Hard-Field THz Tomography

Krikor B. Ozanyan, Senior Member, IEEE, Paul Wright, Mark Stringer and Robert E Miles

Abstract—We report on hard-field tomography measurements in the THz spectral range and subsequent image reconstruction of a phantom subject. At THz wavelengths, the traditional hard-field tomography approach to measure attenuation is hindered by a substantial diffusely scattered component. Consequently, we work in optical density image contrast, as opposed to material density typical in high-energy hard-field modalities, such as x-ray CT. The hard-field component of the signal is extracted with a spatial filter, efficiently suppressing the soft-field contributions from the imaged subject. Using Time-Domain THz Spectroscopy, line integrals of the real part of the refractive index are taken, by measuring the delay of the THz pulse across the subject at 12 angles and 0.5mm steps in the transversal direction for each angle. The delay values are calculated from the location of the first peak in the integrated time-domain waveforms. This is justified by the physics of THz generation with ultrashort pulses in a biased-gap antenna and is shown to be superior to existing alternatives. The resulting tomography projections provide evidence for the hard-field character of the line integrals. The quality of the reconstructed image is interpreted and discussed, together with some limitations and future avenues.

Index Terms—THz, tomography, hard-field, pulse delay, image reconstruction.

I. INTRODUCTION

A widely accepted definition of Hard-Field Tomography (HFT) is based on the probing electromagnetic field propagating along straight lines [1] which allows line integrals of various quantities to be measured. For the simplest case of parallel beams in 2D (‘pencil beam’ arrangement), the mathematical expression of the forward problem in HFT is the 2D Radon transformation [1]:

\[ R(\phi, s) = \int \int f(x, y) \delta(x \sin \phi - y \cos \phi - s) \, dx \, dy , \]

where \( \delta \) is Dirac’s delta function, ensuring that integration is along a chosen set of paths and \( s \) is enumerating the set of parallel beams directed at an angle \( \phi \). While the forward transformation (1) yields a set of measured path integrals \( R(\phi,s) \), the inverse transformation results in an image slice reconstructed to a specific contrast: in x-ray HFT the contrast is typically in material density, but in the general case the contrast is determined by the particular measurand, more specifically by its linear-density function \( f(x,y) \) as a first order approximation. In this work, we are concerned with the imaging of the subject’s refractive index (real part), measured by the propagation delay of an ultrashort THz pulse. Thus, ‘speed-of-light’ tomography is arguably the most direct tomographic modality for the imaging of the refractive index. HFT is most often associated with higher frequency radiation, such as x-rays propagating along straight lines, while the range from DC to radio and microwave frequencies is typical for Soft-Field Tomography (SFT) where field lines are not straight. Furthermore, the HFT modalities’ local character, ensured by the delta function in (1), is often contrasted to the case of SFT, where probing is not local: typically the field distribution within the imaged subject depends on its permittivity at any location. It is worth mentioning here, that straight-line propagation is a weaker indicator of HFT than locality, as demonstrated by the recently introduced Guided-Path Tomography [2-4], which is local but allows propagation along a much wider range of path geometries.

The clear distinction between a hard-field and soft-field modality becomes more difficult as we approach either side of the spectral range between 100 GHz and 10 THz. Because of the experimental complexity and the relative lack of verification in the past, strict definitions and quantification of the HFT/SFT boundaries have been avoided. Nevertheless, tomography work in the THz spectral region, where characteristics of HFT and SFT co-exist, is possible by careful consideration of the dominant scattering mechanisms and the photon propagation trajectories. The solution implemented in this work aims to achieve detection of the HFT component by suppressing the soft-field effects. This is a required and strong justification for the use of well understood and comparatively simple HFT data inversion methods to be utilized, such as the standard filtered backprojection algorithm, the Fourier-slice method, as well as the iterative algebraic method [1].

II. BACKGROUND

By applying THz Time-Domain Spectroscopy (TDS), Zhang et al. [5] acquired Radon transformed data at 100 angular projections from a 30 mm x 50 mm polystyrene phantom containing voids. The THz beam, focused on the subject, was stepped across each angular projection. The transmitted THz field was recorded as a time-domain waveform, from which the 'peak' amplitude and delay, as well as the Fourier amplitude and phase, were independently used to achieve imaging contrast. Under the assumption of a planar THz wave, based on the Rayleigh length being larger than the...
imaged subject size, reconstruction was implemented using the Fourier slice theorem. Subjects made of other materials, such as a dielectric hollow sphere and part of an avian bone, were also imaged. It is important to note that satisfactory reconstruction was observed in the phase and delay-time contrasts only. The quality according to the chosen metrics was similar in these two contrasts.

Tomography images from THz transmission measurements with narrowband THz emission at 2.9 THz have been demonstrated [6,7], apparently in attenuation contrast. Expanded polystyrene phantoms of size between 10 and 15mm (refractive index n=1.02) and various shapes were reconstructed by a standard HFT reconstruction algorithm from 180 angular projections sampled at 0.2mm intervals. The outer contours of a near-circular shape low-density polystyrene object [6], together with the position of an oval void inside that object [7], were reproduced well from just 18 projection angles. The errors in the images were attributed to non-negligible surface refraction, affecting the recorded intensity of the line integrals. The reconstruction of a cylindrical PTFE (polytetrafluoroethylene) phantom [6] yielded a central void artifact. This was attributed to the higher refractive index (n=1.50) of PTFE leading to significant refraction at the surface - corroborating the link between the particular HFT contrast and the physical nature of the measured line integrals.

III. PATH INTEGRALS AND IMAGE CONTRAST

Consider the typical path integral in x-ray Computer Tomography [1]:

\[ N_d = N_s \exp \left( - \int \mu(x) dx \right), \]  

(2)

where \( N_s \) and \( N_d \) are the number of x-ray photons (field intensity, or field amplitude squared), at the source and detector respectively, and \( \mu(x) \) is the attenuation coefficient. For weak attenuation, justified in the x-ray case for materials of moderate density, (2) can be expressed in terms of the drop in number of photons reaching the detector, as a result of a subject introduced in the path:

\[ \Delta N = N_s \int \mu(x) dx \]  

(3)

Eq.(3), implies that HFT measurements on the number of transmitted photons can be inverted to yield the spatial distribution of the attenuation coefficient, which is proportional to the material density. If we now consider the corresponding expression for the THz pulse delay, as a result of a subject introduced in the path, we need to integrate the inverse phase velocity of the material along that path:

\[ \tau = \frac{1}{c} \int n(x) dx \]  

(4)

A standard broadband THz system (0.1-3.0 THz) was used in the TDS experiments, based on 90 fs @ 82 MHz, 800 nm pulses delivered by a Ti:Sapphire laser (Spectra Physics MaiTai). The IR pulse train is split into two optical paths, the pump and probe beams. The pump beam is focused onto a 200
μm gap between biased metal electrodes deposited upon the surface of semi-insulating GaAs. Thus, electron–hole pairs are created by each laser pulse and, being accelerated by the bias field, act as a transient current source for the radiating antenna producing bursts of THz radiation. The latter is collected by a 12.7 mm diameter Au-coated parabolic mirror and directed through an initial 1 mm aperture. The THz beam transmitted through the sample (mounted on the rotational-translational stage in fig.2) passes through two 1mm circular apertures separated by 0.5 m. The detector arrangement is based on electro-optic sampling in a 1 mm thick <110> oriented zinc telluride (ZnTe) crystal, where the THz and IR pulses are combined. Birefringence induced in the ZnTe by the THz electric field effects a polarization rotation of the IR probe pulse, which is then analysed by a Wollaston prism. The optical paths of the THz and IR pulses before the ZnTe crystal are defined so that their relative time of arrival can be controlled by an optical delay line, which allows the complete time-domain waveform of the THz electric field to be mapped. To improve the signal-to-noise ratio, the polarization state of the IR beam during the scan was monitored with analog autobalancing (New Focus Nirvana 2017), eliminating the common-mode signal between the two outputs of the Wollaston prism. The emitter bias was modulated at 10 kHz which allowed the balanced output to be finally processed by a lock-in amplifier (Signal Recovery 7265).

The subject was rotated as a whole to 12 angles, equally spaced at 15°, and for each angle the transmitted THz waveform was recorded by TDS for up to 59 subject displacement positions in steps of 0.5mm. As the processing of the recorded data is solely in the time-domain and does not involve spectral analysis, only a limited region around the pulse, corresponding to less than 8 ps, was recorded.

![Fig. 2. Experimental setup, showing (not to scale) the spatial filter consisting of two 1mm apertures separated by 0.5m. The stage translation and rotation allow line integral measurements of the pulse delay across the phantom.](image)

![Fig 3. Representative f3eld waveform measured by THz TDS for one of the beam positions in Fig.2: with the polystyrene cylinder in (dashed line) and out (solid line) of the beam path.](image)

V. RESULTS AND DISCUSSION

Fig.3 shows the first 7.5 psec of a representative THz waveform, demonstrating the induced time-delay and the changes in the ‘peak’ amplitude effected by the introduction of the subject into the THz beam. Close examination of the full set of waveforms in the angular projection of a homogeneous phantom of n=1.02, reveals poor correlation between the amplitudes of the first positive and first negative peak (or combination thereof) and the intersection length of the particular beam with the phantom. This is corroborated by a separate series of transmission measurements through a variety of Styrofoam shapes, showing persistent loss of signal amplitude at the shapes’ edges and, overall, unpredictable angular projections. Consequently, and in agreement with the findings in [5], HFT reconstructions in amplitude contrast did not produce meaningful results, even in the case of minimal surface refraction, indicating the substantial role of diffraction and diffuse scattering. Therefore, in the light of the arguments in Sections I-III, we discuss here only HFT imaging from delay measurements.

Of utmost importance for the validity of the HFT approach outlined in the previous Sections is to ensure that the diffracted and diffusely scattered THz components are filtered out, so that only line-integral contributions (4) are filtered and used for the reconstruction. We achieve this (see fig.2) by spatial filtering of the received THz beam by the 0.5 m collimator. Thus, ‘softening’ of the hard-field propagation is suppressed and only photons propagating strictly in the forward direction contribute to the forward Radon transform. Geometrically, this is also the shortest path to the detector; hence the term ‘ballistic photons’ can be borrowed from literature on ballistic imaging in the near infrared (diffuse optical tomography [8], time-gated optical tomography [9], shadowgraphy [10] and others). Typically, in these
measurements of the exact time delay for the shortest path filtering (time-gating). In our case, the TDS recording of the THz field and accelerated across the biasing electrodes. The emitted THz field is the temporal derivative of the current density, which also depends on the carrier concentration and mobility. Therefore it is possible, by integrating the photonic field, to obtain a single-maximum shape (or single minimum, depending on the polarity of the recorded signal). This is consistent with the ballistic approach as we work on the first integrated maximum only: the data used for the inverse Radon transform represent the shortest path, both in space and time. Fig. 4 shows the integrated waveforms for a 6.5 mm Styrofoam cylinder fully crossing the ballistic photon path in one direction in 0.5 mm steps. Notably, both, the amplitude and the timing of the single maximum coincide well for the bold and dashed traces representing, respectively, two independent sequences: a) introduction from outside the beam to midpoint, where detected photons travel along the diameter and b) removal from midpoint to outside the beam, which appears as the reverse of a) because of the phantom shape. The maximum delay, measured as the time equivalent between the initial and most remote positions of the single maximum, is in excellent agreement with the delay calculated from the values of the diameter and the refractive index, shown above the graphs. Fig. 5 shows the integrated waveforms for a Styrofoam right prism with an isoscelexan trapezoid base, measuring 9 mm between its parallel sides, fully crossing the ballistic photon path in the direction which is parallel to these sides and perpendicular to its axis. Again, the thickness of the phantom at any step is reproduced very well, but we observe different amplitudes of the solid and dashed maxima, probably due to the different surface quality of the prism’s non-parallel sides, which were cut by hand.

There is no unique approach in the literature to the extraction of the pulse propagation delay from the time-domain response. Possible options are to measure the time shift of the highest peaks, which are easy to identify, or construct a simple function of the positions of the first two peaks. (For clarity we assume that a positive peak is followed by a negative, although this can be reversed depending on the polarity of the recorded signal.) We address this problem on the basis of the physics of THz generation. When the ultrashort pump pulse is focused onto a biased-gap antenna of semiconductor material, photons with energy larger than the bandgap create electron-hole pairs near the surface, which are then separated by the field and accelerated across the biasing electrodes. The emitted THz field is the temporal derivative of the current density, which also depends on the carrier concentration and mobility. Therefore it is possible, by integrating the closest to the origin derivative shape (i.e. the first pair of positive and negative peaks in fig.3) of the THz field, to

Fig 4. Representative integrated waveforms measured by THz TDS of a Styrofoam cylinder, at several translational positions (see Fig.2) while being introduced in (solid lines) and taken out (dashed lines) of the beam

Fig 5. Representative integrated waveforms measured by THz TDS of a Styrofoam trapezoid at several translational positions (see Fig.2) while being introduced in (solid lines) and taken out (dashed lines) of the beam

Strictly speaking, the shape of the integrated THz pulse is not identical to that of the IR pump pulse, since the causal relationship between the two involves other factors stemming from e.g. the carrier dynamics and the radiation coupling in and out of the antenna. As the THz pulse propagates through the components of the system, and through a sample, the integrated shape departs further from that of the initial pump pulse. Therefore, in spite of the obvious justification in the physics of THz generation in a biased-gap antenna, calculating delays with the integrated waveform method should be exercised with caution and the integrated shape should be seen as that of an ‘apparent’ pump pulse, which in first approximation would have generated the recorded THz pulse. Nevertheless, our observations imply strongly that this method is adequate for calculating the delay line integrals for THz HFT. It can be argued that a possible alternative to integration is to calculate the time shift of the first zero value of the waveform, as that would correspond to the maximum of the ‘apparent’ pulse; however, droops and uncertainty in the baseline position of the TDS waveform (see fig.3) introduce substantial additional errors in this case.

The hard-field character of the calculated line integrals is confirmed by the careful inspection of projection data at angles which allow simple interpretation. Fig. 6 shows the delay line integral data points for projections at 0°, 90°, 180° and 270° (as per the inset of fig.1, projecting from bottom to top, right to left, top to bottom and left to right, respectively). The projections show good symmetry under 180° rotation. Most importantly, the 90° and 270° projections demonstrate that the order of the different shapes in the beam path (the circle vs the half-circle’s wedge) has no consequence, which
would not have been the case if the soft-field component was not adequately filtered.

With the above discussion in mind, all acquired projections were integrated, with the exact maxima of the integrated shapes obtained by quadratic approximation, and used to calculate a single numerical value for the pulse delay in each Radon sample. The Radon data inversion was implemented with the iradon function in Matlab 2010, using a ramp filter multiplied by a Hamming window to suppress excessive high-frequency noise. Gridding to Cartesian coordinates was done by nearest-neighbour interpolation on a 40x40 pixel image frame, applying non-negativity constraints and periphery suppression. The reconstructed image is reproduced in fig.7 without any post-reconstruction smoothing or other filtering. The overall shapes, position, separation and size of the phantom’s constituents are reproduced very well, while there is a certain loss of information in that the pixel values within the boundaries of two shapes are not constant, as would be expected for a phantom of homogenous refractive index. The observed artifacts are well understood and are caused by the comparatively small number of projection angles. Most striking is the excellent definition of the half-cylinder’s sharp edge, but this is mainly due to that plane on the phantom coinciding with one of the projection directions. “Glows” around sharp edges, as well as aliasing between adjacent regions of high activity, are expected artifacts. The demonstrated spatial resolution should be judged with a view to the undersampling of the Radon transform in this case, where the 1,600 pixel reconstruction in fig.6 is achieved from only 587 measurements (down to 417 if we exclude line integrals outside the sinogram support). In principle, the utility of a limited set of line integrals of delay can be substantially improved by the application of reconstruction techniques for limited view HFT, such as the sinusoidal Hough transform [3][4].

To investigate further the performance of the integrated waveform method, compared to the alternatives for calculating delay line integrals, from the original waveforms we generated three more sets of line integrals, in which the delay was calculated from the shifts of, respectively, the first peak (maximum), second peak (minimum) and their average (see fig.3). The image reconstructions from these three additional datasets were performed by a procedure identical to the one used with the integrated waveform. The relative quality of the four reconstructed images was assessed by comparison with a binary numerical phantom, synthesized according to the inset of fig.1. Three quantitative measures of reconstruction error were used to evaluate the quality of the four reconstructions:

1. Average error:
   \[
   \text{av}_{\text{err}} = \frac{\sum_{x,y} |f(x,y) - h(x,y)|}{MN},
   \]  
   where \( f \) and \( h \) are the reference and reconstructed images, respectively, with dimensions \( M \times N \). The expression is clearly a simple average error per pixel.

2. Normalized absolute error:
   \[
   \text{norm}_{\text{abs}} = \frac{\sum_{x,y} |f(x,y) - h(x,y)|}{\sum_{x,y} |f(x,y)|},
   \]  
   which emphasizes the effect of many small errors and is invariant with scaling of \( f(x,y) \).

3. Normalized rms error:
   \[
   \text{norm}_{\text{rms}} = \left( \sum_{x,y} \left[ \frac{f(x,y) - h(x,y)}{f'(x,y)} \right]^2 \right)^{1/2},
   \]  
   where \( f' \) is the average value of all the phantom pixels. The expression in (6) is normalized against the image variance and therefore is invariant to the level of activity in an image. The
norm_rms error can become large with only a few large errors in the reconstruction. While av_err is sometimes used in the literature, both norm_rms and norm_abs are the most common objective metrics that are considered to correlate well with perceived image quality [12]. The results from the comparison are shown in Table 1 where, within each error measure, the numbers are normalized to the method yielding the least error. The Table clearly demonstrates that the integrated waveform method outperforms all others, on all three metrics, justifying its choice, as well as the arguments put forward earlier in this Section.

<table>
<thead>
<tr>
<th>Method</th>
<th>Metrics</th>
<th>Average error (4)</th>
<th>Normalised abs. error (5)</th>
<th>Normalised rms error (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrated waveform</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>First maximum</td>
<td>1.304</td>
<td>1.189</td>
<td>1.143</td>
<td></td>
</tr>
<tr>
<td>First minimum</td>
<td>1.800</td>
<td>1.435</td>
<td>1.477</td>
<td></td>
</tr>
<tr>
<td>Average (max., min.)</td>
<td>1.419</td>
<td>1.252</td>
<td>1.240</td>
<td></td>
</tr>
</tbody>
</table>

The error values are normalized toward the best performing method for calculating the delay: by time-integrating the field waveform.

Previously reported THz tomography involves focusing of the THz beam and positioning the sample in the focal plane [5-7]. It has been recognized [5] that this can present problems for the tomography of larger subjects if the Rayleigh length is shorter than the size of the imaged slice and quasi-parallel beam propagation is compromised. Furthermore, collection from focal plane samples is still from a substantial cone (E6.43 in [6,7]), which allows additional soft-field signal on the detector. The ballistic approach to HFT suppresses the soft-field component by a much stricter spatial filter and does not require focusing, thus preserving compliance to the parallel beam geometry independent of sample size.

It should be mentioned that in this work we are justified to utilize TDS for HFT in contrast to refractive index, as the latter has negligible dispersion (see fig. 1) and therefore all the Fourier components of the THz pulse will experience the same delay. With a dispersive refractive index, a typical TDS setup will emphasise the low-energy side of its inherent bandwidth (where the emitted intensity peaks) and care must be taken in the processing and interpretation of results. Furthermore, in that case the ballistic photon filter’s performance will vary with frequency, conditioned by the spectral dependence of the scattering mechanisms.

It is in principle possible to perform similar HFT imaging with CW narrowband THz, e.g. by applying homodyne detection [13]. In cases where the refractive index dispersion is non-negligible, such narrowband HFT can be used to avoid spectral regions where the refractive index varies particularly rapidly, e.g. close to optical resonances, which otherwise will introduce inadequacies in the dataset used for HFT.

VI. SUMMARY AND CONCLUSION

The THz spectral region is characterized by the co-existence of hard-field and soft-field components in the transmitted signal. As a consequence, the transmitted THz field amplitude is observed to reproduce poorly the projection of subjects, which is the basis of HFT imaging. Line integrals of THz pulse delay are more suitable for HFT and can be utilized, under the assumption of negligible scattering and absorption, to image the THz optical density, as opposed to material density in HFT at higher photon energies. In this work we demonstrated refractive index HFT of a complex Styrofoam phantom. We eliminate the soft-field component by retaining only straight-line propagating photons, as required for the validity of HFT image reconstruction. The values of the delay line integrals are calculated from the peak of the integrated time-domain waveforms, which is justified by the physics of THz generation and has been shown to be superior to known alternatives. The hard-field character of the generated projections has been evidenced by the observation that the line integrals depend only on the total optical density of the objects in the path and not on their order. The novelties in our approach are the efficient extraction of the hard-field THz signal component, together with calculation of the path delays from the integrated time-domain waveforms; their combination delivers reliable reconstruction by a HFT filtered backprojection algorithm.

REFERENCES

Krikor B. Ozanyan became a Member (M) of IEEE in 1995 and a Senior Member (SM) in 2003. He received his MSc degree in engineering physics (semiconductors) and PhD degree in solid-state physics in 1980 and 1989 respectively, from the University of Sofia, Bulgaria. He has held previous academic and research posts in the University of Sofia, The Norwegian Institute of Technology (Trondheim, Norway), the University of Hull (UK), and the University of Sheffield (UK), working on projects ranging from Brewster-angle mid-IR spectroscopic ellipsometry and electron confinement in quantum barriers, to the demonstration of the lasing at 333nm from strained MQW ZnCdS/ZnS structures and *in-situ* real-time optical monitoring of growth of III–V semiconductors in MBE and MOCVD machines. His current interests are in the area of optical sensing and indirect imaging (Tomography) by optical modalities, signal processing for optical experiments, and spectroscopy with ultrafast laser sources. He is currently Head of Sensors, Imaging and Signal Processing at the University of Manchester.

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(to be provided later)

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