# Deformation of ${ }^{113} \mathrm{Cs}$ from 

# Proton-emission and Electromagnetic 

## Transition Rates

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Science and Engineering

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## Abstract

This thesis entitled "Deformation of ${ }^{113} \mathrm{Cs}$ from Proton-emission and Electromagnetic Transition Rates" was submitted by Mr Duncan Hodge on the 5th of December 2016 to the University of Manchester for the degree of Doctor of Philosophy.

Studying nuclei beyond the proton dripline can provide valuable information on the structure of nuclei at the limits of stability, where the strong nuclear force starts to be overcome by Coulomb repulsion between protons. Simple experimental observables, such as excitation energies and lifetimes of excited states in these proton-unbound nuclei can provide information on the nuclear wavefunction. Experimental data, such as that presented in this work, can then be used to improve models of nuclear structure at the proton dripline.

This thesis presents data from a recoil-decay tagged differential plunger experiment undertaken at the University of Jyväskylä in 2014. A fusion-evaporation reaction was used to populate excited states in the deformed ground-state proton emitter ${ }^{113} \mathrm{Cs}$. The JUROGAM-RITU-GREAT experimental setup was used to correlate $\gamma$ rays emitted from these excited states with protons emitted from ${ }^{113} \mathrm{Cs}$ and the differential plunger for unbound nuclear states (DPUNS) was placed at the target position to measure the excited state lifetimes.

The lifetime of the $\left(11 / 2^{+}\right)$state in the most intense rotational band of ${ }^{113} \mathrm{Cs}$ was measured to be $\tau=24(6) \mathrm{ps}$, while a limit of $\tau \leq 5 \mathrm{ps}$ was found for the lifetime of the higher energy $\left(15 / 2^{+}\right)$state. The lifetime of proton emission was measured to be $\tau=24.2(2) \mu \mathrm{s}$.

The experimental data were used to test the predictions of a nonadiabatic quasiparticle model for proton-emitting nuclei, which was employed to deduce the deformation of the states in ${ }^{113} \mathrm{Cs}$. Wavefunctions from the nonadiabatic quasiparticle model were used to independently calculate proton-emission rates, $\gamma$-ray transition rates and excited state energies as functions of deformation. The deformation of ${ }^{113} \mathrm{Cs}$ could then be extracted from the intersection of the different theoretical values and experimental observables. A deformation of $\beta_{2}=0.22(6)$ was extracted from the $\left(11 / 2^{+}\right)$excitation energy and lifetime. The deformation values taken from the proton-emission rate and the lifetime limit of the $\left(15 / 2^{+}\right)$state were also consistent with this value. The consistency of the different deformations calculated shows the effectiveness of the nonadiabatic quasiparticle method when used to calculate the properties of deformed ground-state protonemitters.

## Declaration

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## Chapter 1

## Introduction

This thesis presents data on the ground-state proton-emitting nucleus ${ }^{113} \mathrm{Cs}$. Ground-state single-proton emission occurs in odd- $Z$ nuclei where the valence proton is no longer bound by the nuclear potential [1, 2]. Such nuclei are said to lie beyond the proton drip-line, where the $Q$ value for proton emission becomes positive and the nuclear force starts to be overcome by the Coulomb repulsion between protons. Therefore, by studying nuclei at the proton-dripline the limits to the strength of the nuclear force can be deduced. Also of great importance in studying proton-emitting nuclei, is the fact that simple experimental observables, such as the energy and half-life of proton decay may be used to provide information on components of the nuclear wave function [3].

The rate of proton-emission is subject to the width of the centrifugal barrier, which, along with the Coulomb barrier must be tunneled through before the proton can leave the nucleus. As the height of the centrifugal barrier is proportional to the orbital angular momentum of the proton, the nuclear structure of the parent can be inferred from the emission-rate and energy of the proton [4]. The coupling of weakly bound and unbound nuclear states to the continuum may then also be examined. In order for the nuclear wave function to be correctly extracted, a well-developed theoretical model is needed. Experimental data is also needed to refine and test these models to see if the observed nuclear properties can be consistently reproduced.

Ground-state proton emitters have been experimentally identified in several areas of the nuclear chart with masses of $A=109-185$ [6], as shown in Fig. 1.1. Different models then have to be developed for these different regions where


Figure 1.1: The chart of the nuclides, focused on the proton drip-line, where ground-state proton emission has been observed to occur. Proton-emitting isotopes are denoted by the red squares and the location of ${ }^{113} \mathrm{Cs}$, the nucleus studied in this work, is marked with respect to the major shell closures [5].
nuclei may have different shapes. Nuclei with numbers of neutrons or protons that are close to magic numbers, denoted by the black lines in Fig. 1.1 will assume a more spherical shape. Various proton-emission models, which consider the proton as tunneling from a spherically symmetric potential, may then be successfully used to predict proton-decay rates and the excitation energies and spins of states in the decaying nucleus [1, 2, 7]. However, these models do not successfully replicate proton-emission rates of nuclei, which lie away from closed shells. For example, the spherical Wentzel-Kramers-Brillouin (WKB) calculations of Åberg et al. [1, 2], predicted a proton-emission half-life from the $d_{5 / 2}$ state in ${ }^{113} \mathrm{Cs}$ of $\sim 0.9 \mu \mathrm{~s}$, when the spectroscopic factor for the $d_{5 / 2}$ state is included. This is a factor of $\sim 19$ times shorter than the established experimental value [8-12]. This result indicates that ${ }^{113} \mathrm{Cs}$ is deformed, which is in agreement with the global deformation calculations of Möller and Nix [13]. These calculations predict that neutron-deficient caesium isotopes are deformed, even as the $N=50$ shell closure is approached, as shown in Fig. 1.2, and that ${ }^{113} \mathrm{Cs}$ itself has a quadrupole deformation of $\beta_{2}=0.21$. Therefore, proton-emission models which assume a spherical shape for the nucleus are no longer adequate to interpret


Figure 1.2: Ground-state deformations of the neutron-deficient odd-mass Cs isotopes taken from Ref. [13]. All isotopes are shown to be deformed, even those which lie closer to the $N=50$ shell closure.
the experimental energies and half-lives from ${ }^{113} \mathrm{Cs}$. It is then important to develop a model of proton emission which considers the proton decaying through a deformed potential and accounts for the collective excitations that result from the deformation.

A model of proton-emission from a deformed nucleus was provided by Maglione et al. [14], who introduced an exact expression for the decay width of the protonemitting state in a deformed potential. This adiabatic approach considered the proton as occupying a single-particle orbit coupled to a rigid core with a frozen rotational spectrum and an infinite moment of inertia. Although this model successfully replicated the experimental half-lives of many nuclei [15], it did not allow for cases where the core was less rigid and did not account for the strong mixing of states due to the Coriolis interaction. For this scenario where nucleons in the core and the unbound proton may influence each other, the pairing interaction becomes increasingly important. Improved calculations within a nonadiabatic


The proton occupies a single-particle state coupled to a rigid core with infinite moment of inertia

(b)


The proton is a quasiparticle which interacts with the nucleons of the non-rigid core through Coriolis mixing and the pairing interaction


Figure 1.3: The different approaches that may be used to model a deformed proton-emitting nucleus. (a) An adiabatic view of a proton-emitting nucleus, in which the unbound proton occupies a single-particle state coupled to a rigid rotational core. (b) A nonadiabatic view in which the unbound proton is considered a quasiparticle which may interact with nucleons in the core through the pairing and Coriolis interactions. The core, in this case, is treated as less rigid.
quasiparticle approach have since been made that take into account the Coriolis and pairing interactions [16]. The rotational spectrum of the daughter nucleus is also included to provide a more accurate structure for the nuclear core. A summary of the adiabatic and nonadiabatic approaches is shown in Fig. 1.3.

In order to validate the nonadiabatic framework, experimental data from deformed, proton-emitting nuclei, such as ${ }^{113} \mathrm{Cs}[13,14,17]$, are needed. Additionally, ${ }^{113} \mathrm{Cs}$ is predicted to have two candidate proton-emitting Nilsson orbits of $K=1 / 2^{+}$and $K=3 / 2^{+}$which lie very close in energy to each other [15] and are strongly affected by the Coriolis interaction. These strongly mixed orbits provide a good test for the inclusion of the Coriolis interaction in the nonadiabatic quasiparticle model.

Proton emission from ${ }^{113} \mathrm{Cs}$ was first discovered in 1984 by Faestermann et al. [8]. In that work the production cross section of ${ }^{113} \mathrm{Cs}$ was established to be $\sim 30 \mu$ b from the ${ }^{58} \mathrm{Ni}\left({ }^{58} \mathrm{Ni}, \mathrm{p} 2 \mathrm{n}\right)$ reaction at $230 \mathrm{MeV}[9-11]$. This production cross-section provided enough prompt $\gamma$ rays from ${ }^{113} \mathrm{Cs}$ to make the nucleus viable for study using the recoil-decay tagging differential plunger setup of the University of Jyväskylä [18-21]. The nucleus ${ }^{113}$ Cs has previously been studied using this setup without the plunger in an experiment, described in Ref. [12]. This experiment was undertaken to extend and more firmly characterise the level scheme above the proton emitting state first observed in Ref. [22]. Two rotational bands had configurations assigned from aligned angular momentum behaviour, observed band crossings and blocking arguments. The more intense band, band 1, was assigned to be based upon a $g_{7 / 2}[422] 3 / 2$ proton configuration while the second rotational band, band 2 , was assigned to be based upon a $d_{5 / 2}[420] 1 / 2$ proton configuration.

The data presented in this work were collected using the University of Jyväskylä recoil-decay-tagged differential plunger setup in an experiment undertaken in 2014. The objectives of the experiment were to measure the deformation of ${ }^{113} \mathrm{Cs}$ from the excitation energy and lifetime of the $\left(11 / 2^{+}\right)$state in band 1 and the lifetime of proton emission, in order to test the predictions of the nonadiabatic quasiparticle method. The presence of the Differential Plunger for Unbound Nuclear States (DPUNS) [21] at the target position of the recoil-decay tagging setup allowed for the measurement of promptly decaying excited states in ${ }^{113} \mathrm{Cs}$. The lifetime of the $\left(11 / 2^{+}\right)$state in band 1 was measured using DPUNS, while an upper limit was found for the lifetime of the $\left(15 / 2^{+}\right)$state. The experimental $\left(11 / 2^{+}\right)$state lifetime was compared to theoretical lifetime values, which were calculated as a function of deformation using wave functions extracted within the framework of the nonadiabatic quasiparticle approach developed for the protonemission. By observing the deformation at which the theoretical and experimental
half-lives of the $\left(11 / 2^{+}\right)$state intersected, the deformation of the nucleus could be inferred. Experimental proton-emission rates and excited state energies were also compared with theoretical values from the model, calculated for different nuclear deformations. The advantage of using the lifetime of the excited state in $\left(11 / 2^{+}\right)$to attempt to infer the deformation of the nucleus lies in the greater sensitivity of the electromagnetic transition rate calculation to the deformation of the nucleus than the proton-emission rate and excitation energy calculations. Furthermore, the excited state lifetime provides an additional means of calculating the deformation from a common set of nonadiabatic quasiparticle model wave functions within a consistent approach which provides another means of testing the predictive power of the model [16].

The lifetime of the $\left(11 / 2^{+}\right)$state in band 1 of ${ }^{113} \mathrm{Cs}$ was measured to be $\tau=24(6)$ ps and the limit on the lifetime of the $\left(15 / 2^{+}\right)$state was found to be $\tau \leq 5 \mathrm{ps}$. Using the lifetime and excitation energy of the $\left(11 / 2^{+}\right)$state, a deformation of $\beta_{2}=0.22(6)$ was found for ${ }^{113} \mathrm{Cs}$. Consistent deformations of $\beta_{2}>0.19$ and $\beta \sim 0.22$ were also found from the lifetime limit of the $\left(15 / 2^{+}\right)$state and the proton emission rate of ${ }^{113} \mathrm{Cs}$, respectively.

In subsequent chapters, this thesis will present the evolution of proton-emission theory, as well as the properties of $\gamma$-ray decay and the effects and causes of nuclear deformation. The experimental setup and technique, including the measurement of prompt excited state lifetimes, are also discussed in more detail. A comprehensive discussion of the results is presented along with a discussion of the possibility of future experiments which may be undertaken to provide a more accurate value for the deformation of ${ }^{113} \mathrm{Cs}$.

## Chapter 2

## Theory

In this work, data on the proton-emitting nucleus ${ }^{113} \mathrm{Cs}$ is presented. In order to successfully interpret this data a framework is needed to coherently describe the observed properties of the nucleus, such as its half-life and level structure. Such a framework is provided by the nonadiabatic quasiparticle model [16], which was used to extract the wave functions of the proton-emitting state in ${ }^{113} \mathrm{Cs}$. These wave functions could then be used to interpret the observed excitation energies and lifetimes of the excited $\left(11 / 2^{+}\right)$and $\left(15 / 2^{+}\right)$states in ${ }^{113} \mathrm{Cs}$, as well as its proton-emission lifetime, in terms of the deformation of the nucleus. By using the same wave functions to predict the different properties of the nucleus, a consistent method of measuring the nuclear deformation from proton-emission rates, electromagnetic transition rates and excited state energies was achieved.

This Chapter will cover the need for the development of the nonadiabatic quasiparticle model with reference to other proton-emission models. The factors that cause protonemission and the associated nuclear structure concepts are also discussed. Additionally, the properties of $\gamma$-ray decay of excited nuclear states and how it may provide information on nuclear deformation is also covered.

### 2.1 Proton Emission

If the binding energy per nucleon increases by emitting a given particle, then a nucleus will generally be unstable and will eventually decay through the emission of that particle. The $Q$ value defines the amount of energy released in such a decay and will be greater than 0 for these unstable nuclei. In very neutron-deficient nuclei the $Q$ value for proton emission will be high enough, that proton emission can compete with the $\alpha$ and $\beta^{+}$decay modes. These nuclei at the "proton dripline" possess protons that occupy unbound states with energies greater than the depth of the nuclear potential. However, due to the presence of Coulomb and centrifugal barriers the protons are still confined to the nucleus for a period of time before being emitted. The $Q$ value of proton decay consists of a nuclear component and an electronic component in the form,

$$
\begin{equation*}
-S_{p}=Q_{p}=Q_{p}^{n u c}+E S \tag{2.1}
\end{equation*}
$$

where, $S_{p}$ is the proton separation energy, describing the amount of energy that is needed to separate a proton from the nucleus,

$$
\begin{equation*}
Q_{p}^{n u c}=M_{n u c}(N, Z)-M_{n u c}(N, Z-1)-m_{p} \tag{2.2}
\end{equation*}
$$

and

$$
\begin{equation*}
E S=\operatorname{Be}(N, Z)-\operatorname{Be}(N, Z-1) . \tag{2.3}
\end{equation*}
$$

The electron screening component, $E S$, reduces the height of the Coulomb barrier and is given by the difference between the total electron binding energy of the atomic electrons in the parent nucleus, $\operatorname{Be}(N, Z)$ and the daughter nucleus,
$B e(N, Z-1)$. It can be calculated using tabulated electron binding energies [23] or from the approximation

$$
\begin{equation*}
E S=0.49+0.0144 Z^{1.6} \mathrm{keV}, \tag{2.4}
\end{equation*}
$$

which gives an accuracy of $<0.5 \%$ for $42<Z<75$ and drops to $1.6 \%$ at $Z=83$ [3].

Proton-emitting nuclei have been observed in several mass regions of the nuclear chart but are hard to access experimentally due to their short half-lives. Protonemission was first observed in 1970 from a $I=19^{-}$isomeric state in ${ }^{53} \mathrm{Co}$ [24], while ground-state proton emission was first observed in the nucleus ${ }^{151} \mathrm{Lu}$ in 1982 [25]. Over forty proton emitters have since been observed [3]. The data presented in this work provides information on the ground-state proton emitter ${ }^{113} \mathrm{Cs}$ which was first observed in 1984 by Faestermann et al. [8]. The area of the nuclear chart including ${ }^{113} \mathrm{Cs}$ is shown in Fig. 2.1. This figure shows how ${ }^{113} \mathrm{Cs}$ is one of the most neutron-deficient nuclei that can currently be produced and studied. The special importance of studying proton-emitting nuclei lies in the fact that properties of the nuclear wave function can be extracted from the relatively simple observables of the half-life and energy of the proton decay [3].

Various different theoretical approaches have been used to model the emission of protons from nuclei. The proton is commonly treated as tunneling from a central nuclear potential through a potential barrier, consisting of Coulomb and centrifugal components [3]. An example potential experienced by a proton in a nucleus, including the Coulomb and centrifugal barriers, is shown in Fig. 2.2

In Fig. 2.2 the nuclear potential takes the form of a Woods-Saxon potential, subsequently described in section 2.2 . The Coulomb potential behaves differently within and beyond the nuclear radius. For a spherical nucleus the potential takes the form of

$$
\begin{equation*}
V_{C o u}=\frac{Z_{p} Z_{n u c}}{4 \pi \epsilon_{0} R_{C o u}}\left(\frac{3}{2}-\frac{1}{2}\left(\frac{r^{2}}{R_{C o u}^{2}}\right)\right) \text { for } \mathrm{r}<\mathrm{R}_{\mathrm{Cou}} \tag{2.5}
\end{equation*}
$$



Figure 2.1: The neutron-deficient mass $A \sim 110$ area of the nuclear chart including the drip-line nucleus ${ }^{113} \mathrm{Cs}$ which was studied in this work [5].
and

$$
\begin{equation*}
V_{C o u}=\frac{Z_{p} Z_{n u c}}{4 \pi \epsilon_{0} r} \text { for } \mathrm{r}>\mathrm{R}_{\mathrm{Cou}}, \tag{2.6}
\end{equation*}
$$

where $Z_{p}$ is the charge on the single proton, $Z_{n u c}$ is the charge on the nuclear core, $r$ is the distance from the proton to the centre of the nucleus and $R_{\text {cou }}$ is where the nuclear potential becomes infinitesimal. The centrifugal force acts to push nucleons away from the centre of the core [26]. The corresponding potential barrier takes the form of

$$
\begin{equation*}
V_{\text {cent }}=\frac{\hbar^{2}}{2 \mu} \frac{l(l+1)}{r^{2}}, \tag{2.7}
\end{equation*}
$$

where $\mu$ is the reduced mass of the proton and core.
Early proton-emission calculations assumed a spherical shape for the nucleus to theoretically reproduce experimental half-lives and energies [1, 2, 27]. These approaches were based on the consideration of the proton occupying single-particle


Figure 2.2: An example potential experienced by a proton occupying an unbound state in ${ }^{113} \mathrm{Cs}$ with energy, $E_{p}=969 \mathrm{keV}$ [12]. The potential consists of a strong nuclear potential as well as Coulomb and centrifugal components. The radius at which the nuclear contribution drops to zero and the potential felt by the proton is due only to the Coulomb and centrifugal forces is denoted by $R_{\text {cou }}$. The region that the proton is classically forbidden to occupy is denoted by $r_{1}$ and $r_{2}$.
states defined by a mean potential similar to that shown in Fig. 2.2. In order to explain these models, the predictive power and evolution of the single-particle shell model is first described in the next section.

### 2.2 The Single-Particle Shell Model

The single-particle shell model accounts for many properties of the nucleus that were not well described by earlier nuclear structure models [28]. For example,


Figure 2.3: The variation of binding energy per nucleon for a series of nuclei of different mass numbers $A$. Taken from Refs. [29, 30].
earlier models were unable to explain the sharp decreases in the binding energy per nucleon for nuclei with certain numbers of protons or neutrons, as shown in Fig. 2.3. These sudden changes in binding energy have been found at $N$ and $Z$ values of $2,8,20,28,50,82,126$, which are known as the 'magic numbers'. Similar dramatic changes in binding energy are seen at the closures of atomic electron shells. The single-particle shell model then draws on similarities with atomic electron structures by treating nucleons as occupying discrete states within a mean potential [26].

A key difference between the atomic and the nuclear potential is that the atomic potential arises from the Coulomb interaction between the nuclear protons and the atomic electrons, while the nuclear potential arises from the strong force between individual nucleons. A nuclear mean field is then needed as the calculation of the nuclear force acting on an individual nucleon from a many-body calculation is only possible for the lightest nuclei [31]. Orbits within the nuclear potential are sequentially filled by the nucleons according to the energy of each level. In the simplest version of the shell model, the different energy levels are described in
terms of their orbital angular momentum, $l$ and the index $n$, which simply states the number of states with that particular angular momentum. Each $l$ state is degenerate, with the occupancy of each $n, l$ level being $(2 l+1)$. Nucleons within each $l$ orbit occupy sub-orbitals defined by the eigenvalues of the $z$ component of $l, m_{l}$. Nucleons will occupy $m_{l}$ sub-orbitals with values of $m_{l}= \pm 1, \pm 2, \pm 3, \ldots, \pm l$ for each $l$ orbit. There is no energy dependance on which $m_{l}$ sub-orbital is occupied within a given $l$ orbit [26].

The theoretical spacing of the energy levels within the nucleus is largely governed by the form of the chosen nuclear potential. The observed increases in binding energy at the magic numbers should be reflected by large energy gaps after closed shells. The simple harmonic potential is a reasonable starting potential to use as it describes the $n$ and $l$ orbits falling into degenerate multiplets, resulting in a shell structure rather than evenly spaced energy levels [31]. Using a simple harmonic potential the nuclear magic numbers are seen to be replicated at the closed shells at $N, Z=2,8,20$ but higher energy levels do not show the correct shell closures in line with experimental data.

The Woods-Saxon potential may be a more realistic approximation of the mean potential experienced by individual nucleons. The potential provides a finite depth of the well and smooth variation to zero at the well edges, which closely mirrors the charge and mass distribution of the nucleus [26]. The Woods-Saxon potential takes the form of

$$
\begin{equation*}
V(r)=\frac{-V_{0}}{1+\exp [(r-R) / a]}, \tag{2.8}
\end{equation*}
$$

where $V_{0}$ is the depth of the potential, $r$ is the radial distance from the centre of the nucleus, $R$ is the mean radius of the nucleus and $a$ is the skin thickness, describing the change in the nuclear density, within which the value of the potential changes from $0.1 V_{0}$ to $0.9 V_{0}$. A visual comparison between the Simple Harmonic and Woods-Saxon potentials is shown in Fig. 2.4.

Neither the Woods-Saxon or Simple Harmonic potentials reproduce the magic numbers observed experimentally, when the energy levels are described in terms of their $l$ quantum numbers. However, by describing the states in terms of their total angular momentum, $j$, the spacing of energy levels is to agree more succesfully with those observed experimentally. The total angular momentum number


Figure 2.4: Comparison between the different behaviours of the simple harmonic oscillator, HO, and Woods-Saxon, WS, potentials as a function of distance, $r$, from the centre of the nucleus. Taken from Ref. [32].
accounts for the fact that the intrinsic spin of nucleons, $s= \pm \frac{1}{2}$ couples to the angular momentum, $l$, of the nucleon in the energy level it occupies. This gives total angular momentum, $j$, values of $l \pm \frac{1}{2}$. The spin-orbit interaction causes the splitting of the $l$ energy levels, the magnitude of which is proportional to the angular momentum, $l$, of the unsplit state. This is given by equation 2.9,

$$
\begin{equation*}
l . s_{j=l-1 / 2}-l . s_{j=l+1 / 2}=\frac{1}{2}(2 l-1) \hbar . \tag{2.9}
\end{equation*}
$$

Parallel coupling of the angular momentum and intrinsic spin is favoured, so the energy of the higher-spin state is less than the lower-spin state. The inclusion of the spin-orbit term now results in the accurate replication of the experimentally observed magic numbers for both the Simple Harmonic and Woods-Saxon potentials. However, the ordering and spacing of single-particle orbits may vary depending on the mass of the nucleus considered as well as the strength of certain interactions between individual nucleons, which are discussed later in the Chapter.

The $j$ and $l$ values are used to identify individual shell model orbits with the $l$ values described using the spectroscopic notation where, $l=s, p, d, f, g, h \ldots$ corresponds to $l=0,1,2,3,4,5 \ldots$. The evolution of the produced magic numbers when a simple harmonic potential, Woods-Saxon potential and Woods-Saxon
potential with a spin-orbit term are used to predict the nuclear structure are shown in Fig. 2.5.

Excitation of the nucleus may cause nucleons to be raised to higher-energy states within the potential. As the nucleus will naturally tend to its lowest energy state, these states are unstable and the nucleus will eventually decay to its ground state. This decay may proceed via intermediate excited states. The most common means of de-excitation from excited nuclear states is through the emission of $\gamma$ rays. Examining the properties of these $\gamma$ rays can then tell us about the internal structure of the nucleus. The rules governing $\gamma$-ray transitions are described in the next section.

### 2.3 Gamma-ray Decay

Transitions between excited states in the nucleus are largely mediated by $\gamma$-ray decays. When a nucleus decays from an excited state, $I_{i}$, to a lower energy state, $I_{f}$, a photon will commonly be emitted with a certain energy, parity and angular momentum, which is dependent on the properties of $I_{i}$ and $I_{f}$. By characterising the $\gamma$-ray emission from a certain nucleus, initial and final properties of the changing nucleus can be inferred [34].

### 2.3.1 Selection Rules

The properties of $\gamma$ rays emitted from the nucleus are largely dictated by a set of selection rules. Due to the possible vector sums of angular momenta, a $\gamma$ ray effecting a nuclear transition, between two states with angular momenta $I_{i}$ and $I_{f}$, may have angular momentum in the range,

$$
\begin{equation*}
\left|I_{i}+I_{f}\right|>L_{\gamma}>\left|I_{i}-I_{f}\right|, \tag{2.10}
\end{equation*}
$$

where $L_{\gamma}$ is the angular momentum of the $\gamma$ ray, which may have any integer value between these limits [26].


Figure 2.5: The effect of different nuclear potentials and the inclusion of the spin-orbit term on the ordering of single-particle energy levels and predicted magic numbers, as adapted from Ref. [33].

The $\gamma$ rays emitted from the nucleus can be described as either "electric" or "magnetic". Electric $\gamma$ rays change the electric moment of the nucleus while magnetic transitions change the magnetic moment of the nucleus [35]. The $\gamma$ rays are also described by their angular momentum, so that they are labelled as $E L$ or $M L$.

Depending on whether the transition is electric or magnetic, the parity of the radiation field can be calculated by

$$
\begin{equation*}
\pi(M L)=(-1)^{L_{\gamma}+1} \tag{2.11}
\end{equation*}
$$

or

$$
\begin{equation*}
\pi(E L)=(-1)^{L_{\gamma}} \tag{2.12}
\end{equation*}
$$

where $\pi(M L)$ and $\pi(E L)$ indicate the parity change for a magnetic or electric transition, respectively. As shown by equation $2.10, \gamma$-ray decays can proceed with various different angular momenta between the same initial and final states, forming mixed multipolarity transitions. As parity between initial and final states is conserved and the transition rates are dependent on the angular momentum of the emitted $\gamma$ ray (see next section), the most common mixed transitions have multipolarities of E2-M1 [36].

### 2.3.2 Gamma-ray Transition Rates

Transition rates between excited nuclear states can provide valuable nuclear structure information. For example, excited state lifetimes can provide information on the shape of nuclei [37], changes in nuclear structure at certain excitation energies [38] and the amount of collective versus single-particle behaviour in nuclei [39, 40]. The lifetime of a decay from one nuclear state to another can be measured experimentally and compared to theoretical estimates to access this nuclear structure information [41]. The transition probability, $\lambda$, between two states, $I_{i}$ and $I_{f}$, can be described in terms of reduced matrix elements between the two states [42], such that,

$$
\begin{equation*}
\lambda(\sigma L)=\frac{8 \pi(L+1)}{\hbar L((2 L+1)!!)^{2}}\left(\frac{E_{\gamma}}{\hbar c}\right)^{2 L+1} B\left(\sigma L: I_{i} \rightarrow I_{f}\right), \tag{2.13}
\end{equation*}
$$

where $L$ and $E_{\gamma}$ are the angular momentum and energy of the $\gamma$-ray transition, $\sigma$ denotes either an $E$ or $M$ transition and $B\left(\sigma L: I_{i} \rightarrow I_{f}\right)$ is the reduced transition probability, given by [43]

$$
\begin{equation*}
\left.B\left(\sigma L: I_{i} \rightarrow I_{f}\right)=\frac{1}{2 I_{i}+1}\left|\left\langle I_{f}\right| \hat{O}\right| I_{i}\right\rangle\left.\right|^{2}, \tag{2.14}
\end{equation*}
$$

where $\hat{O}$ is either $\hat{Q}$, the electric, or $\hat{M}$, the magnetic, multipole operator depending on whether $\sigma$ is $E$ or $M,\left\langle I_{f}\right| \hat{O}\left|I_{i}\right\rangle$ is the reduced matrix element, governing the decay between states $I_{i}$ and $I_{f}$ by the $\gamma$ ray of angular momentum $L$. The dominance of the double factorial $L$ term in equation 2.13 means that higher multipole $\gamma$ ray decays will have severely hindered decay probabilities compared to lower multipole decays.

The Weisskopf estimates provide lifetime estimates for a single-particle $\gamma$-ray decay by considering the transition to proceed with a single unmixed multipolarity equivalent to $\left|I_{i}-I_{f}\right|$ (known as a "stretched" transition) [44]. For single-particle transitions the rate of decay is inversely proportional to the multipolarity of the emitted $\gamma$ ray, therefore only the stretched transition needs to be considered [33]. The transition probability as stated in 2.13 is used to give the single-particle transition rate with the inclusion of the Weisskopf reduced matrix elements [45, 46], given by

$$
\begin{equation*}
B(\mathrm{Wu}: E L)=\frac{1.2^{2 L}}{4 \pi}\left(\frac{3}{L+3}\right)^{2} A^{\frac{2 L}{3}} e^{2} \mathrm{fm}^{2 \mathrm{~L}} \tag{2.15}
\end{equation*}
$$

for electric transitions and

$$
\begin{equation*}
B(\mathrm{Wu}: M L)=\frac{10}{\pi} 1.2^{2 L-2}\left(\frac{3}{L+3}\right)^{2} A^{\frac{2 L-2}{3}} 2\left(\frac{e \hbar}{2 M c}\right)^{2} \mathrm{fm}^{2 \mathrm{~L}-2}, \tag{2.16}
\end{equation*}
$$

for magnetic transitions. In these equations, $A$ is the mass of the nucleus and $L$ is the multipolarity of the $\gamma$-ray transition. The magnetic reduced matrix elements are given in units of $\left(\frac{e \hbar}{2 M c}\right)^{2} \mathrm{fm}^{2 \mathrm{~L}-2}$, where $M$ is the mass of the nucleon, while the
electric reduced matrix elements are given in units of $e^{2} \mathrm{fm}^{2 \mathrm{~L}}$. Using equations 2.15, 2.16 and 2.13 gives the simple reduced transition probabilities listed in Table 2.1. If the lifetimes of excited nuclear states are seen to significantly differ from these values then a single-particle, spherical picture of the nucleus may not be accurate, as is discussed further in Subsection 2.5.3.

| Multipolarity | $T_{1 / 2}(\mathrm{~s})$ |
| :---: | :---: |
| $E 1$ | $6.76 \times 10^{-6} E_{\gamma}^{-3} A^{-\frac{2}{3}}$ |
| $E 2$ | $9.52 \times 10^{6} E_{\gamma}^{-5} A^{-\frac{4}{3}}$ |
| $E 3$ | $2.04 \times 10^{19} E_{\gamma}^{-7} A^{-2}$ |
| $E 4$ | $6.50 \times 10^{31} E_{\gamma}^{-9} A^{-\frac{8}{3}}$ |
| $M 1$ | $2.20 \times 10^{-5} E_{\gamma}^{-3}$ |
| $M 2$ | $3.10 \times 10^{7} E_{\gamma}^{-5} A^{-\frac{2}{3}}$ |
| $M 3$ | $6.66 \times 10^{19} E_{\gamma}^{-7} A^{-\frac{4}{3}}$ |
| $E 1$ | $2.12 \times 10^{32} E_{\gamma}^{-9} A^{-2}$ |

Table 2.1: Weisskopf single-particle half-life estimates for different $\gamma$-ray multipolarities, $E_{\gamma}$ is the transition energy in units of keV and A is the mass of the nucleus.

Proton-emission rates can also provide information on nuclear structure, such as angular momentum assignments of proton-emitting states. In a similar manner to $\gamma$-ray decay, accurate theoretical models are needed to interpret the experimental data. The next section examines the predictive power of three such models that are based on proton-emission in spherical nuclei.

### 2.4 Spherical Models of Proton Emission

Models of proton-emitting nuclei commonly consider the proton occupying a quasistationary state $[1,2]$, characterised as a long-lived out-of-equilibrium state which is affected by a long-range interaction [47]. Quasistationary states are also defined as having a finite decay width, $\Gamma$, where the decay-width is linked to the half-life, $T_{1 / 2}$, of the state by

$$
\begin{equation*}
T_{1 / 2}=\frac{\hbar \ln (2)}{\Gamma} . \tag{2.17}
\end{equation*}
$$

For the case of spherical proton-emitting nuclei, the exact decay width of the quasistationary state can be calculated.

The wave function of the outgoing proton, $\psi_{l j}^{\text {out }}$, with total and orbital angular momenta of $j$ and $l$, respectively, is given by

$$
\begin{equation*}
\psi_{l j}^{o u t}=N_{l j}\left[G_{l j}+i F_{l j}\right], \tag{2.18}
\end{equation*}
$$

where $N_{l j}$ is a normalisation constant and $G_{l j}$ and $F_{l j}$ are, respectively, the irregular and regular Coulomb wave functions. These wave functions are solutions to the Laplacian in the Coulomb wave equation, which is itself given by the Schrödinger equation including a Coulomb potential. The regular and irregular Coulomb functions give the solutions to the Coulomb equation in the classically allowed and classically forbidden regions, respectively. The normalisation constant, $N_{l j}$ is found by matching the wave function of the outgoing proton, $\psi_{l j}^{o u t}$, to the solution of the Schrödinger equation of the un-emitted proton, $\psi_{l j}$, which obeys the condition $\psi_{l j}(r=0)=0$ and has outgoing boundary conditions at the point, $R$, where the nuclear potential ceases to have an effect on the proton. This wave function defines the proton as occupying a Gamow state, which is the natural wave function of a resonant quasibound state [48, 49]. The decay width of the state is given by the solution for the decay width of a Gamow state which can be solved using several different approaches [50-52]. The expression for the decay width of a proton-emitting state in a spherical nucleus is then

$$
\begin{equation*}
\Gamma(R)=\frac{\hbar^{2} k}{\mu} \frac{R^{2}\left|\psi_{l j}(R)\right|^{2}}{F_{l j}^{2}(R)+G_{l j}^{2}(R)}, \tag{2.19}
\end{equation*}
$$

where $\mu$ is the reduced mass of the proton orbiting the nucleus and $k=\sqrt{2 \mu E} / \hbar$ is the wave number of the proton. In Ref. [51] the Schrödinger equation is solved for the Gamow state, leading to solutions with complex energies, $\mathcal{E}=E_{0}+i \Gamma$, where $E_{0}$ is the energy of the resonance and $\Gamma$ is its decay width. Rather than solving the Schrödinger equation in this way, a variety of approximate methods may be successfully used. Three of these approximate approaches, which treat the proton as occupying a quasistationary state in a spherically symmetric potential are compared below.

### 2.4.1 Distorted Wave Born Approximation (DWBA)

The distorted wave Born approximation (DWBA) provides a rigorous method of calculating the decay width of a proton emitting state if the initial and final wave functions of the proton and core are known. The DWBA considers the emitted proton as undergoing a low-amplitude interaction between a quasibound and unbound channel, with both channels subject to a common potential. The transition amplitude has to be sufficiently small, so transitions with larger decay widths may not be well modelled by the approximation [53].

The parent nucleus wave function, $\Psi_{A+1}$, is taken to be the product of the individual wave functions of the quasibound proton and nuclear core, while $\Psi_{A p}$ is a product of the daughter nucleus, similarly treated as the inert core of the parent nucleus, and the intrinsic wave function of the emitted proton [1, 2].

The radial wave function of the quasibound proton is found by numerically integrating the Schrödinger equation with the one body potential. The quasibound wave function should have a form such that the interior wave function is smoothly joined to the irregular part of the Coulomb function, $G(r)$, which describes the proton wave function as $r \rightarrow \infty$. The depth of the nuclear potential and strength of the spin-orbit interaction can then be altered so that the the quasibound wave function follows this condition.

Using the forms of wave function described the decay width of the proton resonance is

$$
\begin{equation*}
\Gamma=\frac{4 \mu}{\hbar^{2} k}\left|\int_{0}^{\infty} F_{l}(r)\left(V_{N}+\delta V^{C}\right) \phi_{n l j} d r\right|^{2}, \tag{2.20}
\end{equation*}
$$

where $V_{N}$ is the nuclear potential, $\delta V^{C}$ is a factor that accounts for the Coulomb interaction from the finite charge distribution of the nucleus, $\mu$ is the reduced mass of the core and proton and $\phi_{n l j}$ is the wave function of the outgoing proton.

### 2.4.2 Two Potential Approach (TPA)

The Two Potential Approach considers the proton as initially occupying a bound state within a potential $U(r)$. A distorting potential, $W(r)$, is then introduced at


Figure 2.6: Figure taken from Ref. [7] showing the different forms of potential used in the two potential approach: (a) the total potential, $V(r)$, with the proton occupying a state with energy $E$, (b) the stable part of the potential, $U(r)$ with a stationary state of energy $E_{0}$, (c) the deforming potential, $W(r)$, introduced at time $t=0$ and (d) the form of $W(r)$ which tends to zero as $r \rightarrow \infty$. The labels $r_{1}$ and $r_{2}$ denote the classical turning points of the potential and $r_{b}$ indicates the radius at which the potential barrier is at a maximum $[1,2,7]$.
$t=0$, at which point the proton occupies a quasistable state within the potential, $V(r)=U(r)+W(r)$, and eventually decays to the continuum. The forms of the potentials $U(r), V(r)$ and $W(r)$ are shown in Fig. 2.6 [7].

The barrier consists of both Coulomb and centrifugal parts so the barrier height is dependent on the single-particle angular momentum [1, 2]. The potential $U(r)$ commonly takes the form of a simple harmonic potential [7].

The particle will initially occupy the bound eigenstate of the Hamiltonian, $H_{0}$ with potential $U(r)$, therefore,

$$
\begin{equation*}
H_{0} \Phi_{0}=\left(\frac{p_{r}^{2}}{2 \mu}+U(r)\right) \Phi_{0}=E_{0} \Phi_{0} \tag{2.21}
\end{equation*}
$$

where $\Phi_{0}$ is the initial wave function of the proton contained within the potential $U(r)$ with energy $E_{0}$. Once the perturbing potential $W(r)$ is introduced the particle then occupies a resonant state of energy $E$ which is an eigenstate of the full Hamiltonian $H=\frac{p_{r}^{2}}{2 \mu}+V(r)$. As can be seen from Fig. 2.6, the potential $W(r)$ does not disappear at large values of $r$, instead tending to $-V_{B}$. The potential is then slightly altered to the form $\tilde{W}(r)=W(r)+V_{B}$, which is substituted into the Hamiltonian to provide the more realistic behaviour desired. The Fermi Golden Rule [26] can be used to extract the decay width of the quasistable state, which is found to be

$$
\begin{equation*}
\Gamma=\frac{4 \mu}{\hbar^{2} k}\left|\int_{r_{B}}^{\infty} \phi_{n l j}(r) W(r) \chi_{l}(r) d r\right|^{2} \tag{2.22}
\end{equation*}
$$

where $\mu$ is the reduced mass of the nucleus and proton, $k=\sqrt{2 \mu E} / \hbar, \phi_{n l j}$ is the radial component of the wave function $\Phi_{0}$ and $\chi_{l}(r)$ is the radial wave function of the Hamiltonian with the perturbing potential $\tilde{W}(r)$ added. The integral in equation 2.22 can be solved analytically to give a final expression for the decay width of the quasistationary state of

$$
\begin{equation*}
\Gamma=\frac{\hbar^{2}}{\mu k}\left|\phi_{n l j}\left(r_{b}\right)\left[\alpha \chi_{l}\left(r_{b}\right)+\chi_{l}^{\prime}\left(r_{b}\right)\right]\right|^{2}, \tag{2.23}
\end{equation*}
$$

where $\alpha=\sqrt{2 \mu\left(V_{B}-E\right)} / \hbar$ and $r_{b}$ is the barrier radius shown in Fig. 2.6.

### 2.4.3 Wentzel-Kramers-Brillouin (WKB) Approximation

The Wentzel-Kramers-Brillouin (WKB) approximation to proton emission is a quasi-classical approach that considers a proton tunneling through a potential barrier. The WKB approach uses two different wave functions, a propagating form before the potential barrier and a decaying form after the barrier, which are given in terms of plane waves, $\psi \sim e^{-i k r}$. The WKB terms for the decay energy and decay width of the proton decay can be calculated using the quasiclassical
limit of the two-potential approach $[1,2,27]$. The decay energy, $\Delta$, and decay width, $\Gamma$, are given as

$$
\begin{equation*}
\Delta=\frac{N V^{\prime}\left(r_{b}\right)}{16 \alpha^{3}} \exp \left[-2 \int_{r_{1}}^{r_{b}}|k(r)| d r\right] \tag{2.24}
\end{equation*}
$$

and

$$
\begin{equation*}
\Gamma=N \frac{\hbar^{2}}{4 \mu} \exp \left[-2 \int_{r_{1}}^{r_{2}}|k(r)| d r\right], \tag{2.25}
\end{equation*}
$$

where,

$$
\begin{equation*}
k(r)=\sqrt{\frac{2 \mu}{\hbar^{2}}[E-V(r)]}, \tag{2.26}
\end{equation*}
$$

$r_{1}$ and $r_{2}$ are the classical turning points of the potential barrier as shown in Fig. 2.2 and $V^{\prime}\left(r_{b}\right)=\left.d V(r)\right|_{r \rightarrow r_{b}+0}$ where $r_{b}$ is the distance from the centre of the nucleus at which the potential barrier height is at a maximum. $N$ is a normalisation factor given by

$$
\begin{equation*}
N^{-1}=\int_{r_{0}}^{r_{1}} \frac{d r}{k(r)} \cos ^{2}\left(\int_{r_{0}}^{r_{1}} k\left(r^{\prime}\right) d r^{\prime}-\frac{\pi}{4}\right) \tag{2.27}
\end{equation*}
$$

where the limits $r_{0}$ and $r_{1}$ denote the classical region of the potential and the index ' indicates the treatment of the proton as being subject to the perturbing potential, $\tilde{W}(r)$. This normalisation factor only considers the proton as occupying the classically allowed region. The simplicity of the WKB method means that it has been widely used to study proton emitters [54].

### 2.4.4 Comparison of Spherical Models and the Effects of

## the Pairing Interaction

The models described in the last three subsections have all been evaluated using the same parameters by Åberg et al. [1, 2] to predict the lifetime of various
proton-emitting nuclei. The $Q_{p}$ values of proton emission were taken from experiment and the proton is considered as occupying a single-particle level in the parent nucleus. The half-lives predicted for various nuclei, using the different approaches are detailed in Table 2.2 below.

| Nucleus | $\begin{aligned} & \text { Ground State } \\ & \text { or Isomeric Decay } \end{aligned}$ | Proton ang. momentum | DWBA | $\begin{aligned} & \hline T_{1 / 2} \\ & \text { TPA } \end{aligned}$ | WKB | Experiment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{112}$ | Ground State | $l=2$ | $68 \mu$ s | $67 \mu \mathrm{~s}$ | $64 \mu \mathrm{~s}$ | 470(50) $\mu \mathrm{s}$ [55] |
|  | Ground S | $l=2$ | 540 | 540 ns | 510 n | 17.1(2) $\mu \mathrm{s}$ [12] |
| ${ }_{69}^{136} \mathrm{Tm}$ | Isomeric | $l=5$ | 350 ms | 340 ms | 370 m | 200(10) ms [56] |
| $\stackrel{156}{13}$ | Ground State | $l=2$ | 97 ms | 96 ms | 96 ms | 106(4) ms [57] |
| ${ }_{75}^{161} \mathrm{Re}$ | Ground State | $l=0$ | 220 | 220 ms | 210 | 440(10) ms [58] |
| ${ }_{77}^{167}$ Ir | Isomeric | $l=5$ | 2.0 s | 2.0 s | 2.2 s | $7.5(19) \mathrm{s}$ [59] |

Table 2.2: Proton emission lifetimes predicted for various nuclei as calculated by Åberg [1, 2] using the DWBA, TPA and WKB approaches with experimental results also listed for comparison, see text for details.

As can be seen from Table 2.2 the various different approaches all give similar lifetimes when the same parameters are used for the calculations. Additionally, reasonable order-of-magnitude estimates of experimental values are made for some odd-even and odd-odd nuclei as well as for nuclei decaying from the ground state or a low-lying isomeric state. However, some of the experimental half-lives are found to lie far from the theoretical values. As the models consider the the proton moving in a spherical mean field in a single-particle orbit, deviation from this behaviour can explain the discrepancies between theory and experiment.

Although the approach of considering a single particle subject to a mean field leads to the successful prediction of many nuclear properties, the effects of interactions between individual nucleons may alter the spacing of excitation energies of the single-particle orbits. The residual interaction causes the mixing of the wave functions defining individual single-particle orbits so that a nucleon will often have to be described by an admixture of many single-particle wave functions [31]. If there are significant components of several single-particle wave functions then it may be necessary to treat the single particles as quasiparticles, which partially occupy many different single-particle orbits. For proton decay, this mixing may effect the half-life of proton emission as there are different probabilities that a single-particle state is occupied or unoccupied in the daughter nucleus after the decay of the parent [14].

The pairing residual interaction strongly contributes to the mixing of different states and has to be treated in proton-emission models. The effect of the pairing interaction is to attempt to situate paired nucleons as close as spatially possible together while not violating the Pauli exclusion principle [31]. One quality of the pairing interaction is that pairs of nucleons in orbits near the Fermi surface can be scattered to higher orbits. There is then a certain probability of valence nucleons occupying several orbits near the Fermi surface. The gap parameter, $\Delta$, describes the occupation probabilities of excited states, $i$ and $j$, near the Fermi surface as,

$$
\begin{equation*}
\Delta=G \sum_{i, j} U_{i} V_{j} \tag{2.28}
\end{equation*}
$$

where $G$ is the strength of the pairing interaction, $U$ is the emptiness factor given by

$$
\begin{equation*}
U_{i}=\frac{1}{\sqrt{2}}\left[1+\frac{\left(\epsilon_{i}-\lambda\right)}{\sqrt{\left(\epsilon_{i}-\lambda\right)^{2}+\Delta^{2}}}\right]^{\frac{1}{2}} \tag{2.29}
\end{equation*}
$$

and $V$ is the fullness factor given by

$$
\begin{equation*}
V_{i}=\frac{1}{\sqrt{2}}\left[1-\frac{\left(\epsilon_{i}-\lambda\right)}{\sqrt{\left(\epsilon_{i}-\lambda\right)^{2}+\Delta^{2}}}\right]^{\frac{1}{2}} \tag{2.30}
\end{equation*}
$$

$\epsilon_{i}$ are the single-particle energies of states near the Fermi surface and $\lambda$ is the Fermi energy. The factors $V$ and $U$ give the probability that an orbit $i$ is occupied or unoccupied, respectively. The levels about the Fermi surface are occupied over a range of approximately $\Delta$. The level of occupancy of energy levels around the Fermi surface is shown schematically in Fig. 2.7.

Rather than describing single-particle or hole excitations it then becomes easier to describe excitations in terms of quasi-particles, which have a probability of occupying several nuclear states. The energy of a quasi-particle, $E_{i}$, is given in terms of the single-particle energies and range of occupied levels as,


Figure 2.7: Schematic diagram showing the partial occupancy of idealised energy levels about the Fermi surface, denoted by the fullness factor $V_{i}^{2}$, due to pairing [31]. The Fermi surface energy is denoted by $\lambda$.

$$
\begin{equation*}
E_{i}=\sqrt{\left(\epsilon_{i}-\lambda\right)^{2}+\Delta^{2}} . \tag{2.31}
\end{equation*}
$$

Spectroscopic factors can be introduced to account for the pairing interaction in the spherical proton-emission models discussed in this section. The theoretical spectroscopic factor,

$$
\begin{equation*}
S_{p}^{t h}=u_{j}^{2}, \tag{2.32}
\end{equation*}
$$

can be introduced from a single quasiparticle approach [60, 61], where $u_{j}^{2}$ is the probability that the spherical orbit occupied by the proton in the parent nucleus will be unoccupied in the daughter nucleus. The value of $u_{j}^{2}$ should, thus, lie between 0 and 1. The experimental spectroscopic factor can also be calculated from,

$$
\begin{equation*}
S_{p}^{e x p}=T_{1 / 2}^{t h} / T_{1 / 2}^{e x p}, \tag{2.33}
\end{equation*}
$$

where, $T_{1 / 2}^{t h}$ and $T_{1 / 2}^{e x p}$ are the theoretical and experimental half-lives, respectively. If the experimental and theoretical spectroscopic factors agree then the fragmentation of the single-particle strength can be said to cause the observed differences in the proton-emission half-life seen for some nuclei in Table 2.2. This comparison is shown with theoretical half-lives taken from the DWBA approach in Table 2.3.

| Nucleus | $S_{p}^{\text {exp }}$ | $S_{p}^{\text {th }}$ |
| :---: | :---: | :---: |
| ${ }^{112} \mathrm{Cs}$ | $0.145(39)$ | 0.59 |
| ${ }^{51}{ }^{113} \mathrm{Cs}$ | $0.032(3)$ | 0.59 |
| ${ }^{15}$ | ${ }^{14} 6 \mathrm{Tm}$ | $1.77(25)$ |
| ${ }^{69} \mathrm{Tm}$ | 0.64 |  |
| ${ }^{73} \mathrm{Ta}$ | $0.915(164)$ | 0.67 |
| ${ }^{161} \mathrm{Re}$ | $0.514(98)$ | 0.59 |
| ${ }_{15}^{167} \mathrm{Tr}$ | $0.267(86)$ | 0.23 |
| ${ }_{77} \mathrm{Ir}$ |  |  |

Table 2.3: Comparison between experimental and theoretical spectroscopic factors for the proton-emitters listed in Table 2.2. Experimental spectroscopic factors were calculated using half-lives taken from Refs. [12, 55-59] and theoretical half-lives calculated using the DWBA method in Refs. [1, 2]. Theoretical spectroscopic factors are also taken from Refs. [1, 2]. The inconsistency between theory and experiment for the top three results indicates that the DWBA assumption of a spherically symmetric nucleus is not valid.

Table 2.3 shows that only some of the nuclei have consistent experimental and theoretical spectroscopic factors. These nuclei will have lifetimes accurately predicted by the spherical models when the spectroscopic factor takes into account the non-single-particle properties of the proton decay. However, there are some nuclei with inconsistent theoretical and experimental spectroscopic factors, notably ${ }^{113} \mathrm{Cs}$. In these cases the single-particle strength is very strongly fragmented and the tunneling probability is strongly enhanced compared to the spherical potential used in the three approaches detailed in the previous subsections [1, 2]. This is indicative of the parent nucleus deviating from a spherical shape. Therefore, a new theoretical approach encompassing proton emission from a deformed potential is needed to make accurate predictions on the properties of deformed nuclei. Before addressing the models which consider proton emission from a deformed nucleus, the characteristics of deformed nuclei will be described as well as the Nilsson model, which provides a single-particle model for deformed nuclei.

### 2.5 Nuclear Deformation

Nuclei with numbers of protons and neutrons which are significantly different to the magic numbers, will deviate from a spherical shape and have less binding energy per nucleon. This is due to the increased residual interaction between the large number of valence nucleons lying outside of a closed shell. The change in nuclear shape can be represented as a perturbation to the spherical nuclear shape expressed as the series of spherical harmonics, given by equation 2.34:

$$
\begin{equation*}
R(\theta, \phi)=R_{a v}\left(1+\sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda, \mu} Y_{\lambda, \mu}(\theta, \phi)\right), \tag{2.34}
\end{equation*}
$$

where $\lambda$ describes the mode of the resonance, $\alpha$ describes the contribution of each mode to the total shape, $R_{a v}$ is the average nuclear radius and $Y_{\lambda, \mu}(\theta, \phi)$ are the series of spherical harmonics [26]. The dominant mode of deformation is the quadrupole $(\lambda=2)$ mode with a smaller contribution from the octupole deformation mode, $(\lambda=3)$, in some cases (Ref [62] predicts that for ${ }^{113} \mathrm{Cs}$, the quadrupole deformation mode has more than twice the magnitude of the octupole deformation mode) [63, 64]. For a predominantly quadrupole deformation, the number of $\alpha$ terms is reduced. The terms $\alpha_{2,1}=\alpha_{2,-1}=0$, as these terms do not conserve spatial symmetry and due to axial symmetry $\alpha_{2,2}=\alpha_{2,-2}$. The applicable $\alpha$ coefficients are then reduced to $\alpha_{2,0}$ and $\alpha_{2,2}$. These can be simply expressed in terms of the quadrupole deformation, $\beta$, and the deviation from axial symmetry, $\gamma$, as,

$$
\begin{equation*}
\alpha_{2,0}=\beta_{2} \cos (\gamma) \tag{2.35}
\end{equation*}
$$

and

$$
\begin{equation*}
\alpha_{2,2}=\frac{1}{\sqrt{2}} \beta_{2} \sin (\gamma) \tag{2.36}
\end{equation*}
$$

The $\beta$ and $\gamma$ parameters form the basis of the Lund convention of measuring shape changes due to quadrupole deformation [31]. Figure 2.8 shows the shapes associated with the changing of these two parameters.


Figure 2.8: The quadrupole deformation from a spherical shape described by the Lund convention, as taken from Ref. [33]. $\beta$ is the magnitude of quadrupole deformation and $\gamma$ is the deviation from axial symmetry. The different alignments of the rotational and symmetry axis are shown for their associated values of $\gamma$ as well as the changes between prolate and oblate shapes.

The parameters $\epsilon$ and $\delta$ may also sometimes be used to describe axially symmetric nuclear deformation, where $\delta$ is given by

$$
\begin{equation*}
\delta=\frac{\Delta R}{R_{r . m . s}}, \tag{2.37}
\end{equation*}
$$

where $\Delta R$ is the difference between the semi-minor and semi-major axes of the deformed nucleus and $R_{r . m . s}$ is the root mean square radius of the nucleus [65]. The parameter $\epsilon$ is given in terms of an expansion of $\delta$ as,

$$
\begin{equation*}
\epsilon_{2}=\delta+\frac{1}{6} \delta^{2}+\frac{5}{18} \delta^{3} \ldots \tag{2.38}
\end{equation*}
$$

and is, in turn, related to the $\beta_{2}$ parameter by

$$
\begin{equation*}
\beta_{2}=\sqrt{\frac{\pi}{5}}\left[\frac{4}{3} \epsilon_{2}+\frac{4}{9} \epsilon_{2}^{2}+\frac{4}{27} \epsilon_{2}^{3} \cdots\right] . \tag{2.39}
\end{equation*}
$$

The ordering of single-particle levels will be changed in deformed nuclei and the projection of single-particle angular momentum on the symmetry axis of the axially deformed core becomes a good quantum number. These changes have to be accounted for by a deformed model of the nucleus. The Nilsson model provides this framework.

### 2.5.1 The Nilsson Model

The Nilsson model describes the movement of nucleons in a deformed potential and accounts for the variation in energy of the single-particle levels of the spherical shell model with changing nuclear deformation. A nucleon that occupies a single orbit in a deformed nucleus will have a lower energy if the orbit lies closer to the nuclear core. As such, the orientation of a single $j$ orbit will have an effect on its energy, in a way that is not significant when considering spherical nuclei. Therefore, the degeneracy of a $j$ orbital is affected by the deformation of the nucleus [61]. Orbits within the Nilsson model are described by the quantum number $\Omega$, which is the projection of a single $j$ orbit onto the symmetry axis of the nucleus. For the case of there being more than one particle orbiting the nuclear core there will be a total projection of angular momentum onto the symmetry axis of $K$, where $K=\sum_{i} \Omega_{i}$. A classical approximation of the angle of the different $K$-projections may be given by

$$
\begin{equation*}
\Theta=\sin ^{-1}(K / j) . \tag{2.40}
\end{equation*}
$$

## $\Omega$



Figure 2.9: Schematic diagram showing the increased separation of Nilsson single-particle energies with increased projection of the angular momentum of the $g_{7 / 2}$ orbital on the symmetry axis.

This indicates that the energy difference between orbits separated by the same increment of $K$ is greater for high $K$ orbits than for low $K$ orbits, as is shown schematically in Fig. 2.9.

The change in single-particle excitation energy versus quadrupole deformation parameter, $\epsilon_{2}$, is shown graphically in the Nilsson diagram in Fig. 2.10. This shows the Nilsson levels calculated using a deformed simple harmonic potential for protons between $50 \leq Z \leq 82$. This is the area of the proton Nilsson diagram corresponding to the proton number of ${ }^{113} \mathrm{Cs}$, the nucleus examined in this work. A deformed Woods-Saxon potential may also be used to calculate the spacing of the single-particle levels, as was the case for the theoretical model described in Section 2.6.3 [16].

In Fig. 2.10, it can be seen that at $\epsilon_{2}=0$ the $K$-splitting of the spherical orbits occurs and the individual $K$ orbits diverge from each other. The diagram shows how orbits will not cross other orbits with the same $K$ value and parity. The diagram also shows how the deflection and divergence of the orbits at larger deformations leads to individual $K$ orbits lying close in energy to each other, which enhances the configuration mixing of the two states. States will mix more
strongly if they have similar $K$ values. Additionally, the interaction that causes configuration mixing in the Nilsson model is of quadrupole form so mixing tends to occur strongly between configurations which differ by two units of angular momentum [31].

The intruder orbitals, which are orbitals lowered by the spin-orbit interaction into a different major shell will have a different parity to the natural-parity states already present in this major shell. Due to having different parities the naturalparity and intruder $K$ orbits cannot mix so intruder orbitals tend to have pure configurations as they are removed from orbitals with the same parity. By a similar token, high- $K$ states tend to be pure at small deformations as they are rarer in major shells than the low- $K$ states which may emerge from a greater number of orbitals. The high- $K$ states will then not mix strongly with the states that differ by a large value of $K$.

Nilsson orbits are typically described in the following form,

$$
\begin{equation*}
K^{\pi}\left[N, n_{z}, \Lambda\right], \tag{2.41}
\end{equation*}
$$

where $N$ is the principle quantum number of the major shell, $n_{z}$ is the number of nodes of the orbit in the $z$ direction and $\Lambda$ is the projection of the orbital angular momentum onto the symmetry axis $z$. By definition $K=\Lambda+\Sigma$ where $\Sigma$ is the projection of the intrinsic spin of the nucleon onto the symmetry axis.

The Nilsson model provides a framework for single-particle motion in a deformed potential. However, collective modes of excitation will also arise due to the motion of many nucleons in a deformed potential. To describe the angular momentum of a nucleus it then becomes necessary to consider these collective excitations as well as the single-particle angular momentum. Nuclei can undergo both collective vibrations and rotations. The nucleus ${ }^{113} \mathrm{Cs}$ which is studied in this work undergoes nuclear rotation and this excitation mode is examined in the next section.

### 2.5.2 Nuclear Rotation

Excited nuclei with spherically asymmetric charge and mass distributions may rotate. These rotations stem from collective, rather than individual, nucleon


Figure 2.10: Example of the Nilsson diagram for protons $50 \leq Z \leq 82$, which shows the variation in energy of single-particle orbits versus deformation of the nucleus, taken from Ref. [65].


Figure 2.11: Schematic diagram of a deformed nucleus undergoing rotation with frequency $\omega$. The vector $I$ represents the total angular momentum of the nucleus, which can be split into core $R$ and individual particle $J$ components. $K$ is the projection of $I$ onto the symmetry axis while $I_{x}$ is the projection of $I$ onto the rotation axis.
excitations [26, 31]. The angular momentum of the individual and collective excitations of the nucleus can be treated as separate excitations stemming from the rotating nuclear core and the individual nucleons. The total angular momentum of a rotational nucleus is then given by

$$
\begin{equation*}
I=R+J, \tag{2.42}
\end{equation*}
$$

where $I$ is the total angular momentum of the nucleus, $R$ is the contribution of the rotation of the core and $J$ is the angular momentum of valence nucleons. In this approach the presence of the valence nucleons does not have any effect on the behaviour of nucleons in the core and vice versa. This is due to the fact that the motion of the valence nucleons and core take place on significantly different timescales [31]. The core is then described as a rigid rotor, as it will retain its shape and rotational frequency regardless of the excitation of the valence nucleons. This may be referred to as an adiabatic system. A schematic diagram of a rotating deformed nucleus and the different angular momenta describing its behaviour are shown in Fig. 2.11

The energy of rotational excited states in a nucleus is proportional to the angular momentum of the rotating core. The excitation due to the rotation of the core,
$E_{\text {rot }}$ can be, in the first instance, described classically by

$$
\begin{equation*}
E_{\text {rot }}=\frac{1}{2} \mathcal{J} \omega^{2}, \tag{2.43}
\end{equation*}
$$

where $\mathcal{J}$ is the moment of inertia of the core and $\omega$ is its angular frequency. This can also be written semi-classically as,

$$
\begin{equation*}
E_{\text {rot }}=\hbar^{2} \frac{I(I+1)}{2 \mathcal{J}} \tag{2.44}
\end{equation*}
$$

which shows that an increase of angular momentum will result in a smooth increase of the energy of the nucleus. This results in rotational band structures, which have excited states with regular energy spacings and spin differences of $2 \hbar$. The lifetimes of the states in rotational bands can provide information on the deformation of the nucleus.

### 2.5.3 Estimating Nuclear Deformation from $\gamma$-ray Transition Rates

If a nucleus is deformed then it will have a non-zero electric quadrupole moment. The intrinsic quadrupole moment, $Q_{0}$, is proportional to the reduced transition probabilities of $E 2$ transitions seen in rotational band structures. The quadrupole moment of the state can then be related to the quadrupole deformation, $\beta_{2}$, of the nucleus by [31]

$$
\begin{equation*}
Q_{0}=\frac{3}{\sqrt{5 \pi}} Z R^{2} \beta_{2}\left(1+\frac{1}{8} \sqrt{\frac{5}{\pi}} \beta_{2} . .\right), \tag{2.45}
\end{equation*}
$$

where $R$ is the radius of the spherical nucleus of the same volume as the deformed nucleus and $Z$ is the atomic number of the nucleus. As such, the deformation of a nucleus can be estimated from the measured $\mathrm{B}(E 2)$ values. The reduced transition probability of a stretched $E 2$ transition is given by

$$
\begin{equation*}
B(E 2)=\frac{5}{16 \pi} e^{2} Q_{0}^{2}\left|\left\langle J_{i} K 20 \mid J_{f} K\right\rangle\right|^{2}, \tag{2.46}
\end{equation*}
$$

where $\left|\left\langle J_{i} K 20 \mid J_{f} K\right\rangle\right|^{2}$ is a Clebsch-Gordon for a transition between states with spins $I$ and $I-2[66]$.

Proton-emission rates may also provide a measure of nuclear deformation, when deformation is included in the theoretical framework. Models, which describe proton emission from a deformed potential are described in the following sections.

### 2.6 Deformed Models of Proton Emission

Introducing deformation into proton-emission models can remove some of the discrepancies seen between theory and experiment in spherical models, as seen in Subsection 2.4.4. The deformation can be taken into account by evaluating the exact decay width of the proton-emitting state for a deformed potential in a manner similar to that described for spherical models in Section 2.4 [14]. Two different approaches that provide an exact expression for the decay width are described in this section $[14,16]$. The first considers an adiabatic approach where the proton is considered to occupy a single-particle orbit within a deformed potential and is coupled to a rigid nuclear core, with infinite moment of inertia [14]. The second considers a nonadiabatic approach where the core may be less rigid and the pairing interaction and Coriolis interaction (subsequently described in Subsection 2.6.2) may have a significant effect on the ordering of nuclear states and the proton-emission half-life [16].

### 2.6.1 Deformed Adiabatic Single-Particle Approach

The adiabatic approach of Maglione et al. [14] evaluates the decay width of a single nucleon from a single-particle level in a deformed potential. In this case the single proton is coupled to the nuclear core which is treated as a rigid rotor with infinite moment of inertia.

One of the main differences between calculating decay widths in spherical and deformed nuclei is that partial decay widths have to be inferred for different angular momentum projected states [14]. This is due to the fact that the projected angular momentum, $K$, becomes a good quantum number in these nuclei and can be used to describe distinct excited states. Excitations of the nucleus stemming from collective motion also have to be accounted for. Assuming an odd-even nucleus with a single unpaired proton, the angular momentum projected wave function of the parent is given by

$$
\begin{equation*}
\Psi_{m}^{J_{i} M_{i}, K_{i}}=\left(\frac{\hat{J}_{i}}{16 \pi^{2}}\right)^{1 / 2}\left\{\mathscr{D}_{M_{i} K_{i}}^{J_{i}} \chi_{K_{i}}+(-1)^{J_{i}+K_{i}} \mathscr{D}_{M_{i}-K_{i}}^{J_{i}} \chi_{\bar{K}_{i}}\right\}, \tag{2.47}
\end{equation*}
$$

where, $\hat{J}_{i}=2 J_{i}+1, \mathscr{D}$ are the rotation matrices and $\chi$ is the intrinsic singleparticle wave function, which can be expanded in spherical components to

$$
\begin{equation*}
\chi_{K_{i}}(\vec{r})=\sum_{j \geq K_{i}} \alpha_{l j}(r)\left[Y_{l}(\hat{r}) \chi_{1 / 2}\right]_{j K_{i}}, \tag{2.48}
\end{equation*}
$$

where, $Y_{l}$, are spherical harmonics, $\chi_{1 / 2}$ the spin functions and $\alpha_{l j}$ describes the contribution of different $l$ and $j$ orbits to the single-particle wave function.

The wave function of the exit channel will be the product of the daughter wave function and the wave function of the proton's movement relative to the daughter nucleus. The daughter nucleus wave function is

$$
\begin{equation*}
\Psi_{d}^{J_{d} M_{d} K_{d}}=\left(\frac{\hat{J}_{d}}{8 \pi^{2}}\right)^{1 / 2} \mathscr{D}_{M_{d} K_{d}}^{J_{d}} . \tag{2.49}
\end{equation*}
$$

For the case of the daughter nucleus being left in its ground state by the proton emission, as is most likely, the relation $J_{d}=M_{d}=K_{d}=0$ can be assumed. Angular momentum conservation then implies that the outgoing proton has angular momentum $j_{p}=J_{i}=K_{i}$. A spectroscopic factor of $1 /\left(j_{p}+1 / 2\right)$ has to be included to guarantee that the angular momentum of the daughter nucleus is zero.

At large values of $r$, where the nuclear potential vanishes and the Coulomb potential is nearly spherically symmetric, the outgoing proton wave function will adopt a form, similar to that seen in equation 2.18, of

$$
\begin{equation*}
R \chi_{K_{i}}^{\text {out }}(\vec{R})=\sum_{l j} N_{l j}\left[G_{l j}(R)+i F_{l j}(R)\right]\left[Y_{l}(\hat{R}) \chi_{1 / 2}\right]_{j K_{i}} . \tag{2.50}
\end{equation*}
$$

Here $N_{l j}$ are normalisation constants, which are found by matching the outgoing wave function to the internal wave function given in equation 2.48. The probability rate of decay to channel $l j$ is $T_{l j}=\frac{1}{N_{l j}^{2} v}$, where $v$ is the velocity of the proton in the potential.

Using these definitions of the parent, daughter and proton wave functions, the partial decay width corresponding to the decay of the proton channel is found to be

$$
\begin{equation*}
\Gamma_{l_{p} j_{p}}(R)=\frac{\hbar^{2} k}{\mu} \frac{R^{2} \alpha_{l_{p} j_{p}}^{2}(R)}{F_{l_{p} j_{p}}^{2}(R)+G_{l_{p} j_{p}}^{2}(R)} . \tag{2.51}
\end{equation*}
$$

This expression assumes that the deformation of the parent and daughter nuclei remains the same [14].

This model has been used to test the decay width of ${ }^{113} \mathrm{Cs}[14]$ in order to try and replicate the experimental half-life in a way which was not possible for the spherical models discussed in Section 2.4 [1, 2, 7]. Three single-particle excitations are predicted to lie near the Fermi surface for moderate deformations of ${ }^{113} \mathrm{Cs}$ : the $K=1 / 2^{+}$orbital from the $g_{7 / 2}$ shell and the $K=3 / 2^{+}$and $K=5 / 2^{+}$ orbitals from the $d_{5 / 2}$ shell. The half-life, $T_{1 / 2}=\frac{\hbar \ln 2}{\Gamma_{l_{p} j_{p}}}$, of proton emission from each of these orbits was calculated using the deformed adiabatic single-particle method and the results are shown in Fig. 2.12.

As the adiabatic approach does not include the effect of the pairing interaction a spectroscopic factor must be applied to the theoretical values in order to compare with the experimental lifetime. When a spectroscopic factor of $u^{2} \sim 0.5$ is included in the calculation, doubling the theoretical values, Fig. 2.12 shows that there is no orbit for which the experimental half-life, denoted by the dashed lines, is replicated at zero deformation. When the same spectroscopic factor is included in the calculation, the $5 / 2^{+}$state can be discounted as the ground state as at


Figure 2.12: Figure taken from Ref. [14] showing theoretical lifetimes of proton emission from ${ }^{113} \mathrm{Cs}$ calculated from equation 2.51 versus deformation. The lifetime of the decay has been predicted for proton decays from the $K=$ $3 / 2^{+}, 5 / 2^{+}$states from the $d_{5 / 2}$ orbital and the $K=1 / 2^{+}$state from the $g_{7 / 2}$ orbital. The theoretical values are compared to the experimental lifetime value of $16.7(7) \mu$ s from Ref. [11], the limits of which are denoted by the dashed lines. The effects of the pairing interaction are not included in the model and a spectroscopic factor of $u_{j}^{2}=0.5$, which doubles the theoretical values, must be included to compare with the experimental lifetime.


Figure 2.13: The different components of the wave functions making up the $K=1 / 2^{+}$state in ${ }^{113} \mathrm{Cs}$ plotted as a function of deformation, taken from Ref. [14]. The solid line corresponds to the $j=K$ component, the dash-dotted line to the $j=K+1$ orbit, the dashed line to the $j=K+2$ orbit and the dotted line to the $j=K+3$ orbit. The vertical red line marks $\beta=0.08$ to indicate the deformation at which the sharp increase in half-life is seen for the $K=1 / 2^{+}$state in Fig. 2.12. The horizontal red line indicates the point at which the wave functions have greater amplitudes than zero.
the deformation of $\beta_{2} \sim 0.22-0.27$, the state lies too far away from the Fermi surface [14]. This indicates that either the $3 / 2^{+}$or $1 / 2^{+}$state is the ground state of ${ }^{113} \mathrm{Cs}$.

The sharp increase in half-life for the $K_{1 / 2}$ state at $\beta=0.08$, seen in Fig. 2.12 is due to the change in the amplitudes of the different components of the $K=1 / 2^{+}$ state at that deformation. This is shown graphically in Fig. 2.13.

The sign of the amplitude of the $s_{1 / 2}$ orbital is seen to change at the value of the lifetime increase at $\beta=0.08$. Additionally, the $g_{7 / 2}$ component is also
seen to decrease at this deformation. As the square of the half-life is inversely proportional to the square of the wave function, the half-life increases sharply at this point. Due to the sharp increase in the half-life of the decay of the parent to the ground state of the daughter, decay to the first excited state of the daughter may also emerge as an alternative decay path. In order to investigate this further, information on the angular momenta of the daughter and mother nuclei would be needed [14]. As small components of the wave function of the proton-emitting state may have significant effects on its lifetime, it is important to properly consider the mixing that may introduce these small components.

The adiabatic approach considers the proton as having a strong coupling to the nuclear core, so that there is no perturbation to the orbit of the single proton due to the Coriolis interaction. However, in nuclei such as ${ }^{113} \mathrm{Cs}$, the Coriolis interaction is predicted to have a strong effect on the spacing of the different excited states and the mixing of different state wave functions [16]. As such it is important to develop a model that includes these effects. The mechanism behind the Coriolis interaction is covered in the next section, before a model which includes the interaction is introduced.

### 2.6.2 The Coriolis Interaction

The single-particle Nilsson wave function does not possess a fixed angular momentum $J$, but can be projected onto states of many different angular momenta. Therefore, a given Nilsson orbital can be projected onto many different core excitations to form a rotational band. For the case of an odd-even nucleus, the total angular momentum corresponds to the vector sum of the core rotation and odd-particle angular momentum. The Coriolis interaction arises as a result of this coupling of the intrinsic and rotational motions and can be derived from the rotational Hamiltonian,

$$
\begin{equation*}
H=\frac{\hbar^{2}}{2 \mathscr{I}} R^{2}=\frac{\hbar^{2}}{2 \mathscr{I}}(I-j)^{2}=\frac{\hbar^{2}}{2 \mathscr{I}}\left(I^{2}+j^{2}-2 I \cdot j\right), \tag{2.52}
\end{equation*}
$$

where $\mathscr{I}$ is the moment of inertia of the nucleus, $I$ is the total angular momentum of the nucleus, $R$ is the angular momentum of the nuclear core and $j$ is the single-particle angular momentum as shown in Fig. 2.11. Using the raising and
lowering operators, $I_{ \pm}=I_{1} \pm i j_{2}$ and $I_{\mp}=I_{2} \pm i j_{1}$ and replacing the operators by eigenvalues where possible gives

$$
\begin{equation*}
E(J)=\frac{\hbar^{2}}{2 \mathscr{I}}\left[I(I+1)-2 K^{2}+\left\langle j^{2}\right\rangle-\left(I_{+} j_{-}+I_{-} j_{+}\right)\right], \tag{2.53}
\end{equation*}
$$

where

$$
\begin{equation*}
V_{\text {Coriolis }}=\frac{\hbar^{2}}{2 \mathscr{I}}\left(I_{+} j_{-}+I_{-} j_{+}\right) . \tag{2.54}
\end{equation*}
$$

This Coriolis interaction term describes an effect similar to the classical Coriolis effect, which applies to an object travelling with a velocity $v$ on a body rotating with angular velocity $\omega$. When viewed from the rotational frame, the object will appear to be subject to a force, deflecting it from its original path. For the case of nuclei, the Coriolis interaction will alter the projection of the single-particle angular momentum on the symmetry axis, admixing different $K$ values. Due to the properties of the raising and lowering operators, Coriolis mixing only acts between states where $K$ differs by 1 . The Coriolis mixing can have significant effects on the level structure of rotational nuclei. Low $K$ levels from high $j$ spherical orbits have been observed in some rare earth nuclei to have energies shifted by several hundred keV from unmixed energies [31].

For rotational bands based on a $K=1 / 2$ orbit, the energies of the states forming the band including the Coriolis term is given by

$$
\begin{equation*}
E_{\text {rot }}(I)=\frac{-\hbar^{2}}{2 \mathscr{I}}\left[I(I+1)+\delta_{K 1 / 2} a(-1)^{I+1 / 2}\left(I+\frac{1}{2}\right)\right] \tag{2.55}
\end{equation*}
$$

where $a$ is the decoupling parameter given by

$$
\begin{equation*}
a=\sum_{j}(-1)^{j-1 / 2}\left(j+\frac{1}{2}\right) C_{j}^{2} \tag{2.56}
\end{equation*}
$$

and $C_{j}$ denotes the Nilsson wave function describing a particular orbit [31]. The decoupling parameter provides a measure of how easy it is to decouple the motion of the core and valence nucleons. For $a=1$ the energy spacing
of $K=1 / 2,5 / 2,9 / 2$.. states will be exactly the same as those for $0^{+}, 2^{+}, 4^{+}, \ldots$ states of the even-even core. In this case the total angular momentum vector is aligned with the rotation axis and the Coriolis force will not have a strong effect on the projection of the single-particle orbit. This forms the basis of the adiabatic strong-coupling limit where the coupling of the single-particle and rotational motion is strong enough that the perturbation from the Coriolis effect is not significant. However, for nuclei with $K$ states which differ by one unit of angular momentum and lie close in energy, the Coriolis mixing may be strong and needs to be considered. ${ }^{113} \mathrm{Cs}$ is such a nucleus.

### 2.6.3 Deformed Nonadiabatic Quasiparticle Approach

The deformed nonadiabatic quasiparticle approach was developed to include the effect of the Coriolis interaction on proton-emitting nuclei and to account for non-rigidity of the nuclear core [16]. Proton-emitting states in deformed nuclei may be treated as admixtures of various different $K$ states $[14,15]$. As the decay width of the proton emitting state is sensitive to small components of the wave function, it becomes important to properly consider the Coriolis mixing in nuclei where the Coriolis interaction is strong [16]. This is especially important for nuclei where two Nilsson orbits with angular momenta differing by one unit lie very close in energy. These are known as pseudo-spin doublets and may be candidate proton-emitting states, if lying close to the Fermi energy, as is the case for the $K=1 / 2^{+}$and $K=3 / 2^{+}$orbits in ${ }^{113} \mathrm{Cs}[67]$.

Early attempts at including the Coriolis interaction lost the good agreement with experimental data seen in the adiabatic approach [68-70]. The fact that the residual pairing interaction was not included was postulated to be the cause for these discrepancies, as it also affects the mixing between states [16]. A nonadiabatic approach, treating nucleons as quasiparticles rather than particles was then required to properly account for both the pairing residual interaction and Coriolis interaction.

In a similar manner to the adiabatic approach for a nucleus with an even core and odd proton, the total Hamiltonian of the system can be separated into intrinsic and collective parts, $H=H_{i n}+H_{c o l}$. In this case $H_{i n}$, the intrinsic part, includes a deformed spin-orbit term as well as the residual interaction. The collective

Hamiltonian for a core with axial symmetry takes a similar form to that described in Section 2.6.2 of,

$$
\begin{equation*}
H_{c o l}=\frac{\hbar^{2}}{2 \mathscr{I}} \vec{R}^{2}=\frac{\hbar^{2}}{2 \mathscr{I}}\left[I^{2}+j^{2}-2 I_{3} j_{3}-\left(I_{+} j_{-}+I_{-} j_{+}\right)\right] \tag{2.57}
\end{equation*}
$$

where $R$ is the angular momentum of the core, $\mathscr{I}$ is the moment of inertia of the core, $I$ is the total angular momentum of the nucleus and $j$ the single-particle angular momentum. Using the intrinsic frame, the wave function of the parent nucleus, can be expressed in terms of a superposition of the Nilsson single-particle and core wave functions using a basis of states, $\mid I K M>$ as,

$$
\begin{align*}
& \Psi_{I M}=\left(\frac{2 I+1}{16 \pi^{2}}\right)^{1 / 2} \sum_{K>0} a_{K}^{I}\left[\mathscr{D}_{M K}^{I} Y_{K}+(-1)^{I-K} \mathscr{D}_{M-K}^{I} Y_{K}^{-}\right]  \tag{2.58}\\
& =\sum_{K>0} a_{K}^{I}\langle\vec{r} \mid I K M\rangle,
\end{align*}
$$

where $\mathscr{D}_{M K}^{I}$ are the rotational matrices and $a_{K}^{I}$ are the coefficients governing the superposition of the single-particle and core wave functions. The single-particle Nilsson functions $Y_{K}$ are given by

$$
\begin{equation*}
Y_{K}=\sum_{l j} \frac{\phi_{l j}^{K}(r)}{r}\left[\mathbf{Y}_{\mathbf{1}} \times \chi_{\mathbf{s}}\right]_{\mathbf{j}}^{\mathbf{K}}, \tag{2.59}
\end{equation*}
$$

where $Y$ and $\chi_{S}$ are the angular momentum and spin tensors, respectively and $\phi_{l j}^{K}(r)$ is the radial wave function of the proton moving with respect to the core. The Coriolis mixing between states differing by one unit of $K$ can be taken into account from a diagonalisation in the same $\mid I K M>$ basis, giving,

$$
\begin{align*}
& \langle I K+1 M| H_{c r}|I K M\rangle= \\
& \frac{\hbar^{2}}{2 \mathscr{I}} \sum_{l j}[(I-K)(I+K+1)(j-K)(j+K+1)]^{1 / 2} \times \int d r \phi_{l j}^{K+1^{*}}(r) \phi_{l j}^{K}(r) . \tag{2.60}
\end{align*}
$$

The effect of the residual pairing interaction can be introduced by using the Bogoliubov transformation [71] to change from single-particle Nilsson energies, $\epsilon_{K}$, to quasiparticle energies $\tilde{\epsilon}_{K}=\left[\Delta^{2}+\left(\epsilon_{K}-\lambda\right)^{2}\right]^{1 / 2}$ where $\lambda$ is the Fermi energy and $\Delta$ is the pairing gap. After this transformation the Coriolis contribution has to be multiplied by a factor of

$$
\begin{equation*}
f_{u v}=\left(u_{K+1} u_{K}+v_{K+1} v_{K}\right) \tag{2.61}
\end{equation*}
$$

where $u^{2}$ and $v^{2}$ are emptiness and fullness factors describing the probability of a single-particle state being occupied or unoccupied, respectively. The decay width of a proton-emitting state can then be calculated from the probability that the wave function of the form shown in equation 2.58 will decay to the exit channel wave function given by the tensor product of the internal wave function of the daughter nucleus $\Phi_{R M_{R}}$ and the function describing the protons motion relative to the daughter, $\phi_{l j R}^{I}$. The expression for the decay width of a proton escaping with angular momentum $l_{p} j_{p}$ is then,

$$
\begin{align*}
& \Gamma_{l_{p} J_{p}}^{R}(r)=\frac{\hbar^{2} k}{\mu} \frac{2(2 R+1)}{2 I+1} \times \\
& \left|\sum_{K>0} u_{K}^{f}\left\langle j_{p} K R 0 \mid I K\right\rangle \times a_{K}^{I} \frac{\phi_{l_{p} j_{p}}^{K}(r)}{G_{l_{p}}(k r)+i F_{l_{p}}(k r)}\right|^{2} \tag{2.62}
\end{align*}
$$

giving a more generalised form of the adiabatic formula shown in equation 2.51. Here $\left|u_{k}^{f}\right|^{2}$ gives the probability of single-particle proton level being unoccupied in the daughter nucleus.

By using experimental excitation energies of states in the daughter nucleus, its internal wave function can be better defined and the proton decay width can be more accurately evaluated. Using the level scheme of the daughter also accounts for any structures in the daughter nucleus that are not caused by rigid rotation.

The nonadiabatic quasiparticle model removes the disagreement with the strongcoupling limit introduced in the earlier nonadiabatic approaches. For example, the proton decay of the nucleus ${ }^{141} \mathrm{Ho}$, with an experimental lifetime of $T_{1 / 2}=$ $4.1(1) \mathrm{ms}$ [72], was predicted to have a lifetime of $T_{1 / 2}=12.5 \mathrm{~ms}$ in the method of

Ref. [69], which did not consider the effect of the pairing interaction. By contrast, a more accurate lifetime of $T_{1 / 2}=2.6 \mathrm{~ms}$ was predicted by the adiabatic model and the same value was replicated by the nonadiabatic model. The results of this model, when used to calculate energies of excited states and the half-life of proton emission from ${ }^{113} \mathrm{Cs}$, are discussed later in this work.

The correct deformation of the proton-emitting nucleus can be found by changing the level of deformation present in the potential and finding the intersection of the theoretical and experimental half-lives. Similarly, by using the wave functions extracted from the nonadiabatic model to calculate $\gamma$-ray transition rates between excited states of the proton-emitting nucleus, using the approach described in Subsection 2.5.3 [66], the deformation of the nucleus may again be found. Therefore a consistent approach using the same wave functions from the nonadiabatic quasiparticle model can be undertaken to provide the deformation of the nucleus from comparison with both experimental proton-emission and $\gamma$-ray transition rates, in order to test the predictive power of the model.

## Chapter 3

## Experimental Equipment and

## Techniques

This chapter provides an overview of the experimental setup used to obtain the data presented in this work. Data were collected at the Accelerator Laboratory of the University of Jyväskylä using the JUROGAM-RITU-GREAT nuclear spectroscopy apparatus, shown in Fig. 3.1. This setup was originally designed for the identification and analysis of heavy nuclei far from stability [18-20], but has also been widely and successfully used for measurements of nuclei in the mass A~110 region $[12,39,73,74]$. This chapter describes the constituent arrays of the experimental setup as well as the processing of detector signals for analysis.

### 3.1 The K-130 Cyclotron

The K-130 Cyclotron was installed at the University of Jyväskylä in 1993 and provides heavy-ion beams to the JUROGAM-RITU-GREAT experimental setup [76]. Cyclotrons, such as the K-130, make use of the fact that a charged particle travelling in a circular orbit in a uniform magnetic field has a constant orbital period. The radius of the orbit increases as the particle is accelerated, while the


Figure 3.1: Technical drawing of the JUROGAM-RITU-GREAT spectroscopy setup used at the University of Jyväskylä during the experiment described in the present work [75]. The beam from the K-130 Cyclotron impinges on a target surrounded by the JUROGAM-II HPGe array. Recoiling nuclei from the target position will then be transported through RITU, to separate reactants from scattered beam products, before implanting in the GREAT focal plane array.
frequency of the orbit remains the same. As such, a sinusoidal oscillating voltage, with a frequency equal to the orbital frequency of the particles in the cyclotron, can be applied to the accelerating cavities, or dees [77], within the cyclotron. This has the effect of accelerating the charged particles twice within one orbit. The charged particles then take a spiral path of increasing radius as they are further accelerated by the oscillating voltage until they reach the edge of the cyclotron and are ejected as a beam.

Electron-Cyclotron Resonance Ion Sources (ECRIS) are used with the K-130 cyclotron to generate heavy-ion beams in the required charge state. This type of source consists of a plasma, which is contained in a magnetic bottle structure, through which injected ions pass. The electrons in the plasma are resonantly accelerated by a signal applied by a radio-frequency ( RF ) unit, with frequency $\omega_{R F}$, which is equal to the electron-cyclotron resonance frequency, $\omega_{E C R}$, of the form given in equation 3.1

$$
\begin{equation*}
\omega_{E C R}=\frac{q B}{m} \tag{3.1}
\end{equation*}
$$

where $q$ is the charge on the ions in the plasma, $B$ is the magnetic field experienced by these ions and $m$ is the mass of the ion [78]. The resonantly excited electrons then strip atomic electrons from the injected ions, which are extracted from the ion source and are accelerated by the dees [79]. The constant application of the resonant frequency to the electrons in the plasma enables electrons to undergo multiple collisions with the transiting injected ions and also reduces the chance of recombination or electron capture.

The maximum energy per nucleon of accelerated ions is determined by both the bending limit and focussing limit of the cyclotron [78]. The bending limit, $K_{b}$, depends on the magnetic rigidity of the accelerated ions such that,

$$
\begin{equation*}
\frac{T}{A}=\frac{e^{2} \gamma^{2} B_{0}^{2} R_{0}^{2}}{(\gamma+1) m_{u}}\left(\frac{Z}{A}\right)^{2}=K_{b}\left(\frac{Z}{A}\right)^{2} \tag{3.2}
\end{equation*}
$$

where $\gamma$ is the Lorentz factor, $e$ is the electronic charge, $m_{u}$ is the atomic mass unit and $B_{0} R_{0}$ is the magnetic rigidity of a particle with charge number $Z$ and mass number $A$ [80]. The focussing limit, $K_{f}$, depends on the saturation density of the vertical focussing magnets. This limiting factor is given as

$$
\begin{equation*}
\frac{T}{A}=K_{f}\left(\frac{Z}{A}\right)^{2} \tag{3.3}
\end{equation*}
$$

The K-130 cyclotron has a bending limit of 130 MeV and a focussing limit of 90 $\mathrm{keV} / \mathrm{u}$ allowing the production of heavy ion beams with energies of $5-10 \mathrm{MeV} / \mathrm{u}$. These beam energies are high enough to allow fusion-evaporation reactions to take place at the target position, such as that used to gather the experimental data described in this work.

### 3.2 Fusion-Evaporation Reactions

Fusion-evaporation reactions utilise a high energy ion beam which is accelerated onto a target foil. This process will form a compound nucleus if the incident ion has enough kinetic energy to overcome the Coulomb repulsion between the two nuclei. Fusion evaporation dominates at energies that are just enough to overcome the Coulomb barrier, while spallation will dominate at higher energies. The incident energy of the beam will be shared between the nucleons of the compound system. While the average energy is not great enough to result in a nucleon being emitted from the compound system, due to the statistical distribution of the energies among the different nucleons there is a small probability of an individual nucleon being "evaporated". The remaining nuclear core will then be left in a highly excited state, in which it rotates rapidly. Thermal equilibrium of the nucleus is achieved as a result of these nucleons being emitted or high energy $\gamma$-ray decays, with the process taking a time of the order of $\sim 10^{-22} \mathrm{~s}$ [81]. The excited nucleus will then undergo statistical $\gamma$-ray decay before de-exciting through the yrast levels to its ground state. This process is shown in Fig. 3.2. From conservation of energy, the excitation energy of the compound nucleus, $E_{e x}$, is given by the expression:

$$
\begin{equation*}
E_{e x}=E_{c m}+Q, \tag{3.4}
\end{equation*}
$$

where $Q$ is the energy required for the compound nucleus to form and $E_{c m}$ is the kinetic energy passed to the compound nucleus from the reaction. The value of $E_{c m}$ depends on the centre of mass of the reaction between the beam and target nuclei and can be calculated from the difference between the energy of the beam, $E_{B}$ and energy of the recoiling compound nuclei, $E_{R}$, such that,

$$
\begin{equation*}
E_{c m}=E_{B}-E_{R} . \tag{3.5}
\end{equation*}
$$

The kinetic energy transferred to the compound nucleus can then be more simply calculated from the mass of the beam and target and the energy of the beam, as

$$
\begin{equation*}
E_{c m}=E_{B}-\frac{1}{2}\left(M_{T}+M_{B}\right) v_{r}^{2}, \tag{3.6}
\end{equation*}
$$



Figure 3.2: Schematic, adapted from Ref. [82] of the different stages of the formation and de-excitation of a compound nucleus through a fusion evaporation reaction.
where $M_{B}$ and $M_{T}$ are the masses of the beam and target nuclei, respectively. As, the kinetic energy of the beam, $E_{B}=\frac{1}{2} M_{B} v_{B}^{2}$ the velocity term can be removed from equation 3.6 which can then be rearranged to

$$
\begin{equation*}
E_{c m}=E_{B}\left(1-\frac{M_{B}}{M_{T}+M_{B}}\right) \tag{3.7}
\end{equation*}
$$

The angular momentum of the compound nucleus again depends on the centre of mass of the beam and target nuclei involved in the fusion-evaporation reaction. The maximum angular momentum transfer to the compound nucleus occurs when the beam and target nucleus just touch during the reaction, as if the beam and target nuclei were hard spheres. This is known as the sharp-cutoff approximation. Using this classical approximation the maximum angular momentum transfer, $l_{\text {max }}$, can be described as

$$
\begin{equation*}
\hbar l_{\max }=\mu v R, \tag{3.8}
\end{equation*}
$$

where $\mu$ is the reduced mass of the compound system given by

$$
\begin{equation*}
\mu=\frac{A_{T} A_{B}}{A_{T}+A_{B}} \tag{3.9}
\end{equation*}
$$

and $A_{T}$ and $A_{B}$ are the mass numbers of the target and beam nuclei, respectively. The value $v$ is the velocity of the compound nucleus, which can be related to the kinetic energy of the compound nucleus through the difference between the energy transferred to the nucleus in the reaction and the Coulomb barrier of the target nucleus, $V_{c}$, by

$$
\begin{equation*}
\frac{1}{2} \mu v^{2}=E_{c m}-V_{c} . \tag{3.10}
\end{equation*}
$$

$R$ is the sharp cut-off radius and is given empirically by

$$
\begin{equation*}
R=1.36\left(A_{T}^{\frac{1}{3}}+A_{B}^{\frac{1}{3}}\right)+0.5 . \tag{3.11}
\end{equation*}
$$

By substituting the value of $v$ in equation 3.8 using equation 3.10, an expression for $l_{\text {max }}$ can be obtained:

$$
\begin{equation*}
l_{\max }^{2}=\frac{2 \mu R^{2}}{\hbar^{2}}\left(E_{c m}-V_{c}\right) \tag{3.12}
\end{equation*}
$$

Equation 3.12 shows how the maximum angular momentum transfer for a certain centre of mass energy depends strongly on the reduced mass of the compound system. From equation 3.9 this further shows that reactions with symmetric beam and target nuclei masses will result in higher angular momentum transfer [81].

As the recoiling compound nuclei will promptly de-excite by means of $\gamma$-ray emission, a means of detecting $\gamma$ rays at the target position of the University of Jyväskylä setup is needed to measure these decays. This is provided by the JUROGAM-II array.

### 3.3 JUROGAM-II

JUROGAM-II is a high-purity germanium detector array arranged in a $4 \pi$ configuration [18] with a total detection efficiency of $\sim 6 \%$ at 1.3 MeV [12]. The array is located at the target position of the JUROGAM-RITU-GREAT setup at the accelerator laboratory of the University of Jyväskylä, as shown in Fig. 3.3. In the experiment described in this work JUROGAM-II is used to detect $\gamma$ rays emitted by recoiling nuclei at the target position before they enter into the gas-filled separator RITU. The 39 constituent detectors of the array are arranged into four rings, which have discrete angles to the beam axis of $157.6^{\circ}, 133.6^{\circ}, 104.5^{\circ}$ and $75.5^{\circ}$. There are two separate types of germanium detector: 15 Euroball phase 1 detectors [83] and 24 segmented clover germanium detectors [84]. All detectors are surrounded by bismuth-germanate (BGO) Compton-suppression shields to help reduce background from Compton-scattered events. If a signal is detected in the BGO shield then the coincident scattered $\gamma$ ray detected in the germanium crystal can be vetoed.

The phase- 1 detectors are large, coaxial n-type germanium detectors and are tapered, being narrower at the front than back. This ensures that the maximum


Figure 3.3: Photograph of the JUROGAM-II array at the target position of the JUROGAM-RITU-GREAT setup at the University of Jyväskylä. The down-beam end of RITU is shown behind the array [75].
proportion of the total $4 \pi$ solid angle of the array is covered by the phase- 1 detectors. The clover detectors consist of four individual, coaxial, germanium diodes contained within a shared cryostat, configured as shown in Fig. 3.4. The acceptance angle of the individual germanium crystals is smaller than that of the detector as a whole. This reduces the Doppler broadening of incident $\gamma$-ray signals, as discussed in subsection 4.4.5. The Doppler broadening effect is greatest at angles to the beam axis of $\theta=90^{\circ}$, so the clover detectors are used for Rings 3 and 4 , which are situated at angles of $\theta=75.5^{\circ}$ and $\theta=104.5^{\circ}$, respectively. The clover detector setup also allows a detector to have a large active volume by overcoming some of the difficulty and cost of creating large hyper-pure germanium crystals. By using the signals from each crystal, software processing can reduce the Compton background in the $\gamma$-ray spectra produced by each detector. If a $\gamma$ ray is scattered out of a crystal into an adjacent crystal in the same detector, the total energy of the original incident $\gamma$-ray can be reconstructed from the energy sum of coincident events detected separately in the detector crystals. This process is known as add-back and is described in more detail in Subsection 4.4.3.


Figure 3.4: The configuration of individual Ge crystals in the JUROGAM-II clover detectors [84].

### 3.4 RITU

The Recoil Ion Transportation Unit (RITU) is a gas-filled recoil separator that transports nuclei of interest from the target position to focal plane of the JUROGAM-RITU-GREAT setup, while removing scattered beam products [19]. The path that ions take through RITU is determined by the equation

$$
\begin{equation*}
B \rho=\frac{m v}{e q_{a v}}, \tag{3.13}
\end{equation*}
$$

where, $B$ is the strength of the magnetic field in the separator, $\rho$ is the radius of curvature of the ions path, $m$ and $v$ are the mass and velocity of the ion respectively, $q_{a v}$ is the average charge state of the ions and $e$ is the charge of an electron [19].

RITU is filled with helium gas, with which recoiling nuclei undergo charge exchange reactions. An average charge state of the nuclei will result from the balance of probabilities between the recoils losing and gaining electrons in collisions with the gas molecules [85]. The fact that the majority of recoils occupy the average charge state increases the transmission efficiency of the separator compared to vacuum mode separators, in which certain charge states are separated before the focal plane. Although the mass separation of RITU as a gas-filled separator is far inferior to the vacuum mode resolution of $A / \Delta A \sim 100$ [19], the resolution is sufficient to separate the majority of scattered and primary beam nuclei from the fusion-evaporation recoils.

The value of $q_{a v}$ is dependent on both the pressure of helium inside RITU and the velocity of the ions moving through the separator. The effect of different velocities and pressures on the average charge state of ions moving through helium are well known from previous experiments [86]. Using this data the optimum pressure and magnetic fields can be chosen to ensure the best transmission of fusionevaporation recoils and separation of primary and scattered beam products.

The path of ions through RITU is governed by the magnetic fields produced by three quadrupole $(\mathrm{Q})$ and one dipole (D) magnets, which are arranged in a QDQQ configuration, as shown in Fig. 3.5. The quadrupole magnets ensure that the beam remains focussed while passing through RITU. The dipole magnet bends the path of the ions, with the new direction of the ions dependent on equation 3.13. The scattered beam nuclei, having lower mass, will be deflected more by the magnetic field, giving them a distinct path to that of the fusion-evaporation recoils. This allows much of the scattered beam to be directed away from the focal plane while allowing the transport of the desired nuclei. A movable beam stopper can also be inserted into RITU after the dipole magnet to further impede the scattered beam nuclei, as not all of the beam will be deflected away from the focal plane by the dipole magnet alone. The distance from the target position to the focal plane is 5.1 m and RITU can be operated with a maximum beam rigidity of 2.2 Tm [19].

The transmission efficiency through RITU during the commissioning experiment was $\sim 25 \%$ for reaction products from the ${ }^{40} \mathrm{Ar}+{ }^{175} \mathrm{Lu} \rightarrow{ }^{210,211} \mathrm{Ac}+x \mathrm{n}$ reaction [19]. As the tuning of the beam and magnet fields was not optimised at the commissioning stage, this efficiency should be treated as a lower limit. The optimum pressure of the helium gas in the separator in the commissioning experiment was found to be 1.5 mbar, while the pressure used in the experiment described in Chapter 5 was 1.2 mbar.

### 3.5 GREAT

The GREAT spectrometer is positioned at the focal plane of RITU to detect recoiling reaction products that have passed through the separator and their subsequent decays. The arrangement of the various detectors that are subsequently


Figure 3.5: Schematic birds-eye view of the gas-filled recoil separator RITU adapted from Ref. [19]. Recoiling nuclei will be transported from the target chamber at the centre of JUROGAM-II to the detector chamber of GREAT. The quadrupole magnets $Q_{1}, Q_{2}$ and $Q_{3}$ keep the path of the recoiling nuclei focussed, while the dipole magnet $D$ separates recoiling target nuclei from beam products, see text for details.
described is shown in Fig. 3.6. Nuclei exiting RITU initially pass through a multi-wire proportional counter (MWPC). The MWPC is filled with isobutane, which is ionised by incident recoils, and is separated from the low pressure environment of RITU by an aluminised mylar window [20]. The MWPC detects the energy loss of recoils and can be used in tandem with the silicon detectors, described later, to provide information on the time of flight of recoils between RITU and the focal plane [87]. These properties can be used to differentiate recoil events from scattered beam and decay events at the focal plane.

After passing through the MWPC, a nucleus will implant in one of a pair of double-sided silicon strip detectors (DSSDs). The DSSDs provide information on the energy of the implanted recoil as well as its implantation position in the detector. The DSSDs have dimensions of $60 \times 40 \mathrm{~mm}$ and are placed side by side giving a total active area of $4800 \mathrm{~mm}^{2}$. Each detector is split into 60 horizontal strips of width 1 mm on the back side and 40 vertical strips on the front side. This gives each detector 2400 separate pixels to provide positional information. The DSSDs will also provide energy, position and timing information for $\alpha$-, $\beta$ or proton decays of the implanted recoils.


Figure 3.6: Schematic diagram of the GREAT detector setup with views from (a) the down-beam face and (b) the side.

Surrounding the DSSDs is a box of silicon pin diodes. Recoils implanting in the DSSDs will typically implant at depths between $\sim 1-10 \mu \mathrm{~m}[20]$. At these depths there is a significant probability that internal conversion electrons will be able to escape backwards (relative to the recoil direction) from the DSSDs. The pin diodes are placed in a box configuration backwards to the DSSDs as shown in Fig. 3.6. There are 28 separate pins, providing a total geometric efficiency of $\sim 30 \%$.

A planar germanium detector is placed 10 mm behind the DSSDs in the same vacuum chamber. This detector is primarily used for the detection of low-energy $\gamma$ rays and x-rays, as it has an absolute efficiency of $\sim 30 \%$ at $\sim 100 \mathrm{keV}[20]$.


Figure 3.7: The simulated absolute efficiencies of the largest GREAT clover detector and the GREAT planar germanium detector [20].

The detector has an active area of $120 \times 60 \mathrm{~mm}$ detector, a depth of 15 mm , and is segmented into 24 horizontal strips on the front side and 12 vertical strips with widths of 5 mm each. This segmentation allows information on the position of $\gamma$-ray decays to be extracted.

Three clover detectors are placed around the DSSDs at angles of 90 degrees to both the beam axis and each other, as shown in Fig. 3.6. These allow the detection of higher energy $\gamma$ rays with greater efficiency than the planar detector. The clover detector placed vertically above the focal plane has a larger area than the two detectors placed at the sides of GREAT and, as such, has a higher efficiency of $\sim 5 \%$. The simulated absolute efficiencies of the planar detector and the large clover detector are shown in Fig. 3.7, showing the relative detector performances at different energies.

### 3.6 Total Data Readout

Recoil-decay tagging (RDT) is a powerful tool for observing specific nuclear decays by correlating detector events resulting from recoiling nuclei and their associated decay events [88]. In conventional RDT experiments, data is only recorded
when specific trigger conditions are met [89]. The trigger in RDT experiments is typically the implantation of a recoil at the focal plane. Traditionally, in order to detect subsequent decays of the implanted nucleus, a fixed time period (or "time gate") is started by the trigger, in which the decay events are subsequently detected. This time gate will typically be based on the half-life of the desired decays. This conventional setup suffers from the fact that only one event can be recorded at a time, as once the time gate is open any subsequent recoils arriving at the focal plane will not be detected. This large "dead-time" then results in many events being discarded [88].

The Total Data Readout system overcomes these issues by recording and timestamping all detector signals with the use of a 100 MHz global clock. Events are subsequently constructed in the software [90]. This both reduces the dead-time of the setup and allows subsequent optimisation and resorting of the data under different conditions.

Signals from the detectors are initially passed through Constant Fraction Discriminators and shaping amplifiers before being passed into the VXI-D Analogue-to-Digital (ADC) cards. These cards associate a timestamp with each detector output. The ADC conversion and readout time is less than that of the shaping time of the amplifiers. This means that the only losses of detector signals are from pile-up, which is caused by a second detector signal being generated before the first signal is processed and recorded. A module known as the metronome keeps all ADC signals synchronous. The time-stamped data is then passed to the event-builder, which time-orders all events and constructs the events in line with applied software gates [91]. A schematic of the TDR electronics is shown in Fig. 3.8.

### 3.7 Event Construction

The use of the TDR system allows a wide range of detector outputs to be grouped together to form the desired detection events. Raw data from all detectors is formed in to a single stream, which is initially processed by the event builder and time-ordered. A specific set of detector outputs will be used as a trigger. In the case of the work presented here the trigger corresponds to the detection of


Figure 3.8: The electronics in Jyväskylä that comprise the TDR system which converts detector signals into time-stamped events. The signals are passed through the shaping amplifiers and CFDs before being digitised by the ADC and passed to the Event Builder with their associated time-stamps.
an event in any of the DSSD strips. An event time-delay and width is defined in the software. The event width defines a time window, within which any detector signal will have to occur to be included in the event. This width is optimised such that the desired decay events at the focal plane are correlated with prompt decays at the target position, while ensuring that the random correlation rate is kept low to ensure a good event-to-background ratio. The event delay specifies how much earlier timestamped events may be than the trigger to still be included within the event. The delay is usually set so that it corresponds to the time difference between recoils leaving the target position and arriving at the focal plane. This reduces the inclusion of background JUROGAM-II events at the target position resulting from long-lived $\beta$-decays and other random events. Eliminating as much JUROGAM-II background as possible also significantly reduces the amount of data storage required. Figure 3.9 shows typical times at which events at the target position and focal plane happen relative to each other, as well as an example


Figure 3.9: Diagram taken from Ref. [33] showing the time differences between the different detector signals in the JUROGAM-RITU-GREAT setup. The trigger width and delay are displayed to show the typical time scales chosen for both of these quantities.
trigger width and delay.

### 3.8 Data Sorting

The grain software package was used to sort the TDR data collected during the experiment presented in this work [90]. GRAIN allows both online and offline sorting of data into histograms using time and distance correlations between the individual time and detector stamped TDR events. These correlations are defined in a user-created sortcode. A configuration file was used in conjunction with the sortcode to define the trigger conditions for the histogram event construction. The package allows the real-time display of the data being sorted. This allowed
the optimisation of tagging or coincidence conditions included in the sortcode (described in more detail in Chapter 5) to be changed if necessary based on the histogram output.

### 3.9 The Differential Plunger for Unbound States

The differential plunger for unbound nuclear states (DPUNS) was created to perform lifetime measurements on the excited states of exotic nuclei in tandem with recoil-decay tagging [21]. The plunger was primarily designed to measure excited state lifetimes in proton-emitting nuclei with the JUROGAM-RITU-GREAT apparatus [92]. When used with this setup the plunger is placed at the centre of the JUROGAM-II array. The plunger uses a degrader foil, slowing recoiling nuclei from the target but still allowing them to pass from the target position into RITU. Decays from the recoils can then be detected by GREAT and used to tag prompt $\gamma$ rays detected at the target position, as is described in Subsection 4.3.1.

The target and degrader foils are stretched over their respective frames and optically aligned. The process of optical alignment consists of shining a bright light through the gap between the target and degrader foils at different angles and reducing the distance between the two foils. If the foils are aligned then the light will be seen to disappear from the centre of the gap. If the light first disappears from the top, bottom, left or right of the gap then the screws attaching the foil frames to the plunger will have to be tightened or loosened. The minimum target-to-degrader distance is determined by the uniformity of surface and alignment of the two foils [93]. If the foil surfaces are sufficiently smooth and parallel then target-to-degrader distances as low as $\sim 2 \mu \mathrm{~m}$ may be used in DPUNS [21]. Conversely, the longest distances that can be used is determined by the maximum travelling range of the plunger, which is 30 mm . In order to ensure a constant distance between the target and degrader during the experiment, a low-voltage feed back pulse is applied to the degrader foil and the induced voltage on the target foil continually recorded. The voltage is calibrated using a TESA GT43 axial probe with a TT20 electronic micrometer. By measuring the change of distance between the the two foils using the micrometer and reading off the corresponding change in induced voltage using the axial probe, the voltage corresponding to each distance between foils can be recorded. The induced signal is passed via a


Figure 3.10: Technical drawing of DPUNS taken from Ref. [21] showing the locations of the constituent components within the housing of the device. The stepper motor is used to change the distance between the target and degrader foils, while the axial probe provides feedback to ensure that the target-todegrader distance remains constant throughout the experiment.
data acquisition card to a PC using the Köln Plunger control software, developed under the National Instruments Labview framework [94], which continuously adjusts the distance between the target and degrader foils, while an experiment is running, in accordance with the induced signal and the measured calibration. The movement of the foils is achieved through the use of a 45 V high-precision stepping motor. A technical diagram of DPUNS is shown in Fig. 3.10.

The low voltage of the stopper motor allows it to be used in the low-pressure gas environment of RITU. The helium in RITU has the benefit of countering some of the heating effects caused by the impinging of the beam on the target and degrader foils, which could alter the distance between the two foils. To ensure that the beam line is kept at vacuum, a differential pumping system (roots pump) is placed between DPUNS and the beam line at the target position. The roots pump removes approximately $1000 \mathrm{~m}^{3} / \mathrm{hr}$ of helium from its position between the beamline and JUROGAM-II [95]. This removes the need for carbon isolation foils between the beam pipe and RITU. For the ${ }^{98} \mathrm{Mo}\left({ }^{40} \mathrm{Ar}, 4 n\right)$ reaction at 165

MeV with a $0.1 \mu \mathrm{~g} / \mathrm{cm}^{2}$ carbon isolation foil, reactions with the carbon foil are seen to constitute $\sim 30 \%$ of the total reaction products. As such, operation with the roots pump is preferred to reduce contamination during the analysis [21].

## Chapter 4

## Analysis Techniques

The raw data collected by the JUROGAM-RITU-GREAT experimental setup had to be processed using a variety of techniques to ensure coherent and accurate detector outputs. These outputs were affected by corresponding detector efficiencies and individual electronic settings. The $\gamma$ rays emitted from recoiling nuclei were detected with an associated Doppler shift which also needed to be corrected. This chapter describes the techniques used to calibrate the raw detector outputs as well as the tagging techniques utilised to produce the clean spectra used for the lifetime analysis of ${ }^{113} \mathrm{Cs}$. The premise of the Recoil Distance Doppler Shift (RDDS) method of measuring promptly decaying nuclear excited state lifetimes is first discussed as well as the related Differential Decay Curve Method (DDCM) which is used to accurately extract the lifetime information.

### 4.1 The Recoil Distance Doppler Shift Method

The extraction of excited nuclear state lifetimes can provide useful information on many aspects of the nucleus [96], as covered in subsections 2.3.2 and 2.5.3. Nuclear lifetimes between the picosecond and nanosecond range can be measured through the use of differential plunger devices utilising the RDDS method [93]. A schematic of a simple plunger device is shown in Fig. 4.1.


Figure 4.1: A schematic diagram of a plunger device taken from Ref. [93]. A beam is seen to strike a target on the left side of the figure, from which recoiling fusion-evaporation reaction nuclei leave the target with velocity, $v$. The fusion-evaporation recoils de-excite via the emission of $\gamma$ rays, which are detected with an energy that is Doppler-shifted according to the value of $v$ and $\theta$, the angle of the $\gamma$-ray detector with respect to the beam axis. The recoils implant in a stopper foil at a distance, $x$, from the target foil. The $\gamma$ rays emitted from the excited nuclei in the stopper are detected without a Dopplershift to their energy. For a certain $\gamma$-ray decay from an excited state, the variation with $x$ of the ratio of the intensities of the fully shifted and stopped photopeaks can be used to infer the lifetime of the state.

A beam impinges on a target foil, from which recoiling nuclei are emitted. Typically fusion-evaporation reactions are used for plunger experiments, although deep inelastic scattering or Coulomb-excitation reactions can also be used [93, 97, 98]. The excited recoiling nuclei de-excite through $\gamma$-ray emission. These $\gamma$-rays, denoted by $\gamma^{\prime}$ in Fig. 4.1, are detected with a Doppler shift, the magnitude of which is dependent on the velocity of the de-exciting recoil and the angle of the detector to the axis along which the recoil is travelling. The recoils are then conventionally stopped in a second foil placed close to the target foil. The $\gamma$ rays emitted from nuclei which have implanted and stopped in the "stopper" foil, denoted by $\gamma_{0}$ in Fig. 4.1, are detected without a Doppler shift. For a $\gamma$-ray transition depopulating a state with a lifetime in the $10^{-12}-10^{-9} \mathrm{~s}$ range, there will be target-to-stopper distances at which there exists fully shifted and stopped photopeaks with different Doppler-shifted energies and intensities. These peaks


Figure 4.2: Schematic adapted from Ref. [33] showing the change of intensity of the stopped (blue) and fully shifted (green) photopeaks of a $\gamma$-ray transition as the target-to-stopper distance of a plunger is increased. The stopped peak is shown at a higher energy than the fully shifted peak as the $\gamma$ rays are considered to be detected at a backwards angle to the beam axis.
correspond, respectively, to the detection of $\gamma$ rays emitted by recoils before and after implantation in the stopper foil. Increasing the target-to-stopper distance will result in a longer time-of-flight between the foils, so more decays of the state of interest will occur while the recoils are still in flight. Conversely, decreasing the target-to-stopper distance will result in more decays from recoils that have reached the stopper foil. Therefore, the intensities of the two photo-peaks can be varied by changing the distance between the target and stopper foils, as is shown by the schematic in Fig. 4.2.

Instead of a stopper foil, a degrader foil may sometimes be used, as was the case with the plunger device used to collect the data detailed in this work. A degrader foil slows, rather than stops nuclei, recoiling from the target foil, but still results in two distinct recoil velocity regimes and, hence, two different Doppler-shifted $\gamma$-ray energies per transition.


Figure 4.3: Schematic example of a level scheme. If the lifetime of the state $i$ is to be extracted, then the lifetimes of the states $k$ have to be considered.

For an idealised case, the variation in intensity of the shifted and degraded photopeaks of a transition takes the form of a simple exponential decay

$$
\begin{equation*}
I_{S}=\left(I_{S}+I_{D}\right)\left[1-\exp \left(\frac{-x}{v \tau}\right)\right] \tag{4.1}
\end{equation*}
$$

and

$$
\begin{equation*}
I_{D}=\left(I_{S}+I_{D}\right) \exp \left(\frac{-x}{v \tau}\right), \tag{4.2}
\end{equation*}
$$

where $I_{S}$ is the intensity of the shifted photopeak, $I_{D}$ is the intensity of the stopped or degraded photopeak, $x$ is the distance between the two foils of the plunger, $v$ is the velocity of recoils leaving the target foil and $\tau$ is the lifetime of the state being depopulated. By fitting the corresponding function to the variation in the shifted or degraded intensity as a function of distance, the lifetime, $\tau$, of the excited state being depopulated by the detected $\gamma$-ray transition can be extracted.

### 4.1.1 Treatment of Feeding

Although the idealised case for extracting the lifetime of an excited state, as given in equations 4.1 and 4.2 can sometimes be used experimentally, the feeding of the state of interest, $i$, frequently has to be considered. The states, $k$, that feed $i$, as shown in the schematic in Fig. 4.3, often have non-zero lifetimes relative to the
lifetime of $i$. For this more general case the lifetime of $i$ is extracted by solving the set of differential equations which also govern the decay of all of the feeding states, $k$. These are known as the Bateman equations and are given by

$$
\begin{equation*}
\frac{d}{d t} n(t)=-\lambda_{i} \cdot n_{i}(t)+\sum_{k=i+1}^{N} \lambda_{k} \cdot n_{k}(t) \cdot b_{k i} \tag{4.3}
\end{equation*}
$$

where, $n_{i}(t)$ and $n_{k}(t)$ are the number of nuclei in states $i$ and $k$ at time $t, \lambda_{k}$ and $\lambda_{i}$ are the decay constants of levels $k$ and $i$, respectively and $b_{k i}$ are the branching ratios of the levels $k$. The branching ratios are weighting factors used to reflect the greater intensity of some of the transitions that populate level $i$ compared to others. The values of the branching ratios are normalised to fulfill the condition that $\sum_{k} b_{k i}=1$. Substituting in experimental intensities, the solution of the differential equation is then

$$
\begin{equation*}
I_{D}=\left(I_{S}+I_{D}\right) e^{-t \lambda}+\sum_{k=i+1}^{N} M_{k i}\left[\left(\lambda_{i} / \lambda_{k}\right) e^{-t \lambda_{k}}-e^{-t \lambda_{i}}\right] \tag{4.4}
\end{equation*}
$$

where

$$
\begin{equation*}
M_{k i}\left(\lambda_{i} / \lambda_{k}-1\right)=b_{k i} I_{T K}-b_{k i} \sum_{m=k+1}^{N} M_{m k}+\sum_{m=i+1}^{k-1} M_{k m} b_{m i}\left(\lambda_{m} / \lambda_{k}\right), \tag{4.5}
\end{equation*}
$$

the subscript $m$ denotes the levels, which feed the states $k, M_{m k}$ and $M_{k i}$ are the initial population of the states $m$ and $k$, respectively and $I_{T K}$ is the total feeding intensity of the levels $k$.

For structures such as rigid rotational bands the lifetimes of states in the band often reduce quickly at higher angular momenta and energies. For a case where the lifetimes are sufficiently short in the higher energy levels, the feeding can be simply considered as a single populating exponential of the form,

$$
\begin{equation*}
I_{D}=\left(I_{S}+I_{D}\right) \exp \left(\frac{-t}{\tau_{i}}\right)-\left(I_{S k}+I_{D k}\right) \exp \left(\frac{-t}{\tau_{k}}\right) \tag{4.6}
\end{equation*}
$$

where $I_{S k}$ and $I_{D k}$ are the intensities of the shifted and degraded components and $\tau_{k}$ is the lifetime of the transition which depopulates the state above $i$. By separately fitting the intensity of the component peaks of the transitions depopulating levels $i$ and $k$ as a function of distance, the lifetime of $i$ can be calculated.

### 4.2 The Differential Decay Curve Method

The differential decay curve method (DDCM) is an approach of extracting plunger lifetime measurements which has many advantages over the solving of equation 4.4 shown in the previous section [99]. These include the facts that:

- Only experimentally accessible values of $I_{S}, I_{D}, v$ and $d$ are used.
- No assumption is made on the shape of the decay curve, $R(t)$.
- Systematic errors in the consideration of the feeding behaviour of state $i$ can be more easily identified as detailed later in the chapter.

The derivation of the DDCM, for the case of the feeding levels being well known, begins with the simple differential equation detailing the population and depopulation of the state of interest shown in equation 4.3. By integrating this equation, the following is obtained

$$
\begin{equation*}
-n_{i}(t)=-N_{i}(t)+\sum_{k} b_{k i} N_{k}(t) \tag{4.7}
\end{equation*}
$$

from the identities

$$
\begin{equation*}
N_{j}(t)=\int_{t}^{\infty} \lambda_{j} \cdot n_{j}\left(t^{\prime}\right) d t^{\prime} \tag{4.8}
\end{equation*}
$$

and

$$
\begin{equation*}
-n_{i}(t)=\int_{t}^{\infty} \frac{d}{d t^{\prime}} n_{i}\left(t^{\prime}\right) d t^{\prime} \tag{4.9}
\end{equation*}
$$

The index $j=i$ or $k$ depending on whether the state depopulates or populates the state of interest, respectively. The differential of $N(i)$ can be substituted for $n_{i}(t)$ in equation 4.7 to obtain a value of $\lambda$ for the state of interest using the following identities

$$
\begin{equation*}
\frac{d}{d t} N_{i}(t)=\lambda_{i}\left[n_{i}(\infty)-n_{i}(t)\right] \tag{4.10}
\end{equation*}
$$

and

$$
\begin{equation*}
n_{i}(\infty)=0, \tag{4.11}
\end{equation*}
$$

as $-n_{i}(t)$ is then equivalent to $\frac{d}{d t} N_{i}(t) / \lambda_{i}$. The term $N_{i}$ can be related to the experimentally measurable decay curve, $R_{i}$, through the introduction of $\alpha$ coefficients such that $R_{i}=N_{i} / \alpha_{k i}$, where $\alpha_{k i}$ is defined as,

$$
\begin{equation*}
\alpha_{k i}=\frac{\omega_{k}(\Theta) \cdot \epsilon\left(E_{\gamma k}\right)}{\omega_{i}(\Theta) \cdot \epsilon\left(E_{\gamma i}\right)} \tag{4.12}
\end{equation*}
$$

and $\omega_{j}(\Theta)$ and $\epsilon\left(E_{\gamma j}\right)$ give the angular distribution and detection efficiency of transitions $k$ and $i$, respectively. This gives the final form of the DDCM lifetime formulae,

$$
\begin{equation*}
\tau_{i}(t)=1 / \lambda_{i}=\frac{-R_{i}(t)+\sum_{k} \alpha_{k i} b_{k i} R_{k}(t)}{\frac{d}{d t} R_{i}(t)}, \tag{4.13}
\end{equation*}
$$

or in terms of target-to-degrader distance

$$
\begin{equation*}
\tau_{i}(x)=\frac{-R_{i}(x)+\sum_{k} \alpha_{k i} b_{k i} R_{k}(x)}{\frac{d}{d x} R_{i}(x)} \cdot \frac{1}{\langle v\rangle} \tag{4.14}
\end{equation*}
$$

where $\langle v\rangle$ is the velocity of the fully shifted recoils.
All values in the numerator of equation 4.14 are experimentally accessible. However, the differential of the decay curve $\frac{d}{d t} R_{i}(x)$ cannot be directly measured using a two foil plunger. The standard procedure is then to fit an analytical function


Figure 4.4: A schematic of the quantities $\delta R_{i}(x)=-R_{i}(x)+\sum_{k} \alpha_{k i} b_{k i} R_{k}(x)$ and $d / d t R_{i}(x)$ from equation 4.14, based on Ref [99]. The red dashed lines mark the region of sensitivity (R.O.S) where the percentage error on $\delta R_{i}(x)$ and $d / d t R_{i}(x)$ is smallest.
to the experimental values $R_{i}(x)$ found at several target-to-stopper distances and to take the derivative of the fitted function as the denominator. The same relationship in equation 4.14 holds whether intensities of fully shifted or degraded peaks are used to construct the decay curve.

The lifetime of state $i$ can be calculated at any distance where $R_{i}$ and $R_{k}$ are calculated from the values $\delta R_{i}(x)=-R_{i}(x)+\sum_{k} \alpha_{k i} b_{k i} R_{k}(x)$ and $d / d t R_{i}(x)$. The error on $\tau(i)$ will then result from the percentage error on $\delta R_{i}(x)$ and $d / d t R_{i}(x)$. As the absolute error on the experimental measurement is approximately constant across all distances, values of $\tau$ measured in the range of distances over which there is both a maximum value of $R_{i}(x)$ and $d / d t R_{i}(x)$ will have smaller errors. This range of distances is known as the region of sensitivity, and is shown in the schematic diagram in Fig. 4.4


Figure 4.5: The mean lifetime $\tau(i)$ of state $i$ calculated using the DDCM. The lines denoted by (i) shows a case where feeding from a state is not included among actual feeding levels while case (iii) shows the incorrect inclusion of a feeding state with a longer lifetime than the average feeding timing of state $i$. Case (ii) shows the ideal case when $\tau(i)$ is constant and all feeding is correctly considered. Panels (a) and (b) show the cases where the feeding lifetime is considered, respectively, to be longer and shorter than the lifetime of the state i. Based on a Figure in Ref. [99].

The differential decay curve method also allows the easy identification of systematic errors in the assumed behaviour of the transitions populating $i$. Figure 4.5 shows different relationships between the lifetime of the state of interest $i$ and target-to-degrader distance. As $\tau_{i}$ should be independent of target-to-degrader distance the ideal case is that displayed for case (ii) in the figure. Curve (i) indicates that there is an additional feeding level which has not been properly considered in the analysis, so the value of $\delta R_{i}(x)$ is constantly higher than the correct value of $\tau$. Conversely, curve (iii) indicates that a feeding level with a lifetime greater than the actual mean lifetime of the feeding states has been wrongly included in the analysis.

The presence of unobserved side-feeding can be identified in the first instance
from a difference in intensity of the populating and depopulating transitions of the state of interest. The timing behaviour of any unobserved side-feeding can be estimated by fitting a function $F_{i j}^{c}$ to the variation of $\tau_{i}$ with distance where

$$
\begin{equation*}
F_{i j}^{c}=\frac{\tau_{i}-\left(b_{i j} \sum_{k} c_{k} \exp \left(-t / \tau_{k}^{f}\right)\right.}{\frac{d}{d t} R_{i j}} \tag{4.15}
\end{equation*}
$$

where $c_{k}$ and $\tau_{k}^{f}$ are the intensities and lifetimes of the unobserved side-feeding, which can be extracted using this fit.

Alternatively, if there is no sensitivity of $\tau(i)$ to target-to-degrader distance, $x$, or flight time, $\tau$, it is commonly assumed that the timing behaviour of any sidefeeding is the same as the average timing behaviour of the population or depopulation of the state of interest. This assumption is likely to be valid if there are no observed significant structural changes of the nucleus above the level of interest [99]. In this case, the side-feeding may be treated by multiplying the decay curve of the populating transitions $R_{k}$ by a coefficient $C=I_{k} / I_{i}$ in the numerator of equation 4.14, where $I_{k}$ and $I_{i}$ are the total intensities of the states which populate and depopulate the state of interest.

### 4.3 Tagging Techniques

The TDR system allows the correlation of events detected in GREAT with those detected in JUROGAM-II. Due to the high level of background events at the target position, the correlation of events detected in JUROGAM-II with recoils and recoil decays at the focal plane is needed to produce clean prompt $\gamma$-ray spectra. The production of both recoil-tagged and recoil-proton-tagged spectra is discussed below.

### 4.3.1 Recoil Tagging

In the experiment described in this work, the ${ }^{58} \mathrm{Ni}\left({ }^{58} \mathrm{Ni}, p 2 n\right)$ fusion-evaporation reaction was used to produce ${ }^{113} \mathrm{Cs}$ along with other reaction products with similar masses. As well as the desired fusion-evaporation products, some of the
beam impinging on the target was scattered without forming compound nuclei and some beam particles formed fusion-evaporation products through reactions with the degrader foil of DPUNS. In order to provide clean prompt $\gamma$-ray spectra, recoil-tagging was implemented to ensure that all $\gamma$-ray events were correlated with the recoiling beam-on-target reaction products. The time of flight through RITU can be calculated from the recoil velocity measured from the Doppler shift of $\gamma$ rays detected at the target position and the known length of RITU. As individual detector events are time-stamped and the time of flight through RITU is known, prompt $\gamma$ rays can then be correlated with the subsequent implantation of the $\gamma$-ray emitting nuclei in one of the DSSDs of GREAT.

The target recoils detected in GREAT were differentiated from scattered beam and recoils from beam and degrader reaction products that had transited RITU, through the imposition of a two-dimensional gating condition on both the time of flight of particles between the MWPC and the DSSDs and the energy loss in the MWPC. Beam nuclei have a higher velocity than the fusion-evaporation recoils, so a time of flight condition can be used to differentiate the scattered beam that has transited through RITU from the fusion-evaporation recoils. As the degrader foil has a lower $Z$ number than the target foil, a similar condition can also be used to eliminate the degrader fusion-evaporation recoils from the recoil tag.

The energy loss of the reaction products in the MWPC can be approximated using the Bethe-Bloch formula [26]

$$
\begin{equation*}
-\frac{d E}{d x}=\frac{4 \pi}{m_{e} c^{2}} \cdot \frac{n Z^{2}}{\beta^{2}} \cdot\left(\frac{e^{2}}{4 \pi \epsilon_{0}}\right)^{2} \cdot\left[\ln \left(\frac{2 m_{e} c^{2} \beta^{2}}{I \cdot\left(1-\beta^{2}\right)}\right)-\beta^{2}\right], \tag{4.16}
\end{equation*}
$$

where $Z$ is the charge of the reaction product, $\beta=\frac{v}{c}$ where $v$ is the velocity of the recoils entering the MWPC, $\epsilon_{0}$ is the permittivity of free space, $n$ is the electron density of the gas and $I$ describes a mean ionisation potential of the gas. From this formula it can be seen that the energy loss of a particle in the MWPC is strongly dependent on its velocity, while all other terms are constant. The energy loss of a reaction product is then found to be inversely proportional to its velocity. As such, the energy loss of the fusion-evaporation recoils from the beam on target reactions will be higher than that from the beam on degrader reaction products, which in turn will have a higher energy loss than the scattered beam.

Using these facts, a polygonal 2-D coincidence condition or "gate" could be placed over the distinct area corresponding to the higher energy loss and longer time of flight in the histogram shown in Fig. 4.6. All prompt $\gamma$ rays included in recoiltagged spectra had to be detected in coincidence with these events. Note that in Fig. 4.6 the time of flight decreases versus channel number.


Figure 4.6: The two-dimensional histogram of energy loss in the MWPC versus time of flight between the MWPC and DSSDs used to impose the recoiltagging gate used during the analysis. Events included in the recoil gate are surrounded by the green line in the figure. The sharper peak to the right of the highlighted area corresponds to reaction products from beam-on-degrader reactions as well as scattered beam products. The time of flight decreases with increasing channel number.

### 4.3.2 Proton Tagging

${ }^{113} \mathrm{Cs}$ has a very low production cross section of $\sim 30 \mu \mathrm{~b}$ compared to the main channels of the fusion-evaporation reaction listed in Section 5.2. Therefore, a higher degree of selectivity than recoil-tagging was needed to observe its prompt $\gamma$ rays over those of the main channels in the reaction. This selectivity was provided by a two-dimensional energy and decay time gate on the proton-decay of ${ }^{113} \mathrm{Cs}$. Decay events were defined as an event which was in anti-coincidence
with an MWPC signal, detected in a pixel of the DSSD, subsequent to a recoil implant in the same pixel. The recoil in this case was defined by a DSSD signal in delayed coincidence with a JUROGAM-II signal. A 2-D histogram of energy versus decay time was created in order to identify decay products in the DSSD. The decay time was defined as the difference in time between the detection of a recoil and a subsequent decay event, while the energy simply corresponds to the energy of the decay event detected in the DSSD. The two-dimensional histogram is shown in Fig. 4.7 and shows the two-dimensional gate on the protons emitted by ${ }^{113} \mathrm{Cs}$. This encompasses a time range of $10-200 \mu \mathrm{~s}$ and an energy range of $850-1030 \mathrm{keV}$, comprehensively covering the established decay properties of ${ }^{113} \mathrm{Cs}$ of 969(8) keV and 17.1(2) $\mu \mathrm{s}$ [12].


Figure 4.7: The two-dimensional histogram of energy vs decay time of DSSD events detected in anti-coincidence with an MWPC signal. The area corresponding to ${ }^{113} \mathrm{Cs}$ protons is shown enclosed by the red line and corresponds to an energy range of $850-1030 \mathrm{keV}$ and a time range of $10-200 \mu \mathrm{~s}$.

### 4.4 Data Correction

In order to form the accurate recoil-tagged and proton-tagged $\gamma$-ray spectra used to analyse the collected data, it was first necessary to apply a number of corrections to the different detector events. These corrections are detailed below.

### 4.4.1 Energy Calibration

In the experiment described in this work, a variety of different sources were used to calibrate each individual detector to ensure a well defined relationship between the channel number recorded by the ADCs and the actual energy of the detected decays. Accurate calibrations were also required in order to gain match individual detectors to ensure well defined peaks in spectra formed from the output of more than one detector. Germanium detectors were calibrated using mixed ${ }^{133} \mathrm{Ba}$ and ${ }^{152}$ Eu sources while conversion electrons from ${ }^{133} \mathrm{Ba}$ and a triple- $\alpha$, mixed ${ }^{239} \mathrm{Pu}$ ${ }^{241} \mathrm{Am}-{ }^{244} \mathrm{Cm}$ source was used to calibrate the individual strips of the DSSD and the pin diodes. Calibration spectra were formed for each individual detector output. The centroids and intensities of the observed $\gamma$-ray, electron and $\alpha$ particle peaks were then extracted using Gaussian fits. To identify the peaks in each spectrum, the channel number and intensity of the fitted peaks were then compared with peak energies and relative intensities from Refs. [100-102]. A calibration relating channel number to energy could then be made, taking the form of a quadratic polynomial

$$
\begin{equation*}
E=a+b x+c x^{2}, \tag{4.17}
\end{equation*}
$$

where $x$ is the channel number, $E$ the correct peak energy and $a, b$ and $c$ the calibration coefficients. The $c$ coefficient was found to be small for the germanium detectors (of the order $10^{-8}$ ) but there was found to be significant non-linearity in the lower energy regions of the DSSD strips, as is discussed in Section 5.3. Examples of germanium detector and DSSD strip calibration spectra are shown in Fig. 4.8.

### 4.4.2 Efficiency Correction

The measured intensities of the ${ }^{113} \mathrm{Cs} \gamma$-ray transitions had to be corrected to compensate for the efficiencies of the JUROGAM-II detectors at different energies. Data were collected at the start and end of the experiment detailed in this work using a mixed ${ }^{152} \mathrm{Eu}$ and ${ }^{133} \mathrm{Ba}$ source. Efficiency corrections were undertaken for the individual JUROGAM-II rings as well as the entirety of the array.


Figure 4.8: (a) Example germanium detector calibration spectrum of a JUROGAM-II clover detector crystal exposed to a mixed ${ }^{152} \mathrm{Eu}$ and ${ }^{133} \mathrm{Ba}$ source. Also shown is a DSSD strip calibration spectrum focussed on (b) the low-energy range showing internal conversion electrons from a ${ }^{133} \mathrm{Ba}$ source and (c) the high-energy range showing alpha particles from a ${ }^{239} \mathrm{Pu},{ }^{241} \mathrm{Am}$ and ${ }^{244} \mathrm{Cm}$.

The $\gamma$-ray intensities of the photopeaks from the ${ }^{152} \mathrm{Eu}$ and ${ }^{133} \mathrm{Ba}$ sources were recorded and compared to the known intensities from Refs. [100, 103]. The variation of the ratio of the measured and reference intensities versus energy was then fitted with a function of the form

$$
\begin{equation*}
\eta=\exp \left[\left(\left(A+B x+C x^{2}\right)^{-G}+\left(D+E y+F y^{2}\right)^{-G}\right)\right]^{-\frac{1}{G}} \tag{4.18}
\end{equation*}
$$

where $\eta$ is the relative efficiency at energy $E$, which is in $\mathrm{keV}, x=\log (E / 100)$ and $y=\log (E / 1000)$. The $A, B$ and $C$ coefficients describe the detector response at low energies while the $D, E$ and $F$ coefficients describe the detector behaviour at high energies. The $G$ coefficient defines the gradual turning point between the two regions. The efficiency of Ring $2\left(\theta=133.6^{\circ}\right)$ of JUROGAM-II versus energy is shown in Fig. 4.9, fitted by the function in equation 4.18.


Figure 4.9: The relative efficiency versus energy of the EUROGAM phase-1 type detectors in Ring 2 of the JUROGAM-II array.

### 4.4.3 Add-back

Add-back is the process of reconstructing $\gamma$-ray decays which have been Compton scattered between more than one crystal of a segmented detector. Add-back was applied to the JUROGAM-II Clover detector signals in this work. If two signals were detected within 10 ns in adjacent crystals then a single event was constructed consisting of the summed energies of the individual signals. Due to the geometry of the four crystals diagonal correlations were not used, as there is a low probability of a $\gamma$ ray scattering directly between diagonal crystals. These events also fall under the veto of the BGO shields surrounding each detector. For cases where $\gamma$ rays were detected in a crystal but simultaneously detected in the BGO shield, the event was vetoed. This reduced the Compton background in the germanium detector spectra. The common outcomes for a $\gamma$ ray entering a clover germanium detector are shown schematically in Fig. 4.10.


Figure 4.10: Schematic diagram of the four crystals of a clover germanium detector surrounded by a BGO shield and the most common $\gamma$-ray interactions with the detector. (a) The $\gamma$-ray incident on the detector is totally absorbed in the first crystal it enters. (b) The add-back case where the $\gamma$-ray Compton scatters from the first crystal into an adjacent crystal where the scattered $\gamma$ ray is absorbed, the energy signals from the adjacent crystals are summed to provide the correct energy for the incident $\gamma$ ray. (c) The BGO veto case, where the incident $\gamma$ ray is scattered out of the detector and is then detected in the BGO shield. The BGO signal vetoes coincident signals from the germanium detector.

### 4.4.4 Doppler Correction and Recoil Velocity

The energies of $\gamma$ rays emitted by nuclei recoiling from the target foil are shifted by an amount dependent on the recoil velocity. The velocity of the recoiling nuclei must be known in order to correct for the Doppler-shift and create $\gamma$-ray spectra with peaks at the correct energies. Additionally, the velocity must be well known to accurately calculate the lifetime of excited nuclear states using the DDCM. The velocity of the recoils is calculated from the observed Doppler Shift of known $\gamma$-ray energies given by

$$
\begin{equation*}
E=E_{0}(1+\beta \cos \theta) \tag{4.19}
\end{equation*}
$$

where $E$ is the Doppler-shifted energy of the $\gamma$ ray, $E_{0}$ is the true energy of the emitted $\gamma$ ray, $\beta$ is the velocity of the recoils as a fraction of the speed of light and $\theta$ is the angle to the direction of travel at which the $\gamma$ ray is emitted. This
relationship holds for recoil velocities which are a small fraction of the speed of light.

The different rings of JUROGAM-II have well-defined angles, $\theta$, relative to the beam axis. As such, the shift in observed $\gamma$-ray energy as a function of $\theta$ can be used to extract the velocity of the recoils using equation 4.19, if the unshifted $\gamma$ ray energies are known. In this work different ${ }^{113} \mathrm{Cs}$ proton-tagged, JUROGAM-II ring spectra were created with no Doppler correction applied. The data in these spectra were taken at a long target-to-degrader distance of $3000 \mu \mathrm{~m}$ to ensure that each transition had only a fully shifted component. The 166-, 384-, 596-, 658- and 719 keV transitions were identified from their relative intensities [12] and the variation in position of the transition centroids in each of the rings were then separately plotted as a function of $\cos (\theta)$. Linear fits to the variation of centroid position were then used to extract the velocity of the fully shifted recoils for each transition as shown in Fig. 4.19. The final velocity of $\beta=0.038$ (2) was then taken from an average of the individual velocities listed in Table 4.1. This velocity was subsequently used to apply a Doppler correction to all $\gamma$-ray spectra shown in Chapter 5 used during the analysis.

| Transition Energy (keV) | $\chi^{2}$ | $v / c$ |
| :---: | :---: | :---: |
| $166.3(2)$ | 0.22 | $0.0334(11)$ |
| $383.9(2)$ | 2.32 | $0.0392(15)$ |
| $596.2(2)$ | 3.09 | $0.0393(22)$ |
| $658.4(4)$ | 0.10 | $0.0375(6)$ |
| $718.7(4)$ | 4.50 | $0.0427(20)$ |
| Weighted Average |  | $0.0384(24)$ |

Table 4.1: Recoil velocities calculated from the variation of energy versus detector angle for Doppler-shifted ${ }^{113} \mathrm{Cs}$ transitions. The velocity used to apply a Doppler correction to the proton-tagged $\gamma$-ray spectra was taken from a weighted average of these values.

### 4.4.5 Doppler Broadening

The widths of observed $\gamma$ rays emitted from recoils are affected by the recoil velocity. This is due to the finite opening angle of the detectors, with respect to the beam axis. This finite angle effectively allows a range of Doppler shifted energies to be detected in the same detector, resulting in a broader final energy


Figure 4.11: The linear fits to Doppler-shifted energy versus $\cos (\theta)$, where $\theta$ is the angle of the different JUROGAM-II detectors to the beam axis, for the (a) $166-$ - (b) $384-$, (c) $596-$ - (d) $658-$ and (e) $719-\mathrm{keV}{ }^{113} \mathrm{Cs}$ transitions used to extract the fully shifted recoil velocities listed in Table 4.1.
signal. The amount of Doppler broadening can be derived from the differential of equation 4.19 with respect to the angle to the beam axis

$$
\begin{equation*}
\Delta E=\Delta \theta \beta \sin \theta \tag{4.20}
\end{equation*}
$$

where all symbols are the same as in equation $4.19, \Delta E$ is the amount of Doppler broadening and $\Delta \theta$ is the opening angle of the detector. Detectors at $\theta=90^{\circ}$ to the beam axis are affected more by Doppler broadening. In JUROGAM-II this is offset somewhat by having the segmented clover detectors at the angles about $90^{\circ}$. The greater granularity of the clover detectors means that the acceptance angle is reduced to the acceptance angle of one crystal, while the detector as a whole still covers a large area [104]. This is shown schematically in Fig. 4.12. The acceptance angle of the individual clover detector crystals at angles to the


Figure 4.12: Schematic diagram adapted from Ref. [106] showing the Doppler broadening effect on $\gamma$ rays detected by a clover germanium detector at an angle, $\theta$, to the beam axis. The acceptance angle of the individual crystals, $\mathrm{d} \theta$, is shown to be smaller than would be expected for an unsegmented detector.
beam axis of $75^{\circ}$ and $105^{\circ}$ is then $\Delta \theta \sim 5^{\circ}$ while the acceptance angle for the phase 1 detectors at angles of $158^{\circ}$ and $134^{\circ}$ is $\Delta \theta \sim 9^{\circ}$.

In some $\gamma$-ray spectra used during the work presented in this thesis, the width of $\gamma$-ray peaks could not be directly measured due to the low statistics present in the spectra. As the inherent resolution of the germanium detector is approximately constant in the hundreds of keV range [105], the difference in width between two $\gamma$-ray peaks was dominated by the difference in Doppler broadening. Knowing the difference in Doppler broadening at different energies allows the width of low statistics peaks to be calculated using a peak of measurable width and similar energy. This process is described further in Section 5.6.

## Chapter 5

## Results

The lifetime of the $\left(11 / 2^{+}\right)$state in ${ }^{113} \mathrm{Cs}$ was measured using the DPUNS plunger device [21] in a fusion-evaporation experiment at the University of Jyväskylä. Recoil-decay tagging was used to create prompt ${ }^{113} \mathrm{Cs}$ proton-tagged $\gamma$-ray spectra, for each plunger target-to-degrader distance. The $\gamma$ rays in these spectra were correlated with fusion-evaporation recoils arriving at the focal plane and protons emitted from ${ }^{113} \mathrm{Cs}$, which were identified from their distinct decay energies and half-lives. The spectra showed the changing relative intensity of the decay of the $\left(11 / 2^{+}\right)$state before and after the degrader foil of the plunger, as a function of target-to-degrader distance. Gaussian fits were used to extract the intensities of the two photopeaks, corresponding to the decay of the $\left(11 / 2^{+}\right)$state before and after the degrader foil. The extracted intensities were normalised and used to calculate the lifetime of the $\left(11 / 2^{+}\right)$state within the framework of the Differential Decay Curve Method [99]. The conditions used to construct the proton-tagged spectra, the extraction and subsequent normalisation of the photopeak intensities and the calculation of the lifetime of the $\left(11 / 2^{+}\right)$state lifetime are described in more detail in this chapter. The identification of other decay products created by the fusion-evaporation reaction is also covered, as well as the calculation of a limit on the lifetime of the $\left(15 / 2^{+}\right)$state and the energy and half-life of proton emission from ${ }^{113} \mathrm{Cs}$.

### 5.1 Experimental Setup

This section details the specific experimental setup used for the ${ }^{113} \mathrm{Cs}$ experiment at the Accelerator Laboratory of the University of Jyväskylä. The synthesis and excitation of ${ }^{113} \mathrm{Cs}$ was achieved by means of a fusion-evaporation reaction. A $4-\mathrm{pnA}$ beam of ${ }^{58} \mathrm{Ni}$ with charge state $8^{+}$was accelerated by the K-130 cyclotron to an energy of $230-\mathrm{MeV}$ and impinged on a $1.1 \mu \mathrm{~g} / \mathrm{cm}^{2}{ }^{58} \mathrm{Ni}$ target mounted on the DPUNS plunger device. The beam energy was chosen based on Refs. $[8,9]$ to provide the maximum cross section for the creation of ${ }^{113} \mathrm{Cs}$ from the ${ }^{58} \mathrm{Ni}\left({ }^{58} \mathrm{Ni}, p 2 n\right)$ reaction. Estimates for the cross section varied from $5-30 \mu \mathrm{~b}$ [12, 22]. In order to provide the two different recoil velocity regimes needed for the analysis, a $1.5 \mu \mathrm{~g} / \mathrm{cm}^{2}$ natural degrader foil was also mounted on DPUNS to slow down the compound nuclei recoiling from the target.

The JUROGAM-II HPGe array surrounded the target position. During the experiment the array consisted of 14 phase 1 detectors, with 5 placed in Ring 1 and 9 placed in Ring 2 , while there were also 12 clover detectors in each of Rings 3 and 4. The target chamber shared the same helium gas environment of RITU. The gas pressure was adjusted in tandem with the fields generated by the magnets of RITU in order to provide the maximum transmission of the recoiling nuclei and minimum transmission of beam products to the GREAT focal plane. The optimum helium pressure used during the experiment was found to be 1.2 mbar , while the field of the first quadrupole magnet was set at 449 mT , the first dipole magnet at 492 mT and the final two quadrupole magnets at 362 mT and 392 mT , respectively. The GREAT array consisted of the full complement of detectors, as described in Section 3.5. The rate at which recoils implanted in the DSSDs throughout the experiment was approximately $\sim 8.5 \mathrm{kHz}$, while the rate at which protons from ${ }^{113} \mathrm{Cs}$ were detected was $\sim 0.13 \mathrm{~Hz}$. Typical rates for each detector are listed in Table 5.1.

### 5.2 Reaction Products at the Target Position

The main channels from the fusion-evaporation reaction used during the experiment were identified using a recoil-tagged prompt $\gamma-\gamma$ matrix. This matrix was created in GRAIN, with the condition that all events had to be correlated with a

| Detector | Rate $(\mathrm{kHz})$ |
| :---: | :---: |
| JUROGAM-II Phase-1 | 13 |
| JUROGAM-II Clovers | 27 |
| GREAT planar detector strips | 1.5 |
| Top-down oriented GREAT clover detector | 5 |
| Side-ward oriented GREAT clover detectors | 1.6 |
| MWPC | 230 |
| Pins | 6 |
| DSSD y-strips | 9 |
| DSSD x-strips | 11.5 |
| Recoils | 8.5 |

Table 5.1: The event rates for the different detectors in the JUROGAM-RITU-GREAT setup used during the experiment.
recoil implant in the DSSDs. The recoils were differentiated from scattered beam products through the imposition of a two-dimensional gate on the energy-loss of events in the MWPC and the time of flight between the MWPC and DSSD, as described in Subsection 4.3.1. Prompt $\gamma$-ray events included in the matrix also had to be detected within 10 ns of at least one other prompt $\gamma$ ray. By setting gates on $\gamma$-ray energies in this matrix and observing which other transition energies were detected in coincidence, cascades of coincident $\gamma$-rays could be identified. The vast majority of these $\gamma$-ray cascades have been previously observed and attributed to the de-excitation of a particular nucleus. The efficiency-corrected $\gamma$-ray intensities were then used to give an estimate of the relative intensity of each reaction product.

The chosen beam energy was important to ensure the maximum cross section for the production of ${ }^{113} \mathrm{Cs}$ nuclei in the ${ }^{58} \mathrm{Ni}+{ }^{58} \mathrm{Ni}$ fusion evaporation reaction. The main channels of the ${ }^{58} \mathrm{Ni}+{ }^{58} \mathrm{Ni}$ reaction for different beam energies are shown in the spectra below to demonstrate the differences in relative production crosssection of the main reaction products with changing beam energy. The main reaction channels for beam energies of $210-$, $230-$ and $250-\mathrm{MeV}$ are shown in Fig. 5.2. A more comprehensive list of the different reaction products is shown in Table 5.2.

Projections from the JUROGAM II recoil-tagged $\gamma-\gamma$ matrices used to identify the main reaction products for each beam energy are shown in Fig. 5.1. The 210and $250-\mathrm{MeV}$ beam-energy reactions were taken from an experiment undertaken in 2012 using the same experimental setup as that used for the $230-\mathrm{MeV}$ data.


Figure 5.1: Projections from the Doppler-corrected recoil-tagged $\gamma-\gamma$ matrices used to identify the main reaction products from the ${ }^{58} \mathrm{Ni}+{ }^{58} \mathrm{Ni}$ fusionevaporation reaction at energies of (a) 210 MeV , (b) 230 MeV and (c) 250 MeV . Due to de-focussing of the beam for the $210-$ and $250-\mathrm{MeV}$ beam energies, a number of reaction products are present from beam on target frame reactions. These ${ }^{58} \mathrm{Ni}+\mathrm{Al}, A \sim 80$ reaction products are also marked in panels (a) and (c) of the figure.

During this experiment the beam was unfocused or misaligned for a period, resulting in reactions between the aluminium frame of the plunger and the ${ }^{58} \mathrm{Ni}$ beam. This resulted in many reaction products with masses of $A \sim 80$. The $\gamma$ rays emitted by these lower-mass fusion evaporation recoils were much more intense than those from the beam-on-target recoils, as shown in Fig. 5.1. Due to these high intensities, the mass $A \sim 80$ reaction products could not all be removed before reaching the focal plane or entirely excluded from the recoil-tagging gate.
(a)

(b)

(c)


Figure 5.2: The main reaction products from the ${ }^{58} \mathrm{Ni}+{ }^{58} \mathrm{Ni}$ fusionevaporation reaction at beam energies of (a) 210 MeV , (b) 230 MeV and (c) 250 MeV .

| Beam Energy | Exit Channel | Relative Intensity (\%) |
| :---: | :---: | :---: |
| 210 MeV | ${ }^{113} \mathrm{I}$ | 71 |
|  | ${ }^{112} \mathrm{Te}$ | 12 |
|  | ${ }^{110} \mathrm{Te}$ | 8 |
|  | ${ }^{114} \mathrm{Xe}$ | 8 |
|  | ${ }^{113} \mathrm{Xe}$ | 2 |
| 230 MeV | ${ }^{113} \mathrm{I}$ | 54 |
|  | ${ }^{112} \mathrm{Te}$ | 35 |
|  | ${ }^{110} \mathrm{Te}$ | 7 |
|  | ${ }^{114} \mathrm{Xe}$ | 4 |
|  | ${ }^{112} \mathrm{I}$ | 1 |
| 250 MeV | ${ }^{112} \mathrm{Te}$ | 63 |
|  | ${ }^{113} \mathrm{I}$ | 20 |
|  | ${ }^{112} \mathrm{I}$ | 6 |
|  | ${ }^{111} \mathrm{Te}$ | 4 |
|  | ${ }^{114} \mathrm{Xe}$ | 3 |
|  | ${ }^{109} \mathrm{Sb}$ | 3 |
|  | ${ }^{110} \mathrm{Te}$ | 1 |

Table 5.2: The main reaction channels from the ${ }^{58} \mathrm{Ni}+{ }^{58} \mathrm{Ni}$ reaction at different beam energies of $210-$, $230-$ and $250-\mathrm{MeV}$.

### 5.3 Reaction Products at the Focal Plane

The different decay products detected in the DSSDs, as shown in Fig. 5.3 were variously identified from their energies, decay times, prompt $\gamma$-ray coincidences and predicted cross sections. The protons emitted by the decay of ${ }^{113} \mathrm{Cs}$ were identified in Fig. 5.3 (a) from both the known decay energy of $\mathrm{E}_{p}=969(8)$ keV and half-life $\mathrm{T}_{1 / 2}=17.1(2) \mu \mathrm{s}[12]$. The ${ }^{113} \mathrm{Cs}$ proton-tagging gate is shown enclosed by the red line in Fig. 5.3 and covers an energy range of $850-1030 \mathrm{keV}$ and a time range of $10-200 \mu \mathrm{~s}$, encompassing the known energy and decay lifetime. These ranges were used to not only visually encompass all events corresponding to ${ }^{113}$ Cs decay shown in Fig. 5.3 (a), but to also ensure that no other decay products were erroneously included in the gate. The low background in the DSSD at $\sim 1$ MeV meant that events detected within the correct energy range at several halflives were still most likely to have come from the decay of ${ }^{113} \mathrm{Cs}$. Proton-tagged prompt $\gamma$-ray spectra were subsequently created, as shown in Section 5.5, and were found to only contain $\gamma$-ray transitions from ${ }^{113} \mathrm{Cs}$ [12], showing that the proton-tagging gate used did not contain any contaminants.

Approximately 500 protons emitted by ${ }^{109} \mathrm{I}$ and ${ }^{112} \mathrm{Cs}$ were detected in the DSSDs. From a Gaussian fit, the energy of the peak to the left of the ${ }^{113} \mathrm{Cs}$ peak in the


Figure 5.3: (a) Energy versus decay time of decay products detected in the same pixel as a prior recoil implant in the DSSDs of GREAT. (b) The energy projection of the two-dimensional histogram shown in (a). Identified decay products are also listed.
spectrum shown in Fig. 5.3 (b) was found to have an energy of 813(4) keV. This value is consistent with the $813(3) \mathrm{keV}$ proton energy of ${ }^{109} \mathrm{I}[107]$ and also the $810(5) \mathrm{keV}$ proton energy of ${ }^{112} \mathrm{Cs}$ [55]. Setting a gate on the energy of the 813keV peak in Fig. 5.3 (a) and projecting the resultant timing spectrum allowed the half-life of the decays in the peak to be measured. Fitting a decay curve to the timing spectrum shown in Fig. 5.4 returns a half-life value for the $813-\mathrm{keV}$ decay of $158(10) \mu \mathrm{s}$. This value is neither consistent with the known $103-\mu$ s half-life of ${ }^{109}$ I [108] or with the $490 \mu$ s half-life of ${ }^{112} \mathrm{Cs}$ [55], suggesting that the $813-\mathrm{keV}$ peak results from the decay of both nuclei. However, due to the low statistics available and the consistent energies of protons emitted from ${ }^{112} \mathrm{Cs}$ and ${ }^{109} \mathrm{I}$, the relative quantity of each nuclei cannot be determined.

The two broad peaks at the lower end of the energy spectrum in Fig. 5.3 (b)


Figure 5.4: The timing spectrum of the $813-\mathrm{keV}$ decays detected in the DSSD in anti-coincidence with an MWPC signal. Using this figure a half-life value for the decays of $158(10) \mu$ s was extracted. This value is within $5 \sigma$ of the $103 \mu \mathrm{~s}$ half-life of the $813-\mathrm{keV}$ protons emitted by ${ }^{109} \mathrm{I}$ and is over $6 \sigma$ away from the $490 \mu$ s half-life of the $810-\mathrm{keV}$ protons emitted by ${ }^{112} \mathrm{Cs}$. This indicates that the decays observed at $813-\mathrm{keV}$ in the DSSD originate from protons emitted by both ${ }^{109} \mathrm{I}$ and ${ }^{112} \mathrm{Cs}$.
were identified as internal conversion electrons from the decay of the low-lying $\left(11 / 2^{-}\right)$isomeric state in ${ }^{113} \mathrm{Xe}$ and the $\left(7 / 2^{+}\right)$state between the isomer and the ground state [73]. Due to the high statistics of the broad peaks, recoil-decay-tagged prompt $\gamma$-ray spectra could be created to identify the decay events from the corresponding characteristic prompt $\gamma$-ray spectra. Four separate RDT gates with the same time ranges of $\sim 10-40 \mu$ s and energy gates of similar $\sim 40$ keV widths but incrementally higher energies were initially used to create four different prompt $\gamma$-ray spectra. These spectra were used to deduce whether all decay products in the broad peaks were from the same nucleus or whether there was more than one decay product, which would be evidenced by individual peaks in the separate spectra having different relative intensities. All four gates returned spectra containing the same energy $\gamma$-ray events with the same relative intensities, despite some differences in the statistics between each spectrum. All identified prompt $\gamma$-rays were from ${ }^{113} \mathrm{Xe}$ [109], the different decay-tagged spectra are shown in Fig. 5.5.

The fact that all prompt $\gamma$-ray transitions in the decay-tagged spectra come from ${ }^{113}$ Xe strongly indicates that the decay products in the two broad peaks also result from ${ }^{113} \mathrm{Xe}$. The decay of ${ }^{113} \mathrm{Xe}$ proceeds by means of $\beta^{+}$-decay, which can be detected by the DSSDs. Additionally, ${ }^{113} \mathrm{Xe}$ also has an isomeric state depopulated


Figure 5.5: Individual Doppler-corrected recoil-decay tagged prompt $\gamma$-ray spectra with the same time gating condition but different energy gates of similar widths encompassing the whole range of the two broad, low energy peaks shown in Fig. 5.3 (a). Panels (a) to (d) show energy tagging gates of increasing energy. All spectra show only ${ }^{113} \mathrm{Xe}$ transitions, indicating the two broad peaks are due to internal conversion decay of excited states below the low-lying $10-\mu$ s isomer in ${ }^{113} \mathrm{Xe}$ [73].
by low energy transitions, which may decay by the emission of internal-conversion electrons, which can also be detected in the DSSDs. The $\beta^{+}$particles and internal conversion electrons can be differentiated by their vastly different half-lives of 2.8 s [110] and $10 \mu \mathrm{~s}$ [73], respectively. Timing spectra for both low-energy peaks in Fig. 5.3 were formed by setting gates on the energy of each peak and projecting the resultant timing spectra. These spectra were then converted into a logarithmic format and the half-lives of each decay were extracted from linear fits to the variation of the intensity of each peak with time. The timing spectra and associated fits are shown in Fig. 5.6.

The half-lives extracted from both spectra are consistent with the measured $10(4) \mu$ s half-life of the isomeric $\left(11 / 2^{+}\right)$state in ${ }^{113} \mathrm{Xe}$ as measured in Ref. [73].


Figure 5.6: The timing spectra formed by energy gates set in Fig. 5.3 on (a) the lower energy and (b) higher energy bumps shown in Fig. 5.3 around channel 300 . The decay curves of both of these features are consistent with the $10(4) \mu$ s isomeric decay of the $11 / 2^{-}$state in ${ }^{113} \mathrm{Xe}$, indicating that these features result from the internal conversion decay of the isomeric state and $7 / 2^{+}$state in ${ }^{113} \mathrm{Xe}$.

This indicates that the two broad peaks result from internal conversion electrons from decays from the $\left(11 / 2^{+}\right)$or lower-energy states. The fact that there are two transitions between the isomer and the ground state of ${ }^{113} \mathrm{Xe}$, which can account for the fact that there are two broad peaks, also lends weight to the assignment of these decays. The fact that there are counts visible over a large timing range at the energy of the two peaks indicates that the $\beta^{+}$decays may also be detected in the DSSDs within this energy range. It should be noted that the ${ }^{113} \mathrm{Xe}$ peaks have a much broader profile than the internal conversion electron peaks in the source spectrum shown in Fig. 4.8. Additionally, the energy of the two peaks does not correspond to the known $\gamma$-ray energies of the transitions depopulating the $\left(7 / 2^{+}\right)$state or the x -ray energies of ${ }^{113} \mathrm{Xe}$. This indicates that the DSSDs may be very non-linear at low energies. It should be noted that the ${ }^{133} \mathrm{Ba}$ energies were found to be accurate in the calibrated DSSD and that the higher energies
of all other decay products were found to be consistent with known values.
The broad peak in the spectrum around $\sim 3 \mathrm{MeV}$ has the characteristic shape and typical energy range of $\beta$-delayed proton decays [111]. The nuclei ${ }^{112} \mathrm{I}$ and ${ }^{113}$ Xe are tentatively assigned to the $\beta$-delayed proton events, as the limits of the distribution in Fig. 5.3 are consistent with proton energies detected from these nuclei of $\sim 2.5 \mathrm{MeV}-5 \mathrm{MeV}$ [111]. Additionally, ${ }^{112} \mathrm{I}$ is the only $\beta$-delayed proton emitter for which $\gamma$-ray transitions have been observed at the target position, while ${ }^{113} \mathrm{Xe}$ is the only nucleus that has $\gamma$-ray transitions detected at the focal plane. This indicates that ${ }^{112} \mathrm{I}$ and ${ }^{113} \mathrm{Xe}$ are the most intensely populated reaction channels, which may yield $\beta$-delayed proton emission and that detected $\beta$-delayed protons are from these nuclei. The ${ }^{112} \mathrm{I}$ and ${ }^{113} \mathrm{Xe}$ half-lives of 3.4 s and 2.7 s , respectively, are too long to be measured to aid the identification of the decays. This time encompasses several beam pulses from the cyclotron [76] so there is a strong likelihood that a decay event may be correlated with the wrong recoil and hence an incorrect lifetime may be measured. Clean recoil-decay-tagged spectra could then not be created to identify decay products from their prompt $\gamma$ rays as was done for the internal conversion decays of ${ }^{113} \mathrm{Xe}$.

An additional narrower peak, which lies on the $\beta$-delayed proton spectrum, has been measured to have an energy of $3898(3) \mathrm{keV}$. This energy is consistent within $4 \sigma$ of the published value of $3862(10)-\mathrm{keV}$ [112] for $\alpha$ particles emitted by the decay of ${ }^{107} \mathrm{Te}$. The fusion evaporation code PACE [113] predicts that ${ }^{107} \mathrm{Te}$ will be produced with a cross section of 0.39 mb by the ${ }^{58} \mathrm{Ni}+{ }^{58} \mathrm{Ni}$ reaction at 230 MeV . As PACE gives a cross-section for ${ }^{113} \mathrm{Cs}$ of 0.36 mb , which is abundant in the DSSD spectrum it was considered to be possible for $\alpha$-particles from similar cross-section reaction products to also be detected. Therefore, the $3898-\mathrm{keV}$ peak is tentatively assigned to result from the $\alpha$ decay of ${ }^{107} \mathrm{Te}$. Similarly to the $\beta$ delayed proton decays, it was not possible to further characterise the decay events at 3898 keV through $\gamma$-ray coincidences and lifetime measurements, due to the long half-life and low intensity of the decay.

### 5.4 Lifetime and Energy of Proton Emission from

## ${ }^{113} \mathbf{C s}$

The lifetime and energy of ground-state proton emission from ${ }^{113} \mathrm{Cs}$ were measured from the 2D histogram shown in Fig. 5.3. The energy was measured by applying a Gaussian fit to the peak corresponding to the ${ }^{113} \mathrm{Cs}$ proton emission in the energy projection as shown in Fig. 5.3 (b). The centroid of the peak was found to be $950(8) \mathrm{keV}$. This energy is consistent within $3 \sigma$ of the previous measurements of the proton energy (in chronological order) of 980 (80) keV [9], 974(4) keV [114], 959(6) keV [115] and 969(8) keV [12].

A gate was set on the $950-\mathrm{keV}$ proton peak in the 2D histogram and the resultant timing spectrum was projected. This energy gate on the ${ }^{113} \mathrm{Cs}$ protons had a range of $880-1010 \mathrm{keV}$ based on the width of the peak in the energy spectrum seen in Fig. 5.3. The timing spectrum from a background energy gate of 1050-1180 keV was also projected and subtracted from the timing spectrum resultant from the energy gate on the ${ }^{113}$ Cs protons. This background subtraction reduced the number of random timing correlations that were present within the proton energy gate. A background gate with a higher energy than the proton gate was chosen so that ${ }^{113} \mathrm{Cs},{ }^{112} \mathrm{Cs}$ and ${ }^{109}$ I protons, which escaped from the DSSD and only had a portion of their energies detected by the DSSD were not favourably background subtracted. By fitting a decaying exponential function to the number of counts versus time in the background-subtracted proton-emission timing spectrum, the lifetime of proton emission from the ground state of ${ }^{113} \mathrm{Cs}$ was found to be $\tau=$ $24.4(2) \mu \mathrm{s}$. This value is consistent with the known lifetime of $\tau=24.7(3) \mu \mathrm{s}$ previously measured in Ref. [12]. The variation of proton-emission intensity with time and the fitted decay curve are shown in Fig. 5.7.

### 5.5 Construction of Proton-tagged Spectra

To extract the lifetime of an excited state using the DDCM, the variation with target-to-degrader distance of the intensities of the fully shifted and degraded components of a $\gamma$-ray transition depopulating the state of interest must be


Figure 5.7: The intensity versus time of ${ }^{113}$ Cs protons from a timing spectrum tagged by the ${ }^{113}$ Cs proton energy as defined in Fig. 5.3. An exponential fit has been applied to the variation of the proton-emission intensity versus time and gives a lifetime for proton emission of $\tau=24.4(2) \mu \mathrm{s}$.
known. As such, individual $\gamma$-ray spectra corresponding to different target-todegrader distances were created to show the variation of these component intensities for transitions between excited states in ${ }^{113} \mathrm{Cs}$.

The $\left(11 / 2^{+}\right)$excited state was expected to have a measurable lifetime based on Refs. $[13,116]$. As such, the first target-to-degrader distance was chosen based on a calculation of the $\left(11 / 2^{+}\right)$state lifetime in ${ }^{113} \mathrm{Cs}$ by Möller and Nix $[13,116]$ and the expected fully shifted recoil velocity as calculated from the PACE 4 code [113]. The subsequent distances were chosen based on the observed ratio of counts in the two photopeaks of the $384-\mathrm{keV}$ transition at the first target-to-degrader distance. More particularly, these distances were chosen in order to provide a range of target-to-degrader distances over which there is a large variation of intensities of the fully shifted and degraded photopeaks as required to construct the decay curve of the $\left(11 / 2^{+}\right)$state.

Target-to-degrader distances of 135-, 210-, 300-, 590- and $3000-\mu \mathrm{m}$ were chosen to form the decay curve of the $\left(11 / 2^{+}\right)$state. The $300 \mu \mathrm{~m}$ distance was initially chosen based on the theoretical predictions so that the intensity of the $384-\mathrm{keV}$ $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition was evenly distributed between its fully shifted and degraded components. The 135-, 210- and $590-\mu \mathrm{m}$ distances were chosen to provide different ratios of fully shifted and degraded intensities, with measurable intensities in both components. The $3000-\mu \mathrm{m}$ distance was chosen to provide a
limit at which all intensity was present in the fully shifted component, to improve the fit of the decay-curve, which is described in more detail later.

To ensure clean prompt $\gamma$-ray spectra for the measurement of the low cross section ${ }^{113}$ Cs transitions, proton-tagged spectra were created for each target-to-degrader distance. To ensure the separation of the fully shifted and degraded components of the $384-\mathrm{keV}\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition, only events detected in Ring 2 of JUROGAM-II were used to fill these spectra. By using detectors from a single ring of the array with the same polar angle for each detector, the separation and widths of the two components was constant making the extraction of the component intensities easier. Ring 2 was chosen over Ring 1 as it contained nine rather than five detectors. Rings 3 and 4 were not used as, with these rings subtending $90^{\circ}$, the separation of the fully shifted and degraded components was not sufficient for simple fitting of the two component intensities. The proton-tagged JUROGAM-II Ring 2 spectra for the individual target-to-degrader distances are shown in Fig. 5.8, zoomed in on the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition.

More target-to-degrader distances were not used due to the low cross-section of the reaction which restricted the available statistics. Five distances was the greatest number that could be used while ensuring that the photopeaks of the $384-\mathrm{keV}$ transition had enough counts to be discernibly Gaussian and distinct from background in the proton-tagged spectra used during the analysis.

In order to verify the cleanliness of the proton-tagged spectra and to observe any instance of measurable lifetimes evidenced by more than one component per transition, sum spectra corresponding to all target-to-degrader distances used were examined. The $384-\mathrm{keV}$ transition proved to be the only transition which had two photopeaks over the range of target-to-degrader distances used. This sum spectrum is shown in Fig. 5.9.

The intensities of all observed ${ }^{113} \mathrm{Cs}$ spectra were measured from this spectrum and are listed in Table 5.3. The intensities have previously been measured in Ref. [12] and a level scheme constructed. The intensities measured in this work may imply a different ordering of the transitions but the errors on intensities are all consistent with those measured in Ref. [12]. The fact that a single projection had to be used to measure the transitions means that resolving doublets was sometimes problematic. The level scheme of ${ }^{113} \mathrm{Cs}$ built on the basis of the measured


Figure 5.8: Doppler-corrected Proton-tagged JUROGAM-II Ring 2 spectra for the (a) 135- (b) 210- (c) 300- (d) 590- and (e) $3000 \mu \mathrm{~m}$ target-to-degrader distances focussed on the $384-\mathrm{keV}(11 / 2)^{+} \rightarrow(7 / 2)^{+}$transition in ${ }^{113} \mathrm{Cs}$. The fits used to extract the intensity of the fully shifted and degraded components of the transition are shown by the green and red lines, respectively.


Figure 5.9: Doppler-corrected Proton-tagged Ring 2 sum spectrum, consisting of all target-to-degrader distances used, showing the different transitions in ${ }^{113} \mathrm{Cs}$. The $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition proved to be the only transition with two obviously measurable components. These are shown in the insert.
intensities has, therefore, not been reordered from that shown in Ref. [12]. The level scheme is shown in Fig. 5.10

### 5.6 Extraction of Gamma-ray Transition Inten-

## sities

As can be seen from Fig. 5.8 the Gaussian profile of the peaks is not obvious due to the low number of counts in each photopeak. However, from the sum spectrum shown in Fig. 5.9 and the shape of peaks in Fig. 5.1 it becomes apparent that the photopeaks assume this profile when there are sufficient statistics available. As such to extract the intensities of the two photopeaks in the individual distance spectra, Gaussian fits were applied to the photopeaks using the Radware gr3 software [117]. The widths and centroids of the photopeaks were held constant during these fits.

Band 1


Figure 5.10: Level scheme of ${ }^{113} \mathrm{Cs}$ with the ordering of transitions based on Ref. [12] and intensities as measured from Fig. 5.9.

| Energy (keV) | $I_{S} \rightarrow I_{D}$ | Relative Intensity |
| :---: | :---: | :---: |
| $73.7(1)$ | $\left(5 / 2^{+}\right)_{1} \rightarrow\left(3 / 2^{+}\right)$ | $35(4)$ |
| $92.2(1)$ | $\left(7 / 2^{+}\right) \rightarrow\left(5 / 2^{+}\right)_{1}$ | $52(4)$ |
| $167.4(1)$ | $\left(5 / 2^{+}\right)_{2} \rightarrow\left(3 / 2^{+}\right)$ | $17(3)$ |
| $384.1(1)$ | $\left(11 / 2^{+}\right) \rightarrow\left(7 / 2^{+}\right)$ | $100(7)$ |
| $508.0(5)^{*}$ | $\left(27 / 2^{+}\right) \rightarrow\left(23 / 2^{+}\right)$ | $41(6)$ |
| $510.7(3)^{*}$ | $\left(9 / 2^{+}\right) \rightarrow\left(5 / 2^{+}\right)_{2}$ | $82(8)$ |
| $595.9(1)^{*}$ | $\left(15 / 2^{+}\right) \rightarrow\left(11 / 2^{+}\right)$ | $76(7)$ |
| $610.0(4)^{*}$ | $\left(13 / 2^{+}\right) \rightarrow\left(9 / 2^{+}\right)$ | $36(3)$ |
| $616.0(4)^{*}$ | $\left(31 / 2^{+}\right) \rightarrow\left(27 / 2^{+}\right)$ | $24(6)$ |
| $617.9(6)^{*}$ | $\left(29 / 2^{+}\right) \rightarrow\left(25 / 2^{+}\right)$ | $26(6)$ |
| $657.6(1)$ | $\left(25 / 2^{+}\right) \rightarrow\left(21 / 2^{+}\right)$ | $41(5)$ |
| $717.2(3)$ | $\left(17 / 2^{+}\right) \rightarrow\left(13 / 2^{+}\right)$ | $39(4)$ |
| $735.1(6)^{*}$ | $\left(33 / 2^{+}\right) \rightarrow\left(29 / 2^{+}\right)$ | $23(5)$ |
| $737.1(4)^{*}$ | $\left(19 / 2^{+}\right) \rightarrow\left(15 / 2^{+}\right)$ | $37(6)$ |
| $744.9(6)$ | $\left(35 / 2^{+}\right) \rightarrow\left(31 / 2^{+}\right)$ | $19(3)$ |
| $811.2(4)^{*}$ | $\left(21 / 2^{+}\right) \rightarrow\left(17 / 2^{+}\right)$ | $36(4)$ |
| $813.7(5)^{*}$ | $\left(23 / 2^{+}\right) \rightarrow\left(19 / 2^{+}\right)$ | $31(4)$ |
| $819.2(9)^{*}$ | $\left(39 / 2^{+}\right) \rightarrow\left(35 / 2^{+}\right)$ | $19(5)$ |
| $897.3(5)$ | $\left(43 / 2^{+}\right) \rightarrow\left(39 / 2^{+}\right)$ | $9(2)$ |

Table 5.3: The fitted energies and relative intensities of the ${ }^{113} \mathrm{Cs}$ transitions seen in Fig. 5.9. The energies marked by stars were from transitions which formed doublets and could not have centroids reliably extracted. These energies are instead taken from Ref. [12].

In order to extract the width of the fully shifted photopeak of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right) 384-\mathrm{keV}$ transition, a Gaussian fit was first applied to the fully shifted component visible in the sum spectrum of the individual $590 \mu \mathrm{~m}$ and $3000 \mu \mathrm{~m}$ spectra shown in Fig. 5.8 (d) and (e). At these distances the degraded component remained small and did not interfere with the Gaussian fit to the fully shifted component. The centroid of the fully shifted component could be fixed in this fit, as the Doppler-corrected centroid is known to be 384 keV .

In order to extract the width of the degraded photopeak its centroid first had to be extracted. The velocity of the degraded recoils could not be experimentally measured using the same method as was used for the fully shifted recoils due to the absence of a degraded component for the majority of ${ }^{113} \mathrm{Cs}$ transitions. Additionally, no short target-to-degrader distance was used, so there was no proton-tagged plunger spectrum with all of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition intensity contained within the degraded peak. A different technique was used whereby the centroid and width of the degraded peak were extracted from a Gaussian fit to the degraded component of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition in
the spectrum shown in Fig. 5.9. As the fully shifted and degraded peaks are not completely separated and slightly overlap, the previously extracted width and centroid of the fully shifted photopeak were held constant during the fit.

The width and centroid of the fully shifted component of the $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$ $596-\mathrm{keV}$ transition were extracted using the same method as was used for the fully shifted component of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition. The centroid and width of the degraded component of the $596-\mathrm{keV}$ transition could not be directly fitted due to the low number of counts in the degraded components and the lack of a well-defined Gaussian shape. The centroid shift between the fully shifted and degraded components of the $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transition is dependent on the difference in velocities of the fully shifted and degraded recoils. The ratio between the centroids of fully shifted and degraded components will be the same for all transitions as shown by rearranging equation 4.19 to the form

$$
\begin{equation*}
E / E_{0}=(1+\beta \cos (\theta)) . \tag{5.1}
\end{equation*}
$$

For transitions detected in the same ring of detectors in JUROGAM-II the righthand side of equation 5.1 will be constant. The centroid of the degraded component of the $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transition was then deduced to be 604 keV using the known centroid of the fully shifted $596-\mathrm{keV}$ component and the ratio between the fully shifted and degraded centroids of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition.

The width of the degraded component of the $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transition was calculated from the increased Doppler broadening relative to the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition. The Doppler broadening of ${ }^{113} \mathrm{Cs}$ transition components can be calculated using equation 4.20 . The velocity of the degraded recoils was first obtained using equation 4.19 and the shift between the degraded and fully shifted centroids of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition. The velocity of the degraded recoils was found to be $\beta_{d}=0.018(2)$.

The Doppler broadening of the degraded components of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$ and $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transitions were then calculated using equation 4.20 and the increase in Doppler broadening between the two components was found from the difference between the two transitions such that,

$$
\begin{equation*}
\Delta E_{604}-\Delta E_{389}=\left(E_{604}-E_{389}\right)\left(\beta_{d} \sin (\theta)\right) \Delta \theta, \tag{5.2}
\end{equation*}
$$

where, $\Delta E_{389}$ and $\Delta E_{604}$ is the Doppler broadening of the $\gamma$ rays at energies 389and $604-\mathrm{keV}$, respectively and all other symbols have the same meanings as in equation 4.20. The inherent resolution of the JUROGAM-II detectors was assumed to be constant between $384-\mathrm{keV}$ and $596-\mathrm{keV}$, as the germanium detectors do not typically lose resolution until much higher energies [105]. The inherent widths of the degraded components of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$and $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transitions could then be thought to be the same and only the increased Doppler broadening had an effect on the peak widths. Using the measured width of the degraded component of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition and the calculated Doppler broadening, the width of the degraded component of the $\left(15 / 2^{+}\right)$ to $\left(11 / 2^{+}\right)$transition was found to be 5.3 keV using equation 5.2. The centroids and widths of all components are listed in Table 5.4. By fixing these parameters in the fits using GF3, more accurate intensities could be extracted for all components.

| Transition | Component | Centroid (keV) | Width (keV) |
| :---: | :---: | :---: | :---: |
| $\left(11 / 2^{+}\right) \rightarrow\left(7 / 2^{+}\right)$ | Fully Shifted | $384.1(4)$ | $5.0(6)$ |
| $\left(11 / 2^{+}\right) \rightarrow\left(7 / 2^{+}\right)$ | Degraded | $388.9(3)$ | $5.0(6)$ |
| $\left(15 / 2^{+}\right) \rightarrow\left(7 / 2^{+}\right)$ | Fully Shifted | $596.3(5)$ | $5.1(7)$ |
| $\left(15 / 2^{+}\right) \rightarrow\left(11 / 2^{+}\right)$ | Degraded | $604.2(9)$ | $5.3(6)$ |

Table 5.4: The centroids and widths of the fully shifted and degraded components of both the $\left(15 / 2^{+}\right) \rightarrow\left(11 / 2^{+}\right)$and $\left(11 / 2^{+}\right) \rightarrow\left(7 / 2^{+}\right)$transitions.

### 5.7 Normalisation of Gamma-ray Transition In-

## tensities

The measured component intensities had to be normalised to account for the different times at which data was collected at each target-to-degrader distance. There are three methods which may be used to normalise the component intensities for the different distances:

- Normalise the number of counts in each component as a proportion of the total number of counts in the transition at each target-to-degrader distance
- Normalise the component intensities to the number of counts in a total projection spectrum at each target-to-degrader distance
- Normalise the component intensities to the number of counts in one or more transitions from a reaction product in a total projection spectrum at each target-to-degrader distance.

The first method removes information on the timing behaviour of states populating the state of interest. The second method suffers from the fact that the number of background events detected at the target position will increase as the experiment progresses. The third method depends on the beam energy being constant throughout the experiment, as reaction products will be created with different cross sections for different beam energies. As the beam energy is generally constant at the University of Jyväskylä, the third method was used to normalise peak intensities.

The $957-\mathrm{keV}$ peak in the total projection spectrum was used for normalising the ${ }^{113} \mathrm{Cs}$ peak intensities. This peak corresponds to the transition between an $18^{+}$and a $16^{+}$state in ${ }^{112} \mathrm{Te}$ [118] and was chosen due to its high intensity and separation from other transitions in the recoil-tagged total projection spectrum used to measure its intensity. The lifetime of the $957-\mathrm{keV}$ transition is short enough that its intensity is fully contained within the fully shifted component at all target-to-degrader distances [74]. Only this single peak was used as it contained sufficient statistics, such that the error on the intensity was negligible on its own compared to the intensities of the ${ }^{113} \mathrm{Cs}$ transitions as measured in the proton-tagged spectra. The difficulty of choosing another clean transition in the total-projection spectrum was also considered. The intensities of the $957-\mathrm{keV}$ peak are listed below in Table 5.5.

The $300-\mu \mathrm{m}$ distance had fewer counts than all but the $3000-\mu \mathrm{m}$ distance due to the lower beam current at the start of the experiment of $1.5-3 \mathrm{pnA}$. Data was collected at the $3000-\mu \mathrm{m}$ distance for less time than the other target-to-degrader distances as this distance was only used to provide an anchor point for the lifetime fit and the reduction of statistical error was not as important as for the other distances at which the lifetime of the $\left(11 / 2^{+}\right)$state was calculated.

| Target-to-Degrader Distance $(\mu \mathrm{m})$ | Counts in the ${ }^{112} \mathrm{Te} 957-\mathrm{keV}$ Transition |
| :---: | :---: |
| 135 | $69370(1353)$ |
| 210 | $63197(1334)$ |
| 300 | $54266(1220)$ |
| 590 | $67283(1367)$ |
| 3000 | $53299(1366)$ |

TABLE 5.5: The variation in intensity of the $957-\mathrm{keV} 18^{+} \rightarrow 16^{+}$transition in ${ }^{112} \mathrm{Te}$ at different target-to-degrader distances. The intensities were extracted from Recoil-tagged JUROGAM-II Ring 2 spectra at each target-to-degrader distance and were used to normalise the intensities of ${ }^{113} \mathrm{Cs}$ transitions at each distance.

### 5.8 Lifetime of the $\left(15 / 2^{+}\right)$State in ${ }^{113} \mathrm{Cs}$

The lifetime of the $\left(15 / 2^{+}\right)$state could not be measured using the DDCM as the normalised fully shifted and degraded component intensities of the $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transition did not vary with the target-to-degrader distances used in the experiment. The $\left(11 / 2^{+}\right)$state is part of a rotational band structure, the higher spin levels of which are expected to have short lifetimes. Equation 2.46, which can be used to estimate the lifetimes of states in Band 1, if ${ }^{113} \mathrm{Cs}$ is treated as a rigid rotor, gives estimated lifetime values for the $\left(11 / 2^{+}\right),\left(15 / 2^{+}\right)$and $\left(19 / 2^{+}\right)$ states of $38.3 \mathrm{ps}, 3.6 \mathrm{ps}$ and 1.2 ps , respectively. This calculation used the groundstate deformation of $\beta^{2}=0.21$ from Möller and Nix [116] and demonstrates the expected reduction of decay time with increasing spin in the band.

Fig. 5.11 shows the fits used to extract the intensities of the $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$ transition components at each target-to-degrader distance. It can be seen that the $604-\mathrm{keV}$ degraded component of the $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transition has a low intensity for all target-to-degrader distances. Additionally, the degraded component intensity has no correlation to target-to-degrader distance, as is shown in Table 5.6.

As the DDCM could not be used, the lifetime of the $\left(15 / 2^{+}\right)$state was instead estimated from the normalised fully shifted and degraded counts at the $135-\mu \mathrm{m}$ target-to-degrader distance. By treating the lifetimes higher up the rotational band as being negligible compared to the lifetimes lower down the band, a simple exponential decay can be used to extract the upper limit of the lifetime of the $\left(15 / 2^{+}\right)$state from,

$$
\begin{equation*}
\tau \leq-\frac{x}{v \ln \left(\mathrm{I}_{\mathrm{d}} /\left(\mathrm{I}_{\mathrm{s}}+\mathrm{I}_{\mathrm{d}}\right)\right)} \tag{5.3}
\end{equation*}
$$

where $I_{s}$ and $I_{d}$ are the normalised intensities of the fully shifted and degraded components of the $596-\mathrm{keV}$ transition, $v$ is the velocity of the fully shifted recoils, $x$ is the target-to-degrader distance and $\tau$ is the lifetime of the $15 / 2^{+}$state.

The experimental number of counts seen in the degraded component of the $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transition at the shorter target-to-degrader distances is far less than that expected if the $\left(15 / 2^{+}\right)$state had the lifetime value calculated using equation 5.3 and the longer distance intensities shown in Table 5.6. However, if the counts at the $135-\mu \mathrm{m}$ distance are used to calculate the lifetime, then the lifetime is short enough that no counts would be expected at the longer target-todegrader distances, which is consistent with the experimentally observed counts within errors. The lifetime of the $\left(15 / 2^{+}\right)$state calculated in this way is then $\tau<5 \mathrm{ps}$. This is treated as an upper limit, as longer lifetimes would result in a number of counts at longer target-to-degrader distances, which are not experimentally observed. Another reason for treating the lifetime as an upper limit is the lack of treatment of the timing behaviour of the feeding of the $\left(15 / 2^{+}\right)$state. This could artificially inflate the lifetime value of the $\left(15 / 2^{+}\right)$state, as calculated from equation 5.3.

| Target-to-Degrader Distance $(\mu \mathrm{m})$ | $I_{S}$ | $I_{D}$ |
| :---: | :---: | :---: |
| 135 | $43(7)$ | $4(2)$ |
| 210 | $35(6)$ | $2(2)$ |
| 300 | $36(7)$ | $1(1)$ |
| 590 | $37(6)$ | $0(2)$ |
| 3000 | $41(8)$ | $4(2)$ |

Table 5.6: The normalised fully shifted and degraded peak intensities, $I_{S}$ and $I_{D}$, of the $596-\mathrm{keV}\left(15 / 2^{+}\right) \rightarrow\left(11 / 2^{+}\right)$transition in ${ }^{113} \mathrm{Cs}$ at different target-to-degrader distances.

### 5.9 Lifetime of the $\left(11 / 2^{+}\right)$State in ${ }^{113} \mathrm{Cs}$

The intensities of the $384-\mathrm{keV}$ transition components were extracted from the ${ }^{113}$ Cs proton-tagged Ring 2 spectra and normalised as described in Section 5.7. The normalised component intensities are listed in Table 5.7.


Figure 5.11: The fits used to extract the intensities of the fully shifted and degraded components of the $596-\mathrm{keV},\left(15 / 2^{+}\right) \rightarrow\left(11 / 2^{+}\right)$transition in ${ }^{113} \mathrm{Cs}$. The panels show proton-tagged JUROGAM-II Ring 2 spectra corresponding to different target-to-degrader distances of (a) 135-, (b) 210-, (c) 300-, (d) 590and (e) $3000-\mu \mathrm{m}$. The green fit, corresponds to the fully shifted $596-\mathrm{keV}$ peak, while the red fit corresponds to the degraded $604-\mathrm{keV}$ peak. The $610-\mathrm{keV}$ transition in band 2 of ${ }^{113} \mathrm{Cs}$ is also marked.

| Target-to-Degrader Distance $(\mu \mathrm{m})$ | $I_{S}$ | $I_{D}$ |
| :---: | :---: | :---: |
| 135 | $13(5)$ | $34(6)$ |
| 210 | $18(6)$ | $25(6)$ |
| 300 | $24(7)$ | $16(6)$ |
| 590 | $46(7)$ | $6(5)$ |
| 3000 | $52(8)$ | $5(5)$ |

TABLE 5.7: The normalised fully shifted and degraded peak intensities, $I_{S}$ and $I_{D}$, of the $384-\mathrm{keV}\left(11 / 2^{+}\right) \rightarrow\left(7 / 2^{+}\right)$transition in ${ }^{113} \mathrm{Cs}$ at different target-to-degrader distances.

The lifetime of the $\left(11 / 2^{+}\right)$state was extracted using the Differential Decay Curve Method (DDCM), described in Section 4.2. Due to the higher error on the intensities of the degraded components of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition at the longer target-to-degrader distances, the variation of the fully shifted component intensities were used instead of the degraded component intensities. The fully shifted component form of equation 4.14 was then

$$
\begin{equation*}
\tau_{i}(x)=\frac{-I_{s}^{384}(x)+\alpha_{k i} I_{s}^{596}(x)}{\frac{d}{d x} I_{s}^{384}(x)} \cdot \frac{1}{\langle v\rangle} \tag{5.4}
\end{equation*}
$$

where $x$ is the target-to-degrader distance $I_{s}^{384}$ is the intensity of the fully shifted component of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$transition, $I_{s}^{596}$ is the intensity of the fully shifted component of the $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transition, $\alpha_{k i}$ is the ratio of the intensities of the $\left(11 / 2^{+}\right)$to $\left(7 / 2^{+}\right)$and $\left(15 / 2^{+}\right)$to $\left(11 / 2^{+}\right)$transitions and $\langle v\rangle$ is the velocity of the fully shifted recoils.

The $\alpha$ coefficient is a measure of the unobserved side-feeding populating the state of interest. As the coefficient is a constant, the same value is used to calculate the lifetime at all target-to-degrader distances. The $\alpha$ coefficient was then calculated separately at each target-to-degrader distance and an average value was found and used in equation 5.4. The individual $\alpha$ values are listed in Table 5.8. From these, the average coefficient was found to be $\alpha=1.17(23)$. This $\alpha$ coefficient is consistent with the $\alpha=1.00$ (6) value calculated from Ref. [12], which implies no additional side-feeding.

All quantities within equation 5.4 are experimentally measurable with the exception of the differential of the variation of the $384-\mathrm{keV}$ fully shifted component intensity with distance. By fitting a function to the $384-\mathrm{keV}$ transition fully shifted component intensity and taking its differential the lack of a corresponding

| Target-to-Degrader Distance $(\mu \mathrm{m})$ | $\alpha$ |
| :---: | :---: |
| 135 | $1.01(24)$ |
| 210 | $1.14(41)$ |
| 300 | $1.05(41)$ |
| 590 | $1.41(43)$ |
| 3000 | $1.26(40)$ |

Table 5.8: The $\alpha$ coefficients showing the different intensities of the 596- and $384-\mathrm{keV}$ transitions which populate and depopulate the $\left(11 / 2^{+}\right)$state of ${ }^{113} \mathrm{Cs}$.
experimental observable can be overcome. To obtain the value of the differential equation 5.4 can also be rearranged to the form of,

$$
\begin{equation*}
\widetilde{t}(x)\langle v\rangle \frac{d}{d x} I_{s}^{384}(x)=-I_{s}^{384}(x)+\alpha_{k i} I_{s}^{596}(x) \tag{5.5}
\end{equation*}
$$

where $\tilde{t}$ is an initial estimate for $\tau$, known as the taufactor. By simultaneously fitting a polynomial function $f(x)$, with coefficients $a_{0}, a_{1}, \ldots . ., a_{n}$, to the variation of the fully shifted component intensities and the differential of the same function to the RHS of equation 5.5, an optimal function describing $\frac{d}{d x} I_{s}^{384}(x)$ can be found. In this work the program SNAPA $[119,120]$ was used to apply the simultaneous fits. The fit to the intensity of the fully shifted component of the $384-\mathrm{keV}$ transition versus distance took the form of two second-order polynomial fits, which were fitted piecewise to the data. The differentials of the second order polynomials, multiplied by $\tilde{t}$ and divided by $v$, were simultaneously used to fit the RHS of equation 5.5. The $\chi^{2}$ value of the simultaneous fits can be extracted from

$$
\begin{gather*}
\chi^{2}=\sum_{i}\left[\left(\frac{I_{s}^{384}-f^{\left(a_{0}, a_{1}, \ldots, a_{n}\right)}(x)}{\Delta I_{s}^{384}}\right)^{2}+\right. \\
\left.\left(\frac{\left(-I_{s}^{384}(x)+\alpha_{k i} I_{s}^{596}(x)\right)-\tilde{t} \frac{d}{d x} I_{s}^{384}(x)}{\Delta\left(-I_{s}^{384}(x)+\alpha_{k i} I_{s}^{596}(x)\right)}\right)^{2}\right] . \tag{5.6}
\end{gather*}
$$

Assuming that $\tilde{t}$ is constant, equation 5.6 can be solved from the set of linear equations

$$
\begin{gather*}
\frac{\delta}{\delta a_{n}} \chi^{2}=\sum_{i}\left[\left(\frac{I_{s}^{384}-\frac{\delta}{\delta a_{n}} f^{\left(a_{0}, a_{1}, \ldots, a_{n}\right)}(x)}{\Delta I_{s}^{384}}\right)^{2}+\right. \\
\left.\left(\frac{\left(-I_{s}^{384}(x)+\alpha_{k i} I_{s}^{596}(x)\right)-\tilde{t} \frac{d}{d x} \frac{\delta}{\delta a_{n}} I_{s}^{384}(x)}{\Delta\left(-I_{s}^{384}(x)+\alpha_{k i} I_{s}^{596}(x)\right)}\right)^{2}\right]=0 \tag{5.7}
\end{gather*}
$$

Using equation 5.7, the parameters of the polynomial fits, $a_{0}, a_{1}, \ldots . ., a_{n}$, can be extracted for a certain value of $\tilde{t}$. By varying $\tilde{t}$, optimum fit parameters may be extracted which result in a minimum $\chi^{2}$ value, as found from equation 5.6. The lifetime of the $\left(11 / 2^{+}\right)$state at each target-to-degrader distance can then be properly extracted from equation 5.8 , which takes the form of

$$
\begin{equation*}
\tau_{i}(x)=\frac{-I_{s}^{384}(x)+\alpha_{k i} I_{s}^{596}(x)}{\frac{d}{d x} f^{\left(a_{0}, a_{1}, \ldots, a_{n}\right)}} \cdot \frac{1}{\langle v\rangle} . \tag{5.8}
\end{equation*}
$$

Using a weighted average of lifetimes measured at each target-to-degrader distance, a final value for the lifetime of the $\left(11 / 2^{+}\right)$state could be obtained. The individual lifetimes used were from the 135-, 210- and $300-\mu \mathrm{m}$ target-to-degrader distances. These distances all lay within the region-of-sensitivity, in which the errors on individual lifetimes were smallest. The weighted average of these individual lifetimes gave a final value for the lifetime of the $\left(11 / 2^{+}\right)$state of $24(6)$ ps. The fits used to extract the denominator of equation 5.4 and the individual target-to-degrader distance lifetimes are shown in Fig. 5.12.

The lifetime values at all distances are approximately constant, as shown in Table 5.9. This behaviour indicates that the decay of the states which feed the $\left(11 / 2^{+}\right)$ state has been correctly considered; any unobserved side-feeding either has the same timing behaviour, or proceeds much faster than, the decay of the $\left(15 / 2^{+}\right)$ state.


Figure 5.12: Panel (a) shows the lifetimes of the $\left(11 / 2^{+}\right)$state extracted at the $135-$, 210- and $300-\mu \mathrm{m}$ target-to-degrader distances, from which a weighted average was taken to provide the final lifetime value of $\tau=24(6) \mathrm{ps}$. The fits used to extract the lifetime are shown in panels (b) and (c), see text for details. The combined $\chi^{2}$ value for both of the fits was found to be 0.242 .

| Target-to-Degrader Distance $(\mu \mathrm{m})$ | Lifetime, $\tau(\mathrm{ps})$ |
| :---: | :---: |
| 135 | $28(9)$ |
| 210 | $21(10)$ |
| 300 | $21(13)$ |
| Weighted av. | $24(6)$ |

Table 5.9: The lifetimes of the $\left(11 / 2^{+}\right)$state in ${ }^{113} \mathrm{Cs}$ extracted for individual target-to-degrader distances using the DDCM.

## Chapter 6

## Discussion

### 6.1 Calculation of the Deformation of ${ }^{113} \mathrm{Cs}$

Theoretical nonadiabatic quasiparticle model calculations, of the form described in Subsection 2.6.3, were performed to extract the deformation of ${ }^{113} \mathrm{Cs}$ from the experimental results. In these calculations, the experimental rotational spectrum of ${ }^{112} \mathrm{Xe}$ [121] was used as the core to correctly account for any non-rigid rotational behaviour of ${ }^{113} \mathrm{Cs}[16,67]$. Wavefunctions extracted from this model were then fixed and used consistently in both proton emission codes, based on the approaches discussed in Ref. [16], and standard electromagnetic transition-rate calculations, based on Ref. [66]. The half-lives for both proton emission and electromagnetic decays were then predicted using a common set of wavefunctions. The same wavefunctions were also used to calculate the excitation energies of excited states in bands 1 and 2 of ${ }^{113} \mathrm{Cs}$.

### 6.1.1 Deformation of ${ }^{113}$ Cs from Energies of Excited States

The excitation energies of band 1 in ${ }^{113} \mathrm{Cs}$ were calculated using nonadiabatic quasiparticle model wavefunctions and are shown in Fig. 6.1.


Figure 6.1: Excitation energy of the states in band 1 of ${ }^{113} \mathrm{Cs}$, relative to the energy of the $K=3 / 2^{+}$ground state, as a function of quadrupole deformation. Solid lines represent the experimental values and dashed lines the theoretical predictions.

As the $K=3 / 2^{+}$state is the most likely candidate for the ground state of ${ }^{113} \mathrm{Cs}$ $[12,14]$, the excitation energies have been predicted relative to the energy of that state. The deformation of ${ }^{113} \mathrm{Cs}$ for the lower lying states in the band is shown to agree with both the deformation predicted in the simpler adiabatic model of $\beta=0.15-0.2$ [14] and the early configuration mixed calculations of Bugrov which also predicted a deformation of $\beta \sim 0.2$ [17]. However, the deformation appears to reduce, based on the intersection of experimental and theoretical values as the rotational frequency of the nucleus increases. A contrary behaviour is seen for band 2, where the intersection between theory and experiment occurs at increasing deformation as shown in Fig. 6.2.

These discrepancies may be justified based on Fig. 6.3, taken from Ref. [12], which shows the aligned angular momentum of the states in band 1 and band 2


Figure 6.2: Excitation energy of the states in band 2 of ${ }^{113} \mathrm{Cs}$, relative to the energy of the $K=3 / 2^{+}$ground state, as a function of quadrupole deformation. Solid lines represent the experimental values and dashed lines the theoretical predictions.
versus the rotational frequency of the nucleus. The back-bending effects in both bands shown at $\hbar \omega \sim 0.4 \mathrm{MeV}$ are calculated to occur due to the breaking of neutron pairs in the $h_{11 / 2}$ shell [12]. The similarity of the alignment behaviour of both bands lends weight to this assignment for both structures. This change in the neutron configuration then changes the experimental excitation energies relative to the theoretical values, which do not include the effect of the Coriolis force on the neutrons.


Figure 6.3: Figure taken from Ref. [12] showing the aligned angular momentum versus rotational frequency of bands 1 and 2 in ${ }^{113} \mathrm{Cs}$. The back-bend seen at $0.4 \mathrm{MeV} / \hbar$ indicates the underlying configuration of the band is different to that seen at lower excitation energies.

### 6.1.2 Deformation of ${ }^{113} \mathrm{Cs}$ from Lifetimes of Excited States.

The same wavefunctions that were extracted from the nonadiabatic quasiparticle method could also be used to predict electromagnetic reduced transition probabilities [66]. These theoretical values were compared to the experimental lifetimes extracted for the $\left(11 / 2^{+}\right)$and $\left(15 / 2^{+}\right)$states. Figures 6.4 (a) and $6.4(\mathrm{~b})$ show the results of a standard electromagnetic reduced transition probability B(E2) calculation, as described in Subsection 2.5.3, for the lifetimes of the $\left(11 / 2^{+}\right)$and $\left(15 / 2^{+}\right)$states in ${ }^{113} \mathrm{Cs}$ as a function of $\beta_{2}$ deformation. Wavefunctions extracted from the nonadiabatic quasiparticle model for the $\left(11 / 2^{+}\right)$and $\left(15 / 2^{+}\right)$states of ${ }^{113} \mathrm{Cs}$ were used in these transition rate calculations. Also shown in Fig. 6.4(a) is the $24(6) \mathrm{ps}$ lifetime for the $\left(11 / 2^{+}\right)$state from the experimental DDCM analysis, denoted by the red and black dashed lines. From Fig. 6.4(a), it can be seen that the theoretical lifetime for the $\left(11 / 2^{+}\right)$state is only consistent with the experimental $\tau=24(6) \mathrm{ps}$ RDDS experimental lifetime for a quadrupole deformation
parameter, $\beta_{2} \sim 0.22-0.25$. Figure 6.4(b) shows the experimental $\tau<5 \mathrm{ps}$ limit on the $\left(15 / 2^{+}\right)$state. In the Figure, the theoretical values are only seen to overlap with this $<5 \mathrm{ps}$ experimental limit for a quadrupole deformation $\beta_{2}>0.19$, which is consistent with the deformation extracted for the $\left(11 / 2^{+}\right)$state. At deformations $\beta_{2}>0.25$ in the calculation, Band 1 is crossed by the pure [404]9/2 configuration, beyond which the lifetime is no longer reflective of the underlying configuration of Band 1.


Figure 6.4: Predicted lifetimes of the (a) $\left(11 / 2^{+}\right)$and (b) $\left(15 / 2^{+}\right)$excited states as a function of quadrupole deformation in ${ }^{113} \mathrm{Cs}$ calculated using the same quasiparticle wavefunctions used to calculate the proton-emission halflife in Fig. 6.5 (see later). The experimental lifetime of the $\left(11 / 2^{+}\right)$state and its uncertainty are denoted by the red and dashed lines in (a). The $\tau<5 \mathrm{ps}$ limit for the $\left(15 / 2^{+}\right)$state is denoted by the dashed line in (b), as discussed in the text.

### 6.1.3 Deformation of ${ }^{113} \mathrm{Cs}$ from Proton-emission Rates

Employing the same nonadiabatic quasiparticle model wavefunctions used for the $\gamma$-ray transition rates discussed above, the lifetimes for proton emission have been calculated as a function of quadrupole deformation for the $K=1 / 2^{+}, 3 / 2^{+}$and $5 / 2^{+}$states which have been predicted to be candidates for proton emission [14]. Figure 6.5 shows the results of these calculations, along with the experimental proton-emission half-life extracted in this work of $16.9(1) \mu \mathrm{s}$, which is denoted by the solid black line. The theoretical and experimental proton emission half-lives are seen to agree for the $3 / 2^{+}$state at deformation $\beta_{2}=0.08$ and also at $\beta_{2}=$ 0.22 . The smaller $\beta_{2}$ value of 0.08 is discounted as it does not agree with the deformations predicted from the excitation energies of the states of Band 1 , shown in Fig. 6.1, nor with the deformation predicted from the lifetime of the $\left(11 / 2^{+}\right)$ or $\left(15 / 2^{+}\right)$states discussed above. The higher $\beta_{2}=0.22$ value is however, in good agreement with the deformations extracted from the electromagnetic transition rates and also from the excitation energies of the states.

### 6.1.4 Summary of Different Deformation Values

The results from the various methods used to extract deformations for the $\left(11 / 2^{+}\right)$, $\left(15 / 2^{+}\right)$and $\left(3 / 2^{+}\right)$states in ${ }^{113} \mathrm{Cs}$ are summarized in Table 6.1.

| State | Result from | $\beta_{2}$ |
| :---: | :---: | :---: |
| $\left(11 / 2^{+}\right)$ | Excitation energy | $\sim 0.18$ |
| $\left(11 / 2^{+}\right)$ | Lifetime measurement | $0.22-0.26$ |
| $\left(15 / 2^{+}\right)$ | Excitation energy | $\sim 0.15$ |
| $\left(15 / 2^{+}\right)$ | Lifetime measurement | $>0.19$ |
| $\left(3 / 2^{+}\right)$ | Proton Emission lifetime | $\sim 0.22$ |

Table 6.1: A summary of deformation estimates for the $\left(11 / 2^{+}\right),\left(15 / 2^{+}\right)$ and $\left(3 / 2^{+}\right)$states from the various methods used in this work.

From Table 6.1 it can be seen that for the $\left(11 / 2^{+}\right)$state, the range of deformations for the excitation-energy calculation and the $\mathrm{B}(\mathrm{E} 2)$ calculation give an


Figure 6.5: Theoretical proton emission half-lives as a function of quadrupole deformation for low-spin states in ${ }^{113} \mathrm{Cs}$ using nonadiabatic quasiparticle wavefunctions [122]. Also shown is the $16.9(1)-\mu$ s experimental value from this work which only overlaps with the calculated half-life of the $3 / 2^{+}$state.
average and standard deviation, $\beta_{2}=0.22(6)$. This value is dominated by the $\mathrm{B}(\mathrm{E} 2)$ calculation which is the experimental value with the most sensitivity to the deformation of ${ }^{113} \mathrm{Cs}$. The deformation from the proton-emission half-life of $\beta_{2}=0.22$ is consistent with the deformation extracted for the $\left(11 / 2^{+}\right)$state as is the deformation of $\beta_{2}>0.19$ given by the half-life of the $\left(15 / 2^{+}\right)$state.

### 6.2 Reducing the Uncertainty of the Lifetime of the $\left(11 / 2^{+}\right)$State

The deformation of ${ }^{113} \mathrm{Cs}$ extracted from this work confirms that the proton is emitted from the $\left(3 / 2^{+}\right)$ground state. However, the underlying nature of this
$K=\left(3 / 2^{+}\right)$state is not fully understood. The $K=\left(3 / 2^{+}\right)$state is an admixture of many individual states which lie close to the Fermi surface and contribute significant components to the wavefunction. This is shown in Fig. 6.6 where the radial component of the different wavefunctions which form the $K=\left(1 / 2^{+}\right)$and $\left(3 / 2^{+}\right)$states have been calculated within the adiabatic approach described in Section 2.6.1.


Figure 6.6: Figure taken from Ref. [15] showing the radial components, $\alpha_{l, j}$, of the wavefunctions, which form the $K=1 / 2$ and $K=3 / 2$ states, which lie close to the Fermi surface in ${ }^{113} \mathrm{Cs}$. The wavefunctions have been calculated based on a deformation of $\beta_{2}=0.16$ using the single-particle adiabatic method described in Section 2.6.1.

As discussed in Subsection 2.6.3, the lifetime of the proton-emitting state is strongly influenced by even small components of the wavefunction. By reducing the error on the deformation of ${ }^{113} \mathrm{Cs}$, the relative strength of the different components of the wavefunction can be better understood. As the deformation is sensitive to the lifetime of the $\left(11 / 2^{+}\right)$state, by reducing the error on the lifetime of the $\left(11 / 2^{+}\right)$state the error on the deformation can in turn be reduced.

The lifetime of the $\left(11 / 2^{+}\right)$state in ${ }^{113} \mathrm{Cs}$ was extracted within the framework of the DDCM. The DDCM makes use of fully shifted and degraded component
intensities of the transitions populating and depopulating the $\left(11 / 2^{+}\right)$state, as well as the velocity of the fully shifted recoils to extract the lifetime of the state. By reducing the uncertainty on the component intensities and recoil velocity, the uncertainty on the lifetime can be reduced accordingly.

The low cross section of the ${ }^{58} \mathrm{Ni}+{ }^{58} \mathrm{Ni}$ reaction used in the experiment detailed in this work, meant that the fully shifted and degraded components had low intensities and corresponding high statistical error. Additionally, due to the running time of the experiment, only five target-to-degrader distances could be used with only three of these distances being sensitive to the lifetime of the $\left(11 / 2^{+}\right)$state. The final lifetime value was taken from a weighted average of the $\left(11 / 2^{+}\right)$state lifetime calculated at each of these three distances as described in Section 4.2.

Running a similar experiment for a greater amount of time would reduce the error on the final lifetime value, through one of two approaches. Firstly, by increasing the amount of statistics available at each target-to-degrader distance, the statistical error on component intensities could be decreased, reducing the errors on the lifetimes used to calculate the final lifetime value. Secondly, by increasing the number of distances in the region of sensitivity, more individual distance lifetime values could be obtained providing a smaller error on the final average lifetime.

The low statistics of the experiment dominated the error on the lifetime of the state. As such, for an increase in the amount of statistics at each target-todegrader distance a reduction on the lifetime error of the same order of magnitude as the reduction of the statistical error would be expected. For the case of doubling the statistics at each target-to-degrader distance, the statistical error will be reduced by a factor of $\sqrt{2}$ corresponding to the change in the Poisson statistical error. The error on the lifetime of the $\left(11 / 2^{+}\right)$state would then be reduced from 6 to 4 ps.

Instead of doubling the amount of statistics at each of the experimental target-to-degrader distances, more data could instead be collected at additional target-to-degrader distances in the region of sensitivity. The errors on each individual lifetime calculated at the individual target-to-degrader distances, shown in Table 5.9, show a small deviation. Assuming a similar spread of errors on the lifetimes for extra target-to-degrader distance in the region of sensitivity, hypothetical individual lifetimes and errors can be included in the weighted average that provides
the final lifetime value for the $\left(11 / 2^{+}\right)$state. Both theoretical and experimental target-to-degrader distances and the associated individual lifetime values are shown in Table 6.2.

| Target-to-Degrader Distance $(\mu \mathrm{m})$ | Lifetime, $\tau(\mathrm{ps})$ |
| :---: | :---: |
| 135 | $28(9)$ |
| 210 | $21(10)$ |
| 300 | $21(13)$ |
| 96 | $28(9)$ |
| 116 | $21(10)$ |
| 162 | $21(13)$ |
| 248 | $24(11)$ |
| 375 | $24(11)$ |
| Weighted av. | $24(4)$ |

Table 6.2: Experimental and hypothetical lifetimes of the $\left(11 / 2^{+}\right)$state in ${ }^{113} \mathrm{Cs}$ extracted at experimental and theoretical target-to-degrader distances within the region-of-sensitivity.

The proposed additional distances were chosen, based on the lifetime of the $\left(11 / 2^{+}\right)$state, to include a significant number of counts in both the fully shifted and degraded components of the $384-\mathrm{keV}$ transition depopulating the state. As such, all these target-to-degrader distances are expected to provide individual lifetime points within the region of sensitivity. The errors on these lifetimes would then be the same as those seen for the experimental target-to-degrader distances which lie in the region of sensitivity. The experimental lifetimes and errors have been used for the new $96-, 116$ - and $162-\mu \mathrm{m}$ distances with the experimental average lifetime and error used for the remaining 248 - and $375-\mu$ m target-to-degrader distances. Increasing the number of distances in the region of sensitivity at which data is collected gives a reduced error on the final lifetime value of 4 ps , the same reduction in error seen for collecting more data at the existing target-to-degrader distances.

Increasing the amount of target-to-degrader distances may have a greater appeal than collecting more data at the previous experimental target-to-degrader distances, as including a distance shorter than $135-\mu \mathrm{m}$ would provide a better limit on the lifetime of the $\left(15 / 2^{+}\right)$state. If the lifetime of the $\left(15 / 2^{+}\right)$state were the 5 ps upper limit, then $20 \%$ of the intensity of the $596-\mathrm{keV}$ transition depopulating the state would lie within the degraded component at a target-to-degrader distance of $96 \mu \mathrm{~m}$. This would be a measurable 8 counts based on the average intensity of the $596-\mathrm{keV}$ transition collected at each target-to-degrader distance.

If a smaller number of counts were measured in the degraded component then the limit on the lifetime of the $\left(15 / 2^{+}\right)$state could be correspondingly lowered. This target-to-degrader distance is also sensitive to the lifetime of the $\left(11 / 2^{+}\right)$state, so this method provides an opportunity to concurrently improve the measurement of the lifetime of both the $\left(11 / 2^{+}\right)$and $\left(15 / 2^{+}\right)$states.

The amount of statistics that can be gathered in a fusion-evaporation experiment, which lasts a realistic amount of time, using the two foil DPUNS plunger does not provide a dramatic reduction in the error on the lifetime. A triple-foil plunger is currently being developed at the University of Manchester and may provide a better tool for reducing the lifetime error of excited states in such a low cross section nucleus. The addition of a stopper foil means that only one distance between each of the foils needs to be used throughout the experiment. The statistical error is then reduced as the total number of counts does not have to be divided between different target-to-degrader distances. The use of a stopper foil in a triple-foil plunger means that recoil-tagging can no longer be undertaken as recoils can no longer travel from the plunger to focal plane detectors. A higher production cross section of ${ }^{113} \mathrm{Cs}$ would then be needed, which may be provided by radioactive beam facilities. As such, a facility capable of creating higher energy beams and with a different detector setup than the University of Jyväskylä, such as ISOLDE [123] may be needed for future experiments.

## Chapter 7

## Summary and Conclusions

This thesis presents data from a recoil-decay tagged differential plunger experiment, which was performed to measure lifetimes of excited states in ${ }^{113} \mathrm{Cs}$ in order to extract a value for the deformation of the nucleus. The experimental $\gamma$ ray transition rates, as well as the energies of excited states and proton-emission rates, were compared to theoretical values, which were calculated as a function of deformation using wavefunctions from a nonadiabatic quasiparticle model [16]. By checking the consistency of the deformations, deduced from the intersection of the theoretical and experimental values, the validity of the nonadiabatic quasiparticle model could be tested while giving an accurate deformation for ${ }^{113} \mathrm{Cs}$.

The lifetime of the excited $\left(11 / 2^{+}\right)$state in the most intense rotational band of ${ }^{113} \mathrm{Cs}$ was measured to be $\tau=24(6) \mathrm{ps}$ using the differential decay curve method (DDCM). Due to the low production cross-section of ${ }^{113} \mathrm{Cs}$ from the ${ }^{58} \mathrm{Ni}+{ }^{58} \mathrm{Ni}$ fusion-evaporation reaction, only five differential plunger target-todegrader distances could be used. These distances were optimised to measure the lifetime of the $\left(11 / 2^{+}\right)$state, so the DDCM could not be used to extract a value for the significantly shorter lifetime of the $\left(15 / 2^{+}\right)$state. However, the state was found to have a lifetime limit of $\tau \leq 5 \mathrm{ps}$ from intensity matching arguments.

A deformation value of $\beta_{2}=0.22(6)$ was extracted for ${ }^{113} \mathrm{Cs}$ from the lifetime and excitation energy of the $\left(11 / 2^{+}\right)$state. The lifetime limit of the $\left(15 / 2^{+}\right)$state gives a consistent deformation of $\beta_{2}>0.19$, while the experimental proton emission lifetime, of $\tau=24.2(2) \mu \mathrm{s}$, also gives a consistent deformation of $\beta_{2}=0.22$
when compared to the theoretical predictions. Aside from providing a measure for the ground-state deformation of ${ }^{113} \mathrm{Cs}$, the comparison of theoretical and experimental proton-emission rates has more firmly assigned the ground state of ${ }^{113} \mathrm{Cs}$ as having a value of $K=3 / 2$. This is the only state where the theoretical value intersects with the experimental value at a deformation which is not near spherical. This is in line with the other predictions of the nonadiabatic quasiparticle method and earlier predictions of ${ }^{113} \mathrm{Cs}$ as being deformed [13, 14, 17].

The consistency of all of the deformations, calculated using the same set of wavefunctions, underlines the validity of the predictions of the nonadiabatic quasiparticle method. The consistency of the different deformations also shows that the nonadiabatic quasiparticle model has successfully included the effects of the Coriolis interaction, in a way which was not possible for earlier nonadiabatic approaches [16, 68, 69]. The correct treatment of the Coriolis interaction is an important development for predictions concerning ${ }^{113} \mathrm{Cs}$, due to the strong mixing expected between the $K=1 / 2$ and $K=3 / 2$ orbits which lie close to its Fermi surface. The inclusion of the rotational spectrum of the daughter nucleus to provide accurate information on the structure of the core of ${ }^{113} \mathrm{Cs}$ and to account for nonadiabatic behaviour is also seen to be successful.

The inclusion of an electromagnetic transition rate calculation of the excited states provided an additional test for the predictions of the model. Additionally, the sensitivity of $\gamma$-ray transition rates to the deformation of the nucleus provides a more accurate measurement for the deformation of the nucleus than could be extracted just from proton-emission rates and excited state energies.

By reducing the error on the lifetimes of the excited states in ${ }^{113} \mathrm{Cs}$, a smaller range of deformations may be extracted from the nonadiabatic quasiparticle model. This would allow the magnitude of small components of the nuclear wavefunction to be more exactly known. More exact information on the nuclear structure of deformed proton emitters would then be provided, giving greater insight into the role of the residual and Coriolis interactions in mixing different components of the nuclear wavefunction. The errors on the lifetime of the states in this work are unlikely to be reduced using the same experimental setup as was described here, so a new setup may be needed to provide more precise measurements. The new triple foil plunger being developed at the University of Manchester and highenergy fragmentation facilities may provide a means of reducing the experimental lifetime errors.

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