CALCULATION OF DEGREE-DAYS FOR GLACIER-CLIMATE RESEARCH

By ROGER J. BRAITHWAITE, Copenhagen

With 2 figures

ABSTRACT

The conventional way of calculating positive degree-days is by summation of daily temperature series, but this is inefficient in terms of data volume and computing requirements. An alternative method for calculating degree-days from monthly mean temperature is developed from statistical considerations and is tested with data from the Swiss Alps and from West Greenland. For the calculation of seasonal or annual degree-days, the errors in the method are small compared to the errors involved in estimating glacier ablation from degree-day sums. The glaciological implications of the relationship between positive degree-days and mean temperature are also discussed.

1. INTRODUCTION

The importance of air temperature for glacier ablation has been recognized for a long time. In particular, the melting of snow or ice during any particular period of time is assumed to be proportional to the sum of all temperatures above the melting point during that same period. For temperatures measured on the Celsius scale, this sum is called the positive degree-day sum. The concept was applied in glaciology for the first time by Finsterwalder and Schunk (1887) and has been used since by many authors, e.g. Kasser (1959) or Hoinkes and Steinacker (1975) among others.

It is not the purpose of this paper to justify the belief in a relationship between ablation and temperature as this may be controversial; the author has stated his opin-
ions in earlier work (Braithwaite, 1980a, 1981). Rather the purpose of the present paper is to describe a method for calculating positive degree-days which is more convenient than conventional methods, particularly for studies involving long time-series or data at many points. It is hoped thereby to encourage further testing of the ablation-temperature hypothesis.

In principle, degree-days for any desired period can be obtained by a trivial summation of all positive temperatures in that period. For glaciological purposes, the summation should be based upon temperature readings taken several times during the course of the day, e.g. six-hourly synoptic observations, rather than using daily mean temperatures (Arnold and MacKay, 1964). In order to obtain a degree-day total for a single month, one might therefore need 124 individual readings, i.e. 4 temperature observations per day for 31 days. Even if an electronic computer is used for the trivial, but repetitive, arithmetic this approach still involves a very inefficient ratio between desired result and the effort needed to achieve it. Furthermore, detailed synoptic observations are not always available on electronic recording media, especially older records. On the other hand, printed summaries of monthly mean temperatures are readily available for many climate stations, e.g. in national meteorological yearbooks. The method which will be described below is therefore efficient from the point of view of data accessibility as well as data economy.

2. THE METHOD

The degree-day sum for a period of \( N \) days is given by:

\[
Y = \frac{1}{M} \sum_{m=1}^{M} \sum_{n=1}^{N} \alpha_{nm} T_{nm}
\]

where \( \alpha_{nm} \) has either a value of unity or zero according to:

\[
\alpha_{nm} = \begin{cases} 
1.0 & \text{if } T_{nm} > 0^\circ \text{C} \\
0.0 & \text{if } T_{nm} < 0^\circ \text{C} 
\end{cases}
\]

\( T_{nm} \) is the temperature at the \( m \)th observation on the \( n \)th day and \( M \) is the number of observations in the day.

If the temperature is assumed to constitute a stationary random series, the time-summation in Equation (1) can be replaced by an ensemble-summation as follows:

\[
Y = N \sum_{k=0}^{K} f(T_k) T_k
\]

where:

\[
T_k = T_o + k \Delta T
\]

and \( f(T_k) \) is the probability that the temperature lies in an interval of width \( \Delta T \) centred on \( T_k \) and \( K \) is the value of \( k \) such that \( f(T_k) \) becomes zero. \( T_o \) is zero for the computation of positive degree-days.

Strictly speaking, the assumptions of stationarity and randomness cannot be cor-
rect for the T_° series, because air temperatures over many days will exhibit both diurnal fluctuations and seasonal trends. However, it is a hypothesis of the present study that the error in making these assumptions is reasonably small for N values of the order of one month. Alternatively, it is assumed that random fluctuations in temperature during a month are reasonably large compared to any seasonal effects.

For the practical application of Equation (3) the probability function f(T) must be specified. The first, and most obvious, choice is to assume that it is given by the Gaussian or Normal distribution which is characterized by two parameters: the mean value X and the standard deviation S. Under this assumption, monthly degree-days can be calculated as a function of monthly mean temperatures, e.g. in steps of 0.1°C in the range -10 to +10°C for different values of the standard deviation S. If the results are stored in the form of a matrix so that they can be retrieved for each new situation, the calculation only has to be done once, whilst the summation in Equation (1) would have to made separately for each case.

The assumption of a Gaussian distribution of daily mean temperatures is well justified in the ablation season. Care must, however, be taken in the presence of low-level temperature inversions which favor oblique distributions.

3. SOME RESULTS

3.1 GENERAL

Results of the solution of Equation (3), assuming a Gaussian distribution for the temperature, are shown in Table 1 (more detailed tables with a resolution of 0.1°C are available on request from the author). The results are expressed in the form of degree-days per day, i.e. as Y/N, because months have differing lengths of 28, 30, or 31 days.

The relationship between positive degree-days and mean temperature is non-linear and asymptotic both to zero and to the mean temperature at low and high temperatures respectively. There is an intermediate region where Y/N is greater than the mean temperature X. This is where the latter is close enough to the melting point that both negative and positive temperatures occur during the month. Naturally, the width of

<table>
<thead>
<tr>
<th>Mean X (°C)</th>
<th>Standard deviation S</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>-6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>-4</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.3</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>-2</td>
<td>0.0</td>
<td>0.2</td>
<td>0.5</td>
<td>0.8</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>0</td>
<td>0.4</td>
<td>0.8</td>
<td>1.2</td>
<td>1.6</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>+2</td>
<td>2.0</td>
<td>2.2</td>
<td>2.5</td>
<td>2.8</td>
<td>3.1</td>
<td>3.1</td>
</tr>
<tr>
<td>+4</td>
<td>4.0</td>
<td>4.0</td>
<td>4.1</td>
<td>4.3</td>
<td>4.6</td>
<td>4.6</td>
</tr>
<tr>
<td>+6</td>
<td>6.0</td>
<td>6.0</td>
<td>6.0</td>
<td>6.1</td>
<td>6.3</td>
<td>6.3</td>
</tr>
<tr>
<td>+8</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
<td>8.0</td>
<td>8.1</td>
<td>8.1</td>
</tr>
<tr>
<td>+10</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
<td>10.0</td>
</tr>
</tbody>
</table>
this region increases with the standard deviation parameter $S$. Qualitatively, the results in table 1 are on good agreement with those of Boyd (1975) who fitted observed degree-days empirically to a hyperbolic function of mean temperature.

Two examples of the application of the method and an assessment of the results are given below.

3.2 Example 1: Säntis, Swiss Alps

As a first test of the method, positive degree-days were calculated by the conventional method, i.e. by summation of temperature series as in Equation (1), for a thirty year (1946—1975) record of May—October temperatures at Säntis, in north-east Switzerland. Säntis was chosen for the present study because detailed synoptic observations of temperature are readily available, i.e. they are published in the annual numbers of "Annalen der Schweizerischen Meteorologischen Zentral-Anstalt". The station also lies at an elevation of 2500 m a.s.l. so that its data should be fairly representative of air temperature conditions over glaciers in the northern Alps. The resulting positive degree-days per day are plotted in figure 1 together with the calculated $Y/N$ curve (assuming $S = 4^\circ$ C).

![Figure 1: Positive degree-days per day as a function of monthly mean temperature at Säntis, Swiss Alps, 1946—1975. Points represent observed values while the continuous curve is calculated assuming a standard deviation of $4^\circ$ C](image)

There is broad agreement between the observed points and the calculated curve although the curve appears to underestimate in the temperature range of about $-2$ to $+4^\circ$ C. In most cases, these points refer to conditions in the spring or autumn when seasonal trends may be relatively important. However, if one considers the six-month totals of the degree-days for each year, the errors are very small. For example, the 30-year mean of the observed May—October degree-day sums is 733 $d^\circ$ C with a
standard deviation of 126 \textdegree\text{C}. For the same period, the mean error between observed and calculated degree-days was only +3 \textdegree\text{C} with standard deviation 16 \textdegree\text{C}, i.e. errors of less than 1 per cent of the mean and about 2 per cent of the variance respectively. Considering some of the assumptions made in the method, this is a very good agreement.

**EXAMPLE 2: QAMANÅRSSÛP SERMIA, WEST GREENLAND**

Climatological and glaciological observations have been made at Qamanårssûp sermia, West Greenland, since 1979 (Braithwaite and Olesen, 1982; Braithwaite, 1983). Among other things, the programme included maintenance of an automatic climate station at 760 m a.s.l. beside the glacier and measurements of ablation on the glacier at about the same elevation, i.e. 790 m a.s.l.

Figure 2 shows a comparison between observed monthly degree-days at Qamanårssûp sermia and those calculated from the monthly mean temperature assuming $S=4$ \textdegree\text{C}. The agreement between the calculated and the observed values is quite good although there are some disagreements in the region close the zero (note that a non-linear scale has been chosen to give these more prominence). Once again, these errors refer to conditions in the spring and autumn when seasonal trends may be more important. However, the error is insignificant on an annual basis. For example, the observed degree-day sum for the year 1 September 1981 to 31 August 1982 is 592 \textdegree\text{C} while the corresponding calculated value is 605 \textdegree\text{C}, i.e. an overestimation of 2 per cent by the model.
4. DISCUSSION

4.1 ACCURACY OF THE METHOD

From the examples considered, it appears that the method of calculating degree-days from the mean temperature involves some errors. These are probably due to violations of the mathematical assumptions underlying the method, especially with respect to randomness and stationarity of the temperature series.

For seasonal or annual degree-days, the errors are very small compared to the possible errors involved in converting degree-days into ablation. This point can be illustrated by Table 2 which shows monthly positive degree-day factors, obtained by dividing the monthly ice ablation at 790 m a.s.l. on Qamanârssûp sermia by the corresponding positive degree-day total at the automatic climate station. The coefficient of variation of the values in Table 2 is 13 per cent of the mean. This represents the error that would be involved if one assumed a constant positive degree-day factor of 7.1 mm d\(^{-1}\) C\(^{-1}\) for calculating ablation. Compared to this, an error of only one or two per cent in the calculation of degree-day sums is quite negligible. It is also negligible compared to the difference between degree-day factors for ice and for snow.

Table 2: Monthly degree-day factors at 790 m a.s.l. on Qamanârssûp sermia, West Greenland, 1980–1982, i.e. monthly ice ablation divided by monthly degree-days. Units are mm d\(^{-1}\) C\(^{-1}\).

<table>
<thead>
<tr>
<th>Year</th>
<th>June</th>
<th>July</th>
<th>August</th>
<th>Summer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>6.4</td>
<td>6.6</td>
<td>5.9</td>
<td>6.3</td>
</tr>
<tr>
<td>1981</td>
<td>6.9</td>
<td>8.1</td>
<td>6.4</td>
<td>7.1</td>
</tr>
<tr>
<td>1982</td>
<td>7.0</td>
<td>8.5</td>
<td>8.3</td>
<td>7.9</td>
</tr>
<tr>
<td>Mean</td>
<td>6.8</td>
<td>7.7</td>
<td>6.9</td>
<td>7.1</td>
</tr>
</tbody>
</table>

No doubt the proposed method could be further improved by refining some of the statistical assumptions but this hardly seems worthwhile in view of the above considerations.

4.2 EFFICIENCY OF THE METHOD

The efficiency of the proposed method can be illustrated by the example of Braithwaite (1980b) who used the method to estimate the regional distribution of ablation in West Greenland. For that study, data for 2160 individual station-months were considered. The conventional method of calculating degree-days, by summation of temperature series would therefore have required over a quarter million individual temperature readings (124 values per station-month) while the present method required exactly 2160 values of monthly mean temperature (easily extracted from climatological yearbooks for Greenland). The basic calculation, involving summation of Equation (3), only had to be repeated 201 times, i.e. for each 0.1°C temperature increment in the range \(-10\) to \(+10\)°C, while the conventional method would have required a separate summation for each station-month series.
4.3 GLACIOLOGICAL IMPLICATION

The preceding points about the accuracy and efficiency of the proposed method have implications for glacier-climate research. This is because many people would still regard the relation between glacier ablation and positive degree-days as being hypothetical. If an enormous volume of temperature data must be processed to obtain a simple series of degree-day totals, as in the conventional method, there may be reluctance to test the hypothesis in specific cases.

The relation between positive degree-days and mean temperature is non-linear. If ablation is proportional to degree-days, the relation between ablation and mean temperature should also be non-linear as some authors have suggested. For example, Ahlmann (1924) suggested an exponential relation while Krenke (1975) proposed the following equation:

\[ A = (T_s + 9.5)^3 \]

where \( A \) is the annual ablation in mm w.e. and \( T_s \) is the mean temperature over the glacier for the three-month summer June—August. The non-linear equations of Ahlmann and Krenke predict that the ablation increases at an ever-accelerating rate as the summer mean temperature increases, while the degree-day model proposed here suggests that ablation will simply tend to become proportional to summer mean temperature at sufficiently high temperatures. Non-linear equations relating ablation to summer mean temperature also involve an implicit assumption about the length of the ablation season, e.g. a three-month “summer”, while no such assumption is needed for the degree-day approach.

A number of authors have analysed glacier mass balance series by linear regression on summer temperature, among other elements, e.g. Liestol (1967) and Martin (1974). This may seem to be in contradiction with the above statement of a non-linear relation between ablation and summer mean temperature. However, the year-to-year variations in summer mean temperature at any point are only of the order of 1 to 2°C. The strictly non-linear relation between ablation and summer mean temperature will therefore show a kind of “local” linearity. The gradient of this linearized ablation-temperature relation will depend upon the climatic conditions, e.g. it will be higher for glaciers in the Alps than for glaciers in the High Arctic (Braithwaite, 1977) because of the longer summer.

ACKNOWLEDGEMENTS

This paper is published by permission of The Director, The Geological Survey of Greenland. Thorkild Thomsen, The Greenland Technical Organization, kindly processed the data from the automatic climate station at Qamanârussâq sermia.

REFERENCES


Boyd, D. W., 1975: Computing freezing and thawing degree-days from monthly temperatures.
Roger J. Braithwaite


Manuscript received 11 May 1984, revised 2 July 1984

Author’s address: Roger J. Braithwaite, Ph. D.
Gronlands Geologiske Undersøgelse
Øster Voldgade 10
DK-1350 København
Danmark