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Managing uncertainty in multiple-criteria decision making related to sustainability assessment

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Abstract In real life, decisions are usually made by comparing different options with respect to several, often conflicting criteria. This requires subjective judgements on the importance of different criteria by DMs and increases uncertainty in decision making. This article demonstrates how uncertainty can be handled in multi-criteria decision situations using Compromise Programming, one of the Multi-criteria Decision Analysis (MCDA) techniques. Uncertainty is characterised using a probabilistic approach and propagated using a Monte Carlo simulation technique. The methodological approach is illustrated on a case study which compares the sustainability of two options for electricity generation: coal versus biomass. Different models have been used to quantify their sustainability performance for a number of economic, environmental and social criteria. Three cases are considered with respect to uncertainty: (1) no uncertainty, (2) uncertainty in data/models and (3) uncertainty in models and decision-makers' preferences. The results shows how characterising and propagating uncertainty can help increase the effectiveness of multi-criteria decision making processes and lead to more informed decision.

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Introduction

In real life, decisions are usually made by comparing different options with respect to several, often conflicting criteria. In these cases, there is generally no best overall option, as switching from one option to another is likely to result not only in an improvement in some criterion but also in the deterioration of other criteria. Multi Criteria Decision Analysis (MCDA) provides effective techniques for assisting DM(s) in solving such problems (see e.g. Gal et al. 1999).

MCDA assists decision makers (DMs) to rank or prioritise options (or alternatives) analysed. However, before any ranking can be carried out, each option has to be characterised for a number of decision criteria defined, e.g. technical, economic, environmental etc. Various models and tools can be used for these purposes, estimating, for example, costs, environmental impacts and technical performance of the options. Note that the criteria used may be expressed in different units (e.g. monetary units, mass units, dimensionless, etc.).

The ranking of options is then performed by repeatedly asking DMs to elicit their preferences. MCDA methods differ in the way the preferences are handled. Some of the widely used MCDA methods include

(1) the Analytical Hierarchy Process (Saaty 1980),
(2) Compromise Programming (Zeleny 1973),
(3) PROMETHEE—Preference Ranking Organisation METHod for Enrichment Evaluations (Brans and Vincke 1985),

(4) TOPSIS—Technique for Order Preference by Similarity to Ideal Solution (Hwang and Yoon 1981),
(5) Partial order ranking—Hasse diagram technique (Halfon and Reggiani 1986),
(6) Formal Concept Analysis (Wille 1982), and
(7) ELECTERE (I, II, III, IV and TRI) series methods (Roy 1968).

Nowadays, public discussions, policy decisions and scientific results are frequently based on deterministic analysis and without explicit evaluation of the uncertainties involved. Uncertainty exists where there is a lack of knowledge concerning outcomes. Uncertainty may result from an imprecise knowledge of the risk, i.e. where the probabilities and magnitude of either the hazards/failures and/or their associated impacts are uncertain. Note that in MCDA, uncertainty may affect not only criteria estimation but also decision analysis based on these criteria. If criteria estimation is performed using mathematical models, then these criteria may be affected by uncertainty in three different ways: data uncertainty, structural uncertainty and knowledge uncertainty (UKCIP 2003). On the other hand, the MCDA methods may also be affected by the uncertainty in DMs' preferences which are often contradictory, arbitrary and lacking consensus (Mousseau et al. 2003). The combination of these two sources of uncertainty—models and DMs' preferences—is likely to result in compounded uncertainty in the outcome of decision analysis.

The aim of understanding, characterising and propagating uncertainty is to inform the DMs on how likely it is that a different option is selected as a result of that uncertainty. In literature, uncertainty in decision making is mostly handled by using sensitivity analysis (Barron and Schmidt 1988; Ringuest 1997; Triantaphyllou and Sanchez 1997), or probabilistic sensitivity analysis (Critchfield and Willard 1986; Felli and Hazen 1998; Janssen 1996).

The study presented here characterises and quantifies uncertainty using a probabilistic approach and Monte Carlo simulation. The methodological approach is presented next, which is then followed by an illustration of the methodology on a case study, comparing two options for electricity generation—coal and biomass.

The methodology for managing uncertainty

The methodology developed in this study for characterising and quantifying uncertainty in the decision-making process is shown in Fig. 1. As can be seen from this figure, the framework, i.e. the decision-making process starts by defining the set of options of interest (i.e. potential, alternative solutions) for the analysed problem. At the same time, a set of criteria that will be used to evaluate these

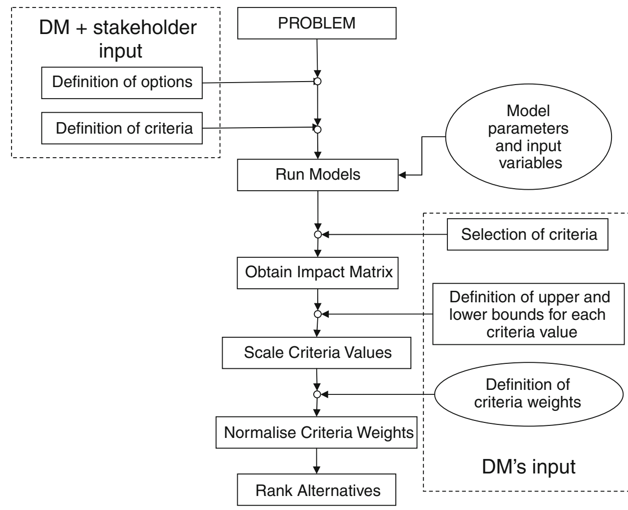

Fig. 1 Methodological approach to characterising and quantifying uncertainty using Compromise Programming

optional solutions is identified too. This preliminary analysis is carried out by a DM (e.g. a programme manager) liaising with stakeholders and supported by the MCDA analyst. The objective is to identify/formulate planning decisions that could be potentially used to solve the problem analysed and, at the same time, to define the quality standards that should be met by the implemented decision. This is an iterative process which eventually results in a list of optional solutions to be considered and the multiple criteria that will be used to evaluate and compare them.

The criteria are then quantified using different simulation and other models and tools, depending on the aim of the analysis. For example, the options can be compared for environmental impacts using Life Cycle Assessment (LCA); capital and operating costs can be estimated for the economic analysis, etc. The output of this stage is an impact matrix, showing the estimated values for each criterion and each option considered. Let N denote the number of selected criteria and M the number of options, the impact matrix $\mathbb{Q}$is an N-by-M matrix, where the element q_{ij} denotes the value of the j-th option for the i-th criterion.

The technique used here to solve the MCDA problem is the Compromise Programming (CP) method (Zeleny 1973). Criteria values are usually quantified in different measurement units. In order to compare them, these values need to be first scaled into comparable units. This operation is known as the normalisation or the standardisation. The standardisation procedure normalises the original criteria values making them dimensionless which, in turn, enables them to be added up. The main issue here is how to choose an adequate value function, or standardisation procedure, as many possible approaches exist: linear scale transformation, interval standardisation, goal standardisation and nonlinear scale transformation (Belton and Stewart 2002). The interval standardisation is used here without the loss of generality for the methodology presented, as it is often done in the Goal Programming literature. Of course, if a DM (e.g. a programme manager) prefers to use some other standardisation technique, this can be done easily.

The normalisation process starts by asking the DMs to specify upper and lower bounds for each criterion analysed (unless these values are obvious, e.g. 0–1 interval for some likelihood value, etc.). The lower bound represents the fully acceptable value, whereas the upper bound represents a non-acceptable value. Note that this implies that each criterion represents a penalty, i.e. something to minimise. If a criterion represents a reward instead, i.e. something needs to be maximised, then it is multiplied by -1 and translated into a penalty (i.e. minimised).

By denoting with v_j and V_j the upper and lower bound of j-th criterion respectively, the impact matrix $\mathbb{Q}$ is transformed into a scaled matrix $\bar{\mathbb{Q}}$, where the element $\bar{q}_{ij}$ is as follows:

$$\bar{q}_{ij} = \begin{cases} \frac{q_{ij}-v_j}{V_j-v_j}, & \text{if } j\text{−th criterion is minimised} \\ -\frac{q_{ij}-v_j}{V_j-v_j}, & \text{if } j\text{−th criterion is maximised} \end{cases} \quad (1)$$

From here on, the scaled criteria are referred to as *losses*. The third and last input required from DMs is to provide, for each criterion, a preference weighting on a scale of their choice (e.g. 0–10), expressing the perceived importance of each criterion. More formally, the DMs produce a vector $\mathbf{w} = (w_1, \ldots, w_N)^T$ where the element w_i is the weight assigned to the i-th criterion. Note that before starting CP, all the weights are normalised so that their sum is equal to 1.

Criteria values and weights are then combined together to quantify the appraisal score for each option. The appraisal score is the *aggregated loss*, and it is obtained using CP:

$$\mathbf{z} = \bar{\mathbb{Q}}^T \cdot \mathbf{w} = \left(\sum_{i=1}^{N} \bar{q}_{i1} w_i, \ldots, \sum_{i=1}^{N} \bar{q}_{iM} w_i \right)^T = (z_1, \ldots, z_M)^T \quad (2)$$

where z_j is the aggregated loss for the j-th option. Based on that, the M options are ranked with respect to their aggregated losses; the preferred one is the one attaining the minimum.

There are two sources of uncertainty affecting CP that are herein considered:

(1) uncertainty in the parameters and input variables of the models used to quantify the criteria; and
(2) uncertainty due the different DM preferences for different criteria.

These two uncertainty sources are represented as ovals in the methodological scheme presented in Fig. 1. Those two sources of uncertainty are here modelled and propagated using a probabilistic approach similar to the one described by Hyde et al. (2004).

In this approach, all the uncertain parameters and input variables of the models quantifying the criteria are defined by means of Probability Distribution Function (PDF). In other words, instead of using one value for each parameter/input variable, a set of possible values and the corresponding probabilities (PDFs) are used. Based on the PDFs, different random instances of parameters/input variables are generated; for each instance, the models are simulated and the impact matrix is obtained. Different instances will normally result in different impact matrices; therefore, the effect of propagating the parameters/input variables uncertainty through the models is to make the impact matrix $\mathbb{Q}$ a random matrix. This iterative way to propagate parameters/input variables uncertainty to the impact matrix uncertainty is known as Monte Carlo simulation (MCS).

MCS propagates the uncertainty all the way to the ranking of the options. In fact, since the impact matrix $\mathbb{Q}$ is a random matrix, then $\bar{\mathbb{Q}}$ is a random matrix too, and hence, the aggregated loss $\mathbf{z} = \bar{\mathbb{Q}}^T \cdot \mathbf{w}$ is a random vector. The question of which option attains the minimum aggregated loss depends on the iteration. In principle, each alternative has a probability p_i^{best}, $i = 1, \ldots, M$ (which may be zero) to be ranked number one, namely best. with the use of MCS, the value of these probabilities $\mathbf{p}^{best} = \left(p_1^{best}, \ldots, p_M^{best}\right)$ can be assessed. Each probability quantifies the level of uncertainty for the corresponding option to be the preferred one. This analysis suggests that the option with the highest probability should be chosen.

The second source of uncertainty considered is the uncertainty due to different DMs' preferences for different criteria. This is the situation where there are K DMs, and each of them has produced a set of different preference weights $\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_K$. In order to apply CP to find the preferred option, there should be only one vector, instead of K. Here, each DM is considered as a representative sample of the criteria weights of a population of stakeholders. Each set of criteria weights can, therefore, be portrayed by a probability $p(\mathbf{w}_1), p(\mathbf{w}_2), \ldots, p(\mathbf{w}_K)$, ensuring that all of the information obtained from the DMs is explicitly incorporated in the decision-making process. In other words, the effect of this second source of uncertainty is that now even the preference weight is a random vector $\mathbf{w} = (w_1, \ldots, w_N)^T$, and hence, different instances of $\mathbf{w}$ can be produced using a random number generator. The effects on the ranking of the options are quantified using MCS, in the same way as described above. The two sources of uncertainty considered can be both simulated either individually or combined.

The following section illustrates the application of this methodology on a case study.

Case study

Description

The case study used here is related to identifying a more sustainable electricity-generating option and compares two alternative options: electricity from coal and electricity from biomass. The question asked is whether is it more sustainable to expand the existing coal power plant (4000 MW) by 40 MW or to build a new biomass plant with the installed capacity of 40 MW.

These two options are compared for a number of economic, environmental and social criteria whose values have been estimated using different models and tools within the PUrE sustainability decision-support framework (Pettit et al. 2005). The models and tools include Life Cycle Assessment (LCA), fate and transport modelling, ecological and health impact assessment. The criteria used and the values obtained from the impact matrix are shown in Table 1. As can be seen from the table, there is no best overall solution: switching from biomass to coal results in the improvement in some criteria and deterioration of some other criteria.

In order to help DMs identify the 'best' option based on their preferences for different criteria, CP has been used as an MCDA tool. The uncertainty is characterised using MCS based approach (Hyde et al. 2004), described in Sect. 2.

The decision criteria considered in the analysis are given in Table 1. Six DMs assuming different roles and affiliations have been asked to make a selection of criteria that are of interest to them (Pettit et al. 2007). After a discussion, a subset of 13 out of 22 initial criteria was selected for the MCDA analysis. The DMs also provided the criteria scaling bounds and weights (see Table 2).

The following three MCDA problems are solved here:

(1) the problem without uncertainty,
(2) the problem with uncertainty in models, and
(3) the problem with uncertainty in models and DM preferences.

The MCDA problem without uncertainty

Since CP requires only one set of criteria weights, the six sets of criteria weights provided by the six DMs are merged

Table 1 Biomass vs. coal impact matrix

Decision criteria	Unit	Biomass	Coal
Environmental criteria[a]			
PM10	μg/m^3	0.196	0.064
SO_2	μg/m^3	0.051	3.551
NO_x	μg/m^3	1.179	7.978
Resource depletion	TJ/year	2	7
Acidification	t SO_2 eq./year	148	397
Eutrophication	t PO_4 eq./year	27	32
Global warming	t CO_2 eq./year	2518	211,840
Summer smog	t ethene eq./year	13	31
Winter smog	t SO_2	8	216
Fresh water ecotoxicity	t DCB[b] eq./year	0	2198
Marine ecotoxicity	t DCB[b] eq./year	3	10
Terrestrial ecotoxicity	t DCB[b] eq./year	0	12
Ecological impact of zinc	min. number of years to reach toxic levels for Zn	3	7
Socio-economic criteria			
Human toxicity	t DCB[b] eq./year	254	9785
Human health impact	additional deaths per year from PM10	0.0048	0.00018
Land use	km^2	10	2
Land use competition	km^2	5	0
Toad transport	km/year	700800	0
Shipping	km/year	0	1620
Rail transport	km/year	0	18000
Cost of energy	£/kWh	2	8.5
Other costs	£/t	0	4

[a] Note that all the values for the environmental criteria except for Ecological impact of zinc have been calculated using Life Cycle Assessment (LCA); Ecological impact of zinc is based on scientific knowledge on its effect on the ecology

[b] *DCB* dichlorobenzene

Table 2 Scaled criteria values and original weights selected by the decision makers (DMs)

Decision criteria	Biomass	Coal	DM1	DM2	DM3	DM4	DM5	DM6
PM10	−0.04	−0.01	8	7	1	8	3	6
SO_2	−0.01	−0.36	8	5	1	8	3	6
NO_x	−0.08	−0.53	8	7	1	8	3	6
Resource depletion	−2	−7	9	10	2	6	10	7
Marine ecotoxicology	−3	−10	9	4	8	6	10	7
Ecological impact of zinc	−3.33	−1.43	8	6	9	0	10	6
Human health impact	−0.02	0	9	8	10	10	3	10
Land use competition	−5	0	10	5	1	0	3	2
Road transport	−7.01	0	10	6	1	6	3	2
Shipping	0	−0.16	9	4	1	6	3	2
Rail transport	0	−0.02	10	6	1	6	7	2
Cost of energy	−2	8.5	9	8	2	7	2	4
Other costs	0	−4	8	8	1	7	2	7

by averaging the values $w_j^1, w_j^2, \ldots, w_j^6$ for each criterion ($j = 1, \ldots, 13$). The models are then run, the impact matrix generated, and the aggregated loss is determined in both cases of alternative options. The obtained aggregated loss of the Biomass alternative is lower that for the Coal; thus, the Biomass option is preferred.

However, by doing so, DMs are not informed about the likelihood for the other alternative to be selected as a result of a change in PUrE input variables/parameters. Such a change is actually likely to happen due to the fact that models are always a simplification of the reality. Another reason why a different alternative may be selected is that CP allows for one set of criteria weights only. Since in this case there are six different sets, in order to reduce them to a single set, the criteria weights assigned by the DMs are averaged thus leading to an aggregated set of criteria weights. However, note that this is not the only way to aggregate multiple sets of criteria weights, and thus it is reasonable to assume that using other methods may result in the selection of a different alternative. As a consequence, uncertainty may cause MCDA to have multiple possible results, and not considering uncertainty is equivalent to considering one possible result only.

The MCDA problem with uncertainty in models

In this instance, the uncertainty in model parameters/input variables is characterised using PDFs with pre-specified parameters. The uncertain model inputs are then propagated to model outputs by using the Monte Carlo Simulation (MCS) method. An instance of all the uncertain inputs/parameters is generated at random at each MCS iteration. The models are then run, the impact matrix is created and scaled, and the aggregated losses are obtained. In other words, for both alternatives, each MCS iteration generates one random aggregated loss. The sampling frequencies of the aggregated losses are shown in Fig. 2. Therefore, at each MCS iteration one of the two alternative options (Biomass or Coal) is selected as a preferred one. Once a number of MCS iterations are performed, it is possible to calculate the relative frequency of each of the two alternatives being the best one. It turns out that in this instance, the loss associated with the Biomass alternative is lower than the one associated with the Coal alternative with a 75% probability. As a consequence, the Biomass alternative is preferred to Coal with 75% confidence. The latter figure is, obviously, important *additional information* which was not available at the time of decision making in the previous case as discussed in the previous section.

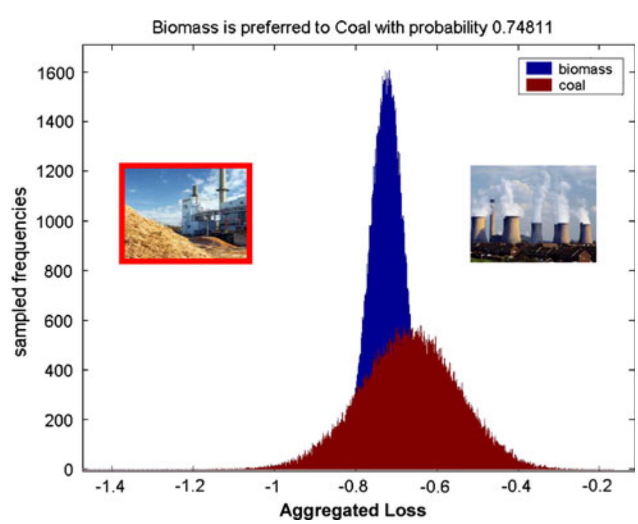

Fig. 2 Results for the MCDA problem with uncertainty in models

The MCDA problem with uncertainty in models and decision maker preferences

As stated above, each DM is likely to have a different perception of importance of the criteria used for evaluating the options analysed. As a consequence, each DM is likely to produce different set of preference weights. On the other hand, only one set of preference weights can be used in the MCDA analysis at a time; hence, the uncertainty now consists of how to derive a single matrix out of many (i.e. as many as the DMs).

Whilst solving the problem without any uncertainty, a single set of criteria weights was obtained from six DM sets by simply averaging the criteria weights for each individual criterion. In this case, each DM is considered as a representative sample of the criteria weights of a population of stakeholders. Each set of criteria weights can therefore, be portrayed by a probability, ensuring that all of the information obtained from the DMs is explicitly incorporated in the decision making process.

In this case, for each DM's set of preference weights, the same probability of 1/6, is assigned. The modelled uncertainty is then propagated along with the uncertainty in the models' parameters using the MCS method. Even though the only difference from the previous case is that in each MCS iteration the set of preference weights is randomly extracted, the results obtained here are different. The resulting distribution of the aggregated losses (Fig. 2) is very different from the previous case (Fig. 3), and so is the outcome of the MCDA. The best alternative is now the Coal and the level of confidence (for this to be the right decision) is equal to 52%. Therefore, taking into account and modelling different sources of uncertainty may result in different decision outcomes.

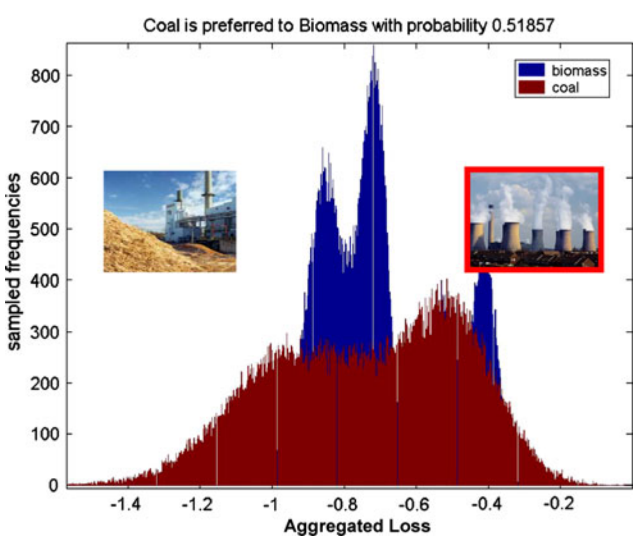

Fig. 3 Results for the uncertainty in models and decision makers' preferences

Conclusions

This article addresses the problem of decision making under uncertainty. The methodology proposed here for characterising and quantifying uncertainty can help DMs to understand better the uncertainty involved in scientific models as well as the decision-making process. It enables the *explicit* quantification of the impact on the decision of uncertainties in impact matrix values and/or DM's preferences. More specifically, it enables the calculation of a probability with which one option is preferred to another. As a consequence, DMs are provided with additional useful information leading to more informed and robust decisions. A further advantage is that the methodology is generic so that it can be applied to different MCDA problems.

However, there are two drawbacks of dealing with uncertainty: (a) additional information is required to characterise the PDFs of uncertain parameters used to calculate the impact matrix values and/or criteria weights and (b) additional computational time is required. Still, this seems a small price to pay for avoiding making wrong or less informed decisions by not taking into account the uncertainties involved in the decision making process.

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References

Barron H, Schmidt P (1988) Sensitivity analysis of additive multi-attribute value models. Oper Res 36(1):122–127

Belton V, Stewart TJ (2002) Multiple criteria decision analysis: an integrated approach. Kluwer Academic Publisher, Dordrecht

Brans JP, Vincke PH (1985) A preference ranking organisation method: the PROMETHEE method for multiple criteria decision making. Manage Sci 31(6):647–656

Critchfield GC, Willard KE (1986) Probabilistic analysis of decision trees using Monte Carlo simulation. Med Decis Making 6(1): 85–92

Felli JC, Hazen GB (1998) Sensitivity analysis and the expected value of perfect information. Med Decis Making 18(1):95–109

Gal T, Stewart TJ, Hanne T (eds) (1999) Multicriteria Decision Making: Advances in MCDM Models, Algorithms, Theory, and Applications, Kluwer

Halfon E, Reggiani MG (1986) On ranking chemicals for environmental hazard. Environ Sci Technol 20:1173–1179

Hwang CL, Yoon KS (1981) Multiple attribute decision making: methods and applications. Springer, Berlin

Hyde KM, Maier HR, Colby CB (2004) Reliability-based approach to multicriteria decision analysis for water resources. J Water Resour Plan Manage 130(6):429–438

Janssen R (1996) Multiobjective decision support for environmental management. Kluwer, Dordrecht, The Netherlands

Mousseau V, Figueira J, Dias L, Gomes da Silva C, Climaco J (2003) Resolving inconsistencies among constraints on the parameters of an MCDA model. Eur J Oper Res 147(1):72–93

Pettit C, Azapagic A, Jefferis S (2005) A framework for sustainable management of urban pollution. Eng Sustain 158(ES3):163–169

Pettit C, Azapagic A, Chalabi Z, Kapelan K, Dorini G (2007) PUrE Risk Workshop Summary Report. Half-day Workshop on: Risk and Uncertainty in Decision Making (held Monday 9th July 2007, Manchester). Joint Report prepared by The University of Manchester, Exeter University and the London School of Hygiene and Tropical Medicine. September 2007

Ringuest JL (1997) Lp-metric sensitivity analysis for single and multi-attribute decision analysis. Eur J Oper Res 98:563–570

Roy B (1968) Classement et choix en presence de points devue multiples (la methode ELECTRE). Revue d'Informatique et de recherché opérationelle 6(8):57–75

Saaty TL (1980) The analytic hierarchy process. McGraw Hill, New York

Triantaphyllou E, Sanchez A (1997) A sensitivity analysis approach for some deterministic multi-criteria decision-making methods. Decis Sci 28(1):151–194

UKCIP (2003) Climate adaption: risk, uncertainty and decision making. UKCIP, Oxford

Wille R (1982) Restructuring lattice theory: an approach based on hierarchies of concepts. In: Rival I (ed) Ordered set. Reidel, Dordrecht–Boston, pp 445–470

Zeleny M (1973) Compromise programming. In: Cochrane J, Zeleny M (eds) Multiple criteria decision making. University of South Carolina Press, Columbia, pp 373–391

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