Global Instability Computations of Separated Flows

R P Logue*, J Gajjar1, and A Ruban1

1 The University of Manchester, Mathematics Dept., Oxford road, Manchester, UK

The primary motivation for the current work is to develop suitable techniques for studying the global instability in many different flows such as that occurring in the subsonic flow past a corner. In this paper we will discuss the methodology and some of the results obtained.

1 Equations and Method

The subsonic triple deck equations where derived in the early seventies for a detailed review see Ruban [1]. The subsonic triple deck equations past a corner are,

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]  
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial^2 u}{\partial y^2}, \]  
\[ \frac{dp}{dx} = -\frac{1}{\pi} \int_{-\infty}^{\infty} a''(w) - f''_w(w) \frac{w-x}{w-x} dw, \]

with the conditions,

\[ u = v = 0 \quad \text{at} \quad y = 0, \]  
\[ u \rightarrow y + a \quad \text{as} \quad y \rightarrow \infty, \]  
\[ u \rightarrow y \quad \text{as} \quad x \rightarrow -\infty, \]  
\[ f_w(x) = \frac{\alpha}{2} (x + \sqrt{x^2 + 0.1^2}), \]

where \( u, v, p \) and \( a \) are the streamwise velocity component, wall normal velocity component, pressure and displacement respectively. Also \( x, y, t, f_w \) and \( \alpha \) are the streamwise spatial coordinate, wall normal coordinate, time, the wall function respectively and scaled corner angle respectively. Equations 1 to 7 are discretized using finite differences in the streamwise direction and a Chebyshev collocation method in the wall normal direction, the finite difference grid points are clustered at the corner. The linear system is solved using a Newton linearization with a preconditioned GMRES technique and typically solves each Newton iteration, using 100 basis vectors, in one iteration.

1.1 Results

It has been known since Korolev [2] that a turning point bifurcation exists for the subsonic triple deck flow over a concave corner. Figure 1 shows the corner angle in terms of separation distance and the two solutions for the various flow quantities. It is evident, from figure 1 that the flow separates at \( \alpha = -5.07 \) and the turning point bifurcation takes place at \( \alpha = -5.58 \). As the upper solution branch develops the separation distance increases and the reattachment point moves further downstream. It is evident that on the upper solution branch a kink forms in the pressure solution and a larger displacement region is formed due to the increased separation region.

The global temporal stability of equations 1 to 7 was carried out by linearizing the equations in terms of the basic flow and a perturbation,

\[ u(x, y, t) = u_b(x, y) + \tilde{u}(x, y)e^{-\lambda t}, \]  
\[ v(x, y, t) = v_b(x, y) + \tilde{v}(x, y)e^{-\lambda t}, \]  
\[ p(x, t) = p_b(x) + \tilde{p}(x)e^{-\lambda t}, \]  
\[ a(x, t) = a_b(x) + \tilde{a}(x)e^{-\lambda t}, \]

* Corresponding author: e-mail: plogue@maths.man.ac.uk, Phone: +0044 161 275 5866.
variables with a subscript $b$ denote the basic flow values (steady state solution), variables with a tilde denote the perturbed flow values and $\lambda$ is an eigenvalue, the resulting eigenvalue problem was solved in ARPACK. The global analysis results are shown in figure 2, the first plot shows the eigenvalues for the flat plate boundary layer and the Tollmien-Schlichting modes are indicated by the presence of two complex conjugate eigenvalues. It is clearly evident that on the lower solution branch there is a real eigenvalue near the imaginary axis and as the flow develops to the upper solution branch the eigenvalue crosses the imaginary axis and becomes unstable.

Fig. 2 Global stability results

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References