Abstract—This paper proposes a new method for optimal placement of Phasor Measurement Units (PMUs) across the weak areas of the power system to monitor the status of the boundary buses during Parallel Power System Restoration (PPSR). The proposed PMU placement methodology is based on a Generalised Integer Linear Programming (GILP) formulation to place PMUs is undertaken using different algorithms. For example, a real-time approach for finding proper system splitting strategies using a three-phase Ordered Binary Decision Diagram (OBDD) method is proposed in [3]. Since this approach takes into consideration only steady-state constraints, an approach considering transient simulations is presented in [4]. In [5], a slow coherency technique is used to group both generators and load buses in a single group considering the closeness of each other. This approach uses the slow eigenbasis theory to collect coherent groups. This methodology determines the closeness of each load bus $j$ to the reference generator $i$. This is obtained by applying the cosine function of the angle between the two row vectors of the eigenbasis matrix corresponding to buses $i$ and $j$.

In [6], an analytical basis for the application of slow coherency method based on two-time scale theory is introduced. It includes a modification of tolerance based slow coherency to create islands in the system by employing an important control strategy to deal with large disturbances. A controlled islanding solution for large power systems verified by dynamic simulations is proposed in [7]. Here a graph representation of the system is used to simplify the structure of the power system. In [8], a method based on the weak coupling concept for identifying groups of slowly coherent generators is presented.

Methodologies for PMUs placement have been mainly proposed for state estimation purposes, i.e. to achieve full network observability during quasi-steady-state operating condition. However, a methodology for PMUs placement to ensure enough information during PPSR is still an unexplored research and practical engineering challenge.

In [9], a technique for identifying placement sites for PMUs in a power system based on incomplete observability is presented. The paper presents a novel concept of depth of unobservability. Initially, it uses the spanning tree of the power system graph and a tree search technique to find the optimal location of PMUs. Then, it extends the modelling to recognise limitations in the availability of communication facilities across the network and pose the constrained placement problem within the framework of simulated annealing. In [10], a Generalised Integer Linear Programming (GILP) formulation to place PMUs is proposed. Different scenarios, including redundant PMU placement, full observability and incomplete observability,
were considered.

In order to solve the problem of the lack of information during PPSR, especially at boundary buses, this paper proposes a PMU placement method based on an ILP formulation across the identified weak areas of the power system. Results for the New England 39-bus test system are firstly presented to validate the proposed solution. In order to validate the new methodology in larger power systems, the results for the IEEE 118-bus test system are presented, as well.

Section II introduces the concept of slow coherency and weak connections. Furthermore, the concept of weak areas, based on the weak connections approach, is defined. Section III presents the ILP formulation to place PMUs across weak areas in the power system in order to have information during the PPSR process. The results of testing the new methodology are presented in Section IV. Finally, in Section V the conclusions drawn from the study are given.

II. SLOW COHERENCY, WEAK CONNECTION AND WEAK AREA

The close relationship between slow coherency and weak connection has been previously stated in [11]. The concept of slow coherency is based on the fact that groups of generators have a tendency to swing together following a disturbance in a multi-machine power system.

Slow coherency analysis solves the problem of theoretically identifying the weakest connections across the power system. These weakest connections are functions of the admittance matrix parameter, machine inertias and the initial rotor angles of the interconnected machines. Therefore, it can be noticed that the boundaries encountered for each island depend upon the inherent structural characteristics of the power system. In addition, these islands must be created considering the availability of intertie lines, a request for the load-generation balance and existence of Blackstart (BS) units within each island.

Based on the singular perturbation form, the slow coherency theory assumes that the state variables of an n-th order system are divided into r slow states and (n-r) fast states, namely y and z, respectively. Where the r slowest states represent r groups with the slow coherency [11]. This can be expressed as follows:

\[
\frac{dy}{dt} = f(y,z,t), \quad y(t_0) = y_0
\]  
\[
\frac{dz}{dt} = G(y,z,t), \quad z(t_0) = z_0
\]

Two assumptions are considered in order to carry out the slow coherency analysis [6]. The first one assumes that the coherent groups of generators are independent of the size of the disturbance; whilst the second considers that the coherent groups are independent of the level of detail used to model the generating units. According to the second assumption, the following simplified classical model of an m-machine power system can be used [12]:

\[
\begin{align*}
\dot{x} &= Ax = (M^{-1}K)x \\
x &= [\Delta \delta_1, \Delta \delta_2, \ldots, \Delta \delta_n]^T \\
A &= M^{-1}K
\end{align*}
\]

\[
M = \text{diag}(2H_1/\omega_1, 2H_2/\omega_2, \ldots, 2H_n/\omega_n)
\]

\[
K_{ij} = \begin{cases} 
-\sum_{l=0, l \neq j}^n K_{ij}, & i \neq j \\
-\sum_{l=0, l \neq i}^n K_{ij}, & i = j
\end{cases}
\]

where \(V_i\) and \(V_j\) are the bus voltage magnitude at bus i and j respectively. \(\delta_i\) is the rotor angle in radians and \(H_i\) is the inertia constant in seconds of the i-th machine; \(G_{ij}\) and \(B_{ij}\) are the real and imaginary entries of the admittance matrix \(Y_{bus}\).

Another concern commonly related to the slow coherency analysis is the weak connection form. In fact, the slow coherency phenomenon occurs in dynamic networks when the connections between areas are weak [11]. A two area system is said to be weakly connected if its dynamic properties can be described by (8):

\[
E \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = E \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + E \begin{bmatrix} A_{11} \ A_{12} \\ A_{21} \ A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\]

where \(x_1\) and \(x_2\) are \(n_1\) and \(n_2\) column vectors, \(A_{11}, A_{12}, A_{21}, A_{22}\) are matrices of order one without any zero entries, \(\tau = (t-t_0)/\kappa\) and \(\kappa\) is a small positive parameter in the slow coherency solution when the external connections are weak or sparse.

In [13], a linear analysis is carried out to prove that by selecting the \(r\) slowest modes, the aggregated system will have the weakest connections between different groups of generators. The weak connection form best states the reason for islanding based on slow coherency grouping. Slow coherency is actually a physical manifestation of a weak connection which is an inherent network characteristic.

In real large scale power systems, there always exist groups of strongly interacting units with weak connections between groups or areas. When a large disturbance occurs, it is imperative to disconnect these weak connections before the slow interaction becomes significant.

Weak connection is a system property which is independent of operating conditions or the degree of modelling. The methodology for system splitting based on weak connections minimises the dynamic coupling between islands according to the following formula:

\[
\min \left( \sum_{y_l \in V_G, y_l \in V_G} \frac{1}{2} \left( \frac{\delta P_{ml}}{\partial \delta} \right) + \frac{1}{2} \left( \frac{\delta P_{ml}}{\partial \theta} \right) \right) \]

where \(y_1\) denotes the dynamic coupling between generators, which is normalised by the inertias and \(y_2\) denotes the dynamic coupling between load buses. In (9), \(V_G^l\) and \(V_G^l\) represent the group of voltages at generator buses within.
islands $l$ and $k$, respectively. $V^l_i$ and $V^k_i$ represent the group of voltages at load buses within islands $l$ and $k$, respectively.

One of the major issues linked to the weak connections theory (9) is the identification of only one possible splitting strategy. To solve this problem and based on the eigenbasis matrix of the power system, this paper defines weak areas as the areas from the weak connections points up to a pre-defined threshold. This methodology provides several options to split the power system into smaller subsystems across these regions. The procedure of identifying the splitting options across the weak areas allows determining different possibilities of splitting strategies across these weak areas before the system reaches the total blackout. The placement of synchronised measurements across the weak areas can be used to ensure sufficient information during PPSR, independently of the splitting strategy perform across these areas.

By computing the eigenbasis matrix, the proposed methodology evaluates the closeness of each bus $j$ with respect to the reference bus $i$. This is obtained by applying the cosine function of the angles between the two row vectors of the eigenbasis matrix corresponding to buses $i$ and $j$. This approach is more clearly explained using the IEEE 9-bus test system shown in Fig. 1 [14]. After applying the slow coherency analysis [11], generators 2 and 3 are determined as coherent generators. Consequently, the other group is only generator 1. By applying the weak connection algorithm (9), it can be determined that the weak connections are lines 4-6 and 4-5. Here, it is assumed that generators 1 and 2 are BS units.

The weak connections algorithm (9) can be further extended to determine the weak areas. This is achieved by defining a threshold, in the eigenbasis matrix, beyond the weak connections of, let say, 20 per cent (10 per cent on each side of the weak connection), as shown in Fig. 1, in which, the load buses are attached to the reference group.

![Fig.1. Single line diagram of the IEEE 9-bus test system with the identified weak area](image)

**III. MINIMAL PMU PLACEMENT ACROSS THE WEAK AREA IN A POWER SYSTEM**

Synchronised measurement technology has become an attractive solution in modern power systems because it provides voltage and current phasors and frequency information across the system, all synchronised with high precision to a common time reference provided by a Global Positioning System (GPS) [9]. In order to obtain information about the actual state of the boundary buses during the restoration process, a methodology to place minimum PMUs across the splitting options, i.e. across the weak areas, is required.

The objective of the PMU placement problem is to achieve this task with a minimal number of devices. In this paper, the number of PMUs that are placed across the identified weak areas is minimised, while ensuring there is sufficient information during PPSR. This problem is solved by an ILP formulation and it takes advantage of zero injection buses to reduce the number of PMUs. Without loss of generality, it is assumed that every PMU has a sufficient number of channels to measure the current phasors through all the branches incident to the corresponding PMU buses (i.e. the buses in which PMUs are installed). This paper does not consider conventional measurements; it is rather a pure PMU solution, taking the advantage of unique high reporting rates possessed by the PMUs.

A binary variable vector $y$, which represents the presence of PMUs across the weak areas in the power system, is defined. Considering this, the $i$-th entry of $y$ is defined as:

$$y_i = \begin{cases} 1, & \text{if a PMU is placed at boundary bus } i \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

For an $n$-bus system separated into $r$ islands, the PMU placement problem can be formulated as follows:

$$\min \sum_{i=1}^{n} c_i \cdot y_i \quad (11)$$

subject to:

$$f(Y) = [1 \ 1 \ \cdots \ 1]^T \quad (12)$$

where $c_i$ is the cost of the PMU installed at bus $i$, $f(Y) = B \times y$ and $B = [b_{ij}]$ is the network connectivity matrix across the identified weak areas defined as follows:

$$b_{ij} = \begin{cases} 1, & \text{if } i=j \\ 1, & \text{if } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

Considering the weak areas shown in Fig. 1 for the IEEE 9-bus system, the PMUs must be placed at buses 4 and 8 to have information during its PPSR. Placing PMUs at these buses allows collecting information about the boundary buses states during the restoration process.

**IV. SIMULATION RESULTS**

The proposed PMU placement method is demonstrated and validated using the New England 39-bus test system and the IEEE 118-bus test systems. To determine the number of coherent generator groups, this paper considers the minimal number of either the first largest gap between two eigenvalues $\lambda_i$ and $\lambda_{i+1}$ [11], where (14) is satisfied, or the number of available BS units:

$$|\lambda_i| \leq |\lambda_{i+1}| \quad i = 1, 2, ..., n \quad (14)$$

**A. Test case 1: New England 39-bus test system**

The single line diagram of the New England 39-bus test system is presented in Fig. 2. This system has 10 synchronous generators, 34 transmission lines, 12 transformers and 19 constant power loads [5]. In this paper,
it is assumed that generators 4, 9 and 10 are BS units. Generator data are provided in the Appendix. From the eigenvalues analysis, it is concluded that the first largest gap between two consecutive values are found between the third and the fourth one. Thus, and taking into account the number of available BS units (three for this test case), the New England 39-bus must be split into three subsystems.

![Fig. 2. Single line diagram of the New England 39-bus test system](image)

Considering the number of areas previously determined, the slow coherency algorithm [11] is performed and the coherent generator groups are shown in Table I.

<table>
<thead>
<tr>
<th>Gen. No. for Group 1</th>
<th>Gen. No. for Group 2</th>
<th>Gen. No. for Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 8, 9</td>
<td>2, 3, 4, 5, 6, 7</td>
<td>10</td>
</tr>
</tbody>
</table>

![Fig. 3. Single line diagram of the New England 39-bus test system with the identified weak areas](image)

![Fig. 4. Single line diagram of the IEEE 118-bus test system](image)
Using the weak connection algorithm (9) described in this paper, the system splitting strategy can be identified. The weak connections are determined as the following transmission lines (also shown in Fig. 2): 1-2, 3-4, 3-18, 8-9 and 17-27. As it can be noticed, using this splitting strategy each island has at least one BS unit.

Once the weak connections are known, and using the obtained power system eigenbasis matrix, a threshold of 15 percent (7.5 per cent on each side of the weak connections) is defined to determine the weak areas. For the New England 39-bus, the weak areas are presented in Fig. 3.

### TABLE II
**Optimal PMU Placement to Improve the PPSR for the New England 39-Bus Test System**

<table>
<thead>
<tr>
<th>System Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>New England 39-bus</td>
</tr>
<tr>
<td>4, 17, 20, 25, 26, 39</td>
</tr>
</tbody>
</table>

As it can be observed, this method to identify the weak areas can provide different splitting options. By considering these weak areas, PMUs are optimally placed while ensuring sufficient information is available during PPSR. This is carried out independent of the splitting strategy across the weak areas. Table II presents the list of buses at which PMUs are placed.

### B. Test case II: IEEE 118-bus test system

The second test system used to demonstrate the efficiency of the proposed methodology is the IEEE 118-bus. The topology of the system is shown in Fig. 4. This test system contains 19 synchronous generators, 177 transmission lines, 9 transformers and 91 constant power loads [15]. Generator data are provided in the Appendix. This paper considers generators 10, 25, 69, 87 and 89 as BS units.

### TABLE III
**Separation of Eigenvalues for the IEEE 118-Bus Test System**

<table>
<thead>
<tr>
<th>Eigenvalues $(\epsilon_i)$</th>
<th>$\epsilon_i = \frac{f_i}{f_{i+1}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ± j 0.00000</td>
<td>0.000000</td>
</tr>
<tr>
<td>2 ± j 0.17276</td>
<td>0.716518</td>
</tr>
<tr>
<td>3 ± j 0.15737</td>
<td>0.800641</td>
</tr>
<tr>
<td>4 ± j 0.19656</td>
<td>0.709788</td>
</tr>
<tr>
<td>5 ± j 0.27692</td>
<td>0.862132</td>
</tr>
<tr>
<td>6 ± j 0.32121</td>
<td>0.911838</td>
</tr>
<tr>
<td>7 ± j 0.35226</td>
<td>0.769798</td>
</tr>
<tr>
<td>8 ± j 0.45760</td>
<td>0.862026</td>
</tr>
<tr>
<td>9 ± j 0.53085</td>
<td>0.725376</td>
</tr>
<tr>
<td>10 ± j 0.73182</td>
<td>0.948589</td>
</tr>
<tr>
<td>11 ± j 0.83885</td>
<td>0.841307</td>
</tr>
<tr>
<td>12 ± j 0.909708</td>
<td>0.797392</td>
</tr>
<tr>
<td>13 ± j 1.25042</td>
<td>0.97812</td>
</tr>
<tr>
<td>14 ± j 1.27839</td>
<td>0.757293</td>
</tr>
<tr>
<td>15 ± j 1.68811</td>
<td>0.865859</td>
</tr>
<tr>
<td>16 ± j 1.94963</td>
<td>0.880022</td>
</tr>
<tr>
<td>17 ± j 2.21543</td>
<td>0.791465</td>
</tr>
<tr>
<td>18 ± j 2.79916</td>
<td></td>
</tr>
<tr>
<td>19 ± j 3.13205</td>
<td></td>
</tr>
</tbody>
</table>

As it can be noticed from Table III, the first largest gap between two consecutive eigenvalues is obtained after the fourth eigenvalue. Therefore, the proposed algorithm splits the system into four subsystems. The coherent generator groups, obtained from the slow coherency algorithm [11], are shown in Table IV.

### TABLE IV
**Coherent Generator Groups for the IEEE 118-Bus Test System Split into 4 Subsystems**

<table>
<thead>
<tr>
<th>Gen. No. for Group 1</th>
<th>Gen. No. for Group 2</th>
<th>Gen. No. for Group 3</th>
<th>Gen. No. for Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>10, 12, 25, 26, 31</td>
<td>46, 49, 54, 59</td>
<td>87</td>
<td>80, 89, 100, 103, 111</td>
</tr>
</tbody>
</table>

Fig. 5. Single line diagram of the IEEE 118-bus test system with the identified weak areas
The weak connections for this test system have been identified (see Table V and dashed line in Fig. 4) and the weak areas across them have been defined as a threshold of 20 percent (10 per cent on each side of the weak connections) as presented in Fig. 5. It is important to understand that the proposed algorithm defines the islands with at least one BS unit. Considering the weak areas, the PMUs must be placed in order to ensure information during PPSR considering all the possible splitting strategies. The optimal placement of these units is presented in Table VI.

The PMU location configuration in Table VI ensures that all the boundary buses are being monitored by at least one PMU (or at least the state of the bus can be obtained from the state estimator). This allows collection of important data regarding frequencies, angles and voltage magnitudes of the boundary buses during the parallel power system restoration and validated in two different test systems. The PMU placement allows the power system operator to speed up the restoration process as more information is available independent of the splitting strategy performed instead of a singular splitting strategy. The proposed optimal placement of PMUs allows the power system operator to consider the weak connection areas across these weak areas. The algorithm was demonstrated and validated in two different test systems. The PMU placement ensures that the operating conditions of all the boundary buses will be known during the parallel power system restoration.

V. CONCLUSION

A methodology to place the minimum number of phasor measurement units across the weak areas to monitor the boundary buses during the parallel power system restoration is presented in this paper. Weak areas across the power system are determined to point out different splitting options instead of a singular splitting strategy. The proposed optimal placement of PMUs allows the power system operator to speed up the restoration process as more information is available independent of the splitting strategy performed across these weak areas. The algorithm was demonstrated and validated in two different test systems. The PMU placement ensures that the operating conditions of all the boundary buses will be known during the parallel power system restoration.

APPENDIX

The appendix presents generator data (on a 100 MVA base) of the three test systems used in this paper.

REFERENCES

**BIOGRAPHIES**

Jairo H. Quirós Tortós (S’08) was born in Limón, Costa Rica. He obtained the B.Sc. and Licentiate degree with honours in Electrical Engineering from the University of Costa Rica, San Pedro, Costa Rica in 2008 and 2009 respectively. Currently he is a Ph.D. student at The University of Manchester, working on power system restoration and intelligent controlled islanding for power system. His main research interests are application of intelligent methods to power system restoration, and controlled islanding, power system monitoring, protection and control, voltage stability assessment, and power system dynamics. At the end of his Ph.D. studies, he hopes to return to the University of Costa Rica as a Professor and Researcher.

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