A negative refractive index metamaterial wave plate for millimetre wave applications

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ABSTRACT

By use of a metamaterial based on the ‘cut wire pair’ geometry, highly birefringent wave plates may be constructed by virtue of the geometry’s ability of having a negative and positive refractive index along its perpendicular axes. Past implementations have been narrow band in nature due to the reliance on producing a resonance to achieve a negative refractive index band and the steep gradient in the phase difference that results. In this paper we attempt to design and manufacture a W-band quarter wave plate embedded in polypropylene that applies the Pancharatnam method to increase the useable bandwidth. Our modelling demonstrates that a broadening of the phase difference’s bandwidth defined as the region 90° ± 2° is possible from 0.6% (101.7 GHz – 102.3 GHz) to 7.8% (86.2 GHz – 93.1 GHz). Our experimental results show some agreement with our modelling but differ at higher frequencies.

Keywords: Negative refractive index, wave plate, Pancharatnam, metal mesh, metamaterial

1. INTRODUCTION

The use of electromagnetic metamaterials to produce negative refractive indices (NRI) is well known and many proposed uses ranging from cloaking devices to “perfect lenses” have been suggested and tested. These applications generally require the metamaterial to be isotropic and have polarisation independent behaviour. However, one case where polarisation dependent i.e. birefringent behaviour is advantageous is in the manufacture of wave plates. Half Wave Plates (HWPs) are normally used to rotate the orientation of linear polarised radiation whereas Quarter Wave Plates (QWPs) convert linear polarised light into circularly polarised light and vice versa. Rotating HWPs find application in Cosmic Microwave Background Radiation (CMBR) experiments where they are used to aid the detection of weak polarised radiation in the CMBR against the much stronger, but generally unpolarised foreground radiation. Polarised radiation from the CMBR is classed into two groups: E-modes and B-modes and it is the detection of the weaker B-modes that has become the next goal in observational cosmology.

Regular birefringent materials such as sapphire [1] as well as metal meshes [2] have been used to create wave plates in the past. The latter have the advantage of being able to be manufactured to larger diameters, are lighter in weight and can be made to work at different frequencies simply by scaling their design.

The phase difference, \( \Delta \phi \), produced by a birefringent wave plate of thickness, \( d \), at a frequency, \( f \), is given by

\[
\Delta \phi = \frac{2 \pi df}{c_0} \cdot \Delta n
\]

where \( c_0 \) is the speed of light in a vacuum and \( \Delta n \) is the difference between the refractive index values along the two orthogonal axes lying across the plate.

By designing a metamaterial with a positive refractive index in one axis and a negative refractive index in the other a wave plate with a birefringence greater than one can be created. These values are much higher than those achievable by natural birefringent materials available at millimetre wavelength, which are generally of the order of ~0.1. By increasing the birefringence, wave plates with sub-wavelength thickness can be created. Attempts at applying NRI to wave plates for this purpose were made by [3] and [4]. The former used crosses to have a NRI in both axes whilst the latter used a ‘cut wire pair’ (CWP) geometry that had NRI in one axis and positive in the other. In both cases, the resulting bandwidths, defined as the region where \( \Delta \phi \) was 90° ± 2°, were narrow, around 0.9% and 0.5% respectively. This was
due to the resonance required to create the NRI band and the steep gradient of $\Delta \phi$ that resulted. We have previously shown [5] that by using the Pancharatnam method [6], it is possible to extend the usable bandwidth of a NRI-HWP despite the steep gradient of $\Delta \phi$ of the single HWPs constituting it. The Pancharatnam method increases the bandwidth of a single plate by cascading different plates with their birefringent axes rotated by specific angles.

In this paper we aimed to apply the same principle to design a QWP that is fully embedded in polypropylene.

2. UNIT CELL GEOMETRY & MANUFACTURE

The cell design is based on the ‘cut wire pair’ (CWP) geometry that has been used to create a negative refractive index previously [7] and was specifically used to create a QWP in [4] owing to the geometry’s high birefringence. The dimensions of the CWP were optimised to work in the W-band (75 GHz – 110 GHz) and with a polypropylene substrate ($\varepsilon_r = 2.2551, \delta = 0.0007$ at 100 GHz [8]). The final dimensions are shown in the caption to Figure 1.

![Figure 1. Face on view of the CWP unit cell. The darker region is the copper and the lighter region is the polypropylene substrate. The dimensions are: $w = 365 \mu m, h = 980 \mu m, l = 827 \mu m, b = 213 \mu m, g_1 = 76 \mu m$ and $g_2 = 76.5 \mu m$. Each grid pair is separated by 94 $\mu m$ of polypropylene. The copper is 2 $\mu m$ in thickness.](image)

Photolithography was used to create individual copper grids consisting of cut wires (also known as capacitive grids in this form) on a 47 $\mu m$ polypropylene substrate. By careful alignment of two of these grids a single QWP consisting of CWP was made. The requisite separation distance between the CWP plates were created by layering plain polypropylene sheets between them. Finally, bonding of the layers to form the final QWP structure took place via mechanical pressing and heating within a vacuum oven.

This QWP’s structure is fully embedded in polypropylene which has some advantages over the air gap structure of the HWP demonstrated in [5]. Being embedded means that plate as a whole is more physically robust and the individual plates and grids cannot become misaligned once fully made.

3. SIMULATION & MODELLING

The individual cells were designed, modelled and optimised with a commercial Finite Element Method (FEM) solver, HFSS [9]. Initially the cell was optimised so that at 92.5 GHz (the centre of the W-band) a phase difference, $\arg(S_{21}^x) - \arg(S_{21}^y) = 90^\circ$, and a magnitude, $|S_{21}^{xy}|^2 \geq 0.9$, was achieved. As will be explained later in this section, this was later altered so that this region occurred at 102 GHz.

In HFSS the embedded CWP were modelled using periodic boundary conditions, hence simulating an infinite 2D array of CWPs. To optimise the QWP we moved our calculations to transmission line modelling. This was necessary to
simulate the interaction between the rotated plates required by the Pancharatnam method but could not be carried out using periodic boundary conditions. To continue with the finite element approach would have required the modelling of the full size plates made from a 2D array of CWPs which would have been prohibitively costly both in terms of CPU time and RAM consumption.

The CWPs were represented using ABCD (Transmission) parameters [10] obtained by converting the S-parameters of a single embedded CWP from the FEM simulations. Propagations matrices were used to represent the polypropylene gaps and matching matrices were included to take into account the propagation of radiation from air into the polypropylene and vice versa [11]. Implementation of the Pancharatnam method was then carried out in the same way as [12][13], here using a combination of four plates with the outer two being used as QWP whilst the inner two behave as a HWP. Our optimising parameters were the rotation angles of the outer and inner QWPs, $\theta_1$ and $\theta_2$, the thickness of the polypropylene between the air and copper surface of the outer QWPs, $d_1$, the polypropylene thickness between the copper surfaces of the outer and inner QWPs, $d_2$ and the polypropylene thickness between the two inner QWPs, $d_3$. These are shown in Figure 2. Optimisation was carried out targeting the flattest achievable $\Delta \phi \sim 90^\circ$ differential phase-shift response across a frequency region greater than 5% bandwidth whilst having $|S_{21}^{x,y}|^2 \geq 0.8$.

![Figure 2. Left: Side view of the full structure of the QWP with four the CWPs embedded in polypropylene. Right: Face on view showing the orientation of the outermost QWP when the fully constructed QWP is rotated at 0°.](image)

Whilst designing the QWP with four plates it became apparent that the region where $\Delta \phi$ became flat at 90° regularly resided at the lower end of the W-band, well below the NRI region where the single QWP were optimised to have a $\Delta \phi$ of 90°. To widen the useable bandwidth laying within the W-band the design was re-optimised so that a single embedded CWP produced a $\Delta \phi$ of 90° at 102 GHz. Continuing with the new design resulted in the optimised parameters for the four plate QWP coming to $\theta_1 = -45^\circ$, $\theta_2 = 17^\circ$, $d_1 = 629 \mu m$ and $d_2 = d_3 = 1320 \mu m$. This gives our QWP a total thickness of 5.61 mm, just under $2\lambda_0$, where $\lambda_0$ is the free space wavelength.

![Figure 3. Graph of the transmission along the x- and y-axes of a single CWP embedded in polypropylene.](image)
The $|S_{21}|^2$ and $\Delta \phi$ of a single CWP embedded in polypropylene are shown in Figure 3 and Figure 4 respectively. Over the entire W-band the transmission in the x-axis is very high, due to the y-orientated CWP being transparent to x-polarised radiation, showing only a small linear decrease from 0.95 to 0.92 as the frequency increases. The y-axis transmission shows a resonance, peaking at 0.95 at 102.8 GHz. The gradient of $\Delta \phi$ increases as the CWP experiences a resonance for
y-polarised radiation and only briefly passes through 90°. A single embedded CWP has a usable bandwidth, defined as where $\Delta \phi$ is $90^\circ \pm 2^\circ$, of 0.6% between 101.7 GHz and 102.3 GHz. The mean transmission in this region is 0.93 and 0.94 in the x- and y-axes respectively.

CWP’s are able to create a negative refractive index band by simultaneously achieving a negative permittivity and permeability through electric resonance in the cut wires themselves and a current loop in the cut wire pairs respectively. Taking a thickness of 300 $\mu$m we calculate the refractive index [14] of the embedded CWP along the x and y axes (Figure 5 and Figure 6). For radiation polarised in the y-axis our CWP displays a NRI band just outside of the W-band above 112.2 GHz whilst the x-axis shows a constant positive refractive index of 1.3.

4. MEASUREMENTS

![Diagram of experimental setup](image)

Figure 7: Schematic of the experimental setup. The horns are set off axis in such a way as to reduce the effect of standing waves but keep the line connecting their centres perpendicular to the surface of the quarter wave plate.

Measurements were carried using a Rhode & Schwarz ZVA40 Vector Network Analyser (VNA) equipped with W-band converter heads and corrugated horns providing almost pure Gaussian beams. The horn to horn distance was set at 50 cm with the QWP placed midway between them. The horns were rotated off axis but positioned such that the line connecting their centres passed through the QWP perpendicularly to reduce the effect of standing waves. Eccosorb was used to minimise other unwanted reflections. All measurements were normalised to the background readings when the QWP was not present. The x- and y-axis readings were taken separately rotating the plate by $90^\circ$. The orientation of the full QWP for the y-axis reading is shown in Figure 2, where as can be seen the outermost QWP rotated by $\theta_1$.

![Graph of transmission data](image)

Figure 8. Modelled and measured transmission data along the x and y axes of the QWP.
Our measurements are shown with the expected results from our transmission line model in Figure 8 and Figure 9. The transmission values follow the general shape of the expected result at the lower end of the W-band but begin to deviate above 95 GHz. We therefore restrict our analysis to the lower half of the W-band. The measured mean transmission between 75 GHz – 95 GHz is 0.66 and 0.75 in the x- and y-axes respectively, whilst the model predicted a mean of 0.76 and 0.74.

The measured phase difference (Figure 9) matches well with the model between 75 GHz and 87 GHz. However, whereas the model predicts a flattening of $90^\circ \pm 2^\circ$ between 86.2 GHz and 93.1 GHz, increasing the bandwidth of a single embedded CWP by 13 times to 7.8%, our measured data show a deviation. It is evident however that the gradient at which this QWP passes through $90^\circ$ is shallower, passing through $90^\circ \pm 2^\circ$ between about 86.9 GHz – 88.1 GHz (a bandwidth of 1.4%), than that of a single embedded pair shown in Figure 4, showing that the effect of Pancharatnam method is present.

The reasons for the discrepancy may be due to manufacturing problem in the individual CWP as well as the angles at which the plates were rotated by.

5. CONCLUSION

We find that when using the Pancharatnam method to increase the usable bandwidth of a CWP QWP whose high birefringence is aided by axes that have refractive indices of opposite signs, the resultant increase in bandwidth occurs lower in frequency to where a single CWP exhibits a $\Delta \phi$ of $90^\circ$. Using four CWP QWP cascaded one after the other with their optic axes rotated to one another, our modelling demonstrates that it is theoretically possible to extend the region where the phase difference is $90^\circ \pm 2^\circ$ from 0.6% (101.7 GHz – 102.3 GHz) to 7.8% (86.2 GHz – 93.1 GHz). Experimentally, the manufactured QWP shows some agreement with modelling between 75 GHz and 95 GHz with the transmission sustaining on average values of 0.66 and 0.75 for x- and y-polarised radiation. The phase difference also shows good agreement with the modelling, but only between 75 GHz and 87 GHz, beyond which the predicted flattening does not occur. However the gradient at the point it passes through $90^\circ$ is less steep showing that in principle the Pancharatnam method works. Further investigations from the manufacture point of view are required in order to achieve the much higher expected performances.

ACKNOWLEDGEMENTS

The first author is funded by a studentship from the Science and Technology Facilities Council (STFC).
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