of the antecedent and consequent attributes, they can be determined by examining the historical data and expert knowledge. Here these initial belief rules are assigned by the researchers as a result of observing the data as given in Figs. 2 and 3.

Similarly, we can obtain the initial belief rules as given in Table 5 for BRB_2. The initial belief rules in Tables 4 and 5 may be qualitatively correct, i.e., the reactor temperature varies with the fed-in flow rate and initial temperature of the reactant in the right trend. However, the initial belief degrees may not be accurate. Therefore, it is necessary to update them using the monitored observation.

4.4. Updating of HMM–BRB model and real-time failure prognosis

In order to predict the hidden failure real-time, the data in Fig. 2 and the simulated observations with influences from the changes of environmental factors (i.e., the values represented by the solid line in Fig. 3) are used to online update the initially constructed HMM–BRB based model, and then predict the probability of T_r to the hidden state '*Failure*'. The process of real-time failure prognosis is implemented using MATLAB.

4.4.1. Set initial parameters of the HMM–BRB based model

The initial belief degrees of BRB_1 and BRB_2 have been listed in Tables 4 and 5. In addition, θ_k^i and $\delta_{j,k}^i$ are assumed to be 1, where $i = 1, 2$, $j = 1, 2$ and $k = 1, \ldots, 6$. Suppose $U(s_1) = 442$ K and $U(s_2) = 471$ K denote the utilities of the two evaluation grades '*Normal*' and '*Failure*', respectively. These two utilities can be determined by experts according to the change range of the reactor temperature. Furthermore, assume that the initial probability vector is $\boldsymbol{\pi}(0) = [0.2, 0.8]^T$, the threshold $P_{\mathrm{th}} = 0.65$, and the prediction step $\Delta t = 5$. Thus, an initial HMM–BRB based model is constructed.

4.4.2. Update and test the HMM–BRB based model

After the input values $[q(t), T_f(t)]$ are transformed and represented in terms of the referential values defined in Tables 2 and 3, the proposed recursive algorithm as given in Eq. (13) is used to update the initially constructed HMM–BRB based model. The detailed transformation processes were developed and discussed by Yang (2001). For example, if $q(1) = 100$, then using the referential values of this attribute as given in Table 2, it is equivalently transformed to $q(1) = \{(\mathrm{S}, 0.67), (\mathrm{M}, 0.33)\}$ because $100 = \mathrm{S} \times 0.67 + \mathrm{M} \times 0.33$. The belief

Table 4
Initial belief rules of BRB_1.

Rule number	q and T_f	Transition action distribution $\{D_{1,1}, D_{1,2}\}$ from S_1
1	L and H	$\{(D_{1,1}, 0), (D_{1,2}, 1)\}$
2	L and L	$\{(D_{1,1}, 0.4), (D_{1,2}, 0.6)\}$
3	M and H	$\{(D_{1,1}, 0.2), (D_{1,2}, 0.8)\}$
4	M and L	$\{(D_{1,1}, 0.6), (D_{1,2}, 0.4)\}$
5	S and H	$\{(D_{1,1}, 0.7), (D_{1,2}, 0.3)\}$
6	S and L	$\{(D_{1,1}, 1), (D_{1,2}, 0)\}$

Table 5
Initial belief rules of BRB_2.

Rule number	q and T_f	Transition action distribution $\{D_{2,1}, D_{2,2}\}$ from S_2
1	L and H	$\{(D_{2,1}, 0.2), (D_{2,2}, 0.8)\}$
2	L and L	$\{(D_{2,1}, 0.1), (D_{2,2}, 0.9)\}$
3	M and H	$\{(D_{2,1}, 0.1), (D_{2,2}, 0.9)\}$
4	M and L	$\{(D_{2,1}, 0.05), (D_{2,2}, 0.95)\}$
5	S and H	$\{(D_{2,1}, 0.2), (D_{2,2}, 0.8)\}$
6	S and L	$\{(D_{2,1}, 0.2), (D_{2,2}, 0.8)\}$

Table 6
Updated belief rules of BRB_1.

Rule number	q and T_f	Transition action distribution $\{D_{1,1}, D_{1,2}\}$ from S_1
1	L and H	$\{(D_{1,1}, 0), (D_{1,2}, 1)\}$
2	L and L	$\{(D_{1,1}, 0.4), (D_{1,2}, 0.6)\}$
3	M and H	$\{(D_{1,1}, 0.6131), (D_{1,2}, 0.3869)\}$
4	M and L	$\{(D_{1,1}, 0.9994), (D_{1,2}, 0.0006)\}$
5	S and H	$\{(D_{1,1}, 0.9993), (D_{1,2}, 0.0007)\}$
6	S and L	$\{(D_{1,1}, 0.9998), (D_{1,2}, 0.0002)\}$

Table 7
Updated belief rules of BRB_2.

Rule number	q and T_f	Transition action distribution $\{D_{2,1}, D_{2,2}\}$ from S_2
1	L and H	$\{(D_{2,1}, 0.3848), (D_{2,2}, 0.6152)\}$
2	L and L	$\{(D_{2,1}, 0.0010), (D_{2,2}, 0.9990)\}$
3	M and H	$\{(D_{2,1}, 0.5700), (D_{2,2}, 0.4300)\}$
4	M and L	$\{(D_{2,1}, 0.2220), (D_{2,2}, 0.7780)\}$
5	S and H	$\{(D_{2,1}, 0.0010), (D_{2,2}, 0.9990)\}$
6	S and L	$\{(D_{2,1}, 0.0010), (D_{2,2}, 0.9990)\}$

degree 0.67 is the matching degree of the input $q(1) = 100$ to the referential value S = 95. Because the referential value L is used in Rule 5 and Rule 6 in Table 4, $\alpha_{1,1}^{k}(1)$ in Eq. (A.7) is 0.67 for $k = 5, 6$. Similarly, $\alpha_{1,2}^{k}(1) = 0.33$ for $k = 3, 4$.

Two updated belief rule bases are listed in Tables 6 and 7, respectively. The estimated value $\widehat{T}_r(t)$ of the reactor temperature $T_r(t)$ is calculated as follows:

$$\widehat{T}_r(t) = [\boldsymbol{\lambda}(t)\boldsymbol{\pi}(t)]^T \boldsymbol{\mu}(t) \tag{30}$$

where the transition probability $\boldsymbol{\lambda}(t)$, the probability vector of the hidden states $\boldsymbol{\pi}(t)$ and the expectation $\boldsymbol{\mu}(t)$ can be determined after the estimated parameter vector $\mathbf{Q}(t)$ is obtained.

Similarly, the estimated values of $T_r(t)$ generated by the initial HMM–BRB based model can also be determined using Eq. (30). As shown in Fig. 4, the estimation generated by the initial HMM–BRB based model cannot match the simulated value generated by the first equation of Eq. (21). However, the updated HMM–BRB model can closely replicate the relationships between q, T_f and T_r. Thus, it can be concluded that using the proposed recursive algorithm and the available information, the updated HMM–BRB based model can simulate the CSTR system well.

4.4.3. Generate the simulated probability to the state 'Failure'

Similar to Eq. (20), the simulated probability of T_r to the state '*Failure*' can be calculated by

$$\mathrm{Prob}\{T_r(t) \in s_2\} = \begin{cases} \frac{T_r(t) - U(s_1)}{U(s_2) - U(s_1)}, & \text{if } U(s_1) < T_r(t) < U(s_2) \\ \frac{U(s_1) - T_r(t)}{U(s_1) - U(s_2)}, & \text{if } U(s_2) < T_r(t) < U(s_1) \end{cases} \tag{31}$$

where $T_r(t)$ is generated by the first equation of Eq. (21).

4.4.4. Predict the failure

After the HMM–BRB based model is updated each time when a simulated observation is available, Eq. (20) is used to predict the failure after Δt steps. The real-time predicted result is given in Fig. 5. From Fig. 5, it is shown that compared with the initial BRB-HMM based

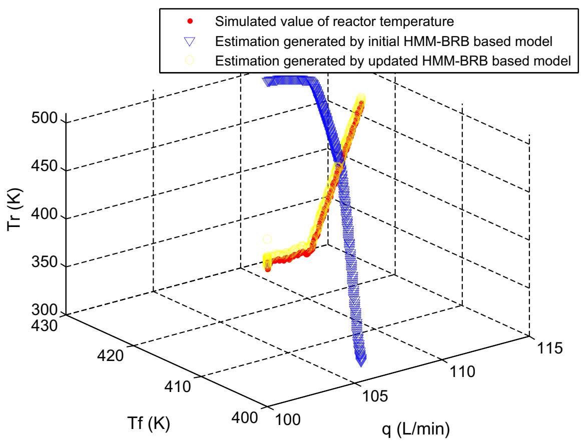

Fig. 4. The simulated value of the reactor temperature and the estimations generated by the initial and updated HMM–BRB based models.

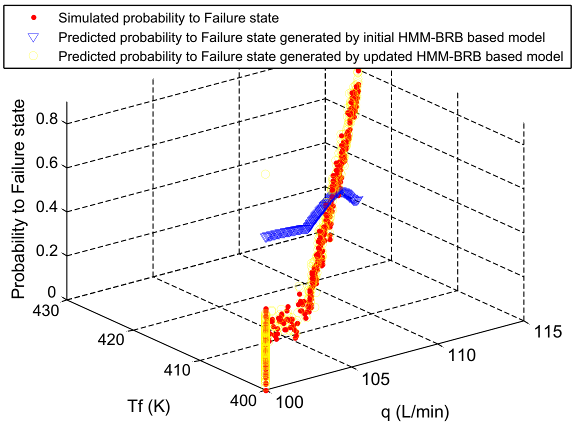

Fig. 5. The simulated probability to '*Failure*' state and the predicted probabilities to '*Failure*' generated by the initial and updated HMM–BRB based models.

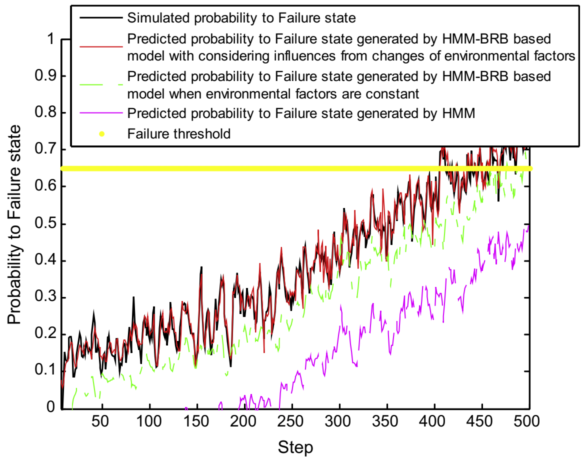

Fig. 6. The simulated probability to '*Failure*' state and the real-time failure prognosis results generated by the updated HMM, the updated HMM–BRB based model under the influences from changes of environmental factors, and the updated HMM–BRB based model when the environmental factors are constant.

model, the predicted probability to '*Failure*' generated by the updated HMM–BRB based model can fit the simulated value more accurately for the same antecedent attributes.

The failure prognosis results are given in Fig. 6 in a time scale in terms of sample interval. In Fig. 6, the solid line denotes the simulated probability to '*Failure*'. The dotted line denotes the real-time failure prognosis result after considering influences from changes of environmental factors.

In this simulation, the inputs are continuous and quantitative. It is noted, however, the inputs can also be qualitative, discrete and symbolic information such as expert judgments.

In order to demonstrate that it is necessary to consider the influences from the changes of environmental factors in failure prognosis, another simulation is carried out.

Assume that the fed-in flow rate and the initial temperature of the reactant are constant in the HMM–BRB based prognosis algorithm. In other words, though the environmental factors are also considered, it is assumed that the change process of the environmental factors as given in Fig. 2 is unknown due to some reasons. In fact, the environmental factors do change. Therefore, the outputs of the HMM–BRB based model are the simulated observations with influences from the changes of environmental factors (i.e., the values represented by the solid line in Fig. 3).

The assumed constant values of the two factors are $q(t) = 100$ and $T_f(t) = 400$ while their simulated values are shown in Fig. 2. Then the initial HMM–BRB based model as constructed in Section 4.4.1 is updated using the above two constant factors as the inputs and the values represented by the solid line in Fig. 3 as the outputs. Finally, Eq. (19) is used to predict the failure and the real-time prognosis result is given by the dash-dotted line in Fig. 6.

Fig. 6 shows that the dotted line is more close to the solid line generated by Eq. (31) than the dash-dotted line. It can be concluded that the proposed HMM–BRB based algorithm can predict the CSTR failure more accurately when influences from the changes of environmental factors are considered.

4.4.5. Comparative study with the classical HMM based failure prognosis algorithm

As mentioned in Section 1, the classical HMM based algorithm can also be used to predict the hidden failure (Baruah and Chinnam, 2003; Zhang et al., 2005). In order to demonstrate the validity of the proposed HMM–BRB based failure prognosis algorithm further, the following comparative study between the proposed algorithm and the classical HMM based failure prognosis algorithm is carried out.

Here the HMM with continuous observation density which includes two hidden states is chosen for failure prognosis of the CSTR. It is also assumed that the monitored observation $\mathbf{y}$ obeys the Gaussian distribution as given in Eq. (3). Note that in the HMM based algorithm, only the simulated observations with influences from the changes of environmental factors (i.e., the values represented by the solid line in Fig. 3) are used. In other words, though the environmental factors do exist, they cannot be considered in the HMM based algorithm. The parameters of the initial HMM model are set as follows:

The two hidden states s_1 and s_2 also denote '*Normal*' and '*Failure*', respectively. Different from the initial HMM–BRB based model, the initial transition probability matrix between the hidden states need to be given directly in the HMM based algorithm. Here it is assumed to be $\lambda(0) = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$. The other parameters which include the initial probability vector of the hidden states $\boldsymbol{\pi}(0)$, the utility of '*Normal*' $U(s_1)$, the utility of '*Failure*' $U(s_2)$, the threshold P_{th} and prediction step Δt are set same to Section 4.4.1.

Then using the values represented by the solid line in Fig. 3 as the outputs of the HMM, the recursive algorithm proposed by LeGland and Mevel (1997) is directly adopted to update the initial HMM. Based on the updated HMM, the failure can also be predicted by using Eqs. (19) and (20). The real-time failure prognosis result is given by the dashed line in Fig. 6.

Fig. 6 shows that among the dotted, dash-dotted and dashed lines, the dashed line generated by the HMM based algorithm is farthest to the solid one. It is demonstrated that the proposed HMM–BRB based algorithm can predict the failure more accurately than the classical HMM based algorithm.

5. Conclusions

In this paper, based on hidden Markov model (HMM) and belief rule base (BRB) methodology, a new HMM–BRB model is firstly proposed to represent a real world system under the influences from changes of environmental factors. Then a recursive algorithm for online updating the HMM–BRB based model is developed on the basis of the recursive algorithms for estimating the parameters of the HMM and the BRB (LeGland and Mevel, 1997; Zhou et al., 2009). Finally, a new HMM–BRB based method for real-time hidden failure prognosis is proposed for condition-based maintenance (CBM) of complex systems. These construct the main contributions of this paper. An experimental case study is examined to demonstrate implementation and potential applications of the proposed real-time failure prognosis method.

There are several features in the proposed failure prognosis method. First of all, the proposed method can predict the hidden failure without exact mathematical models of systems. Secondly, the inputs of a HMM–BRB based model, i.e., the environmental factors, can be either quantitative or qualitative. This is inherited from the BRB methodology (Yang et al., 2006). This ensures that different factors affecting the failure prognosis results can be considered. This feature is also the main contribution of this paper. Finally, once new information becomes available, the proposed method can predict the hidden failure real-time without waiting for all information to be provided, which can greatly save time and be of great practical significance. The above features allow the new real-time failure prognosis algorithm to be applied widely in engineering.

Although the effectiveness of the proposed algorithm has been demonstrated by an experimental case study, its validity and capability in dealing with more practical and complicated problems need to be further tested. Moreover, the hidden states of the HMM can also represent other performance indices of complex systems such as reliability and safety. Therefore, it is possible to use the proposed HMM–BRB based model to solve reliability prediction and safety analysis of the systems. Those need to be studied in future research.

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Appendix A

In Eqs. (17) and (18), the probability vector of the hidden states $\boldsymbol{\pi}(t)$, the partial derivatives $\partial\mathbf{b}(t)^T/\partial Q_d(t)$ and $\mathbf{z}_d(t)$ are needed and can be determined as follows.

(1) $\boldsymbol{\pi}(t)$ is calculated using the recursive algorithm given by LeGland and Mevel (1997).

$$\boldsymbol{\pi}(t+1) = \frac{\boldsymbol{\lambda}(t)^T\mathbf{B}(t)\boldsymbol{\pi}(t)}{(\mathbf{b}(t)^T\boldsymbol{\pi}(t))} \tag{A.1}$$

where $\mathbf{B}(t) = \text{diag}\{b_1(t), \ldots, b_N(t)\}$.

(2) According to $\mathbf{b}(t) = [b_1(t), \ldots, b_N(t)]^T$, Eq. (4) and Eq. (10), $\partial\mathbf{b}(t)^T/\partial Q_d(t)$ is determined by

$$\frac{\partial\mathbf{b}(t)^T}{\partial Q_d(t)} = \begin{cases} \mathbf{0}_N, & d = 1, \ldots, Z \\ \mathbf{e}_i(t), & i = 1, \ldots, N; d = Z + i \\ \boldsymbol{\rho}(t), & d = Z + N + 1 \end{cases} \tag{A.2}$$

where $\mathbf{e}_i(t)$ is a column vector whose dimension is N. The ith element $e_i(t)$ of $\mathbf{e}_i(t)$ is not zero and the others are all zero. $e_i(t)$ $(i = 1, \ldots, N)$ denotes the derivative of $b_i(t)$ with respect to μ_i. $\boldsymbol{\rho}(t) = [\rho_1(t), \ldots, \rho_N(t)]^T$ and $\rho_j(t)$ $(j = 1, \ldots, N)$ denotes the derivative of $b_j(t)$ with respect to σ. The analytical formations of $e_i(t)$ and $\rho_j(t)$ can be obtained according to Eq. (4).

(3) Substituting Eq. (A.1) into Eq. (18), we have

$$\mathbf{z}_d(t+1) = \frac{\partial \boldsymbol{\pi}(t+1)}{\partial Q_d(t+1)} = \Xi(t)\mathbf{z}_d(t) + \widetilde{\Xi}_d(t), \quad d = 1, \ldots, W \tag{A.3}$$

where $\Xi(t)$ and $\widetilde{\Xi}_d(t)$ are the following partial derivatives of $\boldsymbol{\pi}(t+1)$ with respect to $Q_d(t+1)$.

$$\Xi(t) = \boldsymbol{\lambda}(t)^T \left[\mathbf{I} - \frac{\mathbf{B}(t)\boldsymbol{\pi}(t)\mathbf{1}_N^T}{\mathbf{b}(t)^T\boldsymbol{\pi}(t)} \right] \frac{\mathbf{B}(t)}{\mathbf{b}(t)^T\boldsymbol{\pi}(t)} \tag{A.4}$$

$$\widetilde{\Xi}_d(t) = \boldsymbol{\lambda}(t)^T \left[\mathbf{I} - \frac{\mathbf{B}(t)\boldsymbol{\pi}(t)\mathbf{1}_N^T}{\mathbf{b}(t)^T\boldsymbol{\pi}(t)} \right] \frac{\partial \mathbf{B}(t)}{\partial Q_d(t)} \frac{\boldsymbol{\pi}(t)}{\mathbf{b}(t)^T\boldsymbol{\pi}(t)} + \frac{\partial \boldsymbol{\lambda}(t)}{\partial Q_d(t)} \frac{\mathbf{B}(t)\boldsymbol{\pi}(t)}{\mathbf{b}(t)^T\boldsymbol{\pi}(t)} \tag{A.5}$$

In Eqs. (A.4) and (A.5), the probability transition matrix $\boldsymbol{\lambda}(t)$, the partial derivatives $\partial\mathbf{B}(t)/\partial Q_d(t)$ and $\partial\boldsymbol{\lambda}(t)/\partial Q_d(t)$ can be determined as follows.

(i) Let $\mathbf{0}_{N\times N}$ denote $N \times N$ matrix with each entry is 0. From Eqs. (A.1), $\partial\mathbf{B}(t)/\partial Q_d(t)$ satisfies

$$\frac{\partial \mathbf{B}(t)}{\partial Q_d(t)} = \begin{cases} \mathbf{0}_{N\times N}, & d = 1, \ldots, Z \\ \mathrm{diag}\Big\{ \underbrace{0, \ldots, 0}_{j-1}, e_j, \underbrace{0, \ldots, 0}_{N-j} \Big\}, & j = 1, \ldots, N; d = Z + j \\ \mathrm{diag}\{\rho_1(t), \ldots, \rho_N(t)\}. & d = Z + N + 1 \end{cases} \tag{A.6}$$

(ii) From Eqs. (A.4) and (A.5), $\boldsymbol{\lambda}(t) = [\lambda_{i,j}(t)]_{N\times N}$ and $\partial\boldsymbol{\lambda}(t)/\partial Q_d(t)$ are needed. As shown in Eq. (9), the transition probabilities, $\lambda_{i,1}(t), \ldots, \lambda_{i,N}(t)$, are generated by the BRB_*i* using the evidential reasoning (ER) approach which is denoted as O_i, so the transition matrix $\boldsymbol{\lambda}(t)$ is generated by a total of N belief rule bases. The ER approach, which was proposed by Yang and Sen (1994), Yang et al. (2006) and Yang and Xu (2002) based on Dempster–Shafer theory of evidence (Dempster, 1968; Shafer, 1976), decision theory (Huang and Yong, 1981) and fuzzy set theory (Zadeh, 1965), mainly includes two steps.

In the first step, when the inputs $u_1(t), \ldots, u_M(t)$ are available, the activation weight $\omega_k^i(t)$ of the kth belief rule in the BRB_*i* is:

$$\omega_k^i(t) = \frac{\theta_k^i \prod_{m=1}^{M} \left(\alpha_m^k(t)\right)^{\bar{\delta}_m^i}}{\sum_{l=1}^{L} \theta_l^i \prod_{m=1}^{M} \left(\alpha_m^l(t)\right)^{\bar{\delta}_m^i}} \quad \text{and} \quad \bar{\delta}_m^i = \frac{\delta_m}{\max_{m=1,\ldots,M}\{\delta_m\}} \tag{A.7}$$

where $\alpha_m^k(t)(m = 1, \ldots, M, k = 1, \ldots, L)$, which is called the individual matching degree, is the belief degree of the input $u_m(t)$ to the referential value A_m^k in the kth rule of the BRB_*i*. Depending on the nature of an antecedent attribute and data available such as a qualitative attribute using linguistic values, $\alpha_m^k(t)$ could be generated using various ways. A scheme for handling various types of input information has been summarized by Yang (2001) and Yang et al. (2006, 2007). An example for transforming the quantitative information will be given in Section 4.4.

In the second step, according to the ER analytical algorithm (Wang et al., 2006), $\lambda_{i,j}(t)$ in Eq. (9) can be determined as:

$$\lambda_{i,j}(t) = \frac{\xi_i(t) \times \left[\prod_{k=1}^{L}\left(\omega_k^i(t)\beta_{j,k}^i + 1 - \omega_k^i(t)\sum_{s=1}^{N}\beta_{s,k}^i\right) - \prod_{k=1}^{L}\left(1 - \omega_k^i(t)\sum_{i=1}^{N}\beta_{s,k}^i\right) \right]}{1 - \xi_i(t) \times \left[\prod_{k=1}^{L}(1 - \omega_k^i(t)) \right]} \tag{A.8}$$

$$\xi_i(t) = \left[\sum_{j=1}^{N} \prod_{k=1}^{L} \left(\omega_k^i(t)\beta_{j,k}^i + 1 - \omega_k^i(t) \sum_{s=1}^{N} \beta_{s,k}^i \right) - (N-1) \prod_{k=1}^{L} \left(1 - \omega_k^i(t) \sum_{s=1}^{N} \beta_{s,k}^i \right) \right]^{-1} \tag{A.9}$$

(iii) There is $\partial\boldsymbol{\lambda}(t)/\partial Q_d(t) = [\partial\lambda_{i,j}(t)/\partial Q_d(t)]_{N\times N}$ and $\partial\lambda_{i,j}(t)/\partial Q_d(t)$ is calculated by

$$\frac{\partial \lambda_{i,j}(t)}{\partial Q_d(t)} = \begin{cases} \frac{\partial\lambda_{i,j}(t)}{\partial\theta_s^i(t)}, & d = g_1 + s,\ i = 1, \ldots, N,\ s = 1, \ldots, N \\ \frac{\partial\lambda_{i,j}(t)}{\partial\bar{\delta}_m^i(t)}, & d = g_1 + L + m,\ i = 1, \ldots, N,\ m = 1, \ldots, M \\ \frac{\partial\lambda_{i,j}(t)}{\partial\beta_{z,p}^i(t)}, & d = g_1 + L + M + (p-1) \times N + z, p = 1, \ldots, L, z = 1, \ldots, N \\ 0, & d = \text{others} \end{cases} \tag{A.10}$$

where $d = 1, \ldots, W$, $j = 1, \ldots, N$ and $g_1 = (i-1) \times (L + M + L \times N)$. Let $\partial\lambda_{i,j}(t)/\partial Q_d(t)$ denote the value of $\partial\lambda_{i,j}/\partial Q_d$ at time instant t. The detailed algorithm to determine $\partial\lambda_{i,j}/\partial Q_d$ was given by Zhou et al. (2009).

References

Angeli, C., Chatzinikolaou, A., 2002. Prediction and diagnosis of failures in hydraulic systems. Proceedings of the Institution of Mechanical Engineers 216 (2), 293–297.

Baruah, P., Chinnam, R.B., 2003. HMMs for diagnostics and prognostics in machining processes. In: Proceedings of the 57th Society for Machine Failure Prevention Technology Conference, Virginia Beach, VA, April 14–18.

Bunks, C., McCarthy, D., 2000. Condition-based maintenance of machines using hidden Markov models. Mechanical Systems and Signal Processing 14 (4), 597–612.

Cagno, E., Caron, F., Mancini, M., Ruggeri, F., 2000. Using AHP in determining the prior distribution on gas pipeline failures in a robust Bayesian approach. Reliability Engineering & System Safety 67 (3), 275–284.

Chen, D.Y., Trivedi, K.S., 2005. Optimization for condition-based maintenance with semi-Markov decision process. Reliability Engineering & System Safety 90 (1), 25–29.

Chen, M.Z., Zhou, D.H., 2001. An adaptive failure prediction method based on strong tracking filter. Journal of Shanghai Maritime University 22 (3), 35–40 (in Chinese).

Chen, M.Z., Zhou, D.H., Liu, G.P., 2005. A new particle predictor for failure prediction of nonlinear time-varying systems. Developments in Chemical and Engineering and Mineral Processing 13 (3–4), 379–388.
Davis, P., Burn, S., Moglia, M., Gould, S., 2007. A physical probabilistic model to predict failure rates in buried PVC pipelines. Reliability Engineering & System Safety 92 (9), 1258–1266.
Dempster, A.P., 1968. A generalization of Bayesian inference. Journal of the Royal Statistical Society, Series B 30 (2), 205–247.
Dong, M., He, D., 2007. Hidden semi-Markov model-based methodology for multi-sensor equipment health diagnosis and prognosis. European Journal of Operational Research 178 (3), 1177–1187.
Huang, C.L., Yong, K., 1981. Multiple Attribute Decision Making Methods and Applications: A State-Of Art Survey. Springer, Berlin.
Jardin, A.K.S., Lin, D.M., Banjevic, D., 2006. A review on machinery diagnostics and prognostics implementing condition-based maintenance. Mechanical Systems and Signal Processing 20 (7), 1483–1510.
Lee, J.M., Kim, S.J., Hwang, Y., Song, C.S., 2004. Diagnosis of mechanical failure signals using continuous hidden Markov model. Journal of Sound and Vibration 276 (3–5), 1065–1080.
LeGland, F., Mevel, L., 1997. Recursive estimation in hidden Markov models. In: Proceedings of the 36th Conference on Decision and Control, pp. 3468–3473.
Li, Z.N., Han, J., Sun, J.J., He, Y.Y., Chu, F.L., 2007. Failure recognition method based on independent component analysis and hidden Markov model. Journal of Vibration and Control 13 (2), 125–137.
Lin, D., Makis, V., 2002. State and model parameter estimation for transmissions on heavy hauler trucks using oil data. Proceedings of Comadem 2002, 339–348.
Lu, K.S., Saeks, R., 1979. Failure prediction for an on-line maintenance system in a Poisson shock environment. IEEE Transactions on Systems, Man, and Cybernetics 9 (6), 356–362.
Rabiner, L.R., 1989. A tutorial on hidden Markov models and selected applications in speech recognition. Proceedings of the IEEE 77 (2), 257–286.
Rensselaer Polytechnic Institute, 1999. Project links: continuous stirred tank reactor. Available from: <http://www.ibiblio.org/links/devmodules/cstr/index.html>.
Shafer, G., 1976. A Mathematical Theory of Evidence. Princeton University Press, Princeton, NJ.
Tanrioven, M., Wu, Q.H., Turner, D.R., Kocatepe, C., Wang, J., 2004. A new approach to real-time reliability analysis of transmission system using fuzzy Markov model. Electrical Power & Energy Systems 26 (10), 821–832.
Wang, W., 2002. A model to predict the residual life of rolling element bearings given monitored condition monitoring information to data. IMA Journal of Management Mathematics 13, 3–16.
Wang, W., 2007a. A two-stage prognosis model in condition based maintenance. European Journal of Operational Research 182 (3), 1177–1187.
Wang, W., 2007b. A prognosis model for wear prediction based on oil-based monitoring. Journal of the Operational Research Society 58 (7), 887–893.
Wang, W., Christer, A.H., 2000. Towards a general condition based maintenance model for a stochastic dynamic system. Journal of the Operational Research Society 51 (2), 145–155.
Wang, W., Hussin, B., 2009. Plant residual time modeling based on observed variables in oil samples. Journal of the Operational Research Society 60 (6), 789–796.
Wang, W., Zhang, W., 2005. A model to predict the residual life of aircraft engines based on oil analysis data. Naval Research Logistics 52, 276–284.
Wang, Y.M., Yang, J.B., Xu, D.L., 2006. Environmental impact assessment using the evidential reasoning approach. European Journal of Operational Research 174 (3), 1885–1913.
Xu, D.L., Liu, J., Yang, J.B., Liu, G.P., Wang, J., Jenkinson, I., Ren, J., 2007. Inference and learning methodology of belief-rule-based expert system for pipeline leak detection. Expert Systems with Applications 32 (1), 103–113.
Yang, J.B., 2001. Rule and utility based evidential reasoning approach for multi-attribute decision analysis under uncertainties. European Journal of Operational Research 131 (1), 31–61.
Yang, S.K., 2003. A condition-based failure-prediction and processing-scheme for preventive maintenance. IEEE Transactions on Reliability 52 (3), 373–383.
Yang, S.K., Liu, T.S., 1998. A Petri net approach to early failure detection and isolation for preventive maintenance. Quality and Reliability Engineering International 14 (5), 319–330.
Yang, S.K., Liu, T.S., 1999. State estimation for predictive maintenance using Kalman filter. Reliability Engineering & System Safety 66 (1), 29–39.
Yang, J.B., Sen, P., 1994. A general multi-level evaluation process for hybrid MADM with uncertainty. IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans 24 (10), 1458–1473.
Yang, J.B., Xu, D.L., 2002. On the evidential reasoning algorithm for multiple attribute decision analysis under uncertainty. IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans 32 (3), 289–304.
Yang, J.B., Liu, J., Wang, J., Sii, H.S., Wang, H.W., 2006. Belief rule-base inference methodology using the evidential reasoning approach – RIMER. IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans 36 (2), 266–285.
Yang, J.B., Liu, J., Xu, D.L., Wang, J., Wang, H.W., 2007. Optimal learning method for training belief rule based systems. IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans 37 (4), 569–585.
Ying, J., Kirubarajan, T., Pattipati, K.R., Patterson-Hine, A., 2000. A hidden Markov model-based algorithm for failure diagnosis with partial and imperfect tests. IEEE Transactions on Systems, Man, and Cybernetics – Part C: Applications and Reviews 30 (4), 463–473.
Zadeh, L.Z., 1965. Fuzzy sets. Information Control 8 (3), 338–353.
Zhang, X.D., Xu, R., Kwan, C., Liang, S.Y., Xie, Q.L., Haynes, L., 2005. An integrated approach to bearing failure diagnostics and prognostics. In: 2005 American Control Conference, pp. 2750–2755.
Zhou, D.H., Frank, P.M., 1998. Failure diagnostics and failure tolerant control. IEEE Transactions on Aerospace and Electronic Systems 34 (2), 420–427.
Zhou, Z.J., Hu, C.H., Zhou, D.H., 2006. Failure prediction techniques for dynamic systems based on non-analytical model. Information Control 35 (3), 608–613 (in Chinese).
Zhou, Z.J., Hu, C.H., Fan, H.D., Li, J., 2008. Fault prediction of the nonlinear systems with uncertainty. Simulation Modelling Practice and Theory 16 (6), 690–703.
Zhou, Z.J., Hu, C.H., Yang, J.B., Xu, D.L., Zhou, D.H., 2009. Online updating belief-rule-based system for pipeline leak detection under expert intervention. Expert Systems with Applications 36 (3), 7700–7709.
Zhou, Z.J., Hu, C.H., Yang, J.B., Xu, D.L., Chen, M.Y., Zhou, D.H., 2010. A sequential learning algorithm for online constructing belief-rule-based systems. Expert Systems with Applications 37 (2), 1790–1799.