In a rotating two-phase sample of $^3\text{He}$ and magnetic-field stabilized $^3\text{He}$ the large difference in mutual friction dissipation at 0.20$T_c$ gives rise to unusual vortex flow responses. We use noninvasive NMR techniques to monitor spin down and spin up of the $B$-phase superfluid component to a sudden change in the rotation velocity. Compared to measurements at low field with no $A$ phase, where these responses are laminar in cylindrically symmetric flow, spin down with vortices extending across the $AB$ interface is found to be faster, indicating enhanced dissipation from turbulence. Spin up in turn is slower, owing to rapid annihilation of remanent vortices before the rotation increase. As confirmed by both our NMR signal analysis and vortex filament calculations, these observations are explained by the additional force acting on the $B$ phase vortex ends at the $AB$ interface.

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The possibility to stabilize the $A$-phase layer is similar to changing in situ the boundary conditions at one of the end plates of the rotating cylinder. The resulting changes in the dynamics demonstrate that the superfluid boundary conditions are capable of transforming the response from laminar toward turbulent, owing to an effective friction at the $AB$ interface. Experimentally, it has proven difficult to work out a distinction between pinning and other weaker forms of interactions on surfaces. Nevertheless, at the lowest temperatures dissipation and dynamics might well be governed in part by surface interactions. This work provides clues on how to approach these questions with quantitative measurements.

II. PRINCIPLE OF MEASUREMENT

The two-phase liquid $^3$He sample (Fig. 1) is contained in a smooth-walled quartz cylinder which is mounted on a nuclear demagnetization cooling stage and can be rotated at an angular velocity $\Omega$, by rotating the cryostat. The time evolution and the spatial distribution of vortices is surveyed with two NMR detector coils. Two quartz tuning fork oscillators are included for measuring temperature. A small superconducting solenoid around the cylinder provides the axially oriented magnetic field for stabilizing an $A$-phase layer which divides the NMR sample into two identical $B$-phase sections. The differences in the spin-down or spin-up responses of the two-phase and single-phase samples are most noticeable at the lowest temperatures. We concentrate here on measurements at 29 bars liquid $^3$He pressure and $0.20T_c$. This is the minimum temperature for the current setup, limited by a residual heat leak of $\sim 15$ pW through the lower orifice of 0.3 mm diameter.

Vortex structures in the $A$ and $B$ phases are incommensurable, but they do interconnect across the $AB$ interface after the doubly quantized $A$-phase vortices dissociate and terminate in point defects on the phase boundary. The critical velocity of vortex formation is low in the $A$-phase section, and in rotation the $A$ phase is approximately in the equilibrium vortex state, where both the normal and superfluid components are in solid-body rotation with the container, $v_n = \Omega \times r \approx v_s$. In the $B$-phase sections the critical velocity is an order of magnitude higher. Established procedures exist for maintaining here different kinds of rotating states: (i) vortex-free counterflow, where the superfluid fraction is at rest in the laboratory frame ($v_s = 0$) while the normal component is locked to solid-body rotation with the container ($v_n = \Omega \times r$), (ii) a state with a central vortex cluster with a known number of rectilinear vortex lines at the solid-body-rotation density $n = 2\Omega/\kappa$, so that within the cluster $v_s \approx v_n [\kappa = h/(2m\Omega)]$ is the $^3$He circulation quantum, or (iii) the equilibrium vortex state which in usual experimental conditions corresponds to the state with the largest number of rectilinear vortices.

Accordingly, different configurations are possible from where spin-down and spin-up measurements can be started. For simplicity, our spin down starts from the equilibrium vortex state at $\Omega$, which extends through all three sample sections, with vortices interconnected across the two $AB$ interfaces. In the final state the cryostat is at rest, $\Omega = 0$. With decreasing temperature the $^3$He-$B$ mutual friction dissipation $\alpha(T)$ drops increasingly below that of $^3$He-$A$ and its characteristic dynamic response time, $\tau(T) \propto 1/\alpha(T)$, increases. The situation is then encountered where the $A$ phase is essentially already free of vortices while only few vortices have managed to travel to the cylinder wall for annihilation in the $B$-phase sections.

In such a case the $AB$ interface becomes unstable when the difference in the velocities of the $A$ and $B$ superfluid components parallel to the interface reaches a critical value. This superfluid Kelvin-Helmholtz instability has been characterized in a situation where the $A$ phase is in the equilibrium vortex state and the $B$ phase is vortex-free. Here the instability at a rotation velocity $\Omega_{AB}(T, P)$ represents a damped surface wave which allows vortices to escape in one rapid nonequilibrium event across the interface from the $A$- to the $B$-phase side. In the present measurements the change in rotation velocities is kept sufficiently small, $|\Omega_1 - \Omega_2| < \Omega_{AB}(T, P)$, so that the instability does not need to be considered.

For the dynamics, the important feature which emerges with decreasing temperature is the additional initial pull on the $B$-phase vortex ends on the $AB$ interface while the rapidly moving $A$-phase vortices migrate in spiral motion to the cylinder wall for annihilation. This leads to an inhomogeneous $B$-phase vortex distribution, which is explicitly seen in numerical simulation calculations. Let us first describe the qualitative picture which emerges from the simulations although the main features were originally first worked out from the NMR measurements.
III. CALCULATED SPIN-DOWN RESPONSE

The calculation of spin down, of which Fig. 2 is a snapshot, uses the vortex filament method\(^{[20]}\) and makes the following two simplifications: all vortices are considered to be similar and singly quantized, while the AB interface is treated as a plane where mutual friction dissipation changes discontinuously from \(\alpha_A\) to \(\alpha_B\) at \(z = 5\) mm. The normal component is locked to the rotation drive. Starting spin down from equilibrium rotation after a steplike stop of rotation, one finds that the ensuing events are the following: The A-phase vortices spiral rapidly in laminar motion to the cylindrical wall. Their motion exerts a force on the B-phase vortex ends at the AB interface such that the vortex density in the center of the cylinder is initially depleted faster than at larger radii \(r\).

In Fig. 3 the central depletion in the radial distribution of the vortex density \(n(r,z,t)\) is shown 30 s after the abrupt stop of rotation. This spin-down configuration is very different from the laminar two-dimensional spin down in the absence of the A phase, where \(n(t)\) remains constant over the cross section of the cylinder during the spin-down decay. Instead, now the density increases steeply toward the cylinder wall where it rises well above the initial solid-body-rotation value \(n_0\) before it plummets toward zero within a narrow annular shell next to the wall. Close to the center of the cylinder the small maximum is created by the seven innermost vortices, as seen in the inset of the figure. On the whole this means that the superfluid fraction is not in solid-body rotation during the spin-down decay, but relaxes to the final state \(v_{r,B} = 0\) faster in the center than at large radii.

As seen in Fig. 2, neither the A- nor the B-phase vortices remain straight during the spin down; the process becomes three dimensional: the B-phase vortex bundle becomes helically twisted,\(^{[21]}\) owing to the rapid spiral motion of the vortex ends on the AB interface. Differences in the azimuthal precession velocity of the helical vortices as a function of \(z\) introduce reconnections among them and increase dissipation, which speeds up the B-phase response compared to the slow laminar motion of almost straight vortices in the absence of the A-phase layer. On the AB interface the remaining vortices curve parallel to the boundary and extend radially outward, ending perpendicularly on the cylinder wall. This is a B-phase vortex sheet which forms in this calculation because of the difference in the time scale of vortex motion on the two sides of the AB interface. It is analogous to the A-phase vortex sheet covering the AB interface in spin-up measurements when the B phase is initially almost free of vortices. In measurements of the Kelvin-Helmholtz instability\(^{[16]}\) an A-phase vortex sheet is also formed at high temperatures where the AB interface acts as a topological barrier against the penetration of the doubly quantized A-phase vortices. The surprising feature in Fig. 3 is that the vortex density distributions have a rather weak dependence on the distance \(\Delta z\) from the AB interface.

This means that the central region of reduced vortex density extends far from the AB interface. We defer further discussion.
of the numerical calculations to later, after the measurements have been described.

IV. NMR MEASUREMENT TECHNIQUES

In vortex-free rotation at constant \( \Omega \) the \( B \)-phase order-parameter texture is modified by the azimuthally flowing superfluid counterflow (cf) at the velocity \( \mathbf{v}_s = n_0 - \mathbf{v} = \Omega \times \mathbf{r} \). This solid-body-like cf reorients the \( B \)-phase anisotropy axis, which is induced by the NMR-polarization field. The order-parameter distribution is thereby changed, and a large NMR frequency shift appears. In the NMR spectrum the shift is expressed as a so-called cf peak (Fig. 4) which can be calibrated as a function of the experimental variables \( \Omega \), \( T \), and \( P \). The result is a unique trajectory as a function of \( \Omega \) for the peak absorption vs the frequency shift at fixed \( T \) and \( P \), which identifies the solid-body velocity distribution. \cite{13} During the slow spin down of the superfluid fraction after a sudden stop of rotation, when \( \mathbf{v}_s = 0 \) and \( \mathbf{v}_s \) arises from the distribution of vorticity, a cf peak is also formed. If the spin-down response is laminar, as is the case for the single-phase sample, the vortices remain approximately straight and maintain a constant density across the cross section of the cylinder, i.e., they are in the solid-body configuration and the trajectory of this transient cf peak (green triangles in Fig. 4) matches that measured in the vortex-free state at stationary conditions as a function of \( \Omega \). \cite{3,15}

The cf peak recorded during spin down of the two-phase sample follows a different trajectory of peak height vs frequency (blue dots in Fig. 4), with a frequency shift which is larger. During spin down the cf peak is produced by the azimuthal component of flow from the still remaining vortices, which as seen in Fig. 3, are compressed to large radii. A larger cf peak height and larger frequency shift become understandable from this point of view, if compared to the situation where the same vortices would be evenly distributed as in solid-body rotation. \cite{22}

This observation suggests a simple model of how to analyze the changed vortex distribution. Assume that all vortices are compressed into an outer cylindrical shell with the initial vortex density \( n_0 = 2\Omega_0/\kappa \) and that the empty inner region increases gradually in radius \( r_s \). In this cylindrical shell model the azimuthal velocity of the superfluid component is \( \mathbf{v}_s \equiv 0 \) inside the central cylinder \( r < r_s \), while in the outer cylindrical shell \( r_s < r < R \) it is \( \mathbf{v}_s \approx \Omega_0(r^2 - r_s^2)/r \). Here \( \Omega(t = 0) = \Omega_0 \approx 1.0 \text{rad/s} \) is the value extrapolated back to \( t = 0 \), the moment when the cryostat rotation comes to a stop and the cf peak height passes through its maximum value. To compare results with and without the \( A \)-phase layer, we use the normalized solid-body vortex density equivalent \( \Omega_s/\Omega_0 = 1 - (r_s/R)^2 \).

In Fig. 4 the NMR spectra have been calculated for every 20 \( \mu \text{m} \) step increase in \( r_s \). \cite{23} The trajectory of the cf peaks calculated in this way (red lozenges in Fig. 4) agrees closely with that of the measured spin-down response. Thus the simple model with only azimuthal cf appears to capture the dominant features, although it neglects all dependence on distance from the \( AB \) interface and on twisted or tangled vortices. This might be understandable since the distance \( \Delta z \) of the NMR detector coils from the \( AB \) interface is large and their height is short compared to \( \Delta z \).

V. MEASURED SPIN-DOWN RESPONSE

For quantitative analysis, the decreasing cf peaks at \( t \geq 0 \) were fitted by means of the texture calculation procedure to the cylindrical shell model with one free parameter, by adjusting the inner shell radius \( r_s \) and assuming a vortex density outside, \( 2\Omega_0/\kappa \), which is constant as a function of \( r \) and \( t \). The resulting fit in the inset of Fig. 5 reveals a rapidly growing central region where the superfluid component has already come to rest: in 50 s the radius of the vortex-free central region within the NMR detector coil has grown to half of the cylinder radius and at this point \( \sim 25 \% \) of the vorticity has been annihilated.

In Fig. 5 the spin-down responses of the azimuthal flow \( \Omega_s(t)/\Omega_0 \) are compared in the two cases, with and without the \( A \)-phase layer. The measurement is performed by decelerating the rotation drive from \( \Omega_0 \) to zero at \( d\Omega/dt = -0.03 \text{rad/s}^2 \) and the cf peak is recorded as a function of time. With decreasing temperature the cf-peak measurement runs into problems: the cf-peak height is reduced and reaches zero at higher flow velocities. \cite{13} At 0.20\( T_s \) in Fig. 5 the cf peak is lost when the vorticity has dropped by \( \sim 25 \% \) (filled symbols). This explains the blank region where there are no data points. In this region more data (open symbols) can be retrieved by increasing rotation suddenly back to some large value, where the cf peak can be recorded while it decays during spin up. This data can then be extrapolated back to the moment when rotation was increased. Combining the two data sets, the spin-down response is seen to be a smooth monotonic decay, both with and without the \( A \)-phase layer. However, in the former case it is faster and not of the laminar form as in solid-body rotation. \cite{22}

![NMR absorption spectra during spin down. The line shapes (red curves) have been calculated using the cylindrical shell model for the two-phase sample and show the counterflow (cf) peak on the right at large frequency shifts \( f - f_c \) from the Larmor frequency \( f_c \). The decreasing cf peak heights (red lozenges) correspond to increasing radius \( r_s \) of the central vortex-free cylinder in 20 \( \mu \text{m} \) steps, starting from \( r_s = 0 \). The blue curves are from the measurements where the cf peak is monitored continuously by sweeping around its maximum (blue dots) during its decay. The green triangles show the measured peak trajectory in laminar spin down in the absence of the \( A \)-phase layer (Ref. 13). Parameters: \( T = 0.20T_s \), \( P = 29 \text{ bars} \), \( \Omega_i = 1.0 \text{ rad/s} \), and \( d\Omega/dt = -0.03 \text{ rad/s}^2 \).]
of time of the three calculated curves. (Inset) Cylindrical shell model of the dots).

\[ \tau = \Omega_0 \tau_z \]

The fitted curves represent \( \Omega_0(t) = \Omega_0/(1 + t/\tau_z) \), where \( \tau_z = (2\alpha_0)^{-1} \); the solid curve with \( \tau_z = 740 \) s represents the fit to all data of the laminar decay in the absence of the A-phase layer, while the dash-dotted curve is a fit to the laminar tail only of the two-phase-sample data with \( \tau_z = 67 \) s. The numerically calculated solid curves represent the normalized azimuthal velocity \( v_\phi(R)/\Omega_0 R \) with the parameter values from Fig. 2. The three curves represent different distances \( \Delta z \) from the AB interface: red, \( \Delta z = 1 \) mm; blue, \( \Delta z = 5 \) mm; green, \( \Delta z = 10 \) mm. The dashed fit with \( \Omega_0\tau_z = 42 \) represents the laminar tail of the late-time average of the three calculated curves. (Inset) Cylindrical shell model of the two-phase sample, fitted to the measured spin down as a function of time \( t \): (right vertical axis) normalized radius \( r_c/R \) (red triangles) of the vortex-free central cylinder, and (left vertical axis) equivalent normalized solid-body vortex density \( \Omega_0/\Omega_0 = 1 - (r_c/R)^2 \) (blue dots).

difference from the laminar dependence is the feature which quantifies the turbulent dissipation.

VI. MEASURED SPIN-UP RESPONSE

In a similar manner we can determine from the cf peak decay how the azimuthal flow develops during spin up, after a rapid increase of \( \Omega \). We restrict the discussion to the case where the acceleration is started from rest (\( \Omega_z = 0 \)) and is finished at \( \Omega_z < \Omega_{AB}(T, P) \). The spin-up response then depends on the number and configuration of remanent vortices in the B-phase sections in the initial state at rest. The influence of the AB interface is to speed up the removal of remnants and thus also spin up becomes different from that measured for the single-phase sample. In the A-phase section vortices are formed rapidly and independently during the rotation increase. On the AB interfaces they curve radially outward, covering the interface as a vortex sheet. These A-phase vortices connect to B-phase vortices across the AB interface later, when B-phase vortices are formed. There are two main routes along which the B-phase spin up might proceed:

(i) If there is a fair number of remnants evenly distributed along the B-phase section, then the response might result in what appears like a slow axially homogeneous buildup of the vortex density with solid-body distribution, which is carried to completion all the way to the equilibrium vortex state. In this case the cf peak follows the solid-body-rotation trajectory in Fig. 4 (green triangles).

(ii) If there are only few remnants, then more details can be observed. The existing vortices expand axially such that the end of a vortex travels in spiral motion, describing a helix along the cylindrical wall. During the motion reconnection collisions can occur at the wall in which a new vortex loop may form. This type of slow vortex generation usually finally leads to a sudden localized turbulent burst of vortex formation. From the site of the burst subsequent axial expansion proceeds in the form of precessing vortex fronts which move both up and down along the rotating cylinder. The site of the burst and the axial velocity of the front, a function of \( \alpha(T) \) and \( \Omega_z \), then determine how soon information about the burst is transmitted to the NMR detector coils. This type of spin-up process results in a markedly different cf peak response from the previous one, since it also depends on the distance of the turbulent burst from the detector coil.

In Fig. 6 two cases of spin up with and without the A-phase layer are compared. Here the remnants are left-over vortices

FIG. 5. (Color) Normalized spin down of the azimuthal flow \( \Omega_z(t)/\Omega_0 \). The response of the two-phase sample (blue triangles), analyzed as shown in the inset, is compared to that measured in the absence of the A-phase layer (green circles). Solid symbols correspond to cf peaks measured during spin down, while open symbols represent extrapolations from subsequent spin ups (see text). The fitted curves represent \( \Omega_z(t) = \Omega_0/(1 + t/\tau_z) \), where \( \tau_z = (2\alpha_0)^{-1} \); the solid curve with \( \tau_z = 740 \) s represents the fit to all data of the laminar decay in the absence of the A-phase layer, while the dash-dotted curve is a fit to the laminar tail only of the two-phase-sample data with \( \tau_z = 67 \) s. The numerically calculated solid curves represent the normalized azimuthal velocity \( v_\phi(R)/\Omega_0 R \) with the parameter values from Fig. 2. The three curves represent different distances \( \Delta z \) from the AB interface: red, \( \Delta z = 1 \) mm; blue, \( \Delta z = 5 \) mm; green, \( \Delta z = 10 \) mm. The dashed fit with \( \Omega_0\tau_z = 42 \) represents the laminar tail of the late-time average of the three calculated curves. (Inset) Cylindrical shell model of the two-phase sample, fitted to the measured spin down as a function of time \( t \): (right vertical axis) normalized radius \( r_c/R \) (red triangles) of the vortex-free central cylinder, and (left vertical axis) equivalent normalized solid-body vortex density \( \Omega_0/\Omega_0 = 1 - (r_c/R)^2 \) (blue dots).

FIG. 6. (Color) Measured cf peak height as a function of time in spin up. (Solid symbols) Spin up after waiting for 40 min at 0.20\( \Omega_z \), for remanent vortices from a previous spin-down measurement to annihilate in zero rotation. The response lasts much longer in the presence of the A-phase layer (red triangles) than in its absence (blue squares). Owing to the larger number of remnants, the cf signal is smaller in the latter case. (Open symbols) Spin up from a state with \( \sim 120 \) remnants. Spin up lasts longer now without the A-phase layer (blue squares) than in its presence (red triangles). In the latter case a turbulent burst starts the axially propagating vortex motion. The vertical arrow denotes the moment when the vortex front enters the NMR coil. In these measurements rotation is increased from zero to \( \Omega_z = 1.0 \) rad/s at \( d\Omega_z/dt = 0.03 \) rad/s\(^2 \). Time \( t = 0 \) marks the moment when \( \Omega_z \) is reached. The number of remnants is estimated from the cf peak height extrapolated to \( t = 0 \). Note that the cf peak height has here been monitored only during the early part of the spin up, where \( 1 - \Omega_z(t)/\Omega_0 \lesssim 0.25 \), as in the inset of Fig. 5.
from a previous spin-down measurement. In one case a waiting period of 40 min is enforced at zero rotation, to allow for annihilation. This is calculated from the moment when rotation comes to a stop after the preceding spin-down measurement. Owing to the faster spin down in the presence of the A phase, the number and length of remnants is smaller so that the subsequent spin up lasts longer (red solid triangles) than in the absence of the interface (blue solid squares). In the former case new vortices are formed at slow rate, presumably in reconnection collisions at the cylindrical wall.\textsuperscript{20} No indication of a turbulent burst is present, but this might happen any time later after the time span of the data in Fig. 6.

In the second case, when the number of remnants is adjusted to be roughly equal and relatively large, the spin-up response is faster in the presence of the A-phase layer (red open triangles), owing to a turbulent burst of vortex formation which starts the axial motion of vortices along the cylinder. This is signaled by the passage of the vortex front through the NMR coil, indicated by an abrupt decline of the cf peak height (vertical arrow in Fig. 6). Presumably here the turbulent burst is triggered by the complex reconnection processes which take place at and close to the AB interface when A- and B-phase vortices connect across the interface and move at different velocities.

VII. CALCULATION OF SPIN DOWN

Vortex filament calculations have proven instructive for analyzing spin-down responses, while in spin up vortex formation has turned out to be problematic.\textsuperscript{20} At low rotation $\Omega_0 < \Omega_{AB}$ the calculation describes the dynamics properly, in spite of the simplifications, and reproduces the rapid response in the $A$-phase section, the formation of a $B$-phase vortex sheet on the $AB$ interface, and the helically twisted vortices in the $B$ phase, with all the currents which follow from these configurations.\textsuperscript{25} The central depletion in the radial distribution of the vortex density $n(r,z,t)$, as seen in Fig. 3, is the most conspicuous feature. For comparison, the equivalent vortex density in the cylindrical shell model $n(r_{s} \leq r < R,z,t) = n_0$ is illustrated with a square distribution in Fig. 3. Obviously, this is a crude approximation of the calculated density $n(r,z,t)$. However, in the measurements of Figs. 4 and 5 the number of vortices is four times larger and the shell model might provide a better description. Figure 5 supports this expectation, since the calculated and measured spin-down responses are seen to be in fair agreement. In comparison to the crudeness of the shell model, the calculated dependence of the density $n(r,z,t)$ (Fig. 3) or of the spin-down response (Fig. 5) on the distance $\Delta z$ from the $AB$ interface is weak and less significant in the analysis of the measurements.

Figure 7 summarizes some of the further differences in spin down with and without the $A$-phase section. Most important is the much larger frequency of vortex reconnections (red curves) and their spatial distribution. Since intervortex reconnections in the bulk volume feed the turbulence, it is instructive to compare them in the two cases. Roughly half of all reconnections occur within 0.2 mm of the $AB$ interface while the other half is distributed relatively evenly as a function of $z$ above the $AB$ interface. Radially the reconnections increase steeply toward large radii close to the $AB$ interface, while away from the interface the increase is slower. In contrast, in the absence of the $A$-phase section in laminar spin down all reconnections are concentrated to a narrow surface layer on the cylindrical wall, leaving the central bulk volume reconnection free. Here we have a profound difference—in the former case reconnections occur in the bulk volume and on the $AB$ interface giving rise to additional dissipation, while in the latter case they are restricted to the surface layer where the annihilating and reconnecting vortices transfer their angular momentum to the walls.

The response of the total vortex length $L(t)$ underlines in a similar manner the differences between turbulent and laminar behavior (blue curves): in the presence of the $A$ phase $L(t)$ is nonmonotonic with an initial overshoot followed by a subsequent section of rapid turbulent decay and a final slow laminar tail. The overshoot is produced when early in the decay a fraction of the kinetic energy is transformed into an increase in vortices via reconnections. The later rapid decay is reminiscent of one with a $t^{-3/2}$ dependence which generally is associated with the free decay of turbulent vortex tangles with large-scale motions.\textsuperscript{5} In contrast, in the absence of the A phase $L(t)$ decays monotonically with the slow laminar $(1 + t/\tau_{g})^{-1}$ dependence. Overall the comparison of calculation and measurement shows good agreement which proves that differences in vortex velocities across the $AB$ interface adequately explain our results.
VIII. CONCLUSIONS

The two-phase superfluid $^3$He sample, with two vastly different time scales of vortex flow, displays in its spin-down response unusual vortex configurations in the $AB$-phase section: a central region of reduced vortex density, surrounded by a vortex sheet on the $AB$ interface and an outer shell of helically twisted vortices at high density. Reconnections among the twisted vortices and close to the $AB$ interface bring about increased dissipation and a faster turbulent spin-down response. The differences in the responses with and without $A$ phase illustrate how laminar flow transforms to weakly turbulent in the presence of a dissipative planar perturbation which preserves cylindrical symmetry.

This suggests new ways for the study of superfluid surface friction, which has remained an unsettled question: Since the slow spin-down response in the absence of $A$ phase is laminar, the $^3$He-$B$ vortices cannot be pinned on the fused quartz wall and indeed their surface friction has to be weaker than the equivalent of the “effective friction” measured here for the $AB$ interface, where the response is already weakly turbulent. Nevertheless, friction on the surfaces changes the spin-down characteristics, as we well know from the response of a viscous liquid in a cylinder. This work shows that the influence of surface friction can be quantitatively assessed by comparing the responses on a surface with and without a coating of a magnetic-field-stabilized layer of $A$ phase.

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