Network distributed generation capacity analysis using OPF with voltage step constraints
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Abstract
The generation capacity connected in distribution networks is increasing, in large part because of the necessity of siting renewable generation where resources are available. If distributed generation is connected in inappropriate parts of a network, it can cause a significant decrease in the total potential of the network for new generation. An optimal power flow-based method has previously been proposed for assessing network generation capacities. As, for a realistic assessment, it is necessary to include as many of the relevant physical and technical constraints as possible, limits on the voltage step changes on loss of a generator are added to the model here. Results on the variation of the network capacity with the generator power factors and the allowed voltage step window are presented.

Keywords: Optimization methods, Load flow analysis, Power generation planning.

1. Introduction
Worldwide environmental concerns have increased the barriers for new large scale conventional power station developments. Also driven by concerns over security of fuel supply, governments around the world have set targets to diversify their energy mixes in the forthcoming decades. In the UK, incentives are already in place to encourage renewable and combined heat and power developments. However, the use of distribution networks for the connection of Distributed Generation (DG), means that Distribution Network Operators (DNOs) face a number of technical issues such as fault levels, voltage control and power losses relating to DG [1]. This paper describes a method for assessing the DG capacity of a network, taking into account thermal constraints, voltage level constraints, and voltage step constraints on loss of a generator.

Voltage step changes occur when a generator is started up or suddenly disconnected from the network, and limits are typically placed on the maximum step change allowed. Different authorities give a range of limits on voltage steps, for instance in the UK units greater than 5MW have a limit of 3% for infrequent planned switching events or outages and 6% for unplanned outages (e.g. faults) [2], whereas a 5% limit is in common use in the USA [3].

While the process of starting a generator may also lead to step changes in voltage levels, the sudden disconnection of a DG unit from the network due to faults or other causes will be studied in this work. When using a power flow-based model to assess voltage step change, it can be calculated as the difference between the voltage level when the generation unit is connected, and the steady state voltage level with the same network topology but with the generator disconnected.

In order to maximise the potential of a network to support DG, careful planning is required - connection of generation at some buses might result in network sterilisation, significantly reducing the total capacity for DG [4, 5] (the goal is not just to supply the load in the network from the embedded DG, but also to export power to an external transmission network if possible.) This is a particular
concern where connection applications are dealt with on a first-come, first-served basis, without an analysis of the consequences for the network's total capacity.

When assessing a network's capacity for generation, it is necessary to consider all significant technical and physical constraints. This paper therefore builds on earlier work on generation capacity assessment by mathematical optimisation [4], adding maximum voltage step constraints to the capacity assessment model. First, Section 2 introduces a simple two-bus model for analysing the voltage step. In Section 3, the method for using an optimal power flow model to determine a network's capacity for DG is described, including the voltage step constraints and the voltage regulation model applied. Results using a real part of the Scottish distribution network are presented in Section 4 and finally conclusions are drawn.

2. Voltage rise and voltage step

Voltage step changes in distribution networks may be investigated qualitatively using the two bus system shown in Fig. 1. This consists of a grid supply point (GSP) at bus A, and load and generation at bus B.

![Two bus system for voltage step analysis.](image)

Fig. 1: Two bus system for voltage step analysis.

The voltage rise between buses A and B is given approximately by

\[
V_{BA} = (P_{DG} - P_L)R + (Q_{DG} - Q_L)X
\]

It is assumed that the generator is not run in voltage control mode (as generator voltages are typically above nominal because of the real power flow to the grid supply point, the difference from nominal is often referred to as voltage rise rather than level.)

Subtracting this expression with the DG off and on, the voltage step at bus B on loss of the generator, assuming that the voltage at A remains constant, is

\[
V_{BA} = P_{DG}R + Q_{DG}X
\]

Unlike the voltage rise, the step depends on the full output of the generator, and is not mitigated by load at the bus. Also, if the generator is operated at lagging power factor, the reactive flow tends to reinforce the voltage step from the generator real power output, and at leading power factor the reactive flow tends to reduce the voltage step. If the generator consumes enough reactive power, it is possible for the voltage step to be upward.

Due to the nonlinearity of the AC power flow equations, it is necessary to use a full AC power flow model for robust calculation of voltage steps and rises. Nevertheless, this two bus model will be useful in interpreting the results presented later.
3. DG capacity analysis using OPF models

3.1 Previous work
The OPF for DG capacity analysis is based on the concept that the network's capacity for new generation may be found by placing DG expansion sites at the appropriate buses, and using an OPF model to evaluate the maximum total generation which the network can support at these sites [4]. The capacity at each site is a decision variable in the problem, as opposed to a fixed parameter. As is common with DG [6], the generators are assumed to be run in constant power factor mode (i.e. with no voltage control), although alternative operational modes are possible.

The only customised constraint in this model before security constraints are added is the Kirchhoff current law, where the capacity of any DG expansion site must be added to the net power injection at bus \( b \). In the results presented here there is one connection to an external network, via which surplus power may be exported – this will normally be the slack bus. The objective function is simply the total DG real power capacity in the network, i.e. the sum over new generators of the individual capacities. Thermal, voltage level, and Kirchhoff voltage law constraints are included in the usual way.

3.2 Voltage step constraints
Voltage step constraints are included in a similar way to the familiar line outage security constraints. A set of power flow equations is added as extra constraints in the OPF for each generator outage contingency (these differ from the base case only in that the power injection from this generator is zero, and of course in the use of contingency voltage and flow variables.) As the new generation is not run in voltage control mode, the DG connection buses are (P,Q) nodes in the contingency power flows.

The voltage step limit itself is enforced for each generator outage by placing bounds on the deviation of the contingency bus voltages from the base case values.

3.3 Voltage regulation model
Transformer tap settings are used in distribution networks to keep the bus voltages as close to nominal as possible. In this work, therefore, the secondary buses of all transformers are constrained to exactly nominal voltage (a continuous range of tap settings is used in order to retain the continuous optimisation problem.)

3.5 Implementation
The OPF is implemented in the AIMMS optimisation modelling environment [7], and the mathematical program generated is sent to the CONOPT general reduced gradient solver, which has proved absolutely reliable in convergence on a class of much larger SCOPF problems [8], and is reasonably efficient.

A full mathematical specification of the model is available from the first author.

4. Results

4.1 Test Network
The capacity evaluation method is demonstrated on a small section of distribution network in the UK, Fig. 2. This is a subsection of the network presented in the original paper on DG capacity evaluation using an OPF model [4]. The mainly rural network has significant potential for wind, small scale hydro and other renewable developments, and is representative of many UK networks with such potential.
Fig. 2: Test network for the DG capacity analysis OPF, and parameters for the lines and transformers (all quantities are in per unit on a 100 MVA base.) Buses 21, 23 and 26 are 11 kV.

DG expansion sites of unlimited capacity are placed at buses 21, 23 and 26. Bus voltages are constrained between 0.97 and 1.03 p.u. in the base case, apart from the grid supply point at bus 6 which is at nominal voltage.

The maximum DG real power capacity in the network is shown in Fig. 3 for DG power factors fixed at 0.95 lagging, unity and 0.95 leading.

The active inequality constraints, i.e. those which restrict the total DG capacity, are listed in Table I. $V^{(i,j)}(b)$ denotes the (upper,lower) voltage limit at bus $b$ and $f^{+}(l,t)$ the thermal limit on a line or transformer.
Table I: Active constraints in the capacity assessment OPF model.

<table>
<thead>
<tr>
<th>Power factor</th>
<th>Without step constraints</th>
<th>With step constraints</th>
</tr>
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<tbody>
<tr>
<td>0.95 lag</td>
<td>$V^+(b_{22})$, $V^+(b_{24})$, $f^+(t_{20-21})$</td>
<td>$f^+(t_{20-21})$, $V^{S-}(b_{23}, g_{23})$, $V^{S-}(b_{26}, g_{26})$</td>
</tr>
<tr>
<td>Unity</td>
<td>$V^+(b_{25})$, $f^+(t_{20-22})$, $f^+(t_{20-21})$</td>
<td>$f^+(t_{20-21})$, $V^{S-}(b_{23}, g_{23})$, $V^{S-}(b_{25}, g_{26})$</td>
</tr>
<tr>
<td>0.95 lead</td>
<td>$f^+(t_{20-22})$, $f^+(t_{20-21})$, $f^+(t_{25-26})$</td>
<td>$f^+(t_{20-22})$, $f^+(t_{20-21})$, $f^+(t_{25-26})$</td>
</tr>
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When the DG units operate at lagging power factor, the real and reactive flows in the network tend to be in the same direction (as seen earlier in the two bus model). They therefore both contribute to voltage rise at the generators, and upper voltage limits on the 33 kV feeders restrict the generation at buses 23 and 26. At unity and leading power factor, the real and reactive flows are in the opposite directions, reducing voltage rise, and the thermal limits then become more significant. As the lines from 6 to 20 have high capacity and relatively low reactance, the generation at bus 21 is always restricted by the thermal limit of the transformer connecting it to bus 20.

4.2 Voltage Step Constraints

If a limit of 3% is placed on the voltage step at each bus on loss of a generator, the network DG capacities change as in Fig. 2, and the active equality constraints are as listed in Table 2. In addition to the symbols listed in the previous section, $V^{S-}(b, g)$ denotes the upper voltage step constraint at bus $b$ when generator $g$ disconnects (with $V^{S+}(b, g)$ being the lower step constraint). Upward voltage steps are taken to be positive, downward steps are negative. As the voltage steps (unlike voltage rises) are not mitigated by load increases (Section 2), voltage step constraints replace voltage level constraints on the list of active constraints.

Here, the voltage step constraints are significant at lagging and unity power factors, but the result at leading power factor is exactly the same as before. This is again due to the relative directions of the real and reactive flows, as when these are in the same direction their contributions to the voltage step have the same sign, resulting in step constraints becoming active.

4.4 Dependence of Capacity on Step Limit

The variation of the optimal generation capacities with the voltage step limit is shown in Fig. 5, for a range of generator power factors.

The main qualitative change occurs between step limits of 1.5% and 2%, when the number of active step limits decreases from 4 to 2. As the step constraint is relaxed further, voltage level and thermal constraints become the dominant restriction on generation capacity at all power factors. Once more, voltage step constraints become less significant as the power factor moves from lagging to leading. Indeed, at 0.95 leading p.f. the voltage step barely affects the network generation capacity when the step limit is at least 3%.

5. Conclusions

The method for assessing the capacity of networks for new generation has been extended to include voltage step constraints on loss of generators. These are included in a similar manner to the more usual line outage constraints, namely by including contingency power flow equations in the optimisation model.
When all the generators are run at a fixed power factor, voltage step constraints are most significant at lagging power factors, when the real and reactive contributions reinforce each other. In the test network used, thermal constraints become more significant at leading power factor where the voltage step and rise effects are reduced.

While the quantitative results presented are necessarily specific to the particular network, the qualitative trend of voltage step (and level) constraints restricting generation capacity less as the power factor moves from lagging to leading may be expected to generalise to most distribution networks.

References
2. “Recommendations for the connection of embedded generating plant to public distribution systems above 20kV or with outputs over 5MW” (Electricity Networks Association, Engineering Recommendation G75/1, 2002)
3. R. Dugan (personal communication)