DESIGN AND PERFORMANCE ANALYSIS OF COOPERATIVE RELAY SYSTEMS

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Engineering and Physical Sciences

2015

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\[
Q_1(a,b) = \int_0^\infty x \exp \left( \frac{x^2+a^2}{2} \right) I_0(ax) \, dx.
\]
For Nakagami-\( m \) fading, \( m \) is the fading parameter in range of \( \frac{1}{2} \leq m \leq \infty \), \( \Gamma(a,x) \) is the incomplete gamma function defined as
\[
\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t} \, dt,
\]
and \( \Gamma(a) \) is the gamma function.

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Abstract

Cooperative relay systems have emerged as promising techniques to boost the performance of wireless systems. Recent studies have confirmed that interferences, co-channel interferences (CCIs) and self-interferences, have a huge impact on cooperative relay systems and can cause significant performance degradation. Two problems were observed in this research. Firstly, previous studies on performance analysis of Amplify-and-Forward (AF) relay systems in presence of CCIs have only focused on a specific interference channel model. However, in practical design scenarios, such an assumption is not a realistic proposition. Secondly, analyses of overheads introduced by a time-based relay selection protocol in distributed cooperative systems have been based on an over-pessimistic assumption where all packets involved in a collision are destroyed. Nevertheless, collisions due to the protocol overheads cause the system performance to be degraded but this does not mean that the failure of end-to-end transmission certainly occurs.

The thesis aims to analyse the performance of practical cooperative relay systems in the presence of CCIs and self-interferences, by developing exact mathematical methods. A new unified mathematical method for AF relay systems in presence of a random number of arbitrary non-identical CCIs was developed. The obtained new approach derived in terms of a moment generating function of the aggregate interferences’ power led to the derivation of new explicit expressions. The new results greatly simplify evaluation of average error rates over diverse practical interference scenarios. Moreover, a new exact mathematical analysis for distributed cooperative relay systems employing a time-based relay selection protocol based on an accurate interference model was presented. This approach led to the derivation of new exact expressions for the spectral efficiency which accounts for both self-interferences.
and the protocol overheads as well as for different fading scenarios and arbitrary relay locations. This approach provided several advantages over direct approaches, one of which is that it significantly simplified averaging-out the joint random variables involved.
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Acknowledgements

I would like to express my deepest gratitude to my supervisor Dr. Khairi Ashour Hamdi for his guidance, encouragement, in-depth knowledge, and kind support throughout this journey.

Sincere appreciation also to my colleagues, past and present at the research group and in particular Dr. Maksims Abalenkov, Dr. Abubakar Makarfi, Dr. Azwan Mahmood, Dr. Fahimeh Jasbi and Dr. Mohammad Robat Mili. You have helped, encouraged, cheered me from time to time and your acquaintance gave me a rich and rewarding experience as a PhD student.

I would like to thank my dearest friends Cintia, Pablo, Zahra, and Daniel for their support and friendship outside the academic world.

I would also like to thank my sister and best friend, Ayda, for sharing her experience as a former doctorate researcher who constantly inspired me academically, emotionally during this long and tedious process. Also my beloved brother Ghaflan for his kindness and support.

My gratitude also goes to my dear husband Farshad who is always believes in me, being my side, supporting and motivating me throughout difficult times.

Last but not least, I do not have enough words to express my deep and sincere appreciation to my loving parents, Mohammad and Mahedeh, for their guidance, support, encouragement and confidence in me throughout all my life and education.
List of Abbreviations

ADC    Analog-to-Digital Convertor
AWGN   Additive White Gaussian Noise
AF     Amplify-and-Forward
BER    Bit Error Rate
BPSK   Binary Phase-Shift Keying
BS     Base Station
CCI    Co-Channel Interference
CDF    Cumulative Distribution Function
CRS    Cooperative Relay Systems
CSI    Channel State Information
CSMA   Carrier Sense Multiple Access
CTS    Clear-to-Send
DAC    Digital-to-Analog Convertor
DF     Decode-and-Forward
EC     Energy Consumption per Information Bit
EE     Energy Efficiency
FDR    Full-Duplex Relay
HDR    Half-Duplex Relay
i.i.d.  Independent and Identically Distributed
ICSI   Instantaneous Channel State Information
<table>
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<tr>
<td>ICT</td>
<td>Information and Communication Technology</td>
</tr>
<tr>
<td>IFA</td>
<td>Intermediate Frequency Amplifier</td>
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<tr>
<td>LNA</td>
<td>Low Noise Amplifier</td>
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<tr>
<td>LOS</td>
<td>Line of Sight</td>
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<tr>
<td>MGF</td>
<td>Moment Generating Function</td>
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<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>MPSK</td>
<td>M-ary Phase Shift Keying</td>
</tr>
<tr>
<td>MQAM</td>
<td>M-ary Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>MS</td>
<td>Mobile Station</td>
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<tr>
<td>NACK</td>
<td>Negative Acknowledgment</td>
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<tr>
<td>OR</td>
<td>Opportunistic Relaying</td>
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<tr>
<td>OWR</td>
<td>One-Way Relay</td>
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<tr>
<td>PA</td>
<td>Power Amplifier</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>PPP</td>
<td>Poisson Point Process</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
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<tr>
<td>RS</td>
<td>Relay Selection</td>
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<tr>
<td>RTS</td>
<td>Request-to-Send</td>
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<tr>
<td>RV</td>
<td>Random Variable</td>
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<tr>
<td>SC</td>
<td>Selection Cooperation</td>
</tr>
<tr>
<td>SE</td>
<td>Spectral Efficiency</td>
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<tr>
<td>SER</td>
<td>Symbol Error Rate</td>
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<tr>
<td>SINR</td>
<td>Signal-to-Interference-and-Noise Ratio</td>
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<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<td>TWR</td>
<td>Two-Way Relay</td>
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List of Variables

$\beta$  The path loss exponent
$r$    The distance between transmitter and receiver
$N_0$  The power spectral density of AWGN
$g$    The complex channel gain for source-relay
$f$    The complex channel gain for relay-source
$h_I$  The residual loop interference channel
$\bar{\gamma}$  The average SNR or SNIR
$m$    The Nakagami-m fading parameter
$K$    The Rician factor
$\sigma$  The standard deviation
$N_p$  The order Hermite polynomial
$\kappa$  The information bits
$T$    The end-to-end transmission period
$P_t$  The transmission power
$P_n$  The noise power
$P_r$  The received power
<table>
<thead>
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<th>Variable</th>
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<td>$P_c$</td>
<td>The circuit power</td>
</tr>
<tr>
<td>$T_c$</td>
<td>The useful data transmission period</td>
</tr>
<tr>
<td>$\gamma_{th}$</td>
<td>The threshold SNR/SNIR</td>
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<tr>
<td>$d$</td>
<td>The modulation constant</td>
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<td>$\Delta$</td>
<td>The guard interval</td>
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<td>$L_s$</td>
<td>The number of candidate relays</td>
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<td>$C$</td>
<td>The average spectral efficiency in bits/s/Hz</td>
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<tr>
<td>$D$</td>
<td>The reuse distance in cellular</td>
</tr>
<tr>
<td>$G$</td>
<td>The gain for AF relay</td>
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<tr>
<td>$R_s$</td>
<td>The radius of a cellular cell</td>
</tr>
<tr>
<td>$p_s$</td>
<td>The success probability of Bernoulli random variables</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>The mean-square value of received power</td>
</tr>
<tr>
<td>$N$</td>
<td>The number of relay nodes</td>
</tr>
<tr>
<td>$L$</td>
<td>The number of interferers</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>The system parameter of time-based protocol</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>The Poisson process parameter</td>
</tr>
<tr>
<td>$EC$</td>
<td>The energy consumption per unit of information bits</td>
</tr>
<tr>
<td>$R_r$</td>
<td>The data rate</td>
</tr>
<tr>
<td>$B$</td>
<td>The bandwidth</td>
</tr>
<tr>
<td>$t_{\text{min}}$</td>
<td>The best relay’s time</td>
</tr>
<tr>
<td>$T_s$</td>
<td>The time selection period</td>
</tr>
</tbody>
</table>
\( p_h \)  The probability of hidden nodes
\( \xi \)  The collision indicator function
\( \mu \)  The mean of the distribution
\( \mathcal{H} \)  The set of candidate relays
\( T_{tr} \)  The transient mode period
\( T_{sp} \)  The sleep mode period
\( P_{tr} \)  The power consumption for transient mode
\( P_{sp} \)  The power consumption for sleep mode
\( P_{mix} \)  The mixer power consumption
\( P_{sy} \)  The frequency synthesiser power consumption
\( P_{DAC} \)  The DAC power consumption
\( P_{filt} \)  The active filter consumption
\( P_{LNA} \)  The LNA power consumption
\( P_{IFA} \)  The IFA power consumption
\( M \)  The level of modulation
\( \kappa_s \)  The number of symbols
\( T_{sym} \)  The symbol period
\( M_l \)  The link margin
\( G_t \)  The transmitter antenna gain
\( G_r \)  The receiver antenna gain
\( \Lambda \)  The wavelength
ε The peak to average radio
List of Mathematical Notations

\( \exp(x) \) \hspace{1cm} \text{Exponential Function } e^x \\
\text{erfc}(.) \hspace{1cm} \text{Complementary Error Function} \\
\mathbb{E}[.] \hspace{1cm} \text{Expectation of a Random Variable} \\
\Gamma(.) \hspace{1cm} \text{Gamma Function} \\
\Gamma(.,.) \hspace{1cm} \text{Upper Incomplete Gamma Function} \\
\mathcal{M}(.) \hspace{1cm} \text{Moment Generating Function} \\
I_v(.) \hspace{1cm} v\text{th Order Modified Bessel Function of the First Kind} \\
\log_x(.) \hspace{1cm} \text{Logarithmic Function to Base } x \\
\mathcal{N}(\mu,\sigma^2) \hspace{1cm} \text{Normal Distribution with Mean } \mu \text{ and Variance } \sigma^2 \\
\Pr(x) \hspace{1cm} \text{Probability of } x \\
Q(.) \hspace{1cm} \text{Gaussian Q-function} \\
\text{erfc}^2(.) \hspace{1cm} \text{Square complementary error function function} \\
Q^2(x) \hspace{1cm} \text{Square Gaussian Q-function}
Chapter 1

Introduction

1.1 Background

COOPERATIVE relaying system has emerged as a promising technique to enhance coverage area, reliability, and throughput of wireless communication systems. Although three-terminal relay channels have been introduced in 1970s [2], its real benefits to wireless research community came recently [3] following significant advances in signal processing such as diversity. In the cooperative relay system, due to broadcast nature of wireless transmission, some nodes in the network might overhear the transmitted signal from the source. If the direct communication between the source and the destination fails, for example due to channel variations, those nodes that have a copy of the transmitted signal can help to retransmit the source signal toward the destination. The nodes that participate in the transmission are called relays. The relay nodes can operate mainly in two different modes: either digital mode, e.g. Decode-and-Forward (DF), or analogue mode, e.g. Amplify-and-Forward (AF). In a DF scheme, relays first decode and re-encode the received signal from the source then forward it to the destination. However, in an AF scheme, relays simply amplify the received signal and then forward it to the receiver.

Interferences have a huge impact on a wireless system’s performance and cause performance degradation. A cooperative relay system can suffer from co-channel interferences (CCIs) if devices in the neighbourhood transmit at the same time and on the same frequency channel. Besides CCIs, the cooperative relay systems can be also affected by self-interferences. Evaluating the system performance with respect to interferences have been studied in
literature but nevertheless, there remains some major unsolved problems. In this thesis, several new unified mathematical methods are proposed to analyse the performance of cooperative relay systems in presence of interferences. These proposed mathematical models provide solutions relevant for practical implementations.

1.2 Motivation of this Research

The majority of prior studies on CCIs effects on the performance of dual-hop AF relay systems, treated the problem while ignoring the impact of thermal noise \cite{4-11}. There are only limited works considered all thermal noise. Additionally, in all previous related published works, the analyses are limited to a specific interference channel model in which the system is subjected to a given number of homogenous and identically distributed interferers. However, in a practical design, the interferers’ signal can experience different attenuation and the number of interfering signals can be a random variable (RV). As there has been no research considering arbitrary CCIs fading models, this thesis analyses the performance of a dual-hop system in a more general scenario that is close to practical design.

When relays cooperate, multiple paths for transmitting the source signal become accessible. Transmission through multiple paths known as spatial diversity improves quality and reliability of the system. However, benefit of the multiple relay cooperative systems can come at the expense of reduction in energy efficiency (EE) as by having several relay nodes, extra power circuitry can be used by the relaying nodes. Therefore, the energy efficiency of the system can be degraded by increasing the number of relay nodes. Moreover, the benefits of the system come at the expense of a reduction in spectral efficiency (SE) as the relays must transmit on orthogonal channels in order to avoid interfering with the source node and with each other. The problem of inefficiencies, EE and SE, can be excluded with a distributed single relay selection method employing time-based protocols, selection cooperation protocol (SC) and opportunistic relay (OR) protocol.

Time-based relay selection protocols in a cooperative relay systems are prone to self-interferences when relay nodes are hidden from each other.
In practice, additional interferences may also arise due to overheads introduced by the protocols. All previous research have been based on the over-pessimistic assumption that all packets involved in a collision are destroyed [12–15]. However, collisions due to the protocol overheads mainly cause the system performance to be degraded and this does not mean that the failure of end-to-end transmission surely happens. As a result, to achieve an exact performance and more realistic results, it is necessary to evaluate and analyse the distributed cooperative system performance based on an accurate interference model.

Although, unidirectional systems (also known as one-way relay (OWR) cooperative systems) have been studied in the literature, there is a lot of interest on two-way relay (TWR) and full-duplex relay (FDR) cooperative systems because they significantly recover the spectral loss in the unidirectional systems. Recently, applying a relay selection scheme to TWR and FDR relay systems has been the focus of attention, as the systems can provide better efficiency in terms of SE. However, there are very limited related works and as such, in need of further investigation.

Due to exponentially increasing energy consumption of wireless mobile networks, minimizing energy consumption for data transmission becomes one of the most important design considerations for cooperative relay systems. Although both OR and SC protocols are able to reduce the required transmission energy for a successful data transmission but they can consume different levels of energy. Therefore, energy consumption is an appropriate metric for performance evaluation and comparison between OR and SC protocols. The energy efficiency of SC protocols have been investigated in [16]. However, it is not clear whether applying an OR or SC protocol is better in terms of energy consumption. Moreover, in [16] and other previous works, authors considered constant-power transmission in their analysis while in a practical design, power consumption of a cooperative system can vary depending on the information bits.
1.3 Aim and Objectives

The aim of this study is to analyse the performance of practical cooperative relay systems in presence of interferences, CCIs and self-interferences, by introducing new exact and unified analytical approaches. To achieve this aim, the main objectives of this thesis are:

- To develop a new exact mathematical analysis for AF relay systems in presence of a random number of arbitrary non-identical CCIs in order to examine cooperative systems that are close to practical designs.

- To develop a new unified mathematical model for analysing the performance of distributed cooperative systems employing a time-based relay selection protocol. The exact unified expressions are derived for the spectral efficiency based on accurate interference models that accounts for both self-interferences and the overheads introduced by the protocol.

- To analyse the spectral efficiency of distributed two-way relay and full-duplex relay cooperative systems by employing a time-based relay selection protocol.

- To evaluate and compare energy consumption of two proposed time-based relay selection protocols, OR and SC protocols, in distributed cooperative relay systems.

1.4 Key Contributions

The main points of contribution are highlighted in what follows. However, it is worth mentioning that part of the contributions outlined below have been published in conference proceedings, details of which have been outlined in Sec. 1.6. The key contributions are:

- Exact performance analysis of dual-hop AF relay cooperative systems in the presence of a random number of arbitrary non-identical CCIs taking into account the effects of thermal noise at both relay and destination. A new exact mathematical method is developed that leads to the derivation of new explicit expressions in terms of the moment generating function (MGF) of the aggregate interferers’ power. The approach
greatly simplifies performance evaluation of the AF relay systems over diverse practical interference fading models and are examined for different scenarios:

- When useful signals are transmitted over Rayleigh fading channels.
- When useful signals are transmitted over Nakagami-$m$ fading channels.
- When useful signals are transmitted over a Rician and a Nakagami-$m$ fading channel.
- When useful signals are transmitted over composite Nakagami-$m$/log-normal shadowing fading channels.
- When a destination node is in the presence of a Poisson field of interferers.

- Derivation for the average of two common functions. The complementary error function, $\text{erfc}\left(\sqrt{\gamma}\right)$, and the square of the complementary error function, $\text{erfc}^2\left(\sqrt{\gamma}\right)$, based on the new developed analytical approach for the AF relay system in the presence of arbitrary CCIs.

- Exact average error rate analysis of an AF relay system over diverse practical interference fading models for some common digital modulation techniques including
  - M-ary phase shift keying modulation (MPSK).
  - M-ary quadrature amplitude modulation (MQAM).

- Exact performance analysis of distributed cooperative relay systems employing time-based relay selection protocols. To analyse the system performance based on an accurate interference model a new accurate mathematical analysis is presented which leads to derive new exact expressions for the average spectral efficiency. The benefits of this approach include:
  - The new exact expressions deriving by this approach are based on accurate interference models that account for both self-interferences and the protocol overheads as well as different fading scenarios and arbitrary relays’ locations. The explicit expressions are also examined for two different cases; identical and non-identical RVs.
– The approach significantly simplifies the difficult task of obtaining the exact average spectral efficiency in presence of interferences. Direct methods for averaging the RVs will require computing several integral operations with a knowledge of probability density function (PDF) of the RVs. This analysis can be further complicated when the aggregate random interferences are dependent.

– Furthermore, when using direct methods, a very complicated derivation may occur when joint impact of fading and path-loss model $r^{-\beta}$ are considered, where $r$ is the random distance between a transmitter and a receiver and $\beta$ the path-loss exponent. The new mathematical solution proposed in this thesis aids this analysis.

• Derivation of the spectral efficiency bound for a distributed cooperative system employing a time-based relay selection protocol by utilizing Jensen’s inequality. The approach is then extended for TWR and FDR cooperative systems as there are no related works.

• Performance evaluation and comparison between OR and SC protocols in terms of energy consumption. The energy consumption of the protocols are investigated by calculating the transmission power required to send a given number of bits in an MQAM modulation. In order to estimate the total energy, the power of all signal processing at the transmitter and the receiver sides are considered and evaluated for the system operating in multimode.

• Minimizing the total energy consumption required to send a given number of bits in a distributed cooperative relay system by optimizing transmission power and transmission time. By concave-convex optimization, a large amount of energy saving is achievable.

1.5 Structure of Thesis

The rest of this thesis is organised as follows.

Chapter 2 provides relevant theoretical backgrounds in a wireless communication system to review the concepts and theories that will be employed later in this thesis.
Chapter 3 presents a literature review on cooperative relay systems covering the current and key debates. The dominant drive is to outline some major features of cooperative relay systems and relevant analytical techniques to help the reader get a broader perspective on the subject matter of this thesis.

Chapter 4 presents a new exact mathematical analysis of dual-hop fixed-gain AF relay systems in the presence a random number of arbitrary non-identical CCIs taking into account the effects of thermal noise at both relay and destination. For this a non-direct mathematical method is developed leading to the derivation of a new explicit expression for the MGF of SINR. This result then leads to derivation of exact and simple expressions for the average error rates of the AF relay system. Moreover, in this chapter the non-direct mathematical approach is extended for different fading models involving Nakagami-$m$, Rician and composite Nakagami-$m$/log-normal fading channels. The analysis in this chapter concluded by giving a practical example where numerical results are given for bit error rates of a cooperative relay system in heterogeneous cellular networks in the presence of arbitrary and a Poisson field of interferers. Simulation results are provided to demonstrate the accuracy of the new analytical expressions.

Chapter 5 presents a new accurate mathematical analysis on the performance of a time-based relay selection protocol in a distributed cooperative relay system. The new exact unified expressions are derived for the SE based on an accurate interference model. These are used to study the trade-off between the extra protocol overheads required to reduce the amount of the self-interference and the overall system throughput as measured in terms of the SE. The explicit expressions are also presented for two different cases, when RVs are identical and when RVs are non-identical. Additionally in this chapter, bounds for the SE are derived utilising Jensen’s inequality and compared with the developed exact results. The accuracy of the new expressions are validated by Monte-Carlo simulation.

Chapter 6 analyses the SE of two-way relay and full-duplex
relay systems by employing a time-based relay selection protocol based on the developed approach in Chapter 5.

Chapter 7 investigates energy consumption of two proposed timer-based relay selection protocols, OR and SC, by considering the best modulation strategy, MQAM, in terms of green modulation. Additionally, Dinkelbach’s concave-convex algorithm is used to minimize the total energy consumption required to send a given number of bits by optimizing transmission power and transmission time.

Chapter 8 concludes the thesis and considers prospects for the future extension of the work are outlined.
1.6 Publications


Chapter 2

Theoretical Background

2.1 Introduction

Wireless communication systems are the fastest growing technologies in communications industry over the past few decades due to the large demand for mobile access. This chapter covers relevant theoretical backgrounds in a cooperative relay system to review concepts and theories which will be employed later in this thesis. In Sec. 2.2, a brief quantitative description of the main characteristics of wireless channels is presented. In Sec. 2.3, other important theories related to this thesis including error rates and outage probability analysis are discussed.

2.2 Characteristics of Wireless Channels

Propagation environment in wireless communication is very dynamic compared to that in wired communication because of objects within the environment. Furthermore, propagation characteristics change significantly from environment to environment due to the characteristic nature of wave propagation. Therefore, compared with conventional wireline communications, the signal transmitted over wireless channels suffer from i.e., attenuation, phase-shift, and delay. These non-ideal effects are often characterised by three determinants, i.e., path-loss, small-scale fading and large-scale fading [17]. In the next section, a brief description of the characteristics of wireless channels and their impacts on the performance of wireless communication systems are
2.2.1 Deterministic Path-Loss

The strength of the signal attenuates as it travels over wireless channels and, thus it becomes weaker as the propagation distance increases. The amount of reduction in the signal strength with respect to the distance can be characterised by the product of a distance component. A common model is the general inverse power law path-loss model given by \[ P_r \propto r^{-\beta} P_t \] where \( P_r \) and \( P_t \) are the received and transmitted signal power, respectively, \( r \) is the transmitter-receiver separation distance, and \( \beta \) is the path-loss exponent that show how fast the path-loss increases with the transmitter-receiver separation \[19\].

2.2.2 Small-Scale Fading

Wireless channels suffer from attenuation due to diffraction, reflection and scattering, etc. This is because a line of sight (LOS) path between the transmitter and receiver is highly improbable and signals comes from scattering and reflection from various objects such as hills, large buildings, trees, etc. This causes numerous versions of the transmitted signal to arrive at the receiver through different propagation paths at different times \[19\]. This phenomenon of random fluctuations in the received signal is termed as fading. In small scale fading the distinct multipath components have random phases and amplitudes leading to rapid deviations in signal strength, distortion of received signal and loss of data. This form of multipath fading is relatively fast. Depending on the nature of the radio propagation environments, there are different models describing the statistical behavior of the multipath fading envelope. The most common types are Rayleigh fading, Nakagami-\(m\) fading and Rician fading.
2.2.2.1 Rayleigh Fading

Rayleigh fading is a model that can be used to describe a form of fading that occurs when there is no LOS [20]. When there is a large number of paths, where all paths are independent and identically distributed (i.i.d.) circularly symmetric Gaussian variables, the mean of the large number of independent RVs will be approximately Gaussian distributed when the central limit theorem is applied [21]. The central limit theorem states that the distribution of the sum (or average) of a large number of i.i.d. variables will be approximately normal, regardless of the original distribution. The magnitude of normal distribution is Rayleigh distribution thus the channel fading amplitude \( X \) is

\[
f_X (x) = \frac{2x}{\Omega} e^{-\frac{x^2}{\Omega}} \quad x \geq 0
\]

where \( \Omega = \mathbb{E} [X^2] \) and \( \mathbb{E} [.] \) denotes the expectation operator.

In case of communication over fading in absence of interferences, the instantaneous SNR given by \( \gamma = X^2 P_t / N_0 \) is a RV with PDF of \( f(\gamma) \) and average SNR of \( \overline{\gamma} = \Omega P_t / N_0 \), where RV \( X^2 \) is the instantaneous signal power and \( N_0 \) (W/Hz) is the one-sided power spectral density of additive white Gaussian noise (AWGN). The \( f(\gamma) \) is distributed according to an exponential distribution as

\[
f_\gamma (\gamma) = \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}} \quad \gamma \geq 0
\]

where \( \overline{\gamma} = \mathbb{E} [\gamma] \).

The MGF corresponding to this fading model (2.3) is given by (See Appendix A.1 for the definition of MGF)

\[
\mathcal{M}_\gamma (z) = \mathbb{E} [e^{-z\gamma}] = \int_0^\infty e^{-z\gamma} f_\gamma (\gamma) d\gamma = \frac{1}{1 + z\overline{\gamma}}.
\]

2.2.2.2 Generating of Rayleigh RVs

Consider symmetric complex Gaussian RV, \( Z = a + jb \), where \( a \) and \( b \) are Gaussian distributed with \( N(0, \sigma^2) \). Magnitude \( Z, |Z| \), is Rayleigh faded and \( |Z|^2 \) has exponential distribution. In Matlab “randn” function generates unit Gaussian random numbers \( N(0, 1) \) with mean \( \mu = 0 \) and variance \( \sigma^2 = 1 \). To
generate $Z$ component with $\mu$ and $\sigma$, the output of “randn” has to be multiplied with $\sigma$ and added with $\mu$. Therefore, $Z = \mu + \sigma \cdot \text{"randn"} + j \cdot \text{"randn"}$.

2.2.2.3 Nakagami-$m$ Fading

Nakagami-$m$ fading often applies to land-mobile, scintillating ionospheric and indoor-mobile links [18]. The PDF of Nakagami-$m$ is given by [18]

$$f_X(x) = \frac{2mm^2m-1}{\Omega^m \Gamma(m)} e^{-\frac{mx^2}{\Omega}} x \geq 0 \quad (2.5)$$

where $m$ is the Nakagami-$m$ fading parameter in range of $\frac{1}{2} \leq m \leq \infty$ and $\Gamma(.)$ is the gamma function defined as [22]

$$\Gamma(m) = \int_0^\infty x^{m-1} e^{-x} dx. \quad (2.6)$$

In the special case $m = 1$, Rayleigh fading is recovered from (2.5). For $m > 1$, the fluctuations of the signal strength reduce compared to Rayleigh fading.

The corresponding instantaneous power of Nakagami-$m$ fading is gamma distributed, given by [18]

$$f_\gamma(\gamma) = \frac{m^{m\gamma}m-1}{\gamma^m \Gamma(m)} e^{-\frac{m\gamma}{\gamma}} \quad \gamma \geq 0. \quad (2.7)$$

The MGF corresponding to the fading model in (2.7) is given by [18]

$$M_\gamma(z) = \left( \frac{1}{1 + \frac{z^m}{m}} \right)^m. \quad (2.8)$$

2.2.2.4 Generating of Nakagami-$m$ RVs

The Chi-square distribution is the distribution of the sum of squared standard normal variables $N(0, 1)$. Therefore if $X_i$ have normal distributions with mean 0 and variance 1, then $Y_n^2 = \sum_{i=1}^{n} X_i^2$ is distributed as $Y_n^2$ with $n$ degree of freedom and PDF of $f_{Y_n^2}(x) = \frac{1}{2^n \Gamma\left(\frac{n}{2}\right)} x^{\frac{n}{2} - 1} e^{-\frac{x}{2}}$. This makes a $Y_n^2$ distribution
2.2.2.5 Rician Fading

Rician fading is also known as Nakagami-\(n\) fading often used to describe a propagation model that consists of one strong LOS component and many weaker random components. This type of fading typically applies to microcellular urban, suburban land-mobile and picocellular indoor, as well as satellite where a dominant LOS path exists [18]. The channel fading amplitude follows the distribution

\[
fx(x) = \frac{2(1+K)}{\Omega} e^{-Kx} x^{-(1+K)x^2} I_0 \left( 2nx \sqrt{\frac{1+K}{\Omega}} \right) \quad x \geq 0
\]

where \(K\) is the Rician factor and \(I_0(.)\) is the zeroth-order modified Bessel function of the first kind given by [22, Sec. 8.43]

\[
I_0(z) = \sum_{k=0}^{\infty} \left( \frac{z}{2} \right)^k \frac{(k!)^2}{k!}
\]

The corresponding instantaneous power of Rician Fading is given by [18]

\[
f_\gamma(\gamma) = \frac{(1+K)}{\gamma} e^{-K} e^{-\frac{(1+K)\gamma}{\gamma}} I_0 \left( 2 \frac{K(1+K)\gamma}{\sqrt{\gamma}} \right) \quad \gamma \geq 0.
\]

The MGF associated with this fading model is given by [18]

\[
M_\gamma(z) = \frac{1+K}{1+K+z^\gamma} e^{-\frac{Kz^\gamma}{1+K+z^\gamma}}.
\]
the power in direct path $\mu^2$, and power of scattered paths $2\sigma^2$, thus $K = \frac{\mu^2}{2\sigma^2}$. Moreover, $\Omega$ is the total power from both direct and scattered paths, thus, $\Omega = \mu^2 + 2\sigma^2$. Consequently, $\mu$ and $\sigma$ can easily be obtained as $\mu = \sqrt{\Omega K / (K + 1)}$ and $\sigma = \sqrt{\Omega / 2 (K + 1)}$. Thus complex Gaussian RVs is obtained by

$$Z = \sqrt{\frac{\Omega K}{K + 1}} + \sqrt{\frac{\Omega}{2 (K + 1)}} (a + jb)$$

(2.13)

where $|Z|$ gives the Rician distribution and $|Z|^2$ gives the non-central Chi-square. The simulation strategy in Matlab is very similar to the technique used for simulating Rayleigh Fading. The difference here is that in Rician, $Z$ has to be RV with $\mu$ and $\sigma$. Therefore, $Z = \mu + \sigma (\text{"randn"} + j\text{"randn"})$.

2.2.3 Large-Scale Fading

In addition to the impact of multipath fading, the quality of the signal can be affected by shadowing. Shadowing refers to the additional attenuation of signal power due to obstacles in between the transmitter and receiver, where the location and effects of those obstacles are unpredictable [23].

2.2.3.1 Log-normal Shadowing

In addition to path-loss, which is caused by spatial propagation of waves, shadowing is used to model the signal power when the channel is different at different locations, given the same transmitter-receiver separation. In case of communication over shadowing in absence of interferences, the instantaneous SNR given by $\gamma = \frac{SP_t}{N_0}$, where $S$ is shadow. Shadowing can be modeled by a log-normal distribution expressed as

$$f_\gamma (\gamma) = \frac{1}{\sqrt{2\pi}\sigma_\gamma} e^{-\frac{(\ln\gamma - \mu)^2}{2\sigma^2}} \quad \gamma \geq 0$$

(2.14)

where $\mu$ (dB) and $\sigma$ (dB) are the mean and standard deviation of $\ln \gamma$, respectively.

The MGF associated with this fading model is given by [18] (See Appendix
A.2 for the derivation)

\[ M_\gamma (z) \simeq \frac{1}{\sqrt{\pi}} \sum_{I=1}^{N_p} H_{\eta_I} e^{-\exp(\sqrt{2}\sigma_{\eta_I} + \mu)z} \tag{2.15} \]

where \( \eta_I \) are the zeros of the \( N_p \)-order Hermite polynomial, and \( H_{\eta_I} \) are the weight factors of \( N_p \)-order Hermite polynomial and are given by [24, Table 25.10].

### 2.2.3.2 Composite Multipath/Shadowing Fading

A composite multipath/shadowed fading environment contains multipath fading overlaid on lognormal shadowing. This type of fading is often used in congested city centre areas with slow-moving vehicles and pedestrians. This fading model can be applied in land-mobile satellite systems subject to vegetative and/or urban shadowing [18]. As an example consider gamma/lognormal shadowing fading. The PDF of this fading model was presented by Ming-Ju Ho and Stuber, G.L [25] and it is given by

\[ f_\gamma (\gamma) = \int_0^\infty \frac{m^m \gamma^{m-1}}{w^m \Gamma (m)} e^{-\frac{m\gamma}{w}} \frac{1}{\sqrt{2\pi}\sigma w} e^{-\frac{(\ln w - \mu)^2}{2\sigma^2}} dw \quad \gamma > 0 \tag{2.16} \]

Moreover, (2.16) equivalently can be approximated by using Gauss-Hermite integration as [26, eq. (3.68)]

\[ f_\gamma (\gamma) \simeq \frac{1}{\sqrt{\pi}} \sum_{I=1}^{N_p} H_{\eta_I} \left[ \frac{m}{\exp(\sqrt{2}\sigma_{\eta_I} + \mu)} \right]^{m} \gamma^{m-1} \frac{1}{\Gamma (m)} e^{-\frac{m\gamma}{\exp(\sqrt{2}\sigma_{\eta_I} + \mu)}} \quad \gamma > 0. \tag{2.17} \]

The MGF associated with gamma/lognormal shadowing fading model is given by [18, Eq. (2.32)] (See Appendix A.3 for the derivation)

\[ M_\gamma (z) \simeq \frac{1}{\sqrt{\pi}} \sum_{I=1}^{N_p} H_{\eta_I} \left( \frac{1}{1 + z \frac{1}{\exp(\sqrt{2}\sigma_{\eta_I} + \mu)} m} \right)^m. \tag{2.18} \]
2.3 Error Rate Analysis

The higher the error rate, the less reliable connection or data transfer will be over a wireless communication system. Complementary $Q$-function and square of the complementary $Q$-function appear in most formulas of bit error rates (BER) and symbol error rates (SER) of different type of digital modulation over AWGN. These functions simplify the probability of error calculation.

2.3.1 $Q$-function

The $Q$-function is defined as the complement of the cumulative distribution function (CDF) corresponding to the normalised Gaussian RV $X$ with zero mean and unit variance. This function is presented in the form of a semi-infinite integral of the corresponding PDF, as

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \, dt.$$  \hfill (2.19)

The $Q$-function is not that easy to work with as the exponential function in the integral does not lead to a close-form expression. It is because the argument $x$ is in the lower limit of the integrand where the integrand has infinite range. However, an alternative and more useful form of the $Q$-function known as Craig’s formula has been derived in [27] that greatly simplifies computing the $Q$-function. This alternative form of the function is expressed by

$$Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2\theta}} \, d\theta.$$ \hfill (2.20)

The $Q$-function can also be expressed in terms of the complementary error function\footnote{The relation given here is also related to Gaussian $Q$-function, given by $\text{erfc}(x) = 2Q(\sqrt{2x})$ [18].} as

$$\text{erfc}(x) = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2\theta}} \, d\theta$$ \hfill (2.21)

which is another important function in mathematics and physics [18].

The motivation of Craig for deriving the alternative presentation was to simplify the probability of error calculation for AWGN. For example, the
probability of BER for binary phase shift keying (BPSK) using the alternative form in (2.21) can be easily obtained by

\[ p_{\text{BPSK}}(\gamma) = \frac{1}{2} \text{erfc}(\sqrt{\gamma}) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{\gamma}{\sin^2\theta}} d\theta. \]  

(2.22)

### 2.3.2 Square Q-function

The square Q-function is a useful function to evaluate symbol error probability of MQAM over generalised fading channels. Using the classical definition of Q-function, integrals would be significantly complicated as it would be written as a double integral each which has the argument \( x \) is in the lower limit of the integrand. However \( Q^2 \)-function can be written in an alternative and more useful form as [18]

\[ Q^2(x) = \frac{1}{\pi} \int_0^{\pi/4} e^{-\frac{x^2}{2\sin^2\theta}} d\theta. \]  

(2.23)

The square complementary error function function is also given by [18]

\[ \text{erfc}^2(x) = \frac{4}{\pi} \int_0^{\pi/4} e^{-\frac{x^2}{2\sin^2\theta}} d\theta. \]  

(2.24)

### 2.3.3 Average Error Rate Analysis

In the case of communication over fading channels, the instantaneous SNR is a RV with PDF of \( f(\gamma) \). Therefore average error rate in terms of complementary error function from (2.21) can be written as

\[ \mathbb{E}\left[ \text{erfc}(\sqrt{d\gamma}) \right] = \int_0^{\infty} \frac{2}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{d\gamma}{\sin^2\theta}} d\theta f(\gamma) d\gamma = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \left[ \int_0^{\infty} e^{-\frac{d\gamma}{\sin^2\theta}} f(\gamma) d\gamma \right] d\theta \]  

(2.25)

where \( d \) is a constant that depends on a specific modulation/detection combination.

Since a MGF associated with a fading is defined by \( \mathcal{M}_\gamma(s) = \int_0^{\infty} f(\gamma) e^{-s\gamma} d\gamma \), see Appendix (A.3), the inner integral in (2.25) can be written as \( \mathcal{M}_\gamma\left(\frac{d}{\sin^2\theta}\right) \). Therefore,

\[ \mathbb{E}\left[ \text{erfc}(\sqrt{d\gamma}) \right] = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \mathcal{M}_\gamma\left(\frac{d}{\sin^2\theta}\right) d\theta. \]  

(2.26)
Likewise,
\[ E \left[ \text{erfc} \left( \sqrt{d \gamma} \right) \right] = \frac{4}{\pi} \int_{0}^{\pi/4} M_\gamma \left( \frac{d}{\sin^2 \theta} \right) d \theta. \] (2.27)

In chapter 4, (2.26) and (2.27) will be used to analyse the exact average error rates of amplify-and-forward relay systems in presence of arbitrary interferers.

### 2.3.4 Outage Probability

Outage probability is a key performance metric employed in cooperative relay systems for analysing the probability of successful transmission when a selection method is employed in the system. The outage probability is defined as the probability that SNR, \( \gamma \), of a signal falls below a given threshold. The outage probability relative to \( \gamma_{th} \) is thus defined as (e.g [18])

\[ \Pr (\gamma \leq \gamma_{th}) = \int_{0}^{\gamma_{th}} f_\gamma (x) \, dx \] (2.28)

where \( \gamma_{th} \) typically specifies the minimum SNR (threshold) required for acceptable performance and \( f_\gamma (x) \) is the PDF of the SNR.

### 2.4 Chapter Summary

In this chapter, relevant background theories that will be used to investigate and evaluate the new analytical methods were discussed. The characteristic of wireless channels including path-loss, as well as small and large scale fading were presented with some important models describing their statistical behaviour. These models were Rayleigh, Nakagami-\( m \), Rician, lognormal; and composite channel fading models. In this chapter, other important theoretical backgrounds related to this thesis including error rates and outage probability were also presented.
Chapter 3

Cooperative Relay Systems: Literature Review

3.1 Introduction

MULTIPATH fading is one of the pertinent issues in wireless communication systems as discussed in chapter 2. Diversity techniques are used to mitigate the fading phenomenon [28]. In a diversity technique, the same signal is transmitted in replicas over different channel. As a result, the probability that all replicas of the signal fade simultaneously is reduced considerably. The diversity technique are classified into four types namely, time, frequency, polarization and spatial diversity. Among these, spatial diversity with multiple transmitting and receiving antenna are most popular due to its efficiency in terms of system resource usage (power and bandwidth).

Using spatial diversity to combat fading generally needs more than one antenna at a transmitter. The use of multiple antennas at both transmitter and receiver results in a multiple-input multiple-output (MIMO) system providing a significant improvement in terms of throughput and reliability. However, in some wireless applications, implementing MIMO antennas might be impossible due to

- High power consumption: By employing multiple antenna in the system, the transmitter and receiver require more power.
- Design complexity: In MIMO systems, most signal processing algorithms are computationally intensive. Moreover, each antenna requires
a radio-frequency unit and the system needs powerful digital signal processing unit which makes the system more complex in terms of hardware.

- **Antenna size:** Employing several antennas in a MIMO system causes a significant increase in the size of the system. This can be a problem in some wireless applications as antennas can be seen as unsightly. The use of millimeter (mm) wave spectrum for next generation (5G) mobile communication makes it possible to synthesize compact antenna arrays. However, mmwave signals are vulnerable to shadowing which results in intermittent channel quality. Device power consumption is also a key challenge to support a large number of antennas with wide bandwidths [29].

Cooperative relay systems have gained much interest because of their ability to mitigate fading through achieving spatial diversity by forming a virtual MIMO system without using multiple antennas at each transceiver. In a cooperative relay system, a number of neighbouring nodes (relay nodes) are assigned to help a source in forwarding its information to its destination. In this way, a source uses the broadcast nature of wireless to broadcast a desired signal to \( N \) relay nodes through different channels. Consequently, instead of receiving a desired signal through one channel the destination obtains \( N \) copies of the desired signal from relay nodes along \( N \) different channels. For that reason, while one or more copies of the received signal get in a deep fade the others may not. The cooperative relay systems are also viewed as a low energy consumption technique since the long distance transmission is divided into two or more shorter distances transmission which causes reducing of power transmission [30]. The key advantages of cooperative communications can be summarized as follows:

- **Ease of implementation:** The easy implementation of the relaying technique in a cooperative system is its advantage over MIMO systems.

- **Reduce transmission power:** The signal strength is reducing exponentially with distance between the transmitter and the destination. When the distance between transmitter and receiver is too large, the signal attenuation becomes too high due to path loss, which makes the communication between the transmitter and receiver impossible. By employing
relay nodes between the transmitter and the receiver, the distance between the transmitter and receiver is becomes shorter. Therefore, the signal straight can be improved a lot.

- Improve throughput and coverage area: Due to signal strength improvement by employing relay nodes between the transmitter and the receiver, the source can transmit more data in each channel. Therefore, in the cooperative relay systems, not only increases the coverage area but also improves the data rate.

- Reduce cost: Cooperative communications provide more cost effective solutions in many cases. For example, the cellular communication systems are moving to higher frequency. For this reason, the coverage area of each cell is shrinking a lot compared to previews cellular systems. Building more base station can be the solution, however the cost is very high. A low-cost alternative solution is using relay nodes to extend the coverage area.

This chapter presents a review on principles and advanced cooperative relay systems as well as some other major features of cooperative systems and relevant analytical techniques to help the reader get a broader perspective on the subject matter. In Sec. 3.2, the basic principle of a cooperative system is explained. In Sec. 3.3 advanced relay cooperative systems are presented which includes principle performance analysis of two-way relay and full-duplex relay cooperative systems. As a major part of this thesis addresses relay selection aspects, in Sec. 3.4 multiple-relay cooperative system is presented, where the concept includes describing the proposed relay selection methods and a specific interest in single relay selection method employing time-based protocols. In Sec. 3.5 performance measures of interest related to this thesis are introduced.

### 3.2 Fundamentals of Cooperative Systems

A three-terminal network known as a relay channel acts as a fundamental principle for a cooperative relay system. This network was first introduced in [2] and is the earliest model in network information theory which acts as a main building block for large wireless networks. Recently, the subject of relay
channel was resuscitated as a result of considerable advances in signal processing techniques, and since then it has been receiving significant attention in the wireless research community. In [3, 31] the cooperative diversity term used to describe the use of channels to combat effects of multipath fading in direct transmission. The concept of three-terminal communication channel in a cooperative system is shown in Fig. 3.1. The set-up consists of three nodes: a source node $T_1$, a relay node $R$ and a destination node $T_2$. All nodes operate in a half-duplex mode. Let’s consider that terminal $T_1$ wants to transmit data to terminal $T_2$ with help of a relay node $R$. As can be seen from Fig. 3.1, the channel allocation for time-division approach with two orthogonal phases, Phase 1 and Phase 2, which guarantees the half-duplex operation. In Phase 1, $T_1$ broadcasts a signal to $T_2$ and at the same time the information is also received by $R$. In Phase 2, $R$ can help $T_1$ by forwarding or retransmitting $T_1$’s information to $T_2$. Consequently, diversity is provided by this cooperation because even if direct link transmission from $T_1$ to $T_2$ is faded, the information can still be successfully received by $T_2$ with the help of $R$.

### 3.2.1 Relaying Schemes

In a relay cooperative system there are two main relaying schemes which can be employed into the system to process the received information. These schemes can be generally classified into two types: [32, 33]

- Amplify-and-Forward: The AF is a simple relay scheme in which the relay simply amplifies the received signal and forwards a scaled version to the destination. The advantage of this scheme is in its simplicity
as encoding and decoding the signal at the relay node is not needed. However, besides simplicity and low cost, noise amplification at relay is a major drawback.

- **Decode-and-Forward**: In DF, a relay decodes the received signal from the source before forwarding to the destination. After decoding the signal, the relay re-encode the signal and then forwards it to the destination. In contrast to AF relay, DF relay scheme only forwards the useful signal to the destination but with extra time processing.

- **Detect-and-forward**: In this scheme, the relays detect the received signals and forward the detected symbols to the destination.

There are other types of relay schemes proposed in literature, but all of these are mainly created from these two basic schemes, AF and DF [34].

### 3.2.2 One-Way Relay Cooperative Systems

One-way relay (OWR) systems are one of the most popular and simplest techniques for analysing a cooperative system and have been studied extensively in literature. For analysing the OWR system, consider a three-terminal cooperative system as shown in Fig. 3.2, where direct link from $T_1$ to $T_2$ is assumed to be unavailable due to deep fading. From Fig. 3.2, it can be seen that for a full transmission through the relay node $R$, the system needs to operate in four phases (four time slots) including transmission from $T_1 \rightarrow R$, $R \rightarrow T_2$, $T_2 \rightarrow R$ and $R \rightarrow T_1$.

#### 3.2.2.1 Amplify-and-Forward

Consider the OWR system with an AF relay node. In Phase 1, the terminal $T_1$ (source) transmits an information symbol to the relay node $R$. Therefore, the received signal at relay node can be written as

$$y_r = \sqrt{P_1}x_1g + n_1$$

(3.1)

where $g$ is the complex channel gain between $T_1$ and $R$, $x_1$ is the transmit symbol of the source which is assumed to be RV with zero mean and unit energy $\mathbb{E}[|x_1|^2] = 1$; and $P_1$ is the transmit source power. Also, $n_1 \sim \mathcal{N}(0, \sigma_1^2)$
is the AWGN at the relay node. Subsequently, the relay amplifies the received signal from the source with a gain factor $G$ before forwarding the message to the destination. Therefore, the transformed copy of the received signal via the relay $R$ is given by $\mathcal{R}(y_r) = Gy_r$, where

$$G = \sqrt{\frac{P_2}{P_1|g|^2 + \sigma_1^2}}$$

and $P_2$ is the transmit power of the relay node. In Phase 2, the scaled signal, $\mathcal{R}(y_r)$, is forwarded to the terminal $T_2$ (destination). Consequently, the destination receives

$$y_2 = G\sqrt{P_1}x_1f + Gn_1f + n_2$$

where $f$ is the complex channel coefficient between $R$ and $T_2$; and $n_2 \sim \mathcal{N}(0, \sigma_2^2)$ is AWGN at destination. By replacing (3.2) in (3.3), the SNR for source-relay-destination link, $T_1 \rightarrow R \rightarrow T_2$, is obtained by

$$\gamma_{AF} = \frac{|g|^2|f|^2P_2P_1}{P_2|f|^2\sigma_2^2 + P_1|g|^2\sigma_2^2 + \sigma_1^2\sigma_2^2}$$

which with simple arithmetic operations, (3.4) is reduced to

$$\gamma_{AF} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1}$$

where $\gamma_1 = \frac{P_1|g|^2}{\sigma_1^2}$ and $\gamma_2 = \frac{P_2|f|^2}{\sigma_2^2}$.

The capacity for $T_1 \rightarrow R \rightarrow T_2$ link is given by

$$C = \frac{1}{2} \log \left(1 + \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1}\right)$$

where the factor 1/2 follows because of two channel are needed to transmit the information from $T_1$ to $T_2$.

### 3.2.2.2 Decode-and-Forward

In Phase 1 of DF, $T_1$ transmits the message to the relay node $R$. The received signal at relay is given by (3.1). In Phase 2, the relay fully decodes the source message $x_1$ and then transmits the re-encoded message as $\mathcal{R}(y_r) = \tilde{x}_1$ to $T_2$, where $\tilde{x}_1$ is the decoded version of $x_1$. Therefore, the received signal at $T_2$
through the DF relay node is

\[ y_2 = \sqrt{P_2} \tilde{x}_1 f + n_2. \]  \hfill (3.7)

In DF relay systems, communication from \( T_1 \) to \( T_2 \) via the relay node that always decodes and then forwards the received signal is done in two phases, and the system performance is dominated by the worst phase.

The capacity for \( T_1 \rightarrow R \rightarrow T_2 \) link is given by [2]

\[ C = \frac{1}{2} \min \{ \log_2 (1 + \gamma_1), \log_2 (1 + \gamma_2) \} \]  \hfill (3.8)

where \( \gamma_1 = \frac{P_1 |g|^2}{\sigma_r^2} \) and \( \gamma_2 = \frac{P_2 |f|^2}{\sigma_r^2} \).

### 3.3 Advanced Relay Cooperative Systems

Achieving a full information exchange in an OWR system requires four phases, since in each phase only one terminal is used for transmission. Therefore, this cooperative system suffers from spectral loss because of repetitive transmissions from a relay node. However, in advanced cooperative relay systems, two terminals simultaneously transmit in each phase which significantly improves the spectral loss of OWR systems up to four times. For that reason, there is a lot of interest on advanced cooperative systems. Advanced relay cooperative systems can be divided into two-way relay systems (TWR) also known as bidirectional cooperative systems; and full-duplex relay systems (FDR). In TWR, two sources at the same time transmit their messages to...
CHAPTER 3. COOPERATIVE RELAY SYSTEMS: LITERATURE REVIEW

Figure 3.3: Two-Way Relay Cooperative System

each other while in FDR, the relay node receives and transmits a message at the same time in the same frequency.

3.3.1 Two-Way Relay Systems

A two-way relay system is an effective means to increase the SE compared to half-duplex relay systems [35], as two terminals simultaneously transmit their messages to each other. As shown in Fig. 3.3, messages of $T_1$ and $T_2$ with the help of relay $R$ are delivered to $T_2$ and $T_1$, respectively, in two phases. In Phase 1, both $T_1$ and $T_2$ simultaneously transmit their information to the relay node $R$. After receiving the signals, the relay node performs proper signal processing and broadcasts the resulting signal to both $T_1$ and $T_2$ in Phase 2. At each source node, its symbol causes self-interference but it can be canceled clearly by subtracting its own signal [35]. This action makes two-way transmission possible in two phases while one-way transmission requires four phases to exchange the same information. An overview for TWR cooperative systems can be found in [36].

3.3.1.1 Amplify-and-Forward

To see the performance of a TWR system, consider an AF relay node, where the direct link from $T_1$ to $T_2$ is assumed to be unavailable due to deep fading. Moreover all terminals are assumed to operate in a half-duplex mode. In Phase 1, both $T_1$ and $T_2$ transmit their symbols to relay node $R$ at the same time. The received signal at relay node is thus given by

$$y_r = \sqrt{P_1} x_1 g + \sqrt{P_3} x_3 f + n_1$$

(3.9)

where $x_3$ and $x_1$ are the transmit symbol of $T_2$ and $T_1$ nodes, respectively, with unit energy of $\mathbb{E}[|x_3|^2] = 1$ and $\mathbb{E}[|x_1|^2] = 1$. Also, $P_3$ and $P_1$ are the transmit
power of node $T_2$ and $T_1$, respectively. In Phase 2, the relay amplifies the received signal and then forwards scaled $y_r$ (scaled with $G$) to both $T_1$ and $T_2$. The received signal at $T_2$ is thus

$$y_2 = G\sqrt{P_1}x_1gf + G\sqrt{P_3}x_3f^2 + Gn_1f + n_2$$

(3.10)

where the second term $G\sqrt{P_3}x_3f^2$ is the transmitted signal of $T_2$ which is known by $T_2$ and can be removed via self interference cancellation methods [35]. Therefore

$$\tilde{y}_2 = G\sqrt{P_1}x_1gf + Gn_1f + n_2$$

(3.11)

where the gain factor is defined as

$$G = \sqrt{\frac{P_2}{P_1|g|^2 + P_3|f|^2 + \sigma_1^2}}.$$  

(3.12)

By replacing (3.12) into (3.11), the SNR of $T_1 \rightarrow R \rightarrow T_2$ link can be written as

$$\gamma_{1,2}^{AF} = \frac{|g|^2|f|^2P_2P_1}{P_1|g|^2\sigma_2^2 + |f|^2(P_3\sigma_2^2 + P_2\sigma_1^2) + \sigma_1^2\sigma_2^2}.$$  

(3.13)

and similarly, SNR of the $T_2 \rightarrow R \rightarrow T_1$ link is given by

$$\gamma_{2,1}^{AF} = \frac{|g|^2|f|^2P_2P_1}{|g|^2(P_1\sigma_1^2 + P_2\sigma_2^2) + |f|^2(\sigma_1^2\sigma_2^2 + \sigma_1^2\sigma_2^2)}.$$  

(3.14)

The capacity for $T_1 \Rightarrow R \Leftarrow T_2$ is given by

$$C = \frac{1}{2} \log \left( 1 + \frac{|g|^2|f|^2P_2P_1}{P_1|g|^2\sigma_2^2 + |f|^2(P_3\sigma_2^2 + P_2\sigma_1^2) + \sigma_1^2\sigma_2^2} \right).$$  

(3.15)

The transmission still suffers from factor $1/2$ as it needs two time slots for end-to-end transmission. However, the half-duplex constraint can here be used to establish a two-way connection between terminals $T_1$ and $T_2$ and to increase the sum rate of the network.

### 3.3.2 Decode-and-Forward

Consider a TWR communication between terminal $T_1$ and $T_2$ via a half-duplex DF relay $R$. In Phase 1, both terminals $T_1$ and $T_2$ transmit their symbols to
relay node. In this Phase, the relay receives
\[ y_r = \sqrt{P_1} x_1 g + \sqrt{P_3} x_3 f + n_1 \] (3.16)

decodes the symbols \( x_1 \) and \( x_2 \) and transmits \( \tilde{x} = \sqrt{\alpha} P_1 x_1 + \sqrt{(1-\alpha)} P_3 x_2 \) in the Phase 2. The received signals at \( T_1 \) and \( T_2 \) are
\[ y_2 = \tilde{x} f + n_2 \] (3.17)
\[ y_1 = \tilde{x} g + n_1 \] (3.18)

where the relay node uses an average transmit power \( \alpha P_1 \) for forward direction and \( (1-\alpha) P_3 \) for backward direction. Since \( T_1 \) knows \( x_1 \) and \( T_2 \) knows \( x_2 \), these symbols can be subtracted at the respective terminals prior to decoding of the symbol transmitted by the partner terminal.

The capacity for \( T_1 \rightleftharpoons R \rightleftharpoons T_2 \) link is given by
\[ C = \frac{1}{2} \min \left[ \log_2 (1 + \gamma_1), \log_2 (1 + \gamma_2) \right] \] (3.19)

where \( \gamma_1 = \frac{P_1 |g|^2}{\sigma_1^2} \) and \( \gamma_2 = \frac{\alpha P_1 |f|^2}{\sigma_2^2} \).

### 3.3.3 Full-Duplex Relay Systems

A basic principle of wireless communication is that a radio cannot transmit and receive on the same frequency at the same time because of the resultant self-interferences. Consequently, a large number of existing works on cooperative systems have assumed a system with half-duplex operation. Although, to combat half-duplex restriction (e.g. bandwidth loss) several relay selection methods and TWR systems have been proposed and investigated, e.g. \([34,37-40]\) but those systems can still suffer from bandwidth loss.

The author of \([41]\) challenged the half-duplex assumption and discussed the design of a single channel full-duplex wireless transceiver. A full-duplex transmission allows communication in both directions, and unlike half-duplex, it allows this to occur on the same frequency and at the same time. As a result, the system requires only one channel for end-to-end transmission. Therefore, by applying the full-duplex technique in a cooperative
system an enormous bandwidth can be saved. However, this alternative efficiency used for channel bandwidth suffers from loop interference which is due to signal leakage between the relay output and input. In the past, this interference made the practical application and operation of full-duplex very difficult, and even impossible, but, today improvements in antennas and signal processing have made it feasible. For this approach, the use of two spatially-separated antennas, one for the transmitter and the other for receiving in a relay node, loop interference can be reduced significantly [42]. Although it still can suffer from residual loop interference resulting from imperfect isolation or cancellation. The outage performance of a FDR cooperative system in an AF network has been evaluated in [43] and [44]. The achievable transmission rate for full-duplex cooperative system under both the AF and DF networks has been derived in [45]. Applying a relay selection scheme to FDR systems provides a more efficient approach to combine diversity benefits with higher SE. The different relay selection schemes in an AF cooperative system with full-duplex operation have been studied in [46].

3.3.3.1 Amplify-and-Forward

Consider a three terminal cooperative system with $T_1$, $T_2$ and $R$, as shown in Fig. 3.4, where $T_1$ and $T_2$ nodes operate in half-duplex mode and are equipped with a single antenna. While, the relay node is equipped with two antennas, one for receiving and one for transmitting signal, enabling a full-duplex operation at the price of residual loop interference. Direct link from $T_1$ to $T_2$, is assumed to be unavailable.

In Phase 1, $T_1$ broadcasts a signal to the relay node $R$. Simultaneously $R$ forwards a signal from the previous time slot imposing the residual loop
interference from the relay output to the relay input denoted as \( R \). Consequently, the received signal at \( R \) can be written as

\[
y_r = \sqrt{P_1} x_1 g + n_1 + x_I h_I
\]

where \( x_1 \) and \( x_I \) represent the transmit symbol of the source and the regenerated signal at the relay, respectively, which are assumed to be independent RVs with zero mean and unit energy, \( \mathbb{E}[|x_1|^2] = 1 \) and \( \mathbb{E}[|x_I|^2] = 1 \). Also, the RV \( h_I \) denotes the residual loop interference channel. The relay amplifies the received signal from the source with a gain \( G \) before forwarding it to the destination. The gain factor, also called amplification factor, is the extent to which an amplifier boosts the strength of a signal and it is expressed in terms of power. Therefore, the transformed copy of the received signal through the relay \( R \) is

\[
\Re(y_r) = G y_r,
\]

where

\[
G = \sqrt{\frac{P_2}{\sigma_1^2 + P_1 |g|^2 + |h_I|^2}}.
\]

In Phase 2, the relay node \( R \) forwards the scaled signal \( \Re(y_r) \) to terminal \( T_2 \). Meanwhile, \( R \) receives a signal from the next time slot. Thus, the received signal at \( T_2 \) is given by

\[
y_2 = G \sqrt{P_1} x_1 g f + G n_1 f + G x_I h_I f + n_2.
\]

By substituting (3.21) into (3.22), SINR of the \( T_1 \rightarrow \hat{R} \rightarrow T_2 \) link is obtained by

\[
\gamma_{AF} = \frac{|g|^2 |f|^2 P_2 P_1}{P_2 |f|^2 \sigma_1^2 + P_2 |f|^2 |h_I|^2 + \sigma_2^2 (\sigma_1^2 + P_1 |g|^2 + |h_I|^2)}
\]

as SINR is the ratio of the power received signal and the power of the noise. SINR in (3.23) after simple arithmetic manipulations reduce to

\[
\gamma_{AF} = \frac{\gamma_1 \gamma_2}{\gamma_1 + 1 + \gamma_2}
\]

where \( \gamma_I = \frac{|h_I|^2}{\sigma_I^2} \), \( \gamma_1 = \frac{P_1 |g|^2}{\sigma_1^2} \) and \( \gamma_2 = \frac{P_2 |f|^2}{\sigma_2^2} \).
The capacity for $T_1 \rightarrow R \rightarrow T_2$ link is given by

$$C = \log_2 \left( 1 + \frac{\gamma_1}{\gamma_1 + 1} \frac{\gamma_2}{\gamma_2 + 1} \right).$$ \hspace{1cm} (3.25)$$

SNR of the OWR system in half-duplex mode can also be described by the above model, except that the loop interference is eliminated and transmission consumes fourth times the channel resources of the full-duplex mode.

### 3.3.3.2 Decode-and-Forward

The transmission over DF relay follows a similar process as AF discussed in Sec. 3.3.3.1 except that the relay node decodes the signal received from $T_1$ and then transmits it to $T_2$. The capacity for $T_1 \rightarrow R \rightarrow T_2$ link is given by \cite{2}

$$C = \min \left[ \log_2 \left( 1 + \frac{\gamma_1}{\gamma_1 + 1} \right), \log_2 \left( 1 + \gamma_2 \right) \right].$$ \hspace{1cm} (3.26)$$

### 3.4 Multiple-Relay Cooperative Systems

Cooperative diversity goes one step further by considering the contribution of several relay nodes in delivering a source signal to a destination node in order to achieve higher diversity gain \cite{47}. In a cooperative system where $N$ relays participate between the source and the destination link, failure occurs only when $N + 1$ paths (source-destination path plus $N$ source-relay-destination paths) experience deep fading and/or shadowing simultaneously, for which the probability of this phenomenon is very small. Since the long distance transmission is divided into two or more shorter distance transmission that cause reduction in transmission power \cite{30}. However, as \cite{48} stated, having several relay nodes in a cooperative system may not be more energy efficient because of the extra power circuitry used by the relaying nodes. Therefore, the energy efficiency of the system can be degraded by increasing the number of relay nodes. On the other hand, the benefits of a multiple-relay cooperative system also come at the expense of a reduction in SE as the relays must transmit on orthogonal channels in order to avoid interfering with the source node and with each other. \cite{6}. Hence in a cooperative diversity network with $N$ relaying nodes, $2N + 1$ channels are hired, which experiences a bandwidth
penalty. The problem of the inefficient use of the channel resources and extra energy consumption of a multiple-relay cooperative system can be excluded by the use of selective relaying methods.

3.4.1 Distributed Relay Selection Methods

As discussed in earlier section, selective relaying methods are often employed to improve spectral and energy efficiency of a multiple relay cooperative system. Previous studies have shown that selecting only one relay “best” from a number of available relay nodes not only can keep full spatial diversity order but also increase channel capacity. However, how to select the best relay is still an open challenge. Some scholars have proposed a centralised solution where a fixed node decides which relay will be selected to help the source to forward the information. This type of methods needs whole channel state information (CSI), which brings substantial overheads and increase the transmission delay. Therefore, in practical applications that need a short transmission delay, the centralised solution for selecting the best relay cannot be applied. To mitigate the centralised difficulties, several distributed relay selection methods and protocols have been proposed [49]. The simplest solution is choosing the relay randomly, as proposed in [50]. This scheme can reduce the design complexity but can not achieve optimal performance. [37] proposed a solution based on distance. In this method, the relay which has the closest distance to the destination has been chosen to transmit the received signal towards the destination. However, the relay selected based on the distance can not effectively reflect an appropriate channel. For example, communication between transmitters and a receiver with same distances can have significant differences in terms of SNR because of different shadowing, multipath fading and interference experienced between a transmitter and the receiver. The most intuitive solution among distributed relay selection methods to choose the best relay is a method which is based on signal strength measurement and was first introduced by [12]. The distributed method proposed in [12] is a simple method employing time-based relay selection protocols to select the best relay. The time-based distributed protocols have been studied by many scholars e.g. [39, 51–53] and overview on the proposed protocols are given in Sec. 3.4.1.2.
Cooperation with a selected relay can be divided into three stages: direct transmission stage, relay selection stage, and cooperative transmission stage including operation in Phase 1 and Phase 2. In direct transmission stage, source transmits data directly to the destination and neighbouring relay nodes try to receive it. In the selection stage, a single relay among several available neighbouring relay nodes is selected (see, Sec. 3.4.1). In the cooperative transmission stage if destination fails to receive the data from the source during the direct transmission, the selected relay forwards/retransmit the data to the destination. The relay selection stage has a great impact on the system performance and can be done before direct transmission (proactive relay coordination) or after direct transmission (reactive relay coordination), see Fig. 3.5.

**Proactive Coordination:** In a proactive coordination, a specific relay is selected before an information is transmitted from the source and this can potentially result in performance degradation [17]. In proactive relay selection
procedure, the instantaneous channel state information (ICSI) are obtained via Request-to-Send (RTS) and Clear-to-Send (CTS) message exchange. For this, source transmits RTS packet toward relay nodes and the destination. Such transmission can be used for each relay to obtain its signal strength toward the source. The destination responds with CTS packet at the same channel used for communication between relays and destination. The received CTS packet to each relay estimate signal strength from the destination to relay. Due to the reciprocity theorem, the signal straight from the destination to each relay node is equal to the signal straight from each relay node to the destination. This theorem can also be applied for source-relay link. Relays that receive the CTS packets, participate in the selection process and a single relay among the available candidate relays is selected as a best relay. If direct transmission fails, the best relay retransmits the message to the destination. If the selection fails, the source transmits without relay assistance. It is also worth noting that this procedure does not require a specific time synchronization as CTS transmitted from the destination is used to synchronize all relays’ timers. The proactive coordination also presented as a state diagram in Fig. 3.6.

**Reactive Coordination:** In a reactive mode, the relay is selected after information is transmitted from the source and takes place only if the destination is not able to receive the data from the source and asks for assist [17]. In such a case, destination sends a negative acknowledgment (NACK) and the ICSI between the relay and destination is obtained through the NACK. Relays that receive the NACK packets participate in the selection process and the selected “best” relay retransmits the source data to the destination. If no candidates are available for relaying, source retransmits the data itself, since all nodes overhear the direct transmission. The benefit of reactive selection is in the usage of selection diversity at each failed packet. The reactive coordination also presented as a state diagram in Fig. 3.7.

### 3.4.1.2 Time-Based Relay Selection Protocols

As previously discussed, in order to overcome the bandwidth loss and improve the spectral efficiency in a cooperative system, several methods and protocols have been proposed [49]. Among them the distributed method
employing time-based relay selection protocols was introduced as the most intuitive method to improve the bandwidth loss in the system [12]. A time-based distributed protocol relates the packets’ transmission times to their signal strengths. In such a way that as soon as each relay node successfully receives the data from the source starts its own time with an initial value which is inversely proportional to the relay’s signal strengths. The best relay is the one which its time expires first. In case of failure of direct transmission, the best relay comes into action and forwards the received signal toward the destination.

The distributed relay selection method to select the best relay can be approached by two different time-based relay selection protocols, opportunistic relay (OR) and selection cooperation relay (SC) protocols. The OR and SC protocols are two similar but with different modes of operation proposed in [12, 54] and [38] respectively. To provide a better understanding of distributed methods in the proposed protocols, a system model is proposed.

Consider an OWR cooperative system with the reactive coordination as shown in Fig. 3.8, where the direct link from $T_1$ to $T_2$ is assumed to be
unavailable. The system has two end users, namely $T_1$ and $T_2$, where $T_1$ acts as a source and $T_2$ acts as a destination; and $N$ relay nodes. Complex channels from the source to the relay $l$ and from relay $l$ to the destination are denoted by $g_l$ and $f_l$, respectively. In Phase 1, the source broadcasts a signal to $N$ relay nodes and destination, where all relay nodes are in the listening mode. When relay $l$, has a higher SNR than the specific SNR threshold, it is assumed that it can successfully receive the data from the source and it becomes a candidate relay. Then all candidate relays comprise a set denoted as $\mathcal{H}$ to participate in the selection process. Consider $L_s$ as the cardinality of the set $\mathcal{H}$, thus $|\mathcal{H}| = L_s$ where $1 \leq L_s \leq N$. During the transmissions in Phase 1, $g_l$ is estimated at the relay node. Since a reactive coordination system is considered, relay selection takes place only if the destination is not able to receive the data from the source and ask for assist by sending a NACK. The $f_l$ is obtained through the NACK from destination. In Phase 2, a single relay among the available candidate relays in the set $\mathcal{H}$ is selected to forward the received source information to the destination.

The OR and SC are two similar relay selection protocols. In both proposed
protocols the best relay is selected from $L_s$ out of $N$ available nodes based on channel measurements. However, their modes of operation are different: [53]

- SC protocol: in Phase 1, all $N$ nodes listen to the source and those with $SNR_{|g_l|^2} \geq SNR_{th}$ are selected as the relay candidates $L_s$. In the Phase 2, the relay with the highest $|f_l|^2$, where $1 \leq l \leq L_s$, is selected as the best relay and forwards the received data to the destination. Thus

$$h_l = |f_l|^2, \quad h^* = \max h_l, \quad l \in \mathcal{H} \subseteq \{1, ..., L_s\} \quad (3.27)$$

- OR protocol: in Phase 1, the same process as SC is done to select the relay candidates while in Phase 2 the relay with highest $\min(|g_l|^2, |f_l|^2)$ is selected as the best relay. Thus

$$h_l = \min(|g_l|^2, |f_l|^2), \quad h^* = \max h_l, \quad l \in \mathcal{H} \subseteq \{1, ..., L_s\} \quad (3.28)$$

where $\ast$ denotes the “best” (the selected) relay.

Based on the above estimation for the channel measurements of the introduced protocols, SC and OR, each relay $l$ initiates a timer $t_l$ with value inversely proportional to the signal strength metric $h_l$ as

$$t_l = \frac{\lambda}{h_l} \quad (3.29)$$

where $\lambda$ is a system parameter.

The best relay has its time reduced to zero (i.e. to expire) first and it is the
relay that forwards the information toward the destination. In other words, the relay node with \(\max h_l\) is selected as the best relay as it takes the minimum \(t_l\). Consequently,

\[
t_{\text{min}} = \frac{\lambda}{h^*}
\]

which is the best relay’s time.

### 3.5 Performance Measures of Interest

#### 3.5.1 Interference Analysis and Challenges

The noise can be any type of undesired signal that exists in the passband of desired channel, while the interference is when additional signals are added to the signal that we want. Interference is a major source of signal corruption than the noise from circuitry. Wireless communication from a sender to a receiver is significantly affected by the interference generated by other devices. If devices in the neighborhood of the receiver transmit at the same time and on the same frequency channel, their signals interfere at the receiver with the useful signal from the sender and thus obstruct proper reception. Such co-channel interference cause high error rates and system degradation and have a huge impact on system performance. Beside CCIs, cooperative relay systems can also suffer from self-interferences. Therefore, in the design and analysis of modern wireless communication systems, it is essential to study the statistics of the SINR, \(\gamma\), which is defined as

\[
\gamma = \frac{x_0}{\sigma_n^2 + \sum_{l=1}^{L} x_l}
\]

where \(\sigma_n^2\) represents the variance of AWGN, and \((x_0, x_1, ..., x_L)\) is a set of arbitrarily distributed non-negative RVs that represent the received powers of the useful signal and \(L\) is the number of interferers.

#### 3.5.1.1 Poisson Point Process

If no knowledge regarding the location and the number of the source of interferences is available, a typical assumption is that the sources are distributed in a plane according to a poisson point process (PPP). In probability theory,
a PPP is a specific type of random process by which a set of isolated points falling within a region is modeled.

The probability distribution for the number of isolated nodes \( L \), falling within a given region \( \mathcal{A} \), with intensity \( \zeta > 0 \) (terminals/area) is a Poisson distribution with parameter \( \zeta \mathcal{A} \). This means that

\[
\Pr (L = l) = \frac{(\zeta \mathcal{A})^l}{l!} e^{-\zeta \mathcal{A}}. \tag{3.32}
\]

The MGF of PPP is given by

\[
\mathcal{M}_\gamma (z) = e^{-\zeta \mathcal{A}(1-z)}. \tag{3.33}
\]

### 3.5.1.2 Spectral Efficiency

Spectral efficiency refers to the information rate that can be transmitted over a given bandwidth in a specific communication system. The channel capacity, in the Shannon’s sense, provides the maximum achievable transmission rate which the errors are recoverable \([56]\). The SE can be presented by

\[
C = \log_2 (1 + \gamma) \quad \text{[bits/s/Hz]}. \tag{3.34}
\]

As can be seen in (3.34) the SE is given by a logarithmic function of SNR or SNIR, \( \gamma \), which could consists of several RVs. The average value of SE can be obtained by integrating over (3.34). If the expression of \( \gamma \) in (3.34) consists of several RVs, taking average of (3.34) by the known classical methods would be difficult and would require multiple integrations with knowledge of the PDF of \( \gamma \). For example, considering SINR in (3.31), to be defined as \( \gamma = \frac{x_0}{\sigma_n^2 + \sum_{l=1}^{L} x_l} \), thus the average SE is presented as

\[
C = \mathbb{E} \left[ \log_2 \left( 1 + \frac{x_0}{\sigma_n^2 + \sum_{l=1}^{L} x_l} \right) \right] \tag{3.35}
\]

which needs \((L + 1)\)-fold numerical integration operation to average out the
\( L + 1 \) RVs as
\[
C = \int_0^\infty \int_0^\infty \ldots \int_0^\infty \log_2 \left( 1 + \frac{x_0}{\sigma_n^2 + \sum_{l=1}^L x_l} \right) \times f(x_1, x_2, \ldots, x_L) f(x_0) \text{d}x_1 \text{d}x_2 \ldots \text{d}x_L \text{d}x_0 \quad (3.36)
\]

where \( f(.) \) is the PDF. It is clear now that the calculation of (3.34) is not a simple task and needs a long time to evaluate numerically. A unified method for the efficient computation for the averages in (3.34) which reduces the computational complexity has been proposed, using the fact that the RV are independent and identically distributed. This method allows an explicit expression to be obtained for the average SE in terms of the MGF of RVs described as follow:

**Lemma 1.** from [57]

\[
\mathbb{E} \left[ \log_2 \left( 1 + \frac{x_0}{\sigma_n^2 + \sum_{l=1}^L x_l} \right) \right] = \\
\log_2 e \int_0^\infty \frac{1}{z} \left( \mathbb{E} \left[ e^{-z \sum_{l=1}^L x_l} \right] - \mathbb{E} \left[ e^{-z \left( \sum_{l=1}^L x_l + x_0 \right)} \right] \right) e^{-x_0^2 \sigma_n^2} \text{d}z. \quad (3.37)
\]

See Appendix A.6 for the derivation of (3.37).

In chapter 5, the non-direct method proposed in (3.37) is used to evaluate the exact average SE of a distributed relay selection method in a non-ideal cooperative system without the need for the explicit expression of the \( \gamma \)'s PDF.

### 3.5.2 Green Wireless Communication Systems

Today researchers in wireless communications are focusing on energy efficiency because Information and Communication Technology (ICT) already represents about 2% of the total world CO2 emission [58]. It is also expected to increase from 0.53 billion tons (Gt) CO2 in 2002 to 1.43 Gt in 2020 [58]. Mobile communication systems consume 15-20% of entire the ICT energy footprint and energy consumption of mobile networks is growing much faster
than ICT on the whole [58]. This is because at the moment about 5-6 billion people have mobile phones and connect to the internet to share information, work, socialize and shop. To provide these services, there are already more than 8 million base stations (BS) serving the mobile users, with each of them consuming approximately 25MWH per year [59]. The number of mobile users and BSs are increasing each year and it is estimated that by 2020 there could be 50 billion machine-to-machine connections [58]. In addition to the environmental aspect, energy cost also is a significant problem. For that reason, the rising energy cost and carbon footprint of operating mobile networks has become a threat that prompted researchers to explore a new area of research called green wireless communication systems. The purpose of the green is to reduce energy consumption in order to reduce CO2 emissions to cope with climate change and cut energy prices.

3.5.2.1 Energy Consumption

Energy efficiency has always been an important issue in wireless communication systems and its significance seems to be increasing as energy consumption increasingly becomes a global environment problem. Many research and development efforts have been made in the wireless industry, aiming for energy efficient solutions leading to green wireless communications. From the perspective of this thesis, energy consumption of two proposed protocols in a distributed time-based relay selection method in a cooperative system will be analysed. Energy consumption is commonly defined as transmit energy per unit of information bits and is given by [1]

$$EC = \frac{PT}{\kappa} \quad [\text{J/bits}]$$

(3.38)

where $P$ and $T$ are the total power consumption and total time consumed for end-to-end transmission, respectively and $\kappa$ is information bits.

3.6 Chapter Summary

Although three-terminal relay channels has been introduced in the 1970’s, its real development in the wireless research community came recently following significant advances in signal processing such as diversity concepts. After
that cooperative relay systems have been widely used to mitigate the fading phenomenon. This chapter provided a review of the designs and implementation issues associated with different cooperative relay techniques including one-way, two-way and full-duplex relay cooperative systems. As a major part of this thesis addresses relay selection aspects, in this chapter, multiple relay cooperative systems, relay selection methods and specific interest on time-based relay selection protocols were discussed in more details. In addition, the performance measures of interest discussed in this thesis were presented.
Chapter 4

A Unified Analysis of AF Relays in Presence of Arbitrary CCIs

4.1 Introduction

The noise can be any type of undesired signal that exists in the passband of a desired channel. Beside noise, an interference is a major source of signal corruption. The interference is when additional signals are added to the signal. In a cooperative system, communication from a sender to a receiver is significantly affected by the interference generated by other devices. In such a way that if devices in the neighborhood of the receiver transmit at the same time and on the same frequency channel, their signals interfere at the receiver with the useful signal from the sender and thus obstruct proper reception. Such co-channel interference (CCI) cause high error rates and thus have a huge negative impact on the system performance. In general, CCI in a cooperative system can be considered either at the relay, destination, or at both relay and destination node. In a case where a interferer may exists only at the relay node is in the frequency-division relay systems where the relay and destination terminals experience different interferer forms. Considering an interferer at the destination node is mainly relevant to time-division multiple-access (TDMA) systems in which a single time-slot is assigned for each terminal, i.e, orthogonal multiple access is considered. This greatly motivates the model where relay node remains interference-free while the destination is corrupted by many interferers.

Despite of the inherent effect of CCIs and noise in cooperative systems,
most of the conducted research on the AF relay systems have ignored the 
effect of CCIs e.g, [4–11]. However, in practice, the system’s achievable per-
formance is inevitably degraded by CCIs generated by external interfering 
ources. Since an external interference scenario is considered, the majority of 
relevant works treated this by ignoring thermal noise [60–69], and very lim-
ited works considered all thermal noise [70]. Moreover, in all previous related 
published works, e.g. [60–70], the analyses are limited to interference model 
in which the system is subjected to a given number of homogenous and iden-
tically distributed interferers. In [60–62], the performance analysis of a dual-
hop AF relay system over Rayleigh fading channels was studied. By deriving 
expressions for cases where all the signals undergo Nakagami\(-m\) fading chan-
nels, \([63,65,66]\) investigated the system performance. \([71]\) extended the results 
of \([66]\) for the case where fading channels of desired signals and interferers 
were assumed to be Rician and Nakagami\(-m\) distributed respectively. In a 
more general scenario, \([70]\) analysed the system performance considering all 
thermal noise and interferences. In \([70]\), the author derived an approxima-
tion for the BER probability, where all channels were assumed to be subject 
to Rayleigh fading.

To the best of our knowledge, all the aforementioned papers limited the 
study of the performance of AF relay systems to a specific interference model. 
However, in wireless networks the interferers’ signals can experience different 
attenuation, especially in heterogeneous cellular networks. This major limi-
tation motivated current study to derive a new exact mathematical analysis 
for a more realistic scenario. This model considers an AF relay system over 
Rayleigh fading channels in presence of a number of arbitrary non-identical 
interferers at the destination and thermal noise at both source-relay and relay-
destination links. Therefore, the system model considered in this chapter is 
adequate for modeling small as well as large environments. As mentioned 
before considering the interference at the destination node is mainly relevant 
to TDMA relay systems thus the relay node remains interference-free.

The main contribution in this chapter is that a new simple unified math-
ematical method for accurate and efficient evaluation of exact average error 
probabilities of AF relay systems in the presence of a random number of 
arbitrary non-identical interferers at the destination is presented, where the 
effects of thermal noise at both relay and destination are taken into account.
To derive the exact expressions for average error rates, a new non-direct mathematical method is developed which leads to the derivation of a new explicit expression for the MGF of SINR in terms of the MGF of the aggregate interferences’ power. The explicit expression then leads to the derivation of the average of complementary error function and its square appearing in the most formula of bit and symbol error rates of different types of digital modulations, in a simple and an explicit way. Despite the importance of Rician and composite multipath shadowing in micro-cellular mobile, indoor radio and land mobile satellite systems, only few studies considered such channel models for the desired user channels. Consequently, another vital contribution in this chapter is to extend the proposed analytical approach for different fading models involving Nakagami-\( m \), Rician and composite Nakagami-\( m \)/lognormal fading models. The analysis in this chapter concluded by giving a more practical example where numerical results are given for bit error rates of a cooperative relay system in heterogeneous cellular networks in the presence of arbitrary and a Poisson field of interferers. The new results in this chapter greatly simplify performance evaluation of AF relay systems over diverse practical interference scenarios and simulation results are provided to demonstrate the accuracy of the new analytical expressions.

The outline for the rest of this chapter is as follows. The system model is presented in Sec. 4.2. Thereafter, in Sec. 4.3, a simple novel expression for the MGF of SINR in terms of the MGF of arbitrary CCIs is developed. In Sec. 4.4, results for different fading models are presented. An application example for a cellular relay system is provided in Sec. 4.5 and summary of the chapter is presented in Sec. 4.6.

### 4.2 The AF Relay System Model with CCIs

Consider an OWR cooperative system in a three-terminal wireless network as shown in Fig. 4.1, where there is an intermediate AF relay node \( R \) assisting transmission from \( T_1 \) to \( T_2 \). The arrangement is such that \( T_1 \) acts as a source while \( R \) acts as a relay node and \( T_2 \) acts as a destination node. We consider that the signal at \( T_2 \) is corrupted by \( L \) interferers. All nodes are also equipped with a single antenna, and assumed that there is no direct source-destination link due to the large distance between source and destination. As mentioned
in Sec. 3.2.2, the communication from $T_1$ to $T_2$ takes place in two separate phases; transmission from $T_1$ to $R$ (source-relay link), and transmission from $R$ to $T_2$ (relay-destination link).

The source transmits its symbol $\xi_0$ to the relay node with $\mathbb{E}[|\xi_0|^2] = 1$. The received signal at the relay thus can be written as

$$y_r = \sqrt{P_1} \xi_0 g + n_1$$

where $P_1$ is the transmit power of the source, $g$ is the complex channel gain between the source and relay; and $n_1$ is the AWGN at the relay with zero mean and an average power of $\sigma_1^2$.

In the second phase, the relay simply amplifies the received signal with a fixed gain $G$, then forwards it to the destination which suffers from $L$ number of interferers $\xi_l, l = 1, 2, ..., L$ where each interferer has $\mathbb{E}[|\xi_l|^2] = 1$. The received signal at the destination is expressed as

$$y_2 = \left[\sqrt{P_1} \xi_0 h_1 + n_1 \right] G f + n_2 + \sum_{l=1}^{L} \xi_l v_l$$

where $f$ denotes the complex channel gain between the relay and destination, $n_2$ is AWGN at the destination with zero mean and an average power of $\sigma_2^2$; and $v_l, l = 1, 2, ..., L$ are the complex channel gain from the interferers to the destination which are independent and can have arbitrary and non-identical distributions. In a more practical example, $L$ can be a RV as will be seen in Sec. 4.5.3. It is also worth mentioning that for clarity in the presentation the power of interferes are assumed to be one.

From (4.2), it is not difficult to show that SINR at the destination denoted by $\gamma$ can be expressed as

$$\gamma = \frac{P_1 G^2 |g|^2 |f|^2}{\sigma_1^2 |f|^2 + \sigma_2^2 + \sum_{l=1}^{L} |v_l|^2}$$

where $G = \sqrt{\frac{1}{P_1 \mathbb{E}[|g|^2] + \sigma_1^2}}$ and $\mathbb{E}[\cdot]$ denotes the expectation operator.

For convenience, let $i_l = |v_l|^2$, $x = \frac{|f|^2}{\mathbb{E}[|f|^2]}$, $y = \frac{|g|^2}{\mathbb{E}[|g|^2]}$, $a = \frac{\sigma_1^2}{P_1 \mathbb{E}[|g|^2]}$, $b = \frac{(1+a)\sigma_2^2}{\mathbb{E}[|f|^2]}$.
and \( c = \frac{b}{\sigma^2} \). Then (4.3) becomes

\[
\gamma = \frac{xy}{ax + b + c \sum_{l=1}^{L} i_l}. \tag{4.4}
\]

In the rest of this chapter, the unified error rate analysis of an AF relay system over Rayleigh/Rayleigh fading models in the presence of thermal noise and arbitrary CCIIs is presented and thereafter extended to include other fading models.

### 4.3 Rayleigh/Rayleigh Channel Models

Let’s consider Rayleigh fading for both source-relay and relay-destination links. In this case \( x \) and \( y \) become exponentially distributed RVs, see Sec. 2.2.2.1, and thus the MGF associated with these RVs is \( \mathcal{M}_{xy} (z) = (1 + z)^{-1} \).

In order to derive an explicit expression for the MGF of (4.4), let’s first condition on \( x \) and \( i \), where \( i = (i_1, i_2, ..., i_L) \). Therefore,

\[
\mathcal{M}_\gamma (z|x,i) = \mathbb{E} \left[ e^{-z\gamma |x,i} \right] = \mathbb{E} \left[ \frac{-z}{e^{xz + b + c \sum_{l=1}^{L} i_l}} | x, i \right] = \frac{1}{1 + \frac{z x}{x a + b + c \sum_{l=1}^{L} i_l}}. \tag{4.5}
\]
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After some simple algebraic manipulations, (4.5) can be rewritten as

\[
\mathcal{M}_\gamma (z|x, i) = \frac{ax + b + c \sum_{l=1}^{L} i_l}{xa + b + c \sum_{l=1}^{L} i_l + zx} = 1 - \frac{zx}{x (z + a) + c \sum_{l=1}^{L} i_l + b}.
\] (4.6)

Invoking the well-known identity \( \int_0^\infty e^{-su}ds = \frac{1}{u} \) [22, eq. (3.3814)], (4.6) can be expressed in the form of

\[
\mathcal{M}_\gamma (z|x, i) = 1 - z \int_0^\infty e^{-sx(z+a)}e^{-sc\sum_{l=1}^{L} i_l} e^{-sb}ds.
\] (4.7)

The conditions in (4.7) can be removed by the taking expectations with respect to \( x \) and \( i \). Thus the unconditional MGF is

\[
\mathcal{M}_\gamma (z) = 1 - z \int_0^\infty \mathbb{E} [xe^{-sx(z+a)}] \mathbb{E} [e^{-sc\sum_{l=1}^{L} i_l}] e^{-sb}ds.
\] (4.8)

Since RV \( x \) is considered to be exponentially distributed, the expectation, \( \mathbb{E} [xe^{-sx(z+a)}] \), in (4.8) is

\[
\mathbb{E} [xe^{-sx(z+a)}] = \int_0^\infty xe^{-x(sz+sa+1)}dx
\] (4.9)

which can be evaluated by invoking the identity [22, eq. (3.3814)]

\[
u^{-k} = \int_0^\infty \frac{x^{k-1}}{\Gamma (k)}e^{-xu}dx \quad k > 0
\] (4.10)

as,

\[
\mathbb{E} [xe^{-sx(z+a)}] = \frac{1}{(sz + sa + 1)^2}.
\] (4.11)

Substituting (4.11) into (4.8), the following simplified expression is obtained

\[
\mathcal{M}_\gamma (z) = 1 - z \int_0^\infty \frac{1}{(1 + sa + sz)^2} \mathcal{M}_i (sc) e^{-sb}ds
\] (4.12)

where

\[
\mathcal{M}_i (sc) = \mathbb{E} [e^{-sc\sum_{l=1}^{L} i_l}] = \prod_{l=1}^{L} \mathbb{E} [e^{-sci_l}]
\] (4.13)

is the MGF of the accumulated interference at the destination node which depends on the interference scenarios. Specific examples will be given in Sec.
4.5
In the reminder of this section, the new expression, (4.12), is employed to derive simple expressions for the average of some common functions which appear in most formulas of bit and symbol error rates of different types of digital modulation over AWGN.

4.3.1 Average of the Complementary Error Function
As discussed in Sec. 2.3, the complementary error function, \( \text{erfc} \left( \sqrt{d\gamma} \right) \), where \( d \) is a constant and depends on the specific modulation/detection combination, can be written in an alternative form by using the following identity [18, eq. (4.2)]

\[
erfc \left( \sqrt{d\gamma} \right) = \frac{2}{\pi} \int_0^{\pi/2} e^{-\gamma \frac{d}{\sin^2 \theta}} d\theta. \tag{4.14}
\]

Taking the average of (4.14)

\[
\mathbb{E} \left[ \text{erfc} \left( \sqrt{d\gamma} \right) \right] = \frac{2}{\pi} \int_0^{\pi/2} \mathcal{M}_i \left( \frac{d}{\sin^2 \theta} \right) d\theta \tag{4.15}
\]

and substituting (4.12) into (4.15), where \( z = d/\sin^2 \theta \), gives the expression for the average complementary error function as

\[
\mathbb{E} \left[ \text{erfc} \left( \sqrt{d\gamma} \right) \right] = \frac{2}{\pi} \int_0^{\pi/2} \left\{ 1 - \int_0^{\infty} \frac{d \sin^2 \theta}{(1 + sa \sin^2 \theta + sd)^2} \mathcal{M}_i (sc) e^{-sb} d\theta \right\} d\theta. \tag{4.16}
\]

The expression in (4.16) can be further simplified. For this, interchanging the order of integration in (4.16) to get

\[
\mathbb{E} \left[ \text{erfc} \left( \sqrt{d\gamma} \right) \right] = 1 - \int_0^{\infty} \left\{ \frac{2}{\pi} \int_0^{\pi/2} \frac{d \sin^2 \theta}{(1 + sa \sin^2 \theta + sd)^2} d\theta \right\} \mathcal{M}_i (sc) e^{-sb} d\theta. \tag{4.17}
\]

Then the inner integral in (4.17) can be shown to be

\[
\frac{2}{\pi} \int_0^{\pi/2} \frac{\sin^2 \theta}{(1 + sa \sin^2 \theta + s)^2} d\theta = \frac{d}{2\sqrt{sd}(sa + sd + 1)^{3/2}}. \tag{4.18}
\]
Substituting (4.18) into (4.17) gives the following final expression

\[ E \left[ \text{erfc} \left( \sqrt{d\gamma} \right) \right] = 1 - \frac{1}{2} \int_{0}^{\infty} \frac{d}{\sqrt{sd(\text{sa} + sd + 1)^2}} \mathcal{M}_i(s^c) e^{-sb} ds \quad (4.19) \]

which is the average of the complementary error function in terms of the MGF of the CCIs components \((i_1, i_2, ..., i_L)\).

### 4.3.2 Average of the Square of Complementary Error Function

Sec. 4.3.1 can be extended to the square complementary error function. As mentioned in Sec. 2.3.2, SER of MQAM are usually given in terms of square complementary error function, \(\text{erfc}^2 \left( \sqrt{d\gamma} \right)\). For this, recall the alternative form of \(\text{erfc}^2 \left( \sqrt{d\gamma} \right)\) \([18, \text{eq. (4.9)}]\)

\[ \text{erfc}^2 \left( \sqrt{d\gamma} \right) = \frac{4}{\pi} \int_{0}^{\pi/4} e^{-\gamma sd} \sin^2 \theta d\theta. \quad (4.20) \]

Taking the average of (4.20)

\[ E \left[ \text{erfc}^2 \left( \sqrt{d\gamma} \right) \right] = \frac{4}{\pi} \int_{0}^{\pi/4} \mathcal{M}_i \left( \frac{d}{\sin^2 \theta} \right) d\theta \quad (4.21) \]

and then substituting (4.12) in (4.21) and interchanging the order of integration, the following average expression is obtained

\[ E \left[ \text{erfc}^2 \left( \sqrt{d\gamma} \right) \right] = 1 - \int_{0}^{\infty} \left\{ \frac{4}{\pi} \int_{0}^{\pi/4} \frac{d \sin^2 \theta}{(1 + \text{sa}) \sin^2 \theta + sd} \right\} d\theta \times \mathcal{M}_i(s^c) e^{-sb} ds. \quad (4.22) \]

where the inner integral in (4.22) can be evaluated in the closed-form of

\[ \frac{4}{\pi} \int_{0}^{\pi/4} \frac{d \sin^2 \theta}{(1 + \text{sa}) \sin^2 \theta + sd} d\theta = \frac{2d \left( \frac{a}{d} + \frac{1}{sa} + 2 \right) \tan^{-1} \left( \sqrt{\frac{a}{d} + \frac{1}{sd} + 1} \right) - \sqrt{\frac{a}{d} + \frac{1}{sa} + 1}}{\sqrt{sd} \left( \frac{a}{d} + \frac{1}{sd} + 2 \right) \sqrt{(sa + sd + 1)^3}}. \quad (4.23) \]
Substituting (4.23) into (4.22), \( E \left[ \text{erfc}^2 \left( \sqrt{d\gamma} \right) \right] \) can be simplified as

\[
E \left[ \text{erfc}^2 \left( \sqrt{d\gamma} \right) \right] = 1 - \frac{2b}{\pi} \int_0^{\infty} \left( \frac{a}{d} + \frac{1}{sd} + 2 \right) \tan^{-1} \left( \frac{a}{d} + \frac{1}{sd} + 1 \right) - \frac{a}{d} + \frac{1}{sd} + 1 \right) \\
\times \sqrt{sd} \left( \frac{a}{d} + \frac{1}{sd} + 2 \right) (sa + sd + 1)^{3/2} \\
\times M_i (sc) e^{-sb} ds. \tag{4.24}
\]

In summary, (4.12), (4.19) and (4.24) are the new exact and key results over Rayleigh/Rayleigh fading channel models which can be employed to derive average SER and BER expressions of digital modulations for AF relay cooperative systems in presence of arbitrary CCIs. In the next section, the average error rate for two common digital modulation types, MPSK and MQAM, are investigated based on the new exact developed expressions.

### 4.3.3 Error Rate Analysis

#### 4.3.3.1 Average BER of MPSK

BER of BPSK in AWGN is given by [72, pp. 356]

\[
p_{\text{BPSK}} (\gamma) = \frac{1}{2} \text{erfc} \left( \sqrt{\gamma} \right) \tag{4.25}
\]

where \( d = 1 \). Using (4.19) the average of (4.25) is given by

\[
p_{\text{BPSK}} = \frac{1}{2} - \frac{1}{4} \int_0^{\infty} \frac{1}{\sqrt{sd} (sa + s + 1)^{3/2}} M_i (sc) e^{-sb} ds \tag{4.26}
\]

A unified approximation formula for the approximate BER of MPSK for \( M \geq 4 \) in AWGN is given by [73, eq. (12)]

\[
p_{\text{MPSK}} (\gamma) \approx \frac{1}{\log_2 M} \sum_{n=1}^{M/4} \text{erfc} \sqrt{\sin^2 \left( \frac{2n-1}{M} \right) \gamma} \tag{4.27}
\]

where \( \sin^2 \left( \frac{2n-1}{M} \right) = d \).

Replacing (4.19) in (4.27) gives the following expression for the average BER of MPSK in AF relay systems in the presence of arbitrary CCIs

\[
p_{\text{MPSK}} = \frac{M}{4\log_2 M} - \frac{1}{2\log_2 M} \int_0^{\infty} Q(s) M_i (sc) e^{-sb} ds \tag{4.28}
\]
where,
\[
Q(s) = \sum_{n=1}^{M/4} \frac{\sin^2 \left( \frac{\pi 2n-1}{M} \right)}{\sqrt{s \sin^2 \left( \frac{\pi 2n-1}{M} \right) \left( sa + \sin^2 \left( \frac{\pi 2n-1}{M} \right) s + 1 \right)^2}}. \tag{4.29}
\]

### 4.3.3.2 Average SER of MPSK

The conditional SER of MPSK is given by [74, eq. (2)]
\[
p_{\text{MPSK}}(\gamma) = \frac{1}{\pi} \int_{0}^{\pi/\gamma} e^{-\gamma \frac{\sin^2 (\pi/M)}{\sin^2 \theta}} \, d\theta \tag{4.30}
\]
where \(\sin^2 (\pi/M) = d\). Taking average of (4.30) as
\[
p_{\text{MPSK}}(\gamma) = \int_{0}^{\infty} \frac{1}{\pi} \int_{0}^{\pi/\gamma} e^{-\gamma \frac{d \sin^2 \theta}{\sin^2 \theta}} \, d\theta \, d\gamma
= \frac{1}{\pi} \int_{0}^{\pi/\gamma} \mathcal{M}_\gamma \left( \frac{d}{\sin^2 \theta} \right) \, d\theta \tag{4.31}
\]
and applying (4.12), average SER of MPSK can easily be obtained by
\[
p_{\text{MPSK}} = \left( 1 - \frac{1}{M} \right) - \int_{0}^{\infty} Q(s) \, \mathcal{M}_i(s) \, e^{-sb} \, ds \tag{4.32}
\]
where,
\[
Q(s) = \frac{1}{\pi} \int_{0}^{\pi/\gamma} \frac{\sin^2 (\pi/M) \sin^2 \theta}{\left( \sin^2 \theta (1 + sa) + \sin^2 (\pi/M) s \right)^2} \, d\theta. \tag{4.33}
\]
Note that (4.32) reduces into following expression when \(M = 2\)
\[
p_{\text{BPSK}} = \frac{1}{2} - \frac{1}{4} \int_{0}^{\infty} \frac{1}{\sqrt{s (sa + s + 1)^2}} \, \mathcal{M}_i(s) \, e^{-sb} \, ds. \tag{4.34}
\]
which can also be obtained by using (4.19) for the average of \(\frac{1}{2}E \left[ \text{erfc} \left( \sqrt{\gamma} \right) \right]\).

### 4.3.3.3 Average BER of MQAM

General expression for bit error probabilities of generalised square MQAM with Gray coding are giving in [75]. This can be expressed as:
\[
p_{\text{MQAM}}(\gamma) = \sum_{n=0}^{\sqrt{M^2-2}} q_M(n) \, \text{erfc} \left( \frac{\sqrt{3} (2n+1)^2}{2 (M-1) \gamma} \right). \tag{4.35}
\]
where \((3(2n+1)^2/2(M-1)) = d\) and \(q_M(n)\) are some constant that depends on \(M\), and \(\sum_{n=1}^{\sqrt{M}-2} q_M(n) = \frac{1}{2} [75]\). Using (4.19) the average BER of MQAM is given by

\[
p_{\text{MQAM}} = \frac{1}{2} - \frac{1}{4} \int_0^\infty Q(s) \mathcal{M}_i(sc) e^{-sb} ds \tag{4.36}
\]

where

\[
Q(s) = \sum_{n=0}^{\sqrt{M}-2} \left( \frac{3(2n+1)^2}{2(M-1)} \right) q_M(n) \sqrt{s \left( \frac{3(2n+1)^2}{2(M-1)} \right) \left( sa + s \left( \frac{3(2n+1)^2}{2(M-1)} + 1 \right) \right)^{\frac{3}{2}}}.
\tag{4.37}
\]

Furthermore, it can also be verified that quadrature phase shift keying (QPSK) is a special case of (4.36) when \(M = 4\). Therefore, the explicit expression for average BER of QPSK is obtained by

\[
p_{\text{QPSK}} = \frac{1}{2} - \frac{1}{8} \int_0^\infty \frac{1}{\sqrt{\frac{3}{2} (sa + s + 1)^{\frac{3}{2}}}} \mathcal{M}_i(sc) e^{-sb} ds.
\tag{4.38}
\]

### 4.3.3.4 Average SER of MQAM

The conditional SER of Gray-coded square MQAM is [76, eq. (10.32)]

\[
p_{\text{QAM}}(\gamma) = 1 - \left( 1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left( \sqrt{\frac{3}{2} (M-1)} \gamma \right) \right)^2
\tag{4.39}
\]

where \((3/2(M-1)) = d\). The average of (4.39) can be obtained by applying (4.19) and (4.24) in (4.39). Following expression for the average SER of QAM is obtained

\[
p_{\text{QAM}} = 1 - \frac{1}{M} - \int_0^\infty \frac{(1 - \frac{1}{\sqrt{M}}) d}{\sqrt{sd (sa + sd + 1)^{\frac{3}{2}}}} A_M(s) \mathcal{M}_i(sc) e^{-sb} ds
\tag{4.40}
\]

where

\[
A_M(s) = \left\{ 1 - \left( 2 - \frac{2}{\sqrt{M}} \right) \left( \frac{s}{d} + \frac{1}{sd} + 2 \right) \tan^{-1} \left( \sqrt{\frac{s}{d} + \frac{1}{sd} + 1} \right) - \sqrt{\frac{s}{d} + \frac{1}{sd} + 1} \right\}.
\tag{4.41}
\]
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Figure 4.2: Average BER using BPSK modulation with respect to average SNR of 2\textsuperscript{nd} phase, $\text{SNR}_x$, where desired channels consider to be Rayleigh in presence of Nakagami-$m$ CCIs for different number of interferences $L$ and $\text{SNR}_i$, where $\text{SNR}_y = 40\text{dB}$ and $m_I = 2$.

4.3.4 Results

Equations (4.28), (4.32), (4.36) and (4.40) are new exact expressions for bit and symbol error rates of transmission over Rayleigh/Rayleigh fading channels in presence of arbitrary heterogeneous CCIs at the destination.

In Figs. 4.2, 4.3, 4.4 and 4.5, Monte Carlo simulation is used to validate the new analytical methods developed in Sec. 4.3. For the simulation, the instantaneous received power from the desired signals and from the interferers are randomly generated with the knowledge of channel distributions. Generating RVs for different fading scenarios have been explained in details in Sec. 2.2. The generated RVs are first applied in (4.3) to obtain $\gamma$. Afterward RV $\gamma$ is generated in sequence of independent random variables $\{\gamma_1, \gamma_2, \ldots, \gamma_l\}$ which by the law of large numbers, the average of $\{\gamma_1, \gamma_2, \ldots, \gamma_l\}$ is obtained by $\gamma = \frac{\gamma_1 + \gamma_2 + \ldots + \gamma_l}{l}$. Substituting $\gamma$ into complementary error function or its square gives BER/SER of the system based on the Monte Carlo simulation.

Fig. 4.2 examines average BER of BPSK modulation with respect to the average SNR of 2\textsuperscript{nd} phase, $\text{SNR}_x$, for different number of CCIs $L$ and $\text{SNR}$ of
interfering channels, $\text{SNR}_i$, where average SNR of Phase 1 is $\text{SNR}_y = 40\text{dB}$. From Fig. 4.2, it can be seen that by increasing the number of interferences and $\text{SNR}_i$, the BER is increased as the low SNIR $\gamma$ is expected. This figure also compares the exact results with existing results [60–69], where the impact of the thermal noise on the system performance has been neglected and it shows that, in practice, the system results in a higher BER. The average BER and SER of MPSK and MQAM modulation have been displayed in Fig. 4.3 and Fig. 4.4, respectively. In these figures, an example is considered for arbitrary CCIs under Nakagami-$m$ fading with fading parameter of $m_I = 2$ and show that by increasing SNR in Phase 2, $\text{SNR}_x$, the BER of the system is decreasing as SINR $\gamma$ is increasing. Fig. 4.5 verifies the validity of our generalised expressions for the system with arbitrary non-identical CCIs. In this figure, BPSK modulation is considered for a system over Rayleigh/Rayleigh fading in presence of CCIs with different fading models including Rician, Nakagami-$m$, composite Nakagami-$m$/ lognormal shadowing which have been plotted with using a single equation presented in (4.12).
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Figure 4.4: Average SER of different modulation schemes with respect to SNR$_x$ over Rayleigh fading channels and Nakagami-$m$ CCIs, where $L = 2$, SNR$_i = 0$dB, SNR$_y = 40$dB and $m_I = 2$.

Figure 4.5: Average BER using BPSK modulation with respect SNR$_x$ over Rayleigh fading channels and for different CCIs fading models (e.g, Rician, composite Nakagami-$m$/lognormal shadowing), where $L = 2$ and SNR$_i = 0$dB.
4.4 Nakagami and Rician Channel Models

In this section, the analysis in Sec. 4.3 is extended to include more fading models for the desired signal channels.

4.4.1 Nakagami-\textit{m}/Nakagami-\textit{m} Fading

The desired signals’ channel are considered to be Nakagami-\textit{m} fading. As discussed in Sec. 2.2.2.3, such fading scenario finds applicability in land and indoor mobiles where no LOS propagation exists between the source-relay and relay-destination nodes. As far as the Nakagami-\textit{m} fading channel for source-relay link is concerned, \( y \) has a gamma PDF given by [18, eq. (2.21)], see also Sec. 2.2.2.3. Thus,

\[
f_{\text{Nak}}(y) = \frac{m_2^m y^{m_2 - 1}}{\Gamma(m_2)} e^{-m_2 y \text{ } y > 0} \tag{4.42}
\]

where \( m_2 \) is the Nakagami-\textit{m} parameter of the source-relay link and \( \Gamma(.) \) represents the gamma function [18, eq. (2.21)]. It can also be shown from (4.42) that the MGF of the RV \( y \) in this case is given by [18, eq. (2.22)]

\[
\mathcal{M}_y(z) = \left( \frac{1}{1 + \frac{z}{m_2}} \right)^{m_2}. \tag{4.43}
\]

The \( \mathcal{M}_\gamma(z) \), (4.4), can be obtained by taking the MGF with respect to the RV \( y \), conditioned on the RVs \( x \) and \( i \). Therefore, by using (4.43), the conditional MGF of \( \gamma \) is given by

\[
\mathcal{M}_\gamma(z|x, i) = \mathbb{E}[e^{-z\gamma}|x, i] = \left( \frac{1}{1 + \frac{zx}{m_2(xa + b + c \sum_{l=1}^L i_l)}} \right)^{m_2} \tag{4.44}
\]

which by some simple manipulations, it can be expressed as

\[
\mathcal{M}_\gamma(z|x, i) = \left( 1 - \frac{zx}{m_2 x (a + \frac{z}{m_2}) + b + c \sum_{l=1}^L i_l} \right)^{m_2}. \tag{4.45}
\]
Expansion of (4.45), using the binomial theories [77], result in

\[ M_\gamma (z|\bar{i}, x) = 1 + \sum_{k=1}^{\infty} \binom{m_2}{k} \left( -\frac{z}{m_2} \right)^k \frac{x}{x(a + \frac{z}{m_2}) + b + c \sum_{l=1}^{L} \bar{i}_l} \]  

(4.46)

where \( \binom{m_2}{k} \) is a binomial coefficient given by \( \binom{m_2}{k} = \frac{m_2(m_2-1)(m_2-2)\ldots(m_2-k+1)}{k!} \).

Using (4.10), the last term in (4.46) can be written as

\[ \left( \frac{x}{x(a + \frac{z}{m_2}) + b + c \sum_{l=1}^{L} \bar{i}_l} \right)^k = x^k \int_{0}^{\infty} \frac{s^{k-1}}{\Gamma(k)} e^{-sx(a + \frac{z}{m_2}) e^{-sc\sum_{l=1}^{L} \bar{i}_l} e^{-sb} ds}. \]

(4.47)

Therefore

\[ \mathbb{E} \left[ \left( \frac{x}{x(a + \frac{z}{m_2}) + b + c \sum_{l=1}^{L} \bar{i}_l} \right)^k \right] = \int_{0}^{\infty} \frac{s^{k-1}}{\Gamma(k)} \mathbb{E} \left[ x^k e^{-sx(a + \frac{z}{m_2})} \right] \mathbb{E} \left[ e^{-sc\sum_{l=1}^{L} \bar{i}_l} \right] e^{-sb} ds \]

(4.48)

where the expectation \( \mathbb{E} \left[ x^k e^{-sx(a + \frac{z}{m_2})} \right] \) is obtained by averaging out the RV \( x \) over its PDF. As far as Nakagami-\( m \) fading is concerned for relay-destination link, \( x \) has the gamma PDF given by \( f_{Nak}(x) = \frac{x^{m_2-1}}{\Gamma(m_1)m_1^{m_1}} \exp(-m_1x) \), \( x > 0 \) [18, eq. (2.21)], where \( m_1 \) is the fading parameter for the relay-destination link. Consequently, by using (4.10), \( \mathbb{E} \left[ x^k e^{-sx(a + \frac{z}{m_2})} \right] \) can be obtained as

\[ \mathbb{E} \left[ x^k e^{-sx(a + \frac{z}{m_2})} \right] = \frac{\Gamma(m_1 + k)}{\Gamma(m_1)} m_1^{-k} \left( \frac{1}{1 + \frac{s}{m_1} \left( a + \frac{z}{m_2} \right)} \right)^{m_1+k}. \]

(4.49)
Substituting (4.49) into (4.48) to get

$$
E \left[ \left( \frac{x}{x(a + \frac{z}{m_2}) + b + e^{-sc\sum_{i=1}^{L} h_i}} \right)^k \right] =
$$

$$
\int_{0}^{\infty} s^{k-1} \left\{ \frac{\Gamma (m_1 + k)}{\Gamma (m_1) \Gamma (k)} m_1^{-k} \left( \frac{1}{1 + \frac{s}{m_1} (a + \frac{z}{m_2})} \right)^{m_1+k} \right\} M_i(sc) e^{-sb} ds \quad (4.50)
$$

and then applying in (4.46), gives the unconditional $M_\gamma(z)$ expression

$$
M_\gamma(z) = 1 + \sum_{k=1}^{\infty} \binom{m_2}{k} \left( -\frac{z}{m_2} \right)^k
\times \int_{0}^{\infty} s^{k-1} \left\{ \frac{\Gamma (m_1 + k)}{\Gamma (m_1) \Gamma (k)} m_1^{-k} \left( \frac{1}{1 + \frac{s}{m_1} (a + \frac{z}{m_2})} \right)^{m_1+k} \right\} M_i(sc) e^{-sb} ds \quad (4.51)
$$

The expression in (4.51) can be further simplified. For this, first, (4.51) is rearranged to give

$$
M_\gamma(z) = 1 + \int_{0}^{\infty} \left( \frac{1}{1 + \frac{s}{m_1} (a + \frac{z}{m_2})} \right)^{m_1}
\times \frac{1}{s} \left\{ \sum_{k=1}^{\infty} \binom{m_2}{k} \frac{\Gamma (m_1 + k)}{\Gamma (m_1) \Gamma (k)} \left( -\frac{zs}{m_1 m_2 + s (m_2 a + z)} \right)^k \right\} M_i(sc) e^{-sb} ds \quad (4.52)
$$

then the summation term in (4.52) is reduced to

$$
\sum_{k=1}^{\infty} \binom{m_2}{k} \frac{\Gamma (m_1 + k)}{\Gamma (m_1) \Gamma (k)} \left( -\frac{zs}{m_1 m_2 + s (m_2 a + z)} \right)^k
= -\frac{m_1 m_2 sz}{m_1 m_2 + s (m_2 a + z)} \text{}_{2}F_{1} \left( 1 + m_1, 1 - m_2; 2; \frac{sz}{m_1 m_2 + sm_2 a + zs} \right) \quad (4.53)
$$

where $\text{}_{2}F_{1} (a, b; c; A) = \sum_{k=0}^{\infty} \frac{\Gamma(a+k) \Gamma(b+k) \Gamma(c)}{\Gamma(a) \Gamma(b) \Gamma(c+k) k!} A^k$ is the hypergeometric function. See below for the proof of (4.53).
Proof. Consider the summation term with arbitrary \( A, m_1, m_2 \)

\[
\sum_{k=1}^{\infty} \binom{m_2}{k} \frac{\Gamma (m_1 + k)}{\Gamma (m_1) \Gamma (k)} (-A)^k.
\] \hspace{1cm} (4.54)

By applying the upper binomial identity \( \binom{n}{k} = (-1)^k \binom{k-1-n}{k} \) [22] into (4.54), it could be observed that

\[
\sum_{k=1}^{\infty} \binom{m_2}{k} \frac{\Gamma (m_1 + k)}{\Gamma (m_1) \Gamma (k)} (-A)^k = \sum_{k=1}^{\infty} \binom{k-1-m_2}{k} \frac{\Gamma (m_1 + k)}{\Gamma (m_1) \Gamma (k)} A^k
\]

\[
= \sum_{k=0}^{\infty} \binom{k-m_2}{k+1} \frac{\Gamma (m_1 + k + 1)}{\Gamma (m_1) \Gamma (k + 1)} A^{k+1}
\]

\[
= A \sum_{k=0}^{\infty} \frac{\Gamma (k-m_2+1)}{\Gamma (-m_2) \Gamma (k+2)} \frac{\Gamma (m_1 + k + 1)}{\Gamma (m_1)} A^k.
\] \hspace{1cm} (4.55)

After some simple manipulations, the last line in (4.55) can also be expressed as

\[
A \sum_{k=0}^{\infty} \frac{\Gamma (k-m_2+1) \Gamma (m_1 + k + 1)}{\Gamma (-m_2) \Gamma (k+2) \Gamma (m_1)} \frac{A^k}{k!} =
\frac{A}{\Gamma (2)} \frac{\Gamma (1-m_2) \Gamma (m_1 + 1)}{\Gamma (-m_2) \Gamma (m_1)} \sum_{k=0}^{\infty} \frac{\Gamma (k+1-m_2) \Gamma (m_1 + k + 1)}{\Gamma (1-m_2) \Gamma (m_1 + 1)} \frac{\Gamma (2) \ A^k}{\Gamma (k+2) \ k!}.
\] \hspace{1cm} (4.56)

Recall the hypergeometric function defined as [22]

\[
\binom{a}{b} \binom{c}{d} = \sum_{k=0}^{\infty} \frac{\Gamma (a+k) \Gamma (b+k) \Gamma (c) \ A^k}{\Gamma (a) \Gamma (b) \Gamma (c+k) \ k!}
\] \hspace{1cm} (4.57)

(4.56) can be reduced to hypergeometric function. Therefore,

\[
\sum_{k=1}^{\infty} \binom{m_2}{k} \frac{\Gamma (m_1 + k)}{\Gamma (m_1) \Gamma (k)} (-A)^k = -m_1 m_2 A \binom{1}{2} \binom{1+m_1, 1-m_2}{2; A}.
\] \hspace{1cm} (4.58)
Substituting (4.53) in (4.52) yields
\[
M_\gamma (z) = 1 - \int_0^\infty \frac{z}{\left(1 + \frac{s}{m_1}a + \frac{s^2}{m_1 m_2}\right)^{m_1+1}} \times _2F_1 \left(1 + m_1, 1 - m_2; 2; \frac{s^2}{m_1 m_2 + sm_2a + zs}\right) M_i (sc) e^{-sb} ds
\]
which is a new expression for the MGF of SINR of an AF relay system over Nakagami-\(m\) fading in terms of the MGF of the aggregate interference power.

### 4.4.2 Rician/Nakagami-\(m\) Fading

In this part, the investigation is extended to Rician and Nakagami-\(m\) fading for source-relay and relay-destination links respectively. Such fading scenario finds its applicability in land and indoor mobiles where LOS propagation exists between the source and the relay node. As far as Rician distribution is considered for source-relay link, \(y\) has a non-central chi-square distribution given by [18, Eq. (2.16)] (see also Sec. 2.2.2.5)
\[
f_\text{Ric}(y) = (1 + K)e^{-y/2}e^{-y(1+K)/2}I_0 \left(2\sqrt{K (1+K)} y\right).
\]

Therefore, it can be shown that the MGF associated with this fading is [18, Eq. (2.17)]
\[
M_y (z) = \frac{1 + K}{1 + K + z} e^{-\frac{Kz}{1 + K + z}}
\]
where \(K\) is the Rician factor.

To simplify the presentation, let \(t = b + c \sum_{i=1}^L i_i\). Then from (4.61) the conditional \(M_\gamma (z|x, i)\) (conditioned on \(x\) and \(i\)) with respect to \(y\) is given by
\[
M_\gamma (z|x, i) = \mathbb{E} \left[e^{-\frac{y}{ax+t}} | x, i \right] = \frac{1 + K}{1 + K + z} e^{-\frac{K}{1 + K + z} \frac{y}{ax+t}}.
\]
After some simple algebraic manipulations, (4.62) can be expressed as
\[
M_\gamma (z|x, i) = A(z, x, i) - B(z, x, i)
\]
where,
\[
A(z, x, i) = e^{-\frac{K}{1 + K + z} \frac{x}{ax+t}}
\]
and

\[ B(z, x, i) = \frac{zx}{(a + aK + z)x + t(1 + K)} e^{-\frac{Kzx}{(a + aK + z)x + t(1 + K)}}. \] (4.65)

### 4.4.2.1 Computation of the \( A(z) \)

Using the power series representation of the exponential function, which is defined as

\[ e^u = \sum_{k=0}^{\infty} \frac{u^k}{k!} \] (4.66)

(4.64) leads to

\[ A(z, x, i) = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \left( \frac{-Kzx}{(a + aK + z)x + t(1 + K)} \right)^k. \] (4.67)

Now, invoking the identity (4.10) in (4.67) and removing the conditions by averaging out with respect to the RVs \( x \) and \( i \), gives

\[ A(z) = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} \left( -Kz \right)^k \int_0^{\infty} \frac{s^{k-1} e^{-sx(a+aK+z)}}{\Gamma(k)} \mathbb{E} \left[ e^{-st(1+K)} \right] ds \] (4.68)

The expectation \( \mathbb{E} \left[ x^k e^{-sx(a+aK+z)} \right] \) is obtained by averaging out the RV \( x \) over its PDF. As far as Nakagami-\( m \) fading is also concerned for relay-destination link, \( x \) has the gamma PDF with fading parameter of \( m \) given by

\[ f_{Nak}(x) = \frac{m^{m-1} x^{m-1} \exp(-mx)}{\Gamma(m)} \] [18, eq. (2.21)].

Recall the identity in (4.10), the \( \mathbb{E} \left[ x^k e^{-sx(a+aK+z)} \right] \) term in (4.68) can be obtained by

\[ \mathbb{E} \left[ x^k e^{-sx(a+aK+z)} \right] = \frac{\Gamma(m+k)}{\Gamma(m)} m^{-k} \left( \frac{1}{1 + \frac{x}{m} (a + aK + z)} \right)^{m+k}. \] (4.69)

Substituting (4.69) into (4.68) to get

\[ A(z) = 1 + \int_0^{\infty} \left\{ \sum_{k=1}^{\infty} \frac{(-Kz)^k}{k!} \frac{s^{k-1}}{\Gamma(k)} \frac{\Gamma(m+k)}{\Gamma(m)} m^{-k} \left( \frac{1}{1 + \frac{s}{m} (a + aK + z)} \right)^{m+k} \right\} \right\} \times \mathcal{M}_i \left( sc(1 + K) \right) e^{-s(1+K)b} ds \] (4.70)
and then invoking (4.58) to simplify the summation term in (4.70), the following expression is obtained

\[
A(z) = 1 - zK \int_0^\infty \left( \frac{1}{1 + \frac{z}{m} (a (1 + K) + z)} \right)^{m+1} \\
\times \text{I}_{1} \left( m + 1; 2; \frac{-szK}{m + sa (1 + K) + sz} \right) \mathcal{M}_{i} (sc (1 + K)) e^{-s(1+K)k} ds. \tag{4.71}
\]

### 4.4.2.2 Computation of the \( B(z) \)

An alternative representation of (4.65) can be obtained using (4.66) as

\[
B(z, x, i) = \frac{z^x e^{-zKx}}{(a + aK + z) x + t (1 + K)} \\
= \frac{z^x}{(a + aK + z) x + t (1 + K)} \sum_{k=0}^\infty \frac{1}{k!} \left( \frac{-Kz^x}{(a + aK + z) x + t (1 + K)} \right)^{k+1}
\]

which after invoking (4.10) leads to

\[
B(z) = \sum_{k=0}^\infty \frac{(-K)^k z^{k+1}}{k!} \int_0^\infty \frac{s^k}{\Gamma (k+1)} \mathbb{E} [x^{k+1} e^{-zKx}] \\
\times \mathcal{M}_{i} (sc (1 + K)) e^{-s(1+K)k} ds. \tag{4.73}
\]

When \( x \) is a gamma RV, by solving the \( \mathbb{E} [x^{k+1} e^{-zKx}] \) term with respect to \( x \), (4.73) is written as

\[
B(z) = \int_0^\infty \left\{ \sum_{k=0}^\infty \frac{(-K)^k z^{k+1}}{k!} \frac{s^k}{\Gamma (k+1)} \frac{\Gamma (m + k + 1)}{\Gamma (m)} m^{-k-1} \right. \\
\left. \times \left( \frac{1}{1 + \frac{z}{m} (a + aK + z)} \right)^{m+k+1} \right\} \mathcal{M}_{i} (sc (1 + K)) e^{-s(1+K)k} ds. \tag{4.74}
\]
Following method to proof of (4.58), (4.74) can be simplified as

\[ B(z) = z \int_0^\infty \left( 1 + \frac{s}{m} (a + aK + z) \right)^{m+1} \frac{-szK}{m + sa + saK + sz} \times \mathcal{M}_i(s(1+K)) e^{-s(1+K)b} ds. \]  

Combining (4.71) and (4.75) together

\[ \mathcal{M}_\gamma(z) = 1 - z \int_0^\infty \left( \frac{m}{m + sa(1+K) + sz} \right)^{m+1} \mathcal{M}_i(s(1+K)) e^{-s(1+K)b} \times \left\{ K_1 \mathbf{F}_1 \left( m + 1; 2; \frac{-szK}{m + sa(1+K) + sz} \right) 
+ \mathbf{F}_1 \left( m + 1; 1; \frac{-szK}{m + sa(1+K) + sz} \right) \right\} ds \]  

gives the explicit expression for \( \mathcal{M}_\gamma(z) \) over Ricain/Nakagami-\( m \) fading channels in terms of the MGF of CCIs.

### 4.4.3 Composite Nakagami-\( m \)/Lognormal Shadowing

In this part, both source-relay and relay-destination links are considered to be Nakagami-\( m \) faded with lognormal shadowing. This scenario often is observed in land-mobile satellite systems subject to urban shadowing [18]. Since, Nakagami-\( m \) shadow environment is considered for source-relay link, the PDF of \( y \) can be approximately expressed as a composite gamma/longnormal distribution which is given by [26, eq. (3.68)]

\[ f_{\text{Nak/Log}}(y) \simeq \frac{1}{\sqrt{\pi}} \sum_{I=1}^{N_p} H_{\eta_I} \left[ \left( \frac{m_2}{\Phi_I} \right)^{m_2} \frac{y^{m_2-1}}{\Gamma(m_2)} e^{-\frac{m_2^2}{y}} \right] \quad y > 0 \]  

where, \( \Phi_I = \exp \left( \sqrt{2} \sigma_y \eta_I + \mu \right) \). \( H_{\eta_I} \) are the weight factors, \( \eta_I \) are the zeros of the Hermite polynomial, and \( N_p \) is the order of the Hermite polynomial and are given by Table [25.10] of [24]. \( \mu \) (dB) and \( \sigma_y \) (dB) are the mean and standard deviation of \( \ln y \), respectively. The MGF given in this case is given...
by [18, Eq. (2.32)] (See Appendix A.3 for the derivation)

\[
\mathcal{M}_y(z) \simeq \frac{1}{\sqrt{\pi}} \sum_{l=1}^{N_p} H_{\eta l} \left( \frac{1}{1 + \frac{1}{m_2} z} \right)^{m_2}
\]

(4.78)

From (4.78), the MGF of (4.4) with respect to \( y \) conditioned on \( x \) and \( i \), can be obtained by

\[
\mathcal{M}_\gamma(z|x,i) = \frac{1}{\sqrt{\pi}} \sum_{l=1}^{N_p} H_{\eta l} \left( \frac{1}{1 + \frac{1}{m_2} \frac{x \Phi l}{a x + b + c \sum_{i=1}^{L} i}} \right)^{m_2}
\]

\[
= \frac{1}{\sqrt{\pi}} \sum_{l=1}^{N_p} H_{\eta l} \left( 1 - \frac{z \Phi l}{m_2} \frac{x}{a x + \frac{z \Phi l}{m_2} + b + c \sum_{i=1}^{L} i} \right)^{m_2}.
\]

(4.79)

Furthermore, according to the binomial series, \((a + b)^n = \sum_{k=0}^{\infty} \binom{n}{k} a^{n-k} b^k\), it can be shown that the last part of (4.79) admits the following expression

\[
\mathcal{M}_\gamma(z|x,i) = \frac{1}{\sqrt{\pi}} \sum_{l=1}^{N_p} H_{\eta l} \times \left[ 1 + \sum_{k=1}^{\infty} \binom{m_2}{k} \left( -\frac{z \Phi l}{m_2} \right)^k \left( \frac{x}{a x + \frac{z \Phi l}{m_2} + b + c \sum_{i=1}^{L} i} \right)^k \right].
\]

(4.80)

By recalling (4.10)

\[
\mathbb{E} \left[ \left( \frac{x}{a x + \frac{z \Phi l}{m_2} + b + c \sum_{i=1}^{L} i} \right)^k \right] = 
\int_0^{\infty} \frac{s^{k-1}}{\Gamma(k)} \mathbb{E} \left[ x^k e^{-sx(a + \frac{z \Phi l}{m_2})} \right] \mathbb{E} \left[ e^{-s c \sum_{i=1}^{L} i} \right] e^{-sb} ds.
\]

(4.81)

The expectation \( \mathbb{E} \left[ x^k e^{-sx(a + \frac{z \Phi l}{m_2})} \right] \) is obtained by averaging out the RV \( x \) over its PDF. As far as channel for relay-destination link is considered as a composite Nakagami-\(m\)/lognormal distribution, PDF of \( x \) is given by [26, eq.
where, \( \Phi_J = \exp (\sqrt{2\sigma_x} \eta_J + \mu) \). Therefore, the expectation \( \mathbb{E} \left[ x^k e^{-sx(a + \frac{m_2}{m_2})} \right] \) term in (4.81) can be computed by invoking (4.10) as

\[
\mathbb{E} \left[ x^k e^{-sx(a + \frac{m_2}{m_2})} \right] = \frac{1}{\sqrt{\pi}} \sum_{J=1}^{N_p} H_{\eta_J} \Gamma (m_1 + k) \Gamma (m_1) \Gamma (k) \left( \frac{-zs\Phi_J \Phi_I}{m_1 m_2 + s\Phi_J (m_2 a + z\Phi_I)} \right)^k \mathcal{M}_i (sc) e^{-sb} ds
\]  

(4.83)

Now, replacing (4.83) in (4.81) and substituting into (4.80), gives

\[
\mathcal{M}_\gamma (z) = 1 + \frac{1}{\pi} \sum_{J=1}^{N_p} H_{\eta_J} \int_0^\infty \left( \frac{1}{1 + \frac{s\Phi_J}{m_1} (a + \frac{z\Phi_I}{m_2})} \right)^{m_1} \sum_{k=1}^{\infty} \left( \frac{m_2}{k} \right) \Gamma (m_1 + k) \Gamma (m_1) \Gamma (k) \left( \frac{-zs\Phi_J \Phi_I}{m_1 m_2 + s\Phi_J (m_2 a + z\Phi_I)} \right)^k \mathcal{M}_i (sc) e^{-sb} ds
\]  

(4.84)

where \( \frac{1}{\sqrt{\pi}} \sum_{J=1}^{N_p} H_{\eta_J} \sim 1 \) for a large \( N_p \).

Afterward, the summation in (4.84) can be simplified as

\[
\sum_{k=1}^{\infty} \left( \frac{m_2}{k} \right) \Gamma (m_1 + k) \Gamma (m_1) \Gamma (k) \left( \frac{-zs\Phi_J \Phi_I}{m_1 m_2 + s\Phi_J (m_2 a + z\Phi_I)} \right)^k = \frac{-m_1 m_2 z s\Phi_J \Phi_I}{m_1 m_2 + s\Phi_J (m_2 a + z\Phi_I)} \text{$_2$F$_1$} \left( 1 + m_1, 1 - m_2; 2; \frac{zs\Phi_J \Phi_I}{m_1 m_2 + s\Phi_J (m_2 a + z\Phi_I)} \right)
\]  

(4.85)

For the derivation of (4.85) follow the same steps for the proof of (4.53).
Finally, substituting (4.85) into (4.84) gives the expression for $M_\gamma(z)$ as

$$M_\gamma(z) = 1 - \frac{1}{\pi} \sum_{I=1}^{N_p} \sum_{J=1}^{N_p} H_{\eta_I} H_{\eta_J} \int_0^\infty \frac{z \Phi_I \Phi_J}{\left(1 + \frac{z \Phi_I}{m_1} \left(a + \frac{z \Phi_J}{m_2}\right)\right)^{m_1+1}}$$

$$\times {}_2F_1\left(1 + m_1, 1 - m_2; 2, \frac{z s \Phi_I \Phi_J}{m_1 m_2 + s \Phi_J (m_2 a + z \Phi_I)}\right) M_i(s) e^{-s b} ds. \quad (4.86)$$

### 4.4.4 Result

To summarise, performance of AF relay cooperative systems in the presence of arbitrary non-identical interferers at the destination have been analysed in a simple and explicit way. The model considered different fading scenarios for the desired channels such as Nakagami-$m$/Nakagami-$m$, Rician/Nakagami-$m$ and composite Nakagami-$m$/lognormal distributions. Monte Carlo simulations were performed and results presented in Fig. 4.6, which show perfect agreement with the generalised expressions in (4.59), (4.76) and (4.86). In the figure, solid lines and cross markers show the analytical and simulation results, respectively. To plot this figure, the new expressions (4.59), (4.76) and (4.86) were first applied in (4.19) then substituted into the expression (4.26). It is also worth mentioning that interference channel models were assumed to be Rician with fading parameter of $K_I = 1$.

In order to study the influence of SNR variance, a measure that accounts its variability is required. The selected measure of severity of fading is the amount of fading $F$, or fading figure which is associated with the fading PDF defined as $F = \text{var}(\alpha^2) / (\mathbb{E}[\alpha^2])^2$, where $\alpha$ is a RV and $\text{var}(.)$ denotes variance. The amount of fading in Nakagami-$m$ is given by $F_m = (1/m)$ [18], in the Rayleigh distribution as a special case $m = 1$ and in the limit as $m \to \infty$, Nakagami-$m$ fading channel coverage to non-fading AWGN channel. Therefore in the system with a higher Nakagami-$m$ fading parameter, the BER of the system is lower as the fading parameter is reduced. The amount of fading in Rician model is given by $F_K = (1 + 2K) / (1 + K)^2$ [18]. The Rician distribution spans the range from Rayleigh fading when $K = 1$ to non-fading when $K = \infty$. When, $m > 1$, equating $F_m$ and $F_K$ gives one-to-one mapping between the parameter $m$ and parameter $K$ allowing Nakagami-$m$ distribution to closely approximate to Rician distribution.
Figure 4.6: Average BER using BPSK modulation for different desired channel 
fading models in the presence of Rician interferences where Rician factor for 
interference fading is $K_I = 1$, $L = 2$ SNR$_y = 40$dB and SNR$_i = 0$dB. 

given by $m = (1 + K)^2 / (1 + 2K)$. The amount of fading in lognormal dis-
tribution and composite Nakagami-$m$/lognormal distribution are given by 
$F_\sigma = \exp (\sigma^2/\xi) - 1$ and $F_{m\sigma} = ((1 + m)/m) \exp (\sigma^2/\xi) - 1$ [18], respectively, 
where $\xi = 4.3429$ [18] and they can reduced to $F_m$ in the absent of $\sigma$. Con-
sequently, the results presented in Fig. 4.6 for different fading can also be 
validated by comparing the amount of fading $F$ of different fading distribu-
tions.

4.5 The Cellular Relay System Model (Application 
Example)

As an important practical example, the expressions developed in the previous 
sections are applied for the performance analysis of a cellular relay network. 
For this, consider a hexagonal grid cellular network in the down link scenario 
in which a serving cell is surrounded by six co-channel interfering cells, one 
interference in each sector, in the first tier, see Fig. 4.7. Also consider the 
worst-case scenario in the down link such that the base stations (BSs) are
located in the centre of each cell, a mobile station (MSs) allocated as a relay is located in the serving cell; and a user’s MS allocated as the destination being located on the rightmost boundary of the serving cell subjected to a number of interferers from the BS of the neighbouring cells.

### 4.5.1 Channel and Geometry Model

In the cellular relay system, desired signal channels are affected by path-loss, Nakagami-\(m\) fading and lognormal shadowing; and interferer’s signal channels disturbed by arbitrary slow fading with path-loss. The path-loss is modeled as \(r^{-\beta}\) for the purposes of mathematical tractability, where \(r\) is the distance between transmit and receive terminals and \(\beta\) is the path-loss exponent.

SINR of an AF relay system in a cellular network experienced by arbitrary CCIs and path-loss at the destination is given by (4.4), where \(a = \frac{\sigma_1^2}{P_1 \mathbb{E}[|g|^2] r_s^{-\beta}}\), \(b = \frac{(1+a)\sigma_2^2}{\mathbb{E}[|f|^2] r_r^{-\beta}}\), and \(i_l = P_l q_l r_l^{-\beta}\). \(r_s\) is the distance between the BS in the serving cell and the relay, \(r_r\) is the distance between the relay and the destination, \(r_l, l = 1, \ldots, L\) represent the distance between the destination and \(l\)th BS in neighboring cells and can be random as will be seen in Sec. 4.5.3. \(P_l\) is the transmit power allocated to the interferers, \(q_l\) is the fading modeled as.
arbitrary slow fading; and \( \delta_l \) is the indicator function that denotes the status (active/idle) of each channel in the neighbouring cells.

In order to calculate the distances, consider the serving cell with radius of \( R_s \) and co-channel reuse distance of \( D \). Then the co-channel reuse factor, \( R_u \), is defined as \( D/R_s \). When the destination is located at the right most edge of the serving cell, the distance from BS in the serving cell to the destination is \( r_d = R_s \). The distance between the BS and the relay in the serving cell can be expressed as \( r_s = f_o R_s \), where \( 0 < f_o \leq 1 \). Therefore, the location of the relay can be presented as polar coordinate \( (f_o \theta_r, \theta_r) \), where \( \theta_r \) is the angle between the BS-relay and BS-destination link in the serving cell. Thus \( r_r \) can be easily obtained from \( r_r = R_s \sqrt{f_o^2 - 2f_o \cos \theta_r + 1} \).

To find an expression for \( r_l \), \( l = 1, ..., L \), consider \( (D, \theta_l) \) to be polar coordinates of the \( l \)th neighbouring BS, where \( \theta_l = \theta (l - 1) \) and \( \theta \) is the angle between \( D \) and the BS-destination link in the serving cell. Consequently, \( r_l = \sqrt{D^2 - 2D R_s \cos \theta_l + R_s^2} \).

### 4.5.2 Effects of Interferers’ Activities

Following the same steps as Sec. 4.4.3, the expression in (4.86) is obtained with

\[
\mathcal{M}_i (s) = \mathbf{E} \left[ e^{-s \sum_{l=1}^{L} P_l q_l \delta_l r_l^{-\beta}} \right] = \prod_{l=1}^{L} \mathbf{E} \left[ e^{-s P_l q_l \delta_l r_l^{-\beta}} \right] \tag{4.87}
\]

where RVs \( \delta_l \) are considered as binary RVs that present \( l \)th channel’s status (busy/active) interfering destination. This is because in reality not all channels in the cellular system are occupied at all time. Thus

\[
\delta_l = \begin{cases} 
1, & \text{if } l \text{th channel is active} \\
0, & \text{otherwise}
\end{cases}
\]

where \( \Pr (\delta_l = 1) = p_s \) and \( \Pr (\delta_l = 0) = 1 - p_s \). Therefore,

\[
\mathbf{E} \left[ e^{-s P_l q_l \delta_l r_l^{-\beta}} \right], \text{ in (4.87) can be obtained by averaging out with respect to the RVs of } \delta_l \text{ and } q_l \text{ as}
\]

\[
\mathbf{E} \left[ e^{-s P_l q_l \delta_l r_l^{-\beta}} | q_l \right] = 1 - p_s + p_s \phi \left( P_l r_l^{-\beta} \right) \tag{4.88}
\]
Proof. For the proof of (4.88), consider \( \mathbb{E} [A^\alpha] \) where \( A \) is an arbitrary term and \( RV \) \( L \) has Binomial distribution defined as \( \alpha \sim (L, p_s) \). From Appendix A.1 for discrete RVs, the moment of \( A^L \) is given by

\[
\mathbb{E} [A^\alpha] = \sum_{l=0}^{L} \binom{L}{l} p_s^l (1 - p_s)^{L-l} = \sum_{l=0}^{L} \binom{L}{l} (p_s A)^l (1 - p_s)^{L-l}.
\]

Suppose that \( L \) is a positive integer, the binomial series formula gives

\[
\sum_{l=0}^{L} \binom{L}{l} (u)^l (v)^{L-l} = (u + v)^L
\]

(4.89)

Hence, letting \( u = p_s A \) and \( v = 1 - p_s \), we have

\[
\mathbb{E} [A^\alpha] = [p_s A + 1 - p_s]^L.
\]

(4.90)

In case of Bernoulli RV, \( L = 1 \). Thus

\[
\mathbb{E} [A^\alpha] = 1 - p_s + p_s A.
\]

(4.91)

4.5.3 Poisson Point Process

The system model in Sec. 4.5 is modified as in a practical wireless network the number of interfering signals and distances may vary randomly around the destination. To facilitate the statistical analysis of the network interference, the PPP model is adopted. Using similar analysis to the derivation of the \( \mathcal{M}_\gamma (z) \) in a cellular network Sec. 4.5, a similar result with the MGF of aggregate interferences, \( \mathcal{M}_i (s) = \mathbb{E} \left[ e^{-s \sum_{l=1}^{L} P_l q_l r_l^{-\beta}} \right] \) is obtained where number of interferes \( L \) and distances \( r_l = (r_1, r_L) \) are RVs.

Consider the interferences generated within a circular area of \( \pi R^2 \), where \( R \) is the radius of the area. By taking the limit as \( R \to \infty \), \( \mathcal{M}_i (s) \) can be shown as

\[
\lim_{R \to \infty} \mathcal{M}_i (s) = \mathbb{E} \left[ e^{-s \sum_{l=1}^{L} P_l q_l r_l^{-\beta}} \right]
\]

(4.92)
where \( L \) is distributed as a Poisson RV with density \( \zeta [\text{Interferer}/m^2] \), average \( \zeta \pi R^2 \) and the probability mass function of (see Sec. 3.5.1.1)

\[
\text{Pr}(L = l) = \frac{(\zeta \pi R^2)^l e^{-\zeta \pi R^2}}{l!}.
\]

(4.93)

On the other hand, when the interfering users are equally likely to be anywhere within the circular area, the PDF of the distances of the interferers to the destination node \((r_l, l = 1, 2, ..., L)\) in the circle of area \( \pi R^2 \) is given by [78]

\[
f(r) = \begin{cases} 
\frac{2r}{R^2} & 0 < r < R \\
0 & \text{Otherwise}
\end{cases}.
\]

(4.94)

In order to compute (4.92), it is first conditioned on \( L \) and thus use the fact that the RVs \((\delta_l, i_l, r_l, l = 1, 2, ..., L)\) are mutually independent. Therefore, for the total interference defined as \( \sum_{l=1}^{L} P_l q_l \delta_l r_l^{-\beta} \), the MGF in (4.92) is represented as

\[
\lim_{R \to \infty} M_i(s|L) = \prod_{l=1}^{L} \mathbb{E}\left[e^{-sP_l q_l \delta_l r_l^{-\beta}}\right] = \left(\mathbb{E}\left[e^{-sP_1 q_1 \delta_1 r_1^{-\beta}}\right]\right)^L
\]

(4.95)

where the expectation in the last equation is with respect to \( \delta_1, q_1 \) and \( r_1 \).

Using (4.93) and removing the condition on \( L \) in (4.95) leaves

\[
\lim_{R \to \infty} M_i(s) = \sum_{l=0}^{\infty} \frac{(\zeta \pi R^2)^l e^{-\zeta \pi R^2}}{l!} \left(\mathbb{E}\left[e^{-sP_1 q_1 \delta_1 r_1^{-\beta}}\right]\right)^l
\]

(4.96)

which can be simplified to give

\[
\lim_{R \to \infty} M_i(s) = e^{-\zeta \pi R^2 \left(1 - \mathbb{E}\left[e^{-sP_1 q_1 \delta_1 r_1^{-\beta}}\right]\right)}.
\]

(4.97)

Taking the expectation outside the bracket in the exponent of (4.97), \( \mathbb{E}\left[1 - e^{-sP_1 q_1 \delta_1 r_1^{-\beta}}\right] \), the expectation can be obtained by averaging out \( r \) over the circle (whose PDF is given in (4.94)) and conditioned on the RVs \((\delta_1, q_1)\). Therefore, the expectation in (4.97) becomes

\[
\mathbb{E}\left[1 - e^{-sP_1 q_1 \delta_1 r_1^{-\beta}}|\delta_1, q_1\right] = \int_{0}^{2R} \frac{2r}{R^2} \left[1 - e^{-sP_1 q_1 \delta_1 r_1^{-\beta}}\right] dr.
\]

(4.98)
Now, with (4.98), the exponent of (4.97), can be evaluated in the limit as $R \to \infty$,

$$
\lim_{R \to \infty} \zeta \pi R^2 \mathbb{E} \left[ 1 - e^{-s P_1 q_1 \delta_1 r_1^{-\beta}} \right] = \zeta \pi \int_0^\infty 2r \left[ 1 - e^{-s P_1 q_1 \delta_1 r_1^{-\beta}} \right] dr. \tag{4.99}
$$

Invoking the following integral identity (proven by using the rules of the integration by parts)

$$
\int_0^\infty \left( 1 - e^{-ar}\right) 2rdr = a^2 \int_0^\infty \beta r^{-\beta+1} e^{-r} dr = a^2 \Gamma \left( 1 - \frac{2}{\beta} \right) \tag{4.100}
$$

into (4.99), it can be written as

$$
\lim_{R \to \infty} \zeta \pi R^2 \mathbb{E} \left[ 1 - e^{-s P_1 q_1 \delta_1 r_1^{-\beta}} \right] = \zeta \pi \mathbb{E} \left[ s P_1 q_1 \delta_1 \right]^2 \Gamma \left( 1 - \frac{2}{\beta} \right). \tag{4.101}
$$

Finally by removing conditions on $q_1$ and $\delta_1$ in (4.101) and then applying in (4.97), the following closed-form expression for $M_i(s)$ is obtained

$$
M_i(s) = e^{-p_s \zeta \pi \mathbb{E} \left[ \frac{2}{q_1^2} \right] \Gamma \left( 1 - \frac{2}{\beta} \right)} \tag{4.102}
$$

where $\mathbb{E} \left[ \frac{2}{q_1^2} \right]$ is the average taken with respect to the arbitrary fading distribution for interfering signals.

### 4.5.4 Results

The exact average BER of BPSK has been evaluated with respect to the MGF of the end-to-end SINR of the cellular network in the down-link based on (4.86), and results in Sec. 4.3.3. In the cellular relay system, desired signal channels are affected by path-loss, Nakagami-$m$ fading and lognormal shadowing; and interferer’s signal channels disturbed by arbitrary slow fading with path-loss. The path-loss is modeled as $r^{-\beta}$ for the purposes of mathematical tractability, where $r$ is the distance between transmit and receive terminals and $\beta$ is the path-loss exponent. A composite multipath/shadowed fading is often for a scenario in congested downtown areas with slow-moving pedestrians. This type of composite fading is also used in land-mobile satellite systems subject
to vegetative and/or urban shadowing [25]. In this section, a typical macrocellular network is assumed in which transmit power is 43dBm [79] and $R_u = 5$. The SNR for relay-destination link $SNR_x$ is varied between 0dB and 40dB. In all figures, $10^5$ Monte-Carlo simulation is used to validate the new analytical method and assess the accuracy of the proposed system model for the cellular network. In the simulation, local mean power values are randomly generated with lognormal distribution $\mu = 0$dB and the shadow standard deviation for desired signals $\sigma_y = \sigma_x$. The instantaneous received power values for the desired signals are randomly generated using gamma distribution with shape factors of $m_1 = m_2 = 2$ and path-loss exponent $\beta = 3.76$ which is empirically measured for shadowed urban cellular environment. Moreover, for accuracy in all figures $N_p = 20$ and also $Pr (\delta_l = 1) = 0.5$ are considered.

Fig. 4.8 shows the exact average BER of the system against $SNR_x$, in the presence of Rician distribution for interfering channels with Rician factor of $K_I = 2$ for different shadow parameters $\sigma$ for the desired signals. In this figure $60^\circ$ sectorized cell acknowledged thus serving cell has been surrounded by 1 co-channel interfering cell. As can be seen from the figure by increasing the shadow parameter in the system the BER of the system is increasing. This is because by increasing $\sigma$, amount of fading $F_{m_\sigma}$ is increased, see Sec. 4.4.4. Fig. 4.9 shows the exact average BER of the system against $SNR_x$, in the presence of different CCI fading models including Nakagami-\textit{m}, Rician and lognormal distributions and for different $f_o$. It can be seen that by increasing the distance between the BS in the serving cell and the relay, the BER is decreasing as signal-to-noise ratio in the first phase is increasing. Fig. 4.9 also confirms that as far as the calculation of the exact BER is concerned, accurate BER can be obtained for arbitrary interferences. Fig. 4.10 illustrates average BER of BPSK modulation against $SNR_x$, in presence of lognormal CCI with different number of CCI at the destination, where in the downlink the number of CCI at the destination are equal to 6, 2 and 1 for omnidirectional, $120^\circ$ sectorized and $60^\circ$ sectorized cell, respectively. From the figure it is apparent that by increasing number of CCI at the destination the BER of the system is increased.

Fig. 4.11 displays the exact average BER against to the $SNR_x$ in case of the presence of Poisson field of Nakagami-\textit{m} interferences for several values
Figure 4.8: Average BER of BPSK modulation with respect to $\text{SNR}_x$, in presence of Rician CCIs for different shadowing parameters for desired signal channels where, $\sigma_Y = \sigma_X$, $m_1 = m_2 = 2$, $K_I = 1$, $f_0 = 0.2$, $L = 1$ and $R_u = 5$.

of $\zeta$, where $m_I = 2$. As Nakagami-$m$ distribution is considered for interfering channels, from (4.11) and (4.100), and using [80, eq. (3.382.4)], $\mathbb{E}\left[\frac{2}{q_1}\right]$ is evaluated as

$$
\mathbb{E}\left[\frac{2}{q_1}\right] = \int_0^\infty q_1^2 \frac{m_I^{m_I}}{\Gamma(m_I)} q_1^{m_1-1} e^{-m_I q_1} dq_1 = \frac{\Gamma\left(m_I + \frac{2}{\beta}\right)}{m_I^{\frac{2}{\beta}} \Gamma(m_I)}. \tag{4.103}
$$

From Fig. 4.11, it is observed that the contribution of the interferences to the system is reducing for the lower density $\zeta$ [Interferer$/m^2$] as the distance from the interferer users to the desired transmitter $R$ is increased.
Figure 4.9: Average BER of QPSK/QAM modulation in a cellular network with respect to the SNR$_x$, and for different CCIs fading models and $f_o$, where $m_1 = m_2 = 2$, $\mu = 0$, $\sigma_y = \sigma_x = 2$dB and $L = 6$.

Figure 4.10: Average BER of BPSK modulation against SNR$_x$, in presence of lognormal CCIs with different number of CCIs at the destination where $f_o = 0.6$, $m_1 = m_2 = 2$, $\mu = 0$, $\sigma_y = \sigma_x = 2$dB.
In this chapter a new simple unified method for accurate and efficient evaluation for average error rate probabilities of an AF relay system in the presence of a random number of arbitrary non-identical interferers was developed. This work was in contrast to previous published works which have been limited to specific interference model such that the destination node was subjected to a given number of homogenous and identically distributed interferers. For this, first a non-direct mathematical method was developed leading the derivation of new explicit expressions for the MGF of SINR in terms of the MGF of the aggregate interference power. The new simple and explicit expressions derived were in terms of single integrals that led to the derivation of averages of some common functions, like the complementary error function and its square. These averages appear in most formula of bit and symbol error rates of different types of digital modulation schemes. Based on the developed results, the exact average BER and SER of two common digital modulation types, MPSK and MQAM, were then derived. The analytical approach then was extended for different fading scenarios for the desired user involving Nakagami-$m$/Nakagami-$m$, Rician/Nakagami-$m$ and
composite Nakagami-$m$/lognormal fading models. As an important application example, numerical results were given for BER of a cooperative relay system for a down-link relay cellular network in the presence of arbitrary and Poisson field of interferers. Monte Carlo simulation was used to validate the numerical results from the new expressions developed in this chapter.
Chapter 5

Performance Degradation of Distributed Cooperative Relay Systems Due to Self-Interference

5.1 Introduction

A time-based relay selection protocol, see Sec. 3.4.1, is prone to self-interference when relay nodes are hidden from each other. Practically, in the absence of hidden nodes, additional interferences may arise due to the overheads introduced by the protocol. That means that if two or more relays have comparable signal levels, the relays’ time can be expired within the best relay’s time. Therefore, they cannot sense the transmission of the best relay, which may result in collisions. Until now, only few studies have been carried out to investigate the impact of overheads on the performance of a cooperative system [12–15]. An initial study of the protocol overhead has been examined in [12]. In [13] the overhead effects have been investigated and an optimal timer scheme proposed to maximise the probability of successful selection. In [14] the impact of the overhead required for relay selection on SE has been investigated for a specific channel fading scenario. [15] extends the optimal timer design in [13] to maximise the net throughput with due consideration of the overheads.

From the review of previous researchers’ work, it is evident that the performance of distributed systems employing a time-based relay selection protocol has been studied from two main perspectives. The first perspective is that the
impact of overheads simply have been ignored [39,51–53,81–83]. The second perspective is that the researchers have had over-pessimistic assumption in the assessment of the probability of collisions. In such a way that, in their assumption all packets involved in a collision are destroyed [12–15]. In this case, they only investigated a collision that may arise from the second best relay and causes a loss of all packets. However, collisions due to the protocol overheads mainly cause the system performance to be degraded and this does not mean that the failure of end-to-end transmission surely happens. As a result, to achieve an exact performance and a more realistic result, it is necessary to evaluate and analyse the impact of all possible collisions which could occur between the best relay and all participating relay candidates. Therefore, an accurate interference model is needed. Moreover, in the aforementioned works [12–15], the protocol overheads are assumed to be independent of the system being modelled which in practical design scenarios, such an assumption is not a realistic proposition.

The main contribution in this chapter is that a new accurate mathematical analysis on the performance of distributed cooperative communication systems employing time-based relay selection protocols is presented. New exact unified expressions are derived for the SE based on an accurate interference model which accounts for both self-interference and the overheads introduced by the protocol. The new accurate expressions also account for different fading scenarios as well as arbitrary relays’ locations. The explicit expressions are examined for two different cases; identical and non-identical RVs. The scenarios are; presence of hidden nodes and/or with protocol overheads.

It should be emphasised that obtaining the exact average SE of the system in presence of the interferences is a very difficult task. This is because the conventional direct methods involves averaging out the aggregate random interference and other RVs by computing several integral operations which also requires a knowledge of PDFs of the RVs, see Sec. 3.5.1.2 for more details. This analysis can be further complicated when using the general random path-loss model $r^{-\beta}$ and specially when the aggregate random power interferences are dependent. Therefore, to overcome these weaknesses, a new mathematical solution that aids the analysis is proposed. This leads to the derivation of new exact expressions for the SE in terms of the MGF of the random interferences’ power. Additionally, in this chapter, a bound for
the spectral efficiency is derived by invoking Jensen’s inequality. This is then compared with the derived exact results. The accuracy of the new expressions are validated by Monte-Carlo simulations.

In the rest of the chapter, in Sec. 5.2, the system model is presented. In Sec. 5.3, explicit expressions are derived for the SE where RVs are considered to be identical while in Sec. 5.4 the explicit expressions for the SE in the case of non-identical RVs are derived. In Sec. 5.5, a bound of SE over Rayleigh fading channels is presented. In Sec. 5.6, the simulations results are discussed and summary of the chapter is presented in Sec. 5.7.

5.2 The Distributed Relay Selection Model

5.2.1 The Network Model

Consider an OWR cooperative system that employs a time-based relay selection protocol to select the best relay, see Sec. 3.4.1. The distributed cooperative relay system with one source, one destination and $N$ DF relay nodes, as shown in Fig. 5.1. In the system model, direct link from the source to the destination is assumed to be unavailable due to a long distance assumption. The cooperative system operates in two phases. In Phase 1, the source broadcasts a signal to $N$ relay nodes. If relay $l$, where $1 \leq l \leq N$, has a higher signal-to-noise ratio (SNR) than the specific SNR threshold, it is assumed that the relay can successfully receive the signal from the source and it becomes a candidate relay to participate in cooperative transmission. Let $p_s$ denotes the probability that $l$th relay successfully receives the signal from the source given by $p_s = \Pr(\gamma_{sl} \geq \gamma_{th})$, where $\gamma_{sl}$ is the SNR at relay $l$ and $\gamma_{th}$ is the threshold SNR. All successful relays in Phase 1 (candidate relays) then comprise in a decoded set denoted by $\mathcal{H}$. Consider $L_s$ as the cardinality of the decoded set $\mathcal{H}$, hence $|\mathcal{H}| = L_s$. In Phase 2, the time-based selection cooperation (SC) protocol (which has been discussed in Sec. 3.4.1.2) is used to select a single relay among $L_s$ candidate relays to forward the received signal to the destination. The relay selection protocol to select the best relay will be briefly discussed in Sec. 5.2.2.
5.2.2 The Time-Based Relay Selection Protocol

In Phase 1, each relay $l$ in the decoded set $\mathcal{H}$ starts its own timer with an initial value $t_l$. By considering SC protocol, see Sec. 3.4.1, $t_l$ is inversely proportional to the estimated channel power gain of the link from itself to the destination. Therefore, each relay $l$’s time is defined as $t_l = \frac{\lambda}{x_l}$, where $\lambda$ is a system parameter, $1 \leq l \leq L_s$ and $x_l$ is the power channel gain from relay $l$ to the destination. The best relay has a timer reduced to zero first and it is the relay that forwards the information toward the destination. In other words, the relay node with $\max(x_1, \ldots, x_{L_s})$ is selected as the best relay as it takes the minimum of $t_l$. Thus the best relay’s time is defined as $t_{\text{min}} = \frac{\lambda}{\max(x_1, \ldots, x_{L_s})}$.

Note that in this chapter, RVs $x_l$, $l = 1, 2, \ldots, L_s$ are arbitrary non-negative RVs with the PDF that are denoted by $f_{x_l}(x)$; and the AWGN terms of all links are represented by $\sigma_n$.

In order to make sure that the timers at other relay nodes do not expire before the signal is transmitted by the best relay a guard interval $\Delta$ is required. This is because it takes some time for the best relay to prepare the signal for transmission. During the best relay’s time and during the guard interval there is no data transmission which decreases the system performance. The sum of the best relay’s time and guard interval is denoted by $T_s = t_{\text{min}} + \Delta$. 

---

**Figure 5.1:** The system model
5.2.3 Self-interferences

A noise can be any type of undesired signal that exists in the passband of desired signal. Beside noise, a self-interference can be a major source of signal corruption than the noise from circuitry. Such interference cause high error rates and system degradation and have a huge impact on the system performance. Distributed relay cooperative relay systems are prone to self-interferences when relay nodes become hidden from each other. When the time-based relay selection protocol is employed, additional interferences may also arise from the relay nodes that are not hidden from each other. This is because the overheads introduced by the protocol. To provide a better comprehension of the joint effects of the self-interference phenomena, hidden nodes and the protocol overheads, on the system performance, in the following sections, we will individually discuss each phenomenon in detail and then will be extended to consider the joint effect the self-interference phenomena.

5.2.3.1 Hidden Nodes Only

Once the best relay is selected, it starts to transmit the source information to the destination, while all remaining relays in set $\mathcal{H}$ wait for their time to reduce to zero (i.e. to expire). In order to investigate the impact of hidden nodes only on the system performance, we assume an ideal time-based protocol in the sense that the time selection period is negligible $T_s = \Delta + t_{\text{min}} \simeq 0$ compared to useful transmission time $T_e$. In this case, the total end-to-end transmission period $T$ is approximately equal to the useful data transmission time $T_e$, $T \simeq T_e$. Therefore, the best relay starts to forward the received signal as soon as it receives from the source and all relay nodes that are hidden from each other appear as interferences. That is because the hidden nodes can not sense the best relay’s transmission and thus as soon as their timer expired they start to transmit a signal. In this case, the statistics of the SINR, $\gamma$, can be defined as

$$\gamma = \frac{\max \left( x_1, x_2, \ldots, x_L \right)}{\sum_{i \neq j} x_i \psi_i + \sigma_n^2}$$  (5.1)
where \( j = \text{index of maximum power channel gain}, \sigma_n \) is the AWGN term and \( \psi_l \) is an indicator function defined as

\[
\psi_l = \begin{cases} 
1 & \text{relay } l \text{ is hidden} \\
0 & \text{otherwise}
\end{cases}.
\] (5.2)

### 5.2.3.2 The Protocol Overheads

In the time-based distributed relay selection protocol, the relay nodes that are not hidden from each other back off as soon as they hear the best relay’s transmission. However, in the non-ideal protocol, self-interferences not only can arise due to the presence of hidden nodes but may also emerge in the absence of hidden nodes due to the overheads introduced by the protocol. The overheads come from the protocol guard interval \( \Delta \). The probability of having two or more relays’ time expire at the same time is zero. However, in practice, the probability of having two or more relays’ time expire within the same guard interval \( \Delta \) is non-zero. Therefore, when two or more relays have a comparable signal levels, the relay nodes’ time can be expired earlier than the best relay’s time \( t_{\text{min}} \) plus the \( \Delta \). In this circumstance, they can not sense the best relay’s transmission and thus start to transmit a signal resulting in self-interference. Consequently, a collision in the absence of hidden nodes occur only if for relay \( l \), we have \( t_l \leq \Delta + t_{\text{min}} \).

For better understanding consider Fig. 5.2, where there is no hidden nodes. From the figure \( R \) corresponds to the “best” relay. The relays \( R_1, ..., R_{L_s} \) can erroneously be selected as “best” relays if their timer expire within intervals when they can not sense the best relay transmission. That can happen in the interval \([\tau_1, \tau_2]\) for the case of no hidden relay nodes. The statistics of the SINR, \( \gamma \), in the absence of hidden nodes is given by

\[
\gamma = \frac{\max (x_1, x_2, ..., x_{L_s})}{\sum_{l \neq j} x_l \xi_l + \sigma_n^2} \tag{5.3}
\]
Figure 5.2: The middle row corresponds to the best relay. The other relay nodes in top and bottom row could enormously be selected as the best relay if they timer expire within intervals that they cannot hear the best relay transmission. That can happen in interval $[τ_1, τ_2]$ for the case where relay nodes are not hidden from each other or $[τ_1, T]$ for the case where relay nodes are hidden.

where $ξ_l$ is the collision indicator function which is unity when relay $l$ transmits a signal and zero otherwise. Therefore,

$$ξ_l = \begin{cases} 
1 & \text{if relay } l \text{ transmits} \\
0 & \text{otherwise}
\end{cases} \quad (5.4)$$

As discussed earlier, in the absence of hidden nodes only if $\frac{λ}{x_l} ≤ ∆ + \frac{λ}{\max(x_1, x_2, ..., x_L)}$, relay $l$ will transmit a signal. Therefore, $ξ_l$ in (5.4) can be written as

$$ξ_l = \begin{cases} 
1 & \frac{λ}{x_l} - \frac{λ}{\max(x_1, x_2, ..., x_L)} ≤ ∆ \\
0 & \text{otherwise.}
\end{cases} \quad (5.5)$$

On the other hand, in the case of hidden relay nodes, the relays $R_1, ..., R_{L_s}$ can erroneously be selected as the best relays if their timer expire in the interval $[τ_1, T_{total}]$, where $T_{total}$ is the total end-to-end transmission time. Therefore, in order to consider both self-interference phenomena, hidden nodes and the protocol overheads, the statistics of the SINR, $γ$, in (5.3) can be
modified into
\[ \gamma = \frac{\max (x_1, x_2, ..., x_{L_s})}{\sum_{l \neq j} x_l \psi_l + \sum_{l=1}^{L_s-1} x_l (1 - \psi_l) \xi_l + \sigma_n^2}. \] (5.6)

5.3 Identical Case

In this section, new exact expressions for the distributed cooperative relay system in terms of SE based on an accurate interference model are derived, where for simplicity of analyses, all RVs are considered to be identical. Moreover, for clarity in presentation, the impact of the self-interference phenomena on the distributed system performance will be investigated individually and then will be extended to investigate the joint effects of self-interference phenomena. In Sec. 5.3.1, we derive a new exact expression in the presence of hidden nodes only. In Sec. 5.3.2, we derive a new exact expression in the absence of hidden nodes in order to investigate the impact of the protocol overheads on the overall system throughput. In Sec. 5.3.3, we derive a new exact expression where the joint effects of both hidden nodes and the protocol overheads are considered.

5.3.1 SE in the Presence of Hidden Nodes Only

In an ideal system model, the average SE from (5.3) is given by
\[ C = \frac{1}{2} \mathbb{E} \left[ \log_2 \left( \frac{\max (x_1, x_2, ..., x_{L_s})}{\sum_{l \neq j} x_l \psi_l + \sigma_n^2} \right) \right]. \] (5.7)

where \( j = \) index of \( \max (x_1, x_2, ..., x_{L_s}) \) and the factor 1/2 comes from the two-phase transmission.

Direct methods to calculate (5.7) would require at least \((2L_s - 1)\)-fold numerical integrations to average out the \(2L_s - 1\) RVs, \(x_l, \psi_l \ l = 1, 2, ..., L_s\), with knowledge of the PDF of the RVs as
\[ C = \int_0^\infty \int_0^\infty \ldots \int_0^\infty \log_2 \left( \frac{\max (x_1, x_2, ..., x_{L_s})}{\sum_{l \neq j} x_l \psi_l + \sigma_n^2} \right) \times f(x_1, x_2, ..., x_{L_s}) f(\psi_1, \psi_2, ..., \psi_{L_s}) \, dx_1 \ldots dx_{L_s} \, d\psi_1 \ldots d\psi_{L_s}. \] (5.8)
which the evaluation of (5.8) can be time-consuming and very complex. Moreover, evaluation of (5.7) in association with (5.8) would be more complicated if the order statistics of RVs were used; because a closed-form expression for the joint PDF of the ordered RVs $x_l l = 1, 2, \ldots, L_s$ would be required. In this paper, we present a new mathematical solution that greatly simplifies the evaluation of (5.7). To proceed further, Lemma 1 is applied and the following theorem is introduced.

**Theorem 1.** Let $\alpha_1, \ldots, \alpha_{r-1}, \alpha_r, \alpha_1, \ldots, \alpha_N$ be a random sample from an absolutely continuous population with CDF $F(x)$ and PDF $f(x)$, and let $\alpha_{(1)} \leq \ldots \leq \alpha_{(r-1)} \leq \alpha_{(r)} \leq \alpha_{(r+1)} \leq \ldots \leq \alpha_{(N)}$ denote the order statistics obtained from this sample. The RVs in sets of $(\alpha_{(1)}, \ldots, \alpha_{(r-1)})$ and $(\alpha_{(r+1)}, \ldots, \alpha_{(N)})$ become conditionally independent if to fix $\alpha_{(r)}$ (e.g. [84]). Therefore, the joint conditional density of the RVs $(\alpha_1, \ldots, \alpha_{r-1}, \alpha_{r+1}, \ldots, \alpha_N)$ for the given fix $\alpha_r = v$ where sets $(\alpha_1, \ldots, \alpha_{r-1}) \leq (\alpha_{r+1}, \ldots, \alpha_N)$, is given by

$$f_{\alpha_1,\ldots,\alpha_{r-1},\alpha_{r+1},\ldots,\alpha_N}(x_1, \ldots, x_{r-1}, x_{r+1}, \ldots, x_N | v) = \prod_{k=1}^{r-1} \frac{f(x_k)}{F(v)} \prod_{k=r+1}^{N} \frac{f(x_k)}{1 - F(v)}$$  

(5.9)

where for each $\alpha$, CDF is introduced as

$$G(x, v) = \Pr(\alpha \leq x | \alpha \leq v) = \frac{F(x)}{F(v)}$$  

(5.10)

and

$$H(x, v) = \Pr(\alpha \leq x | \alpha > v) = \frac{F(x) - F(v)}{1 - F(v)}.$$  

(5.11)

Thus, the corresponding PDFs have the form of

$$g(x, v) = \begin{cases} \frac{f(x)}{F(v)}, & x \leq v \\ 0, & x > v \end{cases} \quad \text{and} \quad h(x, v) = \begin{cases} \frac{f(x)}{1-F(v)}, & x \geq v \\ 0, & x < v \end{cases}.$$  

(5.12)

By considering $\max(x_1, x_2, \ldots, x_{L_s}) = y$, (5.7) in condition on $L_s$ and $y$ can be written as

$$C(L_s, y) = \frac{1}{2} \mathbb{E} \left[ \log_2 \left( 1 + \frac{y}{\sum_{l=1}^{L_s-1} x_l \psi_l + \sigma_n^2} \right) | L_s, y \right].$$  

(5.13)

Invoking Lemma 1 for the efficient computation, (5.13) can be presented
as

\[
C (L_s, y) = \frac{1}{2} \log_2 e \int_0^\infty \frac{e^{-\sigma_n^2 z}}{z} \left( 1 - e^{-zy} \right) \mathbb{E} \left[ e^{-z \sum_{i=1}^{L_s-1} x_i \psi_i} \right] dz. \tag{5.14}
\]

Since RVs \( x_l, l = 1, 2, ..., L_s - 1 \) are conditionally independent for the given \( y \), see Theorem 1, the expectation in (5.14) in condition on \( y \) and \( L_s \) can be written as

\[
\mathbb{E} \left[ e^{-z \sum_{i=1}^{L_s-1} x_i \psi_i} \right] = \prod_{l=1}^{L_s-1} \mathbb{E} \left[ e^{-zx_i \psi_i} \right] \tag{5.15}
\]

where

\[
\mathbb{E} \left[ e^{-zx_i \psi_i} \right] = \int_0^y e^{-z x_i \psi_i} f_{x_i|y} (x|y) dx \tag{5.16}
\]

and the conditional distribution of \( x_l, f_{x_l|y} (x|y) \), for \( 0 \leq x_l < y \) is obtained by (see. (5.12))

\[
f_{x_l|y} (x|y) = \frac{f (x)}{F (y)} \quad 0 \leq x < y. \tag{5.17}
\]

Replacing (5.17) in (5.16) and taking average with respect to \( \psi_i \), (5.15) is obtained by

\[
\mathbb{E} \left[ e^{-z \sum_{i=1}^{L_s-1} x_i \psi_i} | y, L_s \right] = \prod_{l=1}^{L_s-1} \int_0^y e^{-z x_l \psi_i} f (x) \frac{f (y)}{F (y)} dx = \left[ 1 - p_h + p_h \frac{\Psi (0, z) - \Psi (y, z)}{F (y)} \right]^{L_s-1} \tag{5.18}
\]

where \( p_h = \Pr (\psi_i = 1) \) and \( \Psi (\omega, z) = \int_\omega^\infty e^{-zx} f (x) dx \) is the incomplete MGF. The closed form expression of the complementary incomplete MGF, \( \Psi (\omega, z) \), for different fading models are presented in Table 5.1.

Substituting (5.18) into (5.14) yields

\[
C (L_s, y) = \frac{1}{2} \log_2 e \int_0^\infty \frac{e^{-\sigma_n^2 z}}{z} \left( 1 - e^{-zy} \right) \left[ 1 - p_h + p_h \frac{\Psi (0, z) - \Psi (y, z)}{F (y)} \right]^{L_s-1} dz. \tag{5.19}
\]

The PDF of \( y \) based on the order statistics of RVs is obtained by \( f (y) = \)
\( L_s f (y) [F(y)]^{L_s-1} \), thus the condition on \( y \) in (5.19) can be removed by averaging out the RV \( y \) over its PDF, as

\[
C (L_s) = \frac{1}{2} \log_2 e \int_0^\infty \int_0^\infty \frac{e^{-z\sigma_n^2}}{z} \left[ 1 - p_h + p_h \frac{\Psi (0, z) - \Psi (y, z)}{F(y)} \right]^{L_s-1} \times (1 - e^{-zy}) f (y) \, dz \, dy \tag{5.20}
\]

On the other hand, since RVs are considered to be identical, \( L_s \) has binomial distribution defined by \((N, p_s)\). For this reason, by averaging out respect to RV \( L_s \), we can also remove the condition on \( L_s \) and arrive at the following expression

\[
C = \frac{N p_s}{2} \log_2 e \int_0^\infty \frac{e^{-z\sigma_n^2}}{z} A(z) \, dz \tag{5.21}
\]

where

\[
A(z) = \int_0^\infty (1 - e^{-zy}) [1 - p_s + p_s \{ F (y) (1 - p_h) + p_h (\Psi (0, z) - \Psi (y, z)) \}]^{N-1} \times f (y) \, dy. \tag{5.22}
\]

The expression in (5.21) is the new exact expression for the distributed cooperative system in the presence of hidden nodes only which can also accounts for different fading models. For example, in the case of Rayleigh fading with unity mean, \( f (x) = e^{-x} \) thus \( \Psi (y, z) = \frac{1}{1+z} \exp (-y (z + 1)) \) and \( \Psi (0, z) = \frac{1}{1+z} \). Similar expressions for other fading models can be obtained from Table.

For Rayleigh fading channel, the expression in (5.21) can be further simplified. For this, consider the term in the square brackets in (5.22) which can be written as

\[
[1 - p_s + p_s \{ F (y) (1 - p_h) + p_h (\Psi (0, z) - \Psi (y, z)) \}]^{N-1}
= \left[ 1 - p_s + p_s \left( (1 - e^{-y}) (1 - p_h) + p_h \frac{1 - e^{-y(1+z)}}{1 + z} \right) \right]^{N-1}
= \left[ 1 - p_s p_h \frac{z}{1 + z} - p_s (1 - p_h) e^{-y} - p_s p_h e^{-y(1+z)} \right]^{N-1}
= \left[ -p_s (1 - p_h) e^{-y} - p_s p_h \frac{e^{-y(1+z)}}{1 + z} + 1 + z (1 - p_s p_h) \right]^{N-1} \tag{5.23}
\]
By using Trinomial theorem, (5.23) can be simplified into

\[
\left[-p_s (1 - p_h) e^{-y} - p_s p_h \frac{e^{-y(1+z)}}{1+z} + \frac{1 + z (1 - p_s p_h)}{1+z}\right]^{N-1} = \\
\sum_{l_1=0}^{N-1} \sum_{l_2=0}^{N-1-l_1} \frac{(N-1)!}{l_1!l_2! (N-1-l_1-l_2)!} \left(-p_s p_h \right)^{l_1} \frac{1}{1+z} \\
\times \left(-p_s (1 - p_h)^{l_2} \left(1 + z (1 - p_s p_h)\right)^{N-1-l_1-l_2} \frac{1}{1+z}\right) e^{-y(l_1(1+z)) e^{-y_l}}. \tag{5.24}
\]

Substituting (5.24) in (5.21), gives

\[
C = \frac{Np_s \log_2 e}{2} \sum_{l_1=0}^{N-1-l_1} \sum_{l_2=0}^{N-1-l_1} \frac{(N-1)!}{l_1!l_2! (N-1-l_1-l_2)!} \left(-p_s (1 - p_h)^{l_2} (-p_s p_h)^{l_1} \right) \\
\times \int_0^\infty e^{-y} \left(1 + z (1 - p_s p_h)^{N-1-l_1-l_2} \left(1 + \frac{1}{1+z}\right)^{N-1-l_2}\right) \\
\times \int_0^\infty \left(e^{-y} - e^{-((1+z)y)}\right) e^{-y(l_1(1+z)+l_2)} dy dz. \tag{5.25}
\]

which by solving the inner integral it can be reduced to

\[
C = \frac{Np_s \log_2 e}{2} \sum_{l_1=0}^{N-1-l_1} \sum_{l_2=0}^{N-1-l_1} \left(-p_s (1 - p_h)^{l_2} (-p_s p_h)^{l_1} (N-1)! \right) \\
\times \int_0^\infty e^{-y} \left(1 + z (1 - p_s p_h)^{N-1-l_1-l_2} \left(1 + \frac{1}{1+z}\right)^{N-1-l_2}\right) \\
\times \left\{\frac{1}{1 + l_1 (1+z) + l_2} - \frac{1}{(1 + l_1) (1+z) + l_2}\right\} dz. \tag{5.26}
\]

### 5.3.2 SE in the Absence of Hidden Nodes

To analyse performance degradation of a distributed system due to the protocol overheads only, consider the system with the absence of hidden nodes. Consequently, from (5.3), the average SE with respect to the RVs \((x_l, \xi_l, y, L_s)\) is given by

\[
C = \frac{1}{2} \mathbb{E} \left[ \frac{T_e}{\Delta + \frac{A}{y} + T_e} \log_2 \left( 1 + \frac{y}{\sum_{l \neq j} x_l \xi_l + \sigma_n^2} \right) \right]. \tag{5.27}
\]
where \( y = \max (x_1, x_2, \ldots, x_{L_s}) \).

Let us normalise the useful packet transmission time such that \( T_e = 1 \). Then (5.27) in conditions on \( y \) and \( L_s \) can be rewritten as

\[
C(y, L_s) = \frac{1}{2} \mathbb{E} \left[ \frac{y}{\lambda + y (1 + \Delta)} \log_2 \left( 1 + \frac{y}{\sum_{l=1}^{L_s-1} x_l \xi_l + \sigma_n^2} \right) | y, L_s \right]. \tag{5.28}
\]

Following similar analysis to that in Sec. 5.3.1, instead of (5.14) the following expression is obtained

\[
C(L_s, y) = \frac{1}{2} \log_2 e \frac{y}{\lambda + y (1 + \Delta)} \int_0^\infty \frac{e^{-z\sigma_n^2}}{z} (1 - e^{-yz}) \mathbb{E} \left[ e^{-z \sum_{l=1}^{L_s-1} x_l \xi_l} | L_s \right] dz \tag{5.29}
\]

where

\[
\mathbb{E} \left[ e^{-z \sum_{l=1}^{L_s-1} x_l \xi_l} | L_s, y \right] = \prod_{l=1}^{L_s-1} \mathbb{E} \left[ e^{-zx_l \xi_l} | y \right] = \left[ \mathbb{E} \left[ e^{-zx_l \xi_l} | y \right] \right]^{L_s-1} \tag{5.30}
\]

To evaluate \( \mathbb{E} \left[ e^{-zx_l \xi_l} | y \right] \), recall (5.5) and then rearrange the form of expression into the form of \( x_l \geq \frac{y}{1 + y \frac{\Delta}{\lambda}} \). Thus

\[
\mathbb{E} \left[ e^{-zx_l \xi_l} | y \right] = \int_0^{\frac{y}{1 + y \frac{\Delta}{\lambda}}} f_{x_l | y} (x | y) \, dx + \int_{\frac{y}{1 + y \frac{\Delta}{\lambda}}}^\infty e^{-zx_l} f_{x_l | y} (x | y) \, dx \tag{5.31}
\]

which by invoking (5.17) in (5.31), we get

\[
\mathbb{E} \left[ e^{-zx_l \xi_l} | y \right] = \frac{1}{F(y)} \left\{ 1 - \Psi \left( \frac{\lambda y}{\lambda + \Delta y}, 0 \right) - \Psi (y, z) + \Psi \left( \frac{\lambda y}{\lambda + \Delta y}, z \right) \right\}. \tag{5.32}
\]

Applying (5.32) in (5.30) and then substituting into (5.29), the following expression is obtained

\[
C(L_s, y) = \frac{1}{2} \log_2 e \frac{y}{\lambda + y (1 + \Delta)} \times \int_0^\infty \frac{e^{-z\sigma_n^2}}{z} (1 - e^{-yz}) \left[ 1 - \Psi \left( \frac{\lambda y}{\lambda + \Delta y}, 0 \right) - \Psi (y, z) + \Psi \left( \frac{\lambda y}{\lambda + \Delta y}, z \right) \right]^{L_s-1} F(y) \, dz. \tag{5.33}
\]
By averaging out the RVs $y$ and $L_s$ over their PDF to remove the conditions in (5.33), we arrive at the exact expression for the SE in (5.34),

$$ C = \frac{1}{2} L_p s \log_2 e \int_0^\infty \frac{e^{-z\sigma_n^2}}{z} A(z) \, dz $$  

(5.34)

where

$$ A(z) = \int_0^\infty \frac{y (1 - e^{-zy})}{\lambda + y (1 + \Delta)} \left[ 1 - p_s \left\{ \Psi(y, z) + \Psi\left(\frac{\lambda y}{\lambda + \Delta y}, 0\right) - \Psi\left(\frac{\lambda y}{\lambda + \Delta y}, z\right) \right\} \right]^{N-1} 
\times f(y) \, dy. $$  

(5.35)

### 5.3.3 Joint Effects of Protocol Overheads and Hidden Nodes

In order to consider the combined effects of hidden nodes and the protocol overheads into the system performance, (5.27) is modified to include hidden relay nodes as follow:

$$ C = \frac{1}{2} \mathbb{E} \left[ \frac{1}{\Delta + \frac{y}{\lambda} + 1} \log_2 \left( 1 + \frac{y}{\sum_{l=1}^{L_s-1} x_l \psi_l + \sum_{l=1}^{L_s-1} x_l (1 - \psi_l) \xi_l + \sigma_n^2} \right) \right] $$  

(5.36)

where the Equation (5.36) can also be rewritten as

$$ C = \frac{1}{2} \mathbb{E} \left[ \frac{1}{\Delta + \frac{y}{\lambda} + 1} \log_2 \left( 1 + \frac{y}{\sum_{l=1}^{L_s-1} x_l [\psi_l + (1 - \psi_l) \xi_l] + \sigma_n^2} \right) \right]. $$  

(5.37)

By following the same steps in Sec. 5.3.2, similar result in (5.29) is obtained with

$$ \mathbb{E} \left[ e^{-xz_l [\psi_l + (1 - \psi_l) \xi_l]} | y \right] = p_h \mathbb{E} \left[ e^{-xz_l} | y \right] + (1 - p_h) \mathbb{E} \left[ e^{-xz_l \xi_l} | y \right] $$  

(5.38)

where $\mathbb{E} \left[ e^{-xz_l} | y \right] = F(y)^{-1} (\Psi(0, z) - \Psi(y, z))$ and $\mathbb{E} \left[ e^{-xz_l \xi_l} | y \right]$ is given by (5.32). Replacing (5.38) in (5.29) and then taking average respect to RVs $y$
and $L_s$, we arrive at (5.34) with
\[
A(z) = \int_0^\infty \frac{y(1-e^{-zy})}{\lambda+y(1+\Delta)} \left[ 1 - p_s + p_s \{ p_h \{ \Psi(0, z) - \Psi(y, z) \} + \right. \\
(1-p_h) \left\{ 1 - \Psi \left( \frac{\lambda y}{\lambda+\Delta y}, 0 \right) - \Psi(y, z) + \Psi \left( \frac{\lambda y}{\lambda+\Delta y}, z \right) \} \right]^{N-1} f(y) \, dy.
\]
(5.39)

### 5.3.4 Random Path-Loss

Consider the random variable $x_l$ modeling fading and path-loss as $x_l = r_l^{-\beta} g_l$, where $g_l$ is a small-scale fading with exponential distribution and $r_l$ is the distance between relay $l$ and the destination.

Suppose the distance $r_l$ is random generated in a disc of radius $D$ with the following PDF [78]
\[
f_{r_l}(r) = \begin{cases} 
\frac{2r}{D^2} & 0 < r < D \\
0 & \text{Otherwise}
\end{cases}.
\]
(5.40)

In order to compute the PDF of RV $x_l$, first condition $x_l$ on $r$. Thus
\[
f_{x_l}(x|r) = r^\beta e^{-r^\beta x}.
\]
(5.41)

Then by removing the condition on $r$ in (5.41) as $f(x) = \int_0^D r^\beta e^{-r^\beta x} \frac{2r}{D^2} \, dr$, the PDF $f(x)$ is obtained by
\[
f(x) = \frac{2x^{-\frac{2}{\beta}} \left( \Gamma \left( \frac{2}{\beta} \right) - \Gamma \left( \frac{2}{\beta}, D^\beta x \right) \right)}{\beta D^2} = 2 \frac{\Gamma \left( \frac{\beta+2}{\beta} \right) - \Gamma \left( \frac{\beta+2}{\beta}, x \right)}{\beta} x^{-\frac{2+2}{\beta}}
\]
(5.42)

where the distance is uniform in a unit circle in addition to Rayleigh fading, and $\Gamma(.)$ and $\Gamma(.,.)$ represent the gamma function and the upper incomplete gamma function, respectively [22].

### 5.4 Non-Identical Case

#### 5.4.1 SE in Presence of Hidden Nodes Only

Consider non-identical RVs $(v_1, \ldots, v_N)$ that are independent and present power channel gain of successful and unsuccessful relay nodes in Phase 1.
The joint PDF of RVs \((v_1, \ldots, v_N)\) is the product of the individual PDF’s, thus

\[
  f(v_1, v_2, \ldots, v_N) = \prod_{i=1}^{N} f(v_i).
\]  

(5.43)

Instead of looking for the distribution of the order statistics, the space \((v_1, \ldots, v_N)\) is partitioned into \(N\) different regions, where in the region \(A_j = \{(v_1, \ldots, v_N) : v_i < v_j \forall i \neq j, v_j > 0\}\). The required integration for the distributed system in presences of hidden nodes only is evaluated as follow;

\[
  C = \frac{1}{2} \sum_{j=1}^{N} \int_{A_j} f(v_1, \ldots, v_N) \log_2 \left( 1 + \frac{v_j}{\sum_{i \neq j} v_i \psi_i + \sigma_n^2} \right) dx
\]

(5.44)

where \(f(v_1, \ldots, v_N) = f(v_j) \prod_{i \neq j} f(v_i)\).

In order to take the status of successful relays in the first phase (status of relay nodes in set \(\mathcal{H}\)) into account, let

\[
  v_i = \begin{cases} 
    x_i & \text{if relay } i \text{ is successfull in Phase 1} \\
    0 & \text{otherwise}
  \end{cases}
\]

(5.45)

where \(x_i\) is the RV representing the successful power channel gain of the relay \(i\) in set \(\mathcal{H}\). Then,

\[
  f_{v_i}(v_i) = (1 - p_i) \delta(x_i) + p_i f_i(x_i)
\]

(5.46)

where \(\delta(.)\) is the dirac delta function and \(p_i\) is the probability that relay \(i\) successfully receive the signal from the source in the first phase given by \(p_i = \Pr(\gamma_i \geq \gamma_{(u)})\).

Consequently, \(f(v_1, \ldots, v_N)\) in (5.44) is written as

\[
  f(v_1, \ldots, v_N) = [(1 - p_j) \delta(x_j) + p_j f_j(x_j)] \prod_{i \neq j} [(1 - p_i) \delta(x_i) + p_i f_i(x_i)]
\]

(5.47)

where \(0 \leq x_i < x_j \forall i \neq j, \ x_j > 0\).
Applying (5.47) into (5.44) to get

\[ C = \frac{1}{2} \sum_{j=1}^{N} \int_{0}^{\infty} p_j f(x_j) \times \left[ \int_{0}^{x_j} \cdots \int_{0}^{x_j} \log_2 \left( 1 + \frac{x_j}{\sum_{i \neq j} x_i \psi_i + \sigma_n^2} \right) \prod_{i \neq j}^{N} [(1 - p_i) \delta(x_i) + p_i f_i(x_i)] dx_i \right] dx_j \]

(5.48)

and then Invoking Lemma 1 yields

\[ C = \frac{1}{2} \log_2 e \sum_{j=1}^{N} \int_{0}^{\infty} p_j f_j(x_j) \int_{0}^{x_j} \cdots \int_{0}^{x_j} \int_{0}^{\infty} \int_{0}^{\infty} e^{-\frac{\sigma_n^2}{2} z (1 - e^{-zx_j})} e^{-z \Sigma_{i \neq j} x_i \psi_i} \times \prod_{i \neq j}^{N} [(1 - p_i) \delta(x_i) + p_i f_i(x_i)] dx \] dxdx_jdz.

(5.49)

Interchanging the order of integrations in (5.49), as

\[ C = \frac{\log_2 e}{2} \sum_{j=1}^{N} \int_{0}^{\infty} \int_{0}^{\infty} e^{-\frac{\sigma_n^2}{2} z} \int_{0}^{\infty} p_j f(x_j) (1 - e^{-zx_j}) \times \left[ \int_{0}^{x_j} \cdots \int_{0}^{x_j} \prod_{i \neq j}^{N} e^{-z \Sigma_{i \neq j} x_i \psi_i} [(1 - p_i) \delta(x_i) + p_i f_i(x_i)] dx_i \right] dx_jdz \]

(5.50)

the \((N - 1)\) fold integrations inside the bracket reduces to

\[ \int_{0}^{x_j} \cdots \int_{0}^{x_j} \prod_{i \neq j}^{N} e^{-z \Sigma_{i \neq j} x_i \psi_i} [(1 - p_i) \delta(x_i) + p_i f_i(x_i)] dx_i = \prod_{i \neq j}^{N} \int_{0}^{x_j} e^{-x_i \psi_i} [(1 - p_i) \delta(x_i) + p_i f_i(x_i)] dx_i = \int_{0}^{x_j} e^{-x_i \psi_i} f_i(x_i) dx_i \]

(5.51)

and the inner integral can be computed by removing RV \(\psi_i\) as

\[ \int_{0}^{x_j} e^{-x_i \psi_i} f_i(x_i) dx_i = F(x_j) (1 - p_h) + p_h \int_{0}^{x_j} e^{-x_i} f_i(x_i) dx_i. \]

(5.52)
CHAPTER 5. PERFORMANCE DEGRADATION OF DISTRIBUTED CRS

Substituting (5.52) into (5.51) and then applying in (5.50) gives

$$C = \frac{\log_2 e}{2} \sum_{j=1}^{N} p_j \int_{0}^{\infty} \frac{e^{-z\sigma_n^2}}{z} A(z) \, dz$$

(5.53)

where

$$A(z) = \int_{0}^{\infty} (1 - e^{-zx}) \prod_{i \neq j}^{N} \left[ 1 - p_i + p_i \left( F(x_j) (1 - p_h) + p_h \int_{0}^{x_j} e^{-x_i} f_i(x_i) \, dx_i \right) \right] \times f_j(x_j) \, dx_j. \tag{5.54}$$

The equation (5.53) is the new exact expression for the system in the presence of hidden nodes only and in the case of non-identical RVs. When the links becomes identical, (5.53) reduces into the identical case in (5.21), which validates our analysis.

### 5.4.2 In the Absence of Hidden Nodes

The required integration for the average capacity of the system in the absence of hidden nodes for non-identical case is,

$$C = \frac{1}{2} \sum_{j=1}^{N} \int_{A_j} f(v_1, \ldots, v_N) \frac{1}{\Delta + \frac{v_j}{\sigma_n^2}} 1 \log_2 \left( 1 + \frac{v_j}{\sum_{i \neq j} v_i \xi_i + \sigma_n^2} \right) \, dv. \tag{5.55}$$

Following the same steps in Sec. 5.4.1, (5.55) can be simplified into

$$C = \frac{\log_2 e}{2} \sum_{j=1}^{N} \int_{0}^{\infty} \frac{e^{-z\sigma_n^2}}{z} \int_{0}^{\infty} p_j f(x_j) \frac{x_j (1 - e^{-zx_j})}{\lambda + x_j (1 + \Delta)}$$

$$\times \left[ \prod_{i \neq j}^{N} \int_{0}^{x_j} e^{-zx_i} f_i(x_i) (1 - p_i) \delta(x_i) + p_i f_i(x_i) \, dx_i \right] \, dx_j \, dz \tag{5.56}$$

where the inner integral in (5.56) is computed as

$$\int_{0}^{x_j} e^{-zx_i} [ (1 - p_i) \delta(x_i) + p_i f_i(x_i) ] \, dx_i = 1 - p_i + p_i \int_{0}^{x_j} e^{-zx_i} f_i(x_i) \, dx_i \tag{5.57}$$
and \( \int_0^{x_j} e^{-x_j \xi_i} f_i(x) \, dx \) can be evaluated by following the similar steps presented for (5.31) as

\[
\int_0^{x_j} e^{-x_j \xi_i} f_i(x) \, dx = 1 - \Phi \left( \frac{\lambda x_j}{\lambda + \Delta x_j}, 0 \right) - \Phi(x_j, z) + \Phi \left( \frac{\lambda x_j}{\lambda + \Delta x_j}, z \right) .
\] (5.58)

Substituting (5.58) into (5.57) and then applying in (5.56) gives the following exact unified expression for the SE of the system by considering the protocol overhead effects only

\[
C = \frac{\log_2 e}{2} \sum_{j=1}^{N} p_j \int_0^{\infty} \frac{1}{z} e^{-z \sigma_n^2} A(z) \, dz
\] (5.59)

where

\[
A(z) = \int_0^{\infty} x_j \left( 1 - e^{-x_j} \right) \frac{(\Delta + 1) x_j + \lambda}{x_j + \lambda} \times \prod_{i \neq j}^N \left[ 1 - p_i \left( \Phi \left( \frac{\lambda x_j}{\lambda + \Delta x_j}, 0 \right) + \Phi(x_j, z) - \Phi \left( \frac{\lambda x_j}{\lambda + \Delta x_j}, z \right) \right) \right] f_j(x_j) \, dx_j.
\] (5.60)

When the links become identical, (5.59) reduces into the identical case, (5.34), which validates our analysis.

### 5.4.3 Joint Effects of Protocol Overheads and Hidden Nodes

By considering the joint effects we will have

\[
C = \frac{1}{2} \sum_{j=1}^{N} \int_{A_j} f(v_1, \ldots, v_N) \frac{1}{\Delta + \frac{1}{v_j} + 1} \times \log_2 \left( 1 + \frac{v_j}{\sum_{i \neq j} v_i [\psi_i + (1 - \psi_i) \xi_i] + \sigma_n^2} \right) \, dv
\] (5.61)
We obtain similar results as (5.56), with

\[
\int_0^{x_j} e^{-zx_i(1-\psi_i)\xi_i} (1 - p_i) \delta_i(x_i) + p_i f_i(x_i) \, dx_i = \\
1 - p_i + p_i \left( \int_0^{x_j} e^{-zx_i\psi_i} f_i(x_i) \, dx_i + \int_0^{x_j} e^{-zx_i(1-\psi_i)\xi_i} f_i(x_i) \, dx_i \right) = \\
1 - p_i + p_i \left[ p_h \int_0^{x_j} e^{-zx_i\xi_i} f_i(x_i) \, dx_i + (1 - p_h) \int_0^{x_j} e^{-zx_i\xi_i} f_i(x_i) \, dx_i \right].
\]

(5.62)

5.5 Bound Analysis for SE

Consider the special case of the model in Sec. 5.3.2, in which all RVs \((x_1, x_2, \ldots, x_{L_s})\) have exponential distribution with \(\mathbb{E}[x_i] = 1\). Then rearrange the RVs in decreasing order based on the order statistic such that \(x_1 > x_2 > \cdots > x_{L_s}\), where \(\max(x_1, x_2, \ldots, x_{L_s}) = x_1 = y\). In the approach of bounded SE, RVs \((t_{\min}, \xi_i)\) are acknowledged to be independent from \((x_2, x_3, \ldots, x_{L_s})\) RVs, where \(l \neq 1\) and \(2 \leq l \leq L_s\). For this, \(\xi_i\) and \(t_{\min}\) can be replaced by their average.

5.5.1 Bounded SE in Presence of Hidden Nodes Only

By setting a lower bound to the \(C\) via Jensen’s inequality [85, pp. 409] it asserts that

\[
\frac{1}{2} \mathbb{E} \left[ \log_2 \left( 1 + \frac{y}{\sum_{l=2}^{L_s-1} x_l\psi_l + \sigma_n^2} \right) \bigg| L_s \right] \geq \\
\frac{1}{2} \mathbb{E} \left[ \log_2 \left( 1 + \frac{y}{\sum_{l=2}^{L_s} x_l\psi_l + \sigma_n^2} \right) \bigg| L_s \right] \tag{5.63}
\]

where \(\mathbb{E}[\psi_l|L_s] = p_h\) and by using the property that the average of an indicator function is the probability of its event, then \(p_h = \text{Pr}(\psi_l = 1|L_s)\).

Invoking Lemma 1 in (5.63), the \(C\) conditioned on \(L_s\) can be expressed as

\[
\frac{1}{2} \mathbb{E} \left[ \log_2 \left( 1 + \frac{y}{\sum_{l=2}^{L_s} x_l\psi_l + \sigma_n^2} \right) \bigg| L_s \right] = \\
\frac{1}{2} \log_2 \int_0^{\infty} \frac{e^{-z\sigma_n^2}}{z} \mathbb{E} \left[ e^{-z\sum_{l=2}^{L_s} x_l\psi_l} \bigg| L_s \right] - \mathbb{E} \left[ e^{-z(y + \sum_{l=2}^{L_s} x_l\psi_l)} \bigg| L_s \right] \, dz. \tag{5.64}
\]
Fading Type | PDF expressions | Incomplete MGF, 
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh</td>
<td>( f(x) )</td>
</tr>
<tr>
<td>Rician</td>
<td>( \frac{1+K}{\Omega} \exp \left( -K - \frac{(1+K)x}{\Omega} \right) \times I_0 \left( 2 \sqrt{K(K+1)} \frac{x}{\Omega} \right) )</td>
</tr>
<tr>
<td>Nakagami-(m)</td>
<td>( \left( \frac{m}{\Omega} \right)^m \frac{m^{-m-1}}{\Gamma(m)} \exp \left( -\frac{m}{\Omega} x \right) )</td>
</tr>
</tbody>
</table>

Table 5.1: The PDF and incomplete MGF for different fading models. in the table \( \Omega \) is the average power. For the Rician fading, \( K \) is the Rician factor, \( I_0(x) \) is the zeroth-order modified Bessel function of the first kind, and \( Q_1(a, b) \) is the first order Marcum function defined as \( Q_1(a, b) = \int_0^\infty x \exp \left( \frac{x^2+a^2}{2} \right) I_0 (ax) \, dx \). For Nakagami-\(m\) fading, \( m \) is the fading parameter in range of \( \frac{1}{2} \leq m \leq \infty \), \( \Gamma(a, x) \) is the incomplete gamma function defined as \( \Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} \, dt \), and \( \Gamma(a) \) is the gamma function.

### 5.5.1.1 Computation of the MGFs

The problem in (5.64) is still complicated because the ordered exponential random variables \( x_l, l = 2, \ldots, L_s \) are not independent. However, it can be shown that by using a non-direct method similar to that used in [86, eg. (12)], the joint MGFs in (5.64) can be computed. The method used in [86] is based on the Sukhatme’s theorem, which states that the order statistics of a sample of independent exponential RVs can be expressed as the unorder statistics of a sample of independent exponential RVs with appropriate means [87, Th. 4.6.1]. Therefore,

\[
\mathbb{E} \left[ e^{-z \sum_{l=2}^{L_s} x_l p_h} | L_s \right] = \phi \left( \phi(z \sum_{l=2}^{L_s} p_h), \phi(z \sum_{l=2}^{L_s-1} p_h) \right) \\
= \prod_{l=2}^{L_s} \frac{1}{1 + \frac{L_s-1}{L_s} z p_h}.
\]  (5.65)
On the other hand, 
\[
E \left[ e^{-z(y + \sum_{l=2}^{L_s} x_l p_h)} \middle| L_s \right] = \phi \left( z, z p_h, z p_h, \ldots, z p_h \middle| L_s \right) = \left( \frac{1}{1 + z} \right) \prod_{l=2}^{L_s} \frac{1}{1 + \frac{1}{l} z + \frac{l-1}{l} z p_h}. \tag{5.66}
\]

By replacing (5.65) and (5.66) into (5.64) and then averaging out with respect to \(L_s\) to remove the condition on \(L_s\), we arrive at 
\[
E \left[ \log_2 \left( 1 + \frac{y}{\sum_{l=2}^{L_s} x_l p_h + \sigma^2_n} \right) \right] = \frac{1}{2} \sum_{L_s=1}^{L} \Pr \left( L_s = |\mathcal{H}| \right) 
\times \log_2 \int_0^{\infty} e^{-z\sigma^2_n} \left( \prod_{l=2}^{L_s} \frac{1}{1 + \frac{1}{l} z p_h} - \left( \frac{1}{1 + z} \right) \prod_{l=2}^{L_s} \frac{1}{1 + \frac{1}{l} z + \frac{l-1}{l} z p_h} \right) dz. \tag{5.67}
\]

where \(\Pr \left( L_s = |\mathcal{H}| \right) = \binom{L}{L_s} (1 - p_s)^{L_s} (p_s)^{L-L_s}\).

**5.5.2 Bounded SE in Absence of Hidden Nodes**

Approximating the SE by its average via Jensen’s inequality [85, pp. 409] results in the following bound for SE. Thus,
\[
\frac{1}{2} E \left[ \log_2 \left( 1 + \frac{y}{\sum_{l=1}^{L_s-1} x_l \psi_l + \sum_{l=1}^{L_s-1} x_l (1 - \psi_l) \xi_l + \sigma^2_n} \right) \right] \
\geq \frac{1}{2} \left( \Delta + \mathbb{E}[t_{\min}|L_s] + 1 \right) \mathbb{E} \left[ \log_2 \left( 1 + \frac{y}{\sum_{l=2}^{L_s} x_l \psi_l + \sigma^2_n} \right) \middle| L_s \right] \
\geq \frac{1}{2} \left( \Delta + \mathbb{E}[t_{\min}|L_s] + 1 \right) \mathbb{E} \left[ \log_2 \left( 1 + \frac{y}{\sum_{l=2}^{L_s} x_l \psi_l + \sigma^2_n} \right) \middle| L_s \right]. \tag{5.68}
\]

where \(\psi_l = \mathbb{E}[\xi_l|L_s]\) and by using the property that the average of an indicator function is the probability of its event, then \(\psi_l = \Pr (\xi_l = 1|L_s) = \Pr \left( \frac{\Delta}{z_l} \leq \Delta + \frac{\Delta}{z_l}|L_s \& l \neq 1 \right)\).

Invoking Lemma 1 into (5.68), the \(C\) conditioned by \(L_s\) can be expressed
as
\[
\mathbb{E} \left[ \log_2 \left( 1 + \frac{y}{\sum_{l=2}^{L_s} x_l u_l + \sigma_n^2} \right) \mid L_s \right] = \\
\log_2 \int_0^{\infty} \mathbb{E} \left[ e^{-z \sum_{l=2}^{L_s} x_l u_l \mid L_s} \right] - \mathbb{E} \left[ e^{-z (y + \sum_{l=2}^{L_s} x_l u_l \mid L_s} \right] e^{-z \nu} dz. \tag{5.69}
\]

### 5.5.2.1 Computation of the Overheads Probability

To compute the probability of overheads, \( u_l \), let’s consider
\[
\text{Pr} \left( \frac{\lambda}{u_l} \leq \Delta + \frac{1}{y} L_s \& l \neq 1 \right) = \text{Pr} \left( t_l \leq \Delta + t_{\text{min}} \mid L_s \& l \neq 1 \right).
\]
Using joint distribution of two order statistics, see Appendix A.5, gives
\[
\text{Pr} \left( t_l \leq \Delta + t_{\text{min}} \mid L_s \right) = \int \int_{A} f_{t_{\text{min}}, t_l} (u, u_l) du du_l. \tag{5.70}
\]
where \( A \) is the region of integration defined as \( A = \{ t_{\text{min}} < t_l \ & t_l \leq t_{\text{min}} + \Delta \} \).

As can be seen from (5.70), the joint PDF of RVs \( t_{\text{min}} \) and \( t_l \) based on \( L_s \) ordered RVs are needed which from Appendix A.5 is given by
\[
f_{t_{\text{min}}, t_l} (u, u_l) = \frac{L_s!}{(l-2)! (L_s-l)!} [F (u_l) - F (u)]^{l-2} [1 - F (u_l)]^{L_s-l} f (u_l) f (u). \tag{5.71}
\]
where \( f (u) \) and \( f (u_l) \) are the PDFs of \( t_{\text{min}} \) and \( t_l \) respectively. \( F (u) \) and \( F (u_l) \), are the respective corresponding CDFs of \( t_{\text{min}} \) and \( t_l \) in non-order statistics obtained by
\[
F (u_l) = \text{Pr} \left( t_l \leq u_l \right) = \text{Pr} \left( \frac{\lambda}{x_l} \leq u_l \right) = \begin{cases} 
\text{Pr} \left( x_l \geq \frac{\lambda}{u_l} \right) = e^{-\frac{\lambda}{u_l}} & u_l > 0 \\
0 & u_l \leq 0
\end{cases}.
\tag{5.72}
\]
In the same way, CDF of \( t_{\text{min}} \) is given by \( F (u) = e^{-\frac{\lambda}{u}} \). Moreover,
\[
f (u_l) = \frac{\partial F (u_l)}{\partial u_l} = \begin{cases} 
\lambda u_l^{-2} e^{-\frac{\lambda}{u_l}} & u_l > 0 \\
0 & u_l \leq 0
\end{cases} \tag{5.73}
\]
Likewise, $f(u) = \lambda u^{-\frac{3}{2}} e^{-\frac{u}{2}}$ as all RVs are assumed to be exponentially distributed with unit mean.

By replacing (5.71) into (5.70), the overheads probability can be written as

$$
\Pr (t_l \leq t_{\min} + \Delta \mid L_s) = \frac{L_s!}{(l-2)! (L_s-l)!} \times \int_0^\infty \int_{u_l} f (u_l) \left[ F (u_l) - F (u) \right]^{l-2} \left[ 1 - F (u_l) \right]^{L_s-l} f (u) \, du \, du_l \tag{5.74}
$$

which can be calculated easily by sketching the regions of integration of $f_{t_{\min}, t_l} (u, u_l)$ for $t_{\min} < t_l$ & $t_l \leq t_{\min} + \Delta$ as

$$
\Pr (t_l \leq t_{\min} + \Delta \mid L_s) = 1 - \frac{L_s!}{(l-2)! (L_s-l)!} \times \int_0^\infty \int_{u_l}^{u_l-\Delta} f (u) f (u_l) \left[ F (u_l) - F (u) \right]^{l-2} \left[ 1 - F (u_l) \right]^{L_s-l} \, du \, du_l. \tag{5.75}
$$

### 5.5.2.2 Computation of the Minimum Time Average

The exception time of the best relay, $E [t_{\min} \mid L_s]$, can be obtained by averaging out the RV $t_{\min}$ over its PDF as

$$
E [t_{\min} \mid L_s] = \int_0^\infty u f_{t_{\min}} (u) \, du \tag{5.76}
$$

where $f_{t_{\min}} (u)$ is PDF of $t_{\min}$. Since RVs are ordered as $y > x_2 > ... > x_{L_s}$, the ordered sequence of the timer RVs from $t_l = \frac{\lambda}{x_l}$ can be written as $t_{\min} < t_2 < ... < t_{L_s}$. Therefore, the CDF of $t_{\min}$, $F_{t_{\min}} (u)$, is obtained as follows:

$$
\Pr (t_{\min} \leq u \mid L_s) = 1 - \Pr (t_{\min} \geq u) = 1 - \prod_{l=1}^{L_s} \left[ \Pr (t_l \geq u) \right] = 1 - \left[ 1 - F (u) \right]^{L_s} \tag{5.77}
$$

where $F (u) = \exp (-\lambda/u)$, see (5.72). Taking the derivation of the result in (5.77) with respect to $u$, gives PDF of $t_{\min}$

$$
f_{t_{\min}} (u) = \frac{\partial F_{t_{\min}} (u)}{\partial u} = L_s [1 - F (u)]^{L_s-1} f (u). \tag{5.78}
$$
Substituting (5.78) into (5.76) gives the following expression for $E[t_{\min}|L_s]$,

$$E[t_{\min}|L_s] = \int_0^\infty u L_s [1 - F(u)]^{L_s-1} f(u) \, du. \quad (5.79)$$

### 5.5.2.3 Computation of the MGFs

By using the non-direct method (similar to that used in [86, eg. (12)]), the expectation terms in (5.69) can be obtained by

$$E[e^{-z \sum_{l=2}^{L_s} x_i L_s}] = \phi \left( \begin{array}{c} 0 \\ \frac{1}{z} z v_2, z v_3, \ldots, z v_{L_s} \end{array} \right)_L = \prod_{l=2}^{L_s} \frac{1}{1 + \frac{1}{z} v_l}. \quad (5.80)$$

Likewise,

$$E[e^{-z(y + \sum_{l=2}^{L_s} x_i L_s)}] = \phi \left( \begin{array}{c} z \\ \frac{1}{z} z v_2, z v_3, \ldots, z v_{L_s} \end{array} \right)_L = \left( \frac{1}{1 + z} \right) \prod_{l=2}^{L_s} \frac{1}{1 + \frac{1}{z} L_s + \frac{1}{z} v_l}. \quad (5.81)$$

Replacing (5.80) and (5.81) in (5.69) and then averaging out with respect to $L_s$ to remove the condition on $L_s$ gives the upper bound of the SE as

$$C = \frac{1}{2} \sum_{L_s=1}^N \frac{\Pr(L_s = |\mathcal{H}|)}{\Delta + E[t_{\min}] + 1} \times \log_2 \int_0^\infty \frac{\prod_{l=2}^{L_s} \frac{1}{1 + \frac{1}{z} v_l} - \left( \frac{1}{1 + z} \right) \prod_{l=2}^{L_s} \frac{1}{1 + \frac{1}{z} L_s + \frac{1}{z} v_l} e^{-z^2} \, dz. \quad (5.82)$$
Figure 5.3: Exact SE of the time-based relay selection method in a cooperative relay system over Rayleigh fading channels as a function of a number of relay nodes $N$ and in the absence of hidden nodes with respect to different $\frac{\Delta h}{\lambda}$.

5.6 Results

Expressions obtained for the SE of the distributed cooperative relay system in Sec. 5.3, Sec. 5.4 and Sec. 5.5 were plotted and then validated with simulations. Solid lines in the figures show the theoretical results and markers show the simulation results.

Fig. 5.3 presents the SE of a distributed cooperative relay system in the absence of hidden nodes over Rayleigh fading channels. In Fig. 5.3, the exact numerical values of the SE were obtained from (5.34). It can be seen that the SE is decreasing by increasing $\frac{\Delta h}{\lambda}$. This is because by increasing $\frac{\Delta h}{\lambda}$ the impact of probability of overheads on the system performance is increased, see (5.5), which causes the reduction in the SE. The probability of the overheads can be also increased by increasing the number of relay nodes in the system. Therefore, in a system with higher $\frac{\Delta h}{\lambda}$, by increasing the number of relay nodes the SE of the system starts to decrease as the protocol overheads dominate the system performance. This is why in Fig. 5.3, the SE of the system with $\frac{\Delta h}{\lambda} = 5$ at the beginning is increasing due to the small number of relay nodes in the
network and then decreasing by increasing the number of rely nodes. Additionally, the analytical result obtained from (5.34) is validated for arbitrary channel fading models in Fig. 5.4. In the figure the exact SE has been plotted as a function of number of relay nodes over diverse practical interference fading models by using a single simple formula, (5.34), where Table 5.1 has been used for the incomplete MGF of Rayleigh, Rician and Nakagami-\(m\) distributions. The solid lines in Fig. 5.5 and Fig. 5.6 illustrate the bound of the analysis for the system in presence of hidden and in the absence of hidden nodes. The bounded SE in Fig. 5.5 and Fig. 5.6 have been plotted from (5.67) and (5.82), respectively, where the analytical result of collision probabilities driven in (5.75) has been validated by Fig. 5.7. The figures also compare the bounded SE with the exact SE and show that the practical cooperative relay systems have a higher SE. Fig. 5.6 can also be used to compare the exact results of this thesis with existing results in [14] which analysed the system performance in terms of the bounded SE.

In practice, the system parameter \(\lambda\) can not be made very large. This is because by increasing \(\lambda\) the speed of relay selection defined as \(t_l = \frac{\lambda}{z_l}\) can be significantly reduced and thus the channel can be changed. On the other hand, \(\lambda\) is needed to be as large as possible to reduce the overheads probability. Therefore, there is a trade-off between collision indicator, (5.5), and speed of relay selection. That means that \(\lambda\) is needed to be as large as possible to reduce collision probabilities and at the same time as small as possible to quickly select the best relay before the channel changes. By having small time selection the system can have higher useful data transmission, \(T_e = T - T_s\), where \(T_s = \frac{\lambda}{z_l} + \Delta\), and thus higher SE is achievable. However, at the same time the probability that relay nodes collide with the best relay is increased which cause the reduction in SE. Fig. 5.8 shows a system in which end-to-end transmission is set as \(T = 10\)msec and \(\gamma_{th} = 27\)dBm presenting a middle communication environment \((p_s \approx 0.6)\). As can been seen from Fig. 5.8, in the small network \(N < 6\), by increasing \(\frac{\lambda}{z_l}\) the SE increases despite of the increased probability of overheads. That is because in the small network the impact of probability of overheads are low, while time selection \(T_s\) plays a significant role in determining system performance. Therefore, SE increases with increased \(\frac{\lambda}{z_l}\). In contrast, in a large sized network \(N \geq 6\), the probability of collisions are the main factor in determining the SE. Consequently, SE
decreases as the number of relay nodes is increased.

Figs 5.9 and 5.10 respectively show the impact of the probability of hidden nodes \( p_h \) and successful transmission probability \( p_s \) on the system performance. These figures also validate the numerical results presented in (5.21) and (5.26) as they perfectly match with Mont Carlo simulation. As can be seen from Fig. 5.9, by increasing the probability of hidden nodes, collision probabilities increases and thus the SE is decreased. In contrast, in a small sized network, the system with a higher probability of successful transmission in the first phase, has a higher SE as the effect of probability of hidden nodes is low. However, this impact increases for a large sized network and causes decreasing SE, see Fig. 5.10. The SE of the distributed relay selection method over Rayleigh fading channels with different fading parameters as function of \( \frac{\lambda}{\chi} \) has been plotted in Fig. 5.11 which also validate the expression derived in (5.59).
Figure 5.5: The exact and upper bound of SE over Rayleigh fading channels as function of the number of relay nodes $L$ where $p_h = 0.5$ and $p_s \approx 0.4$.

Figure 5.6: The exact and lower bound of SE over Rayleigh fading channels as function of the number of relay nodes $N$, where $\frac{\lambda}{\lambda} = 2$. 
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Figure 5.7: Probability of the overheads introduced by the time-based relay selection protocol as function of $\frac{\Delta}{\lambda}$.

Figure 5.8: Exact SE of the distributed cooperative relay system over Rayleigh fading channels in the absence of hidden nodes, as function of a number of relay nodes $N$ and $\frac{\Delta}{\lambda}$, where $\gamma_{th} = 27$dBm and $T = 10$ msec.
Figure 5.9: Exact SE of the distributed cooperative relay system over Rayleigh fading channels as a function of a number of hidden relay nodes $N$ and $P_h$, where $P_s = 0.6$.

Figure 5.10: Exact SE of the distributed cooperative relay system over Rayleigh fading channels as a function of number of hidden relay nodes $N$ with different $P_s$ ($P_s = 0.37, 0.6, 0.8$), where $p_h = 0.6$. 
Figure 5.11: Exact SE of the distributed cooperative system in the absence of hidden relay nodes and over Rayleigh fading channels as a function of $\frac{\Delta}{\lambda}$ and different fading parameters, where $N = 5$.

5.7 Chapter Summary

In this chapter, the exact performance of a distributed relay cooperative system employing a time-based relay selection protocol was analysed. For this a new accurate mathematical method proposed to aid the investigation. The accurate mathematical method led to the derivation of new exact expressions for the spectral efficiency considering both self-interferences and overheads introduced by the protocol; as well as accounting for different fading scenarios and arbitrary relay locations. These new exact expressions were used to study the trade-off between the extra protocol overheads required to reduce the amount of self-interferences and the overall system throughput as measured in terms of the SE. The expressions were also examined when the RVs were either identical or non-identical. Additionally, a bound for the spectral efficiency was derived and compared with the derived exact results.

Generally, results confirm that the distributed relay system’s performance is degraded with interferences either in presence or absence of hidden relay nodes. Moreover, observations showed that in a small sized network, time selection plays the main role to design an efficient system in terms of SE,
while in a large sized network the protocol overheads have higher impact on the system performance.
Chapter 6

Advanced Relay Cooperative Systems

6.1 Introduction

This chapter is the extension of Chapter 5 to analyse the SE of distributed TWR and FDR systems by employing a time-based relay selection protocol. As previously discussed in Sec. 3.3, TWR and FDR systems are attracting considerable interest as they significantly recover the spectral loss in an OWR system. Applying a relay selection scheme in TWR and FDR systems have received much more interest as they can provide more efficient systems. The relay selection problem for TWR and FDR have not been much studied in compare to OWR systems, and only few researches have been carried out [46, 88–90]. Authors in [90] investigated several relay-selection techniques including none selection, optimal selection, max-min selection and max-sum selection methods for a TWR system, where effects of relay selection protocol overheads have simply been ignored. In [88, 89], CDF of the worst end-to-end SNR of two users in a TWR system was evaluated and the worst received SNR was maximized. The different relay selection schemes in an AF cooperative system with full-duplex operation was studied in [46] but without considering the loop interferences in the single relay selection scheme. In this chapter, Chapter 5 is extended to evaluate the SE of TWR and FDR systems and investigate the impact of the overheads introduced by the time-based relay selection protocol on the system performance.
In Sec. 6.2, SE of both TWR and FDR employing a time-based relay selection protocol systems are investigated. In Sec. 6.3 the results are discussed and in Sec. 6.4, summary of the chapter is presented.

### 6.2 SE of Advanced Cooperative Systems

Aims to investigate the SE of a time-based relay selection protocol in non-ideal distributed TWR and FDR systems, in sense that in practice the implementation of a time-based relay selection protocol is not ideal due to the possible overheads, see Sec. 5.2.3. In order to simplify the analyses, all RVs assumed to be independent and have exponential distribution with mean value of one. Moreover, the impact of the overheads on the SE are investigated based on the over-pessimistic assumption. Consequently, the average SE is approximated as

$$ C = \frac{R_r}{B} \sum_{L_s = 1}^{N} T_e \left[ 1 - \Pr(\text{overhead}) \right] \Delta + \mathbb{E}[t_{\min}] + T_e \Pr(|\mathcal{H}| = L_s) \Pr(\text{success}) \tag{6.1} $$

where $R_r$ is the data rate, $B$ is the bandwidth, $\Pr(|\mathcal{H}| = L_s)$ is the probability that the size of the set $\mathcal{H}$ is $L_s$, $\Pr(\text{success})$ is the successful transmission probability through the best relay in Phase 2. $\Pr(\text{overhead})$ is the protocol overheads defined in Sec. (5.5.2.1). In the assessment of the overheads probability, the interference that may arise between the best and the second best relay is investigated. For that reason, the overheads probability in (5.75) is reduced to

$$ \Pr(\text{overhead}|L_s) = \Pr(t_2 \leq t_{\min} + \Delta | L_s) = 
1 - \frac{L_s!}{(L_s - 2)!} \int_{\Delta}^{\infty} F(u - \Delta) \left[ 1 - F(u) \right] L_s^{-2} f(u) \, du. \tag{6.2} $$

### 6.2.1 The Distributed TWR System Model

Consider a TWR system described in Sec. 3.3.1. The system with $N$ AF relay nodes and two end users, $T_1$ and $T_2$, where both act as a source. The channel fading amplitude from $T_1$ to relay $l$ and from relay $l$ to $T_2$ are denoted by $g_l$ and $f_l$ respectively. For convenience in the analyses, RVs are denoted by $|g_l|^2 = y_l$ and $|f_l|^2 = x_l$ which all are exponentially distributed with mean one. During the transmission of each block, the system allocates a duration $\frac{T_e}{2}$ for
transmission from the source nodes to the relay nodes $T_1 \rightarrow R_l \leftarrow T_2$; and also the system allocates another duration $\frac{T_e}{2}$ for the transmission from the relay nodes to the source nodes $T_1 \leftarrow R_l \rightarrow T_2$. Consequently, transmission from one source to another source consumes twice of the channel resources in a FDR system and half of the channel resources in an OWR system.

From (6.1), the average SE of the TWR system is given by

$$C_{TWR} = \frac{1}{2} R_e \sum_{L_s=1}^{N} \frac{T_e [1 - \Pr (\text{overhead})]}{\Delta + \mathbb{E}[t_{\min}]} + T_e \Pr (|\mathcal{H}| = L_s) \Pr (\text{success})$$

where the factor $1/2$ is to account for the two phase transmission required for end-to-end transmission, transmission between $T_1 \rightleftharpoons R_l \rightleftharpoons T_2$ links. The two-way connection between terminals $T_1$ and $T_2$ is used to increase the sum rate of the network. See also Sec. 3.3.1 for details.

From Sec. 3.3.1.1, SNR at $T_1$ and $T_2$, respectively, are given by

$$\gamma_{1,l,2} = \frac{P_l P x_l y_l}{1 + (P + P_l) x_l + P y_l}$$

and

$$\gamma_{2,l,1} = \frac{P_l P x_l y_l}{1 + (P + P_l) y_l + P x_l}$$

where $P$ and $P_l$ are the power budget of the sources and relay $l$, respectively; and all noise powers are assumed to be equal to one.

Since in a TWR transmission there are two communication tasks with their own end-to-end SNR, the worst end-to-end SNR of two users, $T_1$ and $T_2$, is required. Therefore, the timer of each relay $l$ in set $\mathcal{H}$ is defined as

$$t_l = \frac{\lambda}{\min (\gamma_{1,l,2}, \gamma_{2,l,1})}.$$ 

Due to SNRs in (6.4) and (6.5) and by assuming channel reciprocity, it can be shown that [91]

$$\gamma_{1,l,2} \geq \gamma_{2,l,1} \iff y_l \leq x_l \quad \& \quad \gamma_{1,l,2} \leq \gamma_{2,l,1} \iff y_l \geq x_l$$
thus, (6.6) can be written as

\[ t_l = \frac{\lambda}{\min (y_l, x_l)} \]  

(6.8)

which is exactly the same as OR protocol in an OWR system.

As previously discussed in 3.4.1.2, the relay with the maximum worst SNR is selected as the best relay and it is the relay that transmits the source signal to the destination in Phase 2. As a result, from (6.8) for the best relay, \( l^* \), the following argument can be given,

\[ l^* = \arg \max_l \min (y_l, x_l) . \]  

(6.9)

In a two-way AF relay system, end-to-end transmission fails if the signal power either in a source-relay link or in source-relay-destination link falls below some threshold. For that reason, it is necessary to have a successful transmission in both Phase 1 and Phase 2.

### 6.2.1.1 Successful Transmission in Phase 1

The number of candidate relays \( L_s \) has binomial distribution defined as \( L_s \sim (N, p_s) \), where \( p_s \) is the probability that each relay node successfully receive the signal transmitted from the source. Therefore, the probability of getting exactly \( L_s \) success candidate relays in \( N \) trails in Phase 1 is given by the following probability of mass function

\[ \Pr (|\mathcal{H}| = L_s) = \binom{N}{L_s} (1 - p_s)^{L_s} (p_s)^{N-L_s} . \]  

(6.10)

where \( \binom{N}{L_s} = \frac{N!}{L_s!(N-L_s)!} \). Since in Phase 1, both scours, \( T_1 \) and \( T_2 \), at the same time broadcast their own data to relay nodes, the probability of successful transmission \( p_s \) is given by

\[ p_s = \Pr (\gamma_{1,l} \geq \gamma_{th,r}) \Pr (\gamma_{2,l} \geq \gamma_{th,r}) = e^{-2\gamma_{th,r}} \]  

(6.11)

where \( \gamma_{1,l} \) is the SNR between \( T_1 \) and relay \( l \) given by \( \gamma_{1,l} = P_{y_l} \gamma_{1,l} \); \( \gamma_{2,l} \) is the SNR between relay \( l \) and \( T_2 \) where, \( \gamma_{2,l} = P_{x_l} \gamma_{2,l} \); and \( \gamma_{th,r} \) is the threshold SNR at the relay nodes.
Applying (6.11) in (6.10), gives
\[
Pr(|\mathcal{H}| = L_s) = \binom{N}{L_s} (1 - e^{-2\gamma_{th,r}})^{L_s} (e^{-2\gamma_{th,r}})^{N-L_s}
\]
which is the probability of successful transmission in Phase 1.

### 6.2.1.2 The Protocol Overhead Probability

Consider \( z_l = \min(y_l, x_l) \). By using the order statistics of RVs for given RVs \( t_l = \frac{\lambda}{z_l}, l = 1, 2, \ldots, L_s \) in increasing order as \( t_{\min} < t_2 < \ldots < t_{L_s} \), see Appendix A.5 for further details, the overhead probability is obtained from (6.2). Recalling (6.2), \( f(u) \) and \( F(u) \) are required which are obtained as follows:

\[
F(u) = Pr(t_l \leq u) = Pr\left(y_l \geq \frac{\lambda}{u}\right)Pr\left(x_l \geq \frac{\lambda}{u}\right) = e^{-2\lambda u}
\]
and PDF, \( f(u) \), is given by

\[
f(u) = \frac{\partial F(u)}{\partial u} = \frac{2\lambda}{u^2} e^{-2\lambda u}.
\]

By substituting (6.13) and (6.14) into (6.2), the protocol overhead probability is obtained by the following expression

\[
Pr(\text{overhead}|L_s) = 1 - \frac{2\lambda L_s!}{(L_s - 2)!} \int_{\Delta}^{\infty} e^{-2\lambda u} \left[1 - e^{-\frac{2\lambda u}{(u-\Delta)}}\right]^{L_s-2} e^{-\frac{2\lambda u}{(u-\Delta)}} du.
\]

### 6.2.1.3 Expectation Time of the Best Relay

Recalling (5.79) to compute the expectation of the best relay’s timer, \( \mathbb{E}[t_{\min}|L_s] \), the PDF and CDF of RV \( t_l \) are required given by (6.14) and (6.13), respectively. Thus,

\[
\mathbb{E}[t_{\min}|L_s] = 2L_s\lambda \int_0^{\infty} u^{-1} \left[1 - e^{-\frac{2\lambda u}{u}}\right]^{L_s-1} e^{-\frac{2\lambda u}{u}} du.
\]
6.2.1.4 Successful Transmission in Phase 2

In Phase 2, the successful transmission based on the highest worse end-to-end SNR for a given non-empty $|H|$ can be evaluated as

$$\Pr (\text{success}|L_s) = \Pr (\gamma_{1,l^*,2} \geq \gamma_{th,d}) \Pr (\gamma_{2,l^*,1} \geq \gamma_{th,d})$$

which can be equivalent to

$$\Pr (\text{success}|L_s) = \Pr (\max_{l} \min (\gamma_{1,l^*,2}, \gamma_{2,l^*,1}) \geq \gamma_{th,d}) = 1 - \prod_{l=1}^{L_s} F_{\gamma_l}(\gamma_{th,d})$$

where $\gamma_{th,d}$ is the threshold SNR at the destination node and $\gamma_l = \min (\gamma_{1,l^*,2}, \gamma_{2,l^*,1})$.

In what follows, the CDF of $\gamma_l$, denoted by $F_{\gamma_l}(\gamma_{th,d})$, is needed to be computed. By assuming that all relays have the same power, $F_{\gamma_l}(\gamma_{th,d})$ is derived from [92] as

$$F_{\gamma_l}(\gamma_{th,d}) = 1 - \frac{k_l(\gamma_{th,d})}{\gamma_{th,d}} \exp \left( \frac{-\theta_l(\gamma_{th,d})}{\gamma_{th,d}} \right) K_1 \left( \frac{k_l(\gamma_{th,d})}{\gamma_{th,d}} \right) + \frac{\gamma_{th,d}}{\gamma_{th,d}} \exp \left( \frac{-\theta_l(\gamma_{th,d}) + k_l(\gamma_{th,d})}{\gamma_{th,d}} \right)$$

where $\theta_l(\gamma_{th,d}) = \left( 1 + \frac{2\sigma^2}{\gamma_{th,d}} \right) \gamma_{th,d} / P$, $k_l(\gamma_{th,d}) \triangleq 2 \sqrt{\frac{P\gamma_{th,d}}{\gamma_{th,d}} \left( 1 + \left( 1 + \frac{\gamma_{th,d}}{\gamma_{th,d}} \right) \gamma_{th,d} \right)}$, and $K_1 (.)$ is the modified first-order Bessel function of the second kind.

6.2.2 The Distributed FDR System Model

Consider a FDR system described in Sec. 3.3.3, the system with $N$ relay nodes, one source ($T_1$) and one destination ($T_2$). The $T_1$ and $T_2$ nodes operate in half-duplex mode and are equipped with a single antenna. However, each relay node is equipped with two antennas, one receiving antenna and one transmitting antenna, enabling full-duplex operation at the price of residual loop interference. For convenience, consider RVs $y_l = |g_l|^2$, $x_l = |f_l|^2$ and $|h_{1l}|^2 = x_l$, which have exponential distributions with means one. The power budgets for source $P_1$ and relay nodes $P_2$ are denoted by $P = P_1 = P_2$. Moreover, all power of AWGN are assumed to be equal to one, $\sigma^2_1 = \sigma^2_2 = 1$. 
In a DF network, SINR at destination through $R_l$ is given by (see Sec. 3.3.3.2 for the derivation)

$$\gamma_{DF}^l = \min \left( \frac{\gamma_{1,l}}{\gamma_{I,l} + 1}, \gamma_{I,l} + 1, \gamma_{l,2} \right)$$  \hspace{1cm} (6.20)

where $\gamma_{1,l} = P_{y_l}, \gamma_{I,l} = i_l$ and $\gamma_{l,2} = P_{x_l}$.

In an AF network, SINR at the destination through $R_l$ is obtained by (see Sec. 3.3.3.1 for the derivation)

$$\gamma_{AF}^l = \frac{\gamma_{1,l}}{\gamma_{I,l} + 1} \gamma_{l,2} \gamma_{1,l} \gamma_{I,l} + 1 + \gamma_{l,2} + 1.$$ \hspace{1cm} (6.21)

In the FDR cooperative system, the time-based OR selection protocol presented in (3.28) can be modified as

$$t_l = \lambda \min \left( \frac{\gamma_{1,l}}{\gamma_{I,l} + 1}, \gamma_{l,2} \right) = \lambda \min \left( \frac{P_{y_l} + 1}{P_{x_l}}, P_{x_l} \right)$$ \hspace{1cm} (6.22)

in order to consider loop interferences in the relay selection scheme.

From 3.4.1.2, the best relay, $l^*$, in the FDR system is defined as

$$l^* = \arg \max_i \min \left( P_{y_l} + 1, P_{x_l} \right).$$ \hspace{1cm} (6.23)

Since in a FDR system two terminals are simultaneously used for transmission in each phase, the average SE for a FDR cooperative system from (6.1) is given by

$$C_{FD} = \frac{R_r}{B} \sum_{L_s=1}^N \frac{T_e [1 - \Pr (\text{overhead})]}{\Delta + \mathbb{E}[t_{\min}] + T_e} \Pr (|H| = L_s) \Pr (\text{success})$$ \hspace{1cm} (6.24)

which consumes half of the channel resources in a TWR system and a quarter of the channel resources in an OWR system.

### 6.2.2.1 Successful Transmission in Phase 1

Recalling (6.10) to obtain $\Pr (|H| = L_s)$, the successful transmission probability of the FDR in Phase 1, $p_s$, is required. For this first by condition on $\gamma_{I,l}$ and
then removing the condition by taking the average with respect to RV $\gamma_{I,l}$ over its PDF, $f_{\gamma_{I,l}}(s)$, the following expression is obtained

$$p_s = \Pr \left( \frac{\gamma_{1,l}}{\gamma_{I,l} + 1} \geq \gamma_{th,r} | \gamma_{I,l} = s \right) = \int_0^\infty \Pr \left( \gamma_{1,l} \geq \gamma_{th,r} (s + 1) \right) f_{\gamma_{I,l}}(s) \, ds$$

$$= \frac{e^{-\gamma_{th,r}}}{1 + \gamma_{th,r}}. \quad (6.25)$$

Therefore,

$$\Pr (|H| = L_s) = \binom{N}{L_s} \left( 1 - \frac{e^{-\gamma_{th,r}}}{1 + \gamma_{th,r}} \right)^{L_s} \left( \frac{e^{-\gamma_{th,r}}}{1 + \gamma_{th,r}} \right)^{N-L_s} \quad (6.26)$$

### 6.2.2.2 The Protocol Overhead Probability

Consider $z_l = \min \left( P_{i_l+1}, P_{y_l} \right)$. By using the order statistics of RVs to order the given RVs $t_l = \frac{\lambda}{z_l}, l = 1, 2, \ldots, L_s$ in increasing order as $t_1 < t_2 \ldots < t_{L_s}$, the overhead probability is obtained from (6.2), where $f(u)$ and $F(u)$ are calculated as follows:

$$F(u) = \Pr \left( t_l \leq u \right) = \Pr \left( z_l \geq \frac{\lambda}{u} \right) = \Pr \left( \frac{y_l}{i_l+1} \geq \frac{\lambda}{P_{u}} \right) \Pr \left( x_l \geq \frac{\lambda}{P_{u}} \right) \quad (6.27)$$

which by conditioning on RV $i_l$, the CDF is given by

$$F(u) = \Pr \left( x_l \geq \frac{\lambda}{P_{u}} \right) \int_0^\infty \Pr \left( \frac{y_l}{i_l+1} \geq \frac{\lambda}{P_{u}} | i_l = s \right) f_{i_l}(s) \, ds = \frac{ue^{-\frac{2\lambda}{P_{u}}}}{u + \lambda}. \quad (6.28)$$

On the other hand, PDF of $t_l$ from (6.28) is given by

$$f(u) = \frac{\partial F(u)}{\partial u} = \frac{\lambda e^{-\frac{2\lambda}{P_{u}}}}{u^2 + \lambda u} + \frac{2\lambda e^{-\frac{2\lambda}{P_{u}}}}{P (u^2 + \lambda u)}. \quad (6.29)$$

Substituting (6.28) and (6.29) into (6.2), gives the probability of the overhead introduced by OR protocol in the FDR as

$$\Pr (\text{overhead}|L_s) = 1 - \frac{L_s!}{(L_s-2)!} \int_\Delta \frac{\left( \frac{u}{u-\Delta} \right)^{2\lambda}}{P (u^2 + \lambda u)} \left[ \frac{\lambda e^{-\frac{2\lambda}{P_{u}}}}{u^2 + \lambda u} + \frac{2\lambda e^{-\frac{2\lambda}{P_{u}}}}{P (u^2 + \lambda u)} \left[ 1 - \frac{ue^{-\frac{2\lambda}{P_{u}}}}{u + \lambda} \right] \right]^{L_s-2} \, du. \quad (6.30)$$
6.2.2.3 Expectation Time of the Best Relay

Using (5.79) to compute the expectation of the best relay’s time, $E[t_{\min}|L_s]$, PDF and CDF of RV $t_i$ are needed given by (6.29) and (6.28), respectively. Thus replacing (6.28) and (6.29) into (5.79) gives

$$E[t_{\min}|L_s] = \int_0^\infty u L_s (P + 2) \lambda e^{-\frac{2u}{P}} \left[ 1 - \frac{ue^{-\frac{u}{P}}}{u + \lambda} \right]^{L_s-1} du. \quad (6.31)$$

6.2.2.4 Successful Transmission in Phase 2

Transmission via DF: SINR of end-to-end transmission through the best relay in a DF relay network is

$$\gamma_{DF}^{l*} = \max \gamma_l \quad (6.32)$$

where $\gamma_l = \min \left( \frac{\gamma_{1,l}}{\gamma_{l,l+1}}, \gamma_{1,2} \right)$. Therefore, the probability of successful transmission in Phase 2 is given by

$$\Pr (\text{success}|L_s) = \Pr (\gamma_{DF}^{l*} \geq \gamma_{th,d}) = \Pr (\max \gamma_l \geq \gamma_{th,d}) = 1 - \prod_{l=1}^{L_s} \Pr (\gamma_l \leq \gamma_{th,d}) \quad (6.33)$$

where $\Pr (\gamma_l \leq \gamma_{th,d})$ is obtained by conditional probability, similar to the way derived for (6.25), as

$$\Pr (\gamma_l \leq \gamma_{th,d}) = 1 - \left[ \Pr \left( \frac{\gamma_{1,l}}{\gamma_{l,l+1}} \geq \gamma_{th,d} \mid \gamma_{l,l} \right) \Pr \left( \gamma_{l,2} \geq \gamma_{th,d} \mid \gamma_{1,l} \right) \right] = 1 - e^{-\frac{2\gamma_{th,d}}{1 + \gamma_{th,d}}} \quad (6.34)$$

Transmission via AF: SINR of end to end transmission through the best relay in an AF relay network is

$$\gamma_{AF}^{l*} = \max \left( \frac{\gamma_{1,l}}{\gamma_{l,l+1} + \gamma_l} \right). \quad (6.35)$$
From (6.35), the end-to-end successful transmission is computed as follows:

\[
\Pr (\text{success}|L_s) = \Pr \left( \max \left( \frac{\gamma_{1,l}}{\gamma_{l+1,l} + 1} \right) \geq \gamma_{th,d} \right) = 1 - \prod_{l=1}^{L_s} \Pr \left( \frac{\gamma_{1,l}}{\gamma_{l+1,l} + 1} \leq \gamma_{th,d} \right) = 1 - \left[ F_{\gamma_{eq}} (\gamma_{th,d}) \right]^{L_s}. \quad (6.36)
\]

The CDF of \( \gamma_{eq} \) in (6.36) can be obtained by conditional probability, conditioned on \( \gamma_l \), as

\[
F_{\gamma_{eq}} (\gamma_{th,d}) = \Pr \left( \frac{\gamma_{1,l}}{\gamma_{l+1,l} + 1} \leq \gamma_{th,d} \right) = \int_{\gamma_{th,d}}^{\infty} F_V (\omega) f_{\gamma_1} (s) ds + F_{\gamma_1} (\gamma_{th,d}) \quad (6.37)
\]

where \( V = \frac{\gamma_{1,l}}{\gamma_{l+1,l}} \), \( \omega = \frac{\gamma_{th,d}(s+1)}{s - \gamma_{th,d}} \) and \( F_V (\cdot) \) is the CDF of \( V \).

In order to compute \( F_V (\omega) \) in (6.37), first \( V \) is conditioned on \( \gamma_{l,l} \) and then the condition is removed by taking the average with respect to RV \( \gamma_{l,l} \) over its PDF defined as \( f_{\gamma_{l,l}} (s) \). Thus

\[
F_V (\omega) = \int_{0}^{\infty} \Pr \left( \frac{\gamma_{1,l}}{\gamma_{l+1,l}} \leq \omega | \gamma_{l,l} = s \right) f_{\gamma_{l,l}} (s) ds
\]

\[
= \int_{0}^{\infty} F_{\gamma_{l,l}} (\omega (s + 1)) f_{\gamma_{l,l}} (s) ds = 1 - \frac{e^{-\omega}}{\omega + 1} \quad (6.38)
\]

where \( f_{\gamma_{l,l}} (s) = \exp (-s) \) and \( F_{\gamma_{l,l}} (\omega (s + 1)) = 1 - \exp (-\omega (s + 1)) \).

On the other hand, \( F_{\gamma_{l}} (\gamma_{th,d}) \) is obtained by

\[
F_{\gamma_{l}} (\gamma_{th,d}) = \Pr \left( \min \left( \frac{\gamma_{1,l}}{\gamma_{l+1,l} + 1}, \frac{\gamma_{2,l}}{\gamma_{l+1,l} + 1} \right) \leq \gamma_{th,d} \right) = 1 - \frac{e^{-2\gamma_{th,d}}}{1 + \gamma_{th,d}} \quad (6.39)
\]

and \( f_{\gamma_{l}} (s) \) is the PDF of \( \gamma_l \) which from (6.39) can be written as

\[
f_{\gamma_{l}} (s) = \frac{e^{-2s}}{(s + 1)^2} + \frac{2e^{-2s}}{s + 1}. \quad (6.40)
\]
Figure 6.1: The overhead probability introduced by the time-based relay selection protocol in a distributed TWR system as a function of $\frac{\lambda}{\Delta}$.

Substituting (6.38), (6.39) and (6.40) into (6.37) gives the following expression for CDF of $\gamma_{eq}$

$$F_{\gamma_{eq}}(\gamma_{th,d}) = 1 - \int_{\gamma_{th,d}}^{\infty} e^{-\frac{\gamma_{th,d}(s+1) + 2s}{s - \gamma_{th,d}}} ds \left( s + 1 \right)^2$$

$$+ \int_{\gamma_{th,d}}^{\infty} 2e^{-\frac{\gamma_{th,d}(s+1) + 2s}{s - \gamma_{th,d}}} ds \left( s + 1 \right). \quad (6.41)$$

6.3 Results

Expressions obtained for the SE in Sec. 6.2.1 and 6.2.2 have been plotted with developed analytical results and then validated with simulations. For all results, $\Delta = 5 \times 10^{-5}$ [40], $T = 10\text{msec}$ and $R_r/B = 1 \text{bits/s/Hz}$ denoting the highest achievable SE in the systems. In addition, $\gamma_{th,r} = \gamma_{th,d} = 27\text{dBm}$ presenting a middle communication environment, unless otherwise stated.

As previously discussed in Chapter 5, the overhead probability, $Pr(t_2 \leq t_{\text{min}} + \Delta \mid L_s) = Pr\left(\frac{1}{x_2} \leq \frac{1}{x_1} + \frac{\Delta}{\lambda} \mid L_s\right)$, and the time selection $T_s =$
\frac{\lambda}{x_1} + \Delta both are related to \lambda and \Delta. Therefore, there is an important trade-off in the choice of \frac{\lambda}{\Delta} to design a system with higher SE. This means that by increasing \frac{\lambda}{\Delta}, time selection \( T_s \) is increased thus the SE of the system is decreased based on \( T_e = T - T_s \). However, by increasing \frac{\lambda}{\Delta}, the probability of overheads goes down and hence the SE is increased. Figures 6.1, 6.2, 6.3 and 6.4 show the impact of \frac{\lambda}{\Delta} on the overhead probability and \( T_s \) for a given number of relay nodes \( N \). In Fig. 6.1 and Fig. 6.2, the numerical results for a TWR system are plotted from Sec. 6.2.1.2 and Sec. 6.2.2.3; while in Fig. 6.3 and Fig. 6.4 the numerical results for a FDR system are represented from Sec. 6.2.2.2 and Sec. 6.2.2.3. It is worth mentioning that for the Monte Carlo simulations, the instantaneous received power values for the desired signals are randomly generated using expectational distribution with mean one. The method of generating the exponential RVs were discussed in Sec. 2.2.2.2. As can be seen in Fig. 6.1 and Fig. 6.3 the probability of overhead decreases with respect to \frac{\lambda}{\Delta} and increases with increasing the number of candidate relays \( N \). However, as shown in Fig. 6.4 and Fig. 6.2, the expected selection time \( T_s \) is increasing with respect to \frac{\lambda}{\Delta}, and decreases with increasing the number of candidate relays.

Figure 6.2: Expected selection process time, \( T_s \), respect to \frac{\lambda}{\Delta} in a distributed TWR system.

\text{Expected Selection Time, } T_s \text{ (second)}

\begin{tabular}{|c|c|c|}
\hline
N=2 & N=10 & N=20 \\
\hline
\end{tabular}
Fig. 6.5, and Fig. 6.6 illustrate the SE of TWR and FDR system, respectively, as a function of a number of relay nodes $N$ and show the effect of the protocol overheads and time selection period on the system throughput. As it can be seen from the figures, in a small sized network, time selection $T_s$ plays the main role thus SE is increased by decreasing $\lambda$ and increased by increasing number of relay nodes. However, in a large sized system, the overhead probability dominates on the time selection $T_s$ and thus by the increasing number of relays, the SE of the system is reduced. Moreover, as can be observed in Fig. 6.6, a FDR system with AF relay nodes is more efficient compared to the system with DF relay nodes. The SE of a TWR system as a function of the number of relay nodes with respect to threshold SNR at the relay and the destination node, $\gamma_{th,r}$ and $\gamma_{th,d}$, is illustrated in Fig. 6.7. The figure shows that, with lower $\gamma_{th,d}$ a higher SE is achievable. Therefore, the SNR in the second phase plays the main role to design a system with a higher SE. This efficiency can be also increased by increasing $\gamma_{th,r}$ for a large sized network ($N > 5$) and by decreasing $\gamma_{th,r}$ for a small sized network. The reason is that in a large sized network by increasing the threshold SNR at the relay node $\gamma_{th,r}$, the number of the successful relay nodes in Phase 1 is decreased which causes the reduction in probability of collisions and increasing SE of the system. However, in a small sized network ($N \leq 5$) to achieve a higher SE, we need to reduce $\gamma_{th,r}$ as much as possible in order to increase the SNR in Phase 1.
Figure 6.3: The overhead probability introduced by the time-based relay selection protocol in a distributed FDR system as function of $\frac{\lambda}{\Delta}$.

Figure 6.4: Expected time selection $T_s$ of distributed method as function of $\frac{\lambda}{\Delta}$ in a distributed FDR system.
Figure 6.5: Spectral efficiency of a TWR system as a function of number of relay nodes with respect to $\frac{\lambda}{\Delta}$, where $\gamma_{th,r} = \gamma_{th,d} = 20$dBm.

Figure 6.6: Spectral efficiency of a distributed FDR system as function of number of relays, $N$. 
Figure 6.7: Spectral efficiency of a TWR system as a function of number of relay nodes with respect to $\gamma_{th,r}$ and $\gamma_{th,d}$, where $\frac{\lambda}{\Delta} = 10$.

### 6.4 Chapter Summary

This chapter was the extension of Chapter 5. In this chapter the analytical performance of a time-based distributed method in advanced relay cooperative systems, two-way relay and full-duplex relay systems, in terms of SE were evaluated. The system performances were analysed for a non-ideal system by evaluating over-pessimistic overheads probability. Observations showed that, by increasing the number of relay nodes, the probability of collision increased, which caused a reduction in the SE of the system. Therefore, in a large sized systems, to provide a system with higher SE, $\frac{\lambda}{\Delta}$ is needed to increase. On the other hand, in a small sized system, the selection time duration $T_s$ dominates the collision probability. As a result, by making $T_s$ as small as possible, a system with a higher SE can be obtained.
Chapter 7

Energy Consumption of Distributed Relay Selection Protocols

7.1 Introduction

Energy consumption is a critical factor in wireless communication systems and the importance is increasing due to global warming. It is well known that when relay nodes in a cooperative system are used to split the direct transmission for source-destination link into two or more hops, the total energy consumption of the network is expected to be significantly reduced. This is because the transceiver distances are smaller than the direct link thus transmit power and path-loss effects on the system performance are greatly decreased [93]. However, it is not always the case because of the extra transceiver circuit energy consumed by the relay nodes [94, 95].

Relay selection (RS) schemes have received a lot of attention due to their capability to improve the system performance to achieve higher transmission quality in terms of diversity, throughput and energy. As discussed in Sec. 3.4.1, among relay selection schemes [37, 50, 54, 96, 97], the best worst channel selection [54] was introduced as a well designed method to select the best relay from $L_s$ relay nodes. This distributed relay selection method can be approached by two different protocols, OR and SC, that are two similar timer-based protocols but with different relay selection mechanism. The energy efficiency of a cooperative relay system employing SC protocol has been
investigated in [16]. However, it is not clear whether applying OR or SC protocol is better in terms of energy consumption. Moreover, in [16] and more previous works on energy efficiency of cooperative systems, authors considered a constant power transmission in their analyses.

In this chapter, energy consumption of two proposed protocols, OR and SC, in an OWR system with calculating the transmission power required to send a number of bits are investigated. As [1] demonstrated, in a small distance, applying MQAM modulation in a non-cooperative system consumes less energy compared to other modulation schemes. Therefore, in the system model, MQAM modulation is considered based on the fact that the link between each node will be smaller compared to a direct link. Consequently, by using MQAM modulation, transmission power required to send a given number of bits are calculated and used to obtain the total energy consumption of OR and SC protocols. The Dinkelbach’s concave-convex algorithm also is used to minimize the total energy consumption by optimizing transmission power and transmission time. Through the optimization, a large amount of energy saving is achievable.

In the rest of the chapter, in Sec. 7.2 the system model is presented. In Sec. 7.3, the energy consumption analysis for OR and SC protocols are presented. In Sec. 7.4, the optimization problem for minimizing the energy consumption is discussed. In Sec. 7.5, the results are presented, and in Sec. 7.6 chapter summary is given.

### 7.2 The Distributed RS Protocols Model

Consider a system model described in Sec. 3.4.1.2. A cooperative relay system with two end users, namely T₁ and T₂ and N DF relay nodes between T₁ and T₂. The time-based relay selection protocols, SC and OR, are applied in the cooperative relay system to select the best relay. Channels from the source to each relay l, and from relay l to the destination indicated as gᵢ and fᵢ, respectively are assumed to be i.i.d. Rayleigh fading. Consider the RVs to be defined as \( y_l = |g_l|^2 \) and \( x_l = |f_l|^2 \). Therefore, the PDF of the RVs \( y_l \) and \( x_l \) follow to be an exponentially distributed, as illustrated in Sec. 2.2.2.1, with parameters \( \lambda_1 = \frac{1}{E[y]} \) and \( \lambda_2 = \frac{1}{E[x]} \) respectively. Thus \( f(y) = \lambda_1 \exp(-\lambda_1 y) \) and \( f(x) = \lambda_2 \exp(-\lambda_2 x) \).
7.3 Energy Consumption of Time-Based Protocols

Aim to investigate energy consumption of two proposed protocols, OR and SC by calculating transmission power required to send a given number of bits in a MQAM modulation scheme. Note that in this chapter, the distributed selection method is assumed to be ideal in the sense that $T_s = \Delta + \frac{1}{\gamma} \sim 0$. In this case, energy consumption per information bit is given by

$$EC = \sum_{L_s=1}^{N} \frac{E_c(L_s)}{\kappa} \Pr(\text{success}|L_s) \Pr(|H| = L_s)$$  \hspace{1cm} (7.1)

where $E_c$ is the total energy consumption of the system, $\Pr(|H| = L_s)$ is the probability that the size of the decoded set $H$ is $L_s$, see Sec. 3.4.1 for details; and $\Pr(\text{success}|L_s)$ is the probability of successful transmission in Phase 2 required to be calculated for both OR and SC protocols.

7.3.1 Computation of the $E_c$

7.3.1.1 Circuit Power Assessment

In order to estimate the total energy consumed in a wireless system, all circuit power of the system are required to be evaluated. Consider two wireless nodes, a transmitter and a receiver. In this assessment all signal processing at the transmitter and the receiver sides need to be considered. However, to simplify the estimation, energy consumption of the base-band signal processing such as digital modulation, source coding and pulse shaping are neglected due to small power consumption [1]. Fig. 7.1 shows a block diagram of a transceiver. As can be seen in the figure, on the transmitter side, Tx, signal is first converted to an analog signal by digital-to-analog convertor (DAC), then filtered by a low pass filter, then modulated and mixed, then filtered again; and finally amplified by a power amplifier (PA) before transmission to the receiver. At the receiver side, Rx, the received signal is filtered first and then down converted by a mixer, filtered again before going through the Intermediate Frequency amplifier (IFA); and finally converted to a digital signal via an analog-to-digital convertor (ADC).

To save energy consumption in a wireless system, the system is assumed to operate in the multimode. Multimode operation provides a significant savings
of energy. This is because when there is no transmission in the system, sleep mode can be set up to save a large amount of power. A transceiver considered on the multimode basis is included in three modes, active mode (when there is a signal to transmit and all circuits work), sleep mode (when there is no signal to transmit and some circuits work) and transient mode (when transceiver switches from sleep to active mode or vice versa).

Assume a source has \( \kappa \) bits to transmit with a deadline for transmission indicated by \( T \) and a useful data transmission period demonstrated by \( T_e \). A transceiver spends \( T_e \leq T \) to communicate and transmits these bits to the receiver and then returns to sleep mode, where more circuits are shut down for saving the energy for duration of \( T_{sp} \). The transient duration from active mode to sleep mode is small enough to be negligible, while transient duration from sleep mode to active mode \( T_{tr} \) is not that short. By considering time duration of all modes, the whole transmission duration in multimode can be obtained by \( T = T_e + T_{tr} + T_{sp} \). The total energy consumption required to send \( \kappa \), bits then is given by [98]

\[
E_c = P_e T_e + P_{tr} T_{tr} + P_{sp} T_{sp} = (P_t + P_c) T_e + P_{tr} T_{tr} + P_{sp} T_{sp}
\]

(7.2)

where \( P_{tr} \) and \( P_{sp} \) are the power consumption values for transient and sleep mode respectively. \( P_e \) is the power consumption value for the active mode and represented by \( P_e = P_t + P_c \), where \( P_t \) is the transmission power and \( P_c \) is the total circuit power consumption at the transmitter side Tx. The total circuit power \( P_e \) also is defined by \( P_e = P_{ct} + P_{cr} \), where \( P_{ct} \) is the total circuit power
in Tx side and $P_{ct}$ is the total circuit power at the receiver side Rx. The circuit power at the transmitter side $P_{ct}$, as shown in the Fig. 7.1, consists of the DAC power consumption $P_{DAC}$, the frequency synthesiser power consumption $P_{sy}$, the mixer power consumption $P_{mix}$, the active filter consumption $P_{filt}$; and the power amplifier consumption $P_{amp}$ defined by $P_{amp} = \varrho P_t$, where $\varrho = \frac{\varepsilon}{\zeta} - 1$, $\varepsilon$ is the peak to average radio and $\zeta$ is the drain efficiency [1]. In addition, the circuit power at the receiver $P_{cr}$ consists of the ADC power consumption $P_{ADC}$, the mixer power consumption $P_{mix}$, the active filter consumption $P_{filt}$, the low noise amplifier (LNA) power consumption $P_{LNA}$, the IFA power consumption $P_{IFA}$. Consequently, the total circuit power in the transmitter $P_{ct}$ and the receiver $P_{cr}$ sides can be written, respectively, as

$$P_{ct} = P_{syn} + P_{mix} + P_{filt} + \varrho P_t + P_{DAC}$$

and

$$P_{cr} = P_{LNA} + P_{filt} + P_{mix} + P_{IFA} + P_{ADC}.$$ 

Moreover, the power consumption in the transient mode $P_{tr}$, is only needed to include the power consumed by frequency synthesisers $P_{sy}$ [1]. Consequently, by recalling (3.38), the energy consumption from (7.2) is obtained by

$$E_c = P_t T_e + P_c T_e + 2P_{sy} T_{tr} + P_{sy} T_{sp}. \quad (7.3)$$

### 7.3.1.2 Energy Consumption Assessment

Based on the assessment in the previous Sec. 7.3.1.1, this section evaluates the amount of energy consumed in a multiple relay cooperative system with a single relay selection strategy. The total energy consumption of each phase, Phase 1 and Phase 2 as well as energy consumption of selection and sleep period are separately calculated and then added together to obtain the total energy consumption of the system.

In Phase 1, the system allocates the duration $\frac{T_e}{2}$ for source-relays transmission. During this period, source is in transmission mode, destination in sleep mode and relay nodes are in the receive modes. Therefore the total energy consumed in Phase 1 can be obtained by

$$E_1 = \left(P_t + P_{ct}^s + NP_{cr}^r + P_{sp}^d\right) \frac{T_e}{2} \quad (7.4)$$
where $P_t$ and $P_{ct}$, respectively, are transmit power and circuit power of the 
source where they are in transmission mode. $P_{cr}$ is the circuit power of each 
relay node where it is in the receive mode and $P_{sp}^d$ is the power consumed 
by the destination in the sleep mode where it is neither transmitting nor re-
ceiving. When Phase 1 completed, time $T_s$ is spent to select the best relay. 
At the same time all $L_s$ relays are in the receive mode to detect the signal 
broadcasted by the best relay. Therefore, energy consumed in this process can 
be given by

\[ E_s = (L_s P_{cr}^r + P_{sp}^s + P_{sp}^d + (N - L_s) P_{sp}^r) T_s \]  

(7.5)

where, $P_{sp}^s$ and $P_{sp}^r$ are the power consumed by the source and the relay nodes 
in the sleep mode, respectively. When the selection process is completed and 
the best relay is selected, duration $\frac{T_e}{2}$ is allocated for Phase 2. In this phase, 
the best relay transmits the signal and destination receives the signal in the 
receive mode. Therefore, the energy consumed is

\[ E_2 = (P_{sp}^s + (N - 1) P_{sp}^r + P_t + P_{ct}^r + P_{cr}^d) \frac{T_e}{2} \]  

(7.6)

where $P_{ct}^r$ is circuit power of the relay node in transmission mode respectively. 
$P_{cr}^d$ is the circuit power consumed by the destination node when it is in the 
receive mode.

In addition, before starting to transmit the next block of information, turning 
both source and destination nodes into sleep mode can save a significant 
amount of energy. However, there is a small amount of consumed energy in 
sleep mode given by

\[ E_{sp} = (P_{sp}^s + P_{sp}^d) (T - T_e - T_s) . \]  

(7.7)

As a result, total energy consumption, $E_{ct}$, in a relay selection cooperative 
system is obtained by $E_c = E_1 + E_s + E_2 + E_{sp}$. Consequently, recalling (7.4), 
(7.5), (7.6) and (7.7) gives
\[ E_c = \left( L_s P_{cs}^r + P_{sp}^s + P_{sp}^d + (N - L_s) P_{sp}^r \right) T_s + \left( 2P_t + P_{ct}^s + NP_{cs}^r + P_{ct}^d \right) \frac{T_e}{2} + \left( P_{sp}^s + (N - 1) P_{sp}^r + P_{ct}^r + P_{ct}^d \right) \frac{T_e}{2} + \left( P_{sp}^s + P_{sp}^d \right) (T - T_e - T_s). \] (7.8)

### 7.3.2 Computation of the Transmit Power, \( P_t \)

Take MQAM as the system design to obtain transmission power required to send a given number of bits. The number of bits per symbol is given by \( b = \log_2 M \), where \( M \) is the level of modulation and \( b \) is the number of bits in each symbol. The number of symbols, \( \kappa_s \), needed to send \( \kappa \) bits in the MQAM can be obtained by \( \kappa_s = \frac{\kappa}{b} \). Moreover, if the symbol period is denoted by \( T_{sym} \), \( \kappa_s \) can also be defined as \( \kappa_s = \frac{T_e}{T_{sym}} \). Therefore,

\[ \frac{T_e}{T_{sym}} = \frac{\kappa}{b}. \] (7.9)

The number of bits per symbol from (7.9) in terms of symbol period \( T_{sym} \), useful transmission duration \( T_e \) and \( \kappa \), can be obtained by

\[ b = \frac{\kappa T_{sym}}{T_e}. \] (7.10)

On the other hand, considering square pulses, the symbol period \( T_{sym} \) is obtained by \( T_{sym} = \frac{1}{2B} \) [1], where \( B \) is the signal bandwidth between two nodes. Hence, (7.10) can be written as

\[ b = \frac{\kappa}{2BT_e}. \] (7.11)

In a Rayleigh fading channel, the average error probability can be derived as [99]

\[ P_e \approx \frac{1}{2} \left( 1 - \sqrt{\frac{d \overline{\gamma}}{1 + d \overline{\gamma}}} \right) \] (7.12)

where \( d = \frac{3}{2(M-1)} \) for MQAM and \( \overline{\gamma} \) is the average SNR defined as \( \overline{\gamma} = \)
\[ \gamma f(\gamma) \, d\gamma, \text{ where } f(\gamma) = \frac{1}{\gamma} e^{-\gamma/\eta} \text{ is the SNR distribution in a Rayleigh fading channel. From (7.12), } \gamma \text{ is given by} \]

\[ \gamma \approx \left( \frac{1}{1 - 2P_e} \right)^2 \approx \frac{1}{4dP_e} \approx \frac{1}{4M^{-1}P_e} = \frac{2^b - 1}{6P_e} \]  

(7.13)

where the fact \( P_e \ll 1 \) was applied [1]. Acknowledging \( \gamma = \frac{P_r}{B N_0} \), where \( P_r \) is the received power and \( N_0 \) is the power spectrum density of AWGN, the required transmission power for the target average probability of error \( P_e \) is obtained by \( P_t = P_r L_d \). Therefore,

\[ P_t = \frac{(2^b - 1) BN_0 L}{6P_e} \]  

(7.14)

where \( L \) is the path-loss model between the transmitter and receiver separated by distance \( r \) with the path-loss exponent of \( \beta \). \( L \) is given by \( L = \frac{P_t}{P_r} = M_t r^\beta L_1 \), where \( M_t \) is the link margin counted for the effects of the hardware. \( L_1 \) is the gain factor at distance \( r = 1\text{m} \) given by \( L_1 = \frac{(4\pi r)^2}{G_t G_r \Lambda^2} \), where \( G_t \) and \( G_r \), respectively, are antenna gains in the transmitter and the receiver; and \( \Lambda \) is the wavelength.

### 7.3.3 Computation of the \( \Pr(|\mathcal{H}| = L_s) \)

Recalling (6.10) in order to obtain \( \Pr(|\mathcal{H}| = L_s) \), the probability of successful transmission in Phase 1, \( p_s \), is required. Successful probability in Phase 1 is given by \( p_s = \Pr(L_s = 1) = \Pr(\gamma_r \geq \gamma_{th,r}) \), where \( \gamma_r \) is the SNR at the relay nodes and \( \gamma_{th,r} \) is the threshold SNR. As performance of both OR and SC protocols in Phase 1 to select the relay candidates are same (see. 3.4.1), \( p_s \) for both protocols is given by

\[ p_s = \Pr(\gamma_r \geq \gamma_{th,r}) = \Pr\left( \frac{y_l P_t}{P_n} \geq \gamma_{th,r} \right) = e^{-\lambda_1 \delta_r} \]  

(7.15)

where \( \delta_r \) is the threshold power channel gain given by \( \delta_r = \left( \frac{\gamma_{th,r} P_n}{P_t} \right) \). Therefore,

\[ \Pr(|\mathcal{H}| = L_s) = \binom{N}{L_s} \left( 1 - e^{-\lambda_1 \delta_r} \right)^{L_s} \left( e^{-\lambda_1 \delta_r} \right)^{N-L_s}. \]  

(7.16)
CHAPTER 7. EC OF DISTRIBUTED RS PROTOCOLS

7.3.4 Employing SC Protocol

By employing SC protocol in the distributed relay selection method, in Phase 2 the relay candidates set their own time based on the definition in (3.27). Therefore, the relay that expires first has the maximum power channel gain from itself to the destination among the other relays and is selected as a best relay. By using order statistics of given RVs $x_l, l = 1, 2, ..., L_s$, the order statistics are also RVs where the values are defined by sorting the RVs in decreasing order as $x_1 > x_2 > \ldots > x_{L_s}$. That means that $x_1 = \max x_l$ and $x_{L_s} = \min x_l$. Therefore, (3.27) can be modified into $h^* = \max x_1 = x_1$. Consequently,

$$
\Pr (\text{success}|L_s) = \Pr (\gamma_{l^*} \geq \gamma_{th,d}) = 1 - \Pr (x_1 \leq \delta_d)
$$

$$
= 1 - \prod_{l=1}^{L_s} \Pr (x_l \leq \delta_d) = 1 - \left[1 - e^{-\lambda_2 \delta_d}\right]^{L_s} \quad (7.17)
$$

where $\gamma_{l^*}$ is the SNR at the destination via the best relay defined as $\gamma_{l^*} = \frac{h^* P_t}{P_n} = \frac{\max x_l P_t}{P_n} = \frac{x_1 P_t}{P_n}$ and $\gamma_{th,d} = \delta_d P_t$, where $\delta_d$ is the threshold channel gain at the destination.

7.3.5 Employing OR Protocol

By employing OR protocol in the distributed relay selection method, in Phase 2, the relay candidates set their own time based on (3.28). The relay with the maximum function $h_l$, where $h_l = \min (y_l, x_l)$ expires first and selected as the best relay among $L_s$ relaying candidates. Therefore, $\Pr (\text{success}|L_s)$ in Phase 2 can be evaluated by

$$
\Pr (\text{success}|L_s) = \Pr (\gamma_{l^*} \geq \gamma_{th,d}|\gamma_r \geq \gamma_{th,r}) = \Pr (\max \min (y_l, x_l) \geq \delta_d|y_l \geq \delta_r). \quad (7.18)
$$

In order to calculate (7.18), lets first consider $z_l = \min (y_l, x_l)$, hence

$$
\Pr (z_l \geq \delta_d|y_l \geq \delta_r) = \Pr (y_l \geq \delta_d|y_l \geq \delta_r) \Pr (x_l \geq \delta_d)
$$

$$
= \begin{cases} 
    e^{-(\lambda_2 \delta_d + \lambda_1 (\delta_r - \delta_d))}, & \delta_d \geq \delta_r \\
    e^{-\lambda_2 \delta_d}, & \delta_d < \delta_r.
\end{cases} \quad (7.19)
$$
Then consider, \( u = \max z_l \). By using the order statistics of RVs, for given RVs \( z_l, l = 1, 2, ..., L_s \) in decreasing order as \( z_1 > z_2 > ... > z_{L_s} \), gives

\[
\Pr (u \geq \delta_d) = 1 - \prod_{l=1}^{L_s} \Pr (z_l \leq \delta_d) = \begin{cases} 
1 - \left[ 1 - e^{-(\lambda_2 \delta_d + \lambda_1 (\delta_r - \delta_d))} \right]^{L_s} & \delta_d \geq \delta_r \\
1 - \left[ 1 - e^{-\lambda_2 \delta_d} \right]^{L_s} & \delta_d < \delta_r \end{cases}
\]

(7.20)

7.4 Optimization Problem

As shown in (7.8), \( E_c \) is a function of \( P_t \) and \( T_e \). Therefore, \( E_c \) can be minimised by jointly optimizing the transmission power \( P_t \) and transmission time \( T_e \) as follows:

\[
\begin{align*}
\text{minimize} & \quad E_c \\
\text{subject to} & \quad T_{\text{min}} \leq T_e \leq T
\end{align*}
\]

(7.21)

If we take the fact into the account that \( b = \frac{\kappa}{2B T_e} \). Therefore, the object and subject of the problem in (7.21) can be reformulated as

\[
\begin{align*}
\text{minimize} & \quad E_c \\
\text{subject to} & \quad b_{\text{min}} \leq b \leq b_{\text{max}}, \quad b \in \mathbb{R}^+ 
\end{align*}
\]

(7.22)

where the upper bound on \( b \) corresponds to the lower bound on \( T_e \) given by \( b_{\text{max}} = \frac{\kappa}{2B T_{\text{min}}} \) and the lower bound on \( b \) is given by \( b_{\text{min}} = \max \left\{ \left[ \frac{\kappa}{2B T} \right], 2 \right\} \). This is because for MQAM the number of bits per symbol is defined as \( b = \log_2 M \), see Sec. 7.3.2. Moreover, \( E_c \) from (7.14) and (7.11) is presented in terms of \( b \) as

\[
E_c = \left( \frac{A \left( 2^b - 1 \right) + O + W}{b} \right) + V
\]

(7.23)

where \( A = \left( \frac{N_0 L_s \kappa}{6 P_r} \right), \ O = \left( \frac{(N+3)(P_{\text{cr}} + P_{\text{dr}})^\kappa}{2B} \right), \ W = \left( \frac{(N-5)(P_{\text{sp}} + P_{\text{dp}})^\kappa}{2B} \right) \) and \( V = \left( L_s \left( P_{\text{cr}} + P_{\text{dr}} \right) + (N - L_s - 1) \left( P_{\text{sp}} + P_{\text{dp}} \right) \right) T_s + \left( P_{\text{sp}} + P_{\text{dp}} \right) T . \)

If \( b \) is defined over real numbers, it can be proved that, \( E_c \) is a convex
function over $b$ for $b \geq 2$ by showing that $\frac{\partial^2 E_c}{\partial b^2} \geq 0$. Therefore,

$$\minimize \left( \frac{A \left( 2^b - 1 \right) + O + W}{b} \right) + V$$

subject to

$$b - b_{\text{max}} \geq 0$$

$$b_{\text{max}} - b \geq 0$$

(7.24)

Since all the constraints are simple linear constraints, the resulting optimization problem in (7.24) can be solved using the Dinkelbach concave-convex optimization algorithm. Dinkelbach [100] delineated an algorithm for solving fractional programming problems for concave and convex functions. He treated the following problems:

$$q = \max_{b \in s} \left( \frac{\mathcal{R}(b)}{\mathcal{D}(b)} \right)$$

(7.25)

$$\mathcal{F}(q) = \max_{b \in s} \left( \mathcal{R}(b) - q\mathcal{D}(b) \right)$$

(7.26)

where $s$ is a compact and connected subset of $E^n$ which is the Euclidean space of dimension $n$. $\mathcal{R}(b)$ and $\mathcal{D}(b)$ were assumed to be concave and convex respectively and $\mathcal{F}(q_n)$ be a concave function. The Dinkelbach concave-convex fractional program method has been described in Algorithm 7.1.

By applying Algorithm 7.1 in the optimization problem in (7.24), it can be described by

$$q = \max \left\{ - \left( \frac{A \left( 2^b - 1 \right) + O + W}{b} \right) \mid b \geq 2 \right\}$$

(7.27)

$$\mathcal{F}(q_n) = \max \left\{ - \left( A \left( 2^b - 1 \right) + O + W \right) - q_n b \mid b \geq 2 \right\}$$

(7.28)

where $q$ is treated as a parameter, corresponding to $-E_c$ in problem (7.24). Moreover, to simplify the optimization problem $V$ is ignored due to the small value and $q_1 = 0$ at starting point of $b_1 = 2$. The subproblem then

$$\mathcal{F}(0) = \max \left\{ - A \left( 2^b - 1 \right) - O - W \mid b \in s \right\}$$

(7.29)

where the solution based on the values, $q_1 = 0$ and $b_1 = 2$, is $\mathcal{F}(0) = | -2.3181| \geq 0.001 (= \varepsilon)$. Now, determine $q_2 = \frac{\lambda_{(b_1)}}{D{(b_1)}} = \frac{A(2^{b_1} - 1) + O + W}{b_1} = -1.1590$
and then solve

$$\mathcal{F}(-1.1590) = \max \left\{ -A(2^b - 1) - O - W + 1.1590b|b|s \right\}. \quad (7.30)$$

The subproblem in (7.30) leads to $b_2 = 15.2101$ and $\mathcal{F}(-1.1590) = |16.1143| \geq 0.001 (= \varepsilon)$. The solution is contributed until to end up with

$$g_b = \frac{A(2^{b_5} - 1) + O + W}{b_5} = -0.1610$$

and gives

$$\mathcal{F}(-0.1610) = \max \left\{ -A(2^b - 1) - O - W + 0.1610b|b|s \right\} \quad (7.31)$$

where $b_5 = 12.4289$ and $\mathcal{F}(-1.1590) = |-2.31 \times 10^{-4}| \leq 0.001 (= \varepsilon)$ which is the optimal point of the proposed system at $r = 10m$.

### 7.5 Results

In this section, numerical results are presented in order to show the performance difference between SC and OR protocols in terms of energy consumption. A specific numerical example with the simulation parameters summarized in Table 7.1 is used. The simulation and theory of outage probability of OR selection protocol is shown in Fig. 7.2. From the figure it can be seen that by increasing the $\delta_d$, outage probability is increased. However, this probability is decreased by increasing the $\delta_r$ and the number of relaying candidates $L_s$. The plot of $EC$ over $b$ for bandwidth $B = 5kHz$ is shown in Fig. 7.3. The vertical axis is the energy consumption per information bit $\kappa$ (in term of dB related to millijoule). The horizontal axis is the number of bits transmitted per symbol. From the figure it can be seen that OR consumes less energy than SC. Moreover, by increasing distance $r$, both OR and SC, consume almost the same energy up to $b = 10$. This is because by increasing the number of bits and distances the transmit power of the system is increased. On the other, it can be observed from Fig. 7.3 that by increasing the number of bits, lets consider for $r = 10m$, the energy consumption is decreasing for $b \leq 12$. This is because by increasing the number of bits, the effective transmission period is decreasing $T_e = \frac{\kappa}{2bb}$ thus the system consumes less power. However, for $b > 12$ the energy consumption of the both OR and SC are increasing as the transmit power of the system is increased by increasing number of bits. Therefore, there is a trade-off between transmit power of the system and the speed.
Algorithm 7.1 Dinkelbach Method

1. Start the algorithm in (7.26) with \( q_1 = 0 \) at starting point of \( b \), then go to Step 2 with \( n = 1 \).

2. By means of any method of concave, solve the following problem

\[
\mathcal{F}(q_n) = \max \{ \mathcal{R}(b) - q_n \mathcal{D}(b) \mid b \in s \}
\]

and denote any solution point by \( b_n \). Then go to Step 3.

3. If \( \mathcal{F}(q_n) \geq \varepsilon \), where \( \Delta \geq 0 \). Then go to Step 4.

4. Evaluate \( q_{n+1} = \frac{\mathcal{R}(b_n)}{\mathcal{D}(b_n)} \). Then go to Step 2 and replacing \( q_n \) by \( q_{n+1} \).

5. If \( \mathcal{F}(q_n) \leq \varepsilon \) : STOP

of useful data transmission. In addition, from Fig. 7.3 it can be observed that \( b_{opt} \approx 12 \) is the minimum value that energy can be consumed by the system in \( r = 10m \). This point can be easily find by concave-convex optimization problem mentioned in Sec. 7.4. Fig. 7.4 shows that in the case that \( \delta_r > \delta_d \), both OR and SC methods consume same amount of energy while in the case of \( \delta_d \geq \delta_r \), OR is more efficient than SC method.
### System Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth ($B$)</td>
<td>$5KHz$</td>
</tr>
<tr>
<td>BER ($P_e$)</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Information bit ($\kappa$)</td>
<td>$2kb$</td>
</tr>
<tr>
<td>Transmission duration ($T$)</td>
<td>$1ms$</td>
</tr>
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<td>Noise ($N_0$)</td>
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</tr>
<tr>
<td>Path loss exponent ($\beta$)</td>
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<td>Circuit Power ($P_c$)</td>
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</tr>
<tr>
<td>Idle Power ($P_{sp}$)</td>
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</tr>
<tr>
<td>Gain factor ($L_1$)</td>
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</tr>
<tr>
<td>Link margin ($M_l$)</td>
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</tr>
<tr>
<td>Number of Nodes ($N$)</td>
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<tr>
<td>Modulation</td>
<td>MQAM</td>
</tr>
<tr>
<td>Channel Model</td>
<td>Rayleigh Fading &amp; Path-Loss</td>
</tr>
</tbody>
</table>

Table 7.1: The System Parameters [1]

![Figure 7.2: The outage probability in Phase 2 by applying OR protocol in a distributed cooperative system.](image-url)
Figure 7.3: Total energy consumption versus bits/symbol in a distributed cooperative relay system employing time-based relay selection protocols, SC and OR, where $\delta_r = 0.8$, $\delta_d = 2$ and $N = 20$.

Figure 7.4: Total energy consumption versus $\delta_d$ in a distributed cooperative relay system employing time-based relay selection protocols, SC and OR, where $\delta_r = 1$, $r = 5m$, $b_{opt} = 12$ and $N = 20$. 
7.6 Chapter Summary

In this chapter energy consumption of OR and SC methods required to convey a given number of bits to the receiver by considering MQAM modulation for a DF relay system were studied. It was found that SC and OR protocols consume the same amount of energy in case of $\delta_r > \delta_d$. However in case that $\delta_d \geq \delta_r$, OR is more efficient and consume less energy than SC protocol. Moreover, it was found that there is a trade off between transmit power of the system and speed of useful data transmission. Therefore, for a system with a small number of bits, useful data transmission period play the significant role in determining system performance in terms of energy consumption, while in a system with higher number of bits, transmit power is the main factor. The convex-concave algorithm also was used to minimize the total energy consumption by optimizing time and power transmission. The results showed that by the optimization a large amount of energy saving is achievable.
Chapter 8

Conclusions and Future Work

8.1 Conclusions

The study presented in this thesis has focused on the problems and challenges of cooperative relay techniques in the presence of interferences, CCIs generated by external interfering sources and self-interferences, in terms of error rates and spectral efficiency, as well as energy consumption. In this thesis, new unified mathematical methods have been developed that produced accurate and simplified expressions for exact analysis of cooperative relay systems and different methods that have recently being proposed to enhance the cooperative systems for the next generation.

Chapter 2 presented relevant background theories required in this thesis including small and large scale fading with their statistics behaviors; and error and outage probability analysis. The literature review in Chapter 3, presented a review on principle and advanced relay cooperative systems as well as several challenges that currently the cooperative systems facing in general. For example, spectral loss, relay selection schemes, and energy consumption. In the subsequent chapters an attempt was made to address a few of these issues.

Chapter 4 addresses the problem of CCIs in a dual-hop AF relay cooperative system that are generated over different interferers channel fading models. The exact average error rates of the AF relay system was put to test under arbitrary CCIs at the destination and thermal noise in both source-relay and relay-destination links. Evaluation of exact average error rates involve the need to compute the MGF of SINR. For this a new exact mathematical method
was introduced which led to the derivation of a new explicit expression for
the MGF of SINR in terms of the MGF of the aggregate interferences’ power.
The approach processed was illustrated to be more accurate than currently
existing techniques and greatly simplify performance evaluation of the AF
relay systems over diverse practical interference fading models. The explicit
expression for the MGF of SINR then led to the derivation of the average of
\( \text{erfc} \left( \sqrt{\gamma} \right) \) and \( \text{erfc}^2 \left( \sqrt{\gamma} \right) \), appearing in
the most formula of bit and symbol error rate of different types of digital modulations, in a simple and a explicit
way as they expressed in terms of a single integral solution. These new ex-
pressions then used to derive new expressions for symbol and bit error rates
such as MFSK and MQAM. In the subsequent chapter, the proposed analytical
approach was extended for different fading models for the desired users in-
volving Nakagami-\( m \), Rician and composite Nakagami-\( m \)/log-normal fading
models. At the end, as an important application example, numerical results
were given for bit error rates of a cooperative relay system for a down-link
relay cellular network in the presence of arbitrary and Poisson field of inter-
ferences. Monte Carlo simulation was used to validate the new expressions
developed in this chapter.

Chapter 5 addresses the important problem of self-interferences that are
generated due to hidden nodes and/or due to the overheads introduced by a
time-based relay selection protocol in a distributed cooperative system. Spect-
ral efficiency analysis was put to test and in order to aid the analysis, a
new exact mathematical method proposed which provided several advan-
tages over more traditional approaches. The new exact unified expressions
derived for the spectral efficiency were based on an accurate interference
model that accounts for both self-interferences and the protocol overheads as
well as for different fading scenarios and arbitrary relays’ locations. The new
exact expressions were also examined for non-identical RVs and validated by
identical cases. In the subsequent chapter, Jensen’s inequality used to derive
a upper bound of the spectral efficiency over Rayleigh fading channel and
compared with the developed exact results. Since applying a relay selection
scheme for TWR and FDR relay systems have been focused of more attention
for the next generation and hence Chapter 5 was extended to Chapter 6 to
examine the spectral efficiency of distributed TWR and FDR relay systems
employing time-based relay selection protocols.
In Chapter 7 the amount of energy that time-based distributed relay selection protocols, OR and SC, consume to convey information bits to a receiver by considering a green modulation, MQAM, were derived. The total consumed energy in each protocol was evaluated for a system operated in multimode. In addition, in this chapter, Dinkelbach’s concave-convex algorithm was used to minimize the total energy consumption required to send a given number of bits by optimizing transmission power and transmission time. The result illustrated that by the optimization, a large amount of energy saving is achievable.

8.2 Future Work

In what follows, possible research extensions to the work in this thesis are suggested.

- In chapter 4, an exact approach for analysing performance of AF cooperative relay system in presence of arbitrary interferers at destination was presented. It would be interesting to extend this analysis to
  - Multiple Antenna: Prior works, show that single-antenna relays in cooperative systems do not work very well in the presence of CCIs [101, 102]. Multiple-antenna relay systems can be used to take the advantage of the higher diversity and interference management capability of MIMO communication systems. Therefore, Chapter 4 can be extended for multiple antenna in AF relay cooperative systems to derive new exact expressions for average error rates of the system in presence of a number of arbitrary non-identical interferences at the destination as no research has been done on this subject.
  - TWR Cooperative Systems: Consider SINR for $T_1 \rightarrow R \rightarrow T_2$ link. From (3.11) SINR for a fixed gain TWR system in presence of a number of arbitrary interferers at $T_2$ can be written as
    \begin{equation}
    \gamma = \frac{yx}{ax + b + c \sum_{n=1}^{N} i_n} \quad (8.1)
    \end{equation}
\[ x = |f|^2, \ y = |g|^2, \ a = \frac{\sigma^2}{P_1P_3E[|g|^2]}, \ b = \left( \frac{P_1E[|g|^2] + \frac{\sigma^2}{P_1P_3E[|g|^2]} + a}{E[|f|^2]} \right) \sigma^2, \ \text{and} \ \ c = \frac{b}{\sigma^2}. \] The MFG of (8.1) can be derived by following the new method presented in Chapter 4.

- In Chapter 5, a new accurate mathematical method on the performance of distributed cooperative communication systems in terms of spectral efficiency was presented. This approach can be used for

  - Generalise Selection Diversity Systems: Current and upcoming wireless communication systems need higher received SNR in order to support applications involved with high data rate transmission. This obligation can be met by using spatial diversity combating the multipath fading and increases the received SNR. Generalized selection diversity scheme has been recently proposed to deliver the best performance. According to this scheme, the \( L \) signals with the largest instantaneous SNR are selected out of the \( N \) available. By using the proposed mathematical method in Chapter 5, a new exact approach to performance evaluation of generalized selection diversity in wireless channels in terms of spectral efficiency can be presented. For example, consider RVs \( x_1 > ... > x_{n-1} > x_n > x_{n+1}... > x_N \), where RV \( n = \text{nth max}(x_1, ..., x_N) \). The average spectral efficiency

\[
C(x_n) = E \left[ \log_2 \left( 1 + \frac{\sum_{i=1}^L x_i}{1 + \sum_{i=L+1}^N x_i} \right) \right] \text{ (8.2)}
\]

which by using the traditional method would be very complicated because the joint PDF of the ordered RVs \( x_i, i = 1, 2, ..., L \) and joint PDF RVs \( x_i, i = L + 1, L + 2, ..., N \) are required with about \( N \)-fold numerical integrations. Using Lemma 1 and Theorem 1 and following the proposed method in Chapter 5, can greatly simplify the evaluational of (8.2).

- Exact Performance Analysis of Distributed FDR Cooperative Relay Systems: Consider the system with a time-based relay selection
protocol, thus

\[ C = \frac{1}{2} \mathbb{E} \left[ \frac{y}{\lambda + y (1 + \Delta)} \log_2 \left( 1 + \frac{y}{\sum_{l=1}^{L_s-1} x_l \xi_l + \sigma_n^2} \right) \right] \]  \hspace{1cm} (8.3)

where \( y = \max \left( \frac{x_1}{\nu_{L_s} + 1}, \ldots, \frac{x_{L_s}}{\nu_{L_s} + 1} \right) \).
References


Appendix A

Mathematical Transformation

A.1 Moment Generating Function

Definition: Let $X$ be a RV with PDF $f_X(x)$. The MGF of $X$ denoted by $M_X(z)$, is

$$M_X(z) = E[e^{-zx}], \quad (A.1)$$

provided that the expectation exists for $z$ in some neighborhood of 0 [18]. That is a $t$ such that for all $z$ in $|z| < t$, $E[e^{-zX}]$ exists.

More explicitly, the MGF of $X$ can be written as [18]

$$M_X(z) = \int_{-\infty}^{\infty} e^{-zx} f_X(x) \, dx \quad \text{If } X \text{ is continuous} \quad (A.2)$$

or

$$M_X(z) = \sum_x e^{-zx} \Pr(X = x) \quad \text{If } X \text{ is discrete} \quad (A.3)$$

An important properties of the MGF for independent RVs such as $X$ and $Y$, the moment-generating function satisfies

$$M_{X+Y}(z) = E[e^{-z(x+y)}] = E[e^{-zx}] E[e^{-zy}] = M_X(z) M_Y(z) \quad (A.4)$$

A.2 MGF Derivation of Lognormal Fading

Consider lognormal RV defined as $\gamma = e^n$, where RV $n$ has normal distribution with $\mathcal{N}(\mu, \sigma^2)$ with PDF of $f(n) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(n-\mu)^2}{2\sigma^2}\right)$. The MGF
of $\gamma$ is given by

$$\mathcal{M}_\gamma (z) = \int_{-\infty}^{\infty} e^{-z \exp(n)} \frac{1}{\sqrt{2\pi} \sigma} e^{-(n-\mu)^2 / 2\sigma^2} d\gamma. \quad (A.5)$$

Replacing $\eta = n - \mu / 2\sqrt{\sigma}$, gives

$$\mathcal{M}_\gamma (z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z \exp(n)} e^{-\eta^2} d\eta = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z [\exp(\sqrt{2} \sigma \eta + \mu)]} e^{-\eta^2} d\eta \quad (A.6)$$

where the last part in (A.6), can be approximated by calling Gauss-Hermite integration which [24, eq. (25.4.46)]

$$\int_{-\infty}^{\infty} e^{-t^2} h(t) \, dt \simeq \sum_{I=1}^{N_p} H_{\eta_I} h(t). \quad (A.7)$$

can be approximated as

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z [\exp(\sqrt{2} \sigma \eta + \mu)]} e^{-\eta^2} d\eta \simeq \frac{1}{\sqrt{\pi}} \sum_{I=1}^{N_p} H_{\eta_I} e^{-\exp(\sqrt{2} \sigma \eta_I + \mu)z}. \quad (A.8)$$

### A.3 MGF Derivation of Composite Fading

Consider composite gamma/log-normal shadowing RV defined as $\gamma = ye^n$, where RV $y$ has gamma distribution with PDF of $f(y) = \frac{m^m e^{-my}}{\Gamma(m)} \exp(-my)$ and $n$ is the RV which has normal distribution with PDF of $N(\mu, \sigma^2)$, $f(n) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(n-\mu)^2}{2\sigma^2}\right)$. The MGF of $y$ respect to the RV $x$ in condition on $n$ is given by

$$\mathcal{M}_\gamma (z|n) = E[ye^n|n] = \left(\frac{1}{1 + \frac{z}{m}}\right)^m \quad (A.9)$$

Removing condition on $n$ in (A.9) by averaging out the RV $n$ over its PDF gives

$$\mathcal{M}_\gamma (z) = E\left[\left(\frac{1}{1 + \frac{z}{m}}\right)^m\right] = \int_{-\infty}^{\infty} \left(\frac{1}{1 + \frac{z}{m}}\right)^m \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(n-\mu)^2}{2\sigma^2}\right) \, dn \quad (A.10)$$
Replacing \( \eta = (n - \mu) / \sqrt{2\sigma} \), we get

\[
\mathcal{M}_\gamma(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left( \frac{1}{1 + z \exp(\sqrt{2\sigma \eta + \mu})} \right)^m \exp(-\eta^2) \, d\eta \quad (A.11)
\]

The term \( \int_{-\infty}^{\infty} \exp(-t^2) h(t) \, dt \) can be calculated using Gauss-Hermite integration which can be approximated as \( \int_{-\infty}^{\infty} \exp(-t^2) h(t) \, dt \simeq \sum_{I=1}^{N_p} H_{\eta_I} h(t) \), where \( N_p \) is number of sample points, \( x_I \) and \( H_{\eta_I} \) are the abscissas and weight factors for different values of \( N_p \) provided by look-up table of [24, Table 25.10]. Consequently, (A.11) now becomes

\[
\mathcal{M}_\gamma(z) = \frac{1}{\sqrt{\pi}} \sum_{I=1}^{N_p} H_{\eta_I} \left( \frac{1}{1 + z \exp(\sqrt{2\sigma \eta_I + \mu})} \right)^m. \quad (A.12)
\]

### A.4 Multinomial Theorem

For any positive integer \( m \) and any non-negative integer \( n \), the multinomial formula tells us how a sum with \( m \) terms expands when raised to an arbitrary power \( n \) [103]

\[
(x_1 + x_2 + \ldots + x_k)^n = \sum_{n_1+n_2+\ldots+n_k=n} \binom{n}{n_1, n_2, \ldots, n_k} x_1^{n_1} x_2^{n_2} \ldots x_k^{n_k} \quad (A.13)
\]

where \( k-1 \)-nomial coefficient of order \( n \), \( \binom{n}{n_1, n_2, \ldots, n_k} \), is defined as

\[
\binom{n}{n_1, n_2, \ldots, n_k} := \binom{n}{n_1} \binom{n-n_1}{n_2} \binom{n-n_1-n_2}{n_3} \ldots \binom{n-n_1-n_2-\ldots-n_{k-2}}{n_{k-1}} = \frac{n!}{n_1! n_2! \ldots n_k!}. \quad (A.14)
\]

The (A.13) can also written as [104]

\[
(x_1 + x_2 + \ldots + x_k)^n = \sum_{n_1=0}^{n} \sum_{n_2=0}^{n-n_1} \sum_{n_3=0}^{n-n_1-n_2} \ldots \sum_{n_{k-1}=0}^{n-n_1-n_2-\ldots-n_{k-2}}
\]

\[
\times \binom{n}{n_1, n_2, \ldots, n_{k-1}, n-n_1-\ldots-n_{k-1}} x_1^{n_1} x_2^{n_2} \ldots x_{k-1}^{n_{k-1}} x_k^{n-n_1-\ldots-n_{k-1}} \quad (A.15)
\]
For example for a trinomial theorem as \((x + y + z)^n\), we will have
\[
(x + y + z)^n = \sum_{i=0}^{n} \sum_{j=0}^{n-i} \binom{n}{i, j, n-i-j} x^i y^j z^{n-i-j} \tag{A.16}
\]

### A.5 Order Statistic Random Variables

Let \(X_1, ..., X_n\) be i.i.d, each with density \(f(x)\) and distribution function \(F(x)\). Let \(Y_1 < Y_2 < ... < Y_n\) be the \(X_i\)’s arranged in increasing order, so that \(Y_k\) is the \(k\)-th smallest. In particular, \(Y_1 = \min X_i\) and \(Y_n = \max X_i\).

The distributions of \(Y_1\) and \(Y_n\) can be computed without developing any new machinery. The probability that \(Y_n \leq x\) is the probability \(X_i \leq x\) for all \(i\), which is for identical case
\[
F_{Y_n}(x) = \Pr(Y_n \leq x) = \prod_{i=1}^{n} \Pr(X_i \leq x) = [F(x)]^n \tag{A.17}
\]
Therefore for \(f_{Y_n}(x)\),
\[
f_{Y_n}(x) = \frac{dF_{Y_n}(x)}{dx} = n [F(x)]^{n-1} f(x) \tag{A.18}
\]

Similarly for the probability that \(Y_1 \leq x\)
\[
F_{Y_1}(x) = \Pr(Y_1 \leq x) = 1 - \Pr(Y_1 \geq x) = 1 - \prod_{i=1}^{n} \Pr(X_i \geq x) = 1 - [1 - F(x)]^n \tag{A.19}
\]
Therefore \(f_{Y_1}(x)\)
\[
f_{Y_1}(x) = \frac{dF_{Y_1}(x)}{dx} = n [1 - F(x)]^{n-1} f(x) \tag{A.20}
\]
By the multinomial formula, \(f_{Y_j}(x)\) can be computed as [105]
\[
f_{Y_j}(x) = \frac{n!}{(j-1)! (n-j)!} [F(x)]^{j-1} [1 - F(x)]^{n-j} f(x) \tag{A.21}
\]
and for joint density $f_{Y_j Y_k}(x, y)$ [105]

$$f_{Y_j Y_k}(x, y) = \frac{n!}{(j-1)! (k-j-1)! (n-k)!} \times [F(x)]^{j-1} [F(x) - F(y)]^{k-j-1} [1 - F(y)]^{n-k} f(x) f(y) \quad (A.22)$$

### A.6 Derivation of Lemma 1

**Lemma 1:** [57]

$$\mathbb{E} \left[ \log_2 \left(1 + \frac{x_0}{x+1}\right) \right] = \log_2 \int_0^\infty \frac{1}{z} \left( \mathbb{E} [e^{-zx}] - \mathbb{E} [e^{-z(x+x_0)}] \right) e^{-z} \, dz \quad (A.23)$$

The proof of Lemma 1 is presented as follows, For all $x \geq 0$, recall the identity [106, eq. (4.1.25)]

$$\ln (1 + x) = \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{x}{1+x} \right)^n \quad x \geq 0 \quad (A.24)$$

Then by invoking the identity [22, eq. (8.312.2)]

$$u^n = \int_0^\infty \frac{s^{n-1}}{\Gamma(n)} e^{-s} \, ds \quad n, u > 0 \quad (A.25)$$

(A.25) becomes

$$\ln (1 + x) = \sum_{n=1}^{\infty} \frac{1}{n} \int_0^\infty \frac{s^{n-1}}{\Gamma(n)} e^{-s(x+1)} \, ds$$

$$= \int_0^\infty \sum_{n=1}^{\infty} \left\{ \frac{1}{n} \frac{s^{n-1}}{\Gamma(n)} \right\} e^{-s(x+1)} \, ds$$

$$= \int_0^\infty \left\{ \frac{1}{s} (e^s - 1) \right\} e^{-s(x+1)} \, ds \quad (A.26)$$

where the last term obtained by using the fact that $e^s = \sum_{n=1}^{\infty} \frac{s^n}{n!}$ and $\Gamma(n) = (n-1)!$. 
Substituting $s = tx$, we obtain

$$\ln (1 + x) = \int_0^\infty \frac{1}{t} (1 - e^{-tx}) e^{-t} dt. \quad (A.27)$$

Considering $x = \frac{x_0}{x+1}$, (A.27) can be written as

$$\ln \left(1 + \frac{x_0}{x+1}\right) = \int_0^\infty \frac{1}{t} \left(1 - e^{-t\left(\frac{x_0}{x+1}\right)}\right) e^{-t} dt \quad (A.28)$$

Thereafter, $t = (x+1) z$ is substituted to obtain

$$\ln \left(1 + \frac{x_0}{x+1}\right) = \int_0^\infty \frac{1}{z} \left(1 - e^{-zx_0}\right) e^{-z(x+1)} dz. \quad (A.29)$$

Using a change of logarithmic base, $\ln (x) = \log_e (x)$ and taking the expectation respect to $x_0$ and $x$, we arrive at

$$\mathbb{E} \left[ \log_2 \left(1 + \frac{x_0}{x+1}\right) \right] = \log_e \int_0^\infty \frac{e^{-z}}{z} (\mathbb{E} [e^{-zx}] - \mathbb{E} [e^{-z(x-x_0)}]) dz. \quad (A.30)$$