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Life cycle assessment and multiobjective optimisation

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Abstract

Life cycle assessment (LCA) is a method to identify and quantify the environmental performance of a process or a product from "cradle to grave". Its main potential in environmental decision-making lies in providing a quantitative basis for assessing potential improvements in environmental performance of a system throughout the life cycle. This paper introduces the use of multiobjective system optimisation in LCA as a tool for identifying and evaluating the best possible options for environmental management of the product system. A life cycle of a system is optimised on a number of environmental objective functions, defined in terms of the usual LCA burden or impact categories, and a range of environmental optima is found on the Pareto or non-inferior surface. As a result, possibilities for improving the environmental performance of the system are identified. Since system improvements cannot be carried out on the basis of environmental LCA only, it is also shown in this paper that the compromise between environmental and economic performance can be found on the non-inferior surface. The value of multiobjective optimisation in system analysis lies in providing a set of alternative options for system improvements rather than a single prescriptive solution, thus enabling the choice of the Best Practicable Environmental Option (BPEO) and Best Available Technique Not Entailing Excessive Cost (BATNEEC). This approach is illustrated by application to a real case study of a system producing five borate products. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Life Cycle Assessment; Multiobjective optimisation; Pareto optimum; System analysis; Boron products

Nomenclature

$a_{j,i}$	Input/output coefficient for a process
$bc_{m,i}$	Environmental burden coefficient
B_m	Environmental burden
C	Cost objective function
C^*	Optimum value of the cost function obtained by single objective optimisation
$ec_{k,m}$	Environmental impact coefficient
e_j	Right-hand side coefficient in a constraint (Eq. (2))
E_k	Environmental impact
GWP	Global warming potential objective function
GWP^*	Optimum value of the Global Warming Potential objective function obtained in single objective optimisation
P	Total production objective function
P^*	Optimum value of the total Production objective function obtained in single objective optimisation
p_n	Production level of product *n* defined by the mass output in one year
x_i	Output of activity (operation level of process)
Z	(Economic) Objective function
z_i	Coefficients in economic objective function

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1. Introduction

Life cycle assessment (LCA) is a method to define and reduce the environmental burdens from a product, process or activity by identifying and quantifying energy and materials usage and waste discharges, assessing the impacts of the wastes on the environment and evaluating opportunities for environmental improvements over the whole life cycle [1,2]. Because of its holistic approach to system analysis, LCA is becoming an increasingly important decision-making tool in environmental management. Its main advantage over other, site-specific, methods for environmental analysis, such as Environmental Impact Assessment and Environmental Audit, lies in broadening the system boundaries to include all burdens and impacts in the life cycle of a product or a process, and not focusing on the emissions and wastes generated by the plant or manufacturing site only. As an environmental management tool, LCA has two main objectives. The first is to quantify and evaluate the environmental performance of a product or a process from "cradle to grave" and thus help decision-makers to choose between alternative products and processes. Another objective of LCA is to provide a basis for assessing potential improvements in the environmental performance of a product system. The importance of the latter objective can be twofold, depending on the goal of the LCA study. If LCA is performed in order to compare supply and demand patterns or alternative processes in a system, it can help identify the best possible choices in this respect. However, if LCA is performed for a specific process or a product, then the objective is to inform process and design engineers on how to modify a system to decrease its overall environmental impacts.

This paper illustrates how improvements in the environmental performance of a system can be achieved in the optimum way by using system optimisation. In addition, it is argued that decisions are not made on the basis of environmental performance only and that other factors such as technical and economic, play an important role in decision-making process. Therefore, this paper also demonstrates that the environmental and economic performance of the system can be optimised together to find the best compromise solution for the improvements in the system. Thus, incorporation of life cycle thinking into the design and optimisation procedures establishes a link between the environmental impacts, operation and economics of the system. The methodological developments in this respect are still underway and the published literature on this subject is limited (e.g. [3,4]).

Because of the nature of the decision-making process in the LCA context, the optimisation problem will inevitably be a multiobjective one and, immediately, one can think of at least a dozen different programming techniques for solving such problems. The selection of a particular method will depend on the problem itself and on the decision-making context. In this work, multiobjective Linear Programming (LP) has been chosen as a particular optimisation tool. Its application in the context of LCA is explained and illustrated on a study of the boron products.

2. Life cycle assessment and multiobjective optimisation

An optimisation problem defined as LP model of a system in general has the form:

Maximise

$$Z = \sum_{i=1}^{I} z_i x_i \tag{1}$$

subject to

$$\sum_{i=1}^{I} a_{j,i} x_i \leq e_j \qquad j = 1,2,\ldots,J \tag{2}$$

and

$$x_i \geq 0 \qquad i = 1,2,\ldots,I \tag{3}$$

where Eq. (1) represents an objective function, usually a measure of economic performance (e.g. profit or cost) and Eqs. (2) and (3) are linear constraints in the system, describing material and energy balance relationships, productive capacity, raw material availabilities, quality requirements, market demand and so on. The variables x_i represent quantitative measures of material and energy flows including inputs, flows within the economic system, and outputs. In LP these variables are termed "activities".

In the context of LCA, the general LP model has the same form [5]. However, in LCA, the constraints (2) encompass all activities from extraction of primary materials from earth through processing to final disposal. In addition, the functional output or outputs are also treated as activities. Furthermore, the objective functions include the environmental burdens as well as an economic function, as represented by:

Minimise

$$B_m = \sum_{i=1}^{I} bc_{m,i} x_i \tag{4}$$

where $bc_{m,i}$ is burden m from process or activity x_i. The objective functions can also be defined as the environmental impacts:

Minimise

$$E_k = \sum_{m=1}^{M} ec_{k,m} B_m \tag{5}$$

where $ec_{k,m}$ represents the relative contribution of burden B_m to impact E_k, as defined by the "problem oriented" approach to Impact Assessment [6].

At present, LCA is based on linear models of human economic activities and the environment; i.e. environmental burdens and impacts are assumed to be directly proportional to the number of functional units produced [6,7]. This can, in mathematical terms, be expressed by Eqs. (2)–(5) so that it is appropriate to use LP in LCA. Furthermore, the LCA system model which relates the burdens and impacts to the outputs is based on physical and technical relationships. These relationships, which form part of the LP model, include material and thermal balances as well as descriptions of the technical performance of the units and operations in the system. Moreover, modelling a system through LP allows for complex interactions between different parts of a system and so closely describes the behaviour of the system. The value of LP in the Inventory and Impact assessment stages of LCA has already been demonstrated by the authors [3,5,8–12]. This paper illustrates the application of multiobjective LP to analysing and managing the environmental performance of a system, as a part of the Improvement (or Interpretation) stage in LCA.

Depending on the goal of the study, the LCA system expressed in terms of a multiobjective optimisation model can be optimised simultaneously either on environmental burden [Eq. (4)] or impact [Eq. (5)] objective functions. As a result, a range of environmental optima, which define the multidimensional Pareto or non-inferior surface, is obtained. Hence, local and global system improvements are found by first moving the system to conditions on the non-inferior surface, and then moving along the surface. By definition, the Pareto or non-inferior surface is optimal in the sense that none of the objective functions can be improved without worsening the value of some other objective function (see Appendix A). Therefore, trade-offs between objective functions are necessary in order to select the best compromise solution. For example, if CO_2 and SO_2 emissions are optimised simultaneously, the resulting Pareto optimum does not necessarily mean that these parameters are at their minima obtained when the system is optimised on each of them separately. The Pareto optimum, however, does mean that the set of best possible options has been identified for a system in which both emissions should be reduced. Therefore, the value of multiobjective optimisation (MO) in LCA lies in offering a range of alternative solutions; they are all optimal in the Pareto sense, but the choice of the best one will depend on the preferences and constraints imposed on decision-makers.

Multiobjective optimisation on environmental objective functions generates "environmental" optimum solutions and so identifies places in the life cycle of a system where improvements can be made. Hence, MO serves as a tool for managing and improving the environmental performance of product systems. However, environmental improvements in the system are usually directly linked to its economic performance, i.e. there are costs associated with these improvements. It is, therefore, important to establish the trade-offs between cost and environmental objectives by optimising the system on both economic and environmental performance. In general, the economic objective function is related to profit or cost, and can be expressed by Eq. (1). Optimisation on the full set of objectives defined by Eqs. (1) and (4) or Eq. (5) yields a non-inferior surface with a range of optimum solutions which represent the compromise between the objectives of economic and environmental performance. This is of particular importance for the process industries, which face the problem of having to keep production costs down while at the same time complying with environmental legislation.

The application of MO to the Improvement stage of LCA, with emphasis on the approach rather than computational technique, is now illustrated on a real case study of a system producing five borate products.

3. The case study

3.1. System description

The system considered here produces five boron products: 5 mol borate ($Na_2B_4O_7 \cdot 4.67H_2O$), 10 mol borate ($Na_2B_4O_7 \cdot 10H_2O$), boric acid ($H_3BO_3$), anhydrous borax ($Na_2B_4O_7$), and anhydrous boric acid (B_2O_3). A simplified flow diagram of the process is shown in Fig. 1. Further details are given by Azapagic [3] and Azapagic and Clift [12]. Boron minerals, borax ($Na_2B_4O_7 \cdot 10H_2O$) and kernite ($Na_2B_4O_7 \cdot 4H_2O$) are extracted in the mine, crushed and transported to the adjacent plant. 5 and 10 mol borates are produced by dissolving borax and kernite in water; Na–borates are then separated from insolubles, crystallised and dried to produce powder products. Boric acid (BA) is produced in a separate plant, by reacting kernite ore with sulphuric acid; the rest of the process is similar to the 5 and 10 mol production. Anhydrous borax (AB) and anhydrous boric acid (ABA) are made in high-temperature furnaces from 5 mol borate and BA, respectively. All products are then either packed or shipped in bulk. Electric energy and the steam for the

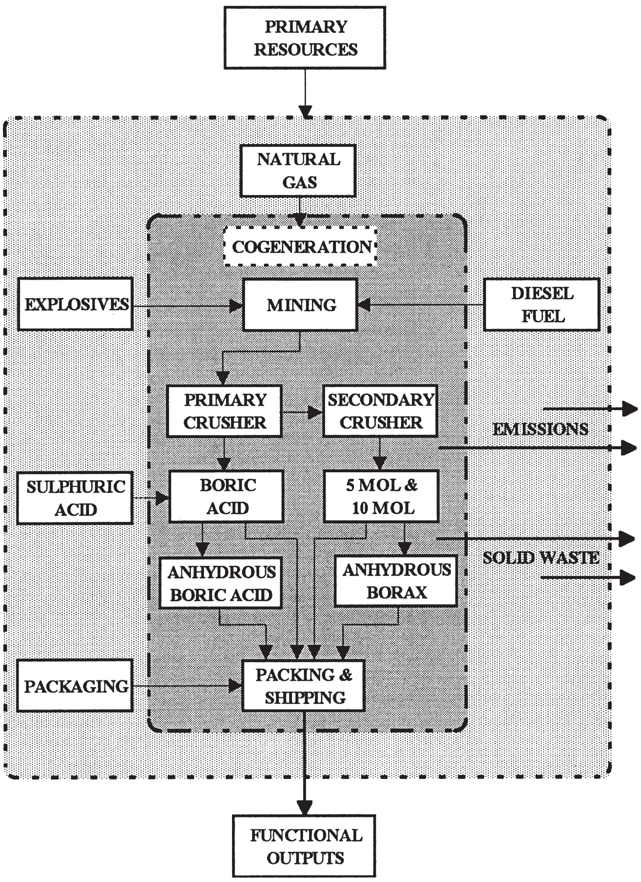

Fig. 1. LCA flow diagram of the boron products system.

system are provided by the on-site natural gas cogeneration facility, which meets all of the electricity and most of the steam demand. The additional steam is, if necessary, provided by the steam plant which is also fired by natural gas. The overburden from the mine and the gangue from the process are stocked in piles; the waste water from the refinery is discharged into self-contained ponds. All activities, from extraction of raw materials to the production of the boron products and materials used, are included in the system described by the model summarised here. However, the use and disposal phases of the products are not considered making this essentially a "cradle-to-gate" study.

Because the study is carried out for internal use within the Company, the functional unit is defined as "operation of the system for one year" corresponding to the total yearly productions of the boron products. The results of LCA, showing the contribution of different products and related processes in the system to the total environmental impacts are published elsewhere [3,5,12]. Since this is clearly a multiple-function system, it also represents an interesting case study for allocation [3,5,8,11].

3.2. Multiobjective optimisation model

The main objective of performing a LCA of the system described above was to identify the possibilities for minimising total environmental burdens and impacts from the system, while maximising production subject to the product demand, and keeping the production costs at a minimum. The objective functions of the LP model, therefore, include environmental burdens and impacts, production costs and total production. The system is described by material balances, subject to products demand and raw material supply, as defined by Eq. (2). Thirty-four environmental burdens and seven environmental impacts identified and quantified in the Inventory and Impact Assessment stages, are defined as objective functions [Eqs. (4) and (5)]. Theoretically, the system can be optimised on all of them. In practice, this is usually not necessary because the impact objective functions represent the aggregated environmental burdens so that their optimisation will also optimise the burdens. The system considered in this paper is, therefore, optimised only on the environmental impact objective functions, defined by Eq. (5) and on the economic objective function, defined as the production costs and given by Eq. (1). Since the definition of the cost function depends on the purpose of the study, which in this case is for internal use within the Company, Eq. (1) includes only operating costs. However, if the study is to be used externally, perhaps to compare the boron products with the alternatives, the full life cycle costing can be included in the model.

In addition, the system is also optimised on total production, defined by the mass outputs of each product in one year:

Maximise

$$P = \sum_{n=1}^{N} p_n \tag{6}$$

The constraint method has been used for solving this MO problem (see Appendix B). The system is first optimised on each objective to identify the feasible region and other functions are ignored. One of the functions is then arbitrarily chosen as an objective function and all other objectives are converted to constraints. A number of optimisations, in which the right-hand sides of the objectives-constraints are varied within the feasible region, are then performed to yield a range of non-inferior solutions.

The results of the individual optimisations on functions (1), (5) and (6), compared to the existing operation of the system, are shown graphically in Fig. 2. The system is optimised first on total production, and the values of all other functions are calculated for the optimum pro-

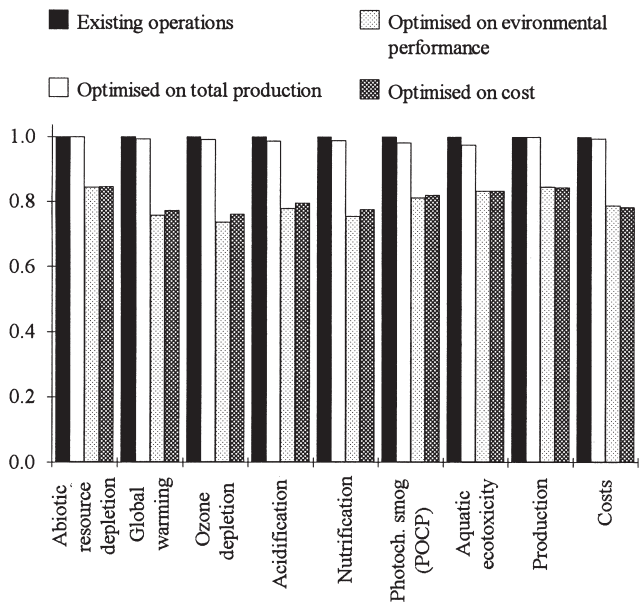

Fig. 2. Comparison of results of individual optimisations.

duction solution. This procedure is repeated for the environmental objectives and the costs, in turn. Optimisation on total production decreases the environmental impacts by 1–2.5% in comparison to the existing operations while the total production increases by 1%. Although these results represent an improvement, they are not significantly different from the existing situation. This outcome is hardly surprising because a real plant is normally operated around the optimum production conditions. These results also directly confirm the validity of the model.

On the other hand, when optimisation on environmental impacts [Eq. (5)] is performed, the impacts and the costs are reduced by, on average, 20% while total production is reduced by 15%. These results also showed that optimisation on one environmental objective optimises all other impacts. The reason for this is that, at the environmental optimum solution, only products with the least impacts are produced and these are 5 mol, 10 mol and boric acid for all impact categories (see Table 1). Furthermore, optimisation on the cost function gives almost the same results as the environmental optimisation because of the properties of this system—the products with the least environmental impacts have the least production costs. Again, this could be expected, because most of the environmental impacts from the process are related to energy consumption, which constitutes the main component of production costs in this system.

MO has, therefore, been performed on three objectives: total production (*P*), costs (*C*) and Global Warming Potential (*GWP*). The latter is arbitrarily chosen for the environmental optimisation since, as already explained, optimisation on one impact function optimises the values of the others for this particular system. The non-inferior curves, showing trade-offs between cost and *GWP* functions, for constant values of the production, are shown in Fig. 3. To preserve the confidentiality of the data, the optimum values of the objectives have been normalised by dividing them by the optimum values obtained in the single objective optimisations, C^* and GWP^*. The figure shows that the cost objective function does not change significantly with

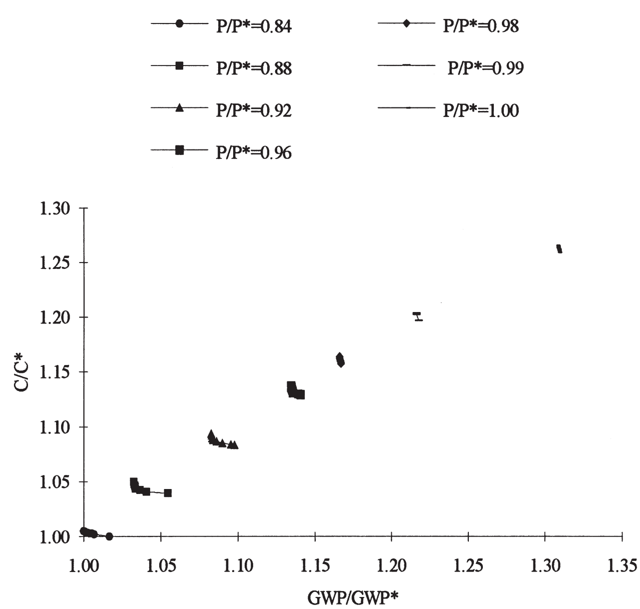

Fig. 3. Non-inferior curves for costs and *GWP* for constant total production rate.

Table 1
Total production for single optimisation problems (index)

Product	Existing operations	Optimised on: total production	Optimised on: environmental impacts	Optimised on: cost
5 mol	100	101.0	89.1	90.4
10 mol	100	102.2	48.0	30.8
Anhydrous borax	100	106.2	0.0	0.0
Anhydrous boric acid	100	98.9	0.0	0.0
Boric acid	100	102.1	93.5	93.5
Total	100	101.0	84.8	84.5

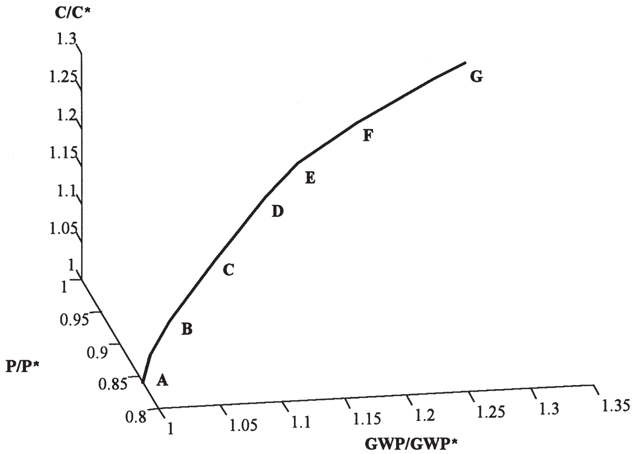

Fig. 4. Non-inferior curve for multiobjective optimisation.

GWP. At the optimum value of the cost objective, *GWP* increases by 1.6% from its minimum; when *GWP* is at the optimum, the costs increase only by 0.5%, relative to the minimum value. The cost objective function can, therefore, be ignored, because optimisation on *GWP* generates solutions that can be approximated as optimal with respect to production costs.

The non-inferior curve obtained in the optimisation on *GWP* and total production objectives is given in Fig. 4. Selected solutions from the non-inferior curve are shown in Fig. 5. Points A and G represent optimum values of *GWP* and *P*, respectively, obtained in the single optimisation problems; all other points are calculated by multiobjective optimisation. At point A, *GWP* is at its minimum, but so is the production (Fig. 5). By moving away from A along the non-inferior curve, both *GWP* and total production increase; at point G, production is at its maximum and the value of *GWP* increases by 24% relative to the solution at point A. The other environmental

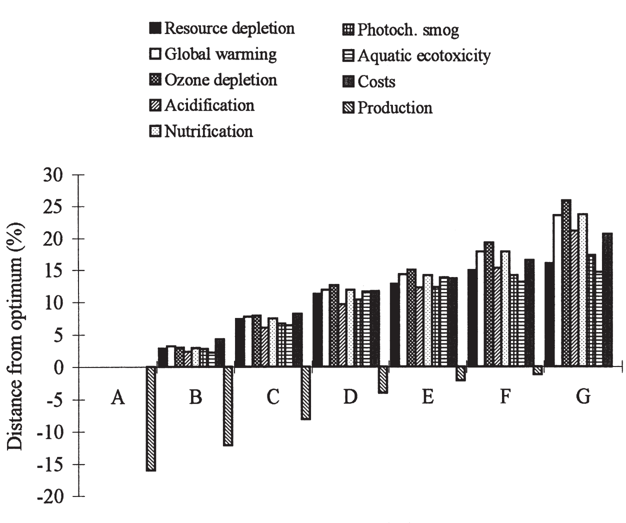

Fig. 5. Graphical presentation of selected non-inferior solutions.

impacts and the costs increase from A to G by, on average, 20%. The environmental impacts at solution B are only 2.5% above the optimum, but the production is 12% below its maximum and the costs increase by 5% (Fig. 5). The Best Practicable Environmental Option (BPEO) at this solution appears to be the maximum production of 10 mol, 90% of 5 mol and 95% of BA, while AB and ABA are not produced (Table 2). At solution C, all objectives are away from their optima by, approximately, 7.5%; the BPEO is similar to the solution at B, except that the production of 5 mol is now only 7% away from its maximum. Solutions at D and E increase the environmental impacts and the costs by 12.5% and 15%, respectively, with production only 4% and 2% less than the maximum value. The solution at F enables almost optimum total production but the environmental impacts increase by 15% for resource depletion, up to 20% for ozone depletion. The costs are 16.5% higher than the optimum. The BPEO is represented by the maximum production of 5 and 10 mol borates and BA, 57% of AB and no production of ABA.

4. Discussion

Obviously, all points on the non-inferior curve in Fig. 4 are optimal in the Pareto sense, and decision-makers can select any solution from A to G, depending on how much of one objective they are prepared to give up in order to gain in another. If all objectives are considered to be of equal importance, then one of the possible ways to choose the best compromise solution is to identify operating conditions at which all objectives differ from their optima by the same percentage. In that case it would be the solution at point C. However, should some objectives be considered more important than the others, any other solution on the non-inferior curve can be chosen as the best compromise. The value of MO in the context of LCA, therefore, lies in offering a range of choices for environmental improvements of the system and so enabling preferences to be identified after analysing all the trade-offs among objectives. Although the choice of the best compromise solution will still imply certain preferences and value judgements, at least the choice will be made from all possible non-inferior solutions. This distinguishes the MO approaches from, for example, the multiattribute utility function method where, due to the way the utility function is assessed, the bulk of non-inferior solutions can be ignored.

Furthermore, optimisation methods can be applied in a wide range of decision-making contexts. In the case of single decision-makers, where there is a group of decision-makers that share the same interests and preferences about the conflicting objectives of a multiobjective problem, the optimisation methods provide information on the trade-offs between different objectives, to show

Table 2
Total production for selected non-inferior solutions (index)

Product	Solution						
	A	B	C	D	E	F	G
5 mol	88.2	87.7	93.0	98.3	100.0	100.0	100.0
10 mol	47.0	100.0	100.0	100.0	100.0	100.0	100.0
Anhydrous borax	0.0	0.0	0.0	0.0	0.0	57.1	100.0
Anhydrous boric acid	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Boric acid	95.5	95.5	95.5	95.5	100.0	100.0	100.0
Total	84.2	87.9	91.9	96.0	97.9	98.8	100.0

explicitly what can be gained and what lost by choosing each alternative. Where there are multiple decision-makers with conflicting interests, this technique can still help to resolve disputes by generating different alternative solutions. Decision makers who understand the trade-offs and the alternatives are more likely to understand the interests of other parties and, therefore, to compromise.

A further reason for choosing this approach is that objectives do not have to be aggregated into a single objective, as is the case with methods which aggregate individual preferences. This is particularly relevant in the LCA context, because it avoids the controversial and debatable concept of aggregation of environmental impacts into a single environmental impact function in the Valuation stage. By being able to trade-off incommensurable objectives, e.g. environmental impacts and economic requirements, this approach avoids the well known problems encountered, for instance, in cost–benefit analysis (e.g. [13]), i.e. reducing individual preferences to a market value or trying to express quality of the environment in financial terms. Thus, the multiobjective approach can also help identify BPEO and Best Available Techniques Not Entailing Excessive Cost (BATNEEC).

5. Conclusions

Multiobjective system optimisation can successfully be combined with LCA as an aid in environmental management of a product system. The proposed approach is illustrated by a real case study of a system producing boron products. The system is simultaneously optimised on a number of environmental objective functions to identify the best compromise solution for improving the system's performance. The results show that the environmental performance of the system can be improved by up to 20% in comparison to the existing operations. Since system improvements cannot be carried out on the basis of environmental LCA only, it is also shown in this paper that the compromise between environmental and economic performance can be found on the non-inferior surface obtained by system optimisation. The advantage of multiobjective optimisation in environmental system management in the context of LCA lies in offering a set of alternative options for system improvements rather than a single optimum solution, and so enables the choice of the BPEO and BATNEEC.

Appendix A

Pareto analysis

Pareto analysis [14] is mainly associated with the "new welfare" economics, which was concerned with the general problem: how should resources be allocated for the production and consumption of goods so as to maximise social welfare? As opposed to "welfare economics" which defined social welfare as a summation of the "utility" or "pleasure" of each individual, the teachings of "new welfare" economics introduced the indifference curve, on which different combinations of social states yield the same level of utility, and Pareto's "optimality" condition was formulated. There are still various interpretations of Pareto's thought, but there is consensus as to what constitutes a Pareto optimum: a social state is Pareto optimal if no individual can be made better off without making at least one other individual worse off. In other words, if such a state is reached it is not possible to increase the utility of some individuals or groups without diminishing that of others. Pareto also argued that interpersonal comparisons of individual utilities could not be made and that maximum utility of a community was not the simple summing of the single individuals' utilities, as the original welfare economists believed.

A Pareto optimum curve, represented by the social–welfare function and related to the utilities U_1 and U_2 of a two-individual society, is shown in Fig. 6. All points on the curve are Pareto optimal since more of individual 2's utility U_2 can be gained only by sacrificing some of the utility U_1, e.g. by moving from B to C in the figure. Point A, below the curve, is not a Pareto optimal social state since both individuals can be made better off by

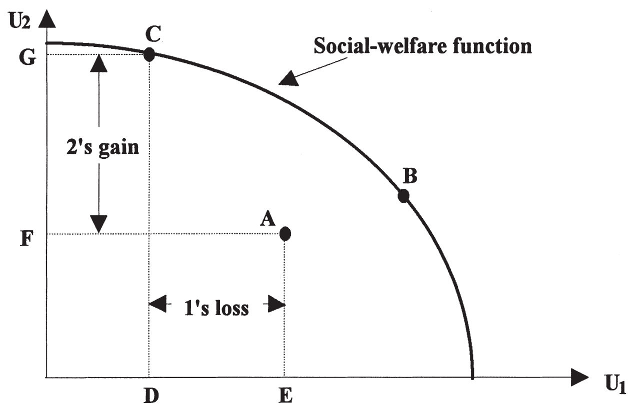

Fig. 6. A welfare frontier for a two-individual society.

moving to state B. In general, there is a continuum of Pareto-optimal social states, and no individual state can be considered better than any other in the absence of further value judgements. Pareto recognised here that when the optimum is reached, movements along the Pareto curve involve resorting to considerations foreign to economics, in order to "decide on grounds of ethics, social utility, or something else, which individuals it is advisable to benefit, which to sacrifice" [15].

The practical use for the concept of Pareto optimality was in evaluation of a movement from a present inferior social state to a new Pareto optimal one. However, some cases, such as movement from A, which is not Pareto optimal, to C, which is, cannot be evaluated in a strict sense since the utilities of the two individuals are not both increased.

Appendix B

The constraint method in multiobjective linear programming

In the constraint method, all objectives but one are converted into the constraints and the problem is then optimised on one objective function only in order to generate the non-inferior solutions. The values of the constrained objectives are varied systematically until all non-inferior solutions are generated. In general, a multiobjective problem with Q objectives can be expressed as:

Maximise

$$F(\mathbf{x}) = [F_1(\mathbf{x}),F_2(\mathbf{x}),\ldots,F_Q(\mathbf{x})] \tag{A2.1}$$

subject to

$$f_j(\mathbf{x}) \leq e_j,\ j = 1,2,\ldots,J \tag{A2.2}$$

and

$$\mathbf{x} \in \mathbf{X} \tag{A2.3}$$

where $f_j(\mathbf{x})$ includes the non-negativity restriction, $\mathbf{x}$ is the I-dimensional vector of decision variables and $\mathbf{X}$ is the feasible decision region. In the constraint method, the problem is transformed into:

Maximise

$$F_h(\mathbf{x}) \tag{A2.4}$$

subject to

$$f_j(\mathbf{x}) \leq e_j,\ j = 1,2,\ldots,J \tag{A2.5}$$

and

$$F_q(\mathbf{x}) \geq \epsilon_q,\ q = 1,2,\ldots,h-1,h+1,\ldots,Q \tag{A2.6}$$

where the hth objective is arbitrarily chosen for maximisation, and all other objective functions of the problem are converted into constraints. In other words, the multiobjective linear programming problem is transformed into a single objective problem, which can be solved by using, for instance, the simplex method for linear problems.

Algorithm for the constraint method [16]

Step 1: Pay-off table

1. Solve Q single-objective optimisation problems to find the optimal solution for each of the Q objectives. Optimal solution for the qth objective is denoted as $\mathbf{x}^q = (x_1^q,x_2^q,\ldots,x_I^q)$.

2. Compute the value of each objective at each of the Q optimal solutions: $F_1(\mathbf{x}^q)$, $F_2(\mathbf{x}^q)$, …, $F_Q(\mathbf{x}^q)$, $q = 1, 2, \ldots, Q$. This gives Q values for each of the Q objectives.

3. Construct a payoff table with rows corresponding to $\mathbf{x}^1$, $\mathbf{x}^2$, …, $\mathbf{x}^Q$ and the columns equal to the number of objectives (Table 3).

4. Identify the largest and the smallest numbers in the qth column and denote them by M_q and n_q, respectively. Repeat for $q = 1, 2, \ldots, Q$.

Step 2: Constraints

Convert a MOLP problem, such as (A2.1–3), to its corresponding constrained problem (A2.4–6).

Step 3: Right-hand side coefficients

The M_q and n_q represent the upper and lower bounds for the qth objective: $n_q \leq \epsilon_q \leq M_q$. Choose the number of different values of ϵ_q and denote it by r.

Step 4: Optimisation

To generate a range of non-inferior solutions, solve the constrained problem in Step 2 for every combination of values for the ϵ_q, $q = 1, 2, \ldots, h-1, h+1, \ldots, Q$, where:

Table 3
Pay-off table for a multiobjective problem

	$F_1(\mathbf{x}^q)$	$F_2(\mathbf{x}^q)$	…	$F_q(\mathbf{x}^q)$
$\mathbf{x}^1$	$F_1(\mathbf{x}^1)$	$F_2(\mathbf{x}^1)$	…	$F_Q(\mathbf{x}^1)$
$\mathbf{x}^2$	$F_1(\mathbf{x}^2)$	$F_2(\mathbf{x}^2)$	…	$F_Q(\mathbf{x}^2)$
…	…	…	…	…
$\mathbf{x}^Q$	$F_1(\mathbf{x}^Q)$	$F_2(\mathbf{x}^Q)$	…	$F_Q(\mathbf{x}^Q)$

$$\epsilon_q = n_q + [t/(r-1)](M_q - n_q), \quad t = 0,1,2,\ldots,(r-1). \tag{A2.7}$$

Nomenclature for Appendix B

M_q	Maximum value of the qth objective function
n_q	Minimum value of the qth objective function
r	Number of different values of ϵ_q
t	Number of multiobjective optimisations
$\mathbf{X}$	Feasible decision region
$\mathbf{x}^q$	Optimal solution of the qth objective
$\mathbf{x}$	I-dimensional vector of decision variables
ϵ_q	Right-hand side of the constrained objective in multiobjective optimisation

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