Analysis and Response Management of Frequency Events in Low Inertia Power Systems

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Negar Shams

School of Electrical and Electronic Engineering
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<th>Description</th>
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<tbody>
<tr>
<td>AUFLS</td>
<td>Adaptive Under Frequency Load Shedding Scheme</td>
</tr>
<tr>
<td>COI</td>
<td>Centre Of Inertia</td>
</tr>
<tr>
<td>DG</td>
<td>Distributed Generation</td>
</tr>
<tr>
<td>DSA</td>
<td>Dynamic Security Assessment</td>
</tr>
<tr>
<td>DT</td>
<td>Decision Tree</td>
</tr>
<tr>
<td>EMS</td>
<td>Energy Management System</td>
</tr>
<tr>
<td>FDR</td>
<td>Frequency Disturbance Recorder</td>
</tr>
<tr>
<td>FNET</td>
<td>Wide Area Frequency Monitoring Network</td>
</tr>
<tr>
<td>GB</td>
<td>Great Britain</td>
</tr>
<tr>
<td>KNN</td>
<td>K Nearest Neighbours</td>
</tr>
<tr>
<td>NERC</td>
<td>North American Electricity Reliability Corporation</td>
</tr>
<tr>
<td>OCGT</td>
<td>Open Cycle Gas Turbines</td>
</tr>
<tr>
<td>PCA</td>
<td>Principle Component Analysis</td>
</tr>
<tr>
<td>PMU</td>
<td>Phasor Measurement Unit</td>
</tr>
<tr>
<td>PV</td>
<td>Photo Voltaic</td>
</tr>
<tr>
<td>RoCoF</td>
<td>Rate of Change of Frequency</td>
</tr>
<tr>
<td>SCADA</td>
<td>Supervisory Control And Data Acquisition</td>
</tr>
<tr>
<td>SFR</td>
<td>Simplified Frequency Response</td>
</tr>
<tr>
<td>SI</td>
<td>System Identification</td>
</tr>
<tr>
<td>SIPS</td>
<td>System Integrity Protection Scheme</td>
</tr>
<tr>
<td>SMT</td>
<td>Synchronised Measurement Technology</td>
</tr>
<tr>
<td>UFLS</td>
<td>Under Frequency Load Shedding Scheme</td>
</tr>
<tr>
<td>WAMC</td>
<td>Wide Area Monitoring Control</td>
</tr>
<tr>
<td>WAMPAC</td>
<td>Wide Area Monitoring Protection And Control</td>
</tr>
<tr>
<td>WAMS</td>
<td>Wide Area Monitoring Systems</td>
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</table>
### Nomenclature

#### Chapter 1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \hat{f}(t) )</td>
<td>vector containing RoCoF measurements of limited number of generators at time ( t )</td>
</tr>
<tr>
<td>( H )</td>
<td>vector containing the inertia constant values of the generators in the system</td>
</tr>
<tr>
<td>( \bar{I}(t) )</td>
<td>vector containing the current phasors of currents flowing through the buses equipped with PMUs at time ( t )</td>
</tr>
<tr>
<td>( LF )</td>
<td>load flow results containing the voltages and angles of the buses as well as active power flowing in and out of each bus</td>
</tr>
<tr>
<td>( P_e(t) )</td>
<td>vector containing active power measurements from limited number of generators at time ( t )</td>
</tr>
<tr>
<td>( \bar{V}(t) )</td>
<td>vector containing the voltage phasors of the buses equipped with PMUs at time ( t )</td>
</tr>
<tr>
<td>( Z )</td>
<td>impedance matrix of the system</td>
</tr>
</tbody>
</table>

#### Chapter 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta f )</td>
<td>frequency deviation</td>
</tr>
<tr>
<td>( \Delta P )</td>
<td>size of active power imbalance</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>torque imbalance</td>
</tr>
<tr>
<td>( dm )</td>
<td>mass particle</td>
</tr>
<tr>
<td>( E_{\text{kin}} )</td>
<td>kinetic energy stored within the rotating mass of a power system plant</td>
</tr>
<tr>
<td>( f(t) )</td>
<td>frequency (used in voltage phasor estimation at time ( t ) )</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>nominal frequency</td>
</tr>
<tr>
<td>( f_{0m} )</td>
<td>rated rotating frequency of a machine</td>
</tr>
<tr>
<td>( f_{\text{COI}}(0^+) )</td>
<td>rate of change of frequency of the centre of inertia shortly following the active power disturbance</td>
</tr>
<tr>
<td>( f_{\text{COI}}(t) )</td>
<td>frequency of the Centre of Inertia at time ( t )</td>
</tr>
<tr>
<td>( f_{\text{COI}}(t) )</td>
<td>rate of change of frequency of the Centre of Inertia at time ( t )</td>
</tr>
<tr>
<td>( f_{\text{fl}} )</td>
<td>steady-state frequency at full load</td>
</tr>
<tr>
<td>( f_{\text{e}} )</td>
<td>estimated frequency at discrete time index</td>
</tr>
<tr>
<td>( f_{\text{r}} )</td>
<td>rotating frequency of the machine</td>
</tr>
<tr>
<td>( f_{\text{nt}} )</td>
<td>steady-state frequency at no load</td>
</tr>
<tr>
<td>( f_{\text{s}} )</td>
<td>sampling frequency</td>
</tr>
<tr>
<td>( H )</td>
<td>inertia constant</td>
</tr>
<tr>
<td>( h(x) )</td>
<td>a vector of non-linear functions</td>
</tr>
<tr>
<td>( H_i )</td>
<td>( H ) constant of the ( i^{th} ) generator on its own base ( S_i )</td>
</tr>
<tr>
<td>( H_{i,\text{sys}} )</td>
<td>( H ) constant of the ( i^{th} ) generator on the system MVA base</td>
</tr>
<tr>
<td>( H_{\text{sys}} )</td>
<td>total system inertia constant</td>
</tr>
<tr>
<td>( J )</td>
<td>moment of inertia of the rotating mass</td>
</tr>
<tr>
<td>( K_D )</td>
<td>damping constant</td>
</tr>
<tr>
<td>( M )</td>
<td>total mass of the rotating body</td>
</tr>
<tr>
<td>( m_k )</td>
<td>number of samples in one ( T_w )</td>
</tr>
<tr>
<td>( n )</td>
<td>number of synchronous generating units</td>
</tr>
<tr>
<td>( r )</td>
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<td>mechanical power input of synchronous generator (i)</td>
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<td>$\bar{Y}_i$</td>
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<td>$\dot{f}_{\text{COI}}(t)$ calculated using limited number monitored generators ($S$)</td>
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<td>(H_{\mathbf{R}})</td>
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<td>inertia of the region (i) ((R_i))</td>
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<td>(H_{\mathbf{Sys}})</td>
<td>total system inertia constant</td>
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<td>Synchronising Power Coefficient between synchronous generator (i) and node (u) when system is reduced to all internal synchronous generator nodes and bus (u)</td>
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<td>metric for evaluating location detection precision (RI method performance metric)</td>
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<td>metric for evaluating location detection precision (RI method performance metric)</td>
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<td>number of regions/clusters</td>
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<td>(o)</td>
<td>generator medoid of a region</td>
</tr>
<tr>
<td>(P_d)</td>
<td>active power disturbance size</td>
</tr>
<tr>
<td>(P_d^u)</td>
<td>disturbance size for disturbance location at bus (u)</td>
</tr>
<tr>
<td>(P_{est})</td>
<td>disturbance size estimate</td>
</tr>
<tr>
<td>(q)</td>
<td>bus with minimum mismatch value at (t_d)</td>
</tr>
<tr>
<td>(R)</td>
<td>Number of monitored generators</td>
</tr>
<tr>
<td>(R)</td>
<td>the set which includes all regions membership information</td>
</tr>
<tr>
<td>(R_{\text{es}}(t))</td>
<td>a set containing all RoCoF estimate matrices at time (t)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$R_i$</td>
<td>set of generators making up region $i$</td>
</tr>
<tr>
<td>$R_{i,\text{est}}(t)$</td>
<td>RoCoF estimate matrix for monitored generator $i$ at time $t$</td>
</tr>
<tr>
<td>$R_{i,\text{ex,}\text{ej}}(t)$</td>
<td>element $j\text{n}$ of $R_{i,\text{est}}(t)$ matrix which represents RoCoF estimate of generator $j$ at time $t$ using generator $i$ RoCoF assuming disturbance has taken place at bus $n$ and time $t$</td>
</tr>
<tr>
<td>$RL$</td>
<td>right location metric (RI method performance metric)</td>
</tr>
<tr>
<td>$RL_{W1}$</td>
<td>metric for evaluating location detection precision (RI method performance metric)</td>
</tr>
<tr>
<td>$RL_{W2}$</td>
<td>metric for evaluating location detection precision (RI method performance metric)</td>
</tr>
<tr>
<td>$RL_{W3}$</td>
<td>metric for evaluating location detection precision (RI method performance metric)</td>
</tr>
<tr>
<td>$R_{\text{mis}}(t)$</td>
<td>mismatch vector at time $t$</td>
</tr>
<tr>
<td>$R_{\text{mis,}\text{n}}(t)$</td>
<td>element $n$ (corresponding to bus $n$) of mismatch vector $R_{\text{mis}}(t)$</td>
</tr>
<tr>
<td>$S$</td>
<td>set of monitored generators</td>
</tr>
<tr>
<td>$t_d$</td>
<td>disturbance time</td>
</tr>
<tr>
<td>$t_d^+$</td>
<td>shortly following the disturbance time</td>
</tr>
<tr>
<td>$\bar{V}(t)$</td>
<td>vector containing the voltage phasors of the buses equipped with PMUs at time $t$</td>
</tr>
<tr>
<td>$Z$</td>
<td>impedance matrix</td>
</tr>
<tr>
<td>$\gamma(t)$</td>
<td>mismatch index based on $B(t)$</td>
</tr>
<tr>
<td>$\gamma_{\text{cut}}$</td>
<td>threshold on $\gamma(t)$ (coming from the disturbance detection decision tree)</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>reporting rate of the measurements (20ms)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>short time frame following the disturbance occurrence for which generators RoCoF clustering (regioning) is carried out</td>
</tr>
<tr>
<td>$\zeta(t)$</td>
<td>mismatch index based on $R_{\text{mis}}(t)$</td>
</tr>
<tr>
<td>$\zeta_{\text{cut}}$</td>
<td>threshold on $\zeta(t)$ (coming from the disturbance detection decision tree)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>multiplier used to determine the DLC</td>
</tr>
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</table>

**Chapter 6**

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<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta f(s)$</td>
<td>frequency deviation in Laplas form</td>
</tr>
<tr>
<td>$\Delta f_f$</td>
<td>final frequency deviation</td>
</tr>
<tr>
<td>$\Delta f_l$</td>
<td>limit for a certain maximum frequency deviation</td>
</tr>
<tr>
<td>$\Delta P(t)$</td>
<td>summation of active power change at generators terminal compared to their scheduled value at time $t$</td>
</tr>
<tr>
<td>$\Delta \omega(t)$</td>
<td>incremental turbine rotating speed</td>
</tr>
<tr>
<td>$a_1$</td>
<td>first numerator coefficient of $G(s)$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>second numerator coefficient of $G(s)$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>first denominator coefficient of $G(s)$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>second denominator coefficient of $G(s)$</td>
</tr>
<tr>
<td>$b_3$</td>
<td>third denominator coefficient of $G(s)$</td>
</tr>
<tr>
<td>$c_i$</td>
<td>regression weight for $i^{th}$ previous sampled output</td>
</tr>
<tr>
<td>$D$</td>
<td>load damping factor</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>delay</td>
<td>delay in active power response initiation</td>
</tr>
<tr>
<td>$d_i$</td>
<td>regression weight for $i^{th}$ previous sampled input</td>
</tr>
<tr>
<td>DLC</td>
<td>disturbance location candidates</td>
</tr>
<tr>
<td><strong>E</strong></td>
<td>measurement noise matrix</td>
</tr>
<tr>
<td>$E(f_i)^*$</td>
<td>percentage error in estimation of $f_i$ when using a portion of $P_d$ seen by the monitored generators to estimate the $f_{CoI}$ (Error index)</td>
</tr>
<tr>
<td>$E(f_n)$</td>
<td>percentage error in estimation of $f_n$ (Error index)</td>
</tr>
<tr>
<td>$E(f_n)^*$</td>
<td>percentage error in estimation of $f_n$ when using a portion of $P_d$ seen by the monitored generators to estimate the $f_{CoI}$ (Error index)</td>
</tr>
<tr>
<td>$E(H_{Sys})$</td>
<td>percentage error in estimation of $H_{Sys}$ (Error index)</td>
</tr>
<tr>
<td>$E(P_{c,0.8})$</td>
<td>percentage error in estimation of $P_{c,0.8}$ (Error index)</td>
</tr>
<tr>
<td>$E(t_n)$</td>
<td>percentage error in estimation of $t_n$ (Error index)</td>
</tr>
<tr>
<td>$E(x)$</td>
<td>percentage error in estimation of variable $x$</td>
</tr>
<tr>
<td>$e_k$</td>
<td>current time measurement noise</td>
</tr>
<tr>
<td><strong>f</strong></td>
<td>vector containing monitored generators frequency at time $t$</td>
</tr>
<tr>
<td>$f_0$</td>
<td>rated frequency</td>
</tr>
<tr>
<td>$\dot{f}_{CoI}^{max}$</td>
<td>loss of main relays RoCoF threshold</td>
</tr>
<tr>
<td>$\dot{f}_{CoI}(t_d^+)$</td>
<td>RoCoF of Cenntre of Inertia shortly following the disturbance occurance</td>
</tr>
<tr>
<td>$\dot{f}_i$</td>
<td>final (steady state) frequency</td>
</tr>
<tr>
<td>$F_H$</td>
<td>fraction of total power generated by the high pressure turbine</td>
</tr>
<tr>
<td><strong>Fit%</strong></td>
<td>a measure for assessing the accuracy of estimation of a signal in time against its real values (defined in Section 6.7.2)</td>
</tr>
<tr>
<td>$f_n$</td>
<td>frequency nadir</td>
</tr>
<tr>
<td><strong>G</strong></td>
<td>closed loop continuous transfer function of simplified system frequency response</td>
</tr>
<tr>
<td>$\hat{G}(z)$</td>
<td>estimate of discrete transfer function</td>
</tr>
<tr>
<td>$H$</td>
<td>inertia constant</td>
</tr>
<tr>
<td>$H_{i,sys}$</td>
<td>$H$ constant of the $i^{th}$ generator on the system MVA base</td>
</tr>
<tr>
<td>$H_{Sys}$</td>
<td>total system inertia constant</td>
</tr>
<tr>
<td>$k$</td>
<td>current sample time</td>
</tr>
<tr>
<td>$K_m$</td>
<td>mechanical gain</td>
</tr>
<tr>
<td>$L_S$</td>
<td>location of active power response</td>
</tr>
<tr>
<td>$m$</td>
<td>order of $G(z)$ numerator</td>
</tr>
<tr>
<td>$n$</td>
<td>order of $G(z)$ denominator</td>
</tr>
<tr>
<td>$n_w$</td>
<td>number of observations in time for both inputs and outputs samples</td>
</tr>
<tr>
<td>$P_{C,C}$</td>
<td>critical disturbance size for $\Delta f$ frequency level violation</td>
</tr>
<tr>
<td>$P_d$</td>
<td>active power disturbance size</td>
</tr>
<tr>
<td>$P_d(s)$</td>
<td>active power load change of the system in Laplas form</td>
</tr>
<tr>
<td>$P_d^{max}$</td>
<td>size of the system reference incident</td>
</tr>
<tr>
<td>$P_e(t)$</td>
<td>vector containing active power measurements from limited number of generators at time $t$</td>
</tr>
<tr>
<td>$P_{e,i}(t)$</td>
<td>active power output of the $i^{th}$ generator at time $t$</td>
</tr>
<tr>
<td>$P_{est}$</td>
<td>disturbance size estimate</td>
</tr>
<tr>
<td>$P_S$</td>
<td>required active power response size</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$P_{S0}$</td>
<td>difference between disturbance size and critical disturbance size</td>
</tr>
<tr>
<td>$P_{Set}$</td>
<td>vector containing all generators scheduled active power output obtained via last SCADA/EMS update</td>
</tr>
<tr>
<td>$P_{Sm}$</td>
<td>minimum response needed to avoid the frequency nadir falling more than 0.3Hz</td>
</tr>
<tr>
<td>$R$</td>
<td>droop</td>
</tr>
<tr>
<td>$s$</td>
<td>Laplas variable</td>
</tr>
<tr>
<td>$S_{base}$</td>
<td>system MVA base</td>
</tr>
<tr>
<td>$sw$</td>
<td>number of past sliding windows to get to the final estimate of SFR parameter at the current time</td>
</tr>
<tr>
<td>$t_d$</td>
<td>disturbance time</td>
</tr>
<tr>
<td>$t_{d^+}$</td>
<td>shortly following the disturbance time</td>
</tr>
<tr>
<td>$t_n$</td>
<td>frequency nadir time</td>
</tr>
<tr>
<td>$T_R$</td>
<td>reheat time constant</td>
</tr>
<tr>
<td>$u(t)$</td>
<td>current input value</td>
</tr>
<tr>
<td>$u_k$</td>
<td>current input in discrete form</td>
</tr>
<tr>
<td>$u_{k-x}$</td>
<td>$x^{th}$ previous sampled input in discrete form</td>
</tr>
<tr>
<td>$w$</td>
<td>length of the sliding window used for SFR identification</td>
</tr>
<tr>
<td>$Y$</td>
<td>output matrix</td>
</tr>
<tr>
<td>$y(t)$</td>
<td>current output value</td>
</tr>
<tr>
<td>$y_k$</td>
<td>current output in discrete form</td>
</tr>
<tr>
<td>$y_{k-x}$</td>
<td>$x^{th}$ previous sampled output in discrete form</td>
</tr>
<tr>
<td>$\beta$</td>
<td>regression weight matrix</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>estimate of regression weight matrix</td>
</tr>
<tr>
<td>$\Delta f_{COI}(t)$</td>
<td>deviation in $f_{COI}$ from rated frequency at time $t$</td>
</tr>
<tr>
<td>$\epsilon(i, \beta)$</td>
<td>prediction error for $i^{th}$ observation having a regression weight matrix of $\beta$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>sampling time for discrete transfer function</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>regressor matrix</td>
</tr>
<tr>
<td>$\Phi^+$</td>
<td>pseudo inverse of $\Phi$</td>
</tr>
<tr>
<td>$\Phi^T$</td>
<td>transpose of matrix $\Phi$</td>
</tr>
</tbody>
</table>

### Chapter 7

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta f$</td>
<td>frequency deviation</td>
</tr>
<tr>
<td>$a_p$</td>
<td>active power second order voltage dependency coefficient of a ZIP load</td>
</tr>
<tr>
<td>$a_q$</td>
<td>reactive power second order voltage dependency coefficient of a ZIP load</td>
</tr>
<tr>
<td>$b_p$</td>
<td>active power first order voltage dependency coefficient of a ZIP load</td>
</tr>
<tr>
<td>$b_q$</td>
<td>reactive power first order voltage dependency coefficient of a ZIP load</td>
</tr>
<tr>
<td>$c_p$</td>
<td>active power zero order voltage dependency coefficient of a ZIP load</td>
</tr>
<tr>
<td>$c_q$</td>
<td>reactive power zero order voltage dependency coefficient of a ZIP load</td>
</tr>
<tr>
<td>$d$</td>
<td>turbine blade diameter</td>
</tr>
<tr>
<td>$H_i$</td>
<td>$H$ constant of the $i^{th}$ generator on its own base $S_i$</td>
</tr>
<tr>
<td>$H_{Sys}$</td>
<td>total system inertia constant</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$H_{\text{Sys},w%}$</td>
<td>total system inertia when having wind penetration level of $w%$</td>
</tr>
<tr>
<td>$H_{\text{Sys},w%}^*$</td>
<td>increased total system inertia with the addition of synthetic inertia</td>
</tr>
<tr>
<td>$H_w$</td>
<td>inertia constant of a wind turbine in its own power base</td>
</tr>
<tr>
<td>$H_{W,w%}$</td>
<td>total inertia contribution from synthetic inertia control of wind turbines for $w%$ of wind generation penetration level</td>
</tr>
<tr>
<td>$K_{pf}$</td>
<td>active power frequency dependency coefficient of a ZIP load</td>
</tr>
<tr>
<td>$K_{qf}$</td>
<td>reactive power frequency dependency coefficient of a ZIP load</td>
</tr>
<tr>
<td>$P$</td>
<td>active power of ZIP load</td>
</tr>
<tr>
<td>$P_0$</td>
<td>active power load of a ZIP load at rated voltage and frequency</td>
</tr>
<tr>
<td>$P_{\Delta f}$</td>
<td>critical disturbance size for $\Delta f$ frequency level violation</td>
</tr>
<tr>
<td>$P_d$</td>
<td>active power disturbance size</td>
</tr>
<tr>
<td>$P_{s,w%}^{\text{max}}$</td>
<td>maximum fast frequency response requirement for wind penetration level of $w%$</td>
</tr>
<tr>
<td>$P_{\text{Sm}}$</td>
<td>load shedding size for the suggested DSA based AUFLS</td>
</tr>
<tr>
<td>$P_{\text{m}}$</td>
<td>minimum response needed to avoid the frequency nadir falling more than 0.8Hz</td>
</tr>
<tr>
<td>$Q$</td>
<td>reactive power of ZIP load</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>reactive power load of a ZIP load at rated voltage and frequency</td>
</tr>
<tr>
<td>$S_1$</td>
<td>semi adaptive UFLS scheme 1</td>
</tr>
<tr>
<td>$S_2$</td>
<td>semi adaptive UFLS scheme 2</td>
</tr>
<tr>
<td>$S_3$</td>
<td>semi adaptive UFLS scheme 3</td>
</tr>
<tr>
<td>$S_{\text{base}}$</td>
<td>system MVA base</td>
</tr>
<tr>
<td>$S_i$</td>
<td>power base for the $i^{th}$ generator</td>
</tr>
<tr>
<td>$S_{w%,i}$</td>
<td>MVA capacity of generator $i$ replaced by converter connected generation (wind generation)</td>
</tr>
<tr>
<td>$w%$</td>
<td>wind penetration level which is total MVA capacity replaced by wind as a percentage of total system installed capacity</td>
</tr>
<tr>
<td>$w_{%}^*$</td>
<td>equivalent wind generation level after the addition of synthetic inertia</td>
</tr>
</tbody>
</table>

**Chapter 8**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{COI}(0^*)}$</td>
<td>rate of change of frequency of the centre of inertia shortly following the disturbance</td>
</tr>
<tr>
<td>$H_{\text{Sys}}$</td>
<td>total system inertia constant</td>
</tr>
</tbody>
</table>
Abstract

The University of Manchester
Faculty of Engineering and Physical Sciences
Negar Shams
Doctor of Philosophy
Analysis and Response Management of Frequency Event in Low Inertia Power Systems

22/12/2017

Power systems have started to and will continue to go through radical changes which bring up new challenges for their secure operation, as well as opening up opportunities to tackle them using modern technologies, e.g. synchronised measurement technology. Frequency security is rising to be one of the main issues in the future power systems control. Replacing fossil fuel burning synchronous generation with asynchronous renewable generation not only reduces system inertia but also decreases the sources capable of providing primary frequency reserves. The faster larger frequency deviation following frequency events (disturbance) in low inertia systems can be limited by contracting larger volume of governor response, which imposes excessive cost on system operators. Alternatively, a new form of frequency control response supported by wide area monitoring systems can be deployed, which is capable of releasing additional active power faster than conventional primary response. Considering the heightened uncertainty of parameters in future power systems, being adaptive to as many parameters as possible in an online manner would be integral to the success of these fast frequency control services.

The objective of the research presented in this thesis was to create online methods for fast estimation of parameters governing frequency behaviour following any frequency event, with the intent to contribute to the development of faster and more adaptive frequency control actions suitable for future power systems. The research presented in this thesis includes the creation of two novel methods for fast and simultaneous detection of disturbance, and estimation of its size and location using limited synchronised measurements. Furthermore, an online method for continuous frequency security assessment and evaluation of required fast frequency response based on identified simplified frequency response (SFR) model of the system has been proposed. Finally, a novel Adaptive Under Frequency Load Shedding Scheme as a form of fast frequency response is suggested, which manifests the benefits of adapting response necessity and size to the identified SFR and estimated disturbance size.
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I would like to thank my supervisor Prof Vladimir Terzija for giving me the opportunity to pursue my Ph.D. Without his contribution of time, ideas and brilliant guidance I could not have hoped to complete this body of work to its current level. It was with the consultation of his deep knowledge of, and fresh perspective of power systems that allowed me to move forward with this research. His patience and kindness with me helped me to navigate this complex topic. I will carry his professional experience and temperament as an example as I make my next career move.

I would like to express my appreciation for detailed comments and productive suggestions of my examiners Dr. Levi and Dr. Laverty which helped me further improve my thesis.

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My time at the University of Manchester was made enjoyable in large part due to the many friends and colleagues who became a part of my life. I am grateful for all the times spent with my friends; Melios, Rasoul, Melikeh and Abraham.
1. Introduction

This chapter introduces the basic objectives, background and motivation, as well as the specific goals and contributions of the research presented in this thesis. The principal objective of the research is described in Section 1.1. A summary of the main motivation and background of this research is presented in Section 1.2. Section 1.3 lists the primary goals of this research and Section 1.4 describes its contributions. Section 1.5 provides a general overview of the fast adaptive frequency control strategy proposed in this research. Finally, the content of the following chapters are summarised in Section 1.6.

1.1 Objectives

The objective of the research presented in this thesis is to develop building blocks for delivering faster and more adaptive frequency control actions, capable of coping with the frequency security challenges of future power systems. The novel methods presented in this thesis belongs to two different building blocks of the new fast adaptive frequency control strategy. These blocks are:

1. Disturbance Analysis
2. Frequency Response Management

The objectives of the disturbance analysis methods suggested are the accurate, reliable and robust detection of active power disturbances and the estimation of their location and size, with the intention of minimising the speed and the information needed from the system to carry out the procedure.

In this sense, active power disturbance, disturbance, and frequency events are interchangeable terms with all referring to events causing sudden active power imbalance between generation and load (e.g. generators trip, sudden load increase or loss of an interconnector). Also Disturbance time is when active power balance is first perturbed. Disturbance size is equal to the magnitude of the initial active power imbalance observed in the system. Disturbance location is the bus in the system where the tripped generator was connected, load increase took place, or interconnector was lost.
The first objective of the second building block, frequency response management, was to continuously assess the frequency security of the system and to evaluate its requirement regarding services capable of delivering fast frequency response [1][2]. This was done based on identifying the frequency response model [3]. In this sense, frequency security of the system is regarded as the capability of the system to contain the maximum frequency deviation (nadir) within predefined limits when subjected to the maximum infeed loss (system reference incident) [4]. Fast frequency response is a term given to any service capable of injecting active power or cutting down its consumption within 0.5s of triggering [1][2], e.g. fast demand response, electrical storage or synthetic inertia of wind turbines [5].

In a practical sense, the system operator has to arm sources capable of delivering this type of response prior to disturbance occurrence. For example in the Great Britain (GB) network, National Grid has introduced a new form of market for fast demand response termed as Enhanced Frequency Response (EFR), with a response time less than a second [6]. In this market the participant services are paid based on their MW hourly availability, plus an additional payment upon dispatch [6]. Therefore, knowing the frequency security status of the system and the fast frequency requirement provides system operator with valuable information to procure just enough fast frequency response services. Therefore, the first objective of the frequency response management block results in maintaining frequency security of the system, while avoiding the extra cost of over engineered, conservative approaches for fast frequency response procurement.

The second objective of the frequency response management block was to decide the necessity, size and location of the fast frequency response once a disturbance was detected in disturbance analysis block. This way the information about disturbance size and location obtained from the disturbance analysis block, as well as system frequency response model, were utilized to deliver an optimal and reliable fast frequency response decision. The optimality of fast frequency response means taking the minimum response required to avoid the frequency nadir falling below the pre-set threshold. Reliability of fast frequency response means ensuring the response would bring frequency nadir within the acceptable range.
1.2 Background and Motivations

To help deliver the global initiatives of a low carbon future as a part of climate change strategies, many power systems will undergo significant decarbonisation of their electricity generation portfolio [7]. In the case of the United Kingdom, the government has set targets of 37% and 60% reduction in CO₂ emission by the years 2030 and 2050 respectively (all based on emission levels in 1990) [8]. In the electricity sector for the most part, this will be achieved by replacing conventional synchronous thermal generation with renewable generation (e.g. wind and PVs) which are connected through inverters. For example in the GB network, the total amount of renewable generation capacity made up 34% of total installed capacity in 2016 and this amount is expected to increase up to 60% by 2050 [8].

The inertia of synchronous generation limits the initial frequency deviation after a disturbance. However, power electronically connected generations do not contribute inertia to the system, as their rotating mass (the source of inertia) is decoupled from the system or it has no rotating mass at all (e.g. PV). Therefore, the system inertia will be reduced, e.g. GB inertia may be reduced by as much as 45% by 2024/25 [9]. Also in the GB network, the ratings of the third generation of nuclear power plants reactors will increase up to 1600 MW compared to 1320 MW today. This means that the maximum plausible disturbance size (system reference incident) would mount up to 1800MW. Lower inertia and bigger system reference incidents translate into having less time to act (shorter time to nadir) and larger maximum frequency excursion, hence deteriorated frequency security.

Additionally, the resources available to the system operator for frequency control will also be reduced, as the synchronous generation being displaced is responsible for providing the vast majority of frequency control services in modern power systems. The faster, larger deviation in frequency in a reduced inertia system can be corrected to some extent by contracting a larger volume of conventional governor response from the remaining synchronous generation. However, procuring more governor response from conventional generators means having generators operate with larger head-rooms, hence lower load factor and reduced efficiency. Therefore, this option is not cost effective, e.g. the cost of frequency control in GB can add up to £200m – £250m by 2020[9]. Furthermore, the delayed response of conventional governors (response starts in 2s) means that as inertia is further reduced they
will eventually be unable to prevent an unacceptable frequency deviation, regardless of the volume of response.

An alternative to this is the creation of new control services that release additional power into the system more quickly than conventional services. However, the faster release of additional power can put greater stress on the power system, particularly angular-stability in the initial tenths of a second after the initiating event [2]. Furthermore, power injection far from the disturbance location, can potentially lead to unacceptable current flows on the lines and increase the risk of cascading outages. On the other hand, having response in the vicinity of the event can minimise the impact on the normal system operating condition and keeps power flows more intact, hence reducing the operational cost. Moreover, targeted response can reduce the risk of voltage instability and help provide a higher voltage stability margin; as it avoids a large amount of power being transferred over long distances [10][11].

To address these issues, once a disturbance is detected the additional power must be deployed in proportion to both the severity of the initiating event (disturbance size) and the proximity to this event (disturbance location). These fast frequency control services are enabled by new tools for managing power systems in real time (e.g. Wide Area Monitoring Systems (WAMS) [12][13]) and new technologies and approaches (e.g. demand size response, energy storage, electric vehicles and synthetic inertia). For instance, in the GB network, electricity storage capacity is expected to grow rapidly to 6GW by 2020 [8]. The coordination of these new services into a single response will allow the creation of a new form of fast adaptive frequency response [1] that can combat the threat of reduced inertia [2][5]. Due to the need to adapt this fast frequency response to disturbance size and location, methods for estimation of disturbance size and location based on time synchronised measurements (gathered via Phasor Measurement Units (PMU)[13]) have gained interests over the last decade [14][15][16].

Following a disturbance, taking faster actions to arrest frequency decline offers significant advantage to system operators, as exposing system to a longer period of large frequency deviation might potentially lead to secondary contingencies and eventual cascading blackout [17][18]. Furthermore, when the goal is keeping frequency nadir within a certain limit, more prompt control actions deliver the same frequency improvement with less response size. Therefore, in order to maximise the benefits offered by adaptive frequency control actions the speed of different elements involved in its procedure needs to be improved. As it was stated, one of the elements for delivering adaptive frequency control response is identifying
disturbance characteristics; e.g. disturbance time, disturbance size and disturbance location (referred to as disturbance analysis). Therefore, developing methods capable of fast disturbance analysis have great potential benefits particularly in future low inertia systems which have a shorter time to nadir.

Moreover, frequency security is further challenged by the trend towards increased connection of distributed generation (DG). The three main drives of increased DG installation are:

- Future low carbon ambition; supported by providing open access to the market for small, independent providers of low carbon energy. [7]
- Amplified system load and the need to manage the impending transmission network congestion issues; reducing the investment in transmission reinforcement schemes supported by increased connection of DGs [17]
- Reduced cost of some DG technologies (e.g. PV modules [19])

A set of threshold are defined for off-nominal frequency and voltages in distribution networks IEEE standard 1547, which dictates the mandatory trip of distribution energy resources (DER). IEEE standard 1547 requirements, were originally driven by safety and protection/control coordination of distribution systems, and did not consider the possibility of high penetration of DERs in the system. As DER capacity continues to increase, sympathetic DER tripping due to a Bulk Supply Point (BPS) contingency could become significant enough to negatively impact bulk system reliability.

The increased experienced RoCoF following an active power disturbance caused by reduced inertia can often lead to cascading outages of DGs due to the operation of their loss of main RoCoF relays.

For instance, in the GB network by the years 2024/2025, 92% of the time the initial RoCoF would be greater than the existing RoCoF relays settings (0.125 Hz/s)[9][20]. As many DGs are equipped with RoCoF relays, having an increased number of DGs connections in low inertia systems can further undermine the frequency security of the system as they potentially add to power imbalance by their disconnection.

Also, proliferation of DGs and their variety of technologies would dramatically contribute to complexity, volatility and uncertainty in power system operation and control. Likewise, parameters governing frequency dynamics would have a heightened degree of uncertainty.
Therefore, having a continuous real-time estimate of parameters affecting frequency dynamics can considerably benefit future low inertia power systems. If implemented well, this could provide valuable inputs to design more optimal fast frequency control and protection systems, which could enhance not only system frequency security but also operational economics. For instance, continuous estimate of frequency response model (SFR) helps having informed procurement of fast frequency services using knowledge of real-time prevailing system conditions and frequency security limits.

### 1.3 Goals

The principal goal behind the research presented in this thesis was developing a strategy for delivering fast and adaptive frequency control. Achieving this main goal can be broken down into the following sub-goals:

- Create a method for fast and reliable detection of active power disturbances
- Create a method for fast and precise estimation of the location of active power disturbances
- Create a method for fast and accurate estimation of the size of active power disturbances
- Estimate the parameters of the simplified frequency response (SFR) model of the system
- Create an approach to assess the frequency security statues of the system as well as its fast frequency response requirements based on the estimated SFR parameters
- Create a method capable of taking fast adaptive decisions about the necessity and the size of the fast frequency response required following an active power disturbance using estimated disturbance size and location as well as estimated SFR parameters.
- Validate the methods created using simulated power system models in PowerFactory under varied conditions.
- Demonstrate the benefits of the created method for adaptive fast frequency response in the form of a new Adaptive Under Frequency Load Shedding Scheme (AUFLS) compared to other forms of AUFLS under varied conditions.
1.4 Road map of the Suggested Fast Adaptive Frequency Control Strategy

Before moving on to elaborate on the different building blocks of the suggested fast frequency control strategy in the following chapters, it is important to give a brief general overview of how the different blocks interact and what their roles are. In this sense, the rate of data exchange as well as information passed through the data channels between different modules are summarised in Figure 1-1.

\[ P_e(t) \] is a vector containing active power measurements from limited number of generators at time \( t \). \( \dot{f}(t) \) is a vector containing RoCoF measurements of limited number of generators at time \( t \). \( Z \) is the impedance matrix of the system. \( LF \) is the load flow results containing the voltages and angles of the buses as well as active power flowing in and out of each bus. \( H \) is a vector containing the inertia constant values of the synchronous generators in the system, where all the inertia constant are defined in a common MVA base. \( \vec{V}(t) \) is a vector containing the voltage phasors of the buses equipped with PMUs at time \( t \). \( \vec{I}(t) \) is a vector containing the current phasors of currents flowing through the buses equipped with PMUs at time \( t \). \( \vec{V}(t) \) and \( \vec{I}(t) \) are used to improve the performance of the state estimation and topology estimation application within the Energy Management System (EMS) [21] (this part is not the focus of this research). State estimation helps building network real time model using a combination of real time measurements and static network data. In case of

![Figure 1-1: Overview of the suggested fast adaptive frequency control strategy](image-url)
conventional state estimation, network real-time modeling can be grouped into two stages [22]:

1) The processing of logical data (i.e. the statuses of switching devices that can influence the network connection), this stage is called topology estimation.

2) The processing of analog data (e.g., power flow, power injection, and voltage magnitude measurements). This stage assumes a known topology.

The solid blue lines in Figure 1-1 represent the data exchange rate of 20ms, which is consistent with PMUs reporting rate. The dashed blue lines represent the data exchange rate of few minutes, which is consistent with the SCADA/EMS rate of update (in some systems it can also be in the order of few seconds). The purple solid lines are only triggered and exchange information when a disturbance is detected in the disturbance analysis block. The / sign separates the two methods and their inputs used for disturbance analysis, one being Synchronising Power Coefficient based method (SPC method) (Chapter 3) and the other Regional Inertia based method (RI method) (Chapter 4 and 5).

As discussed in Section 1.1, the suggested fast adaptive frequency control strategy in this thesis consists of two main building blocks; disturbance analysis and frequency response management.

As mentioned in Section 1.1, the frequency response management block has two main objectives. The first objective is the assessment of the frequency security of the system plus the evaluation of its requirement for fast frequency response. In Figure 1-1, this is the role of the module shown as “Dynamic Frequency Security Assessment Based on System Identification” (Chapter 6). The second objective of the frequency response management block is making decisions regarding the necessity, size and location of the fast frequency response. In Figure 1-1, this is the role of the module shown as “Adaptive Response” (Chapter 7). In Figure 1-1, disturbance detection, localization and size estimation is the role of the module shown as “Disturbance Analysis”.

The “Disturbance Analysis” module utilises PMU measurements and information received from SCADA to continuously scan the power systems for the symptoms of active power
disturbance. The SPC and RI methods are both novel methods suggested in this thesis capable of carrying out this role.

The “Dynamic Frequency Security Assessment Based on System Identification” module utilizes the information from PMU measurements and SCADA to estimate the parameters of the simplified frequency response model (SFR) of the system. Using the identified SFR of the system, this module then calculates the variable referred to as “Critical Disturbance Size” (defined in section 6.4.1). The frequency security status of the system against its reference incident is evaluated using “Critical disturbance Size” and sent back to SCADA. If the system is deemed frequency insecure, the minimum fast frequency response required to keep the frequency security of the system in the case of system reference incident is reported to SCADA.

The “Adaptive Response” module is only activated when a disturbance is detected in the “Disturbance Analysis” module. This module utilizes the information received from the “Disturbance Analysis” and “Dynamic Frequency Security Assessment Based on System Identification” modules, as well as SCADA to decide the necessity, size and location of fast frequency response.

1.5 Contributions

The main contributions made during this research are:

- Creation of an online method (SPC method) capable of simultaneous and fast disturbance detection, localisation and size estimation (Disturbance Analysis) relying on active power output measurements of generators rather than their RoCoFs. The SPC method does not require information on system inertia and it uses much reduced number of PMUs which makes it economically and computationally desirable in real time operation. (Chapter 3)
- Creation of a new partitioning (regioning) method for power systems based on the similarity of frequency behaviour of synchronous generators shortly following the
disturbance. Defining the concept of regional inertia based on the regions’ boundaries and its potential application for disturbance size estimation (Chapter 4). Suggestion of an alternative approach to get the same regions without requiring frequency measurements (offline regioning).( Chapter 4)

- Creation of an online method (RI method) capable of simultaneous and fast disturbance detection, localisation and size estimation (Disturbance Analysis). The RI method uses a greatly reduced number of PMUs which makes it economically and computationally desirable in real time operation. (Chapter 5)

- Creation of an online method for the continuous assessment of frequency security of the system as well as the minimum fast frequency response required to be procured. The method is based on estimated SFR parameters by applying the system identification technique to frequency and power variations due to small load fluctuations existing in normal operating conditions. Following a disturbance occurrence, the identified SFR helps to predict frequency behaviour once the disturbance size is estimated. This way, the delays inherent in other frequency predictive frequency control methods [17][23] are reduced, which is a key factor in low inertia systems where the available time for action is limited. (Chapter 6)

- Creation of a new form of AUFLS (as an example of fast adaptive frequency response) based on both estimated disturbance size and identified SFR. The suggested AUFLS outperforms the other forms of AUFLS which it has been compared to. (Chapter 7)

1.6 Outline of the Thesis

Chapter 2-Literature Review
This chapter gives a brief description of elements involved in frequency control of power systems. It then reviews the methods suggested in the literature for disturbance detection and the estimation of disturbance size and location, and points out their shortcomings. Also, this chapter provides an overview of the existing AUFLS methods and lists their drawbacks and areas for improvement. Finally, it lists the benefits of the suggested methods for disturbance analysis and frequency response management.
Chapter 3-Disturbance Analysis Based on Synchronising Power Coefficients
This chapter describes a novel method (SPC method) for disturbance analysis which uses Synchronising Power Coefficients and active power output measurements of a limited number of synchronous generators. A number of case studies are carried out to explore the robustness of the suggested method under varied conditions.

Chapter 4-System Regioning and Regional Inertia
The main method suggested in literature for disturbance size estimation is the traditional swing based method (described in section 2.7). This method requires all synchronous generators RoCoFs and inertias to build up the equation for the RoCoF of the centre of inertia, and utilises that to estimate the disturbance size. However, as synchronous generators may not all be equipped with PMUs, there exists a drive to divide system into regions and assign the so called regional inertias to the representative RoCoF measurements of each region to account for unmonitored generators. For this method to work, the regioning has to be based on the similarity of immediate frequency behaviour of generators following the disturbance. This chapter describes a new method for grouping the synchronous generators in different regions and calculating regional inertias to achieve an accurate estimation of disturbance size with less PMU measurements. It also demonstrates the vulnerability of the traditional swing based method to missing PMU measurement to justify the ambition behind system regioning.

Chapter 5-Disturbance Analysis Based on Regional Inertia
As region boundaries are dependent on disturbance location (proved in chapter 4), there is a possibility of having some regions with no measurements for some particular disturbance locations. Therefore, this chapter first introduces a methodology to estimate the unmonitored regions representative RoCoFs using other RoCoF measurements. This chapter then describes a novel method (RI method) for disturbance analysis and tests its accuracy and reliability under different conditions.

Chapter 6-Dynamic Frequency Security Assessment Based on System Identification
This chapter describes a new method for dynamic frequency security assessment using system identification technique. The new dynamic frequency security assessment uses PMU measurements at generators terminals during normal operating conditions (having small load fluctuations) to identify SFR model by applying least square method to estimate its
parameters. Critical disturbance size is then calculated using SFR parameters and frequency security and the size of required fast frequency response in case of system reference incident is decided. Case studies are carried out to investigate the effect of different parameters in the performance of the method.

Chapter 7-Adaptive Under Frequency Load Shedding Scheme Based on Dynamic Security Assessment
This chapter explores the potential benefits of adapting the necessity and size of fast frequency response (in this case load shedding size) to both disturbance size estimate and critical disturbance size. The comparison of the new suggested AUFLS and other AUFLS methods is carried out for varied disturbance size and under different system inertia values (which is translated to wind generation penetration level).

Chapter 8-Thesis Summary
This chapter summarises the research presented in this thesis and its contribution plus some possible opportunities for further development of this research.

1.7 References


2. Literature Review

Frequency security is rapidly rising to be one of the main issues in controlling future power systems. Factors such as increased integration of converter connected generations (e.g. wind turbine and PVs) and deployment of larger nuclear power plants (with no frequency governing capabilities) have and will continue to contribute to this challenge. As such, relying solely on primary frequency control actions to arrest large frequency deviation might cease to ensure frequency security in a reliable and economical manner. These are the main drives behind the research presented in this thesis, as adaptive and faster frequency control actions are more suited for low inertia and highly uncertain power systems and can help deliver a satisfactory frequency response with lower cost. Having fast and accurate information of the active power disturbance is an enabler of this type of frequency control.

Section 2.1 discusses the frequency control challenges of the future power systems. Section 2.2 summarizes the elements involved in frequency control. Section 2.3 gives an overview of the demand side management potential in distribution networks to support frequency control. Section 2.4 provides an explanation of the droop control characteristic used in primary frequency control stage. Section 2.5 gives a definition of System Integrity Protection Schemes (SIPS). Section 2.6 defines Under Frequency Load Shedding Schemes (UFLS) and provides an overview of different types of it suggested in the literature and points out the potential areas of improvements. Section 2.7 describes the method typically used for fast estimation of active power deficit (disturbance size) following the disturbance and discusses some of its shortcomings. Section 2.8 lists and discusses different methods used for frequency and RoCoF estimation. An overview of the methods used for disturbance localisation as well as their drawbacks is presented in Section 2.9. Section 2.10 discusses the methods suggested in the literature for disturbance detection and the challenges of each. Section 2.11 lists the improvements of the novel methods suggested in this thesis for disturbance detection and estimation of disturbance size and location (disturbance analysis). Also Section 2.11 presents the benefits of the suggested method for frequency response management which incorporates a new form of Adaptive UFLS (AUFLS). Finally the chapter is summarised in Section 2.12.
2.1 Chapter Introduction

Reduced inertia due to the replacement of conventional synchronous generation by asynchronously connected renewable generation poses new challenges for the future of power systems frequency control. These evolutions in power generation mix stems from the carbon emission reduction initiatives. During the past decade, a surge in the number of installed wind turbines has been experienced in a number of countries e.g. Denmark, Germany and Spain [1][2]. Other countries, such as Ireland and Great Britain have set renewable energy targets in multi-year programmes. In case of Ireland the programme is “Delivering a Secure, Sustainable Electricity System” (DS3)[3]. The goal of DS3 programme is to achieve renewable energy targets for 2020 while operating the system in a secure manner. Under the EU Renewable Energy Directive, Ireland’s target is to have 16% of the country’s total energy consumption to come from renewable sources by 2020. Consequently, Ireland government has set a 12% renewable heat target, 10% renewable transport target and 40% renewable electricity target [3]. Wind power is the main contributor for Ireland’s renewable target. EirGrid and SONI as the Transmission operators in Ireland and Northern Ireland carried out facilitation of renewable studies in which they identified the maximum allowable level of renewable generation to be 50% and 55% in 2010 and 2015 respectively [3]. This limit is commonly referred to as System Non Synchronous Penetration Limit (SNSP). SNSP is a real-time measure of the percentage of generation that comes from non-synchronous sources, such as wind generation, relative to the system demand [4]. The goal is to achieve SNSP of 75% by the year 2020 [3]. As Ireland and Northern Ireland system are isolated and weakly interconnected, this large share of non-synchronous generation would pose a significant challenge as well as an opportunity to Ireland’s TSOs to lead the way in the integration of non-synchronous renewable generation [5].

In the case of UK energy policies, the key targets have been set for 37% and 60% reduction in CO₂ emissions by the years 2030 and 2050 respectively (all based on 1990 emission level in) [1]. In the UK electricity network, decarbonisation is mainly carried out by shutting down the carbon emission intensive generation technologies; e.g. open cycle gas turbines (OCGT) and coal plants and replacing them with wind turbines [7]. At the same time, the ratings of the third generation of nuclear power plants reactors will increase up to 1600 MW compared to 1320 MW today. This means, the maximum size of the active power deficit due to a single contingency (system reference incident) would mount up to 1800MW. Lower inertia and a
bigger system reference incident translate into having less time to act and larger maximum
frequency excursion. Contracting larger volume of primary response in order to achieve
higher ramp up rate is one way of tackling this frequency security problem. However, this
solution has shown to be rather costly as more generators have to operate with larger head
room; e.g. cost of frequency control in Great Britain can rise to £200m-£250m by 2020[8].
Another more economical way to face the ensuing heightened frequency issues is developing
new approaches for faster and more adaptive forms of frequency control (e.g. Adaptive
Under Frequency Load Shedding (AUFLS)) [9][8]. Fast and reliable methods for detection of
disturbance and estimation of disturbance size and location are fundamental to the success of
these new forms of frequency controls, termed as fast frequency response beginning to act
within 0.5s following the disturbance [9].

2.2 Elements of Existing Frequency Control

The frequency of a power system is determined by the rotational speed of the synchronous
generation connected to the system and this is a direct measure of the active power balance in
that system. Therefore, if there is an excess of generated power the frequency will rise and if
there is an excess of load it will fall. The role of frequency control is to ensure that the
deviations in frequency that are caused by disturbances to the active power balance remain
within a set of predefined limits and through this ensure the system retains frequency
stability, one of the three forms of power system stability [10]. When experiencing
significant active power disturbances, preventing large frequency deviation is critical as it can
lead to secondary contingencies and end in cascading blackout [9].

Frequency control can take two main roles:

- Continuous, responsible for managing the small changes in frequency caused by the
  small continuous changes that occur in the load or generation
- Occasional, responsible for managing the large change in frequency caused by
  contingencies (e.g. the loss of a generator)[9].

Furthermore, frequency control is commonly divided into three stages, which can be briefly
described as follows [9][11]:
- Primary Frequency Control, the role of which is to arrest the frequency decline after an imbalance between generation/load has occurred, i.e. brings the system frequency to a new, off-nominal steady state.
- Secondary Frequency Control, the role of which is to return the system frequency to its nominal value.
- Tertiary Frequency Control, the role of which is to optimally dispatch and restore the desired level of primary and secondary frequency control services cost effectively.

Finally, it is common practice to complement these stages of frequency control with emergency frequency control actions to accommodate extreme contingencies; these may include Under Frequency Load Shedding (UFLS) as a form of System Integrity Protection Schemes (SIPS).

As an example, the primary response in GB (Great Britain) must begin within 2 seconds, be complete within 10 seconds and then be sustained for 30 seconds. This primary response is automatically initiated by local control in the generator governors, whilst the secondary control in GB is manually performed between 30 seconds and 30 minutes after the disturbance and includes both secondary and tertiary control actions. Most power systems have different limits and requirements for their frequency control [12][13][9].

2.3 Demand Side Management and Distribution Network Contribution to Frequency Control:

Utilising demand response poses as an appealing alternative for controlling system frequency as appose to expensive generation side controllers [14]. Utilising loads as a flexible resource and additional degree of freedom to contribute to transmission capacity and frequency control issues is not an entirely new concept. Nevertheless, the complexity of implementing real time monitoring and control for many distributed small sized loads has been the main obstacle to cultivate the full potential of demand response. However, the recent developments in affordable global communication infrastructure, decentralised control schemes and embedded systems makes Demand Side Management (DSM) an ever increasing viable option in many utilities [15].
In line with the future goals for increased SNSP limits in many countries, and in order to avoid excessive reserve costs [16], in distribution networks many converter connected generators such as wind farms and a selected categories of loads (demands) need to be able to provide fast frequency response. This notion has prompted National Grid to announce a new market for Enhanced Frequency Response (EFR) providers [17], which can include energy storage, interconnectors, wind farms synthetic inertial response and demand response. Several papers have been published with the focus on contribution of wind farms to frequency control [18][19]. Demand control can be carried out via appropriate frequency sensors which can then turn on and off household appliances such as refrigerators, freezers and water heaters, as these loads are in a considerable volume and are capable of instantaneous switching without much impact on the costumers [20][21]. This class of loads are often referred to as Thermostatically Controlled Loads (TCL), the regular operating cycle of these loads can be altered without noticeable impact on their controlled temperature [21]. The TCLs depending on locally measured frequencies have the potential to contribute immensely to the frequency response market [22]. In the UK the value of frequency response provided by all the refrigerators is estimated as £222 million per year with the cost of reduction in CO$_2$ as $28 million per year [23]. In case of voltage-dependent loads e.g., electric heating and lighting (especially LED lightings), small motors (e.g. fans, ovens, dishwasher and dryers) demand can be continuously controlled (both increase and decrease) through modulation of their supply voltage [24][25].

2.4 Droop Characteristic

Primary frequency control stage is based on adapting generating units active power output to the variations in system frequency. In order to avoid conflicts between different generating units over imposing their own speed settings, generators governors are set to follow a droop characteristic instead of being isochronous governors. Droop characteristic assists with the stable load division between two or more units operating in parallel [11]. Figure 2-1 shows a control block diagram of a speed governor with steady-state feedback loop (Droop control).
Figure 2-1: Diagram of speed governor with droop control

Where, $Y$ is the valve/gate position, $\Delta Y$ is the control signal which is the change in the valve/gate position. $K$ is the gain of the integrator. $R$ is the droop. $\omega_r$ is the measured rotor speed. $\omega_0$ is the reference speed. $\Delta \omega_r$ is the change in measured rotor speed compared to the reference speed.

Droop ($R$) is defined as follows [11]:

$$R = \frac{\text{percent of frequency change}}{\text{percent of power output change}} \times 100 = \left( \frac{f_n - f_f}{f_0} \right) \times 100$$  \hspace{1cm} (2.1)

Where $f_n$ is steady-state frequency at no load, $f_f$ is steady-state frequency at full load and $f_0$ is the nominal frequency.

In other words in droop control, the change in the active power output has a direct linear relationship with the steady state frequency variation with the slop of $1/R$ (Figure 2-2). For example, a 4% droop means that 5% frequency deviation (equal to 2Hz for $f_0$=50Hz) would cause a 100% change in valve position or active power output.
In Figure 2-2, $\omega_{nl}$ is steady-state rotating speed at no load, $\omega_{fl}$ is steady-state rotating speed at full load and $\omega_0$ is the nominal rotating speed of the turbine. As is shown in Figure 2-2 the pu values of frequencies and rotating speeds are equal. In case more than one generator are equipped with speed droop governor, the final unique frequency following an active power disturbance is reached in a way that the load change is shared reversely proportional to the droops between the generators. When having two generating units with droop values of $R_1$ and $R_2$ the amount of load picked up by each generating unit ($\Delta P_1$, $\Delta P_2$) is calculated as follows:

$$\Delta P_1 = \frac{\Delta f}{R_1}$$
$$\Delta P_2 = \frac{\Delta f}{R_2}$$

(2.2)

Following an active power disturbance, if the droops of different generating units are nearly equal, the final change in their active power output would be nearly proportional to their ratings [11].
2.5 System Integrity Protection Schemes

When a disturbance occurs, local protection schemes would automatically react to arrest the fast developing emergencies. However with the growing frequency of system-wide disturbances [26], these local protection schemes are becoming less reliable due to their lack of communication and coordination with other system components. In order to tackle this problem System Integrity Protection Schemes (SIPS) can be deployed.

SIPS can be summarised as a set of pre-determined actions that are designed based on offline system studies and are implemented after a very precise set of conditions have been detected in the system [28]. For a scheme to be classed as SIPS the actions implemented must go beyond simply isolating the faulted elements [29].

Many SIPS are armed by one condition and then the pre-determined actions are triggered by a second condition. SIPS can be separated into two general types, Response Based and Event Based. If both the arming and triggering conditions are set as occurrence of an event then the SIPS is called Event Based SIPS and if either of the arming or triggering condition is based on a measured system response then the SIPS is called Response Based SIPS. Response Based SIPS are generally more adaptable whereas Event Based SIPS are quicker as they do not need to wait for system response to be measured [28][29]. Nevertheless, Response-Based SIPS are generally more complex and should be well coordinated with other system components to ensure robustness of the scheme. Particular SIPS can be designed to aid frequency control of a power system during extreme conditions and can be a cost effective solution for preventing system breakdown. These are one of the main families of SIPS currently practiced and are generally termed Under Frequency Load Shedding Schemes (UFLS)[30].

2.6 Under Frequency Load Shedding

In the case of severe frequency excursions, if both inertial and primary frequency response fails to bring frequency back to the normal frequency range, the system operator would try to lower the electrical load of the system by temporarily disconnecting some loads. The automatic procedure of load disconnection triggered by frequency deviation is called Under Frequency load Shedding (UFLS), which can be fixed or adaptive [31][32][33]. The goal of
UFLS is to contain continuous frequency drop by providing aid to balance load and generation, and to avoid total frequency collapse and prolonged system outage.

In traditional fixed UFLS delivered via relays that control some circuit breakers, load shedding step sizes are constant and get activated with fixed delays when the frequency drops below some pre-defined thresholds [32][34]. Historically, these traditional UFLS have been regarded as the last resort against total or partial system blackouts caused by system wide frequency drops. In these schemes, the shedding step size is determined by worst credible contingency which may lead to frequency decay. However, with the increased range of possible operating conditions, e.g. due to the trend towards having a more diverse generation mix set by carbon emission targets [1] [35], the level of uncertainty in system parameters affecting frequency dynamics escalates. As a result, designing UFLS for the worst case scenario can lead to significant over shedding in almost all cases, as the worst case scenario is more extreme and less likely to happen. Without a change in the design of fixed UFLS, this would force system operators to hold more loads in fast response mode, imposing significant operational cost [36]. Furthermore, as low inertia leads to faster frequency drop, waiting for frequency threshold violations to trigger load shedding steps might fail to arrest frequency nadir in a timely manner. Also, load shedding relays responsible for fixed UFLS are installed in constant locations in the system and in substations where their connected loads are less vital. Meaning, the shedding location is fixed which can push the system to the verge of voltage instability where a large amount of power is transferred through long distances [37]. On the other hand, having load shedding in the vicinity of disturbance location can reduce the risk of voltage instability and help to provide higher voltage stability margins for the system following the disturbance [38].

Therefore, in order to achieve an economical and secure form of UFLS capable of coping with the future volatile and low inertia power systems, there exists a need for the development of UFLS adaptive to as many system frequency dynamics and disturbance parameters as possible and able to evaluate shedding necessity in a faster fashion. In this sense, disturbance parameters can be disturbance size, location, time and system frequency dynamics parameters can include total system inertia, load voltage and frequency sensitivities and damping. The invention of Wide Area Monitoring Systems (WAMS) and availability of synchronised measurement can support these new adaptive forms of UFLS. Many Adaptive UFLS (AUFLS) have been suggested in the literature [33][39].
In [33], a novel AUFLS is suggested which is adaptive to the disturbance size. The instantaneous frequencies and rate of change of frequencies of generators are calculated using non-recursive Newton type algorithm applied to voltage phasors, and are then utilised to estimate the magnitude of disturbance based on the swing equation. In this method the shedding size is equal to the size of disturbance and is distributed evenly in a different number of steps activated based on frequency level violation. Based on [33], many AUFLS have been proposed [39][40][37][36]. Although this method shows a promising performance compared to fixed UFLS, there is a potential for further tuning of the shedding size to get closer to the minimum shedding that is required to keep the frequency nadir above a certain threshold. Furthermore, the triggering procedure can be improved to account for low inertia scenarios where time for action is limited.

A local method for the triggering of AUFLS based on both voltage and frequency at each bus is proposed in [38] and [41]. In these methods, two indices one derived from local voltage and one from local frequency are mapped into a two dimensional region detection unit to determine the necessity of load shedding. In case of having the loci of local voltage and frequency indices in detection region for longer than a pre-set time (50ms), the local load shedding is triggered. By considering the local voltage reaction, these methods achieve a faster and more selective triggering mechanism for their AUFLS. Nevertheless, the shedding size is non-adaptive like traditional fixed UFLS.

In [42], the triggering is based on either having frequency or RoCoF falling out of predefined limits. Yet for this triggering to be independent of the location of disturbance, frequency and RoCoF of the centre of inertia need to be calculated. As shown in Section 2.7.3, calculation of the frequency and RoCoF of the centre of inertia requires all generators PMU measurements, which impose a stringent condition for the type of AUFLS triggering method suggested in [42].

Some methods proposed the inclusion of voltage dependent load modelling for improvement of disturbance size estimation and to account for load response to the disturbance [43][37][42]. In [44], the instantaneous voltage reductions of buses are used as a measure of proximity to disturbance location and the load shedding amount is distributed accordingly. However, as load voltage sensitivity is different throughout the network, depending on
voltage deviation to localise the disturbance is not reliable and can lead to unsuitable shedding distribution.

AUFLS suggested in [42] uses the power flow tracing approach [45] applied to both active and reactive power of the loads in order to determine the shedding amounts at different load buses. Proportions of loads supplied by the lost generator are calculated using the power flow tracing technique. Nevertheless, in this load shedding distribution technique the network has been assumed to be radial.

Recently some efforts have been made in designing AUFLS based on the predicted post-disturbance frequency behaviour [29][46][47][48]. As frequency trace following a disturbance is governed by not only disturbance size but also by system frequency dynamic characteristics [49], these methods are indirectly adaptive to system frequency dynamics as well as the disturbance size.

In [46], a few-seconds-in-advance frequency prediction is used to determine the necessity and the size of shedding. Following the disturbance occurrence, the frequency prediction is updated by constantly fitting a second-order polynomial to three points of frequency measurements. This method bases its shedding size calculation solely on frequency information; therefore, it does not require total inertia value, load characteristics and accurate estimation of centre of inertia rate of change of frequency (RoCoFCOI) to estimate the disturbance size. As accurate calculation of RoCoFCOI is highly sensitive to missing PMU measurements, the independence of the suggested method to RoCoFCOI makes it attractive for practical implementation, where not all generators are equipped with PMUs. However, as mentioned in their analysis, due to the delays associated with frequency prediction, the proposed method fails to provide timely decisions for extreme disturbances having large frequency gradient and short time to frequency nadir, which is most likely the case in low inertia systems.

As pointed out, having information on the size, location and time of disturbance can deliver valuable inputs to improve the performance of AUFLS as well as other forms of fast frequency services [9]. The next sections provide a review of existing methods for disturbance detection and estimation of disturbance size and location.
2.7 Magnitude of Disturbance Estimation Using Swing Equation

For many adaptive frequency control schemes the first step is the estimation of the amount of response, which must be proportional to the disturbance size. This section describes the swing equation based method typically used to estimate the disturbance size. Based on this method, many AUFLS have been proposed assuming that an accurate disturbance size estimate will be available soon after the disturbance has occurred [37][39]. This method is referred to as the swing based, traditional or RoCoF based method throughout this thesis.

2.7.1 Definition of Inertia

Following any frequency variation, kinetic energy of rotating masses helps slow down the frequency dynamics, making it easier to regulate. The inertia of a power system is reflective of the kinetic energy stored within the rotating masses of the system. The kinetic energy stored \(E_{\text{kin}}\) within the rotating mass of a power system plant item can be defined as follows [50]:

\[
E_{\text{kin}} = \frac{J \omega_m^2}{2} = \frac{1}{2} J (2\pi f_m)^2
\]

(2.3)

where \(J\) is the moment of inertia of the rotating mass in kg.m\(^2\), \(\omega_m\) is the rotating speed of the machine in rad/s and \(f_m\) is the rotating frequency of the machine in Hz.

The moment of inertia \(J\) for a rotating mass is calculated as follows:

\[
J = \int_0^M r^2 dm
\]

(2.4)

where \(M\) is the total mass of the rotating body (in this case generator and turbine) and \(r\) is the distance of the mass particle \((dm)\) from the rotation axis.

The inertia constant \(H\) for a synchronous machine is calculated as follows [50]:
\[ H = \frac{E_{\text{kin}}}{S_B} = J(2\pi f_m)^2 \frac{2}{2S_B} \]  \hspace{1cm} (2.5)

where \( S_B \) is the rated power of the generator.

The presence of the current rotating frequency of the machine in (2.5)(2.7) leads to varied inertia constant during frequency fluctuations. Therefore, it is common to assume the rotor frequency as constant (equal to its rated frequency \((f_0)\)) in (2.5) and to derive a fixed \( H \) constant value for each generator which is usually in the range of 2-10s \([51]\). The \( H \) constant is equal to the time that a synchronous generator can continue to supply its rated power solely based on its stored kinetic energy \([52]\).

### 2.7.2 Swing Equation

During stable conditions there is equilibrium between opposing forces. This means that under steady state conditions for synchronous electrical machines the mechanical input torque \((T_m)\) is in balance with output electrical torque \((T_e)\) and the speed remains constant; hence the frequency will also remain constant. In the event of an active power mismatch this balance would be upset and the frequency would start to increase or decrease. The equation defining frequency and power dynamics in power systems is the swing equation \([53]\). For \( t=0^+ \) after the disturbance the frequency response model is equal to the swing equation (2.6).

\[ \dot{f}(t) = \frac{1}{2H} (T_m - T_e - K_D\Delta f(t)) \]  \hspace{1cm} (2.6)

In (2.6) the term \( K_D\Delta f(t) \) reflects the impact of frequency sensitive loads on the frequency deviation and \( H \) is the inertia constant (the link between inertia constant and swing equation is further discussed in Appendix A \([50]\)). \( K_D \) is the load active power change due to frequency variation and is referred to as damping constant. Since the parameters are all defined in pu in (2.6), the per unit active power and per unit torques have the same value as long as frequency deviation remains relatively small.
2.7.3 Swing Equation of Multi-machine Systems

During steady state the frequency is the same throughout the system. However, during transient time synchronous generators rotors speeds would be different, hence relative rotor movements (“swings”). If the system is small signal stable relative rotor movements would die out meaning the frequency of all generators converges.

In order to rewrite the swing equation for a multi-machine system (with \( n \) generating units) an equivalent generating unit is used to model the average behaviour of all the generating units at the time where they are not all the same \((t=0^+)\). This equivalent generating unit is called the Centre of Inertia (COI) [50]. In case of a small signal stable system, after convergence of all frequencies the COI frequency and RoCoF would be the same as frequency and RoCoF of any other synchronous generator in the system.

As the \( H \) constant of synchronous machines are typically normalised based on their own ratings, prior to combining all synchronous generators into a single equivalent generating unit they need to be redefined with reference to a common base. Therefore, all generator \( H \) constants are transformed into the system MVA base \((S_{\text{base}})\) as follows:

\[
H_{i,\text{sys}} = H_i \left( \frac{S_i}{S_{\text{base}}} \right) \quad (2.7)
\]

\( S_{\text{base}} \) is the common power system base, and \( S_i \) is the power base for the \( i^{th} \) generator. \( H_i \) is the \( H \) constant of the \( i^{th} \) generator on its own base \( S_i \). \( H_{i,\text{sys}} \) is the \( H \) constant of the \( i^{th} \) generator on the system base \( S_{\text{base}} \).

By applying the method used in [50] the COI frequency \((f_{\text{COI}}(t))\), COI RoCoF \((\dot{f}_{\text{COI}}(t))\) and total system inertia constant \((H_{\text{sys}})\) in \( S_{\text{base}} \) are given as follows:

\[
f_{\text{COI}}(t) = \frac{\sum_{i=1}^{n} f_i(t)H_{i,\text{sys}}}{\sum_{i=1}^{n} H_{i,\text{sys}}} \quad (2.8)
\]
\[
\dot{f}_{COI}(t) = \frac{\sum_{i=1}^{n} \dot{f}_i(t)H_{i,sys}}{\sum_{i=1}^{n} H_{i,sys}} \quad (2.9)
\]

\[
H_{sys} = \sum_{i=1}^{n} H_{i,sys} \quad (2.10)
\]

Shortly following a disturbance occurrence \((t=0^+)\), the frequency deviation \((\Delta f)\) is small, causing \(\Delta T\) (pu) being nearly equal to \(\Delta P\) (pu) (which is the size of active power imbalance); therefore, (2.6) can be rewritten for the whole system as follows [33]:

\[
\Delta P = 2H_{sys}\dot{f}_{COI}(0^+) \quad (2.11)
\]

(2.11) shows the equation used to estimate disturbance size at \((t=0^+)\) using total system inertia \((H_{sys})\) and frequency of COI frequency \((f_{COI})\). Even though the online calculation of COI variables is not a trivial task, it is necessary as the locally measured frequency does not provide reliable information [40]. The inherent local difference is even more visible in the rate of change of frequency at different buses. It is worth mentioning that the difference in frequency response throughout the whole system is even more pronounced for larger grids.

### 2.7.4 Challenges of Swing Equation Based Disturbance Size Estimation

As shown, the disturbance size estimation using swing equation (2.11) needs the following inputs:

- RoCoF of the Centre of Inertia shortly following the disturbance\((\dot{f}_{COI}(0^+))\)
- The total system inertia \((H_{sys})\).

The first input \((\dot{f}_{COI}(0^+))\) requires the RoCoF of every generator to be measured using PMUs as part of a Wide Area Monitoring System (WAMS) [54]. Due to the current high capital cost per PMU installation [55], this requirement is quite onerous. To add to the matter,
the number of installed PMUs might need to be double the number of operating synchronous generators as redundancy will be required in practice to ensure availability of reliable PMU readings from each generator at any point in time.

In an attempt to utilise less PMU measurement for disturbance size estimation, in [56], limited generators frequency were monitored to represent different areas of the system and reproduce \( \dot{f}_{\text{COI}}(0^+) \) and estimate the disturbance size. However, the problem with this approach is the strong dependency of the optimal RoCoF measurement points capable of representing \( \dot{f}_{\text{COI}}(0^+) \) on the disturbance location. As a result, fixed choice of representing RoCoF measurements can potentially lead to a significant error in disturbance size estimation for some specific disturbance locations. Also [57] suggests the existence of a generator in the system whose frequency shows the behaviour of the frequency of the centre of inertia. Nevertheless, the procedure for choosing that specific generator is not studied. Likewise, the choice of a fixed measurement point to closely replicate \( \dot{f}_{\text{COI}}(0^+) \) for all disturbance locations is unlikely. This issue makes the suggested limited frequency measurement based disturbance size estimation unreliable.

Another requirement is the availability of inertia constant values for all operating synchronous generators at any point in time to construct equation (2.9) for calculation of \( \dot{f}_{\text{COI}}(0^+) \). The acquisition of such values might be difficult, especially with the increased variability in the generation mix and committed generating units.

Furthermore, the real time measurement of RoCoF under stressed operating conditions (e.g. following an active power disturbance or fault) is a challenge [43]. PMUs may calculate frequency and RoCoF as the first and second derivatives of the synchrophasor angle of the voltage [58], however such an approach is sensitive to noise [59]. In addition, [43] shows that during the initial stage of the transient period the RoCoF measurements of PMUs are inconsistent with the RoCoFs determined from generators shaft speeds. This inconsistency is attributed to the influence of other transient phenomena such as the sudden voltage angle change of the generators terminal due to the change in their magnetic reluctance from sub transient to transient [43]. Following an active power disturbance, the voltage drop at the terminal of the synchronous generator has a similar effect on magnetic flux as a three phase fault near the generator. This change in magnetic flux is translated to a transition from
generator’s sub-transient reactance \( (x''_{d}) \) to transient reactance \( (x'_{d}) \) and finally to steady state reactance \( (x_{d}) \). In classical power systems model, synchronous generator is modelled as a constant voltage source with internal voltage \( E \) with a changing reactance between \( E \) and generator’s terminal voltage \( (V) \). Figure 2-3 depicts the relational phasor diagram of a synchronous generator during transient time.

![Figure 2-3: Phasor diagram representing generator’s reactance change from sub transient \( (x''_{d}) \) to transient \( (x'_{d}) \) during transient time](image)

In Figure 2-3, \( V'' \) and \( V' \) are the generator’s terminal voltage during sub-transient and transient time respectively. \( I \) is generator’s load current. \( \phi'' \) and \( \phi' \) are the angle difference between voltage and current phasors at the terminal of the generator during sub-transient and transient time respectively. As \( I \) is considered constant during this transient time, and as is shown in Figure 2-3, the change from \( x''_{d} \) to \( x'_{d} \) causes a sudden change in the terminal voltage phasor angle. This sudden change in voltage phasor angle of the generators terminal affects the PMU estimation of RoCoF shortly following an active power disturbance, which in return leads to erroneous estimation of \( \dot{f}_{coi}(0^+) \) and subsequent inaccurate disturbance size estimation. These factors make fast and accurate estimation of generator RoCoF challenging in the aftermath of a disturbance. Different methods for estimation of frequency and RoCoF are discussed in Section (2.8).

The shift from a relatively small number of thermal mainly similar generating units to a wide range of new generating technologies introduces uncertainty in the total system inertia.
Therefore, with $H_{Sys}$ being a key input for swing based disturbance size estimation, accurate and reliable online inertia estimation methods have gained interest over the last decade [57][60][61][62]. In [60], frequency, load and plant outage data over a period of several years on Western Coordination Council (WECC) system was utilised to from a linear fit between total system load and total inertia constant. The total inertia constant for each case was calculated using the active power output of the disconnected plant and maximum experienced RoCoF observed at a single generator. However, the rarity of large active power events as well as negligence of the effect of disturbance location distance to the RoCoF measurement point limits the capability of this method to accurately estimate the total system inertia. In [61], an online $H_{Sys}$ estimation is demonstrated in which the active power and frequency measurements of all synchronous generating units following a disturbance are required. This method is not suitable as the availability of online measurements of all synchronous generators is not practiced in reality. In [62] the total system inertia of the Japanese power system was estimated using recorded frequency disturbances. In this method, a fifth order polynomial was fitted to the measured frequency in order to filter out the oscillatory component of a single frequency measurement caused by inter-area power oscillations. However, the effect of the disturbance location was not studied. A method for simultaneous estimation of disturbance time and total system inertia is suggested in [57]. The method assumes a homogenous distribution of the rate of change of frequency for all generating units and estimates $H_{Sys}$ using frequency and active power measurements from one location. Although accurate for relatively small networks, the proposed method loses accuracy as network size is increased and local frequency behaviour diverges.

However, despite the growing efforts to achieve fast and accurate estimation of system inertia, practical real time inertia estimation still remains an area of on-going research [63]. This serves as a challenge for the practicality of any disturbance size estimation method based on swing equation.

Another slower form of disturbance size estimation is achieved by first identifying the bus location of the disturbance based on angle measurements and then extracting SCADA information regarding the active power output of the generator connected to that bus [64]. This method is not suitable for applications where the fast estimation of disturbance size is required, as the SCADA rate of report is in the order of few seconds [65] and might fail to provide the estimation before frequency nadir time particularly in low inertia scenarios.
2.8 Methods for Estimation of Frequency

Frequency and Rate of Change of Frequency RoCoF are one of the main inputs in power systems control and protection applications. As modern power systems are being operated closer to their operational limits, a real time wide area monitoring, protection and control (WAMPAC) system has become the key for unlocking the full potential utilisation of the power networks in a secure and reliable manner [66]. Synchronised measurement technology (SMT) is the main enabler of WAMPAC, as it provides time synchronised measurements from widely dispersed locations. In SMTs, a global time base, typically Coordinated Universal Time (UTC) has to be communicated to all synchronised measurement points via some form of telecommunication media, either dedicated (e.g. fibre optics) or shared (e.g. Internet)[67]. The main time dissemination technologies are: Global Positioning Systems or generally speaking Global Navigation Satellite Systems (GNSSs), network time transfer protocols, serial time codes and Pulse Per Second (PPS) [68]. GPS represents the best time-dissemination technology for synchrophasor applications, due to its good compromise between performances and costs and its’ relatively high availability and coverage. Other forms of recognised GNSS systems are GLONASS and Galileo [69].

As explained in section 2.7, real time synchronised frequency and RoCoF measurements are required for estimation of active power disturbance using swing based method. Therefore, this section presents an overview of the main methods used for frequency estimation.

Frequency is by nature defined for a truly periodical signal which is stationary. In other words, frequency is a steady state term [70]. The frequency in power systems is attributed to the rate of change of voltage and current sinusoidal signal phase. However in power systems, during switching and transient events, currents and voltage phasors are not periodic [70]. Meaning in these conditions, frequency cannot be measured based on its definition for periodic signals. Many different efforts have been made to overcome this issue [71][72][73].

Zero crossing detection is the most common frequency measurement method [74]. In zero crossing frequency measurement method, frequency is measured as the inverse of the time between full cycles by looking at the zero crossings of the voltage signal at the same direction (from positive to negative or negative to positive) [70]. The time between cycles can be measured over longer periods to make it more robust against noise. Zero crossing frequency
measurement is relatively immune to harmonic distortion as long as it does not cause extra zero crossings. Low-pass filters can improve the accuracy of the zero crossing frequency measurements by reducing the harmonic content. Switching events can cause a sudden shift in the voltage magnitude which in turn imposes a sudden shift in zero crossing and a large error in frequency measurement. Using some heuristic approaches, these wrong measurement points should be detected and filtered out rather than being averaged [70].

In [72], voltage phasor and frequency estimation has been solved as an unconstrained optimisation problem using Newton’s iterative method. In this method the phase voltage at time \( t \) (\( v(t) \)) is expressed in the following form:

\[
v(t) = h(x(t), t) + \mathcal{N}(t)
\]  

(2.12)

Where \( \mathcal{N}(t) \) is a zero mean random noise, \( x(t) \) is a time varying parameter vector and \( h(x(t), t) \) can be written as follows [72]:

\[
h(x(t), t) = V_0(t) + V(t) \sin[\omega(t)t + \phi(t)]
\]  

(2.13)

Where \( V(t) \) is the peak value of the voltage, \( V_0(t) \) is the magnitude of the DC component, \( \omega(t) \) is the angular velocity and \( \phi(t) \) is the phase angle. Therefore, parameter vector \( x(t) \) is:

\[
x(t) = [V_0(t) \ V(t) \ \omega(t) \ \phi(t)]^T
\]  

(2.14)

By assuming frequency and phase angle nearly constant during each time step, the following expression can be deduced for the frequency at time \( t \):

\[
f(t) = \frac{1}{2\pi} \frac{d}{dt}[\omega(t)t + \phi(t)]
\]  

(2.15)

The discrete representation of the voltage samples can be used as follows:

\[
v(k) = h(x_k, t_k) + \mathcal{N}_k \quad k = 1, 2, \ldots
\]

\[
h(x_k, t_k) = V_0k + V_k \sin(\omega_k t_k + \phi_k)
\]  

(2.16)
Where $\vartheta_k$, $V_{0k}$, $V_k$, $\omega_k$, $\phi_k$ and $t_k$ are discrete form of $\vartheta(t)$, $V_0(t)$, $V(t)$, $\omega(t)$, $\phi(t)$ and $t$ for discrete time index $k$.

Assuming sampling rate of $f_s=1/T_s$, $t_k$ is equal to $kT_s$. For a finite time window ($T_w$), $m_k$ number of samples can be recorded where:

$$T_w = m_kT_s \tag{2.17}$$

Having $m_k$ number of samples, the measurement vector ($V$) of size $m_k \times 1$, a vector of non-linear functions $h(x)$ of size $m_k \times 1$ and an error vector ($\vartheta$) of size $m_k \times 1$ can be constructed. As the number of unknowns are four ($V_{0k}$, $V_k$, $\omega_k$, $\phi_k$), as long as the $m_k$ is bigger than four the following set of equations are over determined and can be solved using either Newton iterative method [72] or Least Square Error method [75].

$$V = h(x) + \vartheta \tag{2.18}$$

Once (2.18) is solved, and based on (2.15) frequency at each discrete index $k$ ($f_k$) can be estimated as follows:

$$f_k = \frac{1}{2\pi T_s} \left[ (\omega_k t_k + \varphi_k) - (\omega_{k-1} t_{k-1} + \varphi_{k-1}) \right] \tag{2.19}$$

In power systems, generators rotors spin at a certain rate. Assuming an accurate shaft speed or position sensor is applied, the rate of change of the angular position of the rotor (angular velocity) can serve as an excellent proxy for a generator’s frequency [70]. This is due to the rotors inertia, which cause the angular velocity to be a clean signal with a high level of signal to noise ratio. This approach for estimation of generators frequency is used in [76], where the p.u. value of the rotors angular velocity is passed through a first order filter and then multiplied by 50 to get the frequency value in Hz. Using the first order Taylor series expansion, the RoCoF is calculated by dividing the difference of two subsequent frequency measurement points by the time difference between them. Similar to this method, in Chapter 4, 5, 6 and 7 of this thesis for each simulation result coming from DlgSILENT, the frequency
is measured from generators rotating speed and their RoCoF is calculated using the rate of change between two subsequent frequency measurement points.

### 2.9 Methods for Localising Power System Disturbances

In low inertia scenarios power systems are subjected to fast and large frequency deviation following active power disturbances. One possible countermeasure for this deteriorated frequency security is contracting larger volumes of primary response (governor response). However, this option is not attractive as it can impose excessive cost, e.g. cost of frequency control in GB can rise to £200m-£250m by 2020[8]. Alternatively, new fast an adaptive frequency control services, e.g. AUFLS, storage and wind turbines can help injecting additional power to the grid faster than conventional services in emergency situations. These fast frequency responses need to be targeted as the fast release of active power far from the origin of disturbance can cause great stress on the power system, particularly angular stability in the initial tenth of a second [9]. Furthermore, the power injection which is not in the vicinity of power loss (disturbance location), can potentially lead to unacceptable current flow on the lines and increase the risk of cascading outages. Whereas having targeted response helps minimise the impact on the normal system operating condition and with keeping the power flows intact, hence reducing the operational cost. Therefore, having fast and reliable information regarding the location of disturbance is a stepping stone towards delivering these new forms of fast frequency services in a cost effective and secure manner.

The power imbalance between load and generation leads to electromechanical transients with an influence on synchronous machines [77]. The electromechanical waves propagate throughout the system from the origin of the disturbance causing location and time dependency of impact seen by different synchronous generators [78]. This effect translates into varied delays in same frequency deviation experienced by generators in different locations. Based on this phenomena many disturbance localisation techniques have been proposed in the literature [79][80][81]. In these methods, the geodetic coordinates of the PMUs are translated into Cartesian coordinates and the following conditions are neglected [82]:

- Actual topology of the power system
- Inertia distribution of the power system (inertias of individual generating units)
- Dependency of electromechanical wave propagation speed on network parameters and the path it travels

In [79] and [80], a triangulation method based on the different frequency wave front arrival time is proposed. Wave front arrival time is defined as the time difference between disturbance time \( t_d \) and the time a measured frequency falls below a specified threshold. As frequency perturbations have been shown to travel at finite speeds [77], the frequency disturbance recorders (FDR) detect wave arrival time at different locations using time synchronised measurements. In these methods, a set of nonlinear over determined equations are written based on having PMUs coordinates, their frequency wave arrival times and electromechanical waves propagation speed as known variables and disturbance time and disturbance location coordinates as unknown variables. These equations have been solved by the least square method to find the Cartesian coordinates of disturbance location and also the disturbance time. For these methods considering the number of unknown parameters, at least four PMUs need to be monitored. However, these methods have some shortcomings:

1. The existence of noise and oscillations in measured PMUs frequencies might affect the precise detection of front wave arrival time at each location
2. The speed of electromechanical wave propagation along different paths can range from 100 to 1000 miles per seconds, depending on the network topology [83]; therefore, the constant propagation speed assumption might fail in providing precise disturbance location
3. Setting the threshold for detection of wave front arrival time is not well studied and it can affect the performance of the method especially considering the increasing volatility of system total inertia value [83].

The first issue can be tackled by applying filtering windows to the measured frequencies which although practical in post mortem analysis, it adds extra delays in fast online applications [84].

In [85] an adaptive disturbance localisation aiming to solve the second issue is proposed. In this method the anisotropy of frequency propagation speed is considered. Anisotropy of propagation speed means difference in speeds along different propagation paths. The speed
of propagations \((v_i)\) are set as unknown variables in each path for the equations connecting wave front arrival times, PMUs coordinates, disturbance time and disturbance location coordinates. This approach makes the set of equations an under determined problem as each PMU measurement has its own propagation speed with respect to the disturbance location. In order to reduce the number of unknowns and transform the problem back into an overdetermined set of equations, the network is partitioned into \(N\) regions. The speed of propagation is assumed constant for disturbances taking place within one region; however this assumption is questionable when the number of PMUs is reduced. The propagation speeds regarding each PMU measurement point and disturbance location makes up \(N\) vectors each with the size of \(M\times1\), with \(M\) being the number of PMUs. Following the detection of a disturbance and calculation of wave front arrival times, the location problem is solved \(N\) times, each giving an estimate of disturbance location within one region. Finally, the location with the lowest error is chosen as the disturbance location. One drawback of this method is the time consuming procedure for updating the propagation speed vectors for all regions. This is done by using an updated snapshot of the system received from SCADA and applying different disturbance types and locations and recording the average propagation speed within each region. Also, it is observed that the propagation speeds are dependent on the size of disturbance. This means trained propagation speed for one disturbance size could result in increased error in localisation if used for another disturbance size. Therefore, the suggestion is made to have disturbance size as an input to the localisation algorithm. Furthermore, it is pointed out that the random partitioning of the network can lead to errors and a more analytical determination of boundaries is required.

In [83], a disturbance detection method based on offline clustering of all synchronous generators into coherent groups is suggested. One PMU measurement is selected to represent each cluster in online analysis stage. Following the detection of disturbance, rotor frequencies of the represented clusters are compared to find the representative with the biggest initial swing. The identified cluster is deemed to include the disturbance location. The clusters are trained offline based on 200 cycles of rotor angle measurements of six separate events. In the online validation stage, the event location is estimated using 20 cycles of measured PMUs data (0.4s). However, this method did not consider the fact that by using the similarity of rotor angle behaviour the resulting clusters are dependent on the disturbance location itself. Therefore, if clusters are trained for a particular disturbance location they do not hold the same boundaries for another disturbance location hence resulting in poor localisation.
performance. This methodology can be improved by training the clusters based on electrical distances (defined in [86]) between generators which are independent of the disturbance location. Even so, the accuracy of the proposed method is highly reliant on the number PMU measurements each representing one cluster. For instance, if a PMU is representing ten generator connected buses, a possible disturbance located at that cluster can be any of these ten buses, which is not a narrow selection. Furthermore, the disturbance localisation is only covering generator trip type as the clusters only include generator buses.

In [87], a decision tree is trained offline based on historical data for generator trip type of events. The decision tree is then used to locate the disturbance location in online applications. The decision tree predictors are the sequence of disturbance detection in different PMU locations using six different algorithms for detection of a specific feature in the frequency wave form. The output classes of the decision tree are the locations of the disturbances. The suggested method is highly dependent on the size of historical data available for training the decision tree. Furthermore, as stated in [87], the localization performance of the method is highly sensitive to the missing data input (missing PMU measurements).

### 2.10 Methods for Detection of Power System Disturbances

Based on the definition given in [88], power system disturbance is described as “A sudden change or sequence of changes in one or more of the power system parameters”. Fast and reliable detection of disturbance in power systems provides a valuable input and mainly triggering signal for initiation of several applications supporting security of power systems, e.g. inertia estimation procedure, fast frequency response activation [61][9]. Currently, detection and analysis of different disturbance types are required in Wide Area Monitoring and Control Systems (WAMCS) which are supported by phasor measurement units (PMUs) installed in different locations and synchronised in time via Global Navigation Satellite System (GNSS), i.e. the GPS, GLONASS, Galileo. The abundance of real-time data gave rise to development of data driven methods for disturbance detection [90][91].

A detection method based on K-Nearest Neighbour (KNN) analysis applied to distance matrix between online measured variables and abnormal classes trained based on historical data is suggested in [92] and [93]. The abnormality related to disturbance condition is
quantified using a threshold on the smallest element on the distance matrix. However, no clear methodology for threshold setting is proposed.

In [94], a threshold is set on standard deviation of frequency measurement in a 10s time window of PMU measurements to detect the disturbance occurrence. In [64] frequency drop divided by time is calculated over a time window of 10s at different frequency measurement locations and the violation of a predefined threshold at any of the monitored locations is used as disturbance occurrence indicator. In [8] a disturbance detection procedure is proposed based on a threshold violation of RoCoF. However, all frequency or RoCoF threshold based disturbance detection methods face a few challenges. One is regarding the setting of the threshold which is subject to change in varied inertia and noise levels, therefore can potentially lead to miss or false detection of the disturbance. In some cases the use of filter windows (moving average window) on frequency or RoCoF is proposed to smooth out the noise and oscillations in the measurements [8], this approach however adds an extra delay in detection procedure and can fail in applications where speed of detection is critical (e.g. fast frequency service activation in low inertia systems). Finally, the dependency of the measured RoCoF to the relative distance between disturbance location and measurement point means the measured RoCoF far from the disturbance location might fail in the detection of the disturbance.

In [57], a disturbance detection method based on convergence properties of real time inertia estimation in a moving window is proposed. In this method, frequency and active power are measured in one location and the division of these two variables at any point in time is compared with its consecutive values in a moving time window. The sufficiently similar output in a window is treated as a measure for disturbance detection. The similarity is evaluated using a threshold on the residue of the moving window. However, the procedure for setting the threshold has been heuristic rather than analytical, therefore not easily repeatable for other systems. Also considering the small size of the network, in this method the initial frequency behaviour following the disturbance has been assumed to be homogenous in all buses. Nevertheless, this assumption does not hold true in larger systems since frequency waves propagate across the system with the speed of several hundred miles per second [95]. Furthermore, as the method is reliant on the properties of moving window of measured variables, disturbance detection can have a delay up to the size of the moving window (up to 0.5s). Even though the delay in detection is relatively small, the method might
fail regarding speed under low inertia systems where time to nadir can be in the order of 2 to 3s [1].

In [96], a Principle Component Analysis (PCA) based method is applied to PMUs frequency measurements to detect islanding conditions. The accuracy of method in distinguishing islanding events from load or generation loss events is examined using real measurements coming from single-phase phasor measurement unit, developed at the Queen’s University Belfast (QUB) as part of the OpenPMU project [97],[98]. In general, the PCA based methods transform set of original correlated variables into a smaller set of uncorrelated ones. The proposed method has the capability of automatically adjusting the islanding detection threshold based on long-term historical data.

In [99], a dimensionality reduction technique is applied to PMU measurement based on PCA. A number of PMUs are chosen as pilot and the principal component of their past $N$ measurements (voltage and frequency) are used in normal operating conditions to be trained for estimating the non-pilot PMUs readings at the current time $t$. The output of this training is a linear regression coefficient matrix. At each time step, the linear regression coefficient matrix and pilot PMUs measurements are used to estimate non-pilot PMUs readings. The difference between estimated non-pilot PMUs measurements and their actual readings is compared against a pre-defined threshold which serves as an indicator for early stage disturbance detection (2 to 3 PMU samples following the disturbance time). Despite the fast execution of the method in [99], all buses need to be equipped with PMUs and no clear methodology is proposed for setting of the threshold.

### 2.11 New Suggested Methods for Disturbance Analysis and Frequency Response Management

#### 2.11.1 Benefits of the Proposed Disturbance Analysis Methods

As discussed, the prerequisite for development of occasional fast and adaptive frequency control services (e.g. AUFLS) capable of coping with varied system conditions and low inertia scenarios are analysing active power disturbance characteristics in a timely manner. Disturbance characteristics include:
- Disturbance time (onset of the disturbance)
- Disturbance location (bus in the network where disturbance takes place)
- Disturbance size (magnitude of the active power imbalance caused by the disturbance)

In this thesis the procedure for identifying all these characteristics is referred to as disturbance analysis (Chapter 1). Two novel disturbance analysis methods are introduced in the following chapters and they are termed as:

- Synchronising Power Coefficient based disturbance analysis (SPC method) (Chapter 3)
- Regional Inertia based disturbance analysis (RI method) (Chapter 4 and 5)

The followings are the benefits of the SPC disturbance analysis method (Chapter 3):

1. The SPC method uses active power outputs of generators instead of their RoCoFs; therefore, it does not suffer the issues with frequency measurements and calculation of COI frequency and RoCoF (mentioned in section 2.7.4).
2. It does not need the estimation of total system inertia or generators individual inertias.
3. It does not rely on all generators being equipped with PMUs; therefore, it is tolerant to missing PMU measurements which makes it more reliable and economical.
4. It has superior speed compared to previous methods as it only requires information from PMUs for one time step following the disturbance (0.02s).
5. Its performance is not deteriorated when the location or the size of disturbance is varied.
6. It is robust against noise in frequency measurements.
7. It can simultaneously detect the disturbance and estimate the disturbance size and location.
8. Threshold setting for disturbance detection is analytical by using a decision tree approach.

The RI disturbance analysis method shares benefit numbers 3 to 8 of SPC disturbance analysis method and in addition it has the following benefits:
1. It uses the already existing and developing synchronised frequency measurements technology and infrastructure, i.e. OpenPMU project [98] and FNET [64] (compared to the suggestion to use synchronised active power measurements for the SPC method (Chapter 3)). FNET consists of many high dynamic precision frequency estimation devices, also known as frequency disturbance recorders (FDR). These FDRs can be used at any 110 V or 220 V wall outlet and transmit measured frequency data remotely via the Ethernet.

2. It does not need frequency arrival time for disturbance localisation; therefore, it does not suffer the issues with correct calculation of frequency arrival time (mentioned in section 2.9).

2.11.2 Benefits of the Proposed Frequency Response Management Method

This thesis suggests a novel method for adaptive fast frequency response management with a holistic approach for planning the requirement for fast frequency response as well as triggering a fast frequency response which is adaptive to:

- Disturbance characteristics
- System frequency response (SFR) characteristics

In this sense, the proposed frequency response management has two stages:

1. Dynamic assessment of frequency security status and determining the fast frequency requirement of the system (carried out continuously in normal system operation)(Chapter 6)

2. Adaptive fast frequency response which could be in the form of AUFLS (carried out once a disturbance is detected in disturbance analysis) (Chapter 7)

The goal of the first stage of the frequency response management is to identify the system frequency response (SFR) model and based on that evaluate the frequency security status and fast frequency response requirement for the worst case scenario (system reference incident). The output of the first stage of frequency response management provides necessary information to system operator to procure right amount of fast frequency response services based on the real-time system operating conditions. This way the extra cost due to
conservative over engineering approach for service arming is reduced while ensuring the frequency security of the system.

The second stage of the frequency response management is activated upon disturbance occurrence and makes a decision on the necessity, size and location of response. The frequency response in the second stage is adaptive to disturbance analysis results (disturbance characteristics) and results from the first stage of the frequency response management (SFR characteristics). This adaptive fast frequency response is an umbrella term that can include different forms of control actions such as: AUFLS, adaptive electrical storage activation or adaptive control of synthetic inertia of wind turbines. In this thesis the AUFLS is chosen to demonstrate the effectiveness of the suggested methodology for adaptive fast frequency response (Chapter 7).

The benefits of the new AUFLS (chapter 7) are as follows:

1. By capturing the frequency response dynamics and disturbance size it can more closely estimate the minimum shedding size needed to avoid the frequency nadir falling below a pre-defined threshold. This results in a reduced risk of over or under shedding.
2. Its trigger mechanism does not rely on frequency or RoCoF threshold violation. Therefore, its triggering is more robust against noise and is faster which makes it more suitable for low inertia systems.
3. Due to its faster activation following the disturbance, it achieves a better frequency response with lower shedding size.
4. The load response is automatically captured when identifying the SFR. Therefore, there is no need to include the effect of voltage sensitivities of loads and voltage drops of buses when calculating the disturbance size.
5. Its triggering is dependent on the detection of disturbance in the disturbance analysis stage; therefore, it is only reliant on few PMU measurements. There is no requirement to have all generators equipped with PMUs to calculate COI frequency or RoCoF for triggering.
6. It does not require the initial frequency trace following the disturbance occurrence to predict the frequency nadir (like methods suggested in [29][46][47]) hence it does not face the problem with speed (mentioned in 2.6) even when time to nadir is short. This
is due to the fact that it uses identified SFR and estimated disturbance size to predict the frequency behaviour.

7. The location of its load shedding is based on the accurate location of disturbance identified in disturbance analysis stage. Therefore, it does not rely on imprecise disturbance location information using voltage drops (like methods suggested in [42] and [44]).

2.12 Chapter Summary

With the ongoing trend towards a sustainable and green energy future, conventional fossil fuelled synchronous generators are being replaced by renewable energy sources with no inertia contribution. As stored kinetic energy in rotating mass of the system helps render slower frequency dynamics, the loss of synchronised rotating energy in the power systems manifests itself as a faster deeper frequency drop caused by the same abrupt active power imbalance (disturbance size). The unprecedented initiatives for wide spread installation of PMUs and development of platforms for utilising their information in decision making procedures have opened new opportunities to tackle frequency security issues (e.g. deployment of fast adaptive demand response in the form of AUFLS). Having fast availability of time synchronised information of different buses helps with identifying the abnormal conditions and the source and severity of them. Following a disturbance occurrence, the frequency response and specially RoCoF is locally variable in initial stages. Therefore, many methods have suggested the monitoring of all synchronous generators to capture overall system frequency behaviour for their applications including disturbance detection or estimation of disturbance size and location. Although the PMU supported information on active power disturbance can help to tailor fast frequency actions, methods suggested in the literature for disturbance detection, localisation and size estimation have extensive potential for improvement; e.g. reducing the strong dependency of their accuracy on the number of PMU measurements or reducing their lengthy buffer window length.

With the increased demand, transmission systems are forced to operate closer to their voltage security limits. The heightened stress on transmission corridors calls for more localised fast adaptive frequency control in case of emergency to avoid voltage instability as well as line overloads. Localised adaptive frequency control can include fast load reductions (AUFLS) in
the vicinity of disturbance. This gives motivation for developing faster and more precise disturbance localisation methods.

Due to the dependency of fast frequency control success to their activation speed in emergency states and particularly in low inertia scenarios, fast and reliable detection of disturbance even in noise polluted conditions becomes critically important.

Frequency trace following the active power disturbance is dictated by both disturbance size and system frequency response model. Therefore, it is widely recognised that accurate disturbance size estimation plays a key role in any kind of adaptive frequency control. Furthermore, as system parameters affecting system frequency dynamics (e.g. inertia) face increasingly more temporal and spatial variations, having a continuously updated understanding of them aid in designing more optimal adaptive frequency response services.

This chapter provided a brief description of, as well as the shortcomings of the methods suggested in the literature for disturbance detection and estimation of disturbance size and location. Also, different AUFLS have been discussed in order to identify the potential areas of improvements. Finally the benefits of the new suggested methods for disturbance analysis and management of fast frequency response in terms of requirement, necessity, size and location of response are listed.

2.13 References


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3. **Disturbance Analysis Based on Synchronising Power Coefficient**

Reduced system inertia decreases the time available for control actions to prevent the system frequency from violating security limits after a disturbance. Therefore, as system inertia is reduced by the shift toward renewable generation that provides little or no inertia, system operators must deploy faster frequency control actions if they are to preserve security and quality of supply. Realising methods for the fast and accurate detection, localization and sizing of an active power disturbance (e.g. the loss/connection of a large generator or load) will be a crucial enabler for the successful implementation of these faster actions. This chapter presents a novel method that can simultaneously estimate the time, size and location of a disturbance using PMU measurements of the active power output of a limited number of generators and the impedance matrix of the system. The term disturbance analysis is given to the detection, localisation and sizing of active power disturbances. The new method is referred to as Synchronising Power Coefficient (SPC) based disturbance analysis.

The immediate power change at the remote generator terminals is combined with the synchronising power coefficient matrix in a two stage process. Stage one uses a decision tree to determine that a disturbance has occurred which then initiates stage two that estimates the size and location of the disturbance. This is based on the level of agreement between the monitored generators. Case studies and sensitivity analysis for the IEEE 39 bus test network are presented to verify the accuracy of the proposed method for varied levels of measurement noise, impedance matrix errors and topology errors for various disturbance sizes and locations.

Section 3.1 provides the motivations for fast disturbance analysis following an active power disturbance as well as giving a short description of the SPC method. Section 3.2 gives an overview of the classical power system model and defines synchronising power coefficients. Section 3.3 discusses the distribution of the active power imbalance between generators that occurs after a disturbance. Section 3.4 presents an example of active power distribution dependency on disturbance location. The methodology for synchronising power coefficient based frequency disturbance analysis is presented in Section 3.5. The results and case studies are discussed in Section 3.6. Finally, the chapter is concluded in Section 3.7.
3.1 Chapter Introduction

The reduction in system inertia, due to the displacement of traditional synchronous generation with converter connected generation, poses a critical threat to the frequency control of power systems in the coming decades [1]. Reduced inertia causes a faster, larger frequency deviation to occur after a sudden generation loss or load increase (i.e. disturbance), which the conventional primary response is too slow to contain within the existing security limits. A solution to this problem is developing new control services that are capable of delivering a fast frequency response [2] (e.g. Adaptive Under Frequency Load Shedding (AUFLS) [3], storage activation [4] and synthetic inertia from wind turbines [5]).

Delivering a response that is adapted to the disturbance size and targeted at the disturbance location would not only aid in arresting the frequency deviation with minimised resource allocation [3], but would also minimise the impact of the action on normal system operation (e.g. power flows). However, achieving the fast and robust triggering (i.e. initiation) of these new frequency control services is a significant challenge. Triggering based on frequency, using a proportional response (i.e. governors) or thresholds (i.e. traditional Under Frequency Load Shedding (UFLS)), is robust, but slow. However, whilst triggering based on the Rate of Change of Frequency (RoCoF) allows a faster response; its effectiveness can be susceptible to mal-operation due to the challenge of quickly estimating RoCoF in disturbed conditions. This vulnerability can be mitigated somewhat by using a combination of RoCoF and frequency, as is seen in many forms of (AUFLS); however, this sacrifices speed of response in order to provide security and reliability. Therefore, an alternative to these methods must be found that can quickly and reliably produce an accurate estimate of the disturbance size and location.

In [3] it was shown that the disturbance size can be calculated using the swing equation with the initial RoCoF of the Centre of Inertia (RoCoF_{COI}) and the total system inertia. Based on this method, many AUFLS have been proposed based on the assumption that an accurate disturbance size estimate will be available soon after the disturbance has occurred [6, 7]. The procedure for calculating the RoCoF_{COI} requires that the RoCoF of every generator must be measured using PMUs as part of a Wide Area Monitoring System (WAMS) [8] and the inertia constant of each generator should be available. Every frequency must be monitored in
order to accommodate the local behaviour of frequency during transient phenomena. This requirement is quite stringent, particularly as redundancy will be required in practice.

Furthermore, the real time measurement of RoCoF under stressed operating conditions is a challenge. PMUs may calculate frequency and RoCoF as the first and second derivatives of the synchrophasor angle, such an approach is sensitive to noise [9]. In addition, [10] shows that during the initial stage of the transient period the RoCoF measurements of PMUs are inconsistent with the RoCoFs determined from generator shaft speeds. These factors make fast and accurate estimation of generator RoCoF challenging in the aftermath of a disturbance. In [11], limited frequency measurements representing different areas of the system were used to reproduce \( R_{\text{CoF}} \) and estimate the disturbance size. However, the optimal RoCoF measurement points capable of representing \( R_{\text{CoF}} \) are heavily dependent on the disturbance location. Furthermore, despite the growing efforts to achieve fast and accurate estimation of system inertia [12, 13], practical real time inertia estimation still remains an area of on-going research [14].

Fast disturbance detection and localisation is a pre requisite for successful initiation of System Integrity Protection Schemes (SIPS) [15]. Threshold violation disturbance detection was carried out in [16] by applying a two second width filter to the recorded RoCoF. In [13] a method for estimating the disturbance time and inertia was introduced for a single generator, or small system. The method used the convergence of a sliding window, which was applied to online estimated inertia calculated using the generators RoCoF. In [17] disturbance localisation was carried out by comparing the arrival time of frequency waves that propagate across the system after disturbance. However, methods reliant on frequency or RoCoF for disturbance detection and localisation can introduce extra delays due to filtering or be vulnerable to false operation caused by noise.

Therefore, given the challenges faced when using online measurements of RoCoF and attempting to determine accurate estimates of system inertia, this chapter presents a novel method for estimating the disturbance size and location that is based upon active power measurements and the impedance matrix of the system. This two stage method detects the disturbance in the first stage and then estimates its size and location in the second stage. Following a disturbance, the active power imbalance created is instantaneously compensated for by the rotating mass of the system (primarily synchronous generators). The initial
distribution of the active power imbalance at each generator terminal is determined by its electrical distance to the disturbance and the size of the disturbance. Therefore, each disturbance location can be thought of as having a unique pattern of how the power imbalance is distributed between the generators that is determined by the impedance matrix. Based on this, in simple terms, the proposed method uses these patterns to determine the disturbance location and size by comparing measurements of active power from a limited number of generators and determining which pattern they correspond to most accurately.

The method improves over existing methods by:

- Using active power measurements of generators rather than frequency and RoCoF
- Avoiding the use of either the system inertia or individual generator inertias
- Only a limited number of generators must be equipped with PMUs.

3.2 Classical Power Systems Model

The simplest form of power system model used for stability studies is the classical power system model [18, 19]. The classical power system model is mainly useful during the period of time for which system dynamics is mainly dependent on the kinetic energy in the rotating masses of the system. In other words, the classic power system model provides acceptable system dynamic representation during the time for which we intend to deploy disturbance size estimation method. In classical power system model the assumptions for synchronous generators are as follows:

1. Mechanical power input is constant \( P_m \)
2. Damping or asynchronous power is neglected
3. The generator is represented by a constant EMF behind the direct axis transient (unsaturated) reactance \( (x'_d) \).
4. The mechanical rotor angle of a synchronous generator can be represented by the angle of the voltage behind transient reactance.

In the classical model the load is usually represented by passive impedances (or admittances), which are based on the initial (pre-disturbance) conditions. These impedances are assumed to be constant during the stability analysis.
The swing equation for a single synchronous machine in a multi machine system (having M synchronous generators) can be written as [20]:

\[
\frac{2H_i}{\omega_r} \frac{d\Delta \omega_i}{dt} + D_i \Delta \omega_i = P_{m,i} - P_{e,i} \quad i = 1, 2, \ldots, M
\]  

(3.1)

\[
P_{e,i} = E_i^2 G_{ii} + \sum_{j \neq i}^M E_i E_j Y_{ij} \cos(\theta_i - \delta_{i0} + \delta_{j0}) = E_i^2 G_{ii} + \sum_{j \neq i}^M E_i E_j (B_{ij} \sin \delta_{ij} + G_{ij} \cos \delta_{ij})
\]  

(3.2)

Where \( H_i \) is inertia constant of the \( i^{th} \) generator in its own MVA base \((S_i)\). \( D_i \) is damping constant of synchronous generator \( i \). \( P_{m,i} \) is mechanical power input and \( P_{e,i} \) is electrical power output of synchronous generator \( i \). \( \omega_r \) is rated rotational speed of the synchronous generator. \( \omega_i \) is rotating speed of synchronous generator \( i \). Also \( \delta_{ij} = \delta_{i0} - \delta_{j0} \), and is equal to angular difference between \( E_i \) and \( E_j \). \( E_i \) is constant voltage behind transient reactance for machine \( i \). \( \delta_{i0} \) is pre-disturbance internal voltage angle of synchronous generator \( i \). \( Y_{ii} = G_{ii} + jB_{ii} \) is a diagonal element of the network admittance matrix \( Y \) when the network is reduced to internal nodes of the synchronous generators (the procedure for network reduction and calculation of \( Y \) matrix is discussed in Section 3.3.2). \( Y_{ij} = G_{ij} + jB_{ij} \) is an off-diagonal element of the network admittance matrix \( Y \). \( \theta_{ij} \) is angle of element \( Y_{ij} \) of network admittance matrix.

Furthermore, it should be noted that prior to the disturbance \( P_{m,i} = P_{e,i} \). Due to the larger time constant of governor and prime mover , it can be assumed that shortly following the disturbance \( P_{m,i} \) still remains the same as \( P_{e,i} \) prior to the disturbance. Therefore, the immediate mismatch of mechanical and electrical power effecting the rotor speed and governing the generator frequency through swing equation shown in (3.1) would be equal to \( \Delta P_{e,i}(0^+) = P_{e,i}(0^+) - P_{e,i}(0) \). Hence, the logical step into studying the effect of active power mismatch is calculation of immediate electrical power change at generators terminal.

### 3.2.1 Synchronising Power Coefficients

Shortly following the active power disturbance the internal voltage magnitude of generators stays constant so the new electrical output of generator \( i \) (\( P_{e,i}(0^+) \)) can be written as follows:
\[ P_{e,i}(0^*) = E_i^2 G_{ij} + \sum_{j=1}^{M} E_i E_j (B_{ij} \sin(\delta_{j0} + \delta_{jA}) + G_{ij} \cos(\delta_{j0} + \delta_{jA})) \]  

where the 0 subscript indicates pre disturbance values.

By assigning \( \delta_{ij} = \delta_{ij0} + \delta_{ijA} \), and considering the small angular change, the following simplification can be applied to (3.3):

\[ \sin \delta_{ij} = \sin \delta_{ij0} \cos \delta_{ijA} + \cos \delta_{ij0} \sin \delta_{ijA} \equiv \sin \delta_{ij0} + \delta_{ijA} \cos \delta_{ij0} \]  

\[ \cos \delta_{ij} \equiv \cos \delta_{ij0} - \delta_{ijA} \sin \delta_{ij0} \]  

By substituting (3.4) and (3.5) in (3.3) \( \Delta P_{e,i}(0^*) \) is estimated as follows:

\[ \Delta P_{e,i}(0^*) = P_{e,i}(0^*) - P_{e,i}(0) = \sum_{j=1}^{M} E_i E_j (B_{ij} \cos \delta_{j0} - G_{ij} \sin \delta_{j0}) \delta_{ijA} \]  

For a given initial condition all parameters in (3.6) are known and the only variable would be \( \delta_{ijA} \). Thus the following:

\[ \Delta P_{e,i}(0^*) = K_{ij} \delta_{ijA} \]  

where \( K_{ij} \) is called the Synchronising Power Coefficient between synchronising machine \( i \) and synchronising machine \( j \), and can be rewritten as:

\[ K_{ij} = \left. \frac{\partial P_{e,i}}{\partial \delta_j} \right|_{\delta_{ij0}} = E_i E_j (B_0 \cos \delta_{j0} - G_{ij} \sin \delta_{j0}) \]  

\( K_{ij} \) is the change of electrical output power of the synchronous machine \( i \) due to the change in the angle between synchronous machine \( i \) and \( j \), assuming all other angles are held constant [20]. An extended definition of Synchronising Power Coefficient is the change of electrical power output of a given synchronous machine due to a change of angles between its internal
voltage (EMF) and any bus, having all other bus angles constant. Meaning, in $K_{ij}$ either of the subscripts can refer to a non-generator node, in which case $E$ would be substituted by $V$ for that node in (3.8).

### 3.3 Power Disturbance Distribution

In this section the effect of a sudden load change ($\Delta P_l$) at some point in the system and the relationship between initial active power changes of different synchronous generators are analysed. For the purpose of simplifying the analysis the reactive power component has been assumed to be negligible. Following a sudden active power change the system would first experience an oscillatory transient before settling to a new steady state condition [20]. By adding node $u$ (load node) to (3.2) and assuming conductance to be nearly zero we will have:

$$P_{x,i} \approx \sum_{j=1}^{M} E_i E_j B^u_{ij} \sin \delta_{ij} + E_i V_u B^u_{iu} \sin \delta_{iu}$$

(3.9)

Where $B^u_{ij}$ is the imaginary part of $Y^u_{ij}$ element of $Y^u$ admittance matrix where superscript $u$ indicates that admittance matrix is calculated for a system which is reduced to the internal generator nodes and disturbance node $u$ (M+1 port network). The procedure for calculation of the $Y^u$ and network reduction is discussed in Section 3.3.2.

The power into node $u$ (the load bus) is:

$$-P_{l,u} \approx \sum_{j=1}^{M} V_u E_j B^u_{uj} \sin \delta_{uj}$$

(3.10)

The negative sign in (3.10) is due to the conventional load difference in the direction of load and generation power. In all these formulas we assume that network has a very high X/R ratio in a way that conductances are all negligible. The machines have been represented by the classical model, constant voltage behind transient reactance. Furthermore, the network has been reduced to internal machine nodes (1,2,..M) and node $u$, where the load change ($\Delta P_l$) is applied. The load change location (node $u$) is shown by subscripts in subscript in ($\Delta P_{l,u}$).
resulting reduced network would have an M+1 port having the configuration shown in Figure 3-1 [20].

![Fig. 3-1: M+1 port Network](image)

The immediate effect of applying (ΔPᵢ) at node u is the change in the angle of bus u whereas its voltage magnitude stays constant. Meaning \( V_u \angle \delta_{u0} \) becomes \( V_u \angle \delta_{u0} + \delta_{u\Lambda} \). Note that the internal angles of machine nodes do not change instantly due to their rotor inertia [20].

### 3.3.1 Linearisation

The equations (3.9) and (3.10) are of nonlinear nature due to sinusoidal factors. However, regarding fast disturbance size estimation, the instant immediately following an active power change is of interest. Meaning the delta changes in power and angles shortly following the active power disturbance are the point of focus. Therefore, the linearised form of (3.9) and (3.10) around \( (0^+ t) \) can be used to determine the ratio of the load change (ΔPᵢ) supplied by each generator for load change taking place at different buses throughout the system (\( \Delta P_{e,j}, i = 1, 2, ..., M \)).

By substituting (3.4) and (3.5) in (3.9) and (3.10) and eliminating the initial values and considering conductance to be zero, the linear equations are estimated as follows:

\[
\Delta P_{e,j} = \sum_{j=1}^{M} \left( E_j E_j^* B_{jj}^u \cos \delta_{j0} \right) \delta_{j\Lambda} + \left( V_u E_j B_{iu}^u \cos \delta_{iu0} \right) \delta_{m\Lambda} = \sum_{j=1}^{M} K_{jj}^u \delta_{j\Lambda} + K_{iu}^u \delta_{m\Lambda} \tag{3.11}
\]
\[-\Delta P_{i,u} = \sum_{j=1}^{M} (V_u E_j B_{uj}^n \cos \delta_{uj\Delta}) \delta_{uj\Delta} = \sum_{j=1}^{M} K_{ju}^u \delta_{uj\Delta} \tag{3.12}\]

These equations are valid for any time \( t \) following the load change and before the initiation of governor action. The superscript \( u \) in \( K_{ij}^u \), \( K_{iu}^u \) and \( K_{ju}^u \) indicates that synchronising power coefficients are calculated for a system which is reduced to the internal generator nodes and disturbance node \( u \) (M+1 port network). The procedure for calculation of the \( Y^u \) and network reduction is discussed in Section 3.3.2.

Furthermore, at instant \( t=0^+ \) we know that \( \delta_{\Delta} = 0 \) for all generators due to the rotor inertias [20]. Thus we can compute (with both \( i \) and \( j \) indicating generators):

\[\delta_{\Delta}^0(0^+) = 0 \tag{3.13}\]

\[\delta_{ui\Delta}^0(0^+) = \delta_{ui\Delta}^0(0^+) - \delta_{ui\Delta}^0(0^+) = -\delta_{ui\Delta}^0(0^+) \tag{3.14}\]

\[\delta_{uj\Delta}^0(0^+) = \delta_{uj\Delta}^0(0^+) - \delta_{uj\Delta}^0(0^+) = \delta_{uj\Delta}^0(0^+) \tag{3.15}\]

Substituting (3.13), (3.14) and (3.15) in (3.11) and (3.12) results in the following:

\[\Delta P_{v,u}^0(0^+) = -K_{iu}^u \delta_{ui\Delta}^0(0^+) \tag{3.16}\]

\[-\Delta P_{l,u}^0(0^+) = \sum_{j=1}^{M} K_{ju}^u \delta_{uj\Delta}^0(0^+) \tag{3.17}\]

Comparing (3.16) and (3.17) for node \( u \) (load node) results in the following:
\[
\Delta P_{i,u}(0^+) = \sum_{j=1}^{M} \Delta P_{r,j}(0^+) \quad (3.18)
\]

The (3.18) is expected since the network has been assumed to be nearly reactive and lossless.

Using (3.16) and (3.17) we can conclude that:

\[
\delta_{u,i}(0^+) = -\Delta P_{r,u}(0^+) / \sum_{j=1}^{M} K_{ju}^u \quad (3.19)
\]

\[
\Delta P_{r,i}(0^+) = \left( K_{iu}^u / \sum_{j=1}^{M} K_{ju}^u \right) \Delta P_{r,u}(0^+) \quad (3.20)
\]

As shown in (3.20), the load change (disturbance size) at bus \( u \) (\( \Delta P_{r,u} \)) is distributed between the synchronous generators according to the synchronizing power coefficients between them and the disturbance location. Therefore, the initial active power imbalance distribution between synchronous generators is dependent on the disturbance location.

### 3.3.2 Admittance Matrix of the Reduced System

In order to prepare the system for calculating the reduced admittance matrix and consequently synchronising power coefficients the following steps needs to be taken [20]:

1. All system data need to be converted to a common MVA base
2. The loads have to be converted to their equivalent admittances using load flow results.

   Assuming a certain load bus has voltage of \( \bar{V}_L \), active power of the load is \( P_L \), reactive power \( Q_L \), and current load of \( I_L \) flowing into the load admittance of \( Y_L = G_L + jB_L \). The shunt admittance at the load bus can be calculated as follows [20]:
\[
Y_\ell = P_\ell / V_\ell^2 - j(P_\ell / V_\ell^2)
\] (3.21)

3. The voltages of the internal nodes of the generators \(E_i \angle \delta_{i0}\) can be calculated using load flow results. The angles of the internal voltages can be computed based on pre-transient terminal voltages \(V \angle \alpha\) as follows. Let the terminal voltage be temporary used as a reference. The terminal current is \(I = I_x + jI_q\), using the relation of \(P + jQ = V I^*\), terminal current can written as \((P - jQ)/V\). With these assumptions, voltage of the internal node of the generator \(E \angle \delta'\) can be calculated as follows:

\[
E \angle \delta' = (V + Qx'_d/V) + j(Px'_d/V)
\] (3.22)

Where, \(x'_d\) is the direct axis transient reactance of the generator.

The pre-transient voltage angle of the internal node of the generator (\(\delta_0\)) can be calculated in system reference as follows:

\[
\delta_0 = \delta' + \alpha
\] (3.23)

4. The system reduced admittance matrix can be calculated for all network conditions, i.e. it could be admittance matrix of only internal generator nodes (\(Y\)) or admittance matrix of the system that is reduced to internal generator nodes and load bus u (\(Y^w\)).

5. The procedure for obtaining the system matrix that is reduced to only internal generator nodes (\(Y\)) is as follows:
   a) Adding equivalent admittance of the loads between load buses and the reference node; defining internal generator nodes and adding their \(x'_d\) values between their internal and terminal nodes.
   b) All impedances are converted to admittances
   c) \(Y_{ii}\) is the summation of all the admittances connected to node \(i\), and \(Y_{ij}\) is the negative value of the admittance between node \(i\) and node \(j\).
d) In the last stage, all nodes except the internal generator nodes need to be eliminated. The procedure for elimination is as follows:

\[
\begin{bmatrix}
I_m \\
0
\end{bmatrix} = \begin{bmatrix}
Y_{mm} & Y_{mr} \\
Y_{mr} & Y_{rr}
\end{bmatrix} \begin{bmatrix}
V_m \\
V_r
\end{bmatrix}
\]

(3.24)

Subscript \( r \) represents none internal generator nodes. Subscript \( m \) represents internal generator nodes. By expanding (3.24):

\[
I_m = Y_{mm} V_m + Y_{mr} V_r \\
0 = Y_{mr} V_m + Y_{rr} V_r
\]

(3.25)

From (3.25) \( V_r \) can be eliminated:

\[
I_m = \left(Y_{mm} - Y_{mr} Y_{rr}^{-1} Y_{rm}\right) V_m
\]

(3.26)

The matrix \( \left(Y_{mm} - Y_{mr} Y_{rr}^{-1} Y_{rm}\right) \) is the matrix of the system which is reduced to only generators internal nodes (Y).

Similar to the approach taken for reduced Y matrix, the admittance matrix of the system which is reduced to generator nodes and load bus \( u \) (Y^u) can be calculated. In this case (3.24) can be rewritten as:

\[
\begin{bmatrix}
I_{m+1} \\
0
\end{bmatrix} = \begin{bmatrix}
Y_{m+1m+1} & Y_{m+1r-1} \\
Y_{m+1r-1} & Y_{r+1r+1}
\end{bmatrix} \begin{bmatrix}
V_{m+1} \\
V_{r-1}
\end{bmatrix}
\]

(3.27)

Where \( m+1 \) subscript represents internal generator nodes and load bus \( u \) and \( r-1 \) subscript represents the rest of the nodes. Similar to (3.25) by expanding (3.27):
\[
I_{m+1} = Y_{m+1m+1}V_{m+1} + Y_{m+1r-1}V_{r-1} \\
0 = Y_{r-1m+1}V_{m+1} + Y_{r-1r-1}V_{r-1}
\] (3.28)

From (3.28), \(V_{r-1}\) can be eliminated:

\[
I_{m+1} = \left( Y_{m+1m+1} - Y_{m+1r-1}Y_{r-1r-1}^{-1}Y_{r-1m+1} \right) V_{m+1}
\] (3.29)

The matrix \(\left( Y_{mm} - Y_{mr}Y_{rr}^{-1}Y_{rm} \right)\) is the matrix of the system which is reduced to generators internal nodes and load bus \(u\) (\(Y^u\)).

In order to calculate synchronous power coefficients for different disturbance locations in the system, the procedure for system reduction should be repeated assuming different load buses.

### 3.4 Example of Power Disturbance Distribution

This example, for the IEEE 39 bus system (Figure 3-2), demonstrates the effect of disturbance location on the initial active power change at generators terminals. The disturbance size (\(\Delta P_l\)) is 40 MW for all cases and the disturbance location is varied (Bus 12, Bus 15 and Bus 38).
Figure 3-2: IEEE standard 39 bus system

Figure 3-3 shows the change in the electrical active power output of Generator 1 for different disturbance locations and the same $P_d$. Figure 3-4 shows the change in the electrical active power output of Generator 7 for different disturbance locations and the same $P_d$.

Figure 3-3: Generator 1 power change following 40MW disturbance at varied locations
As seen in Figure 3-3 and Figure 3-4, shortly following the disturbance time ($t_d$), for a fixed $P_d$, the power changes at the generator terminals are dependent on the disturbance location. Therefore, it can be concluded that an $N$ vector ($d_i$) can be defined for each generator ($i$) that describes the power change observed at that generator for a disturbance at each of the $N$ buses, i.e. the product of $P_d$ and $d_{ij}$ will be equal to $\Delta P_{e,i}$ (the active power change observed at the generator) provided that the disturbance occurred at bus $j$ (the value calculated with any other element of $d_i$ will not be equal to $\Delta P_{e,i}$). The methodology proposed in Section 3.5 is based on the inverse of this process, i.e. using the inverse of $d_{ij}$ and measurements of $\Delta P_{e,i}$ to form a hypothesis of what the disturbance size would be if the disturbance had occurred at the $j^{th}$ bus. Then, by repeating this for every generator and for each bus and then comparing the hypotheses from several generators it is possible to identify the most likely disturbance location, as for the true disturbance location (and corresponding element in $d$) the hypothesis formed for each generator should be in close agreement (and close to the disturbance size), while for the non-disturbance buses the hypotheses will not be in agreement.
**3.5 Methodology**

In this section a novel synchronizing power coefficient based method (SPC) for detection and estimation of the disturbance size and location is introduced. The SPC methods inputs are:

**Input 1)** Active power measurements from a limited number of generators \( (P_e(t)) \) – from PMUs

**Input 2)** Impedance matrix \( (Z) \) and load flow results \( (LF) \) for the normal operating condition – from SCADA/EMS.

Input 1 updates at the reporting rate of the PMUs, e.g. every 20 ms. Input 2 updates at the rate of the SCADA/EMS, which is usually on the order of few seconds. The impedance matrix \( (Z) \) is estimated in EMS as an output of topology estimator and voltage and angles of buses \( (LF) \) are estimated as a part of state estimation in EMS. The SPC method updates with each new set of PMU measurements and uses the most recent \( Z \) and \( LF \). The method has two stages:

**Stage 1)** Disturbance detection

**Stage 2)** Disturbance size and location estimation

Sections 3.5.1 to 3.5.6 will define some key terms and elements used in the SPC method.

### 3.5.1 Generator Power Change Vector \( \Delta P_e(t) \)

In this paper, the active power outputs of a limited set of \( R \) generators \( (S) \) are monitored using PMUs. Let us introduce a Generator Power Change vector denoted as \( \Delta P_e(t) \) for which the \( i^{th} \) element describes the difference between the active power measurements recorded at time \( t \) and at \( t-\Delta t \) at the \( i^{th} \) generator terminals, where \( \Delta t \) is the reporting rate of the PMU measurements (assumed to be 20 ms here). So, \( \Delta P_e(t) \) is:

\[
\Delta P_{e_i}(t) = P_{e_i}(t) - P_{e_i}(t-\Delta t) \quad \forall i \in S
\]  

(3.30)
3.5.2 Generator Disturbance Distribution Matrix D

As shown in Section 3.4, each generator has its own unique disturbance distribution pattern, \(d_i\), which is an \(N\) vector for which the \(j^{th}\) element describes the ratio between the power change at the \(i^{th}\) generator terminals and the disturbance size when the disturbance occurs at the \(j^{th}\) bus. A generator with a larger synchronizing power coefficient for the disturbance location will experience a larger initial change in power (3.20). Each of the \(M\) \(d_i\) vectors can be combined as the row of a new matrix, the Generator Distribution Matrix (\(D\)). \(D\) is an \(M\) by \(N\) matrix, where \(M\) is the number of synchronous generators and \(N\) is the number of buses, and its elements, denoted as \(D_{ij}\) can be calculated as follows, using (3.8) and (3.20) and based on Section 3.3:

\[
D_{ij} = K_{ij} \sum_{l=1}^{M} K_{lj}^j
\]  
(3.31)

The superscript \(j\) in \(K_{ij}^j\) and \(K_{lj}^j\) indicates that synchronising power coefficients are calculated for a system which is reduced to the internal generator nodes and disturbance node \(u\) (M+1 port network). The procedure for calculation of the \(Y_u\) and network reduction is discussed in Section 3.3.2. In order to calculate synchronous power coefficients for different disturbance locations in the system, the procedure for system reduction should be repeated assuming all different load buses.

3.5.3 Synchronising Power Coefficient Matrix K

Synchronizing power coefficient matrix (\(K\)) can be defined as an \(M\) by \(N\) matrix with each column representing a bus (disturbance location) and every row a synchronous generator. Where element \(ij\) of \(K\), is equal to \(K_{ij}^l\), meaning it is equal to synchronising power coefficients between bus \(j\) and synchronous generator \(i\) when system is reduced to the internal generator nodes and bus \(j\) (Section 3.3.2).
3.5.4 Disturbance Estimate Matrix $\hat{P}_d(t)$

The Generator Distribution Matrix ($D$) defines how the disturbance is distributed between the generators. Therefore, based on (3.31) and (3.20), it is possible to calculate the hypothetical disturbance size $P_d$ that would be required to cause specific power change at a generator terminal for every possible disturbance location (i.e. every bus). This creates an $N$ element vector of possible disturbance sizes. By repeating this process for every monitored generator, an $R$ by $N$ matrix, defined here as the Disturbance Estimate matrix $\hat{P}_d(t)$ can be obtained.

$$
\hat{P}_{dij}(t) = \Delta P_{ai}(t) / D_{ij} \quad \forall i \in S, \forall j \in (1, N)
$$

where $t$ is the time the set of measurements of active power changes represent and each element of the matrix represents the disturbance size $P_d$ that would be required at the $j^{th}$ bus to cause the power change observed at $i^{th}$ generator. Each row corresponds to a generator and each column to a bus, so each column of the matrix contains the hypothesized disturbance size for the respective bus for each generator.

In an ideal case (e.g. zero noise, perfect network parameters), the disturbance location will be the column/bus for which the elements/hypotheses are identical (when calculated for the disturbance time). As such, in an ideal case the Disturbance Estimate Matrix offers a simple method for finding the disturbance location. However, a more rigorous approach must be developed to overcome the errors and noise that inevitably exist in power system studies.

3.5.5 Mismatch and Normalized Mismatch Vectors $P_{mis}(t), C(t)$

The mismatch and normalized mismatch vectors are defined here to make it possible to identify the most likely disturbance location in the presence of noise and errors, i.e. in the case where no column of the Disturbance Estimate Matrix contains $R$ identical elements. This can be achieved by measuring how similar the elements of each column are (i.e. how similar the hypothesized disturbance sizes are for each bus) and remembering that the column with the elements that are most similar will likely be the disturbance location.
To this end, the Mismatch vector $\mathbf{P}_{\text{mis}}(t)$ is introduced where its elements ($P_{\text{mis},n}(t)$) are defined as the maximum difference between the disturbance size estimates for the $n^{th}$ bus (i.e. the elements of the $n^{th}$ column of $\hat{\mathbf{d}}(t)$):

$$P_{\text{mis},n}(t) = \max_{i,j \in S} | \hat{P}_{d,in} - \hat{P}_{d,jn} |$$  \hspace{1cm} (3.33)

Where $S$ is the set of monitored generators. Furthermore, in order to make $\mathbf{P}_{\text{mis}}(t)$ independent of the disturbance size a Normalized Mismatch vector $\mathbf{C}(t)$ is defined, the elements of which are calculated as:

$$C_n(t) = P_{\text{mis},n}(t) \left/ \frac{1}{R} \sum_{\mathbf{R} \in S} \hat{P}_{d,in}(t) \right.$$  \hspace{1cm} (3.34)

These vectors have been defined as they exhibit behavior that can be used to determine the disturbance location, even in the presence of noise, as described in the following section. These two vectors are utilized both together for disturbance detection, as the detection using only one of them would result in a less accurate performance as is shown in section 3.5.6 (Table 3-1).

### 3.5.6 Mismatch Indices $\alpha(t)$ and $\beta(t)$

Identifying the disturbance location using fixed threshold on the minimum value of $\mathbf{P}_{\text{mis}}(t)$ or $\mathbf{C}(t)$ are not ideal solutions, due to the effect of noise and errors on the range of elements in $\mathbf{P}_{\text{mis}}(t)$ and $\mathbf{C}(t)$ limiting the robustness of these approaches. In order to solve this issue, two relative mismatch indices $\alpha(t)$ (based on $\mathbf{C}(t)$) and $\beta(t)$ (based on $\mathbf{P}_{\text{mis}}(t)$) are defined:

$$\alpha(t) = \min_{n \in \{1, \ldots, N\}} (C_n(t)) \big/ \max_{n \in \{1, \ldots, N\}} (C_n(t))$$  \hspace{1cm} (3.35)

$$\beta(t) = \min_{n \in \{1, \ldots, N\}} (P_{\text{mis},n}(t)) \big/ \max_{n \in \{1, \ldots, N\}} (P_{\text{mis},n}(t))$$  \hspace{1cm} (3.36)

Dividing the minimum by maximum for the mismatch vectors ($\mathbf{P}_{\text{mis}}(t)$, $\mathbf{C}(t)$), helps alleviating the effect of noise and error when it comes to setting a threshold for disturbance detection.
This is done to turn the absolute minimum to a minimum which is relative to the maximum value. As an example, in case of larger disturbances and higher noise in active power measurements or error in Z matrix, the overall values in mismatch vectors would be larger. Therefore, having a minimum that is relative to the maximum value would help generalising the detection stage.

These two mismatch indices ($\alpha(t)$ and $\beta(t)$), serve as the basis for the detection, localization and sizing of the disturbance, which is described in Section 3.5.7 and 3.5.8.

### 3.5.7 Stage 1- Disturbance Detection

The disturbance detection stage of the SPC method is based on detecting a relatively low level of mismatch between the disturbance size estimates for each of the monitored generators for a specific location $n$ at time $t$ by using the Mismatch indices $\alpha(t)$ (3.35) and $\beta(t)$ (3.36). A decision tree (DT) [21] can be trained offline to establish the mapping relationship between the mismatch indices (as the two predictors) and the status of the system, i.e. if a disturbance has occurred, which is the output of the DT.

**Decision Tree Based Disturbance Detection:**

When designing the DT for disturbance detection three requirements must be satisfied:

1. Detecting when a disturbance occurs
2. Avoiding repeated detection of the disturbance following the initial detection
3. Avoiding false detections under normal conditions.

Therefore, in this paper, the training of the DT is based on a database built offline for a range of disturbance sizes and locations using data from three different time periods ($t < t_d$, $t = t_d$, $t > t_d$). The algorithm for DT training was based on CART algorithm [22], and the matlab function used to apply it was “fitctree”. Each case in the database contains the predictor values ($\alpha(t)$ and $\beta(t)$) and the case output in terms of disturbance occurrence. The training set conditions are:

- $P_d$: varied between 40 to 80 MW with 10 MW steps
- Monitored generators: G1, G7, G8
- Disturbance location: all buses for each disturbance size
- The active power measurements had 1% white noise

The resulting DT is shown in Figure 3-5, where $\alpha_{\text{cut}}$ and $\beta_{\text{cut}}$ are shown in and are 0.0132 and 0.002 respectively. As the disturbance location and size are varied for training the DT, it’s expected to have a relatively robust disturbance detection performance. When it comes to training a DT, apart from aiming to minimize the misclassification it’s also important to avoid overfitting meaning striving to generalize the DT by training it using a wide range of conditions. In order to have a better and even more generalized application of this disturbance detection DT, other conditions such as loading and topology can also be varied and added to the training data points. This is a matter of further improvement on the method. Nevertheless, the trained DT proved to be accurate when examined against error in Z (Case study 4) and topology (Case study 6).

Figure 3-6 shows the $\alpha(t)$ and $\beta(t)$ as the x and y axis and each point corresponds to one simulation case for the IEEE 39 bus system. As it can be seen in Figure 3-6, the time of disturbance ($t=t_d$) can be effectively classified by passing $\alpha(t)$ and $\beta(t)$ to the simple decision tree shown in Figure 3-5. Based on Figure 3-5 and Table 3-1, using only one of the Mismatch indices ($\alpha(t)$ and $\beta(t)$), and putting the threshold on that index would lead to higher misclassification compared to the case of using both of them.

![Decision Tree used for disturbance detection](image-url)
Table 3-1: Disturbance Detection Results Using Different Inputs

<table>
<thead>
<tr>
<th>Algorithm Input</th>
<th>False Detection (t&lt;td)</th>
<th>Missed Detection (t=td)</th>
<th>False Detection (t&gt;td)</th>
</tr>
</thead>
<tbody>
<tr>
<td>α(t)</td>
<td>0.31%</td>
<td>0%</td>
<td>15.43%</td>
</tr>
<tr>
<td>β(t)</td>
<td>0.62%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>α(t) and β(t) (DT)</td>
<td>0.10%</td>
<td>0.05%</td>
<td>0%</td>
</tr>
</tbody>
</table>

3.5.8 Stage 2-Disturbance Size and Location Estimation

Disturbance Location Estimation:

Following the detection of a disturbance in Stage 1, the most likely candidate for the location of the disturbance would be the bus with the lowest mismatch between the monitored generators at td. This can be formulated as follows:

\[
\text{Disturbance Location} = \arg \min_{n \in \{1,..,N\}} (C_n(t_d))
\] (3.37)

However, measurement noise or errors in the Z matrix might cause this simple approach to give a neighboring bus as the disturbance location, particularly when the buses are electrically close. Therefore, in order to ensure that the method chooses a small set of possible locations that will most likely contain the disturbance location, a multiplier (1<λ) is introduced which allows the method to identify a set of buses as Disturbance Location Candidates (DLC) as follows:
Increasing $\lambda$ in (3.38) would increase the likelihood that the true disturbance location is within the set of DLC; however, increasing $\lambda$ will also increase the number of DLCs. Therefore, when choosing $\lambda$, there exists a compromise between ensuring the true disturbance location is within the set of DLC and limiting the number of DLC members. The sensitivity of the method to $\lambda$ values is studied in Section 3.6.9.

**Disturbance Size Estimation:**

Having determined the disturbance time and location the final aspect of the method is to estimate the disturbance size ($P_{est}$). This is estimated as the average of the disturbance estimates from each of the monitored generators for the location that has the smallest element in $C(t)$ at the disturbance time ($t_d$):

$$q = \arg \min_{n \in \{1, \ldots, N\}} n \in \{1, \ldots, N\} \min(C_n(t_d))$$  \hspace{1cm} (3.39)

$$P_{est} = \frac{1}{R} \left( \sum_{g=1}^{G} \hat{P}_{d,gq}(t) \right)$$  \hspace{1cm} (3.40)

A flowchart of the SPC method is given in Figure 3-7. The dashed arrows in the flowchart represent the slower rate of data update (equal to SCADA rate of update) and the solid arrows the faster rate of data update (equal to PMU rate of report).
3.6 Results

3.6.1 Test System and Performance Metrics

Simulations of the IEEE 39-bus test system, as shown in Figure 3-2, are used here to verify the performance of the proposed method (SPC). The following metrics are used for evaluating the performance of the SPC method. In the following simulations, active power disturbance has been modeled as a sudden load increase (Except for Caser study 8). In real
networks, sudden loss of embedded generation can be seen as load increase type of
disturbance. In DigSILENT model of the IEEE 39 bus system, -1MW and +1MW loads were
added at all buses. This way the original system load flow result remained intact. For each
disturbance, depending on the size and location of the disturbance a specific load event has
been defined on 1MW load connected to a particular bus. This approach has been chosen to
enable testing the average performance of the method throughout the system for a constant
disturbance size.

The IEEE 39-bus system model has the following parameters:

Table 3-2: Generators parameter for IEEE 39-bus system

<table>
<thead>
<tr>
<th>Name</th>
<th>Busbar</th>
<th>Rated Voltage (kV)</th>
<th>Apparent Power (MVA)</th>
<th>H (s)</th>
<th>X_d' (p.u.)</th>
<th>Active Power (MW)</th>
<th>Reactive Power (Mvar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gen1</td>
<td>3.00</td>
<td>230.00</td>
<td>1400.00</td>
<td>50.00</td>
<td>0.06</td>
<td>1000.00</td>
<td>74.14</td>
</tr>
<tr>
<td>Gen2</td>
<td>4.00</td>
<td>20.00</td>
<td>1000.00</td>
<td>3.03</td>
<td>0.70</td>
<td>520.83</td>
<td>198.10</td>
</tr>
<tr>
<td>Gen3</td>
<td>34.00</td>
<td>20.00</td>
<td>1000.00</td>
<td>3.58</td>
<td>0.53</td>
<td>650.00</td>
<td>204.97</td>
</tr>
<tr>
<td>Gen4</td>
<td>27.00</td>
<td>20.00</td>
<td>1000.00</td>
<td>2.86</td>
<td>0.44</td>
<td>632.00</td>
<td>109.77</td>
</tr>
<tr>
<td>Gen5</td>
<td>26.00</td>
<td>20.00</td>
<td>1000.00</td>
<td>2.60</td>
<td>1.32</td>
<td>508.00</td>
<td>165.70</td>
</tr>
<tr>
<td>Gen6</td>
<td>30.00</td>
<td>20.00</td>
<td>1000.00</td>
<td>3.48</td>
<td>0.50</td>
<td>650.00</td>
<td>212.26</td>
</tr>
<tr>
<td>Gen7</td>
<td>29.00</td>
<td>20.00</td>
<td>1000.00</td>
<td>2.64</td>
<td>0.49</td>
<td>560.00</td>
<td>101.09</td>
</tr>
<tr>
<td>Gen8</td>
<td>32.00</td>
<td>20.00</td>
<td>1000.00</td>
<td>2.43</td>
<td>0.57</td>
<td>540.00</td>
<td>-0.28</td>
</tr>
<tr>
<td>Gen9</td>
<td>31.00</td>
<td>20.00</td>
<td>1200.00</td>
<td>3.45</td>
<td>0.31</td>
<td>830.00</td>
<td>22.62</td>
</tr>
<tr>
<td>Gen10</td>
<td>33.00</td>
<td>20.00</td>
<td>1000.00</td>
<td>4.20</td>
<td>0.31</td>
<td>250.00</td>
<td>144.77</td>
</tr>
</tbody>
</table>

Table 3-3: Loads parameter for IEEE 39-bus system

<table>
<thead>
<tr>
<th>Name</th>
<th>Busbar</th>
<th>Rated Voltage (kV)</th>
<th>Active Power (MW)</th>
<th>Reactive Power (Mvar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load1</td>
<td>13.00</td>
<td>230.00</td>
<td>322.00</td>
<td>2.40</td>
</tr>
<tr>
<td>Load2</td>
<td>24.00</td>
<td>230.00</td>
<td>500.00</td>
<td>184.00</td>
</tr>
<tr>
<td>Load3</td>
<td>15.00</td>
<td>230.00</td>
<td>224.00</td>
<td>47.20</td>
</tr>
<tr>
<td>Load4</td>
<td>17.00</td>
<td>230.00</td>
<td>281.00</td>
<td>75.50</td>
</tr>
<tr>
<td>Load5</td>
<td>25.00</td>
<td>100.00</td>
<td>628.00</td>
<td>103.00</td>
</tr>
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<td>230.00</td>
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<tr>
<td>Load8</td>
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</tr>
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<td>Name</td>
<td>Busbar i</td>
<td>Busbar j</td>
<td>Rated Line-Line Voltage, (kV)</td>
<td>R1 (Ohm)</td>
</tr>
<tr>
<td>------</td>
<td>----------</td>
<td>----------</td>
<td>------------------------------</td>
<td>----------</td>
</tr>
<tr>
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<td>2</td>
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<td>230.00</td>
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</tr>
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<td>0.69</td>
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<td>230.00</td>
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</tr>
<tr>
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<td>35</td>
<td>36</td>
<td>230.00</td>
<td>0.11</td>
</tr>
<tr>
<td>Line7</td>
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<td>37</td>
<td>230.00</td>
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<td>38</td>
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<td>7</td>
<td>230.00</td>
<td>0.21</td>
</tr>
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<td>230.00</td>
<td>0.42</td>
</tr>
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<td>Line16</td>
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<td>0.37</td>
</tr>
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<td>0.37</td>
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<td>16</td>
<td>18</td>
<td>230.00</td>
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<td>17</td>
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<td>22</td>
<td>230.00</td>
<td>0.42</td>
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<td>28</td>
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<td>1.16</td>
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</table>
Table 3-5: Transformers parameter for IEEE 39-bus system

<table>
<thead>
<tr>
<th>Name</th>
<th>HV-Side Busbar</th>
<th>LV-Side Busbar</th>
<th>HV-Rated Voltage (kV)</th>
<th>LV-Rated Voltage (kV)</th>
<th>Rated Power (MVA)</th>
<th>X1 (p.u.)</th>
<th>R1 (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tx1</td>
<td>2</td>
<td>33</td>
<td>230</td>
<td>20</td>
<td>100</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx2</td>
<td>8</td>
<td>34</td>
<td>230</td>
<td>20</td>
<td>100</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx3</td>
<td>7</td>
<td>6</td>
<td>230</td>
<td>100</td>
<td>100</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx4</td>
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<td>6</td>
<td>230</td>
<td>100</td>
<td>100</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx5</td>
<td>36</td>
<td>4</td>
<td>230</td>
<td>20</td>
<td>100</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx6</td>
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<td>230</td>
<td>100</td>
<td>100</td>
<td>0.01</td>
<td>0.00</td>
</tr>
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<td>100</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx8</td>
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<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx9</td>
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<td>29</td>
<td>230</td>
<td>20</td>
<td>100</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx10</td>
<td>22</td>
<td>30</td>
<td>230</td>
<td>20</td>
<td>100</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx11</td>
<td>19</td>
<td>31</td>
<td>230</td>
<td>20</td>
<td>100</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Tx12</td>
<td>15</td>
<td>32</td>
<td>230</td>
<td>20</td>
<td>100</td>
<td>0.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Stage 1 performance metrics:

In order to enumerate the likelihood of missing a disturbance (i.e. not detecting it) the Missed Detection metric (MD) is defined as the percentage of cases where the method misses the detection of the disturbance. The False Detection metric (FD) is defined as the percentage of cases where the method falsely detects a disturbance. **Stage 2 performance metrics:**

The performance metric used for the disturbance size is $E_{95\%}$ which is equal to 95th percentile of the errors in disturbance size estimation, i.e. $E_{95\%}$ is the value that 95% of all of the reported errors are less than for a given case. Regarding disturbance location estimation two points are important, firstly the accuracy of the localization, i.e. if the true disturbance location is one of the Disturbance Location Candidates (DLC), secondly the precision of the disturbance localization, i.e. how many DLCs are reported by the method. The first point is analyzed using the Right Location metric (RL), which is the percentage of cases where the true location is a DLC; and the second point is studied by three metrics ($L_{W1}$, $L_{W2}$, $L_{W3}$) that measure the percentage of cases where there are 1, 2 and 3 disturbance location candidates (DLCs):

$$L_{W1} = 100 \left( \frac{\text{Number of times where: } |\text{DLC}|=i}{\text{Number of times where: Disturbance Location } \in \text{ DLC}} \right)$$

(3.41)
A worked example is now presented to illustrate the methods execution.

### 3.6.2 Worked Example

In this example the disturbance is a 40 MW load increase at bus 12. PMUs have been placed at generators 1, 7 and 8 (\( S=\{G1,G7,G8\} \)). The time interval for the measurements is 0.02 s (\( \Delta t \)). The \( P_{\text{mis}}(t) \) and \( C(t) \) vectors have been calculated (Figure 3-8 and Figure 3-9) using (3.33) and (3.34) for three times: \( t_d-\Delta t \), \( t_d \) and \( t_d+\Delta t \). The \( \lambda \) in (3.38) is set to 1.8.

**Stage 1**: \( \alpha(t) \) and \( \beta(t) \) are calculated using (3.35) and (3.36) at each time instance (Table 3-6). From Table 3-6 it can be seen that the disturbance is correctly detected at \( t=t_d \) using DT.

<table>
<thead>
<tr>
<th>( \alpha(t_d-\Delta t) )</th>
<th>( \beta(t_d-\Delta t) )</th>
<th>( \alpha(t_d) )</th>
<th>( \beta(t_d) )</th>
<th>( \alpha(t_d+\Delta t) )</th>
<th>( \beta(t_d+\Delta t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>0.032</td>
<td>0.0057</td>
<td>0.0082</td>
<td>0.011</td>
<td>0.021</td>
</tr>
</tbody>
</table>

**Figure 3-8**: \( P_{\text{mis},n}(t) \) values for different time instances and bus numbers

**Figure 3-9**: \( C_n(t) \) values for different time instances and bus number
Stage 2: Figure 3-10 shows $C_n(t)$ values for different times in a two dimensional plot. DLC are chosen by applying (3.38) to $C_n(t_d)$ (values shown as stars in Figure 3-10). In other words, in Figure 3-10 the locations with $C_n(t_d)$ values smaller than the blue line are chosen as DLC members. In this case only bus 12 (the actual disturbance location) is selected. Figure 3-11, shows the $\hat{P}_{dn}(t_d)$ where $i \in S$ and $n \in {1,39}$. As it can be seen in Figure 3-11, the location with the highest level of agreement between the disturbance size estimates for all three generators is bus 12, where all three points almost coincide. Therefore, in (3.39) $q=12$. The disturbance size is then estimated to be 40.69 MW, using (3.40). This is only a 1.72% error in disturbance size estimation.

Figure 3-10: $C_n(t)$ values for different time instances and location
Figure 3-11: Disturbance Estimate at disturbance time \( t_d \) based on different monitored generators (G1, G7, G8) for varied disturbance locations \( \hat{P}_{d,in} (t_d) \)

In the following sections (3.6.3 to 3.6.9) SPC method performance is analysed and compared against variation in different parameters (Case study 1 to 7).

In these case studies, whenever noise is added to the active power measurements of PMUs or error is added to the reactance \( X \) of the lines, the following procedure is carried out:

\[
Y' = Y_{\text{Real}} (1 + \sigma \, \text{Pr}_u)
\]  

(3.42)

Where:

- \( Y_{\text{Real}} \) is the actual value of the parameter before the addition of noise or error
- \( Y' \) is the value of the parameter after the addition of noise or error
- \( \sigma \) is the level of noise or error
- \( \text{Pr}_u \) is a function which extracts a random value from a uniform probability distribution with 0 mean and range from -1 to 1.

For addition of noise in PMU RoCoF measurements, \( Y_{\text{Real}} \) is the RoCoF of each generator at each time instance and \( \sigma \) is the noise level. For addition of error in \( Z \) matrix, \( Y_{\text{Real}} \) is \( X \) of the lines and \( \sigma \) is the error level.
3.6.3 Case Study 1 (Comparison of SPC Method with traditional RoCoF based method in disturbance size estimation)

This section compares the disturbance size estimation error when using a traditional RoCoF based method [3] and for the proposed SPC method when the number of PMU measurements is varied. The conditions are as follows:

- $P_d$: varied between 40 to 80 MW with 10 MW steps.
- The number of monitored generators is varied from 2 to 10 considering all different combinations of monitored generators for each $P_d$ value.
- Disturbance location is set at all different buses for each combination of monitored generators and $P_d$ value.
- Two different levels of white noise (1% and 5%) are added to the active power measurements for the SPC method (using (3.42)).
- For each noise level and each specific disturbance size, disturbance location and set of monitored generators, analyses have been repeated 10 times with different values of noise extracted from uniform distribution (white noise).

The results of this test are presented in Figure 3-12 with a log scale on the y-axis, where each middle horizontal line on the bars represents the median error. The SPC method only shows a significant change in accuracy when two PMUs are used for 5% noise. In contrast, the RoCoF based method shows a significant loss of accuracy when the number of PMUs is reduced from ten. Furthermore, the error in $P_d$ estimation for the SPC method does not rise significantly with increased noise in the measurements (apart from the two PMU case). The median for the traditional method is 10% when eight PMUs are used (two out of ten generators not monitored). In contrast, the SPC method error, even with 5% noise in the measurements, has a smaller median error (3%) when only three PMUs are used (seven out of ten generators are not monitored). However, only when all generators are monitored (10 out of 10), the median error using traditional method gets to 0.5% which is slightly more accurate than SPC method, which has median error of 1.26% and 1% for the 5% and 1% noise respectively.

Therefore, the SPC provides a more reliable and accurate disturbance size estimation when measurements from only a limited number of generators are available, which is likely to be the case in practice.
Figure 3-12: Disturbance size estimate error for varied number of PMU measurements, comparison between traditional and SPC method

3.6.4 Case Study 2 (Impact of number of monitored generators and noise level)

Here, the performance of the SPC method for both stages is analyzed. The conditions are the same as in Case Study 1. Table 3-7 summarizes the results for the two noise levels (1% and 5%) for different numbers of PMU measurements (3 to 5) and all possible combinations of those PMUs.

Table 3-7: Results for Case Study 2, varied number of PMUs and noise in measurements

<table>
<thead>
<tr>
<th>nl</th>
<th>R*</th>
<th>MD</th>
<th>FD</th>
<th>E_{rms}</th>
<th>RL</th>
<th>L_{w1}</th>
<th>L_{w2}</th>
<th>L_{w3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>3</td>
<td>0.34</td>
<td>0.22</td>
<td>2.21</td>
<td>97.62</td>
<td>93.31</td>
<td>4.75</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.20</td>
<td>0.06</td>
<td>2.12</td>
<td>99.63</td>
<td>97.28</td>
<td>2.42</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.15</td>
<td>0.01</td>
<td>2.10</td>
<td>99.97</td>
<td>98.82</td>
<td>1.18</td>
<td>0</td>
</tr>
<tr>
<td>5%</td>
<td>3</td>
<td>4.94</td>
<td>0.93</td>
<td>41.67</td>
<td>92.54</td>
<td>73.38</td>
<td>18.24</td>
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<tr>
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<td>4</td>
<td>1.25</td>
<td>0.24</td>
<td>12.06</td>
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<td>79.43</td>
<td>16.23</td>
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<tr>
<td></td>
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<td>0.76</td>
<td>0.12</td>
<td>3.25</td>
<td>99.86</td>
<td>83.88</td>
<td>13.20</td>
<td>2.91</td>
</tr>
</tbody>
</table>

* nl denotes noise level in generators active power readings of PMUs and R is the number of monitored generators
Based on Table 3-7, the disturbance detection improves (MD and FD both become smaller) with an increased number of PMU measurements. At the same time, the increased noise in the measurements reduces the performance of both stages of the SPC method. However, the effect on RL is not as pronounced for different noise levels as it is for the effect on \( L_{W1} \) values, i.e. the impact on localization accuracy is less than the impact on precision. This means that the SPC method identifies more buses as the potential disturbance location as the noise level is increased (lower \( L_{W1} \) and higher \( L_{W2} \) and \( L_{W3} \)). With increased noise in the PMU measurements, more generators must be monitored to achieve the same level of accuracy in \( P_d \) estimation (E95%). The results shown in Table 3-7 represent all possible combinations of monitored generators (i.e. PMU placements) for each number of PMUs. Therefore, Table 3-7 reflects a conservative estimate of the methods practical performance. This is because it includes results for cases where the generators monitored are close together and the method will perform better when the monitored generators are evenly distributed across the system, e.g. G1, G7 and G8.

In conclusion, despite the reduced accuracy of SPC for the higher noise level, it still yields acceptable results for both stages when only half of the generators are monitored. This characteristic becomes even more valuable when compared with the highly sensitive and vulnerable performance of the traditional method to the number of measurements. The traditional method requires every generator to be monitored to achieve \( E_{95\%} <5\% \) (Case Study 1, Figure 3-12). Furthermore, the sole output of the traditional method is \( P_d \), whereas the SPC method also provides disturbance detection and localization.

### 3.6.5 Case Study 3 (Impact of disturbance size)

Here, two disturbance sizes (40 MW and 80 MW) are analyzed with 1 % white noise in the PMU measurements (noise added using \((3.42)\)). The monitored generators are selected from G1, G7, G8 and G9 and the results are summarized in Table 3-8. Disturbance detection becomes more accurate as the disturbance size is increased (lower MD and FD). Furthermore, the accuracy and precision of the localization also improves with the increased disturbance size (as RL and \( L_{W1} \) increase and \( L_{W2} \) and \( L_{W3} \) decrease). Therefore, the larger and more risky active power disturbances are more accurately
analyzed using the SPC method. As mentioned in Chapter 1, term analysed regarding disturbance refers to detection and estimation of disturbance size and location.

### Table 3-8: Results for Case Study 3, varied disturbance size

<table>
<thead>
<tr>
<th>S</th>
<th>MD</th>
<th>FD</th>
<th>$E_{95%}$</th>
<th>RL</th>
<th>$L_{w1}$</th>
<th>$L_{w2}$</th>
<th>$L_{w3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>5.84</td>
<td>8.15</td>
<td>2.10</td>
<td>96.66</td>
<td>92.92</td>
<td>6.77</td>
<td>0.20</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0.20</td>
<td>0.07</td>
<td>2.08</td>
<td>100</td>
<td>99.79</td>
<td>0.21</td>
<td>0</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>0</td>
<td>2.07</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S</th>
<th>MD</th>
<th>FD</th>
<th>$E_{95%}$</th>
<th>RL</th>
<th>$L_{w1}$</th>
<th>$L_{w2}$</th>
<th>$L_{w3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>3.20</td>
<td>4.745</td>
<td>2.08</td>
<td>98.59</td>
<td>98.07</td>
<td>1.92</td>
<td>0</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>0.06</td>
<td>2.07</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>0</td>
<td>2.07</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

#### 3.6.6 Case Study 4 (Impact of error in Z matrix)

In this case study X of the lines have been subjected to errors with uniform probability distribution (error added using (3.42)). For each error level and each specific disturbance size, disturbance location and set of monitored generators, analyses have been repeated 10 times with different values of error extracted from the uniform distribution.

Based on Table 3-9, increased error in line parameters (from 1% to 5%) worsens the disturbance detection stage of the method. However, even in the presence of errors in the Z matrix, the method’s disturbance detection stage still performs well with three monitored generators (MD and FD <3 %). According to Table 3-5, with fixed monitored generators, both $P_d$ estimation ($E_{95\%}$) and disturbance location estimation (RL) become less accurate with increased error in the Z matrix. With 5 % error in the Z matrix, five generators need to be monitored to achieve $E_{95\%}$ less than 5%, whereas for 1% error in the Z matrix, three generators would suffice. Based on Table 3-9, the increased error in the Z matrix reduces the precision of the disturbance location estimation. This is shown by the reduced percentage of $L_{w1}$ and increased percentage of $L_{w2}$ and $L_{w3}$ for the same set of PMU measurements.

Despite the deteriorated performance of the SPC method when having error in Z matrix, the SPC method achieves acceptable performance for all disturbance analysis stages (detection, localization and sizing) with 3 and 5 generators monitored for Z matrix error of 1% and 5% respectively.
### Table 3-9: Results for Case Study 4, error in Z matrix

<table>
<thead>
<tr>
<th>$S$</th>
<th>MD</th>
<th>FD</th>
<th>$E_{95%}$</th>
<th>RL</th>
<th>$L_{w_1}$</th>
<th>$L_{w_2}$</th>
<th>$L_{w_3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>15.38</td>
<td>9.23</td>
<td>42.39</td>
<td>83.07</td>
<td>80.51</td>
<td>17.43</td>
<td>1.53</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>1.28</td>
<td>0.13</td>
<td>2.51</td>
<td>96.92</td>
<td>83.07</td>
<td>16.92</td>
<td>0</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>0</td>
<td>2.34</td>
<td>98.97</td>
<td>83.59</td>
<td>16.41</td>
<td>0</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>0</td>
<td>0</td>
<td>2.26</td>
<td>99.48</td>
<td>83.84</td>
<td>16.16</td>
<td>0</td>
</tr>
</tbody>
</table>

| $P_d=40\text{MW}$, Noise Level= 5%, Error Level=1% |
|-------------|----|----|------------|----|-----------|-----------|-----------|
| G1,G7       | 31.79 | 9.51  | 64.12      | 66.28 | 65.83    | 24.36     | 6.73     |
| G1,G7,G8    | 2.05  | 0.64  | 48.30      | 88.40 | 64.35    | 20.13     | 9.67     |
| G1,G7,G8,G9 | 0    | 0     | 46.60      | 92.56 | 66.92    | 19.55     | 9.68     |
| G1,G7,G8,G9,G5 | 0   | 0   | 4.53       | 97.50 | 65.83    | 24.36     | 6.73     |

### 3.6.7 Case Study 5 (Impact of monitored set of generators)

The purpose of this case study is to analyze the effect of PMU placement (monitored set of generators) on the method performance. The conditions for the case study are as follows:

- $P_d=40\text{MW}$
- Two sets of monitored generators (G1,G7,G8) and (G4,G5,G3)
- The method is tested for disturbances at every bus
- The active power measurements have 1% white noise (3.42)
- For each disturbance location and set of monitored generators, analyses have been repeated 10 times with different values of noise extracted from uniform distribution (white noise).

Based on Table 3-10, when the monitored generators are close together (i.e. G4,G5,G3) MD, FD, $E_{95\%}$, $L_{W_2}$ and $L_{W_3}$ values all increase while $RL$ and $L_{W_1}$ values become smaller compared to the cases with evenly spread generators (i.e. G1,G7,G8). However, even with the poorly placed PMUs (G4, G5, G3); the SPC method yields an acceptable performance.

### Table 3-10: Results for Case Study 5, varied set of monitored generators

<table>
<thead>
<tr>
<th>$S$</th>
<th>MD</th>
<th>FD</th>
<th>$E_{95%}$</th>
<th>RL</th>
<th>$L_{W_1}$</th>
<th>$L_{W_2}$</th>
<th>$L_{W_3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7,G8</td>
<td>0.20</td>
<td>0.07</td>
<td>2.08</td>
<td>100</td>
<td>99.79</td>
<td>0.21</td>
<td>0</td>
</tr>
<tr>
<td>G4,G5,G3</td>
<td>4.48</td>
<td>0.43</td>
<td>2.31</td>
<td>97.40</td>
<td>86.40</td>
<td>11.9</td>
<td>1.6</td>
</tr>
</tbody>
</table>
3.6.8 Case Study 6 (Impact of topology error)

This case study is carried out to investigate the effect of topology estimation errors on SPC method performance. In this case, the line between bus 9 and bus 24 is disconnected in the model; however, the Z and K matrices are calculated assuming that line 9-24 is connected. The other conditions are as follows:

- \( P_d = 40 \text{ MW} \)
- Disturbance location set at all different buses
- The active power measurements have 1\% white noise ((3.42))
- For each disturbance location analyses have been repeated 10 times with different values of noise extracted from uniform distribution (white noise).

Table 3-11: Results for Case Study 6, error in topology

<table>
<thead>
<tr>
<th>S</th>
<th>MD</th>
<th>FD</th>
<th>( E_{\text{res}} )</th>
<th>RL</th>
<th>( L_{w1} )</th>
<th>( L_{w2} )</th>
<th>( L_{w3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>0</td>
<td>53.97</td>
<td>78.49</td>
<td>52.99</td>
<td>47.43</td>
<td>34.61</td>
<td>6.41</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>3.21</td>
<td>5.07</td>
<td>91.02</td>
<td>50</td>
<td>23.50</td>
<td>14.53</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>0</td>
<td>4.95</td>
<td>92.73</td>
<td>51.71</td>
<td>18.80</td>
<td>14.95</td>
</tr>
</tbody>
</table>

Based on Table 3-11, only monitoring two generators (G1, G7) resulted in unacceptable performance for both stages of the SPC method. According to Table 3-11, when an error exists in the topology estimation, four generators are needed to achieve an acceptable error in disturbance size estimation \( E_{95\%} < 5 \% \). This can be compared to the case with an accurate topology (Table 3-8) for which two monitored generators were sufficient. Nevertheless, even in the presence of a topology error, the SPC method still provides a far better performance than the traditional method with four monitored generators (Figure 3-12).

3.6.9 Case Study 7 (Location detection sensitivity to \( \lambda \))

Setting \( \lambda \) in (3.38) is a compromise between two aspects; firstly the accuracy of the localization (measured with RL), and secondly the precision of the disturbance localization, i.e. how many DLCs are reported by the method (reflected in the \( L_{Wi} \) values). In this case study the effect of varied \( \lambda \) on disturbance localization is analyzed. The conditions are:

- \( P_d \) varied between 40 to 80 MW with 10 MW steps.
- Monitored generators are G1, G7, and G8.
- Disturbance location is set at all different buses.
- Error level in line parameters equals 1% (uniform probability)\((3.42)\).
- PMU active power measurements have 1% white noise \((3.42)\).
- \(\lambda\): varied between 1 to 2.6 with 0.2 steps.

**Figure 3-13**: Location detection results for Case Study 7 \((RL, L_{W1}, L_{W2}, L_{W3})\) for varied \(\lambda\) values (the total height of each column is equal to \(RL\) for that \(\lambda\)).

Figure 3-13 shows the disturbance location estimation results for varied \(\lambda\) values. The total height of each column is equal to \(RL\) (the percentage of accurate detection) and the shading denotes if the correct location was the only location identified \((L_{W1})\) or if two or three locations were identified \((L_{W2} and L_{W3})\). Based on Figure 3-13, by increasing \(\lambda\) from 1, RL and L_{W2} and L_{W3} increase while L_{W1} decreases. The rate of increase in RL value drops as \(\lambda\) increases. The \(\lambda\) value does not significantly change the performance of the disturbance localization stage of the SPC method. Therefore, the method proves to be robust against variation in \(\lambda\) value. As such, the setting of \(\lambda\) can be relatively arbitrary, with the range of 1 to 2.6 as a guideline; with the exact value chosen based on the preference of the user between accuracy and precision of disturbance localization.

### 3.6.10 Case Study 8 (Generator Outage Type of Disturbance)

Previously each disturbance was modeled as load increase in each bus. This was done by adding positive and negative loads with the same size of positive and negative 1MW to keep the power flows intact for pre disturbance conditions. This approach was taken in order to facilitate application of the same disturbance size for each disturbance location. Disturbance
size was then applied by assigning a load increase event to the positive 1MW load in each bus. This type of load increase events can be attributed to fast cloud transients for large PV farms or sudden disconnection of large wind farms, which can be interpreted as sudden load increase rather than generation loss as system topology and inertia would stay unchanged.

A new set of studies are carried out to investigate the performance of the SPC method for generation outages rather than load increase events. In order for the SPC method to detect generation loss outages, the methodology needs to be slightly altered. When experiencing a generation loss, the active power deficit equal to the size of the disconnected generator, is distributed among the other generators. However, in this scenario the system topology and the synchronizing power coefficients are different before and after the disturbance. Nevertheless, the initial simulations showed that using the before disturbance synchronizing power coefficients results in estimated disturbance size which is almost equal to double the size of the active power output of the lost generator. This observation led to the alteration in SPC methodology to accommodate for generator loss events. The new slightly changed version of the SPC methodology is as follows:

Stage 1:

- Disturbance detection is the same as before (section 3.5.7).

Stage 2:

- Disturbance location estimation is the same as before (section 3.5.8).
- Disturbance size estimation would have the following changes:

In case bus \( q \) in (3.39) is a bus which has no synchronous generator connected to it, then the disturbance size estimation formula would be the same as before (section 3.5.8).

In case bus \( q \) in (3.39) is a bus with a synchronous generator connected to it, then the following formula applies:

\[
P_{est}^* = \frac{1}{2R} \left( \sum_{g=1}^{g} \hat{P}_{d,gq}(t) \right)
\]

(3.43)

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Meaning the estimated disturbance size using the previous version of the SPC method ($P_{est}$) has to be divided by two to get to the new estimated disturbance size ($P_{est}^*$). The case study conditions are:

- Monitored generators are varied from 2 to 5.
- Disconnection of generators G2, G3, G4, G6, G10
  RoCoF measurements have 1% white noise.
- $\lambda$ is set to 1.8 in (3.38).

Table 3-12 to Table 3-16, show the results for different generators outages when different number of generators are monitored.

Table 3-12: Generator 2 Outage Results - Case study 8

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>1</td>
<td>39.85</td>
<td>0</td>
<td>37,38</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>37.29</td>
<td>0</td>
<td>37,38</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>36.11</td>
<td>0</td>
<td>37,38</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>1</td>
<td>37.16</td>
<td>0</td>
<td>37,38</td>
</tr>
</tbody>
</table>

Table 3-13: Generator 3 Outage Results - Case study 8

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>1</td>
<td>21.45</td>
<td>0</td>
<td>15,5</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>1</td>
<td>0.12</td>
<td>1</td>
<td>34,4,5,6,8</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>1</td>
<td>0.17</td>
<td>1</td>
<td>34,4,5,6,8</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>1</td>
<td>0.73</td>
<td>1</td>
<td>34,4,5,6,8</td>
</tr>
</tbody>
</table>

Table 3-14: Generator 4 Outage Results - Case study 8

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>1</td>
<td>18.36</td>
<td>0</td>
<td>25,14</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>1</td>
<td>18.93</td>
<td>0</td>
<td>25,14</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>1</td>
<td>18.83</td>
<td>0</td>
<td>25,14</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>1</td>
<td>29.17</td>
<td>0</td>
<td>21,27</td>
</tr>
</tbody>
</table>

Table 3-15: Generator 6 Outage Results - Case study 8

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>0</td>
<td>25.27</td>
<td>1</td>
<td>30,22,20</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>3.47</td>
<td>1</td>
<td>30,22,20</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>1</td>
<td>2.66</td>
<td>1</td>
<td>30,22,20</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>1</td>
<td>0.94</td>
<td>1</td>
<td>30,22,20</td>
</tr>
</tbody>
</table>
Table 3-16: Generator 10 Outage Results - Case study 8

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>1</td>
<td>40.98</td>
<td>0</td>
<td>38</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>0.96</td>
<td>1</td>
<td>33.2</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>0.029</td>
<td>1</td>
<td>33.2</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>0</td>
<td>2.476</td>
<td>1</td>
<td>33.2</td>
</tr>
</tbody>
</table>

Based on the results for disturbance size estimation in Table 3-12 to Table 3-16

Table 3-16, the altered SPC method provides relatively low disturbance size estimation error. However, the disturbance detection and localization stages have significantly worse performance compared to the load disturbance type of events studied in previous case studies.

3.6.11 Case Study 9 (Large Disturbance Case)

In this case study a large disturbance (1000 MW) is analysed with 1 % white noise in the PMU measurements (noise added using (3.33)). The monitored generators are selected from G1, G7, G8 and G9 and the results are summarized in Table 3-17. By comparing Table 3-17 and Table 3-8, it can be seen that for this large disturbance size, disturbance detection has become less accurate (higher MD and FD). Furthermore, compared to small disturbance sizes (40MW and 80MW), the accuracy and precision of the localization have also deteriorated for this large disturbance size (as RL and LW1 have decreased and LW2 and LW3 have increased). The disturbance size estimation error has also increased for this large disturbance compared to Case study 3 (40MW and 80MW disturbance sizes). Nevertheless, despite the worsened performance of the SPC for this large disturbance size, it still provides acceptable accuracy in all stages.

Table 3-17: Results for Case Study 9, Large Disturbance Case

<table>
<thead>
<tr>
<th>S</th>
<th>MD</th>
<th>FD</th>
<th>E95%</th>
<th>RL</th>
<th>Lw1</th>
<th>Lw2</th>
<th>Lw3</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>14.10</td>
<td>10.26</td>
<td>9.19</td>
<td>88.20</td>
<td>80.51</td>
<td>12.82</td>
<td>1.28</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0.513</td>
<td>0</td>
<td>3.97</td>
<td>98.97</td>
<td>85.38</td>
<td>12.82</td>
<td>1.79</td>
</tr>
<tr>
<td>G1,G7, G8,G9</td>
<td>0</td>
<td>0</td>
<td>3.87</td>
<td>99.74</td>
<td>86.92</td>
<td>13.08</td>
<td>0</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>0</td>
<td>0</td>
<td>3.75</td>
<td>100</td>
<td>90.51</td>
<td>9.49</td>
<td>0</td>
</tr>
</tbody>
</table>
3.7 Conclusion

This chapter presents a novel two stage method (SPC) for disturbance analysis which includes fast disturbance detection and estimation of disturbance size and location. The SPC method is based on a limited set of PMU measurements. This disturbance analysis method uses active power changes of the monitored generators and their synchronizing power coefficients to continuously calculate two mismatch indices. In the first stage, disturbance detection is carried out using the mismatch indices as inputs to a decision tree that has been designed offline. If a disturbance is detected, the second stage would localize the disturbance and estimate the disturbance size. The distinctive features of the SPC method are:

- Using active power outputs of generators instead of RoCoFs
- Being independent of the inertia of the system.

A comparison study regarding disturbance size estimation between a traditional swing equation based method and SPC method is performed for a varied number of PMU measurements, disturbance sizes and locations (Case Study 1). The SPC method achieves smaller median error in disturbance size estimation ($E_{\text{median}}=1.2\%$) with three monitored generators than the traditional method does with nine monitored generators ($E_{\text{median}}=1.99\%$) (Figure 3-12).

As a conclusion, the new method provides a more accurate disturbance size estimation with fewer measurements. This merit is even more pronounced considering the likelihood of unavailable PMU readings at all generators, or having erroneous generator RoCoF measured by PMUs at the time of disturbance. Furthermore, through a large number of tests on the IEEE 39 bus system (Case studies 1-7), the robustness and accuracy of the proposed method is proved against various parameters, e.g. disturbance size, location, PMU measurement noise, PMUs placement and error in the system Z matrix and topology.
3.8 References


4. **System Regioning and Regional Inertia**

This chapter describes a new algorithm used for regionalising, meaning grouping synchronous generators, in power systems. The drive behind the new regionalising algorithm is the need for defining a new method for disturbance size estimation that is capable of accounting for unmonitored generators contribution to the average system rate of change of frequency. This is done by partitioning the system into separate regions that can effectively reflect the average system frequency behaviour utilising less PMU measurements. The regional inertias are assigned to representative frequency measurements of each region.

The importance of fast reliable and robust disturbance size estimation is discussed in section 4.1. Section 4.2 shows the vulnerability of the traditional swing equation based disturbance size estimation (Section 2.7) against missing generators frequency measurements. Region, regional inertia and the formulation of the new system regioning is provided in Section 4.3. Section 4.4 describes the clustering method applied for identifying systems regions. Section 4.5 includes examples of the system regioning applied to IEEE 39 bus system. Section 4.6 describes an offline methodology for defining regions which is adaptive to disturbance location. Section 4.7 describes a fixed regioning method that is not adaptive to disturbance location. Finally the chapter is summarised in Section 4.8.

### 4.1 Chapter Introduction

The reduction in system inertia (due to the displacement of synchronous generation that has an inherent, natural inertial response to a power imbalance, with asynchronous generation that has no natural response) will be a critical threat to frequency control in GB in the coming decades [1]. Reduced inertia allows a faster, larger frequency deviation to occur after a disturbance that conventional primary response is too slow to contain. A potential solution to this problem is developing new control services capable of delivering a fast frequency response [2] (e.g. adaptive under frequency load shedding [3][4], storage activation [5], synthetic inertia from wind turbines etc. [6].).
The fundamental step in evaluating the need and the size of fast frequency response is fast and accurate estimation of the disturbance size.

It is proved in the literature that by utilising the Rate of Change of Frequency (RoCoF) of all generators at the time of disturbance given the system total inertia constant $H_{Sys}$ is known, one can determine the actual power imbalance. However, immediately after a sudden power imbalance, frequency is not the same in the whole system, but it has a local character. In other words, it is different from node to node, and from generator to generator. That is why, in a multi-machine system, the calculation of the power imbalance $\Delta P$ cannot be undertaken by using just a single frequency measured in the system. On the contrary, more measurements must be used, if possible measurements at all generators.

Moreover, reduction in the number of large synchronous units and their replacement with intermittent generation that is decoupled from the system by power electronics would also mean that the inertia in the system will become increasingly variable, which is due to the variability of the power dispatch. Therefore, systems frequency dynamics would be different in the individual grid regions/zones depending on the generation dispatch of the zone. The increased regional frequency variation intensifies the need for Wide Area Monitoring Systems (WAMS)[7] and Wide Area Monitoring Control (WAMC)[8] capable of reflecting the local frequency behaviour in the local frequency measures and avoiding the mistakes in event detection and event sizing procedure throughout the entire system.

In this chapter the goal is to define regions including synchronous generators and their regional inertia for the purpose of achieving faster and more accurate disturbance size estimation without having to install PMUs at each generators terminal.

### 4.2 Effect of Losing PMU Measurements

The disturbance size estimated using all generators frequencies and their associated inertias using swing equation has proven to be accurate in the literature [11]. However, in the case of not having PMUs installed in all generators terminals or having unreliable PMU readings at the time of disturbance the accuracy of the (COI) traditional method would drop significantly. This is due to the fact that unmonitored generators contribution to the system average
frequency behaviour would be neglected. Let’s assume set $S$ as the generators equipped with PMU and having available accurate reading at the instant of disturbance ($t_d^*$). The $f_{COI}$ and RoCoF$_{COI}$ calculated using only the monitored generators are expressed as $f_{COI}^S$ and $\dot{f}_{COI}^S$ respectively which are calculated as follows:

$$f_{COI}^S(t_d^*) = \frac{\sum_{i \in S} H_i S_i f_i(t_d^*)}{\sum_{i \in S} H_i S_i}$$  \hspace{1cm} (4.1)

$$\dot{f}_{COI}^S(t_d^*) = \frac{\sum_{i \in S} H_i S_i \dot{f}_i(t_d^*)}{\sum_{i \in S} H_i S_i}$$  \hspace{1cm} (4.2)

The estimated disturbance size using the generators monitored frequencies and the traditional swing based method would be called $P_d^S$, which is calculated as follows:

$$2 H_{Sys} \dot{f}_{COI}^S(t_d^*) = P_d^S$$  \hspace{1cm} (4.3)

where $S$ is the set of monitored generators in the superscription. In the following section (4.3) is used for disturbance size estimation. $H_{Sys}$ is total system inertia constant.

As mentioned at the end of Section 2.8, the RoCoF is calculated for each case study using generators rotating speed coming from DIgSILENT as the frequency signal. For which RoCoFs are obtained by calculating the rate of change between two subsequent frequency output points (time step being 0.01s).

### 4.2.1 Disturbance Size Estimation Error Results with Missing Generators Readings

Figure 4-1 shows the disturbance size estimation error for varied number of generators not monitored.
The cases are obtained by applying varied disturbance size (40MW to 60MW) in different buses in IEEE 39 bus system. Furthermore, the combination of monitored generators is also varied to reflect the average behaviour of disturbance size estimation performance with different number of generators missing.

![Diagram](image)

**Figure 4-1: Disturbance size estimation error for varied number of generators monitored missing**  
*(expressed in log scale)*

As seen in Figure 4-1, disturbance size estimation error is less than 1.5% for 75% of the time when having all generators monitored. With 1 generator missing this value increases to 8.9% and for 3 generators missing it would reach 31.3%. Meaning traditional COI based disturbance estimation fails to give accurate disturbance size estimation when 3 or more generators are not monitored in the IEEE 39 bus system.
4.2.2 Disturbance Size Estimation Error for Large Disturbances with Missing Generators Reading

In this section the error for disturbance size estimation using traditional COI based method is calculated for a large disturbance size (850MW). The location of disturbance is varied as well as the number and combination of the monitored generators. The average errors for different number of monitored generators are presented in Table 4-1.

Table 4-1: Average Disturbance Size Estimation Error for Varied Number of Monitored Generators
(Disturbance Size=850MW)

<table>
<thead>
<tr>
<th>Gen Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Error (%)</td>
<td>90.06</td>
<td>79.73</td>
<td>69.60</td>
<td>59.46</td>
<td>49.33</td>
<td>39.19</td>
<td>29.11</td>
<td>18.98</td>
<td>8.86</td>
<td>1.27</td>
</tr>
</tbody>
</table>

Based on Table 4-1, the average error in disturbance size estimation using the traditional COI based method applied to a large disturbance (850MW) results in average errors which are slightly lower than the average errors calculated for smaller disturbance sizes in Section 4.2.1 (red horizontal lines in Figure 4-1).

4.2.3 Disturbance Size Estimation Error Results Based on Optimal PMU Locations for State Estimation

State estimator utilises information from various sources, e.g. PMUs and SCADA measurements to monitor and estimate the states of a power system. The acquired data from across the system is transmitted to the control system via communication channels and processed using a computer aided tool called Energy Management Systems (EMS) [12], for which state estimator is a main block. Motivated by the high cost of PMU installation and limitation on communication channel band width, many researches have been carried out to set out suitable methodology to determine the optimal PMU locations [13][14][15]. These methods endeavour to find the minimum number of nodes that if monitored can provide full topological observability over the network even in N-1 contingency cases and result in minimum error in state estimation. A power system is called topologically observable when all of its states (voltage magnitudes and phases of all nodes) can be uniquely determined [14]. State estimator estimates the voltage magnitudes and phases of different buses using measurements in the form of power flows, voltage magnitudes and current flows through the branches. Phasor measurements can either help improve the confidence in the available
measurements from remote terminal unites (RTUs) or to replace them. In [13], a set of optimal location for PMU measurements on the 39 bus system is suggested considering random component outages.

The results for disturbance size estimation considering varied number of monitored generators/PMU number is obtained. Disturbance is added in all buses and for disturbance size between 40MW to 60MW and considering the optimal PMU locations retrieved from [13]. The results are presented in Table 4-2. As the original buses for optimal PMU locations are all non-generator buses, the closest generator to each non-generator bus is chosen as the corresponding monitored generator in Table 4-2.

Table 4-2: Summary of results for literature optimal PMU locations

<table>
<thead>
<tr>
<th>PMU Number</th>
<th>Optimal Buses for PMU Based on Literature</th>
<th>Corresponding Monitored Generators</th>
<th>Average Percentage Error in $P_d$ estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>G10</td>
<td>169.51</td>
</tr>
<tr>
<td>2</td>
<td>2,36</td>
<td>G10,G2</td>
<td>150.96</td>
</tr>
<tr>
<td>3</td>
<td>2,36,39</td>
<td>G10,G2,G1</td>
<td>46.7</td>
</tr>
<tr>
<td>4</td>
<td>2,36,39,8</td>
<td>G10,G2,G1,G3</td>
<td>37.28</td>
</tr>
<tr>
<td>5</td>
<td>2,36,39,8,21</td>
<td>G10,G2,G1,G3,G4</td>
<td>29.02</td>
</tr>
<tr>
<td>6</td>
<td>2,36,39,8,21,25</td>
<td>G10,G2,G1,G3,G4,G5</td>
<td>24.7</td>
</tr>
<tr>
<td>7</td>
<td>2,36,39,8,21,25,22</td>
<td>G10,G2,G1,G3,G4,G5,G6</td>
<td>17.25</td>
</tr>
<tr>
<td>8</td>
<td>2,36,39,8,21,25,22,28,15</td>
<td>G10,G2,G1,G3,G4,G5,G6,G7</td>
<td>11.1</td>
</tr>
<tr>
<td>9</td>
<td>2,36,39,8,21,25,22,28,15,22,8,15</td>
<td>G10,G2,G1,G3,G4,G5,G6,G7,G8</td>
<td>5.74</td>
</tr>
</tbody>
</table>

Based on Table 4-2 and as it was expected, the average of the errors in estimated disturbance size falls between the values of error bars shown in Figure 4-1 for each number of monitored generators. In this case, using the literature based optimal locations for PMUs results in even worse average error compared to the average error (middle bars in Figure 4-1) for arbitrary choices of PMU locations. Meaning despite choosing the literature based optimal location for PMUs, the traditional swing based method for disturbance size estimation still fails to provide accurate disturbance size estimate when less than all synchronous generators are monitored.

4.2.4 Disturbance Size Estimation Error Results for Generator Outage Type of Disturbance with Missing Generators Reading

In this section the error for disturbance size estimation using traditional COI based method is calculated for generator outage type of disturbance. Five out of ten synchronous generators in IEEE 39 bus system are disconnected and the size of disturbance is estimated using varied
number of monitored generators for each disconnection. The average errors using different number of monitored generators for each generator outage are presented in Table 4-3.

Table 4-3: Average Disturbance Size Estimation Error for Varied Number of Monitored Generators and Generator Outage Type of Disturbance

<table>
<thead>
<tr>
<th>Disconnected Gen</th>
<th>Gen 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of monitored Gen</td>
<td>1</td>
</tr>
<tr>
<td>Average Error (%)</td>
<td>89.78</td>
</tr>
</tbody>
</table>

Based on the values in Table 4-3, the error in disturbance size estimation using traditional COI based method is generally larger for generator outage type of disturbance compared the load increase type of disturbance (Section 4.2.1). The reason for this increase in error is the fact that the total system inertia is no longer the same following the loss of a synchronous generator.

4.3 System Regioning

Based on the results achieved in section 4.2.1, it can be concluded that there exists a need for defining a new method for disturbance size estimation that is capable of accounting for unmonitored generators contribution to the average system frequency. In order to do that, the intent is to partition the system in separate regions that can effectively reflect the average system frequency behaviour utilising less PMU measurements.
4.3.1 **Definition of Region**

Each region is a predefined group of generators which can be grouped to a single generator with an equivalent inertia and its representative (centroid) frequency.

The set including all sets of generators (Regions) is called \( R \):

\[
R = \{ R_1, R_2, ..., R_{nc} \}
\]  \hspace{1cm} (4.4)

where \( nc \) is the number of regions.

4.3.2 **Definition of the Regional Inertia**

Assuming regions \( R \) are known, each generator in the system is assigned to a particular region. The inertia of the region \( R_i \) or regional inertia \( H_{R_i} \) would be equal to the summation of inertia times apparent power of the generators belonging to each region:

\[
H_{R_i} = \sum_{j \in R_i} H_j S_j \hspace{1cm} i = 1, 2, ..., nc
\]  \hspace{1cm} (4.5)

4.3.3 **Objective of the Region**

The fundamental reason for partitioning system generators into regions is to facilitate estimation of system average frequency and especially RoCoF following a disturbance using only a subset of frequency and RoCoF measurements. By doing so, the goal is to achieve an accurate disturbance size estimate by applying the swing equation to the RoCoF of the centre of regional inertia \( \dot{f}_{CORI} \) instead of \( \dot{f}_{COI} \). In other words, the main objective for system partitioning from frequency behaviour point of view is to minimise the difference between COI and CORI variables.
4.3.4 Definition of Centre of Regional Inertia variables

The representative frequency \( f_{R_i} \) of the region \( R_i \) is effectively defined as the frequency of the generator \( k \) \( f_k \) belonging to region \( R_i \) whose RoCoF \( \dot{f}_k \) has the minimum distance to the RoCoF of other generators belonging to the same region shortly following the disturbance occurrence \( t_d^+ \):

\[
f_{R_i}(t_d^+) = \arg \min_{k \in R_i} \sum_{j \in R_i} \left| \dot{f}_j(t_d^+) - \dot{f}_k(t_d^+) \right|
\]

These representative frequencies \( f_{R_i} \) and regional inertias \( H_{R_i} \) can then be used to construct the Centre of Regional Inertia frequency \( f_{CORI} \) and the Centre of Regional Inertia RoCoF \( \dot{f}_{CORI} \):

\[
f_{CORI}(t_d^+) = \frac{\sum_{i=1}^{nc} H_{R_i} f_{R_i}(t_d^+)}{\sum_{i=1}^{nc} H_{R_i}}
\]

\[
\dot{f}_{CORI}(t_d^+) = \frac{\sum_{i=1}^{nc} H_{R_i} \dot{f}_{R_i}(t_d^+)}{\sum_{i=1}^{nc} H_{R_i}}
\]

Where:

\[
\sum_{i=1}^{nc} H_{R_i} = \sum_{i=1}^{M} H_{\text{base},i} = H_{\text{Sys}}
\]

This means that the summation of regional inertia naturally equals the summation of all generators inertia which is the same as \( H_{\text{Sys}} \).
4.3.5 Formulating the Regioning Problem

The goal is to divide the system into regions based on the inertial behaviour of the generators. Based on the swing equation the time frame for which RoCoF is mainly governed by the inertia is shortly following the disturbance \((t \in (t_d, t_d + \varepsilon))\). Therefore in order to achieve optimal regions the difference between COI and CORI variables needs to be minimised in this time frame following the disturbance:

\[
Min \left[ f_{COI}^d(t) - f_{CORI}^d(t) \right]_{t \in (t_d, t_d + \varepsilon)}
\]  
(4.10)

By substituting (2.9) and (4.8) in (4.10):

\[
Min \left[ \sum_{j=1}^{n} H_j S_j \dot{f}_j(t) - \sum_{j=1}^{n} H_j R_j \dot{f}_j(t) \right]_{t \in (t_d, t_d + \varepsilon)}
\]  
(4.11)

By substituting (4.9) in (4.11):

\[
Min \left[ \sum_{j=1}^{n} \sum_{i \in R_j} H_i S_i \dot{f}_j(t) - \sum_{j=1}^{n} \sum_{i \in R_j} H_i S_i \dot{f}_j(t) \right]_{t \in (t_d, t_d + \varepsilon)}
\]  
(4.12)

\[
Min \left[ \sum_{j=1}^{n} \sum_{i \in R_j} H_i S_i (\dot{f}_j - \dot{f}_j) \right]_{t \in (t_d, t_d + \varepsilon)}
\]  
(4.13)

Therefore based on (4.13), the regioning problem can be reformulated as the search for the best set of regions \((R)\) (4.4) that can minimise the distance between the RoCoF of the
generators ($\dot{f}_i$) belonging to each region ($R_j$) with the representative RoCoF of the region ($\dot{f}_{R_j}$).

The regioning problem would then be:

$$\arg \min_{R} \sum_{t=t_0}^{t_{t+\varepsilon}} \sum_{j=1}^{nc} \sum_{i \in R_j} |\dot{f}_i(t) - \dot{f}_{R_j}(t)|$$

In (4.14) the outer loop step is equal to the time step at which RoCoF is recorded ($\Delta t$). $\Delta t$ is typically equal to 20 ms based on PMU recording rates being equal to 50Hz. The number of times the outer loop is executed is equal to $n_t$ which can be calculated using (4.15):

$$n_t = \left\lceil \frac{\varepsilon}{\Delta t} \right\rceil + 1$$

Now (4.14) can be set as a form of clustering problem having $M$ objects (the total number of generators) in a $n_t$ dimensional space ($n_t$ can also be regarded as the number of variables for each object). Furthermore, the number of regions ($nc$) would be the number of clusters.

However, by not having $nc$ set, the optimal answer for the (4.14) is having each individual synchronous generator as a region which in this case the number of regions ($nc$) would be equal to number of generators ($M$) and the minimum of (4.13) would be zero. Meaning, since each region needs to have at least one PMU readings to calculate the representative frequency of it (4.6), all generators need to be monitored. However assuming having PMUs installed in each generators terminal is not plausible and considering monitoring and control action delays being a function of monitored generators, the answer for the regioning problem in (4.14) would become suboptimal and an extra condition would be included which is the max number of regions ($nc_{max}$):

$$nc \leq nc_{max}$$

(4.16)

The $nc_{max}$ is equal to the number of generators equipped with PMUs.
The fundamentals of clustering algorithms are discussed in the following section.

4.4 Clustering

Clustering is one of the most important and well-known data mining methods. The goal of a cluster algorithm is to group together data objects according to some notion of similarity. Therefore, it assigns data objects to different groups (clusters) so that objects that were assigned to the same groups are more similar to each other than to objects assigned to other clusters [17].

4.4.1 Cluster Centroids definition

The centroid of each cluster is the point that has the smallest distance to the members of that cluster. There are two methods for assigning centroids:

Choosing the average of the members in each cluster as the centroid; in this way the centroid does not necessarily coincide with an actual member in the cluster

Choosing the member in each cluster that has the smallest distance to the average of each cluster; this method is called “medoid” clustering method [18].

4.4.2 Proximity measure

In order to assign members of a data set to different clusters, a measure of proximity that quantifies the notion of “closest” for the specific data under consideration is required. The most common proximity measure used for clustering algorithm is based on Euclidean distance.

The Euclidean distance between two data points \( p = (P_1, P_2, ..., P_n) \) and \( q = (q_1, q_2, ..., q_n) \) in a \( n \) dimension space is defined as follows:
\[ e(\mathbf{p}, \mathbf{q}) = e(\mathbf{q}, \mathbf{p}) = \sqrt{\sum_{i=1}^{n} (q_i - p_i)^2} \] (4.17)

The Euclidean distance matrix can be defined on a data set \( X \) as follows:

\[
E = (e_{ij}) \\
e_{ij} = e(x_i - x_j)
\] (4.18)

### 4.4.3 K-means clustering procedure

The first step in the K-means clustering technique is to randomly choose the first initial centroids amongst the data point, where \( K \) is a pre-defined parameter which is the number of clusters desired. In the second step each point in the data is assigned to a closest centroid based on the Euclidean distance of the data point to different centroids. The centroid of the clusters is then updated by replacing it with the average of the data points belonging to each cluster. We then proceed to repeat the previous steps until no points change cluster or equivalently until the centroids remain the same [17].

In case of having medoid as centroid of the clusters, at each step the centroid is chosen as the closest data point within the cluster to the average of the cluster.

### 4.4.4 RoCoF Based Regioning

One of the well-known applications of clustering is in the summarisation of data. In this sense clusters centroids are used in the process of data analysis rather than the whole data set; reducing the necessity to process or even obtain the whole spectrum of data.

The accuracy of the data summarisation using clustering algorithm is highly dependent on the nature of the data and the similarity level of the members in each cluster.

In this case the goal is obtaining generator clusters based on their similar RoCoF behaviour shortly following the active power disturbance. By doing so, it is possible to utilise the
RoCoF representatives of each cluster to reproduce the swing equation for the average frequency behaviour of the system and consequently estimate the disturbance size.

Similarly the accuracy of disturbance estimation using only frequency representative of the regions (clusters) is dependent on how similar RoCoF of generators are in each region.

Each data point used for the system regioning problem would be a vector of size \( (1 \times n_t) \) (4.15) which would be defined as follows:

\[
x_i = (\hat{f}_i(t_d), \hat{f}_i(t_d + \Delta t), ..., \hat{f}_i(t_d + (n_t - 1)\Delta t))
\]  

(4.19)

The Euclidean distance matrix elements would be as follows:

\[
e_{ij} = \sqrt{\sum_{t=1}^{n_t} \left[ \hat{f}_j(t_d + (l-1)\Delta t) - \hat{f}_j(t_d + (l-1)\Delta t) \right]^2}
\]  

(4.20)

The flowchart for obtaining regions using the K-means cluster algorithm (section 4.4.3) is depicted in Figure 4-2.
Randomly choose $K$ generator RoCoF points (Initial centroids)

Calculate Euclidean distance of all generators RoCoF points with each centroid

Assign generators to the regions based on the smallest Euclidean distance to the centroids

Find the centroids of the newly formed regions

Stop when regions membership no longer changes

**Figure 4-2: Flowchart for Regioning**

### 4.5 Regioning Example

The case study of this example corresponds to the IEEE 39 bus system. The simulations are performed in DigSILENT software. Furthermore, all post simulation analysis regarding clustering and disturbance size estimation is done in Matlab platform. The Single-line diagram of the 39 bus system is presented in Figure 4-3.
4.5.1 Case studies description

The clustering algorithm is applied to RoCoF of all generators in two consecutive time steps following the disturbance time for different disturbance locations. For all simulations the following parameters are considered:

- $\varepsilon = 0.04s$, $\Delta t = 0.02s \Rightarrow n_r = 3$
- $P_d = 850MW$
- $t_d = 2.1s$
- Number of clusters ($nc$) is varied from 1 to 10

Two case studies regioning results are presented in the following sections. The short descriptions of case studies are:

Case Study 1: Disturbance Location is bus 15 and all the other parameters are the same as the ones defined in the previous section

Case Study 2: Disturbance Location is bus 12 and all the other parameters are the same as the ones defined in the previous section
In these case studies, after calculating the regions using RoCoF based clustering, the \( \dot{f}_{CORI} \) shortly following the disturbance (for \( t = t_d^+ \)) is estimated using (4.8). Disturbance size is then estimated as follows:

\[
2H_{Sys} \dot{f}_{CORI} (t_d^+) = P_d
\]  

(4.21)

The inertia of the regions are calculated in both of these case studies using section 4.3.2.

### 4.5.2 Case study 1 results

As stated in section 4.5.1 for this case study disturbance location is bus 15. The generators frequencies and RoCoFs evolution shortly following the disturbance are shown in Figure 4-4 and Figure 4-5.
Based on Figure 4-4 and Figure 4-5, 5 generators RoCoFs shortly after the disturbance can be intuitively clustered in 5 clusters.

The clustering results for the cases depicted in Figure 4-4 and Figure 4-5 using the explained k-mean based methodology (section 4.4) are demonstrated in Figure 4-6 and Figure 4-7. Each points in Figure 4-6 and Figure 4-7 represents one generator RoCoF; and the coordinates correspond to 3 time instances \((t_d, t_d + \Delta t, t_d + 2\Delta t)\).
Figure 4-6: K-means clustering applied to generators RoCoF for case study 1 with 4 clusters

Figure 4-7: K-means clustering applied to generators RoCoF for case study 1 with 5 clusters

Figure 4-8 and Figure 4-9 show the system regioning results with 4 clusters and 5 clusters respectively.
4.5.3 Case study 2 results

As stated in section 4.5.1 for this case study the disturbance location is bus 12. The generators frequencies and RoCoFs evolution shortly following the disturbance are shown in Figure 4-10 and Figure 4-11.
The clustering results for the cases depicted in Figure 4-10 and Figure 4-11 using the explained k-mean based methodology (section 4.4) is demonstrated in Figure 4-12 and Figure 4-13. Each point in Figure 4-12 and Figure 4-13 present one generator RoCoF; and the coordinates correspond to 3 time instances $(t_d, t_d + \Delta t, t_d + 2\Delta t)$. 

Figure 4-10: Generators frequencies for case study 2

Figure 4-11: Generators RoCoFs for case study 2
Figure 4-12: K-means clustering applied to generators RoCoF for case study 2 with 4 clusters

Figure 4-13: K-means clustering applied to generators RoCoF for case study 2 with 5 clusters

Figure 4-14 and Figure 4-15 show the system regioning results with 4 clusters and 5 clusters respectively.
4.5.4 Disturbance Estimation and Clustering Results Summary

The summary of results regarding both case studies for disturbance size estimation error, the regions members plus their centroids (medoids) are presented in Table 4-4.

Based on Table 4-4, the regions members change between the two case studies. As an example generators G8 and G10 are in the same region for case study 2, whereas they split and form two different regions in case study 1. This behaviour is due to the much smaller relative distance to disturbance location seen by G8 compared to G10 when having the disturbance location at bus 15 (case study 1). As a results their RoCoF is no longer
sufficiently similar shortly following the disturbance, therefore they cannot be clustered into one region as oppose to case study 2.

For both case studies the error in disturbance size estimation falls below 4% when having 5 regions (clusters) (Table 4-4). However, for case study 2 having only 4 regions lead to 3.86% which is still less than 4%; whereas in case study 1 having 4 regions lead to 22.36% error. Therefore, it can be concluded that the optimal number of clusters for case study 2 is 4 and for case study 1 is 5.

Table 4-4: Summary of regioning results for case study 1 and case study 2

<table>
<thead>
<tr>
<th>Case Study</th>
<th>Disturbance Location</th>
<th>Number of clusters</th>
<th>Cluster Sets (R)</th>
<th>Cluster medoids</th>
<th>Error in $P_d$ estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>Bus 15</td>
<td>4</td>
<td>{G10} {G9} {G8}</td>
<td>G10,G9,G8,G7</td>
<td>22.36%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>{G10} {G9} {G8}</td>
<td>G10,G9,G8,G7</td>
<td>1.34%</td>
</tr>
<tr>
<td>Case 2</td>
<td>Bus 12</td>
<td>4</td>
<td>{G1} {G9}</td>
<td>G10,G9,G1,G7</td>
<td>3.86%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>{G1} {G9} {G6,G7}</td>
<td>G10,G9,G1,G7</td>
<td>3.13%</td>
</tr>
</tbody>
</table>

A summary of results regarding inertia of the regions their associated generator members is presented in Table 4-5 and Table 4-6 for case study 1 and case study 2 respectively.

Table 4-5: Regional Inertia and Regions members for Case Study 1

<table>
<thead>
<tr>
<th></th>
<th>Regions Members</th>
<th>Regions Inertia(GWs)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4 clusters</td>
<td>{G10}</td>
<td>4.2</td>
<td>{G9}</td>
<td>4.14</td>
<td>2.43</td>
</tr>
<tr>
<td>5 clusters</td>
<td>{G10}</td>
<td>4.2</td>
<td>{G9}</td>
<td>4.14</td>
<td>2.43</td>
</tr>
</tbody>
</table>
### Table 4-6: Regional Inertia and Regions members for Case Study 2

<table>
<thead>
<tr>
<th>Clusters</th>
<th>Regions Members</th>
<th>[G1]</th>
<th>[G9]</th>
<th>[G8,G10,G2,G3,G5]</th>
<th>[G6,G7,G4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 clusters</td>
<td>Regions Inertia(GWs)</td>
<td>70</td>
<td>4.14</td>
<td>15.84</td>
<td>8.98</td>
</tr>
<tr>
<td>5 clusters</td>
<td>Regions Members</td>
<td>[G1]</td>
<td>[G9]</td>
<td>[G8,G10,G2,G3,G5]</td>
<td>[G6,G7] {G4}</td>
</tr>
<tr>
<td></td>
<td>Regions Inertia(GWs)</td>
<td>70</td>
<td>4.14</td>
<td>15.84</td>
<td>6.12 2.86</td>
</tr>
</tbody>
</table>

By comparing Table 4-5 and Table 4-6, it can be concluded that the regional inertia is also dependent on the location of the disturbance. This observation was expected, since the regions boundaries dictate regional inertia and it was seen in Table 4-4 that the boundaries of the regions vary with disturbance location.

Figure 4-16, shows disturbance estimation error results for different number of clusters as the x axis for having disturbance location at different disturbance locations on IEEE 39 bus with varied disturbance size from 750MW to 850MW with 50MW steps.
Based on Figure 4-16, 90% of the times disturbance size estimation error gets to less than 5% when having 5 regions (clusters). Furthermore, increasing the number of clusters to 8 leads to having disturbance size estimation error less than 5% for 100% of the times.

It is apparent in Figure 4-16, the disturbance size estimation error reaches a saturation level from 5 clusters onward; meaning the accuracy of disturbance size estimation does not benefit from extra number of clusters.

Therefore, as one measurement was assumed per cluster (Region), the optimal number of measurements to achieve relatively accurate disturbance size estimation would be equal to 5. This accuracy is achieved only if system is divided into meaningful regions by utilising the clustering algorithm discussed in section 4.3.

On the other hand, based on Figure 4-1, with no regioning it can be concluded that in order to have a same level of accuracy in disturbance size estimation (less than 5%), 9 generators are needed to be monitored out of 10.
This difference in the number of measurement needed for fast and accurate estimation of disturbance size highlights the benefit of partitioning the system into regions based on the clustering method.

4.6 Offline Regioning (Adaptive Regioning)

As shown in section 4.5.4 for two example case studies, regional inertia based disturbance size estimation yields a small error (Table 4-4). This was however dependent on the appropriate selection of region members based on the disturbance location. The applied RoCoF based regioning method (section 4.4.4) used the K-means clustering algorithm on immediate post disturbance RoCoFs of all generators to assign generators in separate clusters (with each cluster forming a region). Following this regioning step, the regional inertia was calculated as the summation of inertias times apparent power base of all generators belonging to each region. However, due to the inputs to this clustering being all generators RoCoF following the disturbance, this approach required all generator to be monitored which contradicts the first reason behind the whole regioning process, which is the missing generators reading (section 4.3).

Furthermore, in clustering based on generators RoCoF traces, the clustering can be done once a disturbance takes place, which adds to the calculation time delay. Therefore, in this section an alternative procedure called offline regioning is introduced. Offline regioning is not based on generators RoCoF readings, therefore its results can be generated offline in a matrix format.

Nevertheless, this offline regioning method needs the information of disturbance location as an input for clustering. Therefore, the offline regioning method is also referred to as the adaptive regioning method as opposed to the fixed regioning method suggested in section 4.7 that does not require disturbance location.

4.6.1 Generators RoCoF at $t_d+ $

Shortly following an active power disturbance at time $t_d$ at bus $u$, the active power imbalance is compensated for by synchronous generators, with each generator picking up a
specific share depending on their synchronising power coefficient with the disturbance location and their inertia. The synchronising power coefficient are calculated for reduced system having internal synchronous nodes and disturbance bus $u$. The formula dictating the initial active power change of generator $i$ is as follows (which has been proved in Chapter 3 section 3.3.1):

$$
\Delta P_{ei}(t_d^+) = \left\{ \frac{K_{iu}^u}{\sum_{j=1}^{M} K_{ju}^u} \right\} P_{d}^u
$$

(4.22)

Where $K_{iu}^u$ is the synchronising power coefficient between generator bus $i$ and disturbance bus $u$ when the system has been reduced to internal generator nodes and bus $u$. Also $P_{d}^u$ is equal to active power disturbance size and superscription is equal to the disturbance location bus.

Based on the swing equation and negligible $\Delta f$ at $t_d^+$ following is derived:

$$
2H_i\dot{f}_i(t_d^+) = \Delta P_{ei}(t_d^+)
$$

(4.23)

By substituting (4.22) in (4.23):

$$
\dot{f}_i(t_d^+) = \left\{ \frac{K_{iu}^u}{2H_i \sum_{j=1}^{M} K_{ju}^u} \right\} P_{d}^u \quad i = 1, 2, ..., M
$$

(4.24)

Based on (4.24) RoCoF of each generator can be estimated at $t_d^+$ based on disturbance size, location, generator inertia and synchronising power coefficients. Furthermore, for a fixed disturbance size and location all generators RoCoFs are relative to one another based on the term in the parenthesis in (4.24). Therefore, as $P_{d}^u$ is constant throughout all generators frequencies, it can be treated as a scaling factor. Meaning, instead of using actual post disturbance RoCoF measurements of generators for clustering, as in 4.4.4, the term in the parenthesis in (4.24) can be used as an input to clustering algorithm regardless of the disturbance size.
4.6.2 Reformulating Regioning for Offline Analysis

Based on (4.24), \( \Psi \) can be defined as \((M \times N)\) matrix. Where \( M \) is the number of synchronous generator and \( N \) is the number of buses. The elements of \( \Psi \) are as follows:

\[
\psi_{ij} = \left( \frac{K^i_j}{2H_{i, sys} \sum_{n=1}^{M} K^j_n} \right) \quad i = 1, 2, \ldots, M \quad j = 1, 2, \ldots, N
\]  

(4.25)

By substituting (4.25) in (4.24):

\[
\dot{f}_i(t_d) = \psi_{iu} P_d^u \quad i = 1, 2, \ldots, M
\]  

(4.26)

Each column of \( \Psi \) represents one disturbance location and each row one synchronous generator.

By substituting (4.25) in (4.14), eliminating the scaling factor \( P_d^u \) and considering the fact that (4.24) holds true for time \( t_d + \varepsilon \), the clustering formulation for regioning problem which was formerly presented in (4.14) can be rewritten as follows:

\[
\arg \min_{R} \sum_{j=1}^{nc} \sum_{i \in R_j} |\psi_{iu} P_d^u - \psi_{R_j} P_d^u| \equiv \arg \min_{R} \sum_{j=1}^{nc} \sum_{i \in R_j} |\psi_{iu} - \psi_{R_j}|
\]  

(4.27)

where \( u \) is the disturbance location bus, \( nc \) is the cluster number, \( R \) is the set of regions (clusters) and \( \psi_{R_j} \) is the centroid rate of change of frequency of the region \( j \) \( (\dot{f}_{R_j}) \) divided by \( P_d^u \).

Therefore based on (4.27), it is expected that for a given disturbance location \( u \), the clustering based on all generators RoCoF traces and column \( u \) of \( \Psi \) renders the same results.

Therefore, the regioning results based on K-means clustering for different disturbance locations can be estimated offline using \( \Psi \) matrix, which needs the synchronising power coefficient matrix ((3.8)) and generators inertia to be calculated.
The following section aims to verify the similarity of results for system regioning using the offline method and the RoCoF trace based method.

### 4.6.3 Cluster Matrix Definition (CL_{nc})

In order to enumerate the regioning results, knowing they are dependent on disturbance location, Cluster matrix (CL_{nc}) is defined as a (M×N) matrix where the superscript \( nc \) is the number of cluster/regions for which the clustering matrix has been formed. In CL_{nc} each row represents one generator and each column represents one disturbance location.

Generators having the same value in each column belong to one region assuming disturbance has taken place in the bus number equal to that column number. Therefore, the regions membership or regions boundaries (\( R \)) for any specific disturbance location (\( u \)) can be interpreted from CL_{nc} as follows:

\[
R_i = \left\{ j \left| CL_{ju}^{nc} = i \right. \right\} \quad i = 1, 2, ..., nc
\]

(4.28)

Where \( j \) is representing the generator number and \( u \) is the location of disturbance. \( R_i \) is region number \( i \), which comprises of all generator numbers that belong to region \( i \). \( i \) can vary from 1 to the number of regions (\( nc \)).

In order to better understand the cluster matrix CL_{nc}, the CL_{6} for 39 bus system is depicted in Figure 4-17, where matrix values are colour coded (with 6 different colours representing 6 regions). Assuming disturbance has taken place at bus 4, in order to find the regions membership, column number 4 should be considered in Figure 4-17 (bottom graph). All generators with the same colour shade in column 4 belong to one region for disturbance at bus 4. This results in the following regions membership:

**Table 4-7: Example of regions membership using CL_{6} of IEEE 39 bus for disturbance at bus 4**

<table>
<thead>
<tr>
<th></th>
<th>( R_1 )</th>
<th>( R_2 )</th>
<th>( R_3 )</th>
<th>( R_4 )</th>
<th>( R_5 )</th>
<th>( R_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>{G10,G8,G4}</td>
<td>{G6,G7}</td>
<td>{G1}</td>
<td>{G5}</td>
<td>{G3}</td>
<td>{G2}</td>
</tr>
</tbody>
</table>
4.6.4 Comparison Between RoCoF Based Regioning and the Offline Regioning

In this section, two regioning methods have been applied to IEEE 39 bus system for different disturbance locations and their $\text{CL}^{nc}$ is calculated for varied cluster number ($nc$).

The two methods used to calculate $\text{CL}^{nc}$:

1. Calculation using $\Psi$. Each column in Clustering matrix $\text{CL}^{nc}$ was calculated offline by applying K means clustering algorithm to the same column in $\Psi$ (4.25).
2. Calculation using immediate post disturbance generators RoCoFs. Each column in $\text{CL}^{nc}$ was calculated by applying disturbance at the bus number equal to the that column and applying the K means clustering algorithm to all generators RoCoFs for two time steps following the $t_d$ ($nc=3$) (section 4.3.4).

The difference between two methods in clustering has been calculated by finding the mismatch of generators region membership between the two methods for all disturbance locations. Therefore, the Percentage Difference was defined as the number of times the $CL^{nc}_{ij}$ was not equal for both methods divided by the total number of elements in $\text{CL}^{nc}$ in this case equal to $10 \times 39 = 390$ times 100. Comparison results for varied number of cluster number are shown in Table 4-8.

Table 4-8: Comparison of RoCoF clustering method and offline method for varied cluster number

<table>
<thead>
<tr>
<th>Cluster Number ($nc$)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage Difference</td>
<td>3.0769</td>
<td>5.3846</td>
<td>5.6410</td>
<td>5.8974</td>
<td><strong>2.0513</strong></td>
<td>10</td>
<td>11.025</td>
<td>8.2051</td>
</tr>
</tbody>
</table>

As seen in Table 4-8, the percentage difference between the two methods is minimised when cluster number ($nc$) equals 6. Therefore, due to the necessity for application of offline clustering (reasons for which was discussed in section 4.6), cluster number equals to 6 ($nc=6$) is used in the following sections in order to have the minimum deviation from the RoCoF based clustering method.

The small difference between the RoCoF based and offline clustering method (3.07% for $nc=6$) (Table 4-8), verify the validity of offline clustering method to be used for system regioning.
Figure 4-17, depicts the CL\textsuperscript{6} matrix (clustering results for nc=6) for both methods. In the checked graphs, each column represents one disturbance location and each row one synchronous generator. For each column of the checked graphs, the generators belonging to the same region (cluster) are marked with the same colour. The red circles mark the different points between the two methods, in this case being 8; therefore the percentage difference between the methods would be 2.0513\% as was presented in Table 4-8.

Figure 4-17: Clustering comparison between RoCoF based method and Offline (using synchronising power coefficient and inertia) method, cluster number equals 6

4.7 Fixed Regioning

As shown in section 4.6, the system regioning can effectively be executed and adapted to disturbance location. However, the disturbance location might not always be available fast enough following a disturbance occurrence. In order to relieve the regioning problem from the necessity of known disturbance location, a fixed regioning method is suggested in this section. For the fixed regioning method, clustering of the generators in different regions is constant for any disturbance location.
Most generators clustering methods suggested in the literature are based on slow coherency theory [19]. The main purpose of these coherency based generators clustering have been optimal dynamic reduction to simplify transient stability analysis [20][21], another more recent application has been in intentional emergency system islanding [22][23]. These methods are focused on the similarity between relative generators angle difference following a fault or disturbance. In [22] the clustering is based on the two weakest damped modes of the system state Laplacian matrix. Each generator is treated as an object with two features one equal to the eigenvector element of that generator for the first slowest mode and the second equal to the eigenvector element of that generator for the second slowest mode. In this method the emphasis is on the weakly damped inter area modes which dictate the angular oscillations following the initial transient and are expanded to time frames in the order of seconds. Consequently, the slow coherency based clustering fails to capture the similarity of generators RoCoF behaviour in the time frame of interest for disturbance size estimation (in the order of ms).

In the fixed regioning method for disturbance size estimation, the regioning should be an optimal clustering for all possible disturbance locations. Meaning, the generators should be clustered based on the similarity of their RoCoF shortly following the disturbance occurrence for all possible disturbance locations. Which leads to having a clustering problem with M (generator number) objects with each object having N (number of buses) feature. Therefore, matrix Ψ (section 4.6.2) can again be used for determining the clustering results but this time each row of Ψ is treated as an object with N feature.

Therefore, in the first step the euclidean distance (4.17) between each two generators object i \(\Psi_i = (\Psi_{i1}, \Psi_{i2}, \ldots, \Psi_{iN})\) and generator j \(\Psi_j = (\Psi_{j1}, \Psi_{j2}, \ldots, \Psi_{jN})\) would be calculated as follows:

\[
ev(i, j) = \sqrt{\sum_{n=1}^{N} (\Psi_{in} - \Psi_{jn})^2}
\]  
(4.29)

After this the regioning results would be calculated using the K-means clustering procedure explained in section 4.4.3 applied to the euclidean matrix defined in (4.29).

The results for fixed regioning applied to 39 bus system are shown in Table 4-9.
## Table 4-9: Fixed regioning results for IEEE 39 bus system

<table>
<thead>
<tr>
<th>Number of Regions</th>
<th>Regions Membership (R)</th>
<th>Regional Inertia values (H(_R))(GWs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>{G1,G2,G3,G4,G5,G6,G7}{G8,G9,G10}</td>
<td>88.19,10.77</td>
</tr>
<tr>
<td>3</td>
<td>{G1,G2,G3,G4,G5}{G6,G7}{G8,G9,G10}</td>
<td>82.07,6.12,10.77</td>
</tr>
<tr>
<td>4</td>
<td>{G1,G8,G10}{G2,G3}{G4,G5,G6,G7}{G9}</td>
<td>76.630,6.61,11.58,4.14</td>
</tr>
<tr>
<td>5</td>
<td>{G1,G8,G10}{G2,G3}{G4,G5}{G6,G7}{G9}</td>
<td>76.630,6.61,5.46,6.12,4.14</td>
</tr>
<tr>
<td>6</td>
<td>{G1,G10}{G2,G3}{G4,G5}{G6,G7}{G8}{G9}</td>
<td>74.2, 6.61,5.46,6.12,4.14,2.43,4.14</td>
</tr>
<tr>
<td>7</td>
<td>{G1,G10}{G2,G3}{G4,G5}{G6}{G7}{G8}{G9}</td>
<td>74.2, 6.61,5.46,3.48,2.64,2.43,4.14</td>
</tr>
<tr>
<td>8</td>
<td>{G1}{G2,G3}{G4,G5}{G6}{G7}{G8}{G9}{G10}</td>
<td>70, 6.61,5.46,3.48,2.64,2.43,4.14,4.2</td>
</tr>
<tr>
<td>9</td>
<td>{G1}{G2,G3}{G4}{G5}{G6}{G7}{G8}{G9}{G10}</td>
<td>70, 6.61,2.86,2.6,3.48,2.64,2.43,4.14,4.2</td>
</tr>
</tbody>
</table>

### 4.8 Chapter Summary

As shown in this chapter, in order to be able to estimate disturbance size using limited number of frequency measurements, it is necessary to adapt the regions boundaries and consequently regional inertia to disturbance location. However, by applying the adaptive regioning procedure and using regional inertia values assigned to representative RoCoF of the regions, it was possible to lose half of the frequency measurements and still get the same accuracy level in disturbance size estimation as having all generators monitored.

Nevertheless, in this process it has been assumed that there always exist RoCoF measurements in each formed region, results for which are presented in Figure 4-17. Meaning, the cases that had a non-monitored region have been eliminated. In other words, the regions are first formed then the monitoring points are picked within each region. However, this is not the case in reality, as the PMU equipped generators are fixed. In order to deal with this issue, the next chapter describes a method to estimate the representative RoCoF of the non-monitored regions.

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4.9 References


5. Disturbance Analysis Based on Regional Inertia

This chapter describes a new method used for disturbance analysis which uses regional inertia and RoCoF representatives of regions (Chapter 4) to detect, localise and estimate the active power disturbance size. As pointed out in section 4.8, considering the dynamic nature of region boundaries based on disturbance location, there is no guarantee that all formed regions include at least one generator equipped with PMUs. Therefore the first logical step is forming a methodology to estimate the RoCoF representative of non-monitored regions using monitored generators RoCoF immediately following a disturbance.

Section 5.1 discusses the importance of fast reliable disturbance analysis as a prerequisite for designing advanced frequency control schemes (e.g. AUFLS) in low inertia power systems. Section 5.2 introduces a methodology for estimating RoCoF of unmonitored generators using monitored generators RoCoF following an active power disturbance. The methodology for the novel regional inertia based disturbance analysis is then described in Section 5.3. Section 5.4 includes the disturbance analysis results for IEEE 39 bus system considering different scenarios and case studies. Finally the chapter is summarised in Section 5.5.

5.1 Chapter Introduction

The ever-growing replacement of conventional generating units by non-synchronously connected source of power generation (e.g. wind farms and photovoltaic power plants) would have a significant impact on system frequency stability in the upcoming decades [1]. A study carried out by Irish regulators identified a serious frequency stability downgrade by wind generation penetration level of 60-70% [2]. In this sense, Under Frequency Load Shedding Schemes (AUFLS) as a subcategory of System Integrity Protection Schemes (SIPS) [3] have been addressed as a plausible solution for arresting frequency decline. Although, the majority of todays practiced UFLS follow traditional fixed approach, with an increased availability of PMU supported real time data, more research is directed towards Adaptive Under Frequency Load Shedding Schemes (AUFLS) [4]. Therefore, the potential role of Wide Area Monitoring Systems (WAMS) in supporting adaptive frequency control schemes in future low inertia systems have gained more interest during recent years [5][6][7].
The term “adaptive” in frequency control can refer to two different type of information:

1. Active power disturbance information, e.g. disturbance time, size and location
2. Equation parameters dictating dynamic system frequency response (SFR) [8]

The first information type can only be available once an active power disturbance takes place and the process for extracting this information is referred to as disturbance analysis. Due to the decreased time available for implementing adaptive frequency control in low inertia systems, fast and accurate disturbance analysis has become an area of ongoing research [9][10][11]. Despite the trend towards wide spread installation of PMUs in the recent years [12], the cost of total system observability via PMUs (e.g. equipping every single generator with one) is yet to be justified [13]. Besides, more input data from PMUs coincide with more communicational delays and complexity, which in return decreases the confidence of TSOs in WAMS supported frequency control schemes, e.g. AUFLS. Hence, disturbance analysis is more desirable when dependent on less PMU measurements.

As mentioned in Chapter 1, disturbance analysis referred to in this thesis produces the following results:

1. Disturbance time
2. Disturbance location
3. Disturbance size

A disturbance analysis method dependent on generator active power measurements of a limited number of generators and synchronising power coefficient matrix was introduced as the SPC method and verified through simulations in Chapter 4. However, despite the appeal of using synchronised active power measurements as input for disturbance analysis as discussed in Chapter 3, at the moment in most power systems infrastructure for analysing synchronised frequency measurements are more developed, e.g. FNET [14]. Therefore, a disturbance analysis method using limited number of frequency measurements as inputs has its own merits.

This Chapter introduces a novel disturbance analysis method based on Regional Inertia which was discussed in Chapter 4.
5.2 Estimation of Missing Generators RoCoFs

In the case of having limited fixed locations for generators equipped with PMU, there is no guarantee that the regions formed using adaptive regioning method (section 4.6) would have at least one PMU reading in each region for all disturbance locations. The reason behind this is the dynamic nature of regions boundaries based on disturbance locations. Furthermore, in the process of making up the $\hat{f}_{\text{CORIF}}$ for disturbance size estimation, all regions need to have a representative RoCoF. Therefore, there exists a need to estimate the missing region representative RoCoFs using the monitored generators RoCoFs.

Assuming an active power disturbance of size $P_d$ at bus $u$ and time $t_d$, Using (4.24) the RoCoF of generators $v$ and $w$ shortly following the disturbance can be calculated as follows:

$$
\hat{f}_v(t_d^+) = \frac{\left(\frac{K_{vu}^u}{\sum_{j=1}^{M} K_{jvu}^u}\right)P_d^u}{2H_{v,sys}} \quad v, w = 1, 2, ..., M
$$

(5.1)

$$
\hat{f}_w(t_d^+) = \frac{\left(\frac{K_{wu}^u}{\sum_{j=1}^{M} K_{jwu}^u}\right)P_d^u}{2H_{w,sys}}
$$

In the case of having generator $v$ RoCoF measured shortly following the disturbance, based on (5.1) generator $w$ RoCoF can be estimated as follows:

$$
\hat{f}_w(t_d + \varepsilon) = \frac{H_{v,sys} K_{vu}^u}{H_{w,sys} K_{wu}^u} \hat{f}_v(t_d + \varepsilon)
$$

(5.2)

Firstly, as seen in (5.2), the synchronising power coefficient terms of generators ($K_{wu}^u$ and $K_{vu}^u$) are in relation to the disturbance location. Therefore, (5.2) can only be used for generators RoCoF estimation when the disturbance location is known.

Secondly, as shown in (5.2), the generators RoCoF estimate using this procedure is only valid shortly following the disturbance occurrence (for $t_d^+$).
As mentioned at the end of Section 2.8, the RoCoF is calculated for each case study using generators rotating speed coming from DIgSILENT as the frequency signal. For which RoCoFs are obtained by calculating the rate of change between two subsequent frequency output points (time step being 0.01s).

**5.2.1 RoCoF Estimate Matrix Definition**

As shown in section 5.2, any synchronous generators RoCoF can be estimated using any other synchronous generators RoCoF given the two following conditions are both met:

1. Disturbance location is known
2. RoCoF estimation is done shortly following the disturbance time

By applying a reverse engineering procedure, the above conditions can be used to detect and localise an active power disturbance. In other words, at any point in time if a generator RoCoF estimated using different monitored generators RoCoF yields relatively similar results for an assumed disturbance location, a disturbance has taken place in the that particular location.

Following this argument, a matrix called the RoCoF estimate matrix ($R_{es}^i(t)$) for each monitored generator $i$ at each time step ($t$) can be calculated. The number of RoCoF estimate matrices calculated at each time PMU readings are being updated (every 20ms), is equal to the number of monitored generators and the subscript $i$ in $R_{es}^i(t)$ represents the monitored generator number. Each $R_{es}^i(t)$ has $M$ (number of synchronous generators) rows and $N$ (number of buses) columns. The general form of RoCoF estimate matrix at time $t$ is as follows:

$$R_{es}^i(t) = \begin{bmatrix} R_{es,11}^i(t) & \cdots & R_{es,1N}^i(t) \\ \vdots & \ddots & \vdots \\ R_{es,M1}^i(t) & \cdots & R_{es,MN}^i(t) \end{bmatrix}_{M \times N} \quad \forall i \in S$$  \hspace{1cm} (5.3)

Where $S$ is the set of monitored generator and $t$ is the time for which the RoCoF estimate matrix is calculated. $R_{es}^i(t)$ is updated when a new set of PMU measurements from generators is available.
Using (5.2) each element of $\mathbf{R}^i_{es}(t)$ matrix is calculated as follows:

$$
R'_{es,jn}(t) = \left( \frac{H_{i,sys}K^n_{jn}}{H_{j,sys}K^n_{in}} \right) \dot{f}_i(t)
$$

(5.4)

Where $\dot{f}_i(t)$ is the RoCoF measurement of monitored generator $i$ at time $t$. $H_{i,sys}$ is generator $i$ inertia in system MVA base and $H_{j,sys}$ is generator $j$ inertia in system MVA base. $K^n_{jn}$ is synchronising power coefficient between generator $j$ and bus $n$ and $K^n_{in}$ is synchronising power coefficient between generator $i$ and bus $n$. Furthermore, the superscript $n$ in $K^n_{jn}$ and $K^n_{in}$ is representing the fact that these synchronising power coefficients are calculated for a system that is reduced to internal synchronous nodes and bus $n$ (Section 3.3.2).

In the case where a disturbance has taken place at time $t$ and bus $n$, the element $R^i_{es,jn}(t)$ would be equal to generator $j$ RoCoF at time $t$.

Generally matrix $\mathbf{R}^i_{es}(t)$ includes a set of generators RoCoF estimates using monitored generator $i$ at time $t$. Each row in $\mathbf{R}^i_{es}(t)$ represents one synchronous generator and each column one hypothetical disturbance location. Meaning in order to calculate column $n$ of $\mathbf{R}^i_{es}(t)$, first it is assumed that a disturbance has taken place at time $t$ and at bus $n$. Secondly each element of that column is calculated as the RoCoF estimate corresponding to that generator number (row number) for that particular disturbance location.

At any point in time the following holds true for matrix $\mathbf{R}^i_{es}(t)$:

$$
R'_{es,jn}(t) = \dot{f}_i(t) \quad \text{for } i = j
$$

(5.5)

Meaning for each $\mathbf{R}^i_{es}(t)$ matrix, a single row where $i=j$ would have constant elements that are equal to the RoCoF readings of generator $i$ at time $t$. In other words, based on (5.4), using a generator RoCoF measurement to estimate its own RoCoF leads to having the estimated RoCoF equal to the measurement.
5.2.2 Estimation of Representative RoCoFs of the Regions

As pointed out in Section 5, following an active power disturbance and in the case of limited generators being equipped with PMUs, it is highly likely that not all regions would have RoCoF readings and instead some might have more than one RoCoF readings. Therefore, there exists a need to define a procedure for calculating representative RoCoF of the regions in these two scenarios.

The representative regional RoCoFs can be put in a vector form \( \tilde{f}_R(t) \) with \( nc \) element, where \( nc \) is the number of regions, and each element \( \tilde{f}_R(t) \) is the RoCoF representative of region \( i \).

\[
\tilde{f}_R(t) = \left[ \tilde{f}_{R_1}(t) \quad \cdots \quad \tilde{f}_{R_{nc}}(t) \right]_{1 \times nc}
\]  

(5.6)

As stated, each region representative RoCoF falls into either of the following two categories:

1. The region has one or more than one RoCoF measurement
2. The region has no RoCoF measurements

In order to estimate the representative RoCoF for these two scenarios the following is suggested:

\[
\tilde{f}_{R_i}(t) = \begin{cases} 
\frac{\sum_{g \in R_i \cap S} \tilde{f}_g(t)}{|R_i \cap S|} & R_i \cap S \neq \emptyset \\
\frac{\sum_{g \in S} R^{e}_{\text{es,eq}}(t)}{|S|} & R_i \cap S = \emptyset 
\end{cases}
\]

(5.7)

Where \( S \) is the set of monitored generators and \( R_i \) is the set of generators belonging to region \( i \), \( q \) is the disturbance location and \( o \) is the medoid generator of region \( i \).

Based on (5.7), when there is at least one RoCoF readings in region \( i \) (the first line in (5.7)), the representative RoCoF of the region would be equal to the average of the readings in that region. On the other hand, in the case where there is no readings in region \( i \) (the second line in (5.40)), the representative RoCoF of the region would be equal to the average of the RoCoF estimate for the generator medoid (o) of that region using all monitored generators.
(g ∈ S) RoCoFs. The RoCoF estimate is done assuming disturbance location is known (bus q).

### 5.3 Methodology

In this section the methodology used for detection, localization and estimation of disturbance size based on Regional Inertia is introduced, which is summarized as Regional Inertia (RI) based disturbance identification method.

The inputs to the RI method are as follows:

**Input 1)** RoCoF measurements from a limited number of generators (\( \hat{f}(i) \)) – from PMUS

**Input 2)** Inertia values of all synchronous generators in a common S base. (H)

**Input 3)** Impedance matrix (Z) and load flow results (LF) for the normal operating condition – from SCADA/EMS.

Input 1 updates at the reporting rate of the PMUs, e.g. every 20ms. Input 2 and 3 update at the rate of the SCADA/EMS system, which is usually in the order of few seconds [18]. The impedance matrix (Z) is estimated in EMS as an output of topology estimator and voltage and angles of buses (LF) are estimated as a part of state estimation in EMS. The RI method updates with each new set of PMU measurements and uses the most recent Z, LF and H. The RI method has two stages:

**Stage 1)** Disturbance detection

**Stage 2)** Disturbance size and location estimation

Stage 1 is a continuous scan of some variables calculated using method inputs, which are further discussed in section 5.3.1 and 5.3.2. Stage 1 is executed at the same rate as PMU measurements readings, which is every 20ms. The goal of this stage is the detection of active
power disturbance, meaning checking whether a disturbance has taken place at the time where the PMU measurements are taken.

Stage 2 of the RI method however, is only executed when a disturbance is detected at stage 1. The goal of this stage is to estimate the location and the size of the disturbance.

5.3.1 Mismatch and Normalised mismatch vectors

As shown in section 5.2.1, the equation (5.2) holds true only for disturbance time and shortly after. Therefore, one procedure for detecting the disturbance would be to explore the validity of equation (5.2). In order to do so, the difference between RoCoF estimated for a single generator using all different monitored generators have to be enumerated.

The RoCoF estimate matrices $R_{i}^{es}(t)$ defined in section 5.2.1 can be used to calculate the difference between RoCoF estimation for any single generator at time $t$. At each time step ($t$) the RoCoF estimation for each generator is done multiple times using every single monitored generators RoCoF. This procedure is done by assuming a disturbance has occurred and varying the hypothetical disturbance location.

The member of family of matrices $R_{i}^{es}(t)$ (section 5.2.1), are characterized by the superscript $i$ which corresponds to the number of the particular monitored generator RoCoF that is used to estimate all other generators RoCoF for that matrix. Therefore, if the set of monitored generators is $S$ and number of monitored generators is $R$, $R$ RoCoF estimate matrices ($R_{i}^{es}(t)$) are calculated at each time step. As discussed in section 5.2.1, each column of $R_{i}^{es}(t)$, stands for one hypothetical disturbance location and each row stands for one synchronous generator. The family of $R_{i}^{es}(t)$ matrices are referred to as $R_{es}(t)$.

$$R_{es}(t) = \{R_{es}^{i}(t)\} | i \in S \}$$

As shown in section 5.2.1, in the case where a disturbance has taken place at time $t$ and bus $n$, and in an ideal condition (e.g zero noise, perfect network parameter), the column $n$ of all matrices in $R_{es}(t)$ would be the same and equal to the actual generators RoCoF at time $t$, for both monitored and unmonitored generators. Meaning, shortly following a disturbance
occurrence and for the hypothetical disturbance location corresponding to the actual disturbance location, all monitored generators give similar estimate of all the other generators. This behaviour can be utilised to detect and localise a disturbance.

However, as the existence of noise in RoCoF measurements and error in system parameter is inevitable, the elements in the column corresponding to disturbance location for matrices in $R_{es}(t)$ would not be exactly the same. Therefore, similarity of elements should be enumerated to serve as an indicator for disturbance detection and later on disturbance localisation. To this end, vector called mismatch vector $R_{mis}(t)$ is defined as follows:

$$R_{mis}(t) = \begin{bmatrix} R_{mis,1}(t) & \cdots & R_{mis,N}(t) \end{bmatrix}_{1 \times N} \quad (5.9)$$

Where the number of elements in $R_{mis}(t)$ is equal to the number of buses ($N$).

Element $R_{mis,n}(t)$ is defined as the average of the maximum difference between monitored generators RoCoF estimate using all monitored generators RoCoF measurements for time $t$ and location $n$. The formula for calculation of $R_{mis,n}(t)$ element is as follows:

$$R_{mis,n}(t) = \frac{1}{R} \sum_{i \in S} \max_{j,k \in S} \left| R_{es,in}^j(t) - R_{es,in}^k(t) \right|$$

$$\quad \quad \quad \quad \quad (5.10)$$

Where $R$ is the number of monitored generators and $S$ is the set of monitored generators.

In an ideal case with no measurement noise or error in system parameter the $R_{mis,n}(t)$ element corresponding to disturbance location would be equal to zero at the disturbance time. Therefore, where zero is detected in an element of $R_{mis}(t)$ vector, a disturbance would be recorded and the location of the disturbance would be the zero element index in $R_{mis}(t)$ vector. Nevertheless, in the presence of noise and error more rigorous approach should be implemented and the relatively small element of $R_{mis}(t)$ should be detected. Furthermore, in order to make $R_{mis}(t)$ independent of the disturbance size/generators RoCoF size, a normalised mismatch vector $B(t)$ is defined:

$$B(t) = \begin{bmatrix} B_1(t) & \cdots & B_N(t) \end{bmatrix}_{1 \times N} \quad (5.11)$$
Where each element is calculated as follows by:

\[
B_n(t) = \frac{1}{R} \sum_{i \in S} \left( \frac{\max_{j,k \in S} \left( R_{st,in}^j(t) - R_{st,in}^k(t) \right)}{\sum_{i \in S} R_{st,in}^i(t) \frac{1}{R}} \right) \quad (5.12)
\]

By comparing (5.12) and (5.10), it can be seen that each element in the mismatch vector \( R_{\text{mis}}(t) \) is just divided by the average of all monitored generators RoCoF estimate to make up element in normalised mismatch vector \( B(t) \).

These two vectors \( R_{\text{mis}}(t) \) and \( B(t) \) are defined as they exhibit behaviour, that can be used to determine disturbance occurrence and disturbance location, which is further discussed in the following sections.

### 5.3.2 Mismatch Indices

Detecting a disturbance and identifying its location by simply using a fixed threshold on the minimum value of \( R_{\text{mis}}(t) \) and \( B(t) \) are not ideal solutions. The problem arises due to the effect of noise and errors on the range of elements on these two mismatch vectors which in return reduces the robustness of fixed threshold approach. In order to solve this issue, two relative mismatch indices \( \zeta(t) \) (based on \( R_{\text{mis}}(t) \)) \( \gamma(t) \) (based on \( B(t) \)) are defined:

\[
\zeta(t) = \min_{n \in \{1, ..., N\}} \left( R_{\text{mis},n}(t) \right) / \max_{n \in \{1, ..., N\}} \left( R_{\text{mis},n}(t) \right) \quad (5.13)
\]

\[
\gamma(t) = \min_{n \in \{1, ..., N\}} \left( B_n(t) \right) / \max_{n \in \{1, ..., N\}} \left( B_n(t) \right) \quad (5.14)
\]

Dividing the minimum by maximum for the mismatch vectors \( R_{\text{mis}}(t) \), \( B(t) \), helps alleviating the effect of noise and error when it comes to setting a threshold for disturbance detection. This is done to turn the absolute minimum to a minimum which is relative to the maximum value. As an example, in case of larger disturbances and higher noise in active power measurements or error in Z matrix, the overall values in mismatch vectors would be
larger. Therefore, having a minimum that is relative to the maximum value would help generalising the detection stage.

These mismatch indices serve as the basis for the detection, localization and sizing of the disturbance, which is described below.

5.3.3 Decision Tree Based Disturbance Detection (Stage 1)

The main idea for disturbance detection stage of the RI method is based on detecting a relatively low level of mismatch between the RoCoF estimate of monitored generators using each of the monitored generators for a specific location \( n \) at time \( t \) by analysing the Mismatch indices \( \zeta(t) \) (5.13) and \( \gamma(t) \) (5.14). A decision tree (DT) [19] can be trained offline to establish the mapping relationship between the mismatch indices (as the two predictors) and the status of the system, i.e. if a disturbance has occurred, which is the output of the DT. When designing the DT for disturbance detection three requirements must be considered:

1) Detecting when a disturbance occurs.

2) Avoiding repeated detection of the disturbance following the initial detection.

3) Avoiding false detections under normal conditions.

Therefore, in this section, the training of the DT is based on a database built offline for a range of disturbance sizes and locations using data from three different time periods \( t<t_d, \ t=t_d, \ t>t_d \). The algorithm for finding the optimal cuts was developed using the CART algorithm [20]. Each case in the database contains the predictor values \( (\zeta(t) \text{ and } \gamma(t)) \) and the case output in terms of disturbance occurrence. The training set conditions are:

- Disturbance size \( (P_d) \): varied between 40 to 80 MW with 10 MW steps
- Monitored generators: Generators 1, 2 and 9
- Disturbance location: all buses for each disturbance size
- The RoCoF measurements had 1% white noise

Meaning mismatch indices \( (\zeta(t) \text{ (5.13) and } \gamma(t) \text{ (5.14)}) \) have been calculated for three different time instances, for each disturbance location (39 buses) and for all disturbance
sizes (40, 50, 60, 70 and 80MW). Furthermore, the procedure is repeated 10 times with different noise values added to monitored RoCoF measurements chosen from a uniformly distributed noise distribution with the range equal to the 1% of the actual monitored RoCoF.

Therefore the number of points analysed for building the DT is equal to \(5850 \times 3 \times 39 \times 5 \times 10 = 5850\).

The resulting DT is shown in Figure 5-1, where \(\gamma_{\text{cut}}\) and \(\zeta_{\text{cut}}\) are shown in Figure 5-1 and are 0.0532 and 0.0015 respectively.

![Decision Tree](image)

**Figure 5-1: Decision Tree used for disturbance detection of RI method**

Figure 5-2, shows the \(\gamma(t)\) and \(\zeta(t)\) as the x and y axis and each point corresponds to one simulation case for the IEEE 39 bus system. It can be seen in Figure 5-2, the time of no disturbance (\(t < t_d\)) can be effectively classified by passing \(\gamma(t)\) and \(\zeta(t)\) to the simple decision tree shown in Figure 5-1. Based on Figure 5-2, using only one of the Mismatch indices (\(\gamma(t)\) and \(\zeta(t)\)), and putting the threshold on that index would lead to higher misclassification compared to the case of using both of them.

Based on Figure 5-2, where \(\gamma(t)\) and \(\zeta(t)\) calculated at time \(t\) falls in the rectangle on the top right corner of Figure 5-2, no disturbance is detected. Otherwise, a disturbance would be detected.

Furthermore, as seen in Figure 5-2, two time instances of \((t = t_d, \ t > t_d)\) can-not be distinguished in this method. Therefore, in order to avoid repeated detection of disturbance in the short
time period following $t_d$, the disturbance detection would be blocked for few time steps following any disturbance detection in time.

Figure 5-2: Disturbance detection classification based on $\gamma(t)$ and $\zeta(t)$ (G1, G2, and G9 are monitored)

Figure 5-3 and Figure 5-4, show the disturbance relative time instance classification results for having 4 generators (G1, G2, G9 and G5) and 2 generators (G1 and G6) monitored respectively.

By comparing Figure 5-2 and Figure 5-3, it can be concluded that increasing the number of monitored generators from three to four, causes the non-disturbance class ($t<t_d$) to further separate from disturbance classes ($t=t_d$ and $t>t_d$). Furthermore by comparing Figure 5-2 and Figure 5-4, it can be seen that reducing the number of monitored generators from three to two caused the non-disturbance class ($t<t_d$) to blend more with the two disturbance classes ($t=t_d$ and $t>t_d$). Meaning, disturbance detection using DT becomes more accurate as the number of monitored generators is increased.
Table 5-1 summarises the disturbance detection results for different number of monitored generators depicted in Figure 5-2, Figure 5-3 and Figure 5-4. The results are shown as the percentage of the times a disturbance is wrongly detected and the percentage of times the detection of a disturbance is missed.

Table 5-1: Disturbance detection results for different number of monitored generators

<table>
<thead>
<tr>
<th>Generators Monitored</th>
<th>G1, G6</th>
<th>G1, G2, G9</th>
<th>G1, G2, G9,G5</th>
</tr>
</thead>
<tbody>
<tr>
<td>False Detection $t &lt; t_d$</td>
<td>29.2308%</td>
<td>3.3333%</td>
<td>0.359%</td>
</tr>
<tr>
<td>Missed Detection ($t = t_d$ &amp; $t &gt; t_d$)</td>
<td>1.4615%</td>
<td>0.0513%</td>
<td>0%</td>
</tr>
</tbody>
</table>
5.3.4 Disturbance Localisation (Stage 2)

Following the detection of a disturbance in Stage 1, the most likely candidate for the location of the disturbance would be the bus with the lowest mismatch between the monitored generators at $t_d$. This can be formulated as follows:

$$\text{Disturbance Location} = \arg \min_{n \in [1,..,N]} R_{\text{mis},n}(t_d)$$  \hspace{1cm} (5.15)

However, measurement noise or errors in the $Z$ matrix or inertias of the generators might cause this simple approach to give a neighboring bus as the disturbance location, particularly when the buses are electrically close. Therefore, in order to ensure that the method chooses a small set of possible locations that will most likely contain the disturbance location, a multiplier ($1 < \lambda$) is introduced which allows the method to identify a set of buses as Disturbance Location Candidates (DLC) as follows:

$$\text{DLC} = \{ n \in [1,..,N] | R_{\text{mis},n}(t_d) < \lambda \left( \min_{n \in [1,..,N]} (R_{\text{mis},n}(t_d)) \right) \}$$  \hspace{1cm} (5.16)

Where by setting $\lambda$ as 1, (5.16) would be equal to (5.15) and DLC would have only one member.

However, increasing $\lambda$ in (5.16) would increase the likelihood that the true disturbance location is within the set of DLC; nevertheless, increasing $\lambda$ will also increase the number of DLC members. Therefore, when choosing $\lambda$, there exists a compromise between ensuring the true disturbance location is within the set of DLC and limiting the number of DLC members.

5.3.5 Disturbance Size Estimation (Stage 2)

Having determined the disturbance time and location the final aspect of the method is to estimate the disturbance size ($P_{\text{est}}$). In order to estimate the disturbance size, the RoCoF of the centre of regional inertia ($\dot{f}_{\text{CORI}}$) (section 4.3.4) should be calculated first.
Following the localisation of a detected disturbance, the monitored generators can be assigned to different regions using the offline regioning results discussed in section 4.6.2. In order to do so, firstly, the location with the minimum element in vector $R_{\text{mis,\text{e}}}(t_d)$ is identified as $q$:

$$q = \arg\min_{n=1,\ldots,N} R_{\text{mis,\text{e}}}(t_d)$$  \hspace{1cm} (5.17)

Secondly, the $q$ column in offline calculated Cluster matrix ($\text{CL}^{nc}$) is used to determine the regions membership ($R$) for the particular disturbance. $R$ is calculated as follows:

$$R = \{R_1, R_2, \ldots, R_{nc}\}$$ \hspace{1cm} (5.18)

$$R_i = \{j | \text{CL}^{nc}_{ij} = i\} \hspace{1cm} i = 1, 2, \ldots, nc$$ \hspace{1cm} (5.19)

Where $nc$ is the number of clusters/regions.

Having $R$, the regional inertia values ($H_R$) (section 4.3.2) are then calculated as follows:

$$H_R = \{H_{R_1}, H_{R_2}, \ldots, H_{R_{nc}}\}$$ \hspace{1cm} (5.20)

$$H_{R_i} = \sum_{j \in R_i} H_{j} S_{\text{base,\text{j}}} \hspace{1cm} i = 1, 2, \ldots, nc$$ \hspace{1cm} (5.21)

In the next step and as discussed in section 5.2.2 ((5.6) and (5.7)), the representative regional RoCoFs vector $\tilde{f}_R(t)$, can be calculated as follows:

$$\tilde{f}_R(t) = \begin{bmatrix} \dot{f}_{R_1}(t) & \cdots & \dot{f}_{R_{nc}}(t) \end{bmatrix}_{\text{vec}}$$ \hspace{1cm} (5.22)

$$\dot{f}_{R_i}(t) = \begin{cases} \sum_{g \in R_i \cap S} \dot{f}_g(t) & R_i \cap S \neq \emptyset \\
\frac{|R_i \cap S|}{\sum_{g \in S} R_{\text{ex,\text{g}}}(t)} & R_i \cap S = \emptyset \end{cases}$$ \hspace{1cm} (5.23)
Having vector $\mathbf{f}_R(t)$ and $\mathbf{H}_R$ as it was discussed in section 4.3.4, RoCoF of the centre of regional inertia ($\dot{f}_{CORI}$) can be calculated as follows:

$$\dot{f}_{CORI}(t) = \frac{\sum_{i=1}^{nc} H_{R_i} \dot{f}_{R_i}(t)}{\sum_{i=1}^{nc} H_{R_i}}$$

(5.24)

By substituting $\dot{f}_{COI}$ by $\dot{f}_{CORI}$ in the swing equation (2.11), for $t=t_d^+$ the disturbance size can be estimated as follows:

$$P_{est} = 2H_{Sys} \dot{f}_{CORI}(t_d^+)$$

(5.25)

Where $H_{Sys}$ is the summation of all generators inertia times their $S_{base}$ as is defined in (4.9).

### 5.3.6 General Flow Chart for Regional Based Disturbance Identification

A flowchart of the RI method is given in Figure 5-5 which summarises the whole RI methodology discussed earlier. The dotted arrows in the flowchart represent the slower rate of data update (up to the order of few minutes) and the solid arrows the faster rate of data update (every 20ms).
The “Calculate $P_{est}$” box shown in Figure 5-5 and indicated as stage 2 (disturbance size estimation) is further expanded and shown in flowchart form in Figure 5-6.

This flowchart can have either three or zero outputs which are presented by circles in Figure 5-5. These three outputs only exist if a disturbance is detected. The outputs are disturbance time ($t_d$), Disturbance Location Candidates (DLC) and disturbance size estimate $P_{est}$. Disturbance time ($t_d$) output is basically the time stamp of the PMU measurements for which a disturbance is detected.
5.4 Results

Simulations of the IEEE 39-bus test system, as shown in Figure 4-3, are used here to verify the performance of the proposed method (RI). The following metrics are used for evaluating the performance of the RI method from statistical point of view.

5.4.1 Worked Example

In this example the disturbance is a 50 MW load increase at bus 12. PMUs have been placed at generators 1, 2 and 9 (S = \{G1, G2, G9\}). The level of white noise in the PMU measurements is 1 %. The time interval for the measurements is 0.02 s (\(\Delta t\)).

The generators inertia \(\mathbf{H}\) and \(S_{\text{base}}\) are shown in Table 5-2.
Table 5-2: IEEE 39 bus generators inertia and $S_{base}$

<table>
<thead>
<tr>
<th>Generators</th>
<th>G1</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
<th>G7</th>
<th>G8</th>
<th>G9</th>
<th>G10</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (pu/s)</td>
<td>50</td>
<td>3.03</td>
<td>3.58</td>
<td>2.86</td>
<td>2.6</td>
<td>3.48</td>
<td>2.64</td>
<td>2.43</td>
<td>3.45</td>
<td>4.2</td>
</tr>
<tr>
<td>$S_{base}$ (GW)</td>
<td>1.4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The $R_{mis}(t)$ and $B(t)$ vectors have been calculated (Figure 5-7 and Figure 5-8) using (5.10) and (5.12) for three time instances: $t_d-\Delta t$, $t_d$ and $t_d+\Delta t$. The $\lambda$ in (5.16) is set to 1.4.

Stage 1: $\gamma (t_d)$ and $\zeta (t_d)$ are calculated using (5.14) and (5.13) at each time instance and the results are shown in Table 5-3. From Table 5-3 it can be seen that the disturbance is correctly detected at $t=t_d$ using DT (section 5.3.3).
Table 5-3: Disturbance detection results for different number of monitored generators

<table>
<thead>
<tr>
<th>γ (t_d−Δt)</th>
<th>ζ (t_d−Δt)</th>
<th>γ (t_d)</th>
<th>ζ (t_d)</th>
<th>γ (t_d+Δt)</th>
<th>ζ (t_d+Δt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4640</td>
<td>0.0607</td>
<td>0.0282</td>
<td>0.0029</td>
<td>0.0295</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

Stage 2: Following the successful detection of disturbance at stage 1 using DT, DLC are chosen by applying (5.16) to $R_{mis,n}(t_d)$ values shown in Figure 5-9. In other words, in, the locations with $R_{mis,n}(t_d)$ values below the dotted line are chosen as DLC members, where the dotted line in marks the value equal to 0.00168 (5.16). In this case only bus 12 (the actual disturbance location) is selected.

![Figure 5-9: $R_{mis,n}(t_d)$ values for different bus locations](image-url)
Figure 5-10: $R_{es}(t_d)$ values for different monitored generators and disturbance locations

Figure 5-10, shows the RoCoF estimate values ($R_{es}(t_d)$) at time $t_d$ using three monitored generators. The top graph in Figure 5-10, shows the RoCoF estimation for generator 1 using all three monitored generators (G1, G2 and G9) and assuming all different possible disturbance locations. The middle graph in Figure 5-10, shows the RoCoF estimation for generator 2 using all three monitored generators (G1, G2 and G9) and assuming all different possible disturbance locations. The bottom graph in Figure 5-10, shows the RoCoF estimation for generator 9 using all three monitored generators (G1, G2 and G9) and assuming all different possible disturbance locations. As it can be seen in Figure 5-10, the location with the highest level of agreement between all three monitored generators RoCoF estimates is bus 12, where all three points almost coincide in all three graphs shown in Figure 5-10. Therefore, in (5.17) q=12. Using section 5.3.5, the regions membership $\mathbf{R}$, regional inertias $\mathbf{H}_R$ and representative RoCoF of the regions $\hat{f}_R(\tau)$ have been calculated and presented in Table 5-4.
Table 5-4: Regions membership, Regional inertias and RoCoF representatives of regions

<table>
<thead>
<tr>
<th>R</th>
<th>$R_1$=G6,G7</th>
<th>$R_2$=G5,G8,G10</th>
<th>$R_3$=G1</th>
<th>$R_4$=G9</th>
<th>$R_5$=G4</th>
<th>$R_6$=G2,G3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_R$(GWs)</td>
<td>6.12</td>
<td>9.23</td>
<td>70</td>
<td>4.14</td>
<td>2.86</td>
<td>6.61</td>
</tr>
<tr>
<td>$\dot{f}_m(t)$ (Hz)</td>
<td>-0.044</td>
<td>-0.3</td>
<td>-0.0034</td>
<td>-0.0224</td>
<td>-0.052</td>
<td>-0.035</td>
</tr>
</tbody>
</table>

Based on Table 5-4, regions 1, 2 and 5 ($R_1$, $R_2$, $R_5$) have no generators monitored, therefore in order to calculate the representative RoCoFs for these regions, the RoCoF needs to be estimated for the medoid of these regions using the monitored generators RoCoFs (section 5.2.2). The medoids of these unmonitored regions are shown as bold in Table 5-4.

The disturbance size is then estimated using the RoCoF of the centre of regional inertia $\dot{f}_{CORI}$ and regional inertias ($H_R$) using (5.24) and (5.25).

Disturbance size is estimated to be 50.33 MW, which is only a 0.655 % error in disturbance size estimation.

### 5.4.2 Performance Metrics

#### Stage 1 performance metrics:

In order to enumerate the likelihood of missing a disturbance (i.e. not detecting it) the Missed Detection metric (MD) is defined as the percentage of cases where the method misses the detection of the disturbance. The False Detection metric (FD) is defined as the percentage of cases where the method falsely detects a disturbance.

#### Stage 2 performance metrics:

The performance metric used for the disturbance size is $E_{med}$ which is equal to the median of errors in disturbance size estimation. Regarding disturbance location estimation two points are important, firstly the accuracy of the localization, i.e. if the true disturbance location is one of the Disturbance Location Candidates (DLC), secondly the precision of the disturbance localization, i.e. how many DLCs are reported by the method. The first point is
analyzed using the Right Location metric (RL), which is the percentage of cases where the true location is a DLC member; and the second point is studied by six metrics:

1) \( L_{W1}, L_{W2}, L_{W3} \) measure the percentage of cases where there are 1, 2 and 3 disturbance location candidates (DLC) (5.26)

2) \( RL_{W1}, RL_{W2}, RL_{W3} \) are equal to the percentage of cases where the number of DLC members is less than 1, 2 and 3 respectively and the disturbance location is a member of DLC (5.27).

\[
L_{W_i} = 100 \left( \frac{\text{Number of times where: } |\text{DLC}|=i}{\text{Number of times where: Disturbance Location } \in \text{DLC}} \right)
\] (5.26)

\[
RL_{W_i} = \left( \sum_{j=1}^{i} \frac{L_{W_j}}{100} \right) \text{ RL}
\] (5.27)

A worked example is now presented to illustrate the methods execution.

### 5.4.3 Comparison of Results for Traditional, Fixed Regional and Adaptive Regional method

In this section a statistical comparison analysis has been carried out between three different methods for disturbance size estimation having a varied number of PMU measurements. The methods are:

1) Traditional disturbance size estimation method using RoCoF of the centre of inertia \( \dot{f}_{cor} \) (section 2.7).

2) Fixed regional method using RoCoF of the centre of regional inertia \( \dot{f}_{cor} \) (section 4.7).

3) Adaptive regional method using RoCoF of the centre of regional inertia \( \dot{f}_{cor} \) which (section 4.6) which was also used at stage 2 of RI method (section 5.3.5).

All methods have been tested under the following conditions:
• Disturbance size ($P_d$): varied between 40 to 80 MW with 10 MW steps
• Monitored generators are varied from 2 to 10 considering all different combinations of monitored generators for each $P_d$ value and disturbance locations
• Disturbance location: all buses for each disturbance size
• The RoCoF measurements had 1% white noise

Furthermore, the procedure is repeated 10 times with different noise values added to monitored RoCoF measurements chosen from uniformly distributed noise distribution with the range equal to the 1% of the actual monitored RoCoF.

In the next step, in order to make the comparison more sensible the best set of monitored generators which yield the smallest median disturbance size estimation error for all disturbance locations have been chosen for all three methods. Therefore, the comparison is carried out for the generators sets which lead to best performance of each method. The best sets for varied number of PMUs and for all three methods are presented in Table 5-5.

Table 5-5: Monitored generators sets for best performance for different methods

<table>
<thead>
<tr>
<th>PMU Number</th>
<th>Traditional</th>
<th>Fixed Regional</th>
<th>Adaptive Regional</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>G5,G9</td>
<td>G1,G9</td>
<td>G1,G6</td>
</tr>
<tr>
<td>3</td>
<td>G1,G6,G9</td>
<td>G1,G6,G9</td>
<td>G1,G2,G9</td>
</tr>
<tr>
<td>4</td>
<td>G1,G3,G9,G10</td>
<td>G1,G2,G4,G9</td>
<td>G1,G2,G6,G9</td>
</tr>
<tr>
<td>5</td>
<td>G1,G2,G3,G8,G10</td>
<td>G1,G2,G4,G7,G9</td>
<td>G1,G2,G6,G9,G10</td>
</tr>
<tr>
<td>6</td>
<td>G1,G2,G3,G8,G9,G10</td>
<td>G1,G2,G4,G7,G8,G9</td>
<td>G1,G2,G3,G4,G6,G9</td>
</tr>
<tr>
<td>7</td>
<td>G1,G2,G3,G4,G8,G9,G10</td>
<td>G1,G2,G4,G6,G7,G8,G9</td>
<td>G1,G2,G3,G4,G6,G7,G9</td>
</tr>
<tr>
<td>8</td>
<td>G1,G2,G3,G4,G5,G6,G7,G10</td>
<td>G1,G2,G4,G6,G7,G8,G9,G10</td>
<td>G1,G4,G5,G6,G7,G8,G9,G10</td>
</tr>
<tr>
<td>9</td>
<td>G1,G2,G3,G4,G6,G7,G8,G9,G10</td>
<td>G1,G2,G4,G5,G6,G7,G8,G9,G10</td>
<td>G1,G3,G4,G5,G6,G7,G8,G9,G10</td>
</tr>
</tbody>
</table>

Based on Table 5-5, there are no particular pattern between different best set of monitored generators among the three methods. Except, for G1 being included in both adaptive regional and fixed regional method best sets for all number of PMUs. Furthermore, G1 and G9 being included in all sets for all three methods when number of PMUs gets bigger than
2. This observation can be attributed to the fact that G1 and G9 have the first and the second biggest inertia (in system base) in IEEE 39 bus system (Table 3-2 in Section 3.6.1).

The median of disturbance size estimation error ($E_{med}$) with the best set of monitored generators (Table 5-5) for all three methods and varied number of PMU measurements are shown in Figure 5-11:

Figure 5-11: Disturbance size median error for varied number of monitored generators using different methods

As it is seen in Figure 5-11, the adaptive regional method has $E_{med}$ smaller than 3% for all number of PMU measurements. Although fixed regional method shows an improvement over the traditional method, both traditional and fixed regional method have $E_{med}$ smaller than 3% only when having 9 out of 10 generators are monitored. Furthermore, both traditional and fixed regional method show a significant loss of accuracy when number of PMUs is reduced from 9. Whereas, the adaptive regional method disturbance size median error shows to be robust against missing PMU measurements. Therefore, the adaptive regional method proves to provide a more reliable and accurate disturbance size estimation when only a limited number of generators are monitored via PMUs, which is likely to be the case in practice.

Nevertheless, the adaptive regional method requires the location of disturbance as an input as opposed to the other two methods. Therefore, in the case where disturbance location is not
estimated, the fixed regional method can be used instead of the traditional method to provide more accurate results.

In the following case studies the adaptive regional disturbance size estimation method is used as the disturbance size estimation (stage 2) of RI method.

5.4.4 Case Study 1 (Impact of RoCoF Measurements Noise Level)

In this case study the impact of RoCoF measurement noise on the performance of the RI method for both stages is analysed. The conditions are as follows:

- Disturbance size ($P_d$): varied between 40 to 80 MW with 10 MW steps
- Monitored generators are varied from 2 to 5 considering the best performing set for adaptive regional method shown in Table 5-5.
- Disturbance location: all buses for each disturbance size
- Two different levels of RoCoF measurements white noise (1 % and 5%)
- $\lambda$ is set to 1.4 in (5.16).

Furthermore, the procedure is repeated 10 times with different noise values added to the monitored RoCoF measurements chosen from uniformly distributed noise distribution with the range equal to 1% and 5% of the actual monitored RoCoF.

Table 5-6 summarizes the results for the two noise levels (1 % and 5 %) regarding disturbance detection (stage 1 of RI method) and disturbance size estimation (stage 2 of RI method).

<table>
<thead>
<tr>
<th>Noise Level</th>
<th>S</th>
<th>MD</th>
<th>FD</th>
<th>$E_{med}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>1,6</td>
<td>1.4615</td>
<td>29.2308</td>
<td>1.9155</td>
</tr>
<tr>
<td></td>
<td>1,2,9</td>
<td>0.0513</td>
<td>3.3333</td>
<td>1.9743</td>
</tr>
<tr>
<td></td>
<td>1,2,5,9</td>
<td>0.0256</td>
<td>0.359</td>
<td>1.4938</td>
</tr>
<tr>
<td></td>
<td>1,2,9,5,6</td>
<td>0</td>
<td>0</td>
<td>1.2762</td>
</tr>
<tr>
<td>5%</td>
<td>1,6</td>
<td>4.359</td>
<td>33.2309</td>
<td>2.5674</td>
</tr>
<tr>
<td></td>
<td>1,2,9</td>
<td>0.2821</td>
<td>6.1258</td>
<td>2.5275</td>
</tr>
<tr>
<td></td>
<td>1,2,5,9</td>
<td>0.0513</td>
<td>0.7179</td>
<td>2.1651</td>
</tr>
<tr>
<td></td>
<td>1,2,9,5,6</td>
<td>0.0513</td>
<td>0.2051</td>
<td>1.7534</td>
</tr>
</tbody>
</table>
Based on Table 5-6, for both noise levels disturbance detection is improved, meaning MD and FD get smaller with an increased number of PMU measurements. Furthermore, again for both noise levels, disturbance estimation gets more accurate, meaning lower $E_{med}$ with an increased number of PMU measurements. At the same time, the increased noise reduces the performance level of both disturbance detection and disturbance size estimation. Nevertheless, $E_{med}$ is still smaller than 3% when having only two generators (G1, G6) monitored for both noise levels (1% and 5%).

Table 5-7 summarises the results for the two noise levels (1 % and 5 %) regarding disturbance location estimation (stage 2 of RI method).

Based on Table 5-7, the disturbance localisation performance deteriorates as noise level is increased. Lower localisation performance means lower $RL$, $L_{w1}$, $R_{lw1}$, $RL_{w2}$, $RL_{w3}$ and higher $L_{w2}$ and $L_{w3}$.

Table 5-7: Disturbance location estimation results-Case study 1

<table>
<thead>
<tr>
<th>Noise Level</th>
<th>S</th>
<th>RL</th>
<th>$L_{w1}$</th>
<th>$L_{w2}$</th>
<th>$L_{w3}$</th>
<th>$R_{lw1}$</th>
<th>$RL_{w2}$</th>
<th>$RL_{w3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>1,6</td>
<td>68.5128</td>
<td>67.6096</td>
<td>31.7539</td>
<td>0.2122</td>
<td>46.3212</td>
<td>68.0767</td>
<td>68.2221</td>
</tr>
<tr>
<td>1,2,9</td>
<td>91.5897</td>
<td>61.5342</td>
<td>13.2699</td>
<td>5.6551</td>
<td>56.3589</td>
<td>68.5128</td>
<td>73.6923</td>
<td></td>
</tr>
<tr>
<td>1,2,5,9</td>
<td>97.5897</td>
<td>73.8833</td>
<td>14.5559</td>
<td>6.8838</td>
<td>72.1024</td>
<td>86.3075</td>
<td>93.0254</td>
<td></td>
</tr>
<tr>
<td>1,2,9,5,6</td>
<td>98.2051</td>
<td>87.0496</td>
<td>8.3551</td>
<td>2.7154</td>
<td>85.4871</td>
<td>93.6922</td>
<td>96.3589</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>1,6</td>
<td>60.9231</td>
<td>60.0168</td>
<td>35.101</td>
<td>2.6094</td>
<td>36.5641</td>
<td>57.9487</td>
<td>59.5384</td>
</tr>
<tr>
<td>1,2,9</td>
<td>82.5641</td>
<td>62.1739</td>
<td>15.7764</td>
<td>5.4658</td>
<td>51.3333</td>
<td>64.3589</td>
<td>68.8717</td>
<td></td>
</tr>
<tr>
<td>1,2,5,9</td>
<td>92.3077</td>
<td>69.6667</td>
<td>17.2222</td>
<td>7.0556</td>
<td>64.3077</td>
<td>80.2051</td>
<td>86.7180</td>
<td></td>
</tr>
<tr>
<td>1,2,9,5,6</td>
<td>96.9744</td>
<td>80.0635</td>
<td>15.1772</td>
<td>3.3316</td>
<td>77.6411</td>
<td>92.3591</td>
<td>95.5899</td>
<td></td>
</tr>
</tbody>
</table>

The improved performance of the disturbance size estimation over the traditional disturbance size estimation method is even more pronounced when comparing the results for all combinations of monitored generators. The results for this comparison are shown in Figure 5-12, where the y axis is in logarithmic scale.
Figure 5-12: Disturbance size estimate error for varied number of PMU measurements, comparison between traditional method and adaptive regional method with 1% and 5% noise in measurements.

The median error ($E_{med}$) (shown by the middle line in each bar in Figure 5-12) for the traditional method is 10% when 8 PMUs are used (2 out of 10 generators not monitored). In contrast, the adaptive regional method, even with 5% noise in the measurements, has a smaller median error (8%) when only 3 PMUs are used (7 out of 10 generators not monitored).

5.4.5 Case Study 2 (Impact of Z matrix Error Level)

In this case study the impact of Z matrix error on the performance of the RI method for both stages is analysed. The conditions are as follows:

- Disturbance size ($P_d$): varied between 40 to 80 MW with 10 MW steps
- Monitored generators are varied from 2 to 5 considering the best performing set for adaptive regional method shown in Table 5-5.
- Disturbance location: all buses for each disturbance size
- $\lambda$ is set to 1.4 in (5.16).
- R and X of the lines have been subjected to errors with uniform probability distribution (1% and 5% white noise).
Furthermore, the procedure is repeated 10 times with error values added to Z matrix elements (chosen from uniformly probability distribution) with the range equal to 1% and 5% of the actual Z element.

Table 5-8 summarizes the results for the two error levels (1% and 5%) regarding disturbance detection (stage 1 of RI method) and disturbance size estimation (stage 2 of RI method).

Table 5-8: Disturbance detection and disturbance size estimation results-Case study 2

<table>
<thead>
<tr>
<th>Error Level</th>
<th>S</th>
<th>MD</th>
<th>FD</th>
<th>E_{med}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,6</td>
<td>0.4615</td>
<td>39.6923</td>
<td>2.5501</td>
<td></td>
</tr>
<tr>
<td>1,2,9</td>
<td>0.4103</td>
<td>7.1282</td>
<td>2.5220</td>
<td></td>
</tr>
<tr>
<td>1,2,5,9</td>
<td>0.2308</td>
<td>1.1282</td>
<td>2.0159</td>
<td></td>
</tr>
<tr>
<td>1,2,9,5,6</td>
<td>0.1538</td>
<td>0.4103</td>
<td>1.4731</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,6</td>
<td>1.2821</td>
<td>66.3077</td>
<td>8.6042</td>
<td></td>
</tr>
<tr>
<td>1,2,9</td>
<td>7.5128</td>
<td>13.5385</td>
<td>7.9137</td>
<td></td>
</tr>
<tr>
<td>1,2,5,9</td>
<td>10.6154</td>
<td>5.0769</td>
<td>6.2057</td>
<td></td>
</tr>
<tr>
<td>1,2,9,5,6</td>
<td>13.8462</td>
<td>3.7436</td>
<td>4.5161</td>
<td></td>
</tr>
<tr>
<td>1,2,9,5,6,10</td>
<td>5.4615</td>
<td>2</td>
<td>3.4469</td>
<td></td>
</tr>
</tbody>
</table>

Based on Table 5-8, the increased Z element error (from 1% to 5%) worsens the disturbance detection stage of the method, meaning lower MD and FD values. In the case of 1% Z matrix error level, the disturbance detection stage still performs well (MD and FD<3%) with only 4 monitored generators. However, for 5% Z matrix error level, even with 5 monitored generators MD is bigger than 3% (5.4615%). Therefore, the disturbance detection stage of RI method is not as reliable and robust to high level of error (more than 1%) in Z matrix elements.

Furthermore, based on Table 5-8, disturbance size estimation decreases in accuracy with increased levels of error in the Z matrix. However, for 1% and 5% error in the Z matrix, E_{med} is less than 5% with 2 and 5 monitored generators respectively. Therefore, the disturbance size estimation stage of RI method proves to be more reliable and robust against Z matrix error compared to the disturbance detection stage.

Table 5-9 summarizes the results for the two error levels (1% and 5%) regarding disturbance location estimation (stage 2 of RI method).
Table 5-9: Disturbance location estimation results—Case study 2

<table>
<thead>
<tr>
<th>Error Level</th>
<th>S</th>
<th>RL</th>
<th>L_{w1}</th>
<th>L_{w2}</th>
<th>L_{w3}</th>
<th>RL_{w1}</th>
<th>RL_{w2}</th>
<th>RL_{w3}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,6</td>
<td>54</td>
<td>69.8006</td>
<td>25.0712</td>
<td>4.0836</td>
<td>37.6923</td>
<td>51.2307</td>
<td>53.4359</td>
<td></td>
</tr>
<tr>
<td>1,2,9</td>
<td>69.7949</td>
<td>71.712</td>
<td>18.9566</td>
<td>5.5107</td>
<td>50.0513</td>
<td>63.2820</td>
<td>67.1282</td>
<td></td>
</tr>
<tr>
<td>1,2,5,9</td>
<td>83.8462</td>
<td>75.841</td>
<td>16.0856</td>
<td>5.0153</td>
<td>63.5897</td>
<td>77.0769</td>
<td>81.2820</td>
<td></td>
</tr>
<tr>
<td>1,2,9,5,6</td>
<td>93.3846</td>
<td>77.3751</td>
<td>18.067</td>
<td>3.0203</td>
<td>72.2564</td>
<td>89.1282</td>
<td>91.9487</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,6</td>
<td>26.359</td>
<td>65.7588</td>
<td>22.5681</td>
<td>8.3658</td>
<td>17.3333</td>
<td>23.2820</td>
<td>25.4872</td>
<td></td>
</tr>
<tr>
<td>1,2,9</td>
<td>41.9487</td>
<td>60.6357</td>
<td>21.1491</td>
<td>9.9022</td>
<td>25.4358</td>
<td>34.3076</td>
<td>38.4615</td>
<td></td>
</tr>
<tr>
<td>1,2,5,9</td>
<td>60.5641</td>
<td>51.9052</td>
<td>24.7248</td>
<td>12.0237</td>
<td>31.4359</td>
<td>46.4102</td>
<td>53.6923</td>
<td></td>
</tr>
<tr>
<td>1,2,9,5,6</td>
<td>70.8718</td>
<td>48.4081</td>
<td>27.3517</td>
<td>13.2417</td>
<td>34.3076</td>
<td>53.6923</td>
<td>63.0769</td>
<td></td>
</tr>
</tbody>
</table>

Based on Table 5-9, the disturbance localisation performance deteriorates as error level in Z matrix elements is increased. Lower localisation performance means lower RL, L_{w1}, RL_{w1}, RL_{w2}, RL_{w3} and higher L_{w2} and L_{w3}. Furthermore, by comparing Table 5-7 with Table 5-9, it can be seen that the disturbance localisation stage of the RI method is much more sensitive to error in Z matrix than it is to noise in RoCoF measurements.

5.4.6 Case Study 3 (Generator Outage Type of Disturbance)

Previously (for all other case studies) each disturbance was modelled as load increase in each bus. This was done by adding positive and negative loads with the same size of positive and negative 1MW to keep the power flows intact for pre-disturbance conditions. This approach was taken in order to facilitate application of the same disturbance size for each disturbance location. Disturbance size was then applied by assigning a load increase event to the positive 1MW load in each bus. This type of load increase events can be attributed to fast cloud transients for large PV farms or sudden disconnection of large wind farms, which can be interpreted as sudden load increase rather than generation loss as system topology and inertia would stay unchanged.

A new set of studies are carried out to investigate the performance of the RI method for generation outages rather than load increase events. In order for the RI method to detect generation loss outages, the methodology needs to be slightly altered. When experiencing a generation loss, the active power deficit equal to the size of the disconnected generator, is distributed among the remaining generators. However, in this scenario the system topology,
inertia and the synchronising power coefficients are different before and after the disturbance. Nevertheless, the initial simulations showed that using the before disturbance synchronising power coefficients and inertia values, results in estimated disturbance size which is almost equal to double the size of the active power output of the lost generator. This observation led to the alteration in RI methodology to accommodate for generator loss events. The new slightly changed version of the RI methodology is as follows:

Stage 1:
- Disturbance detection is the same as before (section 5.3.3).

Stage 2:
- Disturbance location estimation is the same as before (5.3.4).
- Disturbance size estimation would have the following changes:

In case bus \( q \) in (5.17) is a bus which has no synchronous generator connected to it, then the disturbance size estimation formula would be the same as before (section 5.3.5).

In case bus \( q \) in (5.17) is a bus with a synchronous generator connected to it, then the following formula applies:

\[
P_{est}^* = H_{Sys} \hat{f}_{CORI}(t_d^*)
\]

(5.28)

Meaning the estimated disturbance size using the previous version of the RI method \((P_{est})\) has to be divided by two to get to the new estimated disturbance size \((P_{est}^*)\).

The case study conditions are:

- Monitored generators are varied from 2 to 5.
- Disconnection of generators G2, G3, G4, G6, G10
- RoCoF measurements have 1% white noise.
- \( \lambda \) is set to 1.4 in (5.16)
Table 5-10 to Table 5-14, show the results for different generators outages when different number of generators are monitored.

Table 5-10: Generator 2 Outage Results - Case study 3

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>0</td>
<td>121.14</td>
<td>0</td>
<td>1,3,39</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>-2.69</td>
<td>0</td>
<td>1,3,39</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>2.93</td>
<td>0</td>
<td>1,3,39</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>0</td>
<td>-7.59</td>
<td>0</td>
<td>1,3,39</td>
</tr>
</tbody>
</table>

Table 5-11: Generator 3 Outage Results - Case study 3

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>0</td>
<td>5.49</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>-6.25</td>
<td>0</td>
<td>35,36,37,38,4,5,6,8</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>-6.64</td>
<td>1</td>
<td>34,35,36,37,38,4,5,6,7,8</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>0</td>
<td>-5.93</td>
<td>1</td>
<td>34,35,36,37,38,4,5,6,7,8</td>
</tr>
</tbody>
</table>

Table 5-12: Generator 4 Outage Results - Case study 3

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>0</td>
<td>5.704</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>-1.98</td>
<td>1</td>
<td>26,25,23,21,14</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>-2.23</td>
<td>1</td>
<td>26,25,23,21,14</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>0</td>
<td>-37.57</td>
<td>0</td>
<td>21,27</td>
</tr>
</tbody>
</table>

Table 5-13: Generator 6 Outage Results - Case study 3

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>0</td>
<td>3.92</td>
<td>0</td>
<td>22,20</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>-4.79</td>
<td>1</td>
<td>30,22,20</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>-5.47</td>
<td>1</td>
<td>30,28,22,20</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>0</td>
<td>-6.15</td>
<td>1</td>
<td>30,28,22,20</td>
</tr>
</tbody>
</table>

Table 5-14: Generator 10 Outage Results - Case study 3

<table>
<thead>
<tr>
<th>S</th>
<th>Detected?(0,1)</th>
<th>Error(%)</th>
<th>Right Location?(0,1)</th>
<th>DLC (bus numbers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1,G7</td>
<td>0</td>
<td>21.26</td>
<td>0</td>
<td>38</td>
</tr>
<tr>
<td>G1,G7,G8</td>
<td>0</td>
<td>1.73</td>
<td>0</td>
<td>4,5,6,7,8,9,24,34,35,36,37,38</td>
</tr>
<tr>
<td>G1,G7,G8,G9</td>
<td>0</td>
<td>0.35</td>
<td>0</td>
<td>4,5,6,7,8,9,24,34,35,36,37,38</td>
</tr>
<tr>
<td>G1,G7,G8,G9,G5</td>
<td>0</td>
<td>0.13</td>
<td>0</td>
<td>4,5,6,7,8,9,24,34,35,36,37,38</td>
</tr>
</tbody>
</table>

Based on the results for disturbance size estimation in Table 5-10 to Table 5-14, the altered RI method provides relatively low disturbance size estimation error. However, the
disturbance detection and localisation stages have significantly worse performance compared to the load disturbance type of events studied in previous case studies.

5.4.7 Case Study 4 (Large Disturbance Case)

In this case study, RI performance is analysed for a large disturbance size (850MW). The conditions are as follows:

- Monitored generators are varied from 2 to 5 considering the best performing set for adaptive regional method shown in Table 5-5.
- Disturbance location: all buses
- RoCoF measurements have 1% white noise
- $\lambda$ is set to 1.4 in (5.16).

Table 5-15 summarizes the results regarding disturbance detection (stage 1 of RI method) and disturbance size estimation (stage 2 of RI method).

Table 5-15: Disturbance detection and disturbance size estimation results-Case study 4

<table>
<thead>
<tr>
<th>Noise Level</th>
<th>S</th>
<th>MD</th>
<th>FD</th>
<th>$E_{med}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,6</td>
<td>1.4103</td>
<td>31.7949</td>
<td>4.2361</td>
<td></td>
</tr>
<tr>
<td>1,2,9</td>
<td>0.5128</td>
<td>19.2308</td>
<td>3.6259</td>
<td></td>
</tr>
<tr>
<td>1,2,5,9</td>
<td>0</td>
<td>7.6923</td>
<td>2.4056</td>
<td></td>
</tr>
<tr>
<td>1,2,9,5,6</td>
<td>0</td>
<td>4.6154</td>
<td>1.7607</td>
<td></td>
</tr>
</tbody>
</table>

By comparing Table 5-6 and Table 5-15, it can be seen that for this large disturbance size, disturbance detection has become less accurate (higher MD and FD). Furthermore, the disturbance size estimation error has also increased for this large disturbance compared to Case study 1 (with 40MW to 80MW disturbance sizes). Table 5-16 summarises the results for disturbance location estimation (stage 2 of RI method).

Table 5-16: Disturbance location estimation results-Case study 4

<table>
<thead>
<tr>
<th>Noise Level</th>
<th>S</th>
<th>RL</th>
<th>$L_{w1}$</th>
<th>$L_{w2}$</th>
<th>$L_{w3}$</th>
<th>$RL_{w1}$</th>
<th>$RL_{w2}$</th>
<th>$RL_{w3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,6</td>
<td>30.5128</td>
<td>45.3782</td>
<td>36.9748</td>
<td>11.7647</td>
<td>13.8462</td>
<td>25.1282</td>
<td>28.7179</td>
<td></td>
</tr>
<tr>
<td>1,2,9</td>
<td>65.8974</td>
<td>59.1440</td>
<td>19.0661</td>
<td>7.0039</td>
<td>38.9744</td>
<td>51.5384</td>
<td>56.1538</td>
<td></td>
</tr>
<tr>
<td>1,2,5,9</td>
<td>80.5128</td>
<td>67.8344</td>
<td>15.2866</td>
<td>11.1465</td>
<td>54.6154</td>
<td>63.5897</td>
<td>75.8974</td>
<td></td>
</tr>
<tr>
<td>1,2,9,5,6</td>
<td>87.9487</td>
<td>75.2187</td>
<td>14.5773</td>
<td>5.5394</td>
<td>66.1538</td>
<td>78.9744</td>
<td>83.8462</td>
<td></td>
</tr>
</tbody>
</table>
By comparing Table 5-7 and Table 5-16 it can be seen that for this large disturbance size, the accuracy and precision of the localization have deteriorated. Lower localisation performance means lower $RL$, $L_{w1}$, $RL_{w2}$, $RL_{w3}$ and higher $L_{w2}$ and $L_{w3}$. Nevertheless, despite the worsened performance of the RI method for the large disturbance size, it still provides acceptable accuracy in all stages.

5.4.8 Comparison of Results for Traditional, SPC and Adaptive Regional method

In this section a statistical comparison analysis has been carried out between three different methods for disturbance size estimation having a varied number of PMU measurements. The methods are:

4) Traditional disturbance size estimation method using RoCoF of the centre of inertia $\dot{f}_{COI}$ (section 2.7).
5) Synchronising Power Coefficient Based (SPC) method (Chapter 3).
6) Adaptive regional method using RoCoF of the centre of regional inertia $\dot{f}_{CORE}$ (section 4.6) which was also used at stage 2 of RI method (Section 5.3.5).

All methods have been tested under the following conditions:

- Disturbance size ($P_d$): varied between 40 to 80 MW with 10 MW steps
- Monitored generators are varied from 2 to 10 considering all different combinations of monitored generators for each $P_d$ value and disturbance locations
- Disturbance location: all buses for each disturbance size
- The RoCoF measurements had 1% white noise (for adaptive regional method)
- The active power measurements had 1% white noise (for SPC method)

The results of this test are presented in Figure 5-13 with a logarithmic scale on the y axis.
Based on Figure 5-13, both the SPC and Adaptive regional method provide significantly more accurate disturbance size estimation compared to traditional method. However, when all generators are monitored (10 out of 10) the traditional method would have a marginally better performance. Furthermore, the SPC method shows to be slightly more accurate than adaptive regional method for PMU numbers less than 9.

5.5 Chapter Summary

This chapter has presented a novel method for detection, disturbance size and location estimation (disturbance analysis) based on the Regional Inertia concept discussed in Chapter 5. The method uses limited number of generators PMU measurements of RoCoF as well as system impedance matrix $Z$, load flow results ($LF$) and generators inertias ($H$). The method is executed in two stages, the first one dedicated to disturbance detection consisting of a decision tree, the second one dedicated to the estimation of disturbance size and location. The distinctive contributions of this method are as follows:

- Robustness against the loss of PMU measurements (shown in Section 5.4.3).
Simultaneous disturbance detection and estimation of disturbance size and location

- Relying on only one time sample following the disturbance (one time step of PMU measurements), therefore being fast to execute as opposed to methods which need a window of PMU measurements following the disturbance [21].

- Robustness against noise in RoCoF measurements and error in Z matrix (Case studies 1 and 2)

- Usage of already existing synchronised frequency measurements infrastructure (FNET) [14] (compared to the suggestion to use synchronised active power measurements for SPC method (Chapter 3)).

A comparison study was carried out in Section 5.4.3 regarding disturbance size estimation performance between the traditional swing based method, fixed regional based method and the adaptive regional method. The adaptive regional method achieved less than 2% median error in disturbance size estimation when having only 2 out of 10 generators monitored in IEEE 39 bus system. To achieve the same accuracy, traditional swing based and fixed regional method required 8 and 9 monitored generators respectively (Figure 5-11).

### 5.6 References


6. Dynamic Frequency Security Assessment Based on System Identification

This chapter describes a new method for dynamic frequency security assessment using the system identification technique. The new dynamic frequency security assessment uses PMU measurements at generators terminals during normal operating conditions to evaluate whether a particular system is at the risk of minimum frequency level violation for the largest possible infeed loss. Minimum frequency level violation refers to having frequency nadir dropping below a specified level.

The output results of the system identification based dynamic frequency security assessment provide the necessary information to decide the frequency security status, fast frequency response requirements as well as the necessity and the size of fast frequency response in case of disturbance detection.

Section 6.1 discusses the role of dynamic frequency security assessment in controlling the frequency in future low inertia systems. Section 6.2 describes dynamic security assessment and dynamic frequency security assessment. System Frequency Response (SFR) model and transfer function is discussed in Section 6.3. Section 6.4 describes the new approach for dynamic frequency security assessment and defines critical disturbance size term. Section 6.5 includes the methodology behind the system identification techniques. Section 6.6 explains the SFR transfer function identification. Section 6.7 includes the results for SFR identification for different case studies. Section 6.8 investigates the relationship of fast frequency response size with critical disturbance size and response initiation delay. Finally, the chapter is concluded in Section 6.9.

6.1 Chapter Introduction

Traditionally power systems have been planned and operated based on offline security assessments. However, with an increased level of uncertainty introduced by the connection of intermittent sources of generation as well a heightened risk of blackouts, the online dynamic security assessment of the power system has gained great interest in recent years [1][2].
Furthermore, the advent of wide area monitoring systems has paved the path for real time assessment of system security using data analysis techniques [3][4].

The reduction in system inertia due to the displacement of synchronous generation with inertia less electronically connected generation sources, has led to deteriorated system security in terms of frequency. Meaning in the case of active power disturbance occurrence, the system frequency drop rate would be higher, maximum frequency deviation would be larger and it would be reached within a shorter time period.

One way of tackling this frequency security issue is commissioning a larger volume of primary response which leads to tremendous increase in frequency control cost [5], which serves as a burden for further penetration of non-synchronous renewable generation.

Another more economical approach is in advance procurement of some remedial actions in the form of fast frequency response services e.g. demand response, storage activation and synthetic inertia loop of wind turbines. Demand response in this sense is different than classical under frequency load shedding which is carried out at distribution substations using frequency relays. In order to arm the right amount of fast frequency control services to be able to cope with the system reference incident, the fast frequency response requirements of the system needs to evaluated in real time and communicated to the system operator. This can be done using Dynamic Frequency Security Assessment which continuously scans the system and model the overall system frequency response using PMU measurements during normal conditions.

Furthermore, the output of Dynamic Frequency Security Assessment is also useful in system abnormal operating conditions (disturbance condition), where the necessity and the size of fast frequency response has to be decided. In other words, following the detection and sizing of disturbance in disturbance analysis procedure (Chapter 3, Chapter 4, and Chapter 5), the Dynamic Frequency Security Assessment results can help to deliver the optimal response. In this sense, fast frequency response optimality is regarded as minimising response needed to avoid the frequency level violation.
6.2 **Dynamic Security Assessment**

In the event of a system disturbance, such as equipment failure, line outages or sudden changes in load or generation, the various components of the power system respond and hopefully reach a new equilibrium condition that is acceptable according to some criteria. Mathematical analysis of these responses and the new equilibrium condition is called security assessment. Security assessment covers all types of stabilities (Voltage, Transient, and Frequency [6][2]). Static security assessment (SSA) involves evaluation of the expected post-contingency equilibrium condition (steady-state operating point)[7]. Dynamic security assessment (DSA) evaluates the transient performance of the system after the disturbance. DSA has been enabled by advancements in monitoring, communication and computing technologies, e.g. the development of phasor measurement units (PMUs)[8][9]. DSA has been formally defined by the Institute of Electrical and Electronics Engineers (IEEE) as the ability of a power system to withstand sudden contingencies, survive the post-contingency transients and reach an acceptable steady state [2].

6.2.1 **Dynamic Frequency Security Assessment**

Due to the sensitivity of a wide range of system components to frequency fluctuations, system operators aim to keep frequency within pre-defined limits both in normal operation and after a disturbance occurrence. The frequency level regulation is delivered via implementation of different frequency control actions in different time frames leading to variable frequency control stages, e.g. primary, secondary and tertiary. Given the risk of frequency triggered black outs [10][11], and the increased probability of unacceptable frequency deviation following active power disturbance in future low inertia systems, there exists a growing need for assessment of frequency response adequacy in an online manner.

Dynamic Frequency Security Assessment is a sub category of DSA and is defined as: the assessment of the ability of a power system to maintain steady and acceptable frequency following a severe system upset resulting in a significant imbalance between generation and load. This type of stability depends on the ability of the system to maintain equilibrium between generation and load [6].
In order to abide with n-1 security criteria, frequency response adequacy needs to be evaluated for each given power system dispatch considering the largest infeed loss. Meaning dynamic frequency security assessment has to assess the capability of the power system to maintain an acceptable frequency level following occurrence of maximum credible active power imbalance. This is particularly critical with the larger penetration of non-synchronously connected renewable sources since frequency security would be challenged in two forms:

1. A lower amount of kinetic energy stored in the rotating mass of the system leading to less time available before frequency nadir and faster frequency drop (lower inertia)
2. Less resources to provide primary frequency response which were traditionally supported by governor response of conventional generators. This decrease is due to the reduced amount of synchronous generators connected to the system.

In terms of frequency level nadir control, inertia and primary frequency response requirements are very much correlated. Therefore, system operator has to compensate for lower inertia with higher volume of generators in governing mode, which leads to increased cost of operating the system securely. Furthermore, due to the inherent delay in primary frequency response initiation (frequency dead band of governors), no amount of governing response can contain the fast drop of frequency in extremely low inertia systems (e.g. frequency nadir time of 2s). Therefore, new technologies and services capable of providing the so called fast frequency response are expected to gain interest in the following years [5][12]. These include, demand response which can also be categorised as the new form of AUFLS, energy storage and the so called synthetic inertia control loop of wind turbines [13][14]. These services can be used to set up a new frequency reserve market. The logical step for enabling the new frequency service provision is the assessment of the necessity and the amount of service needed to be procured. In this sense two points have to be considered:

1. The maximum credible active power contingency
2. The frequency response dynamics of the system

The first point can be extracted from dispatch as the maximum single generator active power output (system reference incident). The second point can be used to predict how a certain
power system would react to the system reference incident. The dynamics behind system frequency response is discussed in the following section.

## 6.3 Modelling of the System Frequency Response

Models of frequency response are usually designed to reflect different generating technology such as combined cycle units, hydro power generations, steam turbines or wind generations. However, a simplified model of system frequency response (SFR) has been proposed in the literature which is based on the system average frequency behaviour [15]. SFR model averages all machine frequency dynamics into an equivalent single steam turbine synchronous machine. The SFR neglects nonlinearities and only retains the largest time constants in the equations of the generating units of the power system. Moreover, an assumption is made that the generation is dominated by reheat steam turbine generators. Meaning, the generating unit inertia and reheat time constants predominate over the system average frequency response. The model for the response to sudden active power disturbance, modelled as step change, is described using the following equations [15]:

\[
\Delta \omega(t) = \frac{RP_d}{DR + K_m}[1 + \alpha e^{-\omega_m t} \sin(\omega t + \phi)] \quad \text{[pu]} 
\]

(6.1)

Where:

\[
P_d(s) = \frac{P_d}{S} 
\]

(6.2)

\[
\omega_n^2 = \frac{DR}{2HRT_R} 
\]

(6.3)

\[
\varsigma = \omega_n \left( \frac{2HR + T_R(DR + K_m F_H)}{2(DR + K_m)} \right) 
\]

(6.4)

\[
\alpha = \sqrt{\frac{1 - 2\omega_n^2 + T_R^2 \omega_n^2}{1 - \varsigma^2}} 
\]

(6.5)

\[
\omega_r = \omega_n \sqrt{1 - \varsigma^2} 
\]

(6.6)

\[
\phi = \tan^{-1} \left( \frac{\omega R}{1 - \varsigma \omega_n T_R} \right) - \tan^{-1} \left( \frac{\sqrt{1 - \varsigma^2}}{-\varsigma} \right) 
\]

(6.7)
Where $\Delta \omega(t)$ is the incremental turbine rotating speed in pu, which is also equal to generator frequency in pu, $D$ denotes load damping factor, $H$ is the inertia constant, $F_H$ is the fraction of total power generated by the high pressure turbine, $K_m$ is the mechanical gain, $R$ is the droop and $T_R$ is the reheat time constant. $P_d(s)$ is the active power load change of the system in Laplas form. (6.2) shows the step active power change applied to SFR in Laplas format.

The SFR model in Laplas format is shown in Figure 6-1.

\[
P_d(s) \rightarrow \sum \rightarrow \frac{1}{2HS + D} \rightarrow \Delta \omega(s)
\]

\[
\frac{K_m(1 + F_H T_R s)}{R(1 + T_R s)}
\]

**Figure 6-1 : The simplified frequency response model**

Based on Figure 6-1 and considering $\Delta \omega$ equal to $\Delta f$ in pu, the closed loop continuous transfer function of simplified system frequency response is as follows:

\[
G(s) = \frac{\Delta f(s)}{P_d(s)} = \frac{a_1s + a_2}{b_1s^2 + b_2s + b_3}
\]

(6.8)

Where $b_1$ equals 1 and:

\[
a_1 = -\frac{1}{2H} \quad \text{(6.9)}
\]

\[
a_2 = -\frac{1}{2HT_R} \quad \text{(6.10)}
\]

\[
b_2 = \frac{1}{T_R} + \frac{D}{2H} + \frac{K_m F_H}{2HR} \quad \text{(6.11)}
\]

\[
b_3 = \frac{K_m}{2HRT_R} + \frac{D}{2HT_R} \quad \text{(6.12)}
\]

In the case of $P_d(s)$ being regarded as the generator active power change instead of load active power change, the $a_1$ and $a_2$ sign would be positive instead of negative.
Based on final value theorem the steady state frequency deviation ($\Delta f$) following a step change in power balance of the system ($P_d$) would be equal to:

$$\lim_{t \to \infty} \Delta f(t) = \lim_{s \to 0} s\Delta f(s) = \frac{a_c}{b_i} = -\frac{P_d}{K + D}$$  \hspace{1cm} (6.13)$$

It is worth mentioning that the frequency dynamics discussed using SFR parameter of the system reflects the $f_{COI}$ behaviour rather than any individual generator frequency. Furthermore, $P_d$ also refers to system wide active power disturbance (total power imbalance) and $H$ is equal to $H_{Sys}$.

Also the RoCoF of the centre of inertia ($\dot{f}_{COI}$) just immediately following the disturbance can be calculated as follows:

$$\lim_{t \to 0} \Delta f(t) = \lim_{s \to 0} s\Delta f(s) = \frac{a_c}{b_i} = -\frac{P_d}{2H}$$  \hspace{1cm} (6.14)$$

Equation (6.14) is the principle behind most estimation methods for system inertia ($H_{Sys}$) [16][17][18]. Assuming disturbance size ($P_d$) is known and $\dot{f}_{COI}$ can be calculated shortly following the disturbance (at $t_d^+$), $H_{Sys}$ can be estimated as follows:

$$H_{Sys} = \frac{P_d}{2\dot{f}_{COI}(t_d^+)}$$  \hspace{1cm} (6.15)$$

### 6.4 New Approach for Dynamic Frequency Security Assessment

Some dynamic frequency security assessment in the literature suggested the use of total system inertia value as a proxy for evaluating the frequency security level of the system by comparing the minimum RoCoF limit (dictated by loss of main relays settings) and the maximum possible RoCoF experienced following the system reference incident event [20]. The following is assumed as frequency security condition:
\[
\frac{P_d^{\text{max}}}{2f_{\text{COI}}^{\text{max}}} \leq H_{\text{Sys}} \tag{6.16}
\]

where \( P_d^{\text{max}} \) is the size the system reference incident and \( f_{\text{COI}}^{\text{max}} \) is the threshold on RoCoF loss of main relays (typically between 0.1Hz/s to 1Hz/s). All variables in (6.16) are expressed in pu values.

However, despite recent attempts for online estimation of system wide inertia, reliable and accurate estimation of system inertia remains an ongoing research topic [19][16][17]. Furthermore, ensuring the acceptable RoCoF level through conditioning the system inertia does not necessarily guarantee the frequency nadir containment in the acceptable range. This is due to role of other system parameters such as governor characteristics and load damping on frequency behaviour. Therefore, there exists a need to explore more accurate means of dynamic frequency security assessment.

The new suggested dynamic frequency security assessment directly evaluates the capability of the system to maintain the frequency nadir within the pre-defined security limit following the system reference incident disturbance. This is achieved by first estimating the SFR model of the system using online PMU measurements and system identification technique.

In this method the condition for frequency security is set on disturbance size, meaning the maximum credible active power disturbance (reference incident) is compared against the minimum disturbance size that can lead to the violation of frequency security level. This disturbance size is referred to as critical disturbance size and can be extracted from SFR model of the system.

### 6.4.1 Critical Disturbance Size

Critical disturbance size \( P_d^{\text{N}_c} \) is defined in this thesis as the minimum active power disturbance that cause the nadir of \( f_{\text{COI}} \) to violate certain frequency level \( \Delta f \) which is shown as the subscript in \( P_d^{\text{N}_c} \).
The critical disturbance size can be used to determine whether the system has dynamic frequency security (regarding any pre-defined frequency drop level $\Delta f$) against the maximum system reference incident (the loss of the biggest generating unit). This is done by simply comparing the size of the system reference incident $P_{d_{\text{max}}}^d$ with $P_{C}^d$. Therefore the dynamic frequency security condition is:

$$P_{d_{\text{max}}}^d \leq P_C^d$$

(6.17)

Furthermore, where (6.17) is not satisfied for a given system operating condition and system is deemed frequency insecure, the size of fast frequency response that has to be procured in the market and be armed can be decided by the difference between maximum credible disturbance size ($P_{d_{\text{max}}}^d$) and critical disturbance size ($P_C^d$).

Critical disturbance size can be extracted from the simplified frequency response (SFR) transfer function of the system. Therefore, in order to assess the frequency security status and consequently decide the fast frequency requirements of the system, SFR parameters needs to identified for any system operating condition (Figure 6-2).

Assuming the SFR transfer function (6.8) of the system is known/identified, the following steps should be taken to calculate the $P_C^d$.

Firstly, the inverse Laplas of the SFR transfer function needs to be calculated to get to the function for frequency deviation in time:

$$\Delta f(t) = L^{-1}\left[\frac{a_1s + a_2}{b_1s^2 + b_2s + b_3} \frac{P_d}{s}\right]$$

(6.18)
Secondly, the derivative of frequency in time needs to be solved for zero to calculate the frequency nadir time \((t_n)\):

\[
\text{solving } \Delta f(t) = 0 \rightarrow t_n = t \quad (6.19)
\]

Thirdly, the frequency deviation at nadir time being equal to \(\Delta f_i\) should be solved for an unknown disturbance size:

\[
\text{solving } 50\Delta f(t_n) - \Delta f_i = 0 \rightarrow P^{\Delta f}_c = P_d \quad (6.20)
\]

where the solution for (6.20) would be equal to critical disturbance size for \(\Delta f_i \ (P^{\Delta f}_c)\).

Knowing that the critical disturbance size can be easily extracted from the SFR transfer function, the next step is to explore the system identification methodology for online identification of SFR transfer function of the system.

### 6.5 System Identification

Mathematical models governing system dynamics are required in many different areas and they take various forms, including state-space equations, differential equations and transfer functions. The most typical approach for constructing these models is by directly gathering and setting up the physical law based mathematical equations that govern the system dynamics. However, this approach has proved to be rather complex and not easily performed particularly as system states increase and over-parameterisation problems arise [21].

Alternatively, in case of availability of sufficient experimental or operational data, observed input and output data can be used to build the dynamic mathematical model. In the control area this process has been termed as “System Identification” [22].

The basic stages for the system identification problem are as follows:

- Data preparation
- Model structure description
- Setting a criterion for fit between data and the model
- Model validation

In the data preparation stage, the aim is to construct a set of data samples from both inputs and outputs of the model of interest. The input and output data can be either of these three forms:

- Impulse response data
- Step response data
- Ambient response data

The first two data types are more sensitive to noise as they only base the model on a single input output data set. However, with ambient response data the inputs and outputs are recorded for a sequence of events which helps the model identification to average out the noise in the data, therefore reducing the impact of noise [23].

### 6.5.1 Transfer Function Identification

The transfer function identification can be either in continuous form and in laplas space or discrete form and in Z space. The model structure for transfer function identification is described as the number of poles and zeros for both discrete and continuous transfer function forms.

In the case of having discrete and sampled input and output data, the first stage is to construct equations describing the relationship between current output value ($y(t)$) with preceding inputs and outputs values. The notation below is used for $x$ previous sampled inputs ($u(t)$) and outputs ($y(t)$):

$$y_{k-x} = y(t - x\tau) \quad (6.21) \quad u_{k-x} = u(t - x\tau) \quad (6.22)$$

where $\tau$ is the sampling time.
A difference equation describing the current output \( y_k \) as a function of \( n \) past outputs and \( m \) past inputs and current input can be written as follows:

\[
y_k = -c_1 y_{k-1} - c_2 y_{k-2} - \cdots - c_n y_{k-n} + d_0 u_k + d_1 u_{k-1} + \cdots + d_m u_{k-m} + e_k
\]  

(6.23)

where \( e_k \) is the measurement noise at the current time which is uncorrelated with inputs and outputs and \( c_i \) and \( d_i \) values are called regression weights [24]. In (6.23) \( e_k \) is directly entered in the difference equation as white noise term having zero mean. (6.23) is an equation defining a recursion relationship which can be used to predict the output sequence for any given input and estimated model parameter (regression weights).

The matrix equation below can be written when \( n_o \) number of observations in time for both inputs and outputs are present.

\[
\begin{bmatrix}
y_k \\
y_{k-1} \\
\vdots \\
y_{k-n_o+1}
\end{bmatrix} =
\begin{bmatrix}
y_{k-1} & \cdots & y_{k-n_o+1} & u_k & \cdots & u_{k-n_o+1} & u_{k-n_o+1-m} \\
\vdots & \ddots & \vdots & \ddots & \ddots & \vdots & \vdots \\
\vdots & & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & & & \ddots & \ddots & \ddots & \ddots \\
y_{k-n_o+1} & \cdots & y_{k-n_o+1-m} & u_k & \cdots & u_{k-n_o+1-m} & u_{k-n_o+1-m} \\
\end{bmatrix}
\begin{bmatrix}
-c_1 \\
\vdots \\
\vdots \\
\vdots \\
c_n \\
-d_0 \\
\vdots \\
d_m \\
\end{bmatrix} +
\begin{bmatrix}
e_k \\
\vdots \\
\vdots \\
\vdots \\
e_{k-n_o+1}
\end{bmatrix}
\]

\[Y = \Phi \beta + E\]  

(6.24)

where \( \beta \) is regression weights matrix, \( \Phi \) is regressor matrix, \( E \) is the measurement noise matrix and \( Y \) is the output matrix.

The main goal in transfers function identification is to estimate \( \beta \) matrix in a way that minimizes the difference between the model output and real system output data. This problem can be solved by re-stating it in a least square error form as follows [25]:

\[
\beta = \text{arg min}_\beta \| Y - \Phi \beta \|
\]  

(6.26)
Where $\hat{\beta}$ is the estimate of regression weight matrix, and the operator $\parallel \parallel$ is the least square cost function defined as follows:

$$\parallel Y - \phi \beta \parallel = \frac{1}{n_w} \sum_{i=k-n_w+1}^{k} \frac{1}{2} e^2(i, \beta)$$

(6.27)

Where $e(i, \beta)$ is the prediction error for observation $i$ having a regression weight matrix of $\beta$. The prediction error for observation $i$ is:

$$e(i, \beta) = y_i - \hat{y}_i(\beta) = y_i - \phi(i,:) \beta$$

(6.28)

In order to minimize the sum of square errors in (6.27) for the over determined set of equations defined in (6.25) where the number of equation is more than unknown, the solution can be achieved by taking the pseudo inverse of $\Phi (\Phi^+)$ and multiply it by the output matrix [25]:

$$\hat{\beta} = \left[ (\Phi^T \Phi)^{-1} \Phi^T \right] \mathbf{Y}$$

(6.29)

Knowing $\hat{\beta}$, the general form for identified discrete transfer function would be:

$$\hat{G}(z) = \frac{d_0 z^n + d_1 z^{n-1} + \ldots + d_m}{z^n + c_1 z^{n-1} + \ldots + c_n}$$

(6.30)

Following the identification of discrete transfer function using discrete observation of input and output data with time sample $\tau$, based on the Tustin method [26] the continuous transfer function ($G(s)$) can be calculated by substituting $z$ in $G(z)$ with $\frac{\tau s + 2}{2 - \tau s}$. 

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6.6 System Frequency Response Identification

Several researches have emphasised the need to estimate the power systems dynamic parameters using online measurements [27][28][29]. Traditionally, as power systems consisted of a relatively small number of synchronous generators, system frequency dynamics could be estimated by the generators parameters provided by their manufacturer. However, with the ever increasing trend towards the integration of many distributed generators, the former approach fails to guarantee the accurate estimation of system frequency dynamics model. Furthermore, with the increased level of embedded generation in distribution networks, Distribution System Operators (DSOs) face both significant challenges and potentials regarding frequency control capabilities [30]. In other words, TSOs and DSOs need to collaborate closely to provide required level of system frequency security. A stepping stone for enabling distribution networks to participate in frequency control provision is the accurate real time estimation of frequency dynamics of the external grid at the point of common coupling for various system operating conditions. Furthermore, knowing external grid frequency dynamics can help to adapt the RoCoF relay settings of generators in distribution networks to avoid undesirable disconnection of them in an external grid active disturbance event which further deteriorate the frequency stability. These motivations along with expected large deployment of Wide Area Monitoring Systems (WAMS) in future grids lay the foundations for exploring real time measurement based estimation of power systems frequency response dynamics.

Most methods used for the estimation of parameters involved in simplified system frequency response (SFR) have relied on frequency response following a single large disturbance whose characteristics are identified following the disturbance occurrence [16][17]. This approach is based on step response data, since large disturbances are modelled as step change in active power. As stated in [23], the focus on relatively large short duration disturbances is due to the brief desirable signal to noise ratio which leads to more accurate parameter estimation. However, even though these methods can provide accurate results in terms of single generating unit dynamics [23], using one single disturbance response to model SFR and use it to predict the response to the next disturbance is not reliable. This stems from the fact that even though a single generating unit has relatively stationary dynamic response, system wide frequency response is quite volatile, and given the low probability of large disturbance
occurrence, the SFR model estimated from one disturbance might not hold the same for the next disturbance.

Some researchers have suggested the prediction of frequency response following a disturbance using the initial frequency response either by fitting a polynomial or by estimating the transfer function [31][32]. This method, which is referred to in this chapter as Disturbance Triggered SFR Identification method, requires some time window following the disturbance to estimate the rest of the frequency response (e.g. frequency nadir and steady state frequency). In applications concerning frequency nadir prediction and particularly in low inertia systems where frequency nadir reaches quite fast, the delay introduced by the disturbance triggered SFR estimation method can endanger the capability of the system to assess frequency security and react on time.

Therefore, there exists a need for extraction of SFR model parameters from ambient and smaller load and generation fluctuations prior to larger disturbance occurrence. This approach has been deployed in [23] for identification of single pump hydro frequency response.

In this chapter the system identification with least error prediction is applied to PMU measurements of frequency and active power of generators in sliding windows to estimate the SFR transfer function of the system and to utilise SFR parameters to estimate the critical disturbance size in order to assess the frequency security of the system.

### 6.6.1 SFR Parameter Estimation

The first step is to prepare the measured input data and output data for SFR transfer function estimation.

In this method, the input for SFR estimation ($\Delta P(t)$) is the summation of active power change at generators terminal compared to their scheduled value ($P_{set}$) at the last SCADA update, which is stated in pu values. This equals the total power imbalance seen by generators at each time step in pu value. The $\Delta P(t)$ for a system with $M$ monitored generators is:
\[ \Delta P(t) = \frac{1}{S_{\text{base}}} \sum_{i=1}^{M} P_{e,i}(t) - P_{\text{Ref},i} \quad \text{[pu]} \] (6.31)

where the \( P_{e,i}(t) \) is the active power output of generator \( i \) at time \( t \).

The output for SFR estimation is the \( f_{\text{COI}}(t) \) variation in pu at each time step. This is calculated as follows:

\[ \Delta f_{COI}(t) = 1 - \frac{\sum_{i=1}^{M} f_i(t) H_{i,\text{sys}}}{f_0 \sum_{i=1}^{M} H_{i,\text{sys}}} \quad \text{[pu]} \] (6.32)

where \( H_{i,\text{sys}} \) is the inertia of generator \( i \) defined in common system \( S_{\text{base}} \), and \( f_0 \) is equal to 50Hz.

The second step is deciding the model order. Based on Tustin conversion method [26] applied to SFR continuous transfer function (6.8), the discrete transfer function model order for SFR \( (G(z)) \), meaning the numbers of \( n \) and \( m \) in (6.23) would be equal to 2 for both values. Therefore, the discrete SFR transfer function would have the following format:

\[ G(z) = \frac{d_0 z^{-2} + d_1 z^{-1} + d_2}{z^{-2} + c_1 z^{-1} + c_2} \] (6.33)

Based on (6.33), for SFR transfer function identification \( \Phi \) the regression weights matrix would be \( 5 \times 1 \), \( \Phi \) the regressor matrix would be \( n_w \times 5 \) and \( Y \) would be \( n_w \times 1 \). \( n_w \) is the number of observations which is equal to the number of points in the sliding window. The size of the sliding window \( (w) \) is defined as follows:

\[ w = n_w \tau \] (6.34)

Where \( \tau \) is the sampling time of PMU measurement (=0.02s).
At each time step, SFR parameters are estimated for the most current sliding window with size $w$, these estimated parameters are then averaged for the last $sw$ number of past sliding windows to get to the final estimate of SFR parameter at the current time. Then these averaged values for SFR transfer function at each time step are used to calculate the critical disturbance size for any particular $\Delta f (P_c\Delta f)$, the procedure for this is discussed in section 6.4.1.

The flowchart for the online SFR identification and critical disturbance size estimation is depicted in Figure 6-3.
In Figure 6-3, the dashed lines represent slower update rate (SCADA update rate around 1 minute), and solid lines represent the PMU update rate (0.02s). Also $\textbf{H}$ represents the vector containing the inertias of the monitored generators; $\textbf{P}_{\text{set}}$ is the vector containing all generators active power set point values, and $\textbf{P}_e(t)$ and $f(t)$ are vectors containing monitored generators active power output and frequency at time $t$ respectively.

### 6.7 Results

In this section the results for SFR identification and critical disturbance size estimation are presented. The disturbances are modelled as load fluctuations equal to 1% of the size of each load in the system with the average time between consequent load changes being 0.1s. These random load fluctuations were first made in Matlab as time vectors and then fed into DIgSILENT DSL models for load values to help simulate the frequency response results to ambient load changes. All simulations have been done on IEEE standard 39 bus system. A white noise component has been added to both frequency and power measurements with different range of the actual value of the measurements and zero mean.

In the following results section all critical disturbances ($P_c N$) are expressed in pu and $H_{\text{sys}}$ values are expressed in pu.s where the $S_{\text{base}}=100$MW.

Furthermore, the real SFR of the system was calculated by applying a step change in generator power in IEEE 39 bus system.

### 6.7.1 Error Indices Definition

In order to be able to enumerate the performance of the identified SFR compared to the real system response in validation part for the following case studies, different error indices are defined as follows:

- $E(f_n)$: Percentage difference of real frequency nadir drop and estimated frequency nadir drop divided by the real frequency nadir drop
- $E(t_n)$: Percentage difference of real frequency nadir time and estimated frequency nadir time divided by the real frequency nadir time
- \( E(f_f) \): Percentage difference of real frequency final value and estimated frequency final value divided by the real frequency final value
- \( E(P_{C^{0.8}}) \): Percentage difference of \( P_{C^{0.8}} \) for real system SFR and \( P_{C^{0.8}} \) for the identified SFR divided by the \( P_{C^{0.8}} \) for real system SFR
- \( E(H_{Sys}) \): Percentage difference of real \( H_{Sys} \) and \( H_{Sys} \) estimated using identified SFR divided by the real \( H_{Sys} \)

In a general sense \( E(x) \) can be defined as a function to make different Error Indices and is defined as follows:

\[
E(x) = 100 \left( \frac{x - \hat{x}}{x} \right)
\]  

(6.35)

where \( x \) is the real (measured) value and \( \hat{x} \) is the estimated value using identified SFR.

### 6.7.2 First Worked Example

In this example, the SFR is identified using power and frequency measurements of all generators in 39 bus system during small random ambient load fluctuations. After the training time (identification time) of the model, a large disturbance takes place at bus 25 (Figure 3-2) for which the identified SFR model is used to predict the frequency response which can be regarded as the validation part of the model.

The conditions for this first example are as follows:

- Disturbance: load fluctuations equal to 1% of size of each load with normal distribution and the average time between consequent load changes for each load being 0.1s.
- Noise: white noise added to both power and frequency measurements with the range equal to 1% of the actual reading and zero mean.
- Simulation time: 200s
- Large disturbance occurrence time: 120s
- Large disturbance size: 1060 MW
- Large disturbance location: bus 25 (load 5) (Figure 3-2)
- Training time: 120s prior to disturbance occurrence time
- Window length \((w)\): equals 10 s
- Past sliding windows number \( (sw) \): 100 (equals to 2s considering time sample of 0.02s for PMU measurements)

Figure 6-4 shows 20s of input and output measurements for the SFR transfer function identification before large disturbance occurrence. The input being \(\Delta P(t)\) as it was defined in (6.31) and shown by red curve in Figure 6-4. The output being \(\Delta f_{\text{COI}}(t)\) as it was defined in (6.32) and shown by blue curve in Figure 6-4.

The dotted vertical line in Figure 6-4 can be a representative of a current time, and the double arrows show the past \(sw\) number of sliding windows with the size \((w)\). At each time step, the SFR is identified over the last \(w\) size window of data, after that the SFR estimated parameters are averaged for \(sw\) number of past recorded SFR parameters. The averaged SFR parameters would be the estimated SFR parameters for the current time which can then be used to calculate the critical disturbance size \((P_{c\Delta f})\) for the current time.
Figure 6-5 shows the system frequency \( (f_{COI}) \) for both measured and estimated frequency using identified SFR transfer function. As it can be seen in Figure 6-5 the identified SFR succeed to estimate \( f_{COI} \) with relatively low error as the two curves have high \( \text{Fit\%} \) value (94.51). \( \text{Fit\%} \) value between \( y \) and estimated \( \hat{y} \) is calculated as follows:

\[
\text{Fit\%} = 100 \left( 1 - \frac{\|y - \hat{y}\|}{\|y - \text{mean}(y)\|} \right)
\]  \( \text{(6.36)} \)

Table 6-1 shows real SFR parameters and critical disturbance sizes for 5 different frequency drops (0.3Hz, 0.5Hz, 0.8Hz, and 1.2Hz). Table 6-2 shows the identified SFR parameters for this example and critical disturbance sizes using the identified SFR for 5 different frequency drops ((0.3Hz, 0.5Hz, 0.8Hz, 1.2Hz).

**Table 6-1: Real 39 Bus System SFR Parameters and Critical Disturbance Sizes**

<table>
<thead>
<tr>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( P_C^{0.3} )</th>
<th>( P_C^{0.5} )</th>
<th>( P_C^{0.8} )</th>
<th>( P_C^1 )</th>
<th>( P_C^{1.2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.4293e-4</td>
<td>8.2490e-4</td>
<td>1.7158</td>
<td>1.8617</td>
<td>12.0978</td>
<td>20.1630</td>
<td>32.2607</td>
<td>40.3259</td>
<td>48.3911</td>
</tr>
</tbody>
</table>

**Table 6-2: Identified SFR Parameters and Estimated Critical Disturbance Sizes for the First Worked Example**

<table>
<thead>
<tr>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( P_C^{0.3} )</th>
<th>( P_C^{0.5} )</th>
<th>( P_C^{0.8} )</th>
<th>( P_C^1 )</th>
<th>( P_C^{1.2} )</th>
</tr>
</thead>
</table>

By comparing the values in Table 6-1 with Table 6-2, it can be seen that identified SFR has relatively small difference in parameters and critical disturbance sizes with real SFR of the system.

Figure 6-6 shows the entire simulation time of measured and estimated \( f_{COI} \) frequency. The simulation time includes both training time (SFR identification time) prior to a big disturbance occurrence (for \( t<120s \)) and the validation time (for \( t>120s \)). These two time periods are zoomed on in Figure 6-6.
Figure 6-6: Measured $f_{\text{COI}}$ frequency and estimated $f_{\text{COI}}$ frequency using identified SFR for entire simulation time including training and validation periods

Table 6-3 includes the Error Indices (defined in section 6.7.1) for this example.

<table>
<thead>
<tr>
<th>$E(f_n)$</th>
<th>$E(t_n)$</th>
<th>$E(f_f)$</th>
<th>$E(P_{C}^{0.8})$</th>
<th>$E(H_{\text{Sys}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0001</td>
<td>4.5455</td>
<td>2.1866</td>
<td>1.7462</td>
<td>8.5992</td>
</tr>
</tbody>
</table>

Based on Table 6-3, the identified SFR during load fluctuation gives a small error in terms of minimum frequency ($f_n$) and final frequency ($f_f$) estimation ($E(f_n)$ and $E(f_f) < 3\%$). Furthermore, the estimated critical disturbance size $P_{C}^{0.8}$ using identified SFR has less than 2\% (1.7462) error compared to the real $P_{C}^{0.8}$. Also, the estimated $H_{\text{Sys}}$ using identified SFR transfer function parameters (by applying (6.9)) has less than 9\% error (8.5992).

The worked example validated the acceptable performance of the method in providing relatively accurate critical disturbance size (less than 2\% error) which can be used to assess the dynamic frequency security level of the system in an online manner (as discussed in section 6.4).
6.7.3 Case Study 1 (Different Noise Level and Window Length)

In this case study, the sliding window size \( w \) and noise range level have been changed to analyse the sensitivity of SFR transfer function identification and critical disturbance size estimation to these parameters.

All conditions are the same as the first worked example with the following differences:

- Window length \( (w) \): varied between 10s, 20s, 50s and 100s
- Noise: white noise added to both power and frequency measurements with the range equal to 1% and 5% of the actual readings and zero mean.

The results for the identified SFR transfer function parameters and estimated \( H_{Sys} \) (pu.s) for 1% and 5% noise ranges and varied window lengths \( (w) \) are shown in Table 6-4.

**Table 6-4: Identified SFR Parameters and Estimated \( H_{Sys} \) for Case Study 1**

<table>
<thead>
<tr>
<th>Window length</th>
<th>Noise Range</th>
<th>a₁</th>
<th>a₂</th>
<th>b₂</th>
<th>b₁</th>
<th>( H_{Sys} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10s</td>
<td>1%</td>
<td>-5.5279 e⁻⁴</td>
<td>-8.1177 e⁻⁴</td>
<td>1.7991</td>
<td>1.7973</td>
<td>0.9045 e⁺³</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>-5.0288 e⁻⁴</td>
<td>-7.3543 e⁻⁴</td>
<td>1.5567</td>
<td>1.8312</td>
<td>0.9942 e⁺³</td>
</tr>
<tr>
<td>20s</td>
<td>1%</td>
<td>-5.0046 e⁻⁴</td>
<td>-8.0835 e⁻⁴</td>
<td>1.7420</td>
<td>1.8100</td>
<td>0.9991 e⁺³</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>-4.6792 e⁻⁴</td>
<td>-7.3441 e⁻⁴</td>
<td>1.6268</td>
<td>1.7890</td>
<td>1.0808 e⁺³</td>
</tr>
<tr>
<td>50s</td>
<td>1%</td>
<td>-4.9823 e⁻⁴</td>
<td>-8.1481 e⁻⁴</td>
<td>1.7755</td>
<td>1.8073</td>
<td>1.0036 e⁺³</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>-4.5561 e⁻⁴</td>
<td>-7.0288 e⁻⁴</td>
<td>1.6858</td>
<td>1.6982</td>
<td>1.0974 e⁺³</td>
</tr>
<tr>
<td>100s</td>
<td>1%</td>
<td>-4.9925e⁻⁴</td>
<td>-7.2228e⁻⁴</td>
<td>1.6435</td>
<td>1.6655</td>
<td>1.0015e⁺³</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>-4.4829 e⁻⁴</td>
<td>-6.4120 e⁻⁴</td>
<td>1.6033</td>
<td>1.6013</td>
<td>1.1150e⁺³</td>
</tr>
</tbody>
</table>

The results for critical disturbance sizes for different window lengths and noise ranges are summarised in Table 6-5.

**Table 6-5: Critical Disturbance Size Estimates for Case Study 1**

<table>
<thead>
<tr>
<th>Window Length</th>
<th>Noise Range</th>
<th>( P_C^{0.3} )</th>
<th>( P_C^{0.5} )</th>
<th>( P_C^{0.8} )</th>
<th>( P_C )</th>
<th>( P_C^{1.2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10s</td>
<td>1%</td>
<td>11.8865</td>
<td>19.8109</td>
<td>31.6974</td>
<td>39.6217</td>
<td>47.5461</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>12.4527</td>
<td>20.7546</td>
<td>33.2073</td>
<td>41.5091</td>
<td>49.811</td>
</tr>
<tr>
<td>20s</td>
<td>1%</td>
<td>11.9851</td>
<td>19.9752</td>
<td>31.9603</td>
<td>39.9503</td>
<td>47.9404</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>12.6548</td>
<td>21.0913</td>
<td>33.7461</td>
<td>42.1827</td>
<td>50.619</td>
</tr>
<tr>
<td>50s</td>
<td>1%</td>
<td>11.9879</td>
<td>19.9798</td>
<td>31.9677</td>
<td>39.9596</td>
<td>47.9515</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>12.9023</td>
<td>21.5039</td>
<td>34.4062</td>
<td>43.0077</td>
<td>51.6093</td>
</tr>
<tr>
<td>100s</td>
<td>1%</td>
<td>12.1418</td>
<td>20.2363</td>
<td>32.3781</td>
<td>40.4726</td>
<td>48.5671</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>13.1338</td>
<td>21.8896</td>
<td>35.0234</td>
<td>43.7792</td>
<td>52.3551</td>
</tr>
</tbody>
</table>
Error indices for varied noise range and window lengths are shown in Table 6-6.

<table>
<thead>
<tr>
<th>Window length</th>
<th>Noise Range</th>
<th>10s</th>
<th>20s</th>
<th>50s</th>
<th>100s</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E(f_n))</td>
<td>1%</td>
<td>2.0001</td>
<td>1.1704</td>
<td>1.1178</td>
<td>-0.1382</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>-2.5643</td>
<td>-4.1634</td>
<td>-6.0527</td>
<td>-7.7070</td>
</tr>
<tr>
<td>(E(t_n))</td>
<td>1%</td>
<td>4.5455</td>
<td>4.3557</td>
<td>0.1147</td>
<td>4.5454</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>13.6363</td>
<td>4.5455</td>
<td>0.1335</td>
<td>0.1492</td>
</tr>
<tr>
<td>(E(f_l))</td>
<td>1%</td>
<td>2.1866</td>
<td>1.0513</td>
<td>2.0164</td>
<td>-1.8822</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>-9.1605</td>
<td>-7.1278</td>
<td>-6.3483</td>
<td>-9.4011</td>
</tr>
<tr>
<td>(E(P_{c0.8}))</td>
<td>1%</td>
<td>1.7462</td>
<td>0.9314</td>
<td>0.9084</td>
<td>-0.3638</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>-2.9341</td>
<td>-4.6044</td>
<td>-6.6504</td>
<td>-8.5635</td>
</tr>
<tr>
<td>(E(H_{sys}))</td>
<td>1%</td>
<td>8.5992</td>
<td>-0.9583</td>
<td>-1.4106</td>
<td>-1.2025</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>-0.4725</td>
<td>-7.9797</td>
<td>-10.8972</td>
<td>-12.7061</td>
</tr>
</tbody>
</table>

Based on Table 6-6, as expected the increased noise in measurements increases the error in estimated frequency nadir \(f_n\), frequency nadir time\(t_n\), final frequency \(f_l\), critical disturbance size \(P_{c0.8}\) and system inertia \(H_{sys}\). For low noise range (1%) increased window length proved to improve the performance of the method in terms of \(f_n\) and \(P_{c0.8}\) estimation. Whereas, for higher noise range (5%), the smallest window length (10s) achieved the best performance regarding \(f_n\) and \(P_{c0.8}\) estimation. As the window length gets bigger and noise content in the input for transfer function identification increases, the method tends to smooth out the noise more in the process. Therefore in the case of high noise range (5%), for which load fluctuations are less distinguishable from noise, load fluctuations (system real input) get more filtered as the window length increases leading to under estimation of frequency drop, meaning a higher error in estimation of \(f_n\) and \(P_{c0.8}\) (Table 6-6).

In conclusion, given the appropriate window length selection (100s for noise range=1% and 10s for noise range=5%), the method proved to provide acceptable performance regarding critical disturbance size estimation \((E(P_{c0.8})<3\%))\) even in the presence of noise up to 5% (Table 6-6).

### 6.7.4 Case Study 2 (Disturbance Triggered SFR Identification)

As discussed in (section 6.6) Disturbance Triggered SFR Identification method refers to system frequency behaviour prediction by identifying the SFR transfer function of the system once a large disturbance takes place (hence the disturbance triggered term). In this method ambient random load fluctuations are not considered as large disturbance plus the only time
window used for SFR identification is the one following the disturbance occurrence. For applications concerning the prediction of frequency nadir (e.g. fast frequency response activation or adaptation), the size of the time window following the disturbance used for SFR identification and frequency prediction has to be less than the summation of frequency nadir time and some delays associated with fast frequency response adaptation and initiation. Otherwise, the \textit{Disturbance Triggered SFR Identification} method fails to provide necessary information needed to take fast control measures against frequency nadir level violation. Furthermore, in future low inertia systems, less time is available for training SFR and predicting the frequency nadir before frequency nadir time. Therefore this case study is aimed to analyses the effect of time window length used following the disturbance occurrence for identification of SFR parameters and consequently predicting the frequency behaviour in the time following the training time window. In \textit{Disturbance Triggered SFR Identification} time window is stationary and SFR parameters are only estimated once in that window, whereas for ambient load fluctuation SFR identification (section 6.7.2 and section 6.7.3) the time window was sliding with every new set of PMU measurements.

The conditions for this case study are as follows:

- Large disturbance size: 300MW
- Large disturbance occurrence time: 2s
- Large disturbance location: bus 25 (load 5) (Figure 3-2)
- Simulation time: 14s
- The frequency nadir time is 2.2s following the disturbance time.
- Training window length \((w)\) varied between: 0.2s, 0.5s, 1s, 1.5s and 2s.

The results for identified SFR transfer function parameters and estimated \(H_{\text{sys}}\) (pu.s) for varied window lengths following the disturbance occurrence are shown in Table 6-7.

The results for critical disturbance sizes for different window lengths following the disturbance occurrence are summarised in Table 6-8.
Table 6-7: Identified SFR Parameters and Estimated \( \text{H}_{\text{Sys}} \) for Case Study 2

<table>
<thead>
<tr>
<th>Window Length</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( \text{H}_{\text{Sys}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2s</td>
<td>-0.0018</td>
<td>-0.0570</td>
<td>121.2671</td>
<td>3.2860 ( \times 10^8 )</td>
<td>0.2777 ( \times 10^3 )</td>
</tr>
<tr>
<td>0.5s</td>
<td>-0.0081</td>
<td>-0.4702</td>
<td>897.4306</td>
<td>670.2509</td>
<td>0.062 ( \times 10^3 )</td>
</tr>
<tr>
<td>1s</td>
<td>-5.0288 ( \times 10^{-4} )</td>
<td>-7.3543 ( \times 10^{-4} )</td>
<td>1.5567</td>
<td>1.8312</td>
<td>0.9942 ( \times 10^3 )</td>
</tr>
<tr>
<td>1.5s</td>
<td>-4.6792 ( \times 10^{-4} )</td>
<td>-7.3441 ( \times 10^{-4} )</td>
<td>1.6268</td>
<td>1.7890</td>
<td>1.0808 ( \times 10^3 )</td>
</tr>
<tr>
<td>2s</td>
<td>-4.5561 ( \times 10^{-4} )</td>
<td>-7.0288 ( \times 10^{-4} )</td>
<td>1.6858</td>
<td>1.6982</td>
<td>1.0974 ( \times 10^3 )</td>
</tr>
<tr>
<td>2.5s</td>
<td>-4.4829 ( \times 10^{-4} )</td>
<td>-6.4120 ( \times 10^{-4} )</td>
<td>1.6033</td>
<td>1.6013</td>
<td>1.1150 ( \times 10^3 )</td>
</tr>
</tbody>
</table>

Table 6-8: Critical Disturbance Size Estimates for Case Study 2

<table>
<thead>
<tr>
<th>Window Length</th>
<th>( P_{C0.3} )</th>
<th>( P_{C0.5} )</th>
<th>( P_{C0.8} )</th>
<th>( P_{C1} )</th>
<th>( P_{C1.2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2s</td>
<td>2.5412</td>
<td>4.2354</td>
<td>6.7766</td>
<td>8.4708</td>
<td>10.1615</td>
</tr>
<tr>
<td>0.5s</td>
<td>8.7588</td>
<td>14.5980</td>
<td>23.3568</td>
<td>29.1960</td>
<td>35.0352</td>
</tr>
<tr>
<td>1s</td>
<td>12.4527</td>
<td>20.7546</td>
<td>29.0927</td>
<td>36.3659</td>
<td>43.6391</td>
</tr>
<tr>
<td>1.5s</td>
<td>12.6548</td>
<td>21.0913</td>
<td>19.5698</td>
<td>24.4623</td>
<td>29.3548</td>
</tr>
<tr>
<td>2s</td>
<td>12.9023</td>
<td>21.5039</td>
<td>24.3699</td>
<td>30.4624</td>
<td>36.5548</td>
</tr>
<tr>
<td>2.5s</td>
<td>13.1338</td>
<td>21.8896</td>
<td>28.0402</td>
<td>35.0502</td>
<td>42.0602</td>
</tr>
</tbody>
</table>

Error indices for varied window lengths following the disturbance occurrence are shown in Table 6-9.

Table 6-9: Error Indices for Case Study 2

<table>
<thead>
<tr>
<th>Window Length</th>
<th>0.2s</th>
<th>0.5s</th>
<th>1s</th>
<th>1.5s</th>
<th>2s</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(f_{\text{n}}) )</td>
<td>1.5767 ( \times 10^3 )</td>
<td>40.1477</td>
<td>-2.5643</td>
<td>-4.1634</td>
<td>-6.0527</td>
</tr>
<tr>
<td>( E(t_{\text{n}}) )</td>
<td>-127.2727</td>
<td>-127.2727</td>
<td>13.6363</td>
<td>4.5455</td>
<td>0.1335</td>
</tr>
<tr>
<td>( E(f_{\text{f}}) )</td>
<td>1.8396 ( \times 10^3 )</td>
<td>59.0344</td>
<td>-9.1605</td>
<td>-7.1278</td>
<td>-6.3483</td>
</tr>
<tr>
<td>( E(P_{C0.8}) )</td>
<td>78.9941</td>
<td>27.600</td>
<td>-2.9341</td>
<td>-4.6044</td>
<td>-6.6504</td>
</tr>
<tr>
<td>( E(H_{\text{Sys}}) )</td>
<td>71.9303</td>
<td>93.7622</td>
<td>-0.4725</td>
<td>-7.9797</td>
<td>-10.8972</td>
</tr>
</tbody>
</table>

Based on Table 6-9, 0.2s and 0.5s window lengths are not sufficient to provide accurate estimation regarding any of the Error Indices. With 1s window length the error in critical disturbance size estimation and frequency nadir gets less than 3%, however the error in final frequency and frequency nadir time are -9.1605% and 13.636% respectively. By increasing the window length from 1s to 1.5s and 2s, the performance of the Disturbance Triggered SFR Identification method gets worse (all error indices get bigger except for \( E(t_{\text{n}}) \)). This drop in accuracy when moving from 1s to 1.5 and 2s, can be attributed to the fact that 1.5s and 2s window length following the disturbance include some change in slop of frequency drop due...
to inter area oscillations which would have adverse effect on identified SFR and consequently predicted frequency.

Figure 6-7 includes estimated frequency traces for this case study as well as the measured $f_{COI}$ and estimated $f_{COI}$ based on ambient load fluctuation SFR identification with $w=100s$.

![Figure 6-7: Measured $f_{COI}$, estimated $f_{COI}$ using Disturbance Triggered SFR Identification method with varied window length and estimated $f_{COI}$ using ambient load fluctuation SFR identification (Case study 2)](image)

Based on Figure 6-7, and as it was shown in Table 6-9, the estimated frequency based on Disturbance Triggered SFR Identification has the best performance (closest to the measured frequency) when the window length equals 1s (shown by the green curve). However, the frequency estimated by ambient load fluctuation based SFR identification (shown by the light red curve) still outperforms the Disturbance Triggered SFR Identification based frequency estimation.

As a conclusion, using ambient load fluctuations to estimate the SFR transfer function and consequently frequency behaviour following a given disturbance, results in much more accurate prediction of frequency and critical disturbance size compare to using Disturbance Triggered SFR Identification method.
6.7.5 Case Study 3 (Different Number of Generators Monitored)

This case study analyses the effect of missing generators PMU measurements on SFR identification using ambient load fluctuations. The conditions for training the SFR are the same as the condition stated for the first worked example (section 6.7.2). The results for the identified SFRs with varied number of monitored generators are then validated by comparing the estimated frequencies for the validation test against the actual frequency measurement.

The validation test condition is as follows:

- Large disturbance size: 300MW
- Large disturbance occurrence time: 2s
- Large disturbance location: bus 25 (load 5) (Figure 3-2)
- Simulation time: 14s

Table 6-10 shows the generators number and their inertia values in pu.s for IEEE 39 bus system, where $S_{\text{base}}$ equals 100MW. The generators are listed in descending order of their inertia from left to right. In this case study the order of missing generators PMU measurements is from right to left of generators listed in Table 6-9. For instance, in case of having 9 generators monitored out of 10, it means generator 8 is not monitored (based on generators order in Table 6-10).

Table 6-10: Inertia Values of Generators in IEEE 39 Bus System (all expressed in pu.s)

<table>
<thead>
<tr>
<th>Gen no</th>
<th>1</th>
<th>10</th>
<th>9</th>
<th>3</th>
<th>6</th>
<th>2</th>
<th>4</th>
<th>7</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>700</td>
<td>42</td>
<td>41.4</td>
<td>35.8</td>
<td>34.8</td>
<td>30.3</td>
<td>28.6</td>
<td>26.4</td>
<td>26</td>
<td>24.3</td>
</tr>
</tbody>
</table>

The results for identified SFR transfer function parameters, estimated $H_{\text{sys}}$ (pu.s) and critical disturbance size ($P_{C_{0.8}}$) for varied number of monitored generators are shown in Table 6-11.

The results for critical disturbance sizes for different window lengths following the disturbance occurrence are summarised in Table 6-8.
Error indices for varied number of monitored generators are shown in Table 6-12. Furthermore, two methods were used regarding frequency prediction in validation part:

- Predicted frequency using total disturbance size applied to identified SFR transfer function
- Predicted frequency using the portion of disturbance size seen by monitored generators applied to identified SFR transfer function

In Table 6-12, the frequency nadir and final frequency error indices for the first and second method are shown as $E(f_n), E(f_f)$ and $E(f_n)^*, E(f_f)^*$ respectively.

Based on Table 6-12, increase number of monitored generators improves the accuracy of the method regarding all error indices. However, by having 8 number of generators monitored out of 10, error in critical disturbance size estimation falls below 10% ($E(P_c^{0.8})=7.0754$). Also, by comparing $E(f_n), E(f_f)$ and $E(f_n)^*, E(f_f)^*$ values, it can be seen that using portion of the disturbance size seen by the monitored generators to predict the frequency behaviour is much more accurate which results in $E(f_n)^*$ getting less than 3% for only 5 generators monitored.
### Table 6-12: Error Indices for Case Study 3

<table>
<thead>
<tr>
<th>Number of Gens</th>
<th>Parameters</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E(f_n)$</td>
<td>511.0522</td>
<td>267.3278</td>
<td>147.3009</td>
<td>95.0938</td>
<td>62.3704</td>
</tr>
<tr>
<td></td>
<td>$E(f_n)^*$</td>
<td>23.9237</td>
<td>13.3246</td>
<td>-5.0105</td>
<td>-4.5388</td>
<td>-0.6182</td>
</tr>
<tr>
<td></td>
<td>$E(f_a)$</td>
<td>-127.2727</td>
<td>-127.2727</td>
<td>-127.2727</td>
<td>-127.2727</td>
<td>-68.1818</td>
</tr>
<tr>
<td></td>
<td>$E(f_l)$</td>
<td>606.8412</td>
<td>321.7157</td>
<td>179.9854</td>
<td>117.8473</td>
<td>78.0341</td>
</tr>
<tr>
<td></td>
<td>$E(f_f)^*$</td>
<td>43.3501</td>
<td>30.1038</td>
<td>8.0777</td>
<td>6.0680</td>
<td>8.9690</td>
</tr>
<tr>
<td></td>
<td>$E(P_{COI})$</td>
<td>68.1890</td>
<td>59.6897</td>
<td>48.1787</td>
<td>38.0026</td>
<td>31.7786</td>
</tr>
<tr>
<td></td>
<td>$E(H_{sys})$</td>
<td>27.3765</td>
<td>21.3856</td>
<td>13.8435</td>
<td>7.3366</td>
<td>5.4358</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of Gens</th>
<th>Parameters</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E(f_n)$</td>
<td>49.3720</td>
<td>31.9014</td>
<td>18.6985</td>
<td>8.8341</td>
<td>0.5391</td>
</tr>
<tr>
<td></td>
<td>$E(f_n)^*$</td>
<td>-0.4938</td>
<td>3.1638</td>
<td>4.1790</td>
<td>2.2355</td>
<td>0.5391</td>
</tr>
<tr>
<td></td>
<td>$E(f_a)$</td>
<td>-50.000</td>
<td>-27.2727</td>
<td>-13.6364</td>
<td>-4.5454</td>
<td>4.5455</td>
</tr>
<tr>
<td></td>
<td>$E(f_l)$</td>
<td>62.0126</td>
<td>39.6151</td>
<td>22.6163</td>
<td>9.5506</td>
<td>-0.9756</td>
</tr>
<tr>
<td></td>
<td>$E(f_f)^*$</td>
<td>7.9269</td>
<td>9.1970</td>
<td>7.6176</td>
<td>2.9086</td>
<td>-0.9756</td>
</tr>
<tr>
<td></td>
<td>$E(P_{COI})$</td>
<td>21.1318</td>
<td>13.6934</td>
<td>7.0754</td>
<td>5.6268</td>
<td>0.3201</td>
</tr>
<tr>
<td></td>
<td>$E(H_{sys})$</td>
<td>4.2178</td>
<td>1.1911</td>
<td>-0.2277</td>
<td>0.2090</td>
<td>0.0139</td>
</tr>
</tbody>
</table>

*When using portion of $P_d$ seen by the monitored generators to estimate $f_{COI}$.

Figure 6-8 and Figure 6-9 include estimated frequency traces for this case study as well as the measured $f_{COI}$. In Figure 6-8 and Figure 6-9, $f_{COI}$ is predicted by applying the portion of disturbance size seen by monitored generators to identified SFR transfer function.

![Figure 6-8: Measured $f_{COI}$ and estimated $f_{COI}$ using ambient load fluctuation SFR Identification method with varied number of generators monitored (10 to 6) (Case study 3), having disturbance size equal to the pickup share of monitored generators](image-url)
Figure 6-9: Measured $f_{COI}$ and estimated $f_{COI}$ using ambient load fluctuation based SFR Identification method with varied number of generators monitored (5 to 1) (Case study 3), having disturbance size equal to the pickup share of monitored generators.

Figure 6-10 and Figure 6-11 include estimated frequency traces for this case study as well as the measured $f_{COI}$. In Figure 6-10 and Figure 6-11, $f_{COI}$ is predicted by applying the total disturbance size to identified SFR transfer function.

Figure 6-10: Measured $f_{COI}$ and estimated $f_{COI}$ using ambient load fluctuation SFR Identification method with varied number of generators monitored (10 to 6) (Case study 3)
By comparing Figure 6-8 and Figure 6-9 with Figure 6-10 and Figure 6-11, it can be concluded that using portion of the disturbance size seen by the monitored generators to predict the frequency (Figure 6-8 and Figure 6-9) results in much higher accuracy with less monitored generators than the method using total disturbance size applied to identified SFR to predict the frequency (Figure 6-10 and Figure 6-11).

As a conclusion, the ambient load fluctuation based SFR transfer function identification provides relatively accurate estimation of critical disturbance size \(E(P_{c0.8})<10\), Table 6-12) even when only 8 generators are monitored out of 10. Furthermore, \(f_{COI}\) predicted using identified SFR resulted in less than 3% error for frequency nadir estimation \(E(f_n)*<3\), Table 6-12) when only 5 generators were monitored.

6.7.6 Case Study 4 (Generator Outage Frequency Estimates)

This case study examines the accuracy of frequency estimation using the identified SFR for generator outage type of active power disturbances. The simulations are done for six out of ten generators. Frequency is estimated using the disturbance size equal to the active power output of each disconnected generator and the pre disconnection identified SFR parameters. All outages take place at the 2s. The results of this case study are shown in Figure 6-12. The
error indices regarding frequency estimation, i.e. $E(f_n)$, $E(f_i)$ and $E(t_n)$ are presented in Table 6-13.

Figure 6-12: Measured $f_{COI}$ and estimated $f_{COI}$ for different generator outages (Case Study 4)
Table 6-13: Frequency Error Indices for Case Study 4

<table>
<thead>
<tr>
<th>Outaged Generator</th>
<th>G2</th>
<th>G3</th>
<th>G4</th>
<th>G5</th>
<th>G6</th>
<th>G9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(t_n)$</td>
<td>-13.2075</td>
<td>-18.1818</td>
<td>-17.0000</td>
<td>-14.0000</td>
<td>-20.0000</td>
<td>-15.5340</td>
</tr>
</tbody>
</table>

As it can be seen in Figure 6-12, the estimated COI frequencies for generator outages have a larger error with the actual measured COI frequency for generator outage type of disturbance compared to load increase disturbances (red line in Figure 6-8). This fact is also reflected in error indices shown in Table 6-13. The increased error in estimation rises from the fact that in case of generator outages, the SFR parameters of the system are no longer the same as pre disturbance parameters. In other words, the inertia and droop of the lost generator are not taking part in frequency dynamics following its’ outage. Therefore, with the lower inertia and droop the frequency would experience a deeper drop both at nadir and final frequency and nadir is reached faster, hence the negative error indices in Table 6-13. However, despite the increase in frequency estimation error for generator outages, the identified SFR based estimation has relatively low inaccuracy.
6.8 Fast frequency Response Dependency on Disturbance Size, Critical Disturbance size and Response Delay

Assuming critical disturbance size is known and estimated through SFR transfer function identification as discussed in this chapter, the next step is to utilise this value to adapt the fast frequency response following active power disturbance detection. There are two main aspects about fast frequency response that can be decided using critical disturbance size of the system:

- Necessity of response
- Minimum required size of response

The necessity of initiating fast frequency response can be decided by simply comparing the estimated disturbance size with the critical disturbance size of the system for a particular frequency drop size. Having estimated disturbance size larger than $P_a \Delta f$, means maximum frequency drop would be bigger than $\Delta f$, therefore fast frequency response is needed if the drop has to be contained within the $\Delta f$ limit.

In case the fast frequency response is deemed to be necessary, the next step is deciding the size of response. The simple approach is to decide the size of fast frequency response as the difference between estimated disturbance size and the critical disturbance size. However the minimum required response is dependent on the delay in fast frequency response initiation following the disturbance occurrence time. Meaning the faster action would result in a better improvement in frequency nadir even with the same response size. Therefore in this section an experiment is designed to assess the effect of delay in frequency response initiation, disturbance size and critical disturbance size on the minimum required response.

In this experiment the conditions are as follows:

- Disturbance size: varied from 1251MW to 1751MW with 100MW steps
- Critical disturbance size ($P_c^{0.3}$) equals 1210MW (Table 6-1)
- Response initiation delay : varied from 0s to 1.5s with 0.1s steps

For each disturbance and delay the following parameters were calculated:
- $P_{S0}$ which is defined as:

\[ P_{S0} = P_d - P_c \]  

(6.37)

- The minimum response needed for that disturbance size and delay to avoid the frequency nadir falling more than 0.3Hz ($P_{Sm}$). $P_{Sm}$ was calculated empirically for each case by increasing the response with small steps until the frequency violation was avoided.

The initial guess was that for small delays the $P_{Sm}$ would be equal to $P_{S0}$. Also, it was observed that by increasing the delay the $P_{Sm}/P_{S0}$ gets bigger than 1. After plotting the scatter plot for $P_{Sm}/P_{S0}$ values against response initiation delay for varied disturbance size, it was observed that the $P_{Sm}/P_{S0}$ follows an exponential behaviour regarding response initiation delay. Therefore, Matlab multi-regression function [33] was used to find the best exponential fit for $P_{Sm}/P_{S0}$ values. The results for the fitted exponential lines and the real data are shown in Figure 6-13.

![Figure 6-13: Fitted Exponential and real data of $P_{Sm}/P_{S0}$ values for varied disturbance size and delays](image)

In Figure 6-13, the formulas for the fitted exponentials using multi-regression for each line curve (each disturbance size) are as follows:
\[
\frac{P_{Sm}}{P_{S0}} = l_1 e^{delay \times l_2} + 1
\]  \hspace{1cm} (6.38)

Also by applying another regression to \( l_1 \) and \( l_2 \) regarding \( P_{S0} \) values, the following formulas are achieved which adds another layer to the regression:

\[
l_1 = 0.0062P_{S0}^{-0.405} \hspace{1cm} (6.39)
\]

\[
l_2 = 0.5537P_{S0} + 3.0414 \hspace{1cm} (6.40)
\]

As observed in Figure 6-13 and expressed in (6.38),(6.39) and (6.40), minimum required fast frequency response from a given resource (\( P_{Sm} \)) can be estimated using critical disturbance size, disturbance size and the resource initiation delay.

Furthermore, as delay was increased, the minimum required response was increased from \( P_{S0} \) (6.37) in an exponential way. However, with a response initiation delay smaller than 0.5s the minimum required response size was almost equal to \( P_{S0} \). Meaning it is a safe assumption to decide the size of fast frequency response as the difference of estimated disturbance size and critical disturbance size for responses with less than 0.5s delay. Furthermore, based on Figure 6-13, the larger the disturbance size the bigger the effect of delay in minimum required response.

For a general view and as shown in Figure 6-14, the disturbance size estimated in disturbance analysis process is used in conjunction with identified SFR to decide the size and necessity of fast frequency response.

![Figure 6-14: Disturbance and SFR Adaptive Fast Frequency Response Overview](image-url)
Figure 6-15 shows the flowchart depicting the connection between disturbance analysis results (disturbance time ($t_d$), disturbance size estimate ($P_{est}$) and disturbance location candidates (DLC) with SFR identification result (Critical disturbance size ($P_C$) to decide the necessity, size ($P_S$) and location ($L_S$) of fast frequency response. In this flowchart the delay of fast frequency responses are assumed to be less than 0.5s, therefore there is no need to adapt the size of fast frequency response to the delay in response initiation. As a result the response size ($P_S$) is decided using (6.37). The dashed lines in Figure 6-15 shows the update rate equal to SCADA rate of update (in order of minutes) and the solid lines are executed once a disturbance is detected.

Based on Figure 6-15, the response adaptation procedure is triggered once a disturbance is detected in disturbance analysis stage and disturbance time ($t_d$) is calculated. Following the trigger, the response size is decided using the difference between critical disturbance size of the system ($P_C$) and disturbance size estimate ($P_{est}$). The location of response is chosen as the closest fast frequency response resource to the disturbance location candidates (DLC) for which the $P_S$ is available. The closeness of the resources to disturbance location candidates is quantified regarding their electrical distance.
Figure 6-16 shows the same flowchart as Figure 6-15, however in this flowchart the fast frequency response initiation delay is assumed to be bigger than 0.5s, therefore response size is also adapted to response initiation delay using (6.38).

Based on Figure 6-16, the response adaptation procedure is triggered once a disturbance is detected in disturbance analysis stage and disturbance time ($t_d$) is calculated. Following the trigger, the $P_{S0}$ is calculated as the difference between critical disturbance size of the system ($P_C$) and disturbance size estimate ($P_{est}$). The location of response is chosen as the closest fast frequency response resource to the disturbance location candidates (DLC) for which the $P_{S0}$ is available. Following the selection of fast frequency response resource, the delay of the chosen resource is retrieved and then used to calculate the response size ($P_S$) (6.38).
6.9 Chapter Summary

This chapter presented a novel method for a dynamic frequency security assessment of the system using system identification techniques. Parameters of simplified system frequency response transfer function (SFR) were estimated by applying the system identification method to ambient load fluctuations and $f_{COI}$ variation. The term critical disturbance size was defined and calculated using identified SFR. The critical disturbance size was then utilised to decide the frequency security status and fast frequency response requirements. Furthermore, critical disturbance size was proven to be useful in deciding the necessity and size of minimum fast frequency response required following active power disturbance detection.

Case studies applied to IEEE 39 bus system proved the robustness of SFR transfer function and critical disturbance size estimation against varied noise level, window length and number of monitored generators.

The next chapter demonstrates the merits of using critical disturbance size as well as disturbance size in Adaptive Under Frequency Load Shedding schemes (AUFLS) compared to Semi Adaptive schemes adaptive to only disturbance size. In the next chapter a comparison is carried out to investigate the benefits of the fast frequency response adaptation to output result of dynamic frequency security assessment. In this sense the suggested AUFLS is regarded as a form of fast frequency response.

6.10 References


7. **Adaptive Under Frequency Load Shedding Scheme Based on Dynamic Security Assessment**

As discussed in Chapter 6, critical disturbance size estimated using system identification based dynamic frequency security assessment can be used to decide the size of fast frequency response required following an active power disturbance to avoid frequency nadir to fall below a pre-defined threshold. As Under Frequency Load Shedding Schemes (UFLS) can be regarded as a form of fast frequency response, this chapter explores the potential benefits of adapting the necessity and size of load shedding to both disturbance size estimate and critical disturbance size (Adaptive Under Frequency Load Shedding (AUFLS)). Since dynamic frequency security assessment is a prerequisite for this new suggested form of AUFLS, the term Dynamic Security Assessment (DSA) based AUFLS is chosen for it.

The proposed AUFLS scheme proves to achieve a more precise estimate of the minimum shed for a range of system conditions compared to semi adaptive UFLS that are only adaptive to disturbance size. The reduced response size of DSA based AUFLS decreases system operating cost as well as alleviating the risk of over shedding that would expose an already stressed system to an increased risk of secondary contingencies, which might lead to system blackout. Various scenarios with different wind levels have been simulated with results verifying the effectiveness of the method compared to semi adaptive UFLS methods.

Section 7.1 explains the drives behind designing new form of UFLS. Section 7.2 describes the new suggested DSA based AUFLS. Section 7.3 presents different semi adaptive UFLS. Section 7.4 includes the comparison results between the DSA based AUFLS and semi adaptive UFLS under varied wind generation penetration levels. Finally, the chapter is concluded in Section 7.5.

7.1 **Chapter Introduction**

Following an active power disturbance the power system frequency suddenly changes, initially the change is limited by the release of energy stored in the rotating mass of the system. This is due to the coupling between the power system frequency and the
electromagnetic torque of the generators and is known as the “natural” inertial response of the generators [1][2]. This means that the system inertia defines the initial frequency decline; whereas, the governor response (primary response) is the main factor in determining the behaviour of the frequency response after this. However, a combination of governor and inertial response dictates the minimum frequency (nadir) and the time at which it is reached [3][4].

Replacing carbon fuelled energy sources with renewable generation is a key movement towards fulfilling worldwide strategy of reducing carbon emissions as well as maintaining secure and sustainable supply of energy. As an example, in Northern Ireland, a target has been set to supply 40% of electricity consumption from renewable sources by the year 2020 [5]. Similarly, Republic of Ireland and several other European countries have set the same target, whereas both USA and China have their target as 20% [6][7].

Significant penetrations of non-synchronously connected generators, e.g. wind turbines, will cause a significant decrease in the inertia of a power system, which will cause the power system to experience a faster and deeper drop in frequency after any given disturbance. In the case of reduced inertia, the frequency nadir can be kept above the infrequent infeed loss limit (i.e. the minimum allowed system frequency), which is 49.2 Hz for the GB system, using a larger volume of faster governor response delivered with a higher ramp rate. Alternatively, new technologies can be used to deliver fast frequency control services, e.g. fast demand response in the form of fast adaptive Under Frequency Load Shedding Schemes (UFLS) applied to less vital loads, which can help to limit the initial frequency decline [8].

UFLS can either be designed to deliver a fixed response based on the worst case scenario or to adapt the amount of load shed to the system conditions and disturbance sizes. However, the increasing range of possible operating conditions, e.g. due to the more diverse generation mix that must be deployed to meet UK emission targets, will increase the level of uncertainty in system parameters that define the initial frequency response. For example, National Grid has predicted that total system inertia ($H_{Sys}$) may be as low as 50 GVAs by 2025/26 in the Gone Green scenario [9], compared to the approximately 240 GVAs available today [8]. Mitigating the threat posed by reduced inertia has become an area of intense study, an example of which is the National Grid led Network Innovation Competition (NIC) funded Smart Frequency Control Project [8].
These changes in the nature of power systems will mean that designing UFLS for the worst case scenario would lead to considerable over shedding in almost all cases, as the worst case scenario is more extreme and less likely to occur. Without a change in the design of UFLS, this would force system operators to hold more loads in fast response mode.

Semi adaptive UFLS schemes with several shedding steps have been proposed in the literature [10][11][12]. These schemes adapt the amount of load shed to the disturbance size \( P_d \) and shed it based on the violation of fixed frequency thresholds. However, waiting for a threshold violation can increase the degree of risk, especially in low \( H_{sys} \) scenarios where the larger RoCoF means that less time is available to limit the frequency deviation. Therefore, determining the need to shed and the optimal amount of shed almost immediately after the disturbance is particularly beneficial in systems with low/variable inertia.

In this chapter a new Adaptive UFLS is proposed in which shedding is triggered based on the estimated disturbance size \( P_d \) being greater than a Critical Disturbance Size \( P_c^{\Delta f} \). As discussed in Chapter 6, Critical disturbance size \( P_c^{\Delta f} \) is defined as a system parameter which equals the minimum active power disturbance that leads to the frequency nadir violation of a fixed frequency threshold \( \Delta f \). A procedure for estimation of \( P_c^{\Delta f} \) as a part of a new Dynamic Frequency Security Assessment was also described in Chapter 6, where \( P_c^{\Delta f} \) was calculated based on the identified system frequency response parameters.

Therefore a prerequisite for this new UFLS is a dynamic frequency security assessment as a part of online DSA that finds the \( P_c^{\Delta f} \) for the current system conditions using near past generators frequency and power ambient fluctuations. The shedding is performed in one step, which eliminates the accumulated delay of several shedding steps. Furthermore, the amount of shed by the proposed scheme is adapted to not only the disturbance size but also system frequency response dynamics of the system, which is expressed through critical disturbance size.

### 7.2 DSA based Adaptive UFLS

The best parameters for a UFLS scheme, i.e. those that provide the desired frequency change with the minimum amount of load shed, can be selected both during system operation (online) and during system planning (offline). However, due to the ever increasing
uncertainty and volatility in system parameters and the frequency response e.g. disturbance size and inertia, online design of UFLS schemes, which can also be referred to as Adaptive or Semi Adaptive UFLS, has gained more interest in the recent years [10][11][12][13].

The DSA based Adaptive UFLS scheme presented here is designed online and has two main inputs:

1. Critical disturbance size of the system \( P_{c_{\Delta f}} \)
2. Estimated disturbance size \( (P_d) \) shortly following the disturbance detection

Dynamic Frequency Security Assessment results are used to find the Critical Disturbance \( (P_{c_{\Delta f}}) \) for the current system operating conditions e.g. inertia. Furthermore, as discussed in Chapters 3, 4 and 5 the detection and estimation of disturbance size and location can be done by analysing one PMU time step following the disturbance occurrence.

Therefore the first input comes from Dynamic Frequency Security Assessment and the second input comes from disturbance analysis.

### 7.2.1 Critical disturbance size application in AUFLS

The Critical Disturbance Size \( (P_{c_{\Delta f}}) \) is defined here as the minimum active power disturbance that would cause the frequency of the system to fall below a certain frequency threshold. For all the simulations performed in this chapter the frequency drop threshold \( (\Delta f) \) for critical disturbance size was set to 0.8 Hz.

Different parameters affect \( P_{c_{\Delta f}} \), e.g. total system inertia, load characteristics and governors settings. The procedure for estimation of critical disturbance size using identified system frequency response model was explained and proved in chapter 7. As discussed in section 6.8, the triggering condition for the proposed UFLS scheme is having disturbance size \( (P_d) \) bigger than the critical disturbance size:

\[
P_{c_{\Delta f}} \leq P_d \tag{7.1}
\]
As shown in section 6.8, in the case of having response initiation delay of less than 0.5s following the disturbance, the minimum response size to help avoid the frequency nadir violation of a pre-defined level ($\Delta f$) is equal to the difference between $P_d$ and $P_c^{\Delta f}$.

Assuming the delay for the DSA based AUFLS initiation following the disturbance occurrence is 0.1s (less than 0.5s); the load shedding size for the suggested DSA based AUFLS ($P_{S}^{\text{DSA}}$) is calculated as follows:

$$P_{S}^{\text{DSA}} = P_d - P_c^{\Delta f}$$ (7.2)

### 7.3 Semi Adaptive UFLS

Semi Adaptive UFLS schemes are only adaptive to the disturbance size. There are many different Semi Adaptive UFLS scheme suggested in literature which propose different distribution of the load shedding between each step and have various triggering conditions. In [11] more loads are shed in the first steps to restrain the frequency derivative as soon as possible following the disturbance. However as pointed out in [14], the shedding amount would be often excessive for events not as severe as the worst case scenario. In this chapter three different Semi Adaptive UFLS schemes are considered for comparison with DSA based AUFLS proposed here. The number of steps for all Semi Adaptive UFLS is equal to three. Table 7-1 summarizes the frequency thresholds that trigger each shedding step and the amount of shedding as a percentage of disturbance size for all the studied semi adaptive schemes. It worth mentioning that for each load shedding step there is a delay of 0.1 s. In these schemes, the necessity of further response is decided as the frequency keep passing different thresholds, whereas in DSA based AUFLS the response and its size is evaluated in one step. DSA based AUFLS fast decision on the necessity and size of response has the potential to be beneficial especially in low inertia systems where frequency drops in a much faster rate.
Table 7-1: Semi Adaptive UFLS parameters

<table>
<thead>
<tr>
<th>S1</th>
<th>Triggering Frequency</th>
<th>Percentage of $P_d$</th>
<th>Delay of each step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>49.8 Hz</td>
<td>20%</td>
<td>0.1 s</td>
</tr>
<tr>
<td></td>
<td>49.6 Hz</td>
<td>20%</td>
<td>0.1 s</td>
</tr>
<tr>
<td></td>
<td>49.4 Hz</td>
<td>10%</td>
<td>0.1 s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S2</th>
<th>Triggering Frequency</th>
<th>Percentage of $P_d$</th>
<th>Delay of each step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>49.8 Hz</td>
<td>40%</td>
<td>0.1 s</td>
</tr>
<tr>
<td></td>
<td>49.6 Hz</td>
<td>10%</td>
<td>0.1 s</td>
</tr>
<tr>
<td></td>
<td>49.4 Hz</td>
<td>10%</td>
<td>0.1 s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S3</th>
<th>Triggering Frequency</th>
<th>Percentage of $P_d$</th>
<th>Delay of each step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>49.8 Hz</td>
<td>40%</td>
<td>0.1 s</td>
</tr>
<tr>
<td></td>
<td>49.6 Hz</td>
<td>20%</td>
<td>0.1 s</td>
</tr>
<tr>
<td></td>
<td>49.4 Hz</td>
<td>10%</td>
<td>0.1 s</td>
</tr>
</tbody>
</table>

7.4 Simulation results

In this section the performance of the proposed DSA based AUFLS is compared with the semi adaptive UFLS schemes (S1, S2, S3) whose parameters were presented in Table 7-1. All simulations have been performed using DIgSILENT Power Factory and on the IEEE 9 bus system (Figure 7-1). The goal of all of the UFLS schemes is to prevent the frequency from dropping below 49.2 Hz. The simulations are all for an active power disturbance at load C, bus 8 (Figure 7-1). Different levels of wind penetration are studied. $S_{base}$ of the system is 100MVA.

The voltage dependency of the loads is represented using a ZIP model and the frequency dependency is included as a linear gain (7.3) (7.4) [15].
\[ P = P_0 \left[ a_p \left( \frac{V}{V_0} \right)^2 + b_p \left( \frac{V}{V_0} \right)^4 + c_p \right] (1 + K_{pf} \Delta f) \]  \hspace{1cm} (7.3)

\[ Q = Q_0 \left[ a_q \left( \frac{V}{V_0} \right)^2 + b_q \left( \frac{V}{V_0} \right)^4 + c_q \right] (1 + K_{qf} \Delta f) \]  \hspace{1cm} (7.4)

The loads parameters are the same for all three load buses and are given in Table 7-2.

Table 7-2: Load parameters

<table>
<thead>
<tr>
<th>P(MW)</th>
<th>( a_p )</th>
<th>( b_p )</th>
<th>( c_p )</th>
<th>( K_{pf} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>1.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q(MVAR)</th>
<th>( a_q )</th>
<th>( b_q )</th>
<th>( c_q )</th>
<th>( K_{qf} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>-1.2</td>
</tr>
</tbody>
</table>

Before presenting the results for the comparison of different semi Adaptive UFLS and DSA based AUFLS, the approach used to model the wind generation penetration and the effect of increased wind on critical disturbance size and fast frequency response requirements are discussed in the following section.

### 7.4.1 Wind Generation Penetration Level and Critical Disturbance Size Analysis

The increased penetration of wind is modelled in this chapter by replacing the MVA capacity of each synchronous generator with static generators in DigSILENT, therefore the following expression is used for defining the level of wind penetration (w\%) in the system compared to the original case:

\[ w\% = 100 \left( \frac{\sum_{i=1}^{n} S_{w\%,i}}{\sum_{i=1}^{n} S_i} \right) \]  \hspace{1cm} (7.5)

where \( S_i \) is the power system base of synchronous generator \( i \) in case with no wind generation. \( S_{w\%,i} \) is the MVA capacity of generator \( i \) replaced by converter connected generation (wind). w\% represents the total MVA capacity replaced by wind as a percentage of total system installed capacity.
The total system inertia when having wind penetration level of \( w\% \) is shown as \( H_{\text{Sys,}w\%} \) and is calculated as follows:

\[
H_{\text{Sys,}w\%} = \sum_{i=1}^{n} \frac{H_i(S_i - S_{w\%})}{S_{\text{base}}}
\]  

(7.6)

where, \( S_{\text{base}} \) is the common power system base, \( S_i \) is the power base of the \( i^{th} \) generator in no wind scenario, \( H_i \) is the \( H \) constant of the \( i^{th} \) generator on its own base (\( S_i \)).

Inertia constant of each generator can be defined in the common power system base (\( S_{\text{base}} \)) as follows:

\[
H_{i,\text{Sys}} = \frac{H_i S_i}{S_{\text{base}}}
\]  

(7.7)

Where \( H_{i,\text{Sys}} \) represents the inertia constant of generator \( i \) in \( S_{\text{base}} \) of the system.

Table 7-3 shows the \( S_i \) and \( H_i \) values for generators in IEEE 9 bus system with no wind.

<table>
<thead>
<tr>
<th>( S_1 ) (MVA)</th>
<th>( S_2 ) (MVA)</th>
<th>( S_3 ) (MVA)</th>
<th>( H_1 ) (s)</th>
<th>( H_2 ) (s)</th>
<th>( H_3 ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>275</td>
<td>192</td>
<td>128</td>
<td>8.5896</td>
<td>3.333</td>
<td>1</td>
</tr>
</tbody>
</table>

Assuming wind generation penetration is evenly distributed at all generators buses in the IEEE 9 bus system, meaning the \( S_{w\%}/S_i \) being constant for all generator buses, the \( H \) constant reduction in percentage for all generators as well as system inertia reduction would be equal to \( w\% \). Therefore in this case, the wind generation penetration level (\( w\% \)) would have the following relationship with system total inertia value:

\[
w\% = (1 - \frac{H_{\text{Sys,}w\%}}{H_{\text{Sys}}})100
\]  

(7.8)

Where \( H_{\text{Sys,}w\%} \) is the total system inertia for \( w\% \) wind generation penetration and \( H_{\text{Sys}} \) is the total system inertia for case with no wind generation.
Table 7-4 shows the generators inertia constants in system base ($H_{i,Sys}$), system inertia ($H_{Sys,w%}$) and critical disturbance size for 0.8Hz frequency drop ($P_{C0.8}$) for three levels of wind generation penetration (0%, 20% and 40%).

Table 7-4: Wind generation penetration scenario parameters

<table>
<thead>
<tr>
<th>w%</th>
<th>0%</th>
<th>20%</th>
<th>40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{i,Sys}$(pu.s)</td>
<td>23.62</td>
<td>18.90</td>
<td>14.17</td>
</tr>
<tr>
<td>$H_{2,Sys}$(pu.s)</td>
<td>6.40</td>
<td>5.12</td>
<td>3.84</td>
</tr>
<tr>
<td>$H_{3,Sys}$(pu.s)</td>
<td>1.28</td>
<td>1.02</td>
<td>0.77</td>
</tr>
<tr>
<td>$H_{Sys,w%}$(pu.s)</td>
<td>31.30</td>
<td>25.04</td>
<td>18.78</td>
</tr>
<tr>
<td>$P_{C0.8}$(MW)</td>
<td>118</td>
<td>114</td>
<td>104.5</td>
</tr>
<tr>
<td>$S_{w%,1}$(MVA)</td>
<td>0</td>
<td>55</td>
<td>110</td>
</tr>
<tr>
<td>$S_{w%,2}$(MVA)</td>
<td>0</td>
<td>38.4</td>
<td>76.8</td>
</tr>
<tr>
<td>$S_{w%,3}$(MVA)</td>
<td>0</td>
<td>25.6</td>
<td>51.2</td>
</tr>
</tbody>
</table>

Figure 7-2 shows the $P_{C0.8}$ for different wind generation penetration levels for both frequency dependent and frequency independent loads ($K_{pf}$ being zero in (7.3)). As it can be seen in Figure 7-2, $P_{C0.8}$ decreases as the wind generation level increases. In other words, $P_{C0.8}$ has direct relation with total system inertia level. This also translates into needing to take frequency control actions for a smaller value of disturbance size if the system has lower inertia/higher wind generation level. Based on Figure 7-2, $P_{C0.8}$ is lower for frequency independent loads, as the frequency dependent loads contribute to the system total damping ($K_D$ in swing equation (2.6)).
Figure 7-2 : $P_{c,0.8}$ for different level of wind generation penetration (w%) with frequency dependent and frequency independent loads

Furthermore, maximum infeed loss (system reference incident) ($P_{d}^{\text{max}}$) is shown as the red dashed line in Figure 7-2 and is equal to 275MW (equal to the generator 1 maximum output Table 7-3). Since critical disturbance size is smaller than the $P_{d}^{\text{max}}$, system frequency would drop below 49.2 Hz in case of maximum infeed loss. Therefore, system is not frequency secure and needs some fast frequency response to be armed and activated in case of emergency.

The maximum fast frequency requirement for the system with frequency dependent loads (assuming the delay in the response initiation following the disturbance is less than 0.5s), is shown as the purple area covering all level of wind generation penetration. The maximum fast frequency response requirement for wind penetration level of w% is shown as $P_{S,w\%}^{\text{max}}$ and is equal to the difference between $P_{d}^{\text{max}}$ and critical disturbance size for that level of wind penetration (shown as the vertical arrow in Figure 7-2) and formulised as follows:

$$P_{S,w\%}^{\text{max}} = P_{d}^{\text{max}} - P_{c}^{\Delta f}$$  \quad (7.9)
As it can be seen in Figure 7-2, the purple area cross section keeps growing as the wind penetration level increases, meaning the maximum fast frequency response requirement increases as more wind is integrated in the system (inertia reduces).

Other than procuring fast frequency response services (e.g. AUFLS, electrical storage [8][18]) there are three other ways of tackling this frequency security issue and removing the frequency security burden against increased level of renewables:

- Decreasing the maximum infeed loss ($P_{d_{\text{max}}}$) by limiting the maximum output of the biggest generator, by doing that the purple area gets smaller consequently maximum fast frequency response requirements reduces.
- Having more generators in governing mode which shifts the critical disturbance size line higher and makes the purple area smaller, therefore less or no fast frequency response would be required.
- Having wind turbines equipped with synthetic inertia control loop [19][20]. The synthetic inertia contributes to increase the overall inertia of the system which in return help shifting up the critical disturbance size curve and reducing the fast frequency response requirement over varied level of wind generation penetration.

The last option is more desirable as many countries such as Ireland and Denmark have already been practicing the instalment of synthetic inertia on wind turbines [21] ; and some like UK National Grid has outlined the future possibility for enforcing mandatory provision of synthetic inertia from wind farms [22].

The large blades of wind turbines store kinetic energy and by utilising a so called “synthetic inertia” control mechanism the inertia of wind turbines can be accessed [19]. The synthetic inertia of the wind turbines can be comparable of that of a synchronous conventional generator unit. The inertia constant of a wind turbine in its own power base ($H_w$) can be estimated using the following [23]:

$$H_w = 2.63d^{0.12}$$  \hspace{1cm} (7.10)

Where $d$ is the turbine blade diameter in m. In case of 2 MW turbine $d$ would be around 90m resulting in $H_w$ equal to 4.513s in 2MVA $S_{\text{base}}$. Therefore, the $H_w$ of the wind turbine can be roughly estimated as 2.2565s per MVA capacity.
In order to analyse the effect of synthetic inertia on fast frequency response requirement, a simple scenario is investigated in this section. In this part, the previous analysis (Figure 7-2) is repeated but assuming a scenario in which all wind farms are equipped with synthetic inertia control loop. Furthermore, based on (7.10) the synthetic inertia constant is assumed to be equal to 2.256s per MVA.

The total inertia contribution from synthetic inertia control of wind turbines for \( w\% \) of wind generation penetration level is shown as \( H_{W,w\%} \) and is calculated as follows:

\[
H_{W,w\%} = \sum_{i=1}^{n} S_{w\%,i} H_W
\]  

(7.11)

Where \( S_{w\%,i} \) is the MVA capacity of generator \( i \) replaced by non-synchronously connected generation (wind). \( H_W \) is synthetic inertia of the wind in s per MVA of capacity and is equal to 2.256s.

The total system inertia with \( w\% \) wind generation penetration and having the wind farms equipped with synthetic inertia is shown as \( H_{Sys,w\%} \) * and is calculated as follows:

\[
H_{Sys,w\%} = H_{W,w\%} + H_{Sys,w\%} \]  

(7.12)

Where \( H_{Sys,w\%} \) is the total system inertia for \( w\% \) wind generation without the inclusion of synthetic inertia (7.6).

This increased total system inertia with the addition of synthetic inertia \( H_{Sys,w\%} \) * can be perceived as lower wind generation penetration in terms of critical disturbance size calculation. Therefore, in order to be able to plot the critical disturbance size curve after the addition of synthetic inertia control loop and based on (7.8) a new perceived wind generation penetration level is calculated as follows:

\[
w\%* = (1 - \frac{H_{Sys,w\%} \}}{H_{Sys}})100
\]  

(7.13)

where \( H_{Sys} \) is the pre wind integration system total inertia.
Table 7-5, shows the parameters for varied wind generation levels and with the inclusion of synthetic inertia contribution from wind generation.

**Table 7-5: Parameters for varied wind generation penetration levels with synthetic inertia control**

<table>
<thead>
<tr>
<th>w%</th>
<th>$H_{w%}(\text{pu.s})$</th>
<th>$H_{w%}(\text{pu.s})$</th>
<th>$H_{w%*}(\text{pu.s})$</th>
<th>w%*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>31.30</td>
<td>0</td>
<td>31.30</td>
<td>0%</td>
</tr>
<tr>
<td>10%</td>
<td>28.17</td>
<td>1.34</td>
<td>29.51</td>
<td>5.71%</td>
</tr>
<tr>
<td>20%</td>
<td>25.04</td>
<td>2.68</td>
<td>27.72</td>
<td>11.43%</td>
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<td>30%</td>
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<td>4.03</td>
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<td>40%</td>
<td>18.78</td>
<td>5.37</td>
<td>24.15</td>
<td>22.84%</td>
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<tr>
<td>50%</td>
<td>15.65</td>
<td>6.71</td>
<td>22.36</td>
<td>28.56%</td>
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<td>60%</td>
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<td>8.05</td>
<td>20.57</td>
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<td>9.40</td>
<td>18.79</td>
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<td>17</td>
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<td>90%</td>
<td>3.13</td>
<td>12.08</td>
<td>15.21</td>
<td>51.40%</td>
</tr>
</tbody>
</table>

As seen in Table 7-5, the perceived wind generation level after the addition of synthetic inertia control loop ($w\%*$) is lower than the actual wind generation level ($w\%$). This is due to the contribution of synthetic inertia to the total system inertia of the system.

In Figure 7-3, the orange line represents the critical disturbance size values for different wind generation levels after the addition of synthetic inertia. The points on the orange curve are achieved by assigning the critical disturbance size values for $w\%*$ (7.13) to $w\%$ wind generation level.

Based on Figure 7-3, after the addition of synthetic inertia the maximum fast frequency response requirement of the system for different wind generation level is reduced, as seen by the reduced size of the purple area in Figure 7-3 compared to Figure 7-2. The contribution of synthetic inertia to fast frequency response requirements is shown by the green area in varied wind generation penetration level.

In conclusion, the provision of synthetic inertia control from wind turbines can compete or jointly serve with other form of fast frequency response services (e.g. AUFLS and storage) to help relieve the frequency security restriction on the maximum wind generation penetration level in the system.
Addition of Synthetic Inertia
Frequency Dependent Loads
Frequency Independent Loads

Figure 7-3: The effect of addition of synthetic inertia on the maximum fast frequency response requirement of the system \( P_{S,w\%}^{\text{max}} \) for varied level of wind penetration level (w\%).

7.4.2 Comparison of UFLS schemes for 20% wind

Figure 7-4 shows the frequency trace following a disturbance of 135 MW for w\%=20%. Since 135 MW is larger than \( P_{C,0.8} \) (=114MW (Table 7-4)), the frequency would drop below 49.2 Hz with no UFLS action. As it can be seen, all UFLS schemes succeed to prevent a frequency violation. However, the DSA based AUFLS only sheds enough to keep the frequency above 49.2 Hz (sheds 33 MW less than S1 and 46.5 MW less than S2).

Figure 7-5 shows the frequency trace following a disturbance equal 235 MW for w\%=20%. Since 235 MW is larger than \( P_{C,0.8} \) (=114MW), the frequency would drop below 49.2 Hz with no UFLS action. It can be seen that S1 fails to prevent the frequency violation. As before, the DSA based AUFLS only sheds enough to keep the frequency above 49.2 Hz; however in this case it sheds 2.5 MW more than S1 (in order to ensure satisfactory frequency) and 21 MW less than S2.
Figure 7-4: COI Frequency traces with different UFLS schemes, $P_d=135$ MW, $w\%=20$, $H_{Sys,w\%}=25.04s$

Figure 7-5: COI Frequency traces with different UFLS schemes, $P_d=235$ MW, $w\%=20$, $H_{Sys,w\%}=25.04s$
Figure 7-6: Difference of S1 and S2 shedding amount with the DSA based AUFLS for different disturbance sizes applied to IEEE 9 bus system. Wind generation level (w%) is 20% ($H_{sys,w\%} = 25.04s$).

Figure 7-6 shows the difference in the amount of load shed by the DSA based AUFLS and S1 and S2 (Table 7-1) for different disturbance sizes. The maximum active power disturbance $P_{d\text{max}}$ (maximum infeed loss) is equal to 275 MW. The dashed and solid circles in Figure 7-6 mark the disturbance levels at which S1 and S2 would fail to keep the frequency above 49.2 Hz, respectively. For S1 the maximum $P_D$ without frequency violation is 225 MW (Figure 7-6 and Figure 7-7) which is smaller than $P_{d\text{max}}$ (275 MW). Therefore, S1 is not suitable for 20% wind generation penetration. For S2 the maximum $P_D$ without frequency violation is 315 MW (Figure 7-6 and Figure 7-7) which is bigger than $P_{d\text{max}}$ (275 MW), this means S2 is suitable for 20% wind generation penetration.

In Contrast, the DSA based AUFLS does not fail to prevent the frequency falling below 49.2 Hz for any disturbance size and sheds less load in most cases compared to S1 and S2, particularly for the smaller, more probable disturbances (Figure 7-6 and Figure 7-7). However, the DSA AUFLS sheds more than S2 for disturbance sizes between 295 and 315 MW (shown by the dotted circle in Figure 7-6). This can be attributed to the faster response of the DSA based AUFLS limiting the natural load reduction from frequency and voltage-sensitivity. Figure 7-7 shows the absolute shedding amount for DSA based AUFLS, S1 and
S2. Furthermore, $P_{Sm}$ is also shown in Figure 7-7 which is the minimum shedding amount applied 0.1 s following an event which can save frequency from falling below 49.2Hz.

![Figure 7-7: Shedding amount for different disturbance size with various UFLS methods applied to IEEE 9 bus system. Wind generation level ($w\%$) is 20\% ($H_{Sys,w\%}=25.04s$).](image)

As seen in Figure 7-7, DSA based AUFLS ($P_{s}^{DSA}$) can provide a relatively close estimation of the minimum shed needed $P_{Sm}$. As disturbance size gets bigger the natural load response (due to voltage and frequency drop) also gets bigger therefore, the difference between $P_{Sm}$ and $P_{s}^{DSA}$ grows.

### 7.4.3 Comparison of UFLS schemes for 40% wind

Figure 7-8 shows the difference in the amount of load shed by the DSA based AUFLS and S1, S2 and S3 (Table 7-1) for different disturbance size levels for a wind generation level of 40\% ($H=18.78$s) (Table 7-4). The maximum $P_d$ without frequency violation for S1 and S2 is 195 MW and 255 MW respectively, both of which are smaller than $P_{d}^{max}$ (275MW). Therefore S1 and S2 both fail to ensure frequency security with 40\% wind generation.
Therefore, in order to secure the system for disturbance sizes up to \( P_{d}^{\text{max}} \), S3 must be applied to the system. To achieve this, S3 sheds more than S1 and S2 for the whole range of disturbance levels (Figure 7-8). On the other hand, the DSA based AUFLS not only prevents the frequency falling below 49.2 Hz for all disturbance size levels but also sheds less loads in most cases when compared to S1, S2 and S3 (Figure 7-8).

![Graph](image-url)

**Figure 7-8**: Difference of S1, S2 and S3 shedding amount with the DSA based AUFLS for different disturbance sizes applied to IEEE 9 bus system. Wind generation level is 40% (\( H=18.78s \)).

In the absence of DSA based AUFLS a and in the presence of increased wind generation and its associated reduced inertia constant, system operators could be forced to opt for more and more conservative semi adaptive UFLS (moving from S1 to S2 and finally to S3) to maintain system frequency security. In contrast, the DSA based AUFLS can deliver frequency security for increased wind penetration levels and does so with a smaller amount of load shed in the majority of cases.
7.5 Chapter Summary

Due to ever growing incentives for utilizing renewable energy sources, e.g. wind generation, in power systems the system inertia will decrease significantly in the future. This deficit in inertial response will lead to deterioration in frequency security that can either be compensated for by a higher volume of faster governor response or fast frequency response in the form of fast demand response, which in this case is Under Frequency Load Shedding (UFLS) applied to less vital loads. In order for this new frequency response service to be practical the inconvenience to customers should be minimised and a key part of achieving this is optimising the amount of load shed.

The increased volatility in system parameters and frequency behaviour and the importance of optimal load shedding requires the adaptation of UFLS to as many parameters as possible in real time. In this chapter a DSA based AUFLS scheme that is adaptive to both disturbance size and critical disturbance size has been proposed. Dynamic Frequency Security Assessment results are utilised to set the triggering condition for the proposed AUFLS scheme based on the critical disturbance size.

Delayed reaction to a frequency decline during the inertial stage (before frequency minimum) can only be compensated for by an increased volume of response, in other words, faster actions can deliver the same effect with less resource. The proposed DSA based AUFLS scheme provides a fast decision on the need for load shedding, estimates the minimum load shed and then sheds it in one step, which eliminates the accumulated delay of the several shedding steps used by semi adaptive UFLS (Table 7-1).

The proposed AUFLS was tested using dynamic simulations and was found to be suitable for a range of wind generation/inertia levels and disturbance sizes and does not fail if the disturbance size/wind level is increased, unlike the semi adaptive schemes considered here. Furthermore, the proposed DSA based AUFLS sheds less than the semi adaptive schemes in the majority of cases, as their design is inherently more conservative and they shed more for the smaller, more probable disturbances to ensure sufficient response to larger, less probable disturbances.
7.6 References


8. Thesis Summary

This chapter summarizes the intent of the research presented in this thesis (Section 8.1). Also it provides some conclusions regarding the success of the research, which are relative to its original goals (Section 8.2). The contributions made by this research are discussed in Section 8.3. Finally, some recommendations for further development of this research are made in Section 8.4.

8.1 Introduction

The research presented in this thesis was focused on creating methods for enabling the delivery of fast and adaptive frequency control actions (e.g. AUFLS), particularly in low inertia power systems where speed of actions are of paramount importance. The suggested fast adaptive frequency control strategy entails two blocks; Disturbance Analysis and Frequency Response Management. The former is responsible for early detection of active power disturbance and identifying the location and the size of the disturbance. Whereas the latter estimates the system parameters affecting frequency response dynamics (SFR model) and based on that evaluates frequency security and fast frequency response requirement of the system. Moreover, the outputs from the two blocks (disturbance characteristics information and SFR model) can help predict the frequency nadir threshold violation and increase the adaptability level of fast frequency control actions. This would lead to a more accurate decision on the necessity and the size of fast frequency response, hence lower cost and reduced risk of frequency instability.

Fast disturbance analysis allows the system to initiate adaptive control actions before large frequency deviation, where waiting for frequency threshold violation to trigger the response can compromise the success of the control action. Faster delivery of frequency response reduces the size of response required and the risk of secondary contingencies and eventual cascading blackout. Therefore, the effectiveness of these fast frequency services depends highly on the timing and accuracy of the disturbance analysis.
Previous researchers made attempts to predict the frequency behaviour and design frequency control actions based on the predicted frequency. However, their methods required a window of frequency measurements following the disturbance, where having short window would lead to inaccurate prediction and having large window compromised speed of response.

Currently researches into the areas fundamental for delivering a faster more adaptive frequency control actions, are particularly relevant due to the expected frequency security threat posed by the increased connection of renewables in the coming decades. Furthermore, the advent of synchronised measurement technology (SMT) and wide area monitoring, protection and control (WAMPAC) opens opportunities to explore new strategies for power system emergency frequency control using synchronised measurements.

With the increased level of embedded generation in distribution networks, Distribution System Operators (DSOs) face both significant challenges and potentials regarding frequency control capabilities. In other words, TSOs and DSOs need to collaborate closely to provide required level of system frequency security. In this sense, two points worth extra attention: Firstly, having Active Distribution Networks (ADN) which are capable of maintaining frequency stability without or significantly less reliance on transmission network resources, i.e. under islanding conditions. Secondly, having more accurate methods for distinguishing between islanding events and active power disturbance events, this way the sympathetic tripping of a significant amount of distributed generations would be avoided. Based on the methods introduced in this thesis, active power disturbance can be detected and localised on distribution level and the necessity and the minimum required size of fast frequency response can be estimated. These, information can then be used to trigger frequency response from distribution owned resources, i.e. loads, batteries, wind turbines. Furthermore, using the system identification method suggested in Chapter 6, real time frequency dynamics of the external grid at Bulk Supply Points (BSP) of the distribution networks can be estimated. Therefore, the RoCoF relay settings of generators in distribution networks can be adapted to the estimated frequency dynamics at their BSP. This way, the challenge faced by IEEE standard 1547 with fixed trip thresholds can be mitigated, meaning the undesirable disconnection of distribution generators in an external grid active disturbance event can be avoided.
8.2 Conclusions

Fast frequency response is regarded as any form of active power injection or load reduction within half a second of active power disturbance occurrence, aiming to serve as an alternative for increased volume of primary response required in low inertia systems. In this sense, most sensible and abundant literature that can potentially fit this type of response is the ones for AUFLS. The pre-requisite for taking fast and adaptive frequency control actions in emergency states are:

- Fast and reliable detection of disturbance, to help initiating the fast frequency response
- Fast and accurate estimation of disturbance location, to help delivering a more targeted fast frequency response
- Fast and accurate estimation of disturbance size, to help adapting the size of fast frequency response

Therefore Chapter 2 mainly dealt with investigating the existing methods for AUFLS, disturbance detection, disturbance localisation and disturbance size estimation. The review of the AUFLS methods in Chapter 2 revealed the potential room for further tuning of the shedding size in order to get closer to the minimum shedding that is required to keep the frequency nadir above a certain threshold. Furthermore, it was concluded that the triggering procedure for AUFLS can be improved to be more suitable for low inertia scenarios where time for action is limited (section 2.6). Chapter 2 investigated the traditional method used for disturbance size estimation (swing equation based method) and some of its main shortcomings were identified as follows (section 2.7.4):

- Dependence on accurate estimation of \( \dot{f}_{COF}(0^+) \); which requires all synchronous generators RoCoF measurements shortly following the disturbance
- Dependence on accurate estimation of total system inertia \( (H_{Sys}) \)
- Issues regarding accurate RoCoF calculation; RoCoF measurements being inconsistent with RoCoFs determined from generators shaft speed at \( t=0^+ \)

Chapter 2 also discussed the common issues in disturbance detection and disturbance localisation methods, some of which pertaining to the speed of execution, heuristic threshold setting methods and high sensitivity to missing PMU measurements and noise (section 2.9, section 2.10).
In today’s power systems, the synchronised measurements gathered via PMUs in WAMS is progressively rising to become the key enabler in real-time state monitoring, security assessment and adaptive control action design. However despite the unprecedented deployment of PMUs, full system observability is still unrealistic. In practice, cost of building communication infrastructure for supporting SMT and the delay associated with processing large number of synchronised measurements fuel the ambition to develop analysis methods in different fields which are reliant on less PMU measurements, particularly for real time applications. This fact further solidifies the incompetency of the large number of previous methods regarding fast, reliable and practical disturbance analysis (detection/localisation/sizing) for real time applications; e.g. supporting fast adaptive frequency services.

Consequently, one of the main goals of this research was to develop fast, reliable and accurate methodologies for detecting, localising and estimating the size of active power disturbances (disturbance analysis) using limited number of PMU measurements.

Chapter 3 described a novel disturbance analysis method (SPC method) which uses active power outputs of a limited number of generators and the synchronising power coefficients as inputs. The algorithm is designed to operate continuously and decide at each time step (PMU time step) if a disturbance has taken place (stage 1), and if so to simultaneously estimate the size and the location of the disturbance (stage 2). This method uses one time step of PMU measurements (20ms) rather than using a lengthy data buffer, hence it has an improved speed. As SPC method relies on active power distribution shortly following the disturbance (Section 3.3) it is independent of $H_{sys}$ and RoCoF measurements, also it does not require all generators PMU measurements. A comparison between SPC method and traditional swing equation based method proved that SPC provides far more accurate disturbance size estimation with fewer PMU measurements (section 3.6.3). The method robustness was verified by testing it for varied levels of measurement noise, impedance matrix errors and topology errors for various disturbance sizes and locations (section 3.6). Having the same number of PMUs, SPC method achieved better performance for PMUs placed further away from each other than the case where PMUs were placed in the same vicinity (section 3.6.7).

Chapter 4 aimed to ease the reliance of traditional swing based disturbance size estimation method on having all generators equipped with PMUs. The main idea was to divide the
synchronous generators into different regions and have a single RoCoF measurement from each region representing the whole region in the swing equation, therefore reduced number of required PMUs for disturbance size estimation. After defining the regions the regional inertias are defined as the summation of inertias of generators in each region. It was proved that in order to minimise the error in disturbance size estimation, the generators have to be assigned to regions based on the similarity of their RoCoFs shortly following the disturbance occurrence (section 4.3). K-means clustering algorithm was applied to generators’ RoCoF traces to define the regions for varied disturbance locations. It was observed that the regions boundaries were dependent on the location of the disturbance. However, as this regioning method needed all generators RoCoFs following the disturbance, it contradicted the fundamental reason behind regioning, which is missing generators measurements. Furthermore, in this method the regions could only be identified once a disturbance takes place as this approach is based on the observed traces of RoCoFs, which adds to the time delay of the whole procedure.

Therefore, an alternative method for regioning was suggested in Chapter 4 which is done offline and is not based on the generators’ RoCoF measurements and is termed as offline or adaptive regioning (section 4.6). The results of the regioning based on the RoCoFs and offline regioning proved to be consistent (section 0). Nevertheless, the drawback of the offline method was that it required disturbance location as an input, hence the term adaptive regioning. In an attempt to remove the dependency of the offline method on disturbance location, a new method was suggested to define fixed regions (fixed regioning, section 4.7), meaning the regions members were constant for all disturbance locations. Although fixed regioning had better performance compared to the traditional swing based method, in comparison to adaptive regioning method it had significantly less accurate disturbance size estimate (section 5.4.3).

In case of having limited fixed locations for generators equipped with PMUs, there is no guarantee that the regions formed using offline/adaptive regioning method (section 4.6) would have at least one PMU readings in each region for all disturbance locations. Therefore, a methodology for estimating missing regions representative RoCoFs using the monitored generators RoCoFs was suggested in Chapter 5 (section 5.2). Chapter 5 then describes a disturbance analysis method (RI method) which first detects and localises the disturbance and then based on the estimated disturbance location makes up the regions and regional inertias using adaptive regioning (Chapter 4) and estimate the disturbance size. The RI method was
tested for different conditions and its performance proved to be robust; conditions being: number of PMU measurements, disturbance locations, disturbance size, noise in RoCoF measurements and error in Z matrix (section 5.4).

Having created novel methods for fast disturbance detection and estimation of disturbance size and location (disturbance analysis), the next goal of this research was to develop a method capable of continuously assessing the frequency security status of the power system and evaluating the fast frequency response requirements. Chapter 6 presented a method for estimating the SFR parameters of the system by applying system identification techniques to frequency and power deviations due to small (ambient) load fluctuations during normal operating conditions (ambient load fluctuation based SFR identification). The identified SFR was then utilised to assess the frequency security and deciding the size of fast frequency response requirements. The identified SFR was also used to predict the frequency behaviour following a known disturbance size. Different case studies explored the effect of varied parameters (e.g. number of monitored generators, noise in measurements and window length) on the accuracy of the predicted frequency (sections 6.7.3 and 6.7.5). Another form of frequency prediction method called Disturbance Triggered SFR Identification was also analysed and compared with the ambient load fluctuation based SFR identification regarding the accuracy of the predicted frequency (section 6.7.4). In Disturbance Triggered SFR Identification, the SFR parameters are estimated once a large disturbance takes place (hence the disturbance triggered term) and then identified SFR is used with the disturbance size estimate to predict the frequency. Therefore, in case of applications concerning frequency nadir, time available for predicting the frequency behaviour using Disturbance Triggered SFR Identification is limited. As a result and as it was shown in (section 6.7.4), frequency predicted by Disturbance Triggered SFR Identification loses its accuracy when time to nadir (training time window) is limited, hence this method fails to provide reliable inputs for fast adaptive frequency services in low inertia systems. On the other hand, the ambient load fluctuation based SFR identification suggested in Chapter 6 does not face the problem with limited time window available for frequency prediction as it estimates the SFR parameters during normal system operation. Finally in Chapter 6 an experiment was carried out to assess the effects of frequency response initiation delay and disturbance size on the minimum required response to avoid the frequency nadir threshold violation (section 6.8). The results of this experiment helped establishing a multi-regression relationship between critical disturbance size (extracted from identified SFR of the system (section 6.4.1)), disturbance
size, delay in frequency response initiation and the minimum response required to avoid frequency nadir threshold violation. Based on this experiment, it was concluded that for frequency response initiation delays smaller than 0.5s, the minimum frequency response required can be estimated as the difference between disturbance size and critical disturbance size. According to this founding, a novel AUFLS adaptive to both disturbance size and critical disturbance size was suggested in Chapter 7, in this sense AUFLS was regarded as a form of fast adaptive frequency response.

As it was shown in Chapter 6 (section 6.8) faster frequency responses before frequency nadir deliver the same effect with smaller response size compared to slower form of frequency responses. The proposed AUFLS in Chapter 7 (DSA based AUFLS) provides a fast decision on shedding necessity and the minimum shedding required and then sheds it in one step, which eliminates the accumulated delay of the several shedding steps used by other form of AUFLS. In Chapter 7, the DSA based AUFLS was compared against other forms of AUFLS which were only adaptive to disturbance size, hence were referred to as semi adaptive UFLS. The comparison was in terms of the shedding size and the success of the schemes to keep the frequency nadir above a certain threshold for varied disturbance sizes and inertia levels (wind generation levels). It was revealed that with increased wind generation penetration (reduced inertia) system operators would be forced to opt for progressively more conservative semi adaptive UFLS to keep the system frequency secure. In this sense, more conservative semi adaptive UFLS means having their shedding size equal to a larger percentage of the disturbance size. The benefit of the suggested DSA based AUFLS was that it managed to keep the frequency secure even for reduced inertia scenarios (increased wind level) and with smaller amount of load shedding (response size) for the majority of cases.

### 8.3 Contributions

The main contributions of the research presented in this thesis can be described as follows:

- Creation of an online method for fast simultaneous disturbance detection and estimation of disturbance size and location (Disturbance Analysis) based on active power outputs of a limited number of synchronous generators; making it independent of system inertia and RoCoF measurements. (SPC method Chapter 3)
• Creation of a new method for dividing the system into regions (regioning) based on the similarity between synchronous generators RoCoFs shortly following the disturbance. Defining the concept of regional inertia based on the regions boundaries and its potential role in reducing the number of PMU measurements for accurate disturbance size estimation. (Chapter 4)

• Suggestion of an alternative approach to get the same regions without requiring RoCoF measurements (offline regioning). (Chapter 4)

• Creation of an online method capable of fast simultaneous disturbance detection and disturbance size and location estimation using much reduced number of PMU measurements. (RI method Chapter 5)

• Creation of an online method for continuous assessment of frequency security of the system as well as the minimum fast frequency response required to be procured, which is based on estimating SFR parameters using system identification techniques. (Chapter 6)

• Creation of a new form of AUFLS which is adaptive to both disturbance size and identified SFR parameters. (Chapter 7)

### 8.4 Future Developments

The research presented in this thesis has successfully delivered on the majority of its goals and succeeded to make several contributions. Nevertheless, a number of possible areas for further development of this research are discussed in this section.

As it was shown in Chapter 3, SPC disturbance analysis method had worse performance when having PMUs located close to each other. However, the analysis can be expanded to prove that close PMUs can still give accurate results regarding detection, localisation and sizing of the disturbance as long as the disturbance takes place in their vicinity (within the same geographical region). Based on this logic, there is a possibility of exploring decentralised scheme for adaptive frequency response activation (e.g. AUFLS) based on the suggested disturbance analysis method. This can be done by partitioning the network in geographical regions and having disturbance analysis done with at least two generators at each region. In this potential scheme, the communication links for disturbance analysis needs to be built only within the PMUs of each region (minimum two PMUs). This can
significantly reduce the communication infrastructure cost and decrease the delay associated with data communication channels, since in centralised approach all PMUs in the system need to be connected either directly or indirectly. In this approach, disturbance analysis is done separately within each region. In this case, when a disturbance is detected in a region the localisation and the size estimation would only be considered accurate if the estimated location of the disturbance belongs to the same geographical region. Therefore, the fast frequency response resources of each region can be controlled in a decentralised way when a disturbance is detected within each region.

Having decentralised architecture for any WAMS (Wide Area Monitoring System) application, can significantly reduce the cost of their infrastructure development. Due to the stringent requirement on latency and band width for most WAMS applications, in many regions the existing telephone lines, power lines or microwaves have to be replaced by fibre optic channels. This fact coupled with the necessary redundancy in practical applications, doubles their cost. In general telecommunication used to transfer data between PMUs and Phasor Data Concentrators (PDC) can fall into two categories; shared and dedicated. On the transmission networks, fibre optics and pilot wires and on distribution substations lease telephone lines are commonly employed which are all dedicated telecommunication media, hence costly. However, the recent development and experimental analysis on Internet based communication have proved that the end to end delays for this type of data transfer can reach the order of tens of millisecond which meets the delay requirements for many WAMS applications. With the use of emerging 5th generation of mobile networks (5G), the communication latency would be even lower and reliability would be ultra-high. Therefore, Internet as a form of shared telecommunication media is becoming a persuasive option for WAMS considering it's relatively low cost. As the methods suggested in this thesis are all dependent on synchronised measurements, the impact of internet based communication latency on their performance can be examined in a real life platform.

The effect of adding synthetic inertia loop on critical disturbance size and the amount of fast frequency requirements of the system was analysed in Chapter 7 (section 7.4.1). Although, it provided a general conclusion, the procedure for analysis was not based on simulations of modelled DFIGs with synthetic inertia control loop but it was based on analytical estimate of inertial contribution of DFIGs suggested in the literature. Therefore, further more detailed dynamic simulations can be carried out to more precisely determine the contribution of
synthetic inertia and its different parameters on system frequency stability and fast frequency response requirements.

The effects of increasing governors’ speed (primary response) on critical disturbance size and consequently the amount of fast frequency response requirements can be analysed and enumerated thorough estimating SFR parameters for varied governor settings in dynamic simulations using the method suggested in Chapter 6. Furthermore, for high frequency events, the effect of having fast valving schemes on generators can be studied in order to identify their impact on SFR parameters. This way, given the cost of primary frequency response and different fast frequency response resources (electric storage, AUFLS, etc.) the total cost of keeping system frequency security can be optimised.
9. Appendix

9.1 Appendix A-Equation of Motion and Inertia Constant

Whenever there is an unbalance between the torques acting on the rotor of a synchronous machine the net torque causing acceleration/deceleration is [1]:

\[ T_a = T_m - T_e \]  

(A1.1)

In which:

\( T_a \) = accelerating torque in N.m

\( T_m \) = mechanical torque in N.m

\( T_e \) = electromagnetic torque in N.m

The equation of motion therefore can be written as follows:

\[ J \frac{d\omega_m}{dt} = T_a = T_m - T_e \]  

(A1.2)

Where:

\( J \) = combined moment of inertia of generator and turbine, kg m\(^2\)

\( \omega_m \) = angular velocity of the rotor, rad/s

\( t \) = time, s

\( J \) or moment of inertia determines the torque needed for a desired angular acceleration about a rotational axis. It depends on the body’s mass distribution and the axis chosen.

\[ J = \int_0^M r^2 dm \]  

(A1.3)

Where:

\( M \) is the total mass of the rotating body (in this case generator and turbine).
$r$ is the distance of the mass particle($dm$) from the rotation axis.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol/Equation</th>
<th>MKS unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angular displacement</td>
<td>$\theta$</td>
<td>radian (rad)</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>$J = \int r^2 dm$</td>
<td>kg m$^2$</td>
</tr>
<tr>
<td>Angular velocity</td>
<td>$\omega = \frac{d\theta}{dt}$</td>
<td>rad/s</td>
</tr>
<tr>
<td>Angular acceleration</td>
<td>$\alpha = \frac{d\omega}{dt}$</td>
<td>rad/s$^2$</td>
</tr>
<tr>
<td>Torque</td>
<td>$T = J\alpha$</td>
<td>newton-meter (N.m) or J/rad</td>
</tr>
<tr>
<td>Work</td>
<td>$W = \int Td\theta$</td>
<td>J, or W.s</td>
</tr>
<tr>
<td>Power</td>
<td>$P = \frac{dW}{dt} = T\omega$</td>
<td>W</td>
</tr>
</tbody>
</table>

Equation (A1.1) can be normalised in terms of per unit **inertia constant** $H$, defined as the **kinetic energy** ($E_{\text{kin}}$) (A1.4) in watt-seconds at rated rotational speed ($\omega_{\text{nom}}$) divided by the apparent power base ($S_B$) (A1.5):

$$W = E_{\text{kin}} = \int J\alpha d\theta = \int J \frac{d\omega}{dt} d\theta = \int J \omega d\omega = \frac{J\omega_{\text{nom}}^2}{2}$$  \hspace{1cm} (A1.4)

$$H = \frac{1}{2} \frac{J\omega_{\text{nom}}^2}{S_B}$$  \hspace{1cm} (A1.5)

The **moment of inertia** ($J$) in terms of inertia constant $H$ is:

$$J = \frac{2H}{\omega_{\text{nom}}^2} S_{\text{base}}$$  \hspace{1cm} (A1.6)

By substituting (A1.6) in (A1.2) we will have:

$$\frac{2H}{\omega_{\text{nom}}^2} S_{\text{base}} \frac{d\omega}{dt} = T_m - T_e$$  \hspace{1cm} (A1.7)
By rearranging the formula and substituting $T_{\text{base}} = S_{\text{base}}/\omega_{0m}$ the per unit form of (A1.7) would give (A1.9).

$$2H \frac{d}{dt} \left( \frac{\omega_m}{\omega_{0m}} \right) = \frac{T_m - T_e}{S_{\text{base}}/\omega_{0m}}$$  \hspace{1cm} (A1.8)

$$2H \frac{d\omega_m}{dt} = \overline{T_m} - \overline{T_e} = \overline{T_o}$$  \hspace{1cm} (A1.9)

The over lined variables in (A1.9) represent their per unit form.

If $\theta$ is the angular position of the rotor in electrical radians with respect to a synchronously rotating reference and $\theta_0$ is its value at $t=0$:

$$\theta = \omega_m t - \omega_0 t + \delta_0$$  \hspace{1cm} (A1.10)

By taking the derivative over time:

$$\frac{d\theta}{dt} = \omega_m - \omega_0 = \Delta \omega_m$$  \hspace{1cm} (A1.11)

Therefore (A1.9) can be written as follows:

$$2H \frac{d^2\theta}{dt^2} = \overline{T_m} - \overline{T_e} = \overline{T_o}$$  \hspace{1cm} (A1.12)