DISTRIBUTION NETWORK SUPPORTS FOR TRANSMISSION SYSTEM REACTIVE POWER MANAGEMENT

A thesis submitted to The University of Manchester for the degree of Doctor of Philosophy in the Faculty of Engineering and Physical Sciences

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SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING
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# LIST OF ABBREVIATIONS AND SYMBOLS

## Abbreviations

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<th>Description</th>
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<tbody>
<tr>
<td>AVC</td>
<td>automatic voltage control</td>
</tr>
<tr>
<td>AVR</td>
<td>automatic voltage regulator</td>
</tr>
<tr>
<td>CSVC</td>
<td>coordinated secondary voltage control</td>
</tr>
<tr>
<td>DG</td>
<td>distributed generation</td>
</tr>
<tr>
<td>DMS</td>
<td>distribution management system</td>
</tr>
<tr>
<td>DNO</td>
<td>distribution network operator</td>
</tr>
<tr>
<td>DP</td>
<td>dynamic programming</td>
</tr>
<tr>
<td>DSE</td>
<td>distribution state estimation</td>
</tr>
<tr>
<td>EHV</td>
<td>extra high voltage, i.e. 275kV and 400kV</td>
</tr>
<tr>
<td>GA</td>
<td>genetic algorithm</td>
</tr>
<tr>
<td>GSP</td>
<td>grid supply point</td>
</tr>
<tr>
<td>HV</td>
<td>high voltage, i.e. from 33kV to 132kV</td>
</tr>
<tr>
<td>LDC</td>
<td>line drop compensation</td>
</tr>
<tr>
<td>LP</td>
<td>linear programming</td>
</tr>
<tr>
<td>LV</td>
<td>low voltage, i.e. lower than 1kV</td>
</tr>
<tr>
<td>MINLP</td>
<td>mixed-integer nonlinear programming</td>
</tr>
<tr>
<td>MV</td>
<td>medium voltage, i.e. from 1kV to 33kV</td>
</tr>
<tr>
<td>OLTC</td>
<td>on-load tap changer</td>
</tr>
<tr>
<td>OPF</td>
<td>optimal power flow</td>
</tr>
<tr>
<td>PV</td>
<td>photovoltaic generator</td>
</tr>
<tr>
<td>RTU</td>
<td>remote terminal unit</td>
</tr>
<tr>
<td>STATCOM</td>
<td>static synchronous compensator</td>
</tr>
<tr>
<td>SVC</td>
<td>static VAr compensator</td>
</tr>
<tr>
<td>SVR</td>
<td>step voltage regulator</td>
</tr>
<tr>
<td>RTS</td>
<td>Reliability Test System</td>
</tr>
<tr>
<td>TSO</td>
<td>transmission system operator</td>
</tr>
<tr>
<td>TVC</td>
<td>tertiary voltage control</td>
</tr>
</tbody>
</table>
Symbols

\[ \Delta h \] time interval of applying the tap staggering technique

\[ \Delta P_c \] additional active power loss of two parallel transformers due to tap stagger

\[ \Delta P_{\text{loss}} \] active power loss of distribution network caused by tap stagger

\[ \Delta P_{t,n} \] active power loss of distribution network with \( Q_{\text{required}} = n \) MVAr at the time point \( t \)

\[ \Delta Q_c \] additional reactive power absorption of two parallel transformers due to tap stagger

\[ \Delta Q_{\text{lines}} \] additional reactive power consumption of lines due to tap stagger

\[ \Delta Q_{\text{loads}} \] additional reactive power consumption of loads due to tap stagger

\[ \Delta Q_{t,n} \] reactive power absorption measured at GSP with \( Q_{\text{required}} = n \) MVAr at the time point \( t \)

\[ \Delta Q_{t_m} \] total reactive power absorption (in MVAr) provided through the use of tap stagger at the time point \( t_m \)

\[ \Delta Q_{\text{transformers}} \] additional reactive power consumption of transformers due to tap stagger

\[ \Delta TAP \] tap position increment (%) per tap step of OLTC

\[ C_{i,j} \] reactive power absorption that could be individually provided by the \( i^{th} \) pair of parallel transformers with \( 2j \) staggered taps (i.e. \( j \) taps up for one and \( j \) taps down for the other)

\[ C_{\text{reactor}} \] annual cost for TSO to apply reactors to reduce high voltages

\[ C_{\text{stagger}} \] annual cost for TSO to apply the tap staggering technique

\[ C_{VAr} \] unit price of reactive energy in £/MVArh

\[ C_{\Delta \text{gen}}(h) \] cost to compensate generation changes due to tap stagger

\[ C_{\Delta \text{gen}}'(h) \] cost to compensate generation changes due to the use of reactors

\[ C \] network reactive power absorption capability matrix

\[ e_{\text{limit}} \] maximum permitted error (%) between the required and the actual reactive power absorption provided

\[ h \] total hours of using tap stagger in a year, i.e. the utilisation hour

\[ I \] annual equivalent investment cost of installing a reactor

\[ I_1 \] secondary current on Transformer T_1
\( I_2 \) secondary current on Transformer \( T_2 \)
\( I_c \) circulating current around two parallel transformers
\( k \) number of tap steps increased or decreased from the initial tap position
\( M \) maximum allowable tap difference from the initial tap position
\( n_m \) nominal transformer ratio
\( N \) total number of pairs of parallel transformers involved in the tap stagger optimisation
\( N_e \) number of elites
\( N_{pop} \) population size
\( P_{i,j} \) network loss caused by the \( i^{th} \) pair of parallel transformers with \( 2j \) staggered taps (i.e. \( j \) taps up for one and \( j \) taps down for the other)
\( P \) network power loss matrix due to tap stagger
\( Q_{actual} \) actual reactive power absorbed by the downstream distribution network through the use of tap stagger
\( Q_{required} \) additional reactive power absorption required by the upstream transmission network to maintain system voltages
\( r \) discount rate
\( R^2 \) coefficient of determination
\( S \) actual capital investment of installing a reactor
\( T \) total time points in a year
\( TAP_0 \) initial transformer tap position in per-unit
\( TAP_i \) initial tap positions on the \( i^{th} \) pair of parallel transformers in per-unit
\( TAP_{i,max} \) upper limit of the tap positions on the \( i^{th} \) pair of parallel transformers
\( TAP_{i,min} \) lower limit of the tap positions on the \( i^{th} \) pair of parallel transformers
\( TS_{max} \) maximum allowable difference between the tap positions of two parallel transformers without causing overheating and damages
\( V_1 \) primary voltage referred to the secondary side of Transformer \( T_1 \)
\( V_2 \) primary voltage referred to the secondary side of Transformer \( T_2 \)
\( V_p \) transformer primary voltage
\( V_s \) transformer secondary voltage (connected to loads)
$w_1$  weighting coefficient of active power loss

$w_2$  weighting coefficient of tap changer switching operations

$w_3$  weighting coefficient of reactive power absorption

$x_i$  number of tap steps different from the initial tap positions on the $i^{th}$ pair of parallel transformers

$x_{i,j}$  binary integer that determines whether to tap apart the $i^{th}$ pair of parallel transformers

$x$  tap staggering control vector

$y$  economic life of a reactor

$Z_t$  transformer equivalent series impedance in ohms, referred to the secondary side

$Z_{t,pu}$  transformer equivalent series impedance in per-unit

$Z_L$  equivalent load impedance in ohms
LIST OF PUBLICATIONS


ABSTRACT

The University of Manchester

Linwei Chen

A thesis submitted for the degree of Doctor of Philosophy

Distribution Network Supports for Transmission System Reactive Power Management

November 2015

To mitigate high voltages in transmission systems with low demands, traditional solutions often consider the installation of reactive power compensators. The deployment and tuning of numbers of VAr compensators at various locations may not be cost-effective. This thesis presents an alternative method that utilises existing parallel transformers in distribution networks to provide reactive power supports for transmission systems under low demands. The operation of parallel transformers in small different tap positions, i.e. with staggered taps, can provide a means of absorbing reactive power. The aggregated reactive power absorption from many pairs of parallel transformers could be sufficient to provide voltage support to the upstream transmission network.

Network capability studies have been carried out to investigate the reactive power absorption capability through the use of tap stagger. The studies are based on a real UK High Voltage distribution network, and the tap staggering technique has been applied to primary substation transformers. The results confirm that the tap staggering method has the potential to increase the reactive power demand drawn from the transmission grid.

This thesis also presents an optimal control method for tap stagger to minimise the introduced network loss as well as the number of tap switching operations involved. A genetic algorithm (GA) based procedure has been developed to solve the optimisation problem. The GA method has been compared with two alternative solution approaches, i.e. the rule-based control scheme and the branch-and-bound algorithm. The results indicate that the GA method is superior to the other two approaches.

The economic and technical impacts of the tap staggering technique on the transmission system has been studied. In the economic analysis, the associated costs of applying the tap staggering method have been investigated from the perspective of transmission system operator. The IEEE Reliability Test System has been used to carry out the studies, and the results have been compared with the installation of shunt reactors. In the technical studies, the dynamic impacts of tap staggering or reactor switching on transmission system voltages have been analysed. From the results, the tap staggering technique has more economic advantages than reactors and can reduce voltage damping as well as overshoots during the transient states.
DECLARATION

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CHAPTER 1 INTRODUCTION

1.1 Low Carbon Electricity Networks

Climate change, which includes global warming, sea level rise and extreme weather, has been drawing more and more public concern over the years. There is compelling evidence that the major cause of climate change is human activities, for instance burning fossil fuels, transport and industrial processes [1], [2]. The carbon emissions from such activities have exacerbated the greenhouse effect, which has resulted in the observed temperature increases. Concerning the increasing greenhouse gas (GHG) emission issues, the European Council has published the “20 20 by 2020” package, which aims to reduce at least 20% of GHG emissions and to increase the share of renewable energy consumption in the European Union (EU) to 20% [3]. In the United Kingdom (UK), the ‘Climate Change Act of 2008’ also specifies the government’s duty to reduce the net UK carbon account for the year 2050 at least 80% lower than 1990 [4].

In 2013, the UK domestic GHG emissions were 564 MtCO₂e (i.e. metric tonne carbon dioxide equivalent), with 26% coming from the electric power sector [5]. As the electricity generation produces the largest portion of emissions, decarbonising the power sector is the key part of carbon reduction. The renewable energy generation with few GHG emissions is therefore considered as one of the most effective ways to construct low carbon networks. According to the National Renewable Statistics, the UK electricity generated from renewable sources increased by 113% between 2009 and 2013, to reach 53.7 terawatt hours (TWh), taking a 14.9% share of the total electricity generation [6]. The overall wind generation in 2013 was around 28.4 TWh, which was 206% higher than 2009 (as shown in Figure 1-1). For the solar photovoltaic (PV) schemes, the installed capacity increased from 0.027 gigawatt (GW) in 2009 to 2.78 GW in 2013.

Due to the low-carbon incentives offered by the UK government, e.g. Feed-in Tariff
(FiT), large amounts of small-scale generation using renewable resources have been connected to electricity distribution networks [7]. This kind of generation is often known as distributed generation (DG) or embedded generation. The common features of DG are listed as [8]:

- Normally connected to the distribution system.
- Usually smaller than 50-100 MW in capacity.
- Not centrally planned and dispatched by the utility.

![Figure 1-1: Electricity generation by main renewable sources since 2000](Image)

The main reason for introducing DGs is to generate electricity from distributed renewable resources and then to reduce carbon emissions. In addition, as DG is located closer to the load centre or customers, it reduces the costs of transporting electricity via the transmission system. Furthermore, DG is normally of small size hence the construction time is short, and the capital costs are lower than the central generating station [8]-[10].

Traditionally, the distribution system has been designed as a passive network that only receives power from the transmission system. The electric power normally flows from large generating stations, through the transmission and distribution networks, to the loads.
(e.g. industrial, commercial and domestic customers) as shown in Figure 1-2. However, the increasing connections of DGs (e.g. wind farms and solar PV panels) may change this pattern of power flow. During periods of low demand with high DG penetration, some power would be forced back to the upstream network, resulting in a reversed power flow. On the other hand, electric vehicles and heat pumps are now being widely spread due to their low GHG emissions [11], [12]. As a consequence, the electricity demand is likely to increase, and an active network control is required to handle the technical challenges caused by both DGs and new types of loads. Some of the main technical issues associated with active distribution networks are summarised below [8]:

- Network voltage changes caused by DG power injection.
- Protection issues, such as loss-of-mains protection of DGs.
- Power quality, such as harmonic distortion of network voltages by power electronics.

This research project will focus on the voltage rise problem that currently arises in the UK transmission system and will propose a new reactive power management method to address the issue.

![Figure 1-2: Modern electric power system with distributed generation](image)
1.2 Voltage and Reactive Power Control

The electric power system is commonly a three-phase alternating-current (AC) system comprising different voltage levels. To reduce transmission losses, the transmission system is composed of lines with very high voltages. For the UK power system, the typical transmission voltage levels are 400 kilovolts (kV), 275kV and 132kV (in Scotland) [13], [14]. The distribution network voltages are normally 132kV, 33kV, 11kV, 6.6kV and 400 volts (V). The line voltages change continuously with the varying electricity demands. Network operators are responsible for maintaining the voltages high enough to drive the loads while constraining the voltages to prevent equipment breakdown [15]. In general, since the impedances of the network components are predominantly reactive, voltage control is accomplished by managing the production, absorption and flow of reactive power at all levels in the system [16]. The electrical load, e.g. motor, consumes reactive power as well as active power. Network elements both consume and produce reactive power. For instance, power transformers and overhead lines consume reactive power. Lines and underground cables can also generate reactive power due to their shunt capacitance [17]. However, the reactive power produced is only significant at high system voltages or on light loads. During the steady-state operation of a power system, the reactive power generation should match the load plus loss. An excess of reactive power in an area may result in high voltages, or a deficit may lead to low voltages. The relationship between reactive power and voltage is similar to the one between active power and frequency.

Figure 1-3 illustrates a system with various components and devices combined to provide voltage and reactive power (Volt/VAr) control. The network components or devices used for Volt/VAr control may be summarised as follows [18]:

- Generating units, such as synchronous generators.
- Sources or sinks of reactive power, such as passive compensators including shunt reactors and shunt capacitors, and active compensators including synchronous condensers, static VAr compensators (SVCs) and static synchronous compensators.
(STATCOMs).

- Line reactance compensators, such as series capacitors.
- Regulating transformers, such as on-load tap changer transformers.

**Generating units**

The generating units provide the primary voltage control in the system. Synchronous generators are usually equipped with automatic voltage regulators (AVRs), which adjust the field excitation to maintain the scheduled voltages at the terminals of the generators. The control of excitation also determines whether the generators would produce or absorb reactive power.

![Figure 1-3: Voltage and reactive power control with different components and devices in power system](image)

**Reactive power compensation devices**

Due to the high reactance of network elements, reactive power cannot be transmitted easily in the system. Voltage control has to be effected by distributing additional reactive power compensation devices throughout the system [19], [20]. Shunt reactors and capacitors provide passive compensation, which improves voltage profiles and reduces system losses. They are either permanently connected to the transmission and distribution
systems, or switched. Series capacitors are installed to reduce the inductance of transmission lines and to increase the power transfer capability. Synchronous condensers, SVCs and STATCOMs provide active compensation. The produced or absorbed reactive power is automatically adjusted to maintain constant voltages at the connecting buses. The active compensating devices and generating units together establish voltages at specific points in the system. Voltages at other locations are determined by the active and reactive power flows in the circuit. 

**Regulating transformers**

Transformers with tap-changing facilities are also widely used to regulate voltages in the system at various voltage levels. The changes of tap position will affect the terminal voltages and also the reactive power flow through the transformer. There are two types of tap changer: off-load and on-load. The transformers with on-load tap changers (OLTCs) can regulate voltages under varying load conditions without interruption. For the UK transmission system, autotransformers connecting a 400kV or 275kV network to a 132kV or 66kV network are usually equipped with OLTCs [18]. In distribution networks, primary substation transformers (33/11 kV or 33/6.6 kV) are usually fitted with OLTCs [21].

### 1.3 Motivation

One of the important problems concerning the integration of DGs into power systems is the voltage rise issue. At the connection point (or Point of Common Coupling), DG starts to boost the local voltage by injecting real power. However, the current distribution network operator (DNO) policy is based on the “fit and forget” approach, which requires DG to operate at a fixed power factor (e.g. unity power factor) without regulating reactive power to control the voltage [22], [23]. This practice certainly limits the capacity of DG connected and worsens the voltage rise effect. The voltage may sometimes rise over the maximum statutory limit and damage devices at customer sides. In [24], some case studies have demonstrated the voltage rise issue encountered at distribution networks.
The voltage rise problem can also occur in the transmission system. According to National Grid Electricity Ten Year Statements [25] and [26], high voltage situations under low demand conditions are currently increasing in the UK transmission grid. The main reasons include:

- Development of underground cables in transmission and distribution networks.
- Decommissioning of coal generators in specific areas, resulting in a lack of generator voltage control.
- Reduction in reactive power demand during periods of minimum load.

Underground cables commonly have large capacitance, which produces excess reactive power when the system is lightly loaded. The growing connection of large offshore wind farms to the grid would require more submarine cables to be installed, which may also cause voltages to rise if without proper VAr compensation [27]-[29].

In addition, National Grid, the Great Britain (GB) transmission system operator, has reported the gradual decline in reactive power demand during minimum load periods (e.g. summer nights). Figure 1-4 demonstrates the historical trend of minimum active and reactive power demands for the GB transmission system since 2005 [26]. The figure also shows that the ratio between minimum reactive power and active power demands (\(Q/P\) ratio) decreased by almost half from 2005 to 2012. During periods of low demand, National Grid has observed low reactive power demand or even reactive power injection at several Grid Supply Points (GSPs), where distribution networks are connected to the GB transmission system. In some cases, reactive power has been exported from distribution networks back to the transmission grid. The reasons behind this declining trend of reactive power demand during minimum load periods are still uncertain. The development of underground cables in distribution networks, the use of more energy-efficient household appliances and the impact of DGs are some of the possible factors affecting the system VAr demand. National Grid has carried out further investigations to understand the exact cause of the reduction in reactive power demand [30].
The reduction of reactive power demand may result in excess VAr surplus in the system and may cause voltage stability and overvoltage issues [31]. For instance, in 2011-12, there were 165 reported voltage excursions within the GB transmission system, primarily due to the abnormal decline in overnight reactive power demand [32]. Apart from GB, the Romanian transmission network has also experienced voltage rises caused by large reactive power surplus in the system [33]. Currently, available methods to maintain transmission system voltage with excess reactive power flow include [25], [26]:

- Using generating plants to absorb reactive power and control voltages.
- Switching off capacitive lines during periods of low demand.
- Employing additional VAr compensation equipment (e.g. shunt reactor and SVC) to absorb reactive power surplus in the system.

Nowadays, the growing integration of large-scale intermittent renewable resources, e.g. wind power, into the grid has increased the difficulties in balancing generation and demand. As a consequence, voltage problems could become more dynamic and may occur in different time periods or different regions. The placement and tuning of numbers of VAr compensation devices at various locations would prove to be challenging for the transmission system operators (TSOs) [34]. On the other hand, to comply with the forthcoming European Demand Connection Code [35], distribution networks should have...
the capability to maintain a limited range of reactive power consumption at interfaces with transmission systems. The rapid development of smart grid technologies would facilitate the demand side response control in distribution networks. Therefore, considering the VAr management challenges from both the transmission and distribution sides, this study will concentrate on developing a cost-effective means of utilising distribution networks to provide reactive power support for transmission systems.

### 1.4 Research Objectives

To mitigate the aforementioned reactive power surplus in transmission systems during periods of low demand, traditional solutions often involve installing shunt reactors or VAr compensators. The associated costs will be high if numbers of devices need to be placed at various locations in the system. One alternative method is to increase the reactive power consumption of the downstream connected distribution networks. The operation of parallel transformers in different tap positions, i.e. with staggered taps, will introduce a circulating current around the pair. Due to the transformer inductance, the circulating current will draw more reactive power from the upstream network. For each pair, the difference in tap positions should be set within a small range to avoid the transformer overheating. However, the aggregated reactive power absorption from many pairs of parallel transformers within the distribution networks may be sufficient to provide VAr support to the upstream transmission system.

Therefore, the research objective is to develop a reactive power control method that utilises the existing parallel transformers in distribution networks to provide VAr supports for transmission systems. The outcomes may contribute to creating a new business opportunity for distribution networks to participate in the reactive power balancing services in the future. The smart management of distribution networks may also enhance the reliability and stability of the transmission grid. Based on this concept, the main research tasks are listed as follows:

- Literature survey on recent Volt/VAr control development in transmission and
distribution systems, especially on the technologies to reduce reactive power surplus.

- Modelling of typical UK High Voltage (HV) distribution networks, from 132kV GSPs down to 33kV primary substations with parallel transformers.
- Investigation of the reactive power absorption capability of the modelled distribution networks with the tap staggering technique.
- Development of a control algorithm to minimise the network loss and the number of tap switching operations introduced when applying the tap staggering technique.
- Test and comparison of the tap stagger control with other alternative algorithms and distribution network models.
- Modelling of a transmission system to analyse the consequential impacts of applying the tap staggering method.
- Assessment of both economic and dynamic effects of the tap staggering technique on the transmission system.
- Comparison with other VAr compensation methods used to reduce transmission system VAr surplus.

### 1.5 List of Main Contributions to Work

The main contributions of this thesis are given below:

- Propose a new reactive power control method, which utilises the existing parallel transformers in distribution networks, to mitigate high voltages in transmission systems during periods of low demand.
- Assessment of the VAr absorption capability of a real UK HV distribution network with the tap staggering technique.
- Design of an optimal control approach based on genetic algorithm (GA) to coordinate the tap staggering operation of multiple pairs of parallel transformers. An objective function is proposed to minimise the introduced network loss as well as the number of tap switching operations.
- Comparison of the GA-based tap stagger control approach with two alternative methods, i.e. the rule-based control scheme and the branch-and-bound method, using
two UK HV distribution network models.

- Comparison of the impacts of the tap staggering technique and the use of shunt reactors on transmission systems, in terms of economic costs and dynamic performance.

### 1.6 Thesis Outline

The present Chapter 1 briefly introduces the background of power system voltage and reactive power control, describes the research subject and motivation. Detailed research objectives are listed along with the main contributions to the work. The remainder of this thesis is organized as follows:

**Chapter 2** presents a critical literature review of existing voltage and reactive power control techniques in transmission and distribution systems, focusing on the steady-state power flow control. Voltage regulation schemes with the presence of renewable energy generation or DG are described. Different methods for reactive power management in transmission systems are discussed. The optimisation algorithms for voltage and reactive power control are also investigated to understand their working principles, advantages and limitations. From the literature, an alternative reactive power management method to mitigate reactive power surplus in the transmission system is proposed in Chapter 4.

**Chapter 3** describes the fundamental theories related to the proposed reactive power management method, for the purpose of a better understanding of the subsequent chapters. It first presents the basic structure of distribution networks and the relationship between voltage and reactive power control. Tap changer mechanisms and the load flow modelling of the transformer with tap changer are given.

**Chapter 4** proposes a reactive power control method, which utilises the existing parallel transformers in distribution networks, to mitigate high voltages for transmission systems during periods of low demand. The proposed ‘tap staggering’ method aims to use the tap position differences of parallel transformers to introduce virtual inductive loading onto
the upstream system. The mathematical principles of the tap staggering operation are discussed. An optimal control approach for the tap stagger is also proposed, which aims to minimise the active network loss as well as the number of tap switching operations introduced. The following Chapter 5 first presents the feasibility studies of assessing distribution network reactive power absorption capabilities using the tap staggering technique. Chapters 6 and 7 then describe the optimisation studies for the tap stagger with different solution algorithms. Chapter 8 investigates the impacts of tap stagger on the transmission system.

Chapter 5 presents the feasibility studies to investigate the VAr absorption capability of distribution networks through the use of tap stagger. The modelled distribution system is based on a real UK distribution network from the 132kV GSPs down to 33kV primary substations with parallel transformers. Off-line load flow studies are carried out to analyse the overall network VAr consumption as well as the power loss with tap stagger.

Chapter 6 describes the implementation of the tap stagger optimal control. As the optimisation is an integer nonlinear problem, a genetic algorithm (GA) based procedure is developed to find an optimal dispatch for the tap positions of the parallel transformers. Studies are carried out to test the impacts of reactive power requirements, load models and demand levels on the optimisation results. The comparisons with other solution algorithms are given in Chapter 7.

Chapter 7 focuses on the comparisons of the tap stagger control with two other alternative methods, i.e. the rule-based approach and the branch-and-bound algorithm. Two practical UK distribution networks are modelled from real data and used to demonstrate the effectiveness of the three control approaches with different input settings.

Chapter 8 presents both the economic and technical analyses of using the tap staggering method. In the economic analysis, the associated costs of applying the tap staggering technique are investigated through static load flow studies. The IEEE Reliability Test System is used to carry out the studies, and the results are compared with the installation
of shunt reactors. In the technical studies, the dynamic impacts of tap staggering or reactor switching on transmission voltages are analysed.

Chapter 9 draws conclusions from the work described in this thesis. The main findings of the research are discussed, and possible directions for future work are suggested.
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter presents different voltage and reactive power control methods that have been currently applied to or proposed for power systems over the years. The review starts with a discussion of traditional voltage regulation methods used in transmission or distribution networks. This is followed by an overview of recent voltage control approaches developed considering the integration of DG or renewable energy generation into the grid. As this research project is concerned with the voltage problems due to reactive power surplus in transmission systems, the existing reactive power compensation techniques for transmission systems are presented and discussed. The related Volt/VAr optimisation control schemes, which coordinate multiple VAr compensators to optimise network performance, are also described. Finally, a summary is provided to comment on the Volt/VAr control and the associated optimisation algorithms.

2.2 Traditional Voltage Regulation Methods

Both utility equipment and customer equipment that are connected to a power system are designed to operate within a certain voltage range. To avoid any adverse effects on the equipment caused by voltage violations, network operators are responsible for maintaining the system voltages within statutory limits [36].

Since electricity loads continuously change, there must be some means of regulating network voltages to satisfy the requirements. As described in Chapter 1, most voltage control methods are based on the principle of managing reactive power flow in the system. The details are given in the following text.
2.2.1 Transmission system voltage control

2.2.1.1 Automatic voltage regulators

Generating units connected to the transmission grid provide the most elementary voltage control for the system. The AVRs equipped on synchronous generators adjust the field excitation to regulate the voltages at connected buses. Figure 2-1 illustrates the block diagram of a typical AVR closed-loop control [37]. The AVR monitors the generator terminal voltage $V$ and compares it with a desired reference voltage $V_r$. The resulting voltage error is then amplified and used to alter the exciter output, i.e. the generator field current. To stabilise the excitation control system and improve its dynamic performance, a derivative feedback is usually adopted as shown in Figure 2-1. The response time of the AVR control is extremely rapid, within a time frame of a few seconds [38], [39].

![Figure 2-1: Block diagram of a closed-loop automatic voltage regulator [37]](image)

The control of excitation also affects the reactive power production of generators. When underexcited, generators absorb reactive power, and they supply reactive power when overexcited. The excitation system can be controlled to maintain constant reactive power or power factor (PF) instead of constant terminal voltage. However, compared with the AVR controllers, the VAr/PF controllers should generally not be installed on generators that intend to provide voltage support for transmission systems. The VAr/PF controllers will prevent necessary reactive power changes during long periods of system voltage excursions [40].
2.2.1.2 Secondary voltage control

The generators with AVR provide the primary voltage control, which aims to compensate the rapid and random voltage variations at the locally connected buses. To handle slow and large voltage variations (e.g. caused by demand changes), secondary voltage control has been implemented in some European countries, such as France [41], [42] and Italy [43], [44]. The principle of secondary voltage control is to divide a transmission network into several uncoupled zones and to control the voltage profile separately in each zone. A pilot bus is selected as a representative bus for each zone and the bus voltage is controlled via automatic adjustments of the AVR of several generators located in the zone.

The main objectives of secondary voltage control are to maintain the pilot bus voltage at a specified value and to coordinate reactive power sources within a control zone. Figure 2-2 shows how to perform the control [41]. The zone controller located in the regional control centre acquires the pilot bus measured voltage $V_p$ and compares it with a predefined reference $V_r$. According to the voltage error, a control signal is generated and transmitted to each controlling generator inside the zone. For each generator, the control signal is multiplied by a corresponding participation factor $Q_r$ and the result is used as the input to a reactive power control loop. The loop is designed to regulate the generator VAr production by adjusting the AVR voltage reference. The secondary voltage control closes the control loop of the reference settings of AVR at the primary level. It also provides an essential coordination of the reactive power of different controlling generators. The response time is typically in the range of 3 minutes [39].
A key factor for the appropriate functioning of secondary voltage control is the selection of pilot buses. The pilot buses should represent the voltage variations throughout the zones and should be sensitive to generator control actions. In [45] and [46], pilot buses have been chosen so that the expected values of voltage deviations of all load buses, due to random disturbances in all reactive power loads, are minimised in steady-state. The solution procedure consists of two steps: a greedy algorithm to generate an initial set of pilot buses, and a global search to improve the greedy selection [46].

Apart from utilising generator AVRs, the secondary voltage control can also involve the coordination of VAr compensation devices. Wang has proposed a multi-agent based secondary voltage control to improve the voltage responses in system contingencies [47]. Three different types of voltage controllers, i.e. AVR, SVC and STATCOM, formed three individual agents in the system. They shared voltage control tasks based on either the communication between agents or the local estimation. As the multi-agent voltage control can only cover the buses where the voltage controllers are installed, a collaboration protocol has been proposed to extend the coverage to the adjacent locations around voltage controllers [48]. In the studies, two SVCs and two STATCOMs were coordinated to support each other using a ‘request-and-response’ protocol. A learning
fuzzy logic controller was designed to implement the control algorithm for each voltage controller.

2.2.1.3 Coordinated secondary voltage control & tertiary voltage control

The secondary voltage control only maintains the voltage profile inside a control zone. Considering the interaction between zones, the concept of coordinated secondary voltage control (CSVC) has been introduced [38]. The control strategy relies on the partition of a network into regions, which are much larger than zones and include more than one pilot bus. In each control region, the CSVC aims to control the voltages of pilot buses and to maintain the reactive power generation of each controlling generator at a reference value. The control variables are the AVR voltage setpoints and are obtained as the solutions of an optimisation problem, which minimises the sum of pilot bus voltage deviations and generator VAr productions, subject to various network constraints. In [49], a decentralised control approach for CSVC has been presented concerning the VAr compensation from neighbourhood regions. If a control region has voltage violations due to load disturbances, the CSVC controllers located at pilot buses will first employ generators or VAr compensators to regulate the voltages. In a severe contingency when the disturbed region requires more reactive power, the CSVC controllers in neighbourhood regions can transfer their extra reactive power to the affected region. To achieve this control strategy, connection matrices were used to represent the possible coordination between controllers in a region or different regions.

Voltage setpoints of pilot buses can be determined by the tertiary voltage control (TVC). At the national level, the TVC utilises the global information of the transmission grid, and it updates the reference values for the secondary voltage control by solving optimisation problems [43], [50] and [51]. The optimal voltage setpoints for pilot buses are determined to minimise the grid losses while still preserving a sufficiently reactive power margin. In the short term, the optimisation program can be carried out one day in advance based on load forecasting. Alternatively, in the very short term, the program can run every 15-30 minutes using the online state estimation [43], [44].
2.2.2 Distribution system voltage control

As distribution systems have traditionally been designed as passive networks without connections to large generating plants, the voltage regulation often considers the use of reactive power compensation devices or tap changing transformers. The details are given in the following text.

2.2.2.1 Common voltage control techniques for distribution networks

A number of techniques is available for the voltage regulation in distribution networks. Some common methods are listed as [52], [53]:

- Installing reactive power compensation devices (e.g. shunt capacitors).
- Building new substations and primary feeders.
- Increasing the conductor size of existing feeders.
- Rearranging the system, transferring loads.
- Adding series capacitors to the primary feeders.
- Utilising substation transformers with on-load/off-load tap changers.
- Applying voltage regulators on the primary feeders.

The application of reactive power compensation equipment, e.g. shunt capacitor and reactor, synchronous condenser and static var compensator (SVC), is able to provide the reactive power locally when demand is high or to absorb the reactive power when demand is low. They can be installed at different locations in distribution systems to ensure the voltages remain within the permitted range as the loads vary. In addition, the locations and VAr outputs of compensators can be optimised to minimise the network losses and improve the efficiency.

The building of new substations and primary feeders intends to reduce the power flow on each line so that less voltage drop (or rise) is introduced. Similarly, the voltage drop can also be reduced by decreasing the line impedance (e.g. increasing the conductor size or
adding series capacitor). However, all these network reinforcements tend to be very expensive and time consuming.

Both the reactive power compensation and network reinforcement are indirect ways of controlling the voltages in distribution networks. To regulate the voltage directly, an on-load tap changer (OLTC) transformer or a step voltage regulator are often used. The transformer at a distribution substation is usually equipped with an OLTC that automatically controls the secondary voltage. The OLTC transformer provides voltage regulation at the substation level. For feeder regulation, a step voltage regulator is commonly used to boost or step down the voltage on feeder without changing the basic voltage level [18], [53]. Both types of regulation are described below.

2.2.2.2 Substation voltage regulation
The OLTC transformer at the distribution substation provides the voltage regulation under load without interruption. Figure 2-3 illustrates the basic arrangement of substation voltage control. As shown in the figure, the OLTC normally operates in conjunction with an automatic-voltage-control (AVC) relay.

![Diagram of OLTC transformer with AVC relay](image)

Figure 2-3: Basic arrangement of an OLTC transformer with AVC relay [21]
The AVC relay continuously monitors the transformer secondary voltage at the substation and compares the measurement with its reference voltage \( V_{SETPOINT} \). If the measured voltage still exceeds the preset deadband after a specified time delay, a tap changing command will be sent to the OLTC. The tap position will then be altered accordingly by the OLTC so that the transformer secondary voltage can be regulated back to its required range. The AVC relay deadband and time delay are used to reduce the unnecessary tap changing operations caused by transient voltage variations [54].

**Automatic voltage control using line-drop compensation**

The control method described above is only capable of keeping the voltage constant at the local substation. To control the voltage at some remote load centre, the line-drop compensation (LDC) method has been widely adopted [21]. A general schematic of the LDC control is presented in Figure 2-4.

![Figure 2-4: Key elements of the line-drop compensation [21]](image)

With the measurements of the transformer secondary voltage \( V_{BB} \) and current \( I \), the LDC can estimate the actual voltage at the load centre and set a higher voltage reference at the substation to compensate the voltage drops on the line between the transformer and load centre. Figure 2-5 shows the corresponding phasor diagram. The values of \( R \) and \( X \) inside...
the relay are used to simulate the real impedance \( R_L + jX_L \) of the line. The employment of the LDC method allows the OLTC at the substation to control the voltage at the load centre without adding additional communication channels.

\[
\begin{align*}
V_{BB} & \\
I & \\
V_{FB} & \\
IR & \\
jIX & \text{(voltage at load centre)}
\end{align*}
\]

Figure 2-5: Phasor diagram for the LDC method

**Supervisory substation Volt/VAr controller**

A supervisory Volt/VAr controller has been proposed in [55] to improve the performance of the LDC method. This new controller is based on the voltage drop characteristic of each feeder it regulates. Instead of using \( R \) and \( X \), an effective impedance \( Z_{\text{eff}} \) is used to estimate the voltage drop on each feeder. This method is also able to correct the power factor of the substation by integrating capacitor banks.

### 2.2.2.3 Feeder voltage regulation

A distribution substation usually serves one or more radial feeders. In the case where the feeder is very long, the voltage control only at the substation may not be sufficient to keep all the voltages along the feeder within acceptable limits. Therefore, a specified control for the feeder voltages is necessary.

A common device used to maintain feeder voltages is the step voltage regulator (SVR), which consists of an autotransformer and an on-load tap changing mechanism [56]. The voltage change is achieved by varying the taps on the series winding of the autotransformer as illustrated in Figure 2-6. Depending on the polarity of the series winding, the voltage introduced in the series windings is either added to or subtracted from the primary voltage. The polarity is changed using a reversing switch.
Note that the SVR is purely a voltage control device, and it is not designed for voltage transformation. The SVR regulates the voltage on its secondary side. Alternatively, the SVR can be integrated with the LDC to control the voltage at some selected point out on the feeder. Shunt capacitors can also be used together with the SVR to offer feeder voltage regulation. Figure 2-7 demonstrates the voltage control on a feeder by applying SVRs and shunt capacitor. As shown in the figure, two SVRs and a shunt capacitor need to be placed along the feeder in order to bring the entire voltage profile within the limits.
2.3 Voltage Control Methods with Distributed Generation

As mentioned in Chapter 1, the growing integration of renewable energy generation into grids may cause voltage problems on the existing power systems. For transmission networks, the connection of large-scale intermittent renewable resources, e.g. wind power, increases the difficulties in balancing generation and demand. For distribution systems, small variable DGs (e.g. solar PV panels) have been connected to the networks, which may change or even reverse the power flows at the points of connection. The traditional voltage control schemes designed for the passive system with unidirectional power flow may become inadequate [57]. The electricity generation of DGs may also be limited due to the voltage violations caused. Considering the voltage issues, the network management system design will need to be transferred from the traditional ‘passive’ network control to a fully ‘active’ network operation in the future. The following literature reviews present several voltage regulation methods associated with renewable energy generation, concentrating mostly on distribution systems. The control principles can be divided into three main parts: on-load tap changer control, reactive power control and power curtailment. The details are given below.

2.3.1 On-load tap changer control

The traditional substation voltage control with the LDC algorithm is designed for the passive network with the unidirectional power flow. In the active distribution network with DGs injecting current to the feeder, the current measured at the substation is no longer proportional to the load current. As a consequence, the LDC method using current measurements becomes less effective at regulating the voltage at the load centre [21]. Alternative control techniques utilising OLTC transformers have been proposed for the active network.

2.3.1.1 SuperTAPP n+ relay scheme

In [58] and [59], SuperTAPP n+ relay scheme has been introduced to control the voltage in the network with DGs. This type of relay is capable of estimating the output current of
the generator that is connected at some remote point on the feeder. Figure 2-8 shows a distribution network where the SuperTAPP n+ relays control two parallel OLTC transformers at the substation.

Figure 2-8: SuperTAPP n+ relay arrangement [58]

The estimation of the DG current $I_G$ is achieved by the additional current measurement $I_{FG}$ on the feeder with DG and the load share $E_{ST}$ between feeders with DG to those without DG as [59]:

$$E_{TS} = \frac{\text{load on feeders with generators}}{\text{load on feeders without generators}} = \frac{I_1}{I_2}$$  \hspace{1cm} (2-1)

The factor $E_{ST}$ is calculated prior to the connection of DG or when its output is zero. During the operation of DG, $E_{ST}$ is used to estimate the DG current $I_G$ as:

$$I_G = (E_{TS} \cdot (I_{TL} - I_{FG})) - I_{FG}$$  \hspace{1cm} (2-2)

$$I_{TL} = \sum_{n=1}^{N} I_{Tn} = I_{T1} + I_{T2}$$  \hspace{1cm} (2-3)

Based on the estimated $I_G$, the voltage rise at the DG connection point can be evaluated. According to the voltage rise, a reduced voltage reference setting will be determined and sent to the AVC relay to decrease the substation voltage. The voltage profile along the feeder with DG could be regulated back to the permitted limits. The drawback of the SuperTAPP n+ relay scheme is that it requires current measurements for each feeder with
DG. This may increase the control costs and response time when there are multiple feeders connected with DGs.

### 2.3.1.2 Automatic voltage reference setting using state estimation

For the OLTC control, the voltage reference setting of the AVC relay can be determined using the LDC method or the aforementioned SuperTAPP n+ relay scheme. Alternatively, El-Feres and Li have proposed an automatic voltage reference setting technique based on distribution state estimation (DSE) [60]. The DSE is a mathematical minimisation process used to estimate bus voltages in distribution networks. A weighted least square function is commonly adopted to formulate the state estimation as [61], [62]:

\[
\min_x J(x) = \sum_{i=1}^{N_m} \frac{(z_i^{meas} - f_i(x))^2}{\sigma_i^2}
\]  

(2-4)

where,
- \( x \): State vector that consists of network bus voltages.
- \( N_m \): Total number of measurements.
- \( z_i^{meas} \): The \( i^{th} \) measurement value.
- \( f_i(x) \): A function of state variables and it is used to calculate the theoretical value of the \( i^{th} \) measurement.
- \( \sigma_i^2 \): Variance of the \( i^{th} \) measurement.

The solution of Equation (2-4) is a set of state variables that minimises the squares of errors between the measured values and the values calculated from state variables. In other words, the DSE can generate a group of bus voltages that enables the power flow calculated in theory to match the measurements. To reduce the needs of a large number of measurements, the DSE only takes real-time measurements at critical points in the network and combines them with pseudo measurements [63], [64]. The pseudo measurements can be obtained from historical load flow data of substation transformers.
With the DSE, the voltage controller at the substation can observe the voltage changes of the downstream feeders due to DG penetration. The AVC target voltage can then be updated to adjust the substation voltage accordingly. In [60], a real-time closed-loop testing facility has also been developed to investigate the behaviours of the DSE based voltage controller under different load and DG conditions.

2.3.1.3 Impact of wind power variability on transformer tap operations

Due to the fluctuating nature of wind power, the power output from wind turbines may introduce high power flow fluctuations through the substation transformers. This may ultimately lead to an increase in the frequency of tap changes and reduce the lifetime of tap changers. In [65], the corresponding analysis has been carried out for the wind farms connected to a meshed sub-transmission system. The result indicates that the wind power injection has increased the number of tap changing operations. The increase is found to be significant when the wind farm provides local voltage regulation. To reduce the number of tap changes introduced at the substation transformer, a control algorithm utilising the reactive power compensation from local wind turbines has been proposed in [66].

2.3.2 Reactive power control

Reactive power management provides the other way to address the voltage issue due to DG penetration. At the point of connection, voltage variations could be compensated by controlling the reactive power of DG [57]. A range of methods based on reactive power control has been proposed to raise the level of distributed generation that could be connected to the network.

2.3.2.1 Generator reactive power control

In [67], a distributed reactive power generation control has been developed for the DG to alleviate the voltage rise issue (see Figure 2-9). Instead of directly regulating the bus voltage at the connection point, the proposed method aims to minimise the voltage rise by controlling the reactive power absorption of DG. The reference setting of the DG reactive
power \((Q_G)\) is determined according to the output power \((P_G)\) and the feeder impedance \((R + jX)\). Compared with the constant power factor control, the proposed approach can mitigate the voltage rise; however, it is at the expense of higher OLTC tap position volatility and feeder losses.

![Diagram](image)

**Figure 2-9:** Single-line model of distribution network with distributed generator [67]

For transmission systems interconnected with wind generation, the Volt/VAr support provided at the wind farm sides have been studied [68]-[70]. In [68], a secondary voltage control was developed for the transmission grid by coordinating the reactive power output of wind generators with power electronics. To allow wind farms to participate in the reactive power balancing of transmission networks, a multilevel control system was designed in [69]. A generalised reactive power cost model was developed to analyse the technical and economic issues associated with wind farms as reactive power ancillary service providers [70].

**2.3.2.2 Voltage control via VAr compensators**

Apart from generator, synchronous condenser, SVC and STATCOM can also be used for reactive power control. An application of a STATCOM for steady-state voltage and VAr control has been presented in [71]. The STATCOM was installed close to a wind farm and was used to supply reactive power to the wind farm under various operating conditions. An optimal power flow program was developed to minimise network losses by adjusting the voltage of the STATCOM within its reactive power capacity. Simulation results demonstrate that the STATCOM control scheme is able to improve the voltage stability of the network and to allow higher wind power injection than the unity power factor control.
2.3.2.3 Network losses affected by reactive power control

The reactive power flow changes due to VAr compensation may have impacts on the system losses. A significant amount of reactive power compensation may increase the line current, resulting in higher transmission losses in the network. Considering the presence of DGs, an optimisation algorithm has been developed to minimise the network losses while keeping bus voltages within limits [72]. The control variables are the active ($P_{DG}$) and reactive power ($Q_{DG}$) output from each DG. More details about the Volt/VAr optimisation control will be given in Section 2.5.

2.3.3 Power curtailment

Active power curtailment of DG has been proposed as another way to mitigate the voltage rise issue. If the bus voltage at the connection point is approaching the upper limit when demand is low, the generator can reduce its output by a certain amount so that the voltage is still maintained below the limit. This will allow the generator to keep operating rather than being tripped off from the network by the DNO. The generator can increase its output again under normal or heavy load conditions. Therefore, it may be profitable for the DG to curtail some of its output power for a limited period if that allows the DG to connect larger capacity.

2.3.3.1 Power curtailment cost minimisation

Liew and Strbac have developed an optimal power flow (OPF) algorithm to minimise the annual active power curtailment cost of embedded wind generation [57]. The reference settings of the generation curtailment, the reactive power compensation and the OLTC tap positions are combined as the control variables of the OPF algorithm. Case studies show that the proposed control scheme can increase the installed capacity of wind generation without violating voltage limits. The revenue lost due to the power curtailment can be maintained within a reasonable range for low levels of wind penetration (e.g. installed capacity less than 1 MW in the case studies). However, as the penetration level increases, the revenue lost and the curtailed energy increase considerably and these may no longer be acceptable.
2.3.3.2 Active power control of DG

In [73], an active power control approach has been proposed for PVs to provide local voltage regulation. This method is designed for the low voltage (LV) feeder with PV integration as shown in Figure 2-10.

The traditional control based on the maximum power point tracker allows the PV to inject all the power made available by the photovoltaic array. However, the PV may be switched off by the DNO if overvoltages occur at the point of connection. In the proposed method, the traditional control has been integrated with an active power curtailment control, which regulates the active power generation according to the local bus voltage. The PV can be switched from the traditional control mode to the power curtailment control mode, depending on the network operating conditions. Another generation control of DG has been presented in [74]. For this design, the active power control of the generator is used to provide supports for the voltage regulation only when the reactive power control becomes insufficient to keep the voltage as the generator reaches its reactive power limit.

2.4 Reactive Power Management in Transmission Systems

The aim of this project is to develop a reactive power control approach, which can help maintain the voltages of transmission systems during periods of low demand. The existing methods for reactive power management in transmission systems have therefore been reviewed in terms of their methodologies, advantages and limitations. The alternative reactive power control technique will be proposed based on the literature.
2.4.1 Generation and absorption of reactive power

At present, transmission systems employ synchronous generators, capacitors, reactors, synchronous condensers, SVCs and STATCOMs to control the reactive power flow. Synchronous generators are frequently used to produce or absorb reactive power depending on network load levels. Reactive power compensation devices may be installed at substations or on long transmission lines to enhance the voltage and reactive power control. According to the functionality, VAr compensators can be divided into two types: passive and active.

2.4.1.1 Passive compensation

Shunt capacitors and reactors, and series capacitors provide passive compensation. Shunt capacitors generate reactive power and can be placed at various locations throughout the system to boost local voltages or correct power factors. The disadvantage of shunt capacitors is that their reactive power output is proportional to the square of the voltage. Consequently, the reactive power production is reduced at low voltages when the capacitor is likely to be needed most [18]. Series capacitors are connected in series with the line conductors to reduce the equivalent inductive reactance of the line. The connection of series capacitors increases the power that can be transmitted between buses and reduces the effective reactive power loss ($X_f^2$). Nevertheless, as inserting series capacitors will form series-resonant circuits in the system, the introduced subharmonic oscillations that follow any disturbances may lead to rotor hunting and shaft oscillations of generating units [19].

When the electricity demand is low, e.g. during summer nights, the reactive power produced by the capacitive circuits may cause overvoltage issues, especially in cable dominant areas. Shunt reactors are therefore installed to absorb the excess reactive power in the grid [75]. Considering the repairable and aging failures of equipment, the redundancy level of high voltage reactors was determined using a probabilistic benefit/cost analysis technique in [76]. The economic benefit of adding redundant reactors was evaluated using the economic losses due to the unavailability of reactors.
[77], a coordinated control scheme was proposed to combine the automatic switching on of reactors with the switching off of capacitors under light load conditions. This method can help reduce the system VAr surplus if a load shedding scheme is implemented to maintain the system frequency with large disturbances.

2.4.1.2 Active compensation

Synchronous condensers, SVCs and STATCOMs provide active compensation. Compared with passive compensation, the reactive power produced or absorbed by active compensators can be adjusted automatically to control the local voltages. A synchronous condenser is simply a synchronous machine running without a prime mover or a mechanical load [16], [78]. After the unit is synchronised, the field current is adjusted to either generate or absorb reactive power as required by the power system. Synchronous condensers have several limiting features: costs are relatively high; they require substantial foundations and regular maintenance [19]. These drawbacks have prompted the development of alternative types of equipment, using static components, e.g. SVCs and STATCOMs.

The SVC is an integrated system of static electrical components (e.g. capacitors, reactors, transformers and switches) combined together to provide rapid, continuously controllable shunt VAr compensation [79]. Thyristor-controlled reactors (TCRs) and thyristor-switched capacitors (TSCs) are commonly used to implement SVCs [80]-[82]. The STATCOM is based on a voltage-sourced converter and is connected in parallel to the power system through a series inductance (i.e. the transformer leakage) [83], [84]. The inverter generates balanced three-phase sinusoidal voltages at the fundamental frequency, with controllable amplitudes and phase-shift angles. Figure 2-11 illustrates the examples of SVC and STATCOM topologies. Compared with SVCs, STATCOMs can provide better reactive current support for a system whose voltage is severely reduced [19]. In addition, the response speed of a STATACOM (0.01-0.04s) is usually faster than that of an SVC (0.02-0.06s).
2.4.2 Reactive power ancillary service

As aforementioned, sufficient reactive power support is essential for maintaining the voltage profile with varying demands. To ensure a secure and reliable operation of power system, transmission system operators (TSOs) need to procure reactive power ancillary services, which are usually provided by generators and VAr compensation devices [85]. The reactive power providers can receive financial compensation from TSOs with appropriate pricing schemes [86], [87].

2.4.2.1 Generator VAr support

Synchronous generators can produce or absorb reactive power depending on the excitation. The amount of reactive power reserves of generating units is a measure of the system voltage stability. In [88], an optimisation algorithm has been developed to improve the reactive power margins of generators by utilising VAr compensation equipment. From an economic point of view, the costs associated with generator reactive power support have been studied in [89]. These costs consist of three parts: the cost of production or absorption of reactive power, the cost of increased generator losses and the cost of losing the sale opportunity of electricity. The lost opportunity cost (LOC) is introduced when the generator has to reduce its active power output to increase its
reactive power production under contingency situations. A new method has therefore been proposed to accurately calculate the payment for the LOC based on generator capability curves. During periods of low demand, system generators can be set with underexcited operations to absorb the reactive power surplus. However, the reduction of their excitation results in a reduction of the electrical stiffness and an increase in the power angle, which leads to fewer stability margins. Shunt reactors or other VAr compensators can therefore be placed to enhance the system performance at light load operation [90].

2.4.2.2 Other types of VAr service providers
Apart from generators and compensation devices, several studies have also considered the use of wind farms, DGs and loads to support reactive power management in grids [70], [91]-[94]. The variable speed wind turbines with fast-acting power electronic converters can contribute to the reactive power support in transmission systems [95]. A comprehensive cost model has been developed to investigate the payments for wind farms as the potential reactive power ancillary service providers [70]. In terms of DGs, the reactive power generation provided by them may reduce the VAr exchanges at the interfaces between transmission and distribution networks, resulting in less reactive power flow in transmission systems [91]. In [93], assuming DGs to be the VAr service providers, an optimised settlement procedure has been proposed to minimise the total payments for DGs as well as the network power losses and voltage deviations. For electric loads equipped with power electronics, Jelani et al. [94] presented a distributed compensation method by controlling the reactive currents of the loads. From the literature, wind farms, DGs and power electronic loads can be employed to enhance the reactive power control in the system. However, due to the uncertainty and volatility involved with these new techniques, there are still no appropriate incentives or procurement processes established by TSOs to allow them to participate in the VAr balancing services.
2.4.3 Reactive power dispatch

The reactive power dispatch is an optimisation problem that reduces grid congestion with one (or more) objective of minimising active power losses in the transmission system. The solution determines the settings of control variables, e.g. generator reference voltages, transformer tap positions and reactive power outputs of compensation devices. As the reactive power dispatch problem is a mixed-integer nonlinear programming (MINLP) problem, the solution approach is often based on heuristic methods (e.g. particle swarm optimisation, genetic algorithm and differential evolution) to avoid calculating the gradient of the problem.

In [96]-[100], various optimisation algorithms have been proposed to solve the reactive power dispatch problem considering the bus voltage, generator, transformer and line capacity constraints. From the case study results, the optimal dispatch of reactive power in the network can improve the voltage profile as well as minimise the active power losses. To enhance the voltage stability margin, the locations and amounts of switched capacitors were optimised using the backward/forward search algorithm [101].

2.4.4 Distribution network VAr support for transmission system

The growing integration of large-scale intermittent renewable resources into the grid has increased the difficulties in balancing generation and demand. As a consequence, the voltage problems may occur in different time periods or different regions. The placement and tuning of numbers of compensation devices at various locations would prove to be challenging for the system operators [34]. Alternatively, the Volt/VAr support for transmission systems can be provided in distribution networks. In [102] and [103], studies have demonstrated that the strategy of using many, small, distributed SVCs located at distribution buses is more advantageous than a few large bulk SVCs located at transmission buses.
Figure 2-12 shows the single-line diagrams of transmission systems with the two different voltage support schemes. For the voltage support provided at the transmission side (see Figure 2-12a), each SVC is connected through a step-up transformer to regulate the transmission bus voltage $|V_{Rj}|$ at 1.0 pu. In Figure 2-12b, the voltage support at the distribution side is implemented by installing SVCs directly at the load buses to control the load voltage $|V_{Lj}|$ at 1.0 pu. As SVCs can operate at distribution voltage levels, additional step-up transformers are not required for the distribution voltage support scheme.

![Single-line diagrams of transmission systems](image)

Case studies have been carried out to verify the benefits of providing voltage support at the distribution side. From the results, the proposed method can save the costs of step-up transformers, reduce the MVAr requirement for SVCs and the cost to ensure N-1 reliability. Note that a drawback of the distribution network voltage support scheme is that it will increase the transmission line losses slightly. More investigations should be taken to analyse the corresponding costs of energy loss. Nevertheless, the findings have offered the distribution sector an opportunity to provide Volt/VAr support to the transmission sector as an ancillary service. Therefore, based on the concept of utilising
distribution networks for reactive power support, an alternative VAr management technique will be proposed in Chapter 4.

### 2.5 Volt/VAr Optimisation Control

In this research project, the tap staggering technique is adopted to increase the VAr consumptions of parallel transformers in distribution networks so that the reactive power surplus in the transmission system can be reduced during periods of low demand. The tap staggering operation should be optimised concerning the introduced power loss as well as the tap changing operations. Consequently, the existing Volt/VAr optimisation control approaches for distribution networks have been reviewed in terms of their objectives, solution algorithms and effectiveness.

#### 2.5.1 Problem description

The Volt/VAr control in distribution systems usually involves the optimal dispatch of OLTCs and shunt capacitors for the purpose of minimising energy losses while satisfying operating constraints. For instance, considering a distribution network with an OLTC located at the substation and multiple switched shunt capacitors placed along the feeders, the objective is to minimise the system energy loss for the day ahead, which can be formulated as [104]-[106]:

\[
\min \sum_{t=1}^{24} P_{\text{loss}} (Q_t, TAP_t) \cdot \Delta t 
\]  

(2-5)

where \( P_{\text{loss}} \) is the total power loss at hour \( t \) as a function of the control variables \( Q_t \) and \( TAP_t \). \( Q_t \) is a vector whose elements represent the status of each capacitor (on or off). \( TAP_t \) denotes the tap position of the OLTC at hour \( t \) and \( \Delta t \) is the time interval, normally taken as 1 hour.
The objective function is subject to the standard power balancing constraints as well as the following inequality constraints:

\[
V_{\text{min}} \leq V_{i,t} \leq V_{\text{max}}
\]  
\[(2-6)\]

\[
\sum_{t=1}^{24} |TAP_t - TAP_{t-1}| \leq MK_T
\]  
\[(2-7)\]

\[
\sum_{t=1}^{24} (C_{m,t} \oplus C_{m,t-1}) \leq MK_{Cm}
\]  
\[(2-8)\]

where,

- \(V_{i,t}\) is the Voltage of bus \(i\) at the hour \(t\).
- \(V_{\text{min}}, V_{\text{max}}\) are the Minimum and maximum limits of bus voltage.
- \(MK_T\) is the Maximum limit for the daily switching operations of the OLTC.
- \(C_{m,t}\) is the Status of capacitor \(m\) (on or off) at the hour \(t\). The \(\oplus\) represents the logic operator XOR (exclusive or).
- \(MK_{Cm}\) is the Maximum limit for the daily switching operations of capacitor \(m\).

With the knowledge of the daily load forecast, this off-line Volt/Var control problem could be solved using appropriate optimisation algorithms. The solution of Equation (2-5) is a dispatch schedule that decides the OLTC tap position and the on/off status of each shunt capacitor for 24 hours in the next day. In addition to the minimisation of system energy loss, several studies have included the minimisation of bus voltage deviations in the objective function for the voltage regulation purpose [107], [108]. In [109], the reactive power control has also aimed to minimise the reactive energy costs for the voltage support with the presence of DGs.

### 2.5.2 Solution algorithms

Due to the discrete nature of control device settings (e.g. tap positions) and nonlinear load flow constraints, the Volt/Var optimisation problem is usually a mixed-integer nonlinear
programming problem (MINLP). Various approaches have been proposed to solve the MINLP problems. Details of some commonly used algorithms are presented as follows.

2.5.2.1 Method based on discrete-continuous conversion

To handle the discrete control variables in Volt/VAr optimisation, one solution is to convert the discrete variables into continuous variables. For example, in the daily reactive power dispatch problem presented in [110], discrete control variables were first treated as continuous variables. The MINLP problem was then transformed into a nonlinear programming (NLP) problem, which was solved using an interior-point method (IPM). During the IPM iterations, the control variables were rounded off to their nearest discrete values according to a penalty function.

2.5.2.2 Dynamic programming

For the off-line VAr dispatch problem, dynamic programming (DP) is often applied in the search for the optimal solution. In [111], a DP based method was proposed to optimise the dispatching schedule for the OLTC and shunt capacitor at a distribution substation. The schedule would determine the OLTC tap position and the on/off status of the capacitor for every hour of the next day. The DP approach first divided the solution procedure into 24 stages. At each stage (i.e. hour), the possible actions of the control devices were created as states. The purpose was to find a feasible path through the system states at 24 stages, which satisfies the constraints and minimises the value of the objective function. To reduce the computational burden, only the feasible states with lower objective values were stored at each stage.

The optimal VAr control via dynamic programming was also applied to distribution systems with both substation and feeder capacitors [112], [113]. In [113], a coordination scheme of dynamic programming and fuzzy logic was developed for the Volt/VAr control. The problem was decomposed into two sub-problems, i.e. the dispatch of OLTC and capacitors at the substation level and the dispatch of capacitors at the feeder level. The dynamic programming and fuzzy logic control were employed to deal with the two sub-problems, respectively. From the literature, dynamic programming methods are not
suitable for large-scale distribution systems as the state space will increase rapidly with the system size.

2.5.2.3 Global optimisation methods

Apart from dynamic programming, other approaches based on global optimisation methods (e.g. genetic algorithm, particle swarm and simulated annealing) have been proposed for the optimal Volt/VAr control. In [105], a genetic algorithm (GA) based procedure was developed to determine the VAr dispatch schedule with a daily load forecast. During the process of optimisation, the daily load curve was divided into several sequential load levels using the GA and the OLTC tap position may only change when the load transits from one load level to another. This load level partitioning method reduces the unnecessary tap movements while taking into account the daily load changes. In [114], the dispatch schedule for the capacitors in a distribution system was first solved by GA considering the power loss reduction and voltage profile improvement. According to the off-line dispatch schedule obtained, the OLTC reference voltage at the substation was then adjusted in real-time to reduce the unnecessary tap operations. The application of GA was also extended to the harmonic distortion control of the distribution systems serving nonlinear loads [106]. The proposed solution method is based on a hybrid genetic-fuzzy algorithm that employs fuzzy logic to measure the fulfilment of the objective and constraints.

A similar combined approach of evolutionary algorithm (EA) and fuzzy logic was developed to solve the multiobjective optimisation problem in [108], which involved the minimisation of both power losses and voltage deviations. In [115], an annealing algorithm, which is a global search technique like GA and EA, was applied to solve the Volt/VAr control problem. Fuzzy logic variables were used to formulate the objective function with operating constraints, such as the switching operation limits for OLTC and capacitors.
2.6 Summary

Transmission system voltage control has a hierarchical structure with three levels: the primary, secondary and tertiary voltage control. At the primary level, generators with AVR aim to compensate the rapid and random voltage variations at the local buses. The secondary voltage control acquires the regional information and determines the reference settings of the AVRs at the primary level. At the national level, the tertiary voltage control utilises the global information and updates the reference settings for the secondary voltage control by solving optimisation problems. In terms of distribution systems, voltage regulation is implemented at substation and feeder levels. The OLTC transformer equipped with an AVC relay can either control the substation voltage directly or adopt the LDC algorithm to regulate the voltage at some remote point out on a feeder. For the feeder regulation, shunt capacitors and step voltage regulators can be used to boost (or reduce) the voltages on feeders.

Due to the growing DG penetration, various voltage control schemes have been proposed to mitigate the voltage rise issue. They are divided into three aspects: the OLTC control, reactive power control and power curtailment. For the OLTC control, advanced voltage reference setting techniques have been developed concerning the influences caused by DG generation. Several studies have focused on reducing the number of tap changes due to wind power variation. Reactive power control of DG can alleviate the voltage rise at the point of connection. However, this may increase the line current and lead to more losses in the network. In terms of power curtailment, the active power output of DG is controlled according to the local bus voltage. Since constraining the DG power will affect the economic benefit of DG, power curtailment is usually only a viable option when the constraints are expected to be infrequent.

As this research project is concerned with the voltage problems associated with reactive power surplus in transmission systems, the existing reactive power compensation techniques for transmission systems have been reviewed. From the literature, synchronous generators and VAr compensation devices are commonly used to produce or
absorb reactive power in transmission systems. The generator reference voltages, transformer tap positions and VAr outputs of compensation devices can be determined by performing the optimal reactive power dispatch with the objective of minimising network losses. However, with the presence of large-scale intermittent renewable resources nowadays, the placement and tuning of numbers of compensation devices at various locations would prove to be challenging for the system operators. As a consequence, some studies have considered the use of wind farms, DGs and loads to support the reactive power management. In addition, to improve the reactive power control for the transmission system, a strategy of using many, small and distributed SVCs located at distribution buses has been proposed in [102]. The studies have demonstrated the benefits of utilising distribution networks to provide reactive power service for the transmission system.

Finally, the optimal Volt/VAr control in distribution systems has been described and discussed. The control objectives usually include the minimisation of energy losses and bus voltage deviations. Most solution algorithms are based on global optimisation methods (e.g. genetic algorithm) due to their ability to handle discrete variables. From the review, an optimal reactive power control method, which utilises the existing distribution network transformers, will be proposed to provide reactive power absorption service to the transmission system. The details are presented in Chapter 4.
CHAPTER 3
FUNDAMENTALS OF DISTRIBUTION SYSTEMS

3.1 Introduction

As this research project focuses on the investigation of reactive power support provided from distribution networks, a comprehensive understanding of distribution systems is essential. This chapter intends to give the readers basic background knowledge for the purpose of a better understanding of the subsequent chapters. First, the general structure of a distribution system is presented in Section 3.2. The relationship between voltage regulation and reactive power control is then analysed in Section 3.3. A detailed description of on-load tap changer mechanisms and the load flow modelling of a transformer with tap changer are given in Section 3.4.

3.2 Distribution System Structure

The function of an electric power distribution system is to deliver electrical energy from the transmission system to individual consumers and to transform the voltage to a suitable range where necessary. Figure 3-1 illustrates the general structure of a distribution system at different voltage levels, i.e. high voltage (HV), medium voltage (MV) and low voltage (LV) [36]. The distribution system receives electricity from the extra high voltage (EHV) lines of the transmission system and lowers the voltage to the HV level. Small generating stations may be connected to the HV networks, and the HV/MV transforming substations distributed around each HV network supply individual MV networks. The HV and MV networks provide electrical energy direct to large consumers, e.g. industrial customers. However, the majority of consumers (e.g. commercial and domestic customers) are severed by the LV networks. In rural areas, MV overhead lines are often used, and the networks are normally operated in a radial configuration. In urban areas, underground
cables are frequently installed as they are more reliable and more environmentally friendly than overhead lines.

Figure 3-1: General arrangement of a distribution system with different voltage levels [36]
Figure 3-1 only indicates a general distribution system arrangement as the design practice varies from countries to countries. In the UK, the transmission system operates at the EHV levels with 275kV and 400kV. The voltages are transformed down to 132kV at the Grid Supply Points (GSPs), where distribution networks are connected to the transmission system. The distribution HV networks usually operate at 132kV and 33kV. The HV/MV substation shown in Figure 3-1 is often known as the primary substation, which converts the voltage from 33kV to 11kV or 6.6kV. For the UK practice, the primary substation transformers normally operate in parallel, and each has an OLTC equipped to regulate voltages. The details about tap changer mechanisms and the load flow modelling of the transformer with tap stagger are discussed in the following sections.

3.3 Voltage Regulation

One of the most important constraints for distribution system design is the voltage level at the load connection point. The voltage drops on lines or cables may cause voltage violations if without appropriate compensation. As mentioned in the literature reviews, voltage control can be accomplished by managing the production and absorption of reactive power in the system. The relationship between voltage and reactive power is described in this section.

3.3.1 Voltage drop calculations

Considering a line segment having an impedance \( R + jX \) (as shown in Figure 3-2), the voltage drop due to the line impedance can be expressed as:

\[
\Delta V = V_1 - V_2 = (R + jX) \cdot I = (R + jX) \left( \frac{P - jQ}{V_1} \right) = \frac{RP + XQ}{V_1} + j \frac{XP - RQ}{V_1} = \Delta V_r + j \cdot \Delta V_i
\]  

(3-1)

where \( V_I \) is the voltage at the sending end, \( V_2 \) is the voltage at the receiving end (or load centre), \( I \) denotes the current through the line impedance, \( P \) denotes the active power
transmitted out of the sending end and $Q$ for the reactive power. $\Delta V_r$ is the real part of $\Delta V$ and $\Delta V_i$ is the imaginary component that reflects the angle between $V_1$ and $V_2$.

\[
\Delta V \approx \frac{RP + XQ}{V_1}
\]  

(3-2)

According to the equation above, the voltage deviation is related to the power flow ($P$ and $Q$) on the line. The power flow predominantly depends on the system load. When the demand is high, large power flow through the line will lead to a significant voltage drop. For reduced voltage drops, VAr compensation devices can be installed to provide reactive power locally, resulting in less reactive power transmitted on the line. Note that if large $P$ (or $Q$) flows in the opposite direction, the sign of $\Delta V$ may change to negative, which implies the voltage at the receiving end is greater than the sending end voltage. This scenario explains the reason why the voltage at the DG connection point increases.

### 3.3.2 Allowable voltage variations

The distribution system voltage regulation aims to maintain a stable voltage at the consumer terminals and to control the voltage variation at an acceptable level. In the UK, the steady state line-line voltage of the system operating at 132kV or above should be maintained within $\pm 10\%$ of the rated voltage, and $\pm 6\%$ range for the system between 1kV and 132kV [15]. In terms of the system above 50V and below 1kV, voltage is only
permitted with variation between +10% and -6% of the nominal voltage. These requirements need to be satisfied with at least 95% of the 10-minute mean RMS values of the supply voltage during a week [116].

### 3.4 Transformer with On-load Tap Changer

Power transformers equipped with OLTCs have been the main components of electrical networks for decades. The OLTC provides voltage magnitude regulation and/or phase shifting by changing the transformer turns ratio under load without interruption. To alter the transformer turns ratio, the OLTC adds turns to or subtracts turns from either the primary or the secondary winding. The details of the tap changer mechanisms and the load flow modelling of a transformer with OLTC are described in this section.

#### 3.4.1 Tap changer mechanisms

Figure 3-3 illustrates the principle winding arrangement for a three-phase regulating transformer [117]. The OLTC is located at the high-voltage winding with Y connection.

![Figure 3-3: Principle winding arrangement of a regulating transformer in Y-Δ connection [117]]
To change winding turns, the OLTC needs to select the corresponding transformer tap. However, simple switching between two different taps is usually not allowed during the energised operation. This will introduce the high arc current and the momentary loss of system load as shown in Figure 3-4.

![Figure 3-4: Loss of system load with single contact switching][117]

To prevent the interruption of load current, the “make-before-break” concept was introduced [117]. Figure 3-5 illustrates the switching sequence of an OLTC that consists of a tap selector and a diverter switch. During the switching operations (d)-(f), the transition impedance is capable of transferring the load smoothly from one tap to the other without the interruption of load current. Meanwhile, the impedance can limit the circulating current for the period when both taps are used, i.e. operation (e). The voltage between adjacent taps is often referred to as the step voltage $U_s$ as shown in Figure 3-3. It is usually between 0.8% and 2.5% of the transformer rated voltage [117].

According to different forms of transition impedance used, OLTCs can be divided into two types: the resistor-type and reactor-type. Most resistor-type OLTCs are installed inside the transformer tank whereas the reactor-type OLTCs are in a separate compartment, which is usually mounted on the transformer tank externally. Nowadays, the selecting and switching of taps are commonly driven by motors. The total operation time of a tap changing is usually between 3 and 10 seconds. In the future, solid-state technology using fast switching power electronics may be available to OLTCs at low costs [118].

[117] Reference Image
3.4.2 Per-unit model of transformer with tap changer

As the OLTC can alter the transformer turns ratio, a corresponding per-unit model of a transformer with off-nominal turns ratio is essential for load flow calculations. Figure 3-6 shows the single-line diagram of a transformer with the off-nominal turns ratio $a_t$. Given the transformer nominal ratio $b$, the relationship between the rated primary and secondary voltages can be expressed as [119]:

\[ V_{1\text{ rated}} = a_t V_{2\text{ rated}} = b \left( \frac{a_t}{b} \right) V_{2\text{ rated}} = bc V_{2\text{ rated}} \]  

(3-3)
where \( c = a_t/b \) and it denotes the fraction of the transformer nominal ratio, i.e. the tap ratio/nominal ratio. For instance, a transformer with nominal ratio 132kV to 11kV when tapped to give 144kV to 11kV has the fraction \( c = 144/132 = 1.09 \).

\[
\begin{align*}
\text{Figure 3-6: Single-line diagram of transformer (a) with off-nominal turns ratio, and (b) represented as two transformers in series [119]} \end{align*}
\]

Equation (3-3) can be represented by two transformers in series, as shown in Figure 3-6b [119]. The first transformer has the nominal turns ratio \( b \) and the second transformer is considered as an ideal transformer with the turns ratio \( c \). All the real and reactive losses are associated with the first transformer. The resulting per-unit model is illustrated in Figure 3-7a, where the shunt excitation branch is neglected for simplicity. In the figure, \( V_1 \) and \( V_2 \) denote the transformer primary and secondary voltages, respectively. \( Z_{eq} \) denotes the per-unit impedance of the transformer. Alternatively, the per-unit model can be represented by a two-port network. Considering Kirchhoff’s voltage law (KVL), the transformer can be described as:

\[
\begin{align*}
V_1 - I_1 Z_{eq} &= c V_2 \quad (3-4) \\
cl_1 &= -I_2 \quad (3-5)
\end{align*}
\]

where \( I_1 \) and \( I_2 \) denote the currents flowing into the transformer primary and secondary terminals, respectively. Rearrange the equations as:

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
Y_{eq} & -cY_{eq} \\
-cY_{eq} & c^2Y_{eq}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} \quad (3-6)
\]
where \( Y_{eq} = 1 / Z_{eq} \). According to the admittance parameters, the transformer with off-nominal turns ratio can be modelled by a \( \pi \) network as shown in Figure 3-7b. With the \( \pi \) circuit representation, the admittance matrix of a transformer can be easily derived and used to perform load flow calculations in computer programs.

\[
\begin{align*}
Y_{eq} &= \frac{1}{Z_{eq}} \\
V_1 &= Z_{eq} \cdot I_1 + (c)Y_{eq} \cdot I_2 \\
V_2 &= (1 - c)Y_{eq} \cdot I_1 + (|c|^2 - c)Y_{eq} \cdot I_2
\end{align*}
\]

Figure 3-7: Models of transformer with off-nominal turns ratio: (a) per-unit equivalent circuit, and (b) \( \pi \) circuit representation [119]

### 3.5 Summary

A distribution system usually consists of different voltage levels, i.e. HV, MV and LV. The HV and MV networks supply electrical energy direct to large consumers, e.g. industrial customers, while LV networks serve the majority of consumers, e.g. commercial and domestic customers. In general, rural distribution networks are operated in radial configurations and urban distribution networks are arranged in loop or mesh configurations to increase the security of electricity supply. Reactive power compensation devices can be installed to provide reactive power locally, resulting in fewer voltage variations on lines or cables.
The switching mechanism and sequence of a typical OLTC have been described. The “make-before-break” concept has been widely used in the OLTC designs to prevent the interruption of load current. As the OLTC can alter the transformer turns ratio, a corresponding per-unit model of a transformer with off-nominal turns ratio has been derived. The π circuit representation of a transformer with tap changer can be easily processed by the computer program to perform load flow calculations.
CHAPTER 4
TAP STAGGER FOR REACTIVE POWER SUPPORT

4.1 Introduction

With the integration of large-scale intermittent renewable generation, the transmission voltage problem has become more dynamic and may occur in different time periods or different regions. The associated costs will be high if numbers of devices need to be placed at various locations in the system. In addition, the permanent installation of VAr compensators may not be cost-effective for temporary voltage issues. Therefore, this chapter proposes an alternative method, which utilises existing distribution parallel transformers, to provide reactive power absorption services for transmission systems during periods of low demand. The operation of the parallel transformers in small different tap positions, i.e. with staggered taps, can provide a means of absorbing reactive power. The aggregated VAr absorption from many pairs of parallel transformers could be used to mitigate the high voltages in the upstream network. This chapter first presents the structure of the proposed VAr support method using tap stagger. The mathematical principles of the tap staggering operation are then discussed. Following this, an optimal control approach for the tap stagger is proposed and considers the minimisation of the introduced power losses and tap switching operations.

4.2 Proposed Distribution Network VAr Support Method with Tap Stagger

The proposed method of utilising the parallel transformers in a distribution network to provide VAr support is illustrated in Figure 4-1. The distribution network is connected to the transmission system at the GSP. The distribution system has ‘N’ number of primary substations located throughout its high voltage network. At each primary substation, two parallel transformers (e.g. 33/11 kV in the UK) usually operate together, and both have
on-load tap changers (OLTCs) installed, which allow real-time control. Therefore, in this research work, the proposed tap staggering method is applied to the primary substations. However, conceptually the tap staggering technique may be adopted at different voltage levels where there are transformers operating in parallel.

When the transmission system needs extra reactive power absorption during periods of low demand, the Energy Management System (EMS) may send a specific VAr requirement \( Q_{\text{required}} \) to the Distribution Management System (DMS). The Tap Stagger Optimal Control system embedded in the DMS will then issue tap staggering commands. The optimal control system determines how many parallel transformers will be used, with the objectives of minimising the introduced power losses and the number of tap switching operations. The optimisation is based on the measured network states, such as load level, voltage \( V_i \) and transformer tap position \( TAP_i \). The calculated control signals will be sent to the appropriate primary substations via the communication network, and they will be received by the remote terminal units (RTUs), which interface with the AVC relays. According to the tap staggering commands, the AVC relays will instruct the OLTCs of parallel transformers to tap apart. The proposed method may be considered as a demand
response tool that assists the transmission system in managing reactive power flow for some temporary or contingency periods.

Based on Figure 4-1, this research project concentrates on the development of the tap stagger optimal control method as well as the analyses of the tap staggering impacts on the transmission system. The EMS and communication network are not modelled as it is assumed that the VAr absorption requirement and the distribution network states are known in advance. However, for real-time application, the communication plays an essential role in the successful implementation of the proposed method.

4.3 Tap Stagger Mathematic Principles

At modern electric substations, it is common to see the operation of two or more transformers in parallel. This practice reduces the load carried by each transformer and improves the security of the electricity supply. The tap changer of each parallel transformer is normally maintained at the same position to reduce the circulating current between the transformers [21], [120]. However, some substations may need to operate the parallel transformers with offset tap positions, and then a circulation of reactive power occurs between the transformers, resulting in a net absorption of reactive power. This operating mode is known as ‘tap stagger’ and is occasionally applied to transmission substation transformers to introduce inductive loading onto the system [37], [121] and [122].

To explore the potential benefit of utilising distribution networks to support transmission system VAr management, this research project focuses on the application of the tap staggering method at the parallel transformers in distribution networks. The working principles of the proposed tap staggering technique are described in this section.

4.3.1 Parallel operation of two transformers

Although multiple transformers can be used for parallel operation, this research study concentrates on the scenario when two transformers are connected in parallel, which is
the typical arrangement for the UK primary substation transformers. Figure 4-2 shows the single-line diagram of two three-phase transformers arranged in parallel.

As shown in the figure, both the primary and secondary windings of the two transformers are connected to the same set of primary and secondary busbars, respectively. Therefore, the two parallel transformers have the same terminal voltages at the primary and secondary sides. Note that the primary winding of each transformer is equipped with an OLTC. If the two parallel transformers have the same impedances and both tap changers are maintained at the same positions, the power flow through one transformer will be identical to the other.

![Diagram of two parallel transformers](image)

Figure 4-2: Single-line diagram of two parallel transformers

where,

- $T_1$ and $T_2$ Two parallel transformers with OLTCs
- $P_{in1}, Q_{in1}, P_{in2}$ and $Q_{in2}$ Active and reactive power flowing into each transformer
- $P_{out1}, Q_{out1}, P_{out2}$ and $Q_{out2}$ Active and reactive power flowing out of each transformer
- $P_{L1}, Q_{L1}, P_{L2}$ and $Q_{L2}$ Loads connected to the secondary (or LV) busbar
- $CB$ Circuit breaker
4.3.2 Equivalent circuit for tap stagger

The parallel operation of two transformers with staggered taps is illustrated in Figure 4-3. The primary windings of both transformers $T_1$ and $T_2$ are equipped with OLTCs. Initially, both OLTCs were maintained at the same positions. The tap stagger pattern is achieved by tapping down the OLTC on $T_1$ by $k$ steps while tapping up the OLTC on $T_2$ by the same $k$ steps. Figure 4-4 shows the equivalent circuit referred to the transformer secondary sides.

Figure 4-3: Schematic of tap stagger

Figure 4-4: Equivalent circuit of tap stagger (referred to the transformer secondary sides) [121]
4.3.2.1 Transformer circulating current

Assuming both transformers have the same OLTC parameters and the tap staggering starts from the same initial tap position $TAP_0$ (in pu), the primary voltage $V_p$ referred to the secondary side of $T_1$ or $T_2$ (as shown in Figure 4-4) is:

\[
V_1 = \frac{V_p}{n_m(TAP_0 - k\Delta TAP)} \tag{4-1}
\]

\[
V_2 = \frac{V_p}{n_m(TAP_0 + k\Delta TAP)} \tag{4-2}
\]

where,

- $n_m$: Nominal transformer ratio
- $\Delta TAP$: Tap position increment per step of the OLTC
- $k$: The number of tap steps different from the initial position $TAP_0$ (e.g. $k = 1$ indicates that one transformer will increase its position by one tap, and the other will decrease its position by one tap)

The secondary currents of transformer $T_1$ and $T_2$ are [121]:

\[
I_1 = \frac{V_1Z_2 + (V_1 - V_2)Z_L}{Z_1Z_2 + (Z_1 + Z_2)Z_L} \tag{4-3}
\]

\[
I_2 = \frac{V_2Z_1 - (V_1 - V_2)Z_L}{Z_1Z_2 + (Z_1 + Z_2)Z_L} \tag{4-4}
\]

where $Z_1$ and $Z_2$ are the transformer impedances referred to the secondary sides of $T_1$ and $T_2$, respectively. $Z_L$ denotes the equivalent load impedance.
For the usual case of $Z_L >> Z_1$ and $Z_2$,

\[ I_1 \approx \frac{V_1 Z_2 + (V_1 - V_2)Z_L}{(Z_1 + Z_2)Z_L} \]  
(4-5)

\[ I_2 \approx \frac{V_2 Z_1 - (V_1 - V_2)Z_L}{(Z_1 + Z_2)Z_L} \]  
(4-6)

Both $I_1$ and $I_2$ have a common component that is termed as the circulating current:

\[ I_c = \frac{V_1 - V_2}{Z_1 + Z_2} \]  
(4-7)

The remaining components of $I_1$ and $I_2$ are:

\[ I_{L1} = \frac{V_1 Z_2}{(Z_1 + Z_2)Z_L} \]  
(4-8)

\[ I_{L2} = \frac{V_2 Z_1}{(Z_1 + Z_2)Z_L} \]  
(4-9)

When $k$ is small, $I_{L1} \approx I_{L2} = I_L$ hence $I_1 = I_L + I_c$ and $I_2 = I_L - I_c$. The corresponding phasor diagram is shown in Figure 4-5. Due to the introduced circulating current, additional reactive power is consumed by the transformer impedances of $Z_1$ and $Z_2$.

\[ V_s \quad I_c \quad I_1 = I_L + I_c \quad I_2 = I_L - I_c \quad \]  

Figure 4-5: Phasor diagram of tap staggering on transformer $T_1$ and $T_2$
4.3.2.2 Magnitude of circulating current

If \( Z_1 = Z_2 \approx Z_t \) (i.e. the transformer equivalent series impedance in ohms, referred to the secondary side), from the above, the circulating current is given by:

\[
I_c = \frac{V_1 - V_2}{Z_1 + Z_2} = \frac{V_1 - V_2}{2Z_t}
\]

(4-10)

Substituting Equations (4-1) and (4-2) into Equation (4-10),

\[
I_c = \frac{k \cdot \Delta TAP \cdot \frac{V_p}{n_m}}{Z_t(TAP_0^2 - k^2\Delta TAP^2)}
\]

(4-11)

Using the per-unit value to express \( Z_t \),

\[
Z_t = \frac{Z_{t,pu}V_{NOM}}{\sqrt{3}I_{NOM}}
\]

(4-12)

where \( Z_{t,pu} \) is the per-unit value of transformer impedance, \( V_{NOM} \) is the nominal line voltage at the transformer secondary side and \( I_{NOM} \) denotes the transformer full load current. Substituting Equation (4-12) into Equation (4-11), the circulating current can be described as a percentage of the full load current:

\[
\frac{I_c}{I_{NOM}} = \frac{k \cdot \Delta TAP}{Z_{t,pu}(TAP_0^2 - k^2\Delta TAP^2)} \cdot \frac{\sqrt{3}V_p/n_m}{V_{NOM}} \times 100\%
\]

(4-13)

In general, the second term \( \frac{\sqrt{3}V_p/n_m}{V_{NOM}} \) is around 1.0 pu. Applying Equation (4-13) to a 33/11 kV 23 MVA primary substation transformer, with \( Z_{t,pu} = 0.23 \) pu, \( TAP_0 = 1.0 \) pu, \( \Delta TAP = 1.4\% \) per tap and \( k = 1 \), the circulating current will be 6.1\% of the full load current. As the tap stagger is likely to be activated only when system demand is low (e.g. summer nights), the initial current in the transformer is low, e.g. less than 50\% of the
peak load. Therefore, the use of the tap staggering technique is practicable, depending on the transformer capacity.

4.3.2.3 Reactive power absorption & active power loss due to tap stagger

Based on Equation (4-11), the additional reactive power consumption due to the circulating current $I_c$ can be derived as:

$$
\Delta Q_c = 2I_c^2X_t = 2 \frac{X_t(k \cdot \Delta TAP \cdot V_p/n_m)^2}{Z_t^2(TAP_0^2 - k^2 \Delta TAP^2)^2}
$$

(4-14)

where $X_t$ is the transformer leakage reactance. Let $Z_t = R_t + jX_t$, with $R_t$ representing the winding resistance.

From the equation above, the VAr absorption introduced by circulating current will increase with the transformer primary voltage $V_p$ as well as the staggered taps $k$. The aggregated VAr absorption seen at the GSP will be the sum of $\Delta Q_c$ from each substation, plus the additional VAr transmission loss from the transmission supply point down to the primary substations. The additional transformer power loss due to the tap stagger can also be described as:

$$
\Delta P_c = 2I_c^2R_t = 2 \frac{R_t(k \cdot \Delta TAP \cdot V_p/n_m)^2}{Z_t^2(TAP_0^2 - k^2 \Delta TAP^2)^2}
$$

(4-15)

In general, for large power transformers rated more than 500 kVA, the winding resistances are much smaller than the leakage reactances [119], hence $\Delta P_c \ll \Delta Q_c$. The comparison studies between the reactive power absorption and real power loss due to tap stagger are given in Chapter 5.
4.3.2.4 Voltage of downstream network

According to the equivalent circuit shown in Figure 4-4, the transformer secondary voltage can be derived as:

\[ V_s = (I_1 + I_2)Z_L = \frac{V_1 + V_2}{2} \] \hspace{1cm} (4-16)

Substituting Equations (4-1) and (4-2) into Equation (4-16),

\[ V_s = \frac{V_p}{n_m(TAP_0 - k^2 \Delta TAP^2 / TAP_0)} \] \hspace{1cm} (4-17)

![Figure 4-6: Transformer secondary voltage with different staggered taps (with \( \Delta TAP = 1.43 \% \) and \( TAP_0 = 0.9714 \) pu)]

The \( \Delta TAP \) of an OLTC transformer is typically less than 2\%. Therefore, if the primary voltage \( V_p \) stays the same and small staggered taps (i.e. small \( k \)) are applied, the secondary voltage \( V_s \) will remain almost constant, leaving the downstream network voltages and loads unaffected. Figure 4-6 shows an example of the transformer secondary voltage against the difference in tap positions of two parallel transformers. The secondary voltage changes less than 0.05\% up to 4 tap differences, i.e. \( k = 2 \). With the maximum 12 tap differences, the secondary voltage increases by 0.5\%. Therefore, the effect of the proposed tap staggering method on the downstream network voltages is small.
As aforementioned, the difference in tap positions should only be allowed within a small range (e.g. 2 taps up for one transformer and 2 taps down for the other) in order to prevent the transformer overheating and to maintain the secondary voltages constant without affecting the downstream networks. Although each pair of parallel transformers can only provide a limited VAr absorption, the total amount aggregated from many transformers may be sufficient to mitigate the reactive power surplus in the transmission system. Therefore, Chapter 5 will carry out the feasibility studies to investigate and quantify the VAr absorption capability of distribution networks with tap stagger.

### 4.4 Tap Stagger Optimal Control

To provide the VAr absorption service, the distribution network operator needs to decide how many parallel transformers and staggered taps would be used. Consequently, a control method should be developed to address the VAr dispatch problem. For instance, if the transmission system requires a specific reactive power to be consumed during periods of low demand, the tap stagger control will coordinate the available parallel transformers inside the distribution network to absorb reactive power. In addition, the tap stagger control will limit the tap position difference between transformers and it will keep the switching operations of OLTCs as few as possible to reduce the maintenance costs of tap changers.

Therefore, this section proposes an optimal control approach for the tap stagger. The objective is to find an optimal dispatch for the OLTC tap positions of parallel transformers, minimising the active power losses caused as well as the number of tap switching operations introduced. The transformer tap positions should be arranged in a way to achieve the VAr absorption required by the transmission system. In addition, the optimisation is subject to the transformer and OLTC limits. The details of the objective function and constraints are described as follows.
4.4.1 Objective function

The control variables in the tap stagger optimisation are the OLTC tap positions of parallel transformers. The aim is to minimise the network power loss introduced as well as the number of tap switching operations used to achieve the tap stagger, as given in the following function:

\[
\min J = w_1 \cdot \Delta P_{\text{loss}}(x) + w_2 \cdot \sum_{i=1}^{N} 2x_i = J_1 + J_2
\]

where,

- \(x_i\) Number of tap steps different from the initial tap positions on the \(i^{th}\) pair of parallel transformers (e.g. \(x_i = 1\) indicates that one transformer will increase its tap position by one step, and the other will decrease its position by one step; \(x_i = 0\) denotes that this pair of transformers will not be tapped apart). The tap changer movements in a pair are opposite but with the same steps, to keep the transformer secondary voltages constant without affecting the downstream networks.
- \(N\) Total number of pairs of parallel transformers involved in the tap stagger optimisation.
- \(\Delta P_{\text{loss}}(x)\) Active power loss of the distribution network due to tap stagger. It is a nonlinear power flow function of the control vector \(x\). \(x = [x_1 \ x_2 \ ... \ x_N]\). \(\Delta P_{\text{loss}}(x)\) can be calculated by off-line load flow studies with the information of network loading and transformer tap positions.
- \(w_1\) Weighting coefficient of the active power loss.
- \(w_2\) Weighting coefficient of the tap changer switching operations.
4.4.2 Constraints

The objective function is subject to the following inequality constraints:

\[
\frac{|Q_{\text{actual}}(x) - Q_{\text{required}}|}{Q_{\text{required}}} \leq e_{\text{limit}} \quad (4-19)
\]

\[
TAP_{i,\text{min}} \leq TAP_{i} \pm x_{i} \leq TAP_{i,\text{max}} \quad (4-20)
\]

\[
0 \leq x_{i} \leq \frac{TS_{\text{max}}}{2} \quad \text{for } i = 1, ..., N \quad (4-21)
\]

where,

- \( Q_{\text{required}} \): The additional reactive power absorption required by the upstream transmission network to maintain system voltages during periods of low demand.
- \( Q_{\text{actual}}(x) \): The actual reactive power absorbed by the downstream distribution network through the use of tap stagger. It is a nonlinear power flow function of the control vector \( x \). \( Q_{\text{actual}}(x) \) can be determined by calculating the reactive power demand variation at the grid supply point.
- \( e_{\text{limit}} \): Maximum permitted error (%) between the required and the actual reactive power absorption provided.
- \( TAP_{i} \): Initial tap positions on the \( i^{th} \) pair of parallel transformers (assuming the tap positions are the same for both transformers before initiating the stagger).
- \( TAP_{i,\text{max}} \): Upper limit of the OLTC tap positions on the \( i^{th} \) pair of parallel transformers.
- \( TAP_{i,\text{min}} \): Lower limit of the OLTC tap positions on the \( i^{th} \) pair of parallel transformers.
- \( TS_{\text{max}} \): Maximum allowable difference between the tap positions of two parallel transformers without causing overheating and damages.

The solution of Equation (4-18) is a set of control variables that will rearrange the tap positions of the parallel transformers in the network. The new tap position arrangement will result in a group of transformers with staggered taps and then the boost of reactive
power consumption. Figure 4-7 illustrates the block diagram of the tap stagger optimal control. As the control variable $x_i$ is treated as a discrete variable and the calculations of VAr absorption involve nonlinear load flow equations, the tap staggering optimisation problem is an integer nonlinear programming problem. For this project, the problem has been solved using the genetic algorithm, the rule-based approach and the Branch-and-Bound algorithm, respectively. The related solution procedures and result comparisons are described in Chapters 6 and 7.

4.5 Summary

In this chapter, a reactive power control method, utilising the existing parallel transformers in distribution networks, has been proposed. The operation of the parallel transformers with staggered taps provides a means of absorbing reactive power, thus reducing the overvoltages in the upstream transmission system while leaving the downstream networks unaffected. In this research work, the proposed tap staggering
method is applied to the primary substation transformers, which are equipped with OLTCs to allow on-line control.

The equivalent circuit of two parallel transformers with staggered taps has been presented. According to the analysis of the equivalent circuit, the circulating current increases with the staggered taps. As the tap stagger is likely to be adopted only during periods of low demand, the initial current in the transformer is low. Therefore, the transformer current could remain within the limit if the staggered taps are small. In addition, the use of small staggered taps can keep the transformer secondary voltage almost constant, leaving the downstream network voltages and loads unaffected.

Furthermore, an optimal control approach for the tap staggering operation has been proposed. The objective is to find an optimal dispatch for the OLTC tap positions, which minimises the network loss and the number of switching operations introduced when providing the VAr absorption. The implementation of the tap stagger control using the genetic algorithm will be presented in Chapter 6.
CHAPTER 5

NETWORK REACTIVE POWER ABSORPTION CAPABILITY STUDIES

5.1 Introduction

This chapter presents the feasibility studies of assessing the distribution network reactive power absorption through the use of the tap staggering technique. Off-line load flow studies are carried out based on a real UK HV distribution network model. The studies consist of two parts. The first part aims to investigate the maximum potential VAr absorption that can be provided with the consideration of OLTC operating limits and transformer ratings. Both the network reactive power absorption and active power loss due to the tap stagger are analysed. For the concern of transformer health, the second part presents a more conservative study by constraining the number of staggered taps. The details are given as follows.

5.2 Distribution Network Modelling

To investigate the feasibility of using the tap staggering technique, a load flow model of a real UK HV distribution network has been studied. The model has been provided from the ‘Customer Load Active System Services (CLASS)’ project, which is funded by the Office of Gas and Electricity Markets (Ofgem) [123]. The distribution system has been modelled using the Interactive Power System Analysis (IPSA) software [124]. The network data and the IPSA model are shown in APPENDIX A. Figure 5-1 illustrates the basic network components at different voltage levels.

As illustrated in Figure 5-1, the distribution system is configured as a radial network and the main source is from the grid infeed with 400kV and 275kV voltages. The distribution network is operated at four different voltage levels, i.e. 132kV, 33kV, 11kV and 6.6kV.
CHAPTER 5  NETWORK REACTIVE POWER ABSORPTION CAPABILITY STUDIES

From the GSPs, electricity is distributed throughout the 33kV networks. In each 33kV load area, there is at least one primary substation (i.e. 33/11 kV or 33/6.6 kV) connected. The downstream 11kV or 6.6kV networks are modelled as constant power loads and connected to the secondary sides of the primary substations. Some primary substations may also have synchronous generators installed at the secondary sides as DG sources.

Note that there are 354 primary substations located throughout the 33kV networks. Each primary substation consists of two parallel transformers equipped with OLTCs. Therefore, the tap staggering technique has been applied to the primary substation transformers for the following case studies.

5.3 Maximum VAr Absorption Capability Studies

As discussed in Chapter 4, the transformer VAr absorption increases with the number of staggered taps. To fully investigate the potential VAr absorption that can be provided...
through the use of tap stagger, this section focuses on the studies without setting permitted limits on the staggered taps. The maximum tap positions that can be tapped apart will depend on the OLTC operating ranges and transformer ratings. Since there are large numbers of primary substations in the network (i.e. 354 pairs), it can be time-consuming to test every pair of transformers. To reduce the complexity, the following Case 1 has first studied the power flow variations with only one pair of parallel transformers tapped apart. Then in Case 2, the network has been tested again by increasing the transformer pairs with tap stagger. The results are used to estimate the average VAr absorption of the overall system.

5.3.1 Case 1: Reactive power absorption capability of single pair of parallel transformers

As described before, the tap staggering technique has been applied to the parallel transformers at the primary substations. For instance, \( A_{t11} \) and \( A_{t12} \) are two 33/6.6 kV transformers configured as shown in Figure 5-2. The parameters of the two transformers are the same and both OLTCs stay at the same positions. The initial tap position was pre-calculated by the AVC controller, which maintained the transformer secondary voltage \( V_s \) at the set point 1.0 pu. The controller was then disabled to allow the tap staggering operation. The simulation started by tapping down the OLTC on \( A_{t11} \). Meanwhile, the tap changer on \( A_{t12} \) was increased by the same steps to keep \( V_s \) constant. The simulation would be stopped if either one transformer reached its tap position limit or the current of one transformer exceeded the rating. Table 5-1 and Table 5-2 summarise the resulting reactive and active power flow changes, respectively.

As expected, the total reactive power absorbed by the two transformers is lowest when both tap positions are the same. Due to the increase in tap step difference, more circulating currents are produced and then the VAr absorption increases. According to Table 5-1, the large tap difference (i.e. > 4 staggered taps) has reversed the reactive power flow of \( A_{t12} \), resulting in a leading power factor for the transformer. This can be explained by the phasor diagram in Figure 4-5. Note that during the simulation, the
transformer secondary voltage stays almost constant, which is consistent with Equation (4-17). The maximum variation of secondary voltage is 0.48% at the 12 staggered tap steps.

Figure 5-2: Example of two parallel transformers at a primary substation in the IPSA model

where,

- **CB** Circuit breaker
- $P_{in1}$, $Q_{in1}$, $P_{in2}$ and $Q_{in2}$ Active and reactive power flowing into each transformer
- $P_{out1}$, $Q_{out1}$, $P_{out2}$ and $Q_{out2}$ Active and reactive power flowing out of each transformer
- $P_{L1}$, $Q_{L1}$, $P_{L2}$ and $Q_{L2}$ Static loads connected to the secondary busbar,

$$P_{L1} = P_{L2}, \quad Q_{L1} = Q_{L2}.$$  

Based on Table 5-1, the additional reactive power absorption is plotted against the tap step difference in Figure 5-3. As shown in the figure, the additional VAr absorption is zero at the initial point since the same tap positions on both transformers will not create circulating currents. Figure 5-3 also shows the additional VAr absorption that has been calculated using Equation (4-14), with $TAP_0 = 0.9714$ pu, $\Delta TAP = 1.43\%$, $R_t = 0.92\%$ and $X_t = 23\%$. As can be seen from the figure, the simulation and calculation results match
closely, i.e. with errors up to 2%. The errors are caused by the approximation of transformer currents in Equations (4-5) and (4-6).

Table 5-1: Reactive power variations of a single pair of primary substation transformers with tap stagger

<table>
<thead>
<tr>
<th>Tap Steps Apart</th>
<th>$Q_{in1}$ (MVAr)</th>
<th>$Q_{in2}$ (MVAr)</th>
<th>$Q_{out}^a$ (MVAr)</th>
<th>$Q_{absorbed}^a$ (MVAr)</th>
<th>Additional $Q_{absorbed}^b$ (MVAr)</th>
<th>Secondary Voltage (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.092</td>
<td>3.092</td>
<td>5.138</td>
<td>1.046</td>
<td>0</td>
<td>1.00673</td>
</tr>
<tr>
<td>2</td>
<td>4.766</td>
<td>1.466</td>
<td>5.139</td>
<td>1.093</td>
<td>0.047</td>
<td>1.00686</td>
</tr>
<tr>
<td>4</td>
<td>6.490</td>
<td>-0.114</td>
<td>5.139</td>
<td>1.237</td>
<td>0.191</td>
<td>1.00726</td>
</tr>
<tr>
<td>6</td>
<td>8.267</td>
<td>-1.652</td>
<td>5.140</td>
<td>1.475</td>
<td>0.429</td>
<td>1.00794</td>
</tr>
<tr>
<td>8</td>
<td>10.10</td>
<td>-3.148</td>
<td>5.141</td>
<td>1.811</td>
<td>0.765</td>
<td>1.00888</td>
</tr>
<tr>
<td>10</td>
<td>11.992</td>
<td>-4.605</td>
<td>5.143</td>
<td>2.244</td>
<td>1.198</td>
<td>1.01009</td>
</tr>
<tr>
<td>12</td>
<td>13.945</td>
<td>-6.024</td>
<td>5.145</td>
<td>2.776</td>
<td>1.730</td>
<td>1.01156</td>
</tr>
</tbody>
</table>

a. $Q_{out} = Q_{out1} + Q_{out2}$, and $Q_{absorbed} = Q_{in1} + Q_{in2} - Q_{out}$, referred to Figure 5-2

b. Additional reactive power absorption caused by tap stagger (compared to the initial value)

Table 5-2: Active power variations of a single pair of primary substation transformers with tap stagger

<table>
<thead>
<tr>
<th>Tap Steps Apart</th>
<th>$P_{in1}$ (MW)</th>
<th>$P_{in2}$ (MW)</th>
<th>$P_{out}^a$ (MW)</th>
<th>$P_{loss}^a$ (MW)</th>
<th>Additional $P_{loss}^b$ (MW)</th>
<th>Secondary Voltage (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.832</td>
<td>6.832</td>
<td>13.623</td>
<td>0.041</td>
<td>0</td>
<td>1.00673</td>
</tr>
<tr>
<td>2</td>
<td>7.000</td>
<td>6.667</td>
<td>13.623</td>
<td>0.044</td>
<td>0.003</td>
<td>1.00686</td>
</tr>
<tr>
<td>4</td>
<td>7.170</td>
<td>6.502</td>
<td>13.623</td>
<td>0.049</td>
<td>0.008</td>
<td>1.00726</td>
</tr>
<tr>
<td>6</td>
<td>7.342</td>
<td>6.340</td>
<td>13.622</td>
<td>0.060</td>
<td>0.019</td>
<td>1.00794</td>
</tr>
<tr>
<td>8</td>
<td>7.516</td>
<td>6.179</td>
<td>13.623</td>
<td>0.072</td>
<td>0.031</td>
<td>1.00888</td>
</tr>
<tr>
<td>10</td>
<td>7.693</td>
<td>6.020</td>
<td>13.623</td>
<td>0.090</td>
<td>0.049</td>
<td>1.01009</td>
</tr>
<tr>
<td>12</td>
<td>7.872</td>
<td>5.862</td>
<td>13.623</td>
<td>0.111</td>
<td>0.070</td>
<td>1.01156</td>
</tr>
</tbody>
</table>

a. $P_{out} = P_{out1} + P_{out2}$, and $P_{loss} = P_{in1} + P_{in2} - P_{out}$, referred to Figure 5-2

b. Additional active power losses caused by tap stagger (compared to the initial value)
Figure 5-4 indicates the additional active power losses of the transformers from both the simulation and calculation results. According to the results, the transformer power losses increase quadratically as the tap step difference increases. However, as transformers are usually more inductive than resistive, the increase in $P_{loss}$ (i.e. maximum 0.07 MW) is small compared to the increment of $Q_{absorbed}$ (i.e. maximum 1.73 MVAr).

![Graph showing additional reactive power absorption](image1)

Figure 5-3: Additional reactive power absorption of a pair of parallel transformers with tap stagger

![Graph showing additional active power losses](image2)

Figure 5-4: Additional active power losses of a pair of parallel transformers with tap stagger
5.3.2 Case 2: Aggregated reactive power absorption of twenty pairs of parallel transformers

In this case study, another 19 pairs of parallel transformers were selected and tested. The transformers were randomly chosen from either 33/11 kV or 33/6.6 kV primary substations. The selected transformers were uniformly distributed in the 33kV networks so that the simulation result could reflect the overall network characteristics. For each tested primary substation, the difference in tap positions was set at the maximum value (e.g. 10 or 12), without violating the tap position limits and transformer ratings. The sum of the additional $Q_{absorbed}$ (or $P_{loss}$) from each primary substation was then calculated. As there are no VAr compensation devices installed in the distribution system, all the additional reactive power consumption must be supplied from the 400kV grid infeed. Therefore, the power variations at the GSP sites, which connect the grid and the distribution network, were also monitored. Table 5-3 summarises the load flow study results.

Table 5-3: Additional VAr absorption and power loss of distribution system with tap staggered transformers

<table>
<thead>
<tr>
<th>Pairs of parallel transformers$^a$</th>
<th>Additional $Q_{absorbed}$ (MVAr)</th>
<th>Extra Q supplied from 400kV network (MVAr)</th>
<th>Additional $P_{loss}$ (MW)</th>
<th>Extra P supplied from 400kV network (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3.41</td>
<td>5</td>
<td>0.12</td>
<td>0.19</td>
</tr>
<tr>
<td>4</td>
<td>5.6</td>
<td>8.18</td>
<td>0.2</td>
<td>0.36</td>
</tr>
<tr>
<td>6</td>
<td>9.11</td>
<td>13.09</td>
<td>0.31</td>
<td>0.55</td>
</tr>
<tr>
<td>8</td>
<td>11.16</td>
<td>15.88</td>
<td>0.38</td>
<td>0.69</td>
</tr>
<tr>
<td>10</td>
<td>14.86</td>
<td>21.17</td>
<td>0.49</td>
<td>0.93</td>
</tr>
<tr>
<td>12</td>
<td>17.05</td>
<td>24.24</td>
<td>0.56</td>
<td>1.1</td>
</tr>
<tr>
<td>14</td>
<td>19.32</td>
<td>27.27</td>
<td>0.65</td>
<td>1.19</td>
</tr>
<tr>
<td>16</td>
<td>21.49</td>
<td>31.8</td>
<td>0.76</td>
<td>1.42</td>
</tr>
<tr>
<td>18</td>
<td>23.48</td>
<td>34.45</td>
<td>0.85</td>
<td>1.6</td>
</tr>
<tr>
<td>20</td>
<td>25.63</td>
<td>37.2</td>
<td>0.94</td>
<td>1.77</td>
</tr>
</tbody>
</table>

a. with maximum possible staggered taps (e.g. 10 or 12 taps), considering OLTC and transformer ratings
From Table 5-3, the additional $Q_{absorbed}$ increases with the number of transformer pairs using tap stagger. The extra reactive power supplied from the 400kV grid is higher than the transformer consumption. This is due to the reactive power transmission losses between the GSPs and primary substations. In Figure 5-5, the additional $Q$ absorptions at the transformer or the 400kV grid sides are plotted against the transformer pairs. According to the simulation results of the 20 primary substations, trendlines have also been produced in Figure 5-5. Due to the radial configuration of the distribution system, the power flows of primary substations are almost independent of each other. Therefore, the linear fitting method [125], [126] has been used to estimate the relationship between the VAr absorption and the number of transformer pairs. The relationship can be described as:

$$y = \beta_0 + \beta_1 x, \text{ with } R^2 \in (0, 1)$$  \hspace{1cm} (5-1)

where $x$ denotes the pairs of parallel transformers with tap staggered, and $y$ denotes the additional reactive power absorption introduced. $\beta_0$ is the intercept, and $\beta_1$ is the slope, i.e. the rate of change in $y$ per unit change in $x$. $R^2$ is the coefficient of determination, which indicates how well the measured data fit the estimated trendline [126]. An estimated model is most accurate when the coefficient is at or near 1. The details of the derivation of $\beta_0$, $\beta_1$ and $R^2$ are given in APPENDIX B.

As shown in Figure 5-5, the relationship between the VAr absorption and the number of transformers has been estimated as:

$$y_1 = 1.2837x + 0.9005, \text{ with } R^2 = 0.9939$$  \hspace{1cm} (5-2)

$$y_2 = 1.8699x + 1.1445, \text{ with } R^2 = 0.9956$$  \hspace{1cm} (5-3)

where $x$ denotes the number of transformer pairs (with the maximum possible staggered taps), $y_1$ and $y_2$ represent the additional VAr absorption seen from the transformer and the 400kV grid sides, respectively. In Equations (5-2) and (5-3), the values of $R^2$ are higher
than 0.99, which proves the linear estimation is accurate. The slope of the trendline (at the transformer side) is 1.28, which implies a maximum of 1.28 MVAr can be consumed by tapping one pair of primary substation transformers apart. Figure 5-5 also indicates a maximum consumption of 1.87 MVAr/substation at the 400kV grid side (including the VAr absorption from the 132kV GSPs down to the 33kV primary substations). There are 354 primary substations within the distribution network model to which the tap staggering technique can be applied. Consequently, the total potential VAr absorption capability of the overall distribution system would be 662 MVAr.

In terms of active power, the additional $P_{loss}$ at the transformer and 400kV grid sides are plotted against the transformer pairs in Figure 5-6. The additional power loss due to tap stagger could be a maximum of 0.088 MW/substation (observed from the 400kV side). The aggregated loss of the entire distribution network would be 31 MW, corresponding to about 0.6% of the active power flow in the network. Note that this is the largest power loss that would be generated when using the maximum available staggered taps. The loss can be reduced by limiting the tap step difference. A more cautious study with fewer staggered taps is given in the following case.
5.4 Conservative VAr Absorption Capability Studies

From the studies in the previous section, an aggregated VAr absorption of 662 MVAr has been estimated for the overall distribution network. The results have assumed all the primary substation transformers can be set with the maximum staggered taps (e.g. 10 or 12 taps depending on the OLTC and transformer ratings). However in practice, due to network demand variations, the initial OLTC position may not be close to the nominal position, resulting in less headroom for the tap stagger. In addition, a large difference in tap positions will introduce more transformer losses and may lead to overheating. Therefore, considering the transformer health and network losses, a more cautious study of transformers with 4 staggered tap steps (i.e. 2 tap up for one transformer and 2 tap down for the other) has been carried out in this section. The testing procedure was similar to Case 2. However, the 20 pairs of primary substation transformers were only allowed to operate with 4 staggered taps.
Figure 5-7 illustrates the simulation results. From the figure, an average VAr absorption of 0.24 MVAr/substation can be observed from the 400kV grid side and the aggregated VAr consumption of the overall distribution network (with 354 primary substations) will be 85 MVAr. In general, the additional VAr absorption is expected to be needed during periods of low demand, e.g. summer nights. Therefore, assuming there are 4 hours per night over a 90-day period, the total reactive energy absorption would be 30.6 GVArh (with 4 staggered taps). If other distribution networks connected to the same transmission system can also participate in the reactive power service, a significant amount of VAr absorption would be provided to support the transmission system.

![Graph](image)

Figure 5-7: Aggregated Q absorption from 20 pairs of parallel transformers (with 4 staggered taps)

Figure 5-8 shows the aggregated power loss of the distribution network due to the tap stagger. In the case of applying 4 staggered taps, the average loss is 0.012 MW/substation (at the 400kV grid side) and a total loss of 4 MW for the overall distribution system. Compared with the maximum tap staggering study in Case 2 (see Figure 5-6), the use of 4 staggered taps has reduced the power loss by 87%. Since the tap staggering operation is likely to be initiated when the network loading becomes low, the small increase in the network losses may not have significant impacts on the system security. The dynamic performance of the tap staggering method will be discussed in Chapter 8.
5.5 Summary

This chapter has presented the feasibility studies in order to assess the VAr absorption capability of the distribution network through the use of the tap staggering technique. Off-line load flow studies have been carried out based on a real UK HV distribution system model. The model consists of the networks from the 132kV GSPs down to the 33kV primary substations. During the simulations, the tap staggering method has been applied to the parallel transformers at the primary substations.

Case 1 has demonstrated the reactive and real power variations of a single pair of parallel transformers with tap stagger. From the results, both the VAr absorption and power losses of the transformers have increased with the number of staggered taps. The results are also consistent with the theoretical calculation results. In Case 2, 20 pairs of primary substation transformers have been tested again. Based on the results, the curve fitting method has been used to estimate the aggregated VAr absorption of the overall distribution system. According to the estimate, the distribution network can provide a
total 662 MVAr absorption to the 400kV grid if all the primary substation transformers are allowed to operate at the maximum staggered taps. However in practice, due to the variations of network demands, the initial OLTC position may not be at or near the nominal position, resulting in less headroom for the tap stagger. In addition, the large tap difference will introduce more losses and may lead to transformer overloading. Therefore, conservative studies have been carried out considering the transformers with only 4 staggered taps. The results indicate that the aggregated VAr absorption of the distribution network is 85 MVAr, and the introduced network loss is 4 MW, which corresponds to 0.08% of the active power flow in the network.

The network capability studies in this chapter aim to demonstrate the potential of using the tap staggering method to provide the VAr absorption service. The results have confirmed that the distribution network has the potential to support the reactive power management in the transmission system. If each distribution network can offer a certain amount of VAr absorption, the transmission system can utilise these inductive VAr to mitigate the high voltages for some temporary periods. The impacts of the distribution network VAr absorption on the transmission system will be analysed in Chapter 8.
CHAPTER 6
IMPLEMENTATION OF TAP STAGGER CONTROL USING GA

6.1 Introduction

As discussed in the previous chapter, the employment of tap stagger in distribution networks has the potential to provide the VAr absorption service and to increase the utilisation of existing assets. Therefore, this chapter aims to address the issue of how to provide the VAr absorption service when it is needed. For instance, if the upstream transmission grid requires a specific reactive power to be consumed during periods of low demand, the distribution system should be able to decide how many parallel transformers and staggered taps will be used to meet the requirement. It is also important to keep the switching operations of OLTCs as few as possible to reduce the maintenance costs of tap changers.

This chapter presents the implementation of the optimal tap stagger control method proposed in Chapter 4 (see Figure 4-7). The control objective is to find an optimal dispatch for the tap positions of the parallel transformers, minimising the active power losses caused as well as the number of tap switching operations introduced. The solution algorithm is based on Genetic Algorithm (GA), which can solve mixed-integer nonlinear programming (MINLP) problems with fast convergence. The details of the implementation process and testing results are described as follows.

6.2 Network Modelling in OpenDSS

In Chapter 5, an IPSA distribution network model has been provided by the ‘CLASS’ project [123]. The proposed tap stagger optimal control requires the information of the network states, e.g. load levels, voltages and transformer tap positions. However, the
IPSA model does not provide an interface for an external program to access load flow results. Therefore, to have communications between the control algorithm and the network states, the distribution system has been modelled using another power flow software, i.e. the OpenDSS [127].

The Open Distribution System Simulator (OpenDSS) is an open source simulation tool for power flow calculations, harmonics analyses and fault studies in electric distribution systems. Compared with the IPSA, the OpenDSS provides more comprehensive load models and can perform automatic time-series (e.g. daily and yearly) simulations using load profiles. In addition, the OpenDSS can be driven from the MATLAB, where an optimisation algorithm can be developed. Consequently, an interactive system can be implemented between the OpenDSS network model and the MATLAB algorithm.

6.2.1 Network transfer from IPSA to OpenDSS

6.2.1.1 Network modification in IPSA
The IPSA distribution system model studied in Chapter 5 has 354 primary substations and each primary substation transformer usually has 15 available tap positions. The total search space for the tap stagger optimisation is around $15^{354}$, which is a heavy computation burden for a standard personal computer. Therefore, to reduce the burden, the original model has first been simplified. Figure 6-1 shows a screenshot of the modified distribution network in the IPSA. This model includes one of the 132kV GSP networks in the original system as well as the downstream 33kV networks connected to the GSP. As shown in Figure 6-1, the 400kV voltage source generates power to the system. From the GSP, electricity is distributed through two 33kV networks. This radial distribution system has 102 buses, 89 lines and 32 transformers. Figure 6-2 illustrates the corresponding network structure at different voltage levels (i.e. 132kV, 33kV, 11kV and 6.6kV). Note that there are total 11 primary substations in the system and each consists of two parallel transformers with OLTCs. Loads have been modelled as constant power loads and connected to the secondary sides of the primary substation transformers. The total rated load demand for the network is 170 MW and 54 MVAr.
CHAPTER 6  IMPLEMENTATION OF TAP STAGGER CONTROL USING GA

Figure 6-1: Screenshot of the simplified distribution network in IPSA
6.2.1.2 IPSA modelling script

The Windows Notepad editor can open the IPSA network modelling file. Figure 6-3 shows a screenshot of the IPSA script for the modified 102-bus distribution system. This script contains all the information required for the network modelling, e.g. the parameters of lines, transformers and loads. Therefore, a MATLAB based program has been developed to extract these data and convert them into the OpenDSS format.

![Screenshot of the IPSA modelling script for the modified 102-bus distribution system](image.png)
6.2.1.3 OpenDSS modelling files

Since the OpenDSS is a text-based simulation tool, network modelling is achieved by defining circuit description files. Figure 6-4 shows a screenshot of the converted description files from the IPSA script. The modelling files include the Line.dss, Transformer.dss, Load.dss and Tapchanger.dss. These files are sufficient to define an AC load flow in the OpenDSS. The load flow problem will be solved by the Simulation Engine as illustrated in Figure 6-5.

![Figure 6-4: Screenshot of the OpenDSS modelling scripts for the 102-bus distribution system](image)

![Figure 6-5: The OpenDSS structure [128]](image)
6.2.2 Validation of OpenDSS network model

In this section, power flow studies have been carried out to validate the converted OpenDSS model. The simulation results have been compared with the results obtained from the IPSA model.

6.2.2.1 Bus voltage comparisons

Based on the network shown in Figure 6-1, the OpenDSS model has been tested with the following four cases:

Case a: Set the same transformer tap positions for both the OpenDSS and IPSA models. Disconnected the automatic voltage control of the tap changers. Disconnected the distributed generators in Load Area 2. These were the initial conditions for both Case b and Case c.

Case b: Connected a total 23.8 MW distributed generators to Load Area 2 and set the reactive power generation to zero.

Case c: Based on Case b, increased the network load by 50% of its initial consumption.

Case d: Enabled the automatic voltage control of the tap changers. Set the same target voltages in both the OpenDSS and IPSA models.

For each case, both the OpenDSS and the IPSA have calculated the bus voltages. Figure 6-6 plots the absolute errors between the calculated voltages. Table 6-1 also summarises the maximum and average voltage errors, and the standard deviation among all the buses. For Cases a, b and c, the bus voltages from the OpenDSS and IPSA results are almost identical, with average errors less than 0.00003 pu. However for Case d, the errors are higher, i.e. with an average of 0.005 pu. This is due to the automatic tap changer control enabled in Case d. During the load flow simulation, the control would adjust the transformer tap position to maintain the bus voltage within a predefined deadband. As the OpenDSS and IPSA may have different default deadbands, the tap positions may change differently, resulting in different solutions.
CHAPTER 6 IMPLEMENTATION OF TAP STAGGER CONTROL USING GA

6.2.2.2 Tap stagger comparisons

In this study, the tap staggering method has been tested using the primary substation transformers modelled in the OpenDSS and the results have been compared with the IPSA model. The testing procedure described in Section 5.3.1 has been applied to the 11 pairs of primary substation transformers in the network. Figure 6-7 shows an example of the power variations of one primary substation. As shown in the figures, the power variations ...
variations in the OpenDSS and IPSA models are very similar so that the curves overlap each other. All the results above confirm that the converted OpenDSS model is correct.

![Graph](image1)

(a) Additional reactive power absorption of the two transformers

![Graph](image2)

(b) Additional active power losses of the two transformers

Figure 6-7: Example of a pair of primary substation transformers with staggered taps (in IPSA and OpenDSS)

### 6.3 Tap Stagger Control based on Genetic Algorithm

This section presents the implementation of the tap stagger optimal control on the developed OpenDSS distribution network model. The optimisation problem described in Section 4.4 has been solved using the genetic algorithm (GA), which is a global search method based on natural selection [129], [130]. According to the literature review in
Chapter 2, GA has been widely adopted for the optimal Volt/VAr control in distribution systems. GA is capable of dealing with different types of variables, such as continuous, discrete or mixed types. Additionally, it does not need to calculate the gradient of the objective function and can provide the solution with fast convergence. Therefore, the genetic algorithm has been used in this study to solve the tap stagger optimisation problem. The related control variables, fitness function and genetic operators are defined as follows.

### 6.3.1 Control variable encoding

From Equation (4-18), the variable $x_i$ is a control signal that instructs the OLTCs of two parallel transformers to tap apart. A set of such variables will constitute a candidate solution to Equation (4-18). In this GA-based optimisation problem, a candidate solution is termed as an individual and the chromosome of each individual can be represented as a row vector:

$$x = [x_1 \ x_2 \ x_3 \ … \ x_i \ … \ x_N] \quad (6-1)$$

where $x_i$ is an arbitrary integer on the interval $[0, TS_{max} / 2]$ considering the constraint in Equation (4-21). $N$ denotes the total pairs of parallel transformers involved in the tap stagger optimisation. If the maximum allowable tap position difference $TS_{max}$ is set to 4, the size of the search space will be $(1 + TS_{max} / 2)^N = 3^N$. The GA approach will start from a population of randomly generated individuals and produce the next generation based on the current population. Over successive generations, the population may evolve toward an optimal solution.

### 6.3.2 Fitness function

The tap stagger optimal control aims to minimise the power loss and the tap switching operations while achieving the VAr absorption target. Therefore, considering the
objective in Equation (4-18) as well as the fulfilment of the constraint in Equation (4-19), the fitness function (i.e. objective function) of the GA approach can be expressed as:

\[
\min F = w_1 \cdot \Delta P_{\text{loss}}(\mathbf{x}) + w_2 \cdot \sum_{i=1}^{N} 2x_i + w_3 \cdot f(e) 
\]  

(6-2)

where,

\[
f(e) = \begin{cases} 
0, & \text{if } e \leq e_{\text{limit}} \\
 e - e_{\text{limit}}, & \text{otherwise} 
\end{cases} 
\]  

(6-3)

\[
e = \left| \frac{Q_{\text{actual}}(\mathbf{x}) - Q_{\text{required}}}{Q_{\text{required}}} \right| \times 100\%
\]  

(6-4)

\(w_3\) is the weighting coefficient of the reactive power absorption. The penalty function \(f(e)\) is used to measure the violation of the VAr requirement. The weighting factors (i.e. \(w_1\), \(w_2\) and \(w_3\)) can be normalised to find a balance between the loss reduction, the minimisation of tap operations and the provision of reactive power services. Both \(\Delta P_{\text{loss}}(\mathbf{x})\) and \(Q_{\text{actual}}(\mathbf{x})\) can be calculated using a load flow module. The individual with the best fitness is the one with the chromosome to minimise the fitness function.

### 6.3.3 Genetic operators

#### 6.3.3.1 Selection of parents

In each generation, the fitness of every individual in the population is evaluated according to Equation (6-2). Based on the fitness values, the GA uses the current population to create the children for the next generation. The individuals with the best fitness values (i.e. elites) automatically survive to the next generation. The stochastic uniform sampling method [130] has been adopted to select a group of individuals, i.e. known as parents, to produce children. Figure 6-8 illustrates the working principle. The stochastic uniform selection function lays out a line where each individual corresponds to a section of the line. The section length is reciprocal to the fitness value of the individual.
The algorithm moves along the line in an equal step size $s$. At each step, the algorithm allocates a parent from the section it lands on. This method naturally avoids the excessive selection of the individuals with better fitness values and maintains the diversity of the population.

![Diagram of the stochastic uniform sampling method]

**Figure 6-8:** Mechanism of the stochastic uniform sampling method

### 6.3.3.2 Crossover and mutation functions

Based on the selected parents, the children of the next generation are produced through crossover and mutation. Crossover recombines the genes of two individuals while mutation adds new characteristics into the individuals. In this GA approach, the scattered crossover function has been used \[130\]. The function creates a random binary vector and selects the genes from the first parent where the vector is a \('1\)', and the genes from the second parent where the vector is a \('0\)’. For example, if $p1$ and $p2$ are the parents:

$$
\begin{align*}
p1 &= [a \ b \ c \ d \ e \ f \ g \ h] \\
p2 &= [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8]
\end{align*}
$$

and the binary vector is $[1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1]$, the function will produce the child:

$$
\text{child1} = [a \ 2 \ 3 \ d \ 5 \ 6 \ g \ h]
$$

In terms of mutation, a specific function has been developed for this tap stagger optimisation problem. The mutation function randomly generates individuals that will result in the total number of tap changing operations (i.e. $\sum_{i=1}^{N} 2x_i$) equal to or one fewer than the current best solution. The details of the algorithm are given in APPENDIX C. The mutation allows the GA to find more promising solutions rather than being trapped in local minima.
6.3.4 Computational procedure

Figure 6-9 demonstrates the flowchart of the GA-based solution algorithm. As shown in the figure, the algorithm includes a load flow module to calculate the network loss and the actual VAr absorption due to the tap stagger. The initialisation requires the settings of $Q_{\text{required}}$, $e_{\text{limit}}$ and $TS_{\text{max}}$. When evaluating the fitness value, the algorithm will first set the tap positions of OLTCs according to the chromosome $x$. Load flow will then be performed to calculate the values of $\Delta P_{\text{loss}}$ and $Q_{\text{actual}}$. Note that the OLTCs will remain at the maximum or minimum positions if the constraint in Equation (4-20) is violated. The GA process will stop if the maximum number of generations is reached or the best fitness value in the population changes negligibly after several consecutive generations.

Start
Randomly generate an initial population using Equation (6-1)
Calculate the fitness values using Equation (6-2)

Stopping criteria?

No
Selection

Yes
Load flow module

Next generation ($n_{\text{gen}} = n_{\text{gen}} + 1$)

Initialisation
(Set $Q_{\text{required}}$, $e_{\text{limit}}$ and $TS_{\text{max}}$)

$N_{\text{pop}}$ individuals

$x$: Check the constraint of Equation (4-20)

$\Delta P_{\text{loss}}$, $Q_{\text{actual}}$

$N_{\text{pop}} - N_{e}$ children

$N_{e}$ elites

$C$ crossover

$M$ mutation

Output results

End

Figure 6-9: Flowchart of the GA-based solution procedure
6.4 Testing of GA-based Control

The proposed GA-based control approach has been applied to the developed 102-bus distribution network. The algorithm has been programmed using the MATLAB Global Optimisation Toolbox [131] and the full codes are given in APPENDIX C. The OpenDSS has been used to implement the load flow module shown in Figure 6-9.

6.4.1 Parameter assignment

As shown in Figure 6-1, the modelled distribution network has a total 11 pairs of primary substation transformers available for the tap staggering operation, i.e. \( N = 11 \). The network VAr absorption \( (Q_{\text{actual}}) \), due to the tap stagger, is obtained by calculating the reactive power flow variation at the GSP. Based on the Grid Code of National Grid [132], the maximum permitted error \( (e_{\text{limit}}) \) between \( Q_{\text{required}} \) and \( Q_{\text{actual}} \) has been set to 1%. The distribution system loss \( (\Delta P_{\text{loss}}) \), due to the tap stagger is calculated by aggregating the variations of the line and transformer losses. Considering the transformer health and network loss, the maximum tap difference \( (TS_{\text{max}}) \) has been limited to 4 taps throughout the simulations. Therefore, the search space for the GA is \( (1 + TS_{\text{max}} / 2)^{11} = 3^{11} = 177147 \). For Equation (6-2), \( w_1, w_2 \) and \( w_3 \) have been normalised to balance the influences of the power losses, OLTC switching and reactive power absorption on the optimisation result. The details of the GA parameters are given in Table 6-2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of variables (( N ))</td>
<td>11</td>
<td>Population size (( N_{\text{pop}} ))</td>
<td>100</td>
</tr>
<tr>
<td>( e_{\text{limit}} )</td>
<td>1%</td>
<td>Number of elites (( N_e ))</td>
<td>2</td>
</tr>
<tr>
<td>( TS_{\text{max}} )</td>
<td>4</td>
<td>Crossover fraction</td>
<td>0.5</td>
</tr>
<tr>
<td>( w_1 )</td>
<td>1</td>
<td>Maximum generations</td>
<td>100</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>( 1/22 )</td>
<td>Stall generations</td>
<td>20</td>
</tr>
<tr>
<td>( w_3 )</td>
<td>1</td>
<td>Convergence tolerance</td>
<td>( 1\times10^{-6} )</td>
</tr>
</tbody>
</table>
The crossover fraction specifies the proportion of the next generation, other than elite children, that are produced by the crossover. The algorithm will stop if the average change in the fitness function value over Stall generations is less than the Convergence tolerance. These GA parameters have been tuned through trial and error to obtain accurate solutions with less time.

6.4.2 Optimisation results

6.4.2.1 Test with a single reactive power requirement
To demonstrate an example of the optimisation result, the GA approach has solved a dispatch problem with the transmission system VAr requirement $Q_{\text{required}} = 0.5$ MVAr. Figure 6-10 demonstrated the data from the GA when it was running.

As shown in the upper half of the figure, the best fitness value in the population has converged quickly after just one generation. The lower half of Figure 6-10 indicates the average distance between individuals, i.e. known as diversity. Diversity enables the GA to search a larger region of space and it prevents premature convergence. The total
CHAPTER 6 IMPLEMENTATION OF TAP STAGGER CONTROL USING GA

computation time is 8.1s running on an i7-3.4GHz/8GB RAM computer and the final solution is:

\[ x_{\text{optimal}} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 2 \ 0 \ 0 \ 2] \]  

(6-8)

which indicates that there would be 4 staggered taps on both the 8th and 11th pairs of primary substation transformers (see Figure 6-1). The total number of switching operations involved is 8; the introduced \( \Delta P_{\text{loss}} \) is 0.027MW and the error between \( Q_{\text{required}} \) and \( Q_{\text{actual}} \) is 0.95%. The solution \( x_{\text{optimal}} \) determines the arrangement of OLTC tap positions while satisfying the VAr requirement.

6.4.2.2 Tests with different VAr requirements

Due to the random process of the GA, the algorithm may return a different solution each time it runs. Therefore, statistical studies have been undertaken to analyse the accuracy and precision of the algorithm. During the simulations, the value of \( Q_{\text{required}} \) increased from 0.1 MVAr to 2.5 MVAr with a step of 0.1 MVAr. The GA ran ten times for each \( Q_{\text{required}} \) and the statistic results are summarised in Table 6-3.

According to the table, for most \( Q_{\text{required}} \), the standard deviations of the optimisation results over ten runs are zero. This implies that the GA can provide precise control of tap stagger. In the situation where many feasible solutions exist in the search space (e.g. \( Q_{\text{required}} = 1.0 \) and 1.5 MVAr), it is more difficult to find the optimal solutions, resulting in relatively large standard deviations. Note that the maximum limit of the staggered taps \( TS_{\text{max}} \) can constrain the VAr absorption. For instance, at \( Q_{\text{required}} = 2.5 \) MVAr, the algorithm has returned large VAr absorption errors between \( Q_{\text{required}} \) and \( Q_{\text{actual}} \), i.e. an average of 3.095%. It has been observed that all the transformers have already used the maximum staggered taps, i.e. 4 staggered taps. Consequently, no more reactive power can be absorbed through the use of tap stagger. The network can only provide reactive power absorption up to 2.4 MVAr with \( TS_{\text{max}} = 4 \).
Table 6-3: Optimisation results with GA running ten times for each value of $Q_{required}$ (at $e_{limit} = 1\%$ & $TS_{max} = 4$)

<table>
<thead>
<tr>
<th>$Q_{required}$ (MVAr)</th>
<th>Total tap operations</th>
<th>Network loss due to stagger (MW)</th>
<th>Error between $Q_{required}$ and $Q_{actual}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>average</td>
<td>standard deviation</td>
<td>average</td>
</tr>
<tr>
<td>0.1</td>
<td>4</td>
<td>0</td>
<td>0.0094</td>
</tr>
<tr>
<td>0.2</td>
<td>8</td>
<td>0</td>
<td>0.0146</td>
</tr>
<tr>
<td>0.3</td>
<td>6</td>
<td>0</td>
<td>0.0113</td>
</tr>
<tr>
<td>0.4</td>
<td>8</td>
<td>0</td>
<td>0.0198</td>
</tr>
<tr>
<td>0.5</td>
<td>8</td>
<td>0</td>
<td>0.0273</td>
</tr>
<tr>
<td>0.6</td>
<td>12</td>
<td>0</td>
<td>0.0273</td>
</tr>
<tr>
<td>0.7</td>
<td>12</td>
<td>0</td>
<td>0.0264</td>
</tr>
<tr>
<td>0.8</td>
<td>14</td>
<td>0</td>
<td>0.0383</td>
</tr>
<tr>
<td>0.9</td>
<td>16</td>
<td>0</td>
<td>0.0355</td>
</tr>
<tr>
<td>1.0</td>
<td>16.2</td>
<td>0.6325</td>
<td>0.0497</td>
</tr>
<tr>
<td>1.1</td>
<td>20</td>
<td>0</td>
<td>0.0443</td>
</tr>
<tr>
<td>1.2</td>
<td>20</td>
<td>0</td>
<td>0.0532</td>
</tr>
<tr>
<td>1.3</td>
<td>24</td>
<td>0</td>
<td>0.0553</td>
</tr>
<tr>
<td>1.4</td>
<td>24</td>
<td>0</td>
<td>0.0628</td>
</tr>
<tr>
<td>1.5</td>
<td>26.6</td>
<td>0.9661</td>
<td>0.0730</td>
</tr>
<tr>
<td>1.6</td>
<td>28</td>
<td>0</td>
<td>0.0708</td>
</tr>
<tr>
<td>1.7</td>
<td>30</td>
<td>0</td>
<td>0.0799</td>
</tr>
<tr>
<td>1.8</td>
<td>32</td>
<td>0</td>
<td>0.0828</td>
</tr>
<tr>
<td>1.9</td>
<td>34</td>
<td>0</td>
<td>0.0890</td>
</tr>
<tr>
<td>2.0</td>
<td>36</td>
<td>0</td>
<td>0.1028</td>
</tr>
<tr>
<td>2.1</td>
<td>38</td>
<td>0</td>
<td>0.1038</td>
</tr>
<tr>
<td>2.2</td>
<td>40</td>
<td>0</td>
<td>0.1245</td>
</tr>
<tr>
<td>2.3</td>
<td>42</td>
<td>0</td>
<td>0.1237</td>
</tr>
<tr>
<td>2.4</td>
<td>44</td>
<td>0</td>
<td>0.1397</td>
</tr>
<tr>
<td>2.5</td>
<td>44</td>
<td>0</td>
<td>0.1397</td>
</tr>
</tbody>
</table>

Based on Table 6-3, Figure 6-11 illustrates the maximum, minimum and average values of the ten optimisation results at each $Q_{required}$. As shown in Figure 6-11a and Figure 6-11b, the total number of tap switching operations and network power losses increase with the VAr requirement. As the VAr requirement increases, the tap stagger control
needs to use more staggered taps to generate the reactive power absorption, leading to more switching operations and network losses.

Figure 6-11: Optimisation results with different reactive power requirements (with $\epsilon_{\text{init}} = 1\%$ & $TS_{\text{max}} = 4$)
In Figure 6-11c, the VAr absorption errors (between $Q_{\text{required}}$ and $Q_{\text{actual}}$) change randomly with respect to the VAr requirement, and most errors are below the predefined 1% tolerance. However, for $Q_{\text{required}} = 2.5$ MVAr, the errors are higher than 1%. The large errors have been introduced since all the transformers have reached the maximum limit of staggered taps, i.e. 4 staggered taps. The network can only provide reactive power absorption up to 2.4 MVAr under this constraint.

### 6.5 Effects of Different Load Models

As the use of tap stagger will draw more reactive power from the upstream networks, the reactive currents between the 132kV and 33kV networks will increase. The HV voltages of primary substations may drop, depending on the amount of VAr absorption produced by tap stagger. Consequently, the LV voltages of primary substations may decrease as well. The loads connected at the LV sides may change, depending on the relationship between loads and voltages. In the simulations discussed above, the GA approach has solved the VAr dispatch problems considering the network with constant power loads connected at the primary substations. To further understand the impacts of load models on the optimisation results, this section presents the studies with different load models. The network load at each bus has been modelled as the following four static types [133]:

a) Constant power load.

b) Constant current magnitude load, with $P = P_0 \cdot \frac{V}{V_0}$ and $Q = Q_0 \cdot \frac{V}{V_0}$, where $P_0$ and $Q_0$ are the active and reactive components of load when the bus voltage is $V_0$.

c) Constant impedance load, with $P = P_0 \cdot \left(\frac{V}{V_0}\right)^2$ and $Q = Q_0 \cdot \left(\frac{V}{V_0}\right)^2$.

d) 50% constant power load and 50% constant impedance load.

For each load model, the GA has solved the tap stagger optimisation problems with different values of $Q_{\text{required}}$ (i.e. from 0.1 to 2.3 MVAr). The GA parameters have been set according to Table 6-2. The algorithm has run once for each $Q_{\text{required}}$ and the corresponding results are summarised in Table 6-4.
In terms of tap switching operations, the constant power load model has used the lowest number of tap operations for each $Q_{\text{required}}$. This is because that when applying the tap staggering technique, the reactive power flow between the 132kV and 33kV networks will increase. Consequently, the line voltage drops will increase, leading to slightly reduced voltages at both the HV and LV sides of the primary substations (e.g. an average of 0.15% voltage reduction with $Q_{\text{required}} = 2.3$ MVAr). For voltage-dependent load models (i.e. models b, c and d), the load demand will decrease as well, e.g. a maximum of 0.3% VAr demand reduction for the constant impedance load model. As shown in Figure 6-12, the total VAr absorption observed at the GSP ($Q_{\text{actual}}$) consists of the $Q$ variations in line, transformer and load VAr consumptions. The reactive load reduction will counteract the increase of reactive power flow caused by the tap stagger. To achieve the same VAr absorption target, the transformers will need more staggered taps to raise their VAr consumption, which will increase the tap switching operations. However, the variations of the total tap switching operations, due to different load models, are not significant, i.e. within 2 - 6 tap operations.

<table>
<thead>
<tr>
<th>$Q_{\text{required}}$ (MVAr)</th>
<th>Total tap operations</th>
<th>Network loss due to tap stagger (MW)</th>
<th>Error between $Q_{\text{required}}$ and $Q_{\text{actual}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>0.1</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>0.3</td>
<td>6</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>0.5</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>0.7</td>
<td>12</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>0.9</td>
<td>16</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>1.1</td>
<td>20</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>1.3</td>
<td>24</td>
<td>24</td>
<td>28</td>
</tr>
<tr>
<td>1.5</td>
<td>26</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>1.7</td>
<td>30</td>
<td>34</td>
<td>36</td>
</tr>
<tr>
<td>1.9</td>
<td>34</td>
<td>38</td>
<td>42</td>
</tr>
<tr>
<td>2.1</td>
<td>38</td>
<td>42</td>
<td>44</td>
</tr>
<tr>
<td>2.3</td>
<td>42</td>
<td>44</td>
<td>44</td>
</tr>
</tbody>
</table>

Note that at $Q_{\text{required}} = 2.1$ and 2.3 MVAr, the voltage-dependent load models (e.g. of models b, c and d) have generated large errors of VAr absorption. It has been observed
that all the transformers have reached the maximum permitted staggering limit \( T_{S_{\text{max}}} \) and no extra reactive power can be absorbed to meet the VAr requirement. The VAr absorption error can be reduced by increasing \( T_{S_{\text{max}}} \) if allowed.

In addition, Table 6-4 indicates that the voltage-dependent load models have generally resulted in more network losses (caused by tap stagger) than the constant impedance load model. As explained before, the network with voltage-dependent loads will usually use more staggered taps between the transformers to achieve the VAr absorption target. This will further increase the transformer losses as well as the line losses between the 132kV and 33kV networks. Consequently, the total network losses will increase. However, for all the load models, the network losses introduced by tap stagger are much smaller (i.e. 90% less) than the reactive power absorption created.

\[
Q_{\text{actual}} = Q_{\text{loads}} + Q_{\text{transformers}} + Q_{\text{lines}}
\]

Figure 6-12: Reactive power absorption observed at the GSP by using the tap staggering technique

### 6.6 Time-Series Studies with Daily Load Profile

Since the demand for electricity changes continually in real distribution systems, it is necessary to investigate the tap stagger optimisation at different load levels. As network loads increase or decrease, the AVC relay needs to adjust the OLTC tap position to maintain the bus voltage constant. Depending on the changed tap positions, the optimisation result may be different from the previous load level.
To carry out time-series studies, the load profile shown in Figure 6-13 has been used. It is a typical daily load profile provided by the UK Energy Research Centre [134]. As illustrated in the figure, there are 48 load points and each point represents the average load during a half-hour period in a day. The network load is high from 18:00 to 21:00 and decreases during the night. In the following studies, the original load of the 102-bus distribution system has been taken as the peak load. At each time point, the load of each bus has been modified by multiplying the percentage as indicated in Figure 6-13. Both the active and reactive loads share the same profile.

With the daily load profile, time-series studies of the tap stagger optimisation have been carried out. At each time point of the load curve, the tap positions of the primary substation transformers were first calculated by the automatic voltage control to maintain the secondary voltages at 1.0 pu. The tap stagger optimisation was then performed based on the calculated tap positions. The automatic voltage control was disabled during the optimisation process. By setting $Q_{required} = 1.0$ MVAr, $e_{limit} = 1\%$ and $TS_{max} = 4$, Figure 6-14 shows the 24-hour simulation results with different load models (i.e. constant power, constant current magnitude and constant impedance).

In Figure 6-14a, the total number of switching operations used for tap stagger has varied with the load level as expected. During periods of high demand (e.g. from 18:00 to 22:00), the number of tap operations has fallen, since the additional VAr absorption of the lines
(between the 132kV and 33kV networks) due to the tap stagger, has increased (see Figure 6-12). To achieve the same VAr absorption target, the tap stagger control can use fewer staggered taps to reduce the transformer VAr absorption. In terms of the different load models, the variations over the 24-hour period are similar. However, the constant impedance load model has resulted in the largest number of switching operations. This is due to the reactive load demand reduction as explained in Section 6.5. In addition, the result of the constant impedance load model has frequently changed during a day since the load is sensitive to the voltage variation.

Figure 6-14b shows the network loss introduced by the tap staggering operation during the 24-hour period. For all three load models, the network losses introduced have not followed the load profile. During periods of high demand, although the transmission line losses have increased, the transformer losses have decreased as fewer staggered taps have been used. These two factors have counteracted each other, resulting in not significant changes in the total network losses (i.e. within 0.01 MW). In terms of the VAr absorption error between $Q_{\text{required}}$ and $Q_{\text{actual}}$, Figure 6-14c demonstrates that the error has changed randomly between 0% and 1% over the 24-hour period. Figure 6-15 summarises the average, maximum and minimum errors among the results. According to the figures, the VAr absorption errors have been maintained below or close to the predefined 1% tolerance, which satisfies the VAr requirement. From the results, the GA can provide robust control of the tap stagger at different load levels.
CHAPTER 6 IMPLEMENTATION OF TAP STAGGER CONTROL USING GA

(a) Total number of tap switching operations involved

(b) Network power loss introduced

(c) Percentage error between $Q_{\text{required}}$ and $Q_{\text{actual}}$

Figure 6-14: Optimisation results for the 102-bus network with 24-hour study (at $Q_{\text{required}} = 1.0 \text{ MVAr}$ & $\epsilon_{\text{limit}} = 1\%$)

School of Electrical and Electronic Engineering
6.7 Summary

This chapter has presented the implementation of the proposed optimal tap stagger control method. First, a 102-bus HV distribution system has been developed in the OpenDSS, which allows interactions with external control algorithms. The model has been validated against the IPSA model. The results conclude that the developed OpenDSS model is correct and can be used for the following studies.

The objective of the tap stagger control is to find an optimal dispatch for the OLTC tap positions, which minimises the power losses and the number of switching operations introduced when providing the VAr absorption. The solution procedure is based on the genetic algorithm and has been implemented in the MATLAB. The fitness function of the GA approach has been calculated using the OpenDSS load flow model. The control method has been applied to the 102-bus distribution network. With a given VAr requirement $Q_{\text{required}}$, the GA approach can find an optimal solution that minimises both the losses and the number of switching operations while satisfying the operating constraints. The error between the required and the actual VAr absorption can be maintained within the defined tolerance. Note that as the GA uses stochastic processes, the algorithm may return a different solution each time it runs. Statistical studies have
been carried out to analyse the accuracy and precision of the GA method. The results confirm that the developed GA algorithm is capable of providing robust control for different VAr absorption requirements.

In addition, the GA-based control approach has been investigated using different load models. The results indicate that the algorithm will return solutions with more tap switching operations if the network load is more sensitive to the voltage changes (e.g. constant impedance load). This is because that when applying the tap staggering technique, the line voltage drops between the 132kV and 33kV networks will increase. The voltages at both the HV and LV sides of the primary substations will decrease as well, resulting in lower load demands. The reduction in reactive load demands will counteract the increase in reactive power flow caused by the tap stagger. To meet the same VAr requirement, the control algorithm has to use more staggered taps to increase the transformer VAr absorption. However, the results indicate that the variations of tap switching operations, due to different load models, are not significant, i.e. within 2 - 6 tap operations. The results also show that for voltage-dependant load models, the network losses introduced are generally higher than the constant impedance load model when applying the tap staggering technique. However, for all the load models, the network losses introduced by tap stagger are much smaller (i.e. 90% - 95% less) than the reactive power absorption created.

To investigate the GA performance under different load levels, case studies have been carried out considering the network with a daily load profile. At each time point, the OLTC tap positions have first been determined by the AVC relays and then adjusted by the optimisation algorithm to achieve the VAr absorption requirement. The results show that the GA approach can still maintain the VAr absorption error below the limit as the load changes. During periods of high demand, the network can provide the VAr absorption using fewer tap operations since the reactive power consumption of lines, due to tap stagger, increases.
CHAPTER 7
COMPARISON OF GA WITH OTHER SOLUTION METHODS

7.1 Introduction

This chapter presents the comparison study between the tap stagger optimisation using the GA approach and other solution methods. In the study, two alternative control schemes have been developed based on the rule-based approach and the branch-and-bound algorithm, respectively. Both methods utilise the information about the distribution network VAr absorption capability. Sensitivity analysis is carried out to compare all three control approaches using the 102-bus distribution network model presented in Chapter 6. In addition, a new distribution network with 222 buses is modelled to represent a larger network with higher demand. The control methods are investigated in the new model as well. The details of the control principles and the optimisation results are discussed as follows.

7.2 Alternative Control Methods for Tap Stagger

For the optimisation of tap stagger, a rule-based control scheme and a branch-and-bound solution method have been developed so that they can be compared with the GA approach. Both the control methods determine the OLTC tap positions with the knowledge of network VAr absorption capability. As the implementation of the rule-based control is straightforward, the result obtained can establish a baseline for the comparison study. The branch-and-bound algorithm is a well-known method to solve integer programming problems [135] and it is available in the MATLAB Optimisation Toolbox. In addition, the algorithm is a deterministic method, which is different from the random nature of the GA approach. The working principles of the rule-based and the branch-and-bound control methods are described as follows.
7.2.1 Rule-based control scheme

7.2.1.1 Matrix of reactive power absorption capability

When selecting the transformers to provide the VAr absorption service, the rule-based control scheme always starts from the transformers that can absorb the largest reactive power before those with the same staggered taps. This rule can help reduce the total number of tap switching operations used. In order to know which transformers to select first, the rule-based method utilises the information on the VAr absorption capability of each primary substation transformer. The information can be described as a sensitivity matrix:

\[ C = \begin{bmatrix} C_{1,1} & \cdots & C_{1,M} \\ \vdots & \ddots & \vdots \\ C_{N,1} & \cdots & C_{N,M} \end{bmatrix}_{N \times M} \]  

(7-1)

where \( N \) denotes the total pairs of parallel transformers, and \( M = \left\lfloor \frac{TS_{\text{max}}}{2} \right\rfloor \), which is an integer representing the maximum permitted taps to be staggered from the initial position. The matrix element \( C_{i,j} \) indicates the VAr absorption that could be individually provided by the \( i^{th} \) pair of parallel transformers with \( 2j \) staggered taps (i.e. \( j \) taps up for one and \( j \) taps down for the other). The value of \( C_{i,j} \) can be obtained by pre-calculating the reactive load flow change at the GSP with only the \( i^{th} \) pair tap staggering.

7.2.1.2 Control sequence

For a radial distribution network, the power flows of primary substations are usually independent of each other. Therefore, when the network is required to provide reactive power service, the rule-based control scheme can estimate the total VAr absorption (\( Q_{\text{actual}} \)) by summing up the MVArs from each primary substation. Figure 7-1 demonstrates the flowchart and the control steps are described as:

Step 1: Initialise all variables and define \( Q_{\text{required}}, e_{\text{limit}}, TS_{\text{max}} \) and the capability matrix \( C \). Set \( Q_{\text{actual}} = 0, C_{\text{available}} = C \).
Step 2: Determine the maximum VAr increment as:

\[ \Delta Q_{\text{max}} = Q_{\text{required}} (1 + e_{\text{limit}}/100) - Q_{\text{actual}} \]  \hspace{1cm} (7-2)

Find any element in the matrix \( C_{\text{available}} \) greater than \( \Delta Q_{\text{max}} \) and set it to 0. Select the largest element in the modified \( C_{\text{available}} \) and add it to \( Q_{\text{actual}} \). Record the corresponding row and column numbers, i.e. \( i \) and \( j \), as the substation number and the staggered taps, respectively. Update the matrix \( C_{\text{available}} \) by setting every element on the \( i^{th} \) row to 0 (since each pair of parallel transformers can only be selected once for the dispatch of VAr absorption).

Step 3: Calculate the VAr absorption error \( e \) according to Equation (6-4), and if \( e < e_{\text{limit}} \), stop the iteration and go to Step 5.

Step 4: If \( C_{\text{available}} \) still has nonzero elements, go to Step 2. Otherwise, go to Step 5.

Step 5: For \( e < e_{\text{limit}} \), print out the final solution and display ‘Converged’. Otherwise, display ‘Not converged’.

---

Figure 7-1: Flowchart of the rule-based control scheme
Note that in Step 2, the algorithm chooses the largest VAr absorption available in order to speed up the accumulation process and reduce the number of tap switching operations. Although the rule-based control has not included the minimisation of network power loss, the method provides a basic way to dispatch the VAr absorption with tap stagger.

### 7.2.2 Branch-and-bound solution method

#### 7.2.2.1 Linearisation of the tap stagger optimisation problem

Based on the principles of the rule-based control, the branch-and-bound algorithm has been adopted to minimise the network loss as well as the number of switching operations. The algorithm searches for an optimal solution by solving a series of linear programming (LP) relaxation problems [135]. To apply the algorithm, the tap stagger optimisation problem has been linearised. First, the network power loss (due to tap stagger) can be described by a sensitivity matrix:

\[
P = \begin{bmatrix}
P_{1,1} & \cdots & P_{1,M} \\
\vdots & \ddots & \vdots \\
P_{N,1} & \cdots & P_{N,M}
\end{bmatrix}_{N \times M} \tag{7-3}
\]

The matrix element \( P_{i,j} \) indicates the network loss caused by the \( i^{th} \) pair of parallel transformers with \( 2j \) staggered taps (i.e. \( j \) taps up for one and \( j \) taps down for the other). The value of \( P_{i,j} \) can be obtained by pre-calculating the network loss change with only the \( i^{th} \) pair tap staggering.

For a radial distribution network, the objective in Equation (4-18) can be modified as:

\[
\min \sum_{i=1}^{N} \sum_{j=1}^{M} (P_{i,j} + 2j) x_{i,j} \tag{7-4}
\]

where the control variable \( x_{i,j} \) is a binary integer 0 or 1. If \( x_{i,j} = 1 \), the \( i^{th} \) pair of parallel transformers will be tapped apart with \( 2j \) steps (i.e. \( j \) taps up for one and \( j \) taps down for
the other). The coefficient $P_{i,j}$ indicates the corresponding network loss introduced. Since each pair of parallel transformers can only respond to a single command from the control centre, the objective function is subject to:

$$\sum_{j=1}^{M} x_{i,j} \leq 1 \quad \text{for } i = 1, ..., N \quad (7-5)$$

in addition to

$$\frac{|Q_{\text{required}} - \sum_{i=1}^{N} \sum_{j=1}^{M} C_{i,j} x_{i,j}|}{Q_{\text{required}}} \times 100\% \leq e_{\text{limit}} \quad (7-6)$$

$$TAP_{i,\text{min}} \leq TAP_i \pm \sum_{j=1}^{M} j x_{i,j} \leq TAP_{i,\text{max}} \quad (7-7)$$

With Equations (7-4)-(7-7), the non-linear tap stagger optimisation problem can be transformed into a binary integer linear programming problem as:

$$\min f = \min_x f^T x \quad (7-8)$$

$$A \cdot x \leq b \quad (7-9)$$

where $x$ is the solution vector with binary integers and the length is $M \times N$. The coefficients of the objective and constraint functions are:

$$f = \begin{bmatrix}
P_{1,1} + 1 \\
P_{1,2} + 2 \\
\vdots \\
P_{1,M} + M \\
P_{2,1} + 1 \\
\vdots \\
P_{2,M} + M \\
\vdots \\
P_{N,M} + M
\end{bmatrix}_{(M \times N) \times 1} \quad A = \begin{bmatrix}
A_1 \\
A_2 \\
A_3
\end{bmatrix} \quad b = \begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix} \quad (7-10)$$
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with

\[
A_1 = \begin{bmatrix}
1 & \ldots & 1 \\
& \ddots & \\
& & 1 & \ldots & 1
\end{bmatrix}_{N \times (M \cdot N)}
\]

(7-11)

\[
A_2 = \begin{bmatrix}
C_{1,1} & C_{1,2} & \ldots & C_{1,M} \\
-C_{1,1} & -C_{1,2} & \ldots & -C_{1,M} \\
& \ddots & \ddots & \ddots \\
& & -C_{N,1} & \ldots & -C_{N,M}
\end{bmatrix}_{2 \times (M \cdot N)}
\]

(7-12)

\[
A_3 = \begin{bmatrix}
1 & \ldots & M \\
& \ddots & \\
& & 1 & \ldots & M
\end{bmatrix}_{N \times (M \cdot N)}
\]

(7-13)

\[
b_1 = \begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}_{N \times 1}
\]

(7-14)

\[
b_2 = \begin{bmatrix}
Q_{\text{required}}(1 + e_{\text{limit}}/100) \\
-Q_{\text{required}}(1 - e_{\text{limit}}/100)
\end{bmatrix}_{2 \times 1}
\]

(7-15)

\[
b_3 = \begin{bmatrix}
\min(TAP_{1,\text{max}} - TAP_1, -TAP_{1,\text{min}} + TAP_1) \\
\min(TAP_{2,\text{max}} - TAP_2, -TAP_{2,\text{min}} + TAP_2) \\
\vdots \\
\min(TAP_{N,\text{max}} - TAP_N, -TAP_{N,\text{min}} + TAP_N)
\end{bmatrix}_{N \times 1}
\]

(7-16)

7.2.2.2 Solution procedure

As shown in Figure 7-2, the LP based branch-and-bound algorithm first creates a search tree by repeatedly adding the binary constraints \((x_{i,j} = 0 \text{ or } 1)\) to the problem, i.e. known as branching. The added constraints are represented as nodes in the binary tree. At each node, the algorithm solves an LP relaxation problem, where the binary requirement on the variables is replaced with the weaker constraint \(0 \leq x_{i,j} \leq 1\). The LP relaxation problem can be solved using the interior-point method [136]. Depending on the outcome, the algorithm decides whether to branch or to move to another node [137]. The solution to the LP-relaxation problem provides a lower bound for the value of the objective function. If the solution to the LP-relaxation problem is already a binary integer vector, it provides
an upper bound for the objective value. As the search tree grows, the algorithm updates the best solution as well as the lower and upper bounds on the objective function. The bounds serve as the thresholds to cut off unnecessary branches. The maximum number of nodes and the maximum time are used to terminate the process.

The branch-and-bound method could potentially try all the possible combinations of the binary integers. The method searches the global optimum through a deterministic process while the GA approach is based on a random process. The search space for the branch-and-bound method is $2^{M \times N}$. For a large-scale distribution network, the search space will increase significantly compared with the GA approach. However, as the branch-and-bound method does not need to calculate load flows, the computation time may be faster than the GA for small networks.

![Binary search tree](image.png)

**Figure 7-2: Binary search tree [135]**

### 7.3 Comparison Studies for 102-Bus Distribution Network

The two alternative control methods described above have been implemented in the MATA LB. The full codes are given in APPENDIX D. This section presents the tap stagger optimisation studies of using the developed GA approach, the rule-based control scheme and the branch-and-bound method. The results are compared with each other. The 102-bus distribution system developed in the previous chapter has been used to test
the tap stagger control. The studies started from the convergence analysis of each control algorithm under different VAr requirements. To further compare the three control methods, the studies also carried out the sensitivity analysis that investigated the impacts of the input settings (i.e. $e_{\text{limit}}$ and $TS_{\text{max}}$) on the optimisation results. The details are described as follows.

### 7.3.1 Case 1: Convergence analysis with different VAr requirements

The three tap stagger control methods have been applied to the 102-bus distribution system shown in Figure 6-1. The VAr absorption target ($Q_{\text{required}}$) has increased from 0.1 MVAR to 2.5 MVAR in a step of 0.1 MVAR and the algorithms have run once for each target. The maximum permitted absorption error ($e_{\text{limit}}$) and tap difference ($TS_{\text{max}}$) have been set to 1% and 4, respectively. As described before, the rule-based control scheme and the branch-and-bound method can only estimate the value of $Q_{\text{actual}}$ (or $\Delta P_{\text{loss}}$) based on the network VAr absorption matrix $C$ (or the power loss matrix $P$). Therefore, after obtaining the optimal tap positions, the two control methods have performed OpenDSS load flow studies to calculate the actual values of $Q_{\text{actual}}$ and $\Delta P_{\text{loss}}$. Table 7-1 summarises the optimisation results of each control algorithm.

#### 7.3.1.1 Reasons for non-convergence

In Table 7-1, the symbol ‘N/A’ indicates that the algorithm has not converged for the corresponding VAr absorption requirement. Table 7-2 also lists the number of converged solutions during the case study. According to the table, both the branch-and-bound and the GA approaches provide more reliable tap stagger control than the rule-based method does. In the solution process, the rule-based algorithm only attempts to use the transformers with the largest VAr absorptions. This will increase the difficulty in matching $Q_{\text{actual}}$ with $Q_{\text{required}}$. For certain values of $Q_{\text{required}}$, the final mismatch can still exceed the $e_{\text{limit}}$, leading to non-convergence of the iterative process.
Table 7-1: Optimisation results for each value of $Q_{\text{required}}$ using different control algorithms (at $\epsilon_{\text{limit}} = 1\%$ & $TS_{\text{max}} = 4$)

<table>
<thead>
<tr>
<th>$Q_{\text{required}}$ (MVAr)</th>
<th>Total tap operations</th>
<th>Network loss due to stagger (MW)$^a$</th>
<th>Error between $Q_{\text{required}}$ and $Q_{\text{actual}}$ (%)$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rule Based</td>
<td>Branch and Bound</td>
<td>GA</td>
</tr>
<tr>
<td>0.1</td>
<td>N/A</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>0.2</td>
<td>N/A</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>0.3</td>
<td>N/A</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>0.4</td>
<td>N/A</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>0.5</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>0.6</td>
<td>N/A</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>0.7</td>
<td>14</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>0.8</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>0.9</td>
<td>N/A</td>
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<td>16</td>
</tr>
<tr>
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<td>16</td>
<td>16</td>
<td>16</td>
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<td>1.1</td>
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</tr>
<tr>
<td>1.2</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>1.3</td>
<td>N/A</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>1.4</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>1.5</td>
<td>26</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>1.6</td>
<td>30</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>1.7</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>1.8</td>
<td>34</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>1.9</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>2.1</td>
<td>38</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>2.2</td>
<td>N/A</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>2.3</td>
<td>42</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>2.4</td>
<td>N/A</td>
<td>N/A</td>
<td>44</td>
</tr>
<tr>
<td>2.5</td>
<td>N/A</td>
<td>N/A</td>
<td>44</td>
</tr>
</tbody>
</table>

$^a$ For the rule-based and the branch-and-bound methods, the values are calculated by running the OpenDSS load flow model with the obtained optimal tap positions.

In terms of the branch-and-bound method, the approximation error caused by the linearisation of the tap stagger control system may also result in non-convergence. For instance, at $Q_{\text{required}} = 2.4$ MVAr, the network can only achieve the VAr absorption target by setting all the primary substations with 4 staggered taps. The error between $Q_{\text{required}}$ and $Q_{\text{actual}}$ is 0.94% according to the GA result. However, the branch-and-bound method
has calculated the error as 1.16% using Equation (7-6). As the error is higher than the permitted limit 1%, the branch-and-bound cannot find any feasible solution in the search space and therefore has returned ‘N/A’. Note that at $Q_{\text{required}} = 2.5 \text{ MVar}$, both the rule-based and branch-and-bound methods have not converged since all the primary substation transformers have reached the stagger limit.

Table 7-2: Number of converged solutions for the VAr requirement from 0.1 to 2.5 MVar

<table>
<thead>
<tr>
<th>Rule-Based</th>
<th>Branch-and-Bound</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of converged solutions</td>
<td>15</td>
<td>23</td>
</tr>
</tbody>
</table>

7.3.1.2 Comparison of converged solutions

Table 7-3 shows the optimal solutions (i.e. $x$) obtained with different control algorithms at $Q_{\text{required}} = 0.5, 1, 1.5$ and $2 \text{ MVar}$. According to the results, more pairs of transformers and staggered taps need to be used as $Q_{\text{required}}$ increases. The corresponding number of tap switching operations, power loss and absorption error are illustrated in Figure 7-3.

Table 7-3: Tap staggering arrangements of primary substations to provide reactive power absorption

<table>
<thead>
<tr>
<th>NO. of primary sub</th>
<th>Rule-based method</th>
<th>Branch-and-bound method</th>
<th>GA approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{\text{required}}$ (MVar)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$Q_{\text{required}}$ (MVar)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$Q_{\text{required}}$ (MVar)</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
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<td>3</td>
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<td>0</td>
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<tr>
<td>4</td>
<td>0</td>
<td>2</td>
<td>2</td>
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<tr>
<td>5</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
CHAPTER 7 COMPARISON OF GA WITH OTHER SOLUTION METHODS

Figure 7-3: Optimisation results with different control methods for the 102-bus distribution system (with $\varepsilon_{\text{limit}} = 1\%$ and $TS_{\text{max}} = 4$)

(a) Total number of tap switching operations involved
(b) Network power loss introduced
(c) Percentage error between $Q_{\text{required}}$ and $Q_{\text{actual}}$
As shown in Figure 7-3a, both the branch-and-bound and the GA methods have returned the same numbers of tap switching operations. For \( Q_{\text{required}} = 2 \text{ MVar} \), the rule-based control scheme has led to two more switching operations than the other two methods.

In terms of the network loss, both the branch-and-bound and the GA approaches can result in lower losses than the rule-based method (e.g. with average 5% less). The loss reduction is not significant since most primary substation transformer models have similar winding resistances. Therefore, the selection of different transformers for the tap staggering will not have a great impact on the overall network loss. At \( Q_{\text{required}} = 1 \text{ MVar} \), the branch-and-bound method has produced less network loss than the GA approach (see Figure 7-3b). However, according to Figure 7-3c, the corresponding VAr absorption error (between \( Q_{\text{required}} \) and \( Q_{\text{actual}} \)) of the branch-and-bound method is already higher than the predefined limit 1%. This is due to the linearisation error as described before. In contrast, the GA approach can still maintain the absorption error below 1%.

The computation time for the rule-based control scheme, the branch-and-bound method and the GA approach is given in Table 7-4. Although the rule-based and the branch-and-bound methods can provide much faster control than the GA approach, they deeply rely on the linear characteristic of radial networks. However, the GA approach can be applied to meshed distribution networks. In addition, the GA method can achieve more accurate control as it calculates the VAr absorption through load flow studies rather than with the linear approximation.

<table>
<thead>
<tr>
<th>( Q_{\text{required}} ) (MVar)</th>
<th>Rule-Based</th>
<th>Branch-and-Bound</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.42</td>
<td>0.47</td>
<td>8.1</td>
</tr>
<tr>
<td>1.0</td>
<td>0.44</td>
<td>0.48</td>
<td>8.1</td>
</tr>
<tr>
<td>1.5</td>
<td>0.44</td>
<td>0.74</td>
<td>8.4</td>
</tr>
<tr>
<td>2.0</td>
<td>0.46</td>
<td>0.79</td>
<td>8.1</td>
</tr>
</tbody>
</table>

a. on MATLAB Version 7.11 and with i7-3.4 GHz / 8 GB RAM
### 7.3.2 Case 2: Sensitivity analysis with different settings of reactive power absorption accuracy

Case 1 has examined the convergence of each control algorithm with the maximum permitted VAr absorption error \( e_{\text{limit}} = 1\% \). To analyse the impact of \( e_{\text{limit}} \) on the optimisation results, this case study has tested the tap stagger control methods with various settings of \( e_{\text{limit}} \). According to the Grid Code [132], if a generating unit is instructed to provide a specific MVAr output, the generator must achieve that output within a tolerance of +/- 5% of the rated output. Therefore, optimisation studies have been carried out with \( e_{\text{limit}} = 1\%, 2\%, 3\%, 4\% \) and 5\%, respectively. For each setting of \( e_{\text{limit}} \), the control algorithms have been tested with \( Q_{\text{required}} = 0.5, 1, 1.5 \) and 2 MVAr. \( TS_{\text{max}} \) has been set to 4 throughout the simulations. Table 7-5 summarises the simulation results.

Based on the table, Figure 7-4 plots the number of tap switching operations, network loss and VAr absorption error against the setting of \( e_{\text{limit}} \). For each \( e_{\text{limit}} \), the figures indicate the average, maximum and minimum values of the results with the four different VAr requirements (i.e. 0.5, 1, 1.5 and 2 MVAr).

According to Figure 7-4a, the total number of tap operations slightly decreases as \( e_{\text{limit}} \) increases. If the distribution network is allowed to provide the VAr absorption within a larger tolerance, the control algorithms may be able to use smaller staggered taps to satisfy the requirement. However, the reduction in tap switching operations is not significant, e.g. only two tap operations for \( e_{\text{limit}} = 5\% \). Note that the deviation between the maximum and minimum numbers of tap operations at each \( e_{\text{limit}} \) is large since the VAr requirement \( Q_{\text{required}} \) has changed from 0.5 to 2 MVAr. In other words, the number of tap changing operations is more dependent on \( Q_{\text{required}} \) than \( e_{\text{limit}} \).

Figure 7-4b shows the introduced network losses with different settings of \( e_{\text{limit}} \). Compared with the rule-based control scheme, both the branch-and-bound and the GA approaches can reduce the network loss caused by tap stagger. The loss reduction is more effective as \( e_{\text{limit}} \) increases, e.g. up to 18\% reduction at \( e_{\text{limit}} = 5\% \). Since a larger tolerance
of VAr absorption can result in more feasible solutions, the GA algorithm (or the branch-and-bound method) can find more promising solutions that introduce less network loss while satisfying the constraints.

Table 7-5: Optimisation results with different settings of $e_{\text{limit}}$ (with $T_{\text{Smax}} = 4$)

<table>
<thead>
<tr>
<th>$e_{\text{limit}}$</th>
<th>Rule-based method</th>
<th>Branch-and-bound method</th>
<th>GA approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>1%</td>
<td>8</td>
<td>16</td>
<td>26</td>
</tr>
<tr>
<td>2%</td>
<td>8</td>
<td>16</td>
<td>26</td>
</tr>
<tr>
<td>3%</td>
<td>8</td>
<td>16</td>
<td>26</td>
</tr>
<tr>
<td>4%</td>
<td>8</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>5%</td>
<td>8</td>
<td>16</td>
<td>24</td>
</tr>
</tbody>
</table>

Network loss due to stagger (MW)\(^a\)

<table>
<thead>
<tr>
<th>$e_{\text{limit}}$</th>
<th>Rule-based method</th>
<th>Branch-and-bound method</th>
<th>GA approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>1%</td>
<td>0.033</td>
<td>0.052</td>
<td>0.073</td>
</tr>
<tr>
<td>2%</td>
<td>0.027</td>
<td>0.052</td>
<td>0.073</td>
</tr>
<tr>
<td>3%</td>
<td>0.027</td>
<td>0.052</td>
<td>0.073</td>
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<tr>
<td>4%</td>
<td>0.027</td>
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<td>0.070</td>
</tr>
<tr>
<td>5%</td>
<td>0.027</td>
<td>0.052</td>
<td>0.070</td>
</tr>
</tbody>
</table>

Error between $Q_{\text{required}}$ and $Q_{\text{actual}}$ (%)\(^a\)

<table>
<thead>
<tr>
<th>$e_{\text{limit}}$</th>
<th>Rule-based method</th>
<th>Branch-and-bound method</th>
<th>GA approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>1%</td>
<td>0.328</td>
<td>0.773</td>
<td>0.156</td>
</tr>
<tr>
<td>2%</td>
<td>1.592</td>
<td>0.773</td>
<td>0.156</td>
</tr>
<tr>
<td>3%</td>
<td>1.592</td>
<td>0.773</td>
<td>0.156</td>
</tr>
<tr>
<td>4%</td>
<td>1.592</td>
<td>0.773</td>
<td>3.809</td>
</tr>
<tr>
<td>5%</td>
<td>4.117</td>
<td>0.773</td>
<td>3.809</td>
</tr>
</tbody>
</table>

a. For the rule-based and the branch-and-bound methods, the values are calculated by running the OpenDSS load flow model with the obtained optimal tap positions.
CHAPTER 7  COMPARISON OF GA WITH OTHER SOLUTION METHODS

Figure 7-4: Optimisation results for the 102-bus distribution system using different settings of $e_{lim}$ (with $Q_{required} = 0.5$ to 2 MVar & $TS_{max} = 4$)
In terms of the error between $Q_{\text{required}}$ and $Q_{\text{actual}}$, Figure 7-4c indicates that the error increases with $e_{\text{limit}}$. The rule-based algorithm can return relatively small errors than the branch-and-bound and the GA methods since these two methods need to consider the minimisation of network losses. There is a trade-off among the minimisation of tap operations, the loss reduction and the provision of the reactive power service. Note that for $e_{\text{limit}} = 3\%$ and $4\%$, the maximum VAr absorption errors from the branch-and-bound method are slightly over the defined limits. This is because the method estimates the value of $Q_{\text{actual}}$ using linear approximation rather than calculating it through load flow studies.

### 7.3.3 Case 3: Sensitivity analysis with different settings of maximum staggered taps

For the tap stagger control algorithms, there are three input settings, i.e. $Q_{\text{required}}$, $e_{\text{limit}}$ and $TS_{\text{max}}$. The previous Case 1 and Case 2 studies have investigated the impacts of $Q_{\text{required}}$ and $e_{\text{limit}}$ on the optimisation results. Consequently, this case study aims to analyse the performance of the tap stagger control with different settings of $TS_{\text{max}}$ (i.e. the maximum permitted staggered taps). As discussed in Section 4.3, the tap differences between parallel transformers should be limited to prevent the transformers overheating and to maintain secondary voltages constant. In addition, the OLTC of a primary substation transformer usually has 15 to 18 available tap positions. Therefore, considering the transformer rating as well as the headroom for tap staggering, this case study has investigated the control algorithms with $TS_{\text{max}} = 4$, 6 and 8, respectively. For each setting of $TS_{\text{max}}$, the algorithms have been tested with $Q_{\text{required}} = 0.5$, 1, 1.5 and 2 MVar. $e_{\text{limit}}$ has been set to 1% throughout the simulations. Table 7-6 summarises the optimisation results.

According to Table 7-6, the total number of tap operations decreases significantly as $TS_{\text{max}}$ increases. For instance, comparing the results of $TS_{\text{max}} = 4$ and $TS_{\text{max}} = 8$ at $Q_{\text{required}} = 2.0$ MVar, the number of tap switching operations is reduced by 55% for all three algorithms. Figure 7-5a also shows the average, maximum and minimum numbers of the tap changing operations with the four different VAr requirements (i.e. 0.5, 1, 1.5 and 2
MVAr. As $T_{S_{\text{max}}}$ increases, the control algorithms can set the transformers with larger staggered taps. Since the transformer VAr consumption increases quadratically with the staggered taps (see Figure 5-3), the control methods can achieve the VAr absorption target by using fewer transformers but with larger staggered taps. The resulting number of tap operations involved is fewer than when many transformers with small staggered taps are used.

Table 7-6: Optimisation results with different settings of $T_{S_{\text{max}}}$ (with $e_{\text{limit}} = 1\%$)

<table>
<thead>
<tr>
<th>$T_{S_{\text{max}}}$</th>
<th>Rule-based method</th>
<th>Branch-and-bound method</th>
<th>GA approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>16</td>
<td>26</td>
</tr>
<tr>
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<td>6</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>8</td>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$T_{S_{\text{max}}}$</th>
<th>Rule-based method</th>
<th>Branch-and-bound method</th>
<th>GA approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>4</td>
<td>0.033</td>
<td>0.052</td>
<td>0.073</td>
</tr>
<tr>
<td>6</td>
<td>0.021</td>
<td>0.056</td>
<td>0.075</td>
</tr>
<tr>
<td>8</td>
<td>0.021</td>
<td>0.036</td>
<td>0.110</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$T_{S_{\text{max}}}$</th>
<th>Rule-based method</th>
<th>Branch-and-bound method</th>
<th>GA approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>4</td>
<td>0.328</td>
<td>0.773</td>
<td>0.156</td>
</tr>
<tr>
<td>6</td>
<td>0.461</td>
<td>2.700</td>
<td>1.919</td>
</tr>
<tr>
<td>8</td>
<td>0.461</td>
<td>3.179</td>
<td>0.484</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$T_{S_{\text{max}}}$</th>
<th>Rule-based method</th>
<th>Branch-and-bound method</th>
<th>GA approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
<td>$Q_{\text{required}}$ (MVAr)</td>
</tr>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>4</td>
<td>0.951</td>
<td>1.055</td>
<td>0.697</td>
</tr>
<tr>
<td>6</td>
<td>0.461</td>
<td>0.625</td>
<td>0.827</td>
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<tr>
<td>8</td>
<td>0.461</td>
<td>1.007</td>
<td>0.965</td>
</tr>
</tbody>
</table>

a. For the rule-based and the branch-and-bound methods, the values are calculated by running the OpenDSS load flow model with the obtained optimal tap positions.
(a) Total number of tap switching operations involved

(b) Network power loss introduced

(c) Percentage error between $Q_{\text{required}}$ and $Q_{\text{actual}}$

Figure 7-5: Optimisation results for the 102-bus distribution system using different settings of $TS_{\text{max}}$ (with $Q_{\text{required}} = 0.5$ to 2 MVAr \& $e_{\text{limit}} = 1\%$)
Figure 7-5b shows the introduced network losses with different settings of $TS_{\text{max}}$. Compared with the rule-based approach, both the branch-and-bound and the GA methods can reduce the network loss due to tap stagger. The loss reduction is more significant as $TS_{\text{max}}$ increases, e.g. an average of 28% reduction at $TS_{\text{max}} = 8$. The setting of a larger $TS_{\text{max}}$ can allow the transformers to have a wider range of tap staggering. The GA algorithm (or the branch-and-bound method) can find more combinations of the tap staggering arrangements to reduce the power loss. Note that at each $TS_{\text{max}}$, the deviation between the maximum and minimum network losses is quite large (i.e. greater than 300%) since the VAr requirement $Q_{\text{required}}$ has changed from 0.5 to 2 MVAR. The large deviations imply that the network loss introduced by tap stagger is predominated by $Q_{\text{required}}$.

In terms of the error between $Q_{\text{required}}$ and $Q_{\text{actual}}$, Figure 7-5c indicates that the GA approach is capable of maintaining the error below the limit of 1%. However, the rule-based control scheme has returned large errors (i.e. up to 3%) for $TS_{\text{max}} = 6$ and 8. As the rule-based algorithm only attempts to use the transformers with the largest VAr absorptions in the solution process, the uncertainty of the mismatch between $Q_{\text{required}}$ and $Q_{\text{actual}}$ is high. Consequently, the fulfilment of the VAr absorption requirement cannot be guaranteed.

### 7.3.4 Summary of findings

For the 102-bus network model, three case studies have been carried out. Each case study has investigated the impact of one of the control input settings on the optimisation results. The input settings include the VAr absorption target $Q_{\text{required}}$, the tolerance of VAr absorption error $e_{\text{limit}}$ and the limit of maximum staggered taps $TS_{\text{max}}$.

- Case 1 has analysed the convergence of each control algorithm with different values of $Q_{\text{required}}$. The result shows that the GA approach has converged for every $Q_{\text{required}}$ and has maintained the VAr absorption error within the tolerance until the network has reached its absorption limit. The rule-based algorithm can provide the fastest
control while it does not consider the minimisation of network loss and it can lead to non-convergence at certain values of $Q_{\text{required}}$. The branch-and-bound method also has a faster convergence speed than the GA. However, due to the errors caused by the linearisation of the non-linear optimisation problem, the branch-and-bound method may return solutions that violate the VAr absorption tolerance $e_{\text{limit}}$. In addition, both the rule-based and the branch-and-bound methods can only be applied to radial distribution networks while the GA approach can also be used for meshed networks.

- Case 2 has carried out the sensitivity analysis that examined the three control algorithms with different settings of $e_{\text{limit}}$. Both the branch-and-bound and the GA methods can result in lower network loss than the rule-based approach. The loss reduction is more effective as $e_{\text{limit}}$ increases, e.g. up to 18% reduction at $e_{\text{limit}} = 5\%$. The result also shows the VAr absorption error has increased with $e_{\text{limit}}$.

- Case 3 has investigated the impact of $TS_{\text{max}}$ on the optimisation results. For all three algorithms, the total number of tap operations has decreased as $TS_{\text{max}}$ increases (e.g. an average of 55% reduction from $TS_{\text{max}} = 4$ to $TS_{\text{max}} = 8$). The results also indicate that only the GA approach is capable of maintaining the VAr absorption error within the tolerance under different settings of $TS_{\text{max}}$.

- The total number of tap switching operations and the network loss due to tap stagger are more dependent on $Q_{\text{required}}$ than $e_{\text{limit}}$ and $TS_{\text{max}}$. The VAr absorption error changes randomly with $Q_{\text{required}}$ and $TS_{\text{max}}$, but it increases with $e_{\text{limit}}$.

### 7.4 Comparison Studies for 222-Bus Distribution Network

The case studies in the previous section have investigated the tap stagger control methods using the 102-bus distribution network model. The results indicate that the computation time of the rule-based or the branch-and-bound methods is much faster than the GA approach although the GA algorithm can provide more reliable and accurate solutions. To further compare the three control methods, this section presents the optimisation studies in a relatively large distribution system. A distribution network with 222 buses has been modelled based on the distribution system described in Chapter 5. The 222-bus
CHAPTER 7  COMPARISON OF GA WITH OTHER SOLUTION METHODS

distribution network is part of the real UK HV distribution system studied in the ‘CLASS’ project [123]. It contains a 132kV GSP and the entire downstream 33kV networks connected to the GSP. The network data and diagram are given in APPENDIX E.

Compared with the 102-bus distribution network model, the new network model has 28 pairs of primary substation transformers. With $T_{max} = 4$, the search space for the GA approach increases from $3^{11}$ to $3^{28}$, and the search space for the branch-and-bound method increases from $2^{22}$ to $2^{56}$. Comparison studies have been carried out to analyse the convergence and the computation time of each algorithm under this large-scale distribution system. In addition, case studies have been undertaken to investigate the control algorithms at different load levels. The details are discussed as follows.

7.4.1 Case 1: Convergence and computation time analyses with different VAr requirements

The three tap stagger control algorithms have been applied to the 222-bus distribution network and tested with the VAr absorption target from 0.5 MVAr to 7 MVAr in a step of 0.5 MVAr. The algorithms have run once for each target. The values of $e_{limit}$ and $T_{max}$ have been set to 1% and 4, respectively. The GA parameters have been defined according to Table 6-2. As the search space has increased significantly, the maximum solution time has been fixed to 3 minutes for both the GA and the branch-and-bound algorithms. Table 7-7 lists the optimisation results of each control algorithm.

According to the table, the rule-based and the branch-and-bound methods have not converged at $Q_{required} = 7$ MVAr. The GA approach also returned the solution with a large VAr absorption error, i.e. 4.4%. It has been observed that all the transformers have reached the maximum stagger limit, resulting in no more reactive power absorption. The results imply that the network VAr absorption capability is below 7 MVAr with $T_{max} = 4$.

7.4.1.1 Comparison of converged solutions

Figure 7-6 illustrates the numbers of tap switching operations, power losses and VAr absorption errors introduced with different values of $Q_{required}$. As shown in Figure 7-6a,
the three control methods have returned the solutions with the same numbers of tap switching operations. In terms of the network loss caused by tap staggering, both the GA approach and the branch-and-bound method can reduce the loss when compared to the rule-based control scheme (see Figure 7-6b). However, in Figure 7-6c, the VAr absorption error for the branch-and-bound method exceeds the 1% tolerance at $Q_{\text{required}} = 6$ MVAR. In the optimisation process, the branch-and-bound algorithm estimates the value of $Q_{\text{actual}}$ using Equation (7-6) without actually calculating the load flows. The actual network VAr absorption may be very different from the estimated value. Therefore, the branch-and-bound method may obtain a solution that does not actually meet the VAr requirement (i.e. when $Q_{\text{required}} = 2.5, 4.5, 5.5$ and $6$ MVAR). In contrast, the GA approach can find the ‘correct’ solution through load flow calculations.

Table 7-7: Optimisation results of the 222-bus distribution network for different $Q_{\text{required}}$ (with $\epsilon_{\text{limit}} = 1\%$ & $T_{\text{max}} = 4$)

<table>
<thead>
<tr>
<th>$Q_{\text{required}}$ (MVAR)</th>
<th>Total tap operations</th>
<th>Network loss due to staggering (MW)$^a$</th>
<th>Error between $Q_{\text{required}}$ and $Q_{\text{actual}}$ (%)$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rule Based</td>
<td>Branch and Bound</td>
<td>GA</td>
</tr>
<tr>
<td>0.5</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>1.5</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>2.5</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>44</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td>3.5</td>
<td>52</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
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</tr>
<tr>
<td>4.5</td>
<td>68</td>
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</tr>
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<td>5</td>
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<td>76</td>
<td>76</td>
</tr>
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<td>5.5</td>
<td>84</td>
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<td>88</td>
</tr>
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<td>6</td>
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<td>96</td>
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</tr>
<tr>
<td>6.5</td>
<td>108</td>
<td>106</td>
<td>106</td>
</tr>
<tr>
<td>7</td>
<td>N/A</td>
<td>N/A</td>
<td>112</td>
</tr>
</tbody>
</table>

$^a$: For the rule-based and the branch-and-bound methods, the values are calculated by running the OpenDSS load flow model with the obtained optimal tap positions.
CHAPTER 7  COMPARISON OF GA WITH OTHER SOLUTION METHODS

Figure 7-6: Optimisation results with different control methods for the 222-bus distribution system (with $\epsilon_{\text{limit}} = 1\%$ and $TS_{\text{max}} = 4$)
7.4.1.2 Computation time

The computation time for each algorithm is given in Table 7-8. The rule-based algorithm still provides the fastest control but it does not minimise the network loss. The average time for the GA to solve a VAr dispatch problem is 22.3s, which is faster than the average 78.9s of the branch-and-bound method. As the 222-bus network includes 28 primary substations, the search space is $3^{28}$ for the GA, which is significantly less than the $2^{56}$ of the branch-and-bound algorithm. Consequently, the GA approach is more efficient at finding feasible solutions and at identifying the optimal solution. Note that for $Q_{\text{required}} = 3$ MVAr (or 4 MVAr), there are many possible combinations of tap staggering arrangements that can achieve the VAr absorption target. Therefore, the branch-and-bound method has continued to search for the optimal solution and has only terminated the iteration process after reaching the maximum solution time (i.e. 180s).

Table 7-8: Computation time for the 222-bus distribution system studies

<table>
<thead>
<tr>
<th>$Q_{\text{required}}$ (MVAr)</th>
<th>Computation Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rule-Based</td>
</tr>
<tr>
<td>1</td>
<td>0.65</td>
</tr>
<tr>
<td>2</td>
<td>0.66</td>
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<td>3</td>
<td>0.68</td>
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<tr>
<td>5</td>
<td>0.67</td>
</tr>
<tr>
<td>6</td>
<td>0.67</td>
</tr>
<tr>
<td>average</td>
<td>0.67</td>
</tr>
</tbody>
</table>

a. on MATLAB Version 7.11 and with i7-3.4 GHz / 8 GB RAM

7.4.2 Case 2: Comparison of time-series optimisation studies

Case 1 has investigated the three control methods in the 222-bus distribution system with a fixed load level. This case study aims to analyse the performance of each control algorithm with varying load demands. The daily load profile shown in Figure 6-13 has been used. The original load of the 222-bus distribution system has been taken as the
peak load. At each time point, the load of each bus has been modified by multiplying the corresponding percentage indicated on the load profile. Load flow calculations with the automatic voltage control (AVC) have then been performed to determine the initial tap positions of the primary substation transformers. Based on the calculated tap positions, the tap stagger optimal control has been carried out and the AVC has been disabled during the optimisation process. For the rule-based and the branch-and-bound methods, the matrices $C$ and $P$ have been recalculated at each time point. By setting $Q_{\text{required}} = 3$ MVAr, $e_{\text{limit}} = 1\%$ and $T_{S_{\text{max}}} = 4$ throughout the 24-hour simulation, Figure 7-7 shows the optimisation results with the three control methods.

In Figure 7-7a, the total number of tap switching operations varies with the load level. During periods of high demand (e.g. 18:00 to 22:00), the number of tap operations decreases since the additional VAr absorption of the lines (between the 132kV and 33kV networks), due to the tap stagger, increases. To achieve the same VAr absorption target, the control algorithms can use smaller staggered taps to reduce the transformer VAr absorption. For the three control methods, the variations over the 24-hour period are similar. However, the GA approach generally results in larger numbers of tap operations than the other two methods. This is because the GA algorithm needs to consider the fulfilment of the VAr absorption requirement as well as the minimisation of tap operations. Figure 7-7c demonstrates the VAr absorption errors (between $Q_{\text{required}}$ and $Q_{\text{actual}}$) over the 24-hour period and Figure 7-8 summarises the average, maximum and minimum errors among the 24-hour results. As shown in the figures, the GA method can provide the VAr absorption within the $1\%$ tolerance throughout the 24-hour period. In contrast, the other two methods have several VAr absorption errors exceeding the limit at certain time points.

Table 7-9 also lists the number of violations of the VAr requirement over the 24-hour period with different settings of $Q_{\text{required}}$. From the table, the GA algorithm has the least number of violations among all the test results. The studies confirm that the GA approach can provide accurate and robust control of the tap staggering operation under different VAr absorption requirements and at various load levels.
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(a) Total number of tap switching operations involved

(b) Network power loss introduced

(c) Percentage error between $Q_{required}$ and $Q_{actual}$

Figure 7-7: Optimisation results for the 222-bus network using 24-hour load profile (at $Q_{required} = 3$ MVar & $\varepsilon_{limit} = 1\%$)
CHAPTER 7  COMPARISON OF GA WITH OTHER SOLUTION METHODS

Figure 7-8: Reactive power absorption errors of the 24-hour optimisation studies for different solution methods

<table>
<thead>
<tr>
<th>$Q_{\text{required}}$ (MVAR)</th>
<th>Violations ($e &gt; e_{\text{limit}}$)</th>
<th>Computation time (s)$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rule-Based</td>
<td>Branch and Bound</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
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</tr>
<tr>
<td>6</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

$^a$ The total computation time of the 24-hour optimisation study, based on MATLAB Version 7.11 and with i7-3.4 GHz / 8 GB RAM.

Figure 7-7b shows the network loss caused by the tap staggering operation during the 24-hour period. Compared with the rule-based control scheme, the branch-and-bound method has reduced the network loss by an average of 8% and the GA approach has an average reduction of 6%. The loss reductions are not significant since most primary substation transformers have similar winding resistances in the model. Consequently, the selection of different transformers for the tap staggering will not have a great impact on
the overall network loss. Although the branch-and-bound method can produce less loss than the GA approach, the computation time is much longer than the GA approach for this large network model (see Table 7-9).

7.4.3 Summary of findings

For the 222-bus distribution network model, two case studies have been carried out. The first case study has analysed the computation time of each control algorithm in this larger system. The result indicates that the computation time of the GA approach has become faster than the branch-and-bound method since the search space has increased significantly for the branch-and-bound algorithm. The second case study has investigated the performance of each control method at different load levels, i.e. over a 24-hour period of demand changing. Although the GA approach has resulted in slightly more tap operations than the other two methods, the GA can provide the VAr absorption within the defined tolerance throughout the 24-hour period. In contrast, both the rule-based and the branch-and-bound methods have experienced several violations of the VAr absorption requirement in the 24-hour period.

7.5 Summary

This chapter has presented the comparison studies of the tap stagger control using the developed GA approach, the rule-based algorithm and the LP based branch-and-bound method. The latter two methods have utilised the information of distribution network VAr absorption capability (i.e. expressed as a sensitivity matrix). To apply the branch-and-bound method, the tap stagger control problem has been linearised using the network sensitivity matrices $C$ and $P$. All three control approaches have been tested with the 102-bus distribution network model and a relatively large distribution system with 222 buses. Both network models are based on real UK HV distribution systems.

The key outcomes from the comparison studies can be summarised as:
The rule-based control scheme provides the fastest control in both network models. However, it does not consider the minimisation of network loss and may lead to non-convergence.

The branch-and-bound method has a faster convergence speed than the GA approach for the smaller network model (i.e. the 102-bus distribution network). However, due to linearisation errors, the branch-and-bound method may return solutions that violate the VAr absorption tolerance. In addition, for the larger system (i.e. the 222-bus distribution network), the average computation time of the GA approach becomes faster than the branch-and-bound method as the search space of the branch-and-bound algorithm has increased significantly.

The GA approach provides the most accurate control, which maintains the VAr absorption error within the tolerance under various input settings (i.e. $Q_{\text{required}}$, $e_{\text{limit}}$ and $T_{S_{\text{max}}}$) and different load levels (i.e. with the daily load changing).

Both the rule-based and the branch-and-bound methods can only be applied to radial distribution networks since they use the linear approximation to estimate the total VAr absorption. However, the GA approach can also be used for meshed networks.
CHAPTER 8
IMPACTS OF TAP STAGGER ON TRANSMISSION SYSTEM

8.1 Introduction

The studies in the previous chapters have focused on the feasibility and implementation of the tap staggering technique. The reactive power absorption capability of distribution networks through the use of tap stagger has been investigated. The control algorithms of tap stagger have also been developed for distribution networks to provide the VAr absorption service with the minimum number of tap switching operations and network loss introduced. Therefore, based on the concept of utilising distribution networks to support transmission VAr management, this chapter presents a comprehensive analysis of both the economic and technical effects of the tap staggering technique on the transmission system side. In the economic studies, the annual costs of applying the tap staggering technique are investigated from the perspective of transmission system operators. The IEEE Reliability Test System (RTS) load flow model is used to perform the cost calculations under different scenarios. To demonstrate the economic benefit, the results are compared with the use of the other passive compensator, i.e. shunt reactors. In terms of the technical analysis, the transient responses of transmission voltages to the tap staggering or reactor switching are investigated through dynamic studies. The details are described as follows.

8.2 Transmission System Modelling and Tap Stagger Implementation

8.2.1 IEEE reliability test system

The IEEE reliability test system (RTS) model has been used to investigate the impacts of distribution network VAr absorption on the transmission system. The RTS network
comprises 24 buses connected by 33 lines and 5 transformers as shown in Figure 8-1. There are a total 32 generating units, ranging from 12 to 400 MW. The transmission lines are at two voltages, i.e. 138 kV and 230 kV. The system has voltage corrective devices at bus 14 (synchronous condenser) and bus 6 (reactor). The complete network data are available in [138] and the system has been modelled using the MATPOWER [139]. The MATPOWER is a package of MATLAB programming files for solving power flow and optimal power flow problems, especially for transmission systems.

Figure 8-1: Single-line diagram of the IEEE Reliability Test System
8.2.2 Identification of voltage problems

To identify the buses whose voltages are sensitive to load variations, an annual load flow study has been performed. The annual load profile has been produced by combining the weekly, daily and hourly load data from the original RTS system [138]. Figure 8-2 plots the resulting load curve of 52×7×24=8736 hours.

![Figure 8-2: Annual load profile for the RTS system (in percent of annual peak load)](image)

In the RTS model, the default power rating for each load is the annual peak load and each load is modelled as constant power load. The annual peak load for the overall RTS system is 2850 MW. The load curve in Figure 8-2 indicates the hourly demand of the system in the percentage of the annual peak load. The load profile has been applied to all the network loads in order to simulate the power and voltage variations across a year. At each time point, the load of each bus has been modified by multiplying the corresponding percentage indicated on the load profile. Considering the balance between generation and demand, the generating units have also changed their active power outputs according to the load percentage. In addition, the generators have been set to maintain the voltages at the connected buses by regulating their reactive power outputs within their capacities.

From the initial load flow study, only bus 6 and bus 10 have been observed with high voltage violations (e.g. voltages larger than the normal operating limit 1.05 pu [140])
under light load conditions. This is due to the capacitive current produced by the cable between bus 6 and bus 10. Although a 100 MVar shunt reactor has already been placed at bus 6 to absorb the VAr surplus, the voltage still exceeds the limit when demand is low. Figure 8-3 illustrates the annual voltage profiles of buses 6 and 10. As shown in Figure 8-3a, the overvoltage issues at bus 6 have only occurred periodically for 358 hours over a year (8736 hours). This implies that the permanent installation of an additional reactor at bus 6 may not be cost-effective. However, Figure 8-3b shows that bus 10 has more frequent voltage problems, i.e. 2000 hours during a year. The installation of a reactor may need to be considered. The details of how to decide which technique is more cost-effective to mitigate high voltage problems are given in Section 8.4.

![Annual voltage profile of the RTS system](image)

**Figure 8-3:** Annual voltage profile of the RTS system
8.2.3 Implementation of tap stagger using GA

The tap staggering technique can be used to improve the voltage profiles of buses 6 and 10. As load buses (without generators connected) in the RTS system can be considered as GSPs for distribution networks, the implementation of tap stagger can be achieved by equivalently increasing the load consumption at buses 6 and 10. The increments in active and reactive loads should be set to the same values as the power loss and reactive power absorption introduced by the tap stagger, respectively. To determine the corresponding power loss and VAr absorption, the 222-bus distribution network model in Chapter 7 has been used.

Figure 8-4 illustrates the flowchart. The network model was integrated with the annual load profile shown in Figure 8-2 and the tap stagger optimisation study was carried out using the GA approach. The study started from the VAr absorption target \( Q_{\text{required}} \) of 1 MVar. With \( e_{\text{limit}} = 1\% \) and \( TS_{\text{max}} = 6 \) throughout the annual load flow study, the GA calculated the actual network VAr absorption and the introduced network loss at each time point, i.e. \( Q_{\text{actual}}(t) \) and \( \Delta P_{\text{loss}}(t) \). This procedure was repeated for \( Q_{\text{required}} = 2, 3, \ldots, L \) MVar. Considering the 222-bus network absorption limit, \( L \) was set to 10 MVar for this case. Based on the optimisation results, the following power variation matrices were obtained:

\[
\Delta Q(t,n) = \begin{bmatrix}
\Delta Q_{1,1} & \cdots & \Delta Q_{1,L} \\
\vdots & \ddots & \vdots \\
\Delta Q_{T,1} & \cdots & \Delta Q_{T,L}
\end{bmatrix}_{T \times L}
\]

\[
\Delta P(t,n) = \begin{bmatrix}
\Delta P_{1,1} & \cdots & \Delta P_{1,L} \\
\vdots & \ddots & \vdots \\
\Delta P_{T,1} & \cdots & \Delta P_{T,L}
\end{bmatrix}_{T \times L}
\]

where \( T \) denotes the total time points in a year, i.e. 8736. The matrix element \( \Delta Q_{tn} \) indicates the VAr absorption measured at the GSP with the VAr compensation level \( Q_{\text{required}} \) = \( n \) MVar at the time point \( t \). Similarly, the matrix \( \Delta P(t, n) \) indicates the network power losses due to the tap staggering at different time points.
In the RTS system, each load bus can be regarded as a GSP for distribution networks. Therefore, if bus 6 or 10 needs to apply the tap staggering method, the matrices $\Delta Q(t, n)$ and $\Delta P(t, n)$ will be used to determine the load increments, depending on the start time of the staggering and the compensation level ($Q_{\text{required}}$) used. The resulting impacts of the tap stagger on the transmission system voltages and power generation are discussed in Sections 8.3 and 8.4. As the study only involves static load flow calculations, the effects of aggregating VAr absorption at the transmission level should be equivalent to the effects of actually implementing tap stagger at distribution network transformers.

![Flowchart to determine load increments when applying the tap staggering technique to the RTS system](image)

Figure 8-4: Flowchart to determine load increments when applying the tap staggering technique to the RTS system
8.2.4 Cost Analysis for VAr Compensation

The tap staggering technique is expected to provide a more cost-effective means of absorbing VAr surplus than the traditional methods. To prove this potential benefit, the cost for the transmission system operator (TSO) to use the tap staggering technique has been analysed. As shunt reactors are widely installed to absorb the excess reactive power in the grid, the associated costs have also been analysed and compared with the tap staggering technique. The equations used to calculate the costs are described as follows.

8.2.4.1 Tap staggering technique

With the method described in Section 8.2.3, the associated cost for tap stagger can be evaluated from the perspective of the TSO. In general, the TSO procures balancing services to balance demand and supply and to control the system frequency and voltage across the transmission system. As the use of tap stagger will increase the active power demands, more power may need to be generated to maintain the system frequency within the statutory limits. Consequently, the TSO may pay the generators for producing the extra power [141]. Additionally, the TSO shall pay the reactive power service providers for supporting the system voltages. Therefore, considering the VAr supports provided by distribution networks as ancillary services, the annual cost for the TSO to apply the tap staggering technique can be derived as:

\[
C_{\text{stagger}} = C_{\Delta \text{gen}}(h) + C_{\text{VAr}} \sum_{m=1}^{h} \Delta Q_{tm} \cdot h
\]

(8-3)

where \( h \) denotes the total hours of using tap stagger in a year (i.e. the utilisation hour). The first term \( C_{\Delta \text{gen}}(h) \) represents the cost to compensate the generation changes and it can be calculated using the generator cost data. The second term represents the payment for VAr support providers, i.e. the distribution network operators. The payment is calculated based on the unit price of reactive energy \( C_{\text{VAr}} \) (in £/MVArh). \( \Delta Q_{tm} \) denotes the total VAr absorption (in MVAr) provided through the use of tap stagger at the time
point $t_m (m = 1, 2, \ldots, h)$ and $\Delta h$ is the time interval, taken as 1 hour for the following studies.

### 8.2.4.2 Installation of shunt reactor

To compare it with the tap stagger, the cost of employing shunt reactors has also been analysed. For the TSO, a capital investment needs to be paid to purchase and install a reactor. The annual equivalent investment cost $I$ can be calculated as [76]:

$$I = S \frac{r(1 + r)^y}{(1 + r)^y - 1}$$

(8-4)

where $S$ is the actual capital investment, which depends on the reactor rating (in MVAr), $r$ is the discount rate and $y$ is the economic life of the reactor. As the use of shunt reactor could reduce the reactive power flow during periods of low demand, the network power loss would also decrease. The TSO may pay less for the generation used to balance the system frequency. Therefore, without considering the maintenance cost of the reactor, the annual cost for the TSO to use a reactor can be described as:

$$C_{\text{reactor}} = C_{\Delta\text{gen}}'(h) + I$$

(8-5)

where $C_{\Delta\text{gen}}'(h)$ denotes the cost for generation changes, which could be negative [141] as the reactor could reduce the network power loss during periods of low demand.

### 8.3 Effectiveness on Voltage Regulation

In this section, the effectiveness of the tap staggering technique on mitigating high voltages has been demonstrated. First, annual load flow studies have been performed considering the RTS system with the tap staggering method applied to bus 6. At each time point, the tap stagger would only be activated if the corresponding bus voltage exceeded the normal operating limit 1.05 pu. The tap staggering method would increase
the active and reactive loads at bus 6 according to Equations (8-1) and (8-2). As the voltage reduction depends on the amount of VAr absorption, the annual load flow simulation has been repeated for VAr compensation levels ($Q_{\text{required}}$) from 1 MVAR to 10 MVAR. Finally, the same studies were carried out for the RTS system with the tap staggering technique applied to bus 10.

Figure 8-5 illustrates the resulting numbers of annual voltage violations at buses 6 or 10 with various compensation levels. For both buses, voltage violations (i.e. > 1.05 pu) are reduced as the amount of VAr absorption increases. At the compensation level of 6 MVAR, the tap staggering method can eliminate all the high voltage situations at bus 6. However, for bus 10 with more frequent voltage violations, the compensation of 10 MVAR still cannot address all the overvoltage issues. Further studies have shown that bus 10 needs an inductive reactive power of 36 MVAR to eliminate all the high voltage problems. However in practice, due to the limitation of network capability, distribution networks may not be able to provide such large VAr absorption through the use of tap stagger. Therefore, a combined method of employing reactors together with tap stagger may be a solution. The details are discussed in the following economic studies.

![Figure 8-5: Total number of annual voltage violations (i.e. > 1.05 pu) with tap stagger applied at either bus 6 or bus 10](image_url)
8.4 Economic Studies

This section presents the cost analysis of applying the tap staggering technique when high voltage problems arise occasionally or frequently. The results have been compared to the use of shunt reactors. The RTS system has been tested under three different scenarios to demonstrate the economic benefit of applying tap stagger.

8.4.1 Case 1: Cost analysis for bus with occasional voltage issues

As shown in Figure 8-1, the RTS network already has a shunt reactor of 100 MVAR connected at bus 6 to absorb the capacitive reactive power. From the voltage regulation study result shown in Figure 8-5, bus 6 still requires an additional inductive reactive power of 6 MVAR to eliminate all the high voltage problems. Therefore, in this case study, two solution methods have been used to mitigate the high voltages, which are:

**Solution 1:** Installing an additional shunt reactor of 6 MVAR at bus 6,

**Solution 2:** Applying the tap staggering method described in Section 8.2.3 at bus 6 with the compensation level \(Q_{\text{required}}\) of 6 MVAR.

Annual load flow studies have been performed considering the system with the two solution methods separately. Note that for both methods, the VAr compensation would only be activated when bus 6 and/or bus 10 voltages exceeded the normal operating limit 1.05 pu during the simulations. The total utilisation hours for each method are 2000 hrs as bus 10 has experienced the most voltage violations, i.e. for 2000 hrs.

According to Equations (8-3) and (8-5), the cost calculations of both solution methods have considered the impacts of generation changes. Intuitively, the tap staggering method should be less expensive than the use of reactors as no extra capital investment is required. However, since the employment of tap stagger also increases the active power demands, more energy needs to be produced to balance the power flow, resulting in more generation costs. Figure 8-6 indicates the annual energy generation for the RTS system...
with the two solution methods, respectively. As shown in the figure, the generation changes with the utilisation hours of the methods. Solution 1 can reduce the energy generation as the network power loss decreases when the reactor is connected. For solution 2, the energy generation increases with the utilisation hours. However, as the tap staggering method only slightly increases the demands, the increase in energy generation is very small, e.g. 0.003% for 2000 hrs of utilisation.

With Equations (8-3) and (8-5), the annual costs of solutions 1 and 2 have been evaluated. First, the costs to compensate generation changes, i.e. $C_{\Delta gen}(h)$ and $C_{\Delta gen}^{'}(h)$, have been calculated based on the generator cost data from [138]. For solution 1, the capital investment of a high-voltage reactor has been estimated as $37 \times 10^3 \ £/MVAr$ [142]. By applying an economic life of 38 years [76] and a discount rate of 8% to Equations (8-4), the annual equivalent investment cost of a reactor has been estimated as $3182 \ £/MVAr$ per year. In solution 2, the unit price of reactive energy has been assumed as $3.2 \ £/MVArh$ [143]. Considering the compensation level of 6 MVAr, Figure 8-7 plots the annual costs of both methods against the utilisation hour $h$.

According to the figure, the cost of using a reactor starts from the annual equivalent investment $6 \times 3182 = 1.9 \times 10^4 \ £$ and decreases with the number of utilisation hours since the cost of energy generation has been reduced. In contrast, the cost of applying tap

![Figure 8-6: Annual energy generation for the RTS system with either reactor or tap stagger](Image)
stagger starts from 0 and increases with the number of utilisation hours. From the result, the tap staggering technique is more economical than the use of a reactor up to a maximum of 612 utilisation hrs. As bus 6 initially has 358 hrs of voltage violations, the use of tap stagger can eliminate all the voltage issues but cost 40% less than the installation of the reactor as shown in the figure.

![Graph showing cost comparison between installation of reactor and application of tap stagger.](image)

**Figure 8-7:** Annual costs for solutions 1 and 2 in Case 1 (at compensation level of 6 MVAr)

### 8.4.2 Case 2: Cost analysis for bus with frequent voltage issues

From Case 1, the high voltage problems at bus 6 have been solved by applying an additional reactor or the tap staggering method directly at bus 6. However, the adjacent bus 10 still has a large number of voltage violations according to Figure 8-5. To reduce the high voltages, another two methods have been adopted, which are:

- **Solution 3:** Installing shunt reactors at buses 6 and 10,
- **Solution 4:** Applying tap stagger at buses 6 and 10.

In solution 3, both buses 6 and 10 have a shunt reactor installed, with a rating of 6 MVAr. For solution 4, the compensation levels ($Q_{\text{required}}$) at both buses have been set to 6 MVAr. For each method, the VAr compensation at each bus would only be activated if the
corresponding bus had high voltage violations. The annual costs for solutions 3 and 4 have been calculated using the same parameters as in Case 1. From the results shown in Figure 8-8, the tap staggering method is less expensive than the installation of reactors for utilisation hours up to 1069 hrs. However, as stated earlier, bus 10 initially has 2000 hrs of voltage violations, which require 2000 hrs of using tap stagger to reduce the high voltages. Therefore, it is more advisable to install shunt reactors than to apply tap stagger in this case. Note that the placement of the 6 MVAr reactors still cannot completely resolve the voltage issues at bus 10 (see Figure 8-5). The installation of larger rating reactors may be a solution but with higher capital investments. To reduce the cost, a combined method of using both reactor and tap stagger is investigated in the following case study.

![Figure 8-8: Annual costs for solutions 3 and 4 in Case 1 (at compensation level of 6 MVAr)](image)

**8.4.3 Case 3: Combined method of reactor and tap Stagger**

The result of Case 2 indicates that the installation of reactors will be more cost-effective than tap stagger if high voltage situations occur frequently. This case study aims to analyse the cost of applying the combined method of using both reactor and tap stagger. According to load flow studies, an inductive compensation of 4 MVAr at bus 6 together with a compensation of 28 MVAr at bus 10 can eliminate all the voltage violations during
periods of low demand. Therefore, considering the limitation of VAr absorption provided through the use of tap stagger, three solution methods have been investigated:

**Solution 5**: Installing a reactor of 4 MVar at bus 6 & installing a reactor of 28 MVar at bus 10,

**Solution 6**: Applying tap stagger of 4 MVar (i.e. $Q_{required} = 4$ MVar) at bus 6 & installing a reactor of 28 MVar at bus 10,

**Solution 7**: Applying tap stagger of 4 MVar at bus 6 & installing a reactor of 24 MVar and applying tap stagger of 4 MVar at bus 10.

In solution 7, the tap stagger at bus 6 and the reactor at bus 10 had been first used to absorb reactive power surplus when the corresponding bus had voltage violations. If the voltage at bus 10 was still higher than the limit, the tap staggering method for bus 10 would be activated to reduce the voltage further. Table 8-1 summarises the annual costs for the three solution methods along with the utilisation hours of tap staggering.

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Utility hour of tap stagger (hrs)</th>
<th>Annual cost (£/year)</th>
<th>Cost reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solution 5</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(reactors only)</td>
<td>0</td>
<td>0.992 x10^5</td>
<td>0</td>
</tr>
<tr>
<td><strong>Solution 6</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(stagger at bus 6 &amp; reactor at bus 10)</td>
<td>358</td>
<td>0</td>
<td>0.941 x10^5</td>
</tr>
<tr>
<td><strong>Solution 7</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(stagger at bus 6 &amp; reactor + stagger at bus 10)</td>
<td>358</td>
<td>187</td>
<td>0.848 x10^5</td>
</tr>
</tbody>
</table>

From the table, the costs of combined methods (i.e. solutions 6 and 7) are less than the cost of only using reactors (i.e. solution 5). Comparing solutions 5 and 6, it is more economical to apply tap stagger than reactor at bus 6, where high voltage problems occur occasionally. For bus 10, which experiences frequent overvoltages, an integrated method
of using tap stagger and a reactor with small rating can help reduce the VAr compensation cost (e.g. by 15% in solution 7).

### 8.5 Dynamic Performance Studies

In addition to the economic studies, the dynamic impacts of the tap stagger switching on the transmission system have been investigated. The RTS system in Figure 8-1 has been modelled using the DIlgSILENT to perform the transient analyses [144]. To simulate an equivalent distribution network for tap staggering, five pairs of primary substation transformers have been modelled and connected to bus 6 through a 132/33 kV transformer (see Figure 8-9). The line and transformer parameters have been obtained from the 222-bus distribution network model studied in Chapter 7, and 50% of the original load at bus 6 has been distributed evenly through the five primary substations. The resulting demand for each primary substation is around 13 MW, which is close to the default load rating of the 222-bus network. According to the capability studies presented in Chapter 5, five primary substations can provide a total 1 MVar absorption with four staggered taps (i.e. two taps up for one transformer and two taps down for the other).

![RTS Network](image)

**Figure 8-9: Modified RTS system for dynamic studies**

The successive switching of OLTCs in the distribution substations can affect the voltages at the upstream transmission buses. The resulting voltage responses have been compared...
with the voltage fluctuations due to reactor switching. As the studies focus on analysing the transient responses of system voltages, it has been assumed that the tap changing operation or the reactor switching was completed instantaneously. The details of the comparison studies are described as follows.

### 8.5.1 Reactor switching in transmission network

As shown in Figure 8-9, a reactor of 1 MVAr rating is installed at bus 6 to compare the transient voltage response with the tap staggering technique. Initially, the five pairs of primary transformers were operated at the same tap positions (i.e. without tap stagger). The system load demand was set to 60% of the annual peak demand to create high voltage scenarios. The reactor was then switched on to mitigate the high voltage at bus 6.

#### 8.5.1.1 Voltage at bus 6

Figure 8-10 shows the voltage response (i.e. the RMS value) at bus 6 when the reactor was switched on at $t = 1s$ and switched off at $t = 2s$. As illustrated in the figure, the bus voltage has experienced slow damping when the reactor was switched on or off since the reactor does not have resistance. The settling time and voltage overshoot with the reactor switching are summarised in Table 8-2.
8.5.1.2 Voltages at adjacent buses

Figure 8-11 indicates the corresponding voltage responses at buses 2 and 10, which are connected directly to bus 6. The voltage damping introduced at buses 2 and 10 is smaller than the one at bus 6. From Figure 8-10 and Figure 8-11, voltage spikes have occurred at the three buses when the reactor was disconnected at $t = 2s$. According to the results obtained in Table 8-2, the voltage at bus 6 has been observed with the largest overshoot during the transient states.

![Figure 8-11: Buses 2 & 10 voltages with the reactor switching at bus 6](image)

<table>
<thead>
<tr>
<th></th>
<th>Switched on</th>
<th></th>
<th>Switched off</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Settling time (s)</td>
<td>Overshoot (%)</td>
<td>Settling time (s)</td>
</tr>
<tr>
<td>Bus 6</td>
<td>0.99</td>
<td>0.020</td>
<td>0.74</td>
</tr>
<tr>
<td>Bus 2</td>
<td>0.97</td>
<td>0.016</td>
<td>0.81</td>
</tr>
<tr>
<td>Bus 10</td>
<td>0.94</td>
<td>0.011</td>
<td>0.66</td>
</tr>
</tbody>
</table>

8.5.2 Tap staggering in distribution network

The simulation has been repeated by applying the tap staggering technique to the downstream primary substations as shown in Figure 8-9. In order to provide
approximately the same amount of reactive power absorption as the reactor did, the five pairs of parallel transformers used 4 staggered tap steps (i.e. 2 taps up for one transformer and 2 taps down for the other). During the simulation, the transformer tap staggering has been carried out one pair after another rather than simultaneously, considering the communication and control delay in reality. The time interval of tap stagger initiation between each pair has been set to 0.1s.

8.5.2.1 Voltage at bus 6
Figure 8-12 demonstrates the resulting voltage variations at bus 6 when the tap stagger was activated at $t = 1s$ and deactivated at $t = 2s$. Since the tap staggering method has divided the VAr absorption into five stages, the voltage overshoot introduced at each stage is less than the overshoot caused by reactor switching. Table 8-3 shows the average settling time and voltage overshoot of the five staggering stages. From the results, the tap changings of OLTCs in the downstream network have not led to prolonged voltage damping at the upstream transmission bus.

![Figure 8-12: Bus 6 voltage with the application of tap stagger at the five primary substations](image)

8.5.2.2 Voltages at adjacent buses
Figure 8-13 shows the voltage responses at buses 2 and 10 due to the tap stagger. Table 8-3 lists the corresponding settling time and voltage overshoot during the tap changing.
Compared with the use of a reactor, the tap staggering method has significantly reduced the settling time (e.g. by around 90%). In addition, the introduced voltage overshoots by tap stagger are less than the overshoots of reactor switching, especially on bus 6 where the VAr compensation methods are applied. Note that for both methods, the transient voltage overshoots at the three buses are small since the amount of reactive power compensation is not significant, i.e. only around 1 MVar. However, the studies have demonstrated that the use of tap stagger can reduce the voltage overshoots during the switching transient states.

![Figure 8-13: Buses 2 & 10 voltages with the application of tap stagger at the five primary substations](image)

<table>
<thead>
<tr>
<th></th>
<th>Staggering activated</th>
<th>Staggering deactivated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Settling time (s)×</td>
<td>Overshoot (%)×</td>
</tr>
<tr>
<td><strong>Bus 6</strong></td>
<td>0.093</td>
<td>0.010</td>
</tr>
<tr>
<td><strong>Bus 2</strong></td>
<td>0.084</td>
<td>0.008</td>
</tr>
<tr>
<td><strong>Bus 10</strong></td>
<td>0.070</td>
<td>0.007</td>
</tr>
</tbody>
</table>

a. Based on the average value of the five tap staggering stages.
8.5.2.3 Flexible VAr absorption
In Figure 8-12, the voltage at bus 6 is just slightly over the operating limit 1.05 pu. A small amount of VAr absorption would be sufficient to bring the voltage back to the limit. However, the reactor is only capable of consuming fixed reactive power at a certain voltage. In terms of tap stagger, flexible VAr absorption can be achieved by changing the number of transformers with tap stagger. Figure 8-14 illustrates the bus voltages when only three pairs of primary transformers have been operated with tap stagger. As shown in the figure, the voltage at bus 6 has been reduced below 1.05 pu with less VAr consumption, resulting in more efficient VAr compensation than the use of the reactor. Note that the voltage at bus 10 is still over the limit. However, the voltage issue can also be resolved by applying the tap staggering technique to bus 10.

![Figure 8-14: Bus voltages with only three pairs of primary transformers tap staggering](image)

8.6 Summary
In this chapter, both the economic and technical analyses of the tap staggering technique have been presented. The application of tap stagger at primary substation transformers can introduce inductive loading onto the upstream transmission system, mitigating the
high voltages during periods of low demand. The IEEE RTS system has been modelled to compare the economic and technical impacts of using the tap staggering technique and shunt reactors.

In the economic studies, as the use of tap stagger will slightly increase the active demand, more power will need to be generated to balance the power flow. The TSO may need to compensate the generators in order to produce extra power. However, the implementation of tap stagger does not require additional capital investments due to the utilisation of existing assets. Therefore, for the bus with only occasional high voltage issues, results indicate that the tap staggering technique is more cost-effective than the use of reactors. In the system with more frequent voltage violations, a combined method of applying tap stagger and reactor together can be used to reduce the VAr compensation costs, resulting in the deferment of high-cost network reinforcements.

In terms of the transient studies, voltage spikes would be introduced at the transmission buses due to reactor switching. The tap staggering method has been adopted to reduce the settling time and overshoots of the transient voltages. The use of many distributed transformers can divide the VAr absorption into several small steps, reducing the overall dynamic impacts on the transmission system. In addition, the tap staggering method can provide a flexible reactive power absorption service according to the transmission grid VAr requirement.
CHAPTER 9

CONCLUSIONS

This chapter summarises the research studies and discusses possible future work plans.

9.1 Conclusions

The thesis first introduced the background of power system voltage and reactive power control. In recent years, high voltage situations under low demand conditions have been increasing in the UK transmission system. The main reasons include the development of underground cables in transmission and distribution networks and the decommissioning of coal generators in specific areas. In addition, during periods of low demand, National Grid has observed low reactive power demand or even reactive power injection at several GSPs. In some cases, reactive power has been exported from the distribution networks to the transmission system.

The exact cause of this declining trend of reactive power demand during minimum load periods is still uncertain. The development of underground cables in distribution networks, the use of more energy-efficient household appliances and the impact of DGs are some of the possible factors affecting the system VAr demand. On the other hand, to comply with the forthcoming European Demand Connection Code, distribution networks should be able to maintain a limited range of reactive power consumption at GSPs. Therefore, considering the VAr management challenges from both the transmission and distribution sides, this study aims to develop a cost-effective means of utilising distribution networks to provide reactive power absorption services for transmission systems.

Critical reviews of existing Volt/VAr control techniques for both transmission and distribution systems have been carried out. From the literature, generating units and VAr compensation devices are commonly used to produce or absorb the reactive power in
transmission systems. However, with the presence of large-scale intermittent renewable resources nowadays, voltage problems could become more dynamic and may occur in different time periods or different regions. The placement and tuning of numbers of compensation devices at different locations will be challenging for the system operators. The associated costs will also be high if several devices need to be placed at various locations in the system. Some studies have demonstrated the benefits of using many, small and distributed SVCs located at distribution buses to provide reactive power service for the transmission system. Based on this concept, a new reactive power management approach to reduce reactive power surplus in the transmission system has been proposed in this thesis.

The proposed method utilises the existing parallel transformers in distribution networks, to provide reactive power absorption services for transmission systems during periods of low demand. The operation of two parallel transformers in different tap positions, i.e. with staggered taps, will introduce a circulating current around the pair. Due to the transformer inductance, the circulating current will draw more reactive power from the upstream network. For each pair, the difference in tap positions should be limited within a small range to prevent the transformer from overloading and to maintain transformer secondary voltages. The aggregated VAr absorption from many pairs of parallel transformers could be used to mitigate the high voltages in the upstream network. In this research work, the proposed tap staggering method has been applied to the primary substation transformers, which have OLTCs installed. However, conceptually the tap staggering technique may be adopted at different voltage levels where there are transformers operating in parallel.

The ultimate goal of the proposed tap staggering method is to establish a new reactive power ancillary service, which allows distribution networks to use their existing parallel transformers to absorb the temporary VAr surplus. The proposed method may be considered as a smart grid tool to support the reactive power management of the transmission system. This research project has carried out the essential investigations, including the feasibility, control and benefit analyses. The **major contributions** in the
thesis can be concluded as:

- Assessment of the VAr absorption capability of a real UK HV distribution network with the tap staggering technique. The results have confirmed that the method has the potential to increase the reactive power demand drawn from the transmission grid.
- Design of an optimal control approach based on GA to coordinate the tap staggering operation of multiple pairs of parallel transformers. An objective function has been proposed considering the minimisation of the introduced power loss as well as the number of tap switching operations.
- Comparison of the GA-based tap stagger control approach with two alternative methods, i.e. the rule-based control scheme and the branch-and-bound method.
- Comparison of the impacts of the tap staggering technique and the use of shunt reactors on transmission systems, in terms of economic cost and dynamic performance. The results have demonstrated that the tap staggering technique has economic advantages and can reduce voltage overshoots during the transient states.

The details of each contribution are summarised as follows.

1) Network Reactive Power Absorption Capability Assessment

To investigate the feasibility of using the tap staggering technique to provide a reactive power service, network VAr absorption capability studies have been carried out. The studies are based on the off-line load flow analysis of a real UK HV distribution system model. The model consists of the networks from the 132kV GSPs down to the 33kV primary substations. The tap staggering method has been applied to the parallel transformers at the primary substations. The individual primary substation study demonstrates that the VAr absorption and power loss of the parallel transformers will increase quadratically with the number of staggered taps. By applying the tap stagger to multiple primary substations and using the curve fitting method, the study estimates a maximum absorption of 1.87 MVAr/substation could be observed at the GSP side.
However in practice, due to the variations in network demands, the initial OLTC position may not be at or near the nominal position, resulting in less headroom for the tap stagger. In addition, the large tap difference may lead to transformer overload. Therefore, a more conservative study has been carried out considering transformers with only 4 staggered taps. The results indicate an average VAr absorption of 0.24 MVar/substation. If each distribution network can provide a certain amount of VAr absorption, the transmission system can utilise these inductive reactive power to mitigate the high voltages for some temporary periods.

2) Tap Stagger Optimal Control based on GA
The thesis has also proposed an optimal control approach for the tap staggering operation. If the upstream transmission grid requires a specific reactive power to be consumed during periods of low demand, the distribution system should be able to decide how many parallel transformers and staggered taps will be used in order to meet the requirement. The objective function has been defined for the tap stagger control problem, and it has been solved using the genetic algorithm. The objective is to find an optimal dispatch for the OLTC tap positions, which minimises the network loss and the number of switching operations introduced when providing the VAr absorption service. The objective is subject to the constraints of the VAr absorption error, the OLTC operating ranges and the permitted staggered taps.

The GA-based tap stagger control method has been applied to a simplified 102-bus distribution network. With a given VAr requirement, the GA approach can find an optimal solution that minimises the introduced network loss and the number of tap switching operations while satisfying the operating constraints. Statistical studies have been carried out to analyse the accuracy of the VAr absorption provided through the GA method. The results confirm that the developed GA algorithm is capable of providing the VAr absorption within the predefined tolerance under different system requirements.

In addition, the GA-based control approach has been investigated using different load models. The results indicate that the algorithm will return solutions with slightly more tap
switching operations if the network load is more sensitive to the voltage changes. This is because, when applying the tap staggering technique, the line voltage drops between the 132kV and 33kV networks will increase. The voltages at the HV and LV sides of the primary substations will decrease as well, resulting in lower load demands. The reduction in reactive load demands will counteract the increase in reactive power flow caused by the tap stagger. To provide the same VAr absorption at the GSP, the control algorithm has to use more staggered taps to increase the transformer VAr absorption. However, the changes in the number of tap operations due to different load models are not significant, i.e. generally within four tap operations. The GA approach has also been tested with varying load demands, i.e. over a 24-hour period. The result indicates that during periods of high demand, the network can provide the VAr absorption using fewer tap operations since the reactive power consumption of lines (due to the tap stagger) increases.

3) Comparison of GA with Rule-Based and Branch-and-Bound
The GA-based tap stagger control approach has been compared with two alternative solution methods, i.e. the rule-based control scheme and the branch-and-bound algorithm. The latter two methods have utilised the information of distribution network VAr absorption capability (i.e. expressed as a sensitivity matrix). To apply the branch-and-bound method, the tap stagger control problem has been linearised. The three methods have been investigated with different control settings. The results show that the rule-based approach provides the fastest control while it does not consider the minimisation of power losses. For a small system, the branch-and-bound method has a faster convergence speed than the GA approach. However, due to linearisation errors, the branch-and-bound method may return solutions that do not actually provide the VAr absorption within the tolerance. In contrast, the GA approach can produce the most accurate VAr absorption according to the system requirement. In addition, for a larger distribution system (i.e. the 222-bus distribution network), the average computation time of the GA approach becomes faster than the branch-and-bound method as the search space of the branch-and-bound algorithm has increased significantly. Note that, both the rule-based and the branch-and-bound methods can only be applied to radial distribution networks whereas the GA approach can also be used for meshed networks.
4) Impacts of Tap Stagger on Transmission System

Finally, both the economic and technical effects of the tap staggering method on the transmission system have been discussed and compared with the use of shunt reactors. In the economic studies, the cost for the TSO to apply the tap staggering technique consists of the payments for generators as well as DNOs. As the use of tap stagger will slightly increase the active demand, more power will need to be generated to balance the power flow. The TSO may need to compensate the generators in order to produce extra power. However, the implementation of tap stagger does not require additional capital investments. The TSO only needs to pay the DNOs for providing the reactive power ancillary services. Therefore, considering these two factors, studies indicate that the tap staggering technique is more cost-effective than the use of reactors at the bus with only occasional high voltage issues. In the system with more frequent voltage violations, a combined method of applying tap stagger and reactor together can be used to reduce the VAr compensation costs. Consequently, the use of tap stagger may defer the reinforcements of higher rating reactors or other VAr compensators.

In terms of the transient studies, voltage spikes would be introduced at the transmission buses due to the reactor switching. The tap staggering method has been adopted to reduce the settling time and overshoots of the transient voltages. The use of many distributed transformers can divide the VAr absorption into several small steps, reducing the overall dynamic impacts on the transmission system. In addition, the tap staggering method can provide flexible (or adjustable) reactive power absorption according to the transmission grid VAr requirement.

9.2 Suggestions for Future Work

This research project has concentrated on the necessary investigations of the feasibility, implementation and effectiveness of the tap staggering technique. Further studies can be carried out to improve some aspects of the proposed tap staggering method and to design a real-time system for implementing tap stagger optimal control.
Load profiles
For the 24-hour optimisation studies presented in the thesis, a typical daily load profile from literature has been used and both active and reactive demands share the same profile. However, the CLASS project [123] has been undertaking the demand monitoring of real UK primary substations and the measurements obtained can be used to establish more representative load profiles. Further optimisation results can therefore be carried out by adopting separate load profiles for $P$ and $Q$ loads.

Validation of reactive power absorption
This research work has used load flow simulations to investigate the transformer and network VAr absorption capability with tap stagger. Several site trials of the tap staggering technique on real UK primary substations are currently in progress. The measurements taken at the substations and GSPs will be used to validate the VAr absorption capability estimated by simulations.

On-line tap stagger control
The proposed GA-based tap stagger control method has been tested with off-line studies. To further investigate the tap stagger control, a real-time closed-loop system can be developed. Figure 9-1 illustrates the real-time implementation of the tap stagger control.

![Figure 9-1: Real-time implementation of the tap stagger optimal control](image)
First, a distribution network will be modelled using the Real Time Digital Simulator (RTDS) to generate real-time measurements. Since distribution networks usually lack real-time monitoring, state estimation will be developed to calculate and observe the voltage states across the network. According to the estimated voltages, the GA-based solution procedure will then be performed to determine the optimal tap positions for the tap stagger. In addition, the optimisation algorithm will consider the coordination with other existing VAr compensators in the network, such as shunt reactors and capacitor banks. The coordinated VAr control system is expected to provide a fast and stable reactive power service to the upstream transmission system under varying generation and demand conditions.
REFERENCES


REFERENCES

REFERENCES


REFERENCES


REFERENCES


[134] UKERC Energy Data Centre, "Electricity user load profiles (daily) by profile class," 1997. [Online]. Available: [http://data.ukedc.rl.ac.uk/browse/edc/Electricity/LoadProfile](http://data.ukedc.rl.ac.uk/browse/edc/Electricity/LoadProfile)

REFERENCES


APPENDIX A

IPSA Distribution Network Model

The entire distribution network model in IPSA is illustrated in Figure A. This network model is based on the real UK distribution system from the Ofgem funded project – ‘CLASS’ [123].

Due to confidentiality agreements, Table A can only show some basic data of the system model.

Table A: Quantities of different network components

<table>
<thead>
<tr>
<th>Component</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Buses</strong></td>
<td></td>
</tr>
<tr>
<td>400kV</td>
<td>67</td>
</tr>
<tr>
<td>275kV</td>
<td>108</td>
</tr>
<tr>
<td>132kV</td>
<td>586</td>
</tr>
<tr>
<td>33kV</td>
<td>1305</td>
</tr>
<tr>
<td>11kV</td>
<td>388</td>
</tr>
<tr>
<td>6.6kV</td>
<td>413</td>
</tr>
<tr>
<td><strong>Lines</strong></td>
<td>4448</td>
</tr>
<tr>
<td><strong>Transformers</strong></td>
<td>1065</td>
</tr>
<tr>
<td><strong>Static Loads</strong></td>
<td>882 (with a total of 15.4 GW and 2.75 GVar)</td>
</tr>
<tr>
<td>(constant load model)</td>
<td></td>
</tr>
<tr>
<td><strong>GSPs</strong></td>
<td>18</td>
</tr>
<tr>
<td><strong>33kV Load Areas</strong></td>
<td>61</td>
</tr>
<tr>
<td><strong>Primary Substations</strong></td>
<td>354</td>
</tr>
</tbody>
</table>
Figure A: Distribution network model simulated in IPSA+ V1.6.8
APPENDIX B

Derivation of Linear Curve Fitting using Least-Squares Method

The following symbols are used in this Appendix.

- \( x \): Independent variable
- \( y \): Dependent variable
- \( Y_i \): \( i^{th} \) measurement
- \( n \): Number of measurements
- \( \beta_0 \): Intercept of the linear fitting model
- \( \beta_1 \): Slope of the linear fitting model
- \( J \): Objective function of the linear curve fitting
- \( R^2 \): Coefficient of determination

Let the linear estimation model for a set of measurements \((X_i, Y_i, i = 1, 2, \ldots, n)\) be:

\[
y = \beta_0 + \beta_1 x
\]

where \( \beta_0 \) is the intercept, the value of \( y \) when \( x = 0 \), and \( \beta_1 \) is the slope of the line, the rate of change in \( y \) per unit change in \( x \). The least-squares method aims to minimise the sum of squared deviations between the measured \( Y_i \) and the estimate of the measurement. The function can be described as [125], [126]:

\[
\min J = \sum_{i=1}^{n} [Y_i - (\beta_0 + \beta_1 X_i)]^2
\]
Taking partial derivatives of Equation (B-2) with respect to $\beta_0$ and $\beta_1$:

$$\frac{\partial J}{\partial \beta_0} = -2 \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 X_i)$$  \hspace{1cm} (B-3)

$$\frac{\partial J}{\partial \beta_1} = -2 \sum_{i=1}^{n} X_i(Y_i - \beta_0 - \beta_1 X_i)$$  \hspace{1cm} (B-4)

The minimum value of $J$ is obtained when both the derivatives are equal to 0. Therefore,

$$-\beta_0 n - \beta_1 \sum_{i=1}^{n} X_i + \sum_{i=1}^{n} Y_i = 0$$  \hspace{1cm} (B-5)

$$-\beta_0 \sum_{i=1}^{n} X_i - \beta_1 \sum_{i=1}^{n} X_i^2 + \sum_{i=1}^{n} X_i Y_i = 0$$  \hspace{1cm} (B-6)

based on Equations (B-5) and (B-6), the values of $\beta_0$ and $\beta_1$ can be calculated as:

$$\beta_0 = \bar{Y} - \beta_1 \bar{X}$$  \hspace{1cm} (B-7)

$$\beta_1 = \frac{\bar{X} \bar{Y} - \bar{X} \cdot \bar{Y}}{\bar{X}^2 - \bar{X}^2}$$  \hspace{1cm} (B-8)

where $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$, $\bar{Y} = \frac{1}{n} \sum_{i=1}^{n} Y_i$, $\bar{X}^2 = \frac{1}{n} \sum_{i=1}^{n} X_i^2$ and $\bar{X} \bar{Y} = \frac{1}{n} \sum_{i=1}^{n} X_i Y_i$. In order to indicate how well the measured data fit the linear estimation model, the coefficient of determination $R^2$ is often used, which can be defined as [125], [126]:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} [Y_i - (\beta_0 + \beta_1 X_i)]^2}{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}$$  \hspace{1cm} (B-9)

The coefficient of determination ranges from 0 to 1. An estimated model is most accurate when the coefficient is at or near 1.
APPENDIX C

MATLAB Codes for the GA-Based Tap Stagger Optimisation

Main function

clear;
tic;
% driving OpenDSS using Matlab
>DSSStartOK, DSSObj, DSSText] = DSSStartup;

if DSSStartOK
  % Initialization
  fprintf('startingposition.txt');
  fprintf('tapchanger.txt');
  record = textscan(fstarting, '%*s%c%c%c%c%f%n');
  startingpos = cell2mat(record);
  % record the MaxTap, MinTap and NumTaps
  tap = ones(length(startingpos), 3);
  i = 1;
  while ~feof(ftap)
    record = textscan(ftap, '%*s%c%d', 1, 'delimiter', '.');
    % record the NO. of transformer
    i = record{1};
    record = textscan(ftap, '%*s%[^n]', 1);
    % skip the next line
    end
  % change the NumTaps to TapStep(%)
  tap(:, 3) = round((tap(:, 1) - tap(:, 2))./tap(:, 3)*10000)/100;
  % set up the OpenDSS interface variables
  currentpath = cd;
  DSSText.command = ['Compile (currentpath Master.dss)'; % initial network data
  DSSCircuit = DSSObj.ActiveCircuit;
  DSSSolution = DSSCircuit.Solution;
  DSSText.Command = 'Set Normvmaxpu=1.06';
  DSSText.Command = 'Set Normvminp=0.94';
  DSSText.Command = 'Set Maxiter=100';
  DSSText.Command = 'Set Maxcontroliter=30';
  DSSText.Command = 'Set tolerance=0.000001';
  DSSText.Command = 'New Energymeter.m1 Transformer.T33 1';
  DSSText.Command = 'New Monitor.PQ_m1 Transformer.T33 2 Mode=1';
  % Specify the tested primary transformers (A,B,C,D,E,F,G,H,I,J,K)
  testedtrans = [31 30 20 21 22 23 24 25 28 29 26 27 7 6 13 14 11 10 9 8 5 4];
  numberOfVariables = length(testedtrans)/2;
  % Calculate the initial reactive power flow in the network
  for j = 1:numberOfVariables
    DSSCircuit.Transformers.Name = ['T' int2str(testedtrans(2*j-1))];
    % e.g. the first one is from A_33_t11 to A_6_a
DSSCircuit.Transformers.Wdg=1;
DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*j-1));
DSSCircuit.Transformers.Name=[T' int2str(testedtrans(2*j))]; %e.g. the first one is from A_33_t12 to A_6_b
DSSCircuit.Transformers.Wdg=1;
DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*j));
end
DSSSolution.Solve;
if DSSSolution.Converged
    Busname=DSSCircuit.AllBusNames;
    Vpre(:,1)=DSSCircuit.AllNodeVmagPUByPhase(1);
    MyElement=DSSCircuit.CktElements('Transformer.T33');
    Qpre=MyElement.Powers(10)*-3/1.0e3; %the output power from T33 (400/275kV) in MVAr
    Ppre=DSSCircuit.Losses(1)/1.0e6-MyElement.Losses(1)/1.0e6; %the distribution system loss in MW
else
    disp('Not converged!');
end
DSSText.Command='Init';

%Specify the required MVAr absorption and the error allowance (%)
Qref=0.5;
allowanceQ=1.0;
Maxstagger=2;
my_create = @(NVARS,FitnessFcn,options) create_taps(NVARS,options,Maxstagger);
FitnessFcn = @(x)
tapstagger_fitness6b(x,DSSText,DSSCircuit,DSSSolution,startingpos,tap,testedtrans,Qpre,Ppre,Qref,allowanceQ);
%Genetic Algorithm Options Setup
options = gaoptimset(@ga);
options = gaoptimset(options,'PopulationType', 'custom', ...
    'CreationFcn',my_create, ...'
    'CrossoverFcn',@crossover_tapsscattered,
    'CrossoverFraction',0.5,...
    'MutationFcn',[@mutate_tapsmix,1.0],...
    'PlotFcn',[@gaplotbestf,@gaplotdistance],...
    'Generations',100,'PopulationSize',100,...
    'StallGenLimit',20,'Vectorized','on');

Runtimes=1;
Minerror=zeros(Runtimes,1);
Optimtap=zeros(Runtimes,numberOfVariables);
for i=1:Runtimes
    [x,fval,reason,output] = ga(FitnessFcn,numberOfVariables,options);
    Minerror(i,1)=fval;
    Optimtap(i,:)=x;
    disp(i);
end
Resultcount=tabulate(Minerror);
tottapings=sum(Optimtap,2)*2;
fclose(fstarting);
fclose(ftap);
CalculatingTime=toc;
else
    disp('DSS Did Not Start');
end
Fitness function

```matlab
function scores =
tapstagger_fitness6b(x,DSSText,DSSCircuit,DSSSolution,startingpos,tap,testedtrans,Qpre,Ppre,Qref,allowanceQ)
  \%Fitness (objective) function for the tap stagger
  count1=size(x);
  count2=length(testedtrans)/2;

  T33P=zeros(count1(1,1),1);
  T33Q=zeros(count1(1,1),1);
  for i=1:count1(1,1)
    \%tap stagger
    for j=1:count2
      DSSCircuit.Transformers.Name=[\'T\' int2str(testedtrans(2*j-1))]; \%e.g. the first one is from A_33_t11 to A_6_a
      DSSCircuit.Transformers.Wdg=1;
      DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*j-1))-(i,j)*tap(testedtrans(2*j-1),3)/100;
      DSSCircuit.Transformers.Name=[\'T\' int2str(testedtrans(2*j))]; \%e.g. the first one is from A_33_t12 to A_6_b
      DSSCircuit.Transformers.Wdg=1;
      DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*j))+x(i,j)*tap(testedtrans(2*j),3)/100;
    end
    DSSSolution.Solve;
    if DSSSolution.Converged
      MyElement=DSSCircuit.CktElements(\'Transformer.T33\');
      T33P(i,1)=DSSCircuit.Losses(1)/1.0e6-MyElement.Losses(1)/1.0e6;
      T33Q(i,1)=MyElement.Powers(10)*3/1.0e3;
    else
      disp(\'Not converged!\');
    end
    DSSText.Command=\'Init\';
  end

  \%assess the errors between Qabsorbed and Qref
  penaltyA = T33Q(:,1)-Qpre;
  penaltyA = abs(penaltyA-Qref)/Qref*100; \%error in percent(\%)
  for i=1:length(penaltyA)
    if penaltyA(i,1)<allowanceQ
      penaltyA(i,1)=0;
    else
      penaltyA(i,1)=penaltyA(i,1)-allowanceQ;
    end
  end

  \%assess the number of tappings used
  penaltyB = mean(x,2);

  \%assess the increased power losses
  penaltyC = (T33P(:,1)-Ppre)*1.0;
  scores = penaltyA + penaltyB + penaltyC;
```

Creation function

```matlab
function pop = create_taps(NVARS,options,Maxstagger)
  \%Creates a population of tap steps different from the initial positions (0 1 2).
  \% POP = CREATE_PERMUTATION(NVARS,FITNESSFCN,OPTIONS) creates a population
```
% of permutations POP each with a length of NVARS.
%
% The arguments to the function are
% NVARS: Number of variables
% OPTIONS: Options structure used by the GA

totalPopulationSize = sum(options.PopulationSize);
pop = randi([0,Maxstagger],totalPopulationSize,NVARS);

Crossover function

function xoverKids = crossover_tapsscattered(parents,options,NVARS, ...
    FitnessFcn,thisScore,thisPopulation)

CROSSOVERSCATTERED Custom crossover function for tap stagger.
% XOVERKIDS = CROSSOVER_PERMUTATION(PARENTS,OPTIONS,NVARS, ... 
% FITNESSFCN,THISSCORE,THISPOPULATION) crossovers PARENTS to produce 
% the children XOVERKIDS.
%
% The arguments to the function are
% PARENTS: Parents chosen by the selection function
% OPTIONS: Options structure created from GAOPTIMSET
% NVARS: Number of variables
% FITNESSFCN: Fitness function
% STATE: State structure used by the GA solver
% THISSCORE: Vector of scores of the current population
% THISPOPULATION: Matrix of individuals in the current population

nKids = length(parents)/2;

% Allocate space for the kids
xoverKids = zeros(nKids,NVARS);

% To move through the parents twice as fast as the kids are 
% being produced, a separate index for the parents is needed 
index = 1;
% for each kid...
for i=1:nKids
    % get parents
    r1 = parents(index);
    index = index + 1;
    r2 = parents(index);
    index = index + 1;
    % Randomly select half of the genes from each parent
    for j = 1:NVARS
        if(rand > 0.5)
            xoverKids(i,j) = thisPopulation(r1,j);
        else
            xoverKids(i,j) = thisPopulation(r2,j);
        end
    end
end
Mutation function

function mutationChildren = mutate_tapsmix(parents,options,NVARS, ...
    FitnessFcn, state, thisScore,thisPopulation,mutationRate)

    % MUTATE_TAPSMIX Custom mutation function for tap stagger.
    % MUTATIONCHILDREN = MUTATE_PERMUTATION(PARENTS,OPTIONS,NVARS, ...
    % FITNESSFCN,STATE,THISSCORE,THISPOPULATION,MUTATIONRATE) mutate the
    % PARENTS to produce mutated children MUTATIONCHILDREN.
    
    % The arguments to the function are
    % PARENTS: Parents chosen by the selection function
    % OPTIONS: Options structure created from GAOPTIMSET
    % NVARS: Number of variables
    % FITNESSFCN: Fitness function
    % STATE: State structure used by the GA solver
    % THISSCORE: Vector of scores of the current population
    % THISPOPULATION: Matrix of individuals in the current population
    % MUTATIONRATE: Rate of mutation
    
    if nargin < 8 || isempty(mutationRate)
        mutationRate = 0.5; % default mutation rate
    end
    
    mutationChildren = zeros(length(parents),NVARS);
    [~,index]=min(thisScore);
    elite=thisPopulation(index,:);
    reducedtappings=sum(elite)-1;
    %reducedtappings=a*2+b, deliberately allocate the genes as 2,2,...,1,...0
    a=floor(reducedtappings/2);
    b=rem(reducedtappings,2);
    %only numberofparents*mutationRate will be allocated genes as follows
    totalnumber=ceil(length(parents)*mutationRate);
    for i=1:totalnumber
        randomorder = randperm(NVARS);
        for j=1:NVARS
            if randomorder(j)<=a
                mutationChildren(i,j)=2;
            elseif randomorder(j)==a+1
                mutationChildren(i,j)=b;
            end
        end
    end
    % mutationChildren(totalnumber+1:end,:)=randi([0,1],length(parents)-totalnumber,NVARS);
    % the remainings use sum(elite) to allocate the genes as 2,2,...,1,...0
    a=floor(sum(elite)/2);
    b=rem(sum(elite),2);
    for i=totalnumber+1:length(parents)
        randomorder = randperm(NVARS);
        for j=1:NVARS
            if randomorder(j)<=a
                mutationChildren(i,j)=2;
            elseif randomorder(j)==a+1
                mutationChildren(i,j)=b;
            end
        end
    end
APPENDIX D

MATLAB Codes for the Rule-based Control Scheme

Main function

clear;
tic;
fstarting=fopen('startingposition.txt');
ftap=fopen('tapchanger.txt');
record=textscan(fstarting,'%s%c%c%c%f[^n]');
startingpos=cell2mat(record);
%record the MaxTap, MinTap and NumTaps
tap=ones(length(startingpos),3);
while ~feof(ftap)
    record=textscan(ftap,'%d');
    i=record{1};
    record=textscan(ftap,'%d%d%d%d%[^n]');
    tap(i,:)=cell2mat(record);
    record=textscan(ftap,'%s[^n]');
    %skip the next line
end
%change the NumTaps to TapStep(%)
tap(:,3)=round((tap(:,1)-tap(:,2))./tap(:,3)*10000)/100;
%Specify the tested primary transformers (A,B,C,D,E,F,G,H,I,J,K)
testedtrans=[31 30 20 21 22 24 25 28 29 26 27 7 6 13 14 11 10 9 8 5 4];
numberOfVariables=length(testedtrans)/2;
%Read the absorption capacity matrix (at 400kV side)
capmatrix=xlsread('capacitymatrix.xlsx','Additional Qloss');
capmatrixP=xlsread('capacitymatrix.xlsx','Additional Ploss');

%Specify the required MVAr absorption and the error allowance (%)
Qref=2.0;
allowanceQ=1.0;
Maxstagger=2;
%the maximum VAr could be provided at each staggering (among all the tested transformers)
Maxvar=zeros(2,Maxstagger);

Qactual=0.0;
Addvar=0;
Minerror=100;
Optimtap=zeros(1,numberOfVariables);
numofiterations=0;
while (Minerror>allowanceQ)
    %eliminate the used transformer pair
    if(Addvar>0)
        capavailable(Maxvar(2,staggerno),:)=0;
    end
    ...
%try to add VAr to Qactual and slowly accumulate to (1+allowancQ/100)*Qref
indices=find(capavailable>((1+allowanceQ/100)*Qref-Qactual));
capavailable(indices)=0;
%find the max VAr could be provided from 1,2,...staggering
for j=1:Maxstagger
    [Maxvar(1,j) Maxvar(2,j)]=max(capavailable(:,j)); %also recording the row number, which indicates
    %the transformer NO.
end
%add the maximum VAr/pair in order to reduce number of tappings (based on
VAr(2tappings)>4*VAr(1tapping))
    [Addvar,staggerno]=max(Maxvar(1,:));
    if(Addvar==0)
        break;
    end
    Optimtap(1,Maxvar(2,staggerno))=staggerno;
end
Qactual=Qactual+Addvar;
Minerror=abs(Qactual-Qref)/Qref*100;
umofiterations=numofiterations+1;
end
if (Minerror<=allowanceQ)
disp('A solution is found.');
else
    disp('A solution is NOT found.');
end
Resultcount=tabulate(Minerror);
totaltappings=sum(Optimtap,2)*2; %pairs of two transformers
% Ploss=Minobj-totaltappings/2;
fclose(fstarting);
close(ftap);
CalculatingTime=toc;

Function for calculating the VAr absorption capability matrix

clear;
[DSSStartOK, DSSObj, DSSText] = DSSStartup;

if DSSStartOK
    %Initialization
    fstarting=fopen('startingposition.txt');
    ftap=fopen('tapchanger.txt');
    record=textscan(fstarting,'%*s%c%c%c%c%f*[^n]');
    startingpos=cell2mat(record);
    %record the MaxTap, MinTap and NumTaps
    tap=ones(length(startingpos),3);
    i=1;
    while ~feof(ftap)
        record=textscan(ftap,'%s%c%d','delimiter','.
        tape=record{1};
        record=textscan(ftap,'%s%c%c%c%c%c%c%c%c%f*[^n]');
        tap(i,:)=cell2mat(record);
        record=textscan(ftap,'%s[^n]',1); %skip the next line
    end
    %change the NumTaps to TapStep(%)
}
tap(:,3)=round((tap(:,1)-tap(:,2))./tap(:,3)*10000)/100;

% set up the OpenDSS interface variables

currentpath=cd;
DSSText.command=['Compile (' currentpath 'Master.dss)'];
DSSCircuit=DSSObj.ActiveCircuit;
DSSSolution=DSSCircuit.Solution;
DSSText.Command='Set Normvmaxpu=1.06';
DSSText.Command='Set Normvminpu=0.94';
DSSText.Command='Set Maxiter=100';
DSSText.Command='Set Maxcontroliter=30';
DSSText.Command='Set tolerance=0.00001';
DSSText.Command='New Energymeter.m1 Transformer.T33 1';
DSSText.Command='New Monitor.PQ_m1 Transformer.T33 2 Mode=1';

% Specify the tested primary transformers (A,B,C,D,E,F,G,H,I,J,K)
testedtrans=[31 30 20 21 23 22 24 25 28 29 26 27 6 13 14 11 10 9 8 5 4];
numberOfVariables=length(testedtrans)/2;

% Calculate the initial reactive power flow in the network
for i=1:numberOfVariables
    DSSCircuit.Transformers.Name=['T' int2str(testedtrans(2*i-1))];
    DSSCircuit.Transformers.Wdg=1;
    DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*i-1));
    DSSCircuit.Transformers.Name=['T' int2str(testedtrans(2*i))];
    DSSCircuit.Transformers.Wdg=1;
    DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*i));
end
DSSSolution.Solve;
if DSSSolution.Converged
    MyElement=DSSCircuit.CktElements('Transformer.T33');
    Qpre=MyElement.Powers(10)*-3/1.0e3; % the output power from T33 (400/275kV) in MVAR
    Ppre=DSSCircuit.Losses(1)/1.0e6-MyElement.Losses(1)/1.0e6; % the distribution system loss in MW
else
    disp('Not converged!1');
end
DSSText.Command='Init';

Maxstagger=2; % the maximum tap steps increased/decreased
T33P=zeros(numberOfVariables,Maxstagger);
T33Q=zeros(numberOfVariables,Maxstagger);
for i=1:numberOfVariables
    for j=1:Maxstagger
        DSSCircuit.Transformers.Name=['T' int2str(testedtrans(2*i-1))];
        DSSCircuit.Transformers.Wdg=1;
        DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*i-1))-j*tap(testedtrans(2*i-1),3)/100;
        DSSCircuit.Transformers.Name=['T' int2str(testedtrans(2*i))];
        DSSCircuit.Transformers.Wdg=1;
        DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*i))+j*tap(testedtrans(2*i),3)/100;
        DSSSolution.Solve;
        if DSSSolution.Converged
            MyElement=DSSCircuit.CktElements('Transformer.T33');
            T33P(i,j)=DSSCircuit.Losses(1)/1.0e6-MyElement.Losses(1)/1.0e6;
        end
    end
end
T33Q(i,j)=MyElement.Powers(10)*-3/1.0e3;
else
disp('Not converged!2');
end
DSSText.Command='Init';
end

%Initialise the tap position of tested transformers before testing the next transformer
DSSCircuit.Transformers.Name=[T int2str(testedtrans(2*i-1))];
DSSCircuit.Transformers.Wdg=1;
DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*i-1));
DSSCircuit.Transformers.Name=[T int2str(testedtrans(2*i))];
DSSCircuit.Transformers.Wdg=1;
DSSCircuit.Transformers.Tap=startingpos(testedtrans(2*i));
end
APloss = T33P - Ppre;
AQloss = T33Q - Qpre;
xlswrite('capacitymatrix.xlsx',APloss,'Additional Ploss');
xlswrite('capacitymatrix.xlsx',AQloss,'Additional Qloss');
fclose(fstarting);
fclose(ftap);
else
disp('DSS Did Not Start!');
end

MATLAB Codes for the Branch-and-bound Solution Method

clear;
tic;
fstarting=fopen('startingposition.txt');
ftap=fopen('tapchanger.txt');
record=textscan(fstarting,'%*s%c%c%c%f*[^n]');
startingpos=cell2mat(record);
%record the MaxTap, MinTap and NumTaps
tap=ones(length(startingpos),3);
while ~feof(ftap)
    record=textscan(ftap,'%s%c%d',1,'delimiter',','); %record the NO. of transformer
    i=record{1};
    record=textscan(ftap,'%s%c%c%c%c%c%c%f%[^n],1); %record the MaxTap, MinTap and NumTaps
    tap(i,:)=cell2mat(record);
    record=textscan(ftap,'%s%[^n],1); %skip the next line
end
%change the NumTaps to TapStep(%)
tap(:,3)=round((tap(:,1)-tap(:,2))./tap(:,3)*10000)/100;

%Specify the tested primary transformers (A,B,C,D,E,F,G,H,I,J,K)
testedtrans=[31 30 20 21 22 24 25 28 29 26 27 6 13 14 11 10 9 8 5 4];
numberOfVariables=length(testedtrans)/2;
%Read the absorption capacity matrix (at 400kV side)
capmatrix=xlsread('capacitymatrix.xlsx','Additional Qloss');
capmatrixP = xlsread('capacitymatrix.xlsx', 'Additional Ploss');

% haha = zeros(numberOfVariables, 1);
% capmatrix = [capmatrix haha];
% capmatrixP = [capmatrixP haha];

% Specify the required MVAr absorption and the error allowance (%)
Qref = 1.0;
allowanceQ = 1.0;
Maxstagger = 2;

%x (vector): every two bits represent one pair of transformer, 00 for 0, 10 for 1, 01 for 2
% lower bound & upper bound
lb = zeros(numberOfVariables * Maxstagger, 1);
ub = ones(numberOfVariables * Maxstagger, 1);

%f vector for objective = f'x (total tappings needed & Ploss introduced)
% e.g. f1 = [1; 2; 1; 2; 1; 2; ...; 1; 2]
for i = 2:Maxstagger
    for j = i:Maxstagger:length(f1)
        f1(j, 1) = f1(j, 1) + i - 1;
    end
end

%f2 to represent the additional power losses based on capmatrixP
f2 = zeros(numberOfVariables * Maxstagger, 1);
k = 1;
for i = 1:numberOfVariables
    for j = 1:Maxstagger
        f2(k, 1) = capmatrixP(i, j); k = k + 1;
    end
end
f = f1 + f2;

%A matrix for linear constraint Ax <= b,
%A1 for abs(Qactual - Qref) / Qref * 100 <= allowanceQ
A1 = zeros(2, numberOfVariables * Maxstagger);
k = 1;
for i = 1:numberOfVariables
    for j = 1:Maxstagger
        A1(1, k) = capmatrix(i, j); A1(2, k) = -1 * A1(1, k);
        k = k + 1;
    end
end

%A2 for tappings on each transformer <= Maxstagger (upper limit)
%A3 for OLTC on each transformer should be within limits
% tapcount for the total tappings used on each transformer pair
A2 = zeros(numberOfVariables, numberOfVariables * Maxstagger);
A3 = A2;
tapcount = zeros(numberOfVariables, numberOfVariables * Maxstagger);
for i = 1:numberOfVariables
    for j = 1:Maxstagger
        A2(i, Maxstagger * (i - 1) + j) = 1;
        A3(i, Maxstagger * (i - 1) + j) = tap(testedtrans(2 * i - 1), 3) / 100;
    end
end
A = [A1; A2; A3];
%b matrix for linear constraint Ax<=b
b1=[Qref*(1+allowanceQ/100);-1*Qref*(1-allowanceQ/100)];
b2=ones(numberOfVariables,1); %1,2,3,...,Maxstagger can only occur once
b3=zeros(numberOfVariables,1);
for i=1:numberOfVariables
    gaptoupper=tap(testedtrans(2*i-1),1)-startingpos(testedtrans(2*i-1)); %max-startingpos
    gaptolower=-1*tap(testedtrans(2*i),2)+startingpos(testedtrans(2*i)); %min+startingpos
    b3(i,1)=min(gaptoupper,gaptolower);
end
b=[b1;b2;b3];

%starting points for the optimisation
x0=ub;
x0=lb;

% bintprog options
options = optimset('bintprog');
options = optimset(options,'Diagnostics','off','Display','iter','TolFun',1.0e-6,'MaxNodes',10000*numberOfVariables*Maxstagger);

Runtimes=1;
Minobj=zeros(Runtimes,1); %value of objective function
Minerror=zeros(Runtimes,1); %percentage error between Qactual and Qref
Optimtap1=zeros(Runtimes,numberOfVariables*Maxstagger);
Optimtap=zeros(Runtimes,numberOfVariables);
for i=1:Runtimes
    disp(['Iteration No.' num2str(i) ' starts']);
    [x,fval,exitflag,output] = bintprog(f,A,b,[],[],[],options);
    if exitflag>0
        Minobj(i,1)=fval;
        Optimtap1(i,:)=x';
        tapaction=A*x;
        Optimtap(i,:)=(tapcount*x');
        Minerror(i,1)=abs(tapaction(1,1)-Qref)/Qref*100;
    end
    disp(['Iteration No.' num2str(i) ', ' output.message]);
end
Resultcount=tabulate(Minerror);
totaltappings=sum(Optimtap,2)*2; %pairs of two transformers
Ploss=Minobj-totaltappings/2;
fclose(fstarting);
fclose(ftap);
CalculatingTime=toc;
APPENDIX E

222-Bus Distribution Network Model

The 222-bus distribution network model is illustrated in Figure B. This network is part of the large distribution system shown in Figure A. It includes a 132kV GSP and the entire downstream 33kV networks connected to the GSP. The 222-bus network model has first been extracted from the original IPSA network model shown in Figure A and then it has been converted to OpenDSS circuit description files.

Due to confidentiality agreements, Table B can only list some basic data of the 222-bus distribution system model.

| Table B: Quantities of different network components of the 222-bus model |
|---------------------------|-----------------|
| **Buses**                | **Quantity**    |
| 400kV:                    | 1               |
| 275kV:                    | 5               |
| 132kV:                    | 38              |
| 33kV:                     | 120             |
| 11kV:                     | 18              |
| 6.6kV:                    | 40              |
| **Lines**                | 192             |
| **Transformers**         | 76              |
| **Static Loads**         | 66 (with a total of 427 MW and 127MVAr) |
| (constant load model)    |                 |
| **GSPs**                 | 1               |
| **33kV Load Areas**      | 6               |
| **Primary Substations**  | 28              |
Figure B: 222-bus distribution network model simulated in IPSA+ V1.6.8