Investigation of Island Geometry Variations in Bit Patterned Media Storage Systems

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Engineering and Physical Sciences

2011

Yuanjing Shi

School of Computer Science
Contents

List of Figures....................................................................................................................... 5
List of Tables.......................................................................................................................... 10
Abbreviations....................................................................................................................... 11
Abstract............................................................................................................................... 13
Declaration............................................................................................................................ 14
Copyright Statement.......................................................................................................... 15
Preface................................................................................................................................. 16
Acknowledgements............................................................................................................ 16

Chapter 1.......................................................................................................................... 18
Introduction......................................................................................................................... 18
  1.1 Overview of Magnetic Recording............................................................................... 18
    1.1.1 What is Magnetic Recording?............................................................................ 19
    1.1.2 History and Trends of Magnetic Recording...................................................... 21
  1.2 Established Magnetic Recording Techniques......................................................... 23
    1.2.1 Longitudinal Thin Film Recording .................................................................. 23
    1.2.2 Perpendicular Recording................................................................................ 26
  1.3 Prospects for Future Magnetic Recording Techniques........................................... 28
    1.3.1 Bit-Patterned Media ....................................................................................... 29
    1.3.2 Heat Assisted Magnetic Recording ................................................................. 31
    1.3.3 Two-Dimensional Magnetic Recording ........................................................... 33
  1.4 Motivations, Aim and Objectives............................................................................ 34
  1.5 Thesis Structure....................................................................................................... 35

Chapter 2.......................................................................................................................... 37
Replay Process and PRML Read Channel for Magnetic Recording Systems.................. 37
  2.1 The Replay Process in Magnetic Recording Systems............................................. 37
    2.1.1 The GMR Read Head....................................................................................... 37
    2.1.2 The Replay Process from Perpendicular Patterned Media........................... 40
    2.1.2 Three-Dimensional Replay Model for BPM systems.................................... 42
  2.2 PRML Read Channels............................................................................................ 46
2.2.1 Partial Response Polynomials ................................................................. 48
2.2.2 Maximum-Likelihood Detection ............................................................ 52
2.2.3 Viterbi Algorithm used as Maximum-Likelihood Detection .............. 53
2.2.4 Other Detection Strategies ................................................................. 56
2.3 Summary ................................................................................................. 57

Chapter 3 ........................................................................................................... 59
Read Channel Simulations for Bit-Patterned Media ........................................ 59
3.1 Review on Data Recovery Process for Bit-Patterned Media .................. 59
3.2 Read Channel Model for Bit-Patterned Media ........................................ 61
  3.2.1 GPR targets for Bit-Patterned Media ................................................... 62
  3.2.2 Implementation of the Digital Equaliser .............................................. 64
  3.2.3 Implementation of the Viterbi Detector .............................................. 67
  3.2.4 Signal-to-Noise Ratio ........................................................................... 68
3.3 Read Channel Simulator ........................................................................... 69
  3.3.1 Implementation of the Read Channel Model ........................................ 69
  3.3.2 Monte-Carlo Simulation ................................................................. 71
3.4 Read Channel Performance Analysis ..................................................... 73
  3.4.1 What is an Error Event? ................................................................. 74
  3.4.2 Analytical analysis of Read Channel Performance ......................... 76
  3.4.3 Analytical and Numerical Performance Comparison ....................... 78
3.5 Summary ................................................................................................. 80

Chapter 4 ........................................................................................................... 81
Investigation of Island Geometry Variations .................................................. 81
4.1 Island Geometry Variations in BPM Systems ........................................ 81
  4.1.1 Simulation of Island Geometry Variations ........................................ 82
4.2 The Effect of Island Geometry Variations .............................................. 85
  4.2.1 The effect of Island Position Variations ........................................... 85
  4.2.2 The effect of Island Size Variations ................................................. 89
  4.2.3 Island Geometry Variations Performance Comparison .................... 93
  4.2.4 The combined effect of both Position and Size Variations ................ 96
4.3 Summary ................................................................................................. 100

Chapter 5 ........................................................................................................... 101
New Trellis Design for Read Channel of Bit-Patterned Media .................... 101
5.1 The Standard Trellis Design ................................................................. 101
5.2 The Modified Trellis Design ................................................................. 103
5.2.1 Modified Trellis Design for Position Variations ................................................. 104
5.2.2 Performance Improvement with Modified Trellis for Position Variations........ 108
5.2.3 Modified Trellis Design for Size Variations........................................................ 114
5.2.4 Performance Improvement with Modified Trellis for Size Variations.............. 119
5.3 Summary ...................................................................................................................... 123

Chapter 6........................................................................................................................ 125
The Analytical Model of the Read Channel for Bit-Patterned Media.................... 125
6.1 Error Events due to Island Size Variations............................................................... 125
  6.1.1 Size Changes of the Islands involved in Error Events ........................................ 125
6.2 Analytical Analysis of the Standard Trellis with Size Variations...................... 129
  6.2.1 Development of an Analytical Model for the Standard Trellis ......................... 130
  6.2.2 Performance Comparison of the Standard Trellis with Size Variations.......... 134
6.3 Analytical Analysis of the Modified Trellis with Size Variations..................... 136
  6.3.1 Development of an Analytical Model for the Modified Trellis........................ 136
  6.3.2 Performance Comparison of the Modified Trellis with Size Variations .......... 143
6.4 Summary ...................................................................................................................... 148

Chapter 7........................................................................................................................ 149
Conclusions and Future Work ................................................................................... 149
7.1 Conclusions................................................................................................................. 149
7.2 Presented and Published Work .............................................................................. 154
7.3 Future Work................................................................................................................ 155
References ..................................................................................................................... 156
Appendix 1 ....................................................................................................................... 164
Lists of MATLAB functions .......................................................................................... 164
Appendix 2 ....................................................................................................................... 182
MATLAB script for the calculation of BER ................................................................. 182
Appendix 3 ....................................................................................................................... 184
List of Publications........................................................................................................ 184
List of Figures

Fig. 1.1 A schematic overview of perpendicular magnetic recording and replay process showing the GMR read head and inductive write head [13].................................................................20
Fig. 1.2 Trend of areal density of HDDs, where areal density in Megabits/in² is plotted against year of production [18]. ..................................................................................................................22
Fig. 1.3 Longitudinal recording showing that the magnetisation of each data bit is aligned horizontally in relation to the drive's plane [13]. ..........................................................................................23
Fig. 1.4 Longitudinal thin film consists of magnetic grains, which are magnetised into their easy axis direction in plane. Transition boundaries are formed between two opposite magnetisations..........................................................24
Fig. 1.5 Demagnetisation field in longitudinal recording forms at two bits of opposite magnetisation, which destabilise transitions in longitudinal recording [22].................25
Fig. 1.6 Demagnetisation field in perpendicular medium forms at the transition of opposing adjacent bits, which stand with opposite poles that attract each other [22]. ..............27
Fig. 1.7 Future technologies for HDD storage: (a) BPM (b) HAMR (c) MAMR (d) SWR/TDMR [30].................................................................................................................................28
Fig. 1.8 BPM schematics, where bits are defined by isolated single-domain islands. Bit pitch and track pitch are defined during the fabrication process [35].................................29
Fig. 1.9 HAMR schematics, where a laser head is applied to locally heat the medium towards its Curie temperature to allow it to be written by a head field [55]. .........................32
Fig. 1.10 MAMR schematics [30]. ..................................................................................33
Fig. 1.11 Illustration of shingled-writing [40]........................................................................33
Fig. 2.1 Unshielded (a) and shielded (b) MR head, where an unshielded MR head will pick up magnetic flux from all three transitions and shielded head will pick up only the flux of transition beneath it........................................................................................................39
Fig. 2.2 Illustration of a read-back process for perpendicular BPM system. The dotted line shows the signal that will be reached when the GMR sensor moves........................................41
Fig. 2.3 An example of isolated read-back pulse. The amplitude of the replay pulse is
normalised to 1, where PW50 is the pulse width at 50% of its normalised amplitude.

Fig. 2.4 Geometry of the shielded GMR head structure and patterned magnetic medium [67].

Fig. 2.5 Block Diagram of typical PRML channel.

Fig. 2.6 Dipulse response of PR4 system [11].

Fig. 2.7 Geometric view of ML detection [89].

Fig. 2.8 General representation of a trellis diagram.

Fig. 2.9 One trellis representation for PR4 channel.

Fig. 3.1 Block diagram of read channel model. The model consists of a 3-D replay process and PRML recovering process.

Fig. 3.2 Block diagram to determine GPR targets.

Fig. 3.3 Structure of the digital FIR filter.

Fig. 3.4 A 5-tap FIR filter with LMS adaptation of the tap weights.

The algorithm updates the tap weights of the adaptive filter according to the following equation:

Fig. 3.5 Flow chart diagram of the implemented read channel model.

Fig. 3.6 Example of an error event in the trellis of Viterbi detector. The dotted line shows an erroneous path.

Fig. 3.7 Analytical and numerical BER performance comparison for Medium1 with BAR=4.

Fig. 3.8 Analytical and numerical BER performance comparison for Medium2 with BAR=1.

Fig. 4.1 SEM image of fabricated islands. The fabricated islands exhibit the variability in both position and size [52].

Fig. 4.2 Distributions for the variation in island size for four island arrays of nominal island period: 100, 80, 60, and 50 nm [52].

Fig. 4.3 Distributions for the variation in island pitch for four island arrays of nominal island period: 100, 80, 60, and 50 nm [52].

Fig. 4.4 Introduction of jitter in patterned media results in variations in a) the position or b) the size of the islands or c) a combination of both [52].
Fig. 4.5 Effect of island position variations on the replay pulse samples for an isolated island. ..............................................................................................................................................87
Fig. 4.7 BER vs. SNR curves with island position variations present for Medium1 with BAR = 4. .......................................................................................................................................88
Fig. 4.8 BER vs. SNR curves with island position variations present for Medium2 with BAR = 1. .......................................................................................................................................89
Fig. 4.9 Effect of island size variations on the replay pulse samples for an isolated island. The nominal island size is 12.5nm. .......................................................................................................................90
Fig. 4.10 Replay waveforms generated without island geometry variations, with island position variations and with island size variations...............................................................91
Fig. 4.11 BER vs. SNR curves with island size variations present for Medium1 with BAR = 4.................................................................................................................................92
Fig. 4.12 BER vs. SNR curves with island size variations present for Medium2 with BAR = 1.................................................................................................................................93
Fig. 4.13 BER curves for σ variation in island geometry for Medium1 with BAR=4.................94
Fig. 4.14 BER curves for σ variation in island geometry for Medium2 with BAR=1.................94
Fig. 4.15 BER vs. SNR curves in the case of no jitter, σ=1.5nm variations in island position, size and a combination of both for Medium1 with BAR = 4.........................................................95
Fig. 4.16 BER vs. SNR curves in the case of no jitter, σ=2.5nm variations in island position, size variations and a combination of both for Medium2 with BAR =1.............................................96
Fig. 4.17 Effect of both island position and size variations on the replay pulse samples for an isolated island.............................................................................................................................97
Fig. 4.18 Replay waveforms generated without island geometry variations, with combination of island position and size variations. .................................................................................................98
Fig. 4.19 BER vs. SNR curves with both island position and size variations present for Medium1 with BAR = 4..................................................................................................................................99
Fig. 4.20 BER vs. SNR curves with both island position and size variations present for Medium2 with BAR = 1..................................................................................................................................99
Fig. 5.1 Generic diagram of the standard trellis for GPR target in BPM systems. $b_{k+1}$ and $y_{k+1}$ are the input and output symbols at time $k+1$. $y_{k+1}$ is calculated according to the GPR target
chosen and the magnetisation state of the previous bits. ...................................................102

Fig. 5.2 Diagram illustrating the signal contributions from the GPR target. The replay waveform (dashed line) is the superposition of isolated replay pulse (solid line)..............104

Fig. 5.3 Part of the proposed extended trellis structure designed to combat the effects of position jitter in BPM. Shaded blocks indicate additional new states. .........................106

Fig. 5.4 Read channel model with the modified trellis. ..................................................108

Fig. 5.5 The effect of un-equalised ISI..........................................................................109

Fig. 5.6 BER vs. SNR curves with 1.5nm position variations using the standard & modified trellis for Medium1 with BAR=4.................................................................110

Fig. 5.7 BER vs. SNR curves with 2nm position variations using the standard & modified trellis for Medium1 with BAR=4.................................................................111

Fig. 5.8 BER vs. SNR curves with 2.5nm position variations using the standard & modified trellis for Medium1 with BAR=4.................................................................111

Fig. 5.9 BER vs. SNR curves with 2.5nm position variations using the standard & modified trellis for Medium2 with BAR=1.................................................................112

Fig. 5.10 BER vs. SNR curves with 3.0nm position variations using the standard & modified trellis for Medium2 with BAR=1.................................................................113

Fig. 5.11 BER vs. SNR curves with 3.5nm position variations using the standard & modified trellis for Medium2 with BAR=1.................................................................113

Fig. 5.12 Modified trellis structure designed to combat the effects of island size variations. ........................................................................................................115

Fig. 5.13 Island size changes in the modified trellis to determine an optimal BER performance for Medium1 with BAR=4.................................................................117

Fig. 5.14 BER vs. SNR curves to show an optimal BER performance achieved by modified trellis for Medium1 with BAR=4.................................................................118

Fig. 5.15 Numerical BER vs. SNR curves to show an optimal BER performance achieved by modified trellis for Medium2 with BAR=1. .................................................................119

Fig. 5.16 BER vs. SNR curves with 1.5nm size variations using the standard & modified trellis for Medium1 with BAR=4.................................................................120

Fig. 5.17 BER vs. SNR curves with 2.0nm size variations using the standard & modified
trellis with varied size variations for Medium1 with BAR=4..............................121
Fig. 5.18 BER vs. SNR curves with 2.5nm size variations using the standard & modified
trellis for Medium2 with BAR=1................................................................................122
Fig. 5.19 BER vs. SNR curves with 3.0nm size variations using the standard & modified
trellis for Medium2 with BAR=1................................................................................122
Fig. 6.1 Size distributions of the three islands of interest centred on the island corresponding
to the data bit recovered in error. Nominal island size is 7.5nm. Four values of $\sigma$ are
considered: (a) 2.0 nm, (b) 2.5 nm, (c) 3 nm, (d) 3.5 nm..................................................128
Fig. 6.2 Distributions of the noise due to the central island size changes for of $\sigma$ values of: (a)
1.5 nm, (b) 2.0 nm, (c) 2.5 nm. ........................................................................................132
Fig. 6.3 Distributions of the noise due to the neighbouring island size changes, for $\sigma$
values of: (a) 1.5 nm, (b) 2.0 nm, (c) 2.5 nm. ...........................................................................132
Fig. 6.4 BER vs. SNR curves of numerical and analytical evaluation for Medium1. ......135
Fig. 6.5 BER vs. SNR curves of numerical and analytical evaluation for Medium2. ......135
Fig. 6.6 Comparison between numerical and analytical BER for varied island size variations
for Medium1 with BAR=4.................................................................................................144
Fig. 6.7 Comparison between numerical and analytical BER for varied island size variations
for Medium2 with BAR=1.................................................................................................145
Fig. 6.8 Island size changes in the modified trellis to determine an optimal BER performance
for Medium1 with BAR=4.................................................................................................146
Fig. 6.9 Analytical BER vs. SNR curves to illustrate an optimal BER performance achieved
by modified trellis for Medium1 with BAR=4. .................................................................147
Fig. 6.10 Analytical BER vs. SNR curves to illustrate an optimal BER performance achieved
by modified trellis for Medium2 with BAR=1. .................................................................148
List of Tables

Table 2.1 PR Channel polynomials, isolated pulse and impulse response samples [11]. .................................................. 50
Table 2.2: PR channels proposed for the perpendicular recording mode. .......................................................... 51
Table 3.1 Medium characteristics. .................................................................................................................. 73
Table 3.2 Error event statistics for Medium1 of BAR=4. .................................................................................. 76
Table 3.3 Error event statistics for Medium2 of BAR=1. .................................................................................. 76
Table 5.1 List of values of $\alpha$ and $\beta$ corresponding to the known island size change assumed in modified trellis for Medium1 with BAR=4. ................................................................................. 116
Table 5.2 List of values of $\alpha$ and $\beta$ corresponding to the known island size change assumed in modified trellis for Medium2 with BAR=1. ................................................................................. 116
Table 6.1 Error Event Statistics ....................................................................................................................... 126
Table 6.2 Size change statistics for the three islands of interest. ..................................................................... 127
Table 6.3 Distribution statistics for the three islands of interest with a nominal size of 7.5nm. ......................... 129
Table 6.4 Variances of noise due to island size variations for Medium1. .......................................................... 133
Table 6.5 Variances of noise due to island size variations for Medium2. .......................................................... 134
Table 6.6 Values of $\alpha$, $\beta$, $E(d_\alpha)$, $E(d_\beta)$ and $d$ corresponding to island size changes for Medium1. ................................................................................................................................. 141
Table 6.7 Values of $\alpha$, $\beta$, $E(d_\alpha)$, $E(d_\beta)$ and $d$ corresponding to island size changes for Medium2. ................................................................................................................................. 141
## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D</td>
<td>Two-Dimensional</td>
</tr>
<tr>
<td>3-D</td>
<td>Three-Dimensional</td>
</tr>
<tr>
<td>ABS</td>
<td>Air-Bearing Surface</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog-to-Digital Converter</td>
</tr>
<tr>
<td>AFC</td>
<td>Anti-Ferromagnetic Coupled</td>
</tr>
<tr>
<td>APA</td>
<td>Affine Projection Algorithm</td>
</tr>
<tr>
<td>AMR</td>
<td>Anisotropic Magnetoresistive</td>
</tr>
<tr>
<td>ATE</td>
<td>Adjacent Track Erasure</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BAR</td>
<td>Bit-Aspect-Ratio</td>
</tr>
<tr>
<td>BER</td>
<td>Bit-Error-Rate</td>
</tr>
<tr>
<td>BPI</td>
<td>Bit per Inch</td>
</tr>
<tr>
<td>BPM</td>
<td>Bit-Patterned Media</td>
</tr>
<tr>
<td>CGR</td>
<td>Compound Growth Rate</td>
</tr>
<tr>
<td>CTF</td>
<td>Continuous Time Filter</td>
</tr>
<tr>
<td>DRAM</td>
<td>Dynamic Random Access Memory</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite Impulse Response</td>
</tr>
<tr>
<td>GPR</td>
<td>Generalised Partial Response</td>
</tr>
<tr>
<td>GMR</td>
<td>Giant Magnetoresistive</td>
</tr>
<tr>
<td>HDD</td>
<td>Hard Disk Drives</td>
</tr>
<tr>
<td>HGST</td>
<td>Hitachi Global Storage Technologies</td>
</tr>
<tr>
<td>HMAR</td>
<td>Magnetic Recording</td>
</tr>
<tr>
<td>HMS</td>
<td>Head Media Spacing</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-Symbol-Interference</td>
</tr>
<tr>
<td>ITI</td>
<td>Inter-Track-Interference</td>
</tr>
<tr>
<td>JSSD</td>
<td>Jitter-Sensitive-Sequence-Detection</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Definition</td>
</tr>
<tr>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>LDPC</td>
<td>Low-Density Parity-Check</td>
</tr>
<tr>
<td>LMS</td>
<td>Least Mean Square</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
</tr>
<tr>
<td>MMAR</td>
<td>Microwave Assisted Magnetic Recording</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>MRAM</td>
<td>Magnetoresistive Random Access Memory</td>
</tr>
<tr>
<td>MTJ</td>
<td>Magnetic Tunnel Junction</td>
</tr>
<tr>
<td>NLMS</td>
<td>Normalised Least Mean Square</td>
</tr>
<tr>
<td>NPML</td>
<td>Noise-Predictive Maximum Likelihood</td>
</tr>
<tr>
<td>NRZ</td>
<td>Non-Return-to-Zero</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Distribution</td>
</tr>
<tr>
<td>PN-LMS</td>
<td>Power-normalised Least Mean Square</td>
</tr>
<tr>
<td>PR</td>
<td>Partial Response</td>
</tr>
<tr>
<td>PRML</td>
<td>Partial-Response Maximum-Likelihood</td>
</tr>
<tr>
<td>QN</td>
<td>Quasi-Newton Algorithm</td>
</tr>
<tr>
<td>RLS</td>
<td>Recursive Least Square</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscope</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SRAM</td>
<td>Static Random Access Memory</td>
</tr>
<tr>
<td>SUL</td>
<td>Soft Underlayer</td>
</tr>
<tr>
<td>SWR</td>
<td>Shingled Write Recording</td>
</tr>
<tr>
<td>TDMR</td>
<td>Two-Dimensional Magnetic Recording</td>
</tr>
<tr>
<td>TMR</td>
<td>Tunnelling Magnetoresistance</td>
</tr>
<tr>
<td>TPI</td>
<td>Track per Inch</td>
</tr>
<tr>
<td>VGA</td>
<td>Variable Gain Amplifier</td>
</tr>
</tbody>
</table>
Abstract

Bit-Patterned Media (BPM) has been recognised as one of the candidate technologies to achieve an areal density beyond 1Tb/in$^2$ by fabricating single-domain islands out of continuous magnetic media. Though much attention has been focused on the fabrication of BPM, existing lithography techniques demonstrate difficulties in producing uniform islands over large areas cost effectively; the resulting fabricated islands often vary in position and size.

The primary purpose of the research documented in this thesis is to investigate the issue of island geometry variations on the data recovery process from a perpendicular patterned media with head and media configurations optimised to achieve an areal density of 1Tb/in$^2$. In order to achieve the research aim, a read channel model has been implemented as a platform to evaluate the read channel performance numerically. It can be also altered to investigate new read channel designs. The simulated results demonstrate that island geometry variations have a detrimental effect on read channel performance. It has shown that a BPM system can be tolerant to island position variations, but more effort needs to be paid to the effect that island size variations have on the read channel performance.

A new read channel design revolving around the design of a modified trellis has been proposed for use in the Viterbi detector in order to combat the effect of island geometry variations. The modified trellis for island position variations results in extra states and branches compared to the standard trellis, while the modified trellis for island size variations results in only extra branches. The novel read channel designs demonstrate an improved read channel performance in the presence of island geometry variations even with increasing amounts of island position and size variations.

There are two ways to obtain the read channel performance in terms of the bit-error-rate (BER): a) by running a numerical Monte-Carlo simulation to count the number of bits in error at the output of the read channel model and b) using an analytical approach to calculate the BER by approximating the noise into a known distribution. It is shown that both ways demonstrate very similar results, which indicates as long as the distribution of the noise present in read channel model is predictable, the analytical approach can evaluate the BER performance more efficiently, especially when the BER is low. However, the Monte-Carlo simulation is still useful for understanding of the correlation of the errors.

Novel trellis proposed in this work will contribute to the commercial development of BPM in two ways: a) to improve the data recovery process in BPM systems, b) to allow a tolerance of 10% size variations for the existing fabrication techniques.
Declaration

No portion of the work referred to in the thesis has been submitted in support of an application or another degree or qualification of this or any other university of other institute of learning.
Copyright Statement

i. The author of this thesis (including any appendices and/or schedules to this thesis) owns certain copyright or related rights in it (the “Copyright”) and s/he has given The University of Manchester certain rights to use such Copyright, including for administrative purposes.

ii. Copies of this thesis, either in full or in extracts and whether in hard or electronic copy, may be made only in accordance with the Copyright, Designs and Patents Act 1988 (as amended) and regulations issued under it or, where appropriate, in accordance with licensing agreements which the University has from time to time. This page must form part of any such copies made.

iii. The ownership of certain Copyright, patents, designs, trade marks and other intellectual property (the “Intellectual Property”) and any reproductions of copyright works in the thesis, for example graphs and tables (“Reproductions”), which may be described in this thesis, may not be owned by the author and may be owned by third parties. Such Intellectual Property and Reproductions cannot and must not be made available for use without the prior written permission of the owner(s) of the relevant Intellectual Property and/or Reproductions.

iv. Further information on the conditions under which disclosure, publication and commercialisation of this thesis, the Copyright and any Intellectual Property and/or Reproductions described in it may take place is available in the University IP Policy (see http://documents.manchester.ac.uk/DocuInfo.aspx?DocID=487), in any relevant Thesis restriction declarations deposited in the University Library, The University Library’s regulations (see http://www.manchester.ac.uk/library/aboutus/regulations) and in The University’s policy on Presentation of Theses.
Preface

Yuanjing Shi received her first degree in Computer Science from Huazhong University of Science and Technology in 2004. In 2007, Yuanjing Shi received a M.Eng degree in Computer Science from Huazhong University of Science and Technology completing a thesis titled ‘Investigation of Read/Write Channel of Hard Disk Drives’ focusing on read/write channel simulation of traditional longitudinal magnetic recording hard disk drives. From April 2007, Yuanjing Shi was awarded a 3.5-year EPSRC studentship to pursue a PhD with the Nano Engineering Storage Technology Group at the University of Manchester. The research focus of these studies was the investigation of island geometry variations of bit-patterned media storage systems.

Acknowledgements

I would like to express my sincerest gratitude to everyone who has helped and supported me through the course of this work. In particular, I would like to thank my supervisor Dr. Paul Nutter for his constant help, encouragement and guidance through the whole project. I would also like to thank him for spending a lot of time explaining to me the concept of patterned media, repeatedly revising papers, reading my thesis etc. I would also like to thank my advisor Prof. Jim Miles for his guidance and support. Also, I would like to extend my gratitude to EPSRC for their financial support.

I would also like to offer my sincere thanks to all the members of the Nano Engineering Storage Technology Group for their support and being always available to answer any of my questions. I would appreciate the discussions we had during the group meeting and paper club. Without those discussions, I wouldn’t have the opportunity to extend my knowledge in the field of magnetic recording. In particular, I would like to thank Prof. Tom Thomson, Dr. Ernie Hill and Dr Branson Belle. Special thanks to Josephat Kalezhi, Marios Alexandrou, Peng Tian, Chris Morrison and Craig Barton for their friendship and support.
Last but not least, I would like to thank my mother Jun Chen and my father Jingguo Shi for their encouragement and endless support.
Chapter 1

Introduction

A basic background of magnetic recording is outlined in this chapter. It is generally recognised that in order to reach an areal density beyond 1Tb/in², bit-patterned media (BPM) using perpendicular recording may be a future solution [1]. The main aim and objectives of the research documented in this thesis are outlined, followed by a brief description of the structure of the thesis.

1.1 Overview of Magnetic Recording

The preservation of human civilisation is largely contributed to man’s thirst to pass ideas and experiences to their descendents. Art, music and written language are all different kinds of media that allow people to pass knowledge for subsequent generations. It is necessary to ensure that all this information is stored properly and efficiently [2, 3]. During the past thousands of years, the methods for storing durable information have evolved from carving at the very beginning, then printing, and now into digital recording [2]. In a world full of information and electronics, massive amounts of digital data are being created, accessed and shared everyday. People can’t help wondering where all the information comes from, where it all goes, and how to continue enjoying the digital lifestyle. The answer is digital recording [3]. According to the study of University of California-Berkeley in 2003, about 5,421,221EB (one EB is 10¹⁸ bytes – a billion GB) of information has been produced and stored in four physical media: paper, film, magnetic and optical [4]. The estimates show that the storage of new information is growing at a rate of over 30% a year [4] and will expand rapidly in the future [4, 5]. It is estimated that the figure for 2010 is expected to be 170EB [4]. In 2009, a report from the Global Information Industry Centre, University of California by Roger and James [6] gave an estimate of 3.6ZB (one ZB is 10²¹ bytes, or 1,000 EB) for U.S. household information consumption in 2010, which is many
times greater than the findings of the previous study in 2003 [6]. One factor accounting to this discrepancy is that the report includes video games and most TV into its calculation. It is difficult to accurately estimate how much information has been generated so far, as the definition of information and the measuring methods are differed from different studies. It is notice that the in some cases, behaviour is changing so fast that information consumption in January 2008 and December 2008 might be quite different and the study was not always able to make an accurate adjustment for this situation. The report in [6] looked at words, bytes, and the number of hours spent consuming information in the household and indicate that the amount of information has grown faster than we can imagine [4, 6]. As long as there is an ever increasing need for data storage, researchers will never stop exploring ways to meet the need.

Hard disk drives (HDD), which have been the dominant device for secondary storage of data in general purpose computers and other entertainment electronics since the early 1960s [7], are the most common media to meet people’s demand on storage as about 92.22% of information is stored magnetically [4].

1.1.1 What is Magnetic Recording?

Magnetic recording technology is founded on magnetism and electromagnetic induction [8]. The fundamental discoveries about magnetism were first introduced in 1820, when several scientists: Hans Oersted, Andre-Marie Ampere and Michael Faraday, discovered the interchangeable nature of electricity and magnetism. It was demonstrated that magnetic fields can produce an electrical current and vice versa [2]. This phenomenon inspired Oberlin Smith to design a magnetic recording model to record and reproduce sound in 1878, however, he never successfully built the device and left the theory untested [2, 9]. It was not until 1898 that Valdemar Poulsen took the principle further and demonstrated a magnetic recorder [2, 9]. Nowadays, the magnetic recording device is used almost in every facet of our working and leisure activities.

Modern magnetic recording devices basically consist of a read/write head (inductive,
magnetoresistive or giant magnetoresistive) and a magnetic medium [8, 10, 11] as shown schematically in Fig. 1.1. Writing is accomplished by passing current, representing information that is intended to be written, through the write head. The flux caused by the current circulates through the write head pole and propagates from the head pole into the magnetic medium. If the deep gap field that is generated in the vicinity of the gap is sufficiently strong, it will magnetise the recording medium permanently when the write head is moving past the medium [12]. The magnetisations in different directions, up or down in Fig. 1.1, represent the digital ‘0’ or ‘1’ of the information being stored. When the read head passes over the medium, the stray field, which is proportional in magnitude to the medium magnetisation permeates the core and induces a voltage in the giant magnetoresistive (GMR) read head. This voltage after suitable amplification and several signal processing steps will reproduce the original information.

Fig. 1.1 A schematic overview of perpendicular magnetic recording and replay process showing the GMR read head and inductive write head [13, 14].

Depending on whether the magnetisation in the medium is oriented horizontally or vertically to the plane of medium, the magnetic recording can be classified into two modes: longitudinal (horizontal) recording and perpendicular (vertical) recording [14-16]. Fig. 1.1 shows the case of perpendicular recording. Both longitudinal and perpendicular recording will be reviewed in the following sections.
1.1.2 History and Trends of Magnetic Recording

The ultimate goal for magnetic recording is to maximise the areal density, which is defined as the number of bits stored per unit area of the recording medium [17]. One way to recognise the evolution of magnetic recording technology is to look at the development of HDDs, which are the most common storage devices using magnetic recording. Fig.1.2 shows a roadmap of areal density growth in HDD products produced by Hitachi Global Storage Technologies (HGST). Since the first HDD RAMAC invented by IBM was introduced in 1956 [18], an annual compound growth rate (CGR) of the areal density of ~30% was achieved in the 1970s and 1980s. In the 1990s, there was an ever astonishing CGR of 60-100% [11, 13, 15]. However, since 2002 the growth rate has slowed down because of the superparamagnetic effect [19-22], which is also known as thermal instability limit [20, 22]. The superparamagnetic limit arises because in order to increase the areal density, the magnetic grain size must be reduced. However, the grain size cannot be smaller than a lower limit. If the grain size is so small that the anisotropy energy of the grain, \( K_aV \), (where \( K_a \) is the anisotropy energy per unit volume holding the magnetisation state of the grain and \( V \) is the volume of the grain) becomes less than the thermal energy given by \( K_BT \) (where \( K_B \) is Boltzmann’s constant and \( T \) is the absolute temperature), the magnetisation may spontaneously reverse at room temperature [23, 24]. Assuming that the information needs to be stored safely for more than 10 years in a HDD, it was estimated that longitudinal medium with a grain size less than 8-9nm will become thermally unstable at room temperature [11, 12, 25].

In order to delay the onset of the superparamagnetic limit, a number of alternative methods for magnetic recording have been proposed. Commercial products using oriented longitudinal media [26] and anti-ferromagnetic coupled (AFC) media [26-28] demonstrate 2-3 times gain in areal density over traditional longitudinal thin film recording. However, they are unable to reach an areal density beyond 100Gb/in\(^2\) [29]. The recent common technology used in HDD industry is perpendicular recording [14-16], in which an areal density of 612Gb/in\(^2\) has been demonstrated so far [30, 31]. The remarkable improvement...
that the industry has made in increased areal density of the HDD encourages researchers to continue to push the storage boundary beyond the 1Tb/in$^2$ barrier. There are several new technologies that are being explored in order to achieve ultra-high areal density of the HDD in excess of 1Tb/in$^2$, such as heat assisted magnetic recording (HMAR)/microwave assisted magnetic recording (MMAR) [26, 32] and bit-patterned media (BPM) [1, 33-38]. Though it is not clear which technique will be the future solution, the possible commercial HDD with an areal density of 1Tb/in$^2$ could be a combination of the two technologies; utilising HMAR/MMAR writing process to write information on BPM perpendicularly [35]. Apart from the above options of BPM and HAMR/MAMR, there is a new technology that combines shingled write recording (SWR) with two-dimensional (2-D) read-back signal processing to offer a potential areal density beyoond 5Tb/in$^2$ [23, 39, 40].

Apart from HDDs, today’s computers often use other three kinds of storage. Dynamic random access memory (DRAM) has a high density of bits but needs to be constantly refreshed and consumes lots of power [41]. Static random access memory (SRAM), used in caches, is fast to read and write but takes up considerable space on a chip [41]. Flash, unlike SRAM and DRAM, is non-volatile but is has a slow write circle [41]. It has found

Fig. 1.2 Trend of areal density of HDDs, where areal density in Megabits/in$^2$ is plotted against year of production [13].

Apart from HDDs, today’s computers often use other three kinds of storage. Dynamic random access memory (DRAM) has a high density of bits but needs to be constantly refreshed and consumes lots of power [41]. Static random access memory (SRAM), used in caches, is fast to read and write but takes up considerable space on a chip [41]. Flash, unlike SRAM and DRAM, is non-volatile but is has a slow write circle [41]. It has found
more than a niche in memory cards, USB flash drives and MP3 players for general storage and transfer of data between computers and other digital products. Magnetoresistive random access memory (MRAM) is recently become attractive with its full potential to replace DRAM, SRAM, flash, and HDDs or all current electrical and magnetic data storage [41-43]. Some of the advantages of MRAM include unlimited write passes, fast reads and writes and non-volatile data. However, at the moment MRAM suffers from two problems: the density of bits is low and the cost of chips is high [41-43].

1.2 Established Magnetic Recording Techniques

1.2.1 Longitudinal Thin Film Recording

Longitudinal thin film recording [15, 21] was employed exclusively in the HDD industry for nearly 50 years, where the magnetisation of the recorded bit lies horizontally in relation to the plane of the disk as illustrated in Fig. 1.3. In longitudinal recording, the fields between two adjacent bits with opposing magnetisations are separated by a transition region. The traditional recording medium used for longitudinal recording is a continuous, thin, magnetic film supported by a rigid, nonmagnetic disk [21, 26].

Fig. 1.3 Longitudinal recording showing that the magnetisation of each data bit is aligned horizontally in relation to the disk's plane [13, 14].
Chapter 1

Introduction

The thin film consists of weakly coupled magnetic grains, where each grain behaves as a single magnetic particle with a broad distribution in size, shape and anisotropy energy. The magnetisation orientation of these grains are random until a magnetic field created by a write head switches the magnetisation of these grains into their easy axis direction in plane as shown schematically in Fig.1.4. The magnetisation of each bit is determined by the average magnetisation of the grains in it, so the bit boundary follows the grain boundary. When one bit is placed next to another bit with an opposite magnetisation, a transition region [34] is formed to reduce the exchange energy between magnetic grains. The black line in Fig.1.4 shows the magnetic transition region meandering between the grains. The width of the transition region is characterised by the transition parameter $a$, which is an important factor that limits the linear density of bits that can be written on the disk. The zig-zag shape of the transition region affects its effective width and is the source of transition noise [26, 34] in the replay signal (since the read head, having a straight line shape, averages the positive and negative magnetic charges in the zig-zags).

Fig. 1.4 (a) Longitudinal thin film consists of magnetic grains, which are magnetised into their easy axis direction in plane. Transition boundaries are formed between two opposite magnetisations. (b) Transition parameter $a$.

The intrinsic signal-to-noise ratio (SNR) of a magnetic signal can be estimated from [44]
as:

\[ \text{SNR} \approx \frac{B^2 W_{\text{read}}}{a^2 D(1 + \sigma^2)} \]  

where \( W_{\text{read}} \) is the read head width, \( B \) is the bit length, \( \sigma^2 \) is the variance of the grain size distribution, \( D \) is the grain diameter and \( a \) is the transition width. In order to increase the areal density, i.e. to reduce \( B \) while maintaining the SNR, requires reduction of the grain size, transition parameter \( a \) and the variance \( \sigma^2 \). It follows that reducing the grain size results in sharpening (straightening) the transition and thus reduces the transition width. Also, the variance of grain size distribution is reduced because of the reduction of the grain size. However, the superparamagnetic effect sets a lower limit on the grain size as mentioned in Section 1.1.2, which eventually restricts the improvement of longitudinal thin film recording to an areal density beyond 100Gb/in\(^2\) \[29\].

The second limiting factor in longitudinal recording is the demagnetising field \[34\]. As the areal density increases, bits become smaller and are packed closer to each other, so they will experience an increased demagnetising field. The demagnetising field acts like two bar magnets that are placed end-to-end to repel one another, as shown in Fig.1.5. High media coercivity is required to counteract the demagnetisation in order to keep the bits stable under thermal fluctuations (superparamagnetic effect). However, write heads already use the highest saturation magnetisation material currently known, and the ability to greater higher coercivity media is therefore strictly limited.

![Diagram](image.png)

Fig. 1.5 Demagnetisation field in longitudinal recording forms at two bits of opposite magnetisation, which destabilise transitions in longitudinal recording \[22\].
Other factors such as the “side tracks” [34] and the “tracking” [34] also limit the areal density of longitudinal recording. Extra space between two data tracks are reserved because the fringing field at the sides of a tip pole in a write head erases information on neighbouring tracks (called ‘sidetracks’). Since two neighbouring bits do not always have a physical boundary between them (it exists only between two bits of opposite magnetisation), writing or reading a bit is a “blind” process.

Servo information written at the beginning of each data section to help the read/write head get to the “supposed” bit location and therefore the accuracy of the disk rotation and servo will impose another limit of data density [34]. Furthermore, writing/reading servo information is a waste of effective recording space and tracking time [34], which becomes more crucial for higher areal densities.

1.2.2 Perpendicular Recording

The first perpendicular recording disk drive, the Travelstar 5K160 [11, 13] introduced by HGST in 2006 marked the transition from longitudinal recording to perpendicular recording. The announced areal density of the Travelstar 5K160 was 130Gb/in$^2$, which offered 65 million times greater areal density compared to RAMAC. The successful implementation of perpendicular recording into commercial HDD products offers a very positive outlook of its future growth in both capacity and performance. A key feature of perpendicular recording is that unlike longitudinal recording, the magnetic medium for perpendicular recording is designed in such a way that the magnetisation easy axis of the medium is normal to the plane of the film and thus the magnetic bits point up or down vertically to the plane of the disk [14-16] as shown in Fig. 1.1. A write head with a wide gap between the write pole and the return pole is preferred to prevent flux leakage between both poles [19]. The transitions are written with the trailing edge of the write pole. Another feature of perpendicular recording system is the soft underlayer (SUL) [14, 16], which is located underneath the recording layer. The SUL physically belongs to the medium and magnetically belongs to the head since it guides the magnetic flux from the write pole to the return pole [14-16]. The effect of the SUL can be treated as an image of the write pole.
that places the recording medium to be directly in the gap of the write head where the field is more intense. This arrangement theoretically doubles the field and allows higher coercivity media to be used, which in turn are thermally more stable [14, 16, 45]. The presence of the SUL also strengthens the replay signals and helps decrease interference from adjacent tracks [14], though the domain-wall movements in the SUL may lead to noise [15]. The well designed easy axis orientation and the presence of SUL which makes perpendicular recording superior to longitudinal recording [14, 15].

Another advance that perpendicular recording offers over longitudinal recording is that the demagnetisation field in perpendicular recording enhances or stabilises the written magnetisation as shown in Fig. 1.6. This is in contrast to longitudinal recording, where the demagnetisation fields destabilise the magnetisation of the bits at higher areal density.

![Fig. 1.6 Demagnetisation field in perpendicular medium forms at the transition of opposing adjacent bits, which stand with opposite poles that attract each other [22].](image.png)

It should be emphasised that although perpendicular recording delays the onset of superparamagnetic limit of longitudinal recording, there exists a superparamagnetic limit native to perpendicular recording as well [45, 46]. The availability of higher write fields and the possibility of thicker well-aligned media will limit perpendicular recording to an areal density less than 1Tb/in² [21, 45, 47]. Once perpendicular recording reaches its superparamagnetic limit, new technologies will have to take the place of perpendicular recording in HDDs.
1.3 Prospects for Future Magnetic Recording Techniques

There are several new technologies proposed to continue increasing the areal density of magnetic recording systems beyond 1Tb/in². The superparamagnetic limit is fundamentally a trade-off between three competitive parameters: high media SNR, the writeability of the media and the thermal stability of the media. This trade-off is referred as the ‘trilemma’ of magnetic recording [15]. To achieve high media SNR requires the utilisation of small magnetic grains, as it is can be seen from Equation 1.1. The anisotropy energy of the grain is given by $K_uV$, where $K_u$ is defined in Section 1.1.2. Since $K_uV$ scales with the volume of the grain, small magnetic grains reduce the anisotropy energy of the grain to be less than the energy to ensure thermal stability. To maintain thermal stability requires an increase in the magnetic anisotropy energy $K_u$, which increases the required write field. The maximum write field is limited by the current head materials and in fact falls as the write head gets smaller [15, 30]. BPM overcomes the trilemma by isolating the single domain islands (each island consisting either of a single grain or a strongly coupled group of islands) while maintaining thermal stability. HAMR/MAMR approaches the trilemma from the writeability/thermal stability trade-off. Two-dimensional magnetic recording (TDMR) approaches the trilemma by relaxing the requirement for high media SNR through more complex signal processing process [48]. Fig. 1.7 schematically illustrates these three future technologies.

![Fig. 1.7 Future technologies for HDD storage: (a) BPM (b) HAMR (c) MAMR (d) SWR/TDMR [30].](image-url)
1.3.1 Bit-Patterned Media

So far there are many techniques that have been mentioned to delay the onset of superparamagnetism, but it is not clear how far it can be delayed. The general belief is that if the areal density is pushed further to 1Tb/ in\(^2\), techniques based on continuous thin film will need to be transferred to a discrete magnetic material, referred to as bit-patterned media (BPM) [1, 33-38]. BPM has been suggested as a promising technique to replace the continuous thin film media as it will remove or alleviate all the limitations of the continuous media listed in Section 1.2.1. A typical structure of a BPM system is shown in Fig. 1.8. Instead of statistically averaging the magnetisation over many independent grains to form a bit, in BPM, each bit consists of exactly one unit or island. Each island is strongly coupled inside so that the entire element behaves as a single magnetic domain [21, 49]. The islands are lithographically predefined on the recording medium [34] with its magnetic anisotropy (direction-dependent properties of the material) being oriented either parallel or perpendicular to the substrate. Depending on the magnetisation state of the island, for instance magnetisation up could represent 1, and down could represent 0 as in perpendicular recording.

Fig. 1.8 BPM schematics, where bits are defined by isolated single-domain islands. Bit pitch and track pitch are defined during the fabrication process [35].
BPM systems are different from the continuous thin film magnetic recording systems in three ways. First, the superparamagnetic limits can be overcome [33, 34, 38] because the thermal stability in BPM is referred to the volume and anisotropy of isolated magnetic islands rather than grains in continuous magnetic thin film. The development of nanotechnology allows magnetic islands to be patterned as small as a few nanometres, implying areal densities exceeding 1Tbits/in$^2$. Second, transition noise is eliminated because the magnetic bits are defined by the physical location of the islands rather than the boundary between two oppositely magnetised regions in continuous magnetic thin film [11]. Since island position defines where the replay signal is sensed, any variation in island position may eventually be a potential source of noise in replay process [50, 51] of BPM systems. The issue of island position variations in BPM systems will be discussed later as part of this research. Third, servo information can be patterned at fabrication which improves tracking and positioning accuracy [21, 52].

Although many advantages have been reviewed on patterned media compared to continuous magnetic recording media, it has yet to be realised as a commercial product. The requirement for fabricating large areas of magnetic islands cost effectively presents a considerable challenge for current fabrication methods, especially since each disk must be patterned with high resolution (i.e. small islands) [21, 34, 35, 49, 52, 53]. For example, for a BPM system of an areal density of 1Tb/in$^2$, assuming islands with bit-aspect-ratio (BAR) of 1, the island period is approximately 25 nm and the island length and width is about 12.5 nm. The patterning of media for HDD applications also places stringent requirements on the cleanliness and smoothness of the surface [52], which is the other challenge for the fabrication process. There are many lithography techniques that have been used or targeted to be used for the purpose of fabricating bit-patterned recording media, most of the techniques are reviewed in [49, 52, 53]. Conventional optical lithography is limited in resolution so that it will not be possible to use this for fabricating patterned media in nano size [52, 53]. Electron-beam lithography is one of the main lithography techniques used for evaluation and understanding the magnetic or recording properties of the patterned media, but not for mass production due to slow processing time and high cost for each disk [52,
Self-assembly lithography offers a much faster route than electron-beam lithography and can reach higher resolution, such as the sub-12nm resolution required for an areal density beyond 1Tb/in² [49, 52, 53], but it suffers two major difficulties in application to the HDD industry. One difficulty is the control of assembly over a few inches. The second difficulty is the arrangement of the islands in the circumferential geometry [53]. The most likely approach will be the use of nanoimprinting, which is the concept that has been used for the production of CD-ROMs. With nanoimprinting, an expensive master is first made by electron-beam lithography or self-assembly as a mold, and then several steps are followed to duplicate the nano structure from the mold [35, 49, 52, 53]. There are also some challenges to use nanoimprinting for patterning magnetic recording media. To allow the magnetic recording layer to be deposited on a relatively large area (1-3 inches), the substrate bending and roughness would be a serious problem [53]. For magnetic recording, if the magnetic and non-magnetic regions are not in the same surface, the head-disk interface could be a problem [53]. Concluding all these concerns, the challenge for BPM will be an inexpensive fabrication technique with high resolution and high throughput at low cost over a full disk.

1.3.2 Heat Assisted Magnetic Recording

It is generally agreed that the practically achievable areal density of perpendicular recording is between 500Gb/in² and 1Tb/in² or possibly higher if it is combined with other techniques, such as HAMR and BPM [35]. The concept of HAMR is based on the temperature dependence of the magnetic properties of the recording media. Fig. 1.9 illustrates the process of HAMR. Writing is performed in conjunction with a laser that locally heats the medium towards its Curie temperature where the anisotropy of the medium is low and thus only a very small head field is required to switch the medium. The medium is then cooled back quickly to an ambient temperature to maintain the magnetisation. The HAMR system allows the use of high anisotropy materials, which has an extremely high stability at ambient temperature but very small or zero anisotropy at Curie temperature. Comparing HAMR with the longitudinal and perpendicular recording,
HAMR looses the requirement of higher write field and makes writing on high anisotropy material to be possible.

Implementation of HAMR requires considerable changes to the system architecture and the write head, in which a local heat source such as a laser must be integrated. Another issue in a HAMR system is that adjacent tracks will be exposed to high temperatures if the heat spot is greater than the track width, which increases the possibility of thermal instability and bit erasure of the adjacent tracks. Despite these drawbacks, HAMR is still considered as an alternative method to increase the areal density of HDDs[21, 26].

MAMR is another technology that has been proposed recently to achieve an areal density beyond 1Tb/in$^2$. It is worth mentioning that MAMR and HAMR are reviewed separately in [30], here the two technologies are reviewed together as they both target at the writeability of the medium and offer a new degree of freedom in material design. In MAMR, the write field is applied along the easy magnetisation axis of media, and the microwave AC field is applied along the orthogonal direction, which activates the precessional motion of media magnetisation through the ferromagnetic resonance effect [30]. The anticlockwise and clockwise polarised microwave fields assist the magnetisation to change directions to enable the recording of digit binary information.
Fig. 1.10 MAMR schematics shows the excitation of the precessional motion by the microwave AC field can assist the magnetisation reversal in the media. The left figure shows the write component of MARM [30].

1.3.3 Two-Dimensional Magnetic Recording

Two-Dimensional magnetic recording (TDMR) has recently been proposed as a means of extending the existing conventional heads and media into much higher areal densities. TDMR combines the power of shingled write recording (SWR) and 2-D read-back and signal processing. Either technique can be used separately to give large gains and the combination of the two could potentially achieve an areal density around 10Tb/in² [30, 39, 40, 48]. The concept of shingled writing is to heavily overlap tracks written sequentially by a much wider writer. Because of the overlapping, the resulting tracks are much narrower than the original written width as shown in Fig. 1.10.

Fig. 1.11 Illustration of shingled-writing [40].
With SWR, there is no need for a pole-tip as narrow as the track-pitch. SWR has a number of significant advantages, which include higher write field due to large pole and sharp corner-edge field. In addition, adjacent track erasure (ATE), which occurs when a single track is written repetitively and damages an immediately adjacent track, is a nonissue [30, 40]. A huge disadvantage is that “update-in-place” is no longer possible. Since tracks are written sequentially in one direction cross-track, a single track or portion of a track cannot be altered without first recovering information written on the subsequent adjacent tracks. After the target track is updated, the recovered information must be rewritten onto the disk [30, 40].

Aside from the use of shingled-writing to ensure adequate fields, the essence of this new technique is the use of 2-D signal processing [30, 40, 48]. The 2-D signal processing requires either an array head that reads many tracks simultaneously or a single head that is progressively scanned until equivalent information is built up. Since 2-D signal processing recovers information from both along and across track, a powerful 2-D coding scheme is needed. Inter-track-interference (ITI), which is to be strongly avoided in the 1-D signal processing, now becomes an integral and constructive part of the overall detection process [30, 39, 40, 48]. Considering the advantages and disadvantages, TDMR is still an exciting new option for ultra-high areal densities.

1.4 Motivations, Aim and Objectives

The onset of the superparamagnetism will eventually limit granular perpendicular recording from achieving an areal density beyond 1Tb/in² [21, 45, 46] at this time of writing. Several novel techniques are being proposed to tackle the superparamagnetic limit using different approaches. BPM, in which each bit is stored in a single-domain magnetic island [36], has been suggested as a possible technique to overcome the superparamagnetic limit. However, existing lithography techniques (discussed in Section 1.3.1) are imperfect, which result in unavoidable island geometry variations during the fabrication process. The consequence of island geometry variations in the BPM system is the introduction of noise in the replay signal and variations in the amount of inter-symbol-interference (ISI) present.
Also, any island geometry variation will pose a problem for writing information to the island [1].

The basic aim of the research discussed in this thesis is to investigate and mitigate the effect of island geometry variations on read channel performance in BPM storage systems. The reason for doing this research is that the data recovery process is an essential part of any magnetic recording system and the performance of a magnetic recording system relies on a reliable data recovery process. In order to achieve the project aim, the research is carried out by following six major objectives as stated below:

1) The development of read channel model as a platform to investigate the effect of island geometry variations on read channel performance in BPM systems.

2) The implementation of a Monte-Carlo simulation as a numerical approach to evaluate the read channel performance with island geometry variations.

3) The development of an analytical approach to evaluate the read channel performance with island geometry variations.

4) The investigation of novel read channel designs to combat the effect of island geometry variations on read channel performance.

5) The development of an analytical analysis of the novel read channel design to evaluate its performance.

6) Comparisons between the numerical and analytical read channel model performance to evaluate the effectiveness of numerical and analytical approaches.

### 1.5 Thesis Structure

Considering the aim and objectives of this project, the remainder of this thesis will be organised in the following manner:

Chapter 2 introduces the general knowledge of replay process and partial-response maximum-likelihood (PRML) read channel in magnetic recording systems. This chapter aims to provide the background information about signal processing, which will lead to the development of a simulated read channel model to investigate the data recovery process.
Chapter 1

Introduction

from patterned media.

Chapter 3 discusses the implementation of the read channel model for BPM systems, which then enables investigation to analyse the effect of island geometry variations on read channel performance. The evaluation of the read channel performance includes numerical Monte-Carlo simulations and theoretical analysis.

Chapter 4 discusses the effect of island geometry variations in BPM systems. The effect of island position variations, island size variations and the combination of both are explored and discussed in this chapter.

Chapter 5 proposes new trellis designs for use in Viterbi detection in the read channel of BPM systems in order to combat the effect of island geometry variations on read channel performance. The standard trellis structure is also described for a comparison to the modified trellis. The improvements achieved by the modified trellis designs on read channel performance are demonstrated.

Chapter 6 is dedicated to the work on the analytical performance evaluation of the read channel in BPM systems. It proposes an analytical approach to evaluate the read channel performance in the presence of island size variations for both the standard and modified trellis. Performance comparisons between the analytical and numerical results are explored. The analytical approach for performance evaluation of the modified trellis gives a clue on the optimization of the proposed modified trellis design.

Finally, Chapter 7 summarises the main findings and contributions of this project and gives possible suggestions to further improve and develop the research.
Chapter 2

Replay Process and PRML Read Channel for Magnetic Recording Systems

The aim of chapter 2 is to present an overview of the replay and data recovery process in magnetic recording systems. As a part of this overview, every aspect of signal processing involved in the data recovery process is described in the chapter. At the end of this chapter, the reader will have a general concept of the partial-response maximum-likelihood (PRML) read channel that has been widely used in magnetic recording systems for data recovery. This will lead to the establishment of a read channel model for bit-patterned media (BPM) systems as a platform to analyse the data recovery process.

2.1 The Replay Process in Magnetic Recording Systems

The replay process is an essential part of any magnetic recording system. Understanding the replay process is important in order to understand the role of signal processing in magnetic recording systems. A read channel starts with the sensor so initially the concept of the giant magnetoresistive (GMR) read head, which is the read head that has been commonly used in commercial hard disk drives (HDD), is presented. Then the replay process from a perpendicular BPM is illustrated.

2.1.1 The GMR Read Head

The read head is like an antenna, which picks up signals generated by the magnetic field of the medium [12]. It needs to be designed to be as selective as possible so that it can detect as much signal as possible from the magnetisation of each magnetic element that passes underneath it, whilst at the same time ignoring the signals due to the magnetisations of the neighbouring magnetic elements [12]. At the beginning of the 1990s, the annual compound
growth rate (CGR) of areal densities benefited from the introduction of anisotropic magnetoresistive (AMR) heads in HDDs [11]. The advantage of AMR heads was clear, and it allowed extensibility to much higher densities. However, while scaling AMR devices to smaller dimensions the AMR effect does not provide sufficient sensitivity to read small magnetic bits in HDDs [21]. The discovery of GMR effect allowed higher sensitivity so that the GMR heads are widely used in modern disk drives [11, 12].

AMR and GMR heads are based on the same principles that a magnetoresistive material changes its resistivity due to changes of the magnetisation state. The changes of magnetisation can be altered in magnitude by changes in temperature or application of an external applied field [44]. In magnetic recording, if we imagine a coil is wrapped around the MR element and an imaginary current $I$ is passed through the coil, a voltage $U$ on the head output is calculated according to Equation 2.1,

$$U = I \cdot R.$$  

2.1

Since the resistivity $R$ depends on the external magnetic field $H$ (related to magnetic flux of the transition into MR head), the output voltage is proportional to the magnetic field of the media as

$$U = I \cdot R(H).$$  

2.2

This voltage is then filtered and processed by the HDD electronics [12]. GMR heads are an extension of MR heads. The GMR effect was first discovered in 1988 [11] and then incorporated into spin value heads in 1997 in order to obtain higher sensitivity, or equivalently, larger sensor resistance changes for the same change of the external magnetic field [12]. Typically GMR heads are 4-10 times more sensitive compared to the standard AMR sensors [12]. GMR heads are sensitive to the vertical component of the magnetic field for both longitudinal and perpendicular recording. In the case of longitudinal recording, the read head voltage is maximum at the transition, while in the case of perpendicular recording, the output voltage is sensitive to the magnetisation of the bit [12, 55].
The GMR sensor is usually placed between two high-permeability shields in order to realise good linear resolution of closely spaced magnetic transitions, i.e. the shielded GMR element may sense only the flux of the nearest transition. This can be simply illustrated in Fig. 2.1 where transition #2 is being detected by the MR element; however, the unshielded MR will pick up magnetic flux from the adjacent transitions #1 and #3. Since the flux from #1 and #3 oppose that of transition #2, the MR signal will be greatly weakened. The shielded MR element that is shown in Fig. 2.1(b) will only pick up the flux of transition #2 unless the spacing between the transitions is closer than the shield-to-shield gap. The shield-to-shield gap determines the read head resolution.

![Fig. 2.1 Unshielded (a) and shielded (b) MR head, where an unshielded MR head will pick up magnetic flux from all three transitions and shielded head will pick up only the flux of transition beneath it.](image)

The dimension of the GMR sensor is carefully optimised for each particular recording system. For example, the wider sensor has a higher read-back signal but may cause cross-track and track edge noise as it may sense the magnetic flux from the adjacent tracks [11, 12, 44]. The typical cross-track dimensions of the modern GMR sensors are smaller than the physical size of the recorded track [12].

At the time the thesis was written, sensors based on tunnelling magnetoresistance (TMR) devices have started to become available in products. Although TMR or magnetic tunnel junction (MTJ) were first described by Julliere[56] before GMR, it had attracted considerable interest only since 1994 when large magnetoresistance at room temperature
was first observed [57]. TMR sensors are significantly different from GMR sensors, and rely on the TMR effect that spin polarised tunnelling can occur when one ferromagnetic metal is separated from another by a thin insulating barrier [11, 21, 58]. The design of TMR head increases the areal density of HDDs in two ways [59]: the shield gap can be reduced as low as 25nm, which allows higher resolution of bit per inch (BPI) along the track. The use of upper and lower shield is in direct contact with the TMR element, which prevents cross talk with adjacent tracks, thus enabling higher resolution of track per inch (TPI).

2.1.2 The Replay Process from Perpendicular Patterned Media

In longitudinal recording, the read head senses the magnetic flux from the magnetic transition, whereas, in perpendicular recording, the read head senses the field from the magnetisation of the bit. The replay process in the perpendicular BPM system is similar to the replay process in perpendicular recording, with the difference that the read head senses the magnetisation of each magnetic island. The peak of the replay signal is observed when the centre of the read head is exactly above the centre of the island. Negative or positive pulses are generated depending on the orientation of the magnetisation of the islands [12].

The replay process of the perpendicular BPM system is shown in Fig. 2.2, assuming that the magnetisation of the island is ideal. First, when the GMR head approaches magnetic island, the head is not yet exactly above the island but some signal is already picked up and will appear on the read head output (Fig. 2.2(a)). As the head moves on, more and more signal is picked up, the peak of the signal will be reached when the centre of the GMR head is exactly above the island (Fig. 2.2(b)). After the head gradually passes the island, some residual signal is still detected by the head until the head is far enough from the island (Fig. 2.2(c)).
Fig. 2.2 Illustration of a read-back process for perpendicular BPM system. The dotted line shows the signal that will be reached when the GMR sensor moves.

The replay pulse from the read head output due to the magnetisation of an isolated island (island size 12.5nm) is shown in Fig. 2.3. The pulse is usually characterised by the pulse width at 50% of its amplitude. Now consider the perpendicular BPM system that targets ultra high areal densities beyond 1Tb/in², where island separation is much closer than the \( PW_{50} \) parameter of the isolated replay pulse so that the isolated pulses from each island start to overlap. These overlapping effects from the neighbouring islands are called inter-symbol-interference (ISI) [12], which can be controlled and used in the PRML read channel.
Three-Dimensional Replay Model for BPM systems

The replay model is important to explore the replay process for BPM systems. The aim of the replay model is to generate a read-back signal that predicts the read-back signal that would be observed in a real BPM system as accurately as possible. In general, the accuracy of a reciprocity based replay model [60] relies on the precision of the sensitivity function of the read head and the media magnetisation model. The sensitivity function is related to the head media spacing (HMS) and the geometry of the read head, it does not matter which kind of read element (MR, GMR or TMR) is chosen as a read head model [61-63]. Most of the established work has chosen a GMR head for replay process analysis. Hughes [64] first analysed the replay process in BPM systems, where it was assumed that both the media magnetisation and the head field were uniform across the track. In this case, the replay signal was analysed in a two-dimensional (2-D) geometry [65] using a reciprocity integral [11, 44] as a function of along the track only. However, since the shape of the recorded magnetic region is constrained by the geometry of the patterned island and not the recording head, the assumption in the 2-D replay model [65] is that the medium
magnetisation is uniform under the replay head is invalid considering that the width of the patterned element may be comparable or even less than the width of the read sensor [66]. In order to accurately predict the form of the replay signal, a three-dimensional (3-D) replay model is proposed in [66], which is different from the conventional 2-D approach [64, 65, 67, 68] by taking into account the medium magnetisation and the head field across the track.

In this thesis, the 3-D replay model will be adopted as a part of the read channel model to generate the replay samples required for data recovery process. Here, the 3-D replay model assumed a GMR head as the read sensor. The modelling of 3-D replay process is reviewed here. Fig. 2.4 illustrates the patterned magnetic medium with a shielded GMR read head that being modelled in the 3-D geometry.

![Fig. 2.4 Geometry of the shielded GMR head structure and patterned magnetic medium [66].](image)

In Fig. 2.4, the GMR element is of length $2L$ (along track direction), width $2W$ (across track direction), of semi-infinite height, and lies at a distance $d$ above the surface of patterned magnetic medium. The side shields are assumed to be of infinite width, of semi-infinite height, and separated from the GMR element by gaps of $G$ so the total shield-to-shield separation is $2(L+G)$. The medium has thickness $\delta$ and lies at a distance $t$ above any (semi-infinite) magnetic soft underlayer (SUL) of infinite permeability. In the
case of a medium with a SUL and no interlayer between the SUL and the patterned medium is assumed, then $t = d + \delta$. The coordinate system is defined as the along track direction $x$, the across track direction $z$ and the vertical direction $y$ under the air-bearing surface (ABS) (formed by the air under the slider) of the GMR head. One way of simulating the replay process is to calculate the read-back signal according to reciprocity principle [11, 12, 44] and extend it from 2-D to 3-D space.

The reciprocity principle in 2-D space is based on the fact that the mutual inductance between any two objects is equal [11]. Reciprocity is an extremely useful tool for the computation of read-back voltages [44]. Since the playback flux can be expressed as a correlation of the recorded magnetisation with the field from the playback head, the complicated problem of solving for the field due to magnetisation in the presence of a head structure with a complicated shape and a high permeability core is removed [44]. According to the reciprocity principle, the playback voltage $s$ is given by:

$$s \approx \frac{1}{I} \int M H \partial r$$

where $M$ is the magnetisation of the recording layer and $H$ is the sensitivity field generated by the imaginary current $I$. The evaluation of the above integral can often be further simplified because the magnetisation $M$ is aligned predominantly vertically in a perpendicular recording medium, i.e., $M = M_y$, and along the track in a longitudinal recording medium, i.e., $M = M_x$. Then, the reciprocity integral simplifies for the perpendicular recording as:

$$s \approx \frac{1}{I} \int M_y H_y \partial r.$$  \hspace{1cm} (2.4)

According to Equation 2.4, the reciprocity integral over a 3-D space can be expressed by integrating along the cross track coordinate $z$ as:

$$\phi_{\text{ag}}(\bar{x}) = \mu_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{d}^{d+\delta} \frac{\partial \phi(x, y, z)}{\partial y} dy \left[ M_y(x - \bar{x}, z) \right] dx dz \hspace{1cm} (2.5)$$

where $M_y$ is the perpendicular magnetisation component of the medium, $\Phi$ is the scalar
magnetic field potential and $\mu_0$ is the permeability of free space. From Equation 2.5, it can be seen that the magnetisation of both along track (direction $x$) and across track (direction $z$) are considered and the integral is over a 3-D space. With Equation 2.5, evaluating the integral over $y$ gives:

$$\phi_{\text{sig}}(\vec{x}) = \mu_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\phi(x, d, z) \cdot M_y(x - \vec{x}, z) dx - \phi(x, d + \delta, z) \cdot M_y(x - \vec{x}, z) dx] dz. \quad 2.6$$

The reciprocity formula given by Equation 2.6 for the GMR head can be simplified in the form:

$$\phi_{\text{sig}}(\vec{x}) = \mu_0 \left[ \text{IFT} \left[ \hat{M}_y(k_y, k_z) \cdot \hat{\phi}(k_y, d, k_z) \right] - \text{IFT} \left[ \hat{M}_y(k_y, k_z) \cdot \hat{\phi}(k_y, d + \delta, k_z) \right] \right] \quad 2.7$$

where $M_y,M_y^*$ and $\Phi,\Phi^*$ are Fourier transforms pairs, $M_y^*$ is the complex conjugate of $M_y$, $\text{IFT}$ is the inverse Fourier transform operation and $k_x$ is the Fourier transform in $x$. Equation 2.7 indicates that the signal flux is now dependent on the 3-D potential distribution at the top and the bottom of the magnetic medium and the magnetisation distribution in the medium. Now the question of how to derive the signal flux in the GMR sensor becomes the question to determine the scalar potential distribution at the top ($y=d$) and the bottom ($y=d+\delta$) of the magnetic medium.

There are a number of techniques documented for predicting the scalar magnetic potential distribution. In the case of no SUL present, a commonly used expression is that of Potter [65], whereby the GMR head is considered to be two inductive Karlqvist heads [65]. Potter modelled the potential distribution along the ABS ($y=0$) as a linear variation of the potential from the sensor to the side shield across the gap, i.e.

$$\phi(x, 0) = \begin{cases} 
1 & 0 \leq |x| \leq L \\
\frac{L + G - |x|}{G} & L \leq |x| \leq L + G \\
0 & |x| \geq L + G
\end{cases} \quad 2.8$$

Potter’s linear model may not be accurate so Ruigrok[68] proposed a more accurate ABS approximation of the potential distribution to include a curvature term into the linear
potential. Ruigrok’s model imagines the GMR head to be equivalent as a Karqvist head with a thin head, thus the GMR head field is the sum of the Karqvist head field and the thin head field [68]. In terms of potential distributions, his approximation consists of an equally weighted sum of a linear potential and the potential of an infinitely thin head, whereby, the linear variation in potential across the gap is modified, i.e. [68]

\[
\phi(x,0) = \begin{cases} 
1 & 0 \leq |x| \leq L \\
\frac{L + G - |x|}{2G} + \frac{1}{\pi} \arcsin \left( \frac{L + G - |x|}{G} \right) & L \leq |x| \leq L + G. \\
0 & |x| \geq L + G
\end{cases} \tag{2.9}
\]

Using either of these expressions, the potential at any point \(y\) below the ABS in the case of no SUL is given by:

\[
\hat{\phi}(k_x,y,k_z) = \hat{\phi}(k_x,0,k_z) \exp(-ky). \tag{2.10}
\]

In the case of SUL, a technique for predicting the 3-D head field for a perpendicular magnetic recording head with an SUL is followed [69]. If the ABS lies in the plane \(y=0\) and the SUL is at \(y=t\), then the Fourier transforms of the potentials, \(\Phi(x, y, z)\), in these planes are related by [44],

\[
\hat{\phi}(k_x,k_y,k_z) = \hat{\phi}(k_x,0,k_z) \frac{\sinh(k(t-y))}{\sinh(kt)} \tag{2.11}
\]

where \(k = \sqrt{k_x^2 + k_y^2}\) and \(k_x\) and \(k_z\) are the wavenumbers in the \(x\) and \(z\) directions, respectively. The software implementation of the 3-D replay model is detailed in [70].

### 2.2 PRML Read Channels

So far the previous sections have discussed the means to read magnetic transitions or magnetisations as read-back voltage waveforms, but the big question is how to reliably retrieve the digital information from the read-back waveforms. All magnetic systems contain a read channel which aims to reconstruct the digital information initially stored. One significant factor in the growth of areal density of HDDs are the developments in read
channel technology. Advances in coding, equalisation and detection techniques have contributed approximately 50% of the areal density growth in magnetic HDDs [71]. The partial response signalling is first proposed by [72] in data communication systems, in which a controlled amount of ISI is intentionally introduced to improve the information rate. It was in 1970 that Kobayashi and Tang considered the application of partial response (PR) equalisation and maximum likelihood (ML) detection scheme [73], but it was not until 1990’s when a PRML channel was first implemented in a HDD [74]. Later, PRML detection replaced peak detection as the dominant detection scheme in commercial magnetic HDDs because of its simplicity and substantially lower implementation cost [73, 75, 76]. Peak detection works well when recording densities are low, as each transition written on a magnetic medium results in a relatively isolated voltage peak that can be easily distinguished from the noise [11]. However, the peak detection channel has drawbacks which become more serious as the areal density increases. First, ISI can shift the timing of the detected pulse peak enough for read errors to occur when the detected data is decoded. Second, differentiating the incoming signal amplifies high-frequency noise, which degrades the signal-to-noise ratio (SNR) of the read channel and again makes data recovery in HDDs difficult [76].

PRML channels were proposed to overcome the problem that peak detection faces as areal density increases. The PRML channel consists of two parts: PR equalisation and the ML detector [12]. The basic idea of PR is to introduce some controlled amount of ISI into the signal so that the superposition of signals from adjacent transitions becomes predictable [11]. The magnetic recording channels can be transformed into PR channels as long as they satisfy two fundamental properties: (1) the superposition of voltage pulses from adjacent transitions is linear; (2) the shape of the read-back signal from an isolated transition is exactly known and determined [11].

A block diagram of a PRML system is shown in Fig. 2.4. The analog read-back signal from the read head is amplified by a variable gain amplifier (VGA) to have a certain and constant level of amplification. The VGA gets a control signal from a clock and gain recovery system. A continuous time filter (CTF) with a specific frequency response is used
Chapter 2           Replay Process and PRML Read Channel for Magnetic Recording Systems

48

to filter the high frequency signal so that read-back signal is concentrated below the sampling frequency. The reason of removing the high frequency signal is based on the Nyquist theorem that for the PRML channel sampling with \( f=1/T \), the spectrum of the analog read-back signal should be below the frequency \( f_{\text{max}}=1/2T \) to ensure the information content of the signal to be recovered [12]. The signal from the equaliser output is sampled by an analog-to-digital converter (ADC). The sampling is initiated by a clock signal exactly one time per channel bit period. The signal on the ADC output is a stream of digital samples, which are filtered by an additional digital equaliser. The aim of the process is to transform the digital samples into target samples so that the original data can be detected by the ML detector [77, 78]. The ML detector then determines the most likely sequence of data bits written on the disk by comparing the sequence of received samples with the sequence of ideal target samples [77-79].

![Fig. 2.5 Block Diagram of typical PRML channel.](image)

A PRML channel has several advantages over peak detection [80]. With PRML, the transition spacing requirement is relaxed [76] and a lower bit-error-rate (BER) is achieved at higher areal densities as the ML detector will produce a more accurate decision than peak detection [75].

2.2.1 Partial Response Polynomials

The basic idea of PR is to introduce some controlled amount of ISI into the data pattern rather than trying to eliminate it. The controlled amount of ISI is obtained by using a digital equaliser in the PR channel, which transforms the replay samples into a desired
shape. PR channels were originally proposed in digital communication to combat ISI. It is mentioned before that based on the Nyquist theorem, a PRML channel sampling once per channel period $T$ is appropriate to be used as a magnetic recording channel only if the spectrum of the analog read-back signal is concentrated below the frequency $f_{\text{max}}=1/2T$. The frequency spectrum of a linear magnetic recording channel is usually defined as the Fourier transform of its pulse response [11, 12]. The experimental results [11, 12] have demonstrated that the frequency spectrum of the magnetic recording channel is effectively concentrated below $1/2T$, though the tail of this frequency spectrum is still outside the $1/2T$ range. Since the frequency spectrum can be readily equalised by CTF with a cutoff frequency of $1/2T$, the PRML channel is appropriate for magnetic recording systems.

Because of the similarity of the PR4 (a special class of PR targets) spectrum and the frequency response of magnetic recording channels, the PR4 channel was used exclusively in longitudinal recording in the 1990s [11]. A convenient way of describing different PR schemes is given by PR polynomials [11, 12]. These polynomials describe correspondence between (non-return-to-zero) NRZ (i.e. “0” stands for one particular direction of medium magnetisation and “1” for another) input data pattern and ideal samples of the PRML channel. The most common PR targets for longitudinal magnetic recording are defined by the polynomial [81]:

$$F(D) = (1 - D)(1 + D)^N$$  \hspace{1cm} (2.12)

where $N$ is a positive integer, $D$ is the delay operator, $(1-D)$ is the differentiating operator and $(1+D)$ the operator which determines how the transition sample is spread over the neighbouring bit periods [11, 12]. Table 2.1 summarises different PR4 targets for longitudinal recording. For example, according to the polynomial of PR4, the impulse response samples are \{…0 1 0 -1…\}. Fig. 2.6 shows the impulse response of PR4 system.
Fig. 2.6 Impulse response of PR4 system. The dots are the replay samples at each sampling time. The impulse response (solid) is the differentiating of the pulse responses (dashed) [11].

The current sample $y_k$ of the PRML channel based on the current and previous NRZ input data pattern $\{b_k\}$ is described by the following operation:

$$y_k = b_k - b_{k-2}.$$  \hfill (2.13)

EPR4 and $E^2$PR4 targets are higher order PR4 targets, which are referred to as extended PR4. The advantage of using higher order PR4 systems is that the channel frequency response for magnetic systems with higher areal density is much closer to that of EPR4 or $E^2$PR4 than that of PR4. It requires more equalisation to match the signal frequency response into a PR4 response and the ‘boost’ of high frequency components will inevitably amplify noise in the system, which will degrade the BER [11, 12].

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Isolated pulse samples</th>
<th>Impulse response samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>PR4</td>
<td>( (1-D)(1+D) )</td>
<td>...0110...</td>
</tr>
<tr>
<td>EPR4</td>
<td>( (1-D)(1+D)^2 )</td>
<td>...01210...</td>
</tr>
<tr>
<td>$E^2$PR4</td>
<td>( (1-D)(1+D)^3 )</td>
<td>...013310...</td>
</tr>
</tbody>
</table>

Table 2.1 PR Channel polynomials, isolated pulse and impulse response samples [11].
Though the PR4 spectrum matches the frequency response of longitudinal magnetic recording channels well, it is not the case in perpendicular recording channels [82-84]. If a longitudinal read channel is to be used in a perpendicular recording system, the strong low-frequency energy of the perpendicular magnetic recording signal must be filtered out to match the dc-free (1-D) component in the PR4 target polynomial. Filtering this energy out to match a dc-free target eliminates much useful signal energy. Thus, the use of PR4 targets in a perpendicular system suffers a loss in available channel SNR [82-84]. In [82], Ide proposed PR targets which more closely fit the frequency response of perpendicular recording. The PR targets for perpendicular recording have dc on their frequency response [82]. The PR polynomial for perpendicular recording is described as:

\[ F(D) = (1 + D)^K (D^Q - 1)(1 - D). \]  

2.14

These targets take several forms depending on the integer values of the two parameters \( K \) and \( Q \), as illustrated in Table 2.2. The first PR target for perpendicular recording (perp2 target) is also identical to the PR5 target.

<table>
<thead>
<tr>
<th></th>
<th>( K )</th>
<th>( Q )</th>
<th>( D^0 )</th>
<th>( D^1 )</th>
<th>( D^2 )</th>
<th>( D^3 )</th>
<th>( D^4 )</th>
<th>( D^5 )</th>
<th>( D^6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>perp2</td>
<td>1</td>
<td>2</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>perp1</td>
<td>1</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>perp3</td>
<td>2</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>2</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>perp4</td>
<td>2</td>
<td>3</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>perp5</td>
<td>3</td>
<td>2</td>
<td>-1</td>
<td>-2</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: PR channels proposed for the perpendicular recording mode.

While a generally accepted class of PR targets in magnetic recording read channel takes integer coefficients, depending on the operating linear density and head/disk components used, the natural channel response may be significantly different from the prescribed form. In such case, a target response with non-integer valued coefficients, deviating from the prescribed PR form, can result in considerably smaller equalisation loss [85]. The generalised form of PR target, called a generalised partial response (GPR) target offers more accurate channel spectrum matching that can improve the BER performance of the ML detector [84, 85]. For BPM systems, the GPR target can offer more accurate channel spectrum matching even in the presence of island geometry variations [86]. Though the
resulting hardware implementation of ML algorithm may require higher implementation complexity than that corresponding to the integer PR targets, the performance gain can be substantial at high areal densities [85].

**2.2.2 Maximum-Likelihood Detection**

After the PR equalisation process, a ML detector is applied to determine the output sequences based on the input sequences of ML detector. The ML detection is explained in mathematics as estimating a non-random parameter $a$ based on the observations of random variables $z$ provided that $z$ is a function of $a$. The estimations are made in order to maximise the likelihood function $f(z;a)$. For example, the received variables for observation can be expressed as Equation 2.15

$$z = a + x$$  \hspace{1cm} 2.15

where $x$ is additive noise. The estimation for the non-random parameters $a$ based on the observation of $Z$ can be expressed as Equation 2.16 [87]

$$\hat{a}_{ml}(z) = \max_{a} f(z; a)$$  \hspace{1cm} 2.16

where $f(z; a)$ is the likelihood function which is normally defined as the probability density function of $z$. The concept can be extended into estimate a vector of $a$ rather than a single parameter. Assume $a = [a_1, a_2, ..., a_N]$, the maximum likelihood can be extended as Equation 2.17 [87]

$$\hat{a}_{ml}(z_1, z_2, ..., z_n) = \max_{a} f(z_1, z_2, ..., z_n; a).$$  \hspace{1cm} 2.17

ML detection in a PRML read channel in magnetic recording can be generally viewed as Fig. 2.7.
Assuming all the segments are equally probable, the received signal has been compared to all the possible signal sequences. The sequence that best matches to the received sequence will be decided as the estimated sequence so that the transmitting sequence can be recovered from the information of the estimated sequence.

### 2.2.3 Viterbi Algorithm used as Maximum-Likelihood Detection

The Viterbi algorithm was proposed in 1967 [89] as a method of decoding a convolutional code, it has been recognised as an attractive solution to a variety of digital estimation problems that has been widely used in the field of digital communications [90]. Recognising certain similarities between magnetic recording channels and digital communication channels with ISI, Kobayashi [73] and Forney [78] has demonstrated the applicability of Viterbi Algorithm to digital magnetic recording systems.

In certain circumstances, the Viterbi algorithm can be used for ML detection [91]. The Viterbi algorithm makes an estimation of the transmitted sequences based on the received symbol having the least binary Hamming distance or square Euclidean distance from the ideal symbol. Depending on whether binary Hamming distance or square Euclidean distance is used as a measurement method, the Viterbi algorithm can be recognised as ‘hard detection’ or ‘soft detection’[91]. In the Viterbi detector, each estimation or decision that has been made at each state is to choose a symbol sequence \( b \) and to discard another competing symbol sequence \( c \). Let us denote the sequence into \( t+1 \) bits binary vectors:
Assuming the decision is made on the observed receiving sequence:

\[ y = [y_0y_1\ldots y_r]. \]  \hspace{1cm} (2.19)

If the joint properties \( \Pr[y, b] > \Pr[y, c] \), we choose sequences \( b \). The situation can be expressed as:

\[
\Pr(y | b) \Pr(b) > \Pr(y | c) \Pr(c),
\]

where \( \Pr(b) \) is the probability of transmitting symbol \( b \) and \( \Pr(c) \) is the probability of transmitting symbol \( c \). One of the assumptions of the Viterbi algorithm is that each transmitting code word has equal probability, so Equation 2.20 can be simplified to:

\[
\Pr(y | b) > \Pr(y | c).
\]

If \( b \) is the transmitting symbol, \( y \) is the sum of error vector \( e_b \) and \( b \), in ‘hard detection’ error vector is a binary vector,

\[ e_b = [e_{b_0} e_{b_1} \ldots e_{b_r}] \]

and \( y = b + e_b \). As same as the above, if \( c \) is the transmitting symbol, then \( y = c + e_c \). Equation 2.21 is modified into Equation 2.23, assuming that the probability of the error sequence is independent to the transmitting sequence.

\[
\Pr(y | b) = \Pr(b + e_b | b) > \Pr(y | c) = \Pr(c + e_c | c)
\]

Equation 2.23 can be further simplified as:

\[
\Pr(e_b) > \Pr(e_c).
\]

The Viterbi algorithm can be used as ML detection provided that it satisfies two assumptions. One assumption is that all the possible transmitting symbols have equal probability and the other assumption is that the probability of the error sequence is
independent to the transmitting sequence. Thus, the likelihood function of ML detection can use the Hamming distance or the square Euclidean distance as a measurement [91].

ML detection using the Viterbi algorithm can be best understood based on the concept of a state diagram, which describes all the possible states of the magnetic recording system and the transitions between these states. The state diagram consists of two distinct parts: states and transitions. A state represents a unique physical situation at a given time, which may be specified by the current medium magnetisation, and possibly its history, i.e., the medium magnetisation one or several bit periods earlier. A transition is the “event” relating the current state and the next state. The equivalent representation of the state diagram is the trellis, which is obtained by tracing the time sequence of state changes. A general representation of a trellis diagram is illustrated in Fig. 2.8. \( S_k \) = state at time \( k \), \( b \) = input symbol, \( y \) = output symbol and \( S_{k+1} \) = state at time \( k+1 \). The transition in the trellis diagram, from a starting state \( S_k \) is called a branch \( (S_k, S_{k+1}) \). Each branch is labelled with the input symbol \( b \), which causes the corresponding state. In addition, each branch is labelled with the output symbol \( y \), which represents the output of the state machine when the input symbol is known. When the ML detector makes its decisions, it actually “extends” the trellis frame for several consecutive time instants \( k, k+1, k+2 \) etc. and estimate likelihood of possible trajectories in this trellis structure.

![Fig. 2.8 General representation of a trellis diagram.](image)

Taking the PR4 channel as an example, the PR4 polynomial is described as \( F(D) = (1-D)(1+D) \) with coefficients \( g = [g_0, g_1, g_2] = [1, 0, -1] \) and PR4 target has a length of \( L=3 \), so the trellis requires \( 2^{L-1} \) state representations and the output symbols are calculated as:

\[
y(k) = \sum_{i=0}^{2} g(i)b(k-i).
\]

The possible transitions between these states are shown in Fig. 2.9. The input samples and
the output samples are labelled on each branch.

\[
\begin{array}{cccc}
  b_k & b_{k-1} & b_{k+1}/y_{k+1} & b_{k+1} & b_k \\
  00 & 0/0 & 0/-1 & 1/1 & 00 \\
  01 & 1/0 & 0/0 & 0/-1 & 01 \\
  10 & 1/1 & 1/0 & 1/0 & 10 \\
  11 & 1/1 & & & 11 \\
\end{array}
\]

Fig. 2.9 One trellis representation for PR4 channel.

### 2.2.4 Other Detection Strategies

It has been shown in theory and practice that the conventional PRML scheme, i.e. PR equalisation combined with ML detection achieves near the theoretical optimal performance at the areal density measured as \(PW_{50}/T\) (higher areal density means smaller duration of the written bit \(T\) and larger \(PW_{50}/T\) value) in the range 0.8–1.7 [74, 92], where \(PW_{50}\) is the pulse width at the 50% amplitude of the replay response and \(T\) is the duration of the written bit. However, at higher areal density (\(PW_{50}/T \geq 2\)), the channel spectrum may not be well matched to the PR target, which leads to noise enhancement during the equalisation that affects the performance of PRML channel [74, 92]. It is noted that by allowing the PR target polynomial to take non-integer coefficients additional performance gains can be obtained. An alternative method is the introduction of a noise-predictive maximum likelihood (NPML) detector [74, 92], which is proposed by imbedding a noise predictor into the branch metric computation of the Viterbi detector. The predictor in NPML is a digital equaliser with a transfer polynomial \(P(D)\), which equalises the input of
the NPML according to the estimated output sequences so that it can trace the changes of the channel spectrum to offer a more accurate match between the PR target and the channel spectrum. Assuming the PR target is given by the polynomial $F(D)$, as NPML feedbacks its equalised signal the PR target of the overall channel as seen by NPML is represented in a form $G(D)=F(D)(1-P(D))$. As the coefficient of $P(D)$ is non-integer, the polynomial $G(D)$ represents the GPR target of the channel, which explains that use of NPML detector is similar in the performance to the adoption of GPR targets.

In [14], an adaptive NPML is proposed to improve the read channel performance, where the coefficient of the predictor is adapted to the NPML outputs. Considering both the low complexity and effective adaptation, the authors exploit a partial selection method to the Viterbi detector that compares the input sequences with some selected sequences rather than all the possible sequences according to the Viterbi trellis structure. Also, the adaptive update of the coefficients of the predictor starts from a tentative decision value. Though it is not clear from the paper how the tentative decision and partial path selection have been made, it is mentioned by the author that the adaptive NPML may not improve the BER performance if a large delay exists in the adaptive predictor [14]. In [92], the NPML is combined with parity-based post-processing, which in general improves the read channel performance by correcting the errors at the output of the NPML. The robustness and performance of the post-processing scheme depends on the reliability of the mechanism that triggers detection and correction of the errors.

### 2.3 Summary

In this chapter, an introduction of replay process in magnetic recording systems and the PRML read channel is detailed. The development of the 3-D replay model is reviewed as it is essential in the following read channel simulation discussed in Chapter 3 in order to provide more accurate by approximated replay samples of the perpendicular BPM systems. The model is an extension of reciprocity principle from 2-D to 3-D space, and one of the main advantages of its use is that it allows the signal contribution across track to be included in the analysis [66]. The basic concept of the PRML read channel is to equalise
the signal into a pre-defined PR targets (i.e. controlled ISI) and use ML detection to determine the most likely data sequence written on the disk by comparing the sequence of the received samples with the sequence of the possible samples [79]. The PR targets for a magnetic recording system are chosen to match the natural channel response as closely as possible. The Viterbi algorithm is used for ML detection when it satisfies two assumptions: all the possible transmitting symbols have equal probability and the probability of the error sequence is independent to the transmitting sequence. As the areal density of magnetic recording system increases, a family of sequence detection methods are proposed based on NPML detectors in order to improve the read channel performance. For the BPM magnetic recording systems discussed in this thesis, GPR targets are generally accepted to offer a more accurate channel spectrum matching instead of using NPML in the read channel.
Chapter 3

Read Channel Simulations for Bit-Patterned Media

As a part of this research work, simulations of a read channel for bit-patterned media (BPM) systems have been developed and are described in this chapter. The complete read channel model, including the three-dimensional (3-D) replay model adopted from [66] and the partial-response maximum-likelihood (PRML) read channel used to evaluate the read channel performance of perpendicular BPM systems. Analytical analysis of the read channel performance is presented as a comparison with the numerical read channel performance.

3.1 Review on Data Recovery Process for Bit-Patterned Media

The data recovery process for BPM systems has been analysed previously by Hughes [64, 67, 93] at 100Gb/in² areal density and later 1Tb/in² areal density. In [67], the replay responses are calculated according to Potter’s approximation [65], where a two-dimensional (2-D) anisotropic magnetoresistive (AMR) read sensor is assumed, in order to analyse the replay process. In this thesis, considering that the assumption that medium magnetisation across track is uniform may be invalid in the case of BPM, a 3-D replay model which takes into account the effect of the medium magnetisation across the track is used [66]. For an scenario that the dimensions of the read head utilised may be several times larger than track pitch, a “multiple islands per read head” read channel design is proposed in [94], where the output of the read head is a function of the magnetisation of islands from several independently written tracks. In this thesis, a different scenario is considered, where that the dimension of the read head [95] is comparable to the island size; such a scenario ensures the read head will sense the medium magnetisation of one island rather than multiple islands across track at the same time.
Other work of the read channel design of BPM systems has investigated the effect of media and head dimensions on read channel performance [66, 70, 95-97]. Analysis has demonstrated that in terms of island shape, no obvious effect on the read channel performance is observed when using square or circular islands. Reducing the island period will lead to the degradation of the read channel performance due to the increased amount of inter-symbol-interference (ISI) and inter-track-interference (ITI) in the replay waveform. A high bit-aspect-ratio (BAR) media demonstrated an increased amount of ISI, which degrades the read channel performance [70]. Due to closer tracks in BPM systems at ultra-high areal densities, the effect of ITI on data recovery process cannot be ignored [83, 98, 99]. Also, in BPM systems with closely-spaced tracks, read-head offset or track misregistration (TMR) are inevitable and can increase the ITI due to the read head sensing islands in adjacent tracks [98, 99]. The ITI issues together with a read head offset, have been investigated in [70, 98-100]. It has been demonstrated that the introduction of ITI and TMR into the replay waveform resulted in a reduced bit-error-rate (BER) performance [70, 98-100], particularly for the medium with soft underlayer (SUL). In [83, 101], a 2-D generalised partial response (GPR) minimum mean square error (MMSE) equaliser is proposed to improve read channel performance in terms of BER vs. signal-to-noise ratio (SNR) in the presence of ITI and TMR. In [98, 99], the standard trellis is replaced by a new trellis in the read channel model to offer an improved read channel performance in the presence of ITI and TMR.

To date most of the read channel designs that have been proposed have concentrated on combating the effect of ITI and TMR in BPM systems [98, 99, 101], and relatively little effort has been paid to investigating the effect of island geometry variations and the read channel designs to mitigate the effect. Hence, a part of this thesis work will investigate how the island geometry variations affect the read channel performance and how to mitigate such effects. Before the investigation of the effect of island geometry variations on read channel performance, a read channel model is developed and introduced in this chapter.
3.2 Read Channel Model for Bit-Patterned Media

A comprehensive read channel model has been developed in order to predict the performance in terms of BER against SNR for BPM systems. Based on the structure of a typical PRML channel shown in Fig. 2.5, a block diagram of the read channel model for BPM systems is illustrated in Fig. 3.1. First, random data \{-1, 1\} is generated to represent the data that is intended to be stored in the medium, where -1 and 1 represent negative and positive magnetisation of the islands separately and correspond to states of 0 and 1 for the stored data. The 3-D replay model from [66] is adopted as an approximate simulation of replay signals from the read head. It is assumed that the replay signals generated from the 3-D replay model are concentrated below the sampling frequency with a certain level of magnitude so that any analog signal processing is excluded from the read channel model. The implementation of the 3-D replay model is documented in [66, 70]. Media noise due to island geometry variations are introduced in the 3-D replay model for later investigation of their effect on read channel performance. The additive white Gaussian noise (AWGN) is added to the replay signal in order to approximate the electronics noise in BPM systems. The replay signal is sampled with an interval time equal to the island period. The digital equaliser equalises the replay samples into a desired GPR target in order to match the frequency response of the read channel with a desired frequency response (given by the adopted GPR target). The equalised replay samples are finally input to the Viterbi Detector in order to determine the most likely data sequence written on the disk. The implementation of each block in the read channel model will be described later in this chapter and MATLAB (version 7.0.4) is chosen as the development environment of read channel simulation. The read channel performance of BPM systems is measured by identifying the number of the errors by comparing the recorded random data with the recovered data. During the simulation, each frame of random data with length of 4096 is recursively added to the read channel model and the simulation will stop when 50 frames are in error for a specific SNR value. Here, each frame of random data represents information from an individual sector on the hard disk drive (HDD). The length of each sector is defined as 4096 bits on the HDD the length of each frame is chosen accordingly.
The simulation iterates for different values of SNR so that the read channel performance is presented as a BER vs. SNR curve.

![Block diagram of read channel model. The model consists of a 3-D replay process and PRML recovering process.](image)

### 3.2.1 GPR targets for Bit-Patterned Media

It is mentioned in Section 2.2.1 that GPR targets are different from the generally accepted partial response (PR) targets in magnetic recording systems, where for GPR targets the sample values in the read channel response are not limited to integers. In order to find a suitable GPR target for the BPM system being simulated, an approach documented in [85] has been adopted for the read channel model. Fig. 3.2 shows a detailed block diagram of the method to determine the GPR target for a BPM system. Here, $a_k$ is the random data input to the 3-D replay model to generate replay waveforms. Island geometry variations are expressed in terms of edge shifting by $\Delta_k$ the island in the 3-D replay model. AWGN noise $n_k$ is added to the replay waveforms. After sampling, a digital equaliser $F(D)$ is used to reshape the channel frequency response into a desired frequency response. The output sequence $c_k$ of the digital equaliser should closely resemble the samples $d_k$ that are generated through the filtering of $a_k$ by a desired target, $G(D)$. Finally, the Viterbi detector is employed to perform sequence detection on the equalised channel outputs.
The approach shown in Fig. 3.2 is to find the target function $G(D)$ and the equalizer function $F(D)$ simultaneously by minimising the mean square error (MSE) between the equaliser output $c_k$ and the desired samples $d_k$. In the case of integer PR targets, the target function $G(D)$ is known and the MSE between $c_k$ and $d_k$ can be achieved by using an optimum digital equaliser. In the case of GPR targets, the tap weights of the digital equaliser and the targets need to be calculated simultaneously. Let $G = [g_0, g_1, ..., g_{L-1}]’$ and $F = [f_{-K}, f_{-K+1}, ..., f_0, ..., f_K]’$, where the $g_k$ and $f_k$ represent the time domain coefficient of $G(D)$ and $F(D)$ respectively. In addition, let $R$ be an $N$-by-$N$ auto-correlation matrix of the sampled data $s_k$, $A$ be an $L$-by-$L$ auto-correlation matrix of the input channel data $a_k$, and $T$ be an $N$-by-$L$ cross-correlation matrix of the sampled data $s_k$ and the input channel data $a_k$. Then it is straightforward to show that the MSE can be written as:

$$
\varepsilon^2 = F’RF + G’AG - 2F’TG.
$$

To minimize the MSE given by Equation 3.1 with respect to both $F$ and $G$, there are three different cases constraint in order to avoid the trivial solution of $F = 0$ and $G = 0$. First, fix $g_0 = 1$. Secondly, fix the second target sample $g_1 = 1$. The third constraint is to fix the energy $G’G = 1$. For the first two cases, an $L$-element column vector $I$ whose first or second element is 1 (corresponding to $g_0 = 1$ or $g_1 = 1$) and all other elements are zero is introduced. Then, the fixed sample constraint can be expressed in the matrix form as $IG=1$. The equaliser design problem now reduces to the minimization of Equation 3.1 while keeping $IG=1$. The optimal target for $G$ and $F$ are then given by:
\[ G = \lambda (A - TR^{-1}T)^{-1}I \quad 3.2 \]

\[ F = R^{-1}TG \quad 3.3 \]

where \( \lambda \) is the Lagrange multiplier \([85]\) and can be expressed as:

\[ \lambda = \frac{1}{I'(A - TR^{-1}T)^{-1}I}. \quad 3.4 \]

For the third case, \( \lambda \) is simply the minimum eigenvalue of the matrix \( A - TR^{-1}T \), \( G \) is the corresponding normalised eigenvector and \( F \) is again given by Equation 3.3.

The software implementation of the identification of the equaliser tap weights \( F(D) \) and GPR targets \( G(D) \) is based on Equations 3.2, 3.3 and 3.4. The MATLAB function can be found in Appendix 1 – GPR targets.

### 3.2.2 Implementation of the Digital Equaliser

The ultimate goal of the digital equaliser is to shape the replay samples to the desired target values. As a result of equalisation, the equalised samples are as close as the target samples being given by the PR targets, any deviation of the samples from the target values is a potential source of error \([102]\). The finite impulse response (FIR) filter is commonly used as the digital equaliser in a PRML channel in magnetic recording systems \([103]\). The digital FIR is implemented using delay registers and adders as shown in Fig. 3.3, where the Ds represented delay elements (registers) that store the input sample values. Assuming the input samples to the FIR is \( s(k) \), then the corresponding output of the filter \( p(k) \) can be expressed as \([104]\):

\[ p(k) = \sum_{n=0}^{N} h(n)s(k - n) \quad 3.5 \]

where \( h(n) \) is the \( n \)th tap value or “coefficient” of the FIR. In Fig. 3.3, each time a new sample is input, the other samples in the FIR are shifted right. Each sample in the filter is multiplied by a particular tap value, or “coefficient” and then summed together to provide the filter’s output at each sampling time. The process continues over all samples until there
One critical question in designing the digital FIR filter is how many taps should be selected. In general, the more taps are used in FIR filter, the more complex the filter that can be implemented [12]. Another critical question is what the optimum tap weights to use. The answer is an adaptive algorithm is needed to update the filter coefficients. There are various adaptive algorithms used for FIR filters in [51, 105] the performance of some FIR adaptive algorithms is compared. The FIR adaptive algorithms include some widely known adaptive algorithms such as the least mean square (LMS) [51, 105-107] algorithm, the normalised LMS (NLMS) algorithm [51, 105], the power-normalised LMS (PN-LMS) [105] algorithm and the recursive least square (RLS) algorithm [107], and some less well-known algorithm such as Quasi-Newton (QN) algorithm [51, 105] and affine projection algorithm (APA) [51, 105]. The convergence rate of the family of LMS algorithm (i.e. LMS, NLMS and PN-LMS) is slow when the input signal is colored noise or speech [51, 105]. RLS, QN and APA algorithms offer much faster convergence speed than the family of LMS algorithms, but this benefit comes at the cost of high computational complexity [108]. Since the speed of convergence of the LMS algorithm can be improved if the input signal to FIR and its desired output are uncorrelated and the LMS
algorithm is much simpler to implement [103], the LMS algorithm is chosen as the adaptive algorithm being used in the simulation of FIR filter. Fig. 3.4 illustrates a 5-tap FIR structure with LMS adaptive algorithm, which adjusts the filter coefficients by minimising the MSE between the equalised samples and the desired samples [104, 109].

\[ h(k) = h(k-1) + \mu e(k)s(k) \]  

where \( \mu \) is the step size, \( s(k) \) is the input signal and \( e(k) \) is the error sequence between the desired signal and the filtered signal. The equation identifies three parameters that are important for robust LMS adaptation [110]. First, the initial tap weights at \( k=0 \) must be determined. Second, \( \mu \) must be selected. The step size \( \mu \) decides how quickly the taps adapt. A large \( \mu \) results in quicker adaptation but less precise tap weights. A small \( \mu \) results in slower adaptation but more precise tap weights and a lower square error at the output of the FIR. The gearshift algorithm (choosing a varying \( \mu \)) provides a tradeoff between a large and small \( \mu \) by starting with a large \( \mu \) value and then switching to a smaller value for fine-tuning of the tap weights [110]. Third, the error term must be calculated [110].
software implementation of an adaptive FIR filter using the LMS algorithm is based on Equation 3.6 and the choice for $\mu$ adopts the gearshift algorithm. The tap weights are initialised to be arbitrary values. The filter output $p(k)$ with the specific tap weights is calculated according to Equation 3.5. Then the difference between the ideal samples $y(k)$ and $p(k)$ is calculated as the error $e(k)$. The tap weights for the next sample time are updated according to Equation 3.6. The same process is repeated for a number of iterations and stops when the tap weights have converged to the optimal tap weights when the MSE takes the value of $10^{-15}$ (small value makes the tap weights converge to the optimal tap weights as close as possible). $\mu$ is set up to be 0.01 for the first 100 iterations to reduce the MSE quickly and after 100 iterations $\mu$ is reduced to 0.001 in order to obtain more accurate tap weights. The software implementation of the adaptive FIR can be found in Appendix 1 – FIR filter. Accurate equalisation is essential because poor equalisation would produce un-equalised ISI [111], which has a similar effect on that of adding noise and leads to worse BER performance [111]. There is little work that discusses the effect of un-equalised ISI on BPM systems, however, analytical analysis by Nabavi et al [111] concluded that un-equalised ISI is not AWGN and is also different from the coloured Gaussian noise so that the signal processing techniques targeted on mitigating the effect of AWGN and the coloured Gaussian noise may not apply to un-equalised ISI.

### 3.2.3 Implementation of the Viterbi Detector

Following the introduction of Viterbi algorithm and trellis diagram representation of PR targets in Section 2.2.3, the software implementation of a ML detector using sliding window Viterbi algorithm can be summarised as follows:

Assuming there are $M=2^{L-1}$ states for a given GPR target with length $L$.

1. Assume a known state. The path metric (output) of the known state is set to be a small value while the path metric (output) for the rest of the states are large or infinite values.
2. For each state, calculate the path metric for all the possible trajectories leading to it. The path metric is calculated as the sum of square Euclidean distance between the input samples and the expected ideal samples.
where \( p_k \) is the input sample, \( y_k \) is the ideal sample and \( N \) is the length of replay samples.

3. For the trajectories that converge to the same state, select the one with a smaller path metric and discard the others. Trace the trellis and eliminate all the discarded trajectories.

4. Continue this process until only one surviving trajectory is left at some number of steps \( k \) before the last step \( N \). The Viterbi detector stores the magnetisation states corresponding to the surviving path (i.e. input labelled on the trellis) up to step \( N-k \). The Viterbi algorithm continues with step 1, 2 and 3 with an initial known state that all the trajectories converged to at step \( N-k \).

5. The Viterbi algorithm ends at step \( N \) and outputs the magnetisation states corresponding to the surviving path as the recovered data.

The MATLAB function of the Viterbi detector can be found in Appendix 1 – Viterbi detector.

### 3.2.4 Signal-to-Noise Ratio

There are many sources of noise and signal degradation in magnetic recording systems such as: media/head noise, electronics noise, read/write distortions, which cause the definition of SNR to be different \([11, 12]\). The most commonly used definition of SNR is \([11, 12]\):

\[
\text{SNR} = \frac{\text{zero-to-peak signal power}}{\text{noise power}} = \frac{S}{N} = \frac{V_{0-p}^2}{V_{\text{rms},n}^2} \tag{3.8}
\]

where the \( \text{rms} \) noise voltage \( V_{\text{rms},n} \) could broadly include any disturbances such as medium noise, interferences, or distortions. The peak signal voltage can be defined either as the isolated signal voltage peak or as the square wave amplitude recorded at the signal frequency. SNR is most often quoted in the units of dB:

\[
\text{SNR}_{\text{dB}} = 10 \log \frac{S}{N} = 20 \log \frac{V_{0-p}}{V_{\text{rms},n}}. \tag{3.9}
\]
If there are many noise sources, $V_{n,j}$, $j = 1, 2, 3, 4, \ldots$, in a magnetic recording system, then the total average noise power is

$$N = \left( \sum_{j} V_{n,j} \right)^2 = \sum_{j} \langle (V_{n,j})^2 \rangle + \sum_{j,j \neq j} \langle V_{n,j} V_{n,j}^* \rangle. \quad (3.10)$$

If the noise sources are uncorrelated, then the total average noise power is

$$N = \sum_{j} V_{\text{rms},n,j}^2 \quad (3.11)$$

where the $\text{rms}$ noise voltage is defined as

$$V_{\text{rms},n,j} = \sqrt{\langle (V_{n,j})^2 \rangle}. \quad (3.12)$$

### 3.3 Read Channel Simulator

The software implementation of the read channel model and numerical Monte-Carlo simulation running on read channel model is detailed in this section.

#### 3.3.1 Implementation of the Read Channel Model

The complete read channel model that consists of a simulation of the replay process using a conventional GMR read sensor, and the data recovery process in a conventional PRML read channel is written in MATLAB version 7.04. Since each component in the read channel model is implemented as a MATLAB function, a top-level function which calls all the generated functions (representing each component of the read channel model) is written to simulate the data recovery process. Fig. 3.5 shows a flow chart of the read channel simulator. Initially, the random data generator generates 4096 bits of one frame as a sector of information on the HDD, and the simulation starts by feeding the system with a randomly generated frame for a specific SNR value. The generation of the step response for the specific media characteristics needs to be calculated once outside the ‘top-level’ program and is saved and loaded to the 3-D replay model. The details of how the step response is calculated in the 3-D replay model for different media configurations are
introduced in [70]. The step response needs to be recalculated when the media characteristics are changed. The read-back samples are generated based on each frame of the bits and then equalised by a digital FIR to a specific GPR target and finally recovered by Viterbi detector. The number of the bits in error is counted by comparing the recovered data at the output of the Viterbi detector with the recorded data. If one or more bits are in error then the frame is considered in error, and then a new frame of random data feeds the system at the same SNR. The iterative process stops for the specific SNR value when 50 frames (to ensure enough bits are generated to achieve a reliable BER) are in error. The same process is repeated for different SNR values. The raw BER is calculated as the number of bits in error divided by the number of input bits. For each SNR, the BER is calculated accordingly so that BER against SNR curves are generated to represent the read channel performance.

There are two sources of noise that are considered in the read channel model, one is electronics noise (i.e. AWGN) and the other is island geometry variations which manifest themselves as media noise [50, 51] in BPM systems. In the case that island geometry variations exist in the system, they are defined in terms of random shifts of the edges of each island in the 3-D replay model by Nutter et al. [51]. The details of simulating island geometry variations will be explored in the next chapter. The SNR in the simulation model is defined according to Equation 3.9, where $V_{\text{p}}$ is defined as the peak signal of an isolated replay pulse due to an ideal island. For specific SNRs, $V_{n,\text{rms}}$ can be calculated accordingly, thus the additive white Gaussian noise (AWGN) is generated by calling \textit{randn} function which generated a vector of a given length following a Gaussian distribution with mean 0 and variance 1 in MATLAB. The output of the \textit{randn} function is multiplied by $V_{n,\text{rms}}$ in order to enable the vector follow Gaussian distribution with mean 0 and variance $V_{n,\text{rms}}^2$. 


Chapter 3                                    Read Channel Simulations for BPM Systems

The read channel model aims to provide a platform to investigate every aspect that may affect the read channel performance and in the mean time allows new designs of each component to be easily introduced to the standard model. It will be used in the later work on investigation of the effect of island geometry variations on read channel performance. This research work will also present new read channel designs based on the standard read channel model is an effort to improve the read channel performance that has been affected by island geometry variations.

### 3.3.2 Monte-Carlo Simulation

The read channel performance of a magnetic recording system is evaluated as BER performance against SNR; high read channel performance means low BER at low SNR values. Numerical Monte-Carlo simulations were run on the read channel model developed in MATLAB in order to achieve numerical BER performance. It is generally recognised
that in order to get a reliable estimation of BER, the number of the samples provided for estimation should be at least \([112]\):

\[
N > \frac{10}{\text{BER}}
\]

For a BER of \(10^{-5}\), then at least \(10^6\) bits need to be generated for the simulation to ensure the reliability of the result. The Monte-Carlo simulations are performed on a personal computer running windows XP, using Intel Core2 2.66GHz with RAM of 3GB. It takes about 2-3 hours to extract a BER vs. SNR curve with the SNR range from 7dB to 14dB so it can be presumed that more time would be needed when the system incorporates a read channel design of more computation complexity than the standard read channel design. The new read channel designs will be discussed as a part of this PhD thesis work in Chapter 5.

The accuracy of the 3D replay model has been verified in [66, 70]. With the read channel model, two basic tests are used to verify the validity of the read channel model:

1. Provide ideal samples to the PRML read channel model to recover.
2. Provide replay samples with the AWGN to the PRML read channel model to recover.

Each test has been run several times to confirm the accuracy of the result. Because there is no significant variability observed in the results for the same test, it is confirmed that an adequate number of bits are generated to ensure the reliability of the read channel model. For the first test, the result shows no errors are observed. It can be concluded that when there is no noise during the replay process the read channel model will fully recover the data as expected. For the second test, the numerical BER will be compared with the analytical BER as a guidance of validity.

In conventional magnetic recording the size and bit-aspect-ratio (BAR) of the recorded magnetic domains are determined by the dimensions of the recording head, whereas in BPM the fabrication process itself determines these properties. Most approaches to fabricating BPM result in a BAR of 1, i.e., the islands are of equal size along-track and across-track. However, a large BAR is desirable when using modern read/write heads due
to a number of reasons: the ease of head fabrication, the ease of track servo, to improve write head fields, and to obtain acceptable replay waveform SNR [49]. Reference [1] lists a number of design scenarios for BPM at areal densities in the range of 1-5 Tb/in$^2$, with many of the preferred designs having a BAR greater than 1. At any given density, raising the BAR above 1 will decrease one of the dimensions (typically the length along track) of the island, potentially resulting in broader distributions of the island properties, such as island period, island size and magnetic anisotropy [49], which may have an even greater effect on read channel performance. There are two medium configurations used throughout the work documented in this thesis. Both medium characteristics shown in Table 3.1 are configured to support an areal density of 1Tb/in$^2$, but with different BAR. Most of the work is carried out under the investigation of Medium1 with BAR=4, the same experiment is repeated on Medium2 with BAR=1 in order to confirm the validity of the results. The GPR target optimised for Medium 1 and Medium 2 has coefficients [0.25 1 0.25] and [0.1 1 0.1] respectively.

<table>
<thead>
<tr>
<th></th>
<th>BAR</th>
<th>Island Length</th>
<th>Island Width</th>
<th>Island Window</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium 1</td>
<td>4</td>
<td>7.5nm</td>
<td>30nm</td>
<td>15nm</td>
</tr>
<tr>
<td>Medium 2</td>
<td>1</td>
<td>12.5nm</td>
<td>12.5nm</td>
<td>25nm</td>
</tr>
</tbody>
</table>

Table 3.1 Medium dimensions used in BPM system for read channel performance evaluation.

### 3.4 Read Channel Performance Analysis

The performance of the read channel can be evaluated in two ways: one is a Monte-Carlo simulation, which counts the number of bits in error at the output of the read channel model and calculates the ‘real’ BER [70], the other is approximating the BER through the calculation of the probability of the errors [64]. The advantage of the Monte-Carlo simulation is that it can avoid complicated calculation of BER, can look at correlated errors, not just uncorrelated ones, it is easy to understand and can be conveniently implemented in software. Any improvement made to the component of the read channel can be easily incorporated into the simulation model and the improvement of read channel performance can be directly observed from Monte-Carlo simulation. However, a Monte-Carlo simulation is time-consuming at low BER as more bits need to be generated in order to
achieve a reliable estimation of the BER (see Section 3.2.2). An analytical calculation of BER enables a rapid estimation of the BER when it reaches very few bit errors. The disadvantage of the analytical evaluation is that it is complicated to derive. Since noise arises from different sources in the read channel, the distribution of the noise is difficult to be determined so the probability of the errors is difficult to calculate. Considering the advantages and disadvantages of numerical and analytical models, it is useful to develop both methods to evaluate the BER performance of the read channel.

### 3.4.1 What is an Error Event?

The concept of an error event is essential for understanding the analytical calculation of read channel performance [78, 113, 114]. An error occurs in the Viterbi detector when the path metric of the erroneous path through the trellis is smaller than the path metric of the correct path [11]. An error event is defined as a distinct distance between the correct path and the estimated path in the Viterbi detector. In the trellis, the error event occurs when the correct path and the estimated path start to diverge at one state, and it ends when those two paths converge into another state. Fig. 3.6 shows an example of the error event in a trellis for a GPR target of length 3. The solid line shows the correct path and the dashed line shows the erroneous (estimated) path. The Viterbi detector compares the read-back samples, with the path metric to decide the transition from one state to another. It chooses the transition path for which the path metric has the minimum square distance from the replay samples. The correct path (solid line) that the Viterbi detector should take is S1-S1-S1-S1, but because of the presence of noise which varies the replay samples from the ideal case, the Viterbi detector takes the erroneous path S1-S3-S2-S1 (dashed line). An error event starts at magnetization state S1 and ends at S1, between these two states the path goes though two incorrect states. The error event can also be represented by the associated input error sequence as $e_b = b_k - d_k$, where $1 \leq k \leq l(e)$, $l(e)$ is the length of an error event, $e_b(k) = \{-2, 0, 2\}$, $b_k$ is the input data sequence to the PR channel, and $d_k$ is the estimated recovered data sequence from the Viterbi detector. The number of bit errors due to an error event is equal to the number of non-zero coefficients in $e_b$, which is represented
as \( \phi(\epsilon) \) [92]. It can be seen from the definition that the error event that extends from time \( t_1 = k \) to \( t_2 = k+2 \), will not have more than \( v-1 \) consecutive zeros between time \( t_1 \) and \( t_2 - v \), where \( v \) represents the memory in the channel (i.e. the length of the target minus one).

While there are many possible error events in the read channel, a few typical error events may be the dominant source of errors. In order to identify the typical error events, the read channel simulation was run continuously to generate \( 4096 \times 10^3 \) bits for the Viterbi detector to recover, from which the error sequences were identified and recorded. Here, Fig. 3.6 shows the trellis for the channel with the memory of 2 (i.e. the length of the target minus one).

A MATLAB function (shown in Appendix 1 – search for error events) was written to record the error sequences and search all the possible error events. If no more than two consecutive zeros are found in the input error sequences between two non-zero values, then an error event is recognized. For example, a single bit error event can be identified as an input error sequence ‘0 0 +2 0 0’ (assumed the correct input symbol i.e. the magnetisation of the bit is 1 but it is recovered as -1) and a two bits error event can be identified as ‘0 0 +2 -2 0 0’ or ‘0 0 +2 0 -2 0 0’. The distinct error events were searched in the results of the

![Fig. 3.6 Example of an error event in the trellis of Viterbi detector. The dotted line shows an erroneous path.](image-url)
read channel simulation for the case of Medium1 (BAR=4) and Medium2 (BAR=1) where only the AWGN introduced to the read channel model.

The statistical results of the single bit and multiple bit error events are shown in Table 3.2 and Table 3.3 for two media configurations Medium1 and Medium2 respectively with SNR ranging from 7dB to 12dB. Table 3.2 and 3.3 show that in all cases of SNR investigated the majority of error events are single bit in nature, for all the error events (more than 10000 bit errors) recorded, over 70% are single bit error events. Medium2 has demonstrated more significant single bit error events than Medium1. This is because increasing BAR at fixed areal density increases linear density along track (decreases island size and period) and thus increases ISI, which is more likely to cause multiple bit error events. The statistical results shown in Table 3.2 and Table 3.3 enables the analytical calculation of BER performance to be simplified by only taking into account single bit error events for the estimation. The SNR is defined in terms of AWGN in both tables.

<table>
<thead>
<tr>
<th>Error Event</th>
<th>SNR=7dB</th>
<th>SNR=8dB</th>
<th>SNR=9dB</th>
<th>SNR=10dB</th>
<th>SNR=11dB</th>
<th>SNR=12dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bit</td>
<td>73.44%</td>
<td>74.95%</td>
<td>77.40%</td>
<td>78.34%</td>
<td>81.14%</td>
<td>90.14%</td>
</tr>
<tr>
<td>2 bits</td>
<td>22.13%</td>
<td>21.70%</td>
<td>20.42%</td>
<td>20.46%</td>
<td>17.50%</td>
<td>9.86%</td>
</tr>
<tr>
<td>3 bits</td>
<td>3.69%</td>
<td>2.84%</td>
<td>1.90%</td>
<td>1.03%</td>
<td>1.14%</td>
<td>0</td>
</tr>
<tr>
<td>over 3bits</td>
<td>0.74%</td>
<td>0.52%</td>
<td>0.28%</td>
<td>0.18%</td>
<td>0.23%</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.2 Error event statistics for Medium1 of BAR=4.

<table>
<thead>
<tr>
<th>Error Event</th>
<th>SNR=7dB</th>
<th>SNR=8dB</th>
<th>SNR=9dB</th>
<th>SNR=10 dB</th>
<th>SNR=11 dB</th>
<th>SNR=12 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bit</td>
<td>94.58%</td>
<td>96.59%</td>
<td>98.11%</td>
<td>99.16%</td>
<td>99.69%</td>
<td>99.82%</td>
</tr>
<tr>
<td>2 bits</td>
<td>5.12%</td>
<td>3.27%</td>
<td>1.86%</td>
<td>0.84%</td>
<td>0.31%</td>
<td>0.18%</td>
</tr>
<tr>
<td>3 bits</td>
<td>0.3%</td>
<td>0.14%</td>
<td>0.03%</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.3 Error event statistics for Medium2 of BAR=1.

3.4.2 Analytical analysis of Read Channel Performance

The knowledge of error events can be used to estimate the error rate of the Viterbi detector. In the case that there is only AWGN in the channel, the error rate can be readily determined analytically [11]. The input samples of the Viterbi detector can be expressed as:

\[
z(k) = \sum_{i=k-1}^{i=k+1} b(i)g(kT-iT) + n(k) \tag{3.14}
\]
where \( b(i) \in \{-1, 1\} \) denotes the magnetisation of each island, \( g(kT) \) is the symbol-rate samples of the channel pulse response (i.e. GPR target), \( n(k) \) is the noise. Here for the case with only AWGN, \( n(k) \) is AWGN noise sampled at time \( k \). Equation 3.14 can be simplified as:

\[
z(k) = y(k) + n(k)
\]

where \( y(k) \) denotes the desired ideal samples. The input sample sequence to the Viterbi detector is \( z_k \). Suppose the desired ideal sample sequence is \( y_k \) and the estimated erroneous sample sequence is \( \hat{y}_k \), the square Euclidean distance from the input sequence to the correct path is represented by path metric \( m_c \) as:

\[
m_c = \left\| z_k - y_k \right\|^2 = \left\| y_k + n_k - y_k \right\|^2 = \left\| n_k \right\|^2
\]

where \( n_k \) is a vector of noise sequence \( n(k) \). For a specific error event \( \varepsilon \), the square Euclidean distance from the input sequence to the erroneous path is represented by path metric \( m_e \) as:

\[
m_e = \left\| z_k - \hat{y}_k \right\|^2 = \left\| y_k + n_k - \hat{y}_k \right\|^2 = \left\| e_y + n_k \right\|^2
\]

where \( e_y \) is the output error sequence. The error event occurs when \( m_e < m_c \), so the probability for an error event \( \varepsilon \) is

\[
Pr(\varepsilon) = Pr(\left\| n_k + e_y \right\|^2 < \left\| n_k \right\|^2) = Pr(e_y^T n_k < -\frac{1}{2} \left\| e_y \right\|^2) = Pr(e_y^T n_k < -\frac{1}{2} d^2)
\]

where \( d^2 = \left\| e_y \right\|^2 = \sum_k e_y^2(k) \) is defined as square distance. To obtain the probability in Equation 3.18 for a particular error event, we need to know the probability density distribution (PDF) of \( e_y^T n_k \). It is clear that for the read channel with only AWGN, the distribution of \( e_y^T n_k \) is a weighted sum of AWGN samples. Thus, the distribution of \( e_y^T n_k \) is a Gaussian distribution with the mean of 0 and a variance of \( \left\| e_y \right\|^2 \sigma_n^2 \), where \( \sigma_n \) is the standard deviation of \( n(k) \). The probability in Equation 3.18 then can be expressed as:

\[
Pr(e_y^T n_k < -\frac{1}{2} d^2) = Q\left(\frac{\frac{1}{2} d^2}{\left\| e_y \right\| \sigma_n}\right)
\]
where the $Q$-function $Q(x)$ is defined as the probability that a unit-variance zero-mean Gaussian noise exceeds $x$.

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$$

Let $w_H(\epsilon)$ denote the number of bit errors (the hamming weight) in the associated error event $\epsilon$, the probability of bit errors correspond to a specific error event is then obtained by multiplying the probability of the error event by its hamming weight. Therefore, the overall BER in the read channel system is the sum of all the bit error probabilities resulting from all possible error events. The BER is upper-bounded as follows [92, 111, 113-115]:

$$BER \leq \sum_{\epsilon} w_H(\epsilon) \left( \frac{1}{2} \right)^{w_H(\epsilon)} \Pr(\epsilon).$$

The calculation of BER performance is according to Equation 3.19, 3.20 and 3.21 and is implemented in MATLAB. The function can be found in Appendix 2 – BER_Calculation_AWGN.

### 3.4.3 Analytical and Numerical Performance Comparison

For two cases of the medium that have been investigated, it has been verified that with only AWGN in the read channel the dominant error events in Viterbi detector are single bit error events. Here, Equation 3.21 can be simplified to only count the error bits from the single bit error event for BER estimation. The output error sequence $\epsilon_y$ is the convolution of the GPR target and the input error sequences, so according to the GPR targets chosen for the two media, the output error sequences of for the single bit error event for Medium1 is $[0.5 2 0.5]$ and Medium2 is $[0.2 2 0.2]$. The square distance defined in Equation 3.16 in Viterbi detector for Medium1 is 4.5 and Medium2 is 4.08.

Monte-Carlo simulations run on the read channel model to get the numerical result of BER performance for the two media. The BER performance according to the analytical
calculation in Section 3.4.2 is compared with the numerical BER performance Fig. 3.7 and Fig. 3.8.

Fig. 3.7 Analytical and numerical BER performance comparison for Medium1 with BAR=4.

Fig. 3.8 Analytical and numerical BER performance comparison for Medium2 with BAR=1.
The results show that the analytical result under estimates the errors take compare with the numerical result for the two media configurations investigated. The reason the analytical result shows a better BER performance is that multiple bit error events are ignored so that the analytical calculation of BER performance counts less errors than Monte-Carlo simulation. Also as the use of a digital FIR in Monte-Carlo simulation introduces some un-equalised ISI in the simulation, which degrades the BER performance of the read channel. However, un-equalised ISI is not included in the analytical calculation, which means less noise is considered in the analytical calculation than Monte-Carlo simulation. Because of the above reasons, it is clear that analytical BER performance will show a lower BER than the numerical approach. The analytical BER performance indicates an ideal case that if the digital FIR perfectly equalise the samples into the desired GPR target and no multiple errors exist.

### 3.5 Summary

In this chapter, a comprehensive read channel model is developed to evaluate the read channel performance of BPM systems. The flow chart diagram of the read channel model is shown in Fig. 3.5. The simulation model consists of two parts, the 3-D replay model developed by Nutter et al [66] that aims to approximate the replay signal from the perpendicular BPM system and the PRML channel that aims to recover the data and evaluate the read channel performance at various media configurations. The raw BER used as a measurement of read channel performance is calculated numerically as the number of errors divided by the number of input bits. The software implementation of PRML model has been explained in this chapter and MATLAB code can be found in Appendix 1.

In addition, an analytical calculation of BER performance has also been provided for performance evaluation. The analytical BER performance can be used as a guide of the best BER result for an ideal read channel model. MATLAB implementation of the analytical model can be found in Appendix 2.
Chapter 4

Investigation of Island Geometry Variations

The aim of Chapter 4 is to investigate the effect that island geometry variations have on read channel performance in a bit-patterned media (BPM) system. Initially, island geometry variations are introduced. Then, this is followed by an investigation of the effect of island geometry variations on read channel performance. As the island geometry variations can be characterised into island position and island size, the effect of those are explored. Finally, results are summarised.

As discussed in Chapter 3 that there are several issues that affect the data recovery process of BPM systems, such as inter-track-interference (ITI) and track misregistration. This chapter will concentrate on the issue of how island geometry variations will affect the data recovery process in terms of read channel performance.

4.1 Island Geometry Variations in BPM Systems

What are island geometry variations? Island geometry variations arise due to imperfect lithography techniques that result in the fabricated islands having a large variation in their geometry. The result of this variation in island geometry is the introduction of media noise [50, 51] in the replay signal and variations in the amount of inter-symbol-interference (ISI) present, which will affect the read channel performance [51, 116, 117]. In addition, any island geometry variation poses a problem for writing data [1]. Fig. 4.1 shows a scanning electron microscope (SEM) image of sample fabricated islands. The medium produced consists of arrays of islands fabricated in Co/Pt multilayer thin film patterned using electron-beam lithography and Ar ion milling. The complete fabrication process is discussed in [118]. It is shown in Figure 4.1 that the fabricated islands exhibit a variation in both position and size. In practical media, the use of electron-beam lithography or self-assembly techniques often results in island arrays of regular island position, i.e.,
controlled period, but there may still be a severe variation in the island size [118], and the impact of size variations upon bit-error-rate (BER) performance is therefore more of an issue in practical systems [50].

Fig. 4.1 SEM image of fabricated islands. The fabricated islands exhibit the variability in both position and size [51].

4.1.1 Simulation of Island Geometry Variations

As island geometry variations can be identified as variations in position, size and a combination of position and size, the simulation of those variations are varied in the read channel model. In order to simulate island geometry variations, the distributions of such variations from the fabricated islands are being measured and approximated to a known distribution. Island arrays of nominal pitch 100, 80, 60, and 50 nm were produced by Dr Branson Belle (as shown in Fig. 4.1) and then imaged using a LEO 1530 Gemini scanning electron microscope. The resulting images were then processed by Dr Paul Nutter, using the MATLAB image processing toolbox, to determine the approximate size (in terms of the island length along track) and down track pitch of the fabricated islands. Fig. 4.2 and 4.3 illustrate probability density functions for the variation in measured island diameter (Fig. 4.2) and island pitch (Fig. 4.3) for the four island arrays investigated. It is shown in the figures that as the island pitch becomes smaller, i.e. the islands are packed closer, the distributions of
island size and position variations become broader. The distributions show that these variations are Gaussian-like in nature, which agrees with an analysis of island size variations in island arrays of increased diameter and period [119].

Fig. 4.2 Distributions for the variation in island size for four island arrays of nominal island period: 100, 80, 60, and 50 nm [51].

Fig. 4.3 Distributions for the variation in island pitch for four island arrays of nominal island period: 100, 80, 60, and 50 nm [51].
Table 4.1 lists the statistical properties of the island arrays analysed. The results in Table 4.1 show that there is a significant variation in the size and period (pitch) of the fabricated islands. In addition, it indicates that for the fabricated islands as the areal density increases (i.e. the pitch of the fabricated islands is reduced), the calculated standard deviations $\sigma$ increases (i.e. more island geometry variations). Hence demonstrating that island geometry variations will be a significant source of noise in any future ultra-high density storage system incorporating patterned media.

<table>
<thead>
<tr>
<th>Array Nominal Pitch</th>
<th>No. of islands</th>
<th>Mean Island Size</th>
<th>Island Size $\sigma$</th>
<th>Island Period $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100nm</td>
<td>140</td>
<td>39.4nm</td>
<td>2.1nm (2.1%)</td>
<td>1.7nm (1.7%)</td>
</tr>
<tr>
<td>80nm</td>
<td>117</td>
<td>34.1nm</td>
<td>2.6nm (3.2%)</td>
<td>2.0nm (2.5%)</td>
</tr>
<tr>
<td>60nm</td>
<td>150</td>
<td>32.4nm</td>
<td>2.9nm (4.8%)</td>
<td>2.3nm (3.8%)</td>
</tr>
<tr>
<td>50nm</td>
<td>180</td>
<td>28.6nm</td>
<td>3.6nm (7.2%)</td>
<td>2.7nm (5.4%)</td>
</tr>
</tbody>
</table>

Table 4.1 Island size and period statistics for e-beam lithography multilayer islands.

Island geometry variations are introduced into the three-dimensional (3-D) replay model by varying the position of the edge of each island, at which the step responses are superposed. Fig.4.4 illustrates the three cases of island geometry variations which are assumed as down track only. In the case of island position variations, the same amount of variations $\Delta_p$ is applied to each island edge in the same direction, as illustrated in Figure 4.4a), which results in a shift in the island position of $\Delta_p$. In the case of island size variations, equal variations of $\Delta_s$ is applied to each edge but in opposite directions, as illustrated in Figure 4.4b), which results in an increase in the diameter of the island by $2\Delta_s$. Finally, Figure 4.4c) illustrates the case where there is a combination of both position and size variations. Simultaneous position and size variations are generated by randomly shifting the edges of each island. The variations applied to the edge of each island are assumed to be random with a truncated Gaussian distribution, of mean zero and standard deviation $\sigma$ specified in nm, with the edge shift restricted so that there is no island overlapping.
Fig. 4.4 Introduction of jitter in patterned media results in variations in a) the position or b) the size of the islands or c) a combination of both [51].

4.2 The Effect of Island Geometry Variations

Island geometry variations are investigated separately as island position variations, island size variations and the combination of position and size variations below.

4.2.1 The effect of Island Position Variations

Let $\Delta_p$ denote the position shift of the island, then the resulting replay pulse with a position shift can be represented by $h(x-\Delta_p)$, where $h(x)$ denotes the replay pulse due to an ideal island. Differences between the replay pulse and the position-shifted pulse can be expressed approximately as:

$$\Delta h(x) = h(x) - h(x - \Delta_p) = \Delta_p \frac{dh(x)}{dx}.$$  \hspace{1cm} 4.1

It can be seen from Equation 4.1 that the effect of the position shift on the replay samples is related to the local slope of the replay pulse.

Fig. 4.5 illustrates the effect that a fixed amount of $\pm 2.5\text{nm}$ (i.e. $10\%$) island position
variation will have on the replay samples due to an isolated island using Medium2 as an example. A similar trend is observed for Medium1. Fig. 4.5 shows the pulse response due to a positively magnetised island, the pulse response is generated by the superposition of leading and lagging edge response of the island. Sample values are extracted corresponding to the (ideal) position of each island. As the pulse response of the island expands to its neighbouring islands, the sample values, shown as the symbols on the curves of Fig. 4.5, have been taken at points corresponding to the ideal sampling time (in terms of distance is the island centre) of an island, plus an island pitch before and after the island of interest. The circles indicate the ideal replay samples without position variations, the squares represent the replay samples when the island is shifted up along the track, and the triangles represent the sample values when the island is shifted down along the track. It can be seen from Fig. 4.5 that when island position shift is introduced, the central sample, corresponding to the sample at the centre of an island, is at, or close to, the peak of the pulse where the slope is near zero, and so there is a small effect on the sample value. For the samples corresponding to the ideal positions of adjacent islands, i.e. the samples at either side of the central sample, the pulse slope is larger and this will result in a variation in the amount of ISI introduced into the signal from the neighbouring islands.

An analysis was also performed to investigate how position variations affect the read-back waveforms for a random data pattern. The position variation of each island follows a truncated Gaussian distribution (to avoid island overlapping) with zero mean and a standard deviation $\sigma$ (in nm). Fig. 4.6 shows a comparison between the ideal read-back waveform and the read-back waveform when there are $\sigma=2.5\text{nm}$ (i.e. 10% of island period of Medium2) island position variations introduced into the read channel model. It is clear from Fig. 4.6 that island position variations will cause the replay waveform to shift from the nominal position while the shape of the waveform stays almost the same. It can be also seen from Fig. 4.6 that the replay waveform does not change dramatically even for large (10%) island position variations, which indicates that the media noise introduced by the island position variations may not be the major problem during the data recovery process.
Fig. 4.5 Effect of island position variations on the replay pulse samples for an isolated island.

Fig. 4.6 Replay waveforms with and without island position variations.

Fig. 4.7 and Fig. 4.8 show BER vs. signal-to-noise ratio (SNR) curves for different amounts of island position variations for the two media configurations explored. In the case of Medium1, the island position variations introduced to the simulation are 0.9nm
(6% of the island period), 1.5nm (10% of the island period) and 3.5nm (13.3% of the island period). In the case of Medium2, island position variations introduced to the simulation are 1.5nm (6% of the island period), 2.5nm (10% of the island period) and 3.5nm (14% of the island period). In both cases, variations in island position degrade the read channel performance and that the read channel performance degrades as the amount of position variations ($\sigma$) increases. At 10% island position variations for both media, the SNR is degraded by around 1dB for a BER performance around $10^{-5}$. It is demonstrated that in order to maintain a BER performance below $10^{-4}$, a position variation of 13.3% will be tolerated by Medium1 and 14% by Medium2.

Fig. 4.7 BER vs. SNR curves with island position variations present for Medium1 with BAR = 4.
Fig. 4.8 BER vs. SNR curves with island position variations present for Medium2 with BAR = 1.

4.2.2 The effect of Island Size Variations

Fig. 4.9 shows the effect that a fixed amount of island size variation will have on the replay samples due to an isolated island, taking the case of Medium2 as an illustration. The pulse responses are generated according to an isolated island with nominal size of length 12.5nm, an island with +2.5nm (i.e. 10% of the island period) size larger than the nominal size and an island with -2.5nm (i.e. 10% of the island period) size smaller than the nominal size. Replay samples are extracted at the ideal position of the islands. In Fig. 4.9, the circles indicate the replay samples of an island with nominal size, the squares indicate the replay samples of a large size island, and the triangles indicate the replay samples of a small size island. Fig. 4.9 shows that island size variations have an impact on the magnitude of the sample value corresponding to the island of interest. An increase in the island size results in an increase in the signal magnitude and a decrease in the island size results in a decrease in the signal magnitude. These signal amplitude changes are expected since the replay signal is proportional to the flux emanating from the islands and the total amount of
magnetic material present. In the case of the adjacent samples, little change in sample magnitude is observed resulting in a small change in the amount of ISI present.

Fig. 4.9 Effect of island size variations on the replay pulse samples for an isolated island. The nominal island size is 12.5nm.

The effect of island position and size variations on the read-back waveforms is compared in Fig. 4.10 for a random data pattern. The island position and size variations are simulated as truncated Gaussian distribution (to avoid island overlapping) with zero mean and a standard deviation $\sigma$ (in nm). Fig. 4.10 shows that with the same amount of variations of $\sigma=2.5$nm in position and size, the position variations correspond to the shift of the replay waveform, while the size variations are reflected as changes in the signal magnitude. It can be seen clearly from the read-back waveforms that island size variations affect the shape of the waveforms while island position variations does not. The effect of the two different variations of the island geometry (position and size) on the read-back waveforms may result in different effects on the read channel performance.
Chapter 4  
Investigation of Island Geometry Variations in BPM Systems

Fig. 4.10 Replay waveforms generated without island geometry variations, with island position variations and with island size variations.

The effect of island size variations on BER performance is investigated when a varying amount of size variation is introduced into the 3-D replay model. Island size variations are assumed to be truncated Gaussian distributions (to avoid island overlapping) for the Monte-Carlo simulations. In the case of Medium1, island size variations introduced to the simulation are 0.9nm (6% of the island period), 1.5nm (10% of the island period) and 3.5nm (13.3% of the island period). In the case of Medium2, island size variations introduced to the simulation are 1.5nm (6% of the island period), 2.5nm (10% of the island period) and 3.5nm (14% of the island period). In both media configurations, Fig. 4.11 and Fig. 4.12 demonstrate that island size variations have a considerable effect on BER performance in comparison with the case that islands are uniform in size. The BER performance degrades significantly as the island size variations increase, in order to maintain a BER target of $10^{-4}$, island size variations need to be controlled below 10% during the fabrication process. This result is comparable to the result in [50], where a BPM system of 1.5Tb/in$^2$ is simulated and where it was concluded that in order to maintain a BER of about $10^{-4}$, island size variations should not be more than 8% of the island period.
It is worth mentioning that the simulated model in [50] takes into account size variations both along track and cross track, however, this is not the case for the model documented in this thesis where size variations are only introduced in the along track direction in order to simplify the signal generation process. For Medium1 with BAR of 4, the read head width [66, 95] is smaller than the island width so the read head will not sense any change in signal flux due to island size change across the track. For Medium2 with BAR of 1, though the read head width is larger than island width and this can sense changes in signal flux due to island size change across track. It is only island size change along track that has been simulated in order to compare the result with Medium1. It can be concluded from [50] that by taking into account of island size variations along and across track, the BER performance will become worse than the BER performance simulated with island size variations along track only due to a further change in the amplitude of the resulting response.

Fig. 4.11 BER vs. SNR curves with island size variations present for Medium1 with BAR = 4.
4.2.3 Island Geometry Variations Performance Comparison

So far, the effect of each case of island geometry variations on the recovery of recorded data has been investigated. It is also worthwhile to compare how each type of island geometry variations affects the BER performance in different degrees. Fig. 4.13 and Fig. 4.14 show the BER performance vs. the standard deviation, $\sigma$ nm, of the island variations with no other sources of noise present for Medium1 and Medium2 respectively. In the case of just island position variations (not shown) then no errors are detected over the range of $\sigma$ considered. In the presence of just size variations then the number of errors increases with $\sigma$. Consequently, since position variations alone do not introduce any error, the impact of island size variations upon BER performance is identified to be more important than just island position variations. The results shown in Fig. 4.13 and Fig. 4.14 demonstrate the need to investigate the origins of errors in BPM when island size variations are present. In the case of the presence of both position and size variations (dashed line), where each has a standard deviation of $\sqrt{2}\cdot\sigma/2$ giving a total contribution of $\sigma$, then significantly more bit
errors are observed compared with the cases of position and size variations alone. The BER performance drops from $10^{-5}$ to $10^{-3}$ at $\sigma=1.5\text{nm}$ (i.e. 10% of the island period) for Medium1 and from $10^{-6}$ to $10^{-4}$ at $\sigma=2.5\text{nm}$ (i.e. 10% of the island period) for Medium2, if both position and size variations are introduced into the system.

Fig. 4.13 BER curves for $\sigma$ variation in island geometry for Medium1 with BAR=4.

Fig. 4.14 BER curves for $\sigma$ variation in island geometry for Medium2 with BAR=1.
The read channel performance is also compared in the presence of each case of island geometry variations plus AWGN. The comparison is carried out at 10% (i.e. $\sigma=1.5\text{nm}$ for Medium1 and $\sigma=2.5\text{nm}$ for Medium2) island geometry variations and the BER vs. SNR curves are shown in Fig. 4.15 and Fig. 4.16 for the two media configurations explored. Fig. 4.15 and Fig. 4.16 demonstrate that the introduction of island geometry variations, whatever its source has a detrimental effect on the performance of the read channel, and the degradation in performance arising from variations in island size is more severe than that due to variations in island position. The combination of both island position and size variations inevitably affects read channel performance the most. It can be seen that for the two media configurations supporting $1\text{Tb/in}^2$ areal density BPM systems, in order to maintain an acceptable BER performance at $10^{-4}$ [95], island position variations may be tolerant by the BPM system. However, island size variations and a combination of position and size variations degrades the read channel performance below the acceptable BER level, which needs more effort to be controlled during the fabrication process.

![Fig. 4.15 BER vs. SNR curves in the case of no jitter, $\sigma=1.5\text{nm}$ variations in island position, size and a combination of both for Medium1 with BAR = 4.](image-url)
4.2.4 The combined effect of both Position and Size Variations

An isolated replay pulse due to the combination of both island position and size variations is generated by varying each edge of an island by a random value. The combined effect of position and size variations on replay samples due to an isolated island is showed in Fig. 4.17. Fig. 4.17 only shows the replay samples extracted from an isolated island as an illustration. The blue curve shows an ideal replay waveform with a blue circle indicating the ideal replay sample. If there is a variation in the island size (assuming the island size changes smaller than the nominal size), the dashed red curve shows the replay pulse with the square indicating the replay sample. However, when both island position and size variations present the dashed red curve will shift by an amount of position variation shown as the solid red curve, which illustrates the pulse response due to combined position and size variations. Since the replay sample is still extracted at the ideal sampling time (i.e. in terms of distance at the centre of the target island shown as along track distance at 0nm in
Chapter 4

Investigation of Island Geometry Variations in BPM Systems

Fig. 4.17, the resulting replay sample is where the triangle indicates. The combined effect of island position and size variations is unlike the island position and size variation alone, it changes the amplitude of an isolated replay pulse while in the mean time shifts the position of the pulse as shown in Fig. 4.17. The change in the replay sample value due to a combination of position and size variations is expressed as Equation 4.2 in [50], which is expanded to the first-order Taylor series.

\[
\begin{align*}
r(k) &= \sum_{m=-M}^{m=M} \sum_{n=-N}^{n=N} I_{m,n} \left[ H \left( -m + \frac{\Delta T_m}{T_x}, k - n + \frac{\Delta T_n}{T_x}, a \right) + \Delta a_{mn} H' \left( -m + \frac{\Delta T_m}{T_x}, k - n + \frac{\Delta T_n}{T_x}, a \right) \right] + v(k) \\
&= \sum_{m=-M}^{m=M} \sum_{n=-N}^{n=N} I_{m,n} \left[ H \left( -m + \frac{\Delta T_m}{T_x}, k - n + \frac{\Delta T_n}{T_x}, a \right) + \Delta a_{mn} H' \left( -m + \frac{\Delta T_m}{T_x}, k - n + \frac{\Delta T_n}{T_x}, a \right) \right] + v(k)
\end{align*}
\]

where $\Delta T_m/T_x$ is the position shift across the track and $\Delta T_n/T_x$ is the position shift along the track. $\Delta a_{mn}$ is the effect of size variations and $v(k)$ is the AWGN.

Fig. 4.17 Effect of both island position and size variations on the replay pulse samples for an isolated island.

Fig. 4.18 shows the replay waveforms for a random data pattern with no geometry variations and a combination of position and size variations. The blue curve shows the replay waveform with no geometry variations and the red curve shows the replay waveform due to combined position and size variations. It can be seen from the figure that magenta curve changes both shape and the position comparing with the blue curve, which
further demonstrates that the effect of position and size variations on replay waveform exist in both shape and position.

![Replay waveforms](image)

Fig. 4.18 Replay waveforms generated without island geometry variations, with combination of island position and size variations.

The combined effect of position and size variations on BER performance is showed in Fig. 4.19 and Fig. 4.20 for the two media configurations explored. The edges of each island are randomly shifted in order to give a random combination of both position and size as described in Section 4.1.1. The amount of both position and size variations is specified as a truncated Gaussian distribution (to avoid island overlapping), of mean zero and standard deviation $\sigma$ nm, where island position variations and island size variations is of a standard deviation of $\sqrt{2}\cdot\sigma/2$ each. In the case of Medium1, the position and size variations is simulated to give a total contribution with $\sigma$ of 0.9nm (6% of the island period), 1.5nm (10% of the island period) and 3.5nm (13.3% of the island period). In the case of Medium2, the position and size variations is simulated to give a total contribution with $\sigma$ of 1.5nm (6% of the island period), 2.5nm (10% of the island period) and 3.5nm (14% of the island period).
Fig. 4.19 BER vs. SNR curves with both island position and size variations present for Medium1 with BAR = 4.

Fig. 4.20 BER vs. SNR curves with both island position and size variations present for Medium2 with BAR = 1.
It is evident that a combination of both position and size variations has a significant effect on read channel performance. The SNR is degraded by a combination of position and size variations with 6% by 3.5dB for Medium1 and 4dB for Medium2. As the variations of both position and size increases, the BER performance inevitably becomes worse dropping below $10^{-3}$ for the two media configurations investigated.

### 4.3 Summary

This chapter has explored the effect that island geometry variations have on the read channel performance for a 1Tb/in$^2$ BPM system using the two media configurations explored. Initially, island geometry variations are identified as island position variations, island size variations and a combination of both. Island geometry variations are introduced to the 3-D replay model by varying each edge of the isolated island. To introduce position variations, each edge is shifted by the same amount to the same direction. To introduce size variations, each edge is shifted by the same amount but applied to opposite directions. To introduce both position and size variations, each edge is shifted randomly. The effect of the two different variations in island geometry (position and size) on the sample values may enable them to be distinguished on an island-by-island basis, which could be put to use in an advanced detector. Simulation results demonstrate that no matter what kind of island geometry variations are present, they have a detrimental effect on the performance of the read channel, and the degradation in performance arising from variations in island size is more severe than that due to variations in island position. Through comparing the number of errors due to each case of island geometry variations, it is identified the impact of island size variations upon BER performance is more important than the island position variations. However, the combination of both island position and size variations inevitably degrades read channel performance the most.
Chapter 5

New Trellis Design for Read Channel of Bit-Patterned Media

The effect of island geometry variations on read channel performance in bit-patterned media (BPM) systems has been explored in Chapter 4. It has been demonstrated that island geometry variations have a detrimental effect on read channel performance. One question is how can we mitigate the effect of those variations? Following the investigation of the last chapter, this chapter focuses on the development of a new trellis design for Viterbi detection in order to combat the effect of island position and size variations on the read channel performance. Primarily, a general concept of the standard trellis used in read channel in BPM systems is introduced. Based on the structure of the standard trellis, a modified trellis design for position and size variations are explored separately. Finally, a performance comparison using the standard and modified trellis is presented.

5.1 The Standard Trellis Design

The standard trellis is a state diagram that lists all the possible states of the magnetic recording system and the transitions between these states for the Viterbi Detector to recover the data. Based on the information contained in the trellis, it enables the Viterbi detector to reconstruct the recorded sequence from the replay samples which have been ideally equalised into known samples. In BPM systems, the replay samples are equalised to match a generalised partial response (GPR) target that is represented in a general form as:

$$g = [g_{-k} \ g_{-k+1} \ ... \ g_{-1} \ g_0 \ g_1 \ ... \ g_{k-1} \ g_k]$$

where each value of $g$ represents signal weights of islands that contribute to the target island (i.e. the island at time 0). For example, the value $g_{-k}$ represents the signal weight of the previous $k$th island to the current island of interest (subscript 0). The target reveals the information of the signal contributions over the neighbouring islands. For the GPR target
of length $2k+1$, the contributions from $\pm k$ islands are considered. The replay sample of the target island is calculated as:

$$y(k) = \sum_{i=-1}^{1} g(i) b(k-i).$$  \hspace{1cm} 5.2$$

The standard trellis has $2^{2k}$ states, where $2k$ is the memory in the read channel (i.e. the length of the GPR target minus one). Fig. 5.1 shows a generic diagram of the standard trellis with a GPR target of $[g_1 g_0 g_1]$. There are four states shown in the trellis, where each state represents the magnetisation of the current bit and its previous bit.

Fig. 5.1 Generic diagram of the standard trellis for GPR target in BPM systems. $b_{k+1}$ and $y_{k+1}$ are the input and output symbols at time $k+1$. $y_{k+1}$ is calculated according to the GPR target chosen and the magnetisation state of the previous bits.

The states are connected by branches, which are labelled with the input symbol $b$ and the output symbol $y$. With the magnetisation state of either -1 or 1, each state has two branches to the next state. One branch represents the transition from the current state to the next state if the next coming island has a magnetisation of 1, the other branch represents transition from the current state to the next state if the next coming island has a magnetisation of -1. The branch output is calculated according to Equation 5.2.
5.2 The Modified Trellis Design

The read channel performance can be improved in a number of ways, such as an improvement in the digital equaliser, coding scheme or the detector. Previously published work has investigated read channel designs to alleviate the effect of island position variations by applying low-density parity-check (LDPC) coding schemes[116]. However, by identifying the type of island variation prevalent in BPM systems, it may be possible to improve the data recovery process without the need for additional coding schemes. In this section, two modified trellis designs will be proposed to combat the effect of island position and island size variations accordingly. The effect of the two different variations (position and size) on the sample values enables them to be put to use in the construction of the modified trellis.

Like the standard trellis, the modified trellis is constructed according to a specific GPR target. For the GPR targets chosen for Medium1 and Medium2, the signal contribution from the target island (the island of interest) is normalised to 1 and the signal contributions from the neighbouring islands on the target island are limited to the previous and the next island as it is shown in Fig. 5.2. The blue island indicates the target island. The solid blue curves indicate the replay responses due to each island individually. The dashed blue curve is the replay waveform which is the superposition (sum) of the replay pulse. The circle indicates the replay sample due to the target island. It can be seen that the replay sample of the target island results from contributions due to its own and the neighbouring islands.
Fig. 5.2 Diagram illustrating the signal contributions from the GPR target. The replay waveform (dashed line) is the superposition of isolated replay pulse (solid line).

In the case of the presence of island position variations, the constructed modified trellis must take into account the effect of position variations of the neighbouring (previous and next) islands due to the increased or decreased amount of inter-symbol-interference (ISI) which can not be ignored. The read channel needs to know the information of the position variation of each individual island so that such information can be integrated into the states of the trellis. Unlike position variations, size variations of the target island results in a significant magnitude change while little change in magnitude in the amount of ISI is observed from the size variations of the adjacent islands. The magnitude change of the target island can be integrated as the extra branch output rather than extra information in the state. The following sub sections will introduce the modified trellis designs for position and size variations separately.

5.2.1 Modified Trellis Design for Position Variations

The problem of position variations in BPM storage systems is analogous to the problem of transition jitter [46, 120, 121] in conventional magnetic storage systems, and results in a random shift in the position of the recorded information. In Zhang et al [122], a jitter-sensitive-sequence-detection (JSSD) scheme based on realistic transition noise is
proposed for high-density perpendicular magnetic recording channels. In this scheme, the transition noise is estimated jointly with the recorded bits in the Viterbi detector. Following the similarity between transition noise and island position variations, a new trellis design is proposed to combat the effect of position variations in BPM systems. The trellis design takes into account the position shift and makes an estimation based on both the magnetisation of the islands and the amount of island position shift. If we consider a GPR read channel model with only additive white Gaussian noise (AWGN) and position variations, then the symbol-rate (interval of island period) samples $y(k)$ of the partial response (PR) channel can be written as,

$$y(k) = \sum_i b_i g(kT - iT + \Delta_i T) + n(k)$$

where $b_i \in \{-1,1\}$ denotes the magnetisation of each island, $n(k)$ denotes the AWGN, $T$ is the ideal bit period, $g(t)$ is the pulse response due to the magnetisation of each island and is defined by the PR target, $g(kT)$ is the symbol-rate samples of the pulse response, $\Delta_i$ denotes the position shift due to jitter that we assume follows a Gaussian distribution. As it can be seen in Equation 5.3, the replay samples are calculated according to a predefined GPR target sampled at the bit period plus a shift $\Delta_i$ due to island position variations. When using the standard trellis, the branch output from one state to another is determined by the magnetisation state of the islands and the amplitude of the pulse response, including any ISI, at the symbol-rate samples. The most likely input sequence is then chosen by comparing the channel samples with the branch output. However, position variations in BPM storage systems leads to a deviation from the nominal position (symbol-rate in time) of each island, which affects the detection process in selecting the most likely input sequence and introduces errors in the recovered data. It is worth mentioning that the position variation for each island is randomly Gaussian distributed but the modified trellis assumes a fixed amount of position variations for all the islands. The proposed modified trellis design introduces discrete position shift values into the standard trellis structure so that both the magnetisation states and the contributions of position jitter are considered when selecting the most likely input sequence. In the modified trellis, the states are defined by both the magnetisation state of the islands, $b_i$, and the discrete values representing a
position shift of each island, $\Delta_i$.

Fig. 5.3 Part of the proposed extended trellis structure designed to combat the effects of position jitter in BPM. Shaded blocks indicate additional new states. The position shift is assumed to be three discrete values \{-2.5, 0, 2.5\}.

Fig. 5.3 shows part of the modified extended trellis structure with the additional states introduced using this approach (shaded boxes). Also illustrated are some of the possible transitions from the states when $b_k = b_{k-1} = 1$. It can be seen that each state in the modified
trellis represents the magnetisation state of the current island, the magnetisation state of one or more previous islands, as well as discrete jitter values for each island. In general, for a GPR target of length $L$, the isolated pulse extends over $L-1$ bit periods so the state in the modified trellis is required to represent the magnetisation state and the position shift of $L-1$ bits. Assuming island position shift has one of the $Q$ discrete values for each island, and because the magnetisation state of each island is either 1 or -1, the total number of states is given by $2^{L-1}Q^{L-1}$. The branch between two states is defined following the standard trellis with the only difference being that each state has $2Q$ branches. The branch represents the transition from the current state to the next state if the next coming island has a magnetisation of 1 or -1 and the position shift of the next coming island is one of the $Q$ discrete values. The extra states and the branch outputs of the modified trellis are changed as the discrete position shift values are varied. Here, Fig. 5.3 shows an example with a GPR target of length $L=3$ and $Q=3$ discrete values of position shift \{-2.5nm, 0, +2.5nm\} are assumed in the modified trellis. The whole part of the modified trellis will have an additional 28 states, giving a total of 36 states. The branch output, $y_k$, is calculated knowing the effect that any fixed position shift has on the amplitude of the isolated pulse response and the effect of changes in ISI due to the shift of the neighbouring islands. The branch output, $y_k$, from the state to the next state at time $k$ can be calculated using

$$y_k(S_i, S_{i+1}) = \sum_i b_i g(kT - iT + \Delta_i T)$$  \hspace{1cm} (5.4)$$

where the variables have their usual meaning. In the read channel model, the bit period $T$ in Equation 5.4 corresponds to the island period in nm. The values of $g(kT-iT)$, where $\Delta_i=0$, correspond to the samples at the ideal sampling time as indicated by the circles in Fig. 4.5. The values of $g(kT-iT+\Delta_i T)$ correspond to the samples $\pm \Delta_i$ nm away from the ideal samples as indicated by the squares/triangles in Fig. 4.5. For example, transition from state S1 to S7 in Fig. 5.3 represents the next bit (i.e. at time $k+1$) with magnetisation state of 1 have a position shift of 2.5nm and the current bit (i.e. at time $k$) with magnetisation state of 1 have position shift of -2.5 nm. The ideal sample from state S1 to S7 is given by expanding Equation 5.4 as,
\[ y_k = b_{k-1}g(T + \frac{(-2.5)}{25}T) + b_kg(\frac{-2.5}{25}T) + b_{k+1}g(-T + \frac{2.5}{25}T) \]
\[ = b_{k-1}g(T - 0.1T) + b_kg(-0.1T) + b_{k+1}g(-T + 0.1T). \]

For Medium1, the island period is 25nm so \( T=25nm \). \( g(T-0.1T) \) is the replay sample value at 22.5nm, \( g(-0.1T) \) is the replay sample value at -2.5nm and \( g(-T+0.1T) \) is the replay sample value at -22.5nm in Fig. 4.5.

### 5.2.2 Performance Improvement with Modified Trellis for Position Variations

In order to examine the read channel performance of the modified trellis for position and size variations, a standard trellis used for the Viterbi detector has been replaced by a modified trellis as shown in the modified read channel model in Fig. 5.4.

The read channel model shown in Fig. 5.4 excludes the digital equaliser because the un-equalised ISI due to the imperfect equalisation process will add nonlinear noise to the read channel model while the modified trellis is designed for the media noise due to island position variations [123]. The effect of un-equalised ISI to the modified trellis is
unpredictable. Fig. 5.5 shows the BER performance between the equalised signal and the un-equalised signal of Medium2 with position variations of 1.5nm (i.e. 10%). The result shows that the un-equalised ISI results in poorer BER performance. Here, this work only concentrates on investigating the read channel performance of the modified trellis with position variations so the digital equaliser is excluded from the read channel simulation in order to avoid the unwanted effect of un-equalised ISI.

The read channel performance of the modified trellis for position variations is examined for Medium1 and Medium2. Table 4.1 in Section 4.1.1 presenting the standard deviation $\sigma$ of island geometry variations from the fabricated medium [51], has shown that as the island pitch (period) decreases, the standard deviation $\sigma$ of the island position and size increases [51]. If the trend shown in Table 4.1 is followed, it will reach a standard deviation $\sigma$ of 10% island position variations at the dimensions adopted for the two media configurations for 1Tb/in$^2$ recording, so the simulation will adopt an initial Gaussian distributed island position variation of $\sigma=10\%$. Although the position variation is randomly Gaussian distributed, a fixed amount of 10\% (i.e. $\pm1.5\text{nm}$ for Medium1 and $\pm2.5\text{nm}$ for
Medium2) position variation is assumed in the modified trellis. Thus, the discrete levels of position shift \{-1.5\text{nm}, 0, +1.5\text{nm}\} and \{-2.5\text{nm}, 0, +2.5\text{nm}\} are chosen in the modified trellis for Medium 1 and Medium 2 respectively. The minus position shift represents the island shift to its left along the track and the positive position shift represents the island shift to its right along the track. Fig. 5.6, 5.7 and 5.8 show the bit-error-rate (BER) performance by adopting the modified trellis design with the position variations from 1.5nm, 2nm and 2.5nm for Medium1 respectively. The read channel model adopting the modified trellis design offers improved performance as clearly evidenced by the BER vs. signal-to-noise ratio (SNR) curves shown in Fig. 5.6, 5.7 and 5.8. Even as the amount of position variation is increased, the modified trellis still offers a significant improved performance (above 1dB SNR improvement at a BER below $10^{-4}$ for $\sigma \geq 10\%$) compared to the standard trellis.

Fig. 5.6 BER vs. SNR curves with 1.5nm position variations using the standard & modified trellis for Medium1 with BAR=4.
Fig. 5.7 BER vs. SNR curves with 2nm position variations using the standard & modified trellis for Medium1 with BAR=4.

Fig. 5.8 BER vs. SNR curves with 2.5nm position variations using the standard & modified trellis for Medium1 with BAR=4.
Fig. 5.9, 5.10 and 5.11 show the BER performance by adopting modified trellis design with the position variations from 2.5nm, 3nm and 3.5nm for Medium2 respectively. The result illustrates the improvement in BER performance is observed when adopting the modified trellis design compared to the use of the standard trellis with the same amount of position variations present. The results demonstrate that even with increased island position variations, the SNR improvement is still maintained at about 0.8dB for BER below $10^{-4}$ by using the modified trellis design in read channel model.

![BER vs. SNR curves](image)

Fig. 5.9 BER vs. SNR curves with 2.5nm position variations using the standard & modified trellis for Medium2 with BAR=1.
Fig. 5.10 BER vs. SNR curves with 3.0nm position variations using the standard & modified trellis for Medium2 with BAR=1.

Fig. 5.11 BER vs. SNR curves with 3.5nm position variations using the standard & modified trellis for Medium2 with BAR=1.
Concluding from the above simulation results for two different media configurations, it is clearly evidenced that the adoption of the modified trellis in read channel model demonstrates an improved BER performance compared to the standard trellis.

### 5.2.3 Modified Trellis Design for Size Variations

Island size variations are inherent in BPM and significantly inhibit the ability to recover stored information as shown in Section 4.2.2. In the absence of island size variations the input samples to the Viterbi detector are the ideal samples according to a GPR target plus the AWGN. However, in the presence of size variations the magnitude of the read-back samples will deviate from the ideal samples since island size variations manifest themselves as amplitude changes in the replay waveform; this will inevitably reduce the BER performance of the read channel. A modified trellis design for size variations is proposed that changes the standard trellis structure by adding extra branches between two states where the branch output of the extra branches represents changes in the ideal sample amplitude due to island size variations.

For the GPR targets considered for the two media configurations, the replay sample for each bit is composed of signal contributions due to three consecutive islands. Assuming that an island can become either smaller or larger than the nominal size due to size variations, this will result in eight possible combinations for each bit, where the sizes of the three islands of interest can be as \{small, normal, large\}. However, as the variation in the size of the current island dominates the signal amplitude [117], and that variations in the amount of ISI due to size variations of the neighbouring islands is small, then the variations in ISI can be ignored. Hence, the extra branches in the modified trellis take into account the size variations of the target island only. The eight combinations of island sizes discussed are reduced to three combinations representing the cases where the target island of interest is the nominal (ideal) size, where it is smaller and where it is larger.

The modified trellis structure for island size variations is shown in Fig. 5.12, which has the same number of states as the standard trellis for the same target. However, two extra
branches are introduced between each two connected states in order to represent the two extra cases of a “small” and “large” island discussed. The detector will choose the branch for which the branch output is closest to the actual input read-back samples.

\[ y(k) = \begin{cases} 
  g \ast b_k + \alpha b(k) & \text{Large } - \alpha \\
  g \ast b_k & \text{Normal} \\
  g \ast b_k - \beta b(k) & \text{Small } - \beta 
\end{cases} \]

where \( g \) is the GPR target, \( b_k \) is a vector notation of the magnetisation state \{-1,1\} of the islands along a particular track. Here, \( \alpha \) and \( \beta \) are the resulting signal amplitude changes due to the target island (i.e. the island that being read at time \( k \)) becoming larger and smaller respectively. It is important to determine optimum values of \( \alpha \) and \( \beta \) in order to maximise the performance of the modified trellis.

Optimum values of \( \alpha \) and \( \beta \) are determined by running the simulation using the modified
trellis that assumes a fixed amount of island size change and varies $\alpha$ and $\beta$ according to the values shown in Table 5.1 for Medium1 and Table 5.2 for Medium2. For example, if the modified trellis assumes a fixed island size change of $\pm1$nm, the branch output is calculated according to Equation 5.6 by using $\alpha=0.1210$ and $\beta=0.1251$ for Medium1. The values of $\alpha$ and $\beta$ shown in the following tables are determined by measuring the change in the signal amplitude as an island is changed in size (large and small) by a fixed amount.

<table>
<thead>
<tr>
<th>Island Size Change ±(nm)</th>
<th>Signal Amplitude Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
</tr>
<tr>
<td>1</td>
<td>0.1210</td>
</tr>
<tr>
<td>2</td>
<td>0.2380</td>
</tr>
<tr>
<td>3</td>
<td>0.3505</td>
</tr>
<tr>
<td>4</td>
<td>0.4583</td>
</tr>
<tr>
<td>5</td>
<td>0.5613</td>
</tr>
<tr>
<td>6</td>
<td>0.6594</td>
</tr>
<tr>
<td>7</td>
<td>0.7525</td>
</tr>
</tbody>
</table>

Table 5.1 List of values of $\alpha$ and $\beta$ corresponding to the known island size change assumed in modified trellis for Medium1 with BAR=4.

<table>
<thead>
<tr>
<th>Island Size Change ±(nm)</th>
<th>Signal Amplitude Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
</tr>
<tr>
<td>1</td>
<td>0.0624</td>
</tr>
<tr>
<td>2</td>
<td>0.1216</td>
</tr>
<tr>
<td>3</td>
<td>0.1776</td>
</tr>
<tr>
<td>4</td>
<td>0.2303</td>
</tr>
<tr>
<td>5</td>
<td>0.2798</td>
</tr>
<tr>
<td>6</td>
<td>0.3263</td>
</tr>
<tr>
<td>7</td>
<td>0.3697</td>
</tr>
<tr>
<td>8</td>
<td>0.4103</td>
</tr>
</tbody>
</table>

Table 5.2 List of values of $\alpha$ and $\beta$ corresponding to the known island size change assumed in modified trellis for Medium2 with BAR=1.

The optimal values of $\alpha$ and $\beta$ are explored at size variations of $\sigma=1.5$nm (i.e. 10% of island period) for Medium1 without AWGN present. Fig. 5.13 shows the simulation result where the BER performance has been plotted as a function of the fixed island size change assumed in the modified trellis. From Table 5.1 $\alpha$ and $\beta$ are varied according to a known island size change (0 to 7nm) that eventually determines the outputs of the extra branches. The optimum
BER performance is observed for values of $\alpha$ and $\beta$ that are a result of a known size change in the modified trellis of $\approx \pm 4\text{nm}$.

![Graph of BER vs. Island size change in modified trellis](image)

Fig. 5.13 Island size changes in the modified trellis to determine an optimal BER performance for Medium1 with BAR=4.

The optimal BER performance that can be achieved using the modified trellis is further compared in the BPM system with $\sigma=1.5\text{nm}$ island size variations and AWGN present for the case of Medium1. The BER vs. SNR curves are plotted in Fig. 5.14 for different fixed amount of island size changes assumed in the modified trellis. Fig. 5.14 and Fig. 5.13 show the same numerical result that optimal BER performance can be achieved with the use of the modified trellis with a known island size change of $\approx \pm 4\text{nm}$.
Fig. 5.14 BER vs. SNR curves to show an optimal BER performance achieved by modified trellis for Medium1 with BAR=4.

The same simulation has been carried out for the case of Medium2 in order to plot the BER vs. SNR curves for different fixed amounts of island size changes assumed in the modified trellis. It is introduced with $\sigma=2.5\text{nm}$ (i.e. 10% of island period) island size variations and AWGN present for the case of Medium2 in the BPM system. The island size changes assumed in the modified trellis are fixed to 2.5nm, 3.5nm, 4.5nm, 5.5nm and 6.5nm. Fig. 5.15 shows that the BER performance are almost the same for all the island size change assumed in the modified trellis and no predominant optimal BER performance is observed. Since the values of $\alpha$ and $\beta$ set for the modified trellis do not affect the BER performance in a certain degree for the case of Medium2, it is concluded that the values of $\alpha$ and $\beta$ as a result of a known size change in the modified trellis for the case of Medium2 can be chosen to any value listed in Table 5.2.
The read channel performance of the modified trellis for size variations is examined for the two media configurations. It is assumed the island size changes are of fixed ±3.5nm and
±2.5nm in the modified trellis for Medium 1 and Medium 2 respectively. The values are chosen based on the optimal BER performance demonstrated in Fig.5.14 and Fig.5.15.

Fig. 5.16 and Fig. 5.17 show the BER performance by adopting the modified trellis compared to the standard trellis with size variations of 1.5nm and 2nm for Medium 1. It is demonstrated in Fig. 5.16 that about 3dB improvement in SNR at BER performance of $10^{-4}$ for a size variation of 1.5nm. The same amount of improvement in SNR is demonstrated in Fig. 5.17 at BER performance of $10^{-3}$ for a size variation of 2.0nm.

![Fig. 5.16 BER vs. SNR curves with 1.5nm size variations using the standard & modified trellis for Medium1 with BAR=4.](image)

Fig. 5.16 BER vs. SNR curves with 1.5nm size variations using the standard & modified trellis for Medium1 with BAR=4.
Chapter 5                      Investigation of Island Geometry Variations in BPM Systems

Fig. 5.17 BER vs. SNR curves with 2.0nm size variations using the standard & modified trellis with varied size variations for Medium1 with BAR=4.

Fig. 5.18 and Fig. 5.19 show the BER performance by adopting the modified trellis compared to the standard trellis with size variations of 2.5nm and 3nm for Medium2 respectively. The result shows the improvement in BER performance that is observed when adopting the modified trellis design in the presence of size variations, compared to the use of the standard trellis with the same amount of size variations present for both media. The result demonstrates more than 1dB SNR improvement for a BER below $10^{-4}$ by using the modified trellis design. Comparing the improvement achieved using the modified trellis for the two media considered here, the results indicate that a BPM system with high BAR will benefit more from the modified trellis than with low BAR for the head design adopted in 3-D replay model. The complexity of the modified trellis is only three times (extra branches in the modified trellis) greater than that of the standard trellis for the GPR targets chosen. In general, considering the improvement achieved and complexity of the modified trellis, the modified trellis is an attractive choice to alleviate the effect of island size variations.
Fig. 5.18 BER vs. SNR curves with 2.5nm size variations using the standard & modified trellis for Medium2 with BAR=1.

Fig. 5.19 BER vs. SNR curves with 3.0nm size variations using the standard & modified trellis for Medium2 with BAR=1.
5.3 Summary

This chapter has proposed new trellis designs for use in the Viterbi detector of the read channel in BPM systems in order to combat the effect of island position and size variations. The read channel performance is compared with the use of a standard trellis and a modified trellis in the read channel model. The fact that island position and size variations affect replay samples in a different way makes the modified trellis design to be different for each case of island geometry variations. For island position variations, the position shift of the two neighbouring islands affects the replay sample of the target island so the position shift of the neighbouring islands needs to be recorded each time for the calculation of the output sample in the modified trellis design. Since the modified trellis needs to know the history information such as magnetisation state and position shift of the previous islands, it results in both extra states and branches compared to the standard trellis design. For island size variations, the sample magnitude change is mainly due to the island size change of the target island, while island size change of the two neighbouring islands has a minimum effect on the sample magnitude change. Thus, it is not necessary to know any island size change information from the neighbouring islands. The modified trellis only knows information such as magnetisation state of the neighbouring islands, so the number of the states is as the same as the standard trellis. The extra branches introduced into the modified trellis in order to represent the amplitude changes in replay samples due to the size change of the target island.

The improved BER performance introduced by adopting the modified trellis in the read channel model has been demonstrated for media configurations of BAR=1 and BAR=4. The results show that the read channel performance has been improved by the use of modified trellis design compared with the standard trellis design for the same amount of island position and size variations. For example, the SNR improvement is maintained at about 3dB at a BER of $10^{-4}$ for Medium1 with $\sigma=1.5\text{nm}$ (i.e. 10% of island period) size variations. The next chapter will mainly concentrate on the analytical approach of the read
channel performance by using the modified trellis design. The analytical and numerical BER performance will be compared.
Chapter 6

The Analytical Model of the Read Channel for Bit-Patterned Media

In the previous Chapters, numerical Monte-Carlo simulations have been used where the bit-error-rate (BER) performance of the read channel is calculated by counting the number of bits in error at the output of the model. The problem with this approach is that it can be time consuming especially for moderate to high signal-to-noise ratios (SNR) where the BER should be very low. It is recognised in Chapter 5 that island position variations alone hardly produce any error so an analytical BER estimation based on probability calculation of the errors may not apply to island position variations. In this chapter, an analytical approach for performance evaluation in a BPM system where media noise due to island size variations dominates is proposed. The performance of the new trellis design for the island size variations is also analysed according to an analytical model. The analytical approach is developed in order to help predict the BER performance of island size variations efficiently.

6.1 Error Events due to Island Size Variations

The analytical estimation of the BER performance relies on the calculation of the probability of error events due to island size variations. The following analysis will consider the identification of error events in a BPM system for the case of Medium1. The analytical approach can be applied to any media configuration.

6.1.1 Size Changes of the Islands involved in Error Events

The definition of error events is introduced in Section 3.3.1. Typical error events due to the presence of island size variations are found by running the read channel simulation
continuously to generate $4096 \times 10^3$ bits (i.e. 1000 sectors with each sector of 4096 bits) in
the presence of random island size variations for the Viterbi detector to recover. A
MATLAB function (shown in Appendix 1 – search for error events) has been written to
record the error sequences and search all the possible error events. It is identified that for
the generalised partial response (GPR) target of length $L$, if no more than $L-1$ consecutive
zeros are found in the error sequences between two non-zero values, then an error event is
recognised [114].

Distinct error events are found by searching through the output of the read channel
simulation for random island size (length along-track) variations with standard deviation $\sigma$
-ranging from 1.5 nm to 3.5 nm (10% to 23% of nominal island length) for Medium1. The
results are tabulated in Table 6.1, which shows that in all cases that investigated, the single
bit error events account for the majority of the error events observed (over 95% in all
cases).

<table>
<thead>
<tr>
<th>$w_{11}(e_{b})$</th>
<th>Standard Variation of Island Size Variations ($\sigma$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.5nm</td>
</tr>
<tr>
<td>1 bit</td>
<td>100%</td>
</tr>
<tr>
<td>2 bits</td>
<td>0%</td>
</tr>
<tr>
<td>3 bits</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 6.1 Error Event Statistics for different amount of size variations.

Once the error events have been identified the island properties (size and magnetisation
state) corresponding to the islands in the vicinity of each error event were investigated. For
the GPR target chosen, three consecutive islands are considered that are centred on the
island for which the magnetisation state was recovered in error. The eight possible
combinations of magnetisation states for these three islands can be categorised into the
four cases listed in the first column of Table 6.2. In all cases the single bit error was only
observed when the size of the central island (the one recovered in error) was ‘small’
compared to the nominal size of an island along track of 7.5 nm for Medium1. In the case
where the magnetisation state (i.e. ‘+’ represents a positively and ‘-’ represents a
negatively magnetised island) of the neighbouring islands differ from that of the central
island (+ − + or − + −) then an error event is more likely to be observed when they are ‘big’ compared to the nominal island size. Similarly, when the magnetisation state of the neighbouring islands is the same as that of the central island (− − − or + + +) then an error event is more likely to be observed when they are ‘small’ compared to the nominal size.

<table>
<thead>
<tr>
<th>Magnetisation State</th>
<th>Size Change Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>−, +, − or −, +</td>
<td>big, <strong>small</strong>, big</td>
</tr>
<tr>
<td>−, −, − or +, +</td>
<td>small, <strong>small</strong>, small</td>
</tr>
<tr>
<td>−, +, + or +, −</td>
<td>big, <strong>small</strong>, small</td>
</tr>
<tr>
<td>−, −, + or +, −</td>
<td>small, <strong>small</strong>, big</td>
</tr>
</tbody>
</table>

Table 6.2 Size change statistics for the three islands of interest.

Considering the island in error is the target island, the island size distributions of the target island, the previous and the next island are recorded for investigation. Fig. 6.1 shows island size distributions of the three islands involved in all single bit error events recorded when there are varying amount of island size variations present. Fig. 6.1 shows that in all the single bit error events observed, the size of the target island is smaller than the nominal size of 7.5nm. The distribution of the target island size is fitted to Gaussian function as the blue curve shows, but truncated at 0nm as the island size can not be less than 0nm. The size distributions for the neighbouring islands (indicated as previous and next) correspond to two weighted Gaussian distributions as the blue curve shows. The two peaks of the distribution are arising from the “small” (peak is at island size smaller than 7.5nm) and “big” (peak is at island size larger than 7.5nm) cases observed in Table 6.2.
Fig. 6.1 Size distributions of the three islands of interest centred on the island corresponding to the data bit recovered in error. Nominal island size is 7.5 nm. Four values of $\sigma$ are considered: (a) 2.0 nm, (b) 2.5 nm, (c) 3 nm, (d) 3.5 nm.

Table 6.3 summarises mean values ($\mu_d$) and standard deviations ($\sigma_d$) for the distributions for the central islands and a combination of the distributions of the neighbouring islands where an error event occurred, for each case of size variation ($\sigma$) investigated. In the case
of the central island that is recovered in error, as the island size variation ($\sigma$) is increased then the mean value of the island distribution ($\mu_d$) decreases, but the standard deviation ($\sigma_d$) increases. In the case of the neighboring islands, a similar decrease in mean value is observed for the “small” islands, and an increase in mean value is observed for the “big” islands as the size variation increases. In both cases the standard deviation of the distributions increases as the size variation increases.

<table>
<thead>
<tr>
<th>Size Variation $\sigma$ (nm)</th>
<th>Small Neighbours</th>
<th>Big Neighbours</th>
<th>Small Central Island</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_d$ (nm)</td>
<td>$\sigma_d$ (nm)</td>
<td>$\mu_d$ (nm)</td>
<td>$\sigma_d$ (nm)</td>
</tr>
<tr>
<td>2.0</td>
<td>5.0</td>
<td>1.8</td>
<td>10.1</td>
</tr>
<tr>
<td>2.5</td>
<td>4.7</td>
<td>2.0</td>
<td>10.2</td>
</tr>
<tr>
<td>3.0</td>
<td>4.6</td>
<td>2.3</td>
<td>10.3</td>
</tr>
<tr>
<td>3.5</td>
<td>3.8</td>
<td>2.9</td>
<td>10.6</td>
</tr>
</tbody>
</table>

Table 6.3 Distribution statistics for the three islands of interest with a nominal size of 7.5nm.

It can be concluded from the results shown in this section that particular combinations of island size and magnetisation state for a group of three islands are more likely to result in single bit error events and here more importantly when the central island is small (1.6nm when $\sigma = 2$nm, 0.9nm when $\sigma = 3.5$nm) in comparison with the nominal size of 7.5 nm. These results show that island size needs to be carefully controlled in the fabrication process in order to prevent single-bit errors from occurring, thus improving the BER performance. It is also demonstrated a useful conclusion that dominant error events are single bit in nature. Using this conclusion, the BER performance with island size variations can be estimated analytically. The analytical calculation will be introduced in the following sections.

### 6.2 Analytical Analysis of the Standard Trellis with Size Variations

Most of the work related to evaluate the read channel performance in BPM systems has been performed using Monte-Carlo simulations [15, 49, 124]. However, Monte-Carlo simulations are time-consuming, especially for moderate to high SNR where the BER will be very low. Using an analytical approach to estimate the BER is preferable as it can provide a prediction of the BER more efficiently and a way to easily analyse the effects of
different factors on the BER performance. Such analysis can be useful in designing
equalisers and detectors. Analytical BER estimation for partial-response maximum-likelihood (PRML) channels with additive white Gaussian noise (AWGN) has
been investigated in many previous works [78, 84, 85, 92, 113, 114, 125]. Different sources
of noise are considered in different analytical models, in [84, 92, 113, 114, 125]
data-dependent jitter noise and filtered AWGN were considered in the BER analysis of
PRML channels. More recently, authors [111] proposed an analytical approach to evaluate
the read channel performance under a noise condition with inter-track-interference (ITI),
un-equalised inter-symbol-interference (ISI) and AWGN for BPM systems.

Island position variations are not of interest for an analytical BER estimation in this thesis
for two reasons: first, island position variations can be easily controlled by current
fabrication techniques [49], and second, it has been recognised in Chapter 5 that the effect
of island position variations on read channel performance is small compared to that of
island size variations. Even with the presence of 10% position variations in BPM systems,
an acceptable BER performance of $10^{-4}$ can still be achieved. Here, an analytical model
considering size variations is developed for the evaluation of the BER performance in
BPM systems.

6.2.1 Development of an Analytical Model for the Standard Trellis

It is introduced in Section 3.3.2 that the BER performance of a PRML read channel using
the standard trellis can be approximated using Equation 3.21 when there is only AWGN
present [5] and the typical error events are single bit error events. Since it is demonstrated
in a PRML read channel using the standard trellis the single bit error event is also the
typical error event in the presence of island size variations, Equation 3.21 can be used to
calculate the BER performance of a BPM system with island size variations present. In the
case where island size variations are present, for the GPR target chosen for the media
configuration investigated, the noise due to size variations of the target island as well as the
noise due to size variations of two neighbouring islands is considered. As the media noise
due to the island size variation of the neighbouring islands (previous and next) contribute
equally to the noise seen by the target island, the amount of media noise on the target island can be specified as

\[ s(k) = 2s_{-1}(k) + s_0(k) \]  \hspace{1cm} 6.1

where \( s_{-1}(k) \) denotes media noise due to the size variations of the neighbouring islands and \( s_0(k) \) denotes media noise due to the size variations of the target island. The total amount of noise for a BPM system is:

\[ n(k) = v(k) + s(k) \]  \hspace{1cm} 6.2

where \( v(k) \) is AWGN and \( s(k) \) is as in Equation 6.1 and is the media noise introduced by the contribution of island size variations. According to Equation 3.21, the distribution of the noise (i.e. \( n(k) \)) needs to be known. Since the distribution of \( v(k) \) is known as a Gaussian distribution, the question of determining the distribution of \( n(k) \) becomes one of determining the distribution of \( s(k) \). According to Equation 6.1, the distribution of \( s_0(k) \) and \( s_{-1}(k) \) need to be determined separately. The distribution of \( s_0(k) \) and \( s_{-1}(k) \) are obtained in a statistical way by running a read channel simulation to generate a large amount of samples of \( s_0(k) \) and \( s_{-1}(k) \), and the resulting distribution of \( s_0(k) \) and \( s_{-1}(k) \) are plotted. In order to explore the distribution of \( s_0(k) \) (i.e. the signal amplitude changes due to the target island size change), the island period is assumed to be large relative to the island size so the pulse response due to each single island does not span to the neighbouring islands, and there is no ISI present. The replay samples are extracted at the position corresponding to the centre of each island. \( s_0(k) \) is the difference between the ideal replay samples and the replay samples with the media noise. Fig. 6.2 demonstrates distributions of \( s_0(k) \) for the island size variations \( \sigma \) from 1.5nm to 2.5nm. It can be seen from the figure that \( s_0(k) \) can be approximated by a zero-mean Gaussian distribution.
Fig. 6.2 Distributions of the unwanted signal (noise) due to the central island size changes for of $\sigma$ values of: (a) 1.5 nm, (b) 2.0 nm, (c) 2.5 nm.

In order to explore the distribution of $s_1(k)$ (i.e., signal amplitude changes resulting from the neighbouring island size changes affecting the signal amplitude of the target island), first, the island period is assumed to be sufficiently far apart so the ISI combination due to the neighbouring islands could be isolated. Then instead of extracting replay samples at the centre of each island, the replay samples are extracted at the position away from the centre of each island. The distance from the sampling position and the centre of the island is the nominal island period of the medium, which means the replay samples extracted are the residual signals that each island spans to its next/previous island. $s_1(k)$ is the difference between the ideal replay samples and the replay samples with the media noise. Fig. 6.3 demonstrates distributions of $s_1(k)$ for the island size variations $\sigma$ from 1.5nm to 2.5nm. It is shown that the distribution $s_1(k)$ can be also fitted into zero-mean Gaussian distribution so the distribution of $s(k)$ can be treated as a zero-mean Gaussian distribution.

Fig. 6.3 Distributions of the unwanted signal (noise) due to the neighbouring island size changes, for $\sigma$ values of: (a) 1.5 nm, (b) 2.0 nm, (c) 2.5 nm.
The variance (square of the standard deviation $\sigma$) of the distribution shown in Fig. 6.2 and Fig. 6.3 can be derived by MATLAB function `normfit`. It is noticed that the variance given by the MATLAB function is the $rms$ deviation. Equation 6.3 gives the $rms$ deviation of the size variation noise $s(k)$:

$$D_{rms}(s_k) = D_{rms}(s_{0k}) + 4D_{rms}(S_{-1k}). \quad (6.3)$$

The magnetisation state of the islands can be positive or negative so the total variance of the media noise due to either positive or negative magnetisation can be expressed as:

$$D(s_k) = \sqrt{\langle D_{+} \rangle^2 + \langle D_{-} \rangle^2} \quad (6.4)$$

where $\langle D_{+} \rangle$ and $\langle D_{-} \rangle$ are the average variance of the island with positive and negative magnetisation. The $rms$ deviation of $s(k)$ is also expressed as:

$$D_{rms}(s_k) = \sqrt{\langle D_{+} \rangle^2 + \langle D_{-} \rangle^2} \quad (6.5)$$

According to equation 6.3, 6.4 and 6.5, the variance of the media noise can be derived as:

$$D(s_k) = \sqrt{2 \cdot D_{rms}(s_k)} = \sqrt{2 \cdot (D_{rms}(s_{0k}) + 4D_{rms}(S_{-1k}))}. \quad (6.6)$$

$D_{rms}(s_{0k})$ and $D_{rms}(S_{-1k})$ are derived according to the statistical distribution shown in Fig. 6.2 and Fig. 6.3 and the distribution is fitted into a Gaussian distribution by calling the `normfit` function in MATLAB. Table 6.4 lists the $rms$ variance of the distribution of $s_{0}(k)$ and $s_{1}(k)$ with island size variations from 1.5nm to 2.5nm for Medium1. Table 6.5 lists $rms$ variance of the distribution of $s_{0}(k)$ and $s_{-1}(k)$ with island size variations from 2.5nm to 3.5nm for Medium2. The corresponding variance of the media noise $s(k)$ due to island size variations is also listed in the table.

<table>
<thead>
<tr>
<th>Size Variation $\sigma$ (nm)</th>
<th>$D_{rms}(s_{0k})$</th>
<th>$D_{rms}(S_{-1k})$</th>
<th>$D(s_k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5nm</td>
<td>0.0338</td>
<td>0.0028</td>
<td>0.0638</td>
</tr>
<tr>
<td>2.0nm</td>
<td>0.0599</td>
<td>0.0051</td>
<td>0.1135</td>
</tr>
<tr>
<td>2.5nm</td>
<td>0.0927</td>
<td>0.0080</td>
<td>0.1763</td>
</tr>
</tbody>
</table>

Table 6.4 Variances of unwanted signal (noise) due to island size variations for Medium1.
As the read channel model considers the media noise and the AWGN, and both noise are Gaussian distributed so the distribution of the total noise $n(k)$ is Gaussian distributed and the variance is calculated according to Equation 6.7 as below:

$$\sigma_n^2 = D(n_k) = D(v_k) + D(s_k).$$  \hspace{1cm} 6.7

The distribution of the noise needed for the calculation of BER performance according to Equation 3.21 is therefore given by Equation 6.7.

### 6.2.2 Performance Comparison of the Standard Trellis with Size Variations

The calculation of the BER performance is implemented in MATLAB using Equation 3.19, 3.20 and 3.21, where the $\sigma_n^2$ in Equation 3.19 is defined by Equation 6.7. $D(s_k)$ in Equation 6.7 is calculated according to the statistical distribution of $s_{0k}$ and $s_{-1k}$ as it is introduced in Section 6.2.1. The MATLAB script for the calculation of the BER performance can be found in Appendix 2 – BER_Calculation_size_variation. The comparison between the numerical and analytical result for the two media configurations investigated are shown in Fig. 6.4 and Fig. 6.5. The results show that the analytical and numerical BER curves match quite well. The calculation treats the size variation noise as Gaussian distributed, while in the Monte-Carlo simulation the size variation is a truncated Gaussian distribution [117], which could explain the slight difference (not more than 10 bits) between the numerical and analytical results. Both results show that the analytical evaluation is valid in estimating the BER, and can be used further to analyse the factors affecting the BER performance.

<table>
<thead>
<tr>
<th>Size Variation $\sigma$ (nm)</th>
<th>$D_{\text{rms}}(s_{0k})$</th>
<th>$D_{\text{rms}}(s_{-1k})$</th>
<th>$D(s_k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5nm</td>
<td>0.0257</td>
<td>0.0002</td>
<td>0.0376</td>
</tr>
<tr>
<td>3.0nm</td>
<td>0.0370</td>
<td>0.0003</td>
<td>0.0541</td>
</tr>
<tr>
<td>3.5nm</td>
<td>0.0504</td>
<td>0.0004</td>
<td>0.0738</td>
</tr>
</tbody>
</table>

Table 6.5 Variances of unwanted signal (noise) due to island size variations for Medium2.
Fig. 6.4 BER vs. SNR curves of numerical and analytical evaluation for Medium1.

Fig. 6.5 BER vs. SNR curves of numerical and analytical evaluation for Medium2.
6.3 Analytical Analysis of the Modified Trellis with Size Variations

The modified trellis design proposed to combat the effect of island size variations demonstrates an improved BER performance for both media configurations under investigation. Following the previous section, this section proposes an analytical approach to estimate the BER performance of the modified trellis design for island size variations. Using this method, the BER performance can be estimated rapidly and be used as the guidance for the performance estimation of the modified trellis. Also, this method can help determine optimal settings for the proposed modified trellis.

6.3.1 Development of an Analytical Model for the Modified Trellis

The performance analysis of the modified trellis follows the performance analysis of the standard trellis. To estimate the BER performance of the modified trellis is to estimate the probability of errors. The detailed definition of an error event has been introduced in Section 3.4.1 and the modified trellis inherits the definition of the error events in the standard trellis. Comparing the standard trellis with the modified trellis, the difference between the two designs is that the modified trellis introduces two extra branches that aim to capture the signal changes due to statistically large or small islands. The branch output is calculated using Equation 5.6 so that the input samples of the Viterbi detector are compared with more accurate branch output samples to decide the most possible branch to transit. For the modified trellis design, denote $m_e$ as the metric for the erroneous path of an error event $e$, which is the sum of square distance between the input samples and the samples on the erroneous path. Let denote $m_e(k)$ the square distance between the input samples and the samples on the erroneous transition at time $k$, the path metric $m_e$ of a single bit error event from time $k-1$ to $k+1$ is:

$$m_e = m_e(k - 1) + m_e(k) + m_e(k + 1).$$ 6.8
As there are three branches between two states in the modified trellis, \( m_e(k) \) can be expressed as below according to Equation 5.6:

\[
m_e(k) = \begin{cases} 
\left( z(k) + n(k) - (y(k) + \alpha b(k)) \right)^2 \\
\left( z(k) + n(k) - y(k) \right)^2 \\
\left( z(k) + n(k) - (y(k) - \beta b(k)) \right)^2 
\end{cases}
\]

where \( z(k) \) is the ideal replay samples, \( n(k) \) is the total noise including the AWGN and the media noise due to island size variations, \( y(k) \) is the erroneous output by Viterbi detector and \( b(k) \) is the magnetisation state of -1 or 1. The difference between \( z(k) \) and \( y(k) \) is defined as:

\[
\varepsilon_y(k) = z(k) - y(k).
\]

Equation 6.9 can be simplified to:

\[
m_e(k) = \left( \varepsilon_y(k) + n(k) + c(k)b(k) \right)^2, c(k) \in \{\alpha, 0, \beta\}.
\]

According to Equation 6.8, 6.9 and 6.10, the erroneous path metric \( m_e \) is expressed in vector expression as:

\[
m_e = \left\| \varepsilon_y + n_k + C_k b_k \right\|^2
\]

where

\[
C_k = \begin{pmatrix} 
c(k-1) & 0 & 0 \\
0 & c(k) & 0 \\
0 & 0 & c(k+1) 
\end{pmatrix}
\]

Denoting \( m_c \) as the metric for the correct path, which is the sum of square distance between the input samples and the samples of the correct path. The correct path metric \( m_c \) is

\[
m_c = \left\| n_k + C_k b_k \right\|^2.
\]

It is mentioned in Section 3.3.1 that an error event happens when the erroneous path metric is less than the correct path metric so the probability of a particular error event for the modified trellis is:
Chapter 6: The Analytical model of the Read Channel in BPM systems

$$\Pr(\varepsilon) = \Pr(m_y < m_x) = \Pr(e_y^T n_k + e_y^T C_k b_k < -\frac{1}{2} \|e_y\|^2). \quad 6.15$$

To calculate the probability according to Equation 6.15, the probability density distribution (PDF) of $e_y^T n_k + e_y^T C_k b_k$ needs to be derived. It is difficult to determine the distribution of $e_y^T n_k + e_y^T C_k b_k$ directly as $c(k-1), c(k)$ and $c(k+1)$ in $C_k$ vary according to the error events.

An alternative way to determine $C_k$ is to use the expectation of $c(k-1), c(k)$ and $c(k+1)$. Considering a random process $C_t$, $c(t)$ is a discrete random variable, $c(k-1), c(k)$ and $c(k+1)$ are the random variable at time $t=k-1, k$ and $k+1$. Before introducing the method to determine $C_k$ some important definitions of random processes need to be introduced first.

An experiment [87] is specified by the three triple [87] $(S, F, P(\bullet))$, where $S$ is a finite, countable, or noncountable set called the sample space, $F$ is a Borel field [87] specifying a set of events, and $P(\bullet)$ is a probability measure allowing calculation of probabilities of all events.

It is recognised that the Viterbi detector decides the estimated output sequence based on the criterion that the branch output samples have a minimum square Euclidean distance with the input samples. Denoting $[0, d_\alpha], [0, d_0]$ and $[0, d_\beta]$ as the square distance range of each branch, then if the square distance between the input sample and the branch output sample is within the square distance range of that branch, the Viterbi detector will choose the branch as a surviving path. Considering each time each transition between the two states in Viterbi detection is a random event, the result of all the random events is defined as sample space $S$ as:

$$S = \{\zeta: \zeta = (\ldots, d_{-1}, d_0, d_1, d_2, \ldots), d_i \in \{[0, d_\alpha], [0, d_0], [0, d_\beta]\}\} \quad 6.16$$

where $\zeta$ is a random event, the result of the random event $\zeta$ is the square distance $d_i$ within the range of $[0, d_\alpha], [0, d_0]$ or $[0, d_\beta]$. The random process $C_t$ can be defined as a function of $S$ when it satisfies two requirements:

(a) For an arbitrary $c$, $\{\zeta: C(\zeta) \leq c\}$ belongs to an experiment.

(b) The probability of events $\{\zeta: C(\zeta) = +\infty\}$ and $\{\zeta: C(\zeta) = -\infty\}$ are zero.
The random process $C_t$ is described as:

$$C_{t,\zeta} = \begin{cases} 
\alpha, \zeta \in [0, d_a] \\
0, \zeta \in [0, d_a], \ iT \leq t \leq i(T + 1) \\
\beta, \zeta \in [0, d_B] 
\end{cases} \quad \text{6.17}$$

Equation 6.17 can be understood as at arbitrary time $t$, $C_{t,\zeta}$ for a random event $\zeta$ is a discrete random variable which can only be the value $\alpha$, 0 and $\beta$. The PDF of $C_t$ is:

$$f_{C_t}(C_t) = P(C_t = \alpha)\delta(c(t) - \alpha) + P(C_t = 0)\delta(c(t)) + P(C_t = \beta)\delta(c(t) - \beta) \quad \text{6.18}$$

where

$$P(C_t = \alpha) = P\left(\text{During infinitely many transitions, it results in } k_1 \text{ times within } [0, d_a]\right) = \exp\left(-\lambda d_a\right)\left(\frac{\lambda d_a}{k_1!}\right)^{k_1} \quad \text{6.19}$$

Equation 6.19 reflects that $P(C_t=\alpha)$ is approximated as a Poisson distribution, where $\lambda$ is an average number of times per unit square distance. The probability $P(C_t=\beta)$ is analogous to Equation 6.19.

According to Equation 6.18 and 6.19, the expectation of the random process $C_t$ is derived as:

$$E[C(t)] = \int_{-\infty}^{\infty} c_t f_{C_t}(c_t)dc_t = \alpha \exp\left(-\lambda d_a\right)\left(\frac{\lambda d_a}{k_1!}\right)^{k_1} + \beta \exp\left(-\lambda d_B\right)\left(\frac{\lambda d_B}{k_1!}\right)^{k_1} \quad \text{6.20}$$

Equation 6.20 is further simplified to:

$$E[C(t)] \approx \alpha E\left[\exp\left(-\lambda d_a\right)\left(\frac{\lambda d_a}{k_1!}\right)^{k_1}\right] + \beta E\left[\exp\left(-\lambda d_B\right)\left(\frac{\lambda d_B}{k_2!}\right)^{k_2}\right] \quad \text{6.21}$$

$$\approx \alpha \lambda d_a + \beta \lambda d_B.$$

It is clear that $\lambda d_a > 1$ and $\lambda d_B > 1$ if the number of experiments is large enough. Let denote $d$ as the average square distance between the input samples and two extra branch output samples, Equation 6.21 is further simplified to:

$$d = E(C(t)) \approx \alpha \lambda d_a + \beta \lambda d_B > E\left(E(d_a) + E(d_B)\right). \quad \text{6.22}$$
According to Equation 6.22, the average square distance between the input samples and two extra branch outputs can be used to approximate the BER performance instead of $C_k$ in Equation 6.15. Thus, Equation 6.15 can be modified to:

$$\Pr(\varepsilon) \approx \Pr(e_{y}^{T}n_{k} + e_{y}^{T}db_{k} < -\frac{1}{2}\|e_{y}\|^{2}) \approx \Pr(e_{y}^{T}n_{k} + w^{T}b_{k} < -\frac{1}{2}\|e_{y}\|^{2}) \quad 6.23$$

where $w^{T}=e_{y}^{T}d$. In the BPM system the island size change for each island is random and follows a Gaussian distribution. However, the modified trellis assumes that island size change is fixed to a value and any random island size change that is larger or smaller than the fixed value in the modified trellis is rounded to that fixed value. Each value of island size change corresponds to a group value of $\alpha$ and $\beta$ as is shown in Table 5.1 and Table 5.2 for the two media under investigation. Once the modified trellis is set to capture an amount of island size change, the corresponding values of $\alpha$ and $\beta$ are used to calculate the branch output. $E(d_{a})$ and $E(d_{b})$ are calculated as the average square distance for the sample values that are outside the range of $[1-\beta 1+\alpha]$. The truncated (at island size change of +/-7.5nm for Medium1 and +/-12.5nm for Medium2) Gaussian distribution for a minimum 10% island size variations (i.e. 1.5nm and 2.5 nm varied island size for Medium1 and Medium2 respectively) is normally in the range of [-5nm 5nm] for Medium1 and [-8nm 8nm] for Medium2, which correspond to replay samples (the ideal replay samples are normalised to 1) in the range of [0.3451 1.5613] and [0.3947 1.4103] according to the values of $\alpha$ and $\beta$ in Table 5.1 at 5nm and Table 5.2 at 8nm. Denoting the replay samples $X$ are in the range of $[l_{2} l_{1}]$ generally, as replay samples, $E(d_{a})$ and $E(d_{b})$ is calculated as:

$$E(d_{a}) = \begin{cases} 
\sum_{i} (x_{i} - (1+\alpha))^{2} / \text{number of } x_{i} \text{ in } [1+\alpha, l_{i}], & \text{for } x_{i} \text{ satisfy } l_{2} < 1+\alpha \leq x_{i} \leq l_{i} \\
\sum_{i} (x_{i} - (1+\alpha))^{2} / \text{number of } x_{i} \text{ in } [1, l_{i}], & \text{for } x_{i} \text{ satisfy } 1 < x_{i} < l_{i} < 1+\alpha 
\end{cases} \quad 6.24$$
\[ E(d) = \begin{cases} 
\frac{\sum (x_i - (1 - \beta))^2}{\text{number of } x_i \text{ in } [l_2, 1 - \beta]}, & \text{for } x_i \text{ satisfy } l_2 < x_i < 1 - \beta < l_i \\
\frac{\sum ((1 - \beta) - x_i)^2}{\text{number of } x_i \text{ in } [0, l_2]}, & \text{for } x_i \text{ satisfy } x_i < l_2 < 1 - \beta < l_i 
\end{cases} \]

Table 6.6 and Table 6.7 lists values of \( \alpha, \beta, E(d_{\alpha}), E(d_{\beta}) \) and \( d \) for Medium1 and Medium2 respectively.

<table>
<thead>
<tr>
<th>Island Size Change ±nm</th>
<th>Signal Amplitude Change ( \alpha )</th>
<th>Signal Amplitude Change ( \beta )</th>
<th>( E(d_{\alpha}) )</th>
<th>( E(d_{\beta}) )</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1210</td>
<td>0.1251</td>
<td>0.0800</td>
<td>0.1101</td>
<td>0.0951</td>
</tr>
<tr>
<td>2</td>
<td>0.2380</td>
<td>0.2535</td>
<td>0.0452</td>
<td>0.0672</td>
<td>0.0562</td>
</tr>
<tr>
<td>3</td>
<td>0.3505</td>
<td>0.3849</td>
<td>0.0212</td>
<td>0.0340</td>
<td>0.0276</td>
</tr>
<tr>
<td>4</td>
<td>0.4583</td>
<td>0.5189</td>
<td>0.0067</td>
<td>0.0115</td>
<td>0.0091</td>
</tr>
<tr>
<td>5</td>
<td>0.5613</td>
<td>0.6549</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.6594</td>
<td>0.7923</td>
<td>0.1687</td>
<td>0.2639</td>
<td>0.2163</td>
</tr>
<tr>
<td>7</td>
<td>0.7525</td>
<td>0.9307</td>
<td>0.2463</td>
<td>0.4131</td>
<td>0.3297</td>
</tr>
</tbody>
</table>

Table 6.6 Values of \( \alpha, \beta, E(d_{\alpha}), E(d_{\beta}) \) and \( d \) corresponding to island size changes for Medium1.

<table>
<thead>
<tr>
<th>Island Size Change ±nm</th>
<th>Signal Amplitude Change ( \alpha )</th>
<th>Signal Amplitude Change ( \beta )</th>
<th>( E(d_{\alpha}) )</th>
<th>( E(d_{\beta}) )</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0624</td>
<td>0.0657</td>
<td>0.0200</td>
<td>0.0430</td>
<td>0.0629</td>
</tr>
<tr>
<td>2</td>
<td>0.1216</td>
<td>0.1345</td>
<td>0.0118</td>
<td>0.0289</td>
<td>0.0406</td>
</tr>
<tr>
<td>3</td>
<td>0.1776</td>
<td>0.2064</td>
<td>0.0064</td>
<td>0.0179</td>
<td>0.0243</td>
</tr>
<tr>
<td>4</td>
<td>0.2303</td>
<td>0.2811</td>
<td>0.0031</td>
<td>0.0099</td>
<td>0.0130</td>
</tr>
<tr>
<td>5</td>
<td>0.2798</td>
<td>0.3587</td>
<td>0.0013</td>
<td>0.0046</td>
<td>0.0058</td>
</tr>
<tr>
<td>6</td>
<td>0.3263</td>
<td>0.4387</td>
<td>0.0003</td>
<td>0.0016</td>
<td>0.0019</td>
</tr>
<tr>
<td>7</td>
<td>0.3697</td>
<td>0.5210</td>
<td>0.00005</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>8</td>
<td>0.4103</td>
<td>0.6053</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.7 Values of \( \alpha, \beta, E(d_{\alpha}), E(d_{\beta}) \) and \( d \) corresponding to island size changes for Medium2.

In Equation 6.23, the vector term \( w^T b_k \) is the weighted sum of \( b(k) \). Therefore the PDF of \( w^T b_k \) can be obtained by convolving the distribution of weighted \( b(k) \), if the weighted \( b(k) \) is independent from each other. The PDF of \( w^T b_k \) is:

\[
f(w^T b_k) = f(w(k-1)b(k-1) + w(k)b(k) + w(k+1)b(k+1)) \\
= f(w(k-1)b(k-1)) * f(w(k)b(k)) * f(w(k+1)b(k+1)).
\]
Let $v = w(k)b(k)$, where $v$ is a random variable. Assuming equal properties for the two binary input data values, then $v$ is the random variable that can be either -$w(k)$ or $w(k)$ since $b(k)$ can be either -1 or 1. The PDF of weighted $b(k)$ is as:

$$f(w(k)b(k)) = f(v) = \frac{1}{2} \delta(v - w(k)) + \frac{1}{2} \delta(v + w(k)). \tag{6.27}$$

Therefore, the PDF of $w^Tb_k$ can be expressed as:

$$f(w^Tb_k) = \sum_i A_i \delta(w - \Delta_i) \tag{6.28}$$

where $A_i$ and $\Delta_i$ are obtained using Equation 6.26 and 6.27. In Equation 6.23, term $\epsilon^T_j n_k$ is a weighted sum of Gaussian distributed samples and is thus a Gaussian random variable with a variance of $\|\epsilon_j\|^2 \sigma_n^2$, where $\sigma_n$ is the standard deviation of $n(k)$. The overall PDF of $\epsilon^T_j n_k + \epsilon^T_j db_k$ is the convolution of the Gaussian distribution, with a variance of $\|\epsilon_j\|^2 \sigma_n^2$, with the train of weighted impulse functions as in Equation 6.28. The resulting PDF is a train of weighted Gaussian functions with the same variance of $\|\epsilon_j\|^2 \sigma_n^2$, different means $\Delta_i$ and gains $A_i$ as Equation 6.29.

$$f(\epsilon^T_j n_k + \epsilon^T_j db_k) = \frac{1}{\sqrt{2\pi} \|\epsilon_j\|^2 \sigma_n^2} \sum_i A_i \exp \left( \frac{(1/2)\|\epsilon_j\|^2 - \Delta_i}{\|\epsilon_j\|\sigma_n} \right) \tag{6.29}$$

The probability of a particular error event $\epsilon$ in the modified trellis is expressed as:

$$\Pr(\epsilon) = \sum_i A_i Q \left( \frac{(1/2)\|\epsilon_j\|^2 - \Delta_i}{\|\epsilon_j\|\sigma_n} \right) \tag{6.30}$$

The probability of bit error of a particular error event is obtained by multiplying the probability of the error event by the Hamming weight (i.e. numbers of bit errors corresponding to the error event) of that particular error event. The overall BER is obtained by adding up all the bit error probabilities of all possible error events. The BER of the modified trellis can be upper-bounded as follows:
\[
BER \leq \sum_{i} w_i(e) \left( \frac{1}{2} \right) \sum_{j} A_i \mathcal{Q} \left( \frac{1/2}{\|e_j\|\sigma_n} \left( \|e_j\| - \Delta_i \right) \right).
\]

Notice that the modified trellis only takes the size variations of the central island, deviation of \( n_k \) is as below:

\[
\sigma_n^2 = D(v_k) + \sqrt{2} \cdot D_{rms} (s_{0k})
\]

where \( v(k) \) is AWGN and \( s_{0k}(k) \) is defined as the media noise introduced by the contribution of the central island size variations.

### 6.3.2 Performance Comparison of the Modified Trellis with Size Variations

In the modified trellis, it is assumed that a fixed amount of island size change will determine the values of \( \alpha \) and \( \beta \) that eventually determine the outputs of the extra branch. Here, the analytical approach first calculates \( E(d_\alpha) \), \( E(d_\beta) \) and \( d \) according to Equation 6.24, 6.25 and 6.22 with the values of \( \alpha \) and \( \beta \). The results are listed in Table 6.6 and Table 6.7 accordingly for Medium1 and Medium2. Then, the values of \( d \) are used to calculate the \( \Delta_i \) and \( A_i \) according to Equation 6.23, 6.26 and 6.27. Finally, the BER is obtained according to Equation 6.31. The complete MATLAB calculation script can be found in Appendix 2 – BER_Calculation_ext.

The analytical and numerical BER performance are compared in Fig. 6.6 for Medium1, where the media noise due to island size variations from 1.5nm, 2nm and 2.5nm and AWGN are present in the BPM system. A modified trellis assuming a fixed island size change of \( \pm 3.5 \)nm is adopted. It is shown in Fig. 6.6 that the numerical and analytical BER results are in reasonable agreement apart from that the analytical BER performance always demonstrates lower BER performance compared to the numerical BER. This is because Equation 6.15 the average square distance \( d \) are used to replace the actual square distance.
\( C_k \) to estimate the BER performance, which results in less estimated noise in the modified trellis. Thus, it leads to a lower BER performance by analytical approach.

Fig. 6.6 Comparison between numerical and analytical BER for varied island size variations for Medium1 with BAR=4.

The same comparison of the analytical and numerical BER performance is carried out for Medium2 and the BER vs. SNR curves are shown in Fig. 6.7. The modified trellis is assumed to have a fixed island size change of \( \pm 2.5\text{nm} \) and island size variations in the read channel model is random with \( \sigma = 2.5\text{nm}, 3.0\text{nm} \) and \( 3.5\text{nm} \). The same trend has demonstrated in Fig. 6.7 that the numerical and analytical BER results are in reasonably agreement though the analytical result gives a lower BER performance. However, the comparison between the analytical and numerical BER performance for Medium2 confirms that the analytical approach can be used as guide for estimating the BER performance for BPM systems with different media configurations. The analytical approach can help estimate the BER performance of the modified trellis more efficiently compared to the numerical Monte-Carlo simulation, where Monte-Carlo simulation takes about 2-3 hours and the analytical approach only takes a minute.
One of the applications of the proposed analytical approach is to help determine when the optimal BER performance can be achieved with the adoption of the modified trellis. In Section 5.2.3, the numerical Monte-Carlo simulation runs by setting the modified trellis with different fixed volume of size changes to explore the optimum BER performance. Here, the analytical approach calculates the BER performance by providing different values of $E(d_\alpha)$, $E(d_\beta)$ and $d$ listed in Table 6.6 and Table 6.7 accordingly for Medium1 and Medium2. The analytical BER and numerical BER are compared in Fig. 6.8, which shows the case where the island size variation of Medium1 has $\sigma=1.5\text{nm}$ (i.e. 10%) of the nominal island period. The BER has been plotted as a function of the island size change assumed in the modified trellis (Table 6.6) that determines the outputs of the extra branches. It can be concluded from the figure that the numerical and analytical approach indicate the same trend where the optimum BER performance is observed. It is shown that for values of $\alpha$ and $\beta$ that are a result of a known size change in the modified trellis of $\approx\pm4\text{nm}$ gives the best BER performance for both analytical and numerical result. The analytical result is over estimated (lower BER) in comparison with the numerical result.
This is due to the same reason that has mentioned previously, which results in less estimation of the total noise (i.e. the AWGN and media noise) and leads to a lower BER performance by analytical approach, especially when the island size changes assumed in the modified trellis are above 5.5nm.

Fig. 6.8 Island size changes in the modified trellis to determine an optimal BER performance for Medium1 with BAR=4.

Fig. 6.9 illustrates when island size variations ($\sigma=1.5$nm) and AWGN are both present in the BPM system for Medium1, the analytical BER vs. SNR curves are derived by calculating BER performance with different fixed amount of island size changes (1.5nm, 3.5nm, 4nm and 4.5nm) assumed in the modified trellis. The results shown in Fig. 6.9 demonstrate the same results as concluded from Fig. 6.8 that an optimal BER performance is achieved if a modified trellis assuming a fixed island size change of $\approx\pm4$nm is adopted. The result shown in Fig. 6.9 is also analogous to the result shown in Fig. 5.13 derived from the numerical Monte-Carlo simulation.
The optimal BER performance of a modified trellis used for Medium2 is also explored. Fig. 6.10 shows the analytical BER vs. SNR curves when $\sigma=2.5\text{nm}$ (i.e. 10%) island size variations and AWGN are present in BPM system for Medium2. The BER performance is derived by calculating with different fixed amount of island size changes (2.5nm, 3.5nm, 4.5nm, 5.5nm and 6.5nm) assumed in the modified trellis. The result shown in Fig. 6.10 is analogous to the result shown in Fig. 5.14. It is demonstrated that in the case of Medium2, the BER performance achieved by assuming different fixed amount of island size changes in the modified trellis are almost the same. No predominant optimal BER performance can be observed from the figure.
6.4 Summary

Overall, in this chapter a new analytical approach is developed to estimate the probability of error for BPM channels with island size variations present. In order to estimate the BER performance, the origins of errors due to island size variations is investigated. Statistical results have demonstrated that the typical error events due to size variations are single bit error events, and the analytical approach only considers single bit error events. The analytical approach is also further developed to predict the BER performance by adopting the modified trellis design in the read channel modelling. The optimal performance that the modified trellis can offer can be predicted by the analytical approach. This analytical approach provides an efficient way to obtain BER, especially for moderate to high SNR. This analysis can be helpful to design different BPM read channels with island size variations present.
Chapter 7

Conclusions and Future Work

This chapter summarises the work that has been performed in order to achieve the primary aim of the research documented in this thesis. The results presented in previous chapters are also summarised. Finally, possible future work is briefly discussed.

7.1 Conclusions

Research into magnetic recording is aimed at optimising the trade-off described by the ‘trilemma’ of high media signal-to-noise ratio (SNR), the writability of the media and the thermal stability of the media [15]. It is recognised that a conventional magnetic recording system requires the utilisation of small grains to achieve adequate SNR. However, small grains will be subject to thermal activation of the magnetisation. To maintain thermal stability, higher anisotropy media are desired to maintain the recorded data. However, the maximum attainable write head field is limited due to head material constraints [15, 30]. Bit-patterned media (BPM) has been proposed as a potential solution to break the ‘trilemma’ by increasing the magnetic volume from a grain volume to the bit volume to maintain thermal stability. The challenge of implementing a BPM system into a viable magnetic recording system is fabricating uniform island arrays over large areas cost effectively. Island geometry variations due to the imperfect lithography process affect the ability of read channel to recover the recorded information. The primary research aim of this thesis is to investigate and mitigate the effect of island geometry variations on read channel performance in BPM systems.

The data recovery process for BPM systems has been analysed primarily by Hughes [64, 67, 93]. Apart from Hughes’ work [62, 63, 108], a “multiple islands per read head” read channel design is proposed by authors in [94] where it is assumed that the dimensions of the read head utilised may be several times larger than track pitch of the BPM systems with
Chapter 7  Conclusions & Future Work

ultra-high areal density. It is also noticed that other related work has investigated the effect of media and head dimensions on read channel performance in [66, 70, 95-97] and the issues of inter-track-interference (ITI) and track misregistration (TMR) has been investigated [70, 98-100]. However, little effort has been paid to the investigation of the effect of island geometry variations and the read channel designs to mitigate the effect. The work documented in this thesis differs considerably from the previous work in a number of ways.

a) The use of a more accurate three-dimensional (3-D) replay model rather than the two-dimensional (2-D) replay model used in [62, 63, 108]. The 3-D replay model takes into account the shape constrained nature of the media and the giant magnetoresistive (GMR) read head [66].

b) In this thesis, the dimension of the read head [95] is comparable to the island size so the read head only senses the medium magnetisation of one island rather than multiple islands across track at the same time. Such a scenario is different from the scenario in [94].

c) This thesis mainly concentrates on investigation of the effect of island geometry variations on read channel performance and the read channel designs to alleviate such effects.

d) An analytical approach of estimating read channel performance is developed, while most of the previous work uses numerical Monte-Carlo analysis.

Considering the above, it can be demonstrated that the work documented here is novel and may interest read channel designers working on the BPM system as a viable storage system to achieve an areal density above $1\text{Tb/in}^2$. In order to validate and compare the results, two media configurations have been explored to investigate the effect that island geometry variations have on the ability to recover stored information, both correspond to an areal density of $1\text{Tb/in}^2$ but with different bit-aspect-ratio (BAR).

The work was carried out by first implementing a read channel model as a platform to simulate the numerical bit-error-rate (BER) performance. The read channel consists of two
parts, first it is a 3-D replay model developed by Nutter et al [66] is used to simulate the replay process of the BPM systems and provide replay samples for subsequent detection. The second part is the partial-response maximum-likelihood (PRML) read channel to recover the recorded data. Each component (3-D replay model, digital equaliser and Viterbi detector) in the read channel model has been written as a MATLAB function and a top-level function calls all the generated functions following the flow chart of the read channel simulator as shown in Fig. 3.5. The read channel model starts by feeding the system with randomly generated data for a specific SNR value. The 3-D replay model generates the replay samples according to the random data pattern and then equalised by a digital equaliser to the ideal samples (given by the specific GPR target), and finally the equalised samples are input to the Viterbi detector to recover the data. The number of the bits in error is counted by comparing the recovered data at the output of the Viterbi detector with the recorded data. If one or more bits are in error then the frame is considered as in error, and then a new frame of random data feeds the system at the same SNR. The iterative process stops for the specific SNR value when 50 frames (to ensure enough bits are generated to achieve a reliable BER) are in error. The same process is repeated for different SNR values. For each SNR, the BER is calculated accordingly so that BER against SNR curves are generated to represent the read channel performance.

The accuracy of the 3-D replay model has been verified in [66, 70] and the validity of the read channel has been verified in two ways. First, the ability of the read channel to recover data is verified by providing replay samples without the presence of any noise for the read channel to recover. It is shown that the data is recovered as expected without any error. Second, to provide replay samples with the additive white Gaussian noise (AWGN) for the read channel model to recover. The numerical BER performance is verified by comparing with the analytical BER performance. The read channel model can be used to investigate how island geometry variations will affect the read channel performance and can be easily modified to a new read channel design to mitigate such effects.

An investigation was performed to identify the island geometry variations in BPM systems, which concluded that island geometry variations can be categorised as in island position
and size variations. The effect of the island position and size variations on the replay samples is different. For the two media specify configurations investigated, the effect of any island position variation on the replay samples is that the position shift of the target island leaves the pulse shape and size unchanged but shifts the pulse up/down along the track. This has less impact on the replay signal of the target island than the position shift of its two neighbouring islands on the replay signal (i.e. ISI increases). The effect of island size variations on the replay samples is that significant amplitude change is observed due to the size change of the target island while little change in replay signal amplitude is observed due to the size changes of the neighbouring islands, which does not affect inter-symbol-interference (ISI) to any great degree. The combination of both island position and size variations affect the replay samples in the amplitude change and the position shift, which can be approximated by first-order Taylor series [50]. The simulation results in Chapter 4 demonstrate that island geometry variations, whatever the source, have a detrimental effect on the read channel performance. However, the degradation in performance arising from variations in island size is more severe than that due to variations in island position. Interestingly, with the increase of island position variations, the results demonstrate an acceptable BER performance at $10^{-4}$. It is indicated that in order to maintain a BER performance below $10^{-4}$, a position variation of 13.3% can be tolerated by Medium1 and 14% by Medium2. While in order to maintain an acceptable BER performance at $10^{-4}$, island size variations need to be controlled below 10%. The BER performance in the presence of different cases of island geometry variations has been compared. The simulated results show that for 10% geometry variations of island position, size and a combination of both that the SNR is degraded by the island position variations by around 1dB. The SNR is degraded by the island size variations and the combination of both at about 4dB and 6dB respectively. It is demonstrated that a combination of both degrades the BER performance the most.

A new channel design incorporating a novel trellis structures is proposed for use in the Viterbi detector of the read channel for BPM systems in order to combat the effect of island geometry variations. The fact that island position and size variations affect the
replay samples in different ways leads to a different read channel design to combat the
effect of the two cases on read channel performance. For the case of island position
variations, the effect of the position shift of the two neighbouring islands can not be
ignored. The position shift of the island at the current sampling time $k$ needs to be carried
to the next sampling time $k+1$ as the island being read at time $k$ becomes a neighbouring
island of the island that being read at time $k+1$. The position shift of the island is integrated
into the state of the trellis, so the proposed trellis design for island position variations
introduces both extra states and branches compared with the standard trellis design. For the
case of island size variations, as the effect of the size change of the target island dominates
and the effect of the island size change of the two neighbouring islands is small, the
modified trellis design can be simplified to only take into account the amplitude changes
due to the target island only. Since the size information of the target island does not need to
be carried to the next sampling time, the modified trellis design for island size variations
results in only extra branches compared with the standard trellis. The modified trellis
designs for island position and size variations demonstrate improved BER performance for
the two media configurations investigated. Even with more than 10% position variations,
the modified trellis still offers greater than 1dB SNR improvement for Medium1 and about
0.8dB for Medium2 compared to the standard trellis for a BER performance below $10^{-4}$.
With 10% size variation present, the proposed modified trellis design offers 3dB SNR
improvement for Medium1 and above 1dB SNR improvement for Medium2 at a BER
below $10^{-4}$.

Considering Monte-Carlo simulations are sometimes time consuming, especially for
moderate to high SNRs where the BER will be very small, an analytical model has been
developed to estimate the probability of error for a BPM read channel with island size
variations present. Based on the statistical results that the typical error events due to island
size variations are single bit in nature, the analytical estimation considers errors due to
single bit error events only. The numerical and analytical BER performance demonstrate
very similar results, with the analytical BER performance slightly overestimating the BER
performance (lower BER). This is because the calculation takes the average square
distance to replace the actual square distance to estimate the BER performance. Thus, less
noise is estimated in the analytical calculation, which leads a lower BER performance.
However, the errors between the analytical and numerical approach is not more than 10
bits difference in the number of bit errors that has been identified, so it can be used as an
efficient and alternative way to predict read channel performance. The Monte-Carlo
simulation takes about 2-3 hours, while the analytical approach only takes a minute.

Overall, it is clear from this work that BPM systems can tolerate island position variations,
while island size variations will have a significant effect on the data recovery process if
they are not controlled during the fabrication process. An analytical approach to estimate
the BER performance with the presence of island size variations is more efficient than a
numerical approach though the distribution of the media noise due to island size variations
is complex to derive. The numerical Monte-Carlo simulation can be useful to understand
the major error events, the correlation between the errors and island sizes, and the
statistical distribution of the media noise, which the analytical approach couldn’t tell. The
work may interest the hard disc drive industry working on development of BPM systems in
two ways: a) the improvement of signal processing in BPM systems, b) the definition of
the requirements for the fabrication process.

7.2 Presented and Published Work

Some of the work documented in this thesis has been presented and published. The
investigation of island position variations and the modified trellis design for island position
variations has been presented at IEEE International Magnetics Conference (Intermag) in
2008 and published in IEEE Transactions on Magnetics in 2008 [51]. The analysis of error
events due to island size variations has been presented at Intermag 2010 and published in
IEEE transactions on Magnetics in 2010 [126]. The investigation of island size variations
and the modified trellis design for island size variations was presented at 55th Annual
7.3 Future Work

To extend this research work, it would be interesting to investigate the novel read channel design to combat the combined effect of island position and size variations. The analytical approach can be extended to estimate BER performance of BPM systems with combined island position and size variations. In addition, a further evaluation of simulated results against experimental results obtained from the fabricated patterned media would be of great interest. It is noticed that in BPM systems, the magnetic islands are fabricated as isolated single domain that separated by non-magnetic material. The signal information from the non-magnetic area may be used as a part of the information for coding and decoding scheme like the read channels for BPM systems with trench playback [67].

Further, the read channel model can be extended to support an areal density of 5Tb/in², which will involve changing the read head dimensions, modifying the one-dimensional read channel model into two-dimensional.
References


[16] Western Digital, "Perpendicular magnetic recording (PMR)," Technology Papers,
West Digital.


References


References


[70] I. Ntokas, Analysis of data recovery techniques for use with patterned media storage systems, in School of Computer Science, Ph.D thesis, Manchester, The University of Manchester, 2006


[81] E. Kretzmer, "Generalization of a Technique for Binary Data Communication,"
References


[98] S. Nabavi, B. V. K. Kumar, and J. A. Bain, "Mitigating the Effects of Track
References


References


Appendix 1

Lists of MATLAB functions

1. PRML Read Channels

a) GPR targets

function [gpr_coeff, fir_coeff, lagrange] = gpr_target(random, sampled, gpr_len, fir_len, constraint, method)
*********************************************************************
% This is function is used to find the optimum GPR target using the MMSE %
% method under the constraint of g(0)=1.
% Input: random - The random data. sampled - The sampled data after the %
% replay model. fir_len - The length of the FIR filter. gpr_len - The length % of the
% GPR target.
% Output: gpr_coeff - The GPR tap values. fir_coeff - The FIR tap values.
*********************************************************************
% Copyright 2007
% Yuanjing Shi
% School of Computer Science
% The University of Manchester
% shiya@cs.manchester.ac.uk
% Revision 14th May 2007

datalength = length(random);
I_matrix = zeros(1, gpr_len)';

switch constraint
    case '1'
        I_matrix(1,1) = 1;
    case 'centre'
        I_matrix(round(gpr_len/2), 1) = 1;
    case '2'
        I_matrix(2,1) = 1;
end;

% R_matrix.. Calculate the autocorrelation matrix of the sampled data
K = (fir_len-1)/2;
G = (gpr_len-1)/2;
autocorr_sampled = xcorr(sampled);
R_matrix = zeros(fir_len, fir_len);
for i = -K:K;
    R_matrix(i+K+1,:) = autocorr_sampled(datalength+(-K:K)-i);
end;

% A_matrix.. Calculate the autocorrelation matrix of the random data
autocorr_data = xcorr(random);
A_matrix = zeros(gpr_len, gpr_len);
for i = -G:G
    A_matrix(i+G+1,:) = autocorr_data(datalength+(-G:G)-i);
end;

% T_matrix.. Calculate the correlation matrix of the sampled and the random data
corr_ran_sample = xcorr(sampled, random);
T_matrix = zeros(fir_len, gpr_len);
for i = -K:K
    T_matrix(i+K+1,:) = corr_ran_sample(datalength+(-G:G)-i);
end;

switch method
    case '1'
        lagrange = 1/(I_matrix'*inv(A_matrix-T_matrix'*inv(R_matrix)*T_matrix)*I_matrix);
        gpr_coeff = lagrange*inv(A_matrix-T_matrix'*inv(R_matrix)*T_matrix)*I_matrix;
        fir_coeff = inv(R_matrix)*T_matrix*gpr_coeff;
    case '2'
        m = A_matrix-T_matrix'*inv(R_matrix)*T_matrix;
        [gpr_coeff lagrange] = eigs(m,1,0);
        fir_coeff = inv(R_matrix)*T_matrix*gpr_coeff;
end;

b) FIR filter using the LMS algorithms

function [taps, varargout] = fir_lms(ideal_pr_samps, signal, fir_len)
******************************************************************************
% This is a function which optimises the tap weights of the FIR filter % using the LMS algorithm. Inputs: signal: is the sampled signal at the % output of the replay model, and ideal_pr_samps: is the desired signal % optimised by filtering the random input data with the specific PR target. % The function returns an optimised fir taps and minimum squared error as an optional output.
******************************************************************************
% Copyright 2007
% Shi Yuanjing
% School of Computer Science
% The University of Manchester
% shiya@cs.manchester.ac.uk
% Modified version by 27th Nov 2007
% LMS algorithm
% N = length(signal);
% K = (fir_len-1)/2;
taps = zeros(1, fir_len);
taps(K+1) = 1;
taps_offset = floor(fir_len/2);

% convergence
m = 0.01;
% m = 0.001;
for i = 1:N-2*K
    extracted_samps = signal(i+2*K:-1:i);
    filtered = taps*extracted_samps';
    error = ideal_pr_samps(i+K+1) - filtered;
    if (i == N/2)
        m = m/100;
    end;
    taps = taps + m.*error.*extracted_samps;
    mse(i) = error.^2;
end
nout = max(nargout,1)-1;
if (nout == 1)
    varargout{1} = mse;
end;
c) Standard Trellis Diagram

function trellis = trellis_gen(gpr_len, gpr)
*********************************************************************
% This is a function which implements a generalised trellis diagram. %
% Inputs: prfn - is the PR or the GPR polynomial coefficient.
% Output: trells - is a structure which contains a next state table and % a
% previous state table where all the needed information for the generation %
% of a trellis diagram are stored (input, output, nextstate).%
*********************************************************************
% Copyright 2007
% Yuanjing Shi
% School of Computer Science
% The University of Manchester
% shiya@manchester.ac.uk
% Revision 8th May 2007
%format long;
stateSize = gpr_len - 1;
numOfState = 2.^stateSize;
counter = zeros(1,numOfState);
trellis.numOfState = numOfState;
output_array = [];
for s = 1:numOfState
    for b=1:2
        state_num = dec2any(s-1,2,stateSize);
        for i = 1:length(state_num)
            if state_num(i) == 0;
                cur_state(i)=-1;
            else cur_state(i)=1;
            end;
        end;
        if (b-1) == 0
            r = [-1 cur_state];
            input = -1;
        else
            r = [1 cur_state];
            input = 1;
        end;
y = gpr * r';
output = y;
output_array = [output_array y];
nextstate = [b-1 state_num(1:stateSize-1)];
nextstate = any2dec(nextstate,2)+1;
trellis.nst(s,b).input = input;
trellis.nst(s,b).output = output;
trellis.nst(s,b).nextstate = nextstate;

end
end
trellis.counter = counter;
d) Modified trellis diagram for position variations

```matlab
d) Modified trellis diagram for position variations

```function trellis = extended_trellis_gen(pulsefile, prfn, level_values)

% This is a function which implements a extended generalised trellis diagram using jitter-sensitive-sequence-detection scheme.
% Inputs: prfn - is the PR or the GPR polynomial coefficient.
% jitter_levels - is the discrete jitter levels
% std_deviation - is the standard deviation of island each edge
% Output: trellis - is a structure which contains a next state table and a previous state table where all the needed information for the generation of a trellis diagram are stored (input, output, nextstate).
%******************************************************************************
% Copyright 2007
% Yuanjing Shi
% School of Computer Science
% The University of Manchester
% shiya@manchester.ac.uk
% Revision 15th Oct 2007
%--- Initial variables
bits = [-1 1];
%--- Caculate states of extended trellis
stateSize = length(prfn)-1;
jitter_levels = length(level_values);
total_jitter_combin = jitter_levels.^stateSize;
umOfState = (2*jitter_levels).^stateSize;
counter = zeros(1,numOfState);
%generte the target waveform
eval(['load ''',pulsefile,''';']);
g = pulse_L75_nor;
trellis.numOfState = numOfState;
trellis.jitterlevels = jitter_levels;
output_array = [];
for s = 1:numOfState
    bit_state = fix((s-1)./total_jitter_combin);
jitter_state = rem(s-1, total_jitter_combin);
    index = [dec2any(bit_state, 2, stateSize), dec2any(jitter_state, jitter_levels, stateSize)];
    state.bit = bits(index(1:stateSize)+1);
    state.jitter = level_values(index(stateSize+1:end)+1);
    Sample_time = round((-state.jitter/0.1) + (575:150:575+(stateSize-1)*150));
p1 = g(Sample_time);
    for b = 1:2
```
if (b-1) == 0
    r = [-1 state.bit];
    input = -1;
else
    r = [1 state.bit];
    input = 1;
end;
for nxt_l = 1:jitter_levels
    Sample_time = round(-level_values(nxt_l)/0.1)+425;
    p = [g(Sample_time) p1]';
    output = r * p;
    output_array = [output_array output];
    curstate = s;
    nextstate = any2dec([find(bits == r(1))-1 index(1)],
2)*total_jitter_combin + any2dec([nxt_l-1 index(stateSize+1)],
jitter_levels)+1;
    way = (b-1)*jitter_levels + nxt_l;
    trellis.nst(curstate,way).input = input;
    trellis.nst(curstate,way).output = output;
    trellis.nst(curstate,way).nextstate = nextstate;
    counter(nextstate) = counter(nextstate)+1;
    trellis.pst(nextstate,counter(nextstate)).input = input;
    trellis.pst(nextstate,counter(nextstate)).output = output;
    trellis.pst(nextstate,counter(nextstate)).prestate = curstate;
    disp(sprintf('trellis.nst(%d,%d), input:%d, output:%d, 
nextstate:%d',curstate, way,input, output,
trellis.nst(curstate,way).nextstate));
    disp(sprintf('trellis.pst(%d,%d).prestate=%d',nextstate,counter(nextstate),curstate));
end
end

trellis.max_output_value=max(output_array);

e) Modified trellis diagram for size variations

function [trellis combination]=
extended_trellis_gen_branchonly(jitter_types, pulsefile, gpr_len,
level_values, fir)
*********************************************************************
% This is a function which implements a generalised trellis diagram.
% Inputs: prfn - is the PR or the GPR polynomial coefficient.
% Output: trells - is a structure which contains a next state table and % a
previous state table where all the needed information for the
generation of a trellis diagram are stored (input, output, nextstate).

% Copyright 2007
% Yuanjing Shi
% School of Computer Science
% The University of Manchester
% shiya@manchester.ac.uk
% Revision 29th Sept. 2008
eval(['load ''',pulsefile,''';']);
stateSize = gpr_len - 1;
umOfState = 2.^stateSize;
counter = zeros(1,numOfState);
if(strcmp(jitter_types, 'position'))
    [combination total_number]= combination_levels(gpr_len, level_values);
    pulse = pulse_L75_nor;
    centre = round(length(pulse)/2);
    sample_time = round(-combination/0.1) +
    repmat((centre-(gpr_len-1)/2*150:150:centre+(gpr_len-1)/2*150),total_number,1);
    sample_value = pulse(sample_time);
    H_size_variation_centre =
    [repmat(0.021,total_number,1),sample_value,repmat(0.021,total_number,1)];
    H_size_variation_minus(:,2:5) = H_size_variation_centre(:,1:4);
    H_size_variation_positive = H_size_variation_centre(:,2:5);
    H_size_variation_positive(:,5) = 0;
    gpr = [H_size_variation_minus * fir' H_size_variation_centre * fir'
    H_size_variation_positive * fir'];
elseif(strcmp(jitter_types, 'width'))
    [combination total_number]= combination_levels(1,{level_values{1}});
    sample_index = (length(size_variations)+1)/2+ combination/0.5;
    H_size_variation_centre = [repmat([0.021 0.2410],total_number,1),size_variations(sample_index),repmat([0.2410 0.021],total_number,1)];
    H_size_variation_minus(:,2:5) = H_size_variation_centre(:,1:4);
    H_size_variation_positive = H_size_variation_centre(:,2:5);
    H_size_variation_positive(:,5) = 0;
    gpr = [H_size_variation_minus * fir' H_size_variation_centre * fir'
    H_size_variation_positive * fir'];
end;

trellis.numOfState = numOfState;

output_array = {};
for s = 1:numOfState
    for b=1:2
        state_num = dec2any(s-1,2,stateSize);
        for i = 1:length(state_num)
            if state_num(i) == 0;
                cur_state(i)=-1;
            else cur_state(i)=1;
                end;
            end;
        if (b-1) == 0
            r = [-1 cur_state];
            input = -1;
        else
            r = [1 cur_state];
            input = 1;
        end;
        y = gpr * r';
        output = find_duplicate_value(y);
        output_array = {output_array{:} output.value{1}};
        nextstate = [b-1 state_num(1:stateSize-1)];
        nextstate = any2dec(nextstate,2)+1;
        branches = size(output.value{1},1);
        for way = 1:branches;
            trellis.nst(s,branches*(b-1)+way).input = input;
            trellis.nst(s,branches*(b-1)+way).output = output.value{1}(way);
            trellis.nst(s,branches*(b-1)+way).positionshift = output.position(way);
            trellis.nst(s,branches*(b-1)+way).nextstate = nextstate;
            counter(nextstate) = counter(nextstate)+1;
            trellis.pst(nextstate,counter(nextstate)).input = input;
            trellis.pst(nextstate,counter(nextstate)).output = output.value{1}(way);
            trellis.pst(nextstate,counter(nextstate)).prestate = s;
            trellis.pst(nextstate,counter(nextstate)).positionshift = output.position(way);
            disp(sprintf('nst - curstate:%d, input:%d, output:%d, nextstate:%d',s, input, output.value{1}(way),
                trellis.nst(s,way).nextstate));
            disp(sprintf('trellis.pst(%d,%d).prestate= %d',nextstate,
                counter(nextstate),s));
            disp(sprintf(''));
end;
end
trellis.counter = counter;

function [combination total_number] = combination_levels(digits, levels)
level_size = size(levels,2);
diff_combination = 1;
if (level_size ~= digits)
disp('The dimension of the levels should equals to digits');
return;
end;
for i = 1:digits
    diff_combination = length(levels{1,i}) * diff_combination;
end;
for i = 0:diff_combination-1
    dividend = i;
    for j = 1:digits
        level_length = length(levels{1,j});
        residual(j) = rem(dividend,level_length);
        dividend = fix(dividend./level_length);
        combination(i+1,j) = levels{1,j}(residual(j)+1);
    end;
end;
combination = fliplr(combination);
total_number = size(combination,1);

function output = find_duplicate_value(input)
row = size(input,1);
line = size(input,2);
for r = 1:line
    temp = input(:,r);
    j = 1;
    for i = 1:row
        if(temp(i))
            difference = input(:,r)-temp(i);
            duplicate_position = find(abs(difference) < 0.000001 &
abs(difference) >= 0);
            temp([duplicate_position(2:end)]) = 0;
            output.position(j) = duplicate_position;
            j = j+1;
        end;
    end;
end;
on_zero_position = find(temp);
output.value(:,r) = temp([non_zero_position]);
end;

f) Viterbi detector

function [mlsd mlsd_out state out_state ways position] = 
viterbi(filtered_signal, trellis)
*********************************************************************
% This is a function which implements the Viterbi detector.
% Inputs: filtered_signal – the filtered signal received at the output % of
% the FIR filter, % trellis – the trellis structure.
% Output: mlsd_out: is a vector which represents the recovered data.
*********************************************************************
% Copyright 2007
% Yuanjing Shi
% School of Computer Science
% The University of Manchester
% shiya@cs.manchester.ac.uk
% Revision 29th Sept. 2008

% Initialise the state metric at the beginning of the trellis with 0 for
% the first state and any positive large number for the rest of them

format long;
depth = length(filtered_signal)+1;
state_metric = ones(trellis.numOfState, depth);
state_metric = 1000 * state_metric;
state_metric(1, 1) = 0;
state_metric(2, 1) = 0;
for i = 2:depth
    for j = 1:trellis.numOfState
        state_value = zeros(1, trellis.counter(j));
        for way = 1:trellis.counter(j)
            out = trellis.pst(j, way).output;
            f = filtered_signal(i-1);
            e = f-out;
% e = filtered_signal(i-1) - out;
            er = e.^2;
            s = state_metric(trellis.pst(j, way).prestate, i-1);
            state_value(way) = s + er;
        end;
        state_metric(j, i) = min(state_value);
        if(size(p,2)~=1)
preway(j,i-1)= p(unidrnd(size(p,2),1,1));
else
preway(j,i-1) = p;
end;
end;
end;
mlsd_out = zeros(trellis.numOfState, depth-1);
for j = 1:trellis.numOfState
    state=j;
    for i = depth:-1:2;
        out_state(j,i-1) = state;
        op = trellis.pst(state,preway(state,i-1)).input;
        state = trellis.pst(state,preway(state,i-1)).prestate;
        mlsd_out(j,i-1) = op;
    end;
end;
position = find(out_state(1,:)==out_state(2,:)&
out_state(2,:)==out_state(3,:)& out_state(3,:)==out_state(4,:));
mlsd = mlsd_out(1,position);
state = out_state;
out_state = out_state(1,position);
ways = preway((position-1)*trellis.numOfState+out_state);

c) Top-level function (example)

function [gpr, fir, varargout] = top_test_call(varargin)
*********************************************************************
% This is a fuction which calls other functions to generate frames of data
% and calculate BER performance.
% [gpr, fir] = top_test_call(num, gpr_len, fir_len, head_file, constraint,
% method) generates gpr target and fir coefficient using MMSE constraint
% specifies the constraint of MMSE method specifies the method to calculate
% gpr target.
% [gpr, fir] = top_test_call(num, fir_len, head_file) calls the LMS
% equaliser to equalize the signal to a given gpr target with FIR length
% of fir_len. This function returns gpr target and fir taps. It also returns
% running time as an optional output
*********************************************************************
% Copyright 2007
% Shi Yuanjing
% School of Computer Science
% The University of Manchester
% shiya@cs.manchester.ac.uk
% Modified vertion by 27th Nov 2007
%---Initial checks
error(nargchk(3,6,nargin));

%---Set default values
pre_data = [1 -1];
gpr_len = 0;
fir_len = 0;
gpr = [];
fir = [];

%---Placeholder for the signature arguments
sigStr = '';

%---Identify string and numeric arguments
for n = 1:nargin
    if(n>1)
        sigStr(size(sigStr,2)+1) = '/';
    end
    %---Assign the string and numeric flags
    if(ischar(varargin{n}))
        sigStr(size(sigStr,2)+1) = 's';
    elseif(isnumeric(varargin{n}))
        sigStr(size(sigStr,2)+1) = 'n';
    else
        error('Only string and numerical arguments are allowed.');
    end
end

%---Identify parameter signatures and assign values to variables
switch sigStr
    case 'n/n/s'
        num = varargin{1};
        fir_len = varargin{2};
        head_file = varargin{3};
        gpr = [0.25 1 0.25]';
        gpr_len = length(gpr);
    case 'n/n/n/s/s/s'
        num = varargin{1};
        gpr_len = varargin{2};
        fir_len = varargin{3};
        head_file = varargin{4};
        constraint = varargin{5};
        method = varargin{6};
end
for SNR=7:14
    error_frame = 0;
    sum_error_bit = 0;
    frame_count = 0;

    %--- train first to get the gpr target and the fir coefficient
    if(isempty(gpr))
        [gpr, fir] = train_GPR(num, SNR, gpr_len, fir_len, head_file,
            constraint, method);
    end

    %--- Reset the random seed
    rand('state', 1000);
    randn('state', 1000);
    while (error_frame < 50)
        frame_count = frame_count+1;
        %--- Generate random data
        data = [pre_data, gen_rand_data(num)];

        %--- Generate Gaussian distributed jitter
        data_count = length(data);
        jitter_amount = 1.5; % jitter_amount in nm
        %--- Generate Gaussian distributed jitter in the main track
        jitter_main(2*(1:data_count)) = jitter_value_main;
        jitter_main(2*(1:data_count)-1) = -jitter_value_main;
        [soutsig, island_size, island_period, error_count, jitter_change] =
            generate_signal(head_file, data, jitter_main);

        %--- Normalize the signal
        nor_value = 0.5256; % This is the peak amplitude value of the ideal
        nor_value = 0.3324; % This is the peak amplitude value due to an isolated
        island at BAR=4
        soutsig = -1*soutsig/nor_value;

        %--- Define the Vo-p
        Vo_p = 1;

        %Generate noisy signal corrupted by AWGN
        Vrms_n = Vo_p/(10^(SNR./20));
        noise = Vrms_n * randn(1,num+2);
        signal = soutsig+noise;
%--- Equalize the signal through a FIR
ideal = filter(gpr, 1, data);
if (isempty(fir))
    fir = fir_lms(ideal, signal, fir_len);
    save('fir_taps', 'fir');
end
filtered_signal = filter(fir, 1, signal);

%--- Calculate the delay between FIR and GRP
fir_delay = floor((fir_len - gpr_len)/2);

%--- Generate the trellis
trellis = trellis_gen(gpr_len, gpr);
% level_values = {{-3.5 0 3.5}, 0, {-3.5 0 3.5}};
% trellis = extended_trellis_gen_branchonly('width', 'pulse.mat',
gpr_len, level_values, [0 0 1 0 0]);
%
%--- Viterbi detector
% re_data = viterbi(signal(2:data_count), trellis);
% re_data = viterbi(filtered_signal((gpr_len+fir_delay):data_count),
% trellis);

%--- Calculate the viterbi delay
% delay = floor(fir_len/2)+1;

%--- Compare the recovery data with the input data and calculate the error bits
de_data = [pre_data, re_data];
k = length(de_data);
error_bit = length(find(de_data - data(1:k)));
sum_error_bit = sum_error_bit + error_bit;
if (error_bit>0)
    error_frame = error_frame + 1;
    disp(sprintf('SNR: %.2f, Viterbi BER: %ld/%ld (%3.3e)', SNR,
    sum_error_bit, frame_count*num, sum_error_bit/(frame_count*num)));
    fid = fopen('single_size_BAR4.txt', 'a');
    fprintf(fid, 'SNR: %.2f, Viterbi BER: %ld/%ld (%3.3e)', SNR,
    sum_error_bit, frame_count*num, sum_error_bit/(frame_count*num));
    fclose(fid);
end
if (error_frame == 50)
    BER(SNR-6) = sum_error_bit/(frame_count*num);
    save('BER_result.mat', 'BER');
end;
Appendix 1

Lists of MATLAB functions

178

end
disp(sprintf(' '));
fid = fopen('single_size_BAR4.txt', 'a');
fprintf(fid, '\r\n');
fclose(fid);
end
final_time = etime(clock, time);
nout = max(nargout, 1) - 2;
if (nout == 1)
    varargout{nout} = final_time;
end;

2. Search for error events

function [sum_error_bit] = count_error_jitter_value(num, head_file)

% This is a function which counts and records the error events.
% % Copyright 2009
% % Yuanjing Shi
% % School of Computer Science
% % The University of Manchester
% % shiya@cs.manchester.ac.uk
sum_error_bit = 0;
sum_error_multi = 0;
sum_error_single = 0;
gpr = [0.25 1 0.25];
gpr_len = length(gpr);
pre_data = [1 -1];
jitter_amount = 1.5;
jitter_error_multi = [];
data_error_multi = [];
sample_error_multi = [];
state_error_multi = [];
ways_error_multi = [];
jitter_error = [];
data_error = [];
sample_error = [];
state_error = [];
ways_error = [];

for frame_count = 1:1000
    %--- Generate random data
data = [pre_data, gen_rand_data(num)];
%--- Generate Gaussian distributed jitter
data_count = length(data);

%--- Generate Gaussian distributed jitter in the main track
jitter_main = jitter_amount.*randn(1,2.*data_count);
jitter_value_main = (jitter_amount/2).*randn(1, data_count);
jitter_main(2*(1:data_count)) = jitter_value_main;
jitter_main(2*(1:data_count)-1) = -jitter_value_main;
[soutsig, island_size, island_period, error_count, jitter_change] =
generate_signal_change(head_file, data, jitter_main);

%--- Normalize the signal
nor_value = 0.3324; % This is the peak amplitude due to an BAR 4:1 isolated island
nor_value = 0.5526;
soutsig = -1*soutsig/nor_value;
signal = soutsig;

%--- Generate the trellis
trellis = trellis_gen(gpr_len,gpr);

%--- Viterbi detector
[re_data mlsd_out state out_state ways]=
viterbi(signal(2:data_count),trellis);

%--- Compare the recovery data with the input data and calculate the error bits
de_data = [pre_data re_data];
k=length(de_data);
position = find(de_data-data(1:k));
error_bit = length(position);

%--- Count the multiple errors
interval = position(2:error_bit)-position(1:error_bit-1);
multi = find(interval==1|interval==2);
position_multi = [position(multi),position(multi+1)];
position_multi = sort(position_multi);
position_multi = find_duplicate_value(position_multi');
position_multi = position_multi.value{1}';
error_multi = length(position_multi);
sum_error_multi = sum_error_multi + error_multi;

%--- Count the single errors
position(multi) = 0;
position(multi+1)= 0;
position = position(find(position));
error_single = length(position);
sum_error_single = sum_error_single + error_single;

%--- Sum up the errors
sum_error_bit = sum_error_bit + error_bit;
sum_error_ef(frame_count) = sum_error_bit;
save( 'p','position','de_data','data','mlsd_out','position_multi','sum_error_multi','error_multi','jitter_change');

%--- Information to be recorded
jitter = [0 0 jitter_change 0 0];
data = [0 data 0];
signal = [0 signal 0];
out_state = [0 2 out_state 0 0];
ways = [0 ways 0 0];

%--- Record the single errors
jitter_error =
[jitter_error,[jitter(2*(position-2));jitter(2*(position-1));jitter(2*position);jitter(2*(position+1));jitter(2*(position+2));jitter(2*(position+3 ))]];
data_error =
[data_error,[data(position-2);data(position-1);data(position);data(position+1);data(position+2);data(position+3)]];
sample_error =
[sample_error,[signal(position-2);signal(position-1);signal(position);signal(position+1);signal(position+2);signal(position+3)]];
state_error =
[state_error,[out_state(position-2);out_state(position-1);out_state(position);out_state(position+1);out_state(position+2)]];
ways_error =
[ways_error,[ways(position-2);ways(position-1);ways(position);ways(position+1)]];

%--- Record the multiple errors
jitter_error_multi =
[jitter_error_multi,[jitter(2*(position_multi-2));jitter(2*(position_multi-1));jitter(2*position_multi);jitter(2*(position_multi+1));jitter(2*(position_multi+2));jitter(2*(position_multi+3))]];
data_error_multi =
[data_error_multi,[data(position_multi-2);data(position_multi-1);data(position_multi);data(position_multi+1);data(position_multi+2);data(position_multi+3)]];
multi+3)];
    sample_error_multi =
[sample_error_multi,[signal(position_multi-2);signal(position_multi-1);si

gnal(position_multi);signal(position_multi+1);signal(position_multi+2);si

gnal(position_multi+3)]];  
    state_error_multi =
[state_error_multi,[out_state(position_multi-2);out_state(position_multi-

1);out_state(position_multi);out_state(position_multi+1);out_state(positi

on_multi+2)]];  
    ways_error_multi =
[ways_error_multi,[ways(position_multi-2);ways(position_multi-1);ways(pos

ition_multi);ways(position_multi+1)]];  
end;

save('error_info','sum_error_ef','jitter_error','data_error','sample_erro

r','state_error','ways_error');
save('error_info_multi','jitter_error_multi','data_error_multi','sample_e

rror_multi','state_error_multi','ways_error_multi');
Appendix 2

MATLAB script for the calculation of BER

1. The calculation of BER of the standard trellis with no size variations

   function BER = BER_Calculation_AWGN
   diff = [-0.2 -2 -0.2];
   % diff = [-0.5 -2 -0.5];
   min_square = sum(diff.^2);
   d_m = min_square/2;
   Vo_p=1;
   Wh = 1;
   SNR = 7:20;
   sigma_whitenoise = Vo_p./(10.^(SNR./20));
   D_whitenoise_viterbi = min_square*(sigma_whitenoise.^2);
   sigma_total = sqrt(D_whitenoise_viterbi);
   Q_function = 0.5.*erfc((min_square/2)./(sqrt(2).*sigma_total));
   BER = 2*(1/2)^Wh.*Q_function;

2. The calculation of BER of the standard trellis with size variations present

   function BER = BER_Calculation_size_variation
   % D_T=[0.0533 0.0713 0.0894 0.1068 0.1226 0.1367 0.1489 0.1595].^2;  % BAR=4, 15nm island
   % D_0=[0.1838 0.2448 0.3044 0.3608 0.4123 0.4577 0.4974 0.5313].^2;
   D_T=[0.0147 0.0178 0.0211 0.0244 0.0278 0.0311 0.0342].^2;  % BAR=1, 25nm island
   D_0=[0.1603 0.1924 0.2245 0.2562 0.2871 0.3168 0.3448].^2;
   diff = [-0.2 -2 -0.2];
   D_size = (4*D_T+D_0)*sqrt(2);
   % diff = [-0.5 -2 -0.5];
   min_square = sum(diff.^2);
   d_m = min_square/2;
   D_size_viterbi = min_square*D_size;
   Vo_p=1;
   Wh = 1;
   SNR = 7:20;
   sigma_whitenoise = Vo_p./(10.^(SNR./20));
   D_whitenoise_viterbi = min_square*(sigma_whitenoise.^2);
   for i=1:length(D_T)
       sigma_total = sqrt(D_size_viterbi(i)+D_whitenoise_viterbi(i));
   end
   BER = 2*(1/2)^Wh.*Q_function;
3. The calculation of BER of the modified trellis with size variations present

```matlab
function BER = BER_Calculation_ext(d_mean);

D_T=[0.0533 0.0713 0.0894 0.1068 0.1226 0.1367 0.1489 0.1595].^2;  % BAR=4, 15nm island
D_0=[0.1838 0.2448 0.3044 0.3608 0.4123 0.4577 0.4974 0.5313].^2;  % BAR=1, 25nm island
% D_T=[0.0147 0.0178 0.0211 0.0244 0.0278 0.0311 0.0342].^2;  % BAR=1, 25nm island
% D_0=[0.1603 0.1924 0.2245 0.2562 0.2871 0.3168 0.3448].^2;
% diff = [-0.2 -2 -0.2];
D_size = (D_0)*sqrt(2);
diff = [-0.5 -2 -0.5];
min_square = sum(diff.^2);
D_size_viterbi = min_square*D_size;
Vo_p=1;
Wh = 1;
SNR = 7:20;
sigma_whitenoise = Vo_p./(10.^(SNR./20));
D_whitenoise_viterbi = min_square*(sigma_whitenoise.^2);
d_m = min_square/2;
for i=1:length(D_T)
    for j = 1:14
        sigma_total = sqrt(D_size_viterbi(i)+D_whitenoise_viterbi);
        sigma = sqrt(2).*sigma_total;
        Q_1 = 0.5.* (0.125.*erfc((d_m–d_mean(1,j))./sigma)+
        0.125.*erfc((d_m+d_mean(1,j))./sigma)+0.125.*erfc((d_m+d_mean(6,j))./sigm
        a)+0.125.*erfc((d_m+d_mean(6,j))./sigma)+0.25.*erfc((d_m+d_mean(2,j))./si
        gma)+0.25.*erfc((d_m+d_mean(2,j))./sigma));
        Q_function(j) = Q_1;
    end;
    BER(i,:) = 2*(1/2)^Wh.*Q_function;
end;
```
Appendix 3

List of Publications

Journal Publications


Conference Presentations
1. Yuanjing Shi, P. W. Nutter, Jim Miles and Branson Belle, “Understanding Sources of Errors in Bit Patterned Media to Improve Read Channel Performance”, IEEE International Magnetics Conference, May 4-8, 2008


Understanding Sources of Errors in Bit-Patterned Media to Improve Read Channel Performance

Paul W. Nutter, Yuanjing Shi, Branson D. Belle, and Jim J. Miles

School of Computer Science, The University of Manchester, Manchester, M13 9PL, U.K.

The limitations of current lithographic techniques result in a variation of the geometry of the fabricated islands in bit-patterned media. These variations give rise to jitter in the replay waveform that has a detrimental effect on the recovery of stored data. By analyzing experimental bit-patterned media, we show that the presence of lithography jitter can be quantified in terms of variations in the size and position of the islands, which can be seen to be Gaussian-like in nature. In addition, the amount of jitter increases as the periodicity and size of the islands reduces, confirming that lithography jitter will be a significant source of noise in any future storage systems incorporating bit-patterned media. By using a comprehensive read channel model we demonstrate that a novel trellis structure offers improved read channel performance in the presence of island position variations.

Index Terms—Bit-patterned media, jitter, magnetic recording, noise, trellis, Viterbi.

I. INTRODUCTION

BIT-PATTERNED media is seen as one technology suitable for achieving storage densities in excess of 1 Tbit/in² (155 Gb/cm²) in future magnetic recording systems [1], [2]. However, the fabrication of bit-patterned media is technically challenging, and whilst some promising fabrication approaches exist, their applicability at such densities have yet to be proven [2]. More importantly, lithographic techniques are unable to fabricate uniform island arrays over large areas cost effectively, and the resulting island arrays have a large variation in island geometry. The result of this variation in island geometry is the introduction of jitter in the replay signal and variations in the amount of inter-symbol-interference (ISI) present, which has a detrimental effect on the performance of the read channel [3]. In addition, any island geometry variations also pose a problem for recording [4]. Recent work investigating read channel designs for bit-patterned media has shown that the effect of lithography jitter may be alleviated through the application of low-density parity-check (LDPC) coding schemes [3]. However, by identifying the type of island variation prevalent in bit-patterned media, it may be possible to improve the data recovery process without the need for additional coding schemes. Here, we demonstrate that the variation in island geometry can be quantified in terms of variations in island size and position and we investigate the effect that such variations have on the read channel performance in bit-patterned media storage systems. We thus propose a modified trellis design that can be shown to improve the bit-error-rate (BER) performance in the presence of position jitter, without the need for additional coding schemes.

The paper is structured as follows. Section II outlines the analysis of island variations in experimental media. Section III describes the replay and read channel simulations used to perform the data recovery analysis in the presence of lithography jitter, and Section IV outlines the new trellis design proposed to offer improved BER performance.

II. JITTER ANALYSIS OF EXPERIMENTAL MEDIA

In order to understand the nature of the variation in island geometry in bit-patterned media, images of experimental media produced at Manchester have been analyzed. The media produced consist of arrays of islands fabricated in Co/Pt multilayer thin films using electron-beam lithography and Ar ion milling. The complete fabrication process is discussed in [5] and is summarized here. First, the deposited Co/Pt thin film (10 nm/Co/0.4 nm CoPt/1 nm Pt) was coated with a C overcoat in order to form a hard mask for milling. Electron-beam lithography was then used to expose nanostructures in a PMMA resist layer. After development, the patterned resist was then coated with a Ti layer, which was then removed by lift-off to reveal islands of Ti on the C overcoat. Transfer of the pattern to produce a C hard mask was achieved by O₂ plasma etch. Finally, Ar ion milling was used to produce the patterned islands. Island arrays of nominal pitch 100, 80, 60, and 50 nm were produced and then imaged using a 150 kV Gemini-scanning electron microscope. The resulting images were then processed, using the MATLAB image processing toolbox, to determine the approximate size (in terms of the island length along track) and down track pitch of the fabricated islands. Figs. 1 and 2 illustrate probability density functions for the variation in measured island diameter (Fig. 1) and island pitch (Fig. 2) for the four island arrays investigated. Table I lists the statistical properties of the island arrays analyzed. The results in Table I show that there is a significant variation in the size (diameter) and position (pitch) of the fabricated islands. The distributions show that these variations are Gaussian-like in nature, which agrees with an analysis of island size variations in island arrays of increased diameter and period [6]. In addition, as the pitch of the islands is reduced, the calculated standard deviations increase. Hence demonstrating that variations in island size and position will be a significant source of jitter noise in any future ultrahigh-density storage system incorporating patterned media.

Digital Object Identifier 10.1109/TMAG.2008.929516

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Authorized licensed use limited to: The University of Manchester. Downloaded on July 06, 2010 at 16:51:11 UTC from IEEE Xplore. Restrictions apply.
Appendix 3

List of Publications

Fig. 1. Distributions for the variation in island size for four island arrays of nominal island period: 100, 80, 60, and 50 nm.

Fig. 2. Distributions for the variation in island pitch for four island arrays of nominal island period: 100, 80, 60, and 50 nm.

TABLE I

<table>
<thead>
<tr>
<th>Array Nominal Pitch</th>
<th>No. of Islands</th>
<th>Mean Island Diameter</th>
<th>Island Size σ</th>
<th>Island Pitch σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 nm</td>
<td>140</td>
<td>39.42 μm</td>
<td>2.00 μm (2.1%)</td>
<td>1.65 μm (1.7%)</td>
</tr>
<tr>
<td>80 nm</td>
<td>117</td>
<td>34.06 μm</td>
<td>1.55 μm (2.5%)</td>
<td>1.96 μm (2.4%)</td>
</tr>
<tr>
<td>60 nm</td>
<td>150</td>
<td>32.42 μm</td>
<td>2.85 μm (3.8%)</td>
<td>2.29 μm (3.5%)</td>
</tr>
</tbody>
</table>

III. READ CHANNEL SIMULATION

A comprehensive read channel simulation has been developed in order to investigate the effect that the two sources of island jitter have on the ability to recover stored data. The complete read channel model, written in the MATLAB environment, consists of simulations of the replay process using a conventional giant magneto-resistive (GMR) read sensor, and the data recovery process in a conventional partial-response maximum-likelihood (PRML) read channel. A complete description of the simulations is given in [3], [7], and [8].

The replay model uses a 3-D implementation of the reciprocity integral to simulate the replay waveform from the GMR read sensor. The use of a 3-D model allows the 3-D geometry of both the recording medium and the GMR read sensor to be taken into account. In addition, it also allows the effects of the finite track width and inter-track interference to be accounted for, which is important in the case of bit-patterned media, where the written track width is determined by the width of the fabricated islands and not the width of the record head.

The replay waveform due to a track of (assumed) random recorded data is generated by superposing isolated responses due to the leading and lagging edge of each island along the track, which have been generated using the 3-D model with a semi-ininitely long island with a semicircular end. The effect of island jitter is introduced by varying the position (in space) at which the superposition of the step responses takes place. Fig. 3 illustrates the three cases of interest for geometry variations along track only. In the case of variations in the position of an island, then the same amount of jitter, Δp, is applied in the same direction at both edges of each island, as illustrated in Fig. 3(a), which results in a shift in the island position of Δp. In the case of variations in island size, then equal jitter, Δi, is applied to each edge but in opposite directions, as illustrated in Fig. 3(b), which results in an increase in the diameter of the island by 2Δi. Finally, Fig. 3(c) illustrates the case where there is a combination of both position and size variations.

The read channel model consists of a finite-impulse response (FIR) filter followed by a Viterbi maximum-likelihood decoder. The channel data samples are generated by sampling the simulated replay waveform at positions in the waveform corresponding to the position of the ideal centre of each island (at the ideal symbol-rate). The samples are then equalized by the FIR such that the response matches a PR target of [0.1, 1, 0.1]. Finally, the equalized samples are then passed to the Viterbi decoder in order for the recorded data to be recovered.

In the following analysis a patterned medium consisting of a single track of round islands has been assumed, with island diameter and pitch of 12.5 and 25 nm, respectively (1 Tb/in²). The dimensions of the read sensor are the same as those listed in [3], [7].

Following the observations of experimental bit-patterned media we have adopted Gaussian distributions for the variations in island size and position, each with a standard deviation specified as 10% of the island pitch, i.e. σi = 2.5 nm, which
Appendix 3  
Lists of Publications

Fig. 4. BER curves for 1-Thin² bit-patterned medium in the case of no jitter (solid line), 10% island position jitter (dashed line), 10% island size jitter (dotted line), and 10% island size and position jitter (dashed-dotted line).

is appropriate considering the levels of island jitter observed in experimental media. Only island position variations along track are taken into account since the width of the read sensor is much greater than the island width. The read channel performance is evaluated using plots of BER against signal-to-noise ratio (SNR), where the SNR is defined in [3] for the case of additive white Gaussian noise (AWGN) only. Fig. 4 illustrates BER versus SNR curves for the bit-patterned medium investigated using a decoder with a standard trellis for four cases: no jitter (solid); 10% island position jitter (dashed); 10% island size jitter (dotted); and 10% island position and size jitter (dash-dot).

Fig. 4 demonstrates that the introduction of jitter, whatever its source, has a detrimental effect on the performance of the read channel, and the degradation arising from variations in island size is more severe than that due to variations in island position.

IV. MODIFIED TRELLIS DESIGN

The problem of position jitter in bit-patterned media storage systems is analogous to the problem of transition jitter in conventional magnetic storage systems, and results in a random shift in the position of the recorded information. Following this, we have adopted the technique described in [9] in order to propose a new trellis design to combat position jitter in bit-patterned media. The trellis design takes into account the position shift and makes an estimation based on both the state of the recorded bits and the amount of island position shift. If we consider a partial-response (PR) read channel model with only AWGN and position jitter, then the symbol-rate samples of the PR channel can be written as

\[ r(k) = \sum_{\Delta_1} h_0 g(kT - iT + \Delta_1 T) + n(k) \]  

(1)

where \( h_0 \in \{-1, 1\} \) denotes the magnetization of each island, \( n(k) \) denotes the AWGN, \( T \) is the ideal island period, \( g(t) \) is the pulse response due to the magnetization of each island as defined by the PR target, \( g(kT) \) is the symbol-rate samples of the pulse response, and \( \Delta_1 \) denotes the position shift due to jitter that we assume follows a Gaussian distribution. As can be seen in (1), the replay samples are calculated according to a predefined PR target sampled at the bit period plus a shift of \( \Delta_1 \) due to island position variations.

When using the standard trellis, the branch metric from one state to another is determined by the magnetization state of the islands and the amplitude of the pulse response, including any ISI, at the symbol-rate samples. The most likely input sequence is then chosen by comparing the channel samples with the branch metric. However, position jitter in bit-patterned media storage systems leads to a deviation from the nominal position (symbol-rate in time) of each island, which affects the detection process in selecting the most likely input sequence and introduces errors in the recovered data. Since the amount of shift in the position of each island depends on the fabrication process and is uncorrelated, then it is difficult to predict the amount of position shift for each island in advance. One way to overcome this is to approximate the amount of position shift by discrete values chosen over the range that the position jitter values span and to use these values to characterize the ideal output sample. Thus, we have proposed a modified trellis design by introducing discrete jitter values into the standard trellis structure, where we consider both state bits and the contributions of position jitter when selecting the most likely input sequence. In the modified trellis, the states are defined by both the state of the magnetized islands, \( h_k \), and the discrete values representing a position shift of each island due to lithography jitter, \( \Delta_k \). Fig. 5 shows part of the modified extended trellis structure with the additional states introduced using this approach (shaded boxes). Also illustrated are some of the possible transitions from the states when \( b_k = b_{k-1} = 1 \). It can be seen that each state in the
modified trellis represents the magnetization state of the current bit (island), the magnetization state of one or more previous bits, as well as discrete jitter values for each. The discrete jitter values are chosen depending upon the amount of jitter present. Here we have used 10% position jitter, i.e., $\sigma_\Delta = 2.5\ \text{nm}$, so there are three discrete levels of jitter $[-2.5\ \text{nm}, 0, +2.5\ \text{nm}]$. The state branch is defined following the standard trellises with the only difference being that each state has Q branches, where Q is the number of discrete jitter levels, for each possible bit transition. Each branch metric, $\gamma_k$, is calculated knowing the effect that any fixed jitter has on the amplitude of the isolated pulse response and the effect of changes in ISI due to the shift of the neighboring islands. For a PR target of length L, the isolated pulse extends over L-1 bit periods, which requires L-1 bits to represent a state in the modified trellis; in general, the total number of states is given by $2^{L-1}Q^{L-1}$. Here, we use a PR target of length $L = 3$ and $Q = 3$ discrete jitter levels, which results in an additional 28 states, giving a total of 50 states in the modified trellises.

The branch metric, $\gamma_k$, from the state $k$ to the next state at $k+1$ can be calculated using

$$\gamma_k(S_k, S_{k+1}) = \sum_i h_i g(kT - iT + \Delta_i T)$$

(2)

where the variables have their usual meaning. For example, state S1 in Fig. 5 represents a current bit value of 1 with jitter of $-2.5\ \text{nm}$ and a previous bit value of 1 with a jitter value of $-2.5\ \text{nm}$. The ideal sample from state S1 to S7 is given by

$$\gamma_k = b_{k-1}g \left( T - \frac{-2.5}{25} T \right) + b_k g \left( \frac{-2.5}{25} T \right) + b_{k+1}g \left( -2.5 T \right) + b_{k+2}g(0) \gamma_{k+1} = b_{k-1}g(T-0.1T) + b_k g(-0.1T) + b_{k+1}g(-T + 0.1T).$$

(3)

Fig. 6 illustrates the improvement in BER performance that is observed when adopting the modified trellis design in the presence of 10% position jitter (dotted line), compared to the use of the conventional trellis (dashed line) with the same jitter present. The use of the modified trellis design results in more than 6.5 dB improvement in allowable SNR for a BER of $10^{-5}$.

V. CONCLUSION

Through an analysis of experimental bit-patterned media, we have shown that lithography jitter can be characterized by variations in the size and position of islands that are Gaussian in nature. Simulations show that both of these noise components affect the performance of the read channel, but the effect of size variations is more severe. We have investigated a new trellis design that aims to improve the performance of the read channel in the presence of lithography jitter. Initially, we have investigated the alleviation of the effects of island position jitter, and have demonstrated that an improvement in read channel performance can be observed. We hope to extend this work to investigate optimal trellis designs and alternative designs that offer similar improved performance with the addition of island size jitter, as well investigating reduced complexity designs.

ACKNOWLEDGMENT

This work was supported by the Engineering and Physical Sciences Research Council under Grant EP/E017657/1.

REFERENCES


Manuscript received 30 March, 2008. Current version published 17 December, 2008. Corresponding author: P. W. Nutter (e-mail: p.nutter@manchester.ac.uk).
Error Events Due to Island Size Variations in Bit Patterned Media

Yuanqing Shi, Paul W. Nutter, Branson D. Belle, and Jim J. Miles

Nano Engineering & Storage Technology Research Group, School Of Computer Science, The University of Manchester, Manchester, M13 9PL, UK

Control of the variations of island properties is one of the key challenges in fabricating Bit-Patterned Media for future storage systems. The presence on any variation in the size and position of an island has a detrimental effect on the ability to recover recorded data, particularly in the case of variation in island size. By analyzing error events when island size variations are present we have identified that these are more likely to be single-bit in nature. To understand the origins of these error events we have investigated the size and magnetization state of islands in the vicinity where a single-bit error event is encountered. It is shown that these error events occur due to particular combinations of island size and magnetization state for the three islands investigated. In every case the central island, from which the data bit is recovered in error, is small compared to the nominal island size. These results show that size variations must be controlled in the fabrication process in order to maximize the bit-error-rate performance of the read channel.

Index Terms—Bit patterned media, error events, magnetic recording, position variations, read channel, size variations.

I. INTRODUCTION

THE INCREASING demand for higher storage capacities in magnetic disc drives requires a storage density in excess of 1 Tbit/m². It is generally recognised that bit-patterned media (BPM) using perpendicular recording may be a future solution that will permit ultrahigh areal densities in excess of 1 Tbit/m² [1], [2]. In BPM, each information bit is recorded to a patterned, isolated, single-domain magnetic island. One of the problems associated with the fabrication of island arrays for BPM is the variability of island position and island size, both of which have a detrimental effect on the writing and reading of information [1]–[5]. Since variations of island geometry due to imperfect fabrication are unavoidable, the understanding of the effect of such variations upon bit-error-rate (BER) performance is a primary challenge in BPM [2], [5]. The effect of island position, or location, variations on the recovery of recorded data has been studied in [4]. In practical media the use of electron beam lithography or self-assembly techniques often produce island arrays of regular island position, i.e., controlled period, but there may still be a severe variation in the island size [6], and the impact of size variations upon BER performance is therefore important [7].

In conventional magnetic recording the size and bit-aspect ratio (BAR) of the recorded magnetic domains are determined by the dimensions of the recording head, whereas in BPM the fabrication process itself determines these properties. Most approaches to fabricating BPM result in a BAR of 1, i.e., the islands are of equal size along-track and across-track. However, a large BAR is desirable when using modern read/write heads due to a number of reasons: the ease of head fabrication, to improve write head fields, and to obtain acceptable replay waveform signal to noise ratio (SNR) [2]. Reference [5] lists a number of design scenarios for BPM at areal densities in the range of 1–5 Tbit/m², with many of the preferred designs having a BAR greater than 1. However, at any given density, raising the BAR above 1 will decrease one of the dimensions (typically the length along track) of the island, potentially resulting in broader distributions of the island properties, such as island period, island size and magnetic anisotropy [2], which may have an even greater effect on the BER performance of the read channel.

In this paper, the effect of island size variations on the raw BER performance in BPM systems with an island BAR of 4 is explored. We present an analysis of the resulting error events in order to understand the root causes of such errors, with respect to the island size variations and magnetization state. In this analysis the presence of other sources of error, such as inter-track interference (ITI) and additive white Gaussian noise (AWGN) have been removed.

II. READ CHANNEL MODEL

A comprehensive read channel simulation has been developed that allows the investigation of the BER performance of a BPM storage system and, more importantly for the following analysis, allows error events to be identified. The read channel simulation employs a 3-D reciprocity replay model to generate replay signals arising from a large number of islands for a given giant magneto-resistive (GMR) read head design and BPM media design [8]. The replay model allows any variations in island geometry to be easily investigated. Generated replay signal samples are then analysed by a conventional partial-response maximum-likelihood (PRML) read channel, using a Viterbi decoder, to determine recorded data recovered in error.

In the following analysis, single-domain islands of nominal size along-track of 7.4 nm, period along-track of 15 nm with a BAR = 4 (nominal width across-track of 30 nm and track pitch of 40 nm) are assumed, which supports an areal density of 1 Tbit/m². The GMR read head adopted has sensor dimensions: width across-track of 20 nm, length along-track of 4 nm and shield-to-shield spacing of 16 nm [9]. The write process and the thermal stability of stored data are not considered in this paper, and so the magnetic properties of the islands have not been defined. Here we only concentrate on investigating the effect that variations in the position and size of islands have on the raw BER performance of the read channel. The effects of equalization and AWGN have not been included in order to isolate the effects of island geometry variations only. In addition, ITI has also been ignored since the island width across-track is larger

Manuscript received October 30, 2009; revised December 10, 2009; accepted January 12, 2010. Current version published May 10, 2010. Corresponding author: P. W. Nutter (e-mail: p.nutter@mm.ucm.ac.uk).
Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.
Digital Object Identifier 10.1109/TMAG.2010.2041047

0885-8966/$26.00 © 2010 IEEE
Authorized licensed use limited to: The University of Manchester. Downloaded on July 06, 2010 at 17:11:50 UTC from IEEE Xplore. Restrictions apply.
than the width of the GMR sensor. Fig. 1 illustrates a block diagram of the read channel model developed, where a generalized partial response (GPR) target of \((0.25 \pm 0.25)\) is chosen to match the shape of the isolated (ideal) island response.

The read waveform is produced by the superposition of island edge responses at positions defined by each island along the track. From these waveforms sample values are extracted corresponding to the (ideal) position of each island, i.e., at the ideal island period [4]. For example, Fig. 2 illustrates pulse responses generated by the superposition of leading and lagging edge responses. Variations of island position or size are introduced by varying the position at which these edge responses are superposed. In the case of variation in island position, the nominal position of each island is varied by shifting both edges of the island in the same direction (down-track or up-track). In the case of variations in island size an equal shift is applied to each edge of the island, but in opposite directions. Simultaneous position and size variations are generated by randomly shifting the edges of each island. These variations are assumed to be random with a truncated Gaussian distribution, of mean zero and standard deviation \(\sigma\) specified in nm, with the edge shift restricted so that there is no island overlap.

Let \(\Delta_x\) denote the position shift of the island, then the replay pulse due the position shift can be represented by \(h(x - \Delta_x)\), where \(h(x)\) denotes the replay pulse due to an ideal island. Differences between the replay pulse and the position-shifted pulse can be expressed approximately as

\[
\Delta h(x) = h(x) - h(x - \Delta_x) = \Delta_x \frac{dh(x)}{dx}.
\]

It can be seen from (1) that the effect of the position shift on the replay samples is related to the local slope of the replay pulse. Fig. 2 illustrates the effect that an arbitrary amount of island position variation (Fig. 2(a)) and island size variation (Fig. 2(b)) will have on the replay samples due to an isolated island. Here the sample values, shown as the symbols on the curves of Fig. 2, have been taken at points corresponding to the ideal sampling time (in terms of distance from the island centre) of an island, plus an island pitch before and after the island of interest. In both Fig. 2(a) and (b) the circles indicate the ideal replay sample values with no variation in the island geometry. It can be seen from Fig. 2(a) that when island position shift is introduced, the central sample, corresponding to the sample at the centre of an island, is at, or close to, the peak of the pulse where the slope is near zero, and so there is a small effect on the sample value. At the samples either side of the central sample, which correspond to the ideal positions of adjacent islands, the pulse slope is larger and this will result in a variation in the amount of ISI in the waveform observed. Fig. 2(b) shows that island size variations have an impact on the magnitude of the sample value corresponding to the island of interest. An increase in the island size results in an increase in the signal magnitude and a decrease in the island size results in a decrease in the signal magnitude. These signal amplitude changes are expected since the replay signal is proportional to the flux emanating from the islands and the total amount of magnetic material present. In the case of the adjacent samples, little change in sample magnitude is observed resulting in a small change in the amount of ISI present. The effect of the two different variations of the island geometry (position and size) on the sample values may enable them to be distinguished on an island-by-island basis, which could be put to use in an advanced detector.

The effects of island position variations, island size variations, and both island position and size variations on the read channel performance in a high BAR system have been explored. Fig. 3 illustrates the BER performance versus the standard deviation, \(\sigma\) nm, of the island variations with no other sources of noise present. In the case of just island position variations (not shown) then no errors are detected. In the presence of just size variations then the number of errors increases with \(\sigma\). In the case of the presence of both position and size variations (dashed line), where each has a standard deviation of \(\sqrt{2} \cdot \sigma/2\) giving a total contribution of \(\sigma\), then significantly more bit errors are observed compared with the cases of position and size variations alone. The results shown in Fig. 3 demonstrate the need to investigate the origins of errors in BPM when island variations are present. Here, we begin this analysis by considering the identification of error events in the presence of size variations only.

III. IDENTIFICATION OF ERROR EVENTS

The analysis of error events is essential for the performance analysis of read channels [10]. An error event is defined as a distinct distance between the correct path and the estimated path in the maximum-likelihood (ML) Viterbi detector. In the trellis, the error event occurs when the correct path and the estimated path start to diverge at one state, and it ends when those two paths converge into another state. Fig. 4 shows an example of the error event in a trellis for the GPR target used. The Viterbi detector compares the read-back samples, \(y_m\), with the path metric to decide the transition from one state to another. It chooses the transition path for which the path metric has the minimum squared distance from the replay samples. The correct path (solid line) that the Viterbi detector should take is S1-S1-S1-S1, but because of the presence of island geometry variations the replay samples vary from the ideal samples and the Viterbi detector takes the erroneous path S1-S3-S2-S1 (dashed line). An error event starts at magnetization state S1 and ends at S1, between these two states the path goes though
two incorrect states. The error event can also be represented by the associated input error sequence as \( e_k = b_k - d_k \), where \( 1 \leq k \leq l(r) \), \( r \) is the length of an error event, \( e_k (k) = (-2, 0, 2) \), \( b_k \) is the input data sequence to the PR channel, and \( d_k \) is the estimated recovered data sequence from the Viterbi detector. The number of bit errors due to an error event is equal to the number of non-zero coefficients in \( e_k \), which is represented as \( w_0 (e_k) \). It can be seen from the definition that the error event that extends from time \( t_1 = k \) to \( t_2 = k + 2 \), will not have more than \( n - 1 \) consecutive zeros between time \( t_1 \) and \( t_2 - n \), where \( n \) represents the memory in the channel (i.e., the length of the target minus one). While there are many possible error events in the read channel, a few typical error events may be the dominant source of errors. In order to identify the typical error events, the read channel simulation was run continuously to generate \( 4 \times 10^9 \) bits for the Viterbi detector to recover, from which the error sequences were identified and recorded.

Here, the memory of the channel is 2.

If no more than two consecutive zeros are found in the error sequences between two non-zero values, then an error event is recognized. For example, a single bit error event can be indentified as "01200" and a two-bit error event can be indentified as "01202000" or "012002000". The distinct error events were searched in the results of the read channel simulation, with results shown in Table I, for island size (length along-track) standard deviations ranging from 1.5 nm to 3.5 nm (20% to 46% of nominal island length). Table I shows that in all cases of \( \alpha \) investigated the dominant error events are single bit in nature, even as the standard deviation of the size variation is increased (over 95% of all the error events observed).

Once the error events have been identified the island properties (size and magnetization state) corresponding to the islands in the vicinity of each of the error events were investigated. For the GPR target chosen, three consecutive islands are considered that are centered on the island for which the magnetization state was recovered in error. The eight possible combinations of magnetization states for these three islands can be categorized into the four cases listed in the first column of Table II. In all cases the single bit error was only observed when the size of the centre island (the one recovered in error) was "small" compared to the nominal size of an island along track of 7.5 nm. In the case where the magnetization state of the neighboring islands differ from that of the centre island \((- - + \text{ or } + + -)\) then an error event is more likely to be observed when they are "big" compared with the nominal island size. Similarly, when the magnetization state of the neighboring islands is the same as that of the centre island \((- - + \text{ or } + + +)\) then an error event is more likely to be observed when they are "small" compared to the nominal size.

Fig. 4 illustrates distributions for the three islands identified, corresponding to the data bits recovered in error, when there are varying amount of island size variations present. Fig. 5 illustrates that in the error event cases observed the size distribution of the central island is a truncated Gaussian and that the size distributions for the neighboring islands (indicated as previous and next) correspond to two weighted Gaussian distributions arising from the "small" and "big" cases observed in Table II. Table III summarizes mean values \( \mu_k \) and standard deviations \( \sigma_k \) for the distributions of the central island and a combination of the distributions of the neighboring islands where an error event occurred, for each case of size variation \( \alpha \) investigated. In the case of the central island that is recovered in error, as the island size variation \( \alpha \) is increased then the mean value of the island distribution \( \mu_k \) decreases, but the standard deviation \( \sigma_k \)

Authorized licensed use limited to: The University of Manchester. Downloaded on July 06, 2010 at 17:11:50 UTC from IEEE Xplore. Restrictions apply.
Fig. 5. Size distributions of the three islands of interest centered on the island corresponding to the data bit recovered in error. Four values of $\sigma$ are considered: (a) 2.0 nm, (b) 2.5 nm, (c) 3 nm, (d) 3.5 nm (27% to 46% of nominal island length of 7.5 nm).

### TABLE III

<table>
<thead>
<tr>
<th>Size Variation (nm)</th>
<th>Small Neighbors</th>
<th>Big Neighbors</th>
<th>Small Central Island</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d$ ($\mu m$)</td>
<td>$d$ ($\mu m$)</td>
<td>$d$ ($\mu m$)</td>
</tr>
<tr>
<td>2.0</td>
<td>4.98</td>
<td>1.76</td>
<td>10.18</td>
</tr>
<tr>
<td></td>
<td>1.94</td>
<td>1.62</td>
<td>1.17</td>
</tr>
<tr>
<td>2.5</td>
<td>4.67</td>
<td>2.07</td>
<td>10.19</td>
</tr>
<tr>
<td></td>
<td>2.60</td>
<td>1.40</td>
<td>1.39</td>
</tr>
<tr>
<td>3.0</td>
<td>4.55</td>
<td>2.31</td>
<td>10.33</td>
</tr>
<tr>
<td></td>
<td>2.84</td>
<td>1.11</td>
<td>1.66</td>
</tr>
<tr>
<td>3.5</td>
<td>3.75</td>
<td>2.93</td>
<td>10.60</td>
</tr>
<tr>
<td></td>
<td>3.96</td>
<td>0.89</td>
<td>1.88</td>
</tr>
</tbody>
</table>

increases. In the case of the neighboring islands, a similar decrease in mean value is observed for the “small” islands, and an increase in mean value is observed for the “big” islands as the size variation increases. In both cases the standard deviation of the distributions increases as the size variation increases.

### IV. CONCLUSION

In this paper, we have used a read channel simulation to show that fluctuations in the island size in BPM systems have a detrimental effect on the data recovery process, particularly in the case of high BAR islands. Here the analysis of errors shows that the dominant error events are single-bit in nature. Particular combinations of island size and magnetization state for a group of three islands are more likely to cause single-bit error events, more importantly when the central island is small (1.6 nm when $\sigma = 2$ nm, 0.9 nm when $\sigma = 3.5$ nm) in comparison with the nominal size of 7.5 nm. These results show that island size needs to be carefully controlled in the fabrication process in order to prevent single-bit errors from occurring, thus improving the BER performance. Alternatively, coding schemes should be adopted that mitigate the effects of such single-bit error events. Future work will concentrate on determining BER performance using analytical approaches to enable a thorough investigation of the combined effects of position and size variations to understand the origins of the severe degradation in BER performance shown in Fig. 3.

### ACKNOWLEDGMENT

This work was supported by the Engineering & Physical Sciences Research Council under Grant EP/E017674/1, and by the Information Storage Industry Consortium (INSIC) EHDR program.

### REFERENCES


Authorized licensed use limited to: The University of Manchester. Downloaded on July 06, 2010 at 17:11:50 UTC from IEEE Xplor. Restrictions apply.