

Essays on Banking Regulation, Macroeconomic Dynamics and Financial Volatility

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Roy Zilberman
School of Social Sciences
Economics

Contents

Table of Contents	2
List of Tables	5
List of Figures	6
Abstract	7
Declaration	8
Copyright Statement	8
Acknowledgement	9
Introduction	10
1 Supply Shocks and the Cyclical Behaviour of Bank Lending Rates under the Basel Accords	20
1.1 Introduction	20
1.2 The Model	23
1.2.1 Firms	23
1.2.2 Households	25
1.2.3 Commercial Bank	26
1.2.4 Central Bank	31
1.2.5 Market Clearing Conditions	31
1.3 Model Solution under Non-Binding Capital Requirements	32
1.3.1 Financial Market Equilibrium	32
1.3.2 Goods Market Equilibrium	40
1.3.3 General Equilibrium - The Bank Capital Channel Dominating The Collateral Channel	41
1.3.4 General Equilibrium - The Collateral Channel Dominating The Bank Capital Channel	48
1.4 Concluding Remarks	51
1.A Appendix	54
1.A.1 <i>The Effect of Prices on the Loan Rate under Basel I</i>	54
1.A.2 <i>The Effect of Supply Shocks on the Loan Rate under Basel I</i>	55

1.A.3	<i>The Effect of Prices on the Loan Rate under Basel II</i>	56
1.A.4	<i>The Effect Supply Shocks on the Loan Rate under Basel II</i>	58
2	Bank Capital Regulation, Credit Frictions and Macroeconomic Dynamics with Endogenous Default	59
2.1	Introduction	59
2.2	The Model	63
2.2.1	Households	64
2.2.2	Final Good Firm	69
2.2.3	Intermediate Good Firms	70
2.2.4	The Commercial Bank	73
2.2.5	The Transmission Channels of Bank Capital and Risk on the Loan Rate	78
2.2.6	Monetary Policy	79
2.2.7	Market Clearing Conditions	79
2.3	Steady State	80
2.4	The Log-Linearized Model	82
2.5	Calibration	84
2.6	Simulations	86
2.6.1	Supply Shock	86
2.6.2	Monetary Shock	89
2.6.3	Financial Shock	90
2.6.4	The Role of Adjustment Costs and the Risk Weight Channel	93
2.6.5	Permanent Increase in Bank Capital Requirements	93
2.7	Concluding Remarks	96
2.A	Appendix	98
2.A.1	<i>Wage Decision</i>	98
2.A.2	<i>Log-Linearized System</i>	107
3	Loan Loss Provisioning Rules, Procyclicality and Financial Volatility	114
3.1	Introduction	114
3.2	The Model	117
3.2.1	Households	117
3.2.2	Final Good Firm	121
3.2.3	Intermediate Good Firms	121
3.2.4	Capital Good Producer	123
3.2.5	Commercial Bank	125

3.2.6	Central Bank	131
3.2.7	Government	132
3.2.8	Market Clearing Conditions	132
3.3	Steady State	133
3.4	The Log-Linearized Model	136
3.5	Parameterization	139
3.6	Simulations	141
3.6.1	A Shock to Nonperforming Loans with Monitoring Incentive	141
3.6.2	A Shock to Nonperforming Loans with Moral Hazard	144
3.6.3	Collateral Shock	147
3.6.4	Provisions and Collateral Values	148
3.7	Concluding Remarks	149
3.A	Appendix	151
3.A.1	<i>Derivation of the Rental Rate of Capital</i>	151
3.A.2	<i>Derivation of the Loan Rate and Deposit Rate</i>	153
	Summary and Conclusions	156
	Bibliography	160

Final Word Count: 40,000

List of Tables

1.1	Response of the Loan Rate to an Increase in Prices and a Positive Supply Shock under Alternative Regulatory Regimes	39
2.1	Benchmark Parameterization: Parameter Values	85
3.1	Benchmark Parameterization: Parameter Values	139
3.2	Changes in Standard Deviations of Key Variables under Backward and Forward Provisioning Systems with Monitoring Incentive	144
3.3	Changes in Standard Deviations of Key Variables under Backward and Forward Provisioning Systems with Moral Hazard	146

List of Figures

1.1	Macroeconomic Equilibrium	43
1.2	Negative Supply Shock - Case I	45
1.3	Negative Supply Shock - Case II	49
2.1	Negative Supply Shock - Basel I vs. Basel II	87
2.2	Tightening Monetary Policy Shock - Basel I vs. Basel II	89
2.3	Adverse Financial Shock - Basel I vs. Basel II	91
2.4	Adverse Financial Shock with Higher Regulatory Requirements . . .	95
3.1	Shock to Fraction of Nonperforming Loans with Monitoring Incentive	141
3.2	Shock to Fraction of Nonperforming Loans with Moral Hazard	145

Abstract

The recent global financial crisis of 2007-2009 and the subsequent recession have prompted renewed interest into how banking regulation and fluctuations in the financial sector impact the business cycle. Using three different model setups, this thesis promotes a further understanding and identification of the various transmission channels through which regulatory changes and volatility in the financial system link to the real economy.

Chapter 1 examines the effects of bank capital requirements in a simple macroeconomic model with credit market frictions. A bank capital channel is introduced through a monitoring incentive effect of bank capital buffers on the repayment probability, which affects the loan rate behaviour via the risk premium. We also identify a collateral channel, which mitigates moral hazard behaviour by firms, and therefore raises their repayment probability. Basel I and Basel II regulatory regimes are then defined, with a distinction made between the Standardized and Foundation Internal Ratings Based (IRB) approaches of Basel II. We analyze the role of the bank capital and collateral channels in the transmission of supply shocks, and show that depending on the strength of these channels, the loan rate can either amplify or mitigate the effects of productivity shocks. Finally, the impact of the two channels also determines which of the regulatory regimes is most procyclical.

Chapter 2 studies the interactions between bank capital regulation and the real business cycle in a Dynamic Stochastic General Equilibrium (DSGE) framework with financial frictions, along with endogenous risk of default at the firm and bank capital levels. We show that in a model which accounts for bank capital risk and regulatory requirements, the endogenous risk of default produces an accelerator effect and impacts the loan rate and the real economy through multiple channels. Furthermore, the simulations illustrate that a risk sensitive regulatory regime (Basel II) amplifies the response of macroeconomic and financial variables following supply, monetary and financial shocks, with the strength of the key transmission channels depending on the nature of the shock. The impact of higher regulatory requirements (as proposed under Basel III) is also examined and is shown to increase procyclicality in the financial system and real economy.

Chapter 3 studies the interactions between loan loss provisions and business cycle fluctuations in a Dynamic Stochastic General Equilibrium (DSGE) model with credit market imperfections. With a backward-looking provisioning system, provisions are triggered by past due payments (or nonperforming loans), which, in turn, depend on current economic conditions and the loan loss reserves-loan ratio. With a forward-looking system, both past due payments and expected losses over the whole business cycle are accounted for, and provisions are smoothed over the cycle. Numerical experiments based on a parameterized version of the model show that holding more provisions can reduce the procyclicality of the financial system. However, a forward-looking provisioning regime can increase or lower procyclicality, depending on whether holding more loan loss reserves translates into a higher or lower fraction of nonperforming loans.

Declaration

I declare that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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Introduction

The global financial crisis of 2007-2009, triggered by the collapse of the U.S. subprime mortgage market and followed by the worst economic recession since the post war (now officially referred to as the Great Recession), has emphasized the importance of developing macroeconomic models which study the interactions between the financial system and real business cycles. In the aftermath of the crisis, it is now clear that restrictions in lending, higher borrowing costs and financial regulation, all which directly impact the financial system, have resulted in distortions in consumption and investment decisions, both which contribute largely to real sector and output dynamics. As a consequence, a growing number of research papers and policy debates on the role of financial intermediation, credit risk and banking regulation in the transmission of various shocks to the real economy have emerged in the past few years.

Prior to the recent financial turmoil, there have been very few attempts to model banking and study the effects of banking behaviour, financial regulation and banks balance sheets in a general equilibrium framework. In fact, until 2008 most models abstracted from an explicit banking sector but rather focused more generally on the intermediation process of banking, in addition to credit market imperfections arising from two main sources: collateral constraints (Kiyotaki and Moore 1997 and Iacoviello 2005); and agency costs (Carlstrom and Fuerst 1997 and Bernanke, Gertler and Gilchrist 1999). In the first strand of literature, agents are divided between borrowers and lenders, while a financial sector intermediates between these two groups by requiring borrowers to pledge collateral for their loans. Therefore, the volume of lending is directly affected by these collateral constraints. In the second strand of literature, loan monitoring is costly, which in turn induces a wedge between the cost of external finance and the risk free rate (referred to as the external finance premium). Hence, the monitoring costs and the credit market frictions affect the price of loans and result in a financial accelerator effect. Despite the importance of this earlier literature in providing the basic framework for many models which followed, the role of the banking sector is limited to the intermediary process between borrowers and lenders. In other words, banks do not directly lend to borrowers, and the supply side of the credit market or balance sheet effects do not have a profound impact on the agents' behaviour.

The vast majority of the literature since 2008 has therefore diverted the attention to further understanding the role of the banking sector and the explicit balance sheet effects in order to illustrate how changes in the supply side of the

financial system affect the real economy. Gertler and Kiyotaki (2010), by employing an agency problem between borrowers and lenders (as in Bernanke, Gertler and Gilchrist 1999), find that the external finance premium is further amplified with the inclusion of a more explicit banking sector, leading to enhanced borrowing and an exacerbated response of the real business cycle. Cúrdia and Woodford (2010), in a simplified small-scale Dynamic Stochastic General Equilibrium (DSGE) model with a financial sector, show that variations in credit spreads can have meaningful macroeconomic effects. Building a standard medium-scale DSGE model with various types of credit market imperfections, along with liquidity constraints in the banking sector, Christiano, Motto and Rostagno (2010) find that financial factors are crucial determinants in economic fluctuations in the U.S. and the Euro Area. Furthermore, Christiano, Motto and Rostagno (2013) show that shocks originating in the financial sector and specifically fluctuations in risk are the most important drivers of the business cycle. Overall, all the abovementioned models illustrate that changes in the financial system can exacerbate real sector dynamics behaviour, thus leading to increased procyclicality of both financial and real variables following supply, demand and financial shocks.¹

The role of bank capital in the propagation of various shocks and its impact on the real economy has also been subject to increased scrutiny in the past few years. One strand of research emphasizes the importance of bank capital in mitigating asymmetric information problems between banks and their creditors (see Aikman and Paustian 2006 and Meh and Moran 2010). Bank capital in these models arises endogenously due to credit market imperfections, and helps banks to attract loanable funds from depositors, who require banks to fund investment projects through their own net worth (bank capital). Given that net worth is mainly predetermined, then to satisfy the market determined capital requirements, lending and therefore investments must decrease, which in turn has substantial effects on the real economic activity. Hence, bank capital can link between the conditions in the financial system and the business cycle, as well as amplify the impact of various types of shocks.

Bank capital requirements can also arise due to legal enforcement imposed by financial regulators. Since 1988, international banking regulation has been associated with the Basel regulatory standards (Basel Accords) which have raised many concerns regarding whether such type of regulation can increase the degree of procyclicality already inherent in the banking system. As argued by Rosengran (2008), the importance of banks during financial crises is crucial because they are highly

¹In the literature of macrofinance, procyclicality refers to aspects of financial and/or economic policies which can amplify credit and real sector fluctuations, and not necessarily to a positive correlation between two variables.

leveraged and regulated institutions. Therefore, to maintain minimum capital requirements, banks must either raise new capital by selling existing assets or engage in credit rationing. Both these measures can further exacerbate a financial and economic crisis, associated with higher borrowing costs and lower investments and output.

According to the Basel Accords, banks must maintain at least 8% bank capital out of their total risk weighted assets (loans). In the first Basel Accord (Basel I), the risk weight on loans applied equally to all loans in the same particular category and therefore the risk associated with a particular borrower could not be detected by the banks nor the regulators. This, in turn, led banks to engage in regulatory capital arbitrage that undermined the effectiveness of Basel I (Jones 2000).

In 2004, the Basel Committee on Banking Supervision released the current Basel Accord (Basel II) in order to address the main shortcomings of Basel I (Basel Committee on Banking Supervision (BCBS) 2004). The main difference between Basel I and Basel II is that under the latter regime the risk weights on loans and thus bank capital requirements are more sensitive to the banks risk exposure and the evaluation of risk over the business cycle. More specifically, the risk weight on loans under Basel II depends on either the probability of default (The Foundation Internal Rating Based (IRB) approach) or on ratings provided by external rating agencies (The Standardized approach).² These ratings tend to be procyclical and consequently the Standardized approach is often linked with the nature of the business cycle or the output gap in macroeconomic models (see for example Zicchino 2006 and Angeloni and Faia 2013).

It has frequently been argued that Basel II type regulation or more volatile risk weights may aggravate a credit crunch, thereby exacerbating an economic recession and making capital requirements more procyclical (see for example Kashyap and Stein 2004). To illustrate, in the face of an economic recession and a downgrading by the credit risk models in use, banks may have to undertake continuous writedowns, thereby forcing them to raise more bank capital in order to maintain the adequacy requirements. Given that raising bank capital during a recession is costly, an increase in required bank capital may therefore lead to a more intensified decline in lending, resulting in a further amplification of the worsening economic conditions.

Most theoretical models also support this argument, some to a greater extent

²In fact, the Foundation IRB approach of Basel II is a subcategory of the IRB approach. Banks can choose within this approach either the *foundation* format or the *advanced* format. For the purpose of this thesis we consider only the *foundation* approach, where banks calculate their risk weight using the estimated probability of default. For details on the advanced format see Basel Committee on Banking Supervision (2004).

than others. Angelini, Enria, Neri, Panetta and Quagliariello (2011), who employ the Gerali, Neri, Sessa and Signoretti (2010) framework to examine the procyclical nature of Basel II regulation, find that the amplification effect caused by more risk sensitive bank capital requirements is relatively modest. In addition, Covas and Fujita (2010) assume that raising funds through bank capital is more costly than raising it through deposits, with the issuance costs of bank capital increasing during recessions. In their model, Basel II is only slightly more procyclical than Basel I and the differences are mainly observed around the business cycles peaks and troughs.

Other models find that Basel II considerably amplifies the response of the real business cycle. Aguiar and Drumond (2009) and Liu and Seeiso (2012) build upon the Bernanke, Gertler and Gilchrist (1999) framework, and illustrate that the liquidity premium demanded by households to hold bank capital under Basel II further exacerbates the external finance premium channel, which contributes to additional procyclicality. Moreover, in a recent contribution, Angeloni and Faia (2013) study the interactions between bank capital regulation and monetary policy, and show that risk based capital regulation (Basel II) aggravates the cycle and is also welfare detrimental.

In contrast to the abovementioned papers, Agénor and Pereira da Silva (2012) and Agénor, Alper and Pereira da Silva (2012) demonstrate within two different general equilibrium setups, that Basel II may actually be less procyclical than Basel I. These models show that different types of bank capital channels have important implications in terms of explaining the linkages between financial and real variables.

Specifically, in a simple static macroeconomic setup, Agénor and Pereira da Silva (2012) introduce a bank capital channel through a signaling effect of bank capital buffers on the deposit rate. Bank capital is assumed to facilitate deposit collection from households (similar in spirit to Meh and Moran 2010), thereby lowering the deposit rate and increasing households' consumption. Both Basel I and Basel II in their model magnify the procyclical effects of the lending rate following supply shocks, with Basel II inducing less procyclicality compared to Basel I under non-binding capital requirements.

Furthermore, in a DSGE model, Agénor, Alper and Pereira da Silva (2012) introduce a monitoring incentive effect in which total bank capital relative to loans increases incentives for banks to monitor borrowers, thereby reducing the likelihood of default. In their framework, Basel I may also be more procyclical than Basel II following various shocks.

The purpose of Chapter 1 of this thesis is to introduce the monitoring incentive effect of bank capital buffers on the repayment probability (which directly impacts

the behaviour of the loan rate through the risk premium) in a simple macroeconomic framework related to the analysis of Agénor and Pereira da Silva (2012).³ The first chapter is therefore complementary to their paper, exploring how bank capital buffers are transmitted through their direct effect on the financial system rather than their immediate impact on consumption and the real economy. The hypothesis of such a bank capital channel is supported by recent evidence which suggests that banks holding capital above the regulatory minimum requirements charge lower interest spreads on their loans (Fonseca, Gonzalez and Pereira da Silva 2010).

Because total bank capital is fixed given the static nature of the model presented in Chapter 1, the bank capital channel is incorporated in the form of bank capital *buffers* rather than *total* bank capital relative to outstanding loans (which is the case in Agénor, Alper and Pereira da Silva 2012). We also identify a collateral channel, which mitigates moral hazard behaviour by firms, and therefore raises their repayment probability. Therefore, this channel also directly affects the behaviour of the lending rate. The role of both the bank capital and collateral channels in the transmission of supply shocks, as well as the linkages between the financial system and real economy, are studied under the different variants of the Basel accords, with a distinction made between the Foundation IRB and the Standardized approaches of Basel II.

The results of Chapter 1 suggest that depending on the strength of the bank capital and collateral channels, the behaviour of the loan rate can either amplify or mitigate the effect of supply shocks. Finally, the relative impact of these two channels compared to one another determines the degree of procyclicality of the financial system, as well as the procyclicality ranking between the regulatory regimes.

Chapter 2 of this thesis further evaluates the effects of bank capital, regulatory requirements and the financial-real sectors linkages, but from a very different perspective. The model presented in the second chapter is a DSGE framework, with an endogenous formation of risk at both the firm and bank capital levels. The endogeneity of the risk of default at both these levels produces important various linkages between regulatory bank capital requirements, the credit market and the real economy.

The model assumes a representative competitive commercial bank, which collects deposits from households and issues bank capital to satisfy regulatory requirements. Both bank capital and deposits are used to fund loans to firms, which in turn must borrow from the commercial bank to finance the wage bill paid to households. Hence,

³Chapter 1 is a slightly revised, thesis adjusted version of Zilberman (2012), listed as a discussion paper in the Centre for Growth and Business Cycle Research, University of Manchester.

a borrowing cost channel is assumed in which the loan rate affects directly the firms marginal costs and therefore the rate of price inflation. Such type of short term borrowing costs have been utilized and empirically examined in the literature since the contribution of Ravenna and Walsh (2006). These papers include Chowdhury, Hoffmann and Schabert (2006) and Tillmann (2008), among others.⁴

The loan rate behaviour plays an important role in determining the relationship between the credit market and the real business cycle through the borrowing cost channel. Following Agénor, Bratsiotis and Pfajfar (2013), the bank breaks even in each period such that the expected income from lending to firms is equal to the total cost of funding. However, compared to their model, in Chapter 2 risky bank capital is added as an additional source of funding, which must comply with regulatory requirements. The loan rate decision is therefore based on the expected costs of paying back gross interest on deposits and bank capital to households (who own the bank), and the idiosyncratic nature of the borrowers.

As the production of output is subject to idiosyncratic productivity shocks, there is a positive probability of firms defaulting on their loans. The firms must therefore pledge a fraction of output as collateral, which is seized by the commercial bank in case of default (as in Agénor, Bratsiotis and Pfajfar 2013). When setting the loan rate, the bank therefore charges a risk premium (which is a function of risk) over the cost of borrowing from households.

Nevertheless, the probability of default affects the loan rate through two additional channels: The first, referred to as the bank capital default channel, stems from the introduction of a risk of default on bank capital, related to the estimated aggregate default risk at the firm level. The probability of default on bank capital creates an endogenous spread between the rate of return on bank capital and the interest rate on deposits. As bank capital is subject to risk, households demand a higher return for holding this asset such that a no-arbitrage condition between bank capital and deposits prevails. Hence, the model contributes to other models in this literature which include bank capital costs, but abstract from deriving an endogenous wedge between the cost of bank capital and the cost of deposits (see Markovic 2006, Aguiar and Drumond 2009 and Covas and Fujita 2010).

Second, the probability of default also affects the lending rate through the risk weight channel, resulting from the positive relationship between the risk weight on loans (or the bank capital-loan ratio) and the risk of default. This channel is evident

⁴At the same time, other papers, including Rabanal (2007) and Kaufmann and Scharler (2009) find limited evidence of the cost channel transmission mechanism. The issue of the cost channel is therefore still subject to debate, but for the purpose of the second chapter of the thesis, this channel is employed to explain part of the linkages between the financial side and the real economy.

in Foundation IRB approach of Basel II and is shown to amplify the response of the cost of borrowing.

Overall, the results suggest that the endogeneity of the risk of default at the firm and bank capital level produces an accelerator mechanism in the model, and impacts the loan rate through multiple channels. Furthermore, because the loan rate links to the real economy through the borrowing cost channel, all the channels associated with the changes in the probability of default (as mentioned above) also affect real sector dynamics. Hence, risk and bank capital in this model contribute to the standard borrowing cost channel described in Ravenna and Walsh (2006). Finally, the model is simulated following supply, demand and financial shocks, with financial shocks inducing the greatest degree of procyclicality in the financial system.

If indeed credit market frictions, a financial sector and banking regulation result in increased financial and macroeconomic volatility, as most models and evidence suggest, are there tools which can moderate these procyclicality effects? The recent crisis experience has led to a substantial shift in the policy debate, which now not only focuses on the banks' individual solvency captured by bank adequacy requirements, but also on macroprudential tools aiming to prevent and cope with the build-up of financial imbalances, resulting in severe macroeconomic consequences.

The Basel III regulatory standards, scheduled to be fully implemented by 2019, is motivated by increasing the quality of bank assets and raising capital requirements, as well as enforcing macroprudential or countercyclical instruments. The aim of the latter is to enhance financial stability, encourage more restricted lending in economic booms, mitigate systemic risk and allow the financial sector to better absorb losses associated with an eruption of a negative credit cycle.

Among these countercyclical instruments, bank capital buffers have attracted the most interest in policy circles and academic research. Under the Basel III regime for instance, central banks can now impose a countercyclical bank capital buffer ranging from 0 to 2.5 percent of risk-weighted assets; the buffer itself is related to excess growth in credit to the private sector or the loan to GDP ratio, both viewed as good indicators of systemic risk (see Basel Committee on Banking Supervision 2011).

At the same time, the recent financial crisis has renewed calls for central banks to consider more explicitly financial stability objectives in the conduct of monetary policy. That is, during an economic expansion, when credit spreads are low and lending is high, central banks can raise the refinance rate, thus increasing the cost of borrowing and restricting credit growth.

In academic research, a number of recent contributions have studied the per-

formance of countercyclical bank capital rules along with the conduct of a 'lean against the credit cycle' type of monetary policy. These contributions include Angelini, Neri and Panetta (2011), Angeloni and Faia (2013) and Agénor, Alper and Pereira da Silva (2013), among others. These authors find that a combination of a credit-augmented Taylor rule together with a Basel III-type countercyclical rule, may be optimal in achieving macroeconomic and financial stability.

Less attention in the theoretical literature, however, has been given to the use of loan loss provisions as a macroprudential tool despite its importance and impact on the financial system. In practice, loan loss provisions can be classified into two main categories: *a)* specific provisions, which depend on expected losses on loans which have been identified as impaired or nonperforming, that is, if they have not been repaid a certain number of days (usually 90 days) past the due date; and *b)* general provisions, which depend on expected losses on loans which are not necessarily impaired but are likely to be in the future. Specific provisions are governed by International Accounting Standards (IAS) 39, which require domestic banks to adopt an *incurred loss method* of loan loss provisioning.

This implies that specific provisions are set only once a loan loss has been identified, for which a specific documentation can be produced. As a result, general provisions often represent only a small fraction of total provisions. Banks may therefore find it difficult to increase provisions in an economic expansion, even if they correctly judge that borrowers are more likely to default. A possible consequence is that banks may reduce lending in recessions, thereby magnifying the impact of adverse shocks (Beatty and Liao 2011).

The question therefore is: should loan loss provisions take into account expected losses to improve banking sector stability and mitigate procyclicality as well as systemic risk? Indeed, the Basel Committee continues to work with the International Accounting Standards Board (IASB) on the expected loss approach to loan loss provisioning. The view is that if forward-looking provisions can take into account more credit information and anticipate and quantify better the expected losses associated with a loan portfolio, they would provide additional buffers and better incentives to mitigate procyclicality. This is the fundamental idea of *forward looking (statistical or dynamic)* provisioning rules, which have been used for some time in Spain. The Spanish provisioning system requires higher provisions when credit grows more than the historical average, thus linking provisioning to the credit and business cycle. Studies that have attempted to evaluate the performance of Spain's dynamic loan provisioning system include Saurina (2009), Caprio (2010), and Jiménez, Ongena, Peydró and Saurina (2012); all concluded that although the provisioning scheme

allowed banks to enter the downswing associated with the global financial crisis in more robust shape than they would have been otherwise, it is less clear that it had any material effect on the credit cycle or that it helped in any significant way to contain Spain's real estate bubble over the previous decade.

Quite surprisingly, there have been no attempts, as far as we know, to model the use of different types of loan loss provisioning systems in a DSGE framework. One analytical contribution is Bouvatier and Lepetit (2012), who study the impact of loan loss provisioning rules in a partial equilibrium setup. These authors find that a forward-looking provisioning rule performs better than a backward-looking system (incurred loss method) in terms of mitigating procyclicality of the financial system.

However, the framework presented in Bouvatier and Lepetit (2012) is not a full general equilibrium model, and a current issue yet to be studied is the interaction between loan loss provisions, the financial system and the real business cycle. For this purpose, Chapter 3 of this thesis integrates elements from the DSGE framework developed in Agénor, Alper and Pereira da Silva (2013) and the Bouvatier and Lepetit (2012) model to address the effectiveness of various types of loan loss provisioning rules in mitigating financial and macroeconomic volatility.⁵

The distinction in loan loss provisioning rules is made between backward- and forward-looking provisioning systems. In the former, provisions are triggered by past due payments (or the fraction of nonperforming loans), which, in turn, depend on current economic conditions and the loan loss reserves-loan ratio. Forward-looking (statistical) provisioning, by contrast, take into account both past due payments and expected losses over the whole business cycle; provisions are thus smoothed over the cycle and are less affected by the current state of the economy and past due payments.

The solution of the model shows that loan loss provisions affect the financial system and the real economy through various transmission channels. First, loan loss reserves (determined directly by loan loss provisions) can either raise or lower the fraction of non-performing loans in the model, consequently affecting the degree of cyclicity of the loan rate. This channel is referred to as the risk premium channel. Second, a provisioning cost channel is identified, in which loan loss provisions directly result in changes in the lending rate. Finally, a general equilibrium channel of loan loss provisions, arising from the money market equilibrium, affects the bond rate (or the opportunity cost of holding cash), which therefore impacts consumption and real sector dynamics.

⁵Chapter 3 is a slightly revised, thesis adjusted version of Agénor and Zilberman (2013), listed as a discussion paper in the Centre for Growth and Business Cycle Research, University of Manchester.

Numerical experiments, associated with different types of financial shocks, illustrate that holding more loan loss provisions can reduce financial sector procyclicality by inducing a mitigating effect on the loan rate and the fraction of nonperforming loans. In addition, a forward-looking loan loss provisioning system can increase or lower procyclicality, depending on whether holding more loan loss reserves (provisions) translates into a higher or lower fraction of nonperforming loans. These results have useful implications for the ongoing debate on the performance of loan loss provisioning systems and more generally macroprudential rules.

Chapter 1

Supply Shocks and the Cyclical Behaviour of Bank Lending Rates under the Basel Accords

1.1 Introduction

Banking regulation in form of bank capital requirements has gained further attention following the global financial crisis of 2007-2009, triggered by the collapse of the U.S. subprime mortgage market and fuelled by complex financial innovations that have made it difficult for market operators to assess risk. The standards of banking regulation, associated with the Basel Accords, state that banks must meet risk based capital requirements such that the ratio of total bank capital to risk adjusted assets is at least 8%.

Since the adoption of the first Basel accord (Basel I) in 1988, many concerns have been raised concerning the possible procyclical effects caused by such type of banking regulation.¹ For example, during an economic recession accompanied with credit losses incurred by financial intermediaries, the bank capital-loan ratio falls, which forces banks to raise new capital or decrease lending to firms. Assuming that raising capital is very costly during economic downturns, bank capital requirements may therefore lead to a credit crunch and to a further exacerbation of a financial or economic crisis.²

In 2004, the Basel Committee on Banking Supervision released the current Basel Accord (Basel II) in order to address the main shortcomings of Basel I (Basel Com-

¹In the literature of banking regulation, procyclicality refers to aspects of financial and/or economic policies which can amplify credit and real sector fluctuations, and not necessarily to a positive correlation between two variables.

²On the credit crunch that may have occurred in the U.S. in the early 1990's following the implementation of Basel I see Bernanke and Lown (1991) and Peek and Rosengran (1995) for instance.

mittee on Banking Supervision (BCBS) 2004). Most importantly, in Basel I, the risk weight on loans applied equally to all loans in the same particular category and therefore the risk associated with a particular borrower could not be detected by the banks nor the regulators. This, in turn, led banks to engage in regulatory capital arbitrage that undermined the effectiveness of Basel I (Jones 2000). The main difference between Basel I and Basel II is that under the latter regime the risk weights on loans are endogenous, and depend on either the probability of default (The Foundation Internal Rating Based (IRB) approach) or on ratings provided by external rating agencies (The Standardized approach). These ratings tend to be procyclical and consequently the Standardized approach is often linked with the nature of the business cycle or the output gap in macroeconomic models (see for example Zicchino 2006).³

The introduction of increased sensitive risk weights on loans, which may change throughout the business cycle, has led to a broader debate on the procyclical effects of bank capital regulation. In Basel II, the amount of bank capital held by the bank not only depends on the institutional nature of the borrowers but also on the risk imposed by each particular borrower. Moreover, if lending becomes riskier following a negative supply shock for example, banks may be required to hold more capital- or, failing that, to reduce their lending capacity in order to satisfy the more risk sensitive capital requirements. Hence, the increased volatility of the risk weights on loans during an economic recession may result in a more intensified credit crunch, thereby amplifying the economic downturn and making capital requirements more procyclical. The possible increased procyclical effects of the Basel II accord is supported by the models of Aguiar and Drumond (2009), Zicchino (2006) and Tanaka (2002), to name a few.

However, Agénor and Pereira da Silva (2012) and Agénor, Alper and Pereira da Silva (2012) argue that much of the literature examining the effects of Basel II compared to Basel I is based on the analysis of industrialized countries and not middle-income countries, which face more extreme financial market imperfections. These include severe asymmetric information problems fostering collateralized lending, underdeveloped capital markets, limited competition among banks, greater vulnerability to shocks, weak supervision and a limited ability to enforce bank capital regulation (see Agénor and Pereira da Silva 2012 for more details). Both of these models demonstrate that within a general equilibrium framework and accounting for some of the abovementioned credit market imperfections, Basel II may actually

³The risk weighting scheme remains essentially the same under the new proposed regulatory framework - Basel III.

be less procyclical than Basel I.

Specifically, by extending the static framework of Agénor and Montiel (2008), Agénor and Pereira da Silva (2012) introduce a bank capital channel through a signaling effect of bank capital buffers on the deposit rate, which, in turn, impacts directly households' consumption. This analysis demonstrates that such type of bank capital channel has sizable effects on the real economy. More precisely, with a signalling effect of bank capital buffers on the deposit rate, both Basel I and Basel II magnify the procyclical effects of the risk premium following supply shocks, with Basel II inducing less procyclicality compared to Basel I under non-binding capital requirements (the more relevant case in practice, as discussed later).

Furthermore, Agénor, Alper and Pereira da Silva (2012) employ a Dynamic Stochastic General Equilibrium (DSGE) model, and show that it cannot be ascertained a-priori whether Basel II is always more procyclical than Basel I. The bank capital channel in their model assumes that holding more bank capital relative to loans induces banks to screen and monitor borrowers more carefully, which raises the repayment probability and consequently lowers the lending rate through the risk premium. Embedding the bank capital channel through the impact of the bank capital-loan ratio together with the collateral-debt ratio (which mitigates moral hazard behaviour by the borrowers) on the repayment probability, their analysis shows that Basel I may be more procyclical than Basel II following various shocks.

The goal of this chapter is to examine the cyclical effects of Basel I, the Standardized approach of Basel II and the Foundation IRB approach of Basel II in a simple static macroeconomic model, which is related to the analysis of Agénor and Pereira da Silva (2012). However, instead of embedding the bank capital channel through the impact of bank capital buffers on the deposit rate, we employ this channel by positively relating bank capital buffers to the repayment probability and consequently the loan rate. Our model is therefore complementary to their paper, exploring how bank capital buffers are transmitted through their direct impact on the financial system rather than their immediate effect on the deposit rate, consumption and the real economy. The hypothesis of such a bank capital channel is supported by recent evidence which suggests that banks holding bank capital buffers charge lower interest spreads on their loans (Fonseca, Gonzalez and Pereira da Silva 2010).

Because total bank capital is fixed given the short run nature of our model, the bank capital channel is incorporated in the form of bank capital *buffers* rather than *total* bank capital relative to outstanding loans (which is the case in Agénor, Alper and Pereira da Silva 2012). Moreover, the role of this type of bank capital channel

in the transmission of supply shocks is studied under both Basel I and Basel II, with a distinction made between the Foundation IRB and the Standardized approaches. We also analyze the link between bank capital requirements, firms collateral, the repayment probability and the cyclical behaviour of the loan rate. Finally, this model compares between the cyclical effects on the loan rate caused by a negative supply shock under Basel I and the different variants of Basel II.

The rest of the chapter continues as follows. Section 1.2 presents the model, with a detailed examination of the agents behaviour, the different bank capital regulatory regimes, and the market clearing conditions. Section 1.3 provides the solution of the model under non binding capital requirements, and studies the impact of a negative supply shock on the macroeconomic equilibrium and the degree of cyclicity of the loan rate. Finally, section 1.4 summarizes the main results and offers some possible extensions of the analysis.

1.2 The Model

This model follows the static framework proposed by Agénor and Pereira da Silva (2012), but with the incorporation of a bank capital channel similar in spirit to Agénor, Alper and Pereira da Silva (2012). The economy consists of four types of agents: firms, households, a commercial bank and a central bank (which also acts as a regulator) and we now turn to describe their behaviour.

1.2.1 Firms

Firms produce a single, homogenous good using beginning of period capital (which is therefore predetermined) and labour. Firms borrow from the commercial bank in order to finance both their working capital needs, consisting only of labour costs, and investment. Thus, the total costs of firms in producing output comes from paying wages and interest on loans given for employing labour. Financing working capital needs is fully collateralized by the firms' capital stock and thus bears no risk.⁴ Consequently, such loans are provided by the commercial bank at a fixed mark up (normalized to unity) on the cost of borrowing from the central bank, denoted by the refinance rate i^R . In contrast, loans provided for investment financing do carry risk and are priced at a loan rate i^L , set as a mark up over the refinance rate (as shown later in the text).

⁴Loans contracted for working capital needs are short-term in nature, and can be easily monitored by the bank as they are readily observable *ex post* (Besanko and Thakor 1987).

After the homogeneous goods are produced and sold, the firms repay their loans to the commercial bank, with interest, so the loans are single period debt contracts. Finally, the end of period profits are transferred to households, who act as the firms owners.

The firm's total demand for loans (L^F) is given by,

$$L^F = WN + PI, \quad (1.1)$$

where W denotes the nominal wage for employing labour, N the amount of labour employed, P the price of the homogeneous good and I the level of real investment.

The real investment (I) is inversely related to the lending rate (i^L) charged by the commercial bank,

$$I = I(i^L - \pi^e), \quad (1.2)$$

where π^e is the expected (exogenous) rate of inflation and $\frac{dI}{di^L} < 0$.

The production function is assumed to take a typical Cobb-Douglas form,

$$Y = AN^\alpha K_0^{1-\alpha}, \quad (1.3)$$

where $A > 0$ is a shift technology parameter, and K_0 is the predetermined stock of physical capital. The total costs faced by firms when borrowing for working capital needs from the commercial bank are given by WN . Thus, the firm's maximization problem can be written as,

$$\max_N [PY - (1 + i^R)WN - (1 + i^L)PI],$$

subject to,

$$Y = AN^\alpha K_0^{1-\alpha}.$$

Deriving the first order condition with respect to N (the only choice variable) and taking i^R, P, W, i^L and I as given yields the labour demand function,⁵

$$N^d = \left[\frac{\alpha A}{(1 + i^R)(W/P)} \right]^{\frac{1}{1-\alpha}} K_0. \quad (1.4)$$

By substituting (1.4) in (1.3), the total output supply is,

$$Y^s = \left[\frac{\alpha A}{(1 + i^R)(W/P)} \right]^{\frac{\alpha}{1-\alpha}} K_0. \quad (1.5)$$

⁵The investment level is not a choice variable in the profit maximization.

Equations (1.4) and (1.5) show that the demand for labour and the output supply are negatively related to the effective cost of labour, denoted by the term $(1 + i^R) \frac{W}{P}$.

Because the model is short run in nature, the nominal wage is assumed to be fixed at \bar{W} . Thus, the labour demand and output supply equations can be represented as functions of prices, the refinance rate and the technology parameter,

$$N^d = N^d(P, i^R, A), \quad (1.6)$$

$$Y^s = Y^s(P, i^R, A), \quad (1.7)$$

with $N_P^d, Y_P^d > 0$, $N_{i_R}^d, Y_{i_R}^d < 0$ and $N_A^d, Y_A^d > 0$. That is, an increase in prices, lower interest rates (both of which reduce the effective cost of labour) or an upward shift in the technology parameter have an expansionary effect on the labour demand and output supply.

Finally, substituting \bar{W} , (1.2) and (1.6) in (1.1) results in the firms total demand for credit,

$$L^F = \bar{W} N^d(P, i^R, A) + PI(i^L - \pi^e), \quad (1.8)$$

where L^F is negatively related to the lending rate (i^L) for a given price level (P), which is endogenous in the model.

1.2.2 Households

Households consume goods and supply labour inelastically to firms. Furthermore, there are three types of assets available to the households: currency (which bears no interest), bank deposits and equity capital. These three assets are imperfect substitutes and the households hold bank capital as they are assumed to own the bank. Thus, the household's financial wealth (F^H) is defined as,

$$F^H = BILL^H + D + P^E \bar{E}, \quad (1.9)$$

where $BILL^H$ denotes the nominal value of currency holdings, D the nominal quantity of bank deposits and $P^E \bar{E}$ the nominal value of bank capital held by households, with \bar{E} representing the fixed amount of equity capital. As in Agénor and Pereira da Silva (2012), equity prices (P^E) are taken as given because they are determined by the expected value of future dividends, which is exogenous.

The relative demand for currency is assumed to be negatively related to the

interest rate on bank deposits (i^D), thus inversely related to its opportunity cost,

$$\frac{BILL^H}{D} = v(i^D), \quad (1.10)$$

where $v' < 0$. Using (1.9), equation (1.10) can be written as,

$$\frac{D}{F^H - P^E \bar{E}} = h_D(i^D), \quad (1.11)$$

with $h_D = \frac{1}{1+v(i_D)}$ and $h_D' > 0$. Therefore,

$$\frac{BILL^H}{F^H - P^E \bar{E}} = \frac{v(i^D)}{1 + v(i^D)} = h_B(i^D), \quad (1.12)$$

where $h_B' < 0$.

The real consumption expenditure function (C) depends positively on the real labour income $\left(\frac{\bar{W}}{P}N\right)$ and on the beginning of period real value of financial wealth $\left(\frac{F_0^H}{P}\right)$, while being negatively related to the deposit rate. Because profits and interest on deposits are assumed to be distributed at the end of the period, the consumption function is related to the current income composed of wages. Hence,

$$C = \alpha_0 + \alpha_1 \frac{\bar{W}}{P}N - \alpha_2(i^D - \pi^e) + \alpha_3 \frac{F_0^H}{P}, \quad (1.13)$$

with $\alpha_1 \in (0, 1)$ denoting the marginal propensity to consume out of disposable income, and $\alpha_0, \alpha_2, \alpha_3 > 0$.

1.2.3 Commercial Bank

The liabilities of the commercial bank consists of deposits held by households (D), borrowing from the central bank (L^B), and the nominal value of equity capital ($P^E \bar{E}$). The bank's assets are given by the mandatory reserves held at the central bank (RR) and the credit supplied to firms (L^F). Therefore, the bank's balance sheet is,

$$L^F + RR = P^E \bar{E} + D + L^B, \quad (1.14)$$

where the total nominal value of bank capital ($P^E \bar{E}$) is composed of the required regulatory capital ($P^E E^R$) and the capital buffer ($P^E E^E$), which is measured by the ratio of total bank capital to required bank capital. Formally, the bank capital buffer is equal to $\frac{\bar{E}}{E^R}$ (as in Agénor and Pereira da Silva 2012). Moreover, to avoid prohibitive penalties or reputational costs, the bank is assumed to hold a positive

capital buffer such that $\bar{E} \geq E^R$ (also consistent with empirical evidence as shown by Pereira da Silva 2009).

The reserves held at the central bank pay no interest and are set proportionally to the level of deposits,

$$RR = \mu D, \quad (1.15)$$

with $\mu \in (0, 1)$. The total borrowing from the central bank is therefore obtained by combining (1.14) and (1.15),

$$L^B = L^F - (1 - \mu)D - P^E \bar{E}. \quad (1.16)$$

Moreover, the bank must satisfy risk based capital requirements such that bank equity covers at least a given percentage of its loans provided for investment purposes. This capital adequacy requirement, also known as the *Cooke Ratio*, does not apply to loans given for paying working capital needs (which bear no risk) but only to risky loans supplied to firms for investment purposes. Thus, denoting $\sigma > 0$ as the risk weight on investment loans, the capital requirement constraint is,

$$P^E E^R = \rho \sigma P I, \quad (1.17)$$

where $\rho \in (0, 1)$ is the capital adequacy requirement, set at a floor value of 8% under both Basel I and Basel II. Under Basel I, the same risk weight (σ) applied to all loans in the same particular category and therefore σ was set exogenously to a value equal or less than unity (depending on the type / category of loans). Hence, under the old regime, it was not possible to distinguish between risks imposed by different borrowers in the same particular category. On the other hand, under the Foundation IRB approach of Basel II, the risk weight is a function of the repayment probability estimated by the bank because it can be related to the credit default risk;

$$\sigma = (q)^{-\phi_q}, \quad (1.18)$$

where $q \in (0, 1)$ denotes the repayment probability and $\phi_q > 0$.

This specification is similar to Heid (2007) and Tanaka (2002) who relate the risk weight to the probability of default, and to Agénor and Pereira da Silva (2012). In the latter, the risk weight is determined by the risk premium, which, in turn, is negatively related to the repayment probability. Under the Foundation IRB approach of Basel II, banks calculate the estimated risk weight and consequently can shape the capital requirements according to their own private information.

Alternatively, the risk weight under Basel II can be determined by the Stan-

dardized approach, where σ is calculated by external rating agencies. Thus, similar to Zicchino (2006), this approach can be modeled by relating the risk weight to the macroeconomic conditions or the total supply of output relative to its potential value, under the assumption that ratings are procyclical.⁶ Specifically,

$$\sigma = \left(\frac{Y^S}{\bar{Y}} \right)^{-\phi_{Y^S}}, \quad (1.19)$$

where $\phi_{Y^S} > 0$, such that $\frac{\partial \sigma}{\partial (Y^S/\bar{Y})} < 0$. The term \bar{Y} denotes potential output, which is taken as given and normalized to unity in what follows.

The Bank's Optimization Problem

The bank decides on the deposit rate and the lending rate so as to maximize the following real expected profits function (Π^B) subject to the investment function (1.20), the loan demand function (1.21), the total lending from the central bank (1.22) and the capital requirement constraint (1.23),⁷

$$\begin{aligned} \max_{i^D, i^L} E(\Pi^B) = & (1 + i^R) \frac{W}{P} N + q(1 + i^L)I + (1 - q)\kappa Y^S + \mu d - \\ & -(1 + i^D)d - (1 + i^R) \left(\frac{L^B}{P} \right), \end{aligned}$$

subject to,

$$I = I(i^L - \pi^e), \quad (1.20)$$

$$L^F = W N^d(P, i^R, A) + P I(i^L - \pi^e), \quad (1.21)$$

$$L^B = L^F - (1 - \mu)D - P^E \bar{E}, \quad (1.22)$$

$$P^E E^R = \rho \sigma P I, \quad (1.23)$$

where $\kappa \in (0, 1)$ and $d = \frac{D}{P}$ representing the real level of deposits. The first element on the right hand side, $(1 + i^R) \frac{W}{P} N$, denotes the returns of the commercial bank from supplying non-risky loans to finance the firms' working capital needs. The second element, $q(1 + i^L)I$, denotes the expected repayment if there is no default on loans supplied for investment purposes while the third expression, $(1 - q)\kappa Y^S$, is the expected return for the bank if firms default. In case of default the bank

⁶Drumond (2009) shows that these external ratings are indeed procyclical.

⁷Although E^R depends on i^L through the capital constraint, this relationship is not exploited in the bank's maximization problem. Because total bank capital is fixed in this model, any changes in E^R would be fully offset by a change in the capital buffer, which is implausible given the short run nature of the model.

collects collateral pledged by the borrowers, denoted by the term κY^s . Therefore, as pointed out by Agénor and Montiel (2008), κ measures the degree of credit market imperfections. The fourth term, μd , represents the reserve requirements held at the central bank. Because the bank lasts only for one period, μd is given back to the bank at the end of the period and as a result enters positively in the profit maximizing problem. Turning to the bank's costs, the term $(1 + i^D)d$ represents the gross deposit repayment of the bank to households, while $(1 + i^R) \left(\frac{L^B}{P} \right)$ is the gross repayment of central bank loans.

The first order condition of the above bank's maximization problem with respect to i^D is,

$$-d - [(1 + i^D) - \mu - (1 + i^R)(1 - \mu)] \frac{\partial d}{\partial i^D} = 0,$$

or,

$$1 + \frac{i^D}{d} \frac{\partial d}{\partial i^D} = (1 - \mu)i^R \frac{\partial d}{\partial i^D} \frac{i^D}{d}. \quad (1.24)$$

Defining the elasticity of the supply of deposits to households as $\eta^D = \frac{i^D}{d} \frac{\partial d}{\partial i^D}$, treating it as a constant, and rearranging (1.24) results in the rate of return on bank deposits,

$$i^D = \frac{1}{\left(1 + \frac{1}{\eta^D}\right)} (1 - \mu)i^R. \quad (1.25)$$

Hence, the interest rate on bank deposits is set as a constant markup over the refinance rate, adjusted downwards due the implicit costs of holding reserve requirements.

The first order condition with respect to i^L (with q taken as given) yields,

$$qI + [q(1 + i^L) - (1 + i^R)] \frac{\partial I}{\partial i^L} = 0.$$

Defining the interest elasticity of the demand for loans given for investment purposes as $\eta^I = \frac{\partial I}{\partial i^L} \frac{i^L}{I}$ and treating it as constant, then the above equation reduces to,

$$i^L = \frac{1}{\left(1 + \frac{1}{\eta^I}\right)} \left[\frac{1}{q}(1 + i^R) - 1 \right]. \quad (1.26)$$

Therefore, the loan rate is set as a mark up over the refinance rate, with the value of the mark up determined by the risk premium. The risk premium, in turn, is negatively related to the repayment probability.

The Repayment Probability, Collateral and Bank Capital

The repayment probability is now related to two main factors: First, to the firm's collateral relative to (risky) loans given for investment purposes. Following Boot, Thakor and Udell (1991), Bester (1994) and Hainz (2003), collateral reduces borrowers' incentives to engage in risky investments and mitigates moral hazard behaviour. As a result, effective collateral has a positive impact on the repayment probability. Second, the repayment probability depends positively also on the bank capital buffer through a monitoring incentive effect (similar in spirit to Agénor, Alper and Pereira da Silva 2012).⁸

Microeconomic foundations for this monitoring incentive effect are provided by the models of Allen, Carletti and Marquez (2011) and Mehran and Thakor (2009). In the static model of Allen, Carletti and Marquez (2011), excess bank capital held by a monopolistic bank increases its incentives to monitor borrowers, which raises the borrowers' success probability and therefore improves their expected payoff. Mehran and Thakor (2009) show within a dynamic setting that holding bank capital enhances the incentives to monitor borrowers as it raises the future survival probability of the bank. Empirically, the relationship between bank capital and the lending rate is supported by the study of Hubbard, Kuttner and Palia (2002), where the capital structure of the bank determines the rate of return on loans. More specifically, well capitalized banks tend to charge lower lending rates compared to low capitalized banks. Moreover, this effect of an inverse relationship between holding bank capital and loan rates is also highlighted in Coleman, Esho and Sharpe (2006), wherein capital constrained banks charge a higher spread on their loans.

An alternative explanation stems from the idea that banks holding capital buffers are expected to face lower bankruptcy cost, thus allowing them to expand lending by reducing the interest rate charged on loans. In addition, higher capital buffers increase incentives for banks to screen and monitor their borrowers more carefully, thus enabling them to lower the lending rate, which, in turn, leads to an expansionary effect on the economic activity. This idea is supported by Fonseca, Gonzalez and Pereira da Silva (2010), who, by examining the pricing behaviour of more than 2,300 banks in 92 countries over the period 1990-2007, show that bank capital buffers affect the bank lending spreads (or the risk of default). In our model,

⁸In Agénor, Alper and Pereira da Silva (2012) the *total* amount of bank capital relative to outstanding loans induces the positive impact on the repayment probability. However, given that total capital (\bar{E}) in our model is fixed and constant, this type of bank capital channel will not have substantive implications to our results and will not allow a comparison between the different regulatory regimes. The bank capital channel in our model implies that banks only care about the excess capital held *above* the regulatory minimum.

the bank capital channel is embedded into the repayment probability, which ultimately impacts the lending rate (see equation 1.26). This contributes to the model of Agénor and Pereira da Silva (2012), who incorporate this type of bank capital channel through its impact on the deposit rate.

The abovementioned effects on the repayment probability are captured by the following separable linearized equation,

$$q = \varphi_1 \left(\frac{\kappa PY^s}{PI} \right) + \varphi_2 \left(\frac{\bar{E}}{E^R} \right), \quad (1.27)$$

where $\varphi_i > 0 \forall i$, and φ_1, φ_2 denote the elasticities of the repayment probability with respect to the borrowers effective collateral and the bank capital buffer, respectively.

1.2.4 Central Bank

The central bank's liabilities consists of the monetary base (MB) while its liabilities consists of loans provided to the commercial bank (L^B). The balance sheet of the central bank is therefore given by,

$$L^B = MB, \quad (1.28)$$

where the monetary base is defined as the sum of the total currency in circulation ($BILL$), and reserves (RR),

$$MB = BILL + RR. \quad (1.29)$$

The central bank supplies liquidity through a standing facility and sets its monetary policy through the refinance rate, given by a constant rate (i^R). Thus, substituting (1.15) and (1.28) in equation (1.29) results in the total supply of currency,

$$BILL^s = L^B - \mu D. \quad (1.30)$$

1.2.5 Market Clearing Conditions

The market clearing conditions requires the four financial markets (deposits, loans, central bank credit and cash) and the goods market to clear by equating supply and demand. The market for central bank credit is always in equilibrium given the assumption that the central bank fixes the policy rate i^R and inelastically supplies all credit to the commercial bank at that rate. The markets for deposits and loans adjust through quantities, with the commercial bank setting both the deposit rate

and the lending rate. The cash market is cleared through equations (1.10) and (1.30) but this market automatically clears given Walras' law and thus can be ignored.

The equilibrium in the goods market, which determines the price of the domestic good (P), is represented by the following market clearing condition,

$$Y^s = C + I. \quad (1.31)$$

1.3 Model Solution under Non-Binding Capital Requirements

1.3.1 Financial Market Equilibrium

The first step to solve the model under nonbinding capital requirement ($\bar{E} > E^R$) is to find the financial equilibrium condition, obtained by substituting output supply (1.7), the demand for investments (1.2) and the capital requirement constraint (1.17) into the repayment probability (given by 1.27). Setting $\pi^e = 0$ for simplicity, this yields,

$$q = \varphi_1 \left(\frac{\kappa Y^s(P, i^R, A)}{I(i^L)} \right) + \varphi_2 \left(\frac{P^E \bar{E}}{\rho \sigma P I(i^L)} \right). \quad (1.32)$$

Substituting (1.32) in the lending rate equation (1.26) and normalizing $P^E = 1$ results in the following expression,

$$i^L = \frac{1}{\left(1 + \frac{1}{\eta^I}\right)} \left\{ \frac{(1 + i^R)}{\left[\varphi_1 \left(\frac{\kappa Y^s(P, i^R, A)}{I(i^L)} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I(i^L)} \right) \right]} - 1 \right\}. \quad (1.33)$$

That is, the financial equilibrium condition is related to σ , which implies that the cyclicity of the lending rate depends on the nature of the regulatory regime. Solving equation (1.33) gives,

$$i^L = FF^j(P, A, i^R), \quad (1.34)$$

where FF denotes the financial equilibrium function and j stands for the different regulatory regimes ($j = I$ for Basel I, $j = II$ for Basel II, $j = IRB$ for the Foundation IRB approach of Basel II and $j = ST$ for the Standardized approach of Basel II). The total derivative of equation (1.33) with respect to P and A is now calculated in order to find how these variables affect the cyclicity of the lending rate under Basel I, the Foundation IRB approach of Basel II and the Standardized approach of Basel II.

Under Basel I, where σ is exogenous, the effect of P on i^L is (see Appendix

1.A.1 for a complete derivation),

$$FF_P^I = \left(\frac{di^L}{dP} \right)_{FF}^{Basel\ I} = \Delta^{Basel\ I} \left[\varphi_1 \frac{\kappa Y_P^s}{I} - \varphi_2 \frac{\bar{E}}{P^2 \sigma \rho I} \right] \leq 0, \quad (1.35)$$

where,

$$\Delta^{Basel\ I} = - \frac{\frac{(1+i^R)}{(1+\frac{1}{\eta I})}}{\left\{ \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 - \frac{(1+i^R)}{(1+\frac{1}{\eta I})} \left[\varphi_1 \frac{\kappa Y^{sI'}}{I^2} + \varphi_2 \frac{I' \bar{E}}{\rho \sigma P I^2} \right] \right\}} < 0.$$

Similarly, the effect of A on i^L is (see Appendix 1.A.2),

$$FF_A^I = \left(\frac{di^L}{dA} \right)_{FF}^{Basel\ I} = \Delta^{Basel\ I} \varphi_1 \frac{\kappa Y_A^s}{I} < 0. \quad (1.36)$$

A rise in prices under Basel I has an *ambiguous* effect on the lending rate, as long as $\varphi_2 > 0$. On the one hand, an increase in P stimulates real output (by reducing real wages), which increases the effective value of firms collateral relative to risky loans. This, in turn, raises the repayment probability and lowers the loan rate. On the other hand, a rise in P leads to an increase in the nominal value of risky loans and thus to a rise in bank capital requirements, thereby resulting in a lower bank capital buffer. The deterioration in the bank capital buffer reduces the repayment probability and increases the lending rate charged by the commercial bank. In the absence of the bank capital channel ($\varphi_2 = 0$), the loan rate falls unambiguously, which is also the case if the elasticity of the repayment probability with respect to firm's collateral dominates the strength of the bank capital channel. Therefore, the bank capital channel *mitigates* the initial fall in the lending rate following the improvement in firms' collateral (caused by a rise in prices in this example).

A positive supply shock raises output and the value of collateral, without having a direct effect on the level of investment. As a result, a rise in A leads to an *unambiguous* rise in the repayment probability, thereby reducing the loan rate. When examining the effect of productivity shocks under Basel I, the bank capital channel has only a quantitative effect (in terms of the magnitude of the impact), and not a qualitative effect. More precisely, $\Delta^{Basel\ I}$ is lower (in absolute value) if $\varphi_2 > 0$, so the lending rate falls by *less* in the presence of the bank capital channel.

Under Basel II, where σ is endogenous, the effect of prices on the lending rate

is,

$$\frac{di^L}{dP} = \frac{(1 + i^R)}{\left(1 + \frac{1}{\eta^I}\right)} \left\{ \frac{-\varphi_1 \left(\frac{\kappa Y_P^s I - \kappa Y^s I' \frac{di^L}{dP}}{I^2} \right) - \varphi_2 \left(\frac{-[(\sigma_P P + \sigma) \rho I + I' \frac{di^L}{dP} \rho \sigma P] \bar{E}}{(\rho \sigma P I)^2} \right)}{\left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2} \right\},$$

where $\sigma_P = \frac{d\sigma}{dP}$. The risk weight (σ) depends either on the output supply or the repayment probability, which, in turn, is related to both prices and output. Solving the above equation for $\frac{di^L}{dP}$ and using some algebraic manipulations yields (see Appendix 1.A.3),

$$FF_P^{II} = \left(\frac{di^L}{dP} \right)_{FF}^{Basel II} = \Delta^{Basel II} \left\{ \varphi_1 \frac{\kappa Y_P^s}{I} - \varphi_2 \frac{\sigma_P \bar{E}}{\rho P I \sigma^2} - \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} \right\}, \quad (1.37)$$

where $\Delta^{Basel II} = \Delta^{Basel I}$ under the assumption that the initial value of the risk weight under Basel II is equal to the risk weight under Basel I. Substituting (1.37) in (1.35) results in,

$$\left(\frac{di^L}{dP} \right)_{FF}^{Basel II} = \left(\frac{di^L}{dP} \right)_{FF}^{Basel I} - \Delta^{Basel II} \varphi_2 \frac{\sigma_P \bar{E}}{\rho P I \sigma^2}. \quad (1.38)$$

Similarly, the effect of A on i^L under Basel II is (see Appendix 1.A.4),

$$FF_A^{II} = \left(\frac{di^L}{dA} \right)_{FF}^{Basel II} = \Delta^{Basel II} \left\{ \varphi_1 \frac{\kappa Y_A^s}{I} - \varphi_2 \frac{\sigma_A \bar{E}}{\sigma^2 \rho P I} \right\}. \quad (1.39)$$

Again, under the assumption that $\Delta^{Basel I} = \Delta^{Basel II}$, equation (1.39) reduces to,

$$\left(\frac{di^L}{dA} \right)_{FF}^{Basel II} = \left(\frac{di^L}{dA} \right)_{FF}^{Basel I} - \Delta^{Basel II} \varphi_2 \frac{\sigma_A \bar{E}}{\sigma^2 \rho P I}. \quad (1.40)$$

The total effect of prices under Basel II can be decomposed to three effects as implied by equation (1.38). The first two effects are the same as in Basel I, where on the one hand an increase in P stimulates output and lowers the lending rate, while on the other, a rise in P increases the capital requirements, which tends to raise the loan rate. However, under Basel II there is an additional effect of P on the lending rate, stemming from the impact of prices on the risk weight. Under both the Foundation IRB and Standardized approaches of Basel II, the risk weight (σ) depends on the price level.

Similarly, from equation (1.40), supply shocks under Basel II have an additional

effect on the lending rate when compared to Basel I, captured through the impact of A on σ . We now turn to discuss the implications of changes in prices and productivity on the risk weight and the lending rate under the Standardized and the Foundation IRB approaches of Basel II, and examine the role of the bank capital channel following such changes.

Under the Standardized approach of Basel II, where $\sigma = (Y^s)^{-\phi_{Y^s}}$, the effect of prices on the risk weight is,

$$\left(\frac{d\sigma}{dP}\right)^{ST} = -\phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_P^s < 0. \quad (1.41)$$

That is, higher prices increase the supply of output and thus lead to a lower risk weight. Substituting (1.41) in (1.37),

$$FF_P^{ST} = \left(\frac{di^L}{dP}\right)_{FF}^{ST} = \Delta^{Basel II} \left\{ \varphi_1 \frac{\kappa Y_P^s}{I} + \varphi_2 \frac{\bar{E}}{\rho I P \sigma^2} \phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_P^s - \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} \right\} \leq 0. \quad (1.42)$$

Examining equation (1.42), the strength of the bank capital channel (φ_2) has an *ambiguous* impact on the lending rate following changes in prices. Similar to the previous cases, the initial rise in prices results in a higher value of nominal loans and a lower bank capital buffer, thereby leading to a higher lending rate. However, this price increase raises the output supply (by lowering real wages), which directly lowers the risk weight under the Standardized approach. The fall in the risk weight then results in a higher repayment probability and a lower loan rate. Therefore, the increase in output impacts the lending rate through the *collateral channel* (as explained earlier) and via the *bank capital channel*, which operates differently under Basel I and the Standardized approach of Basel II due to the additional impact of prices on the risk weight.

The Standardized approach of Basel II induces a further decrease in the loan rate following a rise in prices compared to Basel I if the sensitivity of the repayment probability to the effective collateral dominates the strength of the bank capital channel relative to the bank capital buffer, $\varphi_1 \frac{\kappa Y_P^s}{I} > \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2}$. Under this condition, the lending rate *falls unambiguously* under Basel I, so the term

$$\Delta^{Basel II} \varphi_2 \frac{\bar{E}}{\rho I P \sigma^2} \phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_P^s < 0 \quad (1.43)$$

(the additional effect of changes in the repayment probability, resulting from changes in output, on the risk weight) *amplifies* the drop in the lending rate in the Standardized approach. If, by contrast, $\varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} > \varphi_1 \frac{\kappa Y_P^s}{I}$, then the lending rate *rises*

unambiguously under Basel I, such that the above term (1.43) *mitigates* the initial rise in the lending rate following an increase in prices under the Standardized approach. If the bank capital channel does not operate ($\varphi_2 = 0$), then following a price increase, the lending rate drops unambiguously via the collateral channel only. Moreover, because of the ambiguous effect of the strength of the bank capital channel on the lending rate, it cannot be concluded whether the bank capital channel amplifies or mitigates the initial fall in the loan rate caused by the rise in prices and the improvement in firms' collateral.

The effect of A on the risk weight under the **Standardized approach** of Basel II, can be directly calculated as follows,

$$\left(\frac{d\sigma}{dA}\right)^{ST} = -\phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_A^s < 0. \quad (1.44)$$

Substituting (1.44) in equation (1.39) yields,

$$FF_A^{ST} = \left(\frac{di^L}{dA}\right)_{FF}^{ST} = \Delta^{Basel II} \left\{ \varphi_1 \frac{\kappa Y_A^s}{I} + \varphi_2 \frac{\bar{E}}{\sigma^2 \rho P I} \phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_A^s \right\} < 0, \quad (1.45)$$

or,

$$\left(\frac{di^L}{dA}\right)_{FF, \varphi_2 > 0}^{ST} = \left(\frac{di^L}{dA}\right)_{FF, \varphi_2 = 0}^{ST} + \Delta^{ST} < 0, \quad (1.46)$$

where $\left(\frac{di^L}{dA}\right)_{FF, \varphi_2 = 0}^{ST} < 0$ and $\Delta^{ST} = \Delta^{Basel II} \varphi_2 \frac{\bar{E}}{\sigma^2 \rho P I} \phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_A^s < 0$. Thus, positive supply shocks lead to an *unambiguous fall* in the loan rate. The effect of A on the repayment probability and the lending rate is captured by two channels influencing directly the cost of borrowing. Specifically, a rise in A increases the effective collateral and directly lowers the risk weight on loans (caused by the output stimulation), both resulting in a lower lending rate. Because of the additional effect of the productivity shock on the risk weight, the Standardized approach induces *additional procyclicality* in the loan rate compared to Basel I.

Without the transmission of the bank capital channel on the repayment probability and the risk weight ($\varphi_2 = 0$), the lending rate falls by *less* when compared to an active bank capital channel. Thus, the bank capital channel, through its impact on the risk weight, *amplifies* the response of the lending rate following supply shocks, as implied from equation (1.46).

To calculate the effects of prices on the risk weight under the **Foundation IRB approach** of Basel II, it is first necessary to determine the impact of prices on the repayment probability. Calculating the derivative of q with respect to P in equation

(1.32) with $P^E = 1$, yields,

$$\frac{dq}{dP} = \varphi_1 \frac{\kappa Y_P^s}{I} - \varphi_2 \frac{\bar{E}}{P^2 \rho \sigma I} \leq 0.$$

Consequently, under the Foundation IRB approach of Basel II, where $\sigma = (q)^{-\phi_q}$, the effect of prices on the risk weight is given by,

$$\left(\frac{d\sigma}{dP} \right)^{IRB} = -\phi_q q^{-\phi_q-1} \left\{ \varphi_1 \frac{\kappa Y_P^s}{I} - \varphi_2 \frac{\bar{E}}{P^2 \rho \sigma I} \right\} \leq 0. \quad (1.47)$$

Thus, in contrast to the Standardized approach, prices have an *ambiguous* effect on the risk weight under the Foundation IRB approach. The initial increase in prices tends to raise the effective collateral pledged by firms, increase the repayment probability and thus lower the risk weight on loans. However, this rise in prices raises the value of nominal investments, increases the capital requirements, lowers the bank capital buffer and reduces the repayment probability, which, in turn, translates into a higher risk weight. In the absence of the bank capital channel ($\varphi_2 = 0$), the rise in prices results unambiguously in a lower risk weight, similar to the Standardized approach.

The total effect of prices on the lending rate is obtained by substituting (1.47) in (1.37),

$$FF_P^{IRB} = \left(\frac{di^L}{dP} \right)_{FF}^{IRB} = \Delta^{Basel II} \left\{ \left(1 + \varphi_2 \phi_q q^{-\phi_q-1} \frac{\bar{E}}{\rho I P \sigma^2} \right) \times \right. \\ \left. \times \left(\varphi_1 \frac{\kappa Y_P^s}{I} - \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} \right) \right\} \leq 0. \quad (1.48)$$

Dividing equation (1.48) by equation (1.35) and using $\Delta^{Basel I} = \Delta^{Basel II}$,

$$\frac{FF_P^{IRB}}{FF_P^I} = \left(1 + \varphi_2 \phi_q q^{-\phi_q-1} \frac{\bar{E}}{\rho I P \sigma^2} \right) > 1,$$

implying that,

$$\left(\frac{di^L}{dP} \right)_{FF}^{IRB} > \left(\frac{di^L}{dP} \right)_{FF}^I. \quad (1.49)$$

Therefore, the additional impact of prices on the risk weight under the Foundation IRB approach leads to increased procyclicality in the loan rate behaviour when compared to Basel I.

The role of the bank capital channel following a rise in prices cannot be determined unambiguously under the Foundation IRB approach. On the one hand, a rise in prices lowers the bank capital buffer, which reduces the repayment probability

and increases the risk weight on loans. These two effects create an upward pressure on the lending rate. On the other hand, the increase in the price level directly increases the repayment probability (through the collateral channel), which directly lowers the risk weight. These two effects result in a downward pressure on the loan rate. Of course, when the bank capital channel is not active ($\varphi_2 = 0$), then the lending rate falls unambiguously, similar to Basel I and the Standardized approach of Basel II.

To examine the total effect of a productivity shock on the lending rate under the **Foundation IRB approach**, we first calculate the derivative of q with respect to A in equation (1.32),

$$\frac{dq}{dA} = \varphi_1 \frac{\kappa Y_A^s}{I}. \quad (1.50)$$

Thus, the impact of A on σ is,

$$\left(\frac{d\sigma}{dA} \right)^{IRB} = -\phi_q q^{-\phi_q-1} \varphi_1 \frac{\kappa Y_A^s}{I} < 0. \quad (1.51)$$

Substituting (1.51) in (1.39) yields,

$$FF_A^{IRB} = \left(\frac{di^L}{dA} \right)_{FF}^{IRB} = \Delta^{Basel II} \left\{ \varphi_1 \frac{\kappa Y_A^s}{I} + \varphi_2 \frac{\bar{E}}{\sigma^2 \rho P I} \phi_q q^{-\phi_q-1} \varphi_1 \frac{\kappa Y_A^s}{I} \right\} < 0, \quad (1.52)$$

or,

$$\left(\frac{di^L}{dA} \right)_{FF, \varphi_2 > 0}^{IRB} = \left(\frac{di^L}{dA} \right)_{FF, \varphi_2 = 0}^{IRB} + \Delta^{IRB} < 0, \quad (1.53)$$

where $\left(\frac{di^L}{dA} \right)_{FF, \varphi_2 = 0}^{IRB} < 0$ and $\Delta^{IRB} = \Delta^{Basel II} \varphi_2 \frac{\bar{E}}{\sigma^2 \rho P I} \phi_q q^{-\phi_q-1} \varphi_1 \frac{\kappa Y_A^s}{I} < 0$. Positive productivity shocks result *unambiguously* in a *lower* lending rate. The impact of A on the loan rate is captured now through two channels: First, higher productivity raises output, increases firms' effective collateral, both which result in a lower loan rate. Second, the rise in the repayment probability, associated with the higher collateral pledged by firms, reduces the risk weight, creating an additional downward pressure on the lending rate. Consequently, both of these channels strengthen one another and lead to a decrease in the loan rate following positive supply shocks. Further, the lending rate reaction is *amplified* under the Foundation IRB approach when compared to Basel I due to the additional effect of collateral on the repayment probability and thus on the risk weight.

Similar to the Standardized approach, in the Foundation IRB approach the bank capital channel *magnifies* the initial fall in the lending rate caused by positive supply shocks.

Comparing between supply shocks under the Foundation IRB approach and the Standardized approach, one should note that supply shocks in the latter *directly* impact the risk weight and thus the lending rate through the direct relationship between the risk weight and the output supply. In the Foundation IRB approach, on the other hand, supply shocks affect the risk weight through the impact of effective collateral on the repayment probability. Therefore, under this approach, productivity shocks *indirectly* impact the risk weight and the lending rate, in contrast to the Standardized approach. Consequently, by subtracting equation (1.52) from equation (1.45), then the Standardized approach induces more procyclicality in the lending rate if $\phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_A^s > \phi_q q^{-\phi_q-1} \varphi_1 \frac{\kappa Y_A^s}{I}$. This implies that the sensitivity of the risk weight with respect to changes in output supply is greater than the sensitivity of the risk weight with respect to changes in the repayment probability, caused by shifts in the firms' effective collateral. We assume that this is indeed the case in what follows.

Table 1.1 summarizes the results presented above and indicates whether the cyclicity in the loan rate is amplified or mitigated with an active bank capital channel following a rise in prices and a positive supply shock.

Table 1.1: Response of the Loan Rate to an Increase in Prices and a Positive Supply Shock under Alternative Regulatory Regimes

	Basel I	Standardized Approach	IRB Approach
Price Effect			
$FF_P^{\varphi_2=0}$	unambiguous fall	unambiguous fall	unambiguous fall
$FF_P^{\varphi_2>0}$	mitigated	ambiguous	ambiguous
Supply Shock Effect			
$FF_A^{\varphi_2=0}$	unambiguous fall	unambiguous fall	unambiguous fall
$FF_A^{\varphi_2>0}$	mitigated	amplified	amplified

Following a rise in prices, the bank capital channel mitigates the initial fall in the loan rate under Basel I, whereas it cannot be concluded whether the bank capital channel magnifies or dampens the drop in the loan rate (caused by the improvement in effective collateral) under both variants of Basel II. Following positive supply shocks and with an active bank capital channel, the impact on the lending rate is mitigated under Basel I (through the quantitative effect on $\Delta^{Basel\ I}$). However, positive productivity shocks have similar qualitative amplifying effects on the loan rate under both variants of Basel II with an active bank capital channel.

1.3.2 Goods Market Equilibrium

The second step to find the general equilibrium is to solve for the goods market equilibrium. Using equations (1.2),(1.6),(1.7),(1.13),(1.25), (1.26) and setting $\bar{W} = 1$, $\pi^e = 0$, $\alpha_0 = 0$ for simplicity, condition (1.31) can be written as,

$$Y^s(P, i^R, A) = \alpha_1 \frac{N^d(P, i^R, A)}{P} - \alpha_2 \left[\frac{1}{\left(1 + \frac{1}{\eta^D}\right)} (1 - \mu) i^R \right] + \alpha_3 \left(\frac{F_0^H}{P} \right) + I(i^L). \quad (1.54)$$

The above expression *does not* directly depend on the regulatory regime and the risk weight (σ), and therefore the equilibrium condition in the goods market is the same under Basel I and both the Foundation IRB and Standardized approaches of Basel II. Solving for i^L yields,

$$i^L = GG(P, i^R, A), \quad (1.55)$$

where GG denotes the goods market equilibrium curve under all regulatory regimes. The impact of P on i^L is,

$$GG_P = \left(\frac{di^L}{dP} \right)_{GG} = \Omega \left(Y_P^s + \alpha_1 \frac{N^d - N_P^d P}{P^2} + \alpha_3 \frac{F_0^H}{P^2} \right) < 0, \quad (1.56)$$

where $\Omega = \frac{1}{P'} < 0$. Studying the effect of A on i^L yields the following,

$$GG_A = \left(\frac{di^L}{dA} \right)_{GG} = \Omega \left(Y_A^s - \alpha_1 \frac{N_A^d}{P} \right) < 0. \quad (1.57)$$

The effect of an increase in prices on the lending rate can be decomposed as follows: First, a rise in prices lowers the real wage, stimulates output, increases labour demand and distributed wage income, all which result in higher consumption. Second, the rise in prices creates a downward pressure on aggregate demand through a negative wealth effect on consumption. As in Agénor and Pereira da Silva (2012), the net effect on consumption depends on the movement of the output supply relative to aggregate demand. Their analysis shows that the effect on the output supply *always* dominates the wage income effect. Therefore, an increase in prices creates excess supply at the initial level of investment, which implies that the lending rate must fall in order to raise investment and restore equilibrium in the goods market. As a result, *higher* prices lead to a *lower* lending rate in the goods market $\left(\left(\frac{di^L}{dP} \right)_{GG} < 0 \right)$

0).

Following a positive productivity shock, the supply side is assumed to dominate the demand side effects ($Y_A^s > \alpha_1 \frac{N_A^d}{P}$). Therefore, in order to eliminate the excess supply in the goods market (given the initial level of investment), the lending rate must fall such that the investment level increases. In this way the equilibrium in the goods market is restored. Consequently, *positive* productivity shocks tend to *lower* the loan rate in the goods market ($\left(\frac{di^L}{dA}\right)_{GG} < 0$).

In the next sections we study the general equilibrium effects of a negative supply shock, with an intuitive graphical solution, and make a distinction between **two cases**: First, the case where the "pure" bank capital channel ($\varphi_2 \frac{\bar{E}}{\rho I \sigma P^2}$) is "strong" and dominates the elasticity of the repayment probability with respect to the collateral pledged by firms ($\varphi_1 \frac{\kappa Y_A^s}{I}$). Second, the scenario where the collateral channel dominates the "pure" bank channel such that $\varphi_1 \frac{\kappa Y_A^s}{I} - \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} > 0$.⁹

1.3.3 General Equilibrium - The Bank Capital Channel Dominating The Collateral Channel

Macroeconomic Equilibrium

In this section we focus on the case where the "pure" bank capital channel ($\varphi_2 \frac{\bar{E}}{\rho I \sigma P^2}$) dominates the collateral channel ($\varphi_1 \frac{\kappa Y_A^s}{I}$). This assumption, in turn, results in the following: First, the effect of prices on the risk weight under the Foundation IRB approach is positive ($\left(\frac{d\sigma}{dP}\right)^{IRB} > 0$), while an inverse relationship exists between the risk weight and prices under the Standardized approach ($\left(\frac{d\sigma}{dP}\right)^{ST} < 0$). Second, from equations (1.35) and (1.48) it can be concluded that a positive relationship prevails between the lending rate and the price level under both Basel I and the Foundation IRB approach ($FF_P^I, FF_P^{IRB} > 0$). In the Standardized approach, it is assumed that the strength of the "pure" bank capital channel dominates *both* the collateral channel, and the additional alternation of the risk weight resulting from changes in prices and consequently output supply ($\varphi_2 \frac{\bar{E}}{\rho I P \sigma^2} \phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_P^s$). That is, $FF_P^{ST} > 0$. As a result, $FF_P^j = \left(\frac{di^L}{dP}\right)^j > 0$ for $j = I, ST, IRB$.

The contribution of this analysis compared to Agénor and Pereira da Silva (2012) is that the financial equilibrium curve can indeed be *upward sloping* if the bank capital channel is dominant. This, of course, is obtained by the way the bank capital channel is incorporated in our model, through the impact of bank capital buffers

⁹The "pure" bank capital channel refers *solely* to the effect of the bank capital buffers on the repayment probability and the lending rate. The other channel associated with bank capital is the impact of the risk weight on the capital buffers and consequently on the financial market equilibrium (observed only in Basel II).

on the repayment probability and consequently on the lending rate. In Agénor and Pereira da Silva (2012), the financial equilibrium *does not* depend on the bank capital channel nor the regulatory regime and hence is *always downward sloping*.

To determine the general equilibrium effects of shocks under **Basel I**, equations (1.34) (for $j = I$) and (1.55) are solved simultaneously for i^L and P . The total effect of prices and productivity shocks on the lending rate in the financial market and goods market can be respectively written as follows,

$$di^L = FF_P^I dP + FF_A^I dA,$$

$$di^L = GG_P dP + GG_A dA.$$

The solution of a shock to A is obtained by solving the following matrix equation,

$$\begin{bmatrix} 1 & -FF_P^I \\ 1 & -GG_P \end{bmatrix} \begin{bmatrix} di^L \\ dP \end{bmatrix} = \begin{bmatrix} FF_A^I \\ GG_A \end{bmatrix} dA,$$

which gives,

$$\left(\frac{di^L}{dA}\right)^{Basel I} = \frac{GG_A FF_P^I - FF_A^I GG_P}{FF_P^I - GG_P} < 0,$$

$$\left(\frac{dP}{dA}\right)^{Basel I} = \frac{GG_A - FF_A^I}{FF_P^I - GG_P} \leq 0.$$

An active and "strong" bank capital channel implies $FF_P^I > 0$. In addition, $GG_P < 0$, $GG_A < 0$ and $FF_A^I < 0$ so $FF_P^I - GG_P > 0$ and $GG_A FF_P^I - FF_A^I GG_P < 0$, which ensures that $\left(\frac{di^L}{dA}\right)^{Basel I} < 0$. In other words, the lending rate falls unambiguously following positive supply shocks, meaning that the change in the loan rate is procyclical. On the other hand, the impact of a supply shock on prices is ambiguous, $\left(\frac{dP}{dA}\right)^{Basel I} \leq 0$, because $GG_A - FF_A^I \leq 0$. In the absence of the bank capital channel, or even when this channel is not "too strong" such that $FF_P^I < 0$, then an ambiguous result is obtained for both $\left(\frac{di^L}{dA}\right)^{Basel I}$ and $\left(\frac{dP}{dA}\right)^{Basel I}$ (see next section for a detailed examination of this case).

Similarly, under the **Standardized approach of Basel II**, solving equations (1.34) for $j = ST$ and (1.55) simultaneously for i^L and P yields,

$$\left(\frac{di^L}{dA}\right)^{ST} = \frac{GG_A FF_P^{ST} - FF_A^{ST} GG_P}{FF_P^{ST} - GG_P} < 0,$$

$$\left(\frac{dP}{dA}\right)^{ST} = \frac{GG_A - FF_A^{ST}}{FF_P^{ST} - GG_P} \leq 0.$$

Figure 1 -
Macroeconomic Equilibrium

To conclude, the lending rate behaviour is *always procyclical* following supply shocks while the impact on prices is *ambiguous*. The difference across the three regulatory regimes is only in terms of magnitudes and not in terms of directions.

The slope of the financial equilibrium curve under Basel I, the Standardized approach and the Foundation IRB approach are denoted respectively by equations (1.35), (1.42) and (1.48), which are rewritten here for convenience,

$$\begin{aligned}
FF_P^I &= \left(\frac{di^L}{dP} \right)_{FF}^{Basel\ I} = \Delta^{Basel\ I} \left[\varphi_1 \frac{\kappa Y_P^s}{I} - \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} \right], \\
FF_P^{ST} &= \left(\frac{di^L}{dP} \right)_{FF}^{ST} = \Delta^{Basel\ II} \left\{ \varphi_1 \frac{\kappa Y_P^s}{I} + \varphi_2 \frac{\bar{E}}{\rho I P \sigma^2} \phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_P^s - \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} \right\}, \\
FF_P^{IRB} &= \left(\frac{di^L}{dP} \right)_{FF}^{IRB} = \Delta^{Basel\ II} \left\{ \left(1 + \varphi_2 \phi_q q^{-\phi_q-1} \frac{\bar{E}}{\rho I P \sigma^2} \right) \left(\varphi_1 \frac{\kappa Y_P^s}{I} - \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} \right) \right\}.
\end{aligned}$$

43

channel, then the slopes of FF^j for $j = I, ST, IRB$ are *positive*, as noted earlier.¹⁰ Moreover, a comparison of (1.35), (1.42) and (1.48) implies that FF^{ST} is *flatter* than FF^I , while FF^{IRB} is *steeper* than FF^I . Intuitively, under Basel II there is an additional effect captured through the relationship between prices and the risk weight. Specifically, under the Standardized approach a rise in prices stimulates output and directly *lowers* the risk weight (as implied from equation 1.41). The fall in the risk weight, in turn, mitigates the initial drop in the bank capital buffer, which *moderates* the increase in the lending rate (at the initial level of investment). As a result, following a rise in prices, the loan rate rises by *less* under the Standardized approach of Basel II compared to Basel I.

By contrast, under the Foundation IRB approach, a rise in prices tends to *increase* the risk weight on loans when the bank capital channel dominates the collateral channel (see equation 1.47). The increase in the risk weight amplifies the fall in the bank capital buffer and leads to a *further* increase in the lending rate, at the initial level of investment. Consequently, the loan rate increases by *more* under the Foundation IRB approach of Basel II compared to Basel I following a hike in prices.

Finally, inspection of equations (1.35), (1.42) and (1.48) shows that in the absence of the bank capital channel the slopes are all equal and downward sloping. More precisely, the lending rate *falls unambiguously* following a rise in prices such that $FF_P^I = FF_P^{ST} = FF_P^{IRB} = \Delta^{Basel\ I} \left[\varphi_1 \frac{\kappa Y_P^s}{I} \right] < 0$. The curve corresponding with the financial equilibrium curve with a non active bank capital channel is denoted by FF^0 in Figure 1.1.¹¹

As shown in the previous section, the goods market equilibrium, labeled as GG , does not depend on the regulatory regime, and its (negative) slope is given by equation (1.56),

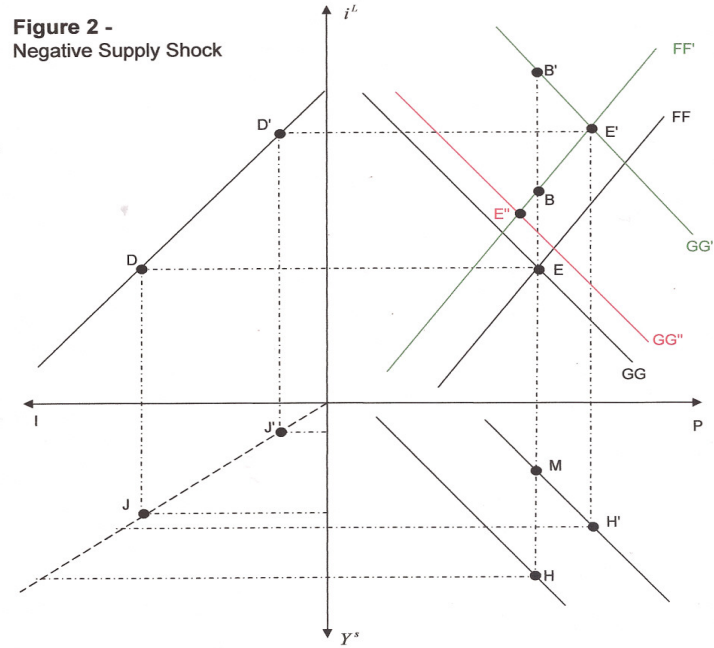
$$GG_P = \left(\frac{di^L}{dP} \right)_{GG}^{I,II} = \Omega \left(Y_P^s + \alpha_1 \frac{N^d - N_P^d P}{P^2} + \alpha_3 \frac{F_0^H}{P^2} \right) < 0.$$

The northeast quadrant exhibits the relationships between the lending rate and the price level in the financial market equilibrium and the goods market equilibrium. The negative relationship between investment and the lending rate is shown in the northwest quadrant, whereas the positive relationship between output supply and the price level is displayed in the southeast quadrant. Using the 45-degree line to

¹⁰Recall that for FF_P^{ST} to be positive it is also assumed that the change in the risk weight, followed by a change in output supply and consequently the repayment probability, is not strong enough to offset the (positive) impact of the "pure" bank capital channel on the lending rate.

¹¹The financial equilibrium curve will also be downward sloping if the *collateral channel* dominates the *bank capital channel*, as examined in the next section.

Figure 1.2: Negative Supply Shock - Case I



report Y^s and I in the southwest quadrant results in the household's consumption (C). The economy's equilibrium is determined at points E, D, H and J .¹²

Negative Supply Shock

A negative supply shock to output, that is, a fall in A , is now examined. The outcomes of such a shock are presented in Figure 1.2; the differences between the three regulatory regimes are only in terms of the slope of the curve FF and therefore only the Basel I regime is considered in order not to complicate and clutter the graph unnecessarily. The differences between the regulatory regimes are pointed out throughout the discussion.

The first effect of a negative supply shock is a drop in output, shown in the southeast quadrant. The supply curve shifts to the right and at the initial level of prices, output falls from point H to point M . As a result, the value of effective collateral pledged by firms decreases (at the initial level of investments), which results in a lower repayment probability. Ultimately, in order to account for the fact that lending is riskier, the FF curve shifts to the left and the lending rate rises from E to B . The fall in output also creates excess demand in the goods market at the initial level of prices. Consequently, from (1.54) the lending rate would need

¹²Naturally, FF^j for $j = I, ST, IRB$ would not normally intersect GG at the same point E . This is used for convenience.

to increase further to bring investments down and restore equilibrium in the goods market. In Figure 1.2, this is shown by the upward movement of the GG curve, such that the loan rate would hypothetically need to rise from point B to point B' .

However, this "overshooting" effect in the behaviour of the lending rate (point B') is not feasible, so the initial rise in the loan rate is not sufficient to eliminate excess demand in the goods market through a fall in investment only. Hence, prices must increase, which (through a negative wealth effect) lower the level of consumption. The higher price level also tends to lower real wages, thus dampening the initial decrease in output; after output falls from H to M , it gradually recovers from M to H' . This rise in output raises effective collateral, thereby mitigating the increase in the lending rate. However, this improvement in effective collateral is not strong enough to offset the impact of the bank capital channel on the lending rate, which has sizable effects in this case. More specifically, the abovementioned rise in the price level (which raises the nominal value of bank loans) lowers the bank capital buffer and reduces the repayment probability, thereby resulting in a *higher* lending rate (from B to E'). The new general equilibrium point therefore corresponds to point E' , where the lending rate is *higher*, investments are *lower*, output is *lower*, and prices are *higher* (compared to the initial equilibrium point E).

Nevertheless, the new general equilibrium can also be characterized by a *higher* lending rate and *lower* prices following a negative productivity shock. This scenario may occur if the FF curve shifts by a large amount (such that the repayment probability and thus the lending rate adjust quickly to changes in the effective collateral, that is φ_1 is high), while the GG curve shifts by a small amount (which happens if the sensitivity of investment to the loan rate is relatively high). Suppose the FF curve shifts to the left by the same amount as before to FF' , but the curve GG moves to the right by a small amount to GG'' . In this case, the new general equilibrium point is characterized by point E'' , where the lending rate is still *higher* (compared to point E) but the price level is *lower*. In sum, in both the abovementioned cases, the loan rate *rises unambiguously* following a negative supply shock ($\frac{di^L}{dA} < 0$), while the impact of such a shock on the price level is *ambiguous* ($\frac{dP}{dA} \leq 0$).

The behaviour of the loan rate is therefore said to be procyclical with respect to supply shocks, such that the lending rate falls during economic upswings and rises during economic recession, thereby exacerbating the initial movement in output. This *unambiguous* result is evident in this model in Basel I and both variants of Basel II. In the absence of a strong bank capital channel (or when the FF curve is *downward* sloping), the lending rate can either increase or decrease following a

negative supply shock (as in the case of Agénor and Pereira da Silva 2012). This depends on the movement of the FF curve relative to the change in the GG curve (see next section for a detailed examination of this case).

With non binding capital requirements and an active bank capital channel, Basel I, the Standardized approach and the Foundation IRB approach all amplify unambiguously the procyclical effects of a negative supply shock in the lending rate. In addition, a negative supply shock under Basel II affects the risk weights in both the Standardized approach and the Foundation IRB approach. The negative supply shock raises the risk weight on loans, increases the bank capital requirements and lowers the bank capital buffer. The deterioration in the bank capital buffer then translates into a lower repayment probability, resulting in an amplified increase in the loan rate compared to Basel I (where the risk weight is constant under a specific loan category).

However, to restore equilibrium in the goods market (following the drop in output), prices must increase (such that $\frac{dP}{dA} < 0$), which, in turn, leads to a further rise in the lending rate through the bank capital channel. Given that $FF_P^{IRB} > FF_P^I > FF_P^{ST} > 0$, then this *additional* rise in the loan rate (followed by the increase in the price level) is the highest under the Foundation IRB approach and the lowest under the Standardized approach. Therefore, combining the effects of A and consequently P on i^L , the following results are obtained: *i*) The Foundation IRB approach is always more procyclical than Basel I following a negative supply shock. *ii*) It cannot be ascertained whether the Foundation IRB approach is more procyclical than the Standardized approach (because $\phi_{Y^s} (Y^s)^{-\phi_{Y^s}-1} Y_A^s > \phi_q q^{-\phi_q-1} \varphi_1 \frac{\kappa Y_A^s}{I}$, so, all else equal, $FF_A^{ST} > FF_A^{IRB}$). *iii*) Whether a supply shock entails more procyclicality under the Standardized approach compared to Basel I cannot be determined unambiguously either.

Alternatively, if $\frac{dP}{dA} > 0$, then following (only) the drop in prices, the loan rate falls by the largest amount under the Foundation IRB approach and by the smallest amount under the Standardized approach. Taking into account the effects of A and thus P on i^L (as before), results in the following: *i*) The Standardized is always more procyclical than the Foundation IRB approach and Basel I following a supply shock. *ii*) Whether the Foundation IRB approach is more procyclical than Basel I cannot be ascertained.

1.3.4 General Equilibrium - The Collateral Channel Dominating The Bank Capital Channel

Macroeconomic Equilibrium

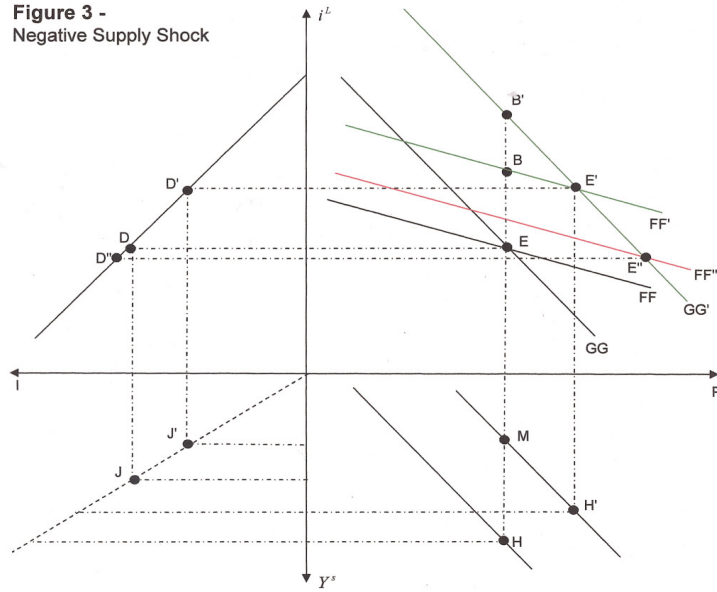
Assuming now that the elasticity of the repayment probability with respect to the borrowers' effective collateral dominates the strength of the bank capital channel, then the slopes of FF^j for $j = I, ST, IRB$ are *negative*. Moreover, a comparison of (1.35), (1.42) and (1.48) implies that FF^I is *steeper* than FF^{ST} and FF^{IRB} , while the ranking between the slopes of FF^{ST} and FF^{IRB} cannot be determined. The goods market equilibrium curve (GG) remains the same and downward sloping under all regulatory regimes as it does not directly depend on the bank capital channel. Finally, because the slopes of the financial equilibrium curves under the Standardized approach and the Foundation IRB approach cannot be ranked, the rest of the discussion focuses on the differences between Basel II (in general) and Basel I.

Negative Supply Shock

A negative supply shock to output is now examined when the collateral channel dominates the bank capital channel. The outcomes of such a shock are presented in Figure 1.3; the difference between Basel I and Basel II is only in terms of the slope of the FF curve and therefore, and as in the previous section, only the Basel I regime is considered in order not to clutter and complicate the graph unnecessarily. The differences between Basel I and Basel II are pointed out throughout the discussion.

As shown in Agénor and Montiel (2008), under standard dynamic assumptions, local stability requires the GG curve to be steeper than the FF curve.

Figure 1.3: Negative Supply Shock - Case II



The first effect of a negative supply shock is a movement of the supply curve (in the southeast quadrant) to the right such that at the initial level of prices, output falls from point H to point M . Thus, the value of collateral is lower, which implies that at the initial level of investment, the curve FF shifts upwards from point E to B . This results in a lower repayment probability and a higher loan rate. As in the previous case, the drop in output leads to excess demand in the goods market (at the initial level of prices) and therefore the lending rate must increase further to bring investment down and restore equilibrium in the goods market. Nonetheless, because a rise to point B' is not feasible, the price level must increase which, in turn, lowers the level of consumption (through a negative wealth effect), but leads to a gradual recovery of output from point M to H' . The rise in output raises the effective collateral pledged by firms and consequently mitigates the rise in the loan rate. Ultimately, the new equilibrium point is characterized by a *higher* lending rate (point E'), *lower* investments, *higher* level of prices and a *lower* level of output (compared to the initial equilibrium point H).

However, it is also possible for the new equilibrium to be characterized by a *lower* loan rate and *higher* prices following a negative productivity shock. This scenario may occur if the GG curve shifts by a large amount (such that investments are not very sensitive to changes in the loan rate), while the FF curve shifts by a small amount (which happens if the lending rate adjusts slowly to changes in effective

collateral). Suppose the GG curve shifts to the right by the same amount to GG' , but the FF curve shifts only slightly to FF'' . As shown in Figure 1.3, the new equilibrium point in this case corresponds with point E'' , where the price level is still *higher* but the loan rate is *lower*. Hence, when the collateral channel dominates the bank capital channel, the lending rate may be either procyclical or countercyclical with respect to productivity shocks ($\frac{di^L}{dA} \leq 0$). In other words, the lending rate may amplify the initial movement in output (procyclicality case when $\frac{di^L}{dA} < 0$), or mitigate the initial drop in output (countercyclicality case when $\frac{di^L}{dA} > 0$).¹³

To make things clearer, note that a negative productivity shock and a higher level of prices have contradicting effects on the lending rate. On the one hand, a negative supply shock leads to a *deterioration* in effective collateral, *lowers* the repayment probability and *increases* the lending rate. On the other hand, the rise in prices associated with the negative productivity shock *improves* the effective collateral pledged by firms, *increases* the repayment probability and thus *lowers* the loan rate. This ambiguity exists regardless of the regulatory regime as it depends solely on the collateral channel, which dominates the bank capital channel in this section.

To investigate how the bank capital channel operates in this setting, the focus again is on how changes in the productivity level and prices impact the nominal value of risky loans and the risk weight. As shown in the financial market equilibrium section, *following (only) a negative supply shock*, Basel II always leads to a further rise in the loan rate compared to Basel I ($|FF_A^{II}| > |FF_A^I|$). This occurs due to the negative relationship between supply shocks and the risk weight under Basel II, as explained earlier.

However, it is important to note that prices play a crucial role in determining which regulatory regime is more procyclical than the other when examining the general equilibrium effects. More specifically, prices must rise to restore equilibrium in the goods market (following the drop in output). The higher prices increase the nominal value of risky loans, reduce the bank capital buffer, lower the repayment probability and raise the loan rate. This is evident in both Basel I and Basel II. Nevertheless, in Basel II, this rise in prices stimulates output, raises effective collateral pledged by firms, increases the repayment probability, which translates into a lower risk weight on loans. The fall in the risk weight mitigates the fall in the

¹³Although not shown and examined in Figure 1.3, it is also possible for the new equilibrium point to exhibit a *higher* lending rate and a *lower* price level (north-west to point E). This happens when the FF curve shifts by a large amount while the GG curve shifts only by a small amount. However, in this section the focus is solely on the cases where prices *increase* following a negative supply shock.

bank capital buffer, thereby dampening the initial increase in the lending rate (at the initial level of investment). Consequently, when examining *only* the effects of the bank capital channel on the lending rate *following a rise in prices*, Basel I may be more procyclical than Basel II.

Taking into account both the collateral channel and the bank capital channel, recall that the lending rate may either *rise* or *fall* following a negative supply shock. In the first scenario where the loan rate is procyclical with respect to supply shocks ($\frac{di^L}{dA} < 0$), both Basel I and Basel II *magnify* the initial rise in the loan rate (caused by the impact of the collateral channel). Nevertheless, when combining the effects of A and consequently P on i^L , it cannot be ascertained whether Basel II is always more procyclical than Basel I in a general equilibrium setup.

Alternatively, if the loan rate is countercyclical with respect to supply shocks ($\frac{di^L}{dA} > 0$), both regulatory regimes *mitigate* the initial drop in the loan rate (led by the improvement in effective collateral), as the increase in prices tends to raise the lending rate through the bank capital channel. Once again, taking into account both the effects of A and consequently P on i^L , it cannot be concluded which regulatory regime is more procyclical than the other when the impact of a negative supply shock is examined in a general equilibrium context.

The results under this section are very similar to the Agénor and Pereira da Silva (2012) paper, but what our model shows is that even with a bank capital channel transmitted through the impact of bank capital buffers on the loan rate, Basel I may still be more procyclical than Basel II.

1.4 Concluding Remarks

This chapter has studied the procyclical effects of bank capital regulation in a simple static macroeconomic model with credit market imperfections. The model combines elements from Agénor and Montiel (2008), Agénor and Pereira da Silva (2012) and Agénor, Alper and Pereira da Silva (2012) and defines Basel I and Basel II regulatory regimes, with a distinction made between the Standardized and the Foundation IRB approaches of Basel II. Under the Standardized approach the risk weight on loans is related to the output supply, while under the Foundation IRB approach, the risk weight is a function of the repayment probability, which, in turn, is embedded in the lending rate charged by the commercial bank. Thus, in contrast to Basel I, the risk weights on loans under both variants of Basel II are endogenous, and are affected by changes in output and prices.

The *bank capital channel* in this model assumes that bank capital buffers induces

the commercial bank to screen and monitor its borrowers more carefully, thus raising the repayment probability and allowing the bank to set a lower loan rate. Empirically, this idea is supported by Fonseca, Gonzalez and Pereira da Silva (2010) and theoretically by the micro founded models of Allen, Carletti and Marquez (2011) and Mehran and Thakor (2009). In our model, and similar in spirit to Agénor, Alper and Pereira da Silva (2012), a reduced form formula relating the repayment probability to total bank capital (or bank capital buffers in our model) is used as a helpful and convenient shortcut to conduct the macroeconomic analysis of this chapter.

This model also illustrates the differences in the transmission processes of the bank capital channel under the various regulatory regimes. Specifically, under both variants of Basel II, a supply shock is not only transmitted through the impact of effective collateral on the loan rate (*the collateral channel*), but also through its effect on the endogenous risk weights. Moreover, changes in the price level (associated with productivity shocks) have a substantial impact on the bank capital channel and the degree of procyclicality of the different regulatory regimes.

Examining the general equilibrium effects, it is shown that when the *bank capital channel dominates the collateral channel*, then the lending rate is *always procyclical* following supply shocks. Under this scenario, it is crucial to know the direction in which prices fluctuate in order to rank between the procyclical effects of the different regulatory regimes. Nevertheless, when the *collateral channel is stronger than the bank capital channel*, the loan rate may be either *procyclical or countercyclical* following supply shocks. In this case the conclusion is that it *cannot* be ascertained whether Basel II is more procyclical than Basel I.

These results can have meaningful policy implications in the wake of the new Basel III agreements, enforcing banks to hold countercyclical bank capital buffers with an aim to mitigate systemic risk and procyclicality. Our findings in this simple setup show that a strong bank capital buffer channel may not necessarily achieve this initial objective, especially when the monitoring incentive effect matters or when the bank capital channel, driven by the cyclical risk weight, is stronger than the collateral effect. The general equilibrium framework presented in this chapter shows that a high sensitivity of the repayment probability with respect to bank capital buffers can actually amplify a negative financial and business cycle, associated with increased borrowing costs, higher prices and falling levels of GDP. Standard results suggest that the relationship between monitoring incentive, bank capital buffers and loan default prevails in banking systems where banks are unable to diversify risk in their balance sheets, or in economies which face more severe financial frictions (see

Fonseca, Gonzalez and Pereira da Silva 2010). The recent crisis experience has clearly highlighted that credit market distortions and financial regulation play a key role in the determination of the real business cycle for both developing and advanced economies. As a result, regulators should carefully assess the impact of bank capital buffers and its interaction with the risk weight channel (affected by both macroeconomic and financial conditions in Basel II) when implementing new regulatory measures.

This analysis can be extended in the following main directions: First, as noted above, the relationship between bank capital buffers, monitoring scrutiny and the repayment probability is of a reduced form and not endogenously derived. A useful and important extension would be to implement microeconomic foundations to derive the bank capital channel in our macroeconomic model, which may build upon the micro foundations of Allen, Carletti and Marquez (2011) and Mehran and Thakor (2009), as already mentioned. Furthermore, holding bank capital buffers can also be motivated by Repullo and Suarez's (2013) model, where banks build capital buffers during good times in order to avoid a significant contraction in lending during a recession. However, this idea can only be implemented in a dynamic setting which leads us to the second useful extension to our model; extending our static framework to a Dynamic Stochastic General Equilibrium (DSGE) model. Extending our model to a DSGE framework with an explicit endogenous derivation of the relationship between bank capital buffers and incentives to monitor and/or the linkage between capital buffers and anticipation of bad times will provide, in our opinion, an original contribution to this line of research.

1.A Appendix

1.A.1 *The Effect of Prices on the Loan Rate under Basel I*

Taking the total derivative of equation (1.33) with respect to P yields the following,

$$\frac{di^L}{dP} = \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ \frac{\left[-\varphi_1 \left(\frac{\kappa Y_P^s I - I' \frac{di^L}{dP} \kappa Y^s}{I^2} \right) - \varphi_2 \left(\frac{-(\rho \sigma P I' \frac{di^L}{dP} + \rho \sigma I) \bar{E}}{(\rho \sigma P I)^2} \right) \right]}{\left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2} \right\},$$

rearranging,

$$\begin{aligned} & \frac{di^L}{dP} \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 \\ &= \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ \varphi_1 \frac{\kappa Y^s I' \frac{di^L}{dP}}{I^2} - \varphi_1 \frac{\kappa Y_P^s}{I} + \varphi_2 \frac{I' \frac{di^L}{dP} \bar{E}}{\rho \sigma P I^2} + \varphi_2 \frac{\bar{E}}{P^2 \sigma \rho I} \right\}, \end{aligned}$$

or,

$$\begin{aligned} & \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 \frac{di^L}{dP} - \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa Y^s I'}{I^2} \frac{di^L}{dP} + \varphi_2 \frac{I' \bar{E}}{\rho \sigma P I^2} \frac{di^L}{dP} \right] \\ &= -\frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa}{I} Y_P^s - \varphi_2 \frac{\bar{E}}{P^2 \sigma \rho I} \right], \end{aligned}$$

taking $\frac{di^L}{dP}$ as a common factor in the left hand side,

$$\begin{aligned} & \left\{ \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 - \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa Y^s I'}{I^2} + \varphi_2 \frac{I' \bar{E}}{\rho \sigma P I^2} \right] \right\} \frac{di^L}{dP} \\ &= -\frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa}{I} Y_P^s - \varphi_2 \frac{\bar{E}}{P^2 \sigma \rho I} \right]. \end{aligned}$$

Finally, solving for $\frac{di^L}{dP}$ results in equation (1.35).

1.A.2 *The Effect of Supply Shocks on the Loan Rate under Basel I*

Computing the total derivative of equation (1.33) with respect to A yields the following,

$$\frac{di^L}{dA} = \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ \frac{\left[-\varphi_1 \left(\frac{\kappa Y_A^s I - \kappa Y^s I' \frac{di^L}{dA}}{I^2} \right) - \varphi_2 \left(\frac{-\rho \sigma P I' \frac{di^L}{dA} \bar{E}}{(\rho \sigma P I)^2} \right) \right]}{\left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2} \right\},$$

rearranging,

$$\left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 \frac{di^L}{dA} = \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ -\varphi_1 \frac{\kappa Y_A^s}{I} + \varphi_1 \frac{\kappa Y^s I' \frac{di^L}{dA}}{I^2} + \varphi_2 \frac{\bar{E} I' \frac{di^L}{dA}}{\rho \sigma P I^2} \right\},$$

or,

$$\begin{aligned} & \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 \frac{di^L}{dA} - \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa Y^s I' \frac{di^L}{dA}}{I^2} + \varphi_2 \frac{\bar{E} I' \frac{di^L}{dA}}{\rho \sigma P I^2} \right] \\ &= -\frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \varphi_1 \frac{\kappa Y_A^s}{I}, \end{aligned}$$

taking $\frac{di^L}{dA}$ as a common factor in the left hand side,

$$\begin{aligned} & \left\{ \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 - \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa Y^s I'}{I^2} + \varphi_2 \frac{I' \bar{E}}{\rho \sigma P I^2} \right] \right\} \frac{di^L}{dA} \\ &= -\frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \varphi_1 \frac{\kappa Y_A^s}{I}. \end{aligned}$$

Finally, solving for $\frac{di^L}{dA}$ results in equation (1.36).

1.A.3 *The Effect of Prices on the Loan Rate under Basel II*

Taking the total derivative of equation (1.33) with respect to P under Basel II, where the risk weight is endogenous, yields,

$$\frac{di^L}{dP} = \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ \frac{-\varphi_1 \left(\frac{\kappa Y_P^s I - \kappa Y^s I' \frac{di^L}{dP}}{I^2} \right) - \varphi_2 \left(\frac{-[(\sigma_P P + \sigma)\rho I + I' \frac{di^L}{dP} \rho \sigma P] \bar{E}}{(\rho \sigma P I)^2} \right)}{\left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2} \right\},$$

where $\sigma_P = \frac{d\sigma}{dP}$, and σ depends either on output supply or the repayment probability, which, in turn, is related to both prices and output. Solving the above equation for $\frac{di^L}{dP}$ and using algebraic manipulations yields,

$$\begin{aligned} & \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 \frac{di^L}{dP} \\ &= \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ -\varphi_1 \frac{\kappa Y_P^s}{I} + \varphi_1 \frac{\kappa Y^s I' \frac{di^L}{dP}}{I^2} + \varphi_2 \frac{(\sigma_P P + \sigma) \bar{E}}{\rho I \sigma^2 P^2} + \varphi_2 \frac{I' \frac{di^L}{dP} \bar{E}}{\rho \sigma P I^2} \right\}, \end{aligned}$$

or,

$$\begin{aligned} & \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 \frac{di^L}{dP} \\ &= \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ -\varphi_1 \frac{\kappa Y_P^s}{I} + \varphi_1 \frac{\kappa Y^s I' \frac{di^L}{dP}}{I^2} + \varphi_2 \frac{\sigma_P \bar{E}}{\rho P I \sigma^2} + \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} + \varphi_2 \frac{I' \frac{di^L}{dP} \bar{E}}{\rho \sigma P I^2} \right\}, \end{aligned}$$

or,

$$\begin{aligned} & \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 \frac{di^L}{dP} - \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa Y^s I' \frac{di^L}{dP}}{I^2} + \varphi_2 \frac{I' \bar{E}}{\rho \sigma P I^2} \frac{di^L}{dP} \right] \\ &= \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ -\varphi_1 \frac{\kappa Y_P^s}{I} + \varphi_2 \frac{\sigma_P \bar{E}}{\rho I P \sigma^2} + \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} \right\}, \end{aligned}$$

taking $\frac{di^L}{dP}$ as a common factor in the left hand side,

$$\begin{aligned} & \left\{ \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 - \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa Y^s I'}{I^2} + \varphi_2 \frac{I' \bar{E}}{\rho \sigma P I^2} \right] \right\} \frac{di^L}{dP} \\ &= \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ -\varphi_1 \frac{\kappa Y_P^s}{I} + \varphi_2 \frac{\sigma_P \bar{E}}{\rho I P \sigma^2} + \varphi_2 \frac{\bar{E}}{\rho I \sigma P^2} \right\}. \end{aligned}$$

Finally, solving for $\frac{di^L}{dP}$ results in equation (1.37).

1.A.4 *The Effect Supply Shocks on the Loan Rate under Basel II*

Computing the total derivative of equation (1.33) with respect to A under Basel II, where the risk weight is endogenous, results in,

$$\frac{di^L}{dA} = \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ \frac{-\varphi_1 \left(\frac{\kappa Y_A^s I - \kappa Y^s I' \frac{di^L}{dA}}{I^2} \right) - \varphi_2 \left(\frac{-\rho P(\sigma_A I + \sigma I' \frac{di^L}{dA}) \bar{E}}{(\rho \sigma P I)^2} \right)}{\left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2} \right\},$$

rearranging,

$$\begin{aligned} & \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 \frac{di^L}{dA} \\ &= \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ -\varphi_1 \frac{\kappa Y_A^s}{I} + \varphi_1 \frac{\kappa Y^s I' \frac{di^L}{dA}}{I^2} + \varphi_2 \frac{\sigma_A \bar{E}}{\sigma^2 \rho P I} + \varphi_2 \frac{I' \frac{di^L}{dA} \bar{E}}{\rho \sigma P I^2} \right\}, \end{aligned}$$

or,

$$\begin{aligned} & \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 \frac{di^L}{dA} - \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa Y^s I'}{I^2} \frac{di^L}{dA} + \varphi_2 \frac{I' \bar{E}}{\rho \sigma P I^2} \frac{di^L}{dA} \right] \\ &= \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ -\varphi_1 \frac{\kappa Y_A^s}{I} + \varphi_2 \frac{\sigma_A \bar{E}}{\sigma^2 \rho P I} \right\}, \end{aligned}$$

taking $\frac{di^L}{dA}$ as a common factor in the left hand side,

$$\begin{aligned} & \left\{ \left[\varphi_1 \left(\frac{\kappa Y^s}{I} \right) + \varphi_2 \left(\frac{\bar{E}}{\rho \sigma P I} \right) \right]^2 - \frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left[\varphi_1 \frac{\kappa Y^s I'}{I^2} + \varphi_2 \frac{I' \bar{E}}{\rho \sigma P I^2} \right] \right\} \frac{di^L}{dA} \\ &= -\frac{(1+i^R)}{\left(1+\frac{1}{\eta^I}\right)} \left\{ \varphi_1 \frac{\kappa Y_A^s}{I} - \varphi_2 \frac{\sigma_A \bar{E}}{\sigma^2 \rho P I} \right\}. \end{aligned}$$

Finally, solving for $\frac{di^L}{dA}$ results in equation (1.39).

Chapter 2

Bank Capital Regulation, Credit Frictions and Macroeconomic Dynamics with Endogenous Default

2.1 Introduction

The global financial crisis of 2007-2009 and the subsequent recession have led to a renewed debate about the nature of bank capital regulation, and to what extent it impacts the credit market and real economy. Since 1988, banking regulation has been associated with the Basel regulatory standards which have raised many concerns regarding whether such type of regulation can increase the degree of procyclicality already inherent in the banking system. According to these standards, banks must maintain at least 8% total bank capital out of their total risk weighted assets (loans). Particular attention has been given to Basel II, adopted in 2004, where risk weights are no longer constant under a specific loan category (as in Basel I), but depend also on the borrower's risk profile. More specifically, under Basel II, banks can calculate risk weights using either the Foundation Internal Ratings Based (IRB) approach or the Standardized approach.¹ The former allows banks to use the estimated probability of default of a specific borrower while the latter depends on credit ratings agencies, whose ratings tend to be procyclical.² Therefore, the Standardized approach is often linked with the nature of the business cycle or the

¹In fact, the Foundation IRB approach of Basel II is a subcategory of the IRB approach. Banks can choose within this approach either the *foundation* format or the *advanced* format. For the purpose of this paper we consider only the *foundation* approach, where banks calculate their risk weight using the estimated probability of default. For details on the advanced format see Basel Committee on Banking Supervision (2004).

²Drumond (2009) shows that these external ratings are indeed procyclical.

output gap in macroeconomic models (see for example Angeloni and Faia 2013).³

Given that risk weights on loans can change throughout the business cycle, Basel II may amplify the procyclical effects compared to Basel I (see Kashyap and Stein 2004). For example, if lending becomes riskier following adverse shocks, banks will have to issue more bank capital to satisfy regulation (which entails additional costs) or, failing that, to engage in credit rationing. Consequently, more volatile risk weights may aggravate a credit crunch, thereby exacerbating an economic recession and making capital requirements more procyclical. A few Dynamic Stochastic General Equilibrium (DSGE) models which support this argument include Aguiar and Drumond (2009), Liu and Seeiso (2012), Covas and Fujita (2010) and De Walque, Pierrard and Rouabah (2010). The first two models, which build upon the Bernanke, Gertler and Gilchrist (1999) framework, show that the liquidity premium for holding bank capital under Basel II further amplifies the external finance premium channel, which contributes to additional procyclicality. Covas and Fujita (2010) assume that raising funds through bank capital is more costly than raising it through deposits, with the issuance costs of bank capital increasing during recessions. In their model, Basel II is only slightly more procyclical than Basel I and the differences are mainly observed around the business cycles peaks and troughs. De Walque, Pierrard and Rouabah (2010) also study the role of bank capital in the transmission of various shocks and show that Basel II amplifies the effects of macroeconomic shocks.⁴

Furthermore, in a related model, Markovic (2006) develops a framework in which banks must issue bank capital to satisfy regulatory requirements, but does not examine the additional cyclical effects of risk sensitive regulation induced by Basel II. However, the model does suggest that various bank capital channels lead to changes in the cost of bank capital, which contribute to the corporate balance sheet channel and amplify the response of monetary shocks.

Other authors focus more on the role of bank capital in the transmission of various shocks, without necessarily examining the impact of financial regulation. Such models, including Aikman and Paustian (2006), Meh and Moran (2010) and Gerali, Neri, Sessa and Signoretti (2010), find that different types of bank capital channels have a sizeable impact in the transmission of supply, monetary and finan-

³The risk weighting scheme remains essentially the same under the new regulatory framework - Basel III.

⁴Nevertheless, in another recent contribution, Agénor, Alper and Pereira da Silva (2012) introduce a monitoring incentive effect in which bank capital increases incentives for banks to monitor borrowers, thereby reducing the likelihood of default. In their framework, Basel I may actually be more procyclical than Basel II following various shocks.

cial shocks, and may act as accelerator or attenuation mechanisms. Specifically, in Aikman and Paustian (2006) and Meh and Moran (2010), bank capital arises endogenously to solve an asymmetric information problem between banks and their creditors. This bank capital channel is shown to amplify and propagate the effects of shocks on output, inflation and investments. Nevertheless, Gerali, Neri, Sessa and Signoretti (2010) show that introducing an imperfectly competitive banking sector, where bank capital is accumulated from retained earnings, actually attenuates the effects of demand shocks but amplifies the impact of supply and financial shocks. Overall, including a banking sector with a role for bank capital (for either market determined or regulatory reasons) has important implications for explaining the transmission process of various shocks.

The contribution of this chapter is to combine insights from the existing studies along with an endogenous formation of risk of default at the firm and bank capital levels, which produce important various linkages between regulatory bank capital requirements, the financial system and the real economy. In particular, this model examines the different transmission channels of bank capital and default risk following supply, monetary and financial shocks when banks are subjected to regulatory requirements in the form of the Basel Accords, all within a DSGE framework. For this purpose, we augment the recent contribution of Agénor, Bratsiotis and Pfajfar (2013) by introducing bank capital risk, regulatory requirements and financial shocks, which directly affect the cost of borrowing and the degree of risk in the financial system.⁵

A key feature of this study is the derivation of the loan rate and probability of default from break even conditions. It is shown that when bank capital is introduced, these variables become endogenously related to the rate on return on bank capital and the bank capital-loan ratio.⁶ The latter, in turn, may be constant (under Basel I), or determined by the probability of default under Basel II. Moreover, when the commercial bank sets the loan rate to firms, it takes into account that some firms may default following unfavourable idiosyncratic shocks. Thus, the lending rate depends on the probability of default also through the finance risk premium, which is set above the weighted average costs of paying back principal plus interest

⁵Angeloni and Faia (2013) also introduce bank capital risk and risk sensitive bank capital requirements in a DSGE model with optimizing banks and bank runs. However, our model differs significantly from their contribution, making comparisons difficult.

⁶Agénor, Alper and Pereira da Silva (2012) and Liu and Seeiso (2012) also relate the loan rate to the rate of return on bank capital and the bank capital-loan ratio. However, in both these models, the loan rate is derived from an optimization problem whereas in our model, the loan rate *and* probability of default are derived from break even conditions, and are endogenously related to one another.

on deposits and bank capital to households. Deposits and bank capital are used to finance the working capital needs of the intermediate good firms and act as liabilities to households, as they are assumed to own the bank.

Furthermore, we identify a bank capital default channel arising from the possibility of banks defaulting on their capital due to the positive default probability at the firm level. This creates an endogenous spread (related to risk) between the rate of return on bank capital and the interest rate on deposits. Hence, this model contributes to Markovic (2006) and Covas and Fujita (2010), who refrain from deriving an endogenous spread linking the cost of bank capital to the cost of deposits. The bank capital default channel in our model is shown to have substantial effects on the business cycle, thus providing another linkage between the financial sector and the real economic activity.

We also compare between Basel I (exogenous risk weight) and the Foundation IRB approach of Basel II (endogenous risk weight) and show that the latter amplifies the response of the key variables following various shocks, consistent with most of the literature and empirical evidence. In addition, the risk weight channel associated with Basel II, which is determined by the risk of default, is shown to be particularly strong when the source of shocks originate from the financial system.

Overall, the endogenous default probability produces an accelerator mechanism in this model, and impacts the loan rate and real economy through various channels; the risk premium channel, the bank capital default channel and the risk weight channel. As in Markovic (2006), we also study the effects of the adjustment cost channel of bank capital (or the cost of adjusting bank capital to its required level), arising from information asymmetry between banks and their shareholders. This channel is shown to have an ambiguous impact on the degree of procyclicality, depending on the strength of the risk weight channel relative to the intermediate good firms credit demand channel. Both the risk weight and the demand for loans translate to direct changes in the volume of bank capital, resulting in adjustment costs alterations borne by households. Finally, and as is proposed under the new regulatory framework (Basel III), we examine a permanent rise in the bank capital-loan ratio and illustrate that raising the regulatory requirements results in increased procyclicality of the key variables.

This chapter proceeds as follows. Section 2.2 presents the model with a detailed analysis of the agents behaviour and the market clearing conditions. The steady state equations and the log-linearized model are presented in sections 2.3 and 2.4, respectively. Section 2.5 provides a discussion on the parameter calibration. Section 2.6 simulates the model following supply, monetary and financial shocks, with

an elaborate explanation on the various transmission channels of the probability of default and bank capital. The role of the adjustment cost channel and the implications of higher regulatory requirements are discussed as well. Finally, section 2.7 concludes and considers some possible extensions of the analysis.

2.2 The Model

Consider an economy consisting of six types of agents: households (who are also labour suppliers), a final good (FG) firm, intermediate good (IG) firms, a competitive labour contractor, a competitive commercial bank and a central bank.⁷

At the beginning of the period and following the realization of aggregate shocks, the representative bank receives deposits from households, issues bank capital to satisfy regulatory requirements and makes a decision on the lending rate. The loan rate decision is based on the idiosyncratic nature of the borrowers (IG firms), and the expected costs of paying back gross interest on deposits and bank capital to households.⁸ At the same time, households choose the level of consumption, deposits and bank capital based on their total income comprised of distributed profits, total returns from holding bank capital and deposits from the previous period, and labour income. Additionally, households are assumed to examine the bank's financial position before investing in new bank capital, and therefore incur additional adjustment (or search) costs. Indeed, if the bank must raise fresh regulatory bank capital, then it sends a bad signal to creditors regarding its financial situation, leading households to require a higher return for holding bank capital. Also at the start of the period and given the going loan rate, the IG firms decide on the level of employment, prices and loans, with the latter used to fund wage payments to households, via the labour contractor.⁹

At the end of the period, the idiosyncratic shocks and hence the firms who default are revealed. As the production of intermediate goods is subject to total

⁷ A single financial intermediary (commercial bank) is assumed in this setup in order to simplify notations. The results would remain unchanged if there were many identical banks which operated in a perfectly competitive environment and earned zero profits.

⁸ As bank capital prices and dividend policies resulting from changes in the price of equity are not modeled in this framework, bank capital in our model is treated more like bank debt rather than equity. In the Basel terminology, bank capital in this model therefore consists of "tier 2" capital and not "tier 1" capital, which consists of equity stock and retained earnings. Nevertheless, there is still an ongoing debate on whether under the new Basel III regulatory rules, banks would also be allowed to hold capital in the form of loss-absorbing debt such as contingent convertible bonds.

⁹ The labour contractor supplies homogeneous labour to IG firms by aggregating the differentiated labour provided from households and paying each one of them its wages. Thus, the labour aggregator simply acts as an intermediary between households and IG firms.

supply and idiosyncratic productivity shocks, there is a positive probability of IG firms defaulting on their loans. The IG firms must therefore pledge a fraction of output as collateral, which is seized by the commercial bank in case of default. We also introduce a probability of default on bank capital, which is determined by the estimated aggregate risk of default at the IG firm's level. In a default scenario, bank capital holders absorb the risk while depositors are insured through the collateral seized by the commercial bank. In this way, the goods market equilibrium is also maintained. Furthermore, because bank capital bears a risk of default, households demand a higher return on bank capital when making their consumption, deposits and bank capital decisions at the beginning of the period.

Finally, at the end of the period the commercial bank pays back gross return to households on deposits and bank capital (given the probability of default), and all profits are distributed to households. We now turn to describe in more detail the behaviour of each agent in the economy.

2.2.1 Households

There is a continuum of households, indexed by $i \in (0, 1)$, who consume, hold deposits, demand bank capital and supply differentiated labour to a labour aggregator.

The objective of each household i is to maximize the following utility function,

$$U_t = E_{i,t} \sum_{s=0}^{\infty} \beta^s \left\{ \frac{[C_{t+s}]^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} - \frac{H_{i,t+s}^{1+\gamma}}{1+\gamma} \right\}, \quad (2.1)$$

where $E_{i,t}$ is the expectations operator, conditional on the information of the i^{th} household available up to period t , and $\beta \in (0, 1)$ denoting the discount factor. The term C_t denotes consumption at time t while $H_{i,t}$ represents the time- t hours worked by household i . The term ς stands for the intertemporal elasticity of substitution in consumption, while γ denotes the inverse of the Frisch elasticity of labour supply.

Households hold bank capital, $V_t \equiv \frac{V_t^{Nom}}{P_t}$, which pays an interest of i_t^V , and bank deposits, $D_t \equiv \frac{D_t^{Nom}}{P_t}$, which bear an interest rate of i_t^D .¹⁰ Hence, total returns from holding bank capital and deposits in period $t-1$ are respectively given by $(1 - \Phi_{t-1}^V)(1 + i_{t-1}^V)V_{t-1} \frac{P_{t-1}}{P_t}$ and $(1 + i_{t-1}^D)D_{t-1} \frac{P_{t-1}}{P_t}$. The term Φ_{t-1}^V denotes the risk of default on bank capital which is derived endogenously later in the text, but taken as given in the household's optimization problem. Also, in subsequent sections we explain why the risk of default on bank capital must be equal to the probability

¹⁰ All variables are defined in real terms where P_t denotes the price of the final good and superscript *Nom* stands for nominal variables.

of firms defaulting on their loans (Φ_t). Deposits, on the other hand, are safe and insured against default risk through the firm's collateral seized by the bank, which is paid to households as a lump sum transfer in times of default.

At the start of period t , households choose the level of deposits and bank capital, which are used by the bank as resources for lending to IG firms. Moreover, following Markovic (2006) and Dib (2010), changing bank capital across periods involves quadratic real adjustment costs of the form $\frac{\Theta}{2} \left(\frac{V_t}{V_{t-1}} - 1 \right)^2 V_{t-1} \frac{P_{t-1}}{P_t}$, with $\Theta > 0$ denoting the adjustment cost parameter.¹¹ These costs can be interpreted as transaction costs associated with payments to brokers for entering the financial markets, or 'search costs' paid to credit rating agencies for collecting information about the bank's balance sheet position prior to bank capital investment decisions. These costs therefore highlight the information asymmetry between banks and their shareholders, and are also assumed to be a deadweight loss for society. Furthermore, the asymmetric information between households (who are also the bank owners) and the financial intermediary justifies the symmetric nature of the quadratic cost function with respect to increasing or decreasing volumes of bank capital. In other words, it is the relative size of the transaction that matters for the adjustment cost as opposed to its sign. The higher the size of the transaction in terms of absolute changes in the volume of bank capital, the higher is the adjustment cost.¹²

Following the realization of the aggregate shocks, each household supplies differentiated labour to the labour aggregator and earns a factor payment of $\frac{W_{i,t}}{P_t} H_{i,t}$ (where $W_{i,t}$ is the nominal wage, and P_t the price of final good). At the end of the period, households receive all profits from IG firms, $J_t^{IG} = \int_0^1 J_{j,t}^{IG} dj$, and pay a lump-sum tax given by $Lump_t$ (in real terms).¹³

Finally, once the value of the final good is realized at the end of the period, the representative household purchases it for consumption purposes.¹⁴ Thus, the

¹¹The decision on bank capital is done at the *beginning* of period t , which is assumed to be arbitrarily close to the *end* of period $t - 1$. Therefore, the adjustment cost function is written in terms of bank capital from period $t - 1$.

¹²With a linear cost function, for example, the adjustment cost for bank capital at the margin would not be affected by the volume of bank capital. Hence, the financial strength of the bank would have no impact on the dynamics of the adjustment cost function and consequently on the financial system.

¹³Households also own the bank and the final good firm but these two economic agents earn zero profits in equilibrium, as noted below.

¹⁴This model abstracts from physical capital accumulation and investments.

household's (real) budget constraint can be written as follows,

$$C_t + D_t + V_t \leq (1 + i_{t-1}^D)D_{t-1}\frac{P_{t-1}}{P_t} + (1 - \Phi_{t-1}^V)(1 + i_{t-1}^V)V_{t-1}\frac{P_{t-1}}{P_t} + \quad (2.2)$$

$$+ \frac{W_{i,t}}{P_t}H_{i,t} + \int_0^1 J_{j,t}^{IG}dj - Lump_t - \frac{\Theta}{2} \left(\frac{V_t}{V_{t-1}} - 1 \right)^2 V_{t-1} \frac{P_{t-1}}{P_t}.$$

Consumption, Savings and Bank Capital Decisions

The Lagrangian associated with maximizing (2.1) with respect to (2.2) takes the following form (with φ_t denoting the Lagrange multiplier),

$$\Lambda = E_t^j \sum_{t=0}^{\infty} \beta^t \left\{ \varphi_t \left[\begin{aligned} & \frac{[C_t]^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} - \frac{H_{i,t}^{1+\gamma}}{1+\gamma} + \\ & (1 + i_{t-1}^D)D_{t-1}\frac{P_{t-1}}{P_t} + (1 - \Phi_{t-1}^V)(1 + i_{t-1}^V)V_{t-1}\frac{P_{t-1}}{P_t} + \frac{W_{i,t}}{P_t}H_{i,t} + \\ & + J_t^{Bank} + \int_0^1 J_{j,t}^{IG}dj - Lump_t - \frac{\Theta}{2} \left(\frac{V_t}{V_{t-1}} - 1 \right)^2 V_{t-1} \frac{P_{t-1}}{P_t} - C_t - \\ & - D_t - V_t \end{aligned} \right] \right\}.$$

The first order conditions with respect to C_t , D_t and V_t (taking the rate of returns and prices as given) are respectively given by,

$$C_t^{-\frac{1}{\varsigma}} - \varphi_t = 0, \quad (2.3)$$

$$E_t \beta^{t+1} \varphi_{t+1} (1 + i_t^D) \frac{P_t}{P_{t+1}} - \beta^t \varphi_t = 0, \quad (2.4)$$

$$\varphi_t \left\{ \Theta \frac{P_{t-1}}{P_t} \left[1 - \frac{V_t}{V_{t-1}} \right] - 1 \right\} = \beta E_t \varphi_{t+1} \frac{P_t}{P_{t+1}} \left\{ \begin{aligned} & \frac{\Theta}{2} \left[1 - \left(\frac{V_{t+1}}{V_t} \right)^2 \right] - \\ & - (1 - \Phi_t^V)(1 + i_t^V) \end{aligned} \right\}. \quad (2.5)$$

From equations (2.3) and (2.4),

$$C_t^{-\frac{1}{\varsigma}} = \beta E_t (1 + i_t^D) \frac{P_t}{P_{t+1}} C_{t+1}^{-\frac{1}{\varsigma}}, \quad (2.6)$$

which is a standard Euler equation determining the optimal consumption path.

Combining equations (2.3), (2.5) and (2.6) yields the relationship between the return on bank capital and the risk free deposit rate (the no-arbitrage condition),

$$(1 - \Phi_t^V)(1 + i_t^V) + \frac{\Theta}{2} \left[E_t \left(\frac{V_{t+1}}{V_t} \right)^2 - 1 \right] = (1 + i_t^D) \left[1 + \frac{P_{t-1}}{P_t} \Theta \left(\frac{V_t}{V_{t-1}} - 1 \right) \right]. \quad (2.7)$$

Therefore, the interest rate on bank capital in the short run depends on the deposit rate, the adjustment costs of changing bank capital across periods and the

probability of default. Because bank capital provides a higher return than deposits, households also want to hold bank capital despite the risk associated with this asset. In addition, if the bank decides to default on part of its bank capital payments to households, bank capital holders must absorb these financial losses. Note that without the adjustment cost and bank capital default channels ($\Theta = 0$ and $\Phi_t^V = 0$), bank capital and deposits become perfect substitutes which yield the same rate of return ($i_t^V = i_t^D$).¹⁵

The Wage Decision

The wage setting environment follows Erceg, Henderson and Levin (2000), and Christiano, Eichenbaum and Evans (2005), where each household i supplies a unique type of labour ($H_{i,t}$) with $i \in (0, 1)$. All these types of labour are then aggregated by a competitive labour contractor into one composite homogenous labour (N_t) using the standard Dixit-Stiglitz (1977) technology given by,

$$N_t = \left(\int_0^1 H_{i,t}^{\frac{\lambda_w-1}{\lambda_w}} di \right)^{\frac{\lambda_w}{\lambda_w-1}}, \quad (2.8)$$

with $\lambda_w > 1$ representing the constant elasticity of substitution between the different types of labour. The i^{th} household therefore faces the following demand curve for its labour,

$$H_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\lambda_w} N_t, \quad (2.9)$$

where W_t denotes the aggregate nominal wage paid for one unit of the composite labour. The zero profit condition for the labour aggregator, obtained by substituting (2.9) in (2.8), yields the economy wide wage equation,

$$W_t = \left[\int_0^1 W_{i,t}^{1-\lambda_w} di \right]^{\frac{1}{1-\lambda_w}}. \quad (2.10)$$

Calvo (1983)-type nominal rigidities is assumed in the wage setting such that in each period a constant fraction of $1 - \omega_w$ workers are able to re-optimize their wages while a fraction of ω_w index their wages according to last period's price inflation rate (π_{t-1}). These non re-optimizing households therefore set their wages according to $W_{i,t} = \pi_{t-1} W_{i,t-1}$. Moreover, if wages have not been set since period t , then at

¹⁵As bank capital prices are not modeled, the no-arbitrage condition does not depend on bank capital gains or losses resulting from changes in its prices. Alternatively, one could consider the rate of inflation as the change in the price of bank capital as this variable is treated in real terms throughout the paper. Other models which do not explicitly model bank capital prices include Aguiar and Drumond (2009), Zhang (2009) and Covas and Fujita (2010).

period $t + s$, the real relative wage for household i becomes $\frac{W_{i,t+s}}{W_{t+s}} = \frac{\Pi^s W_{i,t}}{W_{t+s}}$, where $\Pi^s = \pi_t \times \pi_{t+1} \times \dots \times \pi_{t+s-1}$. Consequently, the demand for labour in period $t + s$ is,

$$H_{i,t+s} = \left(\frac{\Pi^s W_{i,t}}{W_{t+s}} \right)^{-\lambda_w} N_{t+s}. \quad (2.11)$$

Households who are able to optimize their wages in each period set their wage to $W_{i,t}$, which is obtained by maximizing the following function subject to the budget constraint (2.2) and the demand for labour (2.11),

$$\max_{W_{i,t}} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s [\mathbf{U}(C_{t+s}) + \mathbf{V}(H_{i,t+s})],$$

where $\mathbf{U}(C_{t+s}) = \frac{[C_{t+s}]^{1-\varsigma-1}}{1-\varsigma-1}$ and $\mathbf{V}(H_{i,t+s}) = -\frac{H_{i,t+s}^{1+\gamma}}{1+\gamma}$.

The first order condition with respect to $W_{i,t}$ results in,¹⁶

$$E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\frac{\Pi^s W_{i,t}}{P_{t+s}} (\mathbf{U}'_C)_{t+s} - \frac{\lambda_w}{1-\lambda_w} (\mathbf{V}'_H)_{i,t+s} \right] H_{i,t+s} = 0. \quad (2.12)$$

In the case when all households are able to re-optimize ($\omega_w = 0$), equation (2.12) reduces to the condition where the real wages equals to the wage mark-up ($\frac{\lambda_w}{\lambda_w-1}$) multiplied by the marginal rate of substitution between leisure and consumption (MRS_t),

$$\frac{W_t}{P_t} = -\frac{\lambda_w}{\lambda_w-1} \frac{\mathbf{V}'_{H,t+s}}{\mathbf{U}'_{C,t+s}} = \frac{\lambda_w}{\lambda_w-1} MRS_t, \quad (2.13)$$

where $MRS_t = N_t^\gamma C_t^{\frac{1}{\varsigma}}$ and $N_t = H_t$.

In equilibrium all re-optimizing households choose the same wage (W_t^*), and the optimal relative wage in a log-linearized form (denoted by hat) is given by $\widehat{\left(\frac{W_t^*}{W_t}\right)} = \left(\frac{\omega_w}{1-\omega_w}\right) \widehat{\pi_t^W}$, with $\widehat{\pi_t^W} \equiv \widehat{W_t} - \widehat{W_{t-1}}$ denoting the log-linearized wage inflation.

Finally, as in Erceg, Henderson and Levin (2000) the wage inflation equation satisfies,

$$\widehat{\pi_t^W} = \beta E_t \widehat{\pi_{t+1}^W} + \frac{(1-\omega_w)(1-\beta\omega_w)}{(\omega_w)(1+\gamma\lambda_w)} \left[\widehat{MRS_t} - \widehat{\left(\frac{W_t}{P_t}\right)} \right], \quad (2.14)$$

where real wages evolve according to,

$$\widehat{W_t^R} \equiv \widehat{\left(\frac{W_t}{P_t}\right)} = \widehat{\left(\frac{W_{t-1}}{P_{t-1}}\right)} + \widehat{\pi_t^W} - \widehat{\pi_t^P}, \quad (2.15)$$

¹⁶The full derivation of the wage setting environment is provided in Appendix 2.A.1.

with $\widehat{\pi}_t^P \equiv \widehat{P}_t - \widehat{P}_{t-1}$ representing the log-linearized price inflation rate as a deviation from its steady state. The motivation for including sticky wages is twofold: First, sticky wages are necessary to match the sluggish and persistent behaviour of real wages observed in data, and are important for obtaining a persistent response of inflation without relying on implausible values for price stickiness (as in Christiano, Eichenbaum and Evans 2005). Second, wage stickiness is crucial for achieving a co-movement between output, real wages and labour following technology shocks, a feature which is difficult to capture in these class of models (see DiCecio 2009 for further details). Loans in this model are provided for working capital financing, and therefore in order to produce a plausible data consistent positive relationship between loan demand and GDP, it is essential to introduce wage rigidities. As shown later, real wages move countercyclically with the loan rate, and can mitigate a rise in credit risk following adverse shocks associated with higher borrowing costs. The behaviour of real wages is thus important for explaining part of the linkages between the financial system and real economy.

2.2.2 Final Good Firm

A perfectly competitive representative FG firm assembles a continuum of intermediate goods ($Y_{j,t}$ with $j \in (0, 1)$), to produce final output (Y_t) using the standard Dixit-Stiglitz (1977) technology,

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\lambda_p-1}{\lambda_p}} dj \right)^{\frac{\lambda_p}{\lambda_p-1}}, \quad (2.16)$$

where $\lambda_p > 1$ denotes the constant elasticity of substitution between the differentiated intermediate goods. The FG firm chooses the optimal quantities of intermediate goods that maximize its profits, taking as given both the prices of the intermediate goods ($P_{j,t}$) and the final good price (P_t). Therefore, the FG producer maximizes the following profit function,

$$\max_{Y_{j,t}} P_t \left(\int_0^1 Y_{j,t}^{\frac{\lambda_p-1}{\lambda_p}} dj \right)^{\frac{\lambda_p}{\lambda_p-1}} - \int_0^1 P_{j,t} Y_{j,t} dj.$$

Differentiating with respect to $Y_{j,t}$ yields,

$$P_t \frac{\lambda_p}{\lambda_p - 1} \left(\int_0^1 Y_{j,t}^{\frac{\lambda_p-1}{\lambda_p}} dj \right)^{\frac{1}{\lambda_p-1}} \frac{\lambda_p - 1}{\lambda_p} Y_{j,t}^{\frac{-1}{\lambda_p}} - P_{j,t} = 0.$$

Rearranging the above expression results in the demand function for each intermediate good,

$$Y_{j,t} = Y_t \left(\frac{P_{j,t}}{P_t} \right)^{-\lambda_p}. \quad (2.17)$$

Imposing the above zero profit condition (equation 2.17) into equation (2.16) yields the usual definition of the final good price,

$$P_t = \left[\int_0^1 P_{j,t}^{1-\lambda_p} dj \right]^{\frac{1}{1-\lambda_p}}. \quad (2.18)$$

2.2.3 Intermediate Good Firms

A continuum of IG producers, indexed by $j \in (0, 1)$, operate in a monopolistic environment and are subject to Calvo (1983)-type nominal rigidities in the price setting. Each IG firm uses the homogeneous labour supplied by the labour contractor, which, in turn, acts as an intermediary between households and IG firms. Each IG firm j faces the following linear production function,

$$Y_{j,t} = Z_{j,t} N_{j,t}, \quad (2.19)$$

where $N_{j,t}$ is the amount of *homogenous* labour employed by firm j , and $Z_{j,t}$ is the total productivity shock experienced by firm j . Moreover, $Z_{j,t}$ follows the process,

$$Z_{j,t} = A_t \varepsilon_{j,t}. \quad (2.20)$$

The term A_t denotes a common economy wide technology shock which follows the $AR(1)$ process, $A_t = (A_{t-1})^{\xi^A} \exp(\alpha_t^A)$, where ξ^A is the autoregressive coefficient, and α_t^A a normally distributed random shock with zero mean and a constant variance. The expression $\varepsilon_{j,t}$ represents an idiosyncratic shock with a constant variance distributed over the interval $(\underline{\varepsilon}, \bar{\varepsilon})$.

Every firm j must borrow from the representative commercial bank in order to pay households wages in advance. Specifically, let $L_{j,t} \equiv \frac{L_{j,t}^{Nom}}{P_t}$ be the amount borrowed by firm j , then the (real) financing constraint must equal to,

$$L_{j,t} = W_t^R N_{j,t}. \quad (2.21)$$

The Default Space

Financing working capital needs bears risk and in case of default the commercial bank expects to seize a fraction χ_t of the firms' output, with $\chi \in (0, 1)$ denoting the

steady state value of this fraction. The term χ_t is assumed to follow the $AR(1)$ shock process, $\chi_t = (\chi_{t-1})^{\xi^x} \exp(\alpha_t^x)$, where ξ^x denotes the degree of persistence while α_t^x is a random shock with a normal distribution and a constant variance. A shock to effective collateral (χ_t) represents a financial (credit) shock in this model, as it affects the amount the bank can seize in case of default, and therefore is essentially a direct shock to risk.

In the good states of nature, where the firms do not default, each firm pays back the commercial bank principal plus interest on the loans granted. Consequently and in line with the willingness to pay approach to debt contracts, default occurs when the real collateral value pledged by firms is less than the amount that needs to be repaid to the bank at the end of the period. Specifically,

$$\chi_t Y_{j,t} < (1 + i_t^L) L_{j,t}, \quad (2.22)$$

where i_t^L denotes the interest rate on loans granted to IG firms. Similar to Agénor and Aizenman (1998), we assume for simplicity that no IG firm defaults if the economy is at the good state of nature and the level of *collateralized* output is sufficiently high to cover for the loan repayment. It is worth noting that there exists states of nature in which the firms output level exceeds the repayment to the commercial bank, and still firms have an incentive to default. This occurs when $\chi_t Y_{j,t} < (1 + i_t^L) L_{j,t} < Y_{j,t}$, where $\chi_t \in (0, 1)$, implying that the default condition depends on the fraction of output that can be seized by the commercial bank net of state verification and enforcement costs. Hence, χ_t measures the degree of credit market imperfections, affecting directly the threshold point below which default occurs (see below).

A fraction of firms are always expected to default on their loan obligations in the adverse states of nature. As a result, the commercial bank charges a premium over the cost of funds from households as shown in the section deriving the loan rate equation. Moreover, with partial repayment in the bad states of the economy, the commercial bank passes the cost of default to the bank capital investors and insures depositors through the seized collateral. Hence, the commercial bank may default on a fraction of bank capital, leading households to require a risk premium for holding this asset determined by the risk of default at the firm level.

Let $\varepsilon_{j,t}^M$ be the cut-off value below which the IG firm decides to default. Thus, using equations (2.19) and (2.20), the threshold condition is defined as,

$$\chi_t (A_t \varepsilon_{j,t}^M) N_{j,t} = (1 + i_t^L) L_{j,t}. \quad (2.23)$$

Substituting (2.21) and solving the above for $\varepsilon_{j,t}^M$ yields,

$$\varepsilon_{j,t}^M = \frac{1}{\chi_t A_t} (1 + i_t^L) W_t^R. \quad (2.24)$$

The threshold value therefore depends on the fraction of collateral seized from the IG firms in case of default, the real wages, total productivity shocks and the lending rate. However, contributing to Agénor, Bratsiotis and Pfajfar (2013), where the loan rate depends *only* on the risk-free rate and the risk premium, in our model the lending rate becomes dependent *also* on the bank capital-loan ratio and the interest rate on bank capital (as shown in the following sections). Hence, the threshold value ($\varepsilon_{j,t}^M$), from which the probability of default is derived later in the text, is also a function of bank capital and its rate of return.

Pricing of Intermediate Goods

The IG firm solves a two stage pricing decision problem during period t (after shocks are realized). In the *first stage*, each IG producer minimizes the cost of employing labour, taking its (real) effective costs $((1 + i_t^L) \frac{W_t}{P_t})$ as given. Specifically, IG firm j solves,

$$\min_{N_{j,t}} (1 + i_t^L) \frac{W_t}{P_t} N_{j,t},$$

subject to,

$$Y_{j,t} = Z_{j,t} N_{j,t}.$$

The Lagrangian associated with the above minimization problem of the j^{th} IG firm is therefore,

$$\Lambda^{IG} = (1 + i_t^L) \frac{W_t}{P_t} N_{j,t} + \varphi_{j,t}^{IG} (Y_{j,t} - Z_{j,t} N_{j,t}),$$

with $\varphi_{j,t}^{IG}$ denoting the Lagrangian operator. First order conditions with respect to $N_{j,t}$ result in,

$$(1 + i_t^L) \frac{W_t}{P_t} - \varphi_{j,t}^{IG} Z_{j,t} = 0.$$

Because labour is the only input in producing output, the Lagrangian multiplier in the cost minimization problem represents the real marginal cost which equals to,

$$\varphi_{j,t}^{IG} = mc_{j,t} = (1 + i_t^L) W_t^R \frac{1}{Z_{j,t}}. \quad (2.25)$$

In the *second stage*, each IG producer chooses the optimal price for its good. Here Calvo (1983)-type contracts are assumed, where a portion of ω_p firms keep their prices fixed while a portion of $1 - \omega_p$ firms adjust prices optimally given the

going marginal cost and the loan rate (set at the beginning of the period). In other words, ω_p measures the nominal price rigidity and allows for monetary policy to have real macroeconomic effects. Given equation (2.25), the firm's problem is to maximize the following expected discounted value of current and future *real* profits subject to the demand function for each good (equation 2.17). Formally that is,

$$\max_{P_{j,t+s}} E_t \sum_{s=0}^{\infty} \omega_p^s \Delta_{s,t+s} \left[\left(\frac{P_{j,t+s}}{P_{t+s}} \right)^{1-\lambda_p} Y_{t+s} - mc_{t+s} \left(\frac{P_{j,t+s}}{P_{t+s}} \right)^{-\lambda_p} Y_{t+s} \right], \quad (2.26)$$

where $\Delta_{s,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t} \right)^{\varsigma^{-1}}$ is the total discount factor.¹⁷

Denoting P_t^* as the optimal price level chosen by each firm at time t , the first order condition with respect to P_t^* is therefore,¹⁸

$$E_t \sum_{s=0}^{\infty} \omega_p^s \Delta_{s,t+s} \left[(1 - \lambda_p) \left(\frac{P_t^*}{P_{t+s}} \right)^{-\lambda_p} \frac{1}{P_{t+s}} + \lambda_p mc_{t+s} \left(\frac{P_t^*}{P_{t+s}} \right)^{-\lambda_p-1} \frac{1}{P_{t+s}} \right] Y_{t+s} = 0.$$

Solving the above equation with some algebraic manipulations, and using the definition of the total discount factor yields the following expression for the relative price chosen by all IG firms able to adjust their prices at period t ,

$$Q_t = \frac{P_t^*}{P_t} = \left(\frac{\lambda_p}{\lambda_p - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s C_{t+s}^{-\frac{1}{\varsigma}} Y_{t+s} mc_{t+s} \left(\frac{P_{t+s}}{P_t} \right)^{\lambda_p}}{E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s C_{t+s}^{-\frac{1}{\varsigma}} Y_{t+s} \left(\frac{P_{t+s}}{P_t} \right)^{\lambda_p-1}}, \quad (2.27)$$

with $Q_t = \frac{P_t^*}{P_t}$ denoting the relative price chosen by firms adjusting their prices at period t and $pm = \left(\frac{\lambda_p}{\lambda_p - 1} \right)$ representing the price mark-up.

2.2.4 The Commercial Bank

Consider a representative competitive bank which can raise funds through either deposits (D_t), or issuing bank capital (V_t) in accordance with regulation (as explained in the next few sections). Both deposits and bank capital are used to finance the working capital needs of the IG firms and act as liabilities to households. Thus, the

¹⁷The IG firms are owned by the households and therefore each firm's discount value is $\beta^s \left(\frac{C_{t+s}}{C_t} \right)^{\varsigma^{-1}}$. Intuitively, $\left(\frac{C_{t+s}}{C_t} \right)^{\varsigma^{-1}}$ is the marginal utility value (in terms of consumption) of a one unit increase of IG firms profits in period t .

¹⁸The subscript j is dropped because all re-optimizing firms choose the same price so everything becomes time dependent.

bank's balance sheet in real terms can be written as,

$$L_t = D_t + V_t, \quad (2.28)$$

where $L_t \equiv \int_0^1 L_{j,t} dj$ is the aggregate lending to IG firms. Households' deposits and lending from the central bank are assumed to be perfect substitutes (at the margin) for funding loans to IG firms. Consequently, the commercial bank chooses to fund IG firms *only through* deposits and bank capital, and *without* borrowing from the central bank. This also implies that there are no reserve requirements at the central bank, and that the deposit rate (i_t^D) is equal to the policy rate (i_t^R).

Lending Rate Decision

The lending rate is set at the beginning of the period before IG firms engage in their production activity and prior to their labour demand and pricing decisions. As some IG firms may default on their loans at the end of the period due to idiosyncratic shocks, the contractual repayments to the commercial bank are uncertain. The bank breaks-even every period such that the expected income from lending to IG firms is equal to the total costs of borrowing these funds (comprised of deposits and bank capital) from households. Therefore, the bank's expected (period) zero profit condition from lending to IG firms is given by,

$$\begin{aligned} & \int_{\varepsilon_{j,t}^M}^{\bar{\varepsilon}} [(1 + i_t^L) L_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} [\chi_t Y_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t} \\ &= (1 + i_t^V) V_t + (1 + i_t^D) D_t + c V_t, \end{aligned} \quad (2.29)$$

where $f(\varepsilon_{j,t})$ is the probability density function of $\varepsilon_{j,t}$. The first element on the left hand side is the expected repayment to the bank in the non-default states while the second element is the expected return to the bank in the default states. The term $(1 + i_t^V) V_t + (1 + i_t^D) D_t$ is the return to households for holding bank capital and saving deposits. Furthermore, the bank faces an additional linear cost function when issuing bank capital, captured by the term $c V_t$, with $c > 0$. These costs are independent of the state of the economy and reflect steady costs associated with underwriting or issuing brochures for example.¹⁹ By contrast, the rate of return on bank capital is the main driving force of the *total* bank capital costs $(1 + i_t^V + c)$, and endogenously rises following increased financial riskiness and lower output levels, as

¹⁹ These costs are essentially different from the adjustment costs of bank capital faced by households, resulting from asymmetric information between the bank and the households.

shown later in the simulations section.²⁰ More specifically, the cost of bank capital is related to credit risk (equation 2.7), which in turn is endogenous with respect to the macroeconomic conditions (see 2.34 below).

Turning now to the derivation of the loan rate, note that,

$$\begin{aligned} \int_{\underline{\varepsilon}_{j,t}^M}^{\bar{\varepsilon}} [(1 + i_t^L)L_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t} &\equiv \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} [(1 + i_t^L)L_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t} - \\ &\quad - \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} [(1 + i_t^L)L_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t}, \end{aligned}$$

where $\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} [(1 + i_t^L)L_{j,t}] f(\varepsilon_{j,t}) d\varepsilon_{j,t} \equiv [(1 + i_t^L)L_{j,t}]$. Hence, equation (2.29) can be written as,

$$\begin{aligned} &[(1 + i_t^L)L_{j,t}] - \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} [(1 + i_t^L)L_{j,t} - (\chi_t Y_{j,t})] f(\varepsilon_{j,t}) d\varepsilon_{j,t} \quad (2.30) \\ &= (1 + i_t^V + c)V_t + (1 + i_t^D)D_t. \end{aligned}$$

Using the bank's balance sheet (equation 2.28), substituting (2.23) for $\chi_t (A_t \varepsilon_{j,t}^M) N_{j,t} = (1 + i_t^L)L_{j,t}$ and employing the value of output from the production function (equation 2.19) yields,

$$\begin{aligned} &[(1 + i_t^L)L_{j,t}] - \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} [\varepsilon_{j,t}^M - \varepsilon_{j,t}] \chi_t A_t N_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} \quad (2.31) \\ &= (1 + i_t^V + c)V_t + (1 + i_t^D)(L_{j,t} - V_t). \end{aligned}$$

Dividing by $L_{j,t}$ results in,

$$i_t^L = (i_t^V + c) \left(\frac{V_t}{L_{j,t}} \right) + i_t^D \left(1 - \frac{V_t}{L_{j,t}} \right) + \frac{\int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} [\varepsilon_{j,t}^M - \varepsilon_{j,t}] \chi_t A_t N_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t}}{L_{j,t}}. \quad (2.32)$$

Real wages and the amount of labour employed by each firm are identical and therefore the volume of lending by the bank is also the same. Thus, the subscript j is dropped in what follows. Moreover, the threshold value $\varepsilon_{j,t}^M$ depends on the state of the economy (from 2.24) and hence is identical across all firms. Using the expression for $L_{j,t}(= L_t)$ (equation 2.21) and defining $\Delta_t = V_t/L_t$ as the total bank

²⁰From the technical point of view, the additional (exogenous) issuance cost per unit of bank capital (c) is also added to control in a more tractable way for the loan rate steady state value. All other variables which determine the loan rate behaviour are endogenous, and cannot be adjusted without altering the long run values of the other economic variables of the model.

capital-loan ratio, equation (2.32) reduces to,

$$i_t^L = (\Delta_t) (i_t^V + c) + (1 - \Delta_t) (i_t^D) + \frac{\chi_t A_t \int_{\underline{\varepsilon}}^{\varepsilon_t^M} [\varepsilon_t^M - \varepsilon_t] f(\varepsilon_t) d\varepsilon_t}{W_t^R}, \quad (2.33)$$

with $\frac{\chi_t A_t \int_{\underline{\varepsilon}}^{\varepsilon_t^M} [\varepsilon_t^M - \varepsilon_t] f(\varepsilon_t) d\varepsilon_t}{W_t^R}$ denoting the finance premium.

To find an explicit expression for the probability of default it is assumed, as in Agénor, Bratsiotis and Pfajfar (2013), that ε_t follows a uniform distribution over the interval $(\underline{\varepsilon}, \bar{\varepsilon})$. Therefore, its probability density is $1/(\bar{\varepsilon} - \underline{\varepsilon})$ and its mean $\mu_\varepsilon = (\bar{\varepsilon} + \underline{\varepsilon})/2$. The probability of default is given by,

$$\Phi_t = \int_{\underline{\varepsilon}}^{\varepsilon_t^M} f(\varepsilon_t) d\varepsilon_t = \frac{\varepsilon_t^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}. \quad (2.34)$$

Thus, the probability of default depends on the range of the uniform distribution and the threshold value of the idiosyncratic shock (determined by equation 2.24).

Turning now to explain why the IG firms probability of default must equal to the risk of default on bank capital in this setup. At the end of period t the commercial bank decides to default on bank capital if the value of collateral received from IG firms in case of default ($\chi_t Y_{j,t}$) is lower than the bank's total costs, which include repayments to households for holding deposits and bank capital, as well as the additional issuance costs of bank capital,

$$\chi_t Y_{j,t} < (1 + i_t^V) V_t + (1 + i_t^D) D_t + c V_t.$$

As explained earlier, the commercial bank breaks even in each period when setting the loan rate, and supplies loans to identical firms (ex-ante). However, if the IG firms decide to default, which occurs when the cost of loan repayments to the commercial bank is higher than the value of collateral at the end of the period ($\chi_t Y_{j,t} < (1 + i_t^L) L_{j,t}$, thus providing an incentive to default), the bank's break even condition fails (see condition 2.31). That is, the value of pledged collateral for both IG firms and the bank determines the risk of default on credit and bank capital. As the firms must borrow to cover *all* their labour costs, there is full transmission of risk from the asset side to the liability side in the commercial bank's balance sheet. With deposits being insured, the cost of default of firms and the cost of default on bank capital must therefore coincide such that $\Phi_t^V = \Phi_t$. If the value of the bank in case of partial firm default is lower than its total liabilities to households, the bank capital investors bear the cost of default, while depositors retrieve the value of collateral as insurance. As a result of higher bank capital risk, households, who

own the bank and thus know the aggregate risk, charge a higher rate of return on bank capital, making them indifferent between holding deposits and investing in riskier bank capital (see equation 2.7). In a different setup with heterogeneous banks for example, bank capital adequacy requirements (and/or loan loss provisions) can act to mitigate systemic risk, implying different default risks at the firm and bank levels.²¹

Bank Capital Regulation

The representative bank is subject to risk sensitive bank capital requirements imposed by the central bank and set according to the Basel accords. At the beginning of each period the bank must issue a certain amount of capital that covers a given percentage of its loans to IG firms. As explained, lending to IG firms is of a risky nature and therefore the risk weight on loans is defined as ϑ_t . The bank capital requirement constraint in real terms is thus,

$$V_t = \rho \vartheta_t L_t, \quad (2.35)$$

with $\rho \in (0, 1)$ being the capital adequacy ratio, also known as the *Cooke Ratio*. Under the Foundation IRB approach of Basel II (which remains essentially the same under Basel III), the risk weight on loans (ϑ_t) can be related to the default probability of firms estimated by the bank as it is perceived as a measure of credit risk. That is, we exploit the idea that the endogenous default probability of firms is passed to the bank's risk weight on loans according to the following equation,

$$\vartheta_t = \left(\frac{\Phi_t}{\Phi} \right)^q, \quad (2.36)$$

where $q > 0$ represents the elasticity of the risk weight relative to deviations of the probability of default (Φ_t) from its steady state value (Φ). Therefore, in steady state the risk weight on loans is equal to unity, as in Agénor, Alper and Pereira da Silva (2012), but with the exception that in our model the probability of default is endogenous to the characteristics of the borrower.

²¹A model of this nature would be more appropriate for examining how bank regulation can reduce the probability of bank default and the degree of risk transmission from firms to banks.

2.2.5 The Transmission Channels of Bank Capital and Risk on the Loan Rate

Using equations (2.36), (2.35), (2.34) and applying the characteristics of the uniform distribution, the lending rate equation (given by 2.33) reduces to,

$$i_t^L = i_t^D + \left[\rho \left(\frac{\Phi_t}{\Phi} \right)^q \right] (i_t^V - i_t^D + c) + \left(\frac{\chi_t A_t}{W_t^R} \right) \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \Phi_t^2, \quad (2.37)$$

where the term $\frac{1}{2} \left(\frac{\chi_t A_t}{W_t^R} \right) \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \Phi_t^2$ is defined as the finance premium, which itself is also a positive function of the lending rate (from equations 2.24 and 2.34).

Equation (2.37) shows that the lending rate is positively related to the cost of borrowing deposits from households, the finance premium, the bank capital-deposit rate spread and the additional issuance cost of bank capital. The bank capital-deposit rate spread and the extra cost of issuing bank capital, in turn, are set as a proportion of the bank capital-loan ratio, which is determined by the Cooke Ratio and the risk weight on loans (under Basel II and III).

We identify three channels through which the probability of default impacts the loan rate. The first, referred to as the *bank capital default channel*, stems from the bank capital-deposit rate spread, which is determined by the risk of default (see the no-arbitrage condition, equation 2.7). Second, the *risk premium channel*, arising from the positive correlation between the risk of default and the finance premium, which, in turn, directly influences the cost of credit. Third, the probability of default affects the lending rate also through the *risk weight channel*, resulting from the positive relationship between the risk weight on loans and the risk of default. The latter channel is evident in the Foundation IRB approach of Basel II (and Basel III) while the first two channels prevail regardless of the regulatory regime.

Under Basel I, where the risk weight is *independent* of the probability of default and is equal to unity at all times, equation (2.37) becomes,

$$i_t^L = i_t^D + \rho (i_t^V - i_t^D + c) + \left(\frac{\chi_t A_t}{W_t^R} \right) \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \Phi_t^2. \quad (2.38)$$

Thus, in the case of a constant risk weight, the default probability function impacts the lending rate only through the risk premium and the bank capital default channels.

Another key element in this setup is that the probability of default is a function of the loan rate, while the bank capital rate is a function of the probability of default (from the no-arbitrage condition). Hence, an adverse shock, associated with

lower output and hence lower collateral values, leads to a rise in the loan rate and increased risk, which, in turn, raises the bank capital rate. The increase in the bank capital rate then translates into an amplified rise in the loan rate. Therefore, the probability of default, through its relationship with the lending and bank capital rates, aggravates the impact on the rest of the financial and economic variables, leading to an accelerator effect in this model.

2.2.6 Monetary Policy

The central bank targets the short term policy rate (i_t^R) according to the following standard log-linearized Taylor-type policy rule,²²

$$\widehat{i}_t^R = \phi \widehat{i}_{t-1}^R + (1 - \phi) \left[\phi_y \widehat{Y}_t + \phi_\pi \widehat{\pi}_t^P \right] + \epsilon_t^{mp}, \quad (2.39)$$

where \widehat{Y}_t denotes output deviations from its flexible price steady state value, $\widehat{\pi}_t^P \equiv \pi_t^P - \pi^{P,T}$ inflation deviation from its target steady state value ($\pi^{P,T}$), $\phi \in (0, 1)$ the degree of interest rate smoothing and $\phi_y, \phi_\pi > 0$ coefficients measuring the relative weights on output and inflation deviations from their steady states, respectively.²³ Finally, the term ϵ_t^{mp} represents an i.i.d monetary policy shock with zero mean and a constant variance.

2.2.7 Market Clearing Conditions

Equilibrium conditions must ensure that markets for goods, labour, loans, deposits and bank capital clear. The supply of loans by the commercial bank, the supply of deposits by households and bank capital issued in accordance with regulation, are all assumed to be perfectly elastic at the prevailing interest rates and therefore these markets always clear.

The fraction $\chi_t Y_{j,t}$ seized by the bank in case of default is distributed to the households at the end of the period and acts as an insurance policy for deposit holders. Thus, the goods market equilibrium satisfies the condition that realized

²²Using the no arbitrage condition and the fact that the commercial bank does not borrow from the central bank, the deposit rate is equal to the policy rate.

²³Note that output is included in terms of deviations from its steady state value rather than Walsh's (2003) measure of output gap. Our specification is consistent with Faia and Monacelli (2007) and Meh and Moran (2010) among others.

aggregate output is equal to aggregate consumption,²⁴

$$Y_t = C_t. \quad (2.40)$$

2.3 Steady State

The long run steady state values of the endogenous variables are derived below and are denoted by dropping the time subscript.

The steady state value of the deposit rate is calculated from equation (2.6) with $C_t = C_{t+1} = C$ and $P_{t+1} = P_t = P$,

$$(1 + i^D) = \frac{1}{\beta}. \quad (2.41)$$

From equation (2.7) the long term relationship between the bank capital rate and the deposit rate is,

$$(1 + i^V) = \frac{(1 + i^D)}{(1 - \Phi)}. \quad (2.42)$$

Hence, in steady state the return on bank capital is always higher than the risk free return on bank deposits due to the fact that holding bank capital bears risk ($\Phi > 0$).²⁵

With *fully flexible prices* and a constant level of output, the steady state value for the price mark-up (pm) equals the inverse of the marginal cost,

$$pm = \frac{\lambda_p}{\lambda_p - 1} = \frac{1}{mc} > 1. \quad (2.43)$$

The marginal cost at the steady state (mc) is derived from equation (2.25) and the average productivity shock value in steady state ($Z = A\mu_\varepsilon$),

$$mc = W^R \frac{(1 + i^L)}{A\mu_\varepsilon}. \quad (2.44)$$

Combining equations (2.24) and (2.34) results in the long run value of the prob-

²⁴The lost output in case of default, which is interpreted as the *ex post* monitoring costs, is already incorporated in the goods market clearing condition. This is because collateral in this model is given by the level of output (and thus consumption), which already is endogenously related to the probability of default.

²⁵Recall that in this model the probability of default on bank capital is equal to the risk of default at the IG firms' level, $\Phi^V = \Phi$.

ability of default,

$$\Phi = \frac{\frac{1}{\chi A}(1 + i^L)W^R - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}, \quad (2.45)$$

where $\varepsilon^M = \frac{1}{\chi A}(1 + i^L)W^R$ is the steady state level of the idiosyncratic shock threshold point. Using the steady state value for Z , employing the long run value of the marginal cost (from 2.44) and substituting (2.43), equation (2.45) can be written as follows,

$$\Phi = \frac{\frac{1}{\chi}(pm)^{-1}\mu_\varepsilon - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}. \quad (2.46)$$

Hence, the probability of default in the long run is constant and related endogenously to the structural parameters of the model (as in Agénor, Bratsiotis and Pfajfar 2013).

As explained earlier, the risk weight on loans in steady state is $\vartheta = \left(\frac{\Phi}{\bar{\Phi}}\right)^q = 1$. Therefore, from equation (2.37), the long run value of the net loan rate is,

$$i^L = i^D + \rho(i^V - i^D + c) + \frac{1}{2} \left(\frac{\chi A}{W^R} \right) \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} (\Phi)^2. \quad (2.47)$$

The real wages at steady state (where all households are able to re-optimize) is equal to,

$$W^R = (wm)N^\gamma C^{\frac{1}{\zeta}}, \quad (2.48)$$

with $wm = \frac{\lambda_w}{\lambda_w - 1}$ denoting the real wage mark-up.

The steady state value for the economy wide employment demand is given by,

$$N = YZ^{-1}. \quad (2.49)$$

The steady state level of commercial bank lending is,

$$L = W^R N. \quad (2.50)$$

Moreover, the amount of bank capital issued at the steady state is,

$$V = \rho W^R N. \quad (2.51)$$

Finally, the steady state level of output under flexible prices is calculated by combining the production function ($Y = ZN$), goods market clearing condition ($Y = C$) and equations (2.43), (2.44) and (2.48). These substitutions result in,

$$Y = \left(\frac{1}{wm} \right)^{\frac{1}{\gamma + \frac{1}{\zeta}}} \left(\frac{1}{pm} \right)^{\frac{1}{\gamma + \frac{1}{\zeta}}} \frac{(Z)^{\frac{\gamma + 1}{\gamma + \frac{1}{\zeta}}}}{(1 + i^L)^{\frac{1}{\gamma + \frac{1}{\zeta}}}}. \quad (2.52)$$

2.4 The Log-Linearized Model

The log-linearized equations of the model are presented below and are based on the steady state solution presented above. The log-linearized variables are denoted by hat and represent percentage point deviations for price inflation, wage inflation and interest rate variables, and log-deviations around a non-stochastic steady state for the rest of the variables.²⁶ The full derivations of some of the key log-linearized equations are presented in Appendix 2.A.2.

Defining $E_t \widehat{\pi_{t+1}^P} = E_t \widehat{P_{t+1}} - \widehat{P}_t$ as the expected percentage point deviation of inflation from its steady state value (assuming $\pi^{P,T} = 0$) then the log-linearized representation of the Euler Equation is,

$$\widehat{C}_t = E_t \widehat{C_{t+1}} - \varsigma \left[\widehat{i_t^D} - E_t \widehat{\pi_{t+1}^P} \right]. \quad (2.53)$$

Marginal Costs,

$$\widehat{mc}_t = \widehat{i_t^L} + \widehat{W_t^R} - \widehat{Z}_t, \quad (2.54)$$

where real wages evolve according to $\widehat{W_t^R} = \widehat{W_{t-1}^R} + \widehat{\pi_t^W} - \widehat{\pi_t^P}$. The wage inflation is given by $\widehat{\pi_t^W} = \beta E_t \widehat{\pi_{t+1}^W} + \frac{(1-\omega_w)(1-\beta\omega_w)}{(\omega_w)(1+\gamma\lambda_w)} \left[\widehat{MRS_t} - \widehat{W_t^R} \right]$, with the marginal rate of substitution defined as $\widehat{MRS_t} = \frac{1}{\varsigma} \widehat{C}_t + \gamma \widehat{N}_t$.

The aggregate demand for labour, taking into account both the value of the production function and the individual demand for labour faced by each household is,

$$\widehat{N}_t = -\lambda_w \left[\widehat{W_t^R} + \widehat{i_t^L} \right] + \widehat{Y}_t + (\lambda_w - 1) \widehat{Z}_t, \quad (2.55)$$

with the productivity shock following $\widehat{Z}_t = \widehat{A}_t + \widehat{\varepsilon}_t$ in its log-linear form. Hence, the composite labour demand is positively related to output and aggregate supply shocks, and negatively affected by real wages and the lending rate (as long as $\lambda_w > 1$).

The log-linearized probability of default is derived from equations (2.24) and (2.34),

$$\widehat{\Phi}_t = \left(\frac{\varepsilon^M}{\varepsilon^M - \underline{\varepsilon}} \right) \left(\widehat{i_t^L} + \widehat{W_t^R} - \widehat{A}_t - \widehat{\chi}_t \right). \quad (2.56)$$

²⁶Therefore, log-linearized net interest rates are used as an approximation for log-linearized gross interest rates.

The log-linearized lending rate is represented by,

$$\widehat{i}_t^L = \frac{1}{(1+i^L)} \left\{ (1+i^D)\widehat{i}_t^D + \rho \left[(1+i^V)\widehat{i}_t^V - (1+i^D)\widehat{i}_t^D \right] + \rho(i^V - i^D + c)\widehat{\vartheta}_t + \right. \\ \left. + \chi A \frac{\Phi^2}{\widehat{W}^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \left[2\widehat{\Phi}_t - \widehat{W}_t^R + \widehat{A}_t + \widehat{\chi}_t \right] \right\}. \quad (2.57)$$

This is the case where the risk weight is endogenously determined (i.e. Basel II). Under Basel I, equation (2.57) reduces to,

$$\widehat{i}_t^L = \frac{1}{(1+i^L)} \left\{ (1+i^D)\widehat{i}_t^D + \rho \left[(1+i^V)\widehat{i}_t^V - (1+i^D)\widehat{i}_t^D \right] + \right. \\ \left. + \chi A \frac{\Phi^2}{\widehat{W}^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \left[2\widehat{\Phi}_t - \widehat{W}_t^R + \widehat{A}_t + \widehat{\chi}_t \right] \right\}. \quad (2.58)$$

Log-linearizing the rate of interest on bank capital yields,

$$\widehat{i}_t^V = \widehat{i}_t^D + \frac{\Phi}{(1-\Phi)}\widehat{\Phi}_t + \Theta \left[\widehat{V}_t - \widehat{V}_{t-1} \right] - \beta\Theta E_t \left[\widehat{V}_{t+1} - \widehat{V}_t \right]. \quad (2.59)$$

Equation (2.59) shows that the bank capital rate is directly related to the risk-free deposit rate and to the adjustment costs of holding bank capital. Moreover, regardless of the regulatory regime, a rise in the endogenous probability of default increases the interest rate required by households to hold riskier bank capital.

The log-linearized equation for bank capital, issued in accordance with regulation, is,

$$\widehat{V}_t = \widehat{L}_t + \widehat{\vartheta}_t, \quad (2.60)$$

where,

$$\begin{aligned} \widehat{\vartheta}_t &= q\widehat{\Phi}_t \quad (\text{Basel II}), \\ \widehat{\vartheta}_t &= 0 \quad (\text{Basel I}). \end{aligned} \quad (2.61)$$

Examining equations (2.59), (2.60) and (2.61), a rise in the probability of default increases the amount of capital the bank needs to issue in order to satisfy regulatory requirements through the risk weight channel. The change in bank capital, in turn, then impacts its rate of return via the adjustment cost channel. In addition, a rise in the risk weight raises the loan rate charged by the commercial bank, which amplifies the initial increase of the default probability function (see equations 2.56 and 2.57). Hence, the risk weight channel further strengthens both the bank capital default and the adjustment cost channels represented respectively by the terms $\frac{\Phi}{(1-\Phi)}\widehat{\Phi}_t$ and $\Theta \left[\widehat{V}_t - \widehat{V}_{t-1} \right] - \beta\Theta E_t \left[\widehat{V}_{t+1} - \widehat{V}_t \right]$ in equation (2.59)

Finally, log-linearizing equation (2.27) yields the familiar form of the **New Key-**

nesian Phillips Curve (NKPC),

$$\widehat{\pi}_t^P = \beta E_t \widehat{\pi}_{t+1}^P + \frac{(1 - \omega_p)(1 - \omega_p \beta)}{\omega_p} \widehat{mc}_t, \quad (2.62)$$

with \widehat{mc}_t and \widehat{i}_t^L respectively given by equations (2.54) and (2.57) or (2.58). Recall that the probability of default affects the loan rate through the bank capital default channel, the risk premium channel and the risk weight channel (under Basel II and III). Therefore, these channels, together with changes in the deposit rate, also directly impact the marginal costs and thus the rate of price inflation via the borrowing cost channel, which links between the financial system and real economy in this model. Without an endogenous probability of default and a role for bank capital, the loan rate and thus the borrowing cost channel are affected only by deviations in the policy rate and monetary policy shocks. Hence, risk and bank capital in this model contribute to the standard cost channel described in Ravenna and Walsh (2006).

2.5 Calibration

The model is calibrated, where applicable, within the range of the parameters proposed by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003). The baseline calibration numbers are summarized in Table 2.1.

Elaborating now on some parameters unique to this model; First, the adjustment cost parameter for bank capital (Θ) is set at 0.10, which is within the size range of auditing costs arising due to information asymmetry between banks and their shareholders during normal times (see Markovic 2006 and Bernanke, Gertler and Gilchrist 1999).²⁷ Second, the average total productivity level in steady state (A) is set to 1, the idiosyncratic productivity shock's range to (1, 1.36), and the proportion of output the bank seizes from IG firms in case of default (χ) to 97%.²⁸

For the parameters characterizing the bank's behaviour, the bank capital adequacy ratio (ρ) is set to 0.08, in accordance with the Basel Accords,²⁹ while the

²⁷In Markovic (2006), the adjustment cost for bank capital are caused by information asymmetry between banks and their equity holders. Therefore the size of these costs are set to match auditing costs which arise due to information asymmetry between banks and firms. Bernanke, Gertler and Gilchrist (1999) have claimed that plausible values for these auditing costs are between 0 and 0.5. However, during financial crises these costs may be significantly higher.

²⁸This implies that 3% of output is spent on monitoring costs, verifications costs, legal procedures etc.

²⁹The value $\rho = 0.08$ represents to a target value under Basel I and the floor value under Basel II. The impact of a higher ρ is examined in the simulations section.

Table 2.1: Benchmark Parameterization: Parameter Values

Parameter	Value	Description
β	0.99	Discount Factor
ς	0.50	Intertemporal Substitution in Consumption
γ	2.50	Inverse of the Frisch Elasticity of Labour Supply
Θ	0.10	Adjustment Cost Parameter - Bank Capital
λ_w	11.0	Elasticity of Demand - Labour
wm	1.10	Wage Mark-up
ω_w	0.80	Degree of Wage Stickiness
λ_p	6.00	Elasticity of Demand - Intermediate Goods
pm	1.20	Price Mark-up
ω_p	0.75	Degree of Price Stickiness
A	1.00	Average Productivity Parameter
$\bar{\varepsilon}$	1.36	Idiosyncratic Productivity Shock Upper Range
$\underline{\varepsilon}$	1.00	Idiosyncratic Productivity Shock Lower Range
χ	0.97	Proportion of Output seized in case of Default
c	0.50	Cost of Issuing Bank Capital
ρ	0.08	Capital Adequacy Ratio
q	0.08	Elasticity of Risk Weight wrt Default Probability - Basel II
ϕ	0.80	Degree of Persistence in Interest Rate Rule
ϕ_π	1.50	Response of Refinance Rate to Inflation Deviations
ϕ_y	0.10	Response of Refinance Rate to Output Deviations
ξ^A	0.80	Degree of Persistence - Supply Shock
ξ^x	0.80	Degree of Persistence - Financial Shock

Note: Under Basel I, the risk weight on loans is independent on the probability of default such that $q=0$.

elasticity of the risk weight with respect to the default probability (q) is calibrated at 0.08, which is close to the values estimated by Covas and Fujita (2010) and Aguiar and Drumond (2009). This is apparent only under the Foundation IRB approach of Basel II where the risk weight is endogenous and depends on the probability of default. However, under Basel I the risk weight is exogenous and therefore $q = 0$ such that the risk weight is independent of the probability of default and continuously equals to unity. Note that the steady state value of the risk weight (ϑ) is equal to unity under both Basel I (by assumption) and Basel II. Therefore, given the way the risk weight is modeled, switching from one regulatory regime to another does not alter the steady state values. The differences between the regulatory regimes are transparent only in the transitional dynamics of the model.

The above calibration numbers, together with a price mark-up of 20%, generate a steady state default probability (Φ) of 3.82%, a long run value of 5.02% for the return on bank capital (i^V) and a steady state loan rate (i^L) of 5.36%.

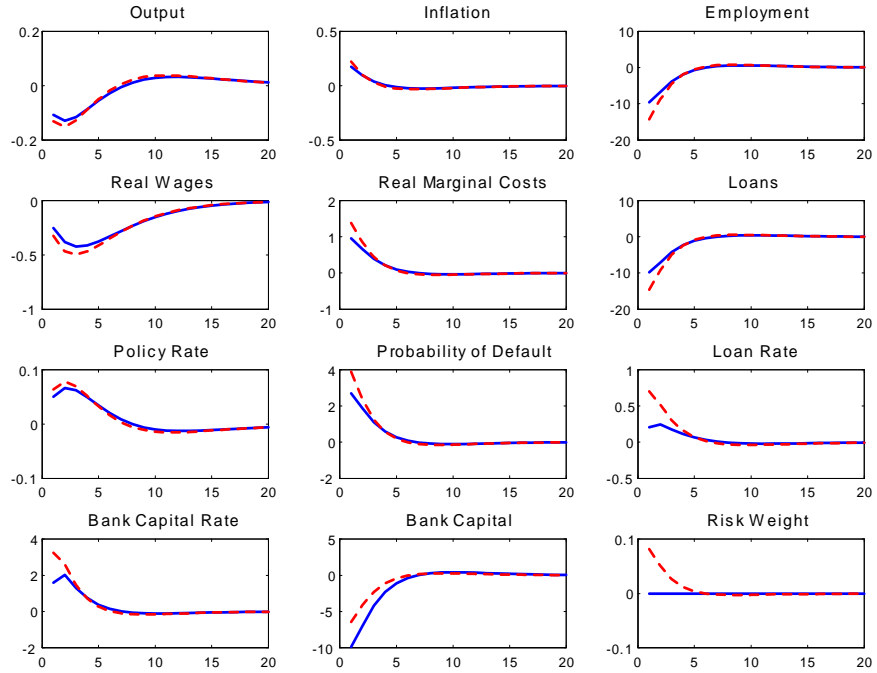
2.6 Simulations

This section examines the cyclical behaviour of the macroeconomic and financial variables following a negative supply shock, a tightening monetary policy shock and an adverse financial shock. Under each shock, a comparison is made between the two regulatory regimes, Basel I (constant risk weight) and Basel II (time varying risk weight). In addition, we study the effects of the adjustment cost channel in this setup and show that this channel may have an ambiguous impact on the degree of procyclicality of the financial system depending on the strength of the risk weight channel. Finally, we also investigate the case where the minimum bank capital requirements (ρ) increase from 8% to 12%, in line with one of the recent proposals of the Basel committee aimed at strengthening the resilience of the banking sector following the 2007-2009 financial crisis (see Basel Committee on Banking Supervision 2011).

2.6.1 Supply Shock

Figure 2.1 shows the impulse response functions of the main variables of the model following a 1% negative technology shock under Basel I ($q = 0$, solid line) and Basel II ($q = 0.08$, dashed line).

Figure 2.1: Negative Supply Shock - Basel I vs. Basel II



Note: Interest rates, inflation rate, the probability of default and the risk weight are measured in percentage point deviations from steady state. The rest of the variables are measured in terms of log-deviations from steady state.

The direct effect of the fall in productivity is a decrease in the level of output and an increase in inflation. Moreover, as output falls, the size of collateral declines as well which raises the probability of default and thus the loan rate through the risk premium channel (as in Agénor, Bratsiotis and Pfajfar 2013). Nevertheless, the increase in the risk perceived by the bank creates an upward pressure on the bank capital rate, via the bank capital default channel, leading the commercial bank to charge a higher lending rate on loans. The negative supply shock also lowers the real wages, resulting in an attenuation effect on the probability of default and consequently on the loan rate and the borrowing cost channel. However, as the lending rate rises, and given the nature of the adverse supply shock, the marginal costs and inflation shoot up while output deteriorates. Moreover, the policy rate rises in response to the increase in prices, which generates an additional upward shift in the bank capital and loan rates, and also lowers further the level of consumption and output via intertemporal substitution (see the Euler equation).

The rise in the loan rate and borrowing costs, coupled with the drop in produc-

tivity and output, reduce the demand for employment, which exerts a downward pressure on the demand for loans. At the same time, as the demand for credit declines, so does the amount of capital the bank needs to issue in each period to satisfy regulation. The fall in bank capital triggers the adjustment cost channel, which mitigates the procyclical effects inherent in the financial system through its impact on the bank capital interest rate (see equation 2.59).

The abovementioned effects are transparent under both regulatory regimes. The main difference between Basel I and Basel II comes from the way the risk weight is calculated. While under Basel I, the bank capital-loan ratio remains fixed at the *Cooke Ratio*, with the Foundation IRB approach of Basel II, the risk weight becomes dependent on the probability of default. Thus, as the perceived risk rises following a negative supply shock (as explained above), the risk weight (or the bank capital-loan ratio) tends to follow and leads to a further increase in the loan rate. From equations (2.56) and (2.57), the loan rate and probability of default are interrelated so a rise in the cost of loans directly increases the probability of default of firms and consequently the expected default on bank capital. The exacerbation in the probability of default reaction strengthens the risk premium channel, generates a further rise in the risk weight and also drives the bank capital default channel, which, in turn, all raise the cost of bank capital and loans. As the cost of borrowing and marginal costs are amplified under Basel II, the drop in employment and credit demand is magnified as well. Nevertheless, because supply shocks impact directly the marginal costs, inflation and thus real wages, the fall in the latter also mitigates the rise in the probability of default and hence weakens the strength of the borrowing cost and risk weight channels. However, given that the borrowing cost channel dominates the dampening effect of real wages, the impact of implementing a more risk sensitive regulatory regime results in output and inflation being slightly more procyclical.³⁰

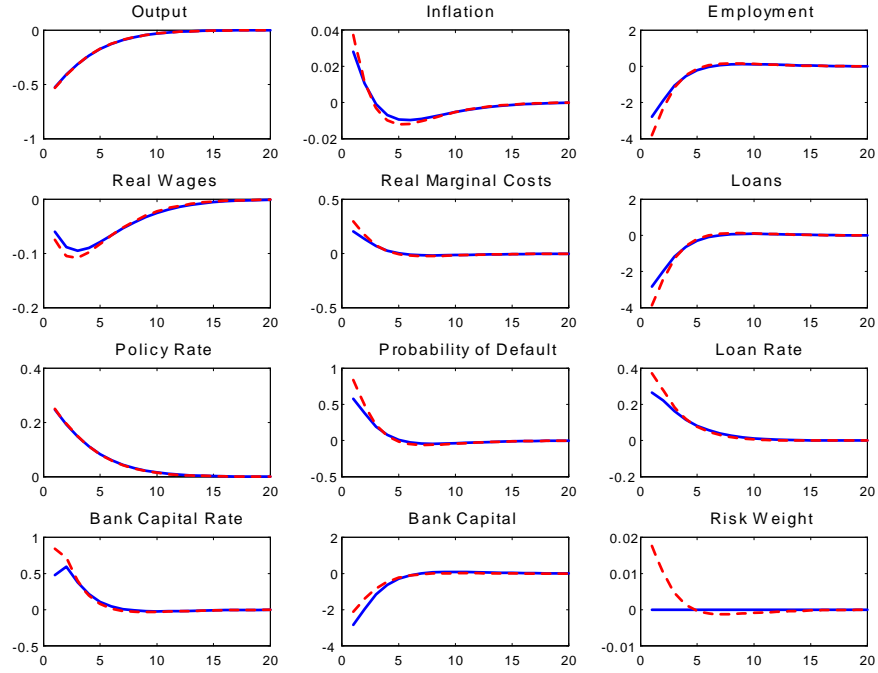
Finally, the rise in the risk weight results in an upward pressure on the volume of bank capital. However, given our calibration, the risk weight channel does not reverse the downward movement in bank capital as the drop in credit dominates the increase in the risk weight. Therefore, the adjustment cost channel still dampens the amplification effects on the economy, but to a much lesser extent due to the presence of the risk weight channel.

³⁰Output and inflation could be made more procyclical under Basel II following supply (and demand) shocks by either: *a*) increasing the steady state probability of default (by lowering χ for example and looking at economies with higher default rates); *b*) raising the value of q . However, the modest amplification effects on output and inflation when switching to a more risk sensitive regulatory regime are consistent with Covas and Fujita (2010) and Angelini, Enria, Neri, Panetta and Quagliariello (2011) for example.

2.6.2 Monetary Shock

Figure 2.2 depicts the impulse response functions of the main variables to a 0.25% increase in the policy rate, given by the Taylor rule in equation (2.39), under Basel I ($q = 0$, solid line) and Basel II ($q = 0.08$, dashed line).

Figure 2.2: Tightening Monetary Policy Shock - Basel I vs. Basel II



An unexpected rise in the risk-free rate has multiple effects in this setup. First, the rise in the policy rate raises the loan rate and the bank capital interest rate. The latter, in turn, increases the cost of liabilities to households, which is passed on to the IG firms with the commercial bank setting an even higher loan rate (compared to the case without bank capital).

Second, the increase in the policy rate directly affects consumption through intertemporal substitution, resulting therefore in a lower level of output in the short run.

Third, the rise in the loan rate and the lower level of output, lead to an increase of the default probability, which triggers both the risk premium and bank capital default channels. The higher risk on loans and bank capital drives up the bank capital interest rate, thus amplifying the movement in the borrowing costs and marginal costs. These magnifying effects of the borrowing cost channel, due to the presence of bank capital, initially raise price inflation following a tightening policy

shock before it starts falling below its steady state value. This scenario is consistent with the price puzzle effect observed in data.

Moreover, the decreasing level of output and the higher costs of borrowing bring down employment and the demand for loans, leading to a fall in the required bank capital. Consequently, and similar to the case of supply shocks, the adjustment cost channel mitigates the rise in the bank capital interest rate, which moderates the procyclical effects on the rest of the variables.

The effects described above are present in both regulatory regimes. However, under Basel II, the rise in risk associated with higher interest rates also generates an increase in the risk weight attached to loans. The presence of the risk weight channel further increases the costs of funds for firms and therefore leads to an amplified rise in the default probability. Moreover, from the perspective of the household, a higher risk of default translates into a higher interest rate on bank capital as holding bank capital becomes riskier. The increase in the cost of bank capital yields an additional rise in the loan rate and marginal costs compared to Basel I.

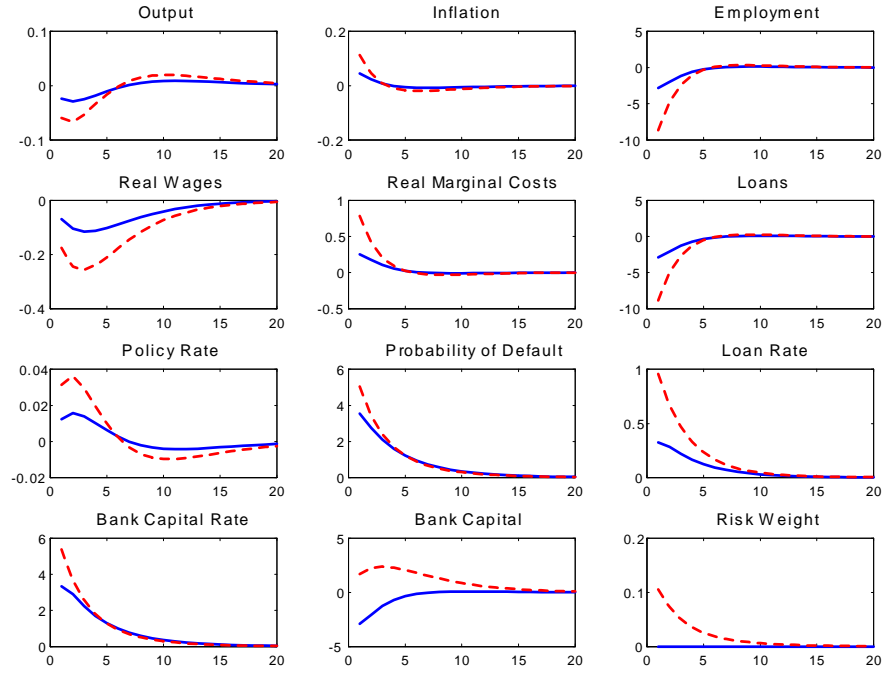
The increase in the policy rate, associated with higher borrowing costs and a lower output level, also reduces employment, loan demand and real wages, and these effects are more profound under Basel II. The fall in real wages, in turn, mitigates slightly the rise in the marginal costs and given the nature of the demand shock, the impact on output and inflation is also dampened.

Finally, the risk weight channel puts an upward pressure on required bank capital but given our calibration, the fall in credit demand dominates such that the level of regulatory bank capital falls. As a result, the rise in the bank capital rate is still mitigated through the adjustment cost channel, but not as much when compared to Basel I.

2.6.3 Financial Shock

Figure 2.3 shows the impulse response functions of the main variables of the model following a 1% adverse financial shock to χ_t under Basel I ($q = 0$, solid line) and Basel II ($q = 0.08$, dashed line).

Figure 2.3: Adverse Financial Shock - Basel I vs. Basel II



See note to Figure 2.1

The direct effect of a negative shock to collateral is a rise in the probability of default and consequently the loan rate through the risk premium channel. The increase in perceived risk by the bank creates an upward pressure on the bank capital rate, via the bank capital default channel, leading the commercial bank to charge an even higher lending rate on loans. Furthermore, as the bank is subject to risk sensitive bank capital regulation, the risk weight on loans increases with the rise in the probability of default, inducing a further amplification effect on the loan rate. Both the bank capital default and risk weight channels lead to a further exacerbation of the loan rate and risk of default compared to the case where the bank is not subjected to risk sensitive bank capital regulation or when there is no role for bank capital.

The increase in the loan rate, coupled with the rise in risk, raises the marginal cost and therefore the rate of price inflation through the borrowing cost channel. Moreover, the policy rate rises in response to the increase in prices, generating an upward shift in the bank capital and loan rates, and leading to a decline in aggregate demand. The rise in borrowing costs reduces also the demand for employment, which

exerts an additional downward pressure on output and lowers the demand for loans. The slight fall in real wages, caused by the rise in inflation, attenuates the rise in the marginal cost following a financial shock, although this mitigation effect is relatively small given that credit shocks do not directly impact the marginal cost, inflation and thus the real wages (unlike supply shocks for example).

Because the risk weight channel induces a further rise in the loan rate and probability of default, this channel also acts to amplify the response of the rest of the key economic variables. Indeed, the loan rate and probability of default link between the financial system and real economy through the borrowing cost channel, which impacts the marginal cost and thus inflation, as explained above.

However, compared to supply and demand shocks, the risk weight channel under credit shocks has a much stronger impact on the key aggregate variables. Intuitively, under supply and demand shocks, the deviations in output, price inflation and real wages stem from both the direct effect of the shocks and the secondary effect associated with the rise in the loan rate. Hence, real wages fall by more under these shocks, leading to a larger dampening effect on the probability of default, the loan rate and consequently the borrowing cost channel. A credit shock, nonetheless, impacts wages and inflation through its effect on the lending rate, which then feeds into the rest of the economy via the borrowing cost channel. In other words, the borrowing cost channel following a financial shock is stronger compared to the case where the economy is hit by productivity and monetary shocks. This implies that the risk weight channel, which impacts directly the loan rate and thus the borrowing cost channel, has a much stronger effect when the source of shocks originate from the financial system.

Given the strength of the risk weight channel following credit shocks, the rise in risk weight now dominates the fall in credit such that the volume of bank capital increases. As a result, the adjustment cost channel acts now to further amplify the response of the key economic and financial variables.

Overall, following supply, demand and credit shocks, the endogenous probability of default produces an accelerator mechanism from the financial sector to the real economy, and impacts the key financial and macro variables through multiple channels. In addition, under this setup, the risk weight channel amplifies the response of key variables compared to the case of a constant risk weight, consequently making Basel II more procyclical than Basel I. This is especially transparent when the economy is hit by credit shocks.

2.6.4 The Role of Adjustment Costs and the Risk Weight Channel

In the previous experiments, following either an adverse supply or tightening monetary shock, the volume of bank capital reduced even with an active risk weight channel. The fall in bank capital triggered the adjustment cost channel and from equation (2.59), this reduction mitigated the rise in the bank capital interest rate. Moreover, due to the impact of the financial side on the real economy, the fall in bank capital moderated the response of the macro variables. Nevertheless, for a financial shock under the same calibrated numbers, bank capital increased due to the strength of the risk weight channel, which dominated the fall in credit demand. As a result, the adjustment cost channel leads to a further amplification effect of the key variables following adverse credit shocks.

It can easily be shown that raising the sensitivity of the risk weight with respect to the probability of default (q) can result in bank capital also increasing following both adverse supply and tightening monetary shocks. Hence, the adjustment cost channel may either have an amplifying or mitigating effect on the economy depending on the strength of the risk weight channel, associated also with the nature of the shock.

Intuitively, if the risk weight channel dominates the credit demand channel of firms, the volume of bank capital increases which translates into an even higher interest rate on bank capital through the adjustment cost channel. As explained in Markovic (2006), the bank sends a bad signal about its financial situation and perceived risk when it needs to raise fresh capital rapidly, which incurs further costs on the households behalf when investing in new bank capital. Hence, the higher auditing costs resulting from the information asymmetry between banks and their shareholders lead households to require a higher return on bank capital. The increase in the bank capital interest rate then feeds into the rest of the economy via the channels described in the previous sections.

To conclude, contributing to Markovic (2006) and Dib (2010), when risk is more carefully assessed under the Basel Accords, the risk weight channel may increase the cyclicity of the volume of bank capital. This leads to undesirable effects when measuring the degree of procyclicality caused by bank capital regulation.

2.6.5 Permanent Increase in Bank Capital Requirements

We now turn to assess the short run economic consequences of imposing a higher bank capital-loan ratio, as is proposed under the new regulatory standards (Basel III). Before discussing the transitional dynamics of an increase in bank capital re-

quirements from 8% to 12% following a negative credit shock, we first briefly describe what happens in steady state to some of the key variables of the model.³¹

Steady State Effects

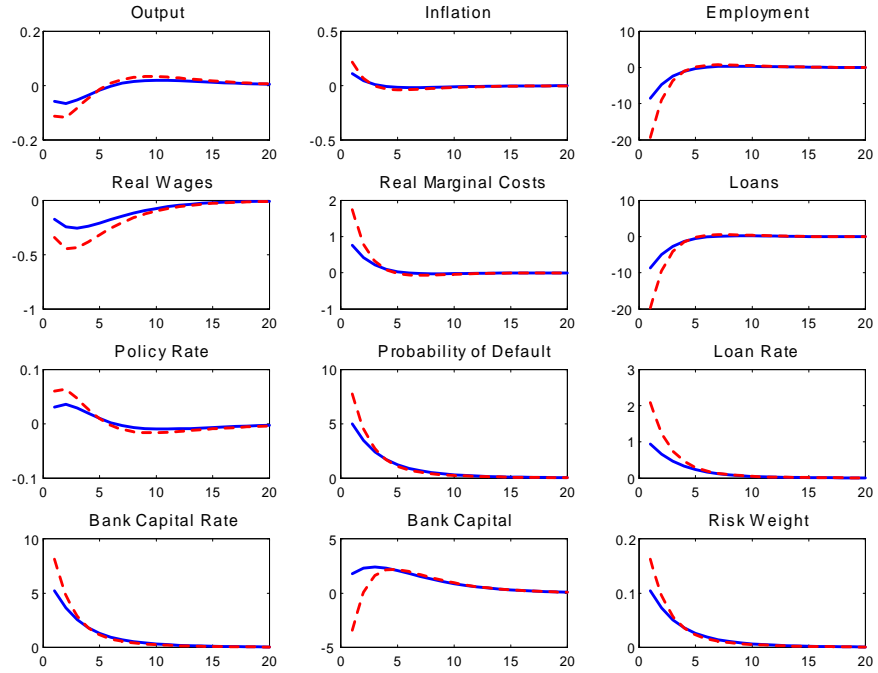
A permanent increase of the bank capital requirement ratio (ρ) from 8% to 12% leads first of all to a higher steady state level of bank capital (relative to loans) and a higher loan rate. The increase in the cost of loans reduces the volume of employment and total borrowing, which also result in a lower level of output. Note that as the probability of default depends on the structural parameters of the model, a permanent rise in bank capital requirements does not change the default probability in the long run. Intuitively, the rise in the loan rate and the borrowing cost channel effect is offset by the drop in real wages such that the marginal cost and thus the default probability in steady state remain constant. Finally, the interest rate on bank capital in the long run also remains the same as this variable depends on the risk free rate and the probability of default, both which depend only on the structural parameters of the model.

Short Run Dynamics

The impact of an adverse credit shock is now examined when the minimum capital requirements increase from 8% (solid line) to 12% (dashed line) under a risk sensitive regulatory regime ($q = 0.08$). The results are presented in Figure 2.4,

³¹We only consider the effects of a negative credit shock to save space. Supply and monetary shocks have very similar implications and if desired, are available upon request.

Figure 2.4: Adverse Financial Shock with Higher Regulatory Requirements



See note to Figure 2.1

The impulse response functions clearly show that raising bank capital requirements result in a stronger response of all key variables when the bank is subjected to 12% bank capital-loan ratio. The increase in the regulatory ratio translates directly into a higher loan rate and probability of default, with the latter amplifying the response of all variables in the economy due to the risk premium, bank capital default and risk weight channels, as illustrated above. Note also that the volume of bank capital now drops with a higher regulatory ratio despite the risk weight deviating by a greater magnitude. Intuitively, the amplification effect on loans is greater than the magnifying effect on the risk weight, leading to a larger fall in the volume of bank capital. As a result, the adjustment cost channel mitigates slightly the amplification effects caused by higher bank capital requirements, although this attenuation effect is negligible in this experiment.

2.7 Concluding Remarks

The purpose of this chapter has been to study the interactions between bank capital regulation, the financial system and the real business cycle in a Dynamic Stochastic General Equilibrium (DSGE) model with credit market frictions. A key feature of this model is the derivation of the bank loan rate and the endogenous probability of default, both which depend on bank capital, from break even conditions. Basel I and Basel II regulatory regimes are defined in terms of the calculation of the risk weight attached to loans. Specifically, in Basel I, the risk weight is constant for each loan in the same particular category (financing working capital needs in our framework) whereas in Basel II, the risk weight is related to the cyclical behaviour of the probability of default, consistent with the Foundation IRB approach.

We show that when bank capital is introduced, the endogenous probability of default produces an accelerator effect and impacts the loan rate through multiple channels: *a)* the risk premium channel, *b)* the bank capital default channel, and *c)* the risk weight channel. While the first two channels prevail regardless of the regulatory regime, the risk weight channel is transparent only under Basel II, and amplifies the response of financial and macroeconomic variables following various shocks, with the strength of this channel depending on the nature of the shock. Moreover, adjustment costs for bank capital are shown to have an ambiguous impact on the degree of procyclicality, depending on the strength of the risk weight channel relative to the credit demand channel. Finally, our simulations suggest that a higher permanent bank capital-loan ratio (as proposed under Basel III) results in a stronger response of the key economic variables following supply, demand and financial shocks.

Despite the undesirable increased procyclicality effects induced by moving towards a more risk sensitive regulatory regime and raising bank capital adequacy requirements, this model does not capture some of the benefits associated with banking regulation. The main objective of bank capital regulation is to promote international banking stability, reduce market power, address market failures (such as externalities) and mitigate asymmetric information (Drumond 2009). However, this model does not incorporate these features as our emphasis is to illustrate how bank capital regulation and financial frictions affect the business cycle as a whole, and to examine the various transmission channels through which the financial system links to the real economy. Addressing the possible benefits and welfare effects of bank capital regulation is an important avenue for future research.

Another important extension would be to explore the role of countercyclical bank capital regulation and monetary policy in promoting financial and macroeconomic

stability. Specifically, it could be assumed that the central bank not only targets inflation and the output gap, but also a financial stability indicator (such as credit spreads, credit growth or the credit to GDP ratio). In addition, we could also implement in this setup a Basel III- type countercyclical regulatory rule which tightens bank capital requirements during good times and loosens requirements during recessions. The Basel Committee on Banking Supervision has suggested that bank capital requirements should be adjusted in response to excess credit growth or the loan to GDP ratio, which it views as good indicators for systemic risk (see Basel Committee on Banking Supervision 2011). Therefore, combining macroprudential policies (a credit augmented Taylor rule for example) with countercyclical bank capital regulation as proposed by Basel III, and examining the possible trade-offs between macroeconomic and financial stability would be an important contribution for this stream of research.

2.A Appendix

2.A.1 Wage Decision

The wage setting environment follows Erceg, Henderson and Levin (2000), Christiano, Eichenbaum and Evans (2005) and Christiano, Motto and Rostagno (2008), where each household i supplies a unique type of labour, $H_{i,t}$ with $i \in (0, 1)$. All these types of labour are then aggregated into one composite labour by a labour aggregator using the standard Dixit-Stiglitz technology,

$$N_t = \left(\int_0^1 H_{i,t}^{\frac{\lambda_w-1}{\lambda_w}} di \right)^{\frac{\lambda_w}{\lambda_w-1}}, \quad (2.63)$$

with $\lambda_w > 1$ representing the constant elasticity of substitution between the different types of labour. The i^{th} household therefore faces the following demand curve for its labour,

$$H_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\lambda_w} N_t, \quad (2.64)$$

where W_t denotes the aggregate wages paid for one unit of the composite labour (N_t). The zero profit condition for the labour aggregator, obtained by substituting condition (2.63) in (2.64), results in the economy wide wage equation,

$$W_t = \left[\int_0^1 W_{i,t}^{1-\lambda_w} di \right]^{\frac{1}{1-\lambda_w}}.$$

It is assumed that in each period a constant fraction of $1 - \omega_w$ workers are able to re-optimize their wages while a fraction of ω_w index their wages according to last period's inflation rate (π_{t-1}). These non re-optimizing households therefore set their wages according to the following rule,

$$W_{j,t} = \pi_{t-1} W_{j,t-1}.$$

With indexed wages, if wages have not been set since period t , then at period $t + s$, the relative real wages for household j will be,

$$\frac{W_{i,t+s}}{W_{t+s}} = \frac{\Pi^s W_{i,t}}{W_{t+s}},$$

where $\Pi^s = \pi_t \times \pi_{t+1} \times \dots \times \pi_{t+s-1}$. Therefore, the demand for labour in period $t + s$ becomes,

$$H_{i,t+s} = \left(\frac{\Pi^s W_{i,t}}{W_{t+s}} \right)^{-\lambda_w} N_{t+s}. \quad (2.65)$$

Substituting the above in the budget constraint yields,

$$C_t + D_t + V_t \leq (1 + i_{t-1}^D)D_{t-1}\frac{P_{t-1}}{P_t} + (1 + i_{t-1}^V)V_{t-1}\frac{P_{t-1}}{P_t} + \frac{\Pi^s W_{i,t}}{P_t} H_{i,t} + \int_0^1 J_{j,t}^{IG} dj - Lump_t - \Theta \frac{1}{2} \left(\frac{V_t}{V_{t-1}} - 1 \right)^2 V_{t-1} \frac{P_{t-1}}{P_t},$$

such that the derivative of C_{t+s} with respect to $W_{j,t}$ is,

$$\frac{\partial C_{t+s}}{\partial W_{i,t}} = \left[\frac{\Pi^s}{P_{t+s}} H_{i,t+s} + \frac{\Pi^s W_{i,t}}{P_{t+s}} \frac{\partial H_{i,t+s}}{\partial W_{i,t}} \frac{H_{i,t+s}}{H_{i,t+s}} \right],$$

or,

$$\frac{\partial C_{t+s}}{\partial W_{i,t}} = \left\{ \frac{\Pi^s}{P_{t+s}} H_{i,t+s} \left[1 + \frac{\partial H_{i,t+s}}{\partial W_{i,t}} \frac{W_{i,t}}{H_{i,t+s}} \right] \right\},$$

or,

$$\frac{\partial C_{t+s}}{\partial W_{i,t}} = \frac{\Pi^s}{P_{t+s}} N_{i,t+s} \left[1 - \lambda_w \left(\frac{\Pi^s W_{i,t}}{W_{t+s}} \right)^{-\lambda_w - 1} \left(\frac{\Pi^s}{W_{t+s}} \right) N_{t+s} \frac{W_{i,t}}{H_{i,t+s}} \right],$$

or,

$$\frac{\partial C_{t+s}}{\partial W_{i,t}} = \frac{\Pi^s}{P_{t+s}} H_{i,t+s} \left[1 - \lambda_w \left(\frac{\Pi^s W_{i,t}}{W_{t+s}} \right)^{-\lambda_w} \frac{N_{t+s}}{H_{i,t+s}} \right].$$

Substituting the demand for labour equation yields,

$$\frac{\partial C_{t+s}}{\partial W_{i,t}} = \frac{\Pi^s}{P_{t+s}} H_{i,t+s} [1 - \lambda_w].$$

Households who are able to optimize their wages in each period set their wage to $W_{i,t}$, which is obtained from the following maximization problem subject to the budget constraint and the demand for labour,

$$\max_{W_{i,t}^*} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s [\mathbf{U}(C_{t+s}) + \mathbf{V}(H_{i,t+s})],$$

where $\mathbf{U}(C_{t+s}) = \frac{[C_{t+s}]^{1-\varsigma-1}}{1-\varsigma-1}$ and $\mathbf{V}(H_{i,t+s}) = -\frac{H_{i,t+s}^{1+\gamma}}{1+\gamma}$. The first order condition with respect to $W_{i,t}^*$ is given by,

$$E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\mathbf{U}'_{C,t+s} \frac{\partial C_{t+s}}{\partial W_{i,t}} + \left(\mathbf{V}'_H \right)_{i,t+s} \frac{\partial H_{i,t+s}}{\partial W_{i,t}} \right] = 0,$$

or,

$$E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\left(\mathbf{U}'_C \right)_{t+s} \frac{\Pi^s}{P_{t+s}} H_{i,t+s} [1 - \lambda_w] - \left(\mathbf{V}'_H \right)_{i,t+s} \lambda_w (W_{i,t})^{-1} H_{i,t+s} \right] = 0,$$

multiply through $W_{i,t} \left(\frac{1}{1-\lambda_w} \right)$,

$$E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\frac{\Pi^s W_{i,t}}{P_{t+s}} \left(\mathbf{U}'_C \right)_{t+s} - \frac{\lambda_w}{1-\lambda_w} \left(\mathbf{V}'_H \right)_{i,t+s} \right] H_{i,t+s} = 0.$$

The above equation is the one showed in the body text which is represented by (2.12). Rearranging results in,

$$E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left(\mathbf{U}'_C \right)_{t+s} \left[\frac{\Pi^s W_{i,t}}{P_{t+s}} \right] H_{i,t+s} = \frac{\lambda_w}{1-\lambda_w} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left(\mathbf{V}'_H \right)_{i,t+s} H_{i,t+s},$$

or,

$$\begin{aligned} & E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left(\mathbf{U}'_C \right)_{t+s} \left[\frac{\Pi^s W_{i,t}}{P_{t+s}} \right] \left(\frac{\Pi^s W_{i,t}}{W_{t+s}} \right)^{-\lambda_w} N_{t+s} \\ &= \frac{\lambda_w}{1-\lambda_w} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left(\mathbf{V}'_H \right)_{i,t+s} \left(\frac{\Pi^s W_{i,t}}{W_{t+s}} \right)^{-\lambda_w} N_{t+s}, \end{aligned}$$

or,

$$\begin{aligned} & E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left(\mathbf{U}'_C \right)_{t+s} \frac{W_{i,t}}{W_t} \left[\frac{\Pi^s W_t}{P_{t+s}} \right] \left(\frac{\Pi^s W_t}{W_{t+s}} \right)^{-\lambda_w} \left(\frac{W_{i,t}}{W_t} \right)^{-\lambda_w} N_{t+s} \\ &= \frac{\lambda_w}{\lambda_w - 1} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left(-\mathbf{V}'_H \right)_{i,t+s} \left(\frac{W_{i,t}}{W_t} \right)^{-\lambda_w} \left(\frac{\Pi^s W_t}{W_{t+s}} \right)^{-\lambda_w} N_{t+s}. \end{aligned}$$

Define $\Upsilon_{i,t} = \frac{W_{i,t}}{W_t}$, $F_{t,t+s} = \frac{\Pi^s W_t}{P_{t+s}}$, $G_{t,t+s} = \left(\frac{\Pi^s W_t}{W_{t+s}} \right)$ so the above equation reduces to,

$$\begin{aligned} & E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left(\mathbf{U}'_C \right)_{t+s} \Upsilon_{i,t} F_{t,t+s} G_{t,t+s}^{-\lambda_w} \Upsilon_{i,t}^{-\lambda_w} N_{t+s} \\ &= \frac{\lambda_w}{1-\lambda_w} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left(-\mathbf{V}'_H \right)_{i,t+s} \Upsilon_{i,t}^{-\lambda_w} G_{t,t+s}^{-\lambda_w} N_{t+s}. \end{aligned}$$

Turning now to log-linearize the above equation with the steady state values

denoted by dropping the time subscript,

$$\begin{aligned}
& \mathbf{U}_C \Upsilon F G^{-\lambda_w} \Upsilon^{-\lambda_w} N E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 + \widehat{(\mathbf{U}'_C)}_{t+s} \right] \left[1 + \widehat{\Upsilon}_{i,t} \right] \left[1 + \widehat{F}_{t,t+s} \right] \times \\
& \times \left[1 - \lambda_w \widehat{G}_{t,t+s} \right] \left[1 - \lambda_w \widehat{\Upsilon}_{i,t} \right] \left[1 + \widehat{N}_{t+s} \right] \\
& = -\frac{\lambda_w}{\lambda_w - 1} \mathbf{V}_N \Upsilon^{-\lambda_w} G^{-\lambda_w} N E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 - \widehat{(\mathbf{V}'_H)}_{i,t+s} \right] \left[1 - \lambda_w \widehat{\Upsilon}_{i,t} \right] \times \\
& \times \left[1 - \lambda_w \widehat{G}_{t,t+s} \right] \left[1 + \widehat{N}_{t+s} \right].
\end{aligned}$$

Note that in steady state $F = \frac{W}{P}$ and $\frac{W}{P} = -\frac{\lambda_w}{\lambda_w - 1} \frac{\mathbf{V}_N}{\mathbf{U}_C} = \frac{\lambda_w}{\lambda_w - 1} MRS$. Moreover,

$\Upsilon = 1$ and therefore the above equation can be written as,

$$\begin{aligned}
& E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 + \widehat{(\mathbf{U}'_C)}_{t+s} \right] \left[1 + \widehat{\Upsilon}_{i,t} \right] \left[1 + \widehat{F}_{t,t+s} \right] \left[1 - \lambda_w \widehat{G}_{t,t+s} \right] \times \\
& \times \left[1 - \lambda_w \widehat{\Upsilon}_{i,t} \right] \left[1 + \widehat{N}_{t+s} \right] \\
& = E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 - \widehat{(\mathbf{V}'_H)}_{i,t+s} \right] \left[1 - \lambda_w \widehat{\Upsilon}_{i,t} \right] \times \\
& \times \left[1 - \lambda_w \widehat{G}_{t,t+s} \right] \left[1 + \widehat{N}_{t+s} \right],
\end{aligned}$$

or,

$$\begin{aligned}
& E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 + \widehat{\Upsilon}_{i,t} + \widehat{(\mathbf{U}'_C)}_{t+s} \right] \left[1 + \widehat{F}_{t,t+s} \right] \left[1 - \lambda_w \widehat{G}_{t,t+s} \right] \left[1 - \lambda_w \widehat{\Upsilon}_{i,t} \right] \left[1 + \widehat{N}_{t+s} \right] \\
& = E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 - \lambda_w \widehat{\Upsilon}_{i,t} - \widehat{(\mathbf{V}'_H)}_{i,t+s} \right] \left[1 - \lambda_w \widehat{G}_{t,t+s} \right] \left[1 + \widehat{N}_{t+s} \right],
\end{aligned}$$

or,

$$\begin{aligned}
& E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 + \widehat{F}_{t,t+s} + \widehat{\Upsilon}_{i,t} + \widehat{(\mathbf{U}'_C)}_{t+s} \right] \left[1 - \lambda_w \widehat{G}_{t,t+s} \right] \left[1 - \lambda_w \widehat{\Upsilon}_{i,t} \right] \left[1 + \widehat{N}_{t+s} \right] \\
& = E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 - \lambda_w \widehat{G}_{t,t+s} - \lambda_w \widehat{\Upsilon}_{i,t} - \widehat{(\mathbf{V}'_H)}_{i,t+s} \right] \left[1 + \widehat{N}_{t+s} \right],
\end{aligned}$$

or,

$$\begin{aligned} & E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 - \lambda_w \widehat{G_{t,t+s}} + \widehat{F_{t,t+s}} + \widehat{\Upsilon_{i,t}} + (\widehat{\mathbf{U}'_C})_{t+s} \right] \left[1 - \lambda_w \widehat{\Upsilon_{i,t}} \right] \left[1 + \widehat{N_{t+s}} \right] \\ &= -\frac{\lambda_w}{1 - \lambda_w} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 + \widehat{N_{t+s}} - \lambda_w \widehat{G_{t,t+s}} - \lambda_w \widehat{\Upsilon_{i,t}} - (\widehat{\mathbf{V}'_H})_{i,t+s} \right], \end{aligned}$$

or,

$$\begin{aligned} & E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 - \lambda_w \widehat{\Upsilon_{i,t}} - \lambda_w \widehat{G_{t,t+s}} + \widehat{F_{t,t+s}} + \widehat{\Upsilon_{i,t}} + (\widehat{\mathbf{U}'_C})_{t+s} \right] \left[1 + \widehat{N_{t+s}} \right] \\ &= E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[1 + \widehat{N_{t+s}} - \lambda_w \widehat{G_{t,t+s}} - \lambda_w \widehat{\Upsilon_{i,t}} - (\widehat{\mathbf{V}'_H})_{i,t+s} \right], \end{aligned}$$

or,

$$E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{F_{t,t+s}} + \widehat{\Upsilon_{i,t}} + (\widehat{\mathbf{U}'_C})_{t+s} \right] = E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[-(\widehat{\mathbf{V}'_H})_{i,t+s} \right],$$

or,

$$\frac{1}{1 - \omega_w \beta} \widehat{\Upsilon_{i,t}} + E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{F_{t,t+s}} + (\widehat{\mathbf{U}'_C})_{t+s} \right] = E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[-(\widehat{\mathbf{V}'_H})_{i,t+s} \right],$$

collecting terms,

$$\frac{1}{1 - \omega_w \beta} \widehat{\Upsilon_{i,t}} = E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[(-\widehat{\mathbf{V}'_H})_{i,t+s} - \widehat{F_{t,t+s}} - (\widehat{\mathbf{U}'_C})_{t+s} \right],$$

multiplying by $(1 - \omega_w \beta)$ yields the following log-linear relationship,

$$\widehat{\Upsilon_{i,t}} = (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[(-\widehat{\mathbf{V}'_H})_{i,t+s} - \widehat{F_{t,t+s}} - (\widehat{\mathbf{U}'_C})_{t+s} \right]. \quad (2.66)$$

From the individual household's utility function,

$$(-\widehat{\mathbf{V}'_H})_{i,t+s} = \gamma \widehat{H_{i,t+s}}. \quad (2.67)$$

Thus, on an average economy scale,

$$(-\widehat{\mathbf{V}'_N})_{t+s} = \gamma \widehat{N_{t+s}}. \quad (2.68)$$

Moreover, from the individual labour demand (equation 2.65) and the definitions of

$\Upsilon_{i,t}$ and $G_{t,t+s}$,

$$\widehat{H_{i,t+s}} = \widehat{N_{t+s}} - \lambda_w \left(\widehat{\Upsilon_{i,t}} + \widehat{G_{t,t+s}} \right). \quad (2.69)$$

Therefore, from (2.67) and (2.69),

$$\left(-\widehat{\mathbf{V}'_H} \right)_{i,t+s} = \gamma \left[\widehat{N_{t+s}} - \lambda_w \left(\widehat{\Upsilon_{i,t}} + \widehat{G_{t,t+s}} \right) \right]. \quad (2.70)$$

Note that by definition,

$$\widehat{F_{t,t+s}} - \widehat{G_{t,t+s}} \equiv \widehat{W_{t+s}} - \widehat{P_{t+s}} \equiv \left(\frac{\widehat{W_{t+s}}}{\widehat{P_{t+s}}} \right).$$

Finally, the population average of the marginal rate of substitution between leisure and consumption is given by,

$$\widehat{MRS_{t+s}} = \left(-\widehat{\mathbf{V}'_N} \right)_{t+s} - \left(\widehat{\mathbf{U}'_C} \right)_{t+s} = \gamma \widehat{N_{t+s}} + \frac{1}{\varsigma} \widehat{C_{t+s}}. \quad (2.71)$$

We now turn to re-write equation (2.66) using the expressions above. First, adding and subtracting the term $(1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \widehat{G_{t,t+s}}$ yields,

$$\widehat{\Upsilon_{i,t}} = (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\left(-\widehat{\mathbf{V}'_H} \right)_{i,t+s} - \widehat{F_{t,t+s}} - \left(\widehat{\mathbf{U}'_C} \right)_{t+s} + \widehat{G_{t,t+s}} - \widehat{G_{t,t+s}} \right].$$

Second, adding and subtracting the expression $(1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\left(-\widehat{\mathbf{V}'_N} \right)_{t+s} \right]$ and using the definition of $\widehat{F_{t,t+s}} - \widehat{G_{t,t+s}}$, then the above can be written as,

$$\begin{aligned} \widehat{\Upsilon_{i,t}} + (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \widehat{G_{t,t+s}} &= (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\begin{aligned} &\left(-\widehat{\mathbf{V}'_H} \right)_{i,t+s} - \left(\widehat{\mathbf{U}'_C} \right)_{t+s} - \\ &\quad - \left(\frac{\widehat{W_{t+s}}}{\widehat{P_{t+s}}} \right) \end{aligned} \right] + \\ &+ (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\left(-\widehat{\mathbf{V}'_N} \right)_{t+s} \right] - \\ &- (1 - \omega \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\left(-\widehat{\mathbf{V}'_N} \right)_{t+s} \right], \end{aligned}$$

collecting terms and rearranging,

$$\begin{aligned} & \widehat{\Upsilon}_{i,t} + (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{G_{t,t+s}} + (-\widehat{\mathbf{V}'_N})_{t+s} \right] \\ = & (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[(-\widehat{\mathbf{V}'_N})_{i,t+s} - (\widehat{\mathbf{U}'_C})_{t+s} - \left(\widehat{\frac{W_{t+s}}{P_{t+s}}} \right) + (-\widehat{\mathbf{V}'_N})_{t+s} \right], \end{aligned}$$

substituting equation (2.68) for $(-\widehat{\mathbf{V}'_N})_{t+s}$, using equation (2.71) for $\widehat{MRS_{t+s}}$, and finally substituting (2.70) for $(-\widehat{\mathbf{V}'_H})_{i,t+s}$ results in,

$$\begin{aligned} & \widehat{\Upsilon}_{i,t} + (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{G_{t,t+s}} + \gamma \widehat{N_{t+s}} \right] \\ = & (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \gamma \left[\widehat{N_{t+s}} - \lambda_w \left(\widehat{\Upsilon}_{i,t} + \widehat{G_{t,t+s}} \right) \right] + \\ & + (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{MRS_{t+s}} - \left(\widehat{\frac{W_{t+s}}{P_{t+s}}} \right) \right], \end{aligned}$$

expanding the brackets,

$$\begin{aligned} & \widehat{\Upsilon}_{i,t} + (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \widehat{G_{t,t+s}} + (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \gamma \widehat{N_{t+s}} \\ = & (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \gamma \widehat{N_{t+s}} - (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \gamma \lambda_w \widehat{\Upsilon}_{i,t} - \\ & - (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \gamma \lambda_w \widehat{G_{t,t+s}} + \\ & + (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{MRS_{t+s}} - \left(\widehat{\frac{W_{t+s}}{P_{t+s}}} \right) \right], \end{aligned}$$

eliminating common factors and rearranging,

$$\begin{aligned} & \widehat{\Upsilon}_{i,t} + \gamma \lambda_w (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \widehat{\Upsilon}_{i,t} + (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \widehat{G_{t,t+s}} + \\ & + \gamma \lambda_w (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \widehat{G_{t,t+s}} \\ = & (1 - \omega_w \beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{MRS_{t+s}} - \left(\widehat{\frac{W_{t+s}}{P_{t+s}}} \right) \right]. \end{aligned}$$

Collecting common terms,

$$\begin{aligned} & (1 + \gamma\lambda_w) \widehat{\Upsilon}_{i,t} + (1 - \omega_w\beta) (1 + \gamma\lambda_w) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \widehat{G}_{t,t+s} \\ = & (1 - \omega_w\beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{MRS}_{t+s} - \widehat{\left(\frac{W_{t+s}}{P_{t+s}} \right)} \right], \end{aligned}$$

or,

$$\begin{aligned} & (1 + \gamma\lambda_w) \left[\widehat{\Upsilon}_{i,t} + (1 - \omega_w\beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \widehat{G}_{t,t+s} \right] \\ = & (1 - \omega_w\beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{MRS}_{t+s} - \widehat{\left(\frac{W_{t+s}}{P_{t+s}} \right)} \right], \end{aligned}$$

or,

$$\begin{aligned} \widehat{\Upsilon}_{i,t} = & \frac{(1 - \omega_w\beta)}{(1 + \gamma\lambda_w)} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{MRS}_{t+s} - \widehat{\left(\frac{W_{t+s}}{P_{t+s}} \right)} \right] - \\ & - (1 - \omega_w\beta) E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \widehat{G}_{t,t+s}, \end{aligned}$$

or,

$$\widehat{\Upsilon}_{i,t} = \frac{(1 - \omega_w\beta)}{(1 + \gamma\lambda_w)} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{MRS}_{t+s} - \widehat{\left(\frac{W_{t+s}}{P_{t+s}} \right)} - (1 + \gamma\lambda_w) \widehat{G}_{t,t+s} \right]. \quad (2.72)$$

Recall, $\Upsilon_{i,t} = \frac{W_{i,t}}{W_t}$; $F_{t,t+s} = \frac{\Pi^s W_t}{P_{t+s}}$, and $G_{t,t+s} = \left(\frac{\Pi^s W_t}{W_{t+s}} \right)$. Also, note that the aggregate wage equation is defined as,

$$W_t^{1-\lambda_w} = \omega_w (\pi_{t-1} W_{t-1})^{1-\lambda_w} + (1 - \omega_w) (W_t^*)^{1-\lambda_w},$$

which after log-linearizing results directly in the relative wage equation,

$$\widehat{\Upsilon}_{i,t} = \widehat{\left(\frac{W_t^*}{W_t} \right)} = \left(\frac{\omega_w}{1 - \omega_w} \right) \widehat{\pi}_t^W, \quad (2.73)$$

where $\widehat{\pi}_t^W \equiv \widehat{W}_t - \widehat{W}_{t-1}$ is the wage inflation.

Now, note that $\widehat{G_{t,t}} = 0$ and $\widehat{G_{t,t+s}} = -\sum_{x=1}^s \widehat{\pi_{t+x}^W}$. Therefore,

$$\begin{aligned} \left(\frac{\omega_w}{1-\omega_w}\right) \widehat{\pi_t^W} &= \frac{(1-\omega_w\beta)}{(1+\gamma\lambda_w)} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{MRS_{t+s}} - \left(\frac{\widehat{W_{t+s}}}{\widehat{P_{t+s}}} \right) \right] + \\ &+ (1-\omega_w\beta) E_t \sum_{s=1}^{\infty} \omega_w^s \beta^s \sum_{x=1}^s \widehat{\pi_{t+x}^W}, \end{aligned}$$

or,

$$\begin{aligned} \frac{1}{(1-\beta\omega_w)} \left(\frac{\omega_w}{1-\omega_w}\right) \widehat{\pi_t^W} &= \frac{1}{(1+\gamma\lambda_w)} E_t \sum_{s=0}^{\infty} \omega_w^s \beta^s \left[\widehat{MRS_{t+s}} - \left(\frac{\widehat{W_{t+s}}}{\widehat{P_{t+s}}} \right) \right] + \\ &+ E_t \sum_{s=1}^{\infty} \omega_w^s \beta^s \sum_{x=1}^s \widehat{\pi_{t+x}^W}. \end{aligned}$$

Finally, using the forward operator, expanding brackets, collecting terms and rearranging results in the final log-linear wage inflation equation,

$$\widehat{\pi_t^W} = \beta E_t \widehat{\pi_{t+1}^W} + \frac{(1-\omega_w)(1-\beta\omega_w)}{(\omega_w)(1+\gamma\lambda_w)} \left[\widehat{MRS_t} - \left(\frac{\widehat{W_t}}{\widehat{P_t}} \right) \right].$$

2.A.2 Log-Linearized System

The log-linearized version of the model is derived below employing standard log-linearization techniques and using the steady state variables presented in the text.

Log-linearizing equation (2.6) from the household's first order conditions,

$$(1 - \frac{1}{\varsigma}\widehat{C}_t) = \beta E_t \left[(1 + i^D)(1 + \widehat{i}_t^D)(1 + \widehat{P}_t)(1 - \widehat{P}_{t+1})(1 - \frac{1}{\varsigma}\widehat{C}_{t+1}) \right],$$

or,

$$(1 - \frac{1}{\varsigma}\widehat{C}_t) = \beta(1 + i^D)E_t \left[1 - \widehat{P}_{t+1} + \widehat{P}_t - \frac{1}{\varsigma}\widehat{C}_{t+1} + \widehat{i}_t^D \right].$$

Defining $E_t\widehat{\pi}_{t+1}^P = E_t\widehat{P}_{t+1} - \widehat{P}_t$ as the expected log-deviation of inflation from its steady state value (assuming $\pi^{P,T} = 0$) then the above equation reduces to,

$$(1 - \frac{1}{\varsigma}\widehat{C}_t) = \beta(1 + i^D) - \beta(1 + i^D)E_t\widehat{\pi}_{t+1}^P - \beta(1 + i^D)\frac{1}{\varsigma}E_t\widehat{C}_{t+1} + \beta(1 + i^D)E_t\widehat{i}_t^D.$$

Using the steady state value of the deposit rate, re-arranging and assuming that households know the actual deposit rate when making their consumption decisions yields the log-linearized Euler equation,

$$\widehat{C}_t = E_t\widehat{C}_{t+1} - \varsigma \left[\widehat{i}_t^D - E_t\widehat{\pi}_{t+1}^P \right].$$

The log-linearized probability of default is derived from equation (2.34),

$$\Phi(1 + \widehat{\Phi}_t) = \frac{\varepsilon^M}{\bar{\varepsilon} - \underline{\varepsilon}} \left[1 + \widehat{\varepsilon}_t^M \right] - \frac{\underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}.$$

Note that $\Phi = \frac{\varepsilon^M}{\bar{\varepsilon} - \underline{\varepsilon}} - \frac{\underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$ so the above equation reduces to,

$$\Phi\widehat{\Phi}_t = \left(\frac{\varepsilon^M}{\bar{\varepsilon} - \underline{\varepsilon}} \right) \widehat{\varepsilon}_t^M,$$

dividing by the steady state value of the default probability,

$$\widehat{\Phi}_t = \frac{\left(\frac{\varepsilon^M}{\bar{\varepsilon} - \underline{\varepsilon}} \right)}{\left(\frac{\varepsilon^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}} \right)} \widehat{\varepsilon}_t^M,$$

or,

$$\widehat{\Phi}_t = \left(\frac{\varepsilon^M}{\varepsilon^M - \underline{\varepsilon}} \right) \widehat{\varepsilon}_t^M.$$

Log-linearizing equation (2.24) results in,

$$\varepsilon^M \left(1 + \widehat{\varepsilon_t^M}\right) = \frac{1}{\chi A} (1 + i^L)(1 + \widehat{i_t^L}) W^R (1 + \widehat{W_t^R}) (1 - \widehat{A_t})(1 - \widehat{\chi_t}),$$

using the steady state value $\varepsilon^M = \frac{1}{\chi A} (1 + i^L) W^R$,

$$\widehat{\varepsilon_t^M} = \widehat{i_t^L} + \widehat{W_t^R} - \widehat{A_t} - \widehat{\chi_t}.$$

Finally, substituting the above in $\widehat{\Phi_t} = \left(\frac{\varepsilon^M}{\varepsilon^M - \underline{\varepsilon}}\right) \widehat{\varepsilon_t^M}$ yields,

$$\widehat{\Phi_t} = \left(\frac{\varepsilon^M}{\varepsilon^M - \underline{\varepsilon}}\right) \left(\widehat{i_t^L} + \widehat{W_t^R} - \widehat{A_t} - \widehat{\chi_t}\right),$$

which is the final log-linearized expression of the probability of default corresponding with equation (2.56).

The log-linearized lending rate is derived from equation (2.37),

$$\begin{aligned} (1 + i^L)(1 + \widehat{i_t^L}) &= (1 + i^D)(1 + \widehat{i_t^D}) + \rho(1 + i^V)(1 + q\widehat{\Phi_t})(1 + \widehat{i_t^V}) - \\ &\quad - \rho(1 + i^D)(1 + q\widehat{\Phi_t})(1 + \widehat{i_t^D}) + \\ &\quad + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} (1 - \widehat{W_t^R})(1 + 2\widehat{\Phi_t})(1 + \widehat{A_t} + \widehat{\chi_t}), \end{aligned}$$

or,

$$\begin{aligned} (1 + i^L)(1 + \widehat{i_t^L}) &= (1 + i^D) + (1 + i^D)\widehat{i_t^D} + \rho(1 + i^V)(1 + \widehat{i_t^V} + q\widehat{\Phi_t}) - \\ &\quad - \rho(1 + i^D)(1 + \widehat{i_t^D} + q\widehat{\Phi_t}) + \\ &\quad + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} (1 + 2\widehat{\Phi_t} - \widehat{W_t^R} + \widehat{A_t} + \widehat{\chi_t}), \end{aligned}$$

or,

$$\begin{aligned} (1 + i^L)(1 + \widehat{i_t^L}) &= (1 + i^D) + (1 + i^D)\widehat{i_t^D} + \rho(1 + i^V) + \rho(1 + i^V)\widehat{i_t^V} + \\ &\quad + \rho(1 + i^V)q\widehat{\Phi_t} - \rho(1 + i^D) - \\ &\quad - \rho(1 + i^D)\widehat{i_t^D} - \rho(1 + i^D)q\widehat{\Phi_t} + \\ &\quad + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} + 2\chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \widehat{\Phi_t} - \\ &\quad - \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \widehat{W_t^R} + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} (\widehat{A_t} + \widehat{\chi_t}). \end{aligned}$$

Substituting the steady state level of the (gross) lending rate (equation 2.47),

$$\begin{aligned}
& \rho(1+i^V) + (1-\rho)(1+i^D) + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} + (1+i^L)\widehat{i}_t^L \\
= & (1+i^D) + (1+i^D)\widehat{i}_t^{\widehat{D}} + \rho(1+i^V) + \rho(1+i^V)\widehat{i}_t^{\widehat{V}} + \\
& + \rho(1+i^V)q\widehat{\Phi}_t - \rho(1+i^D) - \rho(1+i^D)\widehat{i}_t^{\widehat{D}} - \rho(1+i^D)q\widehat{\Phi}_t + \\
& + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} + 2\chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \widehat{\Phi}_t - \\
& - \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \widehat{W}_t^R + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} (\widehat{A}_t + \widehat{\chi}_t),
\end{aligned}$$

or,

$$\begin{aligned}
(1+i^L)\widehat{i}_t^L = & (1+i^D)\widehat{i}_t^{\widehat{D}} + \rho(1+i^V)\widehat{i}_t^{\widehat{V}} + \rho(1+i^V)q\widehat{\Phi}_t - \\
& - \rho(1+i^D)\widehat{i}_t^{\widehat{D}} - \rho(1+i^D)q\widehat{\Phi}_t + \\
& + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \left[2\widehat{\Phi}_t - \widehat{W}_t^R + \widehat{A}_t + \widehat{\chi}_t \right],
\end{aligned}$$

or,

$$\begin{aligned}
(1+i^L)\widehat{i}_t^L = & \rho(1+i^V)\widehat{i}_t^{\widehat{V}} + (1-\rho)(1+i^D)\widehat{i}_t^{\widehat{D}} + \rho q(i^V - i^D)\widehat{\Phi}_t + \\
& + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \left[2\widehat{\Phi}_t - \widehat{W}_t^R + \widehat{A}_t + \widehat{\chi}_t \right],
\end{aligned}$$

or,

$$\widehat{i}_t^L = \frac{1}{(1+i^L)} \left\{ \rho(1+i^V)\widehat{i}_t^{\widehat{V}} + (1-\rho)(1+i^D)\widehat{i}_t^{\widehat{D}} + \rho q(i^V - i^D)\widehat{\Phi}_t + \right. \\
\left. + \chi A \frac{\Phi^2}{W^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \left[2\widehat{\Phi}_t - \widehat{W}_t^R + \widehat{A}_t + \widehat{\chi}_t \right] \right\},$$

which is equation (2.57) presented in the body text.

The log-linearized equation determining the interest rate on bank capital is derived from equation (2.7),

$$\begin{aligned}
& (1+i^V) \left[1 + \widehat{i}_t^{\widehat{V}} \right] - \Phi(1+i^V) \left[1 + \widehat{\Phi}_t + \widehat{i}_t^{\widehat{V}} \right] + \\
& + \frac{1}{2}\Theta \left[1 + 2\widehat{V}_{t+1} - 2\widehat{V}_t \right] - \frac{1}{2}\Theta \\
= & (1+i^D) \left[1 + \widehat{i}_t^{\widehat{D}} \right] + \Theta(1+i^D) \left[1 + \widehat{i}_t^{\widehat{D}} + \widehat{P}_{t-1} - \widehat{P}_t + \widehat{V}_t - \widehat{V}_{t-1} \right] - \\
& - \Theta(1+i^D) \left[1 + \widehat{P}_{t-1} - \widehat{P}_t + \widehat{i}_t^{\widehat{D}} \right],
\end{aligned}$$

or,

$$\begin{aligned} & (1 + i^V) - \Phi (1 + i^V) + (1 + i^V) \widehat{i_t^V} - \Phi (1 + i^V) [\widehat{\Phi_t} + \widehat{i_t^V}] + \Theta [\widehat{V_{t+1}} - \widehat{V_t}] \\ = & (1 + i^D) + (1 + i^D) \widehat{i_t^D} + \Theta (1 + i^D) [\widehat{V_t} - \widehat{V_{t-1}}], \end{aligned}$$

or,

$$\begin{aligned} & (1 + i^V) \widehat{i_t^V} - \Phi (1 + i^V) [\widehat{\Phi_t} + \widehat{i_t^V}] \\ = & (1 + i^D) \widehat{i_t^D} + \Theta (1 + i^D) [\widehat{V_t} - \widehat{V_{t-1}}] - \Theta [\widehat{V_{t+1}} - \widehat{V_t}], \end{aligned}$$

or,

$$(1 - \Phi) (1 + i^V) \widehat{i_t^V} - \Phi (1 + i^V) \widehat{\Phi_t} = (1 + i^D) \widehat{i_t^D} + \Theta (1 + i^D) [\widehat{V_t} - \widehat{V_{t-1}}] - \Theta [\widehat{V_{t+1}} - \widehat{V_t}],$$

using $(1 + i^V) = \frac{(1+i^D)}{(1-\Phi)}$,

$$(1 + i^D) \widehat{i_t^V} - \Phi (1 + i^V) \widehat{\Phi_t} = (1 + i^D) \widehat{i_t^D} + \Theta (1 + i^D) [\widehat{V_t} - \widehat{V_{t-1}}] - \Theta [\widehat{V_{t+1}} - \widehat{V_t}],$$

dividing by $(1 + i^D)$,

$$\widehat{i_t^V} - \Phi \frac{(1 + i^V)}{(1 + i^D)} \widehat{\Phi_t} = \widehat{i_t^D} + \Theta [\widehat{V_t} - \widehat{V_{t-1}}] - \frac{\Theta}{(1 + i^D)} [\widehat{V_{t+1}} - \widehat{V_t}],$$

using the steady state value of $(1 + i^D) = \frac{1}{\beta}$ and $\frac{(1+i^V)}{(1+i^D)} = \frac{1}{(1-\Phi)}$,

$$\widehat{i_t^V} = \widehat{i_t^D} + \frac{\Phi}{(1 - \Phi)} \widehat{\Phi_t} + \Theta [\widehat{V_t} - \widehat{V_{t-1}}] - \beta \Theta E_t [\widehat{V_{t+1}} - \widehat{V_t}],$$

which is equation (2.59) in the body text.

To obtain the New Keynesian Phillips Curve we log-linearize equation (2.27),³²

$$\begin{aligned}
& YC^{-\frac{1}{\varsigma}} E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[1 + \widehat{Y}_{t+s} - \frac{1}{\varsigma} \widehat{C}_{t+s} \right] \left[1 + (\lambda_p - 1) \widehat{P}_{t+s} \right] \times \\
& \times \left[1 + (1 - \lambda_p) \widehat{P}_t \right] \left[1 + \widehat{Q}_t \right] \\
= & YC^{-\frac{1}{\varsigma}} (mc) \left(\frac{\lambda_p}{\lambda_p - 1} \right) E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[1 + \widehat{Y}_{t+s} - \frac{1}{\varsigma} \widehat{C}_{t+s} \right] \times \\
& \times \left[1 + \widehat{mc}_{t+s} \right] \left[1 + \lambda_p \widehat{P}_{t+s} \right] \left[1 - \lambda_p \widehat{P}_t \right],
\end{aligned}$$

or,

$$\begin{aligned}
& E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[1 + (\lambda_p - 1) \widehat{P}_{t+s} + \widehat{Y}_{t+s} - \frac{1}{\varsigma} \widehat{C}_{t+s} \right] \times \\
& \times \left[1 + (1 - \lambda_p) \widehat{P}_t \right] \left[1 + \widehat{Q}_t \right] \\
= & (mc) \left(\frac{\lambda_p}{\lambda_p - 1} \right) E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[1 + \widehat{mc}_{t+s} + \widehat{Y}_{t+s} - \frac{1}{\varsigma} \widehat{C}_{t+s} \right] \times \\
& \times \left[1 + \lambda_p \widehat{P}_{t+s} \right] \left[1 - \lambda_p \widehat{P}_t \right].
\end{aligned}$$

Note that $(mc) \left(\frac{\lambda_p}{\lambda_p - 1} \right) = 1$, so the above reduces to,

$$\begin{aligned}
& E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[1 + \widehat{Q}_t + (1 - \lambda_p) \widehat{P}_t + (\lambda_p - 1) \widehat{P}_{t+s} + \widehat{Y}_{t+s} - \frac{1}{\varsigma} \widehat{C}_{t+s} \right] \\
= & E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[1 - \lambda_p \widehat{P}_t + \lambda_p \widehat{P}_{t+s} + \widehat{mc}_{t+s} + \widehat{Y}_{t+s} - \frac{1}{\varsigma} \widehat{C}_{t+s} \right],
\end{aligned}$$

or,

$$\begin{aligned}
& E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[1 + \widehat{Q}_t + (\lambda_p - 1) (\widehat{P}_{t+s} - \widehat{P}_t) + \widehat{Y}_{t+s} - \frac{1}{\varsigma} \widehat{C}_{t+s} \right] \\
= & E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[1 + \lambda_p (\widehat{P}_{t+s} - \widehat{P}_t) + \widehat{mc}_{t+s} + \widehat{Y}_{t+s} - \frac{1}{\varsigma} \widehat{C}_{t+s} \right],
\end{aligned}$$

³²The value of Q_t at the steady state is 1, $Q = 1$. This is also the value of Q when all firms adjust prices in each period.

Note that $\sum_{s=0}^{\infty} \omega_p^s \beta^s = \frac{1}{1-\omega_p \beta}$ is a sum of a geometric series so,

$$\frac{\widehat{Q}_t}{1-\omega_p \beta} + E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s (\lambda_p - 1)(\widehat{P}_{t+s} - \widehat{P}_t) = E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[\lambda_p (\widehat{P}_{t+s} - \widehat{P}_t) + \widehat{m c_{t+s}} \right].$$

Inserting the expectations operator and collecting common factors yields,

$$\frac{\widehat{Q}_t}{1-\omega_p \beta} = \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[E_t \widehat{P}_{t+s} - \widehat{P}_t + E_t \widehat{m c_{t+s}} \right],$$

or,

$$\frac{\widehat{Q}_t}{1-\omega_p \beta} = \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[E_t \widehat{P}_{t+s} + E_t \widehat{m c_{t+s}} \right] - \frac{\widehat{P}_t}{1-\omega_p \beta},$$

or,

$$\widehat{Q}_t + \widehat{P}_t = (1-\omega_p \beta) \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[E_t \widehat{P}_{t+s} + E_t \widehat{m c_{t+s}} \right].$$

Note that $\widehat{Q}_t + \widehat{P}_t = \widehat{P}_t^* - \widehat{P}_t + \widehat{P}_t = \widehat{P}_t^*$, then the above becomes,

$$\widehat{P}_t^* = (1-\omega_p \beta) \sum_{s=0}^{\infty} \omega_p^s \beta^s \left[E_t \widehat{P}_{t+s} + E_t \widehat{m c_{t+s}} \right].$$

Using recursive substitution,

$$\widehat{Q}_t = (1-\omega_p \beta) \widehat{m c_t} + \omega_p \beta \left[E_t \widehat{Q}_{t+1} + E_t \pi_{t+1}^P \right]. \quad (2.74)$$

Assuming Calvo (1983) type sticky prices, the portion of firms adjusting their prices in each period is randomly selected. Therefore, the average price taken by firms not being able to re-adjust their prices is equal to the average price in period at the end of period t (see Walsh 2003). Hence, from equation (2.18), the average price index in period t is,

$$P_t^{1-\lambda_p} = (1-\omega_p)(P_t^*)^{1-\lambda_p} + \omega_p(P_{t-1})^{1-\lambda_p}.$$

Using the definition of Q_t ,

$$1 = (1-\omega_p)Q_t^{1-\lambda_p} + \omega_p \left(\frac{P_{t-1}}{P_t} \right)^{1-\lambda_p}.$$

Log linearizing the above equation yields,

$$0 = (1 - \lambda_p)(1 - \omega_p)\widehat{Q}_t - (1 - \lambda_p)\omega_p\widehat{\pi}_t^P,$$

or,

$$\widehat{Q}_t = \left[\frac{\omega_p}{1 - \omega_p} \right] \widehat{\pi}_t^P,$$

substituting back in equation (2.74),

$$\left[\frac{\omega_p}{1 - \omega_p} \right] \widehat{\pi}_t^P = (1 - \omega_p\beta)\widehat{mc}_t + \omega_p\beta \left[\frac{1}{1 - \omega_p} \right] E_t\widehat{\pi}_{t+1}^P,$$

multiplying both sides by $\frac{1 - \omega_p}{\omega_p}$ results in the New Keynesian Phillips Curve,

$$\widehat{\pi}_t^P = \frac{(1 - \omega_p)(1 - \omega_p\beta)}{\omega_p}\widehat{mc}_t + \beta E_t\widehat{\pi}_{t+1}^P,$$

where,

$$\begin{aligned} \widehat{mc}_t &= \widehat{i}_t^L + \widehat{W}_t^R - \widehat{Z}_t, \\ \widehat{i}_t^L &= \frac{1}{(1 + i^L)} \left\{ (1 + i^D)\widehat{i}_t^D + \rho \left[(1 + i^V)\widehat{i}_t^V - (1 + i^D)\widehat{i}_t^D \right] + \rho(i^V - i^D + c)\widehat{\vartheta}_t + \right. \\ &\quad \left. + \chi A \frac{\Phi^2}{\bar{W}^R} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \left[2\widehat{\Phi}_t - \widehat{W}_t^R + \widehat{A}_t + \widehat{\chi}_t \right] \right\}. \end{aligned}$$

Chapter 3

Loan Loss Provisioning Rules, Procyclicality and Financial Volatility

3.1 Introduction

The global financial crisis has led to a renewed debate about both the nature and effectiveness of financial regulation, and the extent to which central banks should consider more explicitly financial stability objectives in the conduct of monetary policy. A key issue in this context has been the design of macroprudential instruments that help to mitigate the procyclicality of the financial system, that is, credit booms and busts that exacerbate the inherent cyclicity of lending, and consequently distort investment decisions, either by fuelling excessive growth in credit or restricting access to bank finance.

Among these countercyclical instruments, bank capital buffers have attracted the most interest in policy circles and academic research. Under the Basel III regime for instance, central banks can now impose a countercyclical bank capital buffer ranging from 0 to 2.5 percent of risk-weighted assets; the buffer itself is related to excess growth in credit to the private sector or the loan to GDP ratio, both viewed as good indicators of systemic risk (see Basel Committee on Banking Supervision 2011).¹ In academic research, a number of recent contributions have studied the performance of countercyclical capital rules in New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models; these contributions include Angelini, Neri and Panetta (2011), Suh (2011), and Agénor, Alper and Pereira da Silva (2013).

There has been also much discussion about the use of loan loss provisions as a countercyclical regulatory rule. In general, loan loss provisions can be classified

¹See Drehmann, Borio, Gambacorta, Jiménez and Trucharte (2010), Repullo and Saurina (2011), and Basel Committee on Banking Supervision (2012) for a more detailed discussion.

into two main categories: *a*) specific provisions, which depend on expected losses on loans which have been identified as impaired or nonperforming, that is, if they have not been repaid a certain number of days (usually 90 days) past the due date; and *b*) general provisions, which depend on expected losses on loans which are not necessarily impaired but are likely to be in the future.² In the United Kingdom for instance, general provisions are subjective but banks should take into account past experience and current economic conditions. Specific provisions are governed by International Accounting Standards (IAS) 39, which require domestic banks to adopt an *incurred loss method* of loan loss provisioning: this implies that provisions are set only once a loss has incurred. As a result, general provisions often represent only a small fraction of total provisions. More importantly, some observers have argued that IAS 39 accounting guidelines have been a predominant source of procyclicality in lending standards, because loan loss provisions tend to be essentially *ex post*.³ Indeed, with the incurred loss approach, the recognition of loan losses is delayed until borrowers actually default. Moreover, there are often restrictions on the tax deductibility of provisioning expenses, which tend to affect the cyclicity of bank profits, market valuations, and their funding costs. The result is that it can be difficult for a bank to increase provisions in an economic boom, even if it correctly judges that the future ability of its borrowers to repay has deteriorated. A possible consequence is that banks may reduce lending in recessions, thereby magnifying the impact of negative shocks (Beatty and Liao 2011).

This raises therefore the broader question of redesigning accounting principles (that is, switching from an incurred loss approach to an expected loss approach) to improve banking sector stability and mitigate procyclicality, as well as systemic risk. Indeed, the Basel Committee continues to work with the International Accounting Standards Board (IASB) on the expected loss approach to loan loss provisioning. The view is that if forward-looking provisions can take into account more credit information and anticipate and quantify better the expected losses associated with a loan portfolio, they would provide additional buffers and better incentives to mitigate procyclicality. This is the fundamental idea of *dynamic* provisioning rules, which have been used for some time in Spain.⁴ The Spanish system requires higher

²Perez, Salas-Fumas and Saurina (2008) note that general provisions usually rise during an economic upturn, as banks give out more loans and the demand for credit is high during this period. During a downturn, loans to riskier companies would incur larger loan losses as risks materialize, and therefore higher specific loan-loss provisions follow.

³In the United States under the guidelines of the Generally Accepted Accounting Standards (GAAP), loan loss provisions should be established at a level that is adequate (but not excessive) to cover expected losses related to identified impaired loans and probable credit losses which have been incurred as of the balance sheet date. Thus, they are also essentially *ex post* in nature.

⁴The introduction of countercyclical provisions in Spain was facilitated by the fact that the

provisions when credit grows more than the historical average, thus linking provisioning to the credit and business cycle. This both discourages (although does not eliminate) excessive lending in booms and strengthens the banks for bad times. Studies that have attempted to evaluate the performance of Spain's dynamic loan provisioning system include Saurina (2009), Caprio (2010), and Jiménez, Ongena, Peydró and Saurina (2012); all concluded that although the provisioning scheme allowed banks to enter the downswing associated with the global financial crisis in more robust shape than they would have been otherwise, it is less clear that it had any material effect on the credit cycle or that it helped in any significant way to contain Spain's real estate bubble over the previous decade. Put differently, even though these systems may succeed in making banks more resilient, by increasing their capacity to absorb expected losses, in contrast to capital requirements, they appear to have limited effectiveness when it comes to restraining credit growth.⁵

This chapter contributes to the debate on the performance of loan loss provisioning systems by embedding backward- and forward-looking provisioning rules in a New Keynesian DSGE model with banking. Somewhat surprisingly, there have been so far few attempts in the academic literature to address this issue. One of the few analytical contributions in this area is Bouvatier and Lepetit (2012), but their framework is not a full general equilibrium analysis. The model we develop integrates elements of the DSGE framework developed in Agénor, Alper and Pereira da Silva (2013) with Bouvatier and Lepetit (2012) to study the interaction between bank provisioning rules, credit market imperfections, and business cycles in response to financial shocks. Our analysis considers the extent to which these interactions affect the procyclicality of the financial system as well as real and financial volatility.

The remainder of the chapter is structured as follows. Section 3.2 describes the model with a detailed examination of the agents behaviour, the loan loss provisioning rules and the market clearing conditions. The steady state equilibrium and the log-linearized model are characterized in Sections 3.3 and 3.4, respectively. Parameterization is discussed in Section 3.5. Section 3.6 simulates the model following a positive shock to nonperforming loans and a negative shock to collateral, with an elaborate explanation of the various transmission channels associated with changes in loan loss provisions. We also discuss an alternative definition of the loan value used for calculating provisions. The last section offers some concluding remarks and discusses some possible extensions of the analysis.

design of accounting rules falls under the authority of the Central Bank of Spain.

⁵See Wezel, Chan-Lau and Columba (2012) for a further discussion. As noted in Wezel (2010), several countries in Latin America have introduced dynamic loan provisioning systems in recent years, but their experience is too recent to provide robust conclusions.

3.2 The Model

Consider an economy consisting of eight types of agents: forward-looking optimizing households, a competitive labour aggregator, a final good (FG) firm, a continuum of intermediate good (IG) firms, a capital good (CG) producer, a single commercial bank, a government, and a central bank, which also acts as the bank regulator. The IG firms rent capital from the CG producer and employ differentiated labour from households, via the labour aggregator, to produce a unique good. These intermediate goods are then all combined by the FG firm, who produces a homogeneous final good, which, in turn, can be used for either consumption, investment or government spending.

The commercial bank receives deposits from households, supplies credit to the CG producer for investment financing, decides on the deposit rate and lending rate, and borrows from the central bank to cover any shortfall in funding. The bank receives gross interest payments on investment loans and pays back principal plus interest on households' deposits and loans from the central bank. In addition, the bank holds loan loss reserves. Importantly, the bank closes at the end of each period and a new bank reopens; thus, there is no explicit distinction between stocks (reserves) and flows (current loan loss provisions) in the model.⁶ Provisioning rules are set by the central bank, and can be either backward- or forward-looking. In the former case, provisions are triggered by past due payments (or the fraction of nonperforming loans), which, in turn, depend on current economic conditions, the collateral-loan ratio, and the loan loss provisions-loan ratio. Forward-looking (statistical or dynamic) provisioning, by contrast, take into account both past due payments, as before, and expected losses over the whole business cycle; thus, provisions are smoothed over the cycle and are less affected by the current state of the economy and past due payments. We now turn to a more detailed description of the behaviour of each agent in this economy.

3.2.1 Households

There is a continuum of households of measure 1, who consume, hold deposits and cash, invest in government bonds, and supply differentiated labour to the labour aggregator.

The objective of the representative household is to maximize the following utility

⁶This assumption, which follows Agénor and Alper (2012) and Agénor, Alper, and Pereira da Silva (2012, 2013) helps to simplify the solution of the bank's optimization problem (by turning it into a static problem), without affecting the main insights of the analysis.

function,

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{[C_t]^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} - \frac{H_{i,t}^{1+\gamma}}{1+\gamma} + \eta_x \ln [(M_t^H)^v (D_t)^{1-v}] \right\}, \quad (3.1)$$

where E_t is the expectations operator conditional on the information available in period t , and $\beta \in (0, 1)$ denotes the discount factor. The term C_t denotes consumption at time t while $H_{i,t}$ represents the time- t hours devoted for labour by household i . The term ς stands for the intertemporal elasticity of substitution in consumption while γ denotes the inverse of the Frisch elasticity of labour supply.

Households hold real money balances M_t^H , which yield no return, and real bank deposits, D_t , which bear a net interest rate of i_t^D . The real cash balances combined with real bank deposits generate the composite monetary asset, given by a Cobb-Douglas form: $(M_t^H)^v (D_t)^{1-v}$, where $v \in (0, 1)$. In addition, households invest in one-period (real) government bonds, B_t^H , which yield a net interest of i_t^B . Hence, total gross repayments from holding deposits and government bonds in period $t-1$ (adjusted to real terms in period t) are respectively given by $(1+i_{t-1}^D)D_{t-1}\frac{P_{t-1}}{P_t}$ and $(1+i_{t-1}^B)B_{t-1}^H\frac{P_{t-1}}{P_t}$, with P_t denoting the price of the final good

At the start of period t , each household chooses the level of deposits, cash and government bonds, and supplies differentiated labour to the labour aggregator, for which it earns a factor payment of $\frac{W_{i,t}}{P_t}H_{i,t}$ (where $W_{i,t}$ is the nominal wage).

At the end of period t , households receive all profits from IG firms ($J_t^{IG} = \int_0^1 J_{j,t}^{IG} dj$) and the CG producer (J_t^K) in the form of lump-sum transfers. Moreover, households receive all profits from the commercial bank (J_t^B), and also pay a lump-sum tax given by T_t (in real terms).⁷

Finally, households purchase final output for consumption purposes and therefore the household's (real) budget constraint can be written as follows,

$$\begin{aligned} C_t + D_t + B_t^H + M_t^H \leq & (1+i_{t-1}^D)D_{t-1}\frac{P_{t-1}}{P_t} + (1+i_{t-1}^B)B_{t-1}^H\frac{P_{t-1}}{P_t} + \\ & + M_{t-1}^H\frac{P_{t-1}}{P_t} + \frac{W_{i,t}}{P_t}H_{i,t} + \int_0^1 J_{j,t}^{IG} dj + J_t^K + J_t^B - T_t. \end{aligned} \quad (3.2)$$

Consumption, Deposits, Cash and Government Bonds Decisions

Each household maximizes (3.1) with respect to (3.2). The first order conditions with respect to C_t, D_t, B_t^H and M_t^H (taking interest rates and prices as given) are

⁷Households own the final good firm but this economic agent earns zero profits in equilibrium, as noted below.

respectively given by,

$$C_t^{-\frac{1}{\varsigma}} - \varphi_t = 0, \quad (3.3)$$

$$D_t = \frac{\eta_x(1-v)}{\varphi_t - \beta E_t \left[\varphi_{t+1}(1 + i_t^D) \frac{P_t}{P_{t+1}} \right]}, \quad (3.4)$$

$$\beta E_t \left[\varphi_{t+1}(1 + i_t^B) \frac{P_t}{P_{t+1}} \right] - \varphi_t = 0, \quad (3.5)$$

$$M_t^H = \frac{v\eta_x}{\varphi_t - \beta E_t \left[\varphi_{t+1} \frac{P_t}{P_{t+1}} \right]}, \quad (3.6)$$

with φ_t denoting as the Lagrange multiplier.

Combining equations (3.3) and (3.5) yields the standard Euler equation which determines the optimal consumption path,

$$C_t^{-\frac{1}{\varsigma}} = \beta E_t \left[C_{t+1}^{-\frac{1}{\varsigma}}(1 + i_t^B) \frac{P_t}{P_{t+1}} \right]. \quad (3.7)$$

Using equations (3.3) and (3.5) in (3.4) gives the real demand for deposits,

$$D_t = \frac{\eta_x(1-v)C_t^{\frac{1}{\varsigma}}(1 + i_t^B)}{(i_t^B - i_t^D)}. \quad (3.8)$$

Hence, real deposits depend positively on consumption and the deposit rate, and negatively on the return on government bonds.⁸

Finally, to obtain the real demand for cash, equations (3.3) and (3.5) are substituted in (3.6), which yields,

$$M_t^H = \frac{\eta_x v C_t^{\frac{1}{\varsigma}}(1 + i_t^B)}{i_t^B}. \quad (3.9)$$

Real money balances are therefore positively related with consumption, and negatively with the opportunity cost of holding cash, measured by the rate of return on government bonds.

The Wage Decision

The wage setting environment follows Erceg, Henderson and Levin (2000), and Christiano, Eichenbaum and Evans (2005), where each household i supplies a unique

⁸As in Agénor and Alper (2012), the value of real deposits is assumed to be positive, implying that $i_t^B > i_t^D$. Intuitively, holding deposits yields utility to households and therefore they are willing to accept a lower interest rate on them compared to government bonds.

type of labour ($H_{i,t}$) with $i \in (0, 1)$. All these types of labour are then aggregated by a competitive labour contractor into one composite homogenous labour (N_t) using the standard Dixit-Stiglitz (1977) technology,

$$N_t = \left(\int_0^1 H_{i,t}^{\frac{\theta_w-1}{\theta_w}} di \right)^{\frac{\theta_w}{\theta_w-1}}, \quad (3.10)$$

with $\theta_w > 1$ representing the constant elasticity of substitution between the different types of labour. The i^{th} household therefore faces the following demand curve for its labour,

$$H_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\theta_w} N_t, \quad (3.11)$$

where W_t denotes the aggregate nominal wage paid for one unit of the composite labour. The zero profit condition for the labour aggregator, obtained by substituting (3.10) in (3.11), yields the economy wide wage equation, $W_t = \left[\int_0^1 W_{i,t}^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}}$.

Calvo (1983)-type nominal rigidities is assumed in the wage setting such that in each period a constant fraction of $1 - \omega_w$ workers are able to re-optimize their wages while a fraction of ω_w index their wages according to last period's price inflation rate (π_{t-1}). These non re-optimizing households therefore set their wages according to $W_{i,t} = \pi_{t-1} W_{i,t-1}$. Moreover, if wages have not been set since period t , then at period $t + s$ the real relative wage for household i becomes $\frac{W_{i,t+s}}{W_{t+s}} = \frac{\Pi^s W_{i,t}}{W_{t+s}}$, where $\Pi^s = \pi_t \times \pi_{t+1} \times \dots \times \pi_{t+s-1}$. Consequently, the demand for labour in period $t + s$ is $H_{i,t+s} = \left(\frac{\Pi^s W_{i,t}}{W_{t+s}} \right)^{-\theta_w} N_{t+s}$.

In equilibrium all re-optimizing households choose the same wage (W_t^*), and the optimal relative wage in a log-linearized form (denoted by hat) is given by $\widehat{\left(\frac{W_t^*}{W_t} \right)} = \left(\frac{\omega_w}{1-\omega_w} \right) \widehat{\pi_t^W}$, with $\widehat{\pi_t^W} \equiv \widehat{W_t} - \widehat{W_{t-1}}$ denoting the log-linearized wage inflation. In the absence of wage rigidities ($\omega_w = 0$), the real wage equals to the wage mark-up ($\frac{\theta_w}{\theta_w-1}$) multiplied by the marginal rate of substitution between leisure and consumption (MRS_t). Specifically, $\frac{W_t}{P_t} = \frac{\theta_w}{\theta_w-1} MRS_t$, where $MRS_t = N_t^\gamma C_t^{\frac{1}{\zeta}}$ and $N_t = H_t$.

Finally, as in Erceg, Henderson and Levin (2000) the wage inflation equation is shown to satisfy,

$$\widehat{\pi_t^W} = \beta E_t \widehat{\pi_{t+1}^W} + \frac{(1 - \omega_w)(1 - \beta\omega_w)}{(\omega_w)(1 + \gamma\theta_w)} \left[\widehat{MRS_t} - \widehat{\left(\frac{W_t}{P_t} \right)} \right], \quad (3.12)$$

where real wages evolve according to,

$$\widehat{W_t^R} \equiv \widehat{\left(\frac{W_t}{P_t}\right)} = \widehat{\left(\frac{W_{t-1}}{P_{t-1}}\right)} + \widehat{\pi_t^W} - \widehat{\pi_t^P}, \quad (3.13)$$

with $\widehat{\pi_t^P} \equiv \widehat{P_t} - \widehat{P_{t-1}}$ representing the log-linearized price inflation rate as a deviation from its steady state.

3.2.2 Final Good Firm

A perfectly competitive representative FG firm assembles a continuum of intermediate goods ($Y_{j,t}$ with $j \in (0, 1)$), to produce final output (Y_t) using the standard Dixit-Stiglitz (1977) technology,

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\theta_p-1}{\theta_p}} dj \right)^{\frac{\theta_p}{\theta_p-1}}, \quad (3.14)$$

where $\theta_p > 1$ denotes the constant elasticity of substitution between the differentiated intermediate goods. The FG firm chooses the optimal quantities of intermediate goods ($Y_{j,t}$) that maximize its profits, taking as given both the prices of the intermediate goods ($P_{j,t}$) and the final good price (P_t). This optimization problem yields the demand function for each intermediate good,

$$Y_{j,t} = Y_t \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_p}. \quad (3.15)$$

Finally, imposing the zero profit condition yields the usual definition of the final good price,

$$P_t = \left[\int_0^1 P_{j,t}^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}}. \quad (3.16)$$

3.2.3 Intermediate Good Firms

A continuum of IG producers, indexed by $j \in (0, 1)$, operate in a monopolistic environment and use capital and labour to produce a unique good. The IG firm rents capital from the CG producer at the rate r_t^K , and employs labour from the labour aggregator, for which it pays a real wage of W_t^R . Each IG firm j faces the Cobb-Douglas production function,

$$Y_{j,t} = A_t N_{j,t}^{1-\alpha} K_{j,t}^\alpha, \quad (3.17)$$

where $K_{j,t}$ denotes the amount of capital, $N_{j,t}$ the *homogeneous* amount of labour supplied by the labour aggregator, and $\alpha \in (0,1)$. The term A_t represents a common economy wide technology shock which follows the $AR(1)$ process, $A_t = (A_{t-1})^{v^A} \exp(\epsilon_t^A)$, where v^A is the autoregressive coefficient, and ϵ_t^A a normally distributed random shock with zero mean and a constant variance.

The IG firm solves a two stage pricing decision problem during period t . In the *first stage*, each IG producer minimizes the cost of renting capital and employing labour, taking wages and the rental price of capital as given. Specifically, the IG firm solves,

$$\min_{N_{j,t}, K_{j,t}} [W_t^R N_{j,t} + r_t^K K_{j,t}], \quad (3.18)$$

subject to $Y_{j,t} = A_t N_{j,t}^{1-\alpha} K_{j,t}^\alpha$. Denoting φ_t^{IG} as the Lagrange multiplier, the first order conditions for the above problem, taking the factor prices as given, yield,

$$W_t^R = (1 - \alpha) \frac{Y_{j,t}}{N_{j,t}} \varphi_t^{IG}, \quad (3.19)$$

$$r_t^K = \alpha \frac{Y_{j,t}}{K_{j,t}} \varphi_t^{IG}. \quad (3.20)$$

From the above equations, the optimal capital-labour ratio is,

$$\frac{K_{j,t}}{N_{j,t}} = \left(\frac{\alpha}{1 - \alpha} \right) \left(\frac{W_t^R}{r_t^K} \right). \quad (3.21)$$

Adding (3.19) and (3.20) yields $[(1 - \alpha) + \alpha] \varphi_t^{IG} = \varphi_t^{IG} = [W_t^R N_{j,t} + r_t^K K_{j,t}] / Y_{j,t}$, which implies that φ_t^{IG} is also equal to the real unit marginal cost, $\varphi_t^{IG} = mc_{j,t}$. Dividing the total cost for producing output by the quantity produced, and using equations (3.17) and (3.21), results in the unit real marginal cost,

$$mc_{j,t} = \frac{(W_t^R)^{1-\alpha} (r_t^K)^\alpha}{\alpha^\alpha (1 - \alpha)^{1-\alpha} A_t}. \quad (3.22)$$

In the *second stage*, each IG producer chooses the optimal price for its good. Here Calvo (1983)-type nominal rigidities is assumed, in which a portion of ω_p firms keep their prices fixed while a portion of $1 - \omega_p$ firms adjust prices optimally given the going marginal cost. The firm's problem is to maximize the following expected discounted value of current and future real profits subject to the demand function for each good (equation 3.15), and taking the marginal cost as given. Formally that

is,

$$\max_{P_{j,t+s}} E_t \sum_{s=0}^{\infty} \omega_p^s \Delta_{s,t+s} \left[\left(\frac{P_{j,t+s}}{P_{t+s}} \right)^{1-\theta_p} Y_{t+s} - mc_{t+s} \left(\frac{P_{j,t+s}}{P_{t+s}} \right)^{-\theta_p} Y_{t+s} \right],$$

where $\Delta_{s,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t} \right)^{\varsigma^{-1}}$ is the total discount factor.⁹

Denoting P_t^* as the optimal price level chosen by each firm at time t , and using the definition of the total discount factor, the first order condition of the above problem with respect to P_t^* yields the optimal relative price equation,¹⁰

$$Q_t = \frac{P_t^*}{P_t} = \left(\frac{\theta_p}{\theta_p - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s C_{t+s}^{-\frac{1}{\varsigma}} Y_{t+s} mc_{t+s} \left(\frac{P_{t+s}}{P_t} \right)^{\theta_p}}{E_t \sum_{s=0}^{\infty} \omega_p^s \beta^s C_{t+s}^{-\frac{1}{\varsigma}} Y_{t+s} \left(\frac{P_{t+s}}{P_t} \right)^{\theta_p - 1}}, \quad (3.23)$$

with $Q_t = \frac{P_t^*}{P_t}$ denoting the relative price chosen by firms adjusting their prices at period t and $pm = \left(\frac{\theta_p}{\theta_p - 1} \right)$ representing the price mark-up.

3.2.4 Capital Good Producer

The CG producer owns all physical capital in the economy and uses a linear production function to produce capital goods. In order to produce capital goods, the CG producer spends I_t on the final good. For this purpose, it must pay for these goods in advance and borrows from the commercial bank at the beginning of the period. Thus, the real amount borrowed from the commercial bank (L_t^I) is,

$$L_t^I = I_t. \quad (3.24)$$

Moreover, the net interest rate charged by the commercial bank for funding these investments is denoted by i_t^L , and therefore the total costs faced by the CG producer at the end of period t are represented by $(1 + i_t^L)L_t^I$.

To produce new capital (K_{t+1}), the CG producer uses the investment good together with the existing stock of capital (K_t) from the previous period (net of depreciation). Further, the CG producer incurs adjustment costs in producing new

⁹The IG firms are owned by the households and therefore each firm's discount value is $\beta^s \left(\frac{C_{t+s}}{C_t} \right)^{\varsigma^{-1}}$. Intuitively, $\left(\frac{C_{t+s}}{C_t} \right)^{\varsigma^{-1}}$ is the marginal utility value (in terms of consumption) of a one unit increase of IG firms profits in period t .

¹⁰Subscript j is dropped because all re-optimizing firms choose the same price so everything becomes time dependent.

capital. Hence, capital evolves according to,

$$K_{t+1} = I_t + (1 - \delta_K)K_t - \frac{\Theta_K}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t, \quad (3.25)$$

where $\delta_K \in (0, 1)$ denotes the constant rate of depreciation, and $\Theta_K > 0$ the adjustment cost parameter. The new capital stock is then rented to the IG firms at the rate of r_t^K .

The CG producer chooses the level of capital stock so as to maximize the value of discounted stream of dividend payments to households subject to equation (3.25).¹¹ Specifically, defining $J_t^K = r_t^K K_t - (1 + i_t^L)I_t$ as the CG producer's real profits at the end of period t , the optimization problem can be written as,

$$\max_{K_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \varphi_t \left\{ r_t^K K_t - (1 + i_t^L) \left[K_{t+1} - (1 - \delta_K)K_t + \frac{\Theta_K}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \right] \right\}.$$

Using some algebraic manipulations and substituting equation (3.7), the first order condition of the above maximization yields the arbitrage condition,¹²

$$\begin{aligned} E_t r_{t+1}^K &= (1 + i_t^L) E_t \left\{ \left[1 + \Theta_K \left(\frac{K_{t+1}}{K_t} - 1 \right) \right] (1 + i_t^D) \frac{P_t}{P_{t+1}} \right\} - \\ &\quad - E_t (1 + i_{t+1}^L) \left\{ \left[(1 - \delta_K) + \frac{\Theta_K}{2} \left[\left(\frac{K_{t+2}}{K_{t+1}} \right)^2 - 1 \right] \right] \right\}. \end{aligned} \quad (3.26)$$

Consequently, the rental rate of capital is related to the current and expected loan rates, the adjustment cost of changing capital across periods, the deposit rate (or the opportunity cost of investing in physical capital), the depreciation rate and the inflation rate. Note also that the marginal cost equation (given by 3.22) is related to the rental rate of capital, which in turn depends on the loan rate. As a result, the CG producer passes the cost of credit to the IG firms when setting r_t^K , such that the marginal costs experienced by IG firms depend directly on the lending rate as well.

¹¹As is standard in this class of models, there is no default in equilibrium. We therefore do not account explicitly for repayment problems in the CG producer's optimization program.

¹²See Appendix 3.A.1 for full derivation.

3.2.5 Commercial Bank

Balance Sheet, Loan Loss Reserves and Central Bank Borrowing

At the start of period t the commercial bank collects deposits from households in order to supply credit to the CG producer. The supply of credit is perfectly elastic at the prevailing loan rate and therefore the total amount of lending provided by the bank is given by equation (3.24). To fund any shortfall in funding, the commercial bank borrows an amount of L_t^B from the central bank, for which it pays an interest rate of i_t^R . In addition, the commercial bank holds required reserves (RR_t) at the central bank. Finally, as the loan portfolio takes into account expected loan losses, loan loss reserves (LLR_t) are subtracted from total assets such that $L_t^I - LLR_t$ denote net loans (as in Walter 1991 and Bouvatier and Lepetit 2012).¹³ The bank's balance sheet in real terms is thus,

$$L_t^I - LLR_t + RR_t = D_t + L_t^B. \quad (3.27)$$

Reserves held at the central bank do not bear interest and are set as a fraction of deposits,

$$RR_t = \mu_D D_t, \quad (3.28)$$

where $\mu_D \in (0, 1)$ is the reserve requirement ratio.

The bank must also satisfy regulation in the form of setting loan loss provisions (a flow), which are deducted from current earnings. These provisions (which are defined in detail in the next section) can be based on either a backward- or forward-looking system. Loan loss reserves (a stock) are accumulated in response to changes in loan loss provisions, with the provision value depending on changes in problem loans during the period. However, as noted earlier, there is no distinction in the model between stocks and flows of provisions, and $LLR_t = LLP_t^i$, with LLP_t^i denoting loan loss provisions, and $i = BK, FW$ representing the type of provisioning system (backward or forward looking).¹⁴

Using this result, together with (3.28), borrowing from the central bank can be

¹³In standard accounting, loan loss provisions, are defined as an estimation of probable loan losses for a current year and are charged as an expense, deducted from current profits (although, as noted earlier, these deductions are subject to restrictions in practice). Loan loss reserves, on the other hand, are a balance sheet item which depend on loan loss provisions, accumulated charged off loans, and loan recoveries.

¹⁴A similar approach is used in Agénor, Alper and Pereira da Silva (2013) to model capital requirements; this simplifies considerably the analysis by making the bank's optimization problem static in nature.

determined residually from the balance sheet constraint (3.27),

$$L_t^B = L_t^I - (1 - \mu_D)D_t - LLP_t^i. \quad (3.29)$$

which shows that, fundamentally, loan loss provisions, just like bank capital, are simply an alternative way of funding bank lending operations.¹⁵

Provisioning Rules and Nonperforming Loans

Provisioning rules are set by the central bank. We consider two specifications of loan loss provisions, which depend directly on the fraction of problem loans: First, a *backward-looking* provisioning system, where loan loss provisions are triggered by past due payments. Second, we examine a *forward-looking* (dynamic) provisioning system, where the bank makes provisions based on past due payments and expected losses over the whole business cycle.

We adopt a quasi-reduced form to relate loan loss provisions to the fraction of nonperforming loans, through the following set of factors. First, we impose a negative correlation between provisions and cyclical output, which is the main stylized fact in the determination of specific provisions (see Cavallo and Majnoni 2002, Laeven and Majnoni 2003, and Bikker and Metzmakers 2005).¹⁶ This relationship is consistent with the idea that in periods of economic booms, for instance, profits and cash flows tend to improve such that the fraction of nonperforming loans and thus provisions decrease.

Second, our specification assumes that a rise in collateral values lowers the percentage of problem loans and reduces loan loss provisions, an idea supported for instance by Song (2002) and Davis and Zhu (2009), among others.¹⁷ Intuitively, collateral mitigates moral hazard behaviour by borrowers, thus reducing the likelihood

¹⁵Note that due to timing operations, and the assumption about the bank closing at the end of each period, the loan loss provisions appearing in the bank's balance sheet are estimated at the beginning of the period and represent a flow item.

¹⁶Cavallo and Majnoni (2002) for instance examine empirically the policies of large commercial banks in various countries with regards to their provisions and income smoothing. These authors find that bankers on average smooth their income but do not create enough provisions during good times, implying that banks build provisions during recessions and not before. Therefore, the negative relationship between business cycles and loan loss provisions amplify the effects of a recession.

¹⁷In Davis and Zhu (2009), a rise in property prices (with property acting as collateral) lowers the amount of loan loss provisions. This stems from the idea that collateral is perceived by banks as a reduction of risk, which leads to higher earnings and lower percentage of nonperforming loans. In our model asset prices are not explicitly accounted for, but the intuition remains the same—a negative relationship between collateral (which is positively related to asset prices) and provisions prevails.

of default and lowering the fraction of nonperforming loans.¹⁸ As loan loss provisions are directly positively related to the percentage of problem debt, a negative correlation between provisions and collateral is imposed.¹⁹

Third, the fraction of problem loans (and thus loan loss provisions) is related to the loan loss reserves-lending ratio. On the one hand, this relationship can be negative, which is consistent with the idea that banks have a greater incentive to monitor their borrowers when they hold large loan loss reserves (add backs to regulatory bank capital for example). A greater degree of monitoring improves the ability of the bank to collect the full return on investment loans, thereby reducing the fraction of problem debt and the amount of required loan loss provisions. If indeed loan loss reserves can be treated as add-backs to bank capital (as outlined in the Basel Accords), holding more loan loss reserves relative to total loans allows banks to charge a lower spread on loans, where the spread depends positively on the fraction of nonperforming loans.²⁰ This result is supported by the studies of Barth, Caprio and Levine (2004) and Coleman, Esho and Sharpe (2006), among others. Furthermore, in the context of the recent financial crisis, Cole and White (2012) show a negative correlation between loan loss reserves in 2007 and the probability of bank failure in 2009. Intuitively, loan loss reserves may represent a source of strength against future losses, which can reduce the probability of a banking crisis associated with a higher fraction of nonperforming loans (see Cashin and Duttagupta 2008).

In contrast, the relationship between the fraction of problem debt and loan loss reserves may also be positive as shown in Ng and Roychowdhury (2011). These authors illustrate that during the financial crisis of 2008-2009, an increase in loan loss reserves translated to a higher risk of bank default and a rise in nonperforming loans.²¹ Intuitively, holding more loan loss reserves in the form of bank capital can lead bankers to extend more loans, even during a crisis period, thereby increasing the

¹⁸In principle, collateral could also weaken lenders' incentive to monitor, which could increase the incidence of default. However, most of the empirical evidence shows that collateral indeed reduces the risk of default on the part of borrowers.

¹⁹The effect of collateral on the probability of default (equivalent to the fraction of nonperforming loans in our model) is discussed at length in Agénor and Alper (2012).

²⁰This monitoring incentive effect (discussed in the context of bank capital by Agénor, Alper and Pereira da Silva 2012) is consistent with the fact that general provisions are recognised, up to a limit, as Tier 2 capital under Basel I, and the Standardised approach to Basel II. Under the Internal Rating Based (IRB) approaches all provisions attributable to IRB-rated exposures (including specific provisions) may be used to offset expected losses. Surplus provisions (those in excess of expected losses) may be counted as Tier 2 capital up to a limit of 0.6 percent of credit risk-weighted assets.

²¹Nevertheless, Ng and Roychowdhury (2011) do not include bank characteristics prior to 2007, which could also explain bank failures in the subsequent years if banks had been financially weak for example. Cole and White (2012) do address these issues, thus making a comparison between these studies difficult.

likelihood of moral hazard behaviour and raising the possibility of default. Shrieves and Dahl (1992) also show a positive relationship between higher capital and investments in risky assets. Finally, Jin, Kangaretnam and Lobo (2011) also document a strong positive association between loan loss provisions and the probability of bank failure for financially weak banks between 2007 and 2010. Bank failures, in turn, are positively related with non-performing loans (see Demirgüç-Kunt and Detragiache 2005), which implies that higher loan loss reserves may indeed increase moral hazard behaviour and induce a rise in nonperforming loans.

Given this description, the fraction of non performing loans (J_t) is therefore defined as,

$$J_t = j_0 \left[\frac{Y_t}{Y} \right]^{-\omega_Y} \left[\frac{\kappa_t K_t / L_t^I}{\kappa K / L^I} \right]^{-\omega_K} \left[\frac{LLR_t / L_t^I}{LLR / L^I} \right]^{-\omega_{LR}} \varepsilon_t^J. \quad (3.30)$$

The term $(\frac{Y_t}{Y})$ denotes the output gap (where Y is the steady state value of output under fully flexible prices), and $\omega_Y > 0$ the elasticity with respect to the output gap. Effective collateral is given by a fraction κ_t of the physical capital stock, with $\kappa \in (0, 1)$ denoting the steady state value of this fraction.²² The term κ_t represents a collateral (credit) shock in this model, which is assumed to follow the $AR(1)$ shock process, $\kappa_t = (\kappa_{t-1})^{v^\kappa} \exp(\epsilon_t^\kappa)$, with $v^\kappa \in (0, 1)$ and ϵ_t^κ a random shock with a normal distribution and a constant variance. The term $\omega_K > 0$ is the elasticity with respect to deviations of the effective collateral-loan ratio from its steady state value. The fraction of nonperforming loans is also related to deviations in the loan loss reserves-loan ratio $(\frac{LLR_t / L_t^I}{LLR / L^I})$, where ω_{LR} represents the elasticity with respect to this variable. The elasticity ω_{LR} can be either positive or negative depending on whether loan loss reserves lead to increased bank monitoring or induce moral hazard behaviour, as explained earlier. Furthermore, $j_0 > 0$ represents the steady state fraction of nonperforming loans. The random variable ε_t^J captures nonsystematic shocks to nonperforming loans, that is, shocks that are not directly associated with movements in the output gap, collateral, and loan loss reserves. It follows an $AR(1)$ process, $\varepsilon_t^J = (\varepsilon_{t-1}^J)^{v^J} \exp(\epsilon_t^J)$, where $v^J \in (0, 1)$ and ϵ_t^J a normally distributed random shock with zero mean and a constant variance.²³

Turning now to the different types of provisioning rules, we first model the *backward-looking* provisioning system (denoted by superscript BK), where the bank

²²Steady state values are denoted by dropping the time subscript.

²³In principle (unanticipated) charged-off loans (which are removed from the bank's balance sheet) could also be added to the model and correspond to the fraction of these loans during *previous* periods and recognized during the current period. However, in this model, loans are for one period only and the bank closes at the end of the period. As a result, charged-off loans would fundamentally be no different than nonperforming loans. We therefore abstract from modeling charged-off loans in this chapter. Including charged-off loans would make no difference to the results obtained later in the text.

evaluates its credit risk exposure on current nonperforming loans. Therefore, and similar to Bouvatier and Lepetit (2012), loan loss provisions in a backward-looking system are defined as,

$$LLP_t^{BK} = l_0 J_t L_t^I, \quad (3.31)$$

where l_0 is the steady-state fraction (or average fraction over a whole business cycle) of nonperforming loans ($J_t L_t^I$), which are covered by loan loss provisions in period t . In other words, l_0 is the coverage ratio, measured as loan loss provisions divided by the fraction of nonperforming loans.

The alternative specification for loan loss provisions is the *forward-looking (statistical)* provisioning system (denoted by superscript FW), in which the bank makes provisions related to the current percentage of nonperforming loans (as before) and the evaluation of the latent risk over the whole business cycle. Specifically, following again Bouvatier and Lepetit (2012), statistical loan loss provisions can be written as,

$$LLP_t^{FW} = l_0 J_t L_t^I + \lambda (J - J_t) l_0 L_t^I, \quad (3.32)$$

where $\lambda \in (0, 1)$ denotes the degree of loan loss provisions smoothing under the forward-looking system, and J the steady-state value of the fraction of nonperforming loans (or the long-run evaluation of latent risk by the commercial bank). During an economic expansion, where the short-run value of current nonperforming loans (J_t) is lower than the estimation of the latent risk over the whole cycle (J) the commercial bank can build up statistical provisions. Therefore, taking into account expected losses over the business cycle offsets the short-run impact of problem loans on current provisions.

The Bank's Optimization Problem and Solution

The bank sets *ex ante* the (gross) deposit and lending rates in order to maximize current real profits, defined as,²⁴

$$\max_{(1+i_t^L), (1+i_t^D)} \left\{ \begin{array}{l} [1 - J_t] (1 + i_t^L) L_t^I + J_t (\kappa_t K_t) + \mu_D D_t - \\ - (1 + i_t^D) D_t - (1 + i_t^R) L_t^B - LLP_t^I \end{array} \right\},$$

subject to the loan demand function of the CG producer (3.24), loans from the central bank (3.29), the fraction of nonperforming loans (3.30), and the type of

²⁴Recall that the bank closes at the end of each period making the commercial bank's problem static in nature. Note also that we do not account explicitly for the risk of default. However, this is indirectly captured by the specification of nonperforming loans in (3.30) as a function of the business cycle and the collateral-loan ratio; see Agénor and Alper (2012) for instance for a discussion.

provisioning system LLP_t^i with $i = BK, FW$ (equations 3.31 or 3.32). The bank also internalizes the fact that total lending is negatively related to the cost of borrowing (from 3.24).

The term $(1 - J_t)$ stands for the fraction of loans the commercial bank is able to retrieve with interest (good loans). In case of an increased likelihood of default, when loans move to the nonperforming loans category (problem loans), the bank is partly compensated by seizing collateral, given by the fraction $J_t \kappa_t K_t$. Loan loss provisions are deducted from the bank's profits and denoted by LLP_t^i .

For the variables related to the deposit market; $\mu_D D_t$ is the reserve requirement held at the central bank, and $(1 + i_t^D) D_t$ is the gross repayment to households for holding deposits. Finally, The bank must also repay, with interest, the central bank for its supply of loans and this is given by $(1 + i_t^R) L_t^B$.

In solving the maximization problem, the bank takes as given the fractions of nonperforming loans, the value of collateral, and the refinance rate. The first order conditions with respect to the deposit and loan rates are given respectively by,²⁵

$$(1 + i_t^D) = \frac{1 + (1 - \mu_D) i_t^R}{\left(1 + \frac{1}{\eta_D}\right)}, \quad (3.33)$$

$$[1 - J_t] (1 + i_t^L) = \frac{1}{\left(1 + \frac{1}{\eta_L}\right)} \left\{ (1 + i_t^R) + [1 - (1 + i_t^R)] \frac{\partial LLP_t^i}{\partial L_t^I} \right\}, \quad (3.34)$$

where $\eta_D > 0$ and $\eta_L < 0$ are constant interest elasticities of deposits and loan demand (or investment), respectively.

Equation (3.33) indicates that the deposit rate is set as a markup over the refinance rate, adjusted downwards for the implicit cost of holding reserve requirements. Note that $i_t^D < i_t^R \forall t$, implying that the commercial bank always absorbs all deposits made by households.

Equation (3.34) describes the lending rate equation. Intuitively, there are two channels at play in the determination of the loan rate. The first is a *risk premium channel*, which is related to the fact that the bank receives back only a fraction of its loans; the marginal return on loans is therefore only $(1 - J_t)(1 + i_t^L)$. Equivalently, the bank internalizes the fact that the fraction of nonperforming loans and hence loan loss provisions are positive, and charges a higher loan rate as a result.

The second effect is the (marginal) cost channel, which consists of several components. First, there is a *direct* cost channel associated with changes in the cost of borrowing from the central bank $(1 + i_t^R)$ which the bank “mechanically” passes

²⁵See Appendix 3.A.2 for the full derivation of $(1 + i_t^D)$ and $(1 + i_t^L)$.

on to borrowers. Second, there is what we may call to fix ideas a *provisioning cost channel*, which is related to the term $[1 - (1 + i_t^R)](\partial LLP_t^i / \partial L_t^I)$ in (3.34). This term results from the relationship between central bank borrowing and loan loss provisions (equation 3.29). Intuitively, a one unit increase in lending raises provisions by $\partial LLP_t^i / \partial L_t^I$; this is costly for the bank (provisions reduce profits) and accordingly it adjusts the loan rate upwards. However, at the same time, higher provisions reduce borrowing needs from the central bank. Indeed, from (3.29), because loans from the central bank are determined residually, all else equal any rise in LLP_t^i lowers L_t^B by the same amount. By implication, the cost of borrowing from the central bank, given by $1 + i_t^R$, is reduced by the amount by which provisions increase, $\partial LLP_t^i / \partial L_t^I$, thereby creating a downward pressure on the loan rate. As discussed later, this effect is critical to understand the effect of loan loss provisions on the cyclical behavior of the financial system.

Note that under backward-looking provisioning, the marginal provisioning cost is determined from equation (3.31),

$$\frac{\partial LLP_t^{BK}}{\partial L_t^I} = l_0 J_t. \quad (3.35)$$

On the other hand, under a forward looking provisioning system, the effect of loans on required provisions is derived from equation (3.32),

$$\frac{\partial LLP_t^{FW}}{\partial L_t^I} = (1 - \lambda)l_0 J_t + \lambda l_0 J. \quad (3.36)$$

Thus, the way the provisioning rule determines the behaviour of the loan rate depends on how the fraction of nonperforming loans affects the marginal cost of provisions—which in turn varies across provisioning regimes.

3.2.6 Central Bank

The central bank's assets consist of loans to the commercial bank (L_t^B) and holdings of government bonds (B_t^C), whereas its liabilities are given by currency supplied to households (M_t^S), and required reserves (RR_t). The latter two make up the monetary base. Therefore, the bank's balance sheet (in real terms) is,

$$L_t^B + B_t^C = M_t^S + RR_t. \quad (3.37)$$

Using equation (3.28), then the supply of currency can be written as,

$$M_t^S = L_t^B + B_t^C - \mu_D D_t. \quad (3.38)$$

The central bank targets the short term policy rate (i_t^R) according to the following standard log-linearized Taylor-type policy rule,

$$\widehat{i_t^R} = \phi \widehat{i_{t-1}^R} + (1 - \phi) \left[\phi_Y \widehat{Y_t} + \phi_\pi \widehat{\pi_t^P} \right] + \epsilon_t^M, \quad (3.39)$$

where $\widehat{Y_t}$ denotes output deviations from its flexible price steady state value, $\widehat{\pi_t^P} \equiv \pi_t - \pi^T$ inflation deviation from its target steady state value (π^T), $\phi \in (0, 1)$ the degree of interest rate smoothing and $\phi_Y, \phi_\pi > 0$ coefficients measuring the relative weights on output and inflation deviations from their steady states, respectively. Finally, the term ϵ_t^M represents an i.i.d monetary policy shock with zero mean and a constant variance.

3.2.7 Government

The government spends G_t on the final good, and issues one period risk-free bonds, held by households and the central bank. The government collects all interest income made by the central bank, given by net return on lending to the commercial bank, and net return from investing in government bonds. These variables are respectively denoted by $i_{t-1}^R L_{t-1}^B \frac{P_{t-1}}{P_t}$ and $i_{t-1}^B B_{t-1}^C \frac{P_{t-1}}{P_t}$ (adjusted to real terms in period t). Moreover, the government collects lump-sum taxes from households (T_t). Thus, the government's budget constraint in real terms is,

$$T_t + i_{t-1}^R L_{t-1}^B \frac{P_{t-1}}{P_t} + i_{t-1}^B B_{t-1}^C \frac{P_{t-1}}{P_t} + B_t = (1 + i_{t-1}^B) B_{t-1} \frac{P_{t-1}}{P_t} + G_t, \quad (3.40)$$

where $B_t = B_t^H + B_t^C$, and $(1 + i_{t-1}^B) B_{t-1} \frac{P_{t-1}}{P_t}$ denotes the total gross interest payments to households and the central bank for holding government bonds. Finally, government spending (G_t) is set as a constant fraction of final good output,

$$G_t = \mu_G Y_t, \quad (3.41)$$

with $\mu_G \in (0, 1)$.

3.2.8 Market Clearing Conditions

In what follows the government is assumed to maintain a balanced budget by adjusting lump-sum taxes, while keeping its overall stock of bonds constant at \bar{B} . Moreover, the stock of bonds held by the central bank is also assumed to be constant at \bar{B}^C . Therefore, the value of total bonds held by households is $B^H = \bar{B} - \bar{B}^C$.

In a symmetric equilibrium, households are identical and IG firms produce the same output and set equal prices. Therefore, $K_{j,t} = K_t$, $N_{j,t} = N_t$, $Y_{j,t} = Y_t$ and $P_{j,t} = P_t$ for all $j \in (0, 1)$.

The supply of loans by the commercial bank and the supply of deposits by households are assumed to be perfectly elastic at the prevailing interest rates and therefore markets for loans and deposits always clear.²⁶ The goods market clearing condition is,

$$Y_t = C_t + I_t + G_t, \quad (3.42)$$

where C_t , I_t and G_t are respectively given by equations (3.7), (3.25) and (3.41).

Loans are made in the form of cash. Thus, the equilibrium condition in the currency market is obtained by equating total supply and total demand for money (by households and firms),

$$M_t^S = M_t^H + L_t^I. \quad (3.43)$$

Appropriate substitutions of equations (3.8), (3.9), (3.29) and (3.38) in equation (3.43) results in the money market equilibrium condition,

$$\bar{B}^C = \frac{\eta_x v C_t^{\frac{1}{\zeta}} (1 + i_t^B)}{i_t^B} + D_t + LLP_t^i, \quad (3.44)$$

which can be solved for the equilibrium bond rate (i_t^B). This equation identifies an additional channel through which loan loss provisions operate in the model: because higher provisions for instance tend to reduce borrowing from the central bank, they tend to lower (all else equal) the supply of cash in the economy. To maintain equilibrium of the money market, the bond rate must increase, which therefore affects the dynamics of consumption, through the Euler equation. To fix ideas, this channel will be referred to as the *general equilibrium channel of provisions*.

²⁶ Walras's law ensures that the market for government bonds always clears and therefore this market is ignored.

3.3 Steady State

The long run steady state values of the endogenous variables are derived below and are denoted by dropping the time subscript.

The steady state value of the bond rate (calculated from equation 3.7), is equal to the policy rate in the long run, under the assumption of zero inflation in steady state,

$$(1 + i^B) = (1 + i^R) = \frac{1}{\beta}. \quad (3.45)$$

The equality between i^B and i^R ensures that the commercial bank has no incentive to borrow from the central bank in order to invest in government bonds. The borrowing from the central bank will only be used to fund loans to the CG producer, which is more profitable than investing in government bonds as $i^L > i^B$ (due to the risk involved in loans becoming nonperforming).

The steady state level of the deposit rate is,

$$(1 + i^D) = \frac{1 + (1 - \mu_D)i^R}{\left(1 + \frac{1}{\eta_D}\right)}. \quad (3.46)$$

Deposits and real money balances are respectively given by,

$$D = \frac{\eta_x(1 - v)C^{\frac{1}{\varsigma}}(1 + i^B)}{i^B - i^D}, \quad (3.47)$$

$$M^H = \frac{\eta_x v C^{\frac{1}{\varsigma}}}{1 - \beta}. \quad (3.48)$$

The long run value of wages, when all households are able reoptimize in each period, is represented by,

$$W^R = \left(\frac{\theta_w}{\theta_w - 1} \right) N^\gamma C^{\frac{1}{\varsigma}}. \quad (3.49)$$

The steady state level of output is,

$$Y = AN^{1-\alpha}K^\alpha. \quad (3.50)$$

The long run rental rate on capital is,

$$r^K = (1 + i^L) \left[\frac{1}{\beta} - (1 - \delta_K) \right]. \quad (3.51)$$

The capital-labour ratio in steady state is,

$$\frac{K}{N} = \frac{\alpha}{(1-\alpha)} \frac{W^R}{r^K}. \quad (3.52)$$

With *fully flexible prices* and a constant level of output, the steady state value for the price mark-up (pm) is obtained from equation (3.23) and equals the inverse of the marginal cost,

$$pm = \left(\frac{\theta_p}{\theta_p - 1} \right) = \frac{1}{mc} > 1. \quad (3.53)$$

The marginal cost at the steady state (mc), in turn, is,

$$mc = \frac{(W^R)^{1-\alpha} (r^K)^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha} A}. \quad (3.54)$$

The total amount of lending from the commercial bank in the long run is given by,

$$L^I = I, \quad (3.55)$$

where total investment in steady state is,

$$I = \delta_K K. \quad (3.56)$$

In steady state, loan loss provisions under both the forward and backward looking systems are equal to,

$$LLP = l_0 J L^I. \quad (3.57)$$

Hence, the lending rate equation in the long run is also the same under each provisioning,

$$[1 - J] (1 + i^L) = \frac{1}{\left(1 + \frac{1}{\eta_L}\right)} \{ (1 + i^R) - i^R l_0 J \}. \quad (3.58)$$

The total borrowing from the central bank in steady state is given from equation (3.29),

$$L^B = L^I - (1 - \mu_D) D - LLP. \quad (3.59)$$

The steady state equation for the goods market equilibrium is,

$$Y = C + I + G, \quad (3.60)$$

with Y and I respectively given by equations (3.50) and (3.56), and $G = \mu_G Y$. The

tax equation in the long run is represented by,

$$T = i^B B^H - i^R L^B + G. \quad (3.61)$$

Finally, the money market equilibrium condition in steady state is,

$$\bar{B}^C = \frac{\eta_x v C^{\frac{1}{\varsigma}}}{1 - \beta} + D + LLP, \quad (3.62)$$

which gives the total value of bonds held by the central bank.

3.4 The Log-Linearized Model

The log-linearized equations of the model are based on the steady state solutions and represent percentage point deviations for price inflation, wage inflation and interest rate variables, and log-deviations around a non-stochastic steady state for the rest of the variables.²⁷ The log-linearized variables, denoted by hat, are listed below.

The Euler Equation,

$$\widehat{C}_t = E_t \widehat{C}_{t+1} - \varsigma \left[\widehat{i}_t^B - E_t \pi_{t+1}^P \right].$$

Deposits,

$$\widehat{D}_t = \frac{1}{\varsigma} \widehat{C}_t + \frac{(1 + i^D)}{(i^B - i^D)} \left(\widehat{i}_t^D - \widehat{i}_t^B \right).$$

Money Balances,

$$\widehat{M}_t^H = \frac{1}{\varsigma} \widehat{C}_t - \left(\frac{\beta}{1 - \beta} \right) \widehat{i}_t^B.$$

Wage Inflation,

$$\widehat{\pi}_t^W = \beta E_t \pi_{t+1}^W + \frac{(1 - \omega_w)(1 - \beta \omega_w)}{(\omega_w)(1 + \gamma \theta_w)} \left[\widehat{MRS}_t - \widehat{W}_t^R \right].$$

Marginal Rate of Substitution,

$$\widehat{MRS}_t = \gamma \widehat{N}_t + \frac{1}{\varsigma} \widehat{C}_t.$$

Real Wages,

$$\widehat{W}_t^R = \widehat{W}_t^R + \widehat{\pi}_t^W - \widehat{\pi}_t^P.$$

²⁷Therefore, log-linearized net interest rates are used as an approximation for log-linearized gross interest rates.

Aggregate demand for labour, taking into account both the value of the production function and the individual demand for labour faced by each household,

$$\widehat{N}_t = \frac{1}{(1-\alpha)}\widehat{Y}_t - \frac{\alpha}{(1-\alpha)}\widehat{K}_t + (\theta_w - 1) \frac{1}{(1-\alpha)}\widehat{A}_t - \frac{\theta_w}{(1-\alpha)} \left[(1-\alpha)\widehat{W}_t^R + \alpha \frac{(1+r^K)}{r^K} \widehat{r}_t^K \right].$$

Therefore, as long as $\theta_w > 1$ (which is assumed in this setup), the composite labour demand depends positively on output and aggregate supply shocks, and negatively on physical capital, the rental price of capital and real wages.

Capital-Labour Ratio,

$$\widehat{K}_t - \widehat{N}_t = \widehat{W}_t^R - \frac{(1+r^K)}{r^K} \widehat{r}_t^K.$$

Marginal Costs,

$$\widehat{mc}_t = (1-\alpha)\widehat{W}_t^R + \alpha \frac{(1+r^K)}{r^K} \widehat{r}_t^K - \widehat{A}_t.$$

The **New Keynesian Phillips Curve** (NKPC), which determines the price inflation rate, is given by,

$$\widehat{\pi}_t^P = \beta E_t \widehat{\pi}_{t+1}^P + \frac{(1-\omega_p)(1-\omega_p\beta)}{\omega_p} \widehat{mc}_t.$$

Investment loans,

$$\widehat{L}_t^I = \widehat{I}_t.$$

Deposit Rate,

$$\widehat{i}_t^D = \frac{(1-\mu_D)(1+i^R)}{[(1-\mu_D)(1+i^R) + \mu_D]} \widehat{i}_t^R,$$

with \widehat{i}_t^R defined by the Taylor rule (equation 3.39).

The rental price of capital,

$$\begin{aligned} E_t \widehat{r}_{t+1}^K &= \frac{(1+i^L)}{\beta(1+r^K)} \left[\widehat{i}_t^L + \widehat{i}_t^B - E_t \widehat{\pi}_{t+1}^P + \Theta_K (E_t \widehat{K}_{t+1} - \widehat{K}_t) \right] - \\ &\quad - \frac{(1+i^L)}{(1+r^K)} \left[(1-\delta_K) E_t \widehat{i}_{t+1}^L + \Theta_K E_t (\widehat{K}_{t+2} - \widehat{K}_{t+1}) \right]. \end{aligned}$$

Evolution of capital,

$$E_t \widehat{K}_{t+1} = \delta_K \widehat{I}_t + (1-\delta_K) \widehat{K}_t.$$

Central Bank Borrowing,

$$\widehat{L}_t^B = \frac{1}{L^B} \left\{ L^I \widehat{L}_t^I - (1 - \mu_D) D \widehat{D}_t - \widehat{LLP_t^{LLP^i}} \right\}.$$

Loan loss provisions in the backward and forward looking systems are respectively given by,

$$\begin{aligned} \widehat{LLP_t^{BK}} &= \widehat{J}_t + \widehat{L}_t^I, \\ \widehat{LLP_t^{FW}} &= (1 - \lambda) \widehat{J}_t + \widehat{L}_t^I. \end{aligned}$$

The fraction of nonperforming loans is represented by,

$$\widehat{J}_t = -\omega_Y \widehat{Y}_t - \omega_K \left[\widehat{\kappa}_t + \widehat{K}_t - \widehat{L}_t^I \right] - \omega_{LR} \left[\widehat{LLP_t^i} - \widehat{L}_t^I \right] + \widehat{\varepsilon}_t^J.$$

The loan rate under the *backward looking* provisioning rule, is obtained from equations (3.31) and (3.34),

$$(1 - J)(1 + i^L) \widehat{i}_t^L = \Psi(1 - l_0 J)(1 + i^R) \widehat{i}_t^R + [(1 + i^L) - \Psi i^R l_0] J \widehat{J}_t,$$

where $\Psi \equiv \frac{1}{(1 + \eta_L^{-1})}$. Under the *forward looking* provisioning system, the loan rate equation becomes,

$$(1 - J)(1 + i^L) \widehat{i}_t^L = \Psi(1 - l_0 J)(1 + i^R) \widehat{i}_t^R + [(1 + i^L) - \Psi i^R l_0 (1 - \lambda)] J \widehat{J}_t.$$

From the above equations, and as noted earlier, changes in the fraction of nonperforming loans and the type of provisioning system impact directly the loan rate. In turn, the cost of loans is passed to the rental rate on capital, which changes the marginal cost and the inflation rate. Therefore, the output gap, the collateral channel, the monitoring incentive effect or moral hazard effect (all of which influence the fraction of problem loans) and the type of provisioning system ultimately affect the behaviour of inflation and the real marginal cost.

The goods market equilibrium,

$$Y \widehat{Y}_t = C \widehat{C}_t + I \widehat{I}_t + G \widehat{G}_t,$$

where $\widehat{G}_t = \widehat{Y}_t$.

Taxes,

$$T \widehat{T}_t - G \widehat{G}_t = (1 + i^B) B^H \widehat{i}_{t-1}^B + [G - T - \bar{B}] \widehat{\pi}_t^P - i^R L^B \widehat{L}_{t-1}^B - (1 + i^R) L^B \widehat{i}_{t-1}^R,$$

Finally, the log-linear money market equilibrium, from which $\widehat{i_t^B}$ is obtained, is given by,

$$D\widehat{D_t} + M^H\widehat{M_t^H} + LLP\widehat{LLP_t^i} = 0.$$

Thus, the type of provisioning system impacts the bond rate, which in turn determines changes in consumption and real sector dynamics. Put differently, the provisioning regime affects the response of real variables to shocks not only through the lending rate (which affects investment) but also through the bond rate (which affects consumption).

3.5 Parameterization

The baseline parameterization of the model is summarized in Table 3.1. Parameters that characterize tastes, preferences, technology, adjustment costs, capital depreciation, and the Taylor rule, are all standard in the literature. We therefore focus on in what follows on the parameters that are new to this model.²⁸

²⁸In Table 3.1, LLR refers to loan loss reserves, LLP to loan loss provisions and NPL to non-performing loans.

Table 3.1: Benchmark Parameterization: Parameter Values

Parameter	Value	Description
β	0.99	Discount Factor
ς	0.50	Elasticity of Intertemporal Substitution
γ	3.00	Inverse of the Frisch Elasticity of Labour Supply
η_x	0.035	Preference Parameter for Liquidity Holdings
v	0.30	Share Parameter in Index of Money Holdings
θ_w	21.0	Elasticity of Labour Demand
ω_w	0.75	Degree of Wage Stickiness
θ_p	6.00	Elasticity of Demand for Intermediate Goods
ω_p	0.65	Degree of Price Stickiness
α	0.30	Share of Capital in Intermediate Goods Output
δ_K	0.03	Depreciation Rate of Capital
Θ_K	10.0	Adjustment Cost Parameter for Investment
ω_Y	0.20	Elasticity of Fraction of NPL wrt Output Gap
ω_K	0.01	Elasticity of Fraction of NPL wrt Capital-Loan Ratio
ω_{LR}	0.2,-0.2	Elasticity of Fraction of NPL wrt to LLR-Loan Ratio
l_0	1.00	LLP Coverage Ratio
j_0	0.03	Fraction of NPL in Steady State
κ	0.30	Fraction of Capital Seized in Case of Default
λ	0.80	Smoothing Coefficient in Dynamic Provisioning Rule
μ_D	0.05	Reserve Requirement Ratio
μ_G	0.40	Share of Government Spending in Output
ϕ	0.70	Degree of Persistence in Taylor Rule
ϕ_π	1.50	Response of Policy Rate to Inflation Deviations
ϕ_Y	0.20	Response of Policy Rate to Output Gap

For the variables related to the household, The preference parameter for the composite monetary asset (η_x) is set at 0.035, and the share parameter in the index of cash holding (v) at 0.30. The combination of these values yield a deposit plus cash to output ratio of 0.84, within the range observed for advanced economies.

Referring to the parameters associated with the fraction of nonperforming loans and its relationship with loan loss provisions, we calibrate the elasticity of the percentage of problem loans with respect to the output gap, ω_Y , at 0.2. Although not directly comparable, this value is consistent with the empirical results of Bikker and Metzmakers (2005) using OECD data, and Wong, Fong and Choi (2011) for Hong Kong, both of which focus on the impact of GDP growth on loan loss provisions.

The elasticity of the fraction of problem loans with respect to the collateral-loan ratio (ω_K) is set at 0.01, a relatively low value given that effective collateral appears to have in practice only a moderate effect on nonperforming loans and thus provisions (as documented in Davis and Zhu 2009). The fraction of collateral seized

in case of default (κ) is set at 0.30, which is approximately the value estimated by Cavalcanti (2010).

Given that the loan loss reserves-loan ratio can have opposite effects on the fraction of nonperforming loans, the elasticity ω_{LR} is set to 0.2 in the first part of the simulation section (corresponding therefore to the monitoring incentive effect) and to -0.2 in the second part (corresponding to the moral hazard effect).

For illustrative purposes, the smoothing coefficient in the dynamic loan loss provisions rule (λ) is set at 0.8, and the loan loss provisions coverage ratio (l_0) at 1, implying therefore that provisions fully cover the fraction of nonperforming loans. Furthermore, the fraction of nonperforming loans in steady state (j_0) is equal to 0.03, such that the loan loss provisions-loan ratio is also 3% in the long run.

For the central bank and government related parameters, we set $\mu_D = 0.05$, which is within the range defined by the required reserve ratio in the Eurozone (1%) and the United States (10% maximum, but which can be lower depending on the size of the account). In addition, the government's share of spending out of final output (μ_G) is set to 0.40, the average ratio of the government expenditures-GDP ratio for OECD countries between 2007-2009 (see OECD 2011).

The above parameterization implies that the steady-state values of the loan rate, the bond rate, and the rate of return on physical capital are 6.80%, 1.01%, and 4.28%, respectively. Moreover, the steady-state ratio of private investment to output is 17.5%, private consumption to output 42.5%, and government spending to output 40%. These values are within the range observed for industrialized countries. Finally, all shocks in this model follow an $AR(1)$ process with a persistence coefficient of 0.7.

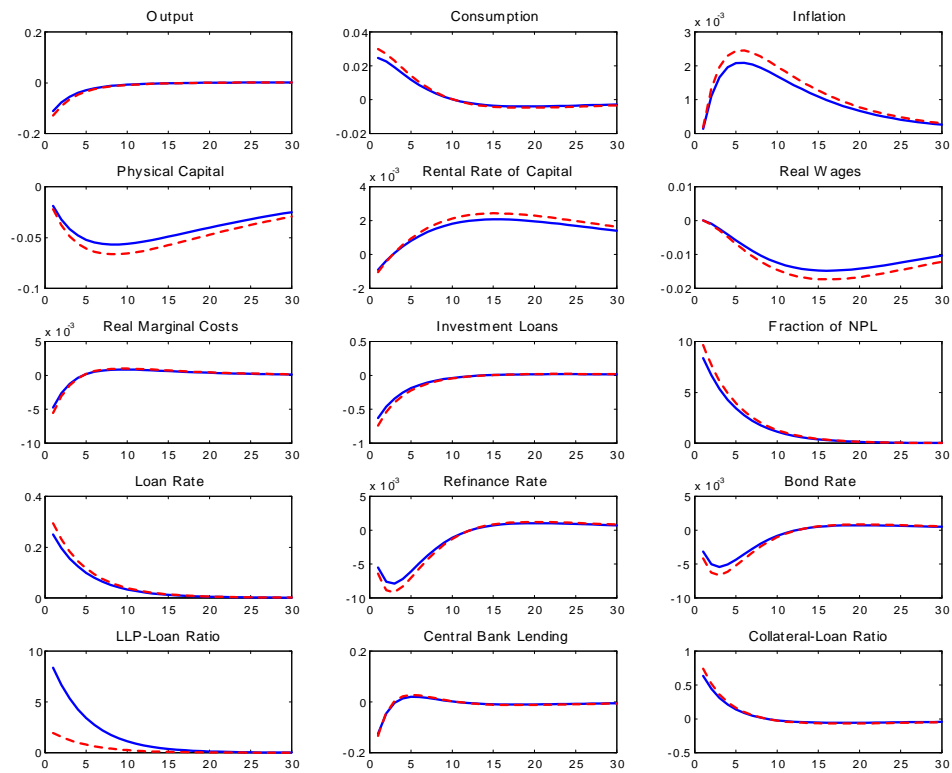
3.6 Simulations

The core experiments examined are financial shocks, taking the form of a rise in the fraction of nonperforming loans and a negative shock to collateral. We compare the performance of backward- and forward-looking provisioning rules ($\lambda = 0$ and $\lambda = 0.8$, respectively) when the loan loss reserves-loan ratio translates into either a lower ($\omega_{LR} = 0.2$) or higher ($\omega_{LR} = -0.2$) fraction of nonperforming loans.

3.6.1 A Shock to Nonperforming Loans with Monitoring Incentive

Figure 3.1 shows the impulse response functions of the main variables of the model following a 10% increase in the fraction of nonperforming loans (ε_t^J) under a backward looking provisioning system (blue line) and a forward looking provisioning system (dashed red line), and with $\omega_{LR} = 0.2$ (monitoring incentive case).

Figure 3.1: Shock to Fraction of Nonperforming Loans with Monitoring Incentive



The direct effect of an exogenous increase in the fraction of nonperforming loans is an immediate rise in the loan rate, stemming from the risk premium channel; the commercial bank sets a higher loan rate when the perception of risk is higher. The rise in the lending rate lowers the level of physical capital and investment loans (through the arbitrage condition (3.26)), which reduce the rental rate of capital and the marginal cost of production. Nevertheless, the increase in the cost of borrowing is also passed along directly to the rental rate of capital, which implies that the drop in the rental rate is short lived. In addition, given the high persistence in real

wages, the drop in marginal costs is very small, implying that its effect on inflation is negligible. As a result, the response of inflation is subdued in the first few periods, followed by a gradual hump shape rise associated with a higher rental rate of capital and cost of borrowing.

The sharp fall in investment induced by the rise in the loan rate, along with the drop in employment and capital stock, leads to a much more volatile negative reaction of output compared to the rise in inflation. Consequently, the policy rate, which is determined by the Taylor rule, falls in response to the decline in output. The deposit rate, set as a mark down of the policy rate, drops as well, which results in a lower demand for deposits, a fall in required reserves and hence, all else equal, an increase in borrowing from the central bank and an expansion in the monetary base. To raise the demand for cash and restore equilibrium in the money market, the bond rate must therefore decrease, which, through intertemporal substitution, results in a higher level of consumption in the short run.

Given our calibration the drop in investment dominates the rise in consumption and output, relative to its steady-state level, drops initially.²⁹ This in turn tends to amplify the response of the fraction of nonperforming loans while the increase in the capital stock-loan ratio acts to mitigate it. Given our calibration and the nature of the financial shock, the net effect is an increase in the fraction of nonperforming loans, thereby leading to a higher loan rate.

Furthermore, to assess how changes in provisions affect the real economy, we identify three channels through which provisions impact the loan rate and real economy. The first channel comes from the relationship between loan loss provisions and central bank borrowing referred to as the provisioning cost channel as explained earlier. The second channel stems from the monitoring incentive effect, in which a higher ratio of loan loss reserves to loans reduces the fraction of nonperforming loans, and therefore mitigates the initial rise in the loan rate following a financial shock. The third channel arises from the money market equilibrium and the relationship between central bank borrowing, loan loss provisions and the supply of cash (the general equilibrium channel of provisions). Specifically, if loan loss provisions increase, then from the commercial bank's balance sheet, borrowing from the central bank decreases. In that sense, provisions (just like bank capital) are simply another way of financing loans. The fall in central bank borrowing reduces the supply of currency by the central bank and therefore, to restore equilibrium in the cash market, the bond rate must increase to lower the demand for money. Hence, the third channel acts to mitigate the drop in the bond rate and therefore creates a downward

²⁹This outcome is quite reasonable, given the nature of the shock

pressure on consumption. Nevertheless, this channel is not strong enough to offset the rise in consumption resulting from the fall in the bond rate. We therefore focus on the first two channels of loan loss provisions.

In our model, following a rise in the percentage of nonperforming loans, the loan loss provisions-loan ratio increases which, from the provisioning cost channel, reduces the cost of borrowing from the central bank, thereby lowering the commercial bank's liabilities. This channel therefore leads to an attenuation effect on the loan rate. The second channel, arising from the monitoring incentive effect, implies that the rise in the loan loss provisions-loan ratio acts also to mitigate the response of the loan rate, through its impact on the fraction of nonperforming loans. The attenuation in perceived risk, in turn, affects the loan rate via the risk premium channel. Hence, both provisioning channels under the backward-looking system act as mitigation mechanisms on the loan rate. The moderate fall in the cost of borrowing increases slightly the demand for investment loans, mitigates the initial drop in the rental rate of capital and the stock of physical capital, thereby moderating the drop in aggregate demand.

With a forward-looking provisioning system, loan loss provisions are smoothed over the cycle such that provisions are less affected by the current fraction of nonperforming loans. Given that raising provisions during a downturn mitigates the procyclicality effects of the financial system, as described above, holding less provisions therefore amplifies the response of the key variables following a financial shock. In particular, smoothing out provisions increases the cost of central bank borrowing, and also creates an upward pressure on the fraction of problem loans, both of which further amplify the response of the lending rate. Hence, a forward-looking provisioning system can *increase* procyclicality compared to a backward-looking provisioning system, in the sense that the rise in the loan rate and the percentage of nonperforming loans is magnified.

Table 3.2 compares the asymptotic standard deviations and the relative standard deviations of key variables of a backward-looking system ($\lambda = 0$), a moderate forward-looking system ($\lambda = 0.4$) and an extreme forward-looking system ($\lambda = 0.8$). The relative standard deviations are calculated with regards to the standard deviations of the backward-looking case. The table confirms the result in which a higher degree of loan loss provisions smoothing increases the volatility of key macro and financial variables. In other words, a backward-looking provisioning system performs better, in terms of mitigating the procyclicality of the financial system, when loan loss provisions reduce the fraction of nonperforming loans.

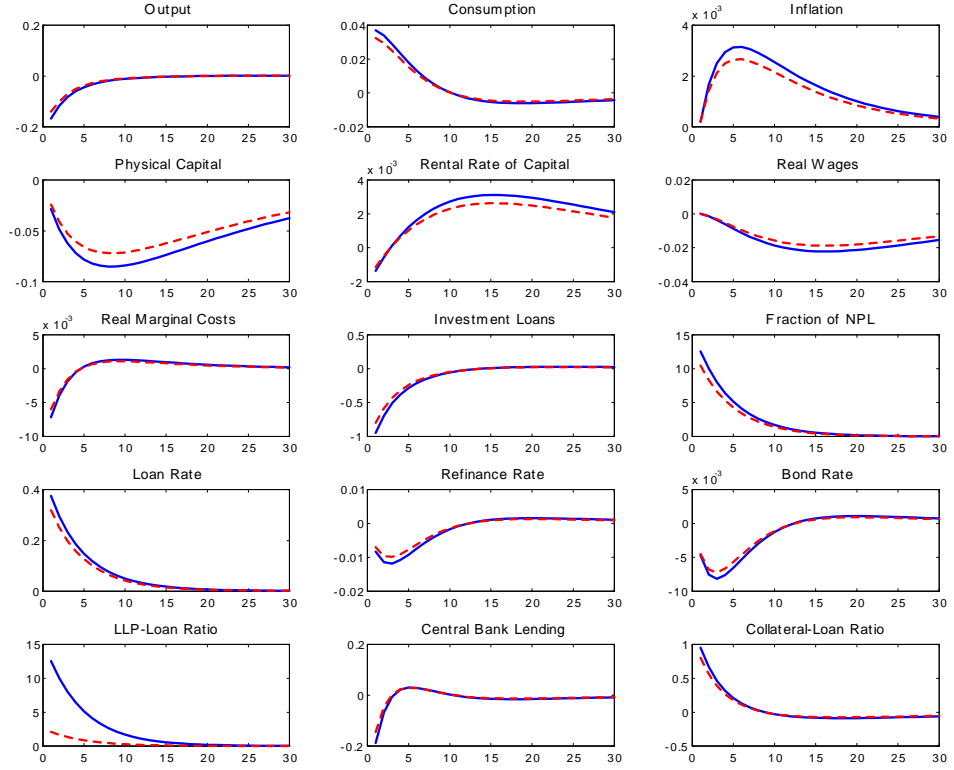
Table 3.2: Changes in Standard Deviations of Key Variables under Backward and Forward Provisioning Systems with Monitoring Incentive

	$\lambda = 0$		$\lambda = 0.4$	Rel. S.D.	$\lambda = 0.8$	Rel. S.D.
Output	0.2442		0.2620	1.0728	0.2825	1.1568
Investment	0.8956		0.9619	1.0740	1.0385	1.1595
Consumption	0.0305		0.0331	1.0852	0.0360	1.1803
Goods Inflation	0.0067		0.0072	1.0746	0.0078	1.1641
Loan Rate	0.3504		0.3770	1.0759	0.4077	1.1635
Fraction of NPL	11.7025		12.5410	1.0716	13.5090	1.1543
Marginal Cost	0.0096		0.0103	1.0729	0.0111	1.1562
Bond Rate	0.0108		0.0118	1.0925	0.0129	1.1944
Collateral-Loan Ratio	0.8850		0.9505	1.0740	1.0261	1.1594
Loan Growth	0.7586		0.8147	1.0739	0.8795	1.1593
Loan-Output Ratio	0.6519		0.7004	1.0743	0.7564	1.1603
Loan-Deposits Ratio	0.5591		0.6323	1.1309	0.7170	1.2824
LLP-Loan Ratio	11.7025		7.5246	0.6429	2.7018	0.2308

3.6.2 A Shock to Nonperforming Loans with Moral Hazard

We now compare between the backward and forward looking provisioning systems when a rise in the loan loss provisions-loan ratio raises the percentage of nonperforming loans, because of moral hazard ($\omega_{LR} = -0.2$). Figure 3.2 illustrates the results following a 10% increase in the fraction of nonperforming loans (ε_t^J), with as before the results corresponding to the backward-looking provisioning regime given by the solid blue line, and those corresponding to the forward-looking provisioning system given by the dashed red line.

Figure 3.2: Shock to Fraction of Nonperforming Loans with Moral Hazard



The general equilibrium effects of the financial shock are similar to those described earlier and therefore we focus here only on the transmission channels associated with provisions. As in the previous case, a rise in the fraction of nonperforming loans directly raises the loan loss provisions-loan ratio which now has two opposite effects on the loan rate. On the one hand, a rise in loan loss provisions relative to loans has a dampening effect on the loan rate, and therefore on aggregate demand, through the provisioning cost channel, as explained above. On the other, the increase in the loan loss provisions-loan ratio now translates into an even higher fraction of nonperforming loans, which through the risk premium channel amplifies the response of the loan rate. Given our calibration, the latter effect dominates the provisioning cost channel, in such a way that the response of all key variables in the economy is amplified when $\omega_{LR} = -0.2$ and the provisioning system is backward looking.

With a forward-looking provisioning system, loan loss provisions are smoothed during a financial distress period, such that the fall in loan loss provisions mitigates the rise in the percentage of problem loans and consequently also the rise in the

loan rate. At the same time, through the negative relationship between the loan rate and the loan loss provisions-loan ratio (from the provisioning cost channel), the smoothing of loan loss provisions amplifies the response of the loan rate and the rest of the key variables. However, the dominating factor comes from the impact of the loan loss provisions-loan ratio on the fraction of nonperforming loans, which through the risk premium channel, mitigates the rise in the cost of borrowing. As a result, a dynamic forward-looking provisioning system can indeed attenuate the response of the key variables following an adverse financial shock, thus reducing the procyclicality effects on the loan rate and the fraction of nonperforming loans, as well as output and inflation.

Table 3.3 shows the asymptotic standard deviations and relative standard deviations of key variables relative to the standard deviation under a backward-looking provisioning system.

Table 3.3: Changes in Standard Deviations of Key Variables under Backward and Forward Provisioning Systems with Moral Hazard

	$\lambda = 0$		$\lambda = 0.4$ Rel. S.D.	$\lambda = 0.8$ Rel. S.D.
Output	0.3668		0.3337 0.9097	0.3062 0.8347
Investment	1.3454		1.2253 0.9107	1.1253 0.8364
Consumption	0.0459		0.0421 0.9172	0.0390 0.8496
Goods Inflation	0.0101		0.0092 0.9108	0.0084 0.8316
Loan Rate	0.5263		0.4802 0.9124	0.4418 0.8394
Fraction of NPL	17.5760		15.9747 0.9087	14.6388 0.8327
Marginal Cost	0.0144		0.0131 0.9097	0.0120 0.8333
Bond Rate	0.0162		0.0150 0.9259	0.0140 0.8641
Collateral-Loan Ratio	1.3294		1.2107 0.9107	1.1119 0.8363
Loan Growth	1.1398		1.0379 0.9105	0.9531 0.8361
Loan-Output Ratio	0.9793		0.8922 0.9110	0.8197 0.8370
Loan-Deposits Ratio	0.8400		0.8054 0.9588	0.7770 0.9250
LLP-Loan Ratio	17.5790		9.5848 0.5452	2.9278 0.1665

As the data indicate, and in contrast to the results presented earlier, moving towards a forward-looking regime that smoothes the reaction of provisions during financial distress reduces the volatility of the key macro and financial variables.

To conclude, the extent to which dynamic forward-looking provisions are either more or less procyclical, and impart greater or lower volatility to financial variables, compared to a backward-looking system, depends on whether a rise in the loan loss

provisions-loan ratio translates into a lower or higher percentage of nonperforming loans.

3.6.3 Collateral Shock

We now consider a negative shock to the fraction of collateral that can be seized in case of default (κ_t). The qualitative effects of a negative 10% shock to κ_t are very similar to the results obtained under a direct shock to the fraction of nonperforming loans, but are different with respect to the degree of volatility and the transmission channels.³⁰

A negative shock to collateral directly reduces the collateral-loan ratio, which leads to a rise in the fraction of problem loans and hence the loan rate via the risk premium channel. In addition, the fall in investment, induced by the higher cost of borrowing, lowers the level of output, which in turn also raises the fraction of nonperforming loans. The higher perception of risk, led by a deterioration in economic activity, amplifies the initial increase in the loan rate caused by lower collateral values. Therefore, as opposed to a direct shock to the fraction of nonperforming loans (discussed in the previous section), both the collateral channel and the decreasing level of output act to raise the loan rate. Nevertheless, because of the small sensitivity of the fraction of nonperforming loans and hence loan loss provisions with respect to collateral, a sharp fall in collateral values has only a mild impact on the behaviour of the loan rate, in contrast to the case of a direct shock to the fraction of problem loans. Therefore, the volatility of all key economic variables is reduced as well.

As the fraction of nonperforming loans rise, so do loan loss provisions. The increase in provisions (relative to loans) affects the loan rate via the channels described in the previous section. Specifically, in the monitoring incentive case, the rise in the loan loss provisions-loan ratio reduces the fraction of nonperforming loans, which through the risk premium channel, mitigates the rise in the loan rate. Moreover, from the provisioning cost channel, the lending rate response is further attenuated. Therefore, a forward-looking provisioning system, which smoothes the response of provisions, exacerbates the response of the loan rate and the other key variables in this setup.

However, if a higher ratio of loan loss provisions to loans increases the fraction of nonperforming loans through the moral hazard effect, the rise in provisions increases

³⁰To save space, we do not present the impulse response functions of a negative shock to collateral as qualitatively, the results are very similar to the case of a shock to nonperforming loans. If required, the impulse response functions showing the effects of a negative collateral shock are available upon request.

the fraction of problem loans, which through the risk premium channel magnifies the response of the loan rate. Hence, a statistical provisioning *mitigates* the response of the lending rate and the other main economic and financial variables of the model.

Overall, the provisioning channels following a collateral shock act similarly to the case where the economy is hit by a direct shock to the fraction of nonperforming loans. However, in terms of volatility, a direct shock to nonperforming loans magnifies volatility in the key variables, given that changes in collateral affect risk and provisions by a small magnitude.

3.6.4 Provisions and Collateral Values

How does collateral affect the determination of loan loss provisions? Under the IASB accounting rules there is no detailed guidance on how collateral should affect provisions. With no international standards, national authorities and bank supervisors have often designed their own regulation on provisions.³¹ In many countries, the value of collateral can be subtracted from required provisions to determine actual provisions.

In the context of the model, this can be captured as follows. Recall that $\kappa_t K_t$ denotes the real value of collateral; thus, if the net value of loans is used to determine provisions, under the backward-looking rule,

$$LLP_t^{BK} = l_0 J_t (L_t^I - \kappa_t K_t),$$

whereas under the forward-looking rule,

$$LLP_t^{FW} = l_0 [J_t + \lambda (J - J_t)] (L_t^I - \kappa_t K_t),$$

instead of (3.31) and (3.32).³²

Given that (3.35) and (3.36) do not change, it is immediately clear that this change in the definition of loan loss provisions has no impact on the loan rate, as given in (3.34). However, the change in definitions does affect the equilibrium condition of the market for cash (3.44), and thus the behaviour of the bond rate. Nevertheless, numerical experiments show that, given our calibration, this effect is quantitatively small. Thus, the previous conclusions about the performance of

³¹In Germany, general guidance is provided for valuing collateral in provisions. In the United Kingdom, collateral is considered in provisions. In the United States, banks should value collateral at its fair market value minus the costs of selling it.

³²For LLP_t^i to be positive in the steady state, it must be assumed that $\kappa < \delta = 0.03$ because loans in this model are provided only for investment purposes and in steady state are equal to $L^I = I = \delta K$.

alternative loan loss provisioning rules remain essentially unchanged.

3.7 Concluding Remarks

The purpose of this chapter has been to study the interactions between loan loss provisions and business cycles in a Dynamic Stochastic General Equilibrium (DSGE) model with credit market imperfections. A key distinction is made between backward- and forward-looking provisioning systems. In the former, provisions are triggered by past due payments (or the fraction of nonperforming loans), which, in turn, depend on current economic conditions and the loan loss reserves-loan ratio. Forward-looking (statistical or dynamic) provisioning, by contrast, take into account both past due payments and expected losses over the whole business cycle; provisions are thus smoothed over the cycle and are less affected by the current state of the economy and past due payments. The solution of the model shows that the type of provisioning system and the fraction of nonperforming loans influence directly the behaviour of both the loan rate and the bond rate (the opportunity cost of holding cash), which in turn determine the degree of cyclicity of financial and real variables in the economy. Numerical experiments with a parameterized version illustrate that holding more loan loss provisions can reduce the procyclicality of the financial system by inducing a mitigating effect on the loan rate and the fraction of nonperforming loans. In addition, a forward-looking loan loss provisioning system can increase or lower procyclicality, depending on whether holding more loan loss reserves translates into a higher or lower fraction of nonperforming loans. These results have useful implications for the ongoing debate on the performance of loan loss provisioning systems and more generally macroprudential rules.

Finally, in the extended version of this chapter, we show in Agénor and Zilberman (2013) that a credit augmented Taylor rule, coupled with a backward-looking provisioning system (as most countries have now) may be quite effective at mitigating real and financial volatility in the financial system and real economy, compared to a forward-looking provisioning system. Intuitively, because a forward-looking provisioning system can either increase or lower real and financial volatility, an augmented Taylor rule that reacts modestly also to a financial indicator, may be superior in terms of minimizing economic and financial fluctuations. At a time when many countries, in the wake of the adoption of the Basel III agreement, are considering the introduction of a variety of countercyclical macroprudential tools, this result is well worth pondering.

The analysis in this chapter can be extended in several directions. First, the

focus of this chapter has been mainly on the direct, cost effect (from the perspective of lenders) of loan loss provisions and their indirect impact on the money market equilibrium. However, another channel that could be explored is the extent to which these provisions may help to mitigate incentives for risk taking by lenders, and the extent to which this could contribute to reducing balance sheet vulnerabilities, thereby reducing their probability of default. Bushman and Williams (2012) for instance, in a study of bank behaviour across 27 countries, found that forward-looking provisioning reflecting timely recognition of expected future loan losses is associated with enhanced risk-taking discipline. Second, it would be useful to model simultaneously capital requirement regimes and loan loss provisioning systems and study how they interact. The common view is that bank capital should cover for *unexpected* credit losses, whereas (dynamic) loan loss provisions are intended to cover *expected* credit losses. However, consistent with the Lucas Critique (1976), introducing either one of those regulatory regimes while the other is present may change the behaviour of banks and thus the effectiveness of both types of tools. This may occur, for instance, if the reasons why banks hold (excess) capital buffers are altered by the introduction of loan loss provisions, and if capital buffers have a signaling effect that translates into changes in their market borrowing costs (as in Agénor, Alper and Pereira da Silva 2012 for instance). Put differently, voluntary capital buffers may be (partial) substitutes for provisions. Thus, understanding the interaction between bank capital requirements and dynamic loan loss provisioning systems, and possibly the optimal combination of these tools to mitigate procyclicality, should be high on the research agenda.

3.A Appendix

3.A.1 *Derivation of the Rental Rate of Capital*

The CG producer chooses the level of capital stock so as to maximize the value of discounted stream of dividend payments to households subject to equation (3.25). Specifically, defining $J_t^K = r_t^K K_t - (1 + i_t^L)I_t$ as the CG producer's real profits in period t , the optimization problem takes the following form,

$$\max_{K_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t \varphi_t \left\{ r_t^K K_t - (1 + i_t^L) \left[K_{t+1} - (1 - \delta_K)K_t + \frac{\Theta_K}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t \right] \right\}.$$

The first order condition with respect to K_{t+1} yields,

$$\begin{aligned} & \beta \varphi_{t+1} r_{t+1}^K - \varphi_t (1 + i_t^L) + \beta \varphi_{t+1} (1 + i_{t+1}^L) (1 - \delta_K) - \\ & - (1 + i_t^L) \varphi_t \Theta_K \left(\frac{K_{t+1}}{K_t} - 1 \right) \frac{K_t}{K_t} - (1 + i_{t+1}^L) \beta \varphi_{t+1} \frac{\Theta_K}{2} \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right)^2 + \\ & + (1 + i_{t+1}^L) \beta \varphi_{t+1} \Theta_K \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right) \frac{K_{t+2}}{K_{t+1}} = 0, \end{aligned}$$

multiplying out,

$$\begin{aligned} & \beta \varphi_{t+1} r_{t+1}^K - \varphi_t (1 + i_t^L) + \beta \varphi_{t+1} (1 + i_{t+1}^L) (1 - \delta_K) - \\ & - (1 + i_t^L) \varphi_t \Theta_K \left(\frac{K_{t+1}}{K_t} - 1 \right) + (1 + i_t^L) \varphi_t \Theta_K - (1 + i_{t+1}^L) \beta \varphi_{t+1} \frac{\Theta_K}{2} \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right)^2 + \\ & + (1 + i_{t+1}^L) \beta \varphi_{t+1} \frac{\Theta_K}{2} 2 \frac{K_{t+2}}{K_{t+1}} - (1 + i_{t+1}^L) \beta \varphi_{t+1} \frac{\Theta_K}{2} + \\ & + (1 + i_{t+1}^L) \beta \varphi_{t+1} \Theta_K \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right)^2 - (1 + i_{t+1}^L) \beta \varphi_{t+1} \Theta_K \frac{K_{t+2}}{K_{t+1}} = 0, \end{aligned}$$

collecting terms,

$$\begin{aligned} & \beta \varphi_{t+1} r_{t+1}^K - \varphi_t (1 + i_t^L) + \beta \varphi_{t+1} (1 + i_{t+1}^L) (1 - \delta_K) - \\ & - (1 + i_t^L) \varphi_t \Theta_K \left(\frac{K_{t+1}}{K_t} - 1 \right) + (1 + i_t^L) \varphi_t \Theta_K + \\ & + (1 + i_{t+1}^L) \beta \varphi_{t+1} \frac{\Theta_K}{2} \left(\frac{K_{t+2}}{K_{t+1}} - 1 \right)^2 - (1 + i_{t+1}^L) \beta \varphi_{t+1} \frac{\Theta_K}{2} = 0, \end{aligned}$$

or,

$$\begin{aligned}\beta\varphi_{t+1}r_{t+1}^K &= \varphi_t(1+i_t^L) \left[1 + \Theta_K \left(\frac{K_{t+1}}{K_t} \right) - \Theta_K \right] - \\ &\quad - \beta\varphi_{t+1}(1+i_{t+1}^L) \left[(1-\delta_K) + \frac{\Theta_K}{2} \left(\frac{K_{t+2}}{K_{t+1}} \right)^2 - \frac{\Theta_K}{2} \right],\end{aligned}$$

or,

$$\begin{aligned}r_{t+1}^K &= \frac{\varphi_t}{\beta\varphi_{t+1}}(1+i_t^L) \left\{ 1 + \Theta_K \left[\left(\frac{K_{t+1}}{K_t} \right) - 1 \right] \right\} - \\ &\quad - (1+i_{t+1}^L) \left\{ (1-\delta_K) + \frac{\Theta_K}{2} \left[\left(\frac{K_{t+2}}{K_{t+1}} \right)^2 - 1 \right] \right\}.\end{aligned}$$

Substituting equation (3.7) and inserting back the expectation operator results in equation (3.26) presented in the body text.

3.A.2 Derivation of the Loan Rate and Deposit Rate

The bank's maximization problem is defined as,

$$\max_{(1+i_t^L), (1+i_t^D)} \sum_{t=0}^{\infty} \beta^t \varphi_t \left\{ \begin{array}{l} [1 - J_t] (1 + i_t^L) L_t^I + J_t (\kappa K_t) + \mu_D D_t - \\ - (1 + i_t^D) D_t - (1 + i_t^R) L_t^B - LLP_t^i \end{array} \right\},$$

subject to,

$$L_t^I = I_t,$$

$$L_t^B = L_t^I - (1 - \mu_D) D_t - LLP_t^i,$$

$$J_t = j_0 \left[\frac{Y_t}{Y} \right]^{-\omega_Y} \left[\frac{\kappa_t K_t / L_t^I}{\kappa K / L^I} \right]^{-\omega_K} \left[\frac{LLR_t / L_t^I}{LLR / L^I} \right]^{-\omega_{LR}} \varepsilon_t^J,$$

$$LLP_t^{BK} = l_0 J_t L_t^I \quad \text{for } i = BK,$$

$$LLP_t^{FW} = l_0 J_t L_t^I + \lambda (J - J_t) l_0 L_t \quad \text{for } i = FW.$$

The first order condition with respect to $(1 + i_t^L)$ yields,

$$\begin{aligned} & \beta^t \varphi_t [1 - J_t] L_t^I + \beta^t \varphi_t [1 - J_t] (1 + i_t^L) \frac{\partial L_t^I}{\partial (1 + i_t^L)} - \\ & - \beta^t \varphi_t (1 + i_t^R) \frac{\partial L_t^B}{\partial L_t^I} \frac{\partial L_t^I}{\partial (1 + i_t^L)} - \beta^t \varphi_t (1 + i_t^R) \frac{\partial L_t^B}{\partial LLP_t^i} \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\partial L_t^I}{\partial (1 + i_t^L)} - \\ & - \beta^t \varphi_t \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\partial L_t^I}{\partial (1 + i_t^L)} = 0, \end{aligned}$$

dividing by $\beta^t \varphi_t$ and noting that $\frac{\partial L_t^B}{\partial LLP_t^i} = -1$ yields,

$$\begin{aligned} & [1 - J_t] L_t^I + [1 - J_t] (1 + i_t^L) \frac{\partial L_t^I}{\partial (1 + i_t^L)} - \\ & - (1 + i_t^R) \frac{\partial L_t^I}{\partial (1 + i_t^L)} + (1 + i_t^R) \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\partial L_t^I}{\partial (1 + i_t^L)} - \\ & - \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\partial L_t^I}{\partial (1 + i_t^L)} = 0, \end{aligned}$$

dividing by L_t^I ,

$$\begin{aligned} & [1 - J_t] + [1 - J_t] (1 + i_t^L) \frac{\partial L_t^I}{\partial (1 + i_t^L)} \frac{1}{L_t^I} - \\ & - (1 + i_t^R) \frac{\partial L_t^I}{\partial (1 + i_t^L)} \frac{1}{L_t^I} + (1 + i_t^R) \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\partial L_t^I}{\partial (1 + i_t^L)} \frac{1}{L_t^I} - \\ & - \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\partial L_t^I}{\partial (1 + i_t^L)} \frac{1}{L_t^I} = 0, \end{aligned}$$

or,

$$\begin{aligned} & [1 - J_t] + [1 - J_t] \frac{\partial L_t^I}{\partial (1 + i_t^L)} \frac{(1 + i_t^L)}{L_t^I} - \\ & - (1 + i_t^R) \frac{\partial L_t^I}{\partial (1 + i_t^L)} \frac{1}{L_t^I} \frac{(1 + i_t^L)}{(1 + i_t^L)} + (1 + i_t^R) \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\partial L_t^I}{\partial (1 + i_t^L)} \frac{1}{L_t^I} \frac{(1 + i_t^L)}{(1 + i_t^L)} - \\ & - \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\partial L_t^I}{\partial (1 + i_t^L)} \frac{1}{L_t^I} \frac{(1 + i_t^L)}{(1 + i_t^L)} = 0, \end{aligned}$$

defining $\eta_L = \frac{\partial L_t^I}{\partial (1 + i_t^L)} \frac{(1 + i_t^L)}{L_t^I}$ as the interest elasticity of the loan demand and treating it as a constant, the above reduces to,

$$\begin{aligned} & [1 - J_t] + [1 - J_t] \eta_L - (1 + i_t^R) \frac{\eta_L}{(1 + i_t^L)} + \\ & + (1 + i_t^R) \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\eta_L}{(1 + i_t^L)} - \frac{\partial LLP_t^i}{\partial L_t^I} \frac{\eta_L}{(1 + i_t^L)} = 0, \end{aligned}$$

multiplying by $(1 + i_t^L)$,

$$\begin{aligned} & [1 - J_t] (1 + i_t^L) + [1 - J_t] \eta_L (1 + i_t^L) - (1 + i_t^R) \eta_L + \\ & + (1 + i_t^R) \frac{\partial LLP_t^i}{\partial L_t^I} \eta_L - \frac{\partial LLP_t^i}{\partial L_t^I} \eta_L = 0, \end{aligned}$$

or,

$$[1 + \eta_L] [1 - J_t] (1 + i_t^L) = \eta_L \left\{ (1 + i_t^R) + \frac{\partial LLP_t^i}{\partial L_t^I} - (1 + i_t^R) \frac{\partial LLP_t^i}{\partial L_t^I} \right\},$$

or,

$$[1 - J_t] (1 + i_t^L) = \left(\frac{\eta_L}{1 + \eta_L} \right) \left\{ (1 + i_t^R) + [1 - (1 + i_t^R)] \frac{\partial LLP_t^i}{\partial L_t^I} \right\},$$

which is equation (3.34) presented in the text.

The first order condition with respect to $(1 + i_t^D)$ yields,

$$\mu_D \frac{\partial D_t}{\partial (1 + i_t^D)} - D_t - (1 + i_t^D) \frac{\partial D_t}{\partial (1 + i_t^D)} + (1 + i_t^R) (1 - \mu_D) \frac{\partial D_t}{\partial (1 + i_t^D)} = 0,$$

dividing by D_t and multiplying the first and last term by $\frac{(1+i_t^D)}{(1+i_t^D)}$,

$$\begin{aligned} & \mu_D \frac{\partial D_t}{\partial (1 + i_t^D)} \frac{1}{D_t} \frac{(1 + i_t^D)}{(1 + i_t^D)} - 1 - (1 + i_t^D) \frac{\partial D_t}{\partial (1 + i_t^D)} \frac{1}{D_t} + \\ & + (1 + i_t^R) (1 - \mu_D) \frac{\partial D_t}{\partial (1 + i_t^D)} \frac{1}{D_t} \frac{(1 + i_t^D)}{(1 + i_t^D)} = 0, \end{aligned}$$

defining $\eta_D = \frac{\partial D_t}{\partial (1 + i_t^D)} \frac{(1 + i_t^D)}{D_t}$ as the interest elasticity of deposits and treating it as a constant, the above reduces to,

$$\mu_D \frac{\eta_D}{(1 + i_t^D)} - 1 - \eta_D + (1 + i_t^R) (1 - \mu_D) \frac{\eta_D}{(1 + i_t^D)} = 0,$$

multiplying by $(1 + i_t^D)$,

$$\mu_D \eta_D - (1 + i_t^D) - \eta_D (1 + i_t^D) + (1 + i_t^R) (1 - \mu_D) \eta_D = 0,$$

or,

$$(1 + i_t^D) (1 + \eta_D) = \eta_D (1 + i_t^R) (1 - \mu_D) + \mu_D \eta_D,$$

or,

$$(1 + i_t^D) = \frac{\eta_D}{(1 + \eta_D)} (1 + i_t^R) (1 - \mu_D) + \frac{\eta_D}{(1 + \eta_D)} \mu_D,$$

or,

$$(1 + i_t^D) = \frac{1}{(1 + \frac{1}{\eta_D})} (1 - \mu_D) (1 + i_t^R) + \frac{1}{(1 + \frac{1}{\eta_D})} \mu_D,$$

which after rearranging results in equation (3.33) presented in the text.

Summary and Conclusions

The recent crisis of 2007-2009 has clearly demonstrated that banking regulation and financial sector volatility translate to substantial real macroeconomic effects, and that incorporating credit market frictions and financial risk into otherwise standard macro models are crucial for explaining the behaviour of real business cycles. Using three different model frameworks, this thesis contributes to the growing literature of macrofinance and financial-real sectors linkages through an investigation into the role of regulatory requirements, bank capital buffers, credit risk and loan loss provisions in the transmission of various types of shocks.

Chapter 1 explores how banking regulation and bank capital buffers impact the financial system and the real economy through their direct effect on the repayment probability and hence the loan rate in a simple macro model. We utilize the monitoring incentive effect, as described by Agénor, Alper and Pereira da Silva (2012), to explain how bank capital buffers can reduce the likelihood of default and lead to lower borrowing costs (the bank capital channel). We also identify a collateral channel, which mitigates moral hazard behaviour by firms, and therefore raises their repayment probability and lowers the loan rate. The model examines the role of the bank capital and collateral channels in the transmission of supply shocks and makes a distinction between the different variants of the Basel Accords.

We show that depending on the strength of the bank capital channel relative to the collateral channel, the loan rate can either amplify or mitigate the effects of productivity shocks. Specifically, with a strong bank capital channel, the loan rate is always procyclical in the sense that it aggravates the effects of supply shocks. Nevertheless, if the collateral channel dominates the bank capital channel, the loan rate may be either procyclical or countercyclical following supply shocks. Finally, the impact of the two channels, along with changes in the price level, determine which of the regulatory regimes is most procyclical. As a result, how we model the interactions between the bank capital channel and the collateral channel, as well as the impact they have on key financial and macroeconomic variables, is important for our understanding of the transmission mechanisms and the role of bank capital regulation and bank capital buffers.

Chapter 2 further investigates the effects of bank capital, regulatory requirements and the financial-real sectors linkages, but from a very different perspective. The model presented in the second chapter is a DSGE framework, with an endogenous formation of risk at both the firm and bank capital levels. The linkages between the financial system and the real business cycle in the model is explained through the borrowing cost channel (introduced by Ravenna and Walsh 2006), which links

the loan rate behaviour to the real marginal costs and hence the rate of price inflation. However, compared to their model, the behaviour of the loan rate and hence the borrowing cost channel is complicated by three additional channels impacting directly the cost of borrowing: *i)* The risk premium channel, resulting from the positive probability of default at the firm level, and which leads the commercial bank to charge a premium over the cost of borrowing from households (similar to Agénor, Bratsiotis and Pfajfar 2013). *ii)* The bank capital default channel, stemming from the introduction of bank capital risk. The probability of default on bank capital creates an endogenous spread between the rate of return on bank capital and the interest rate on deposits. As bank capital is subject to risk, households demand a higher return for holding this asset such that a no-arbitrage condition between bank capital and deposits prevails. Hence, the model contributes to other models in this literature which include bank capital costs, but abstract from deriving an endogenous wedge between the cost of bank capital and the cost of deposits (see Markovic 2006, Aguiar and Drumond 2009 and Covas and Fujita 2010). *iii)* The risk weight channel, determined by the bank capital-loan ratio, which in turn is driven by the cyclical behaviour of the probability of default. The latter channel is evident in the Foundation IRB approach of Basel II (and Basel III) while the first two channels prevail regardless of the regulatory regime.

Overall, the results suggest that the endogenous default probabilities for firms and bank capital produce an accelerator mechanism in the model, and impact the loan rate through multiple channels. Furthermore, because the loan rate links to the real economy through the borrowing cost channel, all the channels associated with the changes in the probability of default (as mentioned above) also affect real sector dynamics. Hence, risk and bank capital in this model contribute to the standard borrowing cost channel described in Ravenna and Walsh (2006). Finally, the model is simulated following supply, demand and financial shocks, with financial shocks inducing the greatest degree of procyclicality in the financial system.

Chapter 2 can provide an important benchmark theoretical model to examine the role of countercyclical bank capital regulation and monetary policy in promoting financial and macroeconomic stability. For example, it could be assumed that the bank capital-loan ratio not only depends on the bank capital adequacy ratio and the risk of default but also on a financial stability indicator such as the loan to GDP ratio, which the Basel Committee on Banking Supervision views as a good indicator for systemic risk. That is, bank capital requirements should be tightened in good times, when the loan to GDP ratio is high, and loosened during recessions when the loan-GDP ratio drops considerably. In the context of the experiments

conducted in the second chapter, in periods of economic recessions accompanied with a deterioration in lending activities, a countercyclical rule loosens bank capital requirements such that the cost of credit falls. The muted response of the loan rate will result in loans falling by a smaller magnitude, as well as inflation rising by less (through the borrowing cost channel). Hence, a macroprudential policy rule, which targets directly bank capital requirements, may mitigate the procyclical effects of the financial system and real economy.

Can monetary policy also promote macroeconomic and financial stability if central banks target a financial indicator (such as credit growth or the credit to GDP ratio), in addition to inflation and the output gap? Suppose again in the context of the second chapter that the central bank sets its policy rate (given by the Taylor rule) also in part to "lean against the credit cycle", and specifically to deviations of the loan-output ratio. To illustrate, during a recession period, the probability of default and loan rate rise, both which further exacerbate the fall in lending to firms and aggravate the drop in the loan-output ratio. Consequently, the policy rate falls and mitigates the initial rise in the lending rate, thereby dampening the decline in credit and the rise in inflation (via the borrowing cost channel), and consumption (via intertemporal substitution). In other words, an augmented Taylor rule may act to reduce the procyclicality effects inherent in the financial system and real economy. At the same time, if lending is a relatively volatile variable, as observed both in data and in Chapter 2, a strong reaction to the loan-output ratio can result in a further drop in the policy rate, which, through intertemporal substitution, can *increase* output beyond its steady state level following adverse shocks, but at the expense of *higher* volatility in this variable.

Therefore, combining macroprudential policies (a credit augmented Taylor rule for example) with countercyclical bank capital regulation as proposed by Basel III, and examining the possible trade-offs between macroeconomic and financial stability would be an important contribution for this stream of research. This is one of the directions in which we are currently extending the second chapter.

Chapter 3 of this thesis is, as far as we know, the first attempt to model the use of loan loss provisions as a macroprudential tool to mitigate financial and macroeconomic volatility in a DSGE model. The third chapter integrates elements from the DSGE framework developed in Agénor, Alper and Pereira da Silva (2013) and the Bouvatier and Lepetit (2012) model to address the effectiveness of various types of loan loss provisioning rules in mitigating procyclicality. The distinction in loan loss provisioning rules is made between backward- and forward-looking provisioning systems. In the former, provisions are triggered by past due payments, which,

in turn, depend on current economic conditions and the loan loss reserves-loan ratio. Forward-looking provisioning, by contrast, take into account both past due payments and expected losses over the whole business cycle; provisions are thus smoothed over the cycle and are less affected by the current state of the economy and past due payments.

The solution of the model shows that loan loss provisions affect the financial system and the real economy through various transmission channels. First, loan loss reserves (determined directly by loan loss provisions) can either raise or lower the fraction of non-performing loans in the model, consequently affecting the degree of cyclicity of the loan rate. This channel is referred to as the risk premium channel. Second, a provisioning cost channel is identified, in which loan loss provisions directly result in changes in the lending rate. Finally, a general equilibrium channel of loan loss provisions, arising from the money market equilibrium, affects the bond rate, which therefore impacts consumption and real sector dynamics.

Numerical experiments, associated with different types of financial shocks, illustrate that holding more loan loss provisions can reduce financial sector procyclicality by inducing a mitigating effect on the loan rate and the fraction of nonperforming loans. In addition, a forward-looking loan loss provisioning system can increase or lower procyclicality, depending on whether holding more loan loss reserves (provisions) translates into a higher or lower fraction of nonperforming loans. These results have useful implications for the ongoing debate on the performance of loan loss provisioning systems and more generally macroprudential rules.

As well as the important extensions discussed earlier in the context of Chapter 2, Chapter 3 also raises a number of interesting policy questions and avenues of future research. For example, a natural extension would be to study the interactions between loan loss provisions, countercyclical bank capital regulation and a "lean against the credit cycle" type of monetary policy in a model which incorporates endogenous credit and bank default. Such a framework should ideally be able to answer the following questions; Does the introduction of loan loss provisions alter the effectiveness of countercyclical regulation and monetary policy? Are loan loss provisions substitutes for bank capital buffers? Can loan loss provisions reduce balance sheet vulnerabilities and reduce the risk of default? What is the optimal combination of loan loss provisions and macroprudential regulation? What are the welfare implications for optimal monetary policy when loan loss provisioning rules and countercyclical regulations are at use? We are currently attempting to address some of these questions in a model which combines elements from Chapter 2 and Chapter 3 of this thesis.

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