A Mechanistic Modelling Approach to Derive Fracture Toughness Properties from Charpy Impact Energy in the Lower Transition Region

A thesis submitted to The University of Manchester for the degree of Engineering Doctorate in the Faculty of Engineering and Physical Sciences

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Robin James Smith

School of Materials
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Summary

A Mechanistic Modelling Approach to Derive Fracture Toughness Properties from Charpy Impact Energy in the Lower Transition Region

Robin James Smith, The University of Manchester, October 2014

This research was undertaken to develop an engineering method for the determination of material fracture toughness, $K_{\text{mat}}$, from Charpy V-notch impact energy, $U_{\text{el+pl,LLD}}$. Current structural integrity assessment methodologies for flawed steel structures are premised upon a knowledge of $K_{\text{mat}}$ for a given material. In circumstances when fracture toughness is not available or possible to obtain for an industrial structure $U_{\text{el+pl,LLD}}$ is commonly used to estimate $K_{\text{mat}}$ using empirical correlations. These correlations are necessarily conservative and are potentially inaccurate for some ferritic steels materials.

The development of a mechanistic engineering procedure for determining fracture toughness using $U_{\text{el+pl,LLD}}$ within the lower ductile-to-brittle transition of fracture behaviour for ferritic steels would significantly reduce this conservatism and provide increased confidence concerning safety assessments. Increased confidence in $K_{\text{mat}}$ calculations using a new mechanistic approach would also provide a functional use for much Charpy impact test data which are available for structural and nuclear grade steels.

The cleavage fracture differences between a single edge notch bend, SE(B), specimen and Charpy specimen were experimentally measured and numerically simulated. A mechanistic correlation was developed using a Weibull stress scaling approach to achieve accurate predictions of cleavage fracture behaviour of SE(B), Charpy V-notch and a range of intermediate geometries between these bounding geometries of interest.

This research has developed a new mechanistic procedure to correlate $U_{\text{el+pl,LLD}}$ with an equivalent $K_J$ corresponding to a SE(B) specimen. The correlation was found to be accurate in the lower ductile-to-brittle transition and provided probabilistic predictions of fracture behaviour. The correlation was validated using quasi-static and dynamic experimental test results and provided a linkage with the existing ASTM E 1921-11 master curve methodology for ferritic steels.

The research has provided recommendations for how the new approach might be applied in practice within an industrial context, and areas for future work that would develop a more generalised approach for a wider range of ferritic materials are highlighted.
Declaration

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1. **Introduction**

Industrial grade ferritic steels are widely used in nuclear power plants which generate electricity, and also components for the extraction and transport of oil and the manufacture of steel products. The presence of crack-like flaws in industrial components may arise from manufacturing methods at the beginning of life or they may develop within the construct-operate-decommission lifecycle. Structural integrity assessments (SIA) assess the significance of such defects and play a crucial role in the assurance of safety. This project concerns the development of structural integrity assessment methods for the purpose of safety assessments of steel structures used by industry.

1.1 **Industrial Background of the Project**

The evaluation of fracture toughness of steel components at the relevant operating temperatures and during adverse loading conditions, such as an accident scenario, is a primary activity during structural integrity assessment of existing industrial structures. Such assessments are essential to demonstrate the ongoing safety of nuclear infrastructure. Improvements in steel processing and manufacturing methods have facilitated the use of steels in applications which are safety critical and require a safe operating lifetime of sixty to eighty years. An increased emphasis on the adequacy of structural integrity assessment procedures is now becoming common and is pertinent for these infrastructure applications. Structural integrity assessment is currently performed using laboratory test data from the parent material extracted from the component or alternatively uses existing material data which is available. Extracting material samples from industrial structures is often expensive or prohibitively difficult to perform safely without an interruption to industrial operations. Circumstances of limited material data necessitates the conduct of assessment using methods to quantify the
uncertainty this introduces. Quantifying the likelihood of failure is therefore an important part of current structural assessment procedures such as R6 (B Energy, 2013) and BS7910 (BSI, 2013). The development of an improved method to quantify material fracture toughness using Charpy impact test data will therefore enable verdicts to be drawn concerning safety over nuclear plant operating lifetime to be reached with confidence. The fracture toughness test (ASTM, 2011a) remains a test of great accuracy, but the difficulties associated with extracting the necessary specimen geometries in industrial scenarios often preclude the use of such data.

1.2 Project Purpose

The outlined deficiency in material data is a hindrance to the accuracy of fracture toughness assessments of industrial structures. This research project has been devised due to previous experience of such cases. Recent advances in statistical and mechanistic properties of ferritic steels (Beremin, 1983)(Wallin, 1984) and Ruggieri and Dodds (1996) have demonstrated improved predictions of fracture toughness in the transition region from relatively small quantities of material data. These approaches have been adopted by practicing materials engineers such as Server et al (1998), Sokolov et al (1998) and implemented as standard procedures for testing and analysis in the ductile to brittle transition for ferritic steels as ASTM E1921-11 (ASTM, 2011b) but they do not directly assist integrity assessment in circumstances where fracture toughness data are difficult to obtain. The type of data more commonly available are Charpy impact test absorbed energy values ($U_{el,pl,LLD}$) at one or several temperatures of interest. These tests are conducted according to standardised procedures such as ASTM E23 (ASTM, 2002) and BS 10045 (BSI, 1990). The Charpy test is straightforward to conduct and relatively inexpensive and hence such material test data are more prevalent than fracture toughness data, additionally, conduct of the Charpy test is a standard delivery requirement for industrial steel products (ASTM, 2000). The current
method to estimate fracture toughness from $U_{el+pl,LLD}$ data is by existing empirically based procedures such as Barsom and Rolfe (1971), Sailors and Corten (1972) and Wallin (1989). These existing estimation procedures are necessarily conservative because of the uncertainties in their empirical basis and limited in their applicability to certain steel types or fracture mechanisms. To prevent this pessimism or potentially significant inaccuracies translating to structural integrity assessment procedures it is necessary to develop an improved theoretical model for fracture toughness prediction from Charpy V-notch $U_{el+pl,LLD}$.

The intention of this thesis is to develop a theoretical model which quantitatively describes the empirically demonstrated differences of test specimen geometry and loading rate which exist between the Charpy pendulum impact test and a single edge notch bend [SE(B)] fracture toughness test. The model will be developed with respect to a structural steel.

Recent developments of ‘local approach’ numerical modelling methods provide a basis for the intended theoretical model to predict fracture toughness from Charpy impact energy. Local approach methods provide an existing means of fracture prediction by linking the macroscopic loading conditions to the fracture mechanisms which operate in the presence of a defect. The numerical model will accurately predict fracture toughness in the lower transition region using local approach methodologies which are verified against a comprehensive experimental investigation.

1.3 Role of Fracture Toughness in Failure Assessment

Structural integrity assessment (SIA) approaches for structures with cracks originated from work which established the criteria for structural failure as one of plastic collapse or brittle fracture. Linear elastic brittle fracture methods were already known from the early work of Irwin (1957). Dowling and Townley (1975) established an interaction diagram to describe to the two governing fracture parameters and therefore
introduced the notion of failure under a combination of the two parameters. The EPRI handbook (Kumar et al, 1983) implemented a failure assessment diagram (FAD) using elastic-plastic fracture methods and this was further developed by researchers to become the most recent guidance: R6 (British Energy, 2005). Figure (1-1) shows a typical FAD which can be used during structural integrity assessments using standard procedures.

**Figure 1-1** The R6 failure assessment diagram and two influences $K_{mat}$ can exert during the assessment of structures with crack defects

The load ratio, $L_r$, is dependent on the material yield properties of a steel, and, similarly, fracture is dependent on the subject steel’s material fracture toughness, $K_{mat}$. $K_I$ is a loading parameter used to assess the proximity to brittle fracture. $K_{mat}$ is evaluated using laboratory testing. Such testing is standardised to ensure accuracy and consistency. $K_{mat}$ is an indispensable feature of current SIA’s, it must preferably be measured using existing standards, but allowance is made for Charpy V-notch $U_{av+p,LLD}$ to be related with $K_{mat}$ using empirical correlations within R6 (2005) and BS 7910 (2013). These correlations were devised to provide safe predictions of $K_{mat}$, but because
of their basis on empirical behaviour, a mechanistic explanation for the physical phenomena is lacking. This represents a discrepancy with the R6 document which is founded upon a shared mechanistic understanding of the plastic collapse and fracture behaviour of structures by engineering professionals. Figure (1-1) highlights two potential influences that \( K_{\text{mat}} \) can exert on engineering critical assessments (ECA) of flawed structures. It is necessary to demonstrate that a structure is safe under all operating loading conditions, therefore an increased understanding and an accurate mechanistic description of such correlations must be considered an essential prerequisite for the advancement of this aspect of structural integrity assessments. A mechanistic solution for calculating fracture toughness from \( U_{\text{el+pl,LLD}} \) will allow the development of the correlation to be undertaken within a material specific framework with the aim of reducing uncertainties associated with such correlations and therefore providing increased options to engineers when undertaking safety case assessments.

1.4 Thesis Structure

This thesis concerns a systematic experimental and numerical investigation of a high strength quenched and tempered structural plate. The experimental work is used to validate a micromechanical model for brittle fracture in the lower transition temperature range of fracture behaviour. The experimental work quantifies probabilistically the geometric differences between SE(B) and Charpy V-notch (CVN) type specimens under quasi-static and dynamic conditions using a range of intermediate specimens.

A literature review was conducted (Chapter 2) for the purpose of assessing the progress made in the field of SIA. Currently available numerical and experimental methodologies which would potentially form a role in the project were reviewed and also the associated fracture mechanics theoretical advances which were of relevance.
Project objectives, aim and a list of knowledge gaps are provided at the end of Chapter 2.

Chapter 3 concerns the research methodology which included the following activities:

- materials characterisation,
- materials test programme for quasi-static and dynamic testing,
- development and validation of a mechanistic approach to Charpy impact data within structural integrity assessments,
- a proposed extension of the mechanistic approach to the entire ductile-to-brittle transition.

The main Results are given in Chapter 4 and these are described quantitatively, within the context of the project objectives. Chapter 5 contains a Discussion of the main Results, given of Chapter 4, in the context of the project aim and objectives and proposals for future work. The Discussion highlights and evaluates the Results so as to establish activities for future research concerning the SIA related objectives of the Thesis. Chapter 6 provides the main Conclusions arising from the work, Future Work arising from the work contained in preceding Chapter of the thesis is provided in Chapter 7.
1.5 References


2. **Literature Review**

The main objective of this literature review is to summarise previous research work relating to prediction of fracture toughness within the ductile to brittle transition temperature regime of ferritic steel behaviour. This will be done in the context of the thesis objective which is to develop a theoretical method for prediction of elastic-plastic material fracture toughness ($K_{mat}$) using Charpy V-notch impact energy ($U_{el+pl,LLD}$).

To accomplish the main literature review objective previous work concerning the physical, theoretical and empirical factors affecting prediction of transition fracture toughness of ferritic steels will be identified and the main findings described within the perspective of this current work. The format of this review is such to include the significant works which have contributed to the formulation of a methodology for conducting this research. The subject has been divided into several components. These components progress from the physical aspects to empirical work and interpretation by statistical methods. Statistical methods provide a bridge with the recent development of ‘local approach’ numerical techniques to predict fracture in steel laboratory specimens. The parts presented in this literature review are the following:

1. Linear elastic fracture mechanics,
2. Elastic-plastic fracture Mechanics,
3. Fracture toughness measurement of ferritic steel components,
4. Charpy impact testing,
5. Correlation of Charpy impact energy and fracture toughness,
6. Assessment methods for cleavage fracture of ferritic steels,
7. Dynamic mechanical properties of ferritic steels,
8. Empirically observed Charpy transition curve shape,
The aims and objectives of the thesis are given in Parts 2.10 and 2.11 of the literature review respectively.

2.1 Linear Elastic Fracture Mechanics

Griffith (1921) showed that linear elastic strain energy, $U$, was related to fracture and the work required to create new surfaces, $W$. $U$ is a measure of the accumulated work during loading of a body:

$$U = \frac{P\Delta}{2} = \frac{C P^2}{2}$$

(2-1)

where, $U$ = the strain energy,

$P$ = the applied load,

$\Delta$ = the load line displacement,

$C = \frac{\Delta}{P}$ = the elastic compliance.

Crack extension by an incremental unit area, $dA$, causes a loss of elastic strain energy in a body under either displacement or load control conditions. According to the work of Griffith (1921):

$$\left. \frac{dU}{dA} \right|_A = \frac{P^2}{2} \left. \frac{dC}{dA} \right|_A = \left. \frac{dU}{dA} \right|_P$$

(2-2)

where, $G =$ the linear elastic strain energy release rate,

$$\frac{dC}{dA} = \text{compliance function.}$$
The following thermodynamic relation was established by Griffith which represented the notion that the work, $W_s$, required to create new surfaces within a material is directly related to a decrease of the strain energy of a body due to crack extension.

\[
\frac{-dU}{dA} = \frac{dW_s}{dA}
\]  

(2-3)

Irwin (1957) introduced the concept of the energy release rate, $G$, using the previous work of Griffith (1921). Hence, the following relationship was given:

\[
G = -\frac{dU}{dA}
\]  

(2-4)

For a plane stress wide plate with a centre crack of length, 2$a$, such as that shown in Figure (2-1), $G$ is given by the following relationship:

\[
G = -\frac{\pi\sigma^2 a}{E}
\]  

(2-5)

where, 

- $\sigma$ = the remotely applied stress,
- $a$ = half the crack length,
- $E$= Young’s modulus of elasticity.

When $G$ is increased, for a linear elastic body under increasing remote loading, the material cohesive surface energy, $\gamma_s$, which is required to balance Expression (2-3), is exceeded when the following condition is met:

\[
G > \frac{dW_s}{dA} = 2\gamma_s
\]  

(2-6)

Hence Expression (2-5) may in theory be used to establish critical applied stresses for engineering use. Metallic materials exhibit significantly higher surface energy than $\gamma_s$ because of plastic flow and hence such analysis is not suitable.
2.1.1  **Linear Elastic Stress Fields**

Early fracture mechanics research introduced a geometric system for postulated cracks in finite bodies loaded under remote stresses. Research focused on specific geometries such as the centre crack plate (CCP), as shown in Figure (2-1).

Several researchers gave solutions to the stress fields at a crack-tip in homogeneous, isotropic, linear elastic bodies. Williams (1952) solution is commonly cited, this worker presented equations for the asymptotic singular linear elastic stress fields of an infinite cracked plate (with no surface tractions on the crack surfaces) using a polar coordinate system centred on the crack tip (Figure 2-2). Williams provided confirmation that the form of the singularity was common to all cracks irrespective of geometric properties. The Williams solution method distinguished between symmetric and asymmetric loading, symmetric loading corresponded to the opening of the crack perpendicular to the crack plane. All numerical analysis assumed that the crack front was straight and smooth.
The mode I crack opening stress field components can be simplified by neglecting the higher order terms present in the Williams (1952) solution. Irwin (1957) provided the following solutions for the linear elastic stresses at a crack tip under symmetric mode I loading in an infinite body under loading using $G$:

\[
\sigma_{xx} = \frac{GE'}{2\pi r} \left[ 1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] \quad (2-7a)
\]

\[
\sigma_{yy} = \frac{GE'}{2\pi r} \left[ 1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] \quad (2-7b)
\]

\[
\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \quad (2-7c)
\]

$E'$ in Expression (2-7) refers to two definitions of Young's modulus: $E'=E$ under conditions of plane stress, and $E'=E/(1-\nu^2)$ under plane strain ($\nu$ is Poisson's ratio).

Equation (2-7) indicates that the opening stress, $\sigma_{yy}$, varies as $1/\sqrt{r}$, a square root singularity as the crack tip is approached and $r\rightarrow0$ (Figure 2-3). This means that infinite stresses are predicted at the crack tip. In practical situations the stresses are limited for
metals due to the presence of plasticity and yielding which causes a departure from elastic behaviour.

![Graph showing linear elastic stress distribution ahead of a crack at θ=0° for a given G](image)

**Figure 2-3 Williams (1952) linear elastic stress distribution ahead of a crack at θ=0° for a given G**

The most common criterion used to quantify yielding of metals is the equivalent stress of von Mises (1913), \( \sigma_{eq} \), given by Expression (2-8). When \( \sigma_{eq} \) exceeds the limit of proportionality, \( \sigma_0 \), of a metal under monotonic loading, a nonlinear stress-strain response will occur which is associated with the movement of dislocations which permit plastic flow of a metal.

\[
\sigma_{eq} = \frac{1}{\sqrt{2}} \left( \left( \sigma_1 - \sigma_2 \right)^2 + \left( \sigma_1 - \sigma_3 \right)^2 + \left( \sigma_2 - \sigma_3 \right)^2 \right)^{1/2}
\]  \hspace{1cm} (2-8)

where, \( \sigma_1, \sigma_2, \sigma_3 \) = each of the three principal normal stresses.
2.1.2 Stress Intensity Factor

The linear elastic stress intensity factor, $K$, is a parameter which can be used to characterise the linear elastic crack tip stresses, strains and displacements. $K$ was also devised to be a function of the remotely applied stress, $\sigma$, and the crack length, $a$, for a through crack in an infinite plate. A geometric factor, $Y$, was thereafter added by engineering professionals, such as Gross et al (1964), to generalise $K$ for application to a range of geometries. The notation and coordinate system adopted for the analysis of stresses and strains in the vicinity of a crack for a linear elastic body is shown in Figure (2-2).

The expression for $K$ under pure mode I loading, $K_I$, is given by the following:

$$K_I = Y\sigma \sqrt{\pi a} \quad (2-9)$$

where,

$K_I =$ the stress intensity factor for mode I loading,

$Y =$ a geometric factor,

$\sigma =$ the remotely applied stress,

$a =$ the crack length.

Irwin (1957) demonstrated that $K$, and $G$ could be related under plane strain conditions by the following relationship and that this relationship was independent of geometry:

$$K^2 = GE' \quad (2-10)$$

where, $\nu =$ Poisson’s ratio,

$E' =$ Young’s modulus of elasticity.

Solutions for $K$ have been compiled by Tada et al (2000) for different cracked structural geometries.
Three modes of loading were defined in relation to $K$, following the implicit dependence of the stress field solution on mode of loading found by Williams (1952), these corresponding to:

1. opening of the crack perpendicular to the crack plane,
2. sliding of the crack faces in the crack plane,
3. transverse shearing of the crack faces.

The stress intensity factors for three individual modes of loading are denoted $K_I$, $K_{II}$ and $K_{III}$ respectively. Mode I is the symmetric form of crack loading which is frequently considered in engineering assessments because it is most easily quantified experimentally and also the most onerous loading mode.

2.2 Elastic Plastic Fracture Mechanics

For a body subject to proportional loading the stress-strain behaviour can be assumed to be nonlinear elastic. Such an assumption was used by Hutchenson (1968) and Rice and Rosenglen (1968) to derive a theoretical stress and strain field solution (commonly termed the HRR solution) for elastic-plastic materials such as metals. At a real crack-tip which is loaded in tension, a zone of large geometric deformation and accompanying large strains exists. The loading is in this region is therefore not proportional. This region is named the process zone as shown in Figure 2-4. Under conditions of small scale yielding (SSY) unstable fracture behaviour ahead of a crack tip may be characterised using elastic-plastic fracture mechanics to quantify the stresses in the region of HRR dominance from which unstable cleavage fracture frequently initiates.
Crack or Notch

Crack or Notch

Region of HRR Applicability
(Elastic-Plastic)

Region of K-field Applicability (LEFM)

Plastic Zone of Finite Crack-Tip Deformation

Figure 2-4 The zones of applicability of linear elastic and elastic plastic fracture parameters

2.2.1 The J-integral

The J-integral, $J$, was introduced by Rice (1968), and is related to the total strain energy of a body, $U$, and crack area $dA$ as follows:

$$J = \frac{dU}{dA}, \quad (2-11)$$

$J$ is the nonlinear elastic energy release rate for quantifying crack driving. Under linear elastic conditions $J$ is equal to $G$ within Expression (2-4).

Under displacement control loading conditions: $J = -\left(\frac{dU}{dA}\right)_a$, quantifies the change of stored strain energy with increasing crack area. Rice (1968) showed that a path integral anti-clockwise around a sharp notch (Figure 2-5), could be used to approximately quantify the $J$. This integral was found to be path independent under conditions of monotonic loading. The mathematical formula for $J$ using the path integral concept was validated by Rice for sharp notches and cracks and is given by Expression (2-12).
\[ J = \int_{\Gamma} \left( W dy - \Gamma_i \frac{\partial u_i}{\partial x} \right) ds, \] \hfill (2-12)

where, \[ W = \text{strain energy density} = \int_{0}^{\varepsilon_{ij}} \sigma_{ij} \varepsilon_{ij}, \]

\( T_i \) = is the surface traction vector or boundary stress on a control volume inside the contour \( \Gamma \), \( T_i = \sigma_i n_i \), \( n_i \) is a unit vector,

\( u_i \) = are the displacement components,

\( s \) = is the distance around the contour.

Figure 2-5  The principle of a contour integral for evaluation of the J-integral around a sharp notch or crack in an infinite body under monotonic loading

2.2.2 Elastic Plastic Stress and Strain Fields

Outside the region of finite deformation identified in Figure 2-4, there exists an annular region for which the stress and strain fields are known. The so-called HRR fields were derived independently by Hutchenson (1968) and Rice and Rosenglen (1968), the derivation of which was based on a nonlinear elastic material for cases of proportional loading and neglecting large crack-tip deformation. The solution produced
by these workers for stresses and strains at a crack under such conditions are given inExpressions (2-13a) and (2-13b) respectively.

\[
\frac{\sigma_{ij}}{\sigma_0} = \left( \frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(n, \theta) \tag{2-13a}
\]

\[
\frac{\varepsilon_{ij}}{\varepsilon_0} = \frac{\alpha}{E} \left( \frac{EJ}{\alpha \sigma_0^2 I_n r} \right)^{\frac{n}{n+1}} \tilde{\varepsilon}_{ij}(n, \theta) \tag{2-13b}
\]

where, \(\sigma_{ij}\) = the relevant component of the stress tensor,
\(\sigma_0\) = the limit of proportionality for the nonlinear elastic material law,
\(\varepsilon_0\) = the strain associated with \(\sigma_0/E\), giving the material law
(Ramberg and Osgood, 1943):

\[
\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left( \frac{\sigma}{\sigma_0} \right)^n \tag{2-14}
\]

and, \(\alpha = a\)
\(l_n\) = an integration constant which depends on a curve fitted \(n\) parameter in Exp. (2-12),
\(r\) = the radial distance from the crack-tip as defined in Fig. (2-2),
\(n\) = the strain hardening exponent.

\(\tilde{\sigma}\) and \(\tilde{\varepsilon}\) in Equation 2-11 are dimensionless functions of \(n\), the strain hardening exponent of Equation 2-12 and \(\theta\), the angle counter-clockwise from the crack-plane as shown in Fig. 2-2. A typical HRR elastic-plastic crack tip stress solution.
at $\theta=0^\circ$ for a steel material of $E=207,000\text{MPa}$, $\sigma_0=415\text{MPa}$ and $n=6.5$ is compared with the linear elastic solution of Section 2.1.1 in Figure (2-6).

![Graph showing stress field comparison](image)

**Figure 2-6** Typical elastic-plastic HRR field stress field solution and linear elastic stress field for $\theta=0^\circ$ at an applied $K_I=60.3\text{MPa}\sqrt{m}$ $J$ equal to $103.2\text{N/mm}$

From Expressions (2-13a) and (2-13b) it is seen that stresses and strains vary in proportion to $r^{\frac{1}{n+1}}$ and $r^{\frac{n}{n+1}}$ respectively in a plastically deformed annulus which is surrounded by another annulus of linear elastic deformation but which exists outside the zone of finite deformation at the crack tip (Figure 2-4). The HRR derivation is based on the assumption of small scale yielding, this is to assume an infinite body with pure mode I opening and no global bending stresses or plasticity encroaching on the crack-tip from external sources. Small geometric deformations are assumed which precludes the description of the effect of crack tip blunting on the stress field in close proximity to the tip.
2.2.3 Crack Tip Opening Displacement

The J-integral was shown to be related to crack tip opening displacement (CTOD), $\delta_t$, by McMeeking (1977), using large deformation plane strain finite element numerical analysis. Shih (1981) investigated the relationship between $\delta_t$ and applied $J$ for cracks in plane strain power law hardening materials. This author defined $\delta_t$ as the distance transverse to the crack plane between crack faces at the 45° intercept position from the deformed crack tip as shown in Figure 2-7.

![Figure 2-7 Definition of crack tip opening displacement, $\delta_t$, by Shih (1981)](image)

Figure 2-8 shows the results of elastic plastic finite element analyse, the work used a large deformation formulation to accurately calculate deformations at the crack tip, and plane strain elements. Analyses results for crack-tip deformation under small scale yielding conditions for a range of hardening exponents, $n$, were provided by Shih (1981).

The Shih (1981) analyses suggested that the J-integral and $\delta_t$ could be calculated from the following relationship:

$$\delta_t = \mu \frac{J}{\sigma_0}$$  \hspace{1cm} (2-15)
Figure 2-8 Deformed crack-tip profile under small scale yielding conditions for a range of hardening exponents (Shih, 1981)

Within Expression (2-15), $\mu$ is Shih’s constant of proportionality, and, $\sigma_0$ the limit of proportionality determined from Equation 2-12.

Values of $\mu$ for use in conjunction with Equation (2-15) were given by Shih (1981) (Table 2-1) using the HRR solution of Expression (2-13), previous workers numerical results, and plane strain SSY analysis to determine displacements behind a crack-tip and also additional numerical analysis of single edge notch bend, SE(B) and CC(P) fracture toughness specimen geometries under large scale yielding (LSY) conditions. A range of material flow properties were considered in accordance with Expression (2-14) This work provided a means of characterising crack tip stress fields using $\delta_t$. Currently, $\delta_t$ is not routinely measurable during experimental testing.
Table 2-1 Values of $\mu$ for calculating $\delta_t$ or $J$ using Expression 2-20 (Shih, 1981)

<table>
<thead>
<tr>
<th>$\delta_t$ Analysis result</th>
<th>$\frac{\sigma_0}{E}$</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRR Expression (2-13)</td>
<td>0.001</td>
<td>0.13</td>
<td>0.27</td>
<td>0.46</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>0.002</td>
<td>0.17</td>
<td>0.31</td>
<td>0.50</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>0.003</td>
<td>0.21</td>
<td>0.36</td>
<td>0.53</td>
<td>0.78</td>
</tr>
<tr>
<td>Plane strain SSY</td>
<td>0.002</td>
<td>0.20</td>
<td>0.32</td>
<td>0.52</td>
<td>0.7</td>
</tr>
<tr>
<td>McMeeking (1977)</td>
<td>0.003</td>
<td>-</td>
<td>0.27-0.30</td>
<td>0.41-0.44</td>
<td>0.55-0.67</td>
</tr>
<tr>
<td>SE(B): Plane strain, LSY</td>
<td>0.002</td>
<td>0.19</td>
<td>-</td>
<td>0.49</td>
<td>0.58-0.65</td>
</tr>
<tr>
<td>CC(P): Plane strain, LSY</td>
<td>0.002</td>
<td>0.22</td>
<td>-</td>
<td>0.64</td>
<td>0.82-0.87</td>
</tr>
</tbody>
</table>

Table 2-1 indicates that the HRR field solution provided agreement with large strain numerical analysis under small scale yielding conditions. For finite values of $n$ the HRR field may also be used to estimate $\delta_t$ under large scale yielding conditions for fracture toughness specimen geometries. McMeeking and Parks (1977) noted that the region of dominance of $J$ reduces considerably with increasing $n$ and vanishes under conditions of no strain hardening ($n=\infty$).

2.2.4 Thickness Effect

Stress triaxiality is a state where tensile stresses exist in all three component directions of stress. These stresses can be defined in the component coordinate system $(X, Y, Z)$ or the principal stress planes $(1, 2, 3)$ therefore:

\[ \sigma_{xx}, \sigma_{yy} \text{ and } \sigma_{zz} > 0 \]

or

\[ \sigma_{11}, \sigma_{22} \text{ and } \sigma_{33} > 0 \]

For a sharp crack SE(B) specimen, subject to mode I three point bending (with coordinates defined as Figure 2-2) $\sigma_{yy}$ coincides approximately with $\sigma_1$ and $\sigma_{xx}$ coincides
approximately with $\sigma_2$. The presence of $\sigma_{zz}$ and absence of significant $\varepsilon_{zz}$ is prerequisite for triaxial stress conditions in laboratory specimens, such conditions quantify the degree of plane strain conditions. A thin specimen, with dimensions $dZ << dX$ and $dY$ is described as being in a state of plane stress.

Stress triaxiality ahead of a crack-tip serves to promote cleavage fracture by elevating the material flow stresses and suppressing yielding in comparison to uniaxial loading conditions (as indicated by Expression (2-8)). The magnitude of $J$, $\delta_t$ and triaxiality, $\sigma_{zz}/(\sigma_{xx}+\sigma_{yy})$, along a SE(B) ($a/W=0.4$) specimen crack-front corresponding to different loading levels, is shown in Figure 2-9 which shows the results of elastic and elastic-plastic finite element analyses by Narasimathan and Rosakis (1988).

The results of Figure (2-9) showed significant elevation of $J$ and $\delta_t$ at the centre of a crack front which whilst related to the crack-front distribution of stress triaxiality, the distribution of $J$ and $\delta_t$ were seen to be also load dependent as the limit load, $P_0$, was approached. Plastic behaviour served to raise the stress triaxiality in comparison to the elastic case. Additional observations from these numerical results was the significant loss of stress triaxiality at the free surfaces of the plate and the diminishing stress triaxialty with distance ahead of the crack-tip ($\sigma_{zz} \approx 0.25(\sigma_{xx}+\sigma_{yy})$ at a distance, $r$, equal to 6.5% of the plate thickness).
Figure 2-9 The variation of $J$, $\delta$, and stress triaxiality along the crack front (through thickness) of a SE(B) specimen by Narasimthan and Rosakis (1988)

2.2.5 Constraint

German and Shih (1981) investigated the effect of a range of hardening exponents when calculating the agreement of the HRR stress and strain field with the quantities which were numerically calculated for common laboratory specimen geometries [SE(B) and CC(P)]. German and Shih found that SE(B) specimens (termed cracked bend bar (CBB) by those authors) had a stress field which at $\theta=0^\circ$ which was dominated by the HRR field to a greater loading intensity than the CC(P) configuration. Figure 2-10 shows results for crack-tip stress distributions at particular loading levels. The level of loading was expressed as $C\sigma_0/J$, where $C$ is a geometric constant, so as $J$ increases this ratio decreases in magnitude. The loading levels plotted correspond to $\sigma_0/J$= 600, 200, 60 and 30. The stress fields were plotted using a dimensionless radius,
\(X \sigma_0 / J\), where \(X = r\) which is consistent with the form of the HRR field solution (Expression 2-13). It was found that the stresses ahead of the crack-tip of the CC(P) geometry departed and fell below the mode I opening stress levels of the plane strain HRR field much more readily than the SE(B) geometry. This is synonymous with a loss of constraint. Loss of constraint occurs when the global stress fields, which are influenced by the specimen geometry and material flow properties and mode of loading, encroach on the crack-tip field as the loading intensity increases and reduce triaxiality.

Shallow fatigue pre-cracked SE(B) specimens of nominal \(a/W = 0.3\) (shallow notch) were found to lose constraint upon loading in comparison to deep crack geometries of \(a/W = 0.5\) (deep notch) by Sumpter (1982) and as such demonstrated an apparent increased resistance to cleavage fracture. Sumpter surmised that different patterns of plastic yielding were responsible for this behaviour (Figure 2-11) because the stress distribution ahead of the crack-tip is less intense for low \(a/W\) ratio SE(B) specimens than the HRR field under SSY and LSY conditions. Such behaviour causes increased critical \(J\) measurements at unstable fracture, \(J_c\), when using experimental procedures (Section 2.3).

Du and Hancock (1988) and O’ Dowd et al (1992) invoked a second parameter (Section 2.6.3) to characterise constraint and quantify the calculated crack-tip stress distributions for SE(B) experimental geometries. Figure 2-12 from Al-Ani and Hancock (1991) shows normalised mode I opening stress fields for \(\theta = 0^\circ\), which were calculated using small strain deformation theory, for a SE(B) of \(a/W = 0.2\) using radial distance, \(X_0 \sigma_0 / J\), where \(X_0\) is the distance from the crack-tip, and the loading parameter \(a \sigma_0 / J\), is normalised using the crack length (\(a\)).
Figure 2-10 A comparison of mode I stress distribution of CC(P) and SE(B) geometries with the theoretical HRR field (German and Shih, 1981)

Figure 2-11 Patterns of plastic yielding identified by Sumpter (1982)
2.2.6 Standardised Fracture Mechanics Test Methods

The measurement of fracture toughness is undertaken using standardised procedures such as ASTM E1820-11 (2011a), ESIS P2-92 (1992) and British Standard BS7448-1 (BSI, 1999). The ESIS (1992) method of calculating $J$ for SE(B) specimens is founded on fracture mechanics test standardisation work of Rice (1973). These procedures dictate the test conduct procedures, specimen dimensions and specimen preparation. Additionally, the calculation method to be adopted for the determination of the critical elastic-plastic $J$-integral, $J_C$, at initiation of ductile crack propagation is provided.

2.3 Fracture Toughness Measurement of Ferritic Steel Components

All the standardised fracture toughness tests use fatigue pre-cracked specimens. The type of specimen of interest for this research work is the single edge
notch bend, SE(B), type. This type of specimen is shown in Figure 2-13 the test configuration is in accordance with the elastic-plastic fracture toughness testing standard ASTM E1820-11 (2011a). Load \((P)\) is applied by a servo-hydraulic ram and measured by a load cell within the loading apparatus. Displacement measurement is undertaken of crack mouth opening displacement (CMOD) using a clip gage which is placed between integral knife edges at the mouth of a machined notch. Load line displacement (LLD) in the plane of the crack may alternatively be measured during the test using a technique such as the comparator bar (Joyce, 1996). The standard SE(B) specimen geometry can vary in cross-section, common configurations use a ratio of width \((W)\) to thickness \((B)\) equal to one or two.

![Diagram of ASTM E1820-11 fracture toughness test configuration using SE(B) specimens](image)

**Figure 2-13 The ASTM E1820-11 (2011a) fracture toughness test configuration using SE(B) specimens**

The span, \(S\), of the SE(B) test specimen is equal to four times its \(W\). Derivation of the J-integral following the ASTM E 1820-11 testing requires the numerical integration of the area under the load versus displacement curve to evaluate the plastic absorbed energy, \(U_p\), as shown in Figure 2-1 Measurement of the J-integral from test specimens is normally undertaken at a quasi-static loading rate. There are two methods of deriving the J-integral from the test recording, both of these approaches use the measured applied load and either CMOD or LLD.
2.3.1 **Critical J-integral Evaluation**

The following method allows evaluation of critical J-integral values \( (J_c) \) corresponding to fracture initiation using load and CMOD data. It originates from the work of Dawes (1979) at The Welding Institute.

![Diagram of Load (P) vs. CMOD/LLD](image)

**Figure 2-14** Determination of the plastic absorbed energy during SE(B) fracture toughness testing to ASTM E1820-11 (2011a) or ESIS (1993)

The J-integral, when calculated in accordance with ASTM E1820-11 (2011a), is evaluated using the following expression for SE(B) specimens of \( 0.05 < \frac{a}{W} \leq 0.55 \):

\[
J = \left( K_i^2 \left( 1 - \nu^2 \right) \right) \frac{E}{Bb_0} + \frac{\eta_{pl} U_{pl}}{Bb_0}
\]  \hspace{1cm} (2-16)

where, \( K_i = \) the linear elastic stress intensity factor,

\( \nu = \) Poisson's ratio.

The constant \( \eta_{pl} \), in Expression (2-14) is an elastic-plastic geometric factor determined from the work of Kirk *et al* (1993), using Expression (2-17).
\[ \eta_{pl} = 3.667 - 2.199(a/W) + 0.437(a/W)^2, \]  

(2-17)

Within Expression (2-17), \( U_{pl} \) is equal to the plastic energy absorbed prior to the point of failure, and,

\( b_0 \) is the ligament length equal to the crack length, \( a \), subtracted from \( W \).

The mode I linear elastic stress intensity factor (\( K_i \)) of Expression (2-16) is determined from the following expression of ASTM (2011a): which incorporates a geometric factor, \( f(a/W) \), which varies with initial fatigue pre-crack depth (\( a \)) and specimen width (\( W \)):

\[ K_I = \left[ \frac{P S}{BW^{1.5}} f(a/W) \right], \]  

(2-18a)

\[ f(a/W) = \frac{3 \left( \frac{a}{W} \right)^{0.5} \left[ 1.99 - \left( \frac{1}{W} \right) \left( 1 - \frac{a}{W} \right) \left( 2.15 - \frac{3.93a}{W} + \frac{2.7a^2}{W^2} \right) \right]}{2 \left( 1 + \frac{2a}{W} \right) \left( 1 - \frac{a}{W} \right)^{1.5}} \]  

(2-18b)

Figure 2-15, given by Rolfe (1977), shows schematically different test results corresponding to tests undertaken at different temperature in the ductile-to-brittle transition. The onset of ductile behaviour can occur at any point preceding maximum load, this is influenced by the size of particle defects in a steel material such as carbides and non-metallic inclusions as illustrated by Anderson (1984) in Figure 2-16. ESIS (1992) and Schwalbe et al. (2002) provided methodologies for calculating \( J \) using load-line displacement methods which are premised on a deeply cracked SE(B) specimen (\( a/W \approx 0.5 \)) using the analysis work of Rice et al. (1973).
Figure 2-15 Schematic of the load versus CMOD test recording for different fracture behaviour types (Rolfe, 1977)

![Figure 2-15 Schematic of the load versus CMOD test recording for different fracture behaviour types (Rolfe, 1977)](image)

Figure 2-16 Comparison of the behaviour of two steels with different defects during fracture toughness testing in the transition region (Anderson, 1984)

![Figure 2-16 Comparison of the behaviour of two steels with different defects during fracture toughness testing in the transition region (Anderson, 1984)](image)

Figure 2-17 shows a comparison between theoretical stress fields predicted using $K$ and $J$ and the associated stress fields (calculated using finite element analysis) of a SE(B) ($B=W=10\text{mm}$, $a/W=0.5$) experiential test geometry for a specimen loading level
corresponding to $K_I = 60.3 \text{MPa}\sqrt{m}$ and $J = 103.2 \text{N/mm}$. The regions of applicability of linear-elastic and elastic-plastic fracture mechanics are clearly evident.

![Figure 2-17 Comparison of theoretical fracture mechanics and calculated SE(B)](image)

$\sigma_{yy}$ distribution at $\theta = 0^\circ$

### 2.3.2 J-R Curve Method

Resistance curves were first calculated using multiple specimens by Landes and Begely (1974), this method required testing in the ductile temperature range and provided the relationship between $J$ and crack length ($\Delta a$) following crack initiation at $J_{IC}$. The J-R curve using a single specimen test requires calculation of a specimen's crack length at intermittent intervals following the onset of stable fracture. The unload compliance method was applied to three point bend specimens (Willoughby and Garwood, 1983) and these authors calculated compliance functions for different crack depths. Methods to calculate J-R curves are provided in ASTM E 1820-11 (2011a) using the unload compliance technique or the normalisation technique investigated by Herrera and Landes (1990). Figure 2-12 from Anderson (1984) shows schematically how the fracture behaviour of different steels is dependent on microstructural defects.
2.3.3 Elevated Loading Rate Method

ASTM E1820-11 (ASTM, 2011a) provides a test methodology for undertaking fracture toughness testing of pre-cracked Charpy specimens at elevated loading rates. This method utilises the instrumented Charpy test machine (Section 2.4.2).

2.4 Charpy Impact Testing

2.4.1 Standard Test Procedures

ASTM E23 (ASTM, 2002) and BS 10045 (1990) codify the existing standard Charpy V-notch testing procedures for evaluation of total absorbed energy, $U_{el+pl,LLD}$, using a Charpy pendulum impact machine. The geometry of the standard Charpy specimen is $B=W=10\text{mm}$ and $L=55\text{mm}$ and the span $(S)$ is equal to four times $W$. The procedures lay down tolerances on the specimen dimensions and machine apparatus dimensions. Verification of the machine for accuracy is undertaken using standardised test pieces. Evaluation of test bias and systematic error may be undertaken using a statistical approach given by Splett et al (2008).

The Charpy impact test can be used to demonstrate a material’s Charpy impact energy transition by conducting tests over a range of temperatures (Figure 2-18). Scatter of measured quantities in the transition region is a common characteristic of the Charpy impact and the fracture toughness test methods (Section 2.3.1). Within the nuclear industry, the Charpy impact test is of particular importance due its utility for monitoring the possible influence of irradiation dose on the transition temperature range and the reduction of upper shelf toughness. There is a significant amount of Charpy impact data available for Reactor Pressure Vessel (RPV) type steels because of the Charpy V-notch specimen being favoured for use in surveillance capsules since the commissioning of early reactors in the 1960’s. Following prolonged radiation exposure of some steel types, the onset of the Charpy and fracture toughness transition regime
for the irradiated steel can occur at an increased temperature. This has a detrimental structural integrity implication for safety assessment and lifetime determination of industrial structures. Figure 2-13 shows the shift in the Charpy $U_{el+pl,LLD}$ transition temperature range caused by irradiation (McGowan et al, 1988). This temperature is commonly denoted as the temperature at which $U_{el+pl,LLD}$ is equal to 41J. The shift $\Delta T_{41J}$ is a positive function of neutron fluence (cumulative received dose). Eason et al (2006) provided guidance on assessing this irradiation shift for integrity assessment of nuclear components subject to ionising radiation.

![Figure 2-18 Charpy impact energy transition curves measured using a Charpy V-notched specimen (McGowan et al, 1988)](image)

2.4.2 Instrumented Test Procedures

The use of instrumented Charpy impact machines is standardised by BS EN ISO 14556 (BSI, 2000). This document contains the methods for determining LLD from the force versus time ($F,t$) test recording using a double integration methodology. The velocity ($v$) at time, $t$, is given by Expression (2-19).
\[
v(t) = v_0 - \frac{1}{m} \int_{t_0}^{t} F(t) \, dt ,
\]

(2-19)

Within (2-19), \( m \) is the mass of the Charpy machine striker, and \( v_0 \) the initial velocity at the instant of contact with the specimen.

The displacement \( s \) is then calculated by a second integration of the force with respect to time according to Expression (2-20).

\[
s(t) = \int_{t_0}^{t} v(t) \, dt .
\]

(2-20)

A schematic load–displacement curve showing the test signal and a moving average line is shown in Figure 2-19 from BS EN ISO 14556. The method of evaluation of dynamic fracture toughness, \( J_{\text{d}} \), from such information is standardised in ISO DIS 26843 (2009) and in ASTM E1820-11 (2011a). These standards adopt the proposal developed by Schindler (1996)(2000) for the lower transition region and lower shelf. The technique for evaluating toughness in these regions of fracture behaviour follows a partitioned J-integral expression, in the same manner as ASTM E1820-11, the quasi-static fracture toughness test standard. LLD is used for the calculation of fracture toughness by these methods. ISO DIS 26843 provides two scenarios where quasi-static methodologies can be employed:

1. When there are no more than three oscillations during the fracture process,
2. When the amplitude of oscillation is less than +/-15% compared with the mean value at the time of fracture, \( t_f \).

Figure 2-20 shows the four types of response at high loading rates, Types I and II are suitable for evaluation by the standard quasi-static methodologies when the above conditions are met and no significant ductile behaviour is observed (\( \Delta a < 0.2 \text{mm} \)).
Figure 2-19 Schematic load versus displacement recording from an instrumented Charpy impact test [adapted from (BSI, 2000)]

Figure 2-20 Four categories of force–displacement response in an instrumented Charpy test given by ASTM E1820-11 (2011a)
2.5 Correlation of Charpy Impact Energy and Fracture Toughness

2.5.1 Empirical Lower Transition Temperature Range Correlations

Direct correlations have been derived using empirical data for specific studies pertaining to the lower shelf and lower transition temperature range for ferritic steels. Several proposed correlations are given between $U_{el+pl,LLD}$ and fracture toughness frequently denoted as a plane strain quantity, $K_{IC}$, but not clearly citing a plane strain fracture toughness test standard. The lack of a common definition of $K_{IC}$ with respect to the data used to develop the correlations increases uncertainty regarding the test standards adopted for individual studies. Many correlations relating to the lower shelf and lower transition of Charpy V-notch behaviour (Numbers 1-9 in Table 2-2) are based upon the following direct form:

$$K_{IC} = a(U_{el+pl,LLD})^b,$$

(2-21)

where,

- $K_{IC}$ = the critical stress intensity factor under mode I loading in units of MPa$\sqrt{m}$,
- $U_{el+pl,LLD}$ = absorbed Charpy V-notch load line energy in units of Joules,
- $a$ and $b$ = material constants derived from experimental data.

Figure 2-21 shows a comparison of some of the existing lower shelf empirical $K_{IC}$-$U_{el+pl,LLD}$ correlations, the lower bound correlation given by BS 7910 (2013) and also experimental data from Wulleart and Server (1980). The scatter of experimental data shows that direct lower shelf and lower transition correlations do not adequately describe the data.
2.5.2 **Empirical Upper Transition Temperature Range Correlations**

The form of expression employed for extending direct correlations to the upper transition have previously taken the form of Expression (2-22):

\[ K_{IC}^2 = a(U_{el+pl,LLD})^b, \]  

Expression (2-22) accounts for a different dependence of \( K_{IC} \) on \( U_{el+pl,LLD} \) in this temperature range for a given ferritic steel material.

2.5.3 **Transition Temperature Range Correlations**

A transition temperature correlation between the temperature corresponding to \( K_{IC}=100MPa/\sqrt{m} \), \( T_{K,100MPa/\sqrt{m}} \), and the 28 Joule Charpy V-notch transition temperature, \( T_{28J} \), was proposed by Sanz (1980). This worker observed an approximate material insensitivity to the temperature dependence of \( K_{IC} \) which enabled such a general correlation based on transition temperatures to be proposed as a preferable method for LEFM analyses. The correlation proposed by Sanz was the following:

\[ T_{K,100MPa/\sqrt{m}} = 1.38 \times T_{28J} - 9^\circ C \]  

Wallin (1989) amended the Sanz concept to include a modified transition temperature correlation using a one to one relationship between \( T_{K,100MPa/\sqrt{m}} \) and \( T_{28J} \) using nuclear parent plate, weld and irradiated materials for validation (Wallin, 1992) and allowing EPFM to be applied. The Wallin (1989) transition temperature correlation was the following:

\[ T_{K,100MPa/\sqrt{m}} = T_{28J} - 18^\circ C \]  

(2-24)
Table 2-2 Lower shelf and transition temperature range correlations between $U_{el+pl,LLD}$ (J) and $K_{IC}$ (MPa√m) for ferritic steels

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Correlation</th>
<th>$\sigma_y$ range (MPa)</th>
<th>$U_{el+pl,LLD}$ Range (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Girenko and Lyndin (1985)</td>
<td>$K_{IC} = 17(U_{el+pl,LLD})^{0.5}$</td>
<td>200-1700</td>
<td>2-150</td>
</tr>
<tr>
<td>2</td>
<td>Imai et al. (1970)</td>
<td>$K_{IC} = 2.5(U_{el+pl,LLD})^{0.5}$</td>
<td>Unknown</td>
<td>5-212</td>
</tr>
<tr>
<td>3</td>
<td>Logan and Crossland (1971)</td>
<td>$K_{IC} = 20.3(U_{el+pl,LLD})^{0.5}$</td>
<td>820-1700</td>
<td>7-37</td>
</tr>
<tr>
<td>4</td>
<td>Sailors and Corten (1973)</td>
<td>$K_{IC} = 14.6(U_{el+pl,LLD})^{0.5}$</td>
<td>410-815</td>
<td>7-70</td>
</tr>
<tr>
<td>5</td>
<td>Barsom and Rolfe (1970)</td>
<td>$(K_{IC})^2 = 45.1(U_{el+pl,LLD})^{1.5}$</td>
<td>270-1700</td>
<td>4-82</td>
</tr>
<tr>
<td>9</td>
<td>Thornby and Ferguson (1976)</td>
<td>$K_{IC} = 15(U_{el+pl,LLD})^{0.534}$</td>
<td>400-600</td>
<td>13-85</td>
</tr>
</tbody>
</table>

Figure 2-21 Variation of existing lower shelf and lower transition direct empirical correlations and the test data for a ferritic steel
Expression (2-24) enabled a relationship between $K_{IC}$ corresponding to a $B=25$mm, $K_{IC,B=25mm}$, and $T_{28J}$ to be established by Wallin for a given Temperature, $T \, (^{\circ}C)$, using a previously empirically observed common material fracture toughness temperature dependence (Wallin, 1991):

$$K_{IC,B=25mm} = 20 + \{11 + 77 \exp(0.019[T - T_{28J} + 18^{\circ}C])\}, \quad (2-25)$$

where, $K_{IC,B=25mm}$ = the linear elastic or elastic-plastic fracture toughness ($K_{IC}$) in units of MPa√m,

$T_{28J}$ in Expression (2-25) is subject to a standard deviation ($\Theta$), the value of $\Theta$ accounts for inaccuracies arising from the inherent practical and physical aspects of applying such a correlation in the presence of limited experimental data of Charpy V-for the temperature range of interest. The significant possibility of reliance upon incomplete transition curves when determining $T_{28J}$ using empirical data for different steel varieties must be viewed as an inherent weakness of the approach given the stochastic nature of brittle fracture (Lin et al., 1986). The present correlation was validated in Wallin (1992) using steels with $\sigma_Y$ varying from 300 to 1,000MPa (Figure 2-22). Figure (2-22) was based upon validation using elastic-plastic fracture mechanics testing test results ($K_{IC}$) and hence this author implied that Expression (2-25) can be applied to describe the temperature dependence of $K_{IC}$. The original correlation derivation had $\Theta=15^{\circ}C$ reported by Wallin (1989) and this work also was validated using a similar range of steels.

Expression (2-25) forms the second correlation for the lower transition temperature region provided in BS 7910. Further details of the terms in Exp. (2-25) are given in Section (2.6.2) in respect of the development of the master curve approach to transition fracture toughness. This correlation if premised upon standard fracture mechanics and Charpy impact test results (and a specimen size correction) as it relates
to ferritic steels and not stainless steels, therefore, in this sense it assumes a common reference temperature correlation exists regardless of material flow properties. This aspect of correlation inherently may introduce errors and a study of the correlation by Bannister (1998), using a range of structural steel data, suggests that the correlation may be subject to larger standard deviations for these materials than stated previously given by Wallin (1989). Figure (2-23) shows the analysis undertaken as part of the SINTAP program (Bannister, 1998). Figure (2-24) illustrates qualitatively the significant approximations necessary during the evaluation of $T_{27J}$ using limited experimental data. Therefore, it is possible that the main source of uncertainties of transition temperature correlations during the evaluation of the $T_{27J}$ values and hence the $\Theta=15^\circ$C of Wallin (1989) would appear to be an artefact of assumptions made during the definition of $T_{27J}$.

**Figure 2-22 Validation of Expression (2-24) transition temperature correlation given by Wallin (1992)***
$T_{27J}$ is a qualifying measure of Charpy impact toughness of steel components under steel product standards such as BS 10025 (2004) and ASTM A20 (2000). Typically these standards require exceedance of a minimum of $U_{el+pl,LLD}=27$ Joules at a particular temperature, and hence evaluation of $T_{27J}$ is likely to be necessary to lower temperatures than the temperature of the test result. Recent work by EricksonKirk et al (2007) proposed a Charpy V-notch curve shape. The implications and quantification of the statistical nature of brittle fracture in the lower ductile to brittle transition are discussed in Section 2.6.

![Figure 2-23 SINTAP analysis of Wallin (1989) correlation showing significantly more scatter than for nuclear grade steels (Bannister, 1998)](image-url)
Figure 2-24 Uncertainties inherent in defining $T_{27J}$ using limited experimental data

2.6 Assessment Methods for Cleavage Fracture of Ferritic Steels

2.6.1 Beremin Model

Landes and Shaffer (1980) used the Weibull distribution (Weibull, 1951) to analyse laboratory test data. This approach required calibration of the model to datasets of measured fracture toughness from laboratory tests and as such was not based on microstructural measurements. The Beremin group (Beremin, 1983) improved this approach by using a two parameter Weibull distribution, calibrated with experimental data, and performing finite element analysis (continuum mechanics) of the experimental geometries. The methodology of these workers assumed the spatial distribution of tensile principal stress within plastically deformed material in the fracture process zone and the size of particles in the fracture process zone at a geometric defect to be the two determinates of macroscopic fracture by cleavage. The main variables of such a statistical approach are shown in Figure 2-25.
The two parameter Weibull statistical distribution was expressed by Beremin (1983) as the following:

$$F(\sigma_w) = 1 - \exp \left( -\left( \frac{\sigma_w}{\sigma_u} \right)^m \right)$$ \hspace{1cm} (2-26)

where, $F(\sigma_w)$ = the cumulative probability of failure by cleavage,

$\sigma_w$ = the Weibull stress,

$\sigma_u$ = the Weibull scale parameter corresponding to a failure probability of 63.2%,

$m$ = the Weibull shape parameter.

The measure used by Beremin (1983) to quantify cleavage fracture toughness in Expression (2-26) was named the Weibull Stress, $\sigma_w$. This measure was obtained by post-processing the finite element calculation results of laboratory tests on notched or pre-cracked specimens. The Weibull stress was determined by Beremin by performing...
an integration over the crack-tip or notch-tip region of plasticity using the following expression:

\[
\sigma_w = \left[ \frac{1}{\lambda_0} \int \sigma_1^m d\lambda \right]^{1/m}
\]  \hspace{1cm} (2-27)

where, \( \lambda_0 \) = reference volume commonly set at 1mm\(^2\) (Gao et al, 1999),

\( \lambda \) = the volume of plastically deformed fracture process zone,

\( \sigma_1 \) = the maximum principal stress in a volumetric element of the fracture process zone.

Work of Gao, Ruggieri and Dodds (1998) has shown that the Weibull stress can be used to calibrate between two datasets of fracture data corresponding to high and low constraint. Horn and Sherry (2010) applied this procedure to notched and cracked geometries. The Beremin (1983) model considers only the maximum principal stress from finite element analyses of geometries where the material is assumed homogenous. There is scope for application of the Beremin model to a crack or notch propagating by a ductile mechanism, the results of such an analysis are to be found in Xia and Shih (1996).

2.6.2  \textbf{ASTM E1921-11 Cleavage Fracture Model}

Wallin (1989) proposed a correlation between \( T_{28J_c} \) and a reference temperature for which \( K_c=100\text{MPa}\sqrt{\text{m}} \) denoted as \( T_0 \) in ASTM E1921-11a (ASTM, 2011b) as described in Section 2.5.3. The \( T_0 \) reference temperature forms a means by which the temperature dependence of fracture toughness, size effects and scatter in the transition temperature range may be analysed using a statistical approach.
Wallin (1984) proposed a cleavage fracture model applicable to ferritic steels to address the innate scatter using a three parameter Weibull distribution to describe the cumulative probability of failure, $P_f$:

$$P_f (K_i) = 1 - \exp \left[ - \left( \frac{K_i - K_{\text{min}}}{K_0 - K_{\text{min}}} \right)^b \right]$$ (2-28)

where,

- $K_i = \text{the plane strain fracture toughness},$
- $K_0 = \text{the value of } K_i \text{ at } P_f=0.63,$
- $K_{\text{min}} = \text{a minimum value of } K_i \text{ below which cleavage crack propagation is impossible, equal to } 20\text{MPa}\sqrt{\text{m}},$
- $b = \text{the slope of the Weibull distribution equal to 4}.$

In Expression (2-28) the value of $b$ was asserted to be equal to four by Wallin (1984), when using the three parameter Weibull distribution, but there was a significant variation of $b$ values determined using small data sets. When defining $K_0$ it was apparent that to achieve a 90% confidence level of +/-10% error of predictions on $K_0$ that approximately 20 fracture toughness tests would be necessary. A thickness correction was proposed by Wallin (1985) for use under cleavage fracture conditions, the current ASTM E1921-11 (2011b) standard requires the user to correct input $K_{Jc}$ data to a standard crack front length, $B_0=25\text{mm}$ using the following:

$$K_{25\text{mm}} = K_{\text{min}} + \left( K_{Jc} - K_{\text{min}} \right) \left( \frac{B}{B_0} \right)^{0.25}$$ (2-29)

where,

- $B = \text{the thickness of the fracture toughness specimen from which the } K_{Jc} \text{ value was measured}.$
The temperature dependence of $K_{jc}$ corresponding to $P_f=0.5$ was proposed to follow a common temperature dependence according to Wallin (1991):

$$K_{jc} = 31 + 77 \exp\left\{0.019(T - T_0)\right\}$$

(2-30)

where, $T_0$ is the reference temperature at which the median $K_j$ value is equal to 100MPa√m.

Expression (2-29) may be applied to $K_j$ data in the temperature regime when failure of ferritic steels is by a transgranular cleavage mechanism. Size limitations are placed upon specimen dimensions to ensure that the physical assumption of the ASTM E1921-11a (2011b) model are satisfied, but permitting measurements of $K_j$ in the ductile-to-brittle transition temperature range. The current size limit specified in ASTM E 1921-11a for the maximum valid $K_{jc}$ measurements, $K_{JC,Limit}$, is the following, corresponding to plane strain conditions:

$$K_{jc,Limit} \leq \sqrt{\frac{Eb_0\sigma_y}{30(1-\nu^2)}}$$

(2-31)

where, $b_0$ = the initial ligament length

$E$ = Young’s modulus of elasticity,

$\sigma_y$ = the material yield strength (often prescribed at 0.2% plastic strain).

The constant of 30 in expression (2-30) is the deformation limit constant and results in smaller specimens being required to produced valid $J_c$ measurements for use with the ASTM E 1921-97 (1997) than ASTM E1820-98 (1998) provided the material strain
hardening exponent is greater than 4.4 (derived from a relationship between $n$, ultimate strength ($\sigma_{\text{UTS}}$) and $\sigma_Y$) as shown in Figure 2-26 (Server et al, 2000).

![Figure 2-26 Differences of size requirements between ASTM E1921-11a and ASTM 1820-98 (Server et al, 2000)](image)

Expressions (2-27), (2-28) and (2-29) may be combined to permit prediction of the material fracture toughness, $K_{\text{mat}}$, which is the value of $K_{\text{lc}}$ value corresponding to a $B=25\text{mm}$ fracture toughness specimen at a predefined $P_f$ at any assessment temperature in the ductile-to-brittle transition:

$$K_{\text{mat}} = 20 + \left\{11 + 77 \exp\left[0.019\left(T - T_0\right)\right]\right\}^{0.25} \left\{\ln\left(\frac{1}{1 - P_f}\right)\right\}^{0.25}$$

(2-32)

where, $T_0$ = the transition temperature range reference temperature calculated using ‘size corrected’ $K_J$ laboratory test data and ASTM E 1921-11a (2011).

The variation of fracture toughness $K_0$ as a function of normalised temperature $(T-T_0)$ and probabilistic predictions of $K_{\text{lc}}$ the model represented by Expression (2-32) are shown for a nuclear grade steel in Figure (2-27) from Wallin (1993).
Figure 2.27 Showing, a.) the variation of $K_0$ with $(T-T_0)$ for a range of ferritic steels materials and b.) the cleavage fracture model including probabilistic predictions (Wallin, 1993)
$K_{mat}$ forms an integral part of assessment procedures for steel structures using BS 7910:2013 (BSI, 2013) and R6 (British Energy, 2005) and hence the accuracy of fracture toughness measurements are of high importance to these endeavours.

2.6.3 **SE(B) Specimen Cleavage Fracture Behaviour**

Sorem *et al* (1991) studied the behaviour of deep and shallow crack SE(B) specimens of $a/W=0.5$ and $0.15$ respectively. They used three dimensional numerical analyses to determine a relationship between CMOD and CTOD and also analysed the calculated laboratory test CTOD values at cleavage fracture using the Weibull distribution. Figure 2-28 shows the results of the Sorem *et al* analyses work.

![Graph showing CTOD values for SE(B) specimens](image)

**Figure 2-28 The effect of crack depth cleavage fracture CTOD values and analysis of Sorem *et al* (1991)**

The SE(B) specimen of $a/W=0.15$ exhibited significantly larger CTOD values ($\approx 2.0-2.5$ times) in comparison to the SE(B) specimen of $a/W=0.5$ at failure at three temperatures in the lower transition regime of a structural steel. The increased crack driving force measurements were attributed to the increased presence of plasticity at the tension
face of the specimen. The deep crack specimens retained contained plasticity, at lower
shelf and higher temperatures in the transition temperature range, the increased CTOD
values for the shallow crack specimens was attributed to loss of stress triaxiality owing
to plastic hinge formation near the tension face of such specimen geometries.

Further work on the effect of constraint on laboratory specimen test results were
used to quantify the effect of constraint loss on measured fracture toughness. Betegon
and Hancock (1991) and Al Ani and Hancock (1991) suggested that the elastic T-stress,
$T$, may be an appropriate parameter to quantify loss of constraint. $T$ represents the
second term in the Williams expansion [Expression (2-7)] and is an elastic stress acting
parallel to the crack-tip (Rice, 1974). Hancock and co-workers used elastic-plastic
stress field analysis ahead of specimen crack tips and modified boundary layer analysis
to study the influence of $T$ on plane strain crack-tip stress fields. Figure 2-29a, from Kirk
et al (1993), shows that the T-stress indexed correctly the experimentally measured
phenomenon of increasing $J_c$ with increasingly compressive $T$ values (consistent with
values of $T$ for shallow crack SE(B) specimens). Such research suggested that the
measured $J_c$ at cleavage fracture was controlled by a second parameter under
conditions of low constraint in addition to $J$ and also that $T$ was potentially an
appropriate parameter. $T$ was related to a biaxiality parameter, $\beta$, by Al Ani and
Hancock (1991):

$$\beta \leq \frac{T \sqrt{\pi a}}{K},$$  \hspace{1cm} (2-33)

where, $T$ = the elastic T-stress,
$K$ = the linear elastic stress intensity factor,

Kirk et al provided the an expression for $\beta$ for three point bend SE(B) specimens which
is given by Expression (2-34).
\[
\beta(\frac{a}{W}) = -0.462 + 0.461 \left( \frac{a}{W} \right) + 2.47 \left( \frac{a}{W} \right)^2,
\]

(2-34)

Expression (2-34) is valid for a large range of crack depths: \(0.025 \leq \frac{a}{W} \leq 0.90\).

Figure 2-29 The effect of crack depth on \(J_c\) at cleavage fracture using a.) \(T\) and b.) \(Q\) (Kirk et al, 1993)

Hancock et al (1993) showed the effect of large deformation on crack tip fields and the influence of the T-stress on the amplitude of these fields using plane strain analyses, these results are reproduced in Figure 2-30.
The $Q$ parameter, $Q$, introduced by O’Dowd and Shih (1991), represented the difference field between the $HRR$ field and calculated stress fields ahead of low constraint crack-tips. Expression (2-35) provides the definition of $Q$. This elastic-plastic parameter can be calculated for specific specimen geometries using numerical analysis and used to quantify loss of constraint, as shown in Figures 2-29b and 2-31 respectively.

Figure 2-30 Triaxial stress distribution plane strain FE analysis and affect of compressive T stresses by Hancock et al (1993)

\[
\frac{\sigma_{ij}}{\sigma_0} = \left( \frac{J}{\alpha \sigma_0^2 I_n r} \right)^{n+1} \tilde{\sigma}_{ij}(n, \theta) + Q \left( \frac{r}{J / \sigma_0} \right)^q \tilde{\sigma}_{ij}(n, \theta)
\]

(2-35)

where, $Q = a$ normalised hydrostatic term,

and, $q = a$ constant, equal to zero for $\theta \leq 90^\circ$. 

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Expression (2-34) is valid for $|\theta| \leq 90^\circ$ and outside of the region of finite strain deformation ($r > J/\sigma_0$). Shih et al (1993) calculated $Q$ for a range of loading levels by undertaking FE analyses and evaluated $Q$ at a common normalised distance which was equal to $\frac{r\sigma_0}{J} = 2$ so as to remove the effects of crack-tip finite deformations on the results. The $Q$ parameter may vary with normalised position ahead of the crack tip for deeply cracked SE(B) specimens.

![Figure 2-31 Q stress-loading relationships for three point bend specimens](Image)

**Figure 2-31 Q stress-loading relationships for three point bend specimens**

[Shih et al (1993)]

Hancock et al (1993) also studied the applicability of $T$ to quantify crack tip constraint under fully plastic conditions and found that this parameter provided reliable ‘ordering’ of the constraint range encompassed by the experimental geometries studied. The T-stress and Q-stress approaches did not provide a microstructural basis to describe the fracture event. Therefore, fracture toughness testing of different geometries was still necessary to quantify the effect of constraint loss on $J_c$ for ferritic steels.
2.6.4 **Constraint Based Fracture Mechanics Assessments**

R6 (2005) provides details concerning fracture mechanics assessments of structures containing defects to assess their structural integrity. Material fracture toughness, $K_{\text{mat}}$, is used as an input for such assessments and is required to correspond to the conditions under which the structure operates. Under conditions when a defect in a structure is under low constraint (low triaxiality or absence of contained plasticity), such as the case of shallow defects arising in a pressure vessel or pipe wall, then constraint based approaches for quantifying a constraint modified $K_{\text{mat}}$ value for a particular material, $K_{\text{mat,c}}$ have been developed. Ainsworth (1995) proposed that $T$ be related to the R6 plastic collapse parameter $L_r$ by the following:

$$\frac{T}{\sigma_Y} = \beta_T L_r,$$

(2-36)

where, $\sigma_Y$ = the material yield stress,

and, $L_r = \frac{P}{P_L}$.

The limit load, $P_L$, is calculated corresponding to a given geometric configuration with crack size ($a$) and $\sigma_Y$ using an elastic-perfectly-plastic analysis. Likewise, $Q$ was related to $L_r$ in a similar manner:

$$Q = \beta_Q L_r,$$

(2-37)

$\beta_Q$ in Expression (2-37) is a function of geometry loading mode and material flow properties. A constraint modified fracture toughness, $K_{\text{mat,c}}$ was therefore defined:

$$K_{\text{mat,c}} = K_{\text{mat}} \left[ 1 + \alpha(-\beta L_r)^p \right],$$

(2-38)
Within Expression (2-38), \( \beta \) can be equal to \( \beta_1 \) or \( \beta_0 \) for a specific geometry, and \( \alpha \) and \( p \) are material constants.

Fracture occurs when \( K > K_{\text{mat,c}} \), and hence the typical failure assessment diagram material failure curve, defined according to \( f(L_r) \), was modified to allow a constraint dependent material failure curve, \( f_c(L_r) \), to be defined by Expression (2-39).

\[
K_r = \frac{K}{K_{\text{mat}}} \leq f_c(L_r) = f(L_r) \left[1 + \alpha (\beta L_r)^p \right],
\]

(2-39)

\( f_c(L_r) \) curves for different using a value of \( p=2 \) are shown for different values of \( \beta \) in Figure 2-32.

2.6.5 Notch Based Fracture Mechanics Assessments

The effect of a finite radius, \( \rho \), at the tip of a defect on measured apparent elastic-plastic fracture toughness, \( K_{\text{mat,\rho}} \), exhibited by such defects was quantified by Horn and Sherry (2012). These authors used a Weibull stress approach (Section 2.6.1) to conduct analyses of plane strain boundary layer models and applied a fracture toughness scaling method to quantify \( K_{\text{mat,\rho}} \) in relation to the standardised high constraint \( K_{\text{mat}} \) value. An elastic parameter, \( \sigma_N \), was used to index constraint for different \( \rho/a \) values. \( \sigma_N \) represented the elastic mode I opening stress at the tip of a notch. The relationship used between \( \sigma_N \) and the collapse parameter \( L_{r,\rho} \) is given in Expression (2-40).

\[
\frac{L_{r,\rho}}{\beta_N} = \frac{\sigma_N}{\sigma_y},
\]

(2-40)

where, \( L_{r,\rho} = \) the load ratio corresponding to the geometry of interest, 
\( \beta_N = \) a geometric parameter.

A relationship was developed to describe the experimentally observed elevation of \( K_{\text{mat,\rho}} \) with increasing \( \rho \) (and constant \( a/W \)). The \( (K_{\text{mat,\rho}}/K_{\text{mat}},\sigma_N) \) relationship was found to be
relatively independent of a wide range of $\sigma_Y$ material property definitions, but the relationship was influenced by the value of the strain hardening parameter $n$ (Expression 2-14). The results of Horn and Sherry (2012) using a Weibull stress model ‘$m$’ parameter equal to 20 are shown in Figure 2-33.

\[
\frac{OB}{OD} = \left[1 + \alpha(-\beta L_r)^{\rho}\right]
\]

$OD\ AC =$ Apparent $K_Y$ constraint assessment allowance for a specific test geometry and material

$CB =$ Associated $L_r$ constraint assessment allowance

$AC =$ Apparent $K_Y$ constraint assessment allowance for a specific test geometry and material

Figure 2-32 Constraint modified failure assessment diagram given by Ainsworth (1995)
Increasing notch $\rho$

Assumption:

$\rho=0\,\text{mm}$,

$\frac{\sigma_N}{\sigma_Y}=30$

$K_{mat,\rho}/K_{mat}=1$

Specimen A:

Cleavage fracture predicted for $n=6$ material

Figure 2-33 The effect of finite notch root radius on $\frac{K_{mat,\rho}}{K_{mat}}$ predicted at cleavage fracture using $\sigma_N$ (Horn and Sherry, 2012)

2.7 Dynamic Mechanical Properties of Ferritic Steels

2.7.1 Tensile Properties

Cambell and Ferguson (1970) showed that the yield strength of mild steels was significantly affected by the strain rate imposed upon the material. These authors used shear test laboratory experiments and provided one of the first systematic studies concerning the flow properties of ferritic steels. Figure 2-34 shows a steel material’s yield stress variation with strain rate and temperature given by Cambell and Ferguson.

Figure 2-34 shows that a linear relationship was found to exist between strain rate and yield stress over a significant part of the strain rate spectrum for mild steels. This behaviour was found to best describe the behaviour of steels at low temperatures.
Figure 2-34 The strain rate dependence of lower yield stress of mild steels as determined by Cambell and Ferguson (1970)

2.8 Empirically Observed Charpy Impact Test Transition Curve Shape

The main difficulty encountered by engineers using Charpy impact data is the non-transferability of the Charpy V-notch $U_{el+pl,LLD}$ measurements to fracture toughness properties and therefore the lack of a direct relationship between the transition temperature range for Charpy V-notch data and fracture toughness data. The lack of a mechanistic correlation between Charpy V-notch data and fracture toughness for a steel material means that such data are only used in structural integrity assessment procedures such as ASTM E1921-11 (2011b) to estimate the value of $T_0$ within the transition region of fracture toughness behaviour. Fracture toughness testing is still necessary to accurately determine more precisely the transition temperature range.
Recent work by EricksonKirk et al (2007) has proposed a common temperature dependence for $U_{el+pl,LLD}$ using the United States Nuclear Regulatory Commission (USNRC) database of RPV steel products in the irradiated and unirradiated conditions. This work focussed on the lower transition of Charpy fracture behaviour and eliminated all data which exhibited greater than 60% ductile fracture appearance. The partitioned $U_{el+pl,LLD}$ data was plotted against a normalised variate $(T - T_{CVE})$, where $T_{CVE}$ represented the temperature at which $U_{el+pl,LLD}$ was equal to 28 Joules. The data of the USNRC is shown for the transition region of temperature dependence in Figure 2-35 for partitioned data.

Figure 2-35 Analysis results from EricksonKirk et al (2007) using partitioned Charpy data and a normalised temperature axis
An exponential curve was fitted to the Charpy V-notch specimen $U_{el+pl,LLD}$ vs. $(T - T_{CVE})$ data by EricksonKirk et al (2007) using the following expression:

$$U_{el+pl,LLD} = 28\{\exp[0.0245(T - T_{CVE})]\}$$  \hspace{1cm} (2.41)

where, $T_{CVE} = \text{the temperature at which the mean measured } U_{el+pl,LLD} \text{ equals } 28 \text{ Joules.}$

2.9 Knowledge Gaps

The knowledge gaps exposed through conduct of a literature review were the following:

1. The uncertainty inherent to Charpy transition curve data and definition of an appropriate reference temperature within this regime of fracture behaviour. Empirical correlations for $U_{el+pl,LLD}$ data to transfer to the material fracture toughness properties of ferritic steels are not sufficient to determine the safety of industrial structures because of this uncertainty.

2. The possibility of numerically quantifying the mechanical differences between the dynamic Charpy V-notch impact test and quasi-static precracked SE(B) specimens at identical temperatures in the transition regime. The efficacy of the Beremin (1983) model to quantify fracture behaviour for cleavage fracture conditions is evident from literature. There is less evidence of its effectiveness for characterising fracture behaviour at temperatures approaching the structurally relevent, $T_b$, ($K_{IC}=100\text{MPa}\cdot\sqrt{m}$) or at $K_J>100\text{MPa}\cdot\sqrt{m}$ where ductile tearing contributes significantly prior to the onset of cleavage fracture. Therefore, a knowledge gap for predicting fracture toughness from Charpy V-notch data exists in the lower-middle and middle transition. By solving this issue
it will be possible to accurately calculate a Charpy V-notch transition curve for
the study material and accompanying fracture toughness prediction curve. This
later numerically generated data can then be validated against the existing
master curve methodology.

3. At the point of applying the correlation to different steels, the extent of parameter
calibration required is not established from existing applications of the Beremin
(1983) model. Therefore, it is necessary to investigate further the possibility of
obtaining a relationship between the Beremin (1983) shape parameter, $m$, and
material properties. A decision needs to be made concerning the completion of
the correlation application to other ferritic steels by this approach or to develop a
relationship between $m$ and microstructural quantities which are readily
available.

2.10 Aims

To develop a mechanistically based procedure for positioning the master curve,
ASTM E1921-11 (2011b), using six $U_{el+pl,LLD}$ data generated in the transition regime of
fracture behaviour.

2.11 Objectives

- Quantify the geometric differences between the SE(B) and the Charpy V-notch
  specimen by experimental testing and numerical modelling.

- Use the Beremin model to quantify the mechanical differences between the
  Charpy impact test and the SE(B) test.
• Form a correlation between Charpy V-notch impact energy and material fracture toughness, $K_{\text{mat}}$, using the Beremin model and other appropriate mechanistic modelling and experimental test results.

• Verify the model and take appropriate actions to achieve the accuracy of the correlation for ferritic steels of structural and nuclear quality.

• Appraise the correlation in view of existing empirical correlations between $U_{\text{el+pl,LLD}}$ and $K_{\text{mat}}$ and develop a procedure to complement the master curve methodology for ductile to brittle transition regime fracture behaviour.
2.12 References


3. Methodology

3.1 Materials Characterisation

3.1.1 Study Material

A ferritic steel study material was used for this work. The steel was previously used for research by Horn and Sherry (2010). The steel used was originally a 35mm thick quenched and tempered steel of classification grade S690Q according to BS 100025-6 (BSI, 2004) which possessed a tempered martensite microstructure, the chemical composition of the steel is provided in Section 4.1. The steel was subsequently heat treated by Horn and Sherry (2010) to achieve a microstructure which exhibited cleavage fracture behaviour at ambient temperatures because of an increase of the ductile-to-brittle transition temperature (the 27 Joule Charpy impact test temperature, $T_{27J}$). This heat treatment consisted of austenitising the material at 1,100°C for 2½ hours then furnace cooling to 650°C and air cooling to room temperature. The micro-structure consisted of coarse ferrite, pearlite and bainite outcrops.

3.1.2 Quasi-Static Tensile Testing

Quasi-static tensile testing of the ferritic steel study material was undertaken using tensile round bar specimens. These specimens were orientated in the longitudinal direction and extracted at a position 3.25mm sub-surface of the steel plate. The tensile specimens had a gauge diameter ($d$) of 11.3mm and gauge length ($l$) of 70mm. The loading rate for this testing was set to achieve a strain rate of $\dot{\varepsilon}=10^{-5}$/s over the strain range at which uniform plastic flow occurred in accordance with the BS10002-1 (BSI, 2001) test standard at the test temperature of $T=0^\circ$C. Temperature of the test specimen was controlled using an insulated chamber and controlled quantities of liquid nitrogen and a thermo-couple attached to the gauge section.
Engineering strain ($e$) was measured using an attachable extensometer and engineering stress ($s$) determined from the machine load cell output recorded during the test. Analysis work was undertaken, the following activities were completed:

- Conversion of the test data from engineering definitions of stress and strain to true stress and strain,
- Curve fitting of a power law hardening expression suitable for the description of the strain hardening behaviour of ferritic steels.

The following formulae were used to convert engineering stress and strain ($s$ and $e$) to true stress and strain ($\sigma$ and $\varepsilon$):

\[
\varepsilon = \ln(1+e), \quad (3-1)
\]

and,

\[
\sigma = s(1+e), \quad (3-2)
\]

The study material tensile properties were curve fitted using a power-law hardening $\sigma$ vs. $\varepsilon$ relationship (Expression 3-3) and one proposed by Ramberg and Osgood (1943) (Expression 3-4) for the purpose of numerical modelling activities.

Curve fitting was undertaken over the strain range encompassing the initial strain hardening to a strain level preceding the onset of necking. A least squares fitting procedure was used to determine the value of $n$ for use within Expressions (3-3) and (3-4). The value of $\sigma_0$ was also adjusted following definition of $n$.

Figure 3-1 shows the curve fitting method used to represent the tensile test data using Expressions (3-3) and (3-4).
True Strain = \ln(1+e) \\

True Stress = s(1+e) (MPa) \\

\begin{align*}
\frac{\varepsilon}{\varepsilon_0} &= \alpha \left( \frac{\sigma}{\sigma_0} \right)^n, \\
\frac{\varepsilon}{\varepsilon_0} &= \frac{\sigma}{\sigma_0} + A \left( \frac{\sigma}{\sigma_0} \right)^n,
\end{align*}

\text{(3-3)} \quad \text{(3-4)}

where, \\
\varepsilon_0 = \text{a reference strain corresponding to } \sigma_0/E, \\
\sigma_0 = \text{a reference stress}, \\
\alpha, \text{ and } A = \text{constants derived from curve fitting}, \\
\text{n} = \text{the strain hardening exponent derived from curve fitting.}
3.1.3 Dynamic Tensile Testing

Dynamic tensile testing was undertaken using the study material. The samples were all extracted from the study material plate in the longitudinal orientation at a position 3.25mm sub-surface. Flat specimens with a gauge section which had dimensions of length=25mm, width=10mm and thickness=2mm, the specimens were electro-discharge machined (EDM) to a tolerance of 0.01mm. Figure 3-2 shows the geometry of the specimens used for the dynamic tensile tests. All specimen gauge dimensions were measured using a digital micrometer prior to testing. Load was measured during each test using the tensile test machine’s load cell for the lower strain rate tests. At higher strain rates the specimen reaction force was acquired using individually calibrated strain gauges in a half bridge arrangement on the grip section of the specimen (Figure 3-2). Displacement of the specimen gauge section was measured using digital image correlation (DIC) for all tests to provide a continuous recording of the strain behaviour of the specimens. The DIC measurements were verified using an attachable extensometer for the lowest strain rate tests. Table 3-1 provides information concerning the dynamic tensile testing strain rates, load acquisition and number of specimens tested. Material stress-strain properties over a range of strain rates (0.001<\dot{\varepsilon}\leq200/s) were measured. Servo-hydraulic machines were used to achieve the required range of strain rate when testing flat tensile samples of the study material. All testing was undertaken at T=0°C. Temperature was controlled using an environmental chamber and thermo-couple attached to the specimen gauge section to within 1°C of the target temperature and stabilised for five minutes prior to each test. A laser trigger was used to control data acquisition at the highest strain rates.
Figure 3-2 Geometry of tensile specimen used for dynamic testing

Table 3-1 Dynamic tensile testing programme

<table>
<thead>
<tr>
<th>$\dot{\varepsilon}$ (s$^{-1}$)</th>
<th>Load measurement</th>
<th>No. specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>Load cell</td>
<td>2</td>
</tr>
<tr>
<td>0.1</td>
<td>Load cell</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>Strain gauges</td>
<td>2</td>
</tr>
<tr>
<td>200</td>
<td>Strain gauges</td>
<td>2</td>
</tr>
</tbody>
</table>

Analysis of the material property parameters derived from curve fitting the experimental dynamic tensile test results was undertaken to quantify the effect upon the power law reference stress $\sigma_0$ in Expression (3-3).

Cowper and Symonds (1957) proposed the following form of expression to describe the dynamic tensile yield stress behaviour of steels:

$$\frac{\sigma_d}{\sigma_s} = 1 + \left( \frac{\dot{\varepsilon}}{D} \right)^{\frac{1}{q}},$$  \hspace{1cm} (3-5)

where,  
\begin{align*}
\sigma_d &= \text{the dynamic yield stress}, \\
\sigma_s &= \text{the quasi-static yield stress}, \\
D, q &= \text{material constants describing the material’s strain rate sensitivity}.
\end{align*}
The material parameter $\sigma_0$, results (corresponding to individual power-law curve fits to tests at a specific $\dot{\varepsilon}$) were used to determine the values of $D$ and $q$ in Expression (3-5) for the study material. Figure 3-3 illustrates the least squares fitting method used to determine $D$ and $q$.

$$\ln\left(\frac{\sigma_0'}{\sigma_0} \right) = 1/q \ln(\dot{\varepsilon}) - Y$$

$$D (\text{s}) = \exp(Yq)$$

$$\sigma_0'/\sigma_0 (\dot{\varepsilon}=D) = 2$$

**Figure 3-3 Method for determining strain rate dependent material yield properties using Expression (3-5)**

A dynamic reference stress, $\sigma_0'$, was therefore defined for use within Expression (3-3) in place of $\sigma_0$ under dynamic loading conditions:

$$\frac{\sigma_0'}{\sigma_0} = 1 + \left( \frac{\dot{\varepsilon}}{D} \right)^q. \quad (3-6)$$

Expression (3-6) and the calibrated parameters were used to extrapolate material flow properties, defined according to Expression (3-3), to $\dot{\varepsilon}=2,000$/s. Analysis of the relationship between material strain hardening behaviour and $\dot{\varepsilon}$ was undertaken, a
least squares linear fit (logarithmic in strain rate) was used to extrapolate the $n$ material parameter, corresponding to the curve fits using Expression (3-3), as shown in Figure 3-4.

![Graph showing the relationship between $n$, $\log(\dot{\varepsilon})$, and $W$. The graph includes a line equation $n = p[\log(\dot{\varepsilon})] + W$.]

Figure 3-4 Method used to extrapolate $n$ in Expression (3-3) to $\dot{\varepsilon} = 2,000$/s

3.1.4 Fracture Toughness

Fracture toughness testing of the study material was undertaken in accordance with the ASTM E 1820-11 (2011a) methodology using sixteen single edge notch bend SE(B) specimens. All specimens were machined at a position 3.25mm below the surface of the steel plate and orientated in the longitudinal direction, the notches were machined through the plate thickness (all SE(B) specimens were of L-T orientation). All specimens had dimensions of thickness ($B$) and width ($W$) equal to 12.5mm (square cross-section) and a 4.4mm fatigue starter notch. All specimens were fatigue pre-cracked at room temperature to a nominal crack depth ratio of $a/W = 0.5$. The loading on
the specimen during fatigue pre-cracking was an initial maximum load of 3.5kN and final maximum load of 2kN. The initial and final stress intensity factors were 25 and 15MPa√m respectively.

The test temperatures were 22°C and 40°C respectively. Crack mouth opening displacement (\(CMOD\)) was measured continuously during each test using a clip gauge mounted onto integral knife edges. An environmental chamber was used to achieve a constant test temperature. The specimens were subject to the test temperature for a minimum of twelve minutes prior to each test. The initial loading and displacement rates during testing in the elastic loading range were 0.25-0.3kN/s and 0.013-0.015mm/s, respectively. The initial length of fatigue pre-crack \(a_0\) was measured following each specimen fracture using a travelling microscope, nine equally spaced \(a_0\) measurements were taken along the crack front.

The measured \(J\) corresponding to the point of cleavage initiation of each test was calculated using the partitioned expression of ASTM E 1820-11 (2011a). Each value of \(J\) at cleavage fracture was converted to an equivalent critical elastic-plastic stress intensity factor, \(K_{Jc}\), using the following plane strain formula:

\[
K_{Jc} = \sqrt{\frac{EJ_c}{(1 - \nu^2)}}
\]  

(3-7)

The fracture toughness data were analysed using the master curve methodology for ferritic steels (ASTM, 2011b). This approach allowed a size corrected (1T specimen size) material fracture toughness, \(K_{mat}\), to be established (corresponding to a predefined failure probability, \(P_f\)) for temperatures corresponding to the lower ductile-to-brittle transition for the study material. The expression used for calculating \(K_{mat}\) was that given by the ASTM standard E1921-11a (ASTM, 2011b) as the following:
where, \( P_f \) = the probability of not exceeding of \( K_{mat} \) at the test temperature,
\( T \) = the test temperature,
\( B \) = specimen thickness and crack front length,
\( T_0 \) = the ASTM E 1921-11 (2011) reference temperature corresponding to a 1T specimen of \( B=25.4\text{mm} \).

3.2 SE(B) Specimen Cleavage Fracture Test Programme

An experimental test programme was undertaken for the purpose of investigating the cleavage fracture behaviour of standard fatigue-pre-cracked SE(B), non-standard U-notch SE(B) and Charpy V-notch specimens in quasi-static three point bending. All specimens were of Charpy specimen outline size, \( B=10\text{mm} \), \( W=10\text{mm} \) and \( L=55\text{mm} \), \( S=4W \) (also referred to 0.4T size when following the notation of ASTM E 1820-11 (2011a). All specimens were machined from 3.25mm below the plate surface in the longitudinal orientation and fatigue starter notches (or machined U and V-notches) were introduced through the plate thickness (L-T orientation). The fatigue starter notches, U-notches and V-notches were machined using EDM. Fatigue pre-cracking of the 0.4T SE(B) specimens was undertaken according to the requirements of ASTM E 1820-11 (2011a) with a maximum and minimum stress intensity factor, \( K \), of 25 and 15MPa\(\sqrt{\text{m}} \) the R-ratio was 0.1 during this test preparation activity.

3.2.1 Specimen Geometries

The specimen geometries were machined with different notch acuity (\( \rho \)), initial notch depth (\( a_0 \)) and notch flank angle (\( \phi \)) to investigate the effect of individual
geometric changes on cleavage fracture behaviour. The range of specimen geometries were devised for the purpose of characterising differences between a fatigue pre-cracked SE(B) \((a/W=0.5)\) specimen and a standard Charpy V-notch specimen.

To achieve this objective, quasi-static testing was undertaken on sixty three point bend specimens at a common cross-head displacement rate of 0.007mm/s. The final test programme incorporated the standard SE(B) fracture toughness specimen and Charpy V-notch specimen. Between these bounding geometries of interest, four intermediate geometries were also tested. Figure 3-5 shows the dimensions of the standard SE(B), non-standard U-notch, non-standard V-notch and Charpy V-notch specimens which were experimentally tested. The geometric attributes of all experimental test geometries and numbers of specimens tested are listed in Table 3-2.

![Test specimen geometries](image)

**Figure 3-5 Test specimen geometries used for the experimental investigation using standard and non-standard SE(B) specimens at a quasi-static loading rate**
Table 3-2  Experimental test programme details for SE(B) specimens at a quasi-static loading rate

<table>
<thead>
<tr>
<th>Specimen Geometry</th>
<th>$a/W$</th>
<th>$\rho$ (mm)</th>
<th>$\phi$ (˚)</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deep pre-crack SE(B)</td>
<td>0.5</td>
<td>0.0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Deep U-notch SE(B)</td>
<td>0.5</td>
<td>0.127</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Shallow U-notch SE(B)</td>
<td>0.2</td>
<td>0.125</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Shallow U-notch SE(B)</td>
<td>0.2</td>
<td>0.25</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Shallow V-notch SE(B)</td>
<td>0.2</td>
<td>0.25</td>
<td>22</td>
<td>10</td>
</tr>
<tr>
<td>Standard Charpy V-notch</td>
<td>0.2</td>
<td>0.25</td>
<td>45</td>
<td>10</td>
</tr>
</tbody>
</table>

3.2.2  Quasi-Static Test Procedure for all Geometries

All specimens described in Section 3.2.1 were tested in three point bending with a span, $S$, equal to $4W$ (in common with common fracture toughness test standards (ASTM, 2011a) and the Charpy impact test standards: BS 10045 (BSI, 1990) and ASTM E 23 (ASTM, 2002). A clip gauge was mounted onto knife edges which were attached onto the notched face of each specimen either side of the notch. The general arrangement of the specimen knife edges for this test programme is shown in Figure 3-6. The knife edge heights and distance from the crack/notch centre line were all measured prior to the experiments using an optical microscope. Each knife edge was a horizontal distance of 2.5mm +/- 0.5mm from the crack/notch centreline of the specimens. The vertical height of each knife edge was 3.0mm +/-0.1mm.

The clip gauge was used to measure notch opening continuously during each test at the knife edge position. All testing was conducted at a test temperature of $\theta=0˚C$ by immersing each specimen in an environmental chamber prior to each test commencing, the temperature was controlled to within 0.5˚C of the target temperature during each test using a thermo-couple attached in the vicinity of the notch tip of each
specimen. Specimens were held at the test temperature for a minimum of 10 minutes prior to the commencement each test. Specimens were loaded at a rate of 0.16kN/s in the elastic loading range in a three point bending arrangement using a servo-hydraulic machine.

![Figure 3-6 The general test arrangement adopted for quasi-static standard and non-standard SE(B) testing](image)

Following testing the initial fatigue pre-crack length of the standard SE(B) specimens was measured at nine locations in accordance with ASTM E 1820-11 (2011). The initial EDM machined notch length of the U-notch and V-notch specimens was determined using an optical microscope (each EDM machined notch was machined with a tolerance of less than 0.01mm). A check was also conducted of each specimen to ascertain if ductile tearing had occurred prior to fracture, all testing was undertaken under conditions such as to minimise ductile tearing ($\Delta a<0.2\text{mm}$) in all geometries.

### 3.2.3 Dynamic Test Procedure for Charpy V-notch Specimens

Instrumented Charpy impact testing of the study material was undertaken at the test $T=0^\circ\text{C}$. Ten specimens were tested using a 450 Joules capacity machine and 2mm radius striker in compliance with the BS 10045 procedures (BSI, 1990) and those for the
instrumented test (BSI, 2000). An environmental chamber was used to control the test
temperature of the specimens to within 1°C of the target temperature for ten minutes
prior to proceeding with each test. The instrumented test record was analysed following
each test to determine the load and LLD at the point of cleavage initiation and evaluate
absorbed load line energy \( (U_{bi,pl,LLD}) \) corresponding to this event.

3.2.4 Fractography

Fracture surface examination concerning experimentally tested specimens was
undertaken using scanning electron microscopy (SEM). SEM analysis was undertaken
on fracture surfaces of quasi-statically tested Charpy V-notch specimens to discern the
distance between the notch tip or ductile tearing crack tip and cleavage initiation. This
work was undertaken to inform the definition of a micro-mechanical model for cleavage
fracture.

3.3 Finite Element Analysis

3.3.1 Quasi-Static SE(B) Models

Three dimensional finite element (FE) modelling was undertaken of the standard
SE(B) \( (a/W=0.5) \), U-notch SE(B) \( (a/W=0.2, \rho=0.25\text{mm}) \) and standard Charpy V-notch
specimen geometries which were tested. These analyses were for the purposes of
developing experimental expressions for evaluating the J-integral (Rice, 1968) and also
micro-mechanical modelling of cleavage fracture behaviour.

The plastic flow behaviour of the study material was modelled using the von
Mises flow rule and isotropic hardening. Material properties from curve fitting of true
stress - true strain quasi-static tensile test experimental data (Section 3.1.2) using the
Ramberg Osgood material law (Expression 3-4). Symmetrical quarter models were
developed for the SE(B) test specimen geometries (as shown in Figure 3-7 for the
shallow U-notch specimen geometry), loading was by imposed displacement of a strip
of nodes at the central support (DY), and a roller support was assigned to the outer
support of the specimen. Variable thickness layers were defined through each model thickness, the thickness of the layers were significantly reduced close to the specimen free surface. This approach was the same as that adopted by Nevalainen and Dodds (1995) for constraint analysis of SE(B) specimens. A concentric arrangement of rings of elements was focussed onto each notch tip centre within each layer for each of the specimen geometries which were analysed. The size of the elements was significantly reduced for the elements as the crack/notch tip was approached in the radial direction. Each element at the crack/notch tip subtended an angle of 9° at the notch tip. The finite element mesh was significantly less refined away from the notch tip regions in accordance with the less steep strain gradients in these regions. The commercial code ABAQUS version 6.11 (Simulia, 2011) was used for the analyses work. One hundred load increments were used for each analysis using an implicit (Full Newton) solution procedure. Details of the three dimensional quasi-static finite element models analysed for this work are given in Table 3-3.

Table 3-3 Details of finite element models used

<table>
<thead>
<tr>
<th>Specimen Geometry</th>
<th>$\frac{a}{W}$, $\rho$ (mm), $\phi$ (°)</th>
<th>No. Elements: Total, Thickness</th>
<th>Deformation Formulation</th>
<th>Element Type</th>
<th>Concentric ring No., Tip Element Size (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard SE(B)</td>
<td>0.5 (nom), 0, 0</td>
<td>15,740, 10</td>
<td>Small and Large</td>
<td>Quadratic 20 node (reduced integration)</td>
<td>60, 1.25</td>
</tr>
<tr>
<td>U-notch SE(B)</td>
<td>0.2, 0.25, 0</td>
<td>22,134, 14</td>
<td>Small and Large</td>
<td>Linear 8 node (full integration)</td>
<td>33, 37.5</td>
</tr>
<tr>
<td>Standard Charpy V-notch</td>
<td>0.2, 0.25, 45</td>
<td>20,692, 14</td>
<td>Large</td>
<td>Linear 8 node (full integration)</td>
<td>33, 37.5</td>
</tr>
</tbody>
</table>

Global model calculations for load (support reaction force) and nodal clip gauge opening were used to validate each model. The quarter total load at each loading
increment was calculated using the vertical reaction force (RF2) at a line of nodes at the loading line position of each quarter symmetry model. The clip gauge opening was calculated by incorporating the attached knife edge into the FE models (Figure 3-7), the height of the knife edge was defined to be at the average experimentally measured height (3mm±0.2mm). The half symmetry displacement of the knife edge tip was used to calculate the total clip opening displacement.

Small strain analyses and the ABAQUS contour J-integral routine was used to determine a converged value of the J-integral at each loading increment. A sharp crack tip with coincident nodes at the crack tip was used when undertaking the analysis of the SE(B) type specimen (the nodes of the elements at the crack-tip were untied). The SE(B) specimen initial crack depth was modelled corresponding to the mean experimentally measured \(a_0\) of the tested SE(B) specimens with nominal crack depth ratio of 0.5. A weighted crack-front average \(J\) value, denoted \(J_{ave}\), was calculated for each load increment.

The standard SE(B) specimen fatigue pre-crack was also modelled with an initial notch tip radius, \(\rho_0\), of 2.5μm for the purposes of large strain modelling to enable application of a micromechanical model for cleavage fracture (Section 3.4). The mesh arrangement used for this analysis is shown in Figure 3-8. Large strain analysis was undertaken of the shallow U-notch SE(B) and Charpy V-notch specimen geometries.

### 3.3.2 Dynamic Charpy V-notch Models

The three dimensional FE model of a quasi-static Charpy V-notch specimen was modified for the purpose of modelling the Charpy impact test. The mesh arrangement at the notch tip region and model symmetry conditions were unchanged from the FE model developed for quasi-static conditions. Dynamic tensile material properties were assigned to the model in addition to the quasi-static tensile properties described in Section 3.1.2. The dynamic tensile properties were implemented using the curve fitting
procedures described in Section 3.1.3 using a power law true stress-true strain relationship (Expression 3-6). Separate power law curve definitions were used to define the dynamic FE model corresponding to each experimental test strain rate.

![Diagram of a shallow U-notch SE(B) specimen](image)

**Figure 3-7** Quarter symmetry finite element model of a shallow U-notch SE(B) specimen ($a/W=0.2$ and $\rho=0.25\text{mm}$) mesh arrangement and boundary conditions

![Diagram of crack tip mesh arrangement](image)

**Figure 3-8** Crack tip mesh arrangement and ligament boundary condition for standard SE(B) specimen
Material strain rate dependent properties were extrapolated to define material flow properties corresponding to $\dot{\varepsilon} = 2,000/s$ so as to calculate the material behaviour at the Charpy V-notch tip under dynamic conditions (Section 3.1.3). This later material property definition was undertaken following preliminary runs of the model using quasi-static material flow properties. Large deformation theory was used for the analysis for the purpose of micro-mechanical modelling of cleavage fracture. The Charpy specimen support condition was defined in accordance with the geometry of the Charpy machine anvil (BSI, 1990) to incorporate the effects of this support contact condition upon global load-displacement behaviour of the specimen during loading (Figure 3-9). The load line displacement loading condition (DY) was applied in conjunction with the time period of the analysis ($\Delta$) to achieve a load line velocity of 5,200mm/s in accordance with the experimental test results (Section 3.2.3). One hundred load increments were used within an implicit solution procedure in common with the quasi-static numerical analysis methodology.

Figure 3-9 Finite element model of dynamic Charpy V-notch specimen showing mesh and boundary conditions
3.3.3 **Boundary Layer Model**

Boundary layer modelling was undertaken (Larsson and Carlsson, 1973). The purpose of this activity was to provide a small scale yielding (SSY) reference state for micro-mechanical calibration activities. The boundary layer model was a semi-circular, plane strain domain with a crack inserted. Figure 3-10a shows the boundary layer finite element model used for this investigation, the geometry was semi-circular because of the use of symmetry conditions at the crack plane ligament (DY=0mm). Symmetry conditions were imposed to the top and bottom faces (DZ=0mm) of the domain.

![Figure 3-10 Boundary layer model used for calculating small scale yielding conditions](image)

**Figure 3-10** Boundary layer model used for calculating small scale yielding conditions a.) general arrangement of mesh global coordinate system and boundary displacement conditions, b.) crack-tip mesh detail with initial root radius, \( \rho_0 \)

The dimensions of the domain were calculated to be sufficiently large so that plasticity at the crack-tip was restricted well within the domain, the maximum radius, \( R_{\text{max}} \), was \( 10^6 \) times \( \rho_0 \), where \( \rho_0 = 2.5 \mu\text{m} \) for this analysis. The plastic zone of deformation was confined to within \( R/20 \) during all increments of the analysis, thereby ensuring the maintenance of SSY conditions at the plastically deformed material around the crack tip.
The mesh arrangement was focussed on the crack-tip with eighty concentric rings of elements (Figure 3-10b), twenty divisions of the outer boundary of the semi-circular domain were employed, one layer of three dimensional 20-node quadratic interpolation brick elements was applied through the domain thickness. The thickness of the boundary layer was 12.5mm, the behaviour of the boundary layer was essentially two dimensional because of the DZ=0mm boundary conditions applied to the model. At the outer nodes of the boundary layer model displacement conditions were applied in an X and Y Cartesian system centred on the crack-tip. These displacement conditions were consistent with the linear elastic asymptotic crack-tip stress and displacement solution (Williams, 1952). The equations used to determine the $u$ and $v$ displacements in the X and Y coordinate system directions respectively were the following:

$$u(R, \theta) = \frac{K(1+\nu)}{E} \sqrt{\frac{R}{2\pi}} \cos\left(\frac{\theta}{2}\right) \left(3 - 4\nu - \cos \theta \right)$$  \hspace{1cm} (3.9a)$$

and,

$$v(R, \theta) = \frac{K(1+\nu)}{E} \sqrt{\frac{R}{2\pi}} \sin\left(\frac{\theta}{2}\right) \left(3 - 4\nu - \cos \theta \right)$$  \hspace{1cm} (3.9b)$$

where, $u$ = the displacement in the X direction at a boundary node, $v$ = the displacement in the Y direction, $R$ = the radius of the outer boundary from the crack-tip (Figure 3-10a), $\theta$ = the angle anti-clockwise from the crack plane (Figure 3-10a), $K$ = the linear elastic stress intensity factor, $E$ = Young’s modulus of elasticity, $\nu$ = Poisson’s ratio.
The boundary conditions were applied incrementally during the analysis, one hundred loading increments were necessary to complete an analysis. Material flow properties derived from quasi-static tensile tests at $\theta=0^\circ C$ were used (Section 3.1.2). A large strain formulation (nonlinear deformation theory) was implemented during the finite element solution procedure.

T-stress displacements were applied to the boundary layer model (where required) prior to the $K$ term. The following displacement terms were used for the T-stress displacements (Rice, 1974):

\[ u(R, \theta) = \tau \left( \frac{1 - \nu^2}{E} \right) RCos\theta \quad (3.10a) \]

and,

\[ v(R, \theta) = -\tau \left( \frac{1 - \nu^2}{E} \right) RSin\theta \quad (3.10b) \]

where, $\tau$ = the elastic T-stress.

### 3.3.4 Method of Calculation of J for SE(B) Models

Applied crack driving force ($J$) during experimental fracture toughness testing of standard SE(B) and low constraint U-notch SE(B) specimens was investigated using numerical analysis and the methods of ASTM E1820-11 (2011a) and ESIS P2-92 (1992).

The procedure adopted to evaluate existing methods for determining $J$ for the standard SE(B) ($a/W=0.5$) geometry and develop a new method for evaluating $J$ for the U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$, $\phi=0^\circ$) geometry was the following:

1. Small deformation theory finite element analysis of each test geometry was conducted using the FE models of the two geometries of interest. The
calculated loading and corresponding \( CMOD \) and \( LLD \) were tabulated and the plastic components of CMOD absorbed energy, \( U_{pl,CMOD} \), and total \( LLD \) absorbed energy \( U_{el+pl,LLD} \) were calculated for each geometry. When using the ASTM E 1820-11 and ESIS P2-92 methods as a basis for the numerical analysis, the \( J \) evaluation equations for SE(B) specimens for these test methods were expressed as:

\[
J = \left(\frac{K_I^2 (1-\nu^2)}{E}\right) + \frac{\eta_{pl} U_{pl,CMOD}}{B b_0} : \text{ASTM E1820-11 (2011a)} \quad (3-11)
\]

and,

\[
J = \frac{\eta_{el+pl} U_{el+pl,LLD}}{B b_0} : \text{ESIS P2/92 (1992)} \quad (3-12)
\]

where, \( \nu \) = Poisson’s ratio, equal to 0.3, and overleaf:

\( E \) = Young’s modulus of elasticity, equal to 207,000MPa,

\( \eta_{pl,CMOD} \) = a plastic rotation factor,

\( U_{pl,CMOD} \) = the area under the plastic portion of the load versus CMOD graph,

\( \eta_{el+pl,LLD} \) = the ESIS (1992) plastic rotation factor,

\( U_{el+pl,LLD} \) = the total area under the load versus LLD curve,

\( B \) = the specimen thickness,

\( b_0 \) = the initial ligament length,

and, \( K_I \) = the linear elastic stress intensity factor for a SE(B) specimen:

\[
K_I = \left[ \frac{PS}{BW^{1/2}} f(a/W) \right] \quad (3-13)
\]
\[ f(a/W) = \frac{3 \left( \frac{a}{W} \right)^{0.5} \left[ 1.99 - \left( \frac{a}{W} \right) \left( 1 - \frac{a}{W} \right) \left( 2.15 - \frac{3.93a}{W} + \frac{2.7a^2}{W^2} \right) \right]}{2 \left( 1 + \frac{2a}{W} \right) \left( 1 - \frac{a}{W} \right)^{1.5}} \]  

(3-14)

where, \( a = \) the crack length.

2. The plastic CMOD absorbed energy, \( U_{pl,CMOD} \), was normalised by the specimen ligament area, \( bB \), as per the ASTM E 1820-11 procedure. The total absorbed LLD energy, \( U_{el+pl,LLD} \), was normalised by \( bB \) following the ESIS P2-92 procedure to measure fracture toughness.

3. Path independent J-integral values, defined remote from the notch tip and not influenced by any specimen free surfaces, were calculated for each specimen geometry. \( J_{ave} \) was calculated for each load increment and equated to total \( J \). The average crack front elastic stress intensity factor, \( K_{ave} \), determined from elastic analyses was used to subtract the elastic component from the total \( J \) to determine the plastic contributions to \( J \) (\( J_{pl} \)).

4. The \( J_{pl} \) values, derived from step 3, and total \( J \) values for each specimen geometry were plotted against the corresponding \( U_{pl,CMOD}/bB \) and \( U_{el+pl,LLD}/bB \) quantities determined during step 2.

5. The linear slope of the \((J, U_{pl}/bB)\) coordinates was calculated using a linear regression line applied to the data from step 4. The form of the equation of the linear fit was therefore the following for the \( \eta_{pl} \) factor in relation to \( U_{pl,CMOD} \):

\[ J_{pl} = \eta_{pl} \frac{U_{pl,CMOD}}{bB}, \]  

(3.15)

where, \( J_{pl} = \) the plastic contribution to the J-integral evaluated using \( U_{pl,CMOD} \);

\( \eta_{pl} = \) calculated plastic rotation factor corresponding to \( U_{pl,CMOD} \).
The form of the equation of the linear fit was the following for the \( \eta_{el+pl} \) factor in relation to \( U_{el+pl,LLD} \):

\[
J_{el+pl,LLD} = \eta_{el+pl} \frac{U_{el+pl,LLD}}{bB},
\]

where, \( J_{el+pl,LLD} \) = the plastic contribution to the J-integral evaluated using \( U_{pl,CMOD} \),

\( \eta_{el+pl} \) = calculated rotation factor corresponding to \( U_{el+pl,LLD} \).

The calibrated \( \eta \) factors from the numerical analysis are given in Table 3-4.

Comparisons of the calibrated experimental \( J \) evaluation methods of Expressions (3-11) and (3-12), denoted \( J_{exp} \), and associated numerical \( J_{ave} \) quantities are shown in Figure 3-11a and 3-11b for the SE(B) (\( a/W=0.5 \)) and shallow U-notch SE(B) (\( a/W=0.2,\  \rho=0.25\text{mm},\ \phi=0^\circ \)) geometries respectively.

**Table 3-4 Derived \( \eta \) factors for measurement of fracture toughness of calibration specimens**

<table>
<thead>
<tr>
<th>Calibration Geometry</th>
<th>( \eta_{pl} )</th>
<th>( \eta_{el+pl} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SE(B) (( a/W=0.5 ))</td>
<td>2.76</td>
<td>1.92</td>
</tr>
<tr>
<td>U-notch SE(B) (( a/W=0.2,\ \rho=0.25\text{mm} ))</td>
<td>3.30</td>
<td>1.44</td>
</tr>
</tbody>
</table>

The calibrated \( \eta_{pl} \) value for the SE(B) specimen (\( a/W=0.5 \)) were in close agreement with the ASTM E 1820-11 (2011) methodology of Expression (3-11), this was confirmed as being accurate (<1% error). Predictions of \( J \) for the SE(B) specimen, using the calibrated \( \eta_{el+pl} \) and Expression (3-12), were in close agreement with theoretical values throughout the loading range examined to this accuracy also.

The calibrated value of \( \eta_{pl} \) for the U-notch SE(B) (\( a/W=0.2,\ \rho=0.25\text{mm} \)) specimen provided less accuracy when calculating \( J \) using Expression (3.11) at low loads (\( J<100\text{N/mm} \)), at higher loading the accuracy was within 1% of the theoretical \( J \) value.
The $J$ evaluated using Expression (3-12) and the calibrated $\eta_{\text{el+pl}}$ value for this geometry had a high accuracy (error <1%) throughout the loading range except at exceptionally low $J$ values of <10N/mm.

Experimental load-displacement results were used in conjunction with numerical analysis to calculate the critical $J$-integral measurements at failure, $J_c$, for the two experimental geometries. A numerical correlation between clip gauge opening displacement and CMOD of the SE(B) specimen ($a/W=0.5$) was developed and Expression (3-11) was used to calculate $J_c$ using the determined $\eta_{\text{pl,CMOD}}$ factor. A correlation between clip gauge opening and LLD for the U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) and Expression (3-12), using $\eta_{\text{el+pl,LLD}}$, was used to calculate non-standard $J_c$ values for this geometry. For the later U-notch SE(B) geometry, $J_{c,\rho}$ was defined, corresponding to this non-standard test geometry.

### 3.3.5 Quasi-Static Stress Field Calculation

The SE(B) ($a/W=0.5$) and U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) specimen geometries were each investigated by undertaking a comparison of the mode I opening stress fields with the theoretical HRR field Expression (2-11a). Nodal mode I opening stress values ($\sigma_{\theta\theta}$) were extracted along paths directly ahead of the SE(B) specimen crack-tip and the U-notch respectively at $\theta=0^\circ$ at different loading levels. Large strain FE analysis results (Section 3.3.1) with power law flow properties for the study material at $T=0^\circ\text{C}$ were used (Section 3.1.2). The radius, $R$, of each stress value was normalised using $R\sigma_{\theta\theta}/J$ to establish self similarity of the HRR stress fields. This analysis was used to verify the SE(B) specimen ($a/W=0.5$) numerical model crack-tip behaviour. The behaviour of the U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) specimen relative to the sharp crack, small deformation, plane strain and infinite medium assumptions of the HRR field (Rice and Rosenglen,1968)(Hutchenson, 1968) was also quantitively assessed.
Analysis of the strain and displacement fields ahead of the crack and notch of these specimens was not undertaken.

![Graph showing J evaluation error using modified ESIS P2/92 (1992) and ASTM E1820-11 (2011a) procedures for a.) SE(B) (a/W=0.5) and b.) non-standard SE(B) (a/W=0.2, ρ=0.25mm)]

**Figure 3-11** Experimental J evaluation error using modified ESIS P2/92 (1992) and ASTM E1820-11 (2011a) procedures for a.) SE(B) (a/W=0.5) and b.) non-standard SE(B) (a/W=0.2, ρ=0.25mm)
3.3.6 Dynamic Stress, Strain and Strain Rate Field Calculation

The mode I opening stress and plastic strain rate ahead of the Charpy V-notch was undertaken using the FE modelling results from dynamic modelling of this specimen geometry (Section 3.3.2). Large strain FE analysis results were used alongside dynamic material flow properties at \( T=0^\circ \text{C} \) (Section 3.1.3). Nodal \( \sigma_{0\theta}, \varepsilon_{0\theta} \) and \( \dot{\varepsilon}_{0\theta} \) quantities were extracted from a line of nodes at \( \theta=0^\circ \) ahead of the notch for evaluation of the field quantities at different \( U_{\text{el+pl,LLD}} \) loading levels.

3.4 Weibull Stress Cleavage Fracture Modelling

A micromechanical model for cleavage fracture (Beremin, 1983) was used to assess the proximity of SE(B) \( (a/W=0.5) \) and U-notch SE(B) \( (a/W=0.5, \rho=0.25\text{mm}) \) to unstable cleavage fracture for the study ferritic steel material. These researchers proposed the scalar Weibull stress, \( \sigma_w \), as a probabilistic indicator of proximity to cleavage fracture for a ferritic steel. During cleavage fracture conditions the Weibull distribution (Weibull, 1939) has been shown to be accurate for describing the probability of failure because a weakest link failure mechanism operates (Batdorf and Crose, 1974)(Matsuo, 1981)(Ruggieri and Dodds, 1996). The expression for \( \sigma_w \) for a sharply cracked or notched fracture geometry subject to external loading is the following:

\[
\sigma_w = \frac{1}{\lambda_0} \left( \sum_{i=1}^{n} \sigma_1^m d\lambda \right), \quad \text{for } \sigma_1 >
\]

The following terms within Expression (3-17) were defined:

\[ \sigma_1 = \text{the maximum principal stress at a material point}, \]
\[ m = \text{the Weibull shape parameter}, \]
\[ \lambda = \text{the crack/notch tip plastic zone}, \]
\[ \lambda_0 = \text{a normalising volume, set at } 1\text{mm}^3, \]
\[ d\lambda = \text{the } p\text{th incremental volume in the plastic zone, } V. \]
The value of the parameter $m$ in Expression (3-17) is a material property which is dependent on microstructural features and was therefore calibrated using material property laboratory data (Section 3.4.1). Large strain FE analyses results were used for the determination of $\sigma_W$ at each loading increment for the specimen geometries analysed. $J$ was calculated at each load increment of finite element model results using Expressions (3-11) and (3-12) in conjunction with the calibrated $\eta$ factors in Table 3-4 for the SE(B) ($a/W=0.5$) and U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) respectively.

### 3.4.1 Calibration of the Cleavage Fracture Model

To calibrate the Weibull shape parameter, $m$, for this investigation the method of Gao et al (1998) was used, this is described here as:

1. Two sets of fracture toughness data corresponding to high and low constraint specimen geometries (SET A and SET B respectively). Each experimental critical fracture toughness measurement, $J_{c,i} \text{ SET A}$ or $J_{c,i} \text{ SET B}$, corresponding to each experimental cleavage fracture event, was positioned on the corresponding SET A or SET B Weibull stress analysis ($\sigma_W$, $J_{\text{ave}}$) trajectory.

2. Using the small scale yielding (SSY) Weibull stress trajectory corresponding to the boundary layer model analysis, the experimental critical fracture toughness results for Types A and B were scaled to the SSY condition for a given value of $m$. Thereby obtaining values of $J_{c,i} \text{ SSY Type A}$ and $J_{c,i} \text{ SSY Type B}$. Figure 3-12 shows schematically the scaling methodology adopted.

3. The sufficiency of the chosen Weibull shape $m$ value was assessed using the following approach:

   A maximum likelihood (ML) expression, $\beta_{\text{SSY}}$, corresponding to each constraint corrected data set and an $m$ dependent error function, $R(m)$ was defined to evaluate the
relative location of the scaled SSY fracture toughness values for each experimental data set. This method was used to assess the adequacy of a given \( m \) parameter value. The expressions for \( \beta_{SSY} \) and \( R(m) \) were the following:

\[
\beta_{SSY} = \left( \frac{1}{N} \sum_{i=1}^{n} J_{i,SSY}^2 \right)^{\frac{1}{2}} : \text{maximum likelihood}, \quad (3.18a)
\]

\[
R(m) = (\beta_{SSY}^{B} - \beta_{SSY}^{A}) / \beta_{SSY}^{A} : \text{error function}, \quad (3.18b)
\]

where,
- \( J_{i,SSY} \) = the constraint corrected fracture toughness \( i^{th} \) value of a test geometry's fracture toughness distribution,
- \( N \) = the total number of data values in a each specimen geometry's set of fracture toughness results,
- \( \beta_{SSY} = \) maximum likelihood expression of fracture toughness for a high constraint, SET A, or low constraint, SET B, test geometry.

4. The procedure steps 2 and 3 were repeated using a different integer for the Weibull shape '\( m \)' value so as to obtain a change of sign of the \( R(m) \) error function.

5. Linear interpolation was used to determine the \( m \) parameter value to one decimal place corresponding to a value of \( R(m)=0 \).

The above procedure was used to calibrate \( m \) for the purpose of scaling \( J_{c,p} \) values, measured using a non-standard U-notch SE(B) \((a'/W=0.2, \rho=0.25\text{mm})\) specimen geometry, to an equivalent \( J_{c} \) corresponding to a standard SE(B) \((a'/W=0.5)\) specimen geometry (Section 3.5.4).
Figure 3-12 J-integral scaling method to the SSY condition used for calibration of the Weibull stress model (Gao et al, 1998)

A second calibration procedure was undertaken, the method adopted was modified from that used for scaling $J$ by undertaking calibration with reference to the SE(B) ($a/W=0.5$) condition in preference to the SSY condition owing to the objectives of the research. $U_{\text{el+pl,LLD}}$ was used as the loading parameter for this calibration activity. A value of Weibull stress model $m$ parameter for the study material which was most accurate for scaling $U_{\text{el+pl,LLD}}$ between the exact range of constraint levels represented by the quasi-statically loaded Charpy V-notch and SE(B) specimen geometries was determined. The $U_{\text{el+pl,LLD}}$ scaling operation using $\sigma_W$ is shown schematically for the study material in Figure 3-13. Two further $m$ parameters were also calibrated for scaling U-notch SE(B) ($a/W=0.5, \rho=0.127\text{mm}$) and U-notch SE(B) ($a/W=0.2$,
\( \rho = 0.25\text{mm} \) specimen geometries to the SE(B) condition under quasi-static loading conditions.

![Graph showing scaling of \( U_{el+pl,LLD} \) between the quasi-static Charpy V-notch specimen and SE(B) specimen using the Weibull stress model.]

**Figure 3-13** Scaling of \( U_{el+pl,LLD} \) between the quasi-static Charpy V-notch specimen and SE(B) specimen using the Weibull stress model

### 3.4.2 Determination of Weibull Scale Parameter, \( \sigma_u \)

A two parameter Weibull distribution predicts the cumulative probability of failure, \( P_i \), under increasing Weibull stress, \( \sigma_w \). The expression for the Weibull probability distribution is the following:

\[
P_i(\sigma_w) = 1 - \exp \left[ - \left( \frac{\sigma}{\sigma_u} \right)^m \right],
\]

(3.19)

where, \( P_i(\sigma_w) = \) the cumulative probability of failure, \( \sigma_w \), is the Weibull stress as defined in Expression (3-17) and \( \sigma_u \) the Weibull scale parameter.
The Weibull scale parameter, $\sigma_u$, positions the calibrated cumulative probability distribution on the $\sigma_w$ ordinate axis; as shown schematically for two sets of data in Figure 3-14. A least square fit was used to minimise the error between the rank probability of failure, $P_{f, \text{rank}}$, for experimental results for the two calibration geometries (Section 3.4.1) and the predicted Weibull stress model distribution for $P_f$. This method permitted a $\sigma_u$ value to be calculated using a combined fit to both experimental data sets; corresponding to both the low and high constraint geometries. When comparing experimental data to calculated micro-mechanistic predictions, experimental data were expressed using the median rank probability of failure (Bombas-Smith, 1973), $P_{f, \text{rank}}$:

$$P_{f, \text{rank}} = \frac{(i - 0.3)}{(N + 0.4)}$$  \hspace{1cm} (3-20)

where, $P_{\text{rank}}$ = the cumulative probability of failure,

$i$ = the ascending rank order of an experimental result for a particular specimen in each dataset,

and, $N$ = the total number of specimens in the dataset.

### 3.4.3 Three Parameter Weibull Stress Cleavage Fracture Model

A three parameter Weibull distribution was applied to calculate the failure probability of the SE(B) specimens, and for comparison to the two parameter predictions from the Beremin model. The formula for calculating the probability of failure for a three parameter Weibull distribution is given by the following:

$$P_f(\sigma_w) = 1 - \exp \left\{ - \left[ \frac{(\sigma_w - \sigma_{w,\text{min}})}{(\sigma_u - \sigma_{w,\text{min}})} \right]^m \right\},$$  \hspace{1cm} (3.21)

where, $\sigma_{w,\text{min}}$ is a minimum value of $\sigma_w$ below which cleavage fracture is not possible.
3.5 Development of an Engineering Procedure for Charpy Specimen Data

Following completion of work concerning a sequential engineering procedure, a direct correlation method was investigated using the ferritic steel study material. This approach was undertaken for the purpose of achieving an improved correlation and reducing the conservatism associated with existing engineering approaches (Appendix A) and also to limit the complexity of a multi-step calculation procedure.

3.5.1 Analysis of 0.4T SE(B) (a/W=0.5) Results

The pre-cracked 0.4T SE(B), of a nominal a/W ratio equal to 0.5, specimen fracture toughness results were analysed using the master curve methodology (ASTM, 2011b). This enabled calculation of a $T_0$ estimate and application of the probabilistic
model to these cleavage fracture results. The purpose of this work was to provide a reference condition by which validation could be conducted on other non-standard SE(B) fracture mechanics models using a Weibull stress modelling approach. The specimen $K_{ic}$ results were not size corrected to the 1T crack front length because all of the associated work concerning development of the engineering procedure concerned non-standard specimens of Charpy outline size ($B=W=10$).

3.5.2 Calculation of J for Deep U-notch SE(B) (a/W=0.5, p=0.25mm) Specimens

Small strain elastic-plastic FE analyses of the U-notch SE(B) (a/W=0.5, p=0.25mm) was undertaken, using a numerical approach similar to that described in Section 3.3.1. The J-integral was calculated numerically using a converged contour integral at nodal positions along the notch front at each loading increment. An average J-integral, $J_{ave}$, corresponding to the notch-front geometry for the specimen was determined using a weighted average of quantities calculated at each notch-front nodal position. The standard ESIS P2-92 (1992) expression for evaluating $J$ when testing SE(B) fracture toughness specimens is valid for crack depth ratios of $0.45< a/W \leq 0.65$, this expression was defined in the following form for application to non-standard U-notch SE(B) specimen geometries:

$$J = \frac{\eta_{LLD} U_{el+pl,LLD}}{B(W-a_0)} ,$$

(3-22)

The terms in Expression (3-22) were defined as the following:

$U_{el+pl,LLD} = \text{the total absorbed load line energy of the test specimen,}$

$\eta_{LLD} = \text{a rotation factor for use when evaluating } U_{el+pl,LLD},$

and,

$a_0 = \text{the initial notch depth.}$
The value of $\eta_{\text{LLD}}$ recommended by ESIS P2-92 for use within Expression (3-22) is equal to 2 for standard SE(B) type fracture toughness specimens. This study required a definition of $\eta_{\text{LLD}}$ to be established for the non-standard U-notch 0.4T SE(B) ($a/W=0.5$, $\rho=0.25\text{mm}$) specimen geometry corresponding to a wide range of relevant ferritic steel material flow properties. The calculated $J_{\text{ave}}$ was used to determine $\eta_{\text{LLD}}$ using an approach which followed that described in Section 3.3.4. This analysis work was repeated for four sets of ferritic steel material which represent feasible limits of yield stress and strain hardening. A power law hardening law (Expression 3-3) was used to define the material flow behaviour. This work was preparatory to micro-mechanical modelling work (Section 3.5.4) so as to assure an accurate method of determining $J$.

The material definition for the $\eta_{\text{LLD}}$ material dependence study of the U-notch SE(B) ($a/W=0.5$, $\rho=0.25\text{mm}$) defined the material yield stress, $\sigma_Y$, at 0.2% plastic strain. The Young’s modulus of elasticity was set to be 200,000MPa. Material parameters were calculated using numerically for the four different pairs of $E/\sigma_Y$ ratios and $n$ values. Material data was therefore processed to determine the corresponding value of $\sigma_0$ to be used for each ($E/\sigma_Y, n$) material property definition. The four material property definitions used for the analyses are given in Table 2-6.

<table>
<thead>
<tr>
<th>$E/\sigma_Y$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>5</td>
</tr>
<tr>
<td>800</td>
<td>5</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>500</td>
<td>20</td>
</tr>
</tbody>
</table>
3.5.3 Material Dependent J Solution for U-notch SE(B) (a/W=0.2, ρ=0.25mm) Specimen

Small strain FE analyses of the U-notch SE(B) (a/W=0.2, ρ=0.25mm), using the numerical approach described in Section 3.3.1, was undertaken for the purpose of undertaking a material dependence study of $\eta_{\text{LLD}}$ within Expression (3.22) for this specimen geometry. This work was completed to establish a suitable expression to be used for calculating $J$ for this specimen geometry, one which inherently accounted for the effect of different material flow property definitions. The development of the $J$ calculation method for this specimen geometry was undertaken to allow micro-mechanical modelling work (Section 3.5.4). The material definition approach for the analyses work was identical to that described in Section 3.5.2. The material property definitions which were used to study the effect of $\sigma_Y$ and $n$ on $\eta_{\text{LLD}}$ for this geometry of interest are given in Table 2.7.

The values of $\eta_{\text{LLD}}$ corresponding to each material property definition in Table 2-7 were analysed for the purpose of defining a material specific expression to describe this quantity within Expression (3-22).

<table>
<thead>
<tr>
<th>$E/\sigma_Y$</th>
<th>$\eta_{\text{LLD}}$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>800</td>
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3.5.4 Application of Weibull Stress Scaling Model to SE(B) and U-notch SE(B) Specimens

A Weibull stress model scaling approach was investigated which incorporated the ASTM E 1921 cleavage fracture model so as to achieve predictions of $P_f$ for non-standard U-notch SE(B) specimens. This analysis was undertaken to assess the combined roles of such a scaling model:

- Cleavage fracture probability of standard SE(B) specimens at a constant temperature described by the ASTM E 1921 methodology,
- The accuracy of the Weibull stress scaling approach when extending ASTM E 1921 to non-standard geometries.

The following expression from ASTM E 1921-11a was used to describe the failure of the 0.4T SE(B) specimens:

$$\ln \left( \frac{1}{1 - P_f} \right) = \left( \frac{(K_{Jf(P_f)} - 20)}{11 + 77 \exp[0.019(T - T_{0.0,4T})]} \right)^4. \quad (3-23)$$

Within Expression (3-23), $K_{Jf(P_f)}$ = the value of crack driving force for the SE(B) specimen, $P_f$ = the corresponding failure probability at T (°C), and $T_{0.0,4T}$ = the value of $T_0$ calculated from $K_{Jc}$ data of Section 3.5.1 for the 0.4T SE(B) geometry.

The Weibull stress model was used to scale $(K_0, P_0)$ predictions of the ASTM E 1921 model to non-standard U-notch SE(B) geometries. These geometries were the U-notch SE(B) $(a/W=0.5, \rho=0.127\text{mm})$ and U-notch SE(B) $(a/W=0.2, \rho=0.25\text{mm})$. The value of $m$ parameter used to undertake the scaling was set to the value of $m$ corresponding to these individual geometries for which calibration was undertaken in Section 3.4.1. One further geometry, a 0.5T U-notch SE(B) $(a/W=0.5, \rho=0.75\text{mm})$ which was previously
tested by Horn and Sherry (2010) using the study material, was also incorporated into the investigation of the scaling method. This allowed the performance of the scaling method over an increased range of constraint because of the larger value of $\rho$ this specimen possessed. This later geometry was outside the test programme undertaken in Section 3.2.2 and hence did not assume any other role in the work except for the purpose of checking of the scaling model during this preliminary work concerning the Weibull stress scaling model described in this Section. The value of $m$ adopted for the scaling to the 0.5T U-notch SE(B) ($a/W=0.5$, $\rho=0.75\text{mm}$) geometry was the value calibrated by Horn and Sherry (2010).

### 3.5.5 Comparison of Weibull Stress Scaling Model with Combined Notch and Constraint Engineering Procedure

Predictions of $(K_J, P_f)$ using the Weibull stress scaling method, described in Section 3.5.4, corresponding to a U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$), were compared to the predictions of $(K_J, P_f)$ made using the combined notch and constraint procedure of Horn and Sherry (2012a)(2012b). The value of $m$ parameter used for application of the Weibull stress scaling method and the engineering procedure of Horn and Sherry was set to the value calibrated using the 0.4T SE(B) ($a/W=0.5$) and Charpy V-notch specimen under quasi-static loading conditions. A constraint dependent apparent fracture toughness, $K_{\text{mat}, c}$ was defined according to Sherry et al (2005a):

$$K_{\text{mat}, c} = \left[ 1 + \alpha \left( -\beta_{\tau} L_{\tau} \right)^k \right], \text{ for } \beta_{\tau} < 0,$$

(3-24)

where, $\alpha, k =$ material constants which are dependent on the material flow properties: $E$, $\sigma_y$ and $n$ (Section 3.1.2), and the calibrated Weibull stress model shape parameter ($m$),
Within (3-24), $K_{\text{mat}}$ equals the material high constraint fracture toughness at the temperature of interest corresponding to a predefined $P_i$ and $\beta_T$ = a biaxiality parameter dependent on geometry.

$K_{\text{mat}}$ was defined as the median material fracture toughness corresponding to a value of $P_i$ equal to 0.5 at $T=0^\circ\text{C}$. An elastic analyses ($E=207,000\text{MPa}$) of a shallow crack 0.4T SE(B) ($a/W=0.2$) specimen was used to evaluate this specimen geometry's $\beta_T$ value using the following expression (Ainsworth, 1995):

$$\beta_T = \frac{\tau_{\text{ave}}}{\sigma_\gamma L_r},$$  

(3-25)

where, $\tau_{\text{ave}}$ = is the average crack-front elastic T-stress acting parallel to the crack plane (MPa),

$L_r$ = the load ratio, equal to $P/P_L$, where $P_L$ is the limit load of the 0.4T SE(B) ($a/W=0.2$) specimen geometry.

$L_r$ was determined using results from an elastic-perfectly-plastic FE analysis, the $\sigma_\gamma$ material property for the study material under quasi-static conditions at the temperature of interest (Section 3.1.2) was used to determine the $P_L$. A constraint modified material failure curve and specimen loading line was constructed as $(K_{\text{mat,c}}/K_{\text{mat}})$ vs. $(\beta_T/\sigma_\gamma)$ and $(K_i/K_{\text{mat}})$ vs. $(\beta_T/\sigma_\gamma)$ on a single plot to describe the cleavage fracture behaviour of the 0.4T SE(B) ($a/W=0.2$) specimen geometry. The value of $K_{\text{mat,c}}/K_{\text{mat}}$ corresponding to the constraint correction was evaluated at the intersection of the specimen loading line and the constraint modified material failure curve. Figure 3-15 shows schematically the methodology adopted for the constraint correction.

A notch modified material failure curve was constructed for a U-notch 0.4T SE(B) ($a/W=0.5$, $\rho=0.25\text{mm}$) specimen geometry. The following expression was used to describe this failure curve for cleavage fracture conditions (Horn and Sherry, 2012a):
The following terms within Expression (3-26) were defined:

\[ \gamma, \ l = \text{material constants dependent on the flow properties } \sigma_Y \text{ and } n \text{ for the ferritic steel under consideration,} \]

\[ \beta_N = \text{a notch geometry parameter which is used to describe the notch geometry,} \]

\[ K_{\text{mat}} = \text{the material fracture toughness at the temperature of interest,} \]

\[ K_{\text{mat,} \rho} = \text{the non-sharp defect modified material fracture toughness.} \]

\[ K_{\text{mat}} \text{ was defined as for the constraint correction. An elastic FE analysis of the U-notch 0.4T SE(B) (}\ a/\ W=0.5, \ \rho=0.25\text{mm}) \text{ specimen was used to evaluate this specimen geometry's } \beta_N \text{ value using the following expression:} \]

\[ \frac{L_{r, \rho}}{\beta_N} = \frac{\sigma_N}{\sigma_Y}, \quad (3-27) \]

where, \( \sigma_N = \text{the elastic stress acting in mode I at the notch front mid-thickness position,} \)

\[ L_{r, \rho} = \text{the load ratio of a U-notch specimen, equal to } P/P_{L, \rho}, \]

and, \( P_{L, \rho} = \text{the limit load of the U-notch 0.4T SE(B) specimen (}\ a/\ W=0.5, \ \rho=0.25\text{mm}).} \]

\( P_{L, \rho} \text{ was determined from elastic-perfectly plastic analyses. } \sigma_Y \text{ was defined as the constraint correction. A notch modified material failure curve and specimen loading line was constructed as } (K_{\text{mat,} \rho}/K_{\text{mat}}) \text{ vs. } (L_{r, \rho} \beta_N) \text{ and } (K_J/K_{\text{mat}}) \text{ vs. } (L_{r, \rho} \beta_N) \text{ on a single plot for} \]
the 0.4T SE(B) \((a/W=0.5, \rho=0.25\text{mm})\) specimen geometry. The notch modified material failure curve which was used to evaluate \(K_{\text{mat,}r}/K_{\text{mat}}\) is shown in Figure 3-16.

![Diagram showing the constraint modified failure curve](image)

**Figure 3-15 Methodology adopted for constraint correction**

A combined constraint and notch material fracture toughness, \(K_{\text{mat,c,}\rho}\), of a U-notch 0.4T SE(B) \((a/W=0.2, \rho=0.25\text{mm})\) was calculated by multiplying the determined \(K_{\text{mat,c}}/K_{\text{mat}}\) and \(K_{\text{mat,}r}/K_{\text{mat}}\) constraint and notch modified fracture toughness factors. This amalgamated single factor was then applied to the ASTM E 1921-11 \((K_J,P_f)\) prediction (Section 3.5.4) to calculate an equivalent constraint and notch corrected \((K_J,P_f)\) prediction corresponding to a U-notch SE(B) \((a/W=0.2, \rho=0.25\text{mm})\). This allowed a comparison to be made with the Weibull stress scaling method predictions of \((K_J,P_f)\) described in Section 3.5.4.
3.5.6 Direct Correlation of Equivalent SE(B) (a/W=0.5) Specimen $K_J$ with Quasi-static Charpy V-notch $U_{el+pl,LLD}$

The quasi-static Charpy V-notch specimen was modelled using the same mesh and boundary conditions as shown in Figure 3-9. The material flow properties used for micromechanical modelling were those given in Section 3.1.2 for the temperature of interest. The correlation was achieved by interpolating $\sigma_w$ values between geometries at each load increment. This method permitted discrete $U_{el+pl,LLD}$ values at each load increment of a quasi-static Charpy V-notch specimen to be equated with the equivalent $J$ value induced in a 0.4T SE(B) specimen ($a/W=0.5$) at an identical $\sigma_w$ value to the quasi-static Charpy V-notch specimen.

The cumulative probability of cleavage failure predictions as a function of Charpy V-notch $U_{el+pl,LLD}$ were calculated using the following steps:
1. The ASTM E1921-11 $P_f$ expression for use with standard pre-cracked fracture toughness specimens was applied:

$$P_f(K_f) = 1 - \exp \left[ - \left( \frac{(K_f - K_{\text{min}})}{(K_0 - K_{\text{min}})} \right)^4 \right], \quad (3.28)$$

where, $K_f = \text{the elastic-plastic equivalent stress intensity factor load parameter}$,

$K_0 = \text{the scale parameter of the } P(K_f) \text{ Weibull distribution}$, corresponding to $P_f=0.63$,

$K_{\text{min}} = \text{a minimum threshold value of fracture toughness, equal to 20MPa}\sqrt{\text{m}} \text{ below which } P_f=0$.

2. The experimental test results (Section 3.2.2) for 0.4T SE(B) ($a/W=0.5$) specimens were used as a basis to define the $K_0$ parameter in Expression (3-28) using the following expression:

$$K_0 = \left[ \sum_{i=1}^{N} \left( \frac{K_{Jc} - K_{\text{min}}}{N} \right) \right]^{0.25} + K_{\text{min}}. \quad (3.29)$$

Within (3.29), $K_{Jc} = \text{the critical elastic-plastic fracture toughness measured using ASTM E 1820-11 (2011)}$ and $N = \text{the number of valid fracture toughness tests undertaken}$.

3. Expression (3-28) was expressed in terms of $U_{el+pl,LLD}$ for application to the Charpy V-notch specimen geometry and this resulted in the following expression for $P_f$ as a function of $U_{el+pl,LLD}$:

$$P_f(U_{el+pl,LLD}) = 1 - \exp \left[ - \left( \frac{U_{el+pl,LLD} - U_{\text{min}}}{(U_0 - U_{\text{min}})} \right)^2 \right], \quad (3.30)$$
Within (3.30), $U_{el+pl,LLD}$ = the Charpy V-notch load line displacement absorbed energy,

$U_0$ = the scale parameter of the $P_f(U_{el+pl,LLD})$ Weibull distribution corresponding to $P_f=0.63,$

$U_{min}$ = a minimum threshold value of $U_{el+pl,LLD}$ below which $P_f=0.$

4. The geometry and material specific $U_0$ value was derived from the $\sigma_W$ analysis results for the quasi-static Charpy V-notch specimen. This was the value of $U_{el+pl,LLD}$ for the Charpy specimen corresponding to a predefined value of $\sigma_W$. This $\sigma_W$ value was calculated using the SE(B) ($a/W=0.5$) specimen ($\sigma_W$, $K_J$) at a value of applied $K_J$ equal to $K_0$. The $U_{min}$ value used for the Charpy V-notch specimen corresponded to a predefined value of $\sigma_W$. This $\sigma_W$ value was calculated for the SE(B) specimen at a value of applied $K_J$ equal to $K_{min}=20\text{MPa}\sqrt{\text{m}}$. A common value of $m$ was used for both geometries in accordance with the value calibrated in Section 3.4.1. This allowed determination of the median $U_{el+pl,LLD}$ prediction for the Charpy V-notch specimen.

5. $P_f$ predictions were determined for the SE(B) specimen at incremental values of $U_{el+pl,LLD}$ corresponding to this particular geometry. A scaling operation was applied to Charpy V-notch specimen $U_{el+pl,LLD}$ values to calculate equivalent $U_{el+pl,LLD}$ values for the SE(B) geometry corresponding an equal $P_f$ value for both geometries. This scaling operation used $\sigma_W$ analyses results for both geometries at a common $m$ value.

6. The SE(B) ($a/W=0.5$) specimen $U_{el+pl,LLD}$ values were thereafter converted to $K_J$ using a correlation between $U_{el+pl,LLD}$ and $K_J$ obtained from the $J$ values calculated using the method described in Section 3.3.4 using numerically
calculated CMOD and load values for this specimen geometry and study material. This correlation is shown schematically in Figure 3-17.

![Graph showing correlation between CMOD and load values](image)

**Figure 3-17 Conversion of equivalent $U_{el+pl,LLD}$ to $K_J$ for a SE(B) ($a/W=0.5$) specimen**

Confidence bounds of $P_f(K_J)$ were generated for the SE(B) ($a/W=0.5$) test results using Expression (3-23) and a MATLAB script (Mathworks, 2011). The $T_0$ value within Expression (3-23) was set to that calculated in Section 3.5.1 using 0.4T SE (B) ($a/W=0.5$) test specimen results at ($T=0°C$). One million randomly generated sets of $N=10$ artificial $U_{el+pl,LLD}$ results were generated and ranked in ascending order. Expression (3-20) was used to calculate the $P_{f,\text{Rank}}$ for each $U_{el+pl,LLD}$ within each dataset. The Monte-Carlo simulation results corresponding to 10, 50 and 90% confidence bounds for the Charpy specimen were calculated. These confidence bounds were scaled to the Charpy V-notch geometry using the $U_{el+pl,LLD}$ value for this geometry associated with a common $\sigma_W$ value determined from each Charpy specimen ($P_{f,\text{Rank}}$, $U_{el+pl,LLD}$) data point corresponding to a specific confidence bound.
3.5.7  Direct Correlation of Equivalent SE(B) (a/W=0.5) Specimen K_J with Dynamic Charpy Impact Test U_{el+pl,LLD}

A direct correlation between \( U_{el+pl,LLD} \) of a dynamically loaded Charpy V-notch specimen and the equivalent \( K_J \) values induced in a 0.4T SE(B) (a/W=0.5) specimen was developed using a Weibull stress scaling method. The dynamic Charpy V-notch FE model was used for this stage of the work, the numerical method for this specimen geometry has been described in Section 3.3.2.

The dynamic correlation method used a cleavage initiation parameter. This parameter was derived from analysis of cleavage initiation site locations in Charpy V-notch specimens (Section 3.2.4). The critical microstructural location of these events was defined numerically on the notch bisector of the Charpy V-notch FE model at a distance ahead of the notch tip, R, equal to \( \psi \) (shown schematically in Figure 3-18).

The primary effect of \( \dot{\varepsilon} \) on cleavage fracture behaviour was assessed within the Weibull stress model using a characteristic strain rate, \( \dot{\varepsilon}_{pl,\psi} \), corresponding to the most probable cleavage initiation site directly ahead of the V-notch tip (Lin and Evans, 1986). Figure 3-19 shows the determination of \( \dot{\varepsilon}_{pl,\psi} \) using numerical analysis of the principal plastic strain rate field (Section 3.3.6). \( \dot{\varepsilon}_{pl,\psi} \) was then applied to develop a modified \( U_{el+pl,LLD} \) scaling scheme for cleavage initiation for the specific purpose of scaling between the dynamic Charpy V-notch specimen (\( V_0=5,200\text{mm/s} \)) and quasi-static SE(B) (a/W=0.5). Section 3.1.3 describes the fitting procedure and material representation of strain rate dependent flow properties using Expressions (3-3) and (3-6) respectively. \( \dot{\varepsilon}_{pl,\psi} \) values at each loading increment of the FE analysis were used to determine a characteristic \( \sigma_0^{\text{dyn}} \) value for a Charpy specimen under impact loading conditions (corresponding to \( V_0=5,200\text{mm/s} \)). The scaling of \( U_{el+pl,LLD} \) using the modified Weibull stress model was made possible by normalising \( \sigma_W \) using the instantaneous dynamic strain rate dependent yield stress \( (\sigma_{Y,\dot{\varepsilon}}) \) corresponding to \( \dot{\varepsilon}_{pl,\psi} \) and the study material dynamic tensile flow properties.
Figure 3-18 Definition of Charpy V-notch NTOD ($\psi$) and the position ahead of the notch for calculation of $\dot{\varepsilon}_{pl,\psi}$ using FE analysis

Figure 3-19 Determination of $\dot{\varepsilon}_{pl,\psi}$ using the distribution of $\dot{\varepsilon}_{pl}$ ahead of a Charpy V-notch
The expression for $\sigma_{Y,2}$ is given by the following:

$$
\sigma_{Y,2} = \sigma_Y \left[ \frac{\sigma_0'}{\sigma_0} \left( \dot{\varepsilon}_{p,pl} \right) \right],
$$

(3.31)

where, $\sigma_Y = \text{the 0.002 strain offset yield stress}$. 

Figure 3-20 shows the normalised $\sigma_W$ trajectories for scaling $U_{el+pl,LLD}$ between the two geometries and loading conditions of interest. Upon scaling dynamic Charpy specimen $U_{el+pl,LLD}$ to an equivalent SE(B) ($a/W=0.5$) $U_{el+pl,LLD}$ value, this quantity was converted to an equivalent $K_J$ for the SE(B) specimen. The equivalent $K_J$ was derived using the same correlation method between $U_{el+pl,LLD}$ and $K_J$ determined from FE analysis of the SE(B) specimen as adopted for the quasi-static correlation. This methodology hence provided a complete routine to develop a $(K_J, U_{el+pl,LLD})$ relationship between dynamic Charpy V-notch $U_{el+pl,LLD}$ and the equivalent $K_J$ induced in a 0.4T size SE(B) specimen.

### 3.5.8 Dynamic Charpy V-notch Testing when Ductile Fracture Behaviour is Present

Charpy V-notch testing was undertaken using an identical method to Section 3.2.3. The test temperatures were 20, 40, 60 and 80°C. The purpose of this laboratory testing was to establish an accurate transition curve for the study material. An investigation was conducted concerning the transition curve shape.

A previously proposed form of expression given by EricksonKirk et al (2007) was fitted to the test data using Expression (3-32):
Figure 3-20 Weibull stress model scaling of $U_{\text{el+pl,LLD}}$ between the dynamic Charpy V-notch specimen and SE(B) specimen using the Weibull stress model

$$U_{\text{el+pl,LLD}} = \alpha + \beta \cdot \exp\left[\gamma (T - T_{\text{CVE}})\right], \tag{3.32}$$

The following terms within Expression (3-32) were defined:

$\alpha, \beta, \gamma$ = empirical constants,

$T_{\text{CVE}}$ = a reference temperature, set to the temperature at which $U_{\text{el+pl,LLD}}$ is equal to 28J.

The analytical curve fitting method concerning the Charpy impact test data over the transition temperature range, including the Charpy V-notch test results from Section (3.2.3), was conducted as follows:
1.) The Charpy impact data set was partitioned; specimen $U_{el+pl,LLD}$ results with brittle fracture appearance, $A_b$, less than 40% were identified and Charpy impact test results at the corresponding temperature were removed from the dataset.

2.) The remaining Charpy V-notch data were plotted as absorbed $U_{el+pl,LLD}$ (in Joules) versus $T$ (°C).

3.) An exponential curve [Expression (3-32)] was then fitted to the plotted Charpy impact data by determining the values of $\alpha$, $\beta$, $\gamma$ by using an iterative solver (Microsoft Excel Solver macro) to minimise the sum of the square error of the predicted curve relative to the partitioned data. $\alpha\geq0$ and $(\alpha+\beta)=28$ according to the definition of $T_{CVE}$.

### 3.5.9 Proposed Method for Correlating Dynamic Charpy V-notch $U_{el+pl,LLD}$ with Equivalent $K_J$ Through the Lower Ductile-to-Brittle Transition

Charpy V-notch transition curve data from Sections 3.2.3 and 3.5.8 and SE(B) fracture toughness analysis data of Section 3.5.1 were used to undertake preliminary work concerning a correlation between $U_{el+pl,LLD}$ and $K_J$ in the transition temperature range. This approach was premised upon the dynamic correlation method described in Section 3.5.7. A method to determine probabilistic predictions for both specimen geometries of interest was developed. A calibration method of the Weibull stress model at temperatures when Charpy impact test specimens exhibit significant ductile fracture behaviour was also incorporated as an integral part of the verification of the micromechanistic approach. The purpose of the work was the formation of an engineering correlation with sufficient flexibility to allow input of Charpy impact test $U_{el+pl,LLD}$ values to be associated to an equivalent $K_J$ corresponding to a predefined $P_f$. The range of applicability within the ductile-to-brittle transition temperature range of Charpy V-notch or SE(B) specimens was not investigated further.
3.6 References


4. Results

4.1 Materials Characterisation

4.1.1 Quasi-Static Tensile Properties

The quasi-static tensile testing data obtained by Horn and Sherry (2010) and the curve fitting results regarding this data for the study material are shown in Figure 4-1.

![Tensile test data for the study material at T=0°C and curve fitting results (Horn and Sherry, 2010)](image)

The material property parameters which were determined from curve fitting using Expressions (3-3) and (3-4) and the tensile testing results are given in Table 4-1. The definition of $\sigma_Y$ in Table 4-1, is the engineering definition of yield stress (defined at the 0.2% plastic strain).
4.1.2 Dynamic Tensile Properties

Dynamic tensile testing results for the study material are shown in Figure 4-2 which shows true stress-true strain data obtained from the tests. The material yielding was continuous in common with the quasi-static tensile test results (Section 4.1.1) and the yield stress increased with increasing $\dot{\varepsilon}$. Complete stress-strain curves for the highest strain rates of 10 and 200/s were obtained for characterising behaviour prior to the onset of necking.

Curve fitting results for the dynamic tensile testing experimental data are shown in Figure 4-2. The tensile strain range over which fitting was undertaken was $0.002<\varepsilon\leq0.08$ for tests corresponding to $\dot{\varepsilon}=0.01$ and 0.1/s respectively, $0.002<\varepsilon\leq0.15$ for the $\dot{\varepsilon}=10$/s tests and $0.04<\varepsilon\leq0.18$ for $\dot{\varepsilon}=200$/s tests. The $\dot{\varepsilon}=200$/s test results were fitted in accordance with the strain and strain rate distribution at the tip of a Charpy V-notch specimen under dynamic loading conditions (Section 4.3.6). The material parameters used for the dynamic material tensile property definition using Expression (3-3) are summarised in Table 4-2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Exp. (3-3)</th>
<th>Exp. (3-4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$ (MPa)</td>
<td>415</td>
<td>230</td>
</tr>
<tr>
<td>$\sigma_Y$ (MPa)</td>
<td>485</td>
<td>430</td>
</tr>
<tr>
<td>$A$</td>
<td>-</td>
<td>0.034</td>
</tr>
<tr>
<td>$n$</td>
<td>6.5</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Figure 4-3 shows the results of logarithmic fitting of $(\frac{\sigma_0}{\sigma_Y}, \dot{\varepsilon})$ data and Table 4-3 lists the values of the material constants which described the material parameter $\sigma_0$ under
dynamic conditions for the study material. The evolution of $\frac{\sigma_0'}{\sigma_0}$ in relation to $\dot{\varepsilon}$ for the study material is given in Figure 4-3.

![Figure 4-2 Results of dynamic tensile testing at $T=0^\circ$C of the study material and curve fitting](image)

**Table 4-2 Dynamic tensile material parameters using Expression (3-3)**

<table>
<thead>
<tr>
<th>$\dot{\varepsilon}$ (s$^{-1}$)</th>
<th>$\sigma_0$ (MPa)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>441</td>
<td>6.5</td>
</tr>
<tr>
<td>0.1</td>
<td>471</td>
<td>6.5</td>
</tr>
<tr>
<td>10</td>
<td>530</td>
<td>7.4</td>
</tr>
<tr>
<td>200</td>
<td>557</td>
<td>7.6</td>
</tr>
</tbody>
</table>

The experimental test curve fitting Expression (3-3) $n$ parameter values were found to marginally increase with strain rate. Expression (3-6) and the material constants shown in Table 4-3 were used to extrapolate material flow properties to $\dot{\varepsilon}=2,000$/s for the
The purpose of numerical modelling is to describe the material behaviour at the Charpy V-notch tip under dynamic conditions (Table 4-4).

![Figure 4-3 Dependence of $\frac{\sigma_0}{\sigma_0'}$ on $\dot{\varepsilon}$ for study material](image)

**Table 4-3 Material constants defining $\sigma_0(\dot{\varepsilon})$ using Expression (3-6)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D (/s)$</td>
<td>178, 290</td>
</tr>
<tr>
<td>$q$</td>
<td>7.01</td>
</tr>
</tbody>
</table>

**Table 4-4 Extrapolated dynamic material tensile parameters at $\dot{\varepsilon}=2,000/s$ using Expression (3-6)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$ (MPa)</td>
<td>619</td>
</tr>
<tr>
<td>$n$</td>
<td>7.8</td>
</tr>
</tbody>
</table>
4.1.3 Quasi-Static Fracture Toughness Properties

The results of fracture toughness testing to characterise the cleavage fracture behaviour of the study material are shown in Figure 4-4. The test temperatures for these laboratory tests were \( T = 20^\circ\text{C} \) and \( 40^\circ\text{C} \), cleavage failure loads were between 6.02-7.87 kN and 4.82-8.17 kN, respectively, at each of these temperatures. The critical J-integral at cleavage initiation \( (J_c) \) ranged between 0.029-0.041 J/mm\(^2\) at \( 20^\circ\text{C} \) and 0.050-0.083 J/mm\(^2\) at \( 40^\circ\text{C} \). The fracture toughness reference temperature \( (T_0) \) value for the steel was found to be 44.6°C when applying the multi-temperature master curve approach of ASTM E1921-11 (2011) to the SE(B) specimen test data. Probability bounds corresponding to \( P_f = 0.2 \) and 0.8 were assigned to the master curve to assess the study material fracture toughness data, these are shown in Figure 4-4.

![Figure 4-4 Results of quasi-static fracture toughness testing (1T size corrected) of the study material in the transition temperature range and ASTM E1921-11 (2011) analysis](image-url)
4.2 SE(B) Specimen Cleavage Fracture Test Programme

4.2.1 Quasi-Static SE(B) Test Geometries

*SE(B) (a/W=0.5, ρ=0mm) and U-notch SE(B) (a/W=0.5, ρ=0.12mm)*

The experimental test results for deeply fatigue pre-cracked Charpy sized SE(B) specimens of nominal crack depth ratio, \(a/W\), equal to 0.5 are shown in Figure 4-5. The test results for deep U-notch specimens of \(a/W=0.5\) and \(ρ=0.12\)mm are shown in Figure 4-6. The mean \(a/W\) ratio of the pre-cracked specimens was determined to be 0.47 following post-test measurement of the initial fatigue crack depths according to ASTM E1820-11 (2011). Larger clip gauge opening measurements at cleavage fracture initiation for the U-notch SE(B) specimens were measured and hence significantly larger absorbed energies were determined at failure for this specimen geometry compared to the pre-cracked SE(B) specimen geometry. The scatter of experimental cleavage fracture load measurements was also significantly reduced when testing the U-notch specimens compared to the pre-cracked geometry. This was attributed to the variation of crack depths measured in pre-cracked specimens compared to the tolerances achieved using EDM machining of the U-notch specimens. Ductile tearing was not evident during examination of the specimen fracture surfaces by optical microscopy (\(Δa<0.2\)mm).

*Shallow U-notch SE(B) (a/W=0.2, ρ=0.25mm), Shallow V-notch 0.4T SE(B) (a/W=0.2, ρ=0.25mm, ϕ=22°) and Charpy V-notch specimens*

The experimental test results for shallow U-notch SE(B) specimens of notch depth to specimen thickness ratio (\(a/W\)) of 0.2 and also shallow V-notch SE(B) specimens with ϕ equal to 22 and 45° are shown in Figure 4-7. All specimens failed by cleavage fracture during a rising load, post test examination of the fracture surfaces using optical microscopy showed that the fracture mode was transgranular cleavage with minimal ductile tearing (\(Δa<0.2\)mm).
Figure 4-5 Load versus clip gauge displacement data for 0.4T SE(B)

\((a/W=0.45-0.5)\)

Figure 4-6 Load versus clip gauge displacement data for U-notch SE(B)

\((a/W=0.5, \rho=0.124\text{mm})\)
Figure 4-7 Load versus clip gauge displacement data for U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$, $\phi=22^\circ$), U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) and Charpy V-notch specimens

4.2.2 Dynamic Charpy V-notch Impact Test Results

Instrumented Charpy impact test results for standard Charpy V-notch specimens which were tested at $V_0=5.3\text{m/s}$ are shown in Figure 4-8. All specimens failed by transgranular cleavage fracture with $\Delta a<0.2\text{mm}$. Significant oscillations were present in the test data due to inertia of the Charpy specimens during deformation. The scatter of experimental cleavage fracture results for these tests was exhibited as variations in $LLD$ at cleavage fracture initiation. The experimentally measured loads at fracture were a function of the position of cleavage fracture initiation relative to the peaks and troughs of the cyclic inertial forces acting on each test specimen. The experimentally measured values of $U_{el+pl,LLD}$ at cleavage fracture are given in Table 4-5. Observation of the
specimens’ fracture surfaces using travelling optical microscope confirmed that the average shear fracture appearance of the specimen set was less than 2%.

Figure 4-8 Load versus load line displacement data for dynamic Charpy V-notch specimens at T=0°C

Table 4-5 Experimentally measured values of dynamic Charpy V-notch $U_{eli+pl, LLD}$ at cleavage initiation

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>$U_{eli+pl, LLD}$ (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6YF5K1</td>
<td>8.8</td>
</tr>
<tr>
<td>6YF5K2</td>
<td>11.9</td>
</tr>
<tr>
<td>6YF5K3</td>
<td>8.6</td>
</tr>
<tr>
<td>6YF5K4</td>
<td>10.4</td>
</tr>
<tr>
<td>6YF5K5</td>
<td>8.3</td>
</tr>
<tr>
<td>6YF5K6</td>
<td>9.2</td>
</tr>
<tr>
<td>6YF5K7</td>
<td>10.1</td>
</tr>
<tr>
<td>6YF5K8</td>
<td>11</td>
</tr>
<tr>
<td>6YF5K9</td>
<td>7.9</td>
</tr>
<tr>
<td>6YF5K10</td>
<td>8.0</td>
</tr>
</tbody>
</table>
4.2.3 Fractography

Results from SEM analysis of fracture surface cleavage initiation sites of quasi-statically tested Charpy V-notch specimens are shown in Figure 4-9. Specimens which exhibited significant ductile tearing (which were discounted from the Section 4.2.2 results section) are shown for comparison purposes. These specimens were calculated as having significantly larger $U_{el+pl,LLD}$ values at cleavage initiation. It was found that cleavage initiation site $R_c$ measurements were between 560-600$\mu m$ for specimens failing by cleavage without significant ductile tearing ($\Delta a<200 \mu m$). Charpy V-notch specimens failing by cleavage fracture preceded by ductile tearing ($\Delta a>200 \mu m$) had cleavage initiation sites observed at positions in the range $97\leq R_c <312 \mu m$.

![Figure 4-9 SEM analysis results of quasi-statically tested Charpy V-notch specimens](image)

Figure 4-9 SEM analysis results of quasi-statically tested Charpy V-notch specimens
4.3 Finite Element Analysis

4.3.1 Quasi-Static FE models

Results of FE analysis of the SE(B) (\(a/W=0.5\)) experimental test geometry are given in Figures 4-5. Figure 4-5 shows that the calculated global quantities of load and clip gauge displacement were in agreement with the experimentally measured values. The FE results for the U-notch SE(B) (\(a/W=0.5, \rho=0.127\text{mm}\)) were found to be in good agreement with the experimental results (Figure 4-6).

Figure 4-7 shows that the FE models calculated values of load versus clip gauge opening displacement were in agreement with the experimental results for the U-notch SE(B) (\(a/W=0.2, \rho=0.25\text{mm}\)) and Charpy V-notch specimen geometry. The calculated loading magnitudes for a given displacement were found to be marginally larger for the Charpy V-notch specimen FE model in comparison to the U-notch SE(B) FE model.

4.3.2 Dynamic Charpy V-notch FE models

The results for FE modelling of a dynamic Charpy V-notch specimen are shown in Figure 4-10 in comparison to the experimental test results (Section 4.2.2) for this test geometry and loading condition. The FE model numerical results of global behaviour, when compared to the experimental \(U_{el+pl,LLD}\) values, had an maximum absolute error of 3.7%. Quasi-static Charpy V-notch model FE results are plotted in Figure 4-10 for comparison with the dynamic case. It was found that the dynamic loading rate increases the rate of increase of \(U_{el+pl,LLD}\) but decreases the experimental LLD values at cleavage fracture in comparison to the quasi-static Charpy V-notch case.
Figure 4-10 Evolution of $U_{el+pl,LLD}$ versus LLD for dynamic Charpy V-notch FE model and experimental data (including quasi-static)

4.3.3 Boundary Layer FE models

Unmodified boundary layer FE modelling results showed agreement between the HRR field and the $\sigma_{90}$ values calculated at $\frac{R\sigma_{90}}{J}$ distances along the $\theta=0^\circ$ symmetry plane of the model. Modified and unmodified boundary layer FE model validation results are shown in Figure 4-11, close agreement of the present study was evident in relation to the results of Sherry *et al* (2005a).
Figure 4-11 Validation results for unmodified and modified boundary layer FE models

4.3.4 J Calculation for SE(B) Specimens

J calculation results indicated that the calibrated $\eta_{pl}$ value was in agreement with the ASTM E 1820-11 (ASTM, 2011) method (Section 3.3.4) for determining $J$ in SE(B) laboratory specimens with a nominal $a/W$ ratio equal to 0.5 and achieved high accuracy (<1% error in relation to the contour integral FE analysis). In comparison, the ESIS P2/92 method (ESIS, 1992), using the calibrated $\eta_{el+pl}$ value, displayed a similar accuracy for the study material flow properties. Results for the U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) specimen indicated that the calibrated ESIS P2/92 method achieved higher accuracy than the calibrated ASTM E1820-11 method throughout the loading range using the material specific $\eta_{el+pl}$ value for this type of non-standard geometry.
4.3.5 **Quasi-Static Stress Fields**

The calculated mode I opening $\sigma_{\theta\theta}$ field for $\theta=0^\circ$ are shown in Figures 4-12a and 4-12b. Agreement with the theoretical HRR field was evident in Figure 4-12a corresponding to loading levels greater than $J> 18.2 \text{ N/mm}$, where $J_{z=0\text{mm}}$ is the maximum contour integral calculated at the centre of the crack-front line ($Z=0\text{mm}$). Figure 4-12b shows the peak stress falls below the HRR at positions along the crack front outside $Z/0.5B=0.3$ (where $Z$ is the distance from the crack-tip centre along the crack line).

The mode I opening stress distribution, $\sigma_{\theta\theta}$ ahead of the notch tip (at $Z=0\text{mm}$) of the U-notch SE(B) ($a/W=0.2$ and $\rho=0.25\text{mm}$) is shown in Figure 4-13. It was seen that the $\sigma_{\theta\theta}$ for $\theta=0^\circ$ is lower than the calculated HRR field. A maximum ratio of $\sigma_{\theta\theta}/\sigma_0$ of 3.3 is achieved during loading, which is approximately $0.8\sigma_0$ below the HRR field value. As expected, the HRR fields do not describe the stress distribution for this specimen geometry.

4.3.6 **Dynamic Stress Fields**

Analysis of the strain rate distribution at the notch tip of a dynamically loaded Charpy V-notch specimen was undertaken and results of this work are shown in Figure 4-14. It was determined that $\dot{\varepsilon}_{p,l,v}$ ranged between approximately 200-250/s during the loading range corresponding to a $\frac{\sigma_0}{\sigma_0}$ value (Section 4.1.2) of approximately 1.36 when fracture behaviour is dominated by cleavage fracture and no significant ductile tearing occurs.

FE analysis post processing results for dynamic stress fields corresponding to mode I opening ($\theta=0^\circ$ at $Z=0\text{mm}$) are given in Figure 4-15. The normalised $\left(\frac{\sigma_0}{\sigma_0}, \frac{R}{\psi}\right)$ dynamic stress fields were of the same magnitude as those calculated for the U-notch SE(B)
(a/W=0.2, ρ=0.25mm). The maximum $\dot{\varepsilon}$ (maximum principal component) at the notch tip (Z=0mm) was found to be approximately 3,000/s throughout the loading range, until $U_\text{el+pl,LLD}$ equalled a maximum of approximately 20 Joules. The maximum principal $\varepsilon_{pl}$ values were found to evolve linearly with $U_\text{el+pl,LLD}$ to a maximum value of approximately 0.65 at the notch tip corresponding to DY=1.4mm. High $\varepsilon_{pl}$ gradients along a $\theta=0^\circ$ trajectory at Z=0mm in close proximity of the Charpy V-notch tip were found to steeply reduce the $\dot{\varepsilon}$ values with distance from the notch tip.

![Graph](image_url)

**Figure 4-12 SE(B) (a/W=0.5) quasi-static stress fields, a.) $\sigma_{90}$ at $\theta=0^\circ$ and b.) the through thickness variation of $\sigma_{90}$**
Figure 4-13 U-notch SE(B) (a/W=0.2, ρ=0.25mm) quasi-static stress fields, σ_{θθ}, at θ=0°

Figure 4-14 $\dot{\varepsilon}_{pl,\psi}$ evolution at a position $\psi$ ahead of the Charpy V-notch tip and associated values of $\sigma'_0$

J_{Z=0mm} (N/mm)

- HRR Z=0mm
- 14.9
- 28.3
- 43.1
- 58.8
- 75.2
- 92.1
- 109.4
- 127.1

$\sigma_{θθ}/\sigma_0$

$\varepsilon = 189.11(U_{el+pl,LLD})^{0.0931}$
4.4 Cleavage Fracture Model

4.4.1 Calibration of the Cleavage Fracture Model

Weibull stress results for Figure 4-16 indicate that the SE(B) specimen 
(a/W=0.5) begins to lose constraint and depart from the SSY condition at a J loading 
level of approximately 20N/mm. The Weibull stress values for the shallow U-notch 
(a/W=0.2 and ρ=0.25mm) are significantly lower throughout the loading range than the 
SE(B) specimen. Departure from SSY occurs immediately in these non-standard 
specimens which is consistent with the stress field calculation results of Section 4.3.5. 
Table 4-6 provides a summary of the calibrated Weibull stress parameters which were 
calibrated using these geometries.
4.4.2 Determination of Weibull Scale Parameter, $\sigma_u$

The calibration activities regarding the two parameter Weibull stress model determined a value of $\sigma_u$ equal to 1,519.4MPa using the calibration method. Table 4-6 summarises the calibrated Weibull stress model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>15.1</td>
</tr>
<tr>
<td>$\sigma_u$ (MPa)</td>
<td>1,519.4</td>
</tr>
<tr>
<td>$\sigma_{W,\min}$ (MPa)</td>
<td>1,031.8</td>
</tr>
</tbody>
</table>

Two parameter Weibull stress model cleavage fracture predictions for the SE(B) ($a/W=0.5$) and U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) specimens in relation to
experimental test results for these specimen geometries are shown in Figure 4-17. The results were found to predict the median $K_{jc}$ value of the failure distribution with good accuracy (less than 5% error of predictions at $P_f=0.5$), as would be expected. The predictions of the model at lower failure probabilities were increasingly pessimistic at lower failure probabilities ($<P_f=0.5$). There was a 27.4% and 29.2% underestimation of the experimental rank probability $K_j$ values at $P_f=0.1$ for the SE(B) and U-notch SE(B) respectively.

![Figure 4-17](image)

**Figure 4-17** Two and three parameter Weibull stress model cleavage failure probability predictions compared with experimental data ($m=15.1$, $\sigma_0=1,519.4$MPa)

### 4.4.3 Three Parameter Weibull Distribution

Figure 4-17 shows the three parameter Weibull stress model predictions using a calibrated $\sigma_{w,min}$ value equal to 1,038.1MPa. The predictions overestimated the experimental rank probability $K_j$ values at both $P_f=0.5$ and 0.1. The overestimation was 23.5 and 16.4% at $P_f=0.5$ and 0.1 respectively for the SE(B) ($a/W=0.5$) specimen.
geometry respectively. The overestimation was 8.2 and 11.3% at $P_f=0.5$ and 0.1, respectively, for the U-notch SE(B) ($a/W=0.5, \rho=0.25\text{mm}$) geometry. The prediction errors of $K_{jc}$ at $P_f=0.1$ were significantly less in magnitude using the three parameter Weibull stress model than the two parameter model (Section 4.4.2). The error in predictions of the $K_{jc}$ at $P_f$ equal to 0.5 were significantly higher, possibly indicating that it is necessary to calibrate $\sigma_U$ and $\sigma_{W,\text{min}}$ together in a unified procedure.

Results from calibrating the Weibull stress model $m$ parameter using $U_{\text{el+pl,LLD}}$ as the loading parameter for the three geometries which were studied are given in Figure 4-18. The $R(m)$ results show that variations in $m$ are necessary to achieve the most accurate predictions of cleavage fracture depending upon the range of constraint represented between the SE(B) ($a/W=0.5$) and the geometry of interest. Table 4-7 summarises these calibration results concerning the Weibull stress model. It was seen that the $m$ parameter calibrated for the U-notch SE(B) ($a/W=0.2, \rho=0.25\text{mm}$) was not affected by the loading parameter used ($J$ or $U_{\text{el+pl,LLD}}$).

<table>
<thead>
<tr>
<th>Low constraint Geometry of interest</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charpy V-notch specimen</td>
<td>16.4</td>
</tr>
<tr>
<td>U-notch SE(B) ($a/W=0.2, \rho=0.25\text{mm}$)</td>
<td>15.1</td>
</tr>
<tr>
<td>U-notch SE(B) ($a/W=0.5, \rho=0.127\text{mm}$)</td>
<td>14.0</td>
</tr>
</tbody>
</table>

The results shown in Table 4-7 confirmed that the calibrated $m$ parameter is not affected by the loading parameter used for calibration ($J$ or $U_{\text{el+pl,LLD}}$).
4.5 Development of an Engineering Procedure for Charpy Specimen Data

4.5.1 Analysis of 0.4T SE(B) (a/W=0.5) Results

Results of fracture toughness testing of 0.4T SE(B) specimens are given in Table 4-8 which provides $a_0/W$ and $K_{jc}$ measurements for each specimen tested. The calculated ASTM E 1921-11 (2011) parameters corresponding to these tests are given in Table 4-9. The value of $T_0$ for this geometry of SE(B) specimen, $T_{0.0.4T \text{ SE(B)}}$ was found to be 31.1°C.

4.5.2 Calculation of J for U-notch SE(B) (a/W=0.5, $\rho=0.25\text{mm}$) Specimens

The analysis results in Table 4-10 show that $\eta_{\text{LLD}}$ is not dependent on material properties for the U-notch 0.4T SE(B) (a/W=0.5, $\rho=0.25\text{mm}$, $\phi=0^\circ$) specimen geometry. A material independent value of $\eta_{\text{LLD}}$ of 1.94 is accurate for this geometry of non-standard test specimen, this value is 3.0% lower than the value recommended in ESIS P2-92 (1992).

![Figure 4-18 Calibration results using $U_{\text{el+pl,LLD}}$ as a loading parameter](image-url)
Table 4-8 Fracture toughness testing results for 0.4T SE(B) \((a/W=0.5)\)

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>(a/W)</th>
<th>(K_{jc}) (MPa√m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6YF5K71</td>
<td>0.479</td>
<td>60.3</td>
</tr>
<tr>
<td>6YF5K73</td>
<td>0.490</td>
<td>71.0</td>
</tr>
<tr>
<td>6YF5K74</td>
<td>0.483</td>
<td>72.9</td>
</tr>
<tr>
<td>6YF5K75</td>
<td>0.452</td>
<td>68.8</td>
</tr>
<tr>
<td>6YF5K76</td>
<td>0.452</td>
<td>81.9</td>
</tr>
<tr>
<td>6YF5K77</td>
<td>0.487</td>
<td>73.7</td>
</tr>
<tr>
<td>6YF5K78</td>
<td>0.450</td>
<td>65.6</td>
</tr>
<tr>
<td>6YF5K79</td>
<td>0.451</td>
<td>81.3</td>
</tr>
<tr>
<td>6YF5K80</td>
<td>0.508</td>
<td>74.8</td>
</tr>
</tbody>
</table>

Table 4-9  ASTM E1921-11 results for 0.4T SE(B) \((a/W=0.5)\)

| ASTM E1921-11 (2011) Parameters for 0.4T SE(B) \((a/W=0.5)\) at \(T=0°C\) |
|-----------------|------------------|
| \(K_{jc,lim}\) (MPa√m) | 125.2 |
| \(K_0\) (MPa√m)       | 73.4  |
| \(K_{jc,med}\) (MPa√m) | 68.8  |
| \(T_{0.0.4T SE(B)}\) (°C) | 31.1  |
Table 4-10 $\eta_{LLD}$ solutions using Expression (3-22) for a range of postulated material flow properties

<table>
<thead>
<tr>
<th>$E/\sigma_Y$</th>
<th>$n$</th>
<th>$\eta_{LLD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>5</td>
<td>1.943</td>
</tr>
<tr>
<td>800</td>
<td>5</td>
<td>1.944</td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>1.936</td>
</tr>
<tr>
<td>500</td>
<td>20</td>
<td>1.935</td>
</tr>
</tbody>
</table>

4.5.3 Material Dependent J Solution for U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$)

The results of material specific analyses work concerning the U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) $\eta_{LLD}$ solutions for use within Expression (3-22) are shown in Figure 4-19. This plot shows that the value of $\eta_{LLD}$ is not significantly dependent on the value of $\sigma_Y$ for a given material property definition, but the value is affected by $n$.

![Figure 4-19 Results of $\eta_{LLD}$ material parameter study for use with Expression (3-22) for a U-notch 0.4T SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$, $\phi=0^\circ$) specimen geometry](image-url)
A solution for $\eta_{LLD}$ which is a function of $n$ for the U-notch 0.4T SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$, $\phi=0^\circ$) specimen geometry was determined by averaging the analyses results corresponding to each value of $n$. A polynomial fit was used to describe the increase of $\eta_{LLD}$ with $n$. Figure 4-20 shows the results of this work.

![Figure 4-20 Material strain hardening property dependent solution for $\eta_{LLD}$ for a U-notch 0.4T SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$, $\phi=0^\circ$) specimen geometry](image)

The following expression was used to calculate the value of $\eta_{LLD}$ for the U-notch 0.4T SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$, $\phi=0^\circ$) for the ferritic steel study material of this work:

$$\eta_{LLD} = 0.00005n^3 - 0.0025n^2 + 0.0477n + 1.2177$$

$$R^2 = 0.9985$$

for $1 \leq n \leq 18$

$$\eta_{LLD} = 1.558$$

for $n > 18$ \hspace{1cm} (4-1)

Using Expression (4-1) it was determined that $\eta_{LLD}$ was equal to 1.436 for the study material. This value of $\eta_{LLD}$ was therefore used to calculate $J_c$ for each experimental U-
notch 0.4T SE(B) \((a/W=0.2, \rho=0.25\text{mm}, \phi=0^\circ)\) test result. \(J_c\) was thereafter converted to the elastic-plastic stress intensity factor, \(K_{jc}\) using the following plane strain formula:

\[
K_{jc} = \sqrt{\frac{J_c E}{1 - \nu^2}},
\]

(4-4)

where,

\(K_{jc}\) = a equivalent elastic plastic stress intensity factor,

\(J_c\) = the critical value of J-integral at cleavage fracture,

\(E\) = Young’s modulus of elasticity,

\(\nu\) = Poisson’s ratio.

### 4.5.4 Application of Weibull Stress Model to U-notch SE(B) Specimens

Figure 4-21 shows the corresponding \(P_f\) predictions using the Weibull stress model calibration results when using a scaling approach derived using three dimensional modelling (Section 3.5.4). The solid lines, corresponding to the calibrated \(m\) parameter, indicate a small underprediction of median failure probability predictions between the present experimental results and the model predictions (approximately 5% for all geometries studied). These predictions become less accurate and increasingly non-conservative at low failure probabilities \((P_f<0.2)\). Predictions for a low constraint geometry studied by Horn and Sherry (2012a) are shown using the calibrated \(m\) parameter of those authors \((m=19.9)\). The dashed lines in Figure 4-21 show the predictions of the Weibull stress model using the Charpy V-notch geometry calibrated \(m\) parameter \((m=16.4)\). These results using \(m=16.4\) showed that the Weibull stress model produces slightly conservative predictions of cleavage fracture (when scaling from the low constraint to the high constraint SE(B) \((a/W=0.5)\) geometry) provided that the geometry is within the range of constraint represented by the calibration geometries. Outside of the range of constraint used for calibration, as indicated by the geometry
studied by Horn and Sherry (2012a)(2012b), the predictions are highly non-conservative and re-calibration is necessary.

![Figure 4-21 Cleavage fracture predictions for U-notch SE(B) geometries](image)

Figure 4-22 shows predictions from the Weibull stress scaling model for the 0.4T SE(B) ($a/W=0.5$) and U-notch SE(B) ($a/W=0.2$, $\rho=0.25$mm). Confidence bounds (0.1 and 0.9) for both low constraint specimen geometries are shown for the purpose of assessing the cleavage fracture probabilistic prediction accuracy. All Charpy V-notch specimen data fell within the confidence bounds. The numerical predictions regarding the shallow U-notch specimen indicate an overestimation of approximately 5% of the experimental $K_I$ values at $P_f=0.5$ for this geometry as a consequence of the $m=16.4$
parameter being calibrated using the quasi-static Charpy V-notch experimental data. The predictions are in good agreement with the bounds of model predictions at lower failure probabilities ($P_f=0.05-0.2$). Results of a combined constraint and notch correction are plotted in Figure 4-22 using the method of Horn and Sherry (2012a)(2012b). The parameters and adjustment factors used for the constraint and notch correction are given in Table 4-11 corresponding to the U-notch SE(B) geometry, further details concerning are given in Appendix A.

**Table 4-11 Combined notch and constraint correction parameters**

<table>
<thead>
<tr>
<th>Material fracture toughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{mat}$ (MPa√m)</td>
</tr>
</tbody>
</table>

**Constraint Correction (Sherry et al, 2005a)**

<table>
<thead>
<tr>
<th>0.4T SE(B) (a/W=0.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{CMOD}$ ASTM E 1820-11</td>
</tr>
<tr>
<td>$\beta_T$</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$k$</td>
</tr>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>$P_L$ (N)</td>
</tr>
</tbody>
</table>

**Notch Correction (Horn and Sherry, 2012a)**

<table>
<thead>
<tr>
<th>0.4T U-notch SE(B) (a/W=0.5, $\rho=0.25$mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{LLD}$</td>
</tr>
<tr>
<td>$\beta_N$</td>
</tr>
<tr>
<td>$\zeta$</td>
</tr>
<tr>
<td>$\lambda$</td>
</tr>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>$P_L$ (N)</td>
</tr>
</tbody>
</table>

**Combined Notch and Constraint Correction (Horn and Sherry, 2012b)**

<table>
<thead>
<tr>
<th>$\eta_{LLD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.44</td>
</tr>
</tbody>
</table>

**Constraint and notch correction factors**

| $[1+\alpha(-\betaTL)]$ | 1.43 |
| $[1+\gamma(\lambda/\beta_N)]$ | 1.56 |
Figure 4-22  U-notch SE(B) \((a/W=0.2\) and \(\rho=0.25\text{mm}\)) cleavage fracture predictions compared to method of Horn and Sherry (2012a)

The analysis results show a significant underprediction of the experimental \(K_{jc}\) results when using the combined notch and constraint correction. This means that scaling of experimental results for the U-notch SE(B) would be significantly non-conservative when calculating equivalent SE(B) \(K_{ij}\) values using the approach of Horn and Sherry (2012a). Therefore, such a model is not appropriate for use with an engineering procedure for deriving equivalent \(K_{jc}\) values from Charpy V-notch specimen data.
4.5.5 **Direct Correlation of Equivalent SE(B) \((a/W=0.5)\) Specimen \(K_J\) with Quasi-static Charpy V-notch Specimen \(U_{el+pl,LLD}\)**

Weibull stress model results for the U-notch SE(B) \((a/W=0.2, \rho=0.25\text{mm})\) and the Charpy V-notch specimen are shown in Figure 4-23. Ten and ninety percent confidence bounds for a sample set of \(N=10\) are shown also in Figure 4-23 and these confirm that the numerical model \((P_f, U_{el+pl,LLD})\) predictions are in agreement with the experimental Charpy V-notch \((P_{f,\text{Rank}}, U_{el+pl,LLD})\) quantities. The error is less than 5\% for \(0.2\leq P_f < 0.8\). At low failure probabilities \((P_f<0.2)\) the predictions of the model are difficult to exactly an error upon, but the experimental data falls within the 10 and 90\% confidence intervals.

The model predictions were shown to be conservative for the U-notch SE(B) \((a/W=0.2, \rho=0.25\text{mm})\). A 9.3\% overprediction of this specimen’s \(U_{el+pl,LLD}\) at cleavage fracture at \(P_f=0.5\) occurred at \(P_f=0.1\) there was a 12.7\% underprediction of the two relevant experimental points [when interpreting linear interpolation of \((U_{el+pl,LLD}, P_{f,\text{Rank}})\) results] in this range of \(P_{f,\text{Rank}}\). The experimental data points corresponding to \(P_f\) values below 0.5 exhibited significant scatter and equalled the 0.1 and 0.9 confidence bounds at values of \(P_{f,\text{Rank}}\) corresponding to 0.452 and 0.067 respectively.

Errors in predictions for the U-notch SE(B) and Charpy V-notch specimen were considered to be relatively insignificant owing to the initial deficiencies of the ASTM E1921-11 methodology adopted to fit the pre-cracked SE(B) \((a/W=0.5)\) test results. The errors when using this method were largest at low \(P_f\) values, corresponding to an 21\% underprediction in comparison to the lowest experimental test results at \(P_{f,\text{Rank}}=0.075\).

Overall, the extension of the ASTM E1921-11 cleavage fracture model to describe U-notch SE(B) and Charpy V-notch specimen fracture behaviour at quasi-static rates of loading was found to provide a good description of the spread of measured \(K_J\) and \(U_{el+pl,LLD}\) quantities. Sources of conservatism or non-conservatism of the Weibull stress scaling approach arose from the specified \(m\) parameter value. The quasi-static
Figure 4-23 Weibull stress model quasi-static predictions using direct correlation method
correlation results are presented as the equivalent $K_{Jc}$ for a 0.4T SE(B) versus Charpy V-notch $U_{el+pl,LLD}$ in Figure 4-24.

4.5.6 Direct Correlation of Equivalent SE(B) (a/W=0.5) Specimen $K_j$ with Dynamic Charpy Impact Test $U_{el+pl,LLD}$

Figure 4-25 shows the numerical Weibull stress predictions for correlating dynamic Charpy V-notch specimen $U_{el+pl,LLD}$ with $K_j$ of a 0.4T SE(B) specimen (using $m=16.0$) in comparison to Charpy impact test experimental data (Section 4.2.2). The $m=16.4$ predictions of the model corresponding to the quasi-static condition, previously given in Section 4.5.5, are also shown for comparison purposes. It was seen that the dynamic
loading condition results in a reduction of experimental and numerically predicted \( U_{el+pl,LLD} \) quantities.

![Graph](image)

**Figure 4-24 Correlation of quasi-static Charpy V-notch \( U_{el+pl,LLD} \) with equivalent \( K_{Jc} \) for a 0.4T SE(B) specimen using material specific approach**

At \( P_f \) values greater than 0.5, the model increasingly predicts higher \( U_{el+pl,LLD} \) values in comparison to the dynamic Charpy V-notch experimental data, the three largest measured \( U_{el+pl,LLD} \) values were close to the 0.1 confidence bound. This indicates conservatism of the dynamic model at higher energies (8-20 Joules) using the calibrated \( m \) parameter. At lower failure probabilities (\( P_f<0.1 \)) the approach is significantly less conservative. Significantly more experimental test results and validation would be required to ascertain the performance of the model at low failure probabilities.
Figure 4-25 Weibull stress model dynamic predictions using direct correlation method

The dynamic Weibull stress correlation work found that, as was the case for the quasi-static correlation, the value of $m$ used for the correlation has a significant affect on accuracy of the approach. The predictions using $m=16$ produced accurate predictions in view of the initial master curve cleavage fracture model limitations. To ascertain the appropriate value of $m$ to be adopted for Charpy V-notch correlations would require increased numbers of dynamic Charpy V-notch test results. The present results indicate that a value of $m=16$ is a suitable value to ascribe for use in dynamic correlations for this study material. Improvements to the accuracy of the correlation at $P_f$ values of less than 0.1 would require amendment of the master curve cleavage fracture model shape.
parameter (set at a value of 4 within the existing E 1921-11 standard). The correlation results are presented as the equivalent $K_{JC}$ for a quasi-static 0.4T SE(B) versus the dynamic Charpy V-notch $U_{el+pl,LLD}$ in Figure 4-26.

![Figure 4-26 Correlation of dynamic Charpy V-notch $U_{el+pl,LLD}$ with equivalent $K_{JC}$ for a 0.4T SE(B) specimen using material specific approach](image)

The validity of the dynamic correlation and its linkage with the master curve methodology is a useful result finding which validates the applicability of the dynamic Charpy V-notch correlation method to other ferritic steels. The accuracy of the correlation methodology when scaling between geometries at other temperatures in the ductile-to-brittle transition was deemed to be largely dependent on the value of $m$ adopted. Further investigation is necessary concerning the fixing of a common temperature-independent and geometry-independent value of $m$, suitable for ferritic steels, which can provide accurate equivalent $K_{JC}$. 
4.5.7 Dynamic Charpy V-notch Testing when Ductile Fracture Behaviour is Present

Results of instrumented Charpy impact testing in the ductile-to-brittle transition for the study material are shown in Figure 4-27 corresponding to values of $T$ equal to 20, 40 and 60°C respectively. The resulting measured $U_{el+pl,LLD}$ values of the laboratory testing are given in Table 4-12.

Figure 4-27 Instrumented Charpy impact testing results at temperatures with ductile fracture behaviour present, a.) 22°C, b.) 40°C and c.) 60°C
Table 4-12 Experimentally measured values of total Charpy V-notch $U_{el+pl,LLD}$ at cleavage initiation

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>T (˚C)</th>
<th>$U_{el+pl,LLD}$ (J)</th>
<th>T (˚C)</th>
<th>$U_{el+pl,LLD}$ (J)</th>
<th>T (˚C)</th>
<th>$U_{el+pl,LLD}$ (J)</th>
<th>T (˚C)</th>
<th>$U_{el+pl,LLD}$ (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
<td>19.9</td>
<td>40</td>
<td>32.7</td>
<td>60</td>
<td>47.3</td>
<td>80</td>
<td>73.2</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
<td>17.3</td>
<td>40</td>
<td>27.7</td>
<td>60</td>
<td>49.6</td>
<td>80</td>
<td>85.4</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>15.1</td>
<td>40</td>
<td>27</td>
<td>60</td>
<td>46.7</td>
<td>80</td>
<td>69.6</td>
</tr>
<tr>
<td>4</td>
<td>22</td>
<td>15</td>
<td>40</td>
<td>24.7</td>
<td>60</td>
<td>53.4</td>
<td>80</td>
<td>74.5</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
<td>13.8</td>
<td>40</td>
<td>28.9</td>
<td>60</td>
<td>39</td>
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<td>62.5</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
<td>15</td>
<td>40</td>
<td>27.6</td>
<td>60</td>
<td>47.1</td>
<td>80</td>
<td>91.5</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
<td>14.4</td>
<td>40</td>
<td>36.5</td>
<td>60</td>
<td>44.3</td>
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<td>80.6</td>
</tr>
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<td>8</td>
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<td>12.9</td>
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<td>27</td>
<td>60</td>
<td>39.5</td>
<td>80</td>
<td>81.5</td>
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<td>9</td>
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<td>21.4</td>
<td>60</td>
<td>53.3</td>
<td>80</td>
<td>81.4</td>
</tr>
<tr>
<td>10</td>
<td>22</td>
<td>14.6</td>
<td>40</td>
<td>23.4</td>
<td>60</td>
<td>43.4</td>
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<td>62</td>
</tr>
</tbody>
</table>

The Charpy impact testing results showed that increasing amounts of ductile fracture behaviour were prevalent at these temperatures. This behaviour was evident as ductile tearing and shear lip formation on the sides and back face of the specimens. The specimens exhibited increasing amounts of $U_{el+pl,LLD}$ with temperature and increasing scatter of results which was attributed to variations in the amounts of ductile tearing. An assessment of broken specimen fracture surfaces using an optical microscope indicated that at $T=20$, 40, 60 and 80˚C, the average shear appearance was approximately 5.0, 9.5, 18.0 and 39.5 percent respectively. Increased variability of measured $U_{el+pl,LLD}$ was recorded for the set of specimens tested at 80˚C.

Figure 4-28 shows the total measured $U_{el+pl,LLD}$ values versus $T$, this figure demonstrated an exponential temperature dependence of $U_{el+pl,LLD}$ which was found to be in close agreement with that found by EricksonKirk et al (2007).

The parameters of Expression (3-32) determined by EricksonKirk et al (2007) using the United States Nuclear Regulatory Commission (USNRC) Database are also provided in Table 4-13.

The values of $T_{CVE}$ calculated for the study material using the present study’s fitted
Figure 4-28 Measured Charpy V-notch impact test transition curve

exponential curve shape and that determined by EricksonKirk et al. are provided in Table 4-14. The values of $T_{CVE}$ calculated for the study material were only 4.5% different providing support to the empirical studies of EricksonKirk et al.

Table 4-13 Expression (3-32) fitting parameters determined for the study material and empirical USNRC database

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Study material</th>
<th>EricksonKirk et al (2007) USNRC database</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0261</td>
<td>0.0245</td>
</tr>
</tbody>
</table>
Table 4-14 Values of $T_{CVE}$ calculated for the study material

<table>
<thead>
<tr>
<th></th>
<th>$T_{CVE}$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>41.4</td>
</tr>
<tr>
<td>EricksonKirk et al (2007)</td>
<td>39.6</td>
</tr>
</tbody>
</table>

Maximum differences of predicted $U_{et+p,LLD}$ between the exponential curve fit for the study material and that proposed by EricksonKirk et al occurred at the lower shelf and intermediate transition regions, they were 10.1% and 6.2% respectively for the temperature range shown in Figure 4-28.
4.6 References


5. Discussion

This study has undertaken experimental testing and analysis work concerning standard SE(B) fracture toughness specimen geometry and new research work concerning non-standard SE(B) geometries. Additionally, Charpy V-notch specimens were tested at quasi-static and dynamic loading rates. The purpose of the work is to achieve the objectives of the thesis concerning the development of an engineering procedure to enable use of Charpy V-notch data to determine material fracture toughness for structural integrity assessments. Numerical work concerning the establishment of a mechanistic foundation for the procedure has been undertaken and the accuracy of predictions are appraised within the context of existing fracture toughness testing standards and structural integrity assessment (SIA) guidance. Two proposed options are recommended for furthering the research towards the objective of understanding theoretically the lower and middle ductile-to-brittle transition regions of Charpy V-notch and SE(B) specimens for ferritic steels. Additionally, the Discussion provides further information concerning how an accurate engineering solution for use by engineers undertaking SIA’s may be obtained using the dynamic Charpy V-notch correlation results.

5.1 SE(B) Specimen Test Programme

The test programme (Section 3.2.1) characterised the effect of individual geometric differences between a pre-cracked SE(B) \((a/W=0.5)\) and Charpy V-notch specimen on fracture behaviour. This programme was conducted and load-clip gauge opening measurements made. This experimental data provided a unique insight concerning the fracture behaviour of these specimen geometries. It was found that each geometric difference contributed to the cleavage fracture \(U_{el+pl,LLD}^{cleavage}\):

- notch depth difference of \(a/W=0.2\) and 0.5,
- notch acuity difference of \(\rho=0.25\text{mm}\) and 0mm (fatigue pre-crack),
- notch flank angle differences of $\phi=45^\circ$ and $0^\circ$.

Figure 5-1 summarises the SE(B) test programme results concerning effect of geometric differences between the SE(B) and Charpy V-notch specimen.

![Graph showing the effects of geometric differences between SE(B) and Charpy V-notch specimens](image)

**Figure 5-1 Effect of individual geometric differences between a SE(B) and Charpy V-notch specimen**

The discrete role of each geometric difference is highlighted by the $(P_{f,\text{Rank}}, U_{\text{el+pl,LLD}})$ experimental results of Figure 5-1. The main observations of this experimental programme can be summarised as follows:

- The effect of notch $\rho$ causes an approximate doubling of $U_{\text{el+pl,LLD}}$ during the transition from a pre-cracked SE(B) ($a/W=0.5$, $\rho=0\text{mm}$) to a U-notch SE(B) ($a/W=0.5$, $\rho=0.127\text{mm}$).
• An approximate trebling of $U_{el+pl,LLD}$ is observed when the notch depth is decreased during the transition of U-notch SE(B) specimens of $\rho=0.127\text{mm}$ from $a/W=0.5$ to $a/W=0.2$ respectively.

• The change of $\rho$ between shallow U-notch SE(B) specimens of $a/W=0.2$ from $\rho=0.127$ to $\rho=0.25\text{mm}$ caused an approximate doubling of $U_{el+pl,LLD}$.

• The effect of changing $\phi$ for shallow U-notch SE(B) specimens of $a/W=0.2$ from $\phi=0^\circ$ to $22^\circ$, and to $45^\circ$ (corresponding to the standard Charpy V-notch specimen geometry) caused further increases of $U_{el+pl,LLD}$, but to a far less extent than the previously observed geometric changes. The shallow V-notch SE(B) specimen and Charpy V-notch specimen exhibited apparent increases in $U_{el+pl,LLD}$ when compared at a failure proximity corresponding to $P_f=0.5$.

• The measured $U_{el+pl,LLD}$ values for the SE(B) specimens of $a/W=0.2$ and $\rho=0.25\text{mm}$ exhibited greater scatter and also therefore an overlap of the experimentally measured $U_{el+pl,LLD}$ at low $P_f$ values ($P_f<0.5$). At higher $P_f$ values ($>0.8$) the difference of $U_{el+pl,LLD}$ between the shallow U-notch SE(B) of $\phi=0^\circ$ and the Charpy V-notch specimen was approximately 25%, which indicates that constraint loss increases the magnitude of the notch flank angle effect on fracture behaviour for these specimen geometries.

These experimental observations motivated work to develop a sequential engineering procedure to quantify the distinct geometric effects on cleavage fracture behaviour of SE(B) and Charpy V-notch specimens.

5.2 Stress Field Analysis of Standard and Non-Standard SE(B) Specimens

The fracture mechanics parameter adopted by testing standards to characterise cleavage fracture behaviour for standard geometries is currently $J$. This parameter was found to be a suitable parameter to describe the stress distribution and ahead of the crack tip in the standard SE(B) specimen (Section 4.3.5). Section 4.3.6 showed that the
U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) was not described by the equivalent $J$ value determined using the numerical estimation scheme. The stresses ahead of the shallow U-notch were significantly lower than the HRR field quantities for Mode I opening. As such, this geometry did not exhibit one parameter characterisation and hence fundamental difficulties are encountered when seeking to characterise $K_{\text{mat}}$ using equivalent $J$ quantities. By comparison of Figures 4-12 and 4-13 it was seen that application of the Q-stress to an elastic-plastic stress analysis of the U-notch specimen geometry would also be problematic. This is because the dimensionless $RJ/\xi_0$ length scale used to characterise the stresses ahead of a crack does not achieve self similarity of the field for U-notch specimens. The U-notch specimen stress fields ahead of the notch tip were seen to be stretched in the notch bisector plane because of the presence of the free surface at the notch tip. This raises a fundamental difficulty when applying a second parameter such as the Q-stress to characterise such stress fields. This difficulty arises because the HRR solution is therefore not valid and hence the Q-stress approach of O’Dowd and Shih (1991), or those based on higher order terms of the HRR field (Yang et al, 1993)(Crane and Anderson, 1993), are not possible. Horn and Sherry (2010) reached similar conclusions and suggested that the Q-stress was not applicable since there was not a characteristic distance ahead of the U-notch where the Q stress could be defined. The U-notch fields of the present work were found to be significantly more ‘broad’ than the sharp crack-tip stress fields and hence the interaction of such fields with global bending stresses also was expected to exercise more influence on the stresses from early stages of loading in finite SE(B) specimen geometries. In the absence of an elastic-plastic solution for U-notch stress fields, characterisation of such fields remains out of reach of the engineering practitioners. These reasons motivated the use of the Weibull stress model to describe cleavage fracture behaviour, and engineering procedure which was developed using this approach is discussed in Section 5.4.
5.3 Difficulties Introducing U-Notch Specimen Geometries into Existing Fracture Toughness Testing Standards

The current basis of fracture mechanics experimental testing is founded on a one parameter assumption, and this predicate has driven the development of such standards. The rules in ASTM E 1921-11 (2011) regarding specimen sizing rules and deformation limits (the $M$ parameter) are borne out of the need for SSY conditions to prevail at a loaded crack tip to enable characterisation of crack driving forces. The Weibull stress toughness scaling model provided a means of circumventing the violation of the SSY assumption which is a feature of U-notch SE(B) specimen fracture behaviour. Horn and Sherry (2012a)(2012b) provided an engineering approach to undertake a ‘correction’ of $K_{\text{mat}}$ to account for the effects of finite $\rho$ on experimentally measured apparent $K_J$ values corresponding to test geometries with such defects. The increased values of $K_{J,\rho}$ (in comparison to standard $K_J$ measurements) for test U-notch geometries was also evident during this work (Section 4.5.4), the context of the present work was to determine the SSY $K_J$ corresponding to the SE(B) geometry and hence a ‘scaling down’ of measured apparent $K_J$ values was inherently necessary. The engineering procedure of Horn and Sherry premised that $K_{\text{mat}}$ should be increased within current failure assessment procedures such as R6 (2005); whereas this work seeks to establish the most reliable correlation of $K_J$ corresponding to the SE(B) geometry. The issue of determining equivalent $K_J$ values corresponding to standard fracture mechanics specimens from non-standard specimens was therefore undertaken.

5.4 Development of an Engineering Procedure to Relate the Cleavage Fracture Behaviour of SE(B) and Charpy V-notch Specimens

The observations arising from the experimental programme concerning Charpy V-notch and SE(B) specimens were deemed to provide significant evidence that individual geometric effect could be individually quantified within an engineering framework. The prospective procedure was conceived to be a multi-step procedure to
convert Charpy V-notch specimen $U_{el+pl,LLD}$ to 0.4T SE(B) specimen $K_J$ to which accounted for the following salient geometric effects on cleavage fracture behaviour:

- a transition of notch depth change from $a/W=0.5$ to 0.2,
- a change of notch acuity from $\rho=0\text{mm}$ to 0.25mm
- a transition of notch flank angle from $\phi=0^\circ$ to 45°.

Initial research efforts were therefore directed towards developing a systematic engineering procedure which was composed of the following steps:

1. A combined notch and constraint correction of SE(B) ($a/W=0.5$) $K_J$ to the equivalent shallow U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) $K_J$.
2. A conversion of constraint and notch corrected U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) $K_J$ to the associated $U_{el+pl,LLD}$.
3. A flank angle correction of U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) of $\phi=0^\circ$ to a standard Charpy V-notch specimen of $\phi=45^\circ$.
4. A loading rate correction of quasi-static Charpy V-notch specimen $U_{el+pl,LLD}$ to the equivalent dynamic $U_{el+pl,LLD}$ corresponding to Charpy impact test results of standard size specimens.

The above sequential procedure was devised for the purpose to establish a new engineering procedure which was capable of rationalising each geometric effect. The quasi-static steps of the outline procedure was amenable to experimental validation using the results from the comprehensive SE(B) test programme undertaken in Section 5.1.

5.4.1 **Step 1: Combined Notch and Constraint Correction**

A combined notch and constraint correction was necessary to quantify the increase of $K_J$ when undertaking a transition from a SE(B)($a/W=0.5$) specimen to a U-notch SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$). Section 3.3.4 developed and validated an accurate
numerical approach to calculate $J$ for both of these specimen geometries using modified $\eta$ factors within existing fracture toughness test standards ASTM E 1820-11 (2011a) and ESIS P2-92 (ESIS, 1992). The analyses shown in Figure 3-11 showed that the error of the FE approach was very low (<2%) over a large range of loading levels which were relevant to cleavage fracture behaviour of the SE(B) specimen geometries. This confirmation allowed continuation of the development, two options were available to undertake the constraint and notch correction activity:

- use of an existing notch and constraint correction methodology given by Horn and Sherry (2012a) (2012b),
- development of a new numerical method to achieve this objective.

The first approach using the approach of Horn and Sherry was implemented using the material flow property data from Section 3.1.2 and also by undertaking supporting FE analysis (as detailed in further detail in Appendix A). The implementation of the procedure of Horn and Sherry to achieve Step 1 of the engineering procedure it was necessary to be in possession of knowledge concerning the Weibull stress model $m$ parameter. Initially, this parameter was set to be a value of 20 for use within the engineering procedure and for validation using experimental results. The value of $m=20$ was employed initially because of previous research of Horn and Sherry (2010) using the study material which established accurate predictions of cleavage fracture for SE(B) specimens of a wide constraint range were achieved using material parameter. It was found that predictions for the U-notch SE(B) ($a/W=0.2, \rho=0.25\text{mm}$) test data were possible when a scaling of the ASTM E1921-11a expression for probability of failure (Expression 3-23) was undertaken using $m=20$. Figure 5-2 shows the initial results for the notch and constraint correction when using a value of $m$ equal to 20 for the study material, it was seen that the scaled results are indeed in good agreement with the shape of the U-notch SE(B) ($a/W=0.2, \rho=0.25\text{mm}$) test results. There was a noticeable underprediction of the experimental results at low failure probabilities of $P_f<0.2$ of
approximately 21% in comparison to the experimental data for both specimen geometries. This disagreement of the mechanistic model and experimental results was apparently because of the inherent assumptions of the ASTM E1921-11 model for cleavage fracture. When applying this model to a specific material, and, in the presence of limited experimental data there is also an uncertainty. The ASTM cleavage fracture model was derived from studies concerning a large group of ferritic steels. For low specimen data sets, the scatter of the shape parameter was found to significantly fluctuate during these studies. Hence, there is also a strong argument that the present study, which used \( N=10 \) for each geometry, does not provide a sound basis to undertake development of the ASTM model. In the case of this study it is sufficient to draw the conclusion that the correlation is therefore accurate to describe the \( P_f \) distribution of steels in the absence of such large data sets. Altering the model parameters to better describe the present study material cleavage fracture properties may in turn lead to a deterioration of the predictions when the model is extended to a wide range of steels used industrially. More research concerning definition of the master curve \( b \) parameter is justified to closer denote this parameter. Currently, only material specific studies have been completed following the development of the master curve cleavage fracture model and therefore, it would be necessary to reassess this model on the basis large studies pertaining to specific steels. This would provide the opportunity to more accurately denote \( b \) for grades of pressure vessel steels and also structural steels. Such a study would certainly be relevance to the understanding of ferritic steel fracture behaviour in the ductile-to-brittle transition regime. The focus of such a study should be the more accurate description of \( P_f \), because during SIA’s the determination of \( K_{\text{mat}} \) values corresponding to low \( P_f \) for study materials is often required.
5.4.2 Limitations of Existing Engineering Approaches for Constraint and Notch Correction of $K_{mat}$

Figure 5-2 provides further details concerning the application of numerical approaches to reconcile $K_J$ between standard and non-standard specimen geometries.

![Graph showing notch and constraint correction results](image)

**Figure 5-2 Notch and constraint correction results using the existing engineering procedure of Horn and Sherry (2012a) and 3D numerical analyses ($m=20$)**

The notch and constraint correction provided adequate estimates of the shallow U-notch specimen equivalent $K_J$ using an $m$ parameter equal to 20. It can be seen from Figure 5-2, that the predictions of a Weibull stress model (Section 3.5.4) using three dimensional models corresponding to the separate effects of notch root radius and notch depth show significant differences with the engineering approach of Horn and
Sherry (2012b). These differences resulted in increased predictions of the equivalent $K_J$ for the shallow U-notch specimen geometry. These results are important for the development of an engineering procedure for correcting $K_J$ with the behaviour of non-standard geometries because they provide an important insight concerning the sources of non-conservatism of the existing engineering approaches. This non-conservatism arises from the requirement to scale too the SE(B) geometry using knowledge gained from the tests concerning the non-standard geometry (in the present research the ultimate objective was the scaling of Charpy V-notch $U_{el+pl,LLD}$). In such a scaling framework, the increased predictions of the three dimensional modelling approach must be associated directly with the deficiencies of the existing notch and constraint correction. Under such circumstances it is valuable to understand the reasons for such inaccuracies. These inaccuracies must be attributed to the assumption of the notch and constraint correction engineering procedure development; these include the use of two dimensional plane strain FE models within a boundary layer geometry. The effects of finite geometry and out-of-plane effects along the notch front were not accounted for. This implies that use of three dimensional analyses is especially warranted in respect of the current research objective. Nevalainan and Dodds (1995) successfully showed that the three dimensional constraint effect can be quantified using three dimensional FE analysis. This study therefore reinforces that finding with respect to non-standard U-notch geometries. Figure 5-2 separately undertakes constraint and notch analyses using 3D models to decompose the separate difference between a standard SE(B) and the shallow U-notch geometry. It was seen in Figure 5-2 that the constraint approach of Sherry et al (2005) provides good predictions in relation to the 3D Weibull stress scaling approach at low failure probabilities, at high $P_f>0.5$, these prediction become less accurate as increased constraint loss occurs in the finite specimen geometry. Conversely, the approach of Horn and Sherry (2012b) was found to provide less accurate predictions throughout the loading range to a $P_f$ of approximately 0.95.
The increased predictions for the notch effect derived from 3D analysis are a possible indication that the assumptions of plane strain and the lack of a significant in-plane bending effect in the boundary layer models of Horn and Sherry (2012b) must be attributed to the source of the inaccuracy. The constraint engineering approach of Sherry et al (2005) was also derived using boundary layer models (and modified boundary layer models). The improved predictions of the constraint correction 3D analysis work are therefore possibly due to the lack of a significant in-plane effect on the \((K_i,P_i)\) analyses results for the SE(B) specimen geometry of \(a/W=0.2\), for the loading range analysed. In contrast, it can be deduced that the results of the notch correction corresponding to 3D analysis show a greater magnitude of differences with the engineering approach because of the presence of a significant in-plane bending effect in the U-notch SE(B) of \(\rho=0.25\text{mm}\) which was used for the work. It was known that notch stress fields provide stress intensification over a larger distance ahead of the notch than crack-tip fields, this behaviour may also provide a reason for why the Weibull stress model scaling approach was able to describe these effects for the finite geometries which form the focus of this study. In summary, it was found that the constraint assessment engineering approach was suited to transfer to finite geometries when behaviour is dominated by cleavage fracture (at the test temperature studied).

The notch assessment engineering approach was found to be less applicable to finite specimen geometries because of the significant in-plane bending effects which occur in SE(B) specimens. Further work is required to understand if these inaccuracies of the notch correction are exhibited for other specimen geometries such as the SE(T) or CC(P).

5.4.3 Step 2: Conversion of \(K_{ij}\) to \(U_{epl,LLD}\) for a Shallow U-notch SE(B) Specimen \((a/W=0.2, \rho=0.25\text{mm})\)

The second step of the prospective engineering procedure consisted of a conversion of the scaled \(K_{ij}\) from Step 1 to the associated \(U_{epl,LLD}\) for this specimen.
geometry. This work was undertaken using a numerical FE method to determine a solution for relating these two quantities using the ESIS P2-92 Expression (3.22). Section 4.5.3 provides the details of this analysis which derived a solution for $\eta_{LLD}$ which was a function of material strain hardening properties. This approach was deemed to be accurate and feasible to implement within the engineering procedure. The applicability of this step of procedure to incorporate a wide range of industrially relevant ferritic steel flow properties was recognised as being a necessary characteristic to be considered when evaluating its utility. The dependence of the $\eta_{LLD}$ solution on the material $n$ parameter within Expression 3-22 could potentially represent an obstacle to engineers in practical circumstances of restricted material data.

5.4.4 Step 3: Flank Angle Correction of a Shallow U-notch SE(B) Specimen ($a/W=0.2$, $\rho=0.25\text{mm}$) to a Standard Charpy V-notch Specimen

Following the work concerning the first two steps of a multi-step procedure (Sections 5.4.1 and 5.4.2) a procedure was instigated for evaluating the effect of the notch flank angle changes from a U-notch SE(B) Specimen ($a/W=0.2$, $\rho=0.25\text{mm}$) specimen to the standard Charpy V-notch specimen. The method adopted for undertaking a flank angle correction was to adopt the Weibull stress scaling model of Section 3.4. A scaling approach was used to quantify the difference of cleavage fracture of a shallow U-notch SE(B) and standard Charpy V-notch specimen at quasi-static rates of loading. Figure 5-3 shows the scaling principle used to achieve predictions of Charpy V-notch $U_{el+p,LLD}$ using shallow U-notch specimen $U_{el+p,LLD}$ values.
Figure 5-3 Principle of Weibull stress based U- to V-notch flank angle correction

The experimental validation of the flank angle correction for the study material was undertaken using experimental data from the quasi-static test programme of Section 3.2.1. Figure (5-4) provides the output of the validation from the flank angle correction, this Figure shows the output prediction of the Step 1 constraint and notch correction and Step 2 conversion of $K_I$ to $U_{el+pl,LLD}$. The predicted difference in $U_{el+pl,LLD}$ using the Weibull stress scaling model was found to be 9.4% less (at $P_f=0.5$) than the average experimental compare to the experimental results, these laboratory test results indicated an average $U_{el+pl,LLD}$ difference of approximately 20.1%. This represented a potentially significant underestimation of the Weibull stress model in relation to the experimental results. The flank angle correction was therefore considered to possibly not be suitable for inclusion in the multi-step procedure. Additionally, the complexity
associated with the definition of multiple $m$ parameter solutions corresponding to this stage of the procedure, as was already present within the Step 1 constraint and notch correction, was a valid argument that the multi-step procedure may potentially not be suitable for undertaking an accurate and viable engineering procedure to satisfy the objectives of this research.

Figure 5-4  Constraint correction comparison of engineering approach and Weibull scaling model approach of this work ($m=20$)

5.4.5  **Material Specific Charpy V-notch $U_{el+pl,LLD}$ Correlation to SE(B) $K_j$ for Evaluating $K_{mat}$**

Following the investigation concerning the initial development of a multistep procedure (Sections 5.4.2, 5.4.3 and 5.4.4), it was deduced that there existed scope to undertake a new numerical approach to the engineering procedure for Charpy V-notch data. This approach was conceived using an approach similar to the flank angle correction work of Section 5.4.4. The potential advantages of this numerical work were
believed to be the ability to accurately accommodate and describe the cleavage fracture behaviour of individual geometries using three dimensional FE analyses. Additionally, it was evident that the non-conservatisms associated with the notch and constraint correction (Section 5.4.2) were not insignificant. Therefore, an approach based upon a calibrated Weibull stress model was introduced. This correlation would seek to accomplish a direct scaling between equivalent and measured quantities of the SE(B) \((a/W=0.5)\) and Charpy V-notch specimen geometries respectively. This approach had the benefits of undertaking the correlation in one operation and had the potential to represent the most accurate approach to satisfy the objectives of this research.

5.4.6 **Accuracy of the Weibull Stress Model Quasi-Static Charpy V-notch Correlation**

When transferring the scaling approach to the Charpy V-notch specimen geometry scaling of \(U_{el+pl,LLD}\) undertaken. This method was found to a consistent method of scaling and hence was suitable for application to the Charpy V-notch specimen under quasi-static conditions. Calibration using the Weibull stress scaling model was undertaken and the value of \(m\) determined was used within a scaling method which incorporated ASTM E 1921-11a (ASTM, 2011b) to describe the probability of cleavage fracture of both SE(B) and the Charpy V-notch specimens. The distribution of failure probability was most accurate at \(P_f=0.5\) and at low failure probabilities deficiencies of the ASTM E 1921-11a cleavage fracture model were apparent. These deficiencies were most likely to arise from the assumptions of the ASTM E 1921-11a cleavage fracture model which assumes a constant Weibull shape parameter, \(b\), of 4. It may be the case that individual steels possess \(b\) values which are significantly different from 4 (Wallin, 1984). Lee et al presented results which suggested that \(b\) increased with decreasing \((T-T_0)\) and suggested a modified form of the master curve methodology which improved low \(P_f\) predictions for the materials studied. Wallin found that errors of the order of 20% upon \(K_0\) were possible for \(N=10\).
The limited experimental data \((N=10)\) used to assess the performance of the quasi-static Charpy V-notch correlation of this work, may also contribute to the uncertainty if seeking to modify the ASTM E 1921-11 cleavage fracture model by introducing the most accurate shape parameter to be applied within the ASTM E 1921-11a. According to the quasi-static Charpy V-notch correlation shown in Figure 4-23, at a value of \(P_l\) less than approximately 0.2 there is a notable deviation from the numerical prediction of the model and the \(P_{l,\text{Rank}}\) experimental data. It was evident that increasing the value of \(b\) alone would not achieve an improved prediction for \(P_{l,\text{Rank}}\) test results less than 0.5. This is because the value of \(m\) within the Weibull stress model would be required to be changed also so as to describe the distribution of \(U_{\text{dev,pl,LLD}}\) corresponding to these test results. This in turn would invalidate the predictions of the model in comparison to the \(P_{l,\text{Rank}}>0.5\). This observation stems from similar observations which were evident in relation to the SE(B) \(P_{l,\text{Rank}}\) data. The SE(B) cleavage fracture behaviour was most accurately characterised at \(P_l\approx0.6\), which is an artefact of the ASTM E 1921-11 definition of \(K_0\). Further work concerning the sensitivity of the Charpy V-notch correlation procedure are necessary regarding the sensitivity of \(P_l\) predictions to \(b\) and the uncertainties of limited datasets. The description of \(K_0\) is of fundamental importance to the master curve methodology and therefore the predictions of the Weibull scaling model developed for this work. A systematic study is therefore pre-requisite concerning errors upon \(K_0\) prior to extension of the methodology. The establishment of this correlation method has permitted future studied to be conducted concerning other ferritic steel study materials which will allow the examination of large specimen datasets, such as the ‘Euro Fracture dataset of Heerens and Hellmann (2002).

The value of \(m=16.4\), calibrated for the quasi-static Charpy V-notch correlation, and the value of \(b=4\) used in Section 4.5.5 provided the best available fit to the \(P_{l,\text{Rank}}\) data which were available. It is arguable that when the \(P_l\) range of interest corresponds to failure behaviour corresponding to \(P_l<0.2\) (a common requirement for industrial
structural integrity assessments) that modification of the \( b \) and \( m \) parameters to best describe this data is a justifiable course of action. The characterisation of the cleavage fracture behaviour of both SE(B) and Charpy V-notch specimen at values of \( P_f \) less than 0.2 would be best done using a significantly larger experimental data set. This underlines the uncertainties inherent when seeking to modify an established cleavage fracture model on the basis of limited data. The quasi-static correlation was developed on the basis of the experimental testing (Section 4.2.1), materials characterisation (Section 4.1.1) and calibration of the micro-mechanical model (Section 4.5.4) concerning the ferritic steel study material. Any modification of the ASTM E 1921-11a model to improve the description of cleavage fracture at industrially relevant \( P_f \) values would inevitably have to account for a range of ferritic steels and therefore would represent a significant undertaking.

### 5.4.7 Physical Meaning of the Weibull Stress Model \( m \) Parameter within the Quasi-Static Charpy V-notch Correlation

The quasi-static Charpy V-notch correlation (Section 4.5.5) was developed using an approach to scale \( U_{el+pl,LLD} \), the accuracy of the approach to quantify \( (P_f, U_{el+pl,LLD}) \) was found to be largely dependent on the value of \( m \) used. The calibration of \( m \) (Section 4.4.1) was undertaken with reference to the Charpy V-notch and SE(B) specimens of interest. This enabled accurate numerical predictions to be made for the study material because the exact range of constraint represented by the correlation was used for the calibration. The physical meaning of the Weibull stress model \( m \) parameter has previously been asserted by Beremin (1983) and Mudry (1987) to be dependent on microstructural features such as the size-frequency characteristics of brittle carbides, or inclusions (Neville and Knott, 1986)(Bowen et al, 1987). This assumption underlies the rationale for use of the Weibull distribution to represent statistically independent brittle features in ferritic steels. Within the context of the developed Charpy V-notch to SE(B) \( K_f \) correlation it is pertinent to state that
this implicitly means that the ‘range of constraint’ represented by the Charpy V-notch to SE(B) specimen numerical correlation is determined by the value of $m$ calibrated. Within the current approach, the Weibull stress is used for ‘scaling’, $P_i$ is determined solely by using the ASTM E 1921-11a model and extending this model to non-standard geometries. Hence an increased value of calibrated $m$ for one material will represent an increased range of constraint for the correlation. This raises a question concerning the physical significance of $m$, because the consequent correlations are significantly affected.

The current quasi-static scaling methodology incorporates the parameters of ASTM E 1921-11a (2011a), as described in Section 4.5.5, and hence inherently fixes the value of $\sigma_U$ in the Beremin (1983) model to $K_0$. This modus-operandi achieved accurate predictions. It remains to be determined experimentally if the proposal of a fixed $m=16-17$ correlation for the Charpy V-notch test correlation (under quasi-static conditions) may be extended to other industrially relevant ferritic steels. If such a finding was to hold, then it would represent a significant advance for fracture toughness scaling correlations between non-standard to standard geometries. Structural integrity assessments of geometry specify plant defect geometries for brittle fracture may also be facilitated as a natural progression by developing suitable guidance for engineers using the Weibull stress scaling approach.

In view of the experience of this work concerning the development of the Charpy quasi-static correlation, when issuing guidance to engineers, it is desirable that the guidance be transparent, the sources of uncertainty are to be highlighted and also that the method not be prohibitively difficult to implement (or require excessively large experimental works). Consequently, the finding that the quasi-static correlation works efficiently for $m=16.4$ necessarily requires further validation concerning other ferritic steels to determine proper bounds on $m$ which can be used. It would be of significant benefit to introduce a correlation which incorporates a constant value of $m$ (and the
associated laboratory fracture test data analysis which is the basis of such a method) this is discussed further in Section 5.4.10 in the context of the dynamic Charpy V-notch correlation which is of relevance to the present objectives of this work.

5.4.8 Relevance of the ASTM E 1921 Deformation Limits when Applied to the Charpy V-notch Specimen

The impossibility of characterisation of U-notch specimen fracture behaviour using $J$ (as described in Section 5.1) provided sufficient evidence that the Charpy V-notch specimen would also not be applicable to established fracture mechanics methods. The Weibull stress scaling method (Section 5.4.1) allowed the calculation of an equivalent $J$ which would be measured during fracture mechanics testing. Current testing standards for large scale yielding, such as ASTM E 1820-11 (2011a), require one parameter characterisation both prior and during circumstances of crack growth. The object of this research was the development of an accurate correlation method which is to serve as an input for structural integrity assessments such as R6 (British Energy, 2005). The contravention of deformation limits hence does not represent an influence on the utility of the proposed Weibull stress scaling methodology because the model adequately reconciles mechanical fields for both standard and non-standard fracture mechanics specimens. The contravention of deformation limits by equivalent ‘scaled’ $K_J$ quantities within the engineering procedure would be a valid cause for consideration during future work.

5.4.9 Conversion of $U_{el+pl,LLD}$ to $K_J$ within the Charpy V-notch Correlation

The conversion of scaled $U_{el+pl,LLD}$ values to $K_J$ for the 0.4T SE(B) specimen geometry (Figure 4-24) was undertaken using FE analysis results for this specimen geometry. It was found that this method worked well and did not represent a significant amount of work. Such a correlation may be determined for a range of material flow
properties for use with a material specific correlation, such an approach was used to
determine a material specific $\eta_{LLD}$ factor (Section 4.5.3).

5.4.10 Weibull Stress Model Dynamic Charpy V-notch Correlation

The dynamic Charpy V-notch correlation (Section used 4.5.6) was found to
provide accurate predictions of $U_{el+pl,LLD}$ for both geometries (SE(B) and Charpy V-
notch). The predictions were valid within the bounds of experimental confidence bounds
because of the $N=10$ experimental data sets adopted for validation of the approach.
Using $m=16$, the correlation was shown to be conservative for $P_f>0.07$ (using the 90% confidence bound). At low failure probabilities, $<P_f=0.07$, experimental $P_{f,Rank}$ data were not available. Validation using more experimental $P_{f,Rank}$ of the scaling model at low $P_{f,Rank}$ values. The predictions at low $P_f$ values indicated that an underprediction was possible when using the 50% confidence bound predictions of the dynamic correlation model to scale $U_{el+pl,LLD}$ to the equivalent SE(B) $U_{el+pl,LLD}$. This potentially indicated that dynamic Charpy V-notch correlation predictions $<0.005$ are not reliable. This observation was similar to that made for the quasi-static correlation in Section 5.1.1 regarding the ASTM E 1921-11a master curve cleavage fracture model parameters.

Figure 5-5 shows dynamic correlation predictions for a range of $m$ values in
relation to quasi-static and dynamic Charpy V-notch test data. It was evident that the scatter and magnitude of $U_{el+pl,LLD}$ predictions at $P_f=0.5$ (and 50% confidence bound) increased with increasing $m$. From this analysis it was evident that a large range of median failure $U_{el+pl,LLD}$ values are given for the range of $m$ studied. If attention is restricted to the median predictions 50% confidence bound predictions of Figure 5-5, $m=16$ was found to be an accurate scaling parameter and deviation from this value by more than $+/-2$ causes significantly inaccurate predictions. Therefore, within the engineering procedure, it was deemed that this value was the most suitable. Other
engineering procedures for constraint such as Minami and Arimochi (2003) proposed that \( m \) varies from 10 to 20 for the purposes of undertaking constraint modified SIA’s, the findings of the present research indicated that such a proposition is not reliable and that \( m \) is required to be specified to a definite integer. It is also possible to state that accurate estimates using \( m=16 \) are within the confidence bounds represented by the data set size \((N=10)\). From Figure 5-5, it can be observed that the Weibull stress model \( m \) parameter is a suitable parameter to describe both the magnitude of median predictions and the scatter of experimental results. The observed development of the shape of the \((P_1, U_{el+pl,LLD})\) predictions with increasing \( m \) is a distinct feature of the dynamic Charpy V-notch correlation; this evolution of model predictions also requires experimental validation. It seems apparent that the \( m \) parameter predicts increasing scatter of predictions within its current role as a scaling parameter. The accuracy of predictions concerning the scatter have to be assessed for other ferritic steels. The presumption that \( m=16 \) is a suitable scaling parameter for all ferritic steels is a significant assumption of the model presented during this work. Hence, further evaluation is necessary using ferritic steels from industrial applications and newer alloy grades. In common with the continuing need to validate and refine the master curve methodology, the development of the Charpy V-notch correlation requires further experimental evidence to validate the model functioning and, importantly, the material independence or otherwise of the \( m \) parameter used within the model.

The evidence for an exponential curve shape for Charpy V-notch transition of this work and EricksonKirk et al (2007) also provided the potential for application of the correlation using \( P_1=0.5 \) predictions rather than undertaking the correlation at the \( P_1 \) value of interest to the structural integrity assessment. This represented a rational methodology to be adopted for a formal engineering procedure because it prevented the transfer of the inaccuracies of the Weibull stress model at low \( P_1 \) values which were previously thought to be a necessary prerequisite for adoption of the Weibull stress
based Charpy V-notch correlation procedure. Instead, future developments of the correlation could viably be accomplished using median predictions. Following the conversion of equivalent median 0.4T SE(B) $U_{el+pl,LLD}$ to an equivalent median $K_{lc}$, the established master curve E 1921-11 probabilistic predictions could be adopted for the purpose of providing the required SIA $K_{mat}$ corresponding to a predefined $P_f$. Such an approach is discussed in more detail in Section 5.5. This approach does not address the physical reasons for the Charpy transition curve shape, and hence it is also of priority to undertake further numerical modelling to ascertain the numerical prediction for the transition curve using the established dynamic Charpy V-notch correlation of this work. Following this work it would be possible to ascertain the performance of the correlation against the empirical evidence of this thesis and EricksonKirk et al.

Research concentrating on the development of the Charpy V-notch dynamic correlation procedure must be undertaken on the basis of existing and future experimental works concerning large data sets. Section 4.5.6 discussed the reasoning for large specimen data sets and the examination of a range of relevant structural and nuclear grade steels. For the case of this present work the datasets would necessarily be composed of SE(B) or other valid fracture toughness specimen geometries for which $K_{lc}$ was evaluated. Additionally, Charpy V-notch test results are required for the same material, such a demand would inevitably represent a significant burden.
Figure 5-5 The effect of the Weibull stress scaling model $m$ parameter on the dynamic Charpy V-notch correlation

5.5 Application of the Charpy V-notch Correlation when Ductile Fracture is Present

The common temperature dependence of $U_{el+pl,LLD}$ within the ductile-to-brittle transition allowed a development of the engineering procedure to be proposed corresponding to this temperature range. Two options are therefore proposed as a future means of developing accurate probabilistic correlations for dynamic Charpy V-notch $U_{el+pl,LLD}$ and equivalent $K_J$ corresponding to a 0.4T SE(B) ($a'/W=0.5$) specimen for this temperature region of fracture behaviour:

1. Extension of the method undertaken during this research to calibrate the model throughout the ductile to brittle transition using the validated approach of Section 4.5.6,
2. Exploitation of the common temperature dependence of the $U_{el+pl,LLD}$ and $K_J$ material property curves to enable material specific correlations to be undertaken at a cleavage fracture reference temperature corresponding to that used for Section 4.5.6 for the study material of this work.

5.5.1 **Option 1: Extension of the Weibull Stress Model Dynamic Charpy V-notch Correlation**

To facilitate the dynamic Charpy V-notch to SE(B) correlation ($U_{el+pl,LLD}$ to $K_J$), described in Section 4.5.6, to be applied by engineers further development of the correlation was found to be required. A possible method was to exploit the common temperature dependence of Charpy V-notch $U_{el+pl,LLD}$ and standardised SE(B) $K_J$ throughout the lower transition region. Prior to introduction of the correlation to engineers, testing of the dynamic correlation method at other temperature was deemed to be necessary. An essential part of the procedure development for the lower-to-upper transition dynamic correlation was therefore to undertake further calibration of the model using the study material. Following calibration activities, verification of the approach using other ferritic steels for which Charpy impact and fracture toughness test data are available was logical and necessary final activity to be completed. The steps required for further calibration are the following:

1. Undertake FE modelling using the dynamic Charpy V-notch specimen FE model developed in Section 3.3.2. Analyses should be performed corresponding to temperatures in the ductile-to-brittle transition temperature range.

2. The materials characterisation activities (Section 3.1.3) characterised the ASTM E 1921-11 $T_0$ value as being equal to 44.6°C for the study material. 0.4T SE(B) ($a/W=0.5$) testing at $T=0$˚C determined $T_0$, corresponding to this geometry, $T_{0.4T SE(B)}$ to be 31.1˚C).
3. The 0.4T SE(B) FE model Weibull stress analysis results corresponding to 
$T=0{^\circ}\text{C}$ are the equivalent of $(T-T_{0.4\text{SE(B)}})=-31.0{^\circ}\text{C}$. This analysis corresponded to 
Charpy V-notch impact testing at the same temperature which were very close to the 
lower shelf and hence can be considered as a baseline condition for establishing the 
correlation at higher temperatures when ductile fracture is prevalent.

4. FE analysis of the Charpy specimen could therefore be undertaken at 
intermittent temperatures throughout the transition and the Weibull stress model 
dynamic correlation approach used to quantify probabilistically the distribution of 
$U_{el+pl,\text{LLD}}$ at each temperature relative to $T_0$.

5. A requirement of this work is the assignment of quasi-static material flow 
properties to the model corresponding to each temperature state. Such analysis of the 
study material has been undertaken in Section 3.1.2 using quasi-static tensile 
properties of Horn and Sherry (2010). Strain rate dependent yield properties of ferritic 
steels may be assumed constant within the ductile-to-brittle temperature range for the 
analyses owing to the behaviour being validated using tensile data for ferritic steels 
derived within the same temperature range.

6. Calibration of the Weibull stress model $m$ parameter is necessary at each 
temperature increment, $\delta T$, for example, $\delta T=20{^\circ}\text{C}$, would correspond to $(T-T_{0.4\text{SE(B)}})=$ 
-40, -20, 0 and 20$^\circ$C for use within the dynamic correlation methodology. This 
calibration is to be undertaken with reference to the 0.4T SE(B) specimen at 
temperature increments of $\delta T=20{^\circ}\text{C}$ using $(T-T_0)=-31.1{^\circ}\text{C}$ as a baseline corresponding 
to 95-100% cleavage fracture. This will allow the use of additional SE(B) fracture 
toughness results to calibrate the dynamic correlation Weibull stress model. The $m$ 
parameter for the dynamic correlation was equal to a value of 16.0 at $(T-T_{0.4\text{SE(B)}})=$ 
-31.1$^\circ$C. At higher normalised temperatures, when ductile fracture becomes increasingly 
evident, the required $m$ parameter necessary to accurately scale the median value of 
Charpy V-notch $U_{el+pl,\text{LLD}}$ for the study material to equivalent SE(B) $K_J$ is likely to
increase. The dependence of \( m \) on \((T - T_{0,0.4T \text{SE(B)}})\) is not known and must therefore be determined using the study material Charpy impact test data set and additional SE(B) test results. Following calibration throughout the lower transition, the dynamic correlation may potentially be extended to other material definitions.

7. Using the calibrated Weibull stress model, determine the distribution of \((P_f, U_{el+pl,LLD})\) at each temperature for which test results exist and the percentage shear appearance of the results is less than 60%.

8. The \((P_f, U_{el+pl,LLD})\) data for the study material may thereafter be related to \((P_f, K_J)\) using the master curve methodology. Following these calibration steps, the resulting theoretical foundation of the procedure may be verified using other ferritic steel test data. Following successful testing of the ductile-to-brittle temperature range correlation, by repeating steps 1-8, a material specific correlation scheme may be generated numerically. A proposed dynamic Charpy V-notch correlation scheme for case of the study material is given by Table 5-1.

Table 5-1 corresponds to \( \delta T = 20 \degree C \) and requires the use of further SE(B) test laboratory results. The material specific table may be used by engineering practitioners undertaking structural integrity assessments to estimate material fracture toughness using a small number of Charpy V-notch specimen results. Available Charpy V-notch impact test \( U_{el+pl,LLD} \) values may be matched with the desired \( P_f \) value to determine a material's \( T_0 \) value.

The approach given in Table 5-1 requires calibration of \( m \) and this represents the most demanding task associated with developing the Option 1 procedure. Experimental data would be required to properly calibrate \( m \) at each temperature increment. Such an undertaking will require significant effort and hence is most effectively undertaken with respect to several different industrial ferritic steels. The successful calibration of such a model will provide a theoretical foundation for the empirically observed Charpy V-notch
curve shape. The extent that ductile tearing is required to be incorporated within the numerical modelling, such work has not been assessed within this research. The description of ductile fracture behaviour of Charpy V-notch and SE(B) specimens has not been established to the author’s knowledge and therefore represents future work concerning this Option 1 of the dynamic Charpy V-notch correlation.

<table>
<thead>
<tr>
<th>Table 5-1 Charpy V-notch material specific dynamic correlation scheme for correlating $U_{el+pl,LLD}$ to $K_J$ of a 0.4T SE(B) ($a/W=0.5$)</th>
</tr>
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<tbody>
<tr>
<td>$T-T_{0,0.4T \text{SE}(B)}$ (°C)</td>
</tr>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>$P_f$</td>
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<tr>
<td>0.01</td>
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<td>0.02</td>
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</table>

Anderson (1984) suggested that modern steels do not exhibit significant tearing behaviour until relatively high in the ductile-to-brittle transition because of improvements in microstructure. A more rational argument could be made if a description for the extent of ductile tearing was established for ferritic steels at a given ($T-T_{0,0.4T \text{SE}(B)}$) temperature index. This would require knowledge of the ductile-tearing behaviour and a coupled ductile fracture approach which was suitable to characterise both the Charpy V-notch specimen and SE(B) specimen fracture behaviours. Such an approach was attempted by Folch and Burdekin (1999) but proper validation of the ductile fracture behaviour was not undertaken for both geometries of interest to enable information for an engineering procedure.
One deficiency of the Option 1 proposed method for developing the correlation is therefore the uncertainties associated with the modelling of ductile fracture behaviour and lack of a generalised description. Figure 5-6 shows the Option 1 calibration scheme, demonstrated using the study material of this thesis, this approach requires calibration of $m$ at each temperature increment $\delta T$ as shown. The value of $m=16.0$ was calibrated in Section 4.5.6. for cleavage dominated behaviour ($A_b<5\%$). A key task of future work would be to ascertain the performance of the model in comparison to experimental for ferritic steel using this value of $m$ parameter. It is probable that this value of $m$ will not be sufficient to accurately predict the $(U_{el+pl,LLD}, K_{IC})$ values, corresponding to $P_f=0.5$, at higher temperatures in the ductile-to-brittle transition. Predictions of these quantities are likely to fall lower than the experimental values, as is shown schematically in Figure 5-6 by the line marked $m=16.0$. This specific hypothesis motivated the conception of the present Option for the engineering procedure development. The $(T-T_{0.0.4T SE(B)})$ level at which $m$ recalibration becomes necessary is not known and will require determination. Prior to this work, it is possible to state that $m=16$ would appear to be a suitable recommendation for the future engineering procedure (for correlation of $U_{el+pl,LLD}$, $K_{IC}$ at $P_f=0.5$). Following the correlation to equivalent $K_{IC}$ of a 0.4T SE(B) specimen, it is then possible to assign confidence bounds to the E 1921-11 master curve methodology and hence obtain probabilistic estimates of $K_{mat}$ for SIA’s. The Option 1 extension of the dynamic correlation is equally applicable to SE(B) and fracture toughness geometries other than the 0.4T size one studied for development of the dynamic Charpy V-notch correlation.

5.5.2 **Option 2: Application of the Weibull Stress Model to Undertake Material Specific Correlations at a Cleavage Fracture Temperature**

The second proposed option would utilise the dynamic Charpy V-notch correlation which was validated in Section 4.5.6 uses a single correlation at a cleavage fracture temperature. A correlation at $(T-T_{CVE}) = -41.4^\circ C$ was used for the case of the
study material of this thesis. In practice, Charpy V-notch $U_{el+pl,LLD}$ material data, $U_{el+pl,LLD,mat}$ are likely to be available at significantly higher temperatures than the one used for the validation which was close to the lower shelf of fracture behaviour (cleavage dominated). The common temperature dependence of the Charpy transition curve validated during this work could be used to calculate an equivalent $U_{el+pl,LLD}$ corresponding to a lower shelf temperature for use with the correlation, $U_{el+pl,LLD,corr}$. This would then enable the correlation to be applied for the material of interest using

![Graph](image)

**Figure 5-6 Proposed dynamic Charpy V-notch calibration scheme (Option 1) for the ductile-to-brittle transition to calculate $K_{mat}$ for a 0.4T SE(B) specimen**

a material specific correlation solutions (similar to Table 5-1). When undertaking in this option the dynamic correlation would only be undertaken at $P_f=0.5$, and hence the tabulated material specific correlations would be much simplified. Additionally, such a correlation method would exploit the strengths of the existing correlation; that of the
high accuracy of the predictions for the study material at the median cumulative probability of failure. Hence this approach would provide a linkage to the median cumulative failure probability curve shape given by EricksonKirk et al (2007) and enable correlations to be made to a material specific $K_{ic}$ for a 0.4T SE(B) specimen corresponding also to this failure probability, $K_{JC,0.4T\ SE(B)}$. Once $K_{JC,0.4T\ SE(B)}$ was established the ASTM E 1921-11a (2011) master curve methodology could be applied to calculate a $K_{mat}$ value for the material of interest corresponding to the structural integrity assessment temperature, $T_{assess}$. Figure 5-7 shows how the Option 2 cleavage fracture temperature correlation procedure would operate. It is seen that $K_{mat}$ values are possible throughout the valid transition temperature range corresponding to the study material and corresponding to any $P_t$ confidence bound relevant to the SIA.

The assumptions of this methodology are evident concerning the transition curve shapes of Charpy V-notch and fracture toughness specimen, but the option has the notable strength of uniting these two empirical devices. A possible disadvantage of this option is that it would require knowledge the material flow properties at or near to the $(T-T_{CVE})=-41.4^\circ\text{C}$ (or a similar predefined lower shelf normalised temperature). In cases when such data are not available, one of two activities would be necessary to implement the dynamic Charpy V-notch correlation:

1. tensile testing of the material of interest at the cleavage fracture correlation temperature,
2. a correlation of existing tensile data for the material of interest corresponding to a temperature removed from the cleavage fracture temperature used for the correlation.

Activity 1 would represent an extra expense to the engineering SIA, and also this activity could also be unviable because of material shortages or safety concerns.

Activity 2 would introduce a significant uncertainty concerning the correlation of material
yield strength and strain hardening behaviour to lower temperatures. Commonly tensile test data is available at temperatures corresponding to steel specifications such as BS 10025-6 (BSI, 2004) and ASTM A20 (ASTM, 2000). These temperatures are usually higher than the lower shelf of Charpy V-notch fracture behaviour and hence the tensile

![Diagram showing the proposed method to calculate $K_{mat}$ using a cleavage fracture temperature dynamic Charpy V-notch correlation (Option 2)]

Figure 5-7 Proposed method to calculate $K_{mat}$ using a cleavage fracture temperature dynamic Charpy V-notch correlation (Option 2)
property correlations would be used to correlate tensile properties to lower temperatures. There is potential to develop specific correlations for certain strengths of steels provided that ample tensile test data is available over the temperature range from specification temperatures to the cleavage fracture temperature of the dynamic correlation. Because activity 2 is the less expensive of the two methods and is potentially the one that would be chosen in practice, further research work is recommended to verify the accuracy of existing correlations in R6 (British Energy, 2005) and BS 7910:2013 (BSI, 2013). Following the evaluation of the low temperature tensile property correlations, it would be possible to ascertain the extent of requirements for further research concerning the establishment of material specific correlations for use with the proposed cleavage fracture temperature dynamic correlation. Such work is likely to be of importance to the overall accuracy and therefore conduct of SIA’s using the proposed dynamic Charpy V-notch correlation. Inaccuracies concerning the definition of the tensile yield strength and strain hardening properties of a material of interest for the correlation using Option 2 will translate to the predicted $K_{\text{mat}}$ values and hence represent a potential source of non-conservatism. Such non-conservatism would be to the detriment of the SIA and could potentially have serious structural safety implications. It is hence of high priority to assess the status of low temperature tensile yield strength correlations in existing standards and literature and additionally to examine the possibility to introduce a temperature dependent strain hardening coefficient (the $n$ parameter of Expression 3-3) correlation. Previous research by Bannister et al (2000) has quantified the $n$ parameter using the ratio of yield and ultimate strengths of structural steels. Further investigation is required concerning the low temperature tensile behaviour of nuclear grade steels, such as those used in nuclear reactor pressure vessels, so as to provide accurate correlation for this pertinent class of steels. A physically based description of the temperature dependence of yield strength and strain hardening could also be investigated as means to provide
theoretical description for this activity to practitioners. There is also potentially scope for undertaking further work concerning the similarity of the strain rate and temperature effect of material flow properties. If such a common relationship were to be elaborated, then this would assist in the present Option 2 correlation, and would also potentially assist in the derivation of an improved dynamic material property definition for undertaking the dynamic Charpy V-notch correlation upon which both options 1 and 2 are based (Section 4.5.6).
5.6 References


6. Relevance to Industry

Following the work conducted during this thesis and the discussion of findings it is possible to highlight the important aspects which are of relevance to industry within the context of the application of fracture mechanics within structural integrity assessments.

6.1 Charpy V-notch Specimen Fracture Behaviour

The experimental work concerning the fracture behaviour of Charpy V-notch specimens found that the individual effect of salient geometric properties of a test piece made a distinct contribution to the cleavage fracture behaviour. The incorporation of ‘non-standard’ geometries into the provision of existing fracture mechanics testing methodologies was found to present significant challenges because of the discrepancies these geometries generate in relation to the existing theoretical basis of fracture mechanics assessments.

This study demonstrated that such specimen geometries (including the standard Charpy V-notch specimen) have definite geometric attributes which influence the cleavage fracture behaviour, these were: crack depth \((a/W)\), notch root radius \((\rho)\) and notch flank angle \((\phi)\). These, whilst not directly applicable to fracture assessments, can be incorporated and quantified using mechanistic modelling. This study uniquely quantified the cleavage fracture behaviour of such specimen geometries using a calibrated mechanistic model. This was found to provide accurate probability of failure predictions over a significant failure range to encompass all the salient geometric differences between an SE(B) \((a/W=0.5)\) and standard Charpy V-notch specimen geometry.
The mechanistic modelling demonstrated that non-standard specimen geometries can be scaled to the equivalent fracture mechanics specimen condition (in the case of this study, the SE(B) specimen). Such an approach represented a powerful method for describing an extraordinarily large range of test specimen geometries which scaled different constraint ranges. Each constraint range corresponding was quantified using the mechanistic approach of thesis work. This inherently has relevance to the Charpy V-notch specimen because the fracture behaviour was found to be described accurately (within the limitations of the limited datasets adopted). Therefore, the predictions of both SE(B) and Charpy V-notch specimens were demonstrated to possess fundamentally common fracture behaviour, as described by the mechanistic model. This finding suggested that association of non-standard geometries with a standard fracture mechanics geometry [such as the SE(B)] is a valid undertaking and hence provided the basis for the development of such an approach for calculating fracture mechanics quantities using these specimens.

6.2 Development of the Mechanistically Based Engineering Procedure for Charpy V-notch Data

Development of an engineering procedure highlighted that existing engineering procedures for constraint and notch effect of $K_{mat}$ were not most suitable for application to the specific task of the Charpy V-notch correlation. This raised significant implications for industrial application when seeking to apply such approaches to structural integrity assessments. The finding that such assessment methods were insufficient for the task of scaling $K_{jc}$ or $U_{el+pl,LLD}$ from a non-standard geometry to an ‘equivalent’ quantity corresponding to a standard SE(B) geometry represented an important step in development of the procedure. The conclusion that three dimensional analyses within a mechanistic framework was the optimum methodology to adopt represented a significant improvement of the capacity to provide accurate probabilistic predictions.
Such a finding changes the form of guidance and linkage it makes with existing
guidance concerning defect assessments in SIA’s when the newly developed Charpy V-
notch correlation eventually is implemented into practice. The definition of model
parameters was found to be of significant importance to the correlation. This study has
demonstrated that accurate predictions are realistically obtainable using material flow
property data alone provided that the mechanistic model is calibrated. The method
adopted for calibration represents a crucial activity within the present work and for
future development (Section 8.2). Extension of the correlation to the dynamic loading
case of an impact loaded Charpy V-notch specimen provided the foundation for
application of the model within a material specific framework to Charpy impact test data.
The ability to characterise fracture behaviour of the Charpy specimen during impact
conditions using material flow properties which are commonly available to engineers
represents a significant advance and an opportunity to extend the model to generalised
guidance. This work has established and undertaken validation activities concerning the
basis for such a methodology. The mechanistic modelling results and experimental
validation work reported from this thesis provide strong grounds for the extension of an
identical approach to include a wide range of ferritic steel sub-groups. Decisions are
required concerning the form of future validation activates and the subject types of
ferritic steels employed for such works to preserve relevance to industrial applications.

Definition of the mechanistic model was devised so as to provide an incorporation of
existing standardised fracture assessment methodologies such as ASTM E 1921-11. In
so doing, the study also highlighted the areas of these existing standards where future
research is required (Section 8.1). Therefore, future improvements to such method for
statistical analysis of the fracture behaviour of ferritic steels may be readily
accommodated into the existing correlation scheme. The strength of the presently
developed mechanistic model lies in its incorporation of such cleavage fracture
methodologies and the scope provided for future refinement of the approach to incorporate specific types of materials. The extension of the mechanistic approach may proceed as further evidence concerning particular steel types. The incorporation of amended cleavage fracture models to inhomogeneous steel materials such as weld microstructures is also a viable option for future application within the structural integrity field.

Future development of the dynamic Charpy V-notch correlation method was recommended using two options; each option had a distinct relevance to industry:

Option 1 was devised to provide a complete theoretical understanding of the ductile-to-brittle transition. This option would provide engineers with a compete tool for describing the fracture behaviour of both the SE(B) and dynamic Charpy V-notch specimen tested according to standardised methods. The method would require calibration, but the results of this work indicate that such a calibration is a realistic undertaking using the approach mechanistic model. The performance of such a correlation offers the potential to significantly increase understanding of ferritic steel fracture behaviour in the ductile to brittle transition. The possibility of introducing micro-mechanistic calibration methods into the approach developed during this research represents a second possible advance which will have significant consequences for physical understanding of fracture and also the basis for the evaluation of material properties by engineering practitioners. The treatment of ductile fracture may require a coupled micro-mechanistic treatment, but such approaches are suitable for integration with the existing Charpy V-notch mechanistic correlation methodology of this work.

The Option 2 development represents a future methodology which links with existing fracture toughness testing and materials characterisation standards for cleavage fracture. This option provides a viable method for engineers to calculate material
fracture toughness using knowledge of material flow properties. The method is hence significantly more transparent than Option 1. This has the advantage of being developed more easily and incorporated into existing guidance for flaw assessment.

The mechanistic approach quantifying probabilistically the cleavage fracture behaviour of ferritic steels represents a suitable tool for engineers to use to determine the material properties of industrial plant. It therefore serves to increase confidence in the use of such correlations by practitioners and also it is more relevant to the existing rationale for the current defect assessment procedures.
7. **Conclusions**

The main objective of this research was the development of an engineering procedure to enable calculation of material fracture toughness using Charpy absorbed impact energy results via a mechanistically based fracture model. This work was for application to ferritic steels in the lower ductile-to-brittle temperature range of fracture behaviour. The objectives concerning the use of Charpy V-notch absorbed impact energy ($U_{el+pl,LLD}$) have been realised for the lower transition region of fracture behaviour when cleavage fracture is dominant. A probabilistic procedure has been developed and validated using a ferritic steel study material and the correlation evaluated within the context of current existing structural integrity assessment technologies.

A mechanistic correlation of dynamic Charpy impact energy with quasi-static SE(B) fracture toughness was achieved using a Weibull stress scaling approach. The correlation methodology was found to be accurate and suitable for extension to develop a material specific procedure for use during industrial structural integrity assessments. The main conclusions are given in the following subsections.

7.1 **Materials Characterisation**

Fracture toughness testing of a heat treated structural steel was evaluated and the $T_0$ value to be 44.6°C using 0.5T SE(B) specimens. The $T_0$ value was increased to above ambient temperatures following the heat treatment of the supplied product and this was attributed to a coarse ferrite, pearlite and bainitic microstructure.

Dynamic tensile testing was undertaken and full stress-strain relations obtained at $T=0\,^\circ\mathrm{C}$ over a range of strain rates from 0.001 to 200/s/ The limit of proportionality was increased by approximately 34% at the highest strain rate compared to quasi-static tensile testing.
7.2 SE(B) Cleavage Fracture Test Programme

A SE(B) cleavage fracture test programme of 0.4T size standard and non-standard specimen geometries was completed and the effect of individual geometric difference between a SE(B) and Charpy V-notch specimen geometry were quantified. It was found that each geometric difference contributed to an increase of the measured $U_{el+pl,LLD}$:

- The change of notch acuity caused an approximate doubling of absorbed energy from a fatigue pre-crack to a U-notch of root radius, $\rho=0.127$mm,
- The decrease of notch depth from $a/W=0.5$ to $a/W=0.2$ caused an approximate trebling of $U_{el+pl,LLD}$ between for U-notch SE(B) specimens of $\rho=0.125$mm,
- The increase of $\rho$ for a shallow U-notch SE(B) ($a/W=0.2$) from 0.125mm to 0.25mm caused an approximate doubling of $U_{el+pl,LLD}$,
- The increase of flank angle, of a Shallow U-notch SE(B) from $\phi=0^\circ$ to $45^\circ$ caused an approximate 25% increase in $U_{el+pl,LLD}$ at higher deformation levels for these geometries.

The $U_{el+pl,LLD}$ at cleavage fracture initiation, corresponding to dynamic Charpy V-notch specimens, was accurately quantified using analysis of the instrumented Charpy impact test results. Fractographic analyses showed that the position of cleavage initiation was situated at distances coinciding with notch tip opening displacement for cleavage fracture without significant ductile tearing. The positions of cleavage fracture were found to be significantly closer to the tip of ductile fracture propagation when significant tearing ($\Delta a>200\mu$m) occurred in Charpy V-notch specimens.
7.3 Finite Element Analysis

A method to calculate an equivalent $J$ corresponding to a shallow U-notch SE(B) specimen ($a/W=0.2$, $\rho=0.25\text{mm}$) was developed using numerical analysis and validated using a numerical contour integral. This method was used to assess the applicability of the Hutchens, Rice and Rosenglen (HRR) fields to this specimen geometry. The HRR stress field solution was found to not be applicable to this non-standard specimen geometry because of fundamental differences of the form of the stress fields.

7.4 Mechanistic Cleavage Fracture Model

A mechanistic model for cleavage fracture was applied and calibrated using a standard 0.4T pre-cracked SE(B) and a shallow U-notch SE(B), the results of the calibration achieved accurate probabilistic predictions when applying a three parameter Weibull stress model to the non-standard U-notch test geometry using a Weibull shape parameter of $m=15.1$. The scale and threshold parameters of the Weibull distribution were found to be required to be calibrated using two sets of fracture toughness data corresponding to low and high constraint geometries.

7.5 Development of an Engineering Procedure for Charpy Impact Data

7.5.1 0.4T SE(B) Fracture Toughness Results

The $T_0$ value for 0.4T standard SE(B) specimens of nominal $a/W=0.5$ was found to be $T_{0,SE(B)0.4T}=31.1^\circ\text{C}$ and a median fracture toughness of $68.8\text{MPa}/\text{m}$ when testing at $(T-T_0)=-44.6^\circ\text{C}$.

7.5.2 Calculation of $J$ for U-notch SE(B) Specimens

Two methods to evaluate equivalent $J$ values for non-standard U-notch SE(B) specimens using the load line absorbed deformation energy were developed using a modification of existing sharp-crack SE(B) formulae:
• For deep U-notch SE(B) \((a/W=0.5)\) specimens \(\eta_{LLD}\) was found to be a material independent value of approximately 1.94 for specimens of \(\rho=0.25\text{mm}\),

• For shallow U-notch SE(B) specimens \((a/W=0.2, \rho=0.25\text{mm})\) a material specific expression for \(\eta_{LLD}\) was developed which was a function of material strain hardening properties.

### 7.5.3 Application of the Weibull Stress Model to U-notch SE(B) Specimens

A Weibull stress scaling model was developed for application to U-notch SE(B) specimens, the method incorporated the probabilistic function of the ASTM E 1921-11 cleavage fracture model. The model was found to provide accurate probabilistic estimates of cleavage fracture for a large range of U-notch constraint conditions when scaling from a standard 0.4T SE(B) specimen. The value of Weibull stress \(m\) parameter required for accurate predictions was found to increase with an increasing constraint range. The probabilistic predictions were found to be less accurate at low failure probabilities. An assessment of the performance of a combined notch and constraint correction found that the method was significantly non-conservative for the purpose of scaling shallow U-notch SE(B) equivalent \(K_{jc}\) quantities to the SE(B) geometry \(K_J\) and hence it was found not to be suitable for use within the engineering procedure.

### 7.5.4 Quasi-Static Correlation of Charpy V-notch \(U_{el+pl,LLD}\) with \(K_J\)

An accurate correlation between quasi-static Charpy V-notch \(U_{el+pl,LLD}\) and the equivalent \(K_J\) of a 0.4T SE(B) specimen was successfully obtained using a calibrated \(m\) value equal to 16.4. The method applied a micro-mechanistic (Weibull stress) approach to scale \(U_{el+pl,LLD}\) quantities between both geometries. It was found that the require \(m\) parameter was unaffected by the quantity scaled (\(U_{el+pl,LLD}\) or \(K_J\)). Predictions at low failure probabilities were found to potentially originate from two sources:

• limitations of the ASTM E 1921-11 cleavage fracture model,
• the uncertainty inherent in the small dataset of \(N=10\).

7.5.5 Dynamic Correlation of Charpy V-notch \(U_{el+pl,LLD}\) with \(K_J\)

A dynamic correlation was developed using a Weibull stress scaling approach to scale dynamic Charpy V-notch \(U_{el+pl,LLD}\) to equivalent \(K_J\) corresponding to a 0.4T SE(B) specimen geometry. The correlation achieved accuracy using a Weibull stress shape parameter equal to \(m=16\). The correlation was found to be accurate within the bounds of experimental scatter imposed by the limited experimental dataset, at lower failure probabilities of less than 0.7% the model was found to be potentially non-conservative. The model successfully reconciled the geometric differences and strains rate differential within the correlation.

7.6 Proposed Extensions of the Dynamic Correlation when Ductile Fracture is Present

Two extensions of the proposed dynamic Charpy V-notch correlation for the purpose of undertaking probabilistic conversions of dynamic Charpy V-notch \(U_{el+pl,LLD}\) to equivalent \(K_J\) SE(B) when ductile fracture were presented and found to be potentially viable:

• Extension of the dynamic Charpy V-notch mechanistic correlation to higher temperatures in the ductile-to-brittle transition using a calibration methodology based on experimental data,

• Exploitation of a common temperature dependence of both \(U_{el+pl,LLD}\) and \(K_J\) in the ductile-to-brittle transition to undertake material specific correlations at a cleavage fracture temperature.
8. Future Work

Following this work concerning the development of a mechanistic method for determining a ferritic steel material’s fracture toughness using Charpy V-notch impact test results, the following future research is recommended. The future work will aid the development of an engineering procedure for the use within existing structural integrity assessments. The purpose of the future work is to improve the accuracy of the mechanistic correlation and also the temperature range and fracture mechanism to which the model is applicable.

8.1 Accuracy of the ASTM E 1921-11 Master Curve Methodology

The existing master curve methodology represented a significant advance in the quantification of ferritic steel fracture behaviour within a probabilistic paradigm. It was found that future research concerning the master curve methodology may concentrate on the examination of several large data sets of fracture toughness data for the purpose of evaluating the extent of dependencies of the methodology on steel microstructure. Such work would represent a valuable exercise for structural integrity engineering practitioners because it could potentially improve low probability of failure calculations using the master curve cleavage fracture model. These are of importance to industrial structural integrity assessments. This work would also provide inherent improvements to the mechanistic methodology developed concerning the fracture behaviour of the Charpy V-notch specimen.

8.2 Validation of the Mechanistic Correlation

Further validation of the mechanistic correlation for calculating SE(B) $K_I$ values using dynamic Charpy impact data is necessary using different study materials of the ferritic steel variety. This would aid in the confirmation of the Weibull stress $m$ parameter used for scaling $U_{el+pl,LDD}$ within the correlation. Such work is of importance
because the model's probabilistic predictions were found to be dependent on this parameter. Such validation requires the presence of both SE(B) (or other valid high constraint fracture toughness specimen data) and Charpy V-notch impact test results. The future work described in Section 8.1 would also inform the improved accuracy of mechanistic correlation at low failure probabilities which are of importance to structural integrity assessments used in industry.

**8.3 Development of the Mechanistic Correlation**

The development of the mechanistic correlation is recommended to widen the application of the correlation on two accounts: 1.) the range of temperatures and fracture mechanisms for which the model is valid, and, 2.) the range of material flow property definitions for which probabilistic solutions are available to engineering practitioners. This development work, concerning the recommended Option 1, would require calibration of the model using additional material data for the study material (or another suitable ferritic steel) at higher temperatures. Additionally, it is possible to calculate an extensive catalogue of cleavage fracture correlations using the existing mechanistic model which has been developed and calibrated during the present work. Extension of the model to other flow property definitions would significantly widen the applicability of the correlation Option 2 and represent a valid extension in the immediate future. It is also recommended to undertake future research concerning the yield and flow behaviour of ferritic steels at low temperatures for use in conjunction with Option 2.

A third research activity, which has importance to the theoretical understanding of the fracture behaviour of ferritic steels, pertains to the combined temperature and strain rate dependence of flow properties of these materials at low temperatures. Further research concerning these characteristics and a possible common relationship specific to ferritic steels would allow incorporation into the model to provide a complete theoretical description.
Appendix A

A.1 Constraint Correction

The procedure for a constraint and notch correction has been conducted using a ferritic steel study material for which a 0.4T SE(B) ($a/W=0.5$) fracture toughness and U-notch 0.4T SE(B) specimen ($a/W=0.2$, $\rho=0.25\text{mm}$) non-standard laboratory test data are available. This section reports work to develop and validate the constraint correction part of the procedure.

A1.1. Numerical Approach

To account for the difference of notch depth between a U-notch 0.4T SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) and the a 0.4T SE(B) ($a/W=0.5$) a constraint correction was conducted. Finite element modelling (FE) of a 0.4T SE(B) specimen ($a/W=0.2$) was undertaken using three dimensional analyses. The constraint correction was undertaken using the T-stress parameter (Cardew et al, 1986)(Kfouri, 1986)(Wang, 1993)(Hadely, 1995) to quantify the effect of change of crack depth ratio ($a/W$) from 0.5 to 0.2 on predicted fracture toughness measurements. Previous experimental works by, for example, Sumpter and Forbes (1992) showed that the measured apparent fracture toughness increases significantly under conditions of lower of stress triaxiality at a crack tip. Deeply cracked SE(B) specimens induce high triaxialty at the tip of the specimen’s fatigue pre-crack, less deeply cracked SE(B) specimens experience a reduction of triaxiality with increasing load as indicated by previous numerical works (McMeeking and Parks, 1979) (Shih and German, 1981).

The purpose of this modelling work was to evaluate the constraint modified material failure diagram for the study material using the methodology developed by Sherry et al (2005). Elastic and elastic-plastic analyses were conducted using ABAQUS version 6.11 (2011) using a small strain formulation to evaluate the T-stress and
contour J-integral at each load increment of the analyses respectively. Von Mises plasticity was used and the Ramberg and Osgood (1943) nonlinear stress-strain relationship using the parameters in Table 3-1 for the study material at $T$=0°C. Reduced integration quadratic twenty node brick elements were used for all FE models. A sharp crack geometry was adopted for the numerical analyses and a straight crack front was also adopted, coincident nodes at the crack tip were allowed to displace independently to allow the formation of a strain singularity for the elastic plastic material. Crack-tip nodes for the elastic analysis were tied together to form a single node consistent with the requirements of a linear elastic singularity. A crack-tip mesh arrangement composed of 40 concentric rings of elements with fans of elements subtending 9° angles at the crack tip point was used. The initial ring of elements at the crack tip had a radius of 5μm. 14 layers of elements through the specimen thickness were used, corresponding to the nodal positions along the crack front. The layers of elements were significantly reduced in size towards the outer free surface in the crack front direction to resolve the changes of strain distribution in this region of the crack front. Approximately 80 load increments were used to complete the elastic-plastic analysis. The mesh arrangement used for the FE model of the 0.4T SE(B) specimen ($a/W$=0.2) is shown in Figure A-1.

A.1.2 Sharp Crack 0.4T SE(B) ($a/W$=0.2, $\rho$=0.0mm, $\phi$=0°) Specimen Model Validation

The global outputs for a 0.4T SE(B) ($a/W$=0.2) elastic-plastic FEM model were validated using experimental data for the study material. This experimental data corresponded to shallow crack SE(B) specimens of 0.5T size according to the ASTM E 1820-11 (2011a) test method used for the work. The laboratory testing of these specimen types was undertaken at $T$=0°C by Horn and Sherry (2010). The range of $a/W$ ratios were determined to be between 0.14 and 0.1, these crack depths were measured using the eight point average technique and a travelling microscope.
Figure A-1 Quarter symmetry finite element model of sharp crack 0.4T SE(B)

(a/W=0.2)

Figure A-2 shows the experimental set of test results for load, P, and CMOD and elastic-plastic FE model results. Elastic-plastic FE model results are given for three a/W ratios, these were 0.1, 0.15 and 0.2 respectively. The 0.4T SE(B) FE model with an a/W ratio of 0.2 was used for the constraint correction. To change the geometry of this FE model to a 0.5T sized SE(B) FE model, a scaling operation was performed on the former geometry's mesh arrangement. The test results spanned a range of reduced a/W ratios, hence it was deemed necessary to conduct FE analyses to cover these a/W ratios. An identical one millimetre radius concentric mesh arrangement at the crack tip was used for all 0.5T SE(B) FE models shown in Figure A-2. Loading of the FE models was by an imposed vertical displacement of a support roller consistent with the position of the loading roller during the experimental test procedure. All variables in the FE models were kept constant with the exception of the differing crack depths. Load and CMOD results were extracted from the FE model output file from nodes at the specimen loading strip and at the crack mouth at the centre of the crack front respectively. Figure
A-2 shows that the FE model for \( a/W = 0.1 \) is in close agreement with the experimental results of Horn and Sherry (2010). The model for \( a/W = 0.15 \) is below the experimental test results but within the range of agreement with the deeper crack test results (\( a/W \) equal to approximately 0.14) from the experimental data set given the inherent uncertainty in measuring fatigue crack depths. Therefore, the accuracy of the FE model for \( a/W = 0.2 \) in relation to the two other models results was affirmed given the constancy of all other variables between FEM models.

![Graph showing experimental results and crack depth ratios](image.png)

**Figure A-2** Experimental shallow crack 0.5T SE(B) test results and finite element model calculations

### A.1.3 Evaluation of the J-integral for Shallow Crack Charpy 0.4T SE(B) (\( a/W = 0.2 \)) Specimens

Numerical calculations of a contour integral, \( J \), for the 0.4T SE(B) specimen geometry were made using a path independent contour around the crack-tip at all nodal positions along the crack front, a weighted average value of the J-integral, \( J_{\text{ave}} \), was...
determined using the positions of each contour along the crack front. Material properties were defined according to those for the study material given in Section 4.1.1. A polynomial fit was made to $J_{\text{ave}}$ versus $U_{\text{el+pl,LLD}}/Bb$ output. Figure A-3 shows the results of this work. The cubic function allowed the following expression to be used for calculating $J_{\text{ave}}$: $0.0000007(U_{\text{el+pl}}/Bb)^3 + 0.0004(U_{\text{el+pl}}/Bb)^2 + 1.1514(U_{\text{el+pl}}/Bb)$ using a modified form of ESIS P2-92 (1992). A comparison of the experimental ASTM E 1820-11 (2011a) $J$ calculations, $J_{\text{ASTM}}$, and the $J$ quantities determined using the ESIS P2 92 (ESIS, 1992) expression, $J_{\text{LLD,ESIS}}$, is shown in Figure A-4. The data in this figure are normalised using $J_{\text{ave}}$ and hence, the figure expresses the relative error of the two different expressions in comparison to the $J_{\text{ave}}$ value derived from FE.

![Graph showing derivation of $J_{\text{ave}}$](image)

\[ J_{\text{ave}} = 0.0000007(U_{\text{el+pl}}/Bb)^3 + 0.0004(U_{\text{el+pl}}/Bb)^2 + 1.1514(U_{\text{el+pl}}/Bb) \]

**Figure A-3** Derivation of $\eta_{\text{LLD}}$ for shallow crack 0.4T SE(B) specimen ($a/W=0.2$)

It can be seen from Fig. (A.3) that both the ASTM E 1820-11 (2011a) and ESIS (1992) methodologies provided accurate estimations of $J$ with an error of approximately 1% and 0.5% respectively for $60 \leq J_{\text{ave}} < 380 \text{N/mm}$. This engineering procedure adopts the ASTM E 1820-11 (2011) methodology for estimating $J$ during the constraint correction.
activity because of the previously recorded dependence of experimental $J_{LLD,ESIS}$ on material yield and strain hardening properties (Kirk and Dodds, 1993). This ensures that material independent measures of fracture toughness can be made in shallow crack SE(B) specimens.

**A.1.4 Methodology for Calculating $\beta_T$ for a 0.4T SE(B) ($a/W=0.2$) Ferritic Steel Specimen**

An elastic analysis of the 0.4T SE(B) ($a/W=0.2$) specimen was conducted using the material properties corresponding to the study material ($E=207,000$MPa).

![Graph](image)

**Figure A-4** Comparison of ASTM E 1820-11 (2011) and Expression (2-7) for calculating the experimental J-integral for a 0.4T SE(B) specimen ($a/W=0.2$)

The purpose of this analysis work was to calculate the T-stress, $T$, using a numerical contour integral approach. The nodal contour integral $T$ quantities at each load increment were averaged at nodal positions along the crack front in a manner similar to that used for calculating $J_{ave}$. A crack front $T$, $T_{ave}$, was therefore determined for the
sharp crack 0.4T SE(B) \((a/W=0.2)\) specimen geometry. This analysis permitted the evolution of the \(T_{ave}\) with load to be ascertained.

The load ratio, \(L_r\) was determined using results from elastic perfectly plastic FEM analyses of the 0.4T SE(B) \((a/W=0.2)\) specimen geometry. A range of steel material yield strengths \((\sigma_Y)\) were defined for the analyses and the Young's modulus, \(E\), was defined as 200,000MPa.

The limit load, \(P_L\), for a shallow crack 0.4T SE(B) specimen \((a/W=0.2)\) may therefore be determined from the following expression:

\[
P_L = 22.413\sigma_Y,
\]

\(P_L\) is in Units of Newtons, and \(\sigma_Y\) in MPa.

The value of \(\beta_T\) for the 0.4T Charpy sized SE(B) \((a/W=0.2)\) specimen geometry was determined to be -0.266 for ferritic steels using this approach. This is in agreement with previous research work by Al-Ani and Hancock (1991) and Kirk et al (1993) which determined the value of \(\beta_T\) for this geometry of SE(B) specimen to be approximately -0.271.

### A.2 Constraint Modified Material Failure Diagram

Construction of the constraint modified material failure curve, shown schematically in Figure A-6, was undertaken using the material flow properties described in Section 4.1.1 for the study material. In addition, knowledge of the Beremin (1983) model shape parameter, \(m\), in Expression (3-17) is currently pre-requisite for using the methodology proposed by Sherry et al (2005) and therefore use of the BS 7910 (2013) constraint modified failure assessment diagram.
The following expression describes the constraint modified material failure curve for cleavage fracture conditions in ferritic steels (Sherry et al, 2005):

\[
\frac{K_{\text{mat,c}}}{K_{\text{mat}}} = \left[1 + \alpha (-\beta_T L_r)^\gamma \right], \text{ for } \beta_T < 0, \quad (A-2)
\]

where, \( \alpha, k \) = material constants which are dependent on the flow properties \( E, \sigma_\gamma \) and \( n \) used in Expression (3-3) and Weibull shape parameter \( (m) \),
\( K_{\text{mat}} \) = the material fracture toughness at the temperature of interest corresponding to a predefined failure probability.

The specimen loading line in Figure A-5 is plotted as the elastic-plastic stress intensity factor, \( K_U \), divided by \( K_{\text{mat}} \) versus \( \beta_T L_r \) for each increment of the analysis.

![Figure A-5 Schematic diagram of the constraint modified material failure curve and increase of fracture toughness under conditions of loss of constraint](image-url)
A.2.1 Influence of $\beta_T$ Parameter on the Constraint Modified Material Failure Curve Diagram Loading Line

A sensitivity study was conducted to assess the influence of $\beta_T$ on the specimen loading line in the constraint modified material fracture diagram for the 0.4T SE(B) ($a/W=0.2$) specimen geometry.

Figure A-6 and Table A-1 show that the value of $\beta_T$ adopted for the constraint correction elevates the value of $K_{\text{mat,c}}$ evaluated from the constraint modified material failure diagram for the sharp crack 0.4T SE(B) ($a/W=0.2$) specimen.
Table A-1 Effect of $\beta_T$ on the ratio $K_{\text{mat,c}}/K_{\text{mat}}$ for the 0.4T SE(B) ($a/W=0.2$) specimen

<table>
<thead>
<tr>
<th>$\beta_T$</th>
<th>$K_{\text{mat,c}}/K_{\text{mat}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.26</td>
<td>1.37</td>
</tr>
<tr>
<td>-0.28</td>
<td>1.42</td>
</tr>
<tr>
<td>-0.30</td>
<td>1.47</td>
</tr>
<tr>
<td>-0.32</td>
<td>1.52</td>
</tr>
</tbody>
</table>

Table A-1 shows that a difference between the present study $\beta_T$ value (-0.266) of 0.02, would therefore correspond to approximately a five percent increase or decrease in the magnitude of $K_{\text{mat}}$ to correct to $K_{\text{mat,c}}$.

A.2.2 Constraint and Ferritic Steel Material Dependent Material Failure Curve (Sherry et al, 2005): Parametric Study of Weibull Stress Model $m$ Parameter

The Weibull stress model shape parameter, $m$, influences the shape of the constraint modified material failure curve. Previous research work has established reliable calibration methodologies for sharp crack fracture toughness specimen geometries (Gao et al, 1998) and U-notch SE(B) geometries (Horn and Sherry, 2010). These approaches require conducting fracture toughness testing of multiple standard or non-standard specimen geometries to ensure accurate calibration of the Weibull stress model parameters. In practical circumstances this may not be feasible to conduct from an engineering standpoint. Therefore, it is desirable to quantify the influence of the Weibull stress model $m$ parameter on the constraint modified fracture toughness to establish if suitable guidance, which is not excessively conservative, may be issued to provide an option for defining the constraint modified material failure curve for ferritic steels without the need for these calibration procedures.

Figure A-7 shows the influence of the value of the Weibull stress $m$ parameter on the constraint modified material failure curve for the sharp crack 0.4T SE(B)
(a/W=0.2) specimen and the study material. Table A-2 provides the $K_{\text{mat,c}}/K_{\text{mat}}$ values calculated for the subject geometry of the constraint correction.

The value of parameter $m$ adopted for a material when using the Sherry et al. (2005) constraint modified material failure diagram has a significant effect on the magnitude of $K_{\text{mat}}$ to $K_{\text{mat,c}}$. The elevation of $K_{\text{mat}}$ is 24 percent greater when $m$ is set to a value of 10 when compared with a value of 5 during a constraint correction. The elevation of $K_{\text{mat}}$ is 17 percent greater when $m$ is set to a value of 20 when compared with a value of 10 during a constraint correction. This result means that lower values of $m$ will establish lower values of $K_{\text{mat,c}}$. Previous research by Gao et al. (1998)(1999) which calibrated the Beremin model and established $10 < m \leq 20$, this is supported by Horn and Sherry (2010) results using both test specimen sharp and non-sharp geometries.

![Effect of Weibull stress model $m$ parameter used in the constraint modified failure assessment diagram for a 0.4T SE(B) (a/W=0.2) specimen](image)

**Figure A-7** Effect of Weibull stress model $m$ parameter used in the constraint modified failure assessment diagram for a 0.4T SE(B) (a/W=0.2) specimen
Table A-2 Effect of Weibull stress model $m$ parameter on the ratio $K_{\text{mat,c}}/K_{\text{mat}}$ for the 0.4T SE(B) ($a/W=0.2$) specimen ($m=20$)

<table>
<thead>
<tr>
<th>$m$</th>
<th>$K_{\text{mat,c}}/K_{\text{mat}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
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</tr>
<tr>
<td>7.5</td>
<td>1.19</td>
</tr>
<tr>
<td>10</td>
<td>1.31</td>
</tr>
<tr>
<td>12.5</td>
<td>1.39</td>
</tr>
<tr>
<td>15</td>
<td>1.43</td>
</tr>
<tr>
<td>17.5</td>
<td>1.45</td>
</tr>
<tr>
<td>20</td>
<td>1.48</td>
</tr>
</tbody>
</table>

A.2.3 Estimation of the Weibull Stress Parameter $m$ Based Upon the Master Curve Methodology

The probabilistic master curve methodology developed by Wallin (1984) and codified as ASTM E 1921-11 (2011b), is premised upon the Weibull statistical distribution. This model calculates the probability of failure by cleavage fracture of 1T size ($B=25.4\text{mm}$) ferritic steel components by the following three parameter Weibull model:

$$P_i = 1 - \exp \left\{ - \left[ \frac{K_{i(i)} - K_{\min}}{K_0 - K_{\min}} \right]^4 \right\}, \quad (A-3)$$

where, $K_i = \text{the elastic-plastic stress intensity factor}$,
$K_0 = \text{the Weibull scale parameter}$,
$K_{\min}$ is a minimum fracture toughness below which cleavage fracture is not possible.

The exponent of 4 in Expression (A-3) is the Weibull shape parameter of the master curve methodology's Weibull model. Therefore, the probability of failure by cleavage is
proportional to $K_j$ raised to the power of 4. Additionally, using the form of the singular HRR stress field solution for a sharp crack in an infinite body under mode I loading given by:

$$\sigma_{ij} \propto C \left( \frac{J}{r} \right)^{\frac{1}{n+1}} \sigma_{ij}(\theta), \quad (A-4)$$

where,

- $J$ = the elastic-plastic J-integral,
- $\sigma_{ij}$ = the component of stress at a material point,
- $C$ = is a coefficient of proportionality,
- $n$ = the power law strain hardening exponent for the material,

and, $\sigma_{ij}(\theta)$ = a factor depending on the polar angle from the crack plane ($\theta$).

Inspection of Expression (3-17) given by Beremin (1983) and adopted by Sherry et al (2005) showed that $\sigma_w$ is related to the maximum principal stress ($\sigma_1$) raised to the power of $m$. The stress field solution in Expression (A-4) suggested the following relationship between $\sigma_1$ and $J$:

$$\sigma_1^{2(m+1)} \propto J^2, \quad (A-5)$$

Using Expression (3-7), $\sigma_1$ in Expression (A-5) can be related to $K_j$ by the following:

$$\sigma_1^{2(m+1)} \propto K_j^4, \quad (A-6)$$

An initial estimate value of the Beremin (1983) $m$ parameter can therefore be established by the following expression under the assumptions of the derivation of the sharp crack stress field solution:

$$m = 2(n+1), \quad (A-7)$$
For the ferritic steel study material, $n$ was determined to be 6.5 using experimental data reported in Section 4.1.1, the Beremin model $m$ parameter can be calculated to be 15 using the approximate Expression (A-7) in this case. This corresponds to a constraint modified increase in $K_{\text{mat}}$ of 43% (Table A-2) using the method of Sherry et al (2005). This approach required experimental validation, therefore laboratory test results for 0.5T, deeply cracked $(a/W=0.5)$ and shallow cracked SE(B) specimens $(a/W<0.15)$ were used to validate this study. These results were chosen because they were undertaken using an identical study material to the present work (Horn and Sherry, 2010).

A.2.4 Experimental Validation of Proposed Weibull Stress Parameter $m$

**Estimation Method**

The experimental results of Section A.1.2 were used to compare the predictions of $m$ using Expression (A-7) and the constraint modified material failure curve of Section (A.2) with experimental constraint dependent test results. $P_{f,\text{Rank}}$ experimental values were plotted against $J$ for SE(B) specimens of nominal $a/W=0.5$ and 0.15 respectively. All the $J$ values were determined using the ASTM E 1820-11 (2011a) methodology and converted to $K_J$ using plane strain formulae for further analysis, the experimental results are shown in Figure A-8.

The ratio of shallow crack SE(B) $(a/W=0.15)$ $K_J$, $K_{J,\text{SE(B)}, a/W=0.15}$ to SE(B) $(a/W=0.5)$ $K_J$, $K_{J,\text{SE(B)}, a/W=0.5}$ was calculated at incremental $P_{f,\text{Rank}}$ values using interpolation of the experimental results. A linear line was fitted through this data to determined this ratio as a function of $P_{f,\text{Rank}}$. The results of this analysis are shown in Figure A-9. Using the equation determined from the linear fit, $K_{J,\text{SE(B)}, a/W=0.15}$/ $K_{J,\text{SE(B)}, a/W=0.5}$ was found to equal approximately 1.9 at, $P_{f,\text{Rank}}=0.5$. This represents a significant fracture toughness increase if such results are to be described by the constraint modified material failure curve approach.
Figure A-8  Experimental validation of constraint modified toughness method

Numerical analysis of the sharp crack 0.5T SE(B) ($a/W=0.15$) was undertaken to construct the constraint modified material failure diagram. This analysis was undertaken following a numerical approach similar to that described in Section A.1.2 for the sharp crack 0.4T SE(B) ($a/W=0.2$) specimen geometry.

The biaxiality parameter, $\beta_T$, for this geometry was calculated to be -0.31. Figure A-10 shows the results of this analysis work. The ratios for $K_{mat,c}/K_{mat}$ for several Weibull stress $m$ parameter values are given in Table A-3. The maximum value of $K_{mat,c}/K_{mat}$ for a value of $m=20$ was found to be 1.82. The value of Weibull stress $m$ parameter for the study material was estimated to be equal to 15 using Expression (A-7), the analysis results of Table A-3 showed that the corresponding $K_{mat,c}/K_{mat}$ ratio was 1.74 using this value of parameter $m$. 

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Figure A-10  Effect of \( m \) parameter used in the constraint modified failure assessment diagram for 0.5T SE(B) \((a/W=0.15)\) specimen

Figure A-9  Ratio of \( J_{p,\text{CMOD}} \) for shallow \((a/W=0.15)\) and deeply \((a/W=0.5)\) cracked 0.5T SE(B) specimens from experimental results of Horn (2010)
Table A-3 Effect of Weibull stress \( m \) parameter on ratio \( K_{\text{mat},c}/K_{\text{mat}} \) for the 0.5T SE(B) \((a/W=0.15)\) specimen

<table>
<thead>
<tr>
<th>( m )</th>
<th>( K_{\text{mat},c}/K_{\text{mat}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.15</td>
</tr>
<tr>
<td>7.5</td>
<td>1.38</td>
</tr>
<tr>
<td>10</td>
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<td>12.5</td>
<td>1.70</td>
</tr>
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<td>15</td>
<td>1.74</td>
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<tr>
<td>17.5</td>
<td>1.76</td>
</tr>
<tr>
<td>20</td>
<td>1.82</td>
</tr>
</tbody>
</table>

A.3 Notch Correction

To correct 0.4T SE(B) \((a/W=0.5)\) \( K_J \) data to a U-notch SE(B) \((a/W=0.2, \rho=0.25\text{mm})\), non-standard, specimen geometry it was necessary to adjust the \( K_{\text{mat}} \) determined from the fracture toughness specimens for the effect of a change of sharp crack to a U-notch. This section details the development work undertaken to apply such an adjustment using the approach of Horn and Sherry (2012a).

A.3.1 Numerical Approach

A finite element model of the deep U-notch 0.4T SE(B) specimen \((a/W=0.5, \rho=0.25\text{mm})\) specimen geometry was used to develop the notch correction part of this procedure. The purpose of this modelling work was to evaluate the notch modified failure diagram for the study material using the methodology developed by Horn and Sherry (2012a). Three dimensional elastic and elastic-plastic analyses were conducted using ABAQUS version 6.11 (Simulia, 2011) using a small strain formulation to evaluate the elastic stress at the tip of the U-notch (at the centre of the crack front) and \( J \) at each load increment of the analyses respectively. Linear full integration elements were used
and the elastic-plastic material properties were those reported in Section 4.1.1. A numerical approach similar to Section 3.1.1 was used. Approximately 60 load increments were used to complete the elastic-plastic analysis. The mesh arrangement for the numerical analysis of the U-notch 0.4T SE(B) specimen \((a/W=0.5, \rho=0.25\text{mm})\) is shown in Figure A-11.

![Quarter symmetry finite element model of U-notch 0.4T SE(B) specimen](image)

**Figure A-11** Quarter symmetry finite element model of U-notch 0.4T SE(B)
\[(a/W=0.5, \rho=0.25\text{mm})\]

### A.3.2 Determination of \(\beta_N\) for a U-notch 0.4T SE(B) Specimen \((a/W=0.5, \rho=0.25\text{mm}, \phi=0^\circ)\)

A linear elastic analysis of the U-notch 0.4T SE(B) \((a/W=0.5, \rho=0.25\text{mm})\) specimen geometry was undertaken for the purpose of determining a relationship between the load and the elastic stress acting perpendicular to the notch tip at a position central to the notch front, \(\sigma_N\). Nodal stress values were extracted, corresponding to the specimen longitudinal direction (the direction of \(\sigma_N\)), at each load increment. Figure A-12 shows the results of this FE analysis, undertaken to determine the relationship between \(\sigma_N\) and applied load for the U-notch 0.4T SE(B) \((a/W=0.5, \rho=0.25\text{mm})\).
A solution for \( \sigma_N \) in U-notch three point bend specimens was proposed for use in defect assessments by Horn and Sherry (2012a). This analytical solution was based on the work of Creager and Paris (1967) and is given by the Expression (A.9).

\[
\sigma_N = \frac{3PS}{2BW^2} \left( 1 + 2Y \sqrt{\frac{a}{\rho}} \right),
\]  

(A-9)

where, \( Y \) = a geometric correction factor.

Figure A-13 compares Expression (A-9) with the FEM analysis results for the U-notch 0.4T SE(B) (\( a/W=0.5, \rho=0.25\)mm). The agreement between FEM and Expression (A-9) was found to be within 0.15%.

**Figure A-12** Comparison of the \( \sigma_N \) relationship with applied load for a U-notch 0.4T SE(B) (\( a/W=0.5, \rho=0.25\)mm) for FEM analysis and an analytical solution
The following linear expression was found to accurately describe the relationship of $\sigma_N$ to $P$:

$$\sigma_N = 0.8206P, \quad \text{(A-10)}$$

$\beta_N$ is related to the elastic mode I opening stress at the tip of a U-notch, $\sigma_N$ by Expression (A-11):

$$\frac{L_r}{\beta_N} = \frac{\sigma_N}{\sigma_Y}, \quad \text{(A-11)}$$

and,

$$L_r = \frac{P}{P_L} \quad \text{(A-12)}$$

where,

$P_L$ = the limit load of the U-notch 0.4T SE(B) specimen ($a/W=0.5$, $\rho=0.25\text{mm}$),

$P_L$ was determined from elastic-perfectly plastic analyses of the U-notch 0.4T SE(B) ($a/W=0.5$, $\rho=0.25\text{mm}$) specimen geometry using several FE models with a range of material $\sigma_Y$ definitions for a steel material ($E=200,000\text{MPa}$). $\sigma_Y$ was defined to be the stress magnitude at which perfectly plastic material behaviour commenced. The results of this numerical work are shown in Figure A-13.

A linear expression was found to accurately fit the numerical results in Figure A-13, $P_L$ can therefore be calculated using the following expression:

$$P_L = 9.182\sigma_Y. \quad \text{(A-13)}$$
Figure A-13 Finite element results for the limit load, $P_L$, for a U-notch 0.4T SE(B) 

(a/W=0.5, ρ=0.25mm) specimen geometry

$\sigma_y$ within Expression (A-13) the yield strength (defined at 0.2% plastic strain) of the ferritic steel material in units of MPa, and, $P_L$ is expressed in units of Newtons in Expression (A-13).

The value of $\beta_N$ was determined to be equal to 0.133 for the U-notch 0.4T SE(B) (a/W=0.5, ρ=0.25mm) specimen geometry using the numerical approach described.

The ratio $K_{mat,\rho}/K_{mat}$ at cleavage fracture for the U-notch 0.4T SE(B) (a/W=0.5, ρ=0.25mm) specimen geometry was found to be approximately 1.94 using $m=20$ within the approach of Horn and Sherry (2012a).

The shape of the non-sharp defect material failure curve is significantly altered by the definition of the Weibull stress $m$ parameter (Expression 3-17). Therefore further
analyses work was completed to establish the effect of the value of $m$ on the $K_{\text{mat},\rho}/K_{\text{mat}}$ at cleavage failure for the study ferritic steel.

**A.3.3 Ferritic Steel Notch Dependent Material Failure Curve (Horn and Sherry, 2012a): Parametric Study Weibull Stress Model $m$ Parameter**

The shape of the non-sharp defect material failure curve is significantly altered by the definition of the Weibull stress $m$ parameter in Expression (3-17). Therefore further analyses work was completed to establish the effect of the value of $m$ on the $K_{\text{mat},\rho}/K_{\text{mat}}$ at cleavage failure for the study ferritic steel. The results of this work are shown in Figure A-14 and the values of $K_{\text{mat},\rho}/K_{\text{mat}}$ on $m$ for the study material and U-notch 0.4T SE(B) ($a/W=0.5, \rho=0.25\text{mm}$) specimen geometry are given in Table A-4.

![Figure A-14](image)

**Figure A-14** Effect of Weibull stress $m$ parameter on the non-sharp defect modified material failure diagram and loading line for a U-notch 0.4T SE(B) ($a/W=0.5, \rho=0.25\text{mm}$) specimen
Table A-4 Effect of Weibull stress $m$ parameter on ratio $K_{\text{mat,c}}/K_{\text{mat}}$ for a U-notch 0.4T SE(B) ($a/W=0.5$, $\rho=0.25\text{mm}$) specimen

<table>
<thead>
<tr>
<th>$m$</th>
<th>$K_{\text{mat,c}}/K_{\text{mat}}$</th>
</tr>
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<tbody>
<tr>
<td>5</td>
<td>1.03</td>
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<tr>
<td>7.5</td>
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<td>10</td>
<td>1.21</td>
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<td>15</td>
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<td>27.5</td>
<td>2.67</td>
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</tbody>
</table>

Figure A-15 shows a significant effect of the value of $m$ on the non-sharp defect modified material failure curves. The ratio $K_{\text{mat,c}}/K_{\text{mat}}$ increases with increasing $m$ and this effect increases in magnitude as $m$ increases, i.e. larger $m$ values greater than 20 produce larger increases in the ratio. This in contrast to the constraint modified failure curve analysed in Section A.2.2 which exhibited a decreasing dependence of the ratio $K_{\text{mat,c}}/K_{\text{mat}}$ with increasing values of $m$. This observation is summarised in Figure A-15 which shows the sensitivity of adjusted material fracture toughness to $m$ for the study material using the two different techniques.

A.4 Combined Constraint and Notch Correction

A combined constraint and notch correction was undertaken using $m=20$, the output represented a combined modified material fracture toughness, $K_{\text{mat,c,p}}$, of a U-notch 0.4T SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) specimen for the ferritic steel study material.
Figure A-15  Effect of Weibull stress $m$ parameter on the constraint and non-sharp defect adjusted material toughness for the study material

The combined correction was undertaken to scale each sharp crack SE(B) $K_J$ test result to an equivalent fracture toughness, $K_{J,c,ρ}$, representative of the U-notch geometry under cleavage fracture conditions. Horn and Sherry (2012b) established the following combined adjustment expression for calculating $K_{mat,c,ρ}$:

$$K_{mat,c,ρ} = K_{mat} \left[ 1 + \alpha (\beta_T L_T) \right]^{1 + \gamma \left( \frac{L_T}{\beta_N} \right)^{-l}} \quad \text{for} \ T / \sigma_f < 0. \quad (A-14)$$

The terms in Expression (A-14) have been previously defined in Section 3.5.5.

Expression (A-14) represents the product of the constraint and notch correction procedure outputs to calculate $K_{mat,c,ρ}$. This considers the two effects of constraint loss and the alteration of the geometry of the defect from a sharp crack to a U-notch as acting independently. This expression can be applied to any ferric steel within the range
of flow properties upon which their conception was based. It therefore allows direct estimation of the $K_{J,\rho}$ of a U-notch 0.4T SE(B) ($a/W=0.2$, $\rho=0.25\text{mm}$) specimen. This later geometry is of interest to the wider scope of this project because of the geometric similarity it holds with the standard Charpy V-notch specimen. The Charpy V-notch specimen has a notch flank angle, $\phi$, equal to 45°. A combined constraint and notch correction factor was derived from the analyses in Sections A.2.2 and A.3.1 for the ferritic steel study material. This factor is derived from the two ratios of constraint and notch modified material fracture toughness, $K_{\text{mat},x}$, using Tables A.2 and A.4, $K_{\text{mat},c,\rho} = 2.84K_{\text{mat}}$.
A.5 References


