Three Essays on Global Yield Curve Factors and International Linkages across Yield Curves

A thesis submitted to The University of Manchester for the degree of Doctoral of Philosophy in the Faculty of Humanities

2014

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Contents

This thesis contains 32,444 words including title page, tables, and footnotes.

Abstract

The University of Manchester

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Doctor of Philosophy (PhD)

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31st March 2014

This thesis presents three essays on global yield curve factors and international linkages across yield curves. The essays represent a contribution to our understanding of the effect of globalization on yields, addressing three topics: modeling global and local yield curve factors, modeling global and local yield curve factors in excess bond returns and a joint model of global macroeconomic and yield curve factors.[1]

The first essay proposes and develops an empirical model of global and local yield curve factors based on three factors proposed by Nelson and Siegel (1987) dynamized and reinterpreted by Diebold and Li (2006) as level, slope and curvature. The results support the existence of a global yield curve composed of global factors which together with local factors describe the yield curve of the USA, Germany and the UK. Specifically, the global factors explain on average 55% of the variance of yields, and impulse response functions indicate that shocks to global factors are larger and last longer than shocks to local factors.

In the second essay, we examine the predictability content of the global and local yield curve factor model to predict excess bond returns one year ahead. We use a rolling window of fifteen years to compare in-sample predictability of our model and two benchmark models: the model proposed by Cochrane and Piazzesi (2005) and the global and local factor model proposed by Dahlquist and Hasseltoft (2011). The results indicate that the global and local yield curve factors from our model predict excess bond returns with an adjusted R^2 up to 59%. We also find that global factors explain explain up to 58% of the forecast error variance when predicting excess bond returns. Moreover, our model outperforms both competing models considering the USA, Germany and the UK.

The third essay proposes and estimates a joint model of global macroeconomic and yield curve factors, which shows the interaction between global yield curve factors and global macroeconomic factors. Our findings show that the influence of macroeconomic factors on yield curve factors is stronger than the influence of yield curve factors on macroeconomic factors.

[1]Wu (2006) indicates that "Recent decades have seen globalization proceed at a rapid pace, tying nations' economies closer together through the freer movement across borders of goods, services, money and ideas. This has brought important changes in the forces that determine interest rates".

Declaration

I, Javier Enrique Sanhueza Gonzalez, declare that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Acknowledgements

I would like to thank my supervisors Ian Garrett and Laura Coroneo for their guidance, time and feedback. I deeply appreciate their guidance and encouragement throughout my studies at the Manchester Business School. Also, I would like to thank to the participants in various PhD committees for their comments and the staff of Manchester Business School for their support.

Finally, I would like to thank my wife Carmen and my family for their invaluable support.

Chapter 1

Introduction

1.1. Motivation

Interest rates are crucial for practitioners and policymakers due to their importance in financial decisions. Interest rates can be described by a yield curve, which in turn can be summarized by three factors: level, slope and curvature as indicated by Diebold and Li (2006). The level is the long-term factor related to the general level of interest rates, the slope is the difference between short and long-term rates and the curvature is a medium-term factor.

Also, globalization integrates economies through exchanging products, services, money and ideas across borders, generating interdependence. In this context, examining the effects of globalization on the yield curve will give us better insight into what determines yields and how yields within a country respond to so-called local factors and global factors which are common across countries.

Chapter 2 addresses the theoretical framework and estimation of the model with global and local yield curve factors. The model is based on the Diebold and Li (2006) dynamization and reparametrization of Nelson and Siegel's (1987) three-factor model (level, slope and curvature). The motivation for this research is the reduced number of studies which address the estimation of models with global yield curve factors and the fact that the existent global yield curve factor model does not consider the global curvature.

The objective is to propose a model based on global and local yield curve factors (level, slope and curvature), in order to deepen our understanding of the mechanism of transmission of shocks to yield curve factors. Also, the objective is to build a global and local yield curve factor model which explains yields of three countries: the USA, Germany and the UK. The countries are selected according to the relative importance of bond markets and the availability of public data.

We estimate global factors as common components between yields of three countries (the USA, Germany and the UK). We study the importance of global and local factors using variance decomposition of the total variance of yields. Global factors explain on average 55% of variance of yields which indicate that global factor are important in explaining yields and should be considered in financial decisions. We investigate the size and extent of shocks to factors by means of impulse response functions using the framework proposed by Sims (1980, 1982). These indicate that the shocks to local and global factors last for about 42 and 72 months, respectively, which highlights the importance of global factors due to the fact that in general the effects of shocks to global factor last longer than shocks to local factors.

We contribute to existing literature due to most yield curve modeling has been conducted in isolation at the country level and we estimate a global and local yield curve factor model which explains the yields of three countries. Also, existing global and local yield curve factor models do not consider curvature which can convey important information about future evolution of interest rates.

Our research is important for policymakers due tothe fact that the influence of global factor could counteract attempts of policymakers to influence the yield curve of the country. Also, financial decisions that do not consider the influence of global factors take the risk that adverse movements in global factors affect the investments.

Chapter 3 uses the global and local yield curve factor model to explore predictability of excess bond returns one year ahead. We investigate whether the model developed in Chapter 2 can better explain excess bond returns than two competing models: Cochrane and Piazzesi (2005) and Dahlquist and Hasseltoft (2011).

The motivation of this research is the lack of studies which consider global and local yield curve factor in excess bond returns. The objectives are to analyze if global components in yield curves across countries also imply global components in excess bond returns and compare the predictive ability of our global and local yield curve factor model with two competing models.

Our factor model predicts excess bond returns with an average adjusted R^2 up to 59% and global factors explain up to 58% of the variance of excess return forecast errors. The aforementioned, means that excess bond returns are well explained by our global and local yield curve factor model. The predictability of the global and local yield curve factor model is not spanned by the factors of Cochrane and Piazzesi (2005) and Dahlquist and Hasseltoft (2011).

We contribute to existing literature due to the fact that previous studies do not consider global and local yield curve factors in excess bond returns. Indeed, previous studies consider the single-factor model developed by Cochrane and Piazzesi (2005, 2008) and linear combinations of the factors from this model.

Our research is important for investors which can take advantage of the predictability of excess bond returns to invest in long-term bonds. Also, our research is important for policymakers which can separate the bond risk premia of expectations of future interest rates.

Chapter 4 expands the global and local yield curve factor model proposed in Chapter 2, to incorporate global macroeconomic factors to study the bidirectional influence of global yield curve factors on global macroeconomic factors and vice versa. The motivation of this research is the two sets of different results in the literature which provide mixed evidence of the influence of interest rates on macroeconomic variables and vice versa.

The objective is to analyze the bidirectional relationship between global and local yield curve factors and global macroeconomic factors. Also, the objective is to investigate whether a joint model of global and local yield curve factors and global macroeconomic factors provides evidence of the influence of yield curve factors on macroeconomic factors, the reverse or both.

We contribute to existing literature addressing the interaction between yield curve factors and macroeconomic factors using global factors which previous studies do not address. Also, we contribute to existing literature considering bidirectional relationship between yield curve factors and macroeconomic factors due to the fact that previous literature provide mixed evidence of the influence of yields on macroeconomic variables and macroeconomic variables on yields. In some respect, we extend the study of Diebold, Rudebusch and Aruoba (2006) which explores the bidirectional influence of yield curve factors and macroeconomic variables for the USA.

We look at the effects that global yield curve factors have on global macroeconomic factors and vice versa for three countries (the USA, Germany and the UK). Our findings indicate that there is a bidirectional interaction between global yield curve factors and global macroeconomic factors with a stronger influence of global macroeconomic factors on global yield curve factors.

Our research is important for policymakers which intend to determine the extent of the influence of global macroeconomic factors on global yield curve factors and vice versa. The aforementioned, is due to the fact that policymakers intend to influence the yield curve through the monetary policy rate in order to control inflation and gross domestic product so they would be interested in the bidirectional interaction between global macroeconomic factors and global yield curve factors.

1.2. Thesis structure

The thesis is structured following the format accepted by the Manchester Accounting and Finance Group, Manchester Business School. In particular, the chapters are in a format suitable for submission for publication in peer-reviewed journals. The thesis contains three original essays in Chapters 2, 3, and 4. Each chapter is self-contained and therefore contains a separate literature review relevant to that chapter. For this reason, the content of each chapter such as equations, tables, figures and footnotes are numbered independently. However, pages, titles, and subtitles are numbered in sequential order throughout the thesis.

The rest of the thesis continues as follows. Chapter 2 proposes and develops a model with local and global yield curve factors. Chapter 3 examines whether the global and local yield curve factors are able to explain excess bond returns better than competing models. Chapter 4 proposes and develops a joint model of global macroeconomic factors and global and local yield curve factors, as well as explains the bidirectional interaction between the factors. Finally, Chapter 5 concludes.

References

[1] Cochrane, J. H., and Piazzesi, M. Bond risk premia. *American Economic Review 95*, 1 (March 2005), 138–160.

[2] Dahlquist, M., and Hasseltoft, H. International Bond Risk Premia. *SSRN eLibrary* (2011).

[3] Diebold, F. X., and Li, C. Forecasting the term structure of government bond yields. *Journal of Econometrics 130*, 2 (February 2006), 337–364.

[4] Diebold, F., Rudebusch, G., and Aruoba, B. The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach. *Journal of Econometrics*, 131 (2006), 309–338.

[5] Nelson, C. R., and Siegel, A. F. Parsimonious modeling of yield curves. *Journal of Business 60*, 4 (October 1987), 473–89.

[6] Sims, C. A. Macroeconomics and Reality. *Econometrica 48*, 1 (January 1980), 1-48.

[7] Sims, C. A. Policy Analysis with Econometric Models. *Brookings Papers on Economics Activity,* 1 (1982), 107-152.

[8] Wu, T. Globalization's effect on interest rates and the yield curve. *Federal Reserve Bank of Dallas Economic Letter 1*, 9 (September 2006), 1–8.

Chapter 2

Modeling global and local yield curve factors

Abstract

We analyze the relationship between the yield curves of three countries, using global and local factors with a focus on dynamic linkages across and between yield curve and factors. We disentangle the latent global and local factors contained in country factors, based on the Diebold and Li (2006) parametrization of Nelson and Siegel's (1987) three factor model (level, slope and curvature) and a quasi-maximum likelihood approach. The results indicate that global factors explain on average 55% of the variance of yields. We study the effects of shocks to the factors, using impulse response functions. These results show that the response of the yields to the shocks to global factors have larger and longer lasting effects than the shocks to local factors.

2.1. Introduction and literature review

Modeling of term structure of interest rates is important for both policymakers and market practitioners since it conveys important information about where the market expects interest rates to be in the future. It is also relevant in securities and portfolio valuation. Most of the yield curve modeling has been conducted in isolation at the country level (Diebold, Li and Yue, 2008). The importance of studying yields at a multi-country level has been highlighted by the recent financial crisis, which has shown that financial markets are globally interconnected and move together. Therefore, it is important to understand the economic linkage of financial markets and in particular the mechanisms by which interest rate shocks are transmitted. Indeed, the Bank for International Settlements (BIS) indicates in its 2009 Report that the financial crisis shows the immense complexity of the modern financial system and the intricate linkage between financial markets, highlighting the need for a good understanding of the links between the yield curves across different countries as it might provide important information for regulators and market participants. In particular, regulators and market participants could benefit from the knowledge of the direction of the movements of global interest rate factors which could adversely affect domestic interest rates in order to take actions to counteract these effects. The main contribution of this paper is to isolate the forces of global comovements from idiosyncratic components for yield curves of different countries. We develop a model that identifies global and local factors for the yield curves of three countries: the USA, Germany and the UK.

Although there are different approaches to estimate yield curves, De Pooter (2007) and BIS (2005) report that the methodological approach developed by Nelson and Siegel (1987) and its extension proposed by Svensson (1994), have been widely used among practitioners and central banks. In particular, the model developed by Nelson and Siegel (1987), NS hereafter, relies on a set of predefined functions (which depend on the maturity and a decay factor), in order to create a fit which is flexible enough to allow to capture the different shapes of yield curves. This is based on factor loadings predefined according to the term to maturity (short, medium and long). Diebold and Li (2006) propose a reparameterization of the model developed by NS, where the coefficients (short, medium and long) are redefined in terms of level, slope and curvature. Although there is some criticism of the NS class of models, since they are not supported by a theoretical framework and are not necessarily arbitrage free, Coroneo, Nyholm and Vidova-Koleva (2011) provide a detailed discussion about how arbitrage-free the NS model actually is. Their conclusions indicate that from a statistical point of view, the factors of the NS model are not different than those of arbitrage-free models (at 99% level of confidence). Additionally, Christensen, Diebold and Rudebusch (2007) develop a theoretical framework in order to estimate an affine arbitrage free NS model (AFNS) maintaining the factor loadings of the NS model and indicate that additional terms that depend on the maturity of the bond are required.

Recently, some papers such as those of Diebold et al. (2008) and Modugno and Nikolaou (2009) have focused on the task of estimating the linkage of yield curves. The former uses a modified version of the NS model in order to estimate level and slope factors for four countries (the UK, the USA, Japan and Germany). The yield curve of these countries is explained by a global yield curve factor model. The model comprises orthogonal factors of two types: global and country-specific factors. Also, Modugno and Nikolaou (2009) evaluate the forecasting power of the international yield curve linkages, using an international yield curve approach for three countries: the UK, the USA and Germany. This methodological approach is based on the NS model's factors and a vector autoregressive (VAR) process estimated by maximum likelihood, where only the same factors for different countries interact with each other. Dahlquist and Hasseltoft (2011) propose to extend the factor model developed by Cochrane and Piazzesi (2005) to an international context, estimating global and local factors for international bonds of the UK, the USA, Germany and Switzerland. The global factor is the weighted average of Cochrane and Piazzesi (2005) factors for each country, where the weights are based on gross domestic product growth. The global factor is closely related to bond risk premia and global macroeconomic conditions.

Previous research on multi-country yield curve estimation could be characterized as global yield curve factors or international linkage of country factors. Although these studies have produced advances in the knowledge of the relationship between the yield curves of different countries, they have limitations. Firstly, studies on global factors do not include the curvature as a global factor, which could be important due to recent evidence that unanticipated movements of curvature factor contain important information on the future evolution of yield curve, output, market prices and inflation (Moench, 2012). Secondly, these studies do not consider interactions between different factors of different countries (e.g., between level and curvature, between level and slope or between slope and curvature), but previous research for single country yield curves (e.g., Diebold and Li, 2006; Moench, 2012) shows that there are interactions between different factors for the same country. In this regard, our preliminary results indicate that there are also important interactions between different factors and different countries.

This paper proposes a model based on a global and local yield curve factors (level, slope and curvature), in order to deepen the understanding of the mechanism of transmission of shocks to these yield curve factors, using impulse response functions. In particular, we build a global and local factor model which explains yields of three countries: the USA, Germany and the UK. Specifically, our global factor model includes level, slope and curvature factors allowing for interactions between the different factors and countries.[1] This interaction between factors differs from the framework proposed by Modugno and Nikolaou (2009) since we estimate a global factor model which allows cross-interactions between factors of different countries. It is different from the model proposed by Diebold et al. (2008) whose factors are orthogonal. Additionally, we use a quasi-maximum likelihood approach which overcomes the difficulties in estimating global factor models (Diebold et al., 2008).[2]

The estimation of our global and local factor model is performed using the quasi-maximum likelihood approach of Doz, Giannone and Reichlin (2006). This approach is developed for estimating dynamic factor models of a large sample size, utilizing the expectation maximization algorithm (hereafter, EM algorithm) and the Kalman filter. The implementation of our model is based on the technical report of Ghahramani and Hinton

[1]Specifically, the cross-factor interaction is between level and slope, level and curvature, and slope and curvature for the USA, Germany and the UK.

[2]Diebold et al. (2008) indicate that under normality assumptions the estimation of the model for a single country is straightforward, but in a multi-country framework estimation by maximum likelihood is "particularly difficult to implement" given the "large number of parameters to be estimated", for this reason they use a Bayesian approach (p. 355).

(1996) who provide a detailed description of the methodological procedures and steps involved in the estimation of parameters of linear dynamical systems (LDS) using the EM algorithm. This technical report is based on the methodological approach developed by Shumway and Stoffer (1982) to estimate the state-space model using the EM algorithm in conjunction with the Kalman smoother.

The results show that global factors explain on average 55% of the total variance of yields, and more specifically, global level factor explains on average 40% of the total variance. Moreover, we track the effects of shocks to both local and global factors on yields using impulse response functions. Our findings indicate that effects of the local and global factor shocks disappear no later than after 42 and 72 months, respectively. In addition, the range of response of the yields to shocks on global factors is larger than the response of yields to local factor shocks. The size and the lasting of effects of shocks to global factors on yields indicate the predominance of global factors on country yields.

The rest of the paper is organized as follows: in Section 2.2, we present a preliminary analysis. Section 2.3 describes the model approach. Section 2.4 details the estimation and Section 2.5 discusses the data and main results. Finally, Section 2.6 concludes.

2.2. Preliminary analysis

In this section, we introduce the NS model reparameterized by Diebold and Li (2006) and provide a generalization of this model to estimate simultaneously the yield curve for three countries: the USA, Germany and the UK. Also, we present evidence of common components between the NS factors of these countries. Specifically, the reparameterized NS model provides an interpretation to factors in the context of dynamic estimation as level, slope and curvature: l_t, s_t and c_t, respectively.

The NS model for each country i is

$$y_{i,t}(\tau) = l_{i,t} + s_{i,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + c_{i,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + e_{i,t}(\tau), \tag{1}$$

where $y_{i,t}(\tau)$ is the yield at time t with maturity τ, λ is the decay factor and $e_{i,t}(\tau)$ is the estimated error of the respective yield.[3] The loadings are 1 for level, which is a long-term factor, $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)$ for slope which is a short-term factor and $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$ which is a medium-term factor.

[3] The decay factor, λ, is fixed at the value of 0.0609 in order to maximize the curvature loadings for the period of 30 months and to reduce the numerical optimization process as suggested by Diebold and Li (2006).

The matrix representation of this model is

$$Y_{i,t} = \Gamma_i F_{i,t} + \varepsilon_{i,t}, \tag{2}$$

where $Y_{i,t}$ is the matrix that stacks the yields of country i for n maturities, Γ_i is the matrix of the NS factor loadings, whose jth row, $\Gamma_{i,j}$, contains the NS factor loadings $\Gamma_{i,j} = \left[\begin{array}{ccc} 1 & \frac{1-e^{-\lambda\tau_j}}{\lambda\tau_j} & \frac{1-e^{-\lambda\tau_j}}{\lambda\tau_j} - e^{-\lambda\tau_j} \end{array} \right]$, $F_{i,t}$ is the vector of factors (level, slope and curvature) and $\varepsilon_{i,t}$ is the vector of errors, for country i at time t.

Accordingly, the generalization of this model for several countries is straightforward, as follows

$$Y_t = \Gamma F_t + \varepsilon_t, \tag{3}$$

where Y_t is the matrix that stacks yields for all the countries at time t. Also, Γ is a block diagonal matrix of factor loadings that contains three identical submatrices, Γ_i, which in turn contain the factors loadings: level, slope and curvature for the three countries. The vector, F_t, contains the three factors (level, slope and curvature) for each one of the countries, as well as ε_t is the vector of errors, all at time t.

To explore the possibility of there being components of the level, slope and curvature that are common across countries, we undertake a preliminary analysis to estimate the NS factor model for the yield curve of the USA, Germany and the UK, in order to obtain the level, slope and curvature factors for each country, using the data provided by central banks.[4] Figure 1 indicates that there is a similar pattern among the three NS factors for the three countries.

[4] The information is obtained directly or provided through the Bank of International Settlement (BIS). The details of the data will be described later in Section 2.5.

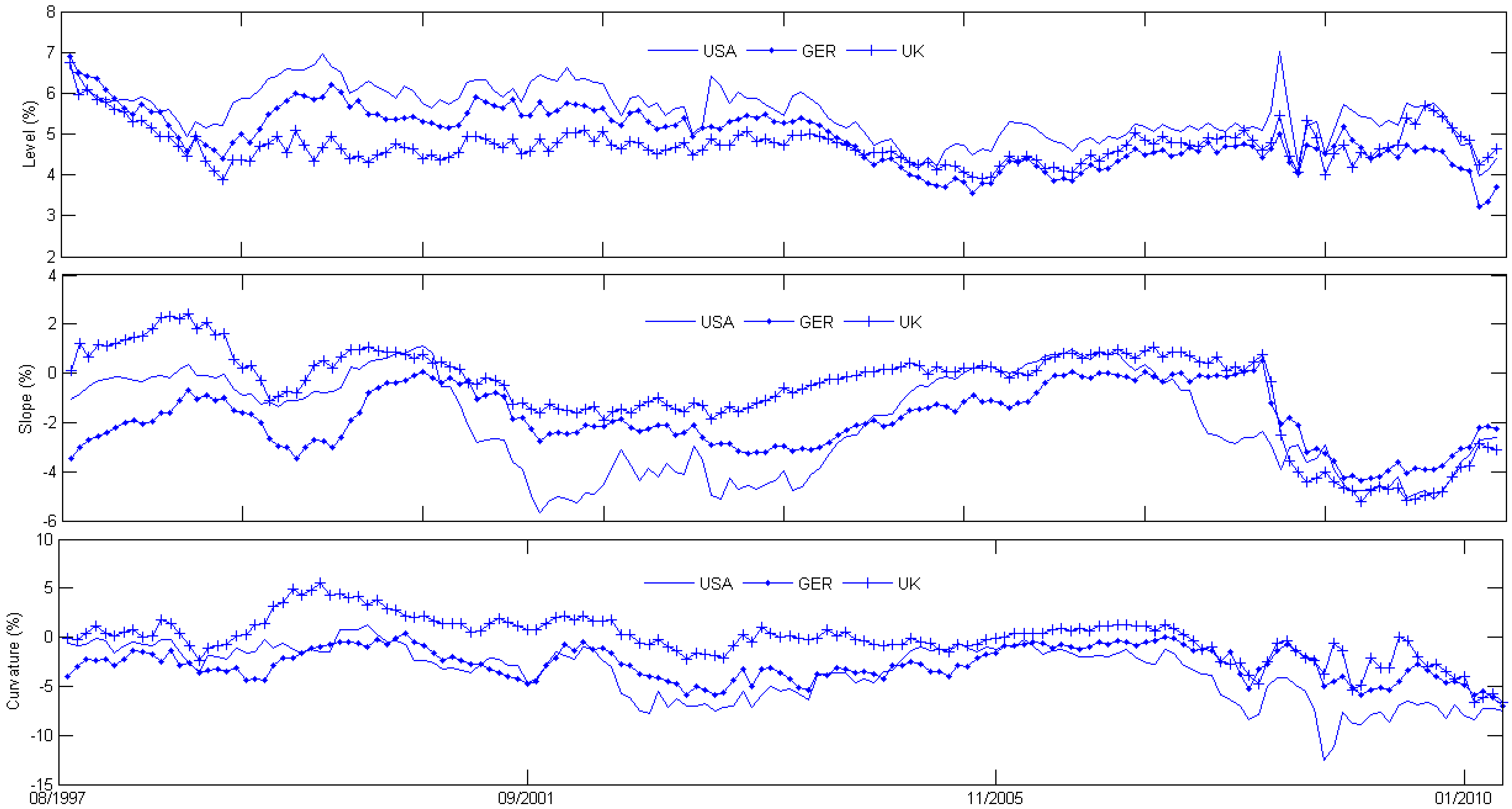

Figure 1: Nelson and Siegel (1987) factors reparameterized by Diebold and Li (2006) as level, slope and curvature, for the USA, Germany and the UK spanning the period 1997:08-2010:05

Figure 1 shows that there is not only a linkage between the same factors (level, slope and curvature) for different countries (the USA, Germany and the UK), but also a similarity between the patterns of these factors, especially between the slope and curvature factors. Table I shows the matrix of correlations between the level, slope and curvature factors. In absolute terms, the minimum correlation among the countries for the same factor is roughly 0.5 and the maximum correlation is 0.9. Moreover, the correlations between the slope and curvature factors range between 0.2 and 0.7, which confirms the similarity observed between slope and curvature. The negative correlation between the level and the slope could be due to the fact that increases in general level of rates reduce the gap between short and long term rates.

Table I

Correlation matrix of the NS factors

		Level			Slope			Curvature		
		USA	GER	UK	USA	GER	UK	USA	GER	UK
	USA	1.00								
Level	GER	0.89	1.00							
	UK	0.48	0.59	1.00						
	USA	-0.25	-0.19	-0.28	1.00					
Slope	GER	-0.30	-0.28	-0.36	0.68	1.00				
	UK	0.01	0.14	-0.11	0.75	0.70	1.00			
	USA	0.17	0.23	-0.10	0.66	0.34	0.72	1.00		
Curvature	GER	0.11	0.06	-0.19	0.63	0.55	0.62	0.73	1.00	
	UK	0.52	0.52	-0.15	0.39	0.22	0.55	0.75	0.69	1.00

Note: This table shows correlation between the factors level, slope and curvature for the USA, Germany and the UK using monthly yield data spanning the period 1997:08-2010:05.

In order to find the common components of the factors level, slope and curvature, we estimate and extract the first principal component for each factor for the three countries: the USA, Germany and the UK. Table II shows the percentage of the variance for each one of the factors explained by the first principal component for each country. The total variance explained for each principal component ranges between 78% and 82% and the details are described below.

Table II

Percentage of variance of the country-specific factors explained by the first principal component

Country	Level	Slope	Curvature
USA	84%	81%	84%
GER	91%	78%	80%
UK	58%	83%	81%
Mean	78%	81%	82%

Note: This table shows estimates of the first principal component for each factor (level, slope and curvature) for the USA, Germany and the UK, and computes the percentage of variance of each factor explained by each principal component, spanning the period 1997:08-2010:05.

These preliminary results support the idea of an international linkage. This would allow us to assess the effect of changes in common factors on the yield curve of each country, as well as the effect of changes in the local factors of one country on the yield curve of other countries.

2.3. The model

The results in the previous section indicate that there are common components across countries. Indeed, these suggest that there are two types of factors: common and local. Hence, we could define a model of factors which considers both global and local factors. Furthermore, by following Coroneo, Giannone and Modugno (2008), we predefine the factor loadings, in order to estimate a dynamic NS factor model, constraining the observation equation. Also, we restrict the matrix of factor loadings to be block diagonal as in Cicconi (2010). In this respect, the factors of the NS model, F_t, described in equation (3), are disentangled as the sum of two orthogonal or independent group of factors: global factors, F_t^G, and local factors, F_t^L, whose matrix β contains the loadings of each country over the global factors, F_t^G, as follows

$$F_t = \beta F_t^G + F_t^L. \tag{4}$$

Replacing equation (4) in equation (3), we have

$$Y_t = \Gamma \left[\beta F_t^G + F_t^L\right] + \varepsilon_t. \tag{5}$$

Therefore, in this specification of the model the global factors along with the local factors define the respective yield curve of each country. In addition, defining $\Gamma\beta = \Gamma^G$ we have the following state space representation of the model

$$Y_t = \Gamma^G F_t^G + \Gamma F_t^L + \varepsilon_t, \tag{6}$$

which we can conveniently rewrite as

$$Y_t = [\Gamma^G \quad \Gamma][F_t^{G'} \quad F_t^{L'}]' + \varepsilon_t. \tag{7}$$

Moreover, the factors follow a VAR of order one

$$[F_t^{G'} \quad F_t^{L'}]' = \Phi \; [F_{t-1}^{G'} \quad F_{t-1}^{L'}]' + w_t, \tag{8}$$

where the matrix Φ is a block diagonal matrix of factor loadings and w_t is the vector of errors at time t.

$$\varepsilon_t \sim N(0, \Sigma), \tag{9}$$

$$w_t \sim N(0, \Omega), \tag{10}$$

where matrices Σ and Ω are the variance-covariance matrices which are independent.

The state space representation of the model described by equations (7) and (8), could be written in a compact way as follows

$$Y_t = \Gamma^T F_t^T + \varepsilon_t, \tag{11}$$

where Γ^T contains the predefined factor loadings, $\Gamma^T = [\Gamma^G \quad \Gamma]$, and F_t^T contains the global and local NS factors, $F_t^T = [F_t^{G'} \quad F_t^{L'}]'$, and as we defined before follows a VAR of order one

$$F_t^T = \Phi F_{t-1}^T + AU_t. \tag{12}$$

The reduced form of errors is $w_t = AU_t$, where the shocks U_t are defined as "primitive" or "fundamental", which are orthogonal and have unit variance. Also, matrix A is defined as $\Omega = AA'$.

The matrix Σ is a diagonal matrix, whereas the matrix Ω is a two block diagonal matrix. The first block contains the variance-covariance errors of global factors and the second block contains the variance-covariance errors of the local factors.

The global and local factors are independent of each other both contemporaneously and across time. This specification means that global factors, F_t^G, only depend on global factors, while local factors only depend on local factors, F_t^L, but both together explain yields of countries. The global factors only interact with each other directly and the local also interact directly (between the different factors and the same country) and indirectly (between different factors and different countries).

2.4. Estimation

2.4.1. Estimation of global and local factors

Factor models do not have a unique solution because they have the rotational indeterminacy problem, so different combinations of factors and factor loadings could provide observationally equivalent solutions with the same likelihood but with different financial or economic implications. Hence, in order to obtain a unique identification of the parameters and unobservable factors in our model, we need to impose restrictions on the factor loadings.

The loadings, β, in equation (4) are not identified so we need to impose some restrictions in order to identify them. First, we constrain the matrix β to be block diagonal, in order to restrict that each global factor only loads in the same global factor, e.g., global level factor only load on global level.[5] Second, the factors are restricted to have a variance-covariance matrix equal to identity matrix.

Also, to estimate the model described by equations (11) and (12), we need to ensure that factor loadings and factors are uniquely identified. Hence, we restrict the factor loadings imposing the NS factor loading restrictions (1 , $\frac{1-e^{-\lambda\tau_j}}{\lambda\tau_j}$ and $\frac{1-e^{-\lambda\tau_j}}{\lambda\tau_j}-e^{-\lambda\tau_j}$). Moreover, due to the fact that the yields of the countries and therefore the global and local factors do not a mean of zero, we demean and standardize the yields. The details of the identification process are described in 2.4.2.

In order to disentangle the global and local factors, the estimation is developed according to the following. Firstly, the three NS factors are estimated for each country, using the predefined factor loadings (Γ) and assuming these factors contain the total effect of both kind of factors (global and local). Secondly, we estimate the loadings of each country over the global factors, $\hat{\beta}$, restricting this matrix to be block diagonal, standardizing the factors and using the quasi-maximum likelihood approach. Thirdly, the latent global ($\hat{l}_G$, $\hat{s}_G$ and

[5] The same is valid for the other factors: slope and curvature.

$\hat{c}_G$) and local factors ($\hat{l}_i$, $\hat{s}_i$ and $\hat{c}_i$) are estimated using the quasi-maximum likelihood approach and imposing orthogonality between both type of factors. In particular, in order to estimate the global and local factors we use a joint estimation procedure. We initialize the estimates of global factors using the standardized first principal components of each factor (estimated with all the countries) as well as we initialize the estimates of local factors using idiosyncratic terms (or error terms). We standardize both (yields and factors) subtracting the mean and dividing by the standard deviation.

The estimation procedure is conducted using quasi-maximum likelihood and the EM algorithm, according to the methodological approach proposed by Doz et al. (2006).

2.4.2. Identification

In factor models, the factor loadings and factors are not generally observable and their estimation does not have a unique solution due to the rotational indeterminacy problem. Therefore, different combinations of factors and factor loadings could provide solutions to the model, but with different economic implications. Henceforth, in order to obtain a unique solution, it is necessary to impose restrictions to identify the model (Moench, 2012).

In particular, the model described by equations (11) and (12) is not identified, and we need to impose some restrictions. Specifically, our identification of the model could be described in two steps: identification of loadings β and identification of Γ^T and factors.

Firstly, in order to identify β we need to go back to equation (4), where the country factors are explained by loadings, β, global factors, F_t^G, and local factors, F_t^L. The matrix, β, is restricted to be block diagonal, hence in this matrix of factor loadings of size 3 by 9 we restrict 18 values to be equal to 0. Moreover, we demean and standardize the country factors and initialize the estimation of factors and factor loadings using the first principal component of country factors, F_t. We choose the positive first principal component to initialize the estimation because of two main reasons. First, previous evidence indicates that factor loadings of countries are positive (Diebold et al., 2008). Second, we are interested in long term relationships, so if there is an inverse relationship between country factors and global factors, it is temporary and not sustainable over time.

Secondly, given that we have already identified β, we can focus our attention on equation (5). We demean and standardize the yields, Y_t, and we use the standardized factors, F_t^T. Also, we restrict the matrix of factor loadings Γ to be a block diagonal matrix, which contains the factor loadings of the NS model, so the matrix Γ is nonsingular. In particular, to illustrate the identification of the model we could consider the case where the matrix Γ is a block diagonal matrix that contains three identical submatrices, Γ^i, each one containing the NS factor loadings and the matrix, P, of size $3K \times 3K$, that rotates the factors, such that $PP' \equiv I$. Therefore, if we rotate the factors of the model described in equations (11) and (12), we have

$$\begin{aligned} Y_t &= \Gamma^T P^{-1} P F_t^T + \varepsilon_t, && (13) \\ P F_t^T &= P \Phi P^{-1} P F_{t-1}^T + P w_t. && (14) \end{aligned}$$

If we replace the terms $\tilde{\Gamma}^T = \Gamma^T P^{-1}$, $\tilde{F}_t^T = P F_t^T$, $\tilde{\Phi} = P \Phi P^{-1}$, $\tilde{F}_{t-1}^T = P F_{t-1}^T$ and $\tilde{w}_t = P w_t$, in equations (13) and (14), it is possible to rewrite the model described by equations (11) and (12) in an equivalent way, with the same likelihood, but with different factor loadings, $\tilde{\Gamma}^T$ and $\tilde{\Phi}$, and factors, $\tilde{F}_t^T$ and $\tilde{F}_{t-1}^T$, as follows

$$\begin{aligned} Y_t &= \tilde{\Gamma}^T \tilde{F}_t^T + \varepsilon_t, && (15) \\ \tilde{F}_t^T &= \tilde{\Phi} \tilde{F}_{t-1}^T + \tilde{w}_t. && (16) \end{aligned}$$

Therefore, from equation (11) we have $Var(Y_t) = \Gamma^T Var(F_t^T) \Gamma^{T\prime} + var(\varepsilon_t)$ and from equation (13) we have $Var(Y_t) = \tilde{\Gamma} Var(\tilde{F}_t^T) \tilde{\Gamma}' + var(\varepsilon_t)$ hence $\Gamma^T \Gamma^{T\prime} = \Gamma^T P^{-1} P^{-1\prime} \Gamma^{T\prime}$ and from our initial definition of $\tilde{\Gamma}^T$ we have $\tilde{\Gamma}^T = \Gamma^T P^{-1}$.

Moreover, from equation (11) we have $\Gamma^T = [\Gamma^G \;\; \Gamma] = [\Gamma\beta \;\; \Gamma] = \Gamma \; [\beta \;\; I]$ where Γ is a block diagonal matrix. Also, I is the identity matrix and β is the matrix of loadings that we already pointed out how to identify in previous paragraphs. Hence, the matrix Γ^T could be represented as a block diagonal matrix, Γ, containing the NS factor loadings, of size $3K \times 3K$, augmented with a full matrix, $\Gamma\beta$, of size $3K \times 3$, which is defined as the multiplication of the loadings and the NS factor loadings.

The rotation described by matrix $\tilde{\Gamma}^T$ should provide an equivalent solution to the model but with different factor loadings, such that $\tilde{\Gamma}^T = \Gamma^T P^{-1} = \Gamma^T P'$ and this equality should keep the same structure, i.e., $[\tilde{\Gamma}^T \beta \;\; \tilde{\Gamma}^T] = \tilde{\Gamma} \; [\beta \;\; I] = \Gamma \; [\beta \;\; I] P'$. Therefore, in order to obtain an observationally equivalent solution, matrix P' should keep the same structure of matrix Γ^T. Also, we know that $\tilde{\Gamma}^T = \Gamma^T P^{-1}$, so $\Gamma \; [\beta \;\; I] P' = \Gamma \; [\beta \;\; I]$ and none of the columns (rows) of Γ^T could be described as a linear combination of the other columns (rows). Hence, this can hold only if $P' = I$, where I is the identity matrix of size $3K \times 3K$. Hence, the solution to the model described by equations (11) and (12) is the same as the solution provided by equations (13) and (14). Therefore, the model is identified and the solution is unique.

2.4.3. Impulse response functions

The impulse response functions (IRF) allow us to track the effect of shocks to the factors on factors and in turn the effect on all the yields of the countries according to the model described by equations (11) and (12).

In order to track the effect of shocks to the factors through the system it is possible to write equation (12) as a moving average representation using the lag operator, L, as follows

$$\begin{aligned}(I - \Phi L)F_t^T &= AU_t, &(17)\\ F_t^T &= (I - \Phi L)^{-1}AU_t, &(18)\end{aligned}$$

where I is the identity matrix. Substituting equation (14) in to equation (11) we can write

$$Y_t = \Gamma^T(I - \Phi L)^{-1}AU_t + \varepsilon_t, \tag{19}$$

hence, the impulse response functions are

$$B(L) = \Gamma^T(I - \Phi L)^{-1}A, \tag{20}$$

and replacing equation (16) in equation (15) we have

$$Y_t = B(L)U_t + \varepsilon_t. \tag{21}$$

The identification of IRF is performed using the approach proposed by Sims (1980) and Sims (1982) which is based on Cholesky decomposition. Specifically, the reduced form of errors in equation (12) indicates that $w_t = AU_t$, but we can estimate w_t, so we need to find A and U_t in order to recover U_t and identify orthogonalized shocks. Hence, using the Cholesky decomposition, $\Omega = AA'$, we define a lower diagonal matrix, A, imposing $K(K-1)/2$ restrictions on matrix A, with K defined as the total number of factors. We determine the ordering of the factors in the decomposition as follows.

Firstly, in our specification there is zero correlation between the shocks to global factors and the shocks to local factors, so the global and local factors are no contemporaneously correlated. Hence, allocating the global factors first or the local factors first, this does not change our analysis nor results.

Secondly, the evidence in Section 2.2 indicates that yields are explained primarily by level, then by slope and finally by curvature. Therefore, we follow the same order in the hierarchy of both global and local factors for the identification of shocks. Hence, if we translate this ordering into the VAR representation, we have the level influence both slope and curvature contemporaneously, as well as the slope influence curvature.

Finally, we rank the countries in descending order by gross domestic product (GDP) to identify shocks. This means that factors of the USA explain those of Germany and the UK, and the factors of Germany explain those of the UK. This hierarchy works only in one direction but not vice versa.

2.4.4. Variance decomposition

The variance decomposition of yields explained by factors requires us to consider equations (11) and (12), as well as the decomposition of matrix $\Omega = AA'$. Specifically, yields are defined by $Y_t = \Gamma^T F_t^T + \varepsilon_t$ and conditional variance of yields is equal to $\left[\Gamma^T \left[AIA'\right] \Gamma^{T\prime}\right]$.

Therefore, the variance of nth yield explained by the ith factor is given by the mathematical expression $\Theta_n^{-1} \left[\Gamma_n^T \left[AI_iA'\right] \Gamma_n^{T\prime}\right]$. Where the matrix Θ_n^{-1} is the inverse of the variance of the yield nth, Γ_n^T is the row nth of the matrix Γ^T and I_i is the matrix with one in the row ith and column ith and zeros in any other coordinate.

2.5. Data and results

The data consist of monthly zero coupon government yields, for 10 maturities from 1 to 10 years for the USA, Germany and the UK, collected from the BIS database and the Bank of England.[6] The data span from August 1997 to May 2010.

Table III reports the results of estimates of global factor loading, β, in equation (4). Overall, the figures of the factor loadings in Table III are around 0.6, with the only exception of the UK which is 0.5. This is consistent with the lowest percentage explained by the first principal component of the level factor (58%) as we can see from Table II.

[6] In the case of BIS database the data set is provided by the central banks of respective countries.

Table III

Factor loadings

Country	Level	Slope	Curvature
USA	0.60	-	-
GER	0.63	-	-
UK	0.50	-	-
USA	-	0.58	-
GER	-	0.57	-
UK	-	0.59	-
USA	-	-	0.59
GER	-	-	0.57
UK	-	-	0.58

Note: This table shows the factor loadings of the USA, Germany and the UK over the global factors, spanning the period 1997:08-2010:05.

Table IV reports the mean absolute error of estimates of the global and local yield curve factor model for 10 maturities (from 1 to 10 years) and three countries (the USA, Germany and the UK). The estimates of the mean absolute error for all the countries indicate that average absolute error is in general around 2%. The average errors are lower for Germany and the UK, which is consistent with the higher influence of the USA and global factors on both countries, in comparison with the influence of these countries over the USA.

Table IV

Mean absolute error

Country	1 yr.	2 yrs.	3 yrs.	4 yrs.	5 yrs.	6 yrs.	7 yrs.	8 yrs.	9 yrs.	10 yrs.
USA	0.06	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.02
GER	0.04	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
UK	0.04	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02

Note: This table shows the mean absolute errors of the global and local yield curve factor model for the USA, Germany and the UK for yields 1 to 10 years. The data span the period 1997:08-2010:05.

Table V indicates a high goodness of fit in the estimation of the dynamic model of global and local yield curve factors. However, there is some loss of accuracy of fit in the longest maturities for the sample period.

Table V

Goodness of fit for the global and local factor model (in percentage)

	1 yr.	2 yrs.	3 yrs.	4 yrs.	5 yrs.	6 yrs.	7 yrs.	8 yrs.	9 yrs.	10 yrs.	Mean
USA	98.1	99.4	99.4	99.1	98.7	98.2	97.7	97.2	96.6	95.9	98.0
GER	98.7	99.6	99.6	99.5	99.5	99.5	99.5	99.4	99.3	99.1	99.4
UK	98.3	99.3	99.3	99.3	99.2	98.9	98.4	97.7	96.3	94.2	98.1

Note: This table shows goodness of fit of the global and local yield curve factor model for the USA, Germany and the UK for yields 1 to 10 years. The data span the period 1997:08-2010:05.

Table VI shows the variance decomposition of global and local factors on average for all the yields. It is noticeable that global factors explain roughly 55% of the total variance of yields, and less than 45% is explained for the local factors and the interaction between them. The factor which explains the most of the variance of yields is global level (40%), followed by global slope. Also, local level explains an important percentage of the total variance of the USA and the UK, but in the case of Germany, local curvature represents the most important local factor.

Table VI

Variance decomposition: one step ahead forecast error variance

	Global			USA			GER			UK		
	l_G	s_G	c_G	l_{USA}	s_{USA}	c_{USA}	l_{GER}	s_{GER}	c_{GER}	l_{UK}	s_{UK}	c_{UK}
USA	43%	1%	16%	23%	1%	13%						
GER	59%	5%	6%	3%	2%	7%	1%	1%	14%			
UK	19%	8%	10%	2%	0%	4%	8%	1%	4%	27%	8%	2%
TOTAL	40%	5%	10%	9%	1%	8%	3%	1%	6%	9%	3%	1%

Note: This table shows variance decomposition of one step ahead forecast error variance for the USA, Germany and the UK based on average of 10 yields (1 to 10 years) per country.

Table VII reports the variance decomposition of one step ahead forecast variance for the USA, Germany and the UK for yields from 1 to 10 years. Global level explains more the longest maturities up to 55%, 84% and 33% of the forecast variance of the USA, Germany and the UK. Conversely, global slope explains more the shortest maturities, explaining up to 7%, 21% and 27% of the forecast variance of the USA, Germany and the UK. Global curvature explains more of the forecast variance for maturities between 2 and 3 years, up to 31%, 10% and 18% for the USA, Germany and the UK, respectively.

Table VII

Variance decomposition: one step ahead forecast error variance

		Global			USA			GER			UK		
		l_G	s_G	c_G	l_{USA}	s_{USA}	c_{USA}	l_{GER}	s_{GER}	c_{GER}	l_{UK}	s_{UK}	c_{UK}
	1 yr.	26%	7%	18%	2%	9%	16%	-	-	-	-	-	-
	2 yrs.	28%	3%	31%	8%	3%	27%	-	-	-	-	-	-
	3 yrs.	31%	1%	28%	15%	1%	24%	-	-	-	-	-	-
U	4 yrs.	37%	1%	22%	20%	0%	19%	-	-	-	-	-	-
S	5 yrs.	43%	0%	17%	25%	0%	15%	-	-	-	-	-	-
A	6 yrs.	48%	0%	13%	28%	0%	11%	-	-	-	-	-	-
	7 yrs.	51%	0%	9%	31%	0%	8%	-	-	-	-	-	-
	8 yrs.	54%	0%	7%	32%	0%	6%	-	-	-	-	-	-
	9 yrs.	55%	0%	5%	33%	0%	5%	-	-	-	-	-	-
	10 yrs.	54%	0%	4%	32%	0%	4%	-	-	-	-	-	-
	1 yr.	20%	21%	6%	0%	1%	18%	5%	0%	14%	-	-	-
	2 yrs.	30%	11%	10%	4%	5%	16%	1%	1%	24%	-	-	-
	3 yrs.	39%	6%	10%	5%	5%	12%	0%	2%	23%	-	-	-
G	4 yrs.	50%	3%	8%	5%	4%	9%	0%	2%	19%	-	-	-
E	5 yrs.	60%	2%	7%	4%	3%	6%	0%	2%	16%	-	-	-
R	6 yrs.	69%	2%	5%	3%	2%	4%	1%	2%	12%	-	-	-
	7 yrs.	76%	1%	4%	2%	1%	3%	1%	1%	10%	-	-	-
	8 yrs.	81%	1%	3%	1%	1%	2%	1%	1%	8%	-	-	-
	9 yrs.	84%	1%	3%	1%	0%	1%	1%	1%	6%	-	-	-
	10 yrs.	84%	1%	2%	1%	0%	1%	1%	1%	5%	-	-	-
	1 yr.	0%	27%	7%	1%	0%	2%	5%	1%	3%	18%	6%	2%
	2 yrs.	2%	19%	18%	1%	1%	10%	8%	2%	7%	12%	15%	4%
	3 yrs.	6%	11%	18%	1%	1%	11%	9%	2%	8%	12%	16%	4%
	4 yrs.	14%	7%	16%	2%	0%	9%	9%	2%	7%	18%	14%	3%
U	5 yrs.	21%	4%	12%	2%	0%	6%	9%	1%	6%	25%	11%	3%
K	6 yrs.	26%	3%	9%	2%	0%	3%	9%	1%	4%	32%	8%	2%
	7 yrs.	31%	2%	7%	2%	0%	2%	8%	0%	3%	38%	6%	1%
	8 yrs.	33%	1%	5%	2%	0%	1%	7%	0%	3%	41%	4%	1%
	9 yrs.	31%	1%	3%	2%	0%	0%	6%	0%	2%	40%	3%	1%
	10 yrs.	27%	1%	2%	1%	0%	0%	5%	0%	1%	34%	2%	0%
	Tot.	40%	5%	10%	9%	1%	8%	3%	1%	6%	9%	3%	1%

Note: This table reports the variance decomposition of one step ahead forecast errors for the USA, Germany and the UK and yields from 1 to 10 years.

Results of the IRF (in Figures 2 to 5) indicate that in general, effects of the local factor shocks over the country yields disappear no later than after 42 months. The average response of the yields to global factor shocks is 4 basis point (bp) for the whole period (72 months) and maturities. Also, the average response of yields for the first 42 months is 6 bp. This reaction is larger than the average response of yields to the local factor shocks, which is lower than 1 bp, with the only exception of the response of yields to shocks to local factors of USA whose response is 1 and 2 bp for 72 months and 42 months, respectively. Furthermore, the effects of shocks to global factors disappear slowly and last for about 72 months. In general, the shortest maturities exhibit a larger response than the longest, and this is especially important for global factor shocks. In general, one year yield for all the countries is more sensitive than longer maturities to shocks to global and local factors, and is thus more sensitive to shocks to global factors than local factors.

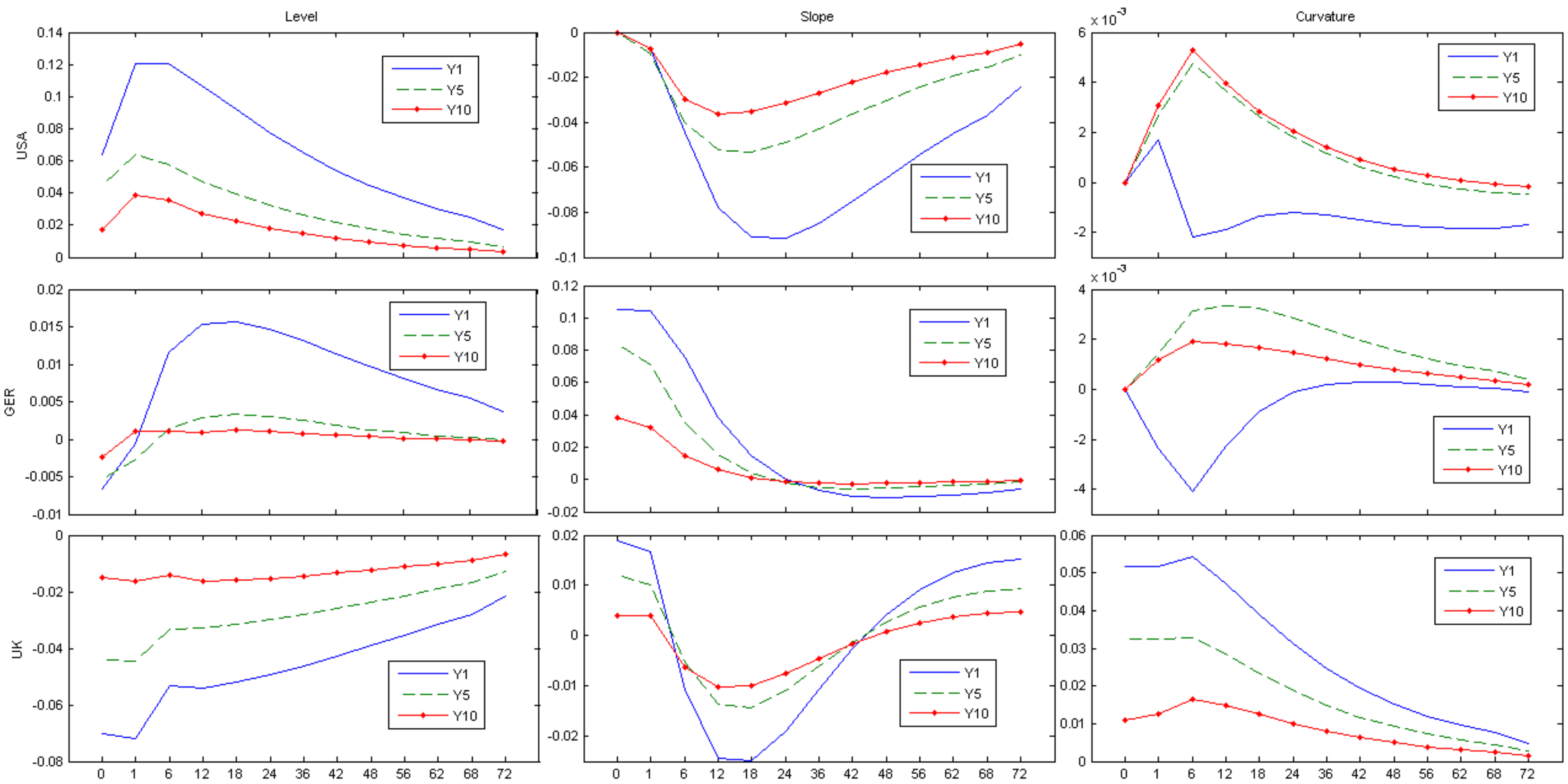

Figure 2. Effects of one standard deviation shock of global factors on yields of the USA, Germany and the UK. The data span the period 1997:8-2010:5.

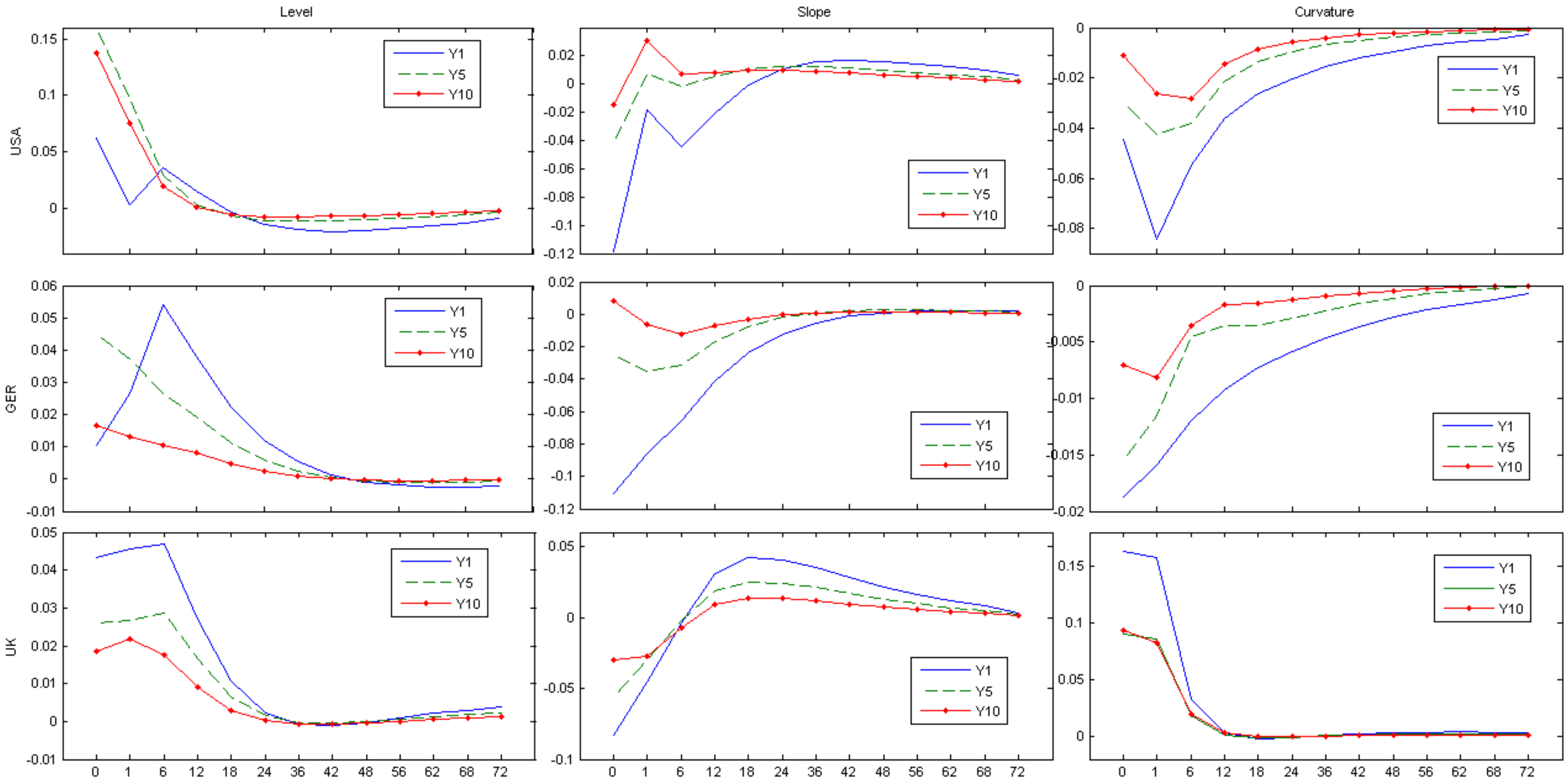

Figure 3. Effects of one standard deviation shock of the USA local factors on yields of the USA, Germany and the UK. The data span the period 1997:8-2010:5.

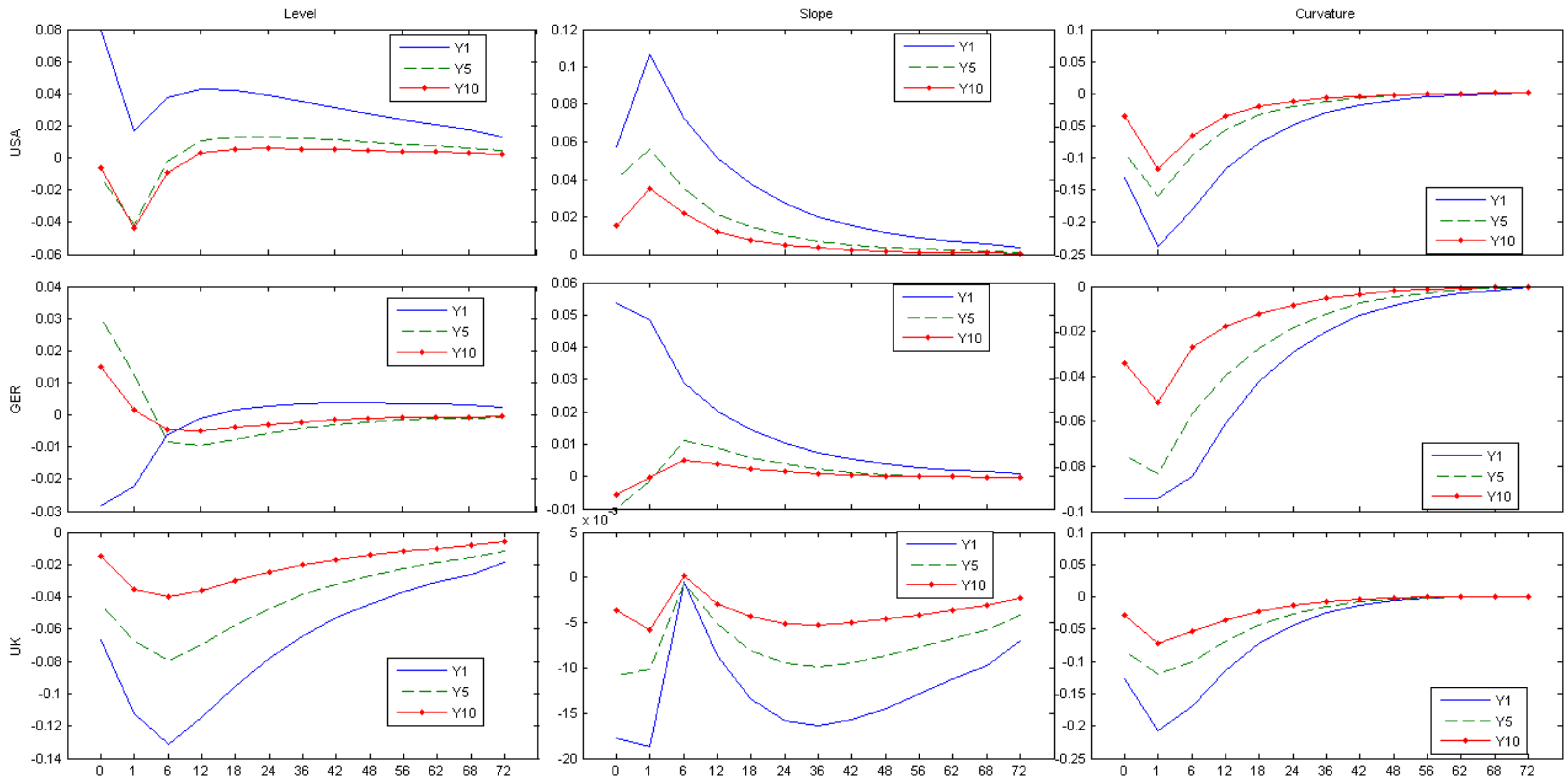

Figure 4. Effects of one standard deviation shock of Germany local factors on yields of the USA, Germany and the UK. The data span the period 1997:8-2010:5.

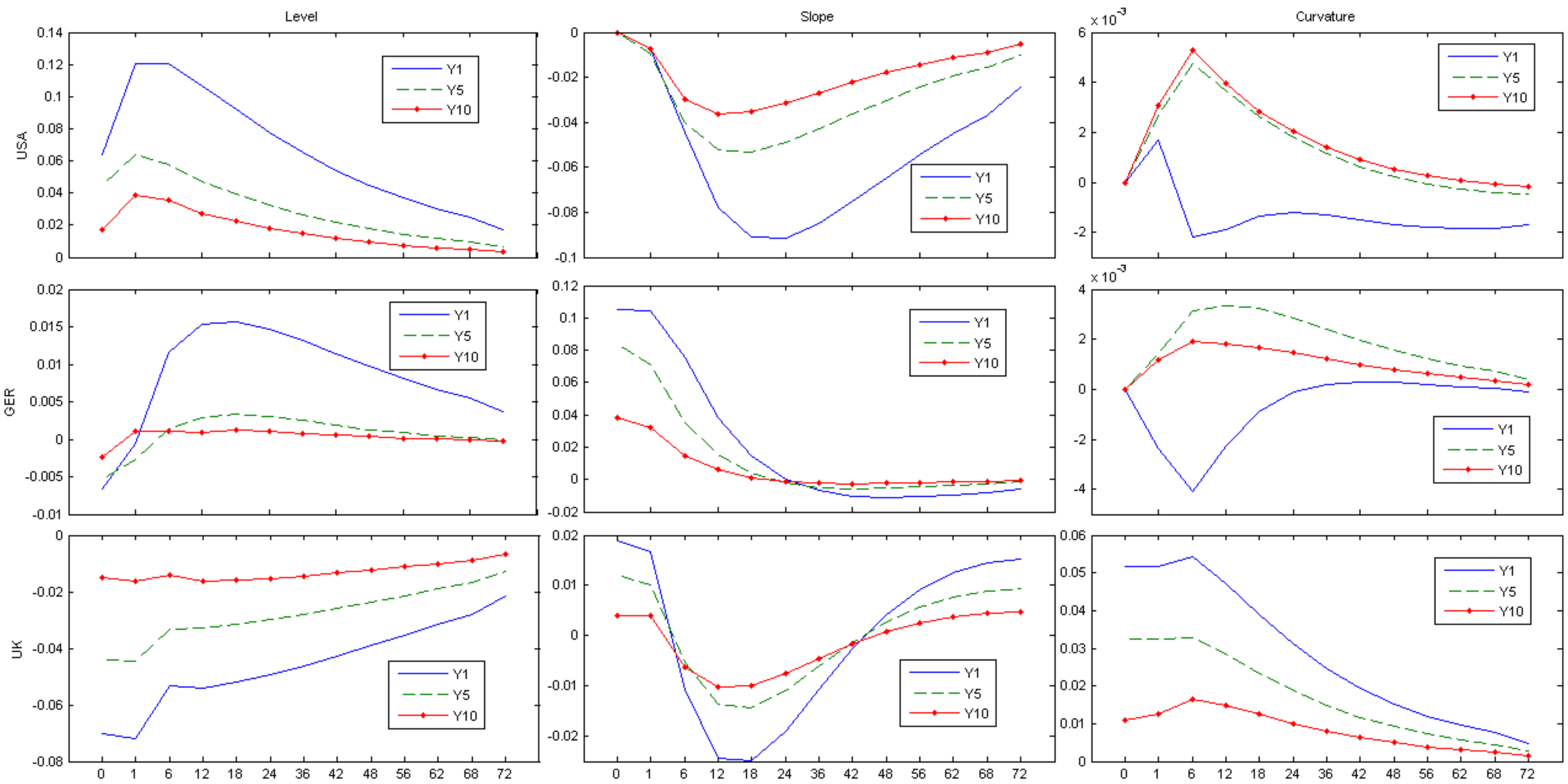

Figure 5. Effects of one standard deviation shock of the UK local factors on yields of the USA, Germany and the UK. The data span the period 1997:8-2010:5.

These results do not support a relative independence of the term structure of the USA with respect to common or global factors. These results differ from those obtained by Diebold et al. (2008) and Modugno and Nikolaou (2009), which could be due to the methodological approach adopted that does not allow for interactions between the factors. Indeed, our results indicate that there is an important linkage of international yield curves due to the global factor effect, as well as cross-factor dynamic interactions of local factors.

2.6. Conclusion and limitations

We proposed a global and local factor model based on the three NS factors (level, slope and curvature) for the USA, Germany and the UK. We estimate this factor model using monthly government zero coupon yields for a sample spanning from August 1997 to May 2010. The variance decomposition indicates that global factors explain on average 55% of the variance of yields, and that the most important factor is global level, which explains 40% of the variance of yields.

Estimates of IRF (in Figures 2 to 5) show that effects of disturbance to local factors disappear at shorter horizons than shocks to global factors, lasting of approximately 42 and 72 months, respectively. Moreover, the size of the effects of global factor shocks are larger than that of local factor shocks. In particular, the shortest yield maturities are more sensitive to shocks, and specifically to shocks on global factors.

Therefore, global factors play an important role in explaining yields on bonds of different maturities across different countries. Indeed, local factors have a limited influence over different countries. These results indicate that a yield curve model can better explain future evolution of yields if it considers global factors and the dynamic interaction of these factors.

Our findings suggest at least two new lines of research. First, the model could be used to explain bond risk premia in long-term bonds for a set of countries since it incorporates dynamic linkages among the factors. Second, this model could be used to forecast the future evolution of the yield curve since it incorporates not only the dynamics of local factors, also of global factors.

References

[1] BIS, Zero-coupon yield curves: technical documentation. *SSRN eLibrary* (2005).

[2] BIS, 79th Annual Report. *Bank for International Settlements* (2009).

[3] BIS, 80th Annual Report. *Bank for International Settlements* (2010).

[4] Christensen, J. H., Diebold, F. X., and Rudebusch, G. D. The affine arbitrage-free class of: Nelson-Siegel term structure models. *SSRN eLibrary* (2007).

[5] Cicconi, C. On Maximum Likelihood Estimation of Dynamic Factor Models in the Presence of Local Factor. *Mimeo* (2010).

[6] Cochrane, J. H., and Piazzesi, M. Bond risk premia. *American Economic Review 95*, 1 (March 2005), 138–160.

[7] Coroneo, L., Giannone, D., and Modugno, M. Forecasting the term structure of interest rates using a large panel of macroeconomic data. *Mimeo* (2008).

[8] Coroneo, L., Nyholm, K., and Vidova-Koleva, R. How arbitrage-free is the nelson-siegel model? *Journal of Empirical Finance 18*, 3 (2011), 393–407.

[9] Dahlquist, M., and Hasseltoft, H. International Bond Risk Premia. *SSRN eLibrary* (2011).

[10] De Pooter, M. Examining the nelson-siegel class of term structure models. *Tinbergen Institute Discussion Papers*, 07-043/4 (June 2007).

[11] Diebold, F. X., and Li, C. Forecasting the term structure of government bond yields. *Journal of Econometrics 130*, 2 (February 2006), 337–364.

[12] Diebold, F. X., Li, C., and Yue, V. Z. Global yield curve dynamics and interactions: a dynamic nelson-siegel approach. *Journal of Econometrics 146*, 2 (October 2008), 351–363.

[13] Doz, C., Giannone, D., and Reichlin, L. A quasi maximum likelihood approach for large approximate dynamic factor models. *Working Paper Series, European Central Bank*, 674 (September 2006).

[14] Ghahramani, Z., and Hinton, G. E. Parameter estimation for linear dynamical systems. Tech. rep., 1996.

[15] Modugno, M., and Nikolaou, K. The forecasting power of international yield curve linkages. *Working Paper Series, European Central Bank*, 1044 (Apr. 2009).

[16] Moench, E. Term structure surprises: the predictive content of curvature, level, and

slope. *Journal of Applied Econometrics 27,* 4 (2012), 574–602.

[17] Nelson, C. R., and Siegel, A. F. Parsimonious modeling of yield curves. *Journal of Business 60*, 4 (October 1987), 473–89.

[18] Shumway, R. H., and Stoffer, D. S. An approach to time series smoothing and forecasting using the em algorithm. *Journal of Time Series Analysis 3*, 4 (1982), 253–264.

[19] Sims, C. A. Macroeconomics and Reality. *Econometrica 48*, 1 (January 1980), 1-48.

[20] Sims, C. A. Policy Analysis with Econometric Models. *Brookings Papers on Economics Activity,* 1 (1982), 107-152.

[21] Svensson, L. E. Estimating and interpreting forward interest rates: Sweden 1992-1994. *SSRN eLibrary* (1994).

Chapter 3

Global and local yield curve factors in excess bond returns

Abstract

We identify the global and local components of the yield curves of three countries using the Nelson and Siegel factors: level, slope and curvature. Using yield curve data from the USA, Germany and the UK for the period 1980 to 2008, we show that the proposed factor model predicts excess bond returns with an average adjusted R^2 up to 59%. Our results also indicate that global factors explain explain up to 58% of the variance of excess return forecast errors. Our global and local yield curve factor model outperforms both the single-factor model proposed by Cochrane and Piazzesi (2005) and the global and local factor model proposed by Dahlquist and Hasseltoft (2011) for the USA, Germany and the UK.

3.1. Introduction and literature review

The predictability of excess bond returns have important economic and financial implications as it provides evidence against the Expectation Hypothesis, which states that long rates are equal to the average of future expected short rates and implies that excess bond returns are not predictable. The deviations from the Expectation Hypothesis can indicate a term premium which change over time (Kim and Orphanides, 2007). In this regard, Cochrane and Piazzesi (2005) propose to study time-varying risk premia for the USA government bonds. They build a single-factor model using one-year yield and a set of four forward rates (from two to five years), and run predictive regressions of one-year excess bond returns of zero-coupon bonds with the same maturities as the forward rates.

Literature on predictability of excess bond returns mainly focuses on the role of local variables on excess bond returns (Fama and Bliss (1987); Campbell and Shiller (1991), among others) and only a few studies pay attention to international and global linkages of interest rates and their role in excess bond returns. The majority of studies on excess bond returns have focused on explaining excess bond returns for one or more countries in an isolated way. In this regard, efforts to understand the global components which explain excess returns have focused on linear combinations of individual country factors from the Cochrane and Piazzesi (2005, 2008) model. For example, Dahlquist and Hasseltoft (2011) construct a global factor by weighting country factors of Cochrane and Piazzesi (2005) by GDP while Hellerstein (2011) does the same using factors of Cochrane and Piazzesi (2008). We examine whether global and local factors, which explain the yield curve, allow us to explain excess bond returns for more than one country. We study predictability of excess bond returns by means of in-sample forecasts, which incorporate global and local factors studied in Chapter 2, to characterize term structure of interest rates as common and idiosyncratic components. Our hypothesis framework relies mainly on two elements. The first aspect is explained in Section 3.2 and is related to the fact that excess bond returns can be described as a linear combination of future and current yields. Therefore, conditional forecast errors of excess bond returns are equivalent to scaled conditional forecast errors of yields for the same horizon. The second refers to the empirical fact that excess bond returns of different countries show similarities in their patterns, as we will see in Section 3.3. Hence, our model of global and local factors can explain, at least in part, excess bond returns. This means that excess bond returns can also be characterized by both local and global factors.

In this paper, we estimate one-year excess bond returns using the following strategy: borrowing money over a one-year term, buying a long term government bond and selling it one year later, and we examine whether the global and local yield curve factors from the model in Chapter 2 explain these excess returns. Considering that there is a global component in zero-coupon yields, as examined in the previous chapter, we propose to analyze if global component in yield curves across countries also imply a global component in excess bond returns, which would allow us to explain one-year excess bond returns and to decompose in-sample predictability in both global and local factors.

We contribute to the existing literature in two ways. First, we provide evidence that yield curve factors play an important role in predictability of excess bond returns, and we indicate the percentage of variance of excess bond return forecast errors explained by each one of these local and global yield curve factors. Second, we compare the predictive ability of our global and local yield curve factors with two competing models using a rolling window based on in-sample forecast of excess bond returns one year ahead. In particular, we use the traditional three yield curve factors proposed by Nelson and Siegel (1987) and reparameterized by Diebold and Li (2006) as level, slope and curvature to build a global and local factor model, for the USA, Germany and the UK. We estimate in-sample forecast using a rolling window of fifteen years of data, which spans the period from December 1980 to May 2008.[1]

[1] Even though the Nelson and Siegel (1987) model is not necessarily arbitrage free, Christensen, Diebold and Rudebusch (2007) propose an affine arbitrage free version of the model maintaining the factor loadings, and they indicate that this improves the forecasting performance. However, Joslin, Singleton and

Our findings indicate that our model predicts one-year excess bond returns for four maturities, from two to five years, with an average adjusted R^2 up to 59%, as well as shocks to global factors account up to 58% of variance of excess bond return forecast errors. Moreover, in-sample forecasts from our model show lower root mean squared errors and mean absolute errors than those from the two competing models: the single-factor model developed by Cochrane and Piazzesi (2005) and the global and local factor model proposed by Dahlquist and Hasseltoft (2011) which uses global factors based on Cochrane and Piazzesi (2005). In addition, using our global and local yield curve factor model our results indicate that the predictability of one-year excess bond returns is not completely spanned either by the Cochrane and Piazzesi (2005) single-factor model or the Dahlquist and Hasseltoft (2011) model.

The rest of the paper is organized as follows: in Section 3.2 introduces notation and definitions, Section 3.3 presents data and preliminary analysis, and Section 3.4 describes global and local yield curve factor model. Section 3.5 shows forecasts of excess bond returns using a rolling window and Section 3.6 details two benchmark models. Section 3.7 reviews the estimation procedure of global and local factors as well as variance decomposition, and Section 3.8 presents main results. Finally, Section 3.9 concludes.

3.2. Notation and definitions

In this section, we introduce the following notation for describing excess bond returns. In general, the log price of a zero-coupon bond, $p_{j,t}^{(\tau)}$, for country j, at time t,with maturity τ, can be written as a function of zero-coupon log yields as follows

$$p_{j,t}^{(\tau)} = -\tau y_{j,t}^{(\tau)}, \tag{1}$$

where $y_{j,t}^{(\tau)}$ is the log yield for country j, at time t and maturity τ.[2]

We can therefore write log yields as

$$y_{j,t}^{(\tau)} = -\frac{1}{\tau} p_{j,t}^{(\tau)}. \tag{2}$$

Zhu (2011) show that in the context of Gaussian term structure models the imposition of no-arbitrage conditions on the Nelson and Siegel (1987) model do not affect its forecasting performance. Moreover, Coroneo, Nyholm and Vidova-Koleva (2011) provide evidence that the Nelson and Siegel (1987) model is not statistically different to a no-arbitrage model.

[2]The zero coupon bond price can be defined as $P_{j,t}(\tau) = e^{-\tau y_{j,t}^{(\tau)}}$.

The one-year log forward rate ($f_{j,t}^{(\tau)}$) for country j, at time t for maturity τ is given by

$$f_{j,t}^{(\tau)} = p_{j,t}^{(\tau-1)} - p_{j,t}^{(\tau)} = \tau y_{j,t}^{(\tau)} - (\tau - 1) y_{j,t}^{(\tau-1)}. \tag{3}$$

The log return, $r_{j,t+1}^{(\tau)}$, for country j at time $t+1$ of buying a bond with maturity τ at time t, holding it for one year and selling it with maturity $\tau - 1$ at time $t+1$, could be defined as function of log price of zero-coupon bonds or log yields as follows

$$r_{j,t+1}^{(\tau)} = p_{j,t+1}^{(\tau-1)} - p_{j,t}^{(\tau)} = \tau y_{j,t}^{(\tau)} - (\tau - 1) y_{j,t+1}^{(\tau-1)}. \tag{4}$$

Hence, excess log return ($rx_{j,t+1}^{(\tau)}$) for country j at time $t+1$ of buying a bond with maturity τ at time t, holding the bond for one year, selling it with maturity $\tau - 1$ at time $t+1$, and financing it with a one year loan, is given by the following equation

$$rx_{j,t+1}^{(\tau)} = r_{j,t+1}^{(\tau)} - y_{j,t}^{(1)} = \tau y_{j,t}^{(\tau)} - (\tau - 1) y_{j,t+1}^{(\tau-1)} - y_{j,t}^{(1)}. \tag{5}$$

Therefore, the excess bond return is a linear combination of future ($y_{j,t+1}^{(\tau-1)}$) and current yields ($y_{j,t}^{(\tau)}$ and $y_{j,t}^{(1)}$). Additionally, the vector of excess bond returns, $rx_{j,t+1}$, for a set of four maturities (from two to five years) is denoted without superscript as

$$rx_{j,t+1} = \begin{bmatrix} rx_{j,t+1}^{(2)} & rx_{j,t+1}^{(3)} & rx_{j,t+1}^{(4)} & rx_{j,t+1}^{(5)} \end{bmatrix}'. \tag{6}$$

The average excess return for country j is given by

$$\overline{rx}_{j,t+1} = \frac{1}{4} \sum_{\tau=2}^{5} rx_{j,t+1}^{(\tau)}. \tag{7}$$

3.3. Data and preliminary analysis

The data consist of nominal monthly zero-coupon government yields for ten maturities from one to ten years for the USA, Germany and the UK. The source is the database constructed by Wright (2011). In particular, we use data spanning the period from December 1980 to May 2008.

Preliminary analysis of data is shown in Table I, which reports means, standard deviations, maximum and minimum values of zero-coupon yields for the USA, Germany and the UK for five maturities. The following table shows a relatively larger standard deviation of yields for the UK for the sample period, which is most pronounced for the shortest maturities.

Table I

Summary of descriptive statistics for monthly zero-coupon yields for three countries

Panel A: USA					
Maturity	1 yr.	2 yrs.	3 yrs.	4 yrs.	5 yrs.
Mean	3.66	3.88	4.11	4.32	4.52
S. D.	1.06	1.04	1.05	1.07	1.10
Maximum	6.59	6.65	7.06	7.31	7.47
Minimum	1.93	2.04	2.19	2.38	2.56
Panel B: Germany					
Maturity	1 yr.	2 yrs.	3 yrs.	4 yrs.	5 yrs.
Mean	5.30	5.45	5.55	5.62	5.66
S. D.	1.03	1.14	1.22	1.27	1.31
Maximum	7.21	7.99	8.39	8.62	8.75
Minimum	3.23	3.32	3.49	3.65	3.77
Panel C: UK					
Maturity	1 yr.	2 yrs.	3 yrs.	4 yrs.	5 yrs.
Mean	4.23	4.46	4.64	4.80	4.94
S. D.	1.67	1.56	1.44	1.33	1.25
Maximum	7.25	7.56	7.66	7.71	7.74
Minimum	1.03	1.33	1.64	1.98	2.33

Note: This table shows descriptive statistics for monthly zero-coupon yields at maturities ranging from one to five years for the USA, Germany and the UK. Statistics include mean, standard deviation as well as maximum and minimum values for the sample period 1980:12-2008:05.

Table II reports the correlation matrix of zero-coupon yields between different countries and shows a cross-country commonality in yield movements with all cross correlations over 0.55.

Table II

Correlation matrix of zero-coupon yields for three countries: the USA, Germany and the UK

		USA					Germany					UK				
Country		1 yr.	2 yrs.	3 yrs.	4 yrs.	5 yrs.	1 yr.	2 yrs.	3 yrs.	4 yrs.	5 yrs.	1 yr.	2 yrs.	3 yrs.	4 yrs.	5 yrs.
	1 yr.	1.00														
	2 yrs.	0.98	1.00													
USA	3 yrs.	0.93	0.99	1.00												
	4 yrs.	0.88	0.96	0.99	1.00											
	5 yrs.	0.83	0.92	0.97	0.99	1.00										
	1 yr.	0.55	0.64	0.68	0.70	0.71	1.00									
	2 yrs.	0.61	0.73	0.79	0.83	0.85	0.95	1.00								
GER	3 yrs.	0.63	0.75	0.83	0.88	0.90	0.90	0.99	1.00							
	4 yrs.	0.64	0.76	0.84	0.89	0.92	0.85	0.96	0.99	1.00						
	5 yrs.	0.63	0.75	0.84	0.89	0.92	0.81	0.94	0.98	1.00	1.00					
	1 yr.	0.51	0.58	0.60	0.60	0.59	0.84	0.81	0.75	0.70	0.66	1.00				
	2 yrs.	0.54	0.63	0.66	0.67	0.67	0.86	0.86	0.82	0.78	0.74	0.99	1.00			
UK	3 yrs.	0.56	0.66	0.71	0.73	0.73	0.86	0.89	0.86	0.83	0.79	0.96	0.99	1.00		
	4 yrs.	0.58	0.69	0.74	0.77	0.77	0.85	0.90	0.89	0.86	0.83	0.93	0.98	1.00	1.00	
	5 yrs.	0.60	0.71	0.77	0.80	0.81	0.84	0.91	0.90	0.88	0.86	0.90	0.96	0.98	1.00	1.00

Note: This table shows cross correlation of monthly zero-coupon yields at maturities ranging from one to five years for the USA, Germany and the UK. The sample period is 1980:12-2008:05.

To shed light on the similarities not only in yields but also in excess returns, Figure 1 shows patterns of one-year excess bond returns for maturities of two and five years, average of excess bond returns for each country, and global average of three countries (the USA, Germany and the UK).

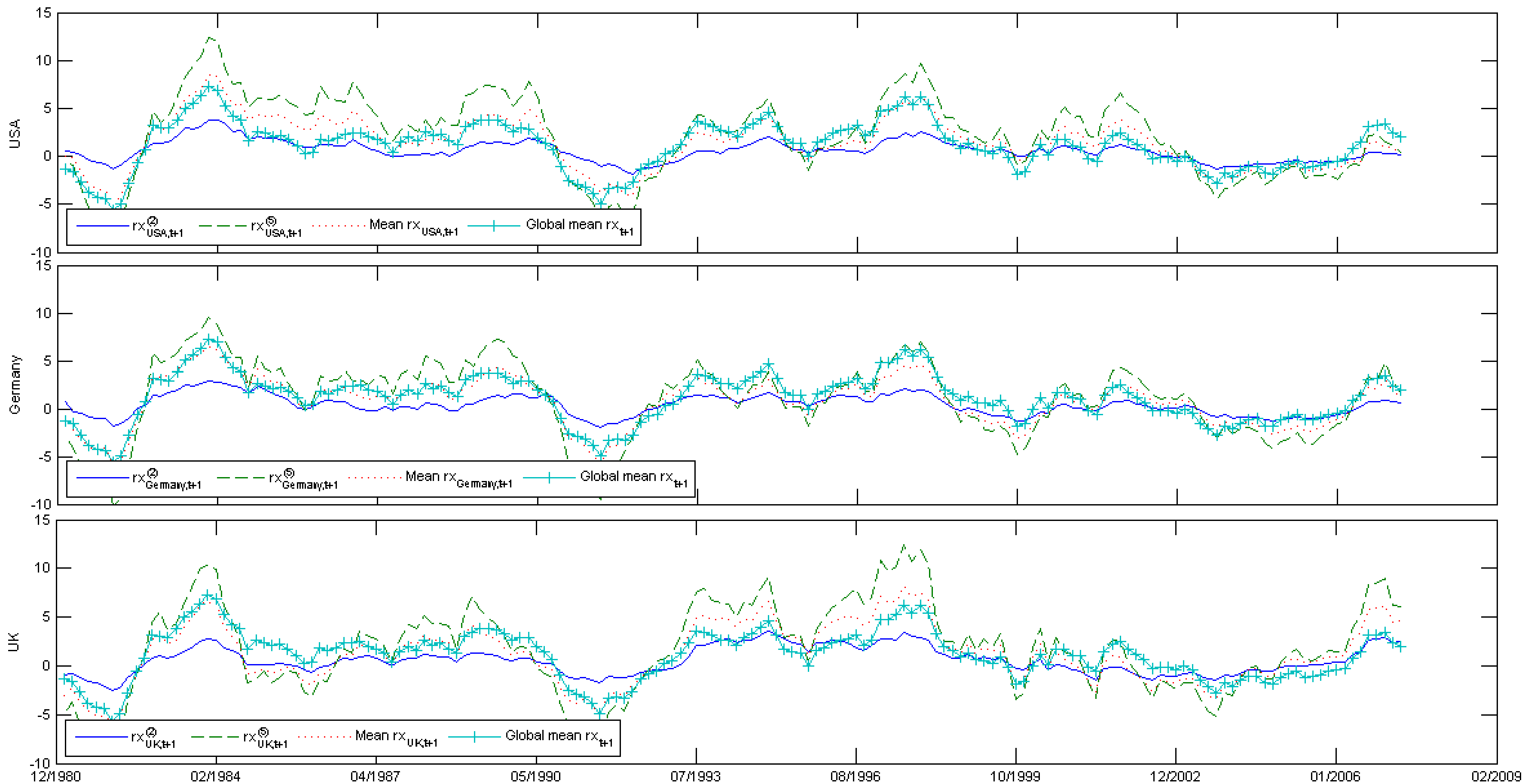

Figure 1: One-year excess bond returns for maturities of two and five years, the average of excess bond returns for each country, and global average of all three countries. The top, middle and bottom plots show excess returns for the USA, Germany and the UK, respectively.

3.4. Global and local yield curve factors model

We use the yield curve factor model from Chapter 2, which describes country yields using two set of orthogonal factors: global and local. The factors are defined according to the model proposed by Nelson and Siegel (1987), and reparameterized by Diebold and Li (2006), as level, slope and curvature. Recall from Chapter 2 that we can write the model for a set of countries in state space form as:

$$Y_t = \Gamma^T F_t^T + u_t, \tag{8}$$

where Y_t is the matrix which stacks log yields for all maturities and countries at time t, Γ^T is a matrix of factor loadings defined as $\Gamma^T = [\Gamma^G \;\; \Gamma]$, with Γ^G being a matrix of global factor loadings, defined as $\Gamma^G = \Gamma\beta$, and in turn Γ is defined as a block diagonal matrix which contains the NS factor loadings. The matrix of loadings, β, rescales the shocks to global factors for each country. The matrix of factors, F_t^T, is defined as $F_t^T = [F_t^{G\prime} \;\; F_t^{L\prime}]'$, where F_t^G and F_t^L are two orthogonal set of global (G) and local factors (L), respectively. The vector u_t contains innovation terms. The factors F_t^T follow a vector autoregressive process of order one according to

$$F_t^T = \Phi F_{t-1}^T + w_t, \tag{9}$$

where Φ is a block diagonal matrix of parameters, w_t is a vector of error terms, $u_t \sim N(0, \Sigma)$ where Σ is a diagonal matrix, and $w_t \sim N(0, \Omega)$ where Ω is a block diagonal matrix, whose first and second block contains the variance-covariance errors of global and local factors, respectively. In addition, u_t and w_t are independent.

In this framework, the global and local yield curve factors (level, slope and curvature) jointly explain the yields of the countries. Hereafter, we refer to our global and local factor model as GLYCF.

Given that factor models do not have a unique solution due to the rotational indeterminacy problem, which could lead to a different combination of parameters and unobservable factors, we impose restrictions on factor loadings. Firstly, in order to identify the loadings on β, we restrict matrix β to be block diagonal. In order to do that, factor loadings for each country only loads in the same global factor, e.g., the global level factor loading of country j only loads on global level and not on level and slope. Secondly, factors are restricted to having a variance-covariance matrix equal to the identity matrix. Thirdly, the matrix of factor loadings, Γ, is restricted to containing the NS factor loadings and finally yields are demeaned and standardized.

3.5. Forecasts of excess bond returns using a rolling window

Following the notation introduced in Sections 3.2 and 3.4, we define excess bond return for horizon h, on a bond with maturity τ, for country j and at time $t+h$ as follows

$$rx_{j,t+h}^{(\tau)} = r_{j,t+h}^{(\tau)} - y_{j,t}^{(h)} = \tau y_{j,t}^{(\tau)} - (\tau - h) y_{j,t+h}^{(\tau-h)} - y_{j,t}^{(h)}. \tag{10}$$

Moreover, conditional forecast of excess bond returns at time t for the same maturity, country and horizon, can be defined as

$$\widehat{rx}_{j,t+h|t}^{(\tau)} = \hat{r}_{j,t+h}^{(\tau)} - y_{j,t}^{(h)} = \tau y_{j,t}^{(\tau)} - (\tau - h) \hat{y}_{j,t+h|t}^{(\tau-h)} - y_{j,t}^{(h)}, \tag{11}$$

where $y_{j,t+h}^{(\tau-h)}$ is the yield with maturity $\tau - h$ at time $t+h$ and $\hat{y}_{j,t+h|t}^{(\tau-h)}$ is the conditional forecast of yield for horizon h at time t, i.e., h months ahead. In addition, the forecast error of excess bond return, $\varepsilon_{j,t+h}^{(\tau)}$, for a bond with maturity τ is obtained subtracting equation (10) from equation (11), as follows

$$\varepsilon_{j,t+h}^{(\tau)} = -(\tau - h)[y_{j,t+h}^{(\tau-h)} - \hat{y}_{j,t+h|t}^{(\tau-h)}]. \tag{12}$$

Therefore, the forecast error of excess bond return for a bond with maturity τ is equivalent to the forecast error of the yield with maturity $\tau - h$ scaled by $-(\tau - h)$. Hence, once we have a conditional forecast of yields, it is straightforward to obtain a conditional forecast of excess bond returns as well as the forecast error of excess bond returns.

Considering that global and local yield curve factor model explains yields across maturities and countries, and that excess bond returns at $t+1$ could be expressed as scaled yields at time t and $t+1$, we estimate the GLYCF model and use it to generate in-sample forecasts of one-year excess bond returns.

We construct the forecast by estimating the GLYCF model using a fifteen-year rolling window.[3] The first rolling window starts in December 1980 up to November 1995 and we proceed as follows:

1. We use fifteen years (180 months) of data to estimate the GLYCF model.

2. We use the estimated parameters and factors at time t to forecast yields one year ahead and estimate the forecast error of excess bond returns. In particular, for the

[3] We use a fifteen-year rolling window due to the number of parameters to be estimated which is equal to 99.

GLYCF model, the matrix of yields at time $t+h$ is defined as Y_{t+h}, and conditional forecast at time t for yields at time $t+h$ (i.e., h-step ahead) is defined as $\hat{Y}_{t+h|t}$. We use the parameters and the factors estimated at time t to forecast the factors at time $t+h$, i.e., $\hat{F}^{(T)}_{t+h|t} = \Phi^h F^{(T)}_t$. Considering that we have a conditional forecast of the factors h-step ahead, $\hat{F}^{(T)}_{t+h|t}$, the conditional forecast of the yields h-step ahead is direct, because we use $\hat{\beta}$ and $\hat{F}^{(T)}_{t+h|t}$ to forecast $\hat{Y}_{t+h|t}$, i.e., $\hat{Y}_{t+h|t} = \Gamma^{(T)}\hat{F}^{(T)}_{t+h|t}$.

3. We move the sample forward one month, maintaining a sample size of 180 months and keeping the same parameters of previous months, we reestimate the factors, the forecast of yields and estimate the forecast errors.

4. We repeat the estimation of factors keeping the same parameters for one year (12 months), and after that we repeat step 1.

5. We repeat the previous steps, excluding the first month of the previous subsample and including the next month

In the case of both competing models: Cochrane and Piazzesi (2005) and Dahlquist and Hasseltoft (2011), we estimate factors and parameters using the same approach that we use for the GLYCF model. We consider the same rolling window, with subsamples of fifteen years each time, producing in-sample forecasts for each model, which means using the factors at time t to forecast excess bond returns at time $t+h$.

3.6. Benchmark models

We analyze the ability of two alternative models to predict excess bond returns. The first model is the well-known single-factor model proposed by Cochrane and Piazzesi (2005) to predict excess bond returns based on one-year yields and forward rates. The second model is proposed by Dahlquist and Hasseltoft (2011), which considers the Cochrane and Piazzesi (2005) single factors, and a linear combination of these factors for different countries to create a global factor.

3.6.1. Cochrane and Piazzesi (2005) single-factor model

The single-factor model proposed by Cochrane and Piazzesi (2005), henceforth CP, explains one-year excess bond returns for country j at time $t+1$ and maturity τ, for $\tau = 2$ to 5 years, using one-year yield and forward rates from two to five years at time t. The general model is

$$rx^{(\tau)}_{j,t+1} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1 y^{(1)}_{j,t} + \beta^{(\tau)}_2 f^{(2)}_{j,t} + ... + \beta^{(\tau)}_5 f^{(5)}_{j,t} + \varepsilon^{(\tau)}_{j,t+1}, \tag{13}$$

where $\beta_0^{(\tau)}$ to $\beta_5^{(\tau)}$are the parameters, $y_{j,t}^{(1)}$ is the one-year yield, $f_{j,t}^{(2)}$ to $f_{j,t}^{(5)}$ are forward rates defined in equation (3), and $\varepsilon_{j,t+1}^{(\tau)}$ is the error term.

Cochrane and Piazzesi (2005) show that one-year excess bond returns can be described using a single-factor model as

$$rx_{j,t+1}^{(\tau)} = b_j^{(\tau)}(\gamma_{j,0} + \gamma_{j,1}y_{j,t}^{(1)} + \gamma_{j,2}f_{j,t}^{(2)} + ... + \gamma_{j,5}f_{j,t}^{(5)}) + \varepsilon_{j,t+1}^{(\tau)}, \tag{14}$$

where $b_j^{(\tau)}$ is the coefficient for bond with maturity τ and $\gamma_{j,i}$ are coefficients for $i = 0$ to 5. The variables can be written more compactly by defining a vector of yield and forward rates including an intercept:

$$f_{j,t} = \begin{bmatrix} 1 & y_{j,t}^{(1)} & f_{j,t}^{(2)} & f_{j,t}^{(3)} & f_{j,t}^{(4)} & f_{j,t}^{(5)} \end{bmatrix}', \tag{15}$$

and a coefficient vector

$$\gamma_j = \begin{bmatrix} \gamma_{j,0} & \gamma_{j,1} & \gamma_{j,2} & \gamma_{j,3} & \gamma_{j,4} & \gamma_{j,5} \end{bmatrix}'. \tag{16}$$

A problem with this model is that coefficients $b_j^{(\tau)}$ and $\gamma_{j,i}$ are not separately identified. For this reason, Cochrane and Piazzesi (2005) propose a two-step approach using the average of excess bond returns, $\overline{rx}_{j,t+1}$, to estimate equation (17) and obtain γ_j

$$\overline{rx}_{j,t+1} = \gamma_j' f_{j,t} + \bar{\varepsilon}_{j,t+1}. \tag{17}$$

From previous steps, the coefficients γ_j are computed. Once we have estimates of γ_j the coefficients $b_j^{(\tau)}$ can be obtained by estimating the following regression

$$rx_{j,t+1}^{(\tau)} = b_j^{(\tau)}(\gamma_j' f_{j,t}) + \varepsilon_{j,t+1}^{(\tau)}. \tag{18}$$

Therefore, the model described by equation (18) is a restricted version of the model described by equation (13), imposing that average value of coefficients is equal to 1 (i.e., $\frac{1}{4}\sum_{\tau=2}^{5} b_j^{(\tau)} = 1$).

3.6.2. Dahlquist and Hasseltoft (2011) global CP factor model

Dahlquist and Hasseltoft (2011) propose to estimate a global model based on the approach of Cochrane and Piazzesi (2005), hereafter GCP, to explain excess returns across countries and maturities. They use gross domestic product (GDP) of each country and the five variables of the CP model (one-year yield and four forward rates from two to five years) to build global and local return-forecasting factors and run predictive regressions to explain excess bond returns. The global return-forecasting factor is a linear combination of country return-forecasting factors weighted by GDP. The country return-forecasting factor is a linear combination of yields and forward rates whose weights are calculated using a two-step regression as in Cochrane and Piazzesi (2005). Their findings indicate these global and local factors are poorly spanned by the first three principal components of yields.

Following the model of Cochrane and Piazzesi (2005), excess bond returns at time $t+1$, for country j and maturity τ, are explained by a vector which contains the one-year yield and the four forward rates, which can be described as follows

$$rx_{j,t+1}^{(\tau)} = b_j^{(\tau)} x_{j,t} + \varepsilon_{j,t+1}^{(\tau)}, \tag{19}$$

where $x_{j,t} = (\gamma_j' f_{j,t})$.

The global return forecasting factor or global CP factor (GCP) is a linear combination of previous CP factors, which in turn are a linear combination of the one-yield and forward rates. The CP factor for country j, at time t, $CP_{j,t}$, is weighted by the average of the relative weight of the GDP of country j, i.e.,

$$w_{j,t} = GDP_{j,t} / \Sigma_{j=1}^{cty} GDP_{j,t}, \tag{20}$$

where cty is equal to the number of countries in the sample, hence

$$GCP_{j,t} = w_{j,t} \Sigma_{j=1}^{cty} CP_{j,t}. \tag{21}$$

The GCP is orthogonalized with respect to each country's CP factors, in order to include it in each predictive regression of excess bond returns.

Including (21) in equation (19) and redefining $CP_{j,t} = x_{j,t}$ as the factor of country j, we have

$$rx_{j,t+1}^{(\tau)} = \alpha_j^{(\tau)} + b_{CP,j}^{(\tau)} CP_{j,t} + b_{GCP,j}^{(\tau)} GCP_{j,t} + v_{j,t+1}^{(\tau)}, \tag{22}$$

where $\alpha_j^{(\tau)}$, $b_{CP,j}^{(\tau)}$, and $b_{GCP,j}^{(\tau)}$ are the parameters, and $v_{j,t+1}^{(\tau)}$ is the error term.

The results of Dahlquist and Hasseltoft (2011) indicate that both return-forecasting factors (CP and GCP) explain a small percentage of yield variation but have a strong power to explain future excess bond returns.

Hellerstein (2011) uses a variant of this model and proposes a joint estimation of the factor model developed by Cochrane and Piazzesi (2008), and a weighted average of the factors for a single country proposed by Cochrane and Piazzesi (2008) using the relative weight of GDP, which represents a global factor. The global factor as well as the three principal components of innovation terms are orthogonalized with respect to each one of the CP factors, in order to run a predictive regression which explain excess bond returns. In this setup, the local return-forecasting factor is based on three months moving average of the five forward rates (from one to five years) to explain excess returns of ten maturities from one to ten years. Her findings suggests that global forecasting factor indicates there are spillover effects and that the information of the global forecasting factor is not spanned by local return forecasting factor or the level, slope and curvature factors.

3.7. Estimation of global and local factors and variance decomposition

The estimation of the global and local yield curve factor model, defined in Section 3.4, is based on the quasi-maximum likelihood approach proposed by Doz, Giannone and Reichlin (2006). Specifically, the estimation is performed by estimating the three NS factors for each country, using the matrix of factor loadings, Γ. Then, we estimate the loadings of each country over the global factors, $\hat{\beta}$, imposing restrictions over this matrix. Global factors (level, slope and curvature) and local factors (level, slope and curvature) are estimated using the quasi-maximum likelihood approach, and imposing orthogonality between both types of factors. The initialization of the estimation of global factors is made using the standardized first principal component of each factor (level, slope and curvature). The estimation of the local factors is initialized using idiosyncratic (error) terms. The yields and both groups of factors are standardized by subtracting the mean and dividing by standard deviation. The model is estimated by quasi-maximum likelihood.

We can estimate the relative contribution of a shock to the lth factor to variance of a τ-period excess bond return forecast (for horizon h) using variance decomposition. This is computed for the GLYCF model, using the fact that forecast error of excess bond returns for a bond with maturity τ is equal to the forecast error of yield with maturity $\tau - h$ scaled by $-(\tau - h)$. Hence, using the impulse response functions we can obtain the variance decomposition of yields, and in turn the variance decomposition of excess bond returns. Our approach relies on the identification schemes of impulse response functions for structural vector autoregressive models (SVARs) proposed by Sims (1980, 1982).

To accomplish this, we use the lag operator to rewrite equation (9) as $(I - \Phi L)F_t^{(T)} = w_t$ and obtain $F_t^{(T)} = (I - \Phi L)^{-1}w_t$. We can write this as a vector moving average (VMA) process, $F_t^{(T)} = \Sigma_{i=0}^{\infty}\Phi^i w_{t-i}$, and orthogonalize the shocks using the Cholesky decomposition of Ω, which is decomposed in the multiplication of a lower triangular matrix, A, and its conjugate transpose, A', i.e., $\Omega = AA'$. Moreover, defining primitive shocks U_t, where $E(U_t) = 0$, $E(U_t U_t') = I$ and $w_{t-i} = AU_{t-i}$, we can redefine $F_t^{(T)}$ using these definitions and the previous VMA representation as $F_t^{(T)} = \Sigma_{i=0}^{\infty}\Phi^i AU_{t-i}$. Also, given that $y_{j,t+h}^{(\tau-h)}$ is function of factors, $F_t^{(T)}$, we could redefine the yields in terms of the impulse response functions, as we discussed in Chapter 2. In particular, we can rewrite equation (8) as $Y_t = \Gamma^{(T)}\Sigma_{i=0}^{\infty}\Phi^i AU_{t-i}$.

Secondly, the forecast error for the factors can be written using the VMA representation, $F_{t+h}^{(T)} - \hat{F}_{t+h|t}^{(T)} = \Sigma_{i=0}^{h-1}\Phi^i AU_{t-i}$ and the error of the h-step ahead forecast of yields can be written as $Y_{t+h} - \hat{Y}_{t+h|t} = \Gamma^{(T)}\Sigma_{i=0}^{h-1}\Phi^i AU_{t-i}$.[4] Then, the mean squared error (MSE) of forecast error is equal to $E[(Y_{t+h} - \hat{Y}_{t+h|t})(Y_{t+h} - \hat{Y}_{t+h|t})'] = \Gamma^{(T)}\Sigma_{i=0}^{h-1}\Phi^i AA'(\Phi^i)'(\Gamma^{(T)})'$. Hence, the MSE can be decomposed in the contribution of shocks to each one of the k factors (with $k = 12$), as follows $\Sigma_{l=1}^{k}[\Gamma^{(T)}\Sigma_{i=0}^{h-1}\Phi^i A_l A_l'(\Phi^i)'(\Gamma^{(T)})']$, where A_l is the lth column of matrix A. Therefore, the contribution of the shock to the lth factor on the variance of the yield with maturity τ for horizon h is given by

$$[\Gamma_m^{(T)}\Sigma_{i=0}^{h-1}\Phi^i A_l A_l'(\Phi^i)'(\Gamma_m^{(T)})']/\Sigma_{l=1}^{k}[\Gamma_m^{(T)}\Sigma_{i=0}^{h-1}\Phi^i A_l A_l'(\Phi^i)'(\Gamma_m^{(T)})'], \tag{23}$$

where $\Gamma_m^{(T)}$ is mth row of matrix $\Gamma^{(T)}$.

Finally, the contribution of the shock to the lth factor on the variance of excess bond return, for a bond with maturity τ, for horizon h is given by the same expression, because $(h-\tau)^2$ is in the numerator and denominator, this means that the detail of yield variance explained by each factor is as in equation (23).

3.8. Results

We present the results of in-sample forecast of excess bond returns one year ahead, based on a rolling window of fifteen years, for three different factor models: CP, GCP and GLYCF. Table IV shows the contribution of each factor to in-sample forecasts of excess bond returns. Table V shows the mean absolute error (MAE) and Table VI depicts root mean squared errors (RMSE) for in-sample forecast errors for the CP, GCP and GLYCF models. The tables summarize average, minimum and maximum statistics for MAE and

[4] A detailed explanation of the variance decomposition is provided by Hamilton (1994), pages 323-340.

RMSE, using a rolling window of fifteen years for the period December 1980 to May 2008 for the USA, Germany and the UK. Figures 2 to 4 show the comparison of the adjusted R^2 ($\bar{R}^2$) in-sample predictability of excess bond returns for four maturities (two to five years), three models and three countries.

Specifically, we present a decomposition of the contribution of shocks to each factor to the variance of excess bond returns forecast errors (for one-year horizon) in Table III. It is noticeable that global factors explain on average over 43%, and up to 58% of the variability in the forecast error of the countries. Also, the global level explains no less than 14% and up to 43% of the variance for all maturities and countries. In this regard, Driessen, Melenberg and Nijman (2003) show that a common level factor for the US, Germany and Japan explains nearly 50% of the variation in bond returns. Moreover, global slope explains up to 35% and global curvature seems to be only important for predictability of excess bond returns for the USA. However, local curvature is important for all the countries explaining on average over 14% and up to 47% of variance.

Table III

Contribution of shocks to each factor on variance of excess bond return forecast errors, using global and local yield curve factor model

Panel A: USA

	Global			Local		
Maturity	Level	Slope	Curvature	Level	Slope	Curvature
2 yrs.	0.17	0.11	0.16	0.12	0.10	0.34
3 yrs.	0.14	0.12	0.30	0.13	0.01	0.30
4 yrs.	0.14	0.11	0.30	0.18	0.01	0.26
5 yrs.	0.15	0.09	0.25	0.26	0.02	0.23
Mean	0.15	0.11	0.25	0.17	0.04	0.28

Panel B: Germany

	Global			Local		
Maturity	Level	Slope	Curvature	Level	Slope	Curvature
2 yrs.	0.37	0.05	0.00	0.03	0.08	0.47
3 yrs.	0.37	0.04	0.00	0.06	0.02	0.51
4 yrs.	0.39	0.04	0.00	0.07	0.03	0.47
5 yrs.	0.43	0.04	0.00	0.09	0.03	0.41
Mean	0.39	0.04	0.00	0.06	0.04	0.47

Table III-*Continued*

Panel C: UK						
	Global			Local		
Maturity	Level	Slope	Curvature	Level	Slope	Curvature
2 yrs.	0.23	0.35	0.05	0.21	0.05	0.11
3 yrs.	0.28	0.27	0.03	0.18	0.09	0.15
4 yrs.	0.33	0.21	0.02	0.16	0.12	0.16
5 yrs.	0.37	0.16	0.02	0.16	0.14	0.15
Mean	0.30	0.25	0.03	0.18	0.10	0.14

Note: This table shows variance decomposition of excess return forecast errors attributed to each factor for three countries (the USA, Germany and the UK), using global and local yield curve factor model described in Section 3.4. The model is defined as $Y_t = \Gamma^T F_t^T + u_t$, where Y_t is the matrix of yields at time t, as well as Γ^T and F_t^T are matrices of global and local factor loadings and factors, respectively. Also, u_t is the vector of error terms. The first column shows the maturity of the bond whose excess bond return is calculated. The next three columns report the percentage explained by global factors: level, slope and curvature. The following three columns show the percentage explained by respective local factors: level, slope and curvature. The data consist of nominal monthly zero-coupon yields provided by Wright (2011), spanning the period 1980:12-2008:05. Panels A, B and C depict the results for the USA, Germany and the UK.

Table IV shows comparative MAE statistics for the CP single-factor model, the GCP factor model and our GLYCF factor model. The table shows lower mean absolute errors of in-sample forecasts using GLYCF model, for all the maturities and countries, than competing models: the CP single-factor model and the GCP model. Moreover, the GLYCF factor model also shows lower minimum and maximum values than both competing models across maturities and countries.

Table IV

Comparative mean absolute error statistics of in-sample forecast for the USA, Germany and the UK

Panel A: USA									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	0.92	0.57	1.16	0.85	0.56	1.02	0.70	0.48	0.86
3 yrs.	1.80	1.14	2.20	1.68	1.13	2.01	1.43	0.93	1.78
4 yrs.	2.53	1.64	3.02	2.39	1.63	2.88	2.07	1.39	2.57
5 yrs.	3.13	2.11	3.77	2.98	2.11	3.60	2.63	1.87	3.22
Panel B: Germany									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	1.03	0.61	1.34	1.02	0.57	1.32	0.83	0.48	1.12
3 yrs.	1.83	1.14	2.34	1.81	1.05	2.31	1.59	0.99	2.01
4 yrs.	2.52	1.65	3.20	2.49	1.52	3.14	2.32	1.51	2.88
5 yrs.	3.18	2.20	3.97	3.12	1.99	3.86	3.01	2.02	3.69
Panel C: UK									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	0.98	0.82	1.28	0.93	0.74	1.13	0.76	0.56	1.11
3 yrs.	1.89	1.60	2.51	1.79	1.46	2.24	1.52	1.11	2.13
4 yrs.	2.68	2.31	3.60	2.52	2.11	3.28	2.24	1.66	3.05
5 yrs.	3.39	3.01	4.58	3.18	2.73	4.22	2.92	2.19	3.91

Note: This table shows mean absolute error (MAE) statistics, computed using a fifteen-year rolling window of one-year excess bond returns using the Cochrane and Piazzesi (2005) model (CP), Dahlquist and Hasseltoft (2011) global factor model (GCP) and global and local yield curve factor model (GLYCF). The first column shows the maturity of the bond whose excess bond return is calculated. The next three columns report the mean, minimum and maximum of the MAE explained by the CP model. The next three columns show the average, minimum, and maximum of the MAE for the GCP model. The last three columns show the same statistics for the GLYCF model. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) spanning the period 1980:12-2008:05. Panels A, B and C depict results for the USA, Germany and the UK.

Furthermore, in Table V we report comparative RMSE statistics for the CP single-factor model, the GCP factor model and our GLYCF factor model. Table V shows similar results to Table IV. In particular, Table V presents lower RMSE of predictions using the GLYCF model, for all maturities and countries, than using either of the competing models. In addition, the GLYCF factor model shows lower minimum and maximum values than both competing models across maturities and countries.

Table V

Comparative root mean squared errors statistics of in-sample forecast for the USA, Germany and the UK

Panel A: USA									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	1.12	0.73	1.33	1.04	0.72	1.21	0.86	0.59	1.01
3 yrs.	2.18	1.42	2.53	2.04	1.41	2.40	1.76	1.24	2.10
4 yrs.	3.08	2.03	3.58	2.91	2.01	3.42	2.57	1.83	3.08
5 yrs.	3.85	2.61	4.48	3.66	2.59	4.32	3.27	2.41	3.94
Panel B: Germany									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	1.29	0.72	1.69	1.28	0.68	1.67	1.02	0.59	1.48
3 yrs.	2.30	1.44	2.92	2.27	1.31	2.89	2.00	1.26	2.62
4 yrs.	3.18	2.15	3.98	3.11	1.90	3.93	2.93	1.96	3.71
5 yrs.	4.00	2.83	4.94	3.89	2.48	4.86	3.82	2.64	4.73
Panel C: UK									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	1.25	1.04	1.53	1.17	0.94	1.39	0.93	0.66	1.37
3 yrs.	2.36	2.00	2.94	2.21	1.82	2.74	1.83	1.32	2.60
4 yrs.	3.33	2.88	4.22	3.11	2.61	4.00	2.69	1.99	3.70
5 yrs.	4.20	3.70	5.41	3.92	3.36	5.19	3.50	2.62	4.71

Note: This table shows root mean squared error (RMSE) statistics computed using a fifteen-year rolling window of one-year excess bond returns using the Cochrane and Piazzesi (2005) model (CP), Dahlquist and Hasseltoft (2011) global factor model (GCP) and global and local yield curve factor model (GLYCF). The first column shows the maturity of the bond whose excess bond return is calculated. The next three columns report the mean, minimum and maximum of the RMSE explained by the CP model. The next three columns show the average, minimum, and maximum of the RMSE for the GCP model. The last three columns show the same statistics for the GLYCF model. The data consist of nominal monthly zero-coupon yields provided by

Wright (2011) spanning the period 1980:12-2008:05. Panels A, B and C depict results for the USA, Germany and the UK.

In Figure 2 we plot the adjusted R^2 ($\bar{R}^2$) of in-sample forecast of one-year excess bond returns using a rolling window of fifteen years for the models proposed by Cochrane and Piazzesi (2005), Dahlquist and Hasseltoft (2011) and our global and local yield curve factor model, for the USA. In Figures 3 and 4 we display $\bar{R}^2$ for the same rolling window and period, for Germany and the UK, respectively. In Figures 2, 3 and 4 we show that the global and local yield curve factor model is able to explain excess bond returns with an average $\bar{R}^2$ over 42%, and up to 59% for all the countries. It is noticeable that the in-sample predictability of excess bond returns for the UK is higher than the predictability for the USA and Germany, because the $\bar{R}^2$, in Figure 4, is always over 30%, and on average is over 50% for all maturities.

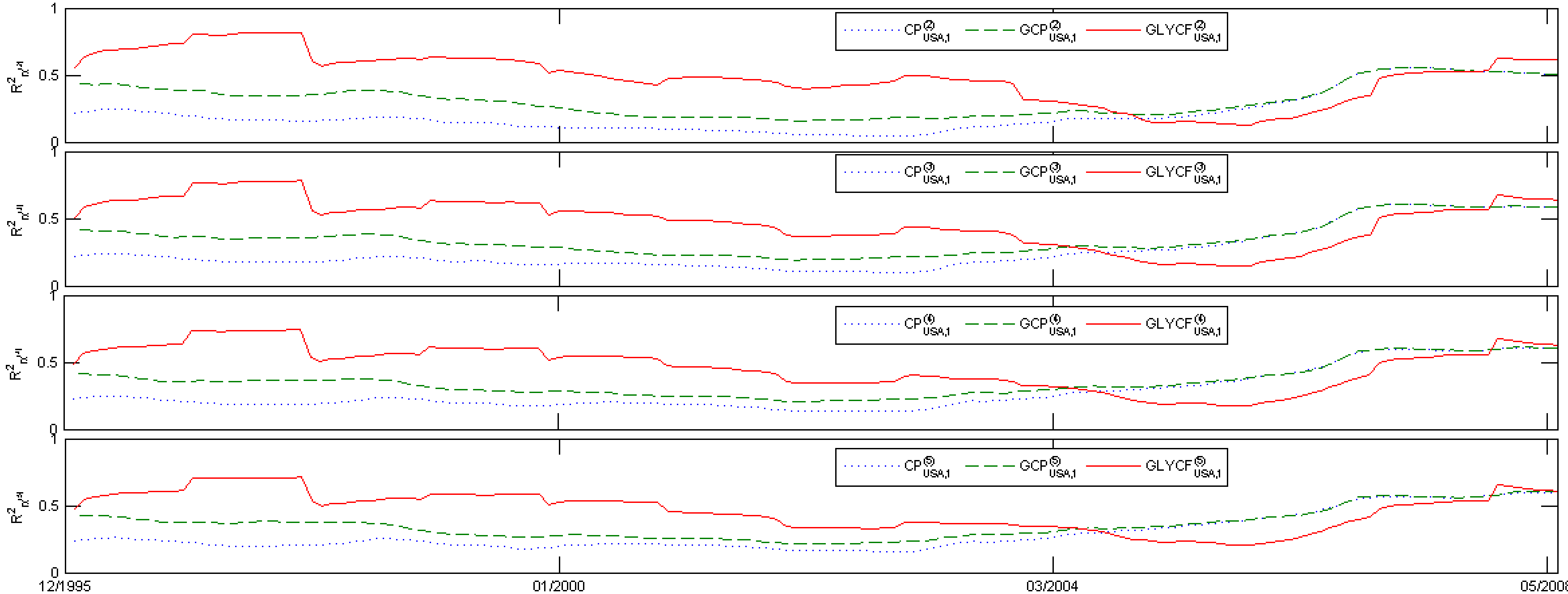

Figure 2: $\bar{\mathrm{R}}^2$ computed using a fifteen-year rolling window of one-year excess bond returns for the USA using three models: Cochrane and Piazzesi (2005), Dahlquist and Hasseltoft (2011) and global and local yield curve factor model. The top plot shows the one-year excess bond return for a bond with two years maturity and the next plots for three, four and five years maturities.

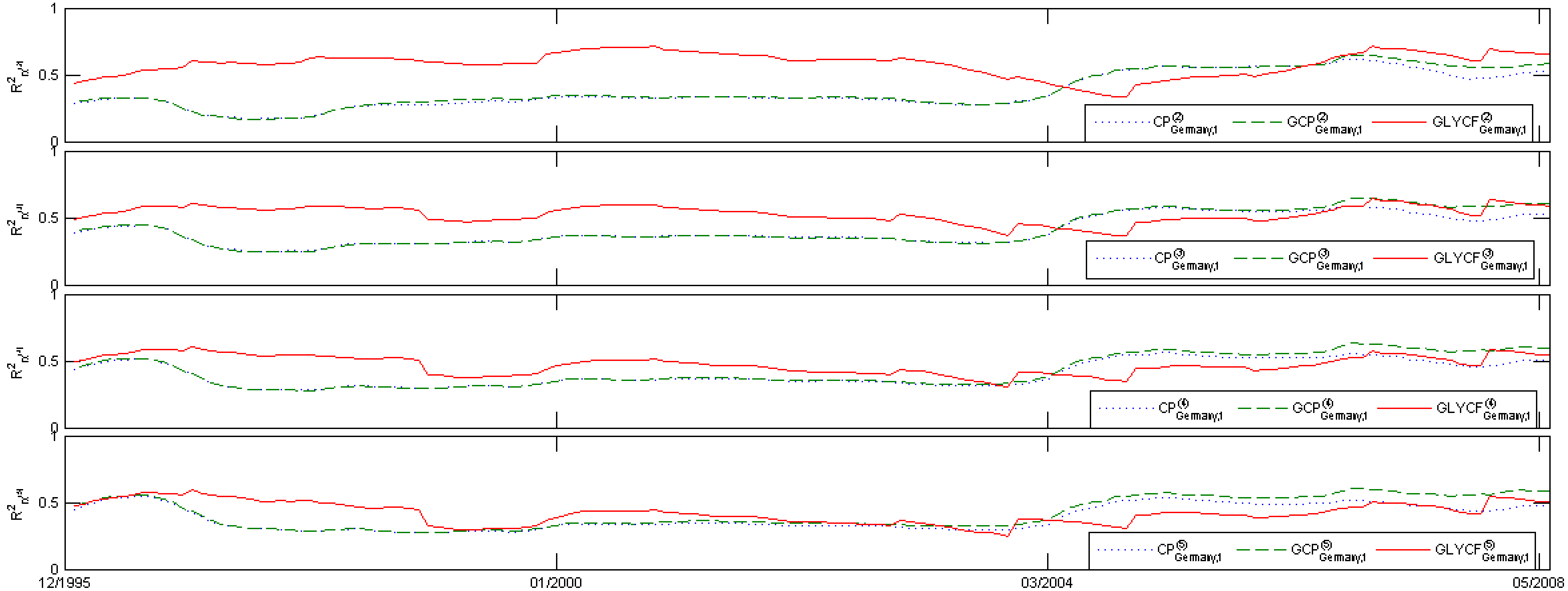

Figure 3: $\bar{R}^2$ computed using a fifteen-year rolling window of one-year excess bond returns for Germany using three models: Cochrane and Piazzesi (2005), Dahlquist and Hasseltoft (2011) and global and local yield curve factor model. The top plot shows the one-year excess bond return for a bond with two years maturity and the next plots for three, four and five years maturities.

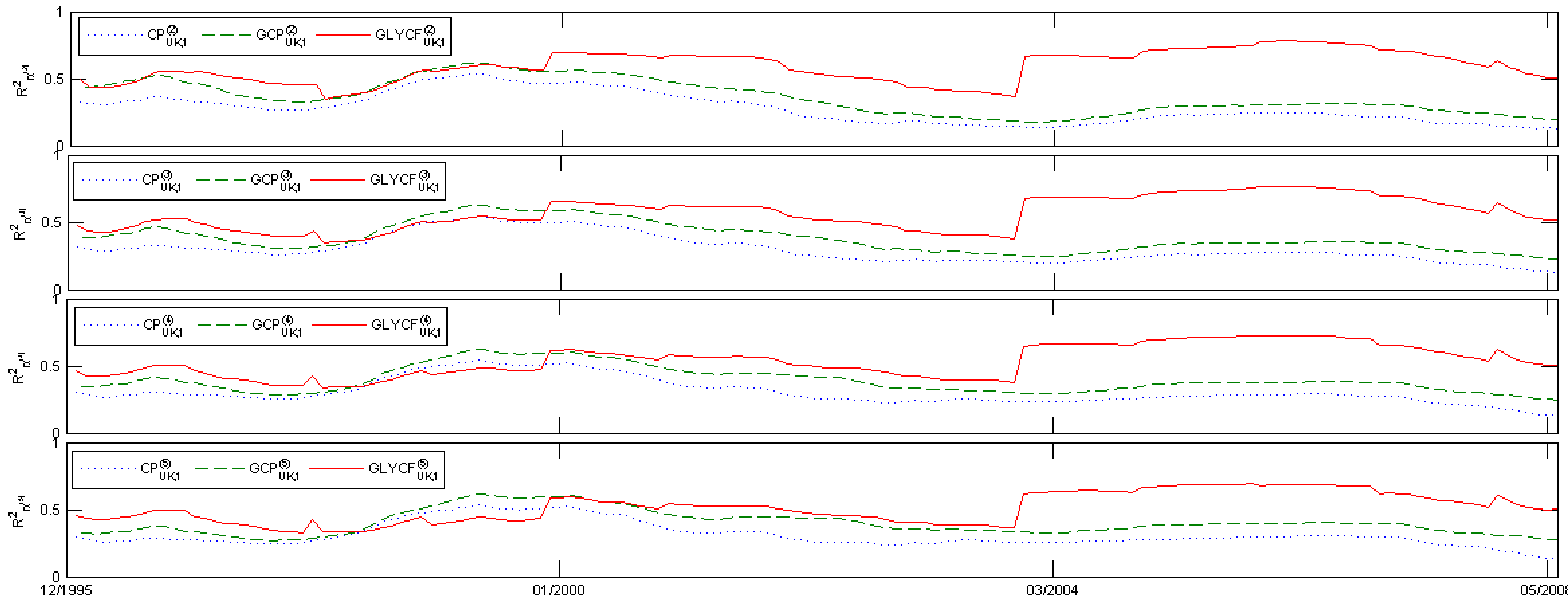

Figure 4: $\bar{R}^2$ computed using a fifteen-year rolling window of one-year excess bond returns for the UK using three models: Cochrane and Piazzesi (2005), Dahlquist and Hasseltoft (2011) and global and local yield curve factor model. The top plot shows the one-year excess bond return for a bond with two years maturity and the next plots for three, four and five years maturities.

Table IX

Comparative adjusted R^2 for the USA, Germany and the UK

Panel A: USA			
Maturity	CP	GCP	GLYCF
2 yrs.	0.21	0.31	0.48
3 yrs.	0.26	0.34	0.48
4 yrs.	0.28	0.35	0.47
5 yrs.	0.29	0.36	0.46
Panel B: Germany			
Maturity	CP	GCP	GLYCF
2 yrs.	0.37	0.39	0.58
3 yrs.	0.40	0.42	0.53
4 yrs.	0.40	0.43	0.47
5 yrs.	0.39	0.42	0.42
Panel C: UK			
Maturity	CP	GCP	GLYCF
2 yrs.	0.28	0.36	0.59
3 yrs.	0.30	0.38	0.57
4 yrs.	0.31	0.39	0.54
5 yrs.	0.31	0.40	0.51

Note: This table shows the average of adjusted R^2 statistics computed using a fifteen-year rolling window of one-year excess bond returns using the Cochrane and Piazzesi (2005) model (CP), Dahlquist and Hasseltoft (2011) global factor model (GCP) and global and local yield curve factor model (GLYCF). The first column shows the maturity of the bond whose excess bond return is calculated. The next three columns report the average of adjusted R^2 for the CP, GCP and GLYCF model. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) spanning the period 1980:12-2008:05. Panels A, B and C depict results for the USA, Germany and the UK.

Overall, the model of Dahlquist and Hasseltoft (2011) adds some predictability to the model of Cochrane and Piazzesi (2005). However, both models exhibit larger MAE and RMSE than the global and local yield curve factor model, and the first two models do not span the levels of predictability ($\bar{R}^2$) shown by the latter, in Figures 2, 3 and 4 for the USA, Germany and the UK, respectively.

Additionally, we test the stability of results using a twenty-year rolling window in Appendix I. The results computed with the twenty-year rolling window do not change qualitatively the conclusions obtained with the fifteen-year rolling window.

3.9. Conclusion and extensions

We use the global and local yield curve factor model based on level, slope and curvature, to explain excess bond returns one year ahead, with average adjusted R^2 up to 0.59. Our results indicate that the global and local factors of term structure of interest rates are important for explaining excess bond returns across countries and maturities. Therefore, excess bond returns could be characterized using global and local factors. Global yield curve factors play an important role because they explain up to 58% of variance of excess bond returns forecast error. In particular, the most important global factor is level, which explains no less than 14% and could explain up to 43% of total variance, followed by global slope which explains up to 35% of the total variance of excess bond returns forecast error.

The fact that global and local yield curve factors are important for explaining excess bond returns, has important implications for both central banks and portfolio managers. For example, central banks of countries more influenced by global factors could see more limitations in the extent of their influence on longest yields, and portfolio managers could detect an opportunity window to exploit predictability of excess bond returns, or anticipate the degree of impact of a movement in global components. Moreover, predictability of excess bond returns provides evidence against Expectation Hypothesis which can indicate time-varying term premia.

The global and local yield curve factor model proposed shows lower mean absolute error and root mean squared error than the single-factor model proposed by Cochrane and Piazzesi (2005) in explaining excess bond returns one year ahead for all countries and maturities. The Dahlquist and Hasseltoft (2011) factor model adds an additional percentage of goodness of fit to the Cochrane and Piazzesi (2005) model, since it integrates global factors into the single-factor model. In this respect, the global and local yield curve factor model outperforms on average to the global and local factor model proposed by Dahlquist and Hasseltoft (2011), for all countries and maturities. Moreover, there is a large percentage of predictability, measured using adjusted R^2, captured by the global and local yield curve factor model, which is not spanned either by the Cochrane and Piazzesi (2005) single-factor model or the Dahlquist and Hasseltoft (2011) model.

Considering that Thornton and Valente (2012) report a decrease in predictability power of the Cochrane and Piazzesi (2005) model for recent periods, one of the possible extensions of our research is to extend the dataset of Wright (2011) to include the most recent periods. This would allow us to test the evolution of the global and local yield curve factor model and the two competing models: the Cochrane and Piazzesi (2005) single-factor model and the Dahlquist and Hasseltoft (2011) factor model.

References

[1] Campbell, J. Y., and R. J. Shiller. Yield Spreads and Interest Rates: A Bird's Eye View. *Review of Economic Studies 58* (1991), 495–514.

[2] Christensen, J. H., Diebold, F. X., and Rudebusch, G. D. The affine arbitrage-free class of Nelson-Siegel term structure models. *SSRN eLibrary,* (2007).

[3] Cochrane, J. H., and Piazzesi, M. Bond risk premia. *American Economic Review 95*, 1 (March 2005), 138–160.

[4] Cochrane, J. H., and Piazzesi, M. Decomposing the yield curve. *Working Paper,* (March 2008).

[5] Coroneo, L., Nyholm, K., and Vidova-Koleva, R. How arbitrage-free is the nelson-siegel model? *Journal of Empirical Finance 18*, 3 (2011), 393–407.

[6] Dahlquist, M., and Hasseltoft, H. International Bond Risk Premia. *SSRN eLibrary,* (March 2011).

[7] Diebold, F. X., and Li, C. Forecasting the term structure of government bond yields. *Journal of Econometrics 130*, 2 (February 2006), 337–364.

[8] Doz, C., Giannone, D., and Reichlin, L. A quasi maximum likelihood approach for large approximate dynamic factor models. *Working Paper Series, European Central Bank*, 674 (September 2006).

[9] Driessen, J., Melenberg, B., and Nijman, T. Common factors in international bond returns, *Journal of International Money and Finance 22*, 5 (October 2003), 629-656.

[10] Fama, E. F., and Bliss, R. R. The information in long-maturity forward rates. *American Economic Review 77*, 4 (September 1987), 680–692.

[11] Hamilton, James D. Time series analysis, *Princeton University Press*, *Princeton*, (1994).

[12] Hellerstein, R. Global Bond Risk Premiums. *FRB of New York Staff Report No. 499,* (2011).

[13] Joslin, S., Singleton, K., Zhu, H. A New Perspective on Gaussian Dynamic Term Structure Models. *Review of Financial Studies 24*, 3 (2011), 926–970.

[14] Kim, D. H., and Orphanides, A. The bond market term premium: what is it, and how can we measure it? *BIS Quarterly Review*, (June 2007), 27–40.

[15] Nelson, C. R., and Siegel, A. F. Parsimonious modeling of yield curves. *Journal of*

Business 60, 4 (October 1987), 473–489.

[16] Sims, C.A. Macroeconomics and Reality. *Econometrica* 48, 1 (January 1980), 1-48.

[17] Sims, C.A. Policy Analysis with Econometric Models. *Brookings Papers on Economics Activity*, 1 (1982),107-152.

[18] Thornton, D.L., and Valente, G. Out-of-sample Predicting Bond Excess Returns with Forward Rates: An Asset-Allocation Perspective. *Forthcoming in the Review of Financial Studies,* (2012).

[19] Wright, J. H. Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset. *American Economic Review 101*, 4 (June 2011), 1514–1534.

Appendix I

Table VII reports mean absolute error (MAE) and Table VIII depicts root mean squared error (RMSE) of excess bond returns using a twenty-year rolling window. The excess bond return predictability results are slightly better but similar to the outcomes reported for the fifteen-year rolling window period. The results using a twenty-year rolling window confirm our previous results using a fifteen-year rolling window.

Table VII

Comparative mean absolute error statistics of in-sample forecast for the USA, Germany and the UK

Panel A: USA									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	0.96	0.86	1.04	0.90	0.85	0.93	0.73	0.65	0.85
3 yrs.	1.87	1.72	1.98	1.74	1.68	1.78	1.44	1.36	1.57
4 yrs.	2.63	2.44	2.74	2.44	2.35	2.52	2.08	1.98	2.22
5 yrs.	3.27	3.03	3.39	3.04	2.93	3.15	2.65	2.48	2.80
Panel B: Germany									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	1.06	0.94	1.22	1.06	0.95	1.21	0.80	0.74	0.95
3 yrs.	1.88	1.68	2.20	1.88	1.68	2.19	1.52	1.40	1.71
4 yrs.	2.59	2.33	3.08	2.57	2.29	3.05	2.21	2.00	2.61
5 yrs.	3.28	2.97	3.92	3.23	2.86	3.85	2.89	2.62	3.50
Panel C: UK									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	1.04	0.96	1.15	1.01	0.94	1.15	0.68	0.51	0.92
3 yrs.	2.00	1.78	2.25	1.93	1.73	2.23	1.37	1.04	1.84
4 yrs.	2.82	2.44	3.25	2.69	2.33	3.16	2.05	1.59	2.72
5 yrs.	3.55	3.01	4.16	3.37	2.86	4.00	2.71	2.17	3.55

Note: This table shows mean absolute error (MAE) statistics computed using a twenty-year rolling window of one-year excess bond returns using the Cochrane and Piazzesi (2005) model (CP), Dahlquist and Hasseltoft (2011) global factor model (GCP) and global and local yield curve factor model (GLYCF). The first column shows the maturity of the bond whose excess bond return is calculated. The next three columns report the average, minimum and maximum of the MAE explained by the CP model. The next three columns show the average, minimum, and maximum of the MAE for the GCP model. The last three columns show the same statistics for the GLYCF model. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) spanning the period 1980:12-2008:05. Panels A, B and C depict results for the USA, Germany and the UK.

Table VIII

Comparative root mean squared errors statistics of in-sample forecast for the USA, Germany and the UK

Panel A: USA									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	1.19	1.09	1.27	1.09	1.04	1.15	0.88	0.80	1.02
3 yrs.	2.31	2.16	2.38	2.13	2.07	2.19	1.74	1.67	1.85
4 yrs.	3.26	3.08	3.30	3.03	2.94	3.10	2.52	2.37	2.62
5 yrs.	4.07	3.89	4.11	3.82	3.71	3.89	3.22	3.03	3.40
Panel B: Germany									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	1.38	1.25	1.56	1.38	1.25	1.56	1.00	0.92	1.20
3 yrs.	2.42	2.24	2.74	2.41	2.24	2.73	1.91	1.80	2.15
4 yrs.	3.31	3.10	3.81	3.29	3.07	3.79	2.79	2.59	3.26
5 yrs.	4.15	3.90	4.84	4.10	3.83	4.79	3.64	3.38	4.37
Panel C: UK									
	CP			GCP			GLYCF		
Maturity	Mean	Min.	Max.	Mean	Min.	Max.	Mean	Min.	Max.
2 yrs.	1.31	1.21	1.43	1.25	1.18	1.40	0.86	0.64	1.20
3 yrs.	2.48	2.25	2.73	2.34	2.13	2.68	1.71	1.28	2.34
4 yrs.	3.48	3.10	3.89	3.28	2.87	3.82	2.53	1.93	3.37
5 yrs.	4.38	3.82	4.99	4.12	3.49	4.87	3.34	2.60	4.34

Note: This table shows root mean squared error (RMSE) statistics computed using a twenty-year rolling window of one-year excess bond returns using the Cochrane and Piazzesi (2005) model (CP), Dahlquist and Hasseltoft (2011) global factor model (GCP) and global and local yield curve factor model (GLYCF). The first column shows the maturity of the bond whose excess bond return is calculated. The next three columns report the average, minimum and maximum of the RMSE explained by the CP model. The next three columns show the average, minimum, and maximum of the RMSE for the GCP model. The last three columns show the same statistics for the GLYCF model. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) spanning the period 1980:12-2008:05. Panels A, B and C depict results for the USA, Germany and the UK.

Chapter 4

A joint model of global macroeconomic and yield curve factors

Abstract

We extend the model we use in previous chapters to study the relationship between global macro factors and yield curve factors. We take the idea of yield curves having global components to see if there are global components in macroeconomic variables. We examine the dynamic interaction between global macroeconomic factors and yield curve factors. Global macroeconomic factors are defined as the common components of industrial production, inflation and monetary policy for three countries, the USA, Germany and the UK. We find evidence of bidirectional interaction between global yield curve factors and global macroeconomic factors. The main results indicate a stronger influence of macroeconomic factors on yield curve factors than the reverse.

4.1. Introduction and literature review

The relationship between interest rates and macroeconomic variables has attracted increasing attention and has been studied recently with two sets of different results.

On the one hand, some studies focus on the influence of domestic macroeconomic variables on the yield curve. In particular, they indicate that monetary policy plays an important role, in determining the term structure of interest rates. Evans and Marshall (1998) use a vector autoregressive (VAR) model with different identification schemes and find evidence that monetary policy shocks mostly affect short-term rates. Wu (2002) finds evidence of a strong relation between slope and monetary policy using also a VAR with six variables and a Taylor rule model, using general moment methods (GMM). Wu (2006b) develops a macro-term structure model and his findings indicate that slope and level are connected to monetary policy and technology shocks, respectively. Hördahl, Tristani and Vestin (2006, 2008) combine a VAR with New-Keynesian models which include shocks to technology and inflation target. Their findings indicate that the model can replicate the sign and size of average excess holding period returns on bonds as well as the variance of yields across the term structure.

The second strand of research finds evidence of the influence of inflation and output on interest rates. Specifically, Ang and Piazzesi (2003) study the empirical relation between macroeconomics and yields using a no-arbitrage VAR with yields and macroeconomic factors related to inflation and real activity. Their findings indicate that these factors account for around 85% of the variance of short-term and medium-term rates, and 40% of long-term rates. Moreover, Evans and Marshall (2007) find evidence that macroeconomic shocks to technology and consumption preferences affect inflation, output and term structure of interest rates as a whole, shifting the level of yield curve. The differences in the findings of Ang and Piazzesi (2003) and Evans and Marshall (2007) can be attributed to the smoothing of interest rates, which suggests that dynamics are important.

Ang, Bekaert and Wei (2008) use a regime switching model and find an important role for expected inflation and inflation risk, with these accounting for 80% of variation in nominal yields. Rudebusch and Wu (2008) find that level and slope factors are linked to inflation and slope factor is linked to output gaps. The papers discussed above are complemented with other studies such as Coroneo, Giannone and Modugno (2008, 2013) who find that inclusion of macroeconomic variables in a model of the yield curve improves the ability to predict yields and excess bond returns. Ludvigson and Ng (2009) also show improvements on predictability of excess bond returns for the USA when macroeconomic factors are included in a factor model.

Estrella and Hardouvelis (1991) and Estrella and Mishkin (1998) study the other side of the interaction between interest rates and macroeconomic variables, i.e., the extent of explanatory or predictive content of interest rates on macroeconomic variables. They find evidence that slope help to predict economic activity and recessions. Also, Ang, Piazzesi and Wei (2006) provide evidence of the influence of short-term rates and yield curve factors on output, due to the fact that inclusion of these variables allows for superior forecastability of gross domestic product (GDP) of the USA.

A few studies consider both sides of predictability. Diebold, Rudebusch and Aruoba (2006) analyze the ability of interest rates to predict movements in macroeconomic variables and vice versa. Their findings indicate that there is stronger evidence of the influence of macro variables on interest rates than the reverse. Moench (2012) studies linkages between a large set of macroeconomic variables and the yield curve and his findings suggest that the curvature factor contains important information about future evolution of the yield curve and output. Bekaert, Cho and Moreno (2010) study interactions between interest rates and macroeconomic variables using a New-Keynesian macro model. Their findings indicate that output responds to real interest rate shocks and the level interest rate factor responds to inflation shocks, and that both curvature and slope factors respond to monetary policy shocks.

In addition to the studies discussed above, another strand of the literature focuses on the international linkage of government bonds and interest rates between different countries, which provide support to the existence of global factors explaining the term structure of different countries. In this regard, Driessen, Melenberg and Nijman (2003) find evidence of a common level factor which explains around 50% of the predictability of international bond returns. Diebold, Li and Yue (2008) go one step further, estimating a global yield curve which contains level and slope factors for Germany, Japan, the USA, and the UK. Their findings indicate that global yield curve factors are economically important and explain an important part of dynamic of yields. The importance of global factors is also highlighted with the strengthening of the linkage between different countries, as shown by Christiansen (2010) who studies volatility-spillover effects between bonds and stocks for the USA and the European Union (EU). She finds evidence of volatility-spillovers across countries in the bond markets and that the introduction of Euro increased the integration of European financial markets.

Byrne, Fazio and Fiess (2012) investigate the decoupling of short and long-term interest rates for the USA and comovements of long-term interest rates between countries using a latent global macroeconomic factor. They find that this factor is connected to global savings glut. Gomez-Biscarri (2008) provides evidence of influence of global factors in major economies examining changes in predictive power of term spreads to predict recessions, he shows that domestic spread has lost its informative content in favor of international spreads of the USA and Germany. Also, Wu (2006a) suggests that globalization plays an important role in the decoupling observed between short-term and long-term interest rates. However, different evidence is provided by Bredin, Hyde and O'Reilly (2010) who use futures markets to investigate domestic and foreign influence of surprise changes in monetary policy over excess bond returns for the USA, Germany and the UK. They find that excess bond returns are more responsive to domestic than to international monetary policy surprises.

Some studies provide evidence of there being common components in macroeconomic variables. In particular, Bagliano and Morana (2009) find evidence of comovements in international macroeconomic variables for the USA, the UK, Canada, and the Euro area. They find comovements in output rates and a common global factor which drives inflation, interest rates and monetary aggregates. These results are endorsed by other research which considers the G-7 countries and investigates similarities and convergence of business cycles between them. In this regard, Canova, Ciccarelli and Ortega (2007) show that a world factor explains 30% of variations in sales, industrial production, output and employment, indicating that there is a world economic cycle which is stronger in contraction periods. Crucini, Kose and Otrok (2011) decompose business cycles using a dynamic factor model for output, fiscal and monetary policy, trade terms and oil prices. Their findings show a large common factor in oil prices, productivity, and terms of trade. A more extensive study is conducted by Mumtaz, Simonelli and Surico (2011) who document evidence of international comovements in output and inflation with increasing importance of regional linkages (Europe, North America, Oceania, Asia and South America). Kose, Otrok and Prasad (2012) conduct a similar study using a dynamic global factor model to extract a global factor from output, consumption and investment. Their findings also provide evidence of a global factor in business cycles with some level of decoupling between industrial and emerging countries. Sousa and Zaghini (2004) find evidence of a global monetary policy factor based on monetary aggregates, using the G5 countries. The existence of a common or global monetary policy is endorsed by Taylor (2013) who indicates that there are monetary policy spillovers between the USA and the rest of the world through two main channels. First, when the USA reduces its rate encouraging banks to provide loans to foreign firms incentive to foreign central banks to reduce the monetary policy rate in order to reduce the risk taking. Second, foreign exchange rates appreciate (USD depreciates against foreign currency) due to inflows of loans denominated in USD, which in turn induces further foreign loans and further appreciation of the foreign currency.

Kaminska, Meldrum and Smith (2013) propose a global no-arbitrage yield curve factor model to study the linkage between interest rates and exchange rates to account for any deviation from uncovered interest rate parity (UIP) for the UK, USA and Euro area. Their findings indicate that it is necessary to use global and local yield curve factors to explain bond yields, while exchange rates movements are explained by monetary policy rate differences and exchange rate risk premia. This kind of linkage between monetary policy rates and exchange rates was previously documented by Lubik and Schorfheide (2007) who find that, among others, the Bank of England includes exchange rates in monetary policy.

Rudebusch (2010) argues that the linkage between the economy and financial markets poses a challenge for researchers since both had been modeled separately. The financial and economic crises of 2008 and 2009 highlighted the importance of these spillovers. Therefore, to account for the feedback between real economy and the financial sector, in a unified framework, requires to reconcile both in a joint model.

Our motivation is driven by the mixed evidence of the influence of macroeconomic factors on yield curve factors and vice versa. In addition, we are interested in exploring the extent of the influence of the global macroeconomic factors on yield curve factors in the context of previous evidence of reinforcing of macroeconomic linkages among countries. Although our study is closely related to the research of Diebold, Rudebusch and Aruoba (2006), who study the bidirectional empirical linkage between the term structure and macroeconomic factors, it differs in some important respects. First, our aim is to propose and estimate a joint macroeconomic and interest rates factor model with global and local factors to explain the role of global macroeconomic factors in the dynamic interaction between macroeconomic and yield curve factors. Second, we study bidirectional linkage not just only one country, but for a set of countries. The variables considered in the sample are monthly yields, from one to ten years, as well as the following macroeconomic variables: monetary policy rates, inflation, industrial production growth, for the USA, Germany and the UK. The results indicate that there is an important correlation between these macroeconomic variables. In this respect, we estimate global macroeconomic factors as common components between the macroeconomic variables of the three countries.

Considering the evidence provided by previous research about the strengthening of the international linkage of financial markets, in this research we address the following questions: Is there a bidirectional relationship between global and local yield curve and global macroeconomic factors? Does a global and local yield curve and macroeconomic factor model provide evidence of the influence of yield curve factors on macroeconomic factors, the reverse or both?

The results indicate that the joint model of global and local yield curve factors and global macroeconomic factors does a good job in explaining the yields and macroeconomic variables of the three countries. Also, the results show that the influence of macroeconomic factors on yield curve factors is stronger than the influence of yield curve factors on global macroeconomic factors. In particular, the influence of all the global macroeconomic factors on the global level is positive, indicating that any increase in these factors will lead to an increase in the global level of rates. Moreover, in the case of macroeconomic factors, the feedback from the global yield curve factor to global macroeconomic factor show small parameters (below 0.09) with the exception of the influence of global level and global slope on global monetary policy rate whose parameters are 0.42 and 0.45.

The rest of the paper is organized as follows. Section 4.2 presents data and preliminary analysis. Section 4.3 describes the models. Section 4.4 discusses the estimation and results, Section 4.5 concludes.

4.2. Data and preliminary analysis

The yield data consist of monthly nominal zero-coupon government yields for 10 maturities from 1 to 10 years. The source is a subset of the database constructed by Wright (2011) which spans the period from December 1980 to May 2008. The macroeconomic data are monthly industrial production index (IPI), monthly consumer price index (CPI) and monthly monetary policy market rate (MPR) for the USA, Germany and the UK, spanning the periods from January 1980 to May 2008 for IPI and CPI and from December 1980 to May 2008 for MPR. Monetary policy rates are the official monthly averages of overnight discount rates.[1] The sources are the Organization for Economic Cooperation and Development (OECD) database for IPI and CPI, and respective central banks for MPR from December 1980 to May 2008.

Table I reports the matrix of correlation between annual industrial production growth, defined as $ln(IPI_t/IPI_{t-12})$, annual inflation, defined as $ln(CPI_t/CPI_{t-12})$, and monetary policy rates.[2] The correlation of Δ IPI ranges between 0.24 and 0.31 for three countries, the correlation of Π ranges between 0.61 and 0.80 and the correlation of MPR ranges between 0.56 and 0.75 for three countries. The strength of the correlations across the countries suggests that there may be a common (global) component to industrial production growth, inflation and monetary policy rates.

Table I

Correlation matrix of macroeconomic variables

	$\Delta\ \text{IPI}^{(t)}_{\text{USA}}$	$\Delta\ \text{IPI}^{(t)}_{\text{GER}}$	$\Delta\ \text{IPI}^{(t)}_{\text{UK}}$	$\Pi^{(t)}_{\text{USA}}$	$\Pi^{(t)}_{\text{GER}}$	$\Pi^{(t)}_{\text{UK}}$	$\text{MPR}^{(t)}_{\text{USA}}$	$\text{MPR}^{(t)}_{\text{GER}}$	$\text{MPR}^{(t)}_{\text{UK}}$
$\Delta\ \text{IPI}^{(t)}_{\text{USA}}$	1.00	0.24	0.31	- 0.16	- 0.27	- 0.27	0.01	- 0.27	- 0.21
$\Delta\ \text{IPI}^{(t)}_{\text{GER}}$	0.24	1.00	0.11	0.04	- 0.31	- 0.03	0.10	- 0.26	0.08
$\Delta\ \text{IPI}^{(t)}_{\text{UK}}$	0.31	0.11	1.00	- 0.01	- 0.11	0.02	0.22	- 0.07	0.14
$\Pi^{(t)}_{\text{USA}}$	- 0.16	0.04	- 0.01	1.00	0.61	0.80	0.74	0.65	0.61
$\Pi^{(t)}_{\text{GER}}$	- 0.27	- 0.31	- 0.11	0.61	1.00	0.63	0.38	0.82	0.39
$\Pi^{(t)}_{\text{UK}}$	- 0.27	- 0.03	0.02	0.80	0.63	1.00	0.77	0.79	0.85
$\text{MPR}^{(t)}_{\text{USA}}$	0.01	0.10	0.22	0.74	0.38	0.77	1.00	0.56	0.75
$\text{MPR}^{(t)}_{\text{GER}}$	- 0.27	- 0.26	- 0.07	0.65	0.82	0.79	0.56	1.00	0.71
$\text{MPR}^{(t)}_{\text{UK}}$	- 0.21	0.08	0.14	0.61	0.39	0.85	0.75	0.71	1.00

Note: This table shows correlation between macroeconomic variables: yearly industrial production growth, yearly inflation and monetary policy rates for the USA, Germany and the UK using monthly data spanning the period 1980:12-2008:05.

[1] In case of Germany, we use the Frankfurt Interbank Offered Rate Overnight.

[2] Also, we define monthly industrial production growth, Δ IPI, as $ln(IPI_t/IPI_{t-1})$ and monthly inflation, Π, as $ln(CPI_t/CPI_{t-12})$.

Table II explores the relationship between the yields and macroeconomic variables estimating the correlation between the yields at time $t-1$ and the macroeconomic variables at time t. The correlation matrix shows in general a negative correlation between short term yields and Δ IPI, a positive correlation between all the yields and Π which ranges between 0.38 and 0.87 as well as a positive correlation between yields and MPR which ranges between 0.55 and 0.98.

Table II

Correlation matrix of yields and macroeconomic variables

	$Y^{(1,\ t\text{-}1)}_{\text{USA}}$	$Y^{(5,\ t\text{-}1)}_{\text{USA}}$	$Y^{(10,\ t\text{-}1)}_{\text{USA}}$	$Y^{(1,\ t\text{-}1)}_{\text{GER}}$	$Y^{(5,\ t\text{-}1)}_{\text{GER}}$	$Y^{(10,\ t\text{-}1)}_{\text{GER}}$	$Y^{(1,\ t\text{-}1)}_{\text{UK}}$	$Y^{(5,\ t\text{-}1)}_{\text{UK}}$	$Y^{(10,\ t\text{-}1)}_{\text{UK}}$
$\Delta\text{IPI}^{(1,\ t)}_{\text{USA}}$	- 0.23	- 0.12	- 0.04	- 0.13	- 0.06	- 0.08	0.03	0.00	- 0.02
$\Delta\text{IPI}^{(1,\ t)}_{\text{GER}}$	- 0.16	- 0.15	- 0.18	0.08	- 0.06	- 0.15	0.08	- 0.03	- 0.09
$\Delta\text{IPI}^{(1,\ t)}_{\text{UK}}$	- 0.01	0.12	0.20	0.23	0.28	0.29	0.29	0.31	0.31
$\Pi^{(1,\ t)}_{\text{USA}}$	0.69	0.65	0.60	0.64	0.64	0.64	0.70	0.66	0.65
$\Pi^{(1,\ t)}_{\text{GER}}$	0.79	0.68	0.60	0.40	0.48	0.50	0.38	0.44	0.46
$\Pi^{(1,\ t)}_{\text{UK}}$	0.84	0.84	0.80	0.86	0.87	0.86	0.76	0.79	0.80
$\text{MPR}^{(1,\ t)}_{\text{USA}}$	0.65	0.72	0.71	0.82	0.83	0.81	0.98	0.92	0.88
$\text{MPR}^{(1,\ t)}_{\text{GER}}$	0.98	0.89	0.81	0.70	0.73	0.73	0.55	0.60	0.63
$\text{MPR}^{(1,\ t)}_{\text{UK}}$	0.77	0.80	0.77	0.97	0.90	0.86	0.75	0.77	0.77

Note: This table shows correlation between selected yields (1, 5 and 10 years) at time $t-1$ and macroeconomic variables at time t: yearly industrial production growth, yearly inflation and monetary policy rates for the USA, Germany and the UK using monthly data spanning the period 1980:12-2008:05.

Table III estimates the correlation matrix between the macroeconomic variables at time $t-1$ and yields at time t. The matrix of correlation shows a generally negative correlation between Δ IPI and short term yields, a positive correlation between Π and all yields which ranges between 0.37 and 0.86 as well as a positive correlation between yields and MPR which ranges between 0.53 and 0.97. These results suggest that there is a linkage between lagged macroeconomic variables and yields as well as a linkage between lagged yields and macroeconomic variables.

Table III

Correlation matrix of macroeconomic variables and yields

	$\Delta\text{IPI}_{\text{USA}}^{(\text{t-1})}$	$\Delta\text{IPI}_{\text{GER}}^{(\text{t-1})}$	$\Delta\text{IPI}_{\text{UK}}^{(\text{t-1})}$	$\Pi_{\text{USA}}^{(\text{t-1})}$	$\Pi_{\text{GER}}^{(\text{t-1})}$	$\Pi_{\text{UK}}^{(\text{t-1})}$	$\text{MPR}_{\text{USA}}^{(\text{t-1})}$	$\text{MPR}_{\text{GER}}^{(\text{t-1})}$	$\text{MPR}_{\text{UK}}^{(\text{t-1})}$
$Y_{\text{USA}}^{(1,\text{ t})}$	- 0.20	- 0.12	- 0.04	0.70	0.77	0.83	0.67	0.96	0.77
$Y_{\text{USA}}^{(5,\text{ t})}$	- 0.12	- 0.14	0.10	0.66	0.67	0.84	0.73	0.88	0.80
$Y_{\text{USA}}^{(10,\text{ t})}$	- 0.04	- 0.18	0.19	0.61	0.59	0.79	0.71	0.80	0.76
$Y_{\text{GER}}^{(1,\text{ t})}$	- 0.09	0.08	0.22	0.64	0.39	0.84	0.82	0.67	0.94
$Y_{\text{GER}}^{(5,\text{ t})}$	- 0.05	- 0.07	0.26	0.65	0.47	0.86	0.83	0.72	0.87
$Y_{\text{GER}}^{(10,\text{ t})}$	- 0.07	- 0.15	0.27	0.65	0.50	0.86	0.81	0.73	0.84
$Y_{\text{UK}}^{(1,\text{ t})}$	0.09	0.08	0.29	0.70	0.37	0.75	0.97	0.53	0.72
$Y_{\text{UK}}^{(5,\text{ t})}$	0.04	- 0.04	0.30	0.67	0.43	0.79	0.91	0.60	0.75
$Y_{\text{UK}}^{(10,\text{ t})}$	0.01	- 0.10	0.29	0.66	0.46	0.80	0.87	0.62	0.75

Note: This table shows correlation between macroeconomic variables: yearly industrial production growth, yearly inflation and monetary policy rates at time $t-1$ and selected yields (1, 5 and 10 years) at time t for the USA, Germany and the UK using monthly data spanning the period 1980:12-2008:05.

4.3. Description of the model

4.3.1. A global and local yield curve factor model and a global macroeconomic factor model

For convenience, we will recap the model from Chapter 2. Recall from Chapter 2 that the relationship between yields and level, slope and curvature is given by (Diebold and Li, 2006)

$$y_{i,t}(\tau) = l_{i,t} + s_{i,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + c_{i,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + e_{i,t}(\tau), \tag{1}$$

where $y_{i,t}(\tau)$ is the yield for country i at time t for maturity τ, λ is the decay factor, $l_{i,t}$ is the level, $s_{i,t}$ is the slope, and $c_{i,t}$ is the curvature, and $e_{i,t}(\tau)$ is error term.[2]

The matrix representation of this model is

$$Y_{i,t} = \Gamma_i F_{i,t} + \varepsilon_{i,t}, \tag{2}$$

[2] The decay factor, λ, is fixed at the value of 0.0609 to maximize the curvature loadings for the period of 30 months.

where $Y_{i,t}$ is the matrix that stacks the yields of country i, Γ_i is the matrix of factor loadings, $F_{i,t}$ is the vector of factors (level, slope and curvature) and $\varepsilon_{i,t}$ is the vector of errors, for country i at time t. The generalization of this model for several countries is defined as follows

$$Y_t = \Gamma F_t + \varepsilon_t, \tag{3}$$

where the matrix Y_t stacks yields for 10 maturities ranging from 1 to 10 years for the USA, Germany and the UK, Γ is the matrix of factor loadings, the matrix F_t, stacks the factors of different countries and ε_t is the vector of errors. The factors, F_t, described by previous equation could be split into two types of factors: global, F_t^G, and local, F_t^L, with different loadings over the global factors, β, as follows

$$F_t = \beta F_t^G + F_t^L. \tag{4}$$

Replacing equation (4) in equation (3), we have

$$Y_t = \Gamma \left[\beta F_t^G + F_t^L\right] + \varepsilon_t. \tag{5}$$

Therefore, the yields for the different countries are described by the global and local yield curve as

$$Y_t = \Gamma^G F_t^G + \Gamma F_t^L + \varepsilon_t, \tag{6}$$

where Γ^G is the multiplication of the loadings, β, and the NS factor loadings, Γ, i.e., $\Gamma\beta = \Gamma^G$, equation (6) can be rewritten as

$$Y_t = [\Gamma^G \quad \Gamma][F_t^{G'} \quad F_t^{L'}]' + \varepsilon_t. \tag{7}$$

The factors follow a VAR of order one

$$[F_t^{G'} \quad F_t^{L'}]' = \Phi^Y \ [F_{t-1}^{G'} \quad F_{t-1}^{L'}]' + w_t, \tag{8}$$

where Φ^Y is a block diagonal matrix of factor loadings, whose blocks contain global and local factor loadings. The superscript, Y, indicates that the matrix of parameters is for yields, and w_t is the vector of errors. The state space representation of the model described by equations (7) and (8) can be written in a compact way as

$$Y_t = \Gamma^Y F_t^Y + \varepsilon_t, \tag{9}$$

$$F_t^Y = \Phi^Y F_{t-1}^Y + w_t, \tag{10}$$

with

$$\begin{aligned} \varepsilon_t &\sim N(0, \Sigma), &(11)\\ w_t &\sim N(0, \Omega), &(12)\end{aligned}$$

where Γ^Y contains factor loadings, $\Gamma^Y = [\Gamma^G \ \ \Gamma]$, and F_t^Y contains global and local factors, i.e., $F_t^Y = [F_t^{G'} \ \ F_t^{L'}]'$.

Considering previous empirical evidence (Bagliano and Morana (2009); Crucini, Kose and Otrok (2011); Kose, Otrok and Whiteman (2008); Mumtaz, Simonelli and Surico (2011); Kose, Otrok and Prasad (2012), among others) which suggests that there are common or global macroeconomic factors across countries, we also propose a model to estimate global factors in the macroeconomic variables:

$$X_t = \Gamma^X F_t^X + \varepsilon_t^X, \tag{13}$$

$$F_t^X = \Phi^X F_{t-1}^X + w_t^X, \tag{14}$$

with

$$\begin{aligned} \varepsilon_t^X &\sim N(0, \Sigma^X), &(15)\\ w_t^X &\sim N(0, \Omega^X), &(16)\end{aligned}$$

where the matrices X_t and F_t^X contain macroeconomic variables and global macroeconomic factors, respectively.[3] Also, Γ^X and Φ^X are the matrices of factor loadings of the observation and state equation, respectively; ε_t^X and w_t^X are error terms. The matrix Γ^X is constrained so that the global macroeconomic variables are independent of each other, e.g. only variables of industrial production growth load on global production growth factor.[4] This is to estimate the size of the load of each country on the common factors. Specifically, Γ^X is expressed as follows

[3] The local effects are included in the error term.

[4] The same applies to the global inflation factor and global monetary policy rate factor.

$$\Gamma^X = \begin{pmatrix} \Gamma^{\Delta\, IPI} & \Gamma^{\Pi} & \Gamma^{MPR} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{24} & b_{25} & b_{26} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & b_{37} & b_{38} & b_{39} \end{pmatrix}',$$

where $\Gamma^{\Delta\, IPI}$, Γ^{Π}, Γ^{MPR} are the matrices of factor loadings of industrial production growth, inflation and monetary policy rate, respectively.

4.3.2. A joint model of global macroeconomic and yield curve factors

In the previous section we treated each model separately (global and local yield curve factor model and global macroeconomic factor model). In this section, we can merge them in one model to provide a joint model of global macroeconomic factors and global and local yield curve factors. In particular, we extend the previous yield curve factor model to characterize the relationship among yield curve factors and macroeconomic factors as follows

$$Z_t = (\Gamma^{YX} F_t^{YX}) + u_t, \tag{17}$$
$$F_t^{YX} = \Phi^{YX} F_{t-1}^{YX} + \eta_t, \tag{18}$$

with

$$u_t \sim N(0, \Sigma^{YX}), \tag{19}$$
$$\eta_t \sim N(0, \Omega^{YX}), \tag{20}$$

where matrix Z_t stacks the yields (Y_t) and macroeconomic variables (X_t), and $\Gamma^{(YX)}$ is the matrix of global and local factor loadings for both macroeconomic and yield curve factors. The matrix F_t^{YX} contains the yields and macroeconomic factors which follow a VAR process of order one, Φ^{YX} is the respective matrix of parameters, u_t and η_t are vectors of errors.

The matrix of factor loadings for the observation equation, Γ^{YX}, can be partitioned as $\Gamma^{YX} = \begin{bmatrix} \Gamma^{yy} & \Gamma^{yx} \\ \Gamma^{xy} & \Gamma^{xx} \end{bmatrix}$, where Γ^{yy}, Γ^{yx}, Γ^{xy}, Γ^{xx} are submatrices of factor loadings on yields and macro variables. They load according to the following: yield factors on yields, macro factors on yields, yields on macro factors and macro factors on macro factors, respectively.

Similarly, the matrix of parameters for the state equation can be partitioned as $\Phi^{YX} =$

$\begin{bmatrix} \Phi^{yy} & \Phi^{yx} \\ \Phi^{xy} & \Phi^{xx} \end{bmatrix}$, where Φ^{yy}, Φ^{yx}, Φ^{xy}, Φ^{xx} are the parameter submatrices of the influence of factors at time $t-1$ on factors at time t, which load according to the following: yield factors on yield factors, macroeconomic factors on yield factors, yield factors on macro factors, and macro factors on macro factors, respectively.

We impose restrictions on both parameter matrices. We impose restrictions on the contemporary influence of macroeconomic factors on yields by constraining elements of the matrix , Γ^{yx}, to be equal to zero. Similarly, we restrict the other side of the contemporary interaction, i.e., between yield factors and macroeconomic variables, constraining the elements of the matrix, Γ^{xy}, to be equal to zero. These restrictions allow us to fit the yields and explain the macroeconomic variables by relying only on the use of the yield curve factors and macroeconomic factors, respectively.

We constrain submatrices $\Phi^{yx\prime}$ and $\Phi^{xy\prime}$, so that global macroeconomic factors at time $t-1$ influence both local and global yield curve factors at time t, but only global yield curve factors at time $t-1$ influence global macroeconomic factors at time t.

4.4. Estimation and results

4.4.1. Estimation

The estimation procedure of the model described by equations (17) to (20) is based on the quasi-maximum likelihood approach proposed by Doz, Giannone and Reichlin (2006, 2012) and the procedure described by Coroneo et al. (2013) for estimating the model using the Expectation Restricted Maximization algorithm (ERM). In particular, the ERM algorithm alternates the estimation of the log-likelihood using the Kalman filter, conditional on the data and parameter estimates of previous steps or initial values, with the update of parameters based on the maximization of the expected log-likelihood with respect to each parameter. Specifically, the model is estimated by quasi-maximum likelihood. The steps are as follows:

1. We estimate three NS factors for each one of the countries using the yields data and predefined NS factor loadings as described in equation (2).

2. We extract the principal components for each one of the set of factors (level, slope and curvature) for the set of countries, obtaining global factor loadings, β, and three global yield curve factors which are treated as the true global yield curve factors to initialize the estimation. Finally, we use the residual of this estimation to initialize the local yield curve factors.

3. We extract the principal components for each one of the macroeconomic variables for the set of countries, obtaining three global macroeconomic factors, treated as the true macroeconomic global factors to initialize the estimation.

4. We use the projection of global and local initial factors at time t on global and local initial factors at time $t-1$ to obtain the initial values for Φ^{YX} and Ω^{YX}.

5. We estimate the log-likelihood conditional on the previous parameters using the Kalman filter. Then we estimate each one of the parameters maximizing the expected log-likelihood.

6. We repeat iteratively the previous step until we obtain convergence.

4.4.2. Results

We estimate the model discussed in Section 4.3.2 using two slightly different data sets: we use both monthly and annual growth in industrial production and monthly and annual inflation. There are two reasons to consider both alternatives. Firstly, twelve-month growth is widely used by previous studies, but monthly rates of output growth and inflation change more than annual growth rates, so these could capture different patterns of the monthly comovements of global macroeconomic factors and global yield curve factors.[5] Secondly, as expected, the Augmented Dickey–Fuller test indicates that annual growth of industrial production and inflation are highly persistent, while monthly growth are not. Figures 1 and 2 show that monthly industrial production growth and monthly inflation change more than annual industrial production growth and annual inflation.

[5] Among others, the studies of Ang and Piazzesi (2003); Diebold, Rudebusch and Aruoba (2006); and Moench (2008) use annual growth.

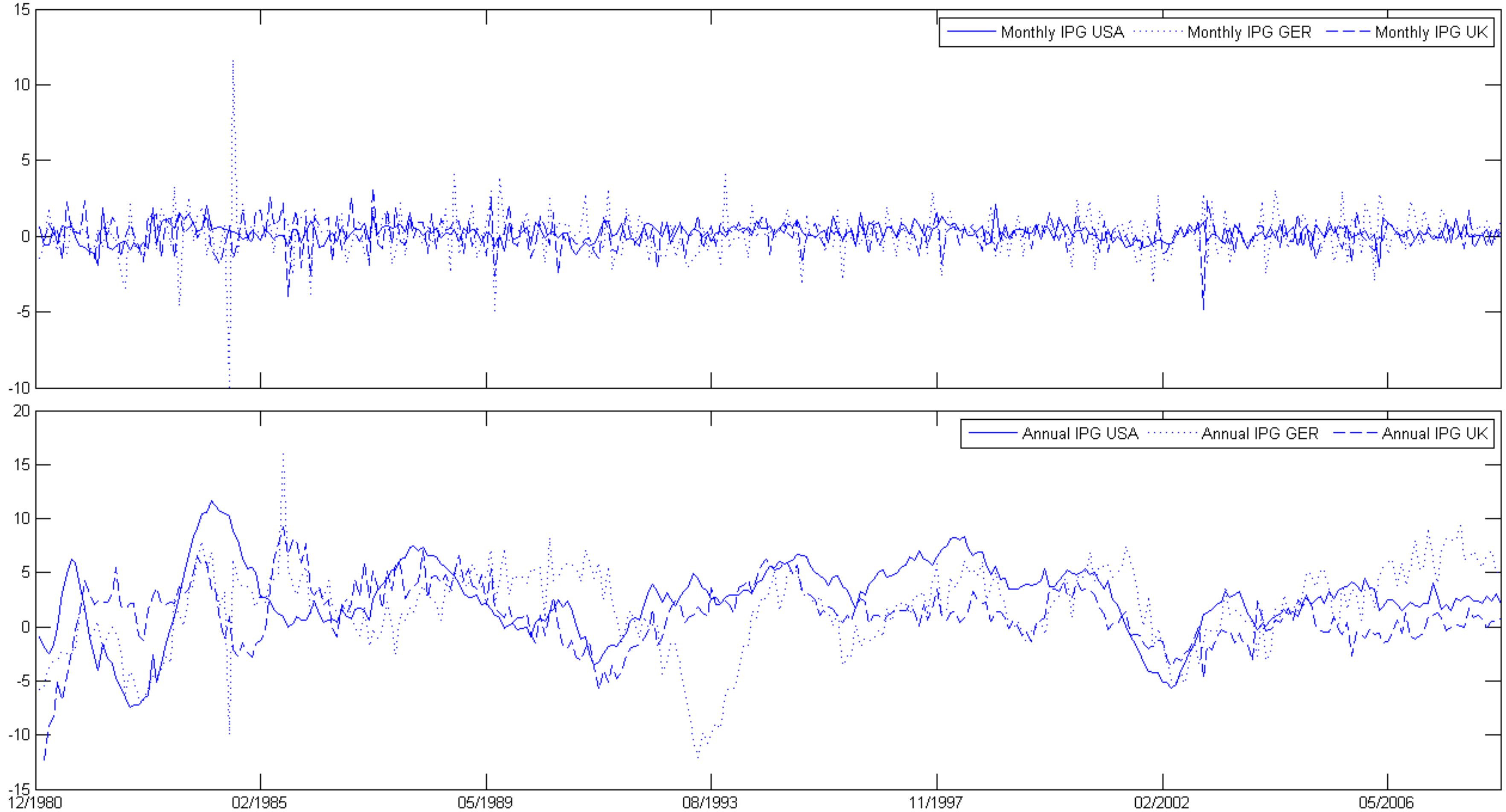

Figure 1: Annual and monthly industrial production growth spanning the period 1980:12-2008:05. The first plot shows the monthly industrial production growth for the USA, Germany and the UK. The second plot shows annual industrial production growth for the USA, Germany and the UK.

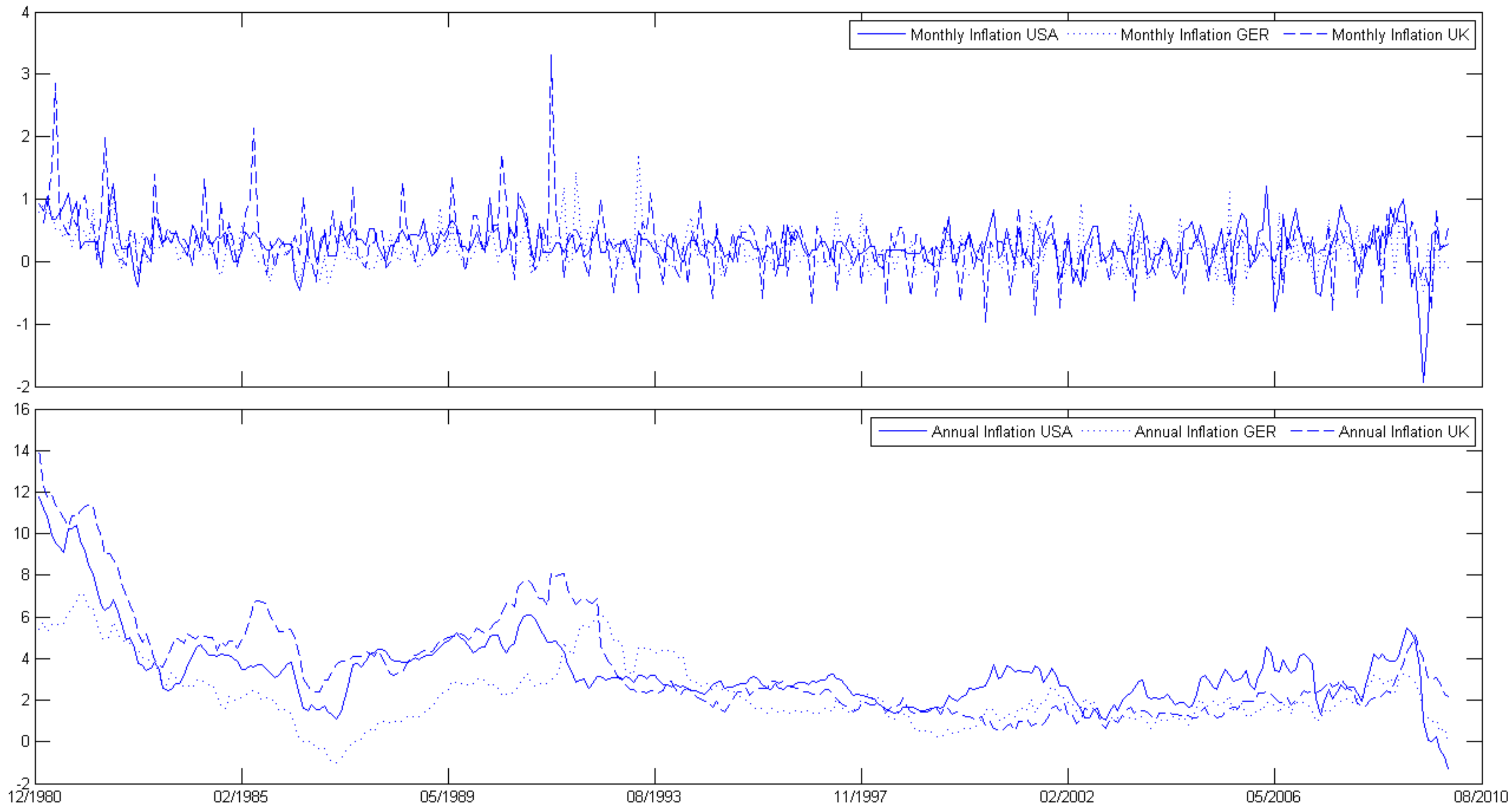

Figure 2: Annual and monthly inflation spanning the period 1980:12-2008:05. The first plot shows the monthly inflation for the USA, Germany and the UK. The second plot shows annual inflation for the USA, Germany and the UK.

Figure 3 shows the macroeconomics variables (industrial production growth, inflation and monetary policy rates) for the USA, Germany and the UK and global macroeconomic factors using twelve-month growth. The global macroeconomic factors are estimated using the joint model of global macroeconomic and yield curve factors. Figure 1 indicates the similarity in the patterns of the macroeconomic variables for different countries and shows that these patterns are followed closely by the global macroeconomic factors (global industrial production growth, global inflation and global monetary policy rates).

Figure 4 shows NS factors reparameterized by Diebold and Li (2006) as level, slope and curvature, for the USA, Germany and the UK and global level, global slope and global curvature. The global NS factors are estimated using the joint model of global macroeconomic and yield curve factors. Figure 2 indicates the similarity in the patterns of the NS factors for different countries and shows that these patterns are followed closely by the global NS factors.

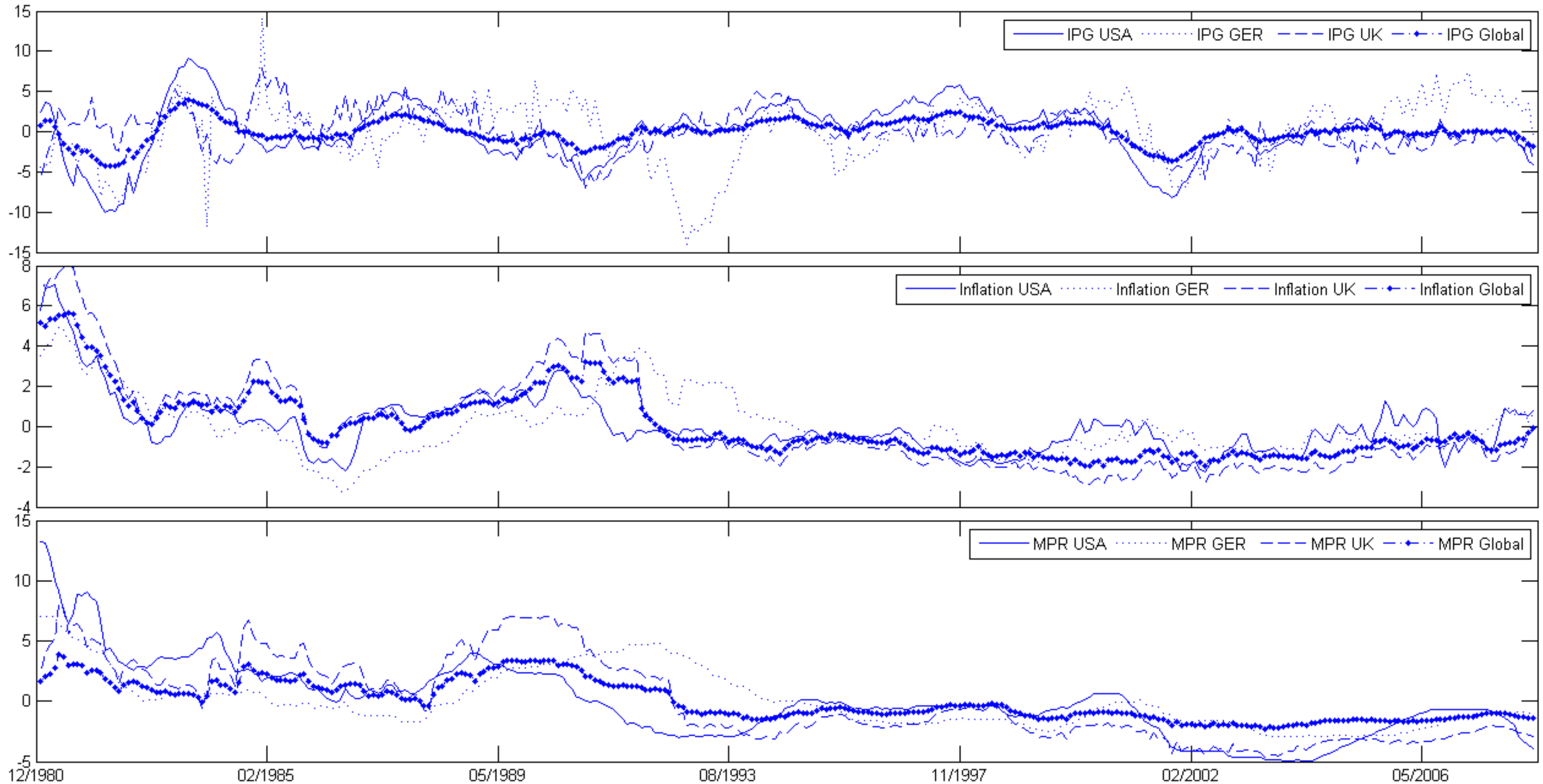

Figure 3: Macroeconomics variables and global macroeconomic factors computed using the joint model of global macroeconomic and yield curve factors for twelve-month growth spanning the period 1980:12-2008:05. The first plot shows the demeaned industrial production growth for the USA, Germany and the UK along with global industrial production growth factor. The second plot shows demeaned inflation for the USA, Germany and the UK along with global inflation factor. The third plot shows demeaned monetary policy rate along with global monetary policy rate factor.

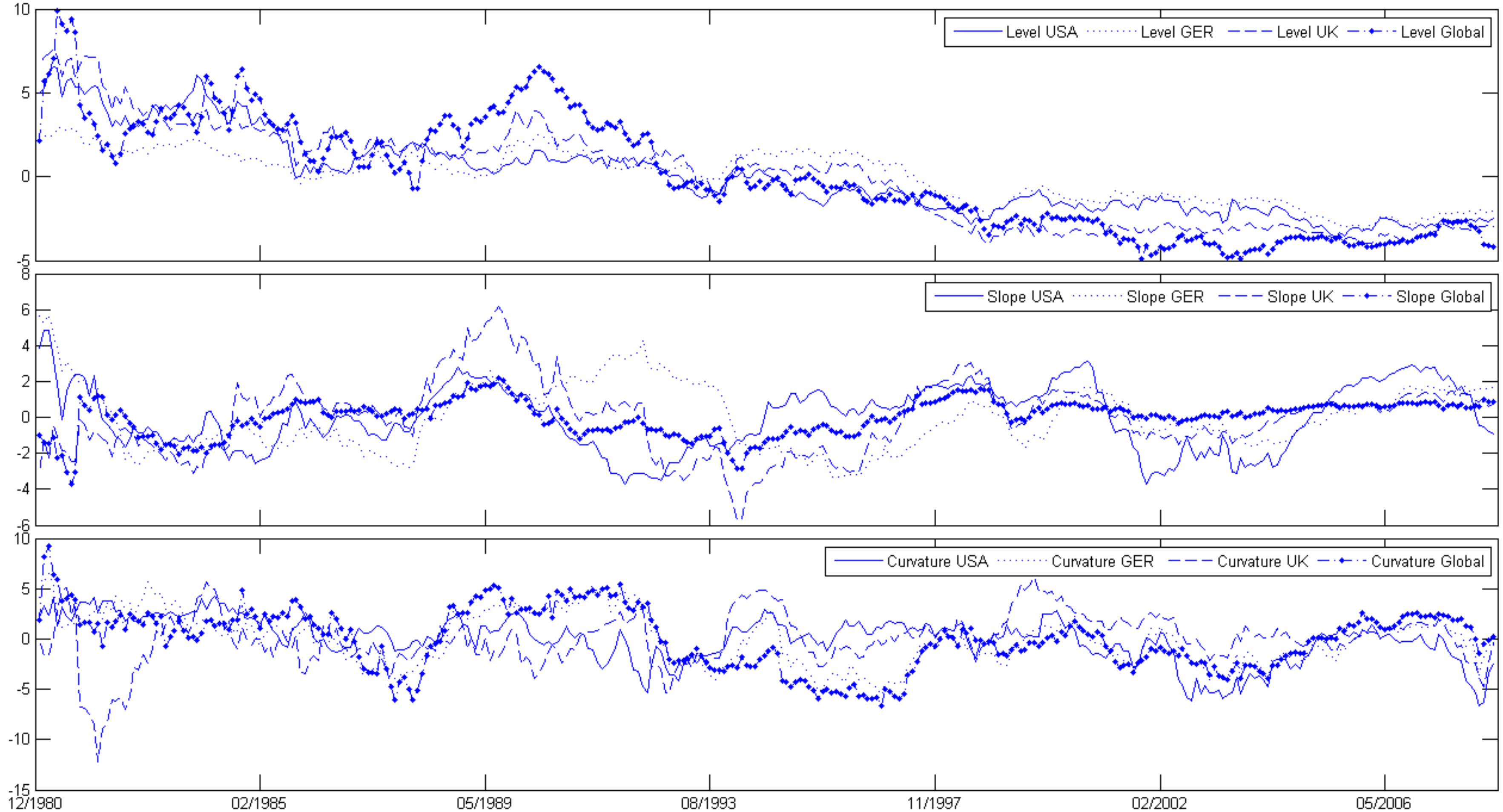

Figure 4: Nelson and Siegel (1987) factors reparameterized by Diebold and Li (2006) as level, slope and curvature, for the USA, Germany and the UK along with global level, global slope and global curvature spanning the period 1980:12-2008:05. The first plot shows the demeaned level for the USA, Germany and the UK along with the global level. The second plot shows demeaned slope for the USA, Germany and the UK along with the global slope factor. The third plot shows demeaned curvature along with the global curvature.

Tables IV and V report the results from estimating equations (17) and (18) using twelve-month growth. Table IV shows that parameters of the influence of global macroeconomic factors on global yield curve factors are relatively larger than the parameters of the influence of the global yield curve factors on global macroeconomic factors. In particular, all parameters of the influence of the macroeconomic factors on the global level factor are positive and the coefficients of global industrial production growth, inflation and monetary policy rates are 0.20, 0.11 and 0.77, respectively, indicating that any increase in these factors will lead to an increase in the global level of yields. The general level of rates should increase when the economy shows signs of overheating, i.e., the level of yields should go up when output and inflation are growing. Moreover, global industrial production growth and global inflation influence negatively on global slope factor (-0.03 and -0.04) and positively on the global curvature (0.06 and 0.21). This means that an increase in these factors will lead to a flattening between short and long-term yields, as well as a more pronounced hump of the yield curve, making the slope less steep (or steeper in the case of a decrease in the macroeconomic factors) and a more pronounced curvature. This interaction could be, to some extent, explained by the increase in inflation risk premium, i.e., an increase in expected inflation which increases the short and long-term rates but reduces the gap between short and long-term rates. The global monetary policy rate influences slope and curvature positively (0.12 and 0.67) which could be explained by tightening in monetary policy (an increase of monetary policy rates) which increases the general level of rates.

The negative parameters of the influence of global inflation and global slope on next period's industrial production growth suggest that an increase in global inflation and global slope causes global industrial production growth to fall in the next period. Global level and slope factors, along with the global monetary policy factor, load positively on the global monetary policy factor of the next period, indicating a strong relationship between global monetary policy rates and the global level factor.

Furthermore, global inflation, global industrial production growth and global monetary policy negatively influence the local levels.

Overall, these tables show a bidirectional interaction between global macroeconomic factors. The global factors are generally persistent (diagonal elements of Table V), for example parameters of the influence of global level, slope and curvature on one step ahead global level, slope and curvature are 0.95, 0.83 and 0.86, respectively. The influence of global macroeconomic factors on yield curve factors of the next period is larger than the influence of global yield curve factors on global macroeconomic factors of the next period, for example the parameters of the influence of global industrial production growth, global inflation and global monetary policy rates on global level is 0.20, 0.11 and 0.77 and the parameters of the influence of global level on global industrial production growth, global inflation and global monetary policy rates are 0.03, 0.09 and 0.42, respectively.

Table IV

Coefficient matrix of observation equation for the USA, Germany and the UK ($\Gamma^{(YX)}$)

	Panel A														
				USA			Germany			UK					
	Global			Local			Local			Local			Global		
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
$Y_{USA}^{(1)}$	0.59	0.24	0.07	1.00	0.71	0.23	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(2)}$	0.59	0.18	0.09	1.00	0.53	0.29	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(3)}$	0.59	0.14	0.09	1.00	0.41	0.29	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(4)}$	0.59	0.11	0.09	1.00	0.32	0.27	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(5)}$	0.59	0.09	0.08	1.00	0.27	0.24	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(6)}$	0.59	0.08	0.07	1.00	0.23	0.21	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(7)}$	0.59	0.07	0.06	1.00	0.19	0.19	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(8)}$	0.59	0.06	0.05	1.00	0.17	0.17	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(9)}$	0.59	0.05	0.05	1.00	0.15	0.15	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(10)}$	0.59	0.05	0.04	1.00	0.14	0.14	-	-	-	-	-	-	-	-	-

Table IV-*Continued*

	Panel B														
				USA			Germany			UK					
	Global			Local			Local			Local			Global		
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
$Y_{GER}^{(1)}$	0.58	0.17	0.18	-	-	-	1.00	0.71	0.23	-	-	-	-	-	-
$Y_{GER}^{(2)}$	0.58	0.13	0.23	-	-	-	1.00	0.53	0.29	-	-	-	-	-	-
$Y_{GER}^{(3)}$	0.58	0.10	0.23	-	-	-	1.00	0.41	0.29	-	-	-	-	-	-
$Y_{GER}^{(4)}$	0.58	0.08	0.21	-	-	-	1.00	0.32	0.27	-	-	-	-	-	-
$Y_{GER}^{(5)}$	0.58	0.06	0.19	-	-	-	1.00	0.27	0.24	-	-	-	-	-	-
$Y_{GER}^{(6)}$	0.58	0.05	0.17	-	-	-	1.00	0.23	0.21	-	-	-	-	-	-
$Y_{GER}^{(7)}$	0.58	0.05	0.15	-	-	-	1.00	0.19	0.19	-	-	-	-	-	-
$Y_{GER}^{(8)}$	0.58	0.04	0.13	-	-	-	1.00	0.17	0.17	-	-	-	-	-	-
$Y_{GER}^{(9)}$	0.58	0.04	0.12	-	-	-	1.00	0.15	0.15	-	-	-	-	-	-
$Y_{GER}^{(10)}$	0.58	0.03	0.11	-	-	-	1.00	0.14	0.14	-	-	-	-	-	-

Table IV-*Continued*

	Panel C														
				USA			Germany			UK					
	Global			Local			Local			Local			Global		
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
$Y_{UK}^{(1)}$	0.64	0.56	0.02	-	-	-	-	-	-	1.00	0.71	0.23	-	-	-
$Y_{UK}^{(2)}$	0.64	0.42	0.02	-	-	-	-	-	-	1.00	0.53	0.29	-	-	-
$Y_{UK}^{(3)}$	0.64	0.32	0.02	-	-	-	-	-	-	1.00	0.41	0.29	-	-	-
$Y_{UK}^{(4)}$	0.64	0.26	0.02	-	-	-	-	-	-	1.00	0.32	0.27	-	-	-
$Y_{UK}^{(5)}$	0.64	0.21	0.02	-	-	-	-	-	-	1.00	0.27	0.24	-	-	-
$Y_{UK}^{(6)}$	0.64	0.18	0.01	-	-	-	-	-	-	1.00	0.23	0.21	-	-	-
$Y_{UK}^{(7)}$	0.64	0.15	0.01	-	-	-	-	-	-	1.00	0.19	0.19	-	-	-
$Y_{UK}^{(8)}$	0.64	0.14	0.01	-	-	-	-	-	-	1.00	0.17	0.17	-	-	-
$Y_{UK}^{(9)}$	0.64	0.12	0.01	-	-	-	-	-	-	1.00	0.15	0.15	-	-	-
$Y_{UK}^{(10)}$	0.64	0.11	0.01	-	-	-	-	-	-	1.00	0.14	0.14	-	-	-

Table IV-*Continued*

	Panel D														
				USA			Germany			UK					
	Global			Local			Local			Local			Global		
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
Δ IPI $_{USA}$	-	-	-	-	-	-	-	-	-	-	-	-	0.66	-	-
Δ IPI $_{GER}$	-	-	-	-	-	-	-	-	-	-	-	-	0.49	-	-
Δ IPI $_{UK}$	-	-	-	-	-	-	-	-	-	-	-	-	0.57	-	-
Π_{USA}	-	-	-	-	-	-	-	-	-	-	-	-	-	0.59	-
Π_{GER}	-	-	-	-	-	-	-	-	-	-	-	-	-	0.54	-
Π_{UK}	-	-	-	-	-	-	-	-	-	-	-	-	-	0.60	-
MPR $_{USA}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.57
MPR $_{GER}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.56
MPR $_{UK}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.61

Note: This table shows twelve yield curve factors and three macroeconomic factors for twelve-month change using the joint model of global macroeconomic and yield curve factors. The first column shows the maturity of the yield and macroeconomic variables explained by the factors. The next three columns report the NS factors multiplied by respective factor loading of each country. The next nine columns show the NS factor loadings for the local yield curve factors and the last three columns report the global macroeconomic coefficients estimated for explaining the macroeconomic variables. The model does not allow for contemporary interactions between macroeconomic and yield curve factors. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) and macroeconomic variables provided by OECD and central banks, spanning the period 1980:12-2008:05.

Table V

Coefficient matrix of state equation for the USA, Germany and the UK (Φ^{YX})

					USA			Germany			UK					
		Global			Local			Local			Local			Global		
		Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
	Global Level	0.95	0.06	- 0.01	-	-	-	-	-	-	-	-	-	0.20	0.11	0.77
		**(0.08)	*(0.03)	(0.05)	-	-	-	-	-	-	-	-	-	**(0.01)	**(0.04)	**(0.04)
	Global Slope	- 0.02	0.83	0.02	-	-	-	-	-	-	-	-	-	- 0.03	- 0.04	0.12
		(0.09)	**(0.03)	(0.06)	-	-	-	-	-	-	-	-	-	**(0.01)	(0.04)	**(0.04)
	Global Curvature	0.03	0.14	0.86	-	-	-	-	-	-	-	-	-	0.06	0.21	0.67
		**(0.01)	**(0)	**(0.01)	-	-	-	-	-	-	-	-	-	**(0)	**(0.01)	**(0.01)
	Local Level	-	-	-	0.84	0.02	0.02	-	-	-	-	-	-	- 0.11	- 0.05	- 0.43
		-	-	-	**(0.02)	**(0)	(0.04)	-	-	-	-	-	-	**(0.01)	**(0.02)	**(0.02)
USA	Local Slope	-	-	-	- 0.00	0.87	0.04	-	-	-	-	-	-	0.03	0.03	- 0.10
		-	-	-	(0.02)	**(0.01)	(0.03)	-	-	-	-	-	-	**(0.01)	**(0.01)	**(0.01)
	Local Curvature	-	-	-	- 0.02	0.11	0.88	-	-	-	-	-	-	- 0.03	- 0.09	- 0.04
		-	-	-	(0.02)	**(0)	**(0.05)	-	-	-	-	-	-	(0.02)	**(0.02)	**(0.02)
	Local Level	-	-	-	-	-	-	0.69	0.08	0.15	-	-	-	- 0.11	- 0.07	- 0.45
		-	-	-	-	-	-	**(0.06)	**(0.02)	**(0.02)	-	-	-	**(0.02)	*(0.04)	**(0.04)
GER	Local Slope	-	-	-	-	-	-	- 0.00	0.92	0.01	-	-	-	0.03	0.03	- 0.10
		-	-	-	-	-	-	(0.08)	**(0.03)	(0.03)	-	-	-	(0.02)	(0.05)	*(0.05)
	Local Curvature	-	-	-	-	-	-	0.65	- 0.19	0.19	-	-	-	- 0.10	- 0.12	- 0.39
		-	-	-	-	-	-	**(0.03)	**(0.01)	**(0.01)	-	-	-	**(0.01)	**(0.02)	**(0.02)

Table V-*Continued*

					USA			Germany			UK					
		Global			Local			Local			Local			Global		
		Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
	Local Level	-	-	-	-	-	-	-	-	-	0.82	- 0.08	- 0.06	- 0.13	- 0.06	- 0.46
		-	-	-	-	-	-	-	-	-	**(0.01)	**(0.01)	**(0)	**(0.01)	**(0.01)	**(0.01)
UK	Local Slope	-	-	-	-	-	-	-	-	-	- 0.11	0.05	- 0.01	0.03	0.05	0.05
		-	-	-	-	-	-	-	-	-	**(0.01)	**(0.01)	(0.01)	**(0)	**(0.01)	**(0)
	Local Curvature	-	-	-	-	-	-	-	-	-	0.02	- 0.03	0.88	0.04	0.02	- 0.19
		-	-	-	-	-	-	-	-	-	**(0)	**(0)	**(0.01)	**(0)	**(0)	**(0)
	Δ IPI Global	0.03	- 0.07	0.03	-	-	-	-	-	-	-	-	-	0.92	- 0.04	- 0.00
		(0.02)	**(0.01)	**(0.01)	-	-	-	-	-	-	-	-	-	**(0.01)	**(0.01)	(0.01)
	Π Global	0.09	0.04	- 0.00	-	-	-	-	-	-	-	-	-	0.00	0.86	0.05
		*(0.05)	**(0.01)	(0.03)	-	-	-	-	-	-	-	-	-	(0.01)	**(0.02)	**(0.02)
	MPR Global	0.42	0.45	0.00	-	-	-	-	-	-	-	-	-	- 0.01	0.04	0.51
		**(0.12)	**(0.04)	(0.08)	-	-	-	-	-	-	-	-	-	(0.02)	(0.06)	**(0.06)

Note: This table shows twelve yield curve factors and three macroeconomic factors for twelve-month change using the joint model of global macroeconomic and yield curve factors. The first column shows the country and the second column indicates the factor explained by factors in the columns. The next fifteen columns report the explanatory global, local yield curve factors or the global macroeconomic factors. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) and macroeconomic variables provided by OECD and central banks, spanning the period 1980:12-2008:05. Asterisks denote significance at the *5% and **1% levels.

Tables VI and VII report the results from estimating equations (17) and (18) using monthly growth. The results in Table VII show that in general the factors are less persistent (smaller coefficients on diagonal elements of Table VII). Specifically, the global industrial production growth and global monetary policy factors load positively on global level factor, anticipating the positive response of global level to an increase in these factors. However, the influence of global monthly inflation on global level factor has the opposite sign when we use annual growth, which could be due to the fact that inflation risk premium and monetary policy are influenced by the cumulative inflation of the annual period. Also, the global monetary policy factor seems to capture the influence of global yield curve factors on global industrial production and global inflation factors.

Table VI

Coefficient matrix of observation equation for the USA, Germany and the UK ($\Gamma^{(YX)}$)

	Panel A														
				USA			Germany			UK					
	Global			Local			Local			Local			Global		
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
$Y_{USA}^{(1)}$	0.59	0.24	0.07	1.00	0.71	0.23	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(2)}$	0.59	0.18	0.09	1.00	0.53	0.29	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(3)}$	0.59	0.14	0.09	1.00	0.41	0.29	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(4)}$	0.59	0.11	0.09	1.00	0.32	0.27	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(5)}$	0.59	0.09	0.08	1.00	0.27	0.24	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(6)}$	0.59	0.08	0.07	1.00	0.23	0.21	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(7)}$	0.59	0.07	0.06	1.00	0.19	0.19	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(8)}$	0.59	0.06	0.05	1.00	0.17	0.17	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(9)}$	0.59	0.05	0.05	1.00	0.15	0.15	-	-	-	-	-	-	-	-	-
$Y_{USA}^{(10)}$	0.59	0.05	0.04	1.00	0.14	0.14	-	-	-	-	-	-	-	-	-

Table VI-*Continued*

	Panel B														
				USA			Germany			UK					
	Global			Local			Local			Local			Global		
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
$Y_{GER}^{(1)}$	0.58	0.17	0.18	-	-	-	1.00	0.71	0.23	-	-	-	-	-	-
$Y_{GER}^{(2)}$	0.58	0.13	0.23	-	-	-	1.00	0.53	0.29	-	-	-	-	-	-
$Y_{GER}^{(3)}$	0.58	0.10	0.23	-	-	-	1.00	0.41	0.29	-	-	-	-	-	-
$Y_{GER}^{(4)}$	0.58	0.08	0.21	-	-	-	1.00	0.32	0.27	-	-	-	-	-	-
$Y_{GER}^{(5)}$	0.58	0.06	0.19	-	-	-	1.00	0.27	0.24	-	-	-	-	-	-
$Y_{GER}^{(6)}$	0.58	0.05	0.17	-	-	-	1.00	0.23	0.21	-	-	-	-	-	-
$Y_{GER}^{(7)}$	0.58	0.05	0.15	-	-	-	1.00	0.19	0.19	-	-	-	-	-	-
$Y_{GER}^{(8)}$	0.58	0.04	0.13	-	-	-	1.00	0.17	0.17	-	-	-	-	-	-
$Y_{GER}^{(9)}$	0.58	0.04	0.12	-	-	-	1.00	0.15	0.15	-	-	-	-	-	-
$Y_{GER}^{(10)}$	0.58	0.03	0.11	-	-	-	1.00	0.14	0.14	-	-	-	-	-	-

Table VI-*Continued*

	Panel C														
				USA			Germany			UK					
		Global			Local			Local			Local			Global	
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
$Y_{UK}^{(1)}$	0.64	0.56	0.02	-	-	-	-	-	-	1.00	0.71	0.23	-	-	-
$Y_{UK}^{(2)}$	0.64	0.42	0.02	-	-	-	-	-	-	1.00	0.53	0.29	-	-	-
$Y_{UK}^{(3)}$	0.64	0.32	0.02	-	-	-	-	-	-	1.00	0.41	0.29	-	-	-
$Y_{UK}^{(4)}$	0.64	0.26	0.02	-	-	-	-	-	-	1.00	0.32	0.27	-	-	-
$Y_{UK}^{(5)}$	0.64	0.21	0.02	-	-	-	-	-	-	1.00	0.27	0.24	-	-	-
$Y_{UK}^{(6)}$	0.64	0.18	0.01	-	-	-	-	-	-	1.00	0.23	0.21	-	-	-
$Y_{UK}^{(7)}$	0.64	0.15	0.01	-	-	-	-	-	-	1.00	0.19	0.19	-	-	-
$Y_{UK}^{(8)}$	0.64	0.14	0.01	-	-	-	-	-	-	1.00	0.17	0.17	-	-	-
$Y_{UK}^{(9)}$	0.64	0.12	0.01	-	-	-	-	-	-	1.00	0.15	0.15	-	-	-
$Y_{UK}^{(10)}$	0.64	0.11	0.01	-	-	-	-	-	-	1.00	0.14	0.14	-	-	-

Table VI-*Continued*

Panel D															
				USA			Germany			UK					
	Global			Local			Local			Local			Global		
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
Δ IPI $_{USA}$	-	-	-	-	-	-	-	-	-	-	-	-	0.59	-	-
Δ IPI $_{GER}$	-	-	-	-	-	-	-	-	-	-	-	-	0.56	-	-
Δ IPI $_{UK}$	-	-	-	-	-	-	-	-	-	-	-	-	0.59	-	-
Π_{USA}	-	-	-	-	-	-	-	-	-	-	-	-	-	0.61	-
Π_{GER}	-	-	-	-	-	-	-	-	-	-	-	-	-	0.55	-
Π_{UK}	-	-	-	-	-	-	-	-	-	-	-	-	-	0.57	-
MPR $_{USA}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.57
MPR $_{GER}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.56
MPR $_{UK}$	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.61

Note: This table shows twelve yield curve factors and three macroeconomic factors for one-month change using the joint model of global macroeconomic and yield curve factors. The first column shows the maturity of the yield and macroeconomic variables explained by the factors. The next three columns report the NS factors multiplied by respective factor loading of each country. The next nine columns show the NS factor loadings for the local yield curve factors and the last three columns report the global macroeconomic coefficients estimated for explaining the macroeconomic variables. The model does not allow for contemporary interactions between macroeconomic and yield curve factors. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) and macroeconomic variables provided by OECD and central banks, spanning the period 1980:12-2008:05.

Table VII

Coefficient matrix of state equation for the USA, Germany and the UK (Φ^{YX})

					USA			Germany			UK					
		Global			Local			Local			Local			Global		
		Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
	Global Level	0.84	0.22	- 0.02	-	-	-	-	-	-	-	-	-	0.03	- 0.49	0.86
		**(0.02)	**(0.01)	(0.02)	-	-	-	-	-	-	-	-	-	**(0)	**(0)	**(0.01)
	Global Slope	- 0.01	0.77	0.03	-	-	-	-	-	-	-	-	-	- 0.02	0.09	- 0.13
		(0.02)	**(0.01)	(0.02)	-	-	-	-	-	-	-	-	-	**(0)	**(0)	**(0.01)
	Global Curvature	0.02	0.22	0.84	-	-	-	-	-	-	-	-	-	0.06	- 0.18	0.53
		**(0)	**(0)	**(0.01)	-	-	-	-	-	-	-	-	-	**(0)	**(0)	**(0)
	Local Level	-	-	-	0.47	- 0.03	0.20	-	-	-	-	-	-	- 0.02	0.31	- 0.51
		-	-	-	**(0.02)	**(0.01)	**(0.04)	-	-	-	-	-	-	**(0.01)	**(0.01)	**(0.02)
USA	Local Slope	-	-	-	0.00	0.91	0.03	-	-	-	-	-	-	- 0.01	- 0.01	0.01
		-	-	-	(0.01)	**(0.01)	**(0.01)	-	-	-	-	-	-	**(0)	**(0)	(0.01)
	Local Curvature	-	-	-	0.20	0.11	0.40	-	-	-	-	-	-	- 0.01	0.11	- 0.35
		-	-	-	**(0.02)	**(0.01)	**(0.04)	-	-	-	-	-	-	(0.01)	**(0.01)	**(0.03)
	Local Level	-	-	-	-	-	-	0.41	- 0.40	- 0.18	-	-	-	- 0.02	0.32	- 0.54
		-	-	-	-	-	-	**(0.01)	**(0.01)	**(0)	-	-	-	*(0.01)	**(0)	**(0.01)
GER	Local Slope	-	-	-	-	-	-	- 0.13	0.08	- 0.11	-	-	-	0.04	- 0.09	0.22
		-	-	-	-	-	-	**(0.02)	**(0.02)	**(0.01)	-	-	-	**(0)	**(0.02)	**(0.03)
	Local Curvature	-	-	-	-	-	-	- 0.03	- 0.30	0.82	-	-	-	- 0.08	0.06	- 0.26
		-	-	-	-	-	-	**(0)	**(0.01)	**(0.01)	-	-	-	**(0)	**(0.01)	**(0.01)

Table VII-*Continued*

					USA			Germany			UK					
		Global			Local			Local			Local			Global		
		Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π	MPR
	Local Level	-	-	-	-	-	-	-	-	-	0.58	0.07	0.04	- 0.02	0.31	- 0.50
		-	-	-	-	-	-	-	-	-	**(0.01)	**(0)	**(0)	**(0)	**(0)	**(0.01)
UK	Local Slope	-	-	-	-	-	-	-	-	-	0.03	0.86	0.04	0.01	- 0.03	- 0.02
		-	-	-	-	-	-	-	-	-	**(0)	**(0.01)	**(0)	**(0)	**(0)	**(0)
	Local Curvature	-	-	-	-	-	-	-	-	-	- 0.04	0.12	0.86	0.01	0.06	- 0.05
		-	-	-	-	-	-	-	-	-	**(0)	**(0)	**(0.01)	**(0)	**(0)	**(0)
	Δ IPI Global	0.37	0.22	0.01	-	-	-	-	-	-	-	-	-	- 0.22	0.11	- 0.53
		**(0.01)	**(0)	(0.01)	-	-	-	-	-	-	-	-	-	**(0.01)	**(0)	**(0)
	Π Global	0.45	0.34	0.03	-	-	-	-	-	-	-	-	-	- 0.03	0.48	- 0.39
		**(0.02)	**(0)	**(0.01)	-	-	-	-	-	-	-	-	-	**(0)	**(0.01)	**(0.01)
	MPR Global	0.45	0.44	0.01	-	-	-	-	-	-	-	-	-	0.00	0.00	0.55
		**(0.03)	**(0.01)	(0.03)	-	-	-	-	-	-	-	-	-	**(0)	(0.01)	**(0.02)

Note: This table shows twelve yield curve factors and three macroeconomic factors for one-month change using the joint model of global macroeconomic and yield curve factors. The first column shows the country and the second indicates the factor explained by factors in the columns. The next fifteen columns report the explanatory global, local yield curve factors or the global macroeconomic factors. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) and macroeconomic variables provided by OECD and central banks, spanning the period 1980:12-2008:05. Asterisks denote significance at the *5% and **1% levels.

We also estimate the model described by equations (17) and (18), using monthly industrial production growth and monthly inflation but excluding MPR, as the MPR seems to capture the influence of rates on global factors. The results are reported in Tables VIII and IX. The parameters of the factors present negative signs, i.e., global inflation and industrial production growth factors have negative influence on the global level factor. The global level factor impacts negatively on global industrial production growth factor by pushing up rates and subsequently the cost of capital. This impact is even larger for the slope due to the fact that an increase in the slope means an increase in short-terms yields, which reinforces the effect of higher rates and higher cost of capital, which could have negative impacts over the global growth industrial production factor. The global level, slope and curvature positively influence the global inflation.

Table VIII

Coefficient matrix of observation equation for the USA, Germany and the UK ($\Gamma^{(YX)}$)

	Panel A													
				USA			Germany			UK				
	Global			Local			Local			Local			Global	
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π
$Y_{USA}^{(1)}$	0.59	0.24	0.07	1.00	0.71	0.23	-	-	-	-	-	-	-	-
$Y_{USA}^{(2)}$	0.59	0.18	0.09	1.00	0.53	0.29	-	-	-	-	-	-	-	-
$Y_{USA}^{(3)}$	0.59	0.14	0.09	1.00	0.41	0.29	-	-	-	-	-	-	-	-
$Y_{USA}^{(4)}$	0.59	0.11	0.09	1.00	0.32	0.27	-	-	-	-	-	-	-	-
$Y_{USA}^{(5)}$	0.59	0.09	0.08	1.00	0.27	0.24	-	-	-	-	-	-	-	-
$Y_{USA}^{(6)}$	0.59	0.08	0.07	1.00	0.23	0.21	-	-	-	-	-	-	-	-
$Y_{USA}^{(7)}$	0.59	0.07	0.06	1.00	0.19	0.19	-	-	-	-	-	-	-	-
$Y_{USA}^{(8)}$	0.59	0.06	0.05	1.00	0.17	0.17	-	-	-	-	-	-	-	-
$Y_{USA}^{(9)}$	0.59	0.05	0.05	1.00	0.15	0.15	-	-	-	-	-	-	-	-
$Y_{USA}^{(10)}$	0.59	0.05	0.04	1.00	0.14	0.14	-	-	-	-	-	-	-	-

Table VIII-*Continued*

	Panel B													
				USA			Germany			UK				
	Global			Local			Local			Local			Global	
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π
$Y^{(1)}_{GER}$	0.58	0.17	0.18	-	-	-	1.00	0.71	0.23	-	-	-	-	-
$Y^{(2)}_{GER}$	0.58	0.13	0.23	-	-	-	1.00	0.53	0.29	-	-	-	-	-
$Y^{(3)}_{GER}$	0.58	0.10	0.23	-	-	-	1.00	0.41	0.29	-	-	-	-	-
$Y^{(4)}_{GER}$	0.58	0.08	0.21	-	-	-	1.00	0.32	0.27	-	-	-	-	-
$Y^{(5)}_{GER}$	0.58	0.06	0.19	-	-	-	1.00	0.27	0.24	-	-	-	-	-
$Y^{(6)}_{GER}$	0.58	0.05	0.17	-	-	-	1.00	0.23	0.21	-	-	-	-	-
$Y^{(7)}_{GER}$	0.58	0.05	0.15	-	-	-	1.00	0.19	0.19	-	-	-	-	-
$Y^{(8)}_{GER}$	0.58	0.04	0.13	-	-	-	1.00	0.17	0.17	-	-	-	-	-
$Y^{(9)}_{GER}$	0.58	0.04	0.12	-	-	-	1.00	0.15	0.15	-	-	-	-	-
$Y^{(10)}_{GER}$	0.58	0.03	0.11	-	-	-	1.00	0.14	0.14	-	-	-	-	-

Table VIII-*Continued*

	Panel C													
				USA			Germany			UK				
	Global			Local			Local			Local			Global	
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π
$Y_{UK}^{(1)}$	0.64	0.56	0.02	-	-	-	-	-	-	1.00	0.71	0.23	-	-
$Y_{UK}^{(2)}$	0.64	0.42	0.02	-	-	-	-	-	-	1.00	0.53	0.29	-	-
$Y_{UK}^{(3)}$	0.64	0.32	0.02	-	-	-	-	-	-	1.00	0.41	0.29	-	-
$Y_{UK}^{(4)}$	0.64	0.26	0.02	-	-	-	-	-	-	1.00	0.32	0.27	-	-
$Y_{UK}^{(5)}$	0.64	0.21	0.02	-	-	-	-	-	-	1.00	0.27	0.24	-	-
$Y_{UK}^{(6)}$	0.64	0.18	0.01	-	-	-	-	-	-	1.00	0.23	0.21	-	-
$Y_{UK}^{(7)}$	0.64	0.15	0.01	-	-	-	-	-	-	1.00	0.19	0.19	-	-
$Y_{UK}^{(8)}$	0.64	0.14	0.01	-	-	-	-	-	-	1.00	0.17	0.17	-	-
$Y_{UK}^{(9)}$	0.64	0.12	0.01	-	-	-	-	-	-	1.00	0.15	0.15	-	-
$Y_{UK}^{(10)}$	0.64	0.11	0.01	-	-	-	-	-	-	1.00	0.14	0.14	-	-

Table VIII-*Continued*

	Panel D													
					USA			Germany			UK			
		Global			Local			Local			Local		Global	
	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π
Δ IPI $_{USA}$	-	-	-	-	-	-	-	-	-	-	-	-	0.59	-
Δ IPI $_{GER}$	-	-	-	-	-	-	-	-	-	-	-	-	0.56	-
Δ IPI $_{UK}$	-	-	-	-	-	-	-	-	-	-	-	-	0.59	-
Π_{USA}	-	-	-	-	-	-	-	-	-	-	-	-	-	0.61
Π_{GER}	-	-	-	-	-	-	-	-	-	-	-	-	-	0.55
Π_{UK}	-	-	-	-	-	-	-	-	-	-	-	-	-	0.57

Note: This table shows twelve yield curve factors and two macroeconomic factors for one-month change using the joint model of global macroeconomic and yield curve factors. The first column shows the maturity of the yield and macroeconomic variables explained by the factors. The next three columns report the NS factors multiplied by respective factor loading of each country. The next nine columns show the NS factor loadings for the local yield curve factors. The last two columns report the global macroeconomic coefficients estimated for explaining the macroeconomic variables. The model does not allow for contemporary interactions between macroeconomic and yield curve factors. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) and macroeconomic variables provided by the OECD and central banks, spanning the period 1980:12-2008:05.

Table IX

Coefficient matrix of state equation for the USA, Germany and the UK (Φ^{YX})

					USA			Germany			UK				
		Global			Local			Local			Local			Global	
		Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π
	Global Level	0.97	0.17	- 0.06	-	-	-	-	-	-	-	-	-	- 0.01	- 0.05
		**(0.01)	**(0.01)	**(0.01)	-	-	-	-	-	-	-	-	-	**(0)	**(0)
	Global Slope	- 0.02	0.70	0.07	-	-	-	-	-	-	-	-	-	0.01	0.13
		*(0.01)	**(0.01)	**(0.01)	-	-	-	-	-	-	-	-	-	**(0)	**(0)
	Global Curvature	0.03	0.25	0.78	-	-	-	-	-	-	-	-	-	- 0.01	- 0.08
		**(0.01)	**(0.01)	**(0)	-	-	-	-	-	-	-	-	-	**(0)	**(0)
	Local Level	-	-	-	0.68	0.06	- 0.05	-	-	-	-	-	-	0.01	0.04
		-	-	-	**(0)	**(0)	**(0)	-	-	-	-	-	-	**(0)	**(0)
USA	Local Slope	-	-	-	0.09	0.83	0.07	-	-	-	-	-	-	- 0.00	- 0.03
		-	-	-	**(0)	**(0.01)	**(0)	-	-	-	-	-	-	**(0)	**(0)
	Local Curvature	-	-	-	- 0.11	0.16	0.80	-	-	-	-	-	-	0.03	0.03
		-	-	-	**(0)	**(0)	**(0.01)	-	-	-	-	-	-	**(0)	**(0)
	Local Level	-	-	-	-	-	-	0.88	0.12	- 0.12	-	-	-	0.01	0.04
		-	-	-	-	-	-	**(0.01)	**(0.01)	**(0.02)	-	-	-	**(0)	**(0.01)
GER	Local Slope	-	-	-	-	-	-	- 0.08	0.80	0.12	-	-	-	- 0.02	- 0.05
		-	-	-	-	-	-	**(0.01)	**(0.01)	**(0.02)	-	-	-	**(0)	**(0.01)
	Local Curvature	-	-	-	-	-	-	0.54	0.68	0.28	-	-	-	0.05	0.14
		-	-	-	-	-	-	**(0.01)	**(0.01)	**(0.02)	-	-	-	**(0)	**(0.01)

Table IX-*Continued*

					USA			Germany			UK				
		Global			Local			Local			Local			Global	
		Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Level	Slope	Curvature	Δ IPI	Π
	Local Level	-	-	-	-	-	-	-	-	-	- 0.12	- 0.32	- 0.05	0.01	0.04
		-	-	-	-	-	-	-	-	-	**(0.03)	**(0.02)	**(0)	(0.01)	**(0.02)
UK	Local Slope	-	-	-	-	-	-	-	-	-	- 0.06	0.36	- 0.05	0.02	- 0.12
		-	-	-	-	-	-	-	-	-	**(0.01)	**(0.02)	**(0.01)	**(0.01)	**(0.01)
	Local Curvature	-	-	-	-	-	-	-	-	-	- 0.21	- 0.11	0.86	- 0.07	0.05
		-	-	-	-	-	-	-	-	-	**(0.01)	**(0.01)	**(0.01)	**(0)	**(0)
	Δ IPI Global	- 0.06	- 0.29	0.03	-	-	-	-	-	-	-	-	-	- 0.20	- 0.02
		**(0.01)	**(0.01)	(0.02)	-	-	-	-	-	-	-	-	-	**(0.01)	**(0)
	Π Global	0.18	0.13	0.03	-	-	-	-	-	-	-	-	-	0.07	0.44
		**(0.02)	**(0.02)	(0.02)	-	-	-	-	-	-	-	-	-	**(0)	**(0)

Note: This table shows twelve yield curve factors and two macroeconomic factors for one-month change using the joint model of global macroeconomic and yield curve factors. The first column shows the country and the second indicates the factors at time t explained by factors at time $t-1$ in the columns. The next twelve columns report the explanatory factor either global or local and the following two columns show the global macroeconomic factors. The data consist of monthly nominal zero-coupon yields provided by Wright (2011) and macroeconomic variables provided by the OECD and central banks, spanning the period 1980:12-2008:05. Asterisks denote significance at the *5% and **1% levels.

Table X reports the results from estimating the proportion of the variance of annual industrial production growth, annual inflation and monetary policy rates explained by each global macroeconomic factor. The explained variance ranges between 0.718 and 0.999, which indicates that the majority of the variance of macroeconomics variables is captured by global macroeconomics factors.

Table X

Variance of macroeconomic variables explained by each global macroeconomic factor

	$\Delta_{\text{IPI}_{\text{USA}}}$	$\Delta_{\text{IPI}_{\text{GER}}}$	$\Delta_{\text{IPI}_{\text{UK}}}$	Π_{USA}	Π_{GER}	Π_{UK}	MPR_{USA}	MPR_{GER}	MPR_{UK}
$\Delta_{\text{IPI}_{\text{G}}}$	0.996	0.925	0.826	–	–	–	–	–	–
Π_{G}	–	–	–	0.835	0.718	0.997	–	–	–
MPR_{G}	–	–	–	–	–	–	0.954	0.904	0.999

Note: This table shows the variance of nine macroeconomic variables explained for three macroeconomic factors using the joint model of global macroeconomic and yield curve factors for the twelve-month change. The first column shows the global macroeconomic factors which explain the macroeconomic variables. The next nine columns report the variance of industrial production growth, inflation and monetary policy rates for the USA, Germany and the UK. The data consist of nominal monthly zero-coupon yields provided by Wright (2011) and macroeconomic variables provided by OECD and central banks, spanning the period 1980:12-2008:05.

The results show that the proposed model is useful in explaining the yields and macroeconomic variables as well as the relationships between yields and macroeconomic factors. The results indicate that the coefficient associated with the influence of global macroeconomic factors on global and local yield curve factors are larger than the coefficient associated with the influence of yield curve factors on global macroeconomic factors.

4.5. Conclusion

We propose and construct a joint model of global and local yield curve factors and global macroeconomic factors. The model shows that there is a bidirectional relationship between yield curve factors and macroeconomic factors. However, the sizes of the coefficients indicate that macroeconomic factors have stronger influences on yield curve factors than the reverse. In general, the variance of annual industrial production growth, annual inflation and monetary policy rates is well explained by global macroeconomic factors.

On the one hand, the influence of global macroeconomic factors on global yield curve factors indicates that global level of interest rate is positively influenced by past values of global inflation, global growth in industrial production and global monetary policy rate. This indicates that these macroeconomic factors affect the global level of interest rate mainly due to the future expected response of central banks to positive inflation and production gap. Specifically, global growth in industrial production, global inflation and global monetary policy rate influence positively the global level one period ahead. The coefficients are 0.20, 0.11 and 0.77, meaning that an increase of 1% in global inflation, global growth in industrial production or global monetary policy rates raises the global level of rates by at least 11 basis points. The influence of global inflation and global growth in industrial production on global slope is negative while their influence on global curvature is positive, which means a reduction in the gap between long and short-term rates as well as an increase in medium term rates.

In addition, global slope positively affects the global level one period ahead, which could be due to anticipation of future movements of global level. This could be to some extent explained by the influence of short-term rates on long-term rates.

On the other hand, influence of global yield curve factors on macroeconomic factors indicates a positive influence of global level and negative influence of global slope on global growth in industrial production one period ahead as well as positive influence of global level and global slope on global inflation and global monetary policy rate one period ahead.

In summary, the results suggest that a joint model of global and local yield curve and global macroeconomic factors is useful in highlighting and explaining the bidirectional relationships between yields and macroeconomic factors. Also, the proposed factor model provides evidence of the influence of the yield curve factors on macroeconomic factors and vice versa. However, the influence of global macroeconomic factors on global and local yield curve factors is stronger than the influence of global yield curve factors on global macroeconomic factors.

References

[1] Ang, A., Bekaert, G., and Wei M. The Term Structure of Real Rates and Expected Inflation *Journal of Finance 63*, 2 (April 2008), 797–849.

[2] Ang, A., and Piazzesi, M. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics 50*, 4 (May 2003), 745–787.

[3] Ang, A., Piazzesi, M., and Wei M., What does the Yield Curve Tell us about GDP Growth?. *Journal of Econometrics 131*, 1–2 (March-April 2006), 359–403.

[4] Bagliano, F. C., and Morana, C. International macroeconomic dynamics:A factor vector autoregressive approach, *Economic Modelling 26*, 2 (March 2009), 432–444.

[5] Bekaert, G., Cho, S., and Moreno, A. New-Keynesian Macroeconomics and the Term Structure. *Journal of Money, Credit and Banking 42,* 1 (February 2010), 33–62.

[6] Bredin, D., Hyde S., O'Reilly G. O. Monetary policy surprises and international bond markets. *Journal of International Money and Finance 29*, 6 (October 2010), 988–1002.

[7] Byrne, J., Fazio, G., and Fiess, N. Interest rate co-movements, global factors and the long end of the term spread. *Journal of Banking and Finance 36*, 1 (2012), 183–192.

[8] Canova, F., Ciccarelli, M., and Ortega, E. Similarities and convergence in G-7 cycles, *Journal of Monetary Economics 54*, 3 (April 2007), 850–878.

[9] Christiansen, C. Decomposing European bond and equity volatility. *International Journal of Finance & Economics 15*, 2 (2010), 105–122.

[10] Coroneo, L., Giannone, D., and Modugno, M. Forecasting the term structure of interest rates using a large panel of macroeconomic data. Mimeo (2008).

[11] Coroneo, L., Giannone, D., and Modugno, M. Unspanned macroeconomic factors in the yield curve. Ecares (2013).

[12] Crucini, M., Kose, M. A., Otrok, C. What are the driving forces of international business cycles? *Review of Economic Dynamics 14*, 1 (January 2011), 156–175.

[13] Diebold, F. X., and Li, C. Forecasting the term structure of government bond yields. *Journal of Econometrics 130*, 2 (February 2006), 337–364.

[14] Diebold, F. X., Li, C., and Yue, V. Z. Global yield curve dynamics and interactions: a dynamic nelson-siegel approach. *Journal of Econometrics 146*, 2 (October 2008), 351–363.

[15] Diebold, F., Rudebusch, G., and Aruoba, B. The Macroeconomy and the Yield Curve:

A Dynamic Latent Factor Approach. *Journal of Econometrics*, 131 (2006), 309–338.

[16] Doz, C., Giannone, D., and Reichlin, L. A quasi maximum likelihood approach for large approximate dynamic factor models. *Working Paper Series, European Central Bank*, 674 (September 2006).

[17] Doz, C., Giannone, D., and Reichlin, L. A quasimaximum likelihood approach for large, approximate dynamic factor models. *The Review of Economics and Statistics, 94*, 4 (2012), 1014–1024.

[18] Driessen, J., Melenberg, B., and Nijman, T. Common factors in international bond returns, *Journal of International Money and Finance 22*, 5 (October 2003), 629–656.

[19] Estrella, A., Hardouvelis, G. The term structure as a predictor of real economic activity. *Journal of Finance 46,* 2 (1991), 555–576.

[20] Estrella, A., Mishkin, F. Predicting U.S. recessions: Financial variables as leading indicators. *Review of Economics and Statistics 80,* 1 (February 1998), 45–61.

[21] Evans, C.L., and Marshall D. Monetary policy and the term structure of nominal interest rates: evidence and theory. *Carnegie-Rochester Conference Series on Public Policy 49*, (1998) 53–111.

[22] Evans, C.L., and Marshall D. Economic determinants of the nominal Treasury yield curve. *Journal of Monetary Economics 54*, 7 (October 2007), 1986–2003.

[23] Gomez-Biscarri, J. Changes in the informational content of term spreads: Is monetary policy becoming less effective? *Journal of Economics and Business 60*, 5 (September–October 2008) 415–435.

[24] Hördahl, P., Tristani, O., Vestin D. A joint econometric model of macroeconomic and term-structure dynamics. *Journal of Econometrics 131*, 1–2 (February 2006), 405–444.

[25] Hördahl, P., Tristani, O., Vestin D. The Yield Curve and Macroeconomic Dynamics. *Economic Journal, Royal Economic Society 118*, 533 (November 2008), 1937–1970.

[26] Kaminska, I., Meldrum, A., and Smith, J. A Global Model of International Yield Curves: No-Arbitrage Term Structure Approach. *International Journal of Finance & Economics* (2013).

[27] Kose, M. A., Otrok, C., and Prasad E. Global Business Cycles: Convergence Or Decoupling?, *International Economic Review 53*, 2 (May 2012), 511–538.

[28] Kose, M. A., Otrok, C., and Whiteman, C. Understanding the evolution of world business cycles. *Journal of International Economics 75*, 1 (May 2008), 110–130.

[29] Lubik, T. A., and Schorfheide F. Do central banks respond to exchange rate movements? A structural investigation. *Journal of Monetary Economics 54*, (2007), 1069–1087.

[30] Ludvigson, S. C., and Ng, S. Macro factors in bond risk premia. *Review of Financial Studies 22*, 12 (2009), 5027–5067.

[31] Moench E. Forecasting the yield curve in a data-rich environment: a no-arbitrage factor-augmented VAR approach. *Journal of Econometrics 146*, (2008), 26–43.

[32] Moench, E. Term structure surprises: the predictive content of curvature, level, and slope. *Journal of Applied Econometrics 27,* 4 (2012), 574–602.

[33] Mumtaz, H., Simonelli, S., Surico, P. International comovements, business cycle and inflation: A historical perspective, *Review of Economic Dynamics 14*, 1, (January 2011), 176–198.

[34] Nelson, C. R., and Siegel, A. F. Parsimonious modeling of yield curves. *Journal of Business 60*, 4 (October 1987), 473–489.

[35] Rudebusch, G. Macro-Finance Models Of Interest Rates And The Economy. *Manchester School, University of Manchester 78,* s1 (September 2010), 25–52.

[36] Rudebusch, G., Wu, T. A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy *Economic Journal, Royal Economic Society 118*, 530 (July 2008), 906–926.

[37] Sousa, J. and Zaghini, A. Monetary Policy Shocks in the Euro Area and Global Liquidity Spillovers. *ECB working paper series 309* (2004).

[38] Taylor, J. B. International Monetary Coordination and the Great Deviation. *Forthcoming in the Journal of Policy Modeling (2013).*

[39] Wright, J. H. Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset. *American Economic Review 101*, 4 (June 2011), 1514–1534.

[40] Wu, T. Monetary policy and the slope factors in empirical term structure estimations. *Federal Reserve Bank of San Francisco Working paper 02–07* (2002).

[41] Wu, T. Globalization's effect on interest rates and the yield curve. *Federal Reserve Bank of Dallas Economic Letter 1*, 9 (September 2006a), 1–8.

[42] Wu, T. Macro factors and the affine term structure of interest rates. *Journal of Money, Credit, and Banking 38, 7* (October 2006b), 1847–1875.

Chapter 5

Conclusion

5.1. Conclusion

This thesis presents three essays that empirically investigate aspects of the yield curve and the link between the yield curve and macroeconomic factors.

We begin in Chapter 2 by proposing and estimating a factor model that decomposes the yield curve in to global and local factors. Using data on yields for the USA, Germany and the UK we examine whether the USA, UK and German yields contain common components, which we label global factors, and if so to what extent these global factors explain the yield curve. We examine the importance of global factors by means of variance decomposition, and examine the effect of shocks on factors using impulse response functions. The majority of variance of yields is explained by global yield curve factors (level, slope and curvature) as well as shocks to global factors lasting longer than the shock to local factors. In particular, the variance of yields explained by global yield curve factors is on average 55% and in turn the global level factor explains 40%. Also, effects of shocks on local and global factors disappear no later than 42 and 72 months, respectively.

Using the model in Chapter 2, Chapter 3 investigates whether the global yield curve factor estimated in Chapter 2 help in forecasting one-period-ahead excess bond returns for three countries (the USA, Germany and the UK) and compare explanatory power of the model with two competing models: the model proposed by Cochrane and Piazzesi (2005) and the model developed by Dahlquist and Hasseltoft (2011). We estimate the model using a rolling window with a horizon of 15 years and produce in sample forecasts of one year excess bond returns. We show that global and local yield curve factor model present higher R^2 than competing models for explaining one year excess bond returns. Also, the model shows lower mean absolute errors and root mean squared errors than both competing models for all the countries and maturities. The global yield curve factors explain over 43% and up to 58% of variance of excess bond return forecast errors. The global level explains no less than 14% and up to 43% of this variance. These results are qualitatively similar when we use a rolling window with a horizon of 20 years.

Chapter 4 investigates the bidirectional relationship between global and local yield curve factors and global macroeconomics factors. We estimate a joint model of yields and macroeconomics variables for three countries (the USA, Germany and the UK). We provide evidence of influence of global yield curve factor on global macroeconomics factors (industrial production growth, inflation and monetary policy rate) and vice versa. In particular, in the case of annual industrial production growth and inflation the coefficients of the influence of global macroeconomic factors on yield curve factors are larger than the coefficient of the influence of global yield curve factors on global macroeconomic factors, which indicates a stronger influence of macroeconomic factors on yield curve factors than the reverse. Specifically, global growth in industrial production, global inflation and global monetary policy rate show a positive influence over the global level one period ahead, which indicates that an increase in these factors can lead to an increase in global level which could be explained by the expected response of monetary policy to the increasing in global inflation and industrial production growth and the direct influence of monetary policy rates.

Our findings have important implications for policymakers and practitioners since shocks to global factors have larger and longer-lasting effects than shocks to local factors, which means that global factors should be considered in policy and financial decisions. In particular, the influence of global factor could counteract attempts of policymakers to influence the yield curve of the country and financial decisions that do not consider the influence of global factors take the risk that adverse movements in global factors affect the investments. Also, our model outperforms the two competing models by predicting one year excess bond returns and indicating that global factors play an important role in determining bond risk premia. It also provides evidence against the Expectation Hypothesis, which states that long-term rates are equal to the average of future expected short-term rates. Furthermore, the study of linkages between global macroeconomic factors and global and local yield curve factors highlights the importance of major global macroeconomics factors which influence the global yield curve factors.

Future research can examine other aspects of international linkage among yield curve factors. Also, research can extend the number of countries considered in order to explore the differences between developing and developed countries. Further research can also extend the number of macroeconomics variables considered in order to obtain additional global macroeconomic factors.