Damage Modelling For Composite Structures

A thesis submitted to the University of Manchester for the degree of Doctor of Philosophy in the Faculty of Engineering and Physical Sciences

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Abstract

Damage Modelling For Composite Structures

Hao Lee, 2015

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Modelling damage in composite materials has played an important role in designing composite structures. Although numerical models for the progressive damage in laminated composites (e.g. transverse cracking, delamination and fibre breakage) have been developed in the literature, there is still a need for further improvement. This thesis aimed at developing damage models suitable for predicting intra-laminar and inter-laminar damage behaviour in fibre-reinforced composite materials. Several approaches such as fracture mechanics and continuum damage mechanics have been adopted for constructing the damage model.

Meso-macro-mechanics analysis was performed to gain an insight into the entire damage process up to the final failure of the composite laminate under various conditions. Cohesive elements were placed in the finite element model to simulate the initiation and propagation of matrix crack and delamination in cross-ply laminates. This helped to understand the direct interactions between damage modes, i.e. whether one damage mode would initiate the other damage mode. The formation of a single matrix crack and its propagation across the layer thickness was also revealed.

A new cohesive zone/interface element model was developed to consider the effect of through-thickness compressive stress on mode II fracture resistance by introducing friction into the constitutive law of the conventional cohesive zone model. Application of the model to practical problem in composite laminates shows that this model can simulate delamination failure more accurately than the cohesive element in ABAQUS.

Damage models based on continuum damage mechanics were proposed for predicting intra-laminar damage and interlaminar damage. Five intra-laminar failure modes, fibre tension, fibre compression, matrix tension, matrix compression and shear failure, were modelled. Damage initiation was predicted based on stress/strain failure criteria and damage evolution law was based on fracture energy dissipation. The nonlinear shear behaviour of the material was considered as well. These models have been implemented into ABAQUS via a user-defined material subroutine and validated against experimental/numerical results available in the literature. The issue related to numerical implementation, e.g. convergence in the softening regime, was also addressed.
Numerical simulation of the indentation test on filament-wound pipe was finally conducted and damages generated in the pipe were predicted using the above developed damage models. The predictions show an excellent agreement with experimental observations including load/indentation responses and multiple delaminations shape and size. Attempt was made to detect damage-induced leakage path in the pipe after indentation.
Declaration

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Chapter 1
Introduction

1.1 Background

Fibre-reinforced composite materials consist of fibre and matrix; these two primary materials, when combined, give superior performance. Although fibre-reinforced composites are considered to be the most significant type of composites in the aerospace applications, they are also used as fundamental materials in disciplines as diverse as civil engineering, automobiles, and sporting equipment. The advantages of composite materials include, but not limit to, high specific stiffness and strength, low thermal expansion, resistant to corrosion, and high resistance to fatigue. However, composite materials are expensive in raw materials and manufacturing, adversely affected by both temperature and moisture. Most importantly, they are weak in the transverse direction where matrices carry the load and the interlaminar strength is low. Therefore, composites are susceptible to impact damage and other damage under complex loading conditions and it is crucial to assess the influence of such a damage event on the behaviour of composite materials.

The reliability of composite materials mostly depends on their adhesive and mechanical properties, and researches on the failure of fibre-reinforced composites have attracted much attention in mechanics fields. In most instances failure is observed as separation of structural components or material parts of components. The failure characterization of composites can best be classified on the different levels of discipline for identity with each failure type. More attentions have been focused on lamina and micromechanical
failure as a result of producing the damage and degradation leading to failure in composite laminates. It appears useful to characterize failure at the micro level by introducing a combination of factors such as fibre breakage, matrix cracking and delamination which may cause the failure of fibre-reinforced composites (Fig. 1.1). Specifically, the local failures in fibre-reinforced composites have three distinct modes: fibre-dominated failure, matrix-dominated failures, and interface/flaw-dominated failures (Vinson & Sierakowski, 2004). Although the failure modes can be either one of them, these fundamental mechanisms would appear to be important features to researchers due to the fact that an individual lamina failure can cause a catastrophic failure within the remaining laminae of the laminate.

Matrix cracking frequently occurs in laminated composites and many efforts have been made to determine its contribution to the failure of such materials. It is now clear that matrix cracking can reduce the matrix-dominated stiffness of laminates and, even more severely, cause the onset of delamination as the cracking propagates to the interface between adjacent layers. The resulting separation of adjacent layers in delamination diminishes the bending stiffness as well as the load-carrying capacity of laminates (Kollár & Springer, 2003).

Figure 1.1: Typical failure modes of composites (Kollár & Springer, 2003).

1.2 Overview of damage and failure analysis in composite materials

Approach behind the validation of failure in composite materials differs from the conventional methodology. While the conventional approaches verify the ability of the structure or the material to resist the damage occurrence, the failure of composite materials can be described by allowable variables associated with the ultimate strength
of the material. For example, it requires the knowledge of five standard plane stresses or stains in material's principal directions to describe the failure of orthotropic materials. However, those failure modes can occur sequentially and are likely to interact, which makes it difficult to model the behaviour of the laminate after initial failure.

Most new concepts build on a progressive damage evaluation approach that is composed of techniques for predicting both crack initiation and propagation. While material failure refers to the complete loss of load carrying capacity that results from progressive degradation of the material stiffness, different failure modes are considered such as fibre rupture in tension, fibre buckling and kinking in compression, matrix cracking under transverse tension and shearing and matrix crushing under transverse compression and shearing. The typical material response for progressive damage is shown in Fig. 1.2, the linear part from original point to point A represents the undamaged constitutive behaviour (e.g. linear elastic: orthotropic, anisotropic, traction, etc.), point A refers to the damage initiation and path A-B represents the damage evolution. For some unidirectional fibre composites, the failure criterion proposed by Hashin & Rotem (1973) has been used to define the damage initiation. Initiation does not actually lead to damage unless post damage-initiation behaviour, i.e. damage evolution law, is also specified. The role of damage evolution is to describe the rate of the material stiffness degradation once the initiation criterion is satisfied. This approach can be implemented in numerical methods such as Finite Element Method.

![Figure 1.2: Typical material response showing progressive damage](image-url)
1.3 Aims and Objectives of the present research

The aim of the present research was to develop damage models suitable for predicting intra-laminar and inter-laminar damage behaviour in fibre-reinforced composite materials.

The specific objectives of the present research were as follows from theoretical and numerical modelling perspective:

1) To perform meso-macro-mechanics analysis on cross-ply composite laminates and to model damage (the matrix crack, delamination, fibre breakage) in the laminate under different loading conditions, such as uniaxial tensile loading, pure bending and combined loading. The purpose of this analysis was to obtain the understanding of the detailed damage initiation and development sequence, interaction between damage modes, and the formation of a single matrix crack.

2) To develop an interface element/cohesive model which includes interfacial friction to take account of effect of compressive through-thickness stress on mode II fracture. The model must be validated based on a comparison between the proposed damage model and relevant experimental results.

3) To develop two continuum damage models for the purpose of modelling inter-laminar damage and intra-laminar damage. The inter-laminar damage model is used to simulate the delamination in laminated composite structures. The intra-laminar damage model is used to simulate the transverse matrix cracking, which can be further combined with the inter-laminar damage model.

4) To implement the continuum damage models into commercial finite element code via a single user-defined subroutine for wide applications to practical engineering problems, and validate the models by comparison against experimental results.
1.4 Layout of the thesis

Chapter 1: Presents an introduction of the research background and the research problem with a statement of the aims and objectives of the research.

Chapter 2: Review different approaches found in the literature, especially those for the simulation of delamination and transverse matrix cracking and several damage models.

Chapter 3: Describes the Cohesive Zone Model (CZM) and the Progressive Damage Model (PDM) available in commercial finite element code ABAQUS. Numerical simulations of damage formation and development sequence in cross ply laminate using the above damage models are then presented.

Chapter 4: Presents a newly developed interface element/cohesive model which takes account of the effect of compressive through-thickness stress and friction on the mode II fracture. Numerical simulations of delaminations in cut- and dropped-ply specimens are also given in this chapter.

Chapter 5: Proposes two continuum damage models for modelling inter-laminar and intra-laminar damage. The inter-laminar damage model is based on the concept of damage surface and further used to simulate the damage behaviours of delamination. The intra-laminar damage model which also accounts for the shear nonlinearity is used to simulate the transverse matrix cracking in laminated composite structure. Three applications used to validate the damage models against experimental data of standard test are presented.

Chapter 6: Presents the application of the damage models to the filament-wound pipes under lateral indentation. The predicted inter-laminar damage and intra-laminar damage are presented and compared with the experimental observations. Leakage path from the simulation results is also revealed in this chapter. Effect of the interaction between damage modes is presented.

Chapter 7: Presents the main findings and conclusions together with suggestions for future work.
Chapter 2

Literature Review

2.1 Introduction

This chapter reviews analytical and numerical damage modelling in crack initiation and propagation and failure criteria in laminated composites. Previous studies on the progressive damage modelling techniques of fibre-reinforced composites are summarized.

2.2 Failure Criteria

The failure criteria are considered to be the conditions for the prediction of the occurrence of material damage. The term failure criterion refers to mathematical equations that predict the states of stress and strain at the onset stage of damage. In conventional design, the structure is advised as safe if the maximum stress or strain is less than its corresponding limit, known as strength-based failure criteria. The simplest strength-based failure criteria are the maximum stress and maximum strain criteria. These criteria are simple inequality conditions relating the internal stresses or strains to the experimental measures of material strength. Having one stress or strain component in each equation, this criterion is simple to use and also indicates the failure mode. However, these criteria provide no interaction between individual tensor components.
of stress or strain, their accuracy is limited. For instance, failure prediction in transverse
tension is not influenced by the presence of longitudinal shear. Another commonly used
failure criterion is Tsai-Wu (1971) criterion, denoted by the following polynomial
equation

\[ F_{ij} \sigma_i \sigma_j + F_i \sigma_i = 1 \quad (i, j = 1, 2, ..., 6) \quad (2-1) \]

where \( F_{ij} \) and \( F_i \) are the experimental measures of the biaxial and uniaxial strengths.
Due to the interaction of the terms in the polynomial equation, this criterion can be
referred as interactive failure criterion. For plane stress state, the Tsai-Wu criterion can
be simplified as follows

\[ F_{11} \sigma_1^2 + 2F_{12} \sigma_1 \sigma_2 + F_{22} \sigma_2^2 + F_{66} \tau_{12}^2 + F_1 \sigma_1 + F_2 \sigma_2 = 1 \quad (2-2) \]

where

\[ F_{11} = \frac{1}{\sigma_{1t}^* \sigma_{1c}^*}, \quad F_{22} = \frac{1}{\sigma_{2t}^* \sigma_{2c}^*}, \quad F_{66} = \frac{1}{\tau_{12}^* \tau_{12}^*}, \quad F_1 = \frac{1}{\sigma_{1t}^*} - \frac{1}{\sigma_{1c}^*}, \quad F_1 = \frac{1}{\sigma_{2t}^*} - \frac{1}{\sigma_{2c}^*} \]

while \( F_{12} \) is the interaction term which can be determined from a combined-stress test.
However, the value of the interaction term is negligible for many engineering
applications due to the sufficient accuracy in the prediction of filamentary composite
materials (Narayanaswami and Adelman, 1977). To identify failure modes, failure
indices can be defined as

\[ H_1 = F_1 \sigma_1 + F_{11} \sigma_1^2, \quad H_2 = F_2 \sigma_2 + F_{22} \sigma_2^2, \quad H_3 = F_{66} \tau_{12}^2 \quad (2-3) \]

The failure indices represent the contribution to different failure modes by the
corresponding stress component of an individual layer of a laminate. For other failure
criteria other than the Tsai-Wu criterion, similar conclusions regarding the failure modes
could be defined. However, the interactive failure criteria such as Tsai-Wu are often
criticised due to their inadequacy of phenomenological basis and origins of theories.
Furthermore, if the failure indices are close to each other, then the predicted failure
mode may not be correct.

Strength-based failure criteria are commonly utilized in the finite element software to
predict failure in composite structures. Over the last decade, many researchers have
proposed continuum-based criteria relating internal stresses and experimentally
measured material strengths to the onset of composite failure. The criteria proposed by
Hashin (1973, 1980), Sun et al. (1996) and Puck and Schürmann (1998, 2002) are a few examples of continuum-based criteria. These criteria predict a variety of failure modes such as fibre tensile failure, fibre compressive failure, matrix tensile failure, matrix compressive failure.

Failure criteria have been gradually modified to include new considerations, e.g. some modifications were adopted to incorporate nonlinear in-plane shear behaviour into the failure predictions. Chang et al. (1984) modified the criterion proposed by Yamada and Sun (1978) to incorporate in-plane shear behaviour. Chang and Lessard (1991) modified Hashin’s criterion (1980) to include not only the through-thickness (in-situ) strength but also a nonlinear shear formulation. Numerous studies showed that a moderate transverse compression increases the apparent shear strength of a ply. Dávila and Camanho (2003) proposed a new set of criteria for matrix fracture under transverse compression which is based on the concepts proposed by Hashin and the fracture plane concept proposed by Puck and used Mohr-Coulomb criterion for taking account of the effective shear stresses acting on the fracture plane. The use of Mohr-Coulomb criterion for the transverse matrix failure has improved the accuracy of the original Hashin matrix failure criterion.

2.3 Analytical and Numerical Damage Modelling In Crack Initiation and Propagation

2.3.1 Energy-based fracture mechanics model

Many investigations have been made into the laminar fracture in fibre reinforced composite materials. Fracture mechanics technique, such as virtual crack closure technique, is one of the widely used methods to simulate crack propagation. The assumption based on Irwin’s (1957) contention is that if a crack extends by a small amount, the energy absorbed in the process is equal to the work required to close the crack to its original length. The energy release rates in mixed fracture mode can be presented from the nodal forces and displacements obtained from the solution of a finite element model proposed by Rybicki and Kanninen (1977). The finite element mesh around the crack tip is shown in Figure 2.1. Therefore,
\[ G_I = \frac{1}{2\Delta c} F^y_{cd} (v_c - v_d) \]  
\[ G_{II} = \frac{1}{2\Delta c} F^x_{cd} (u_c - u_d) \]  
\[ G_{III} = \frac{1}{2\Delta c} F^z_{cd} (w_c - w_d) \]

where \( G_I \), \( G_{II} \) and \( G_{III} \) are the energy release rate for modes I to III, \( F^y_{cd} \), \( F^x_{cd} \) and \( F^z_{cd} \) are the magnitudes of nodal forces at nodes c and d in the y, x and z directions, respectively, required to close them. \( u_c, v_c, w_c \) and \( u_d, v_d, w_d \) are nodal displacements before nodes c and d are pulled together. Two analyses of consecutive configurations are necessary to obtain the nodal forces and relative displacements. In Rybicki and Kanninen’s (1977) calculation, the nodal forces at the crack tip (\( F^x_{ef} \) and \( F^y_{ef} \) in Figure 2.1) can be used to simplify the calculation when \( \Delta c = l_2 \), i.e. the values of nodal forces in equation (2-1), (2-2) and (2-3) can be replaced by the corresponding components of nodal forces \( F^x_{ef} \), \( F^y_{ef} \) and \( F^z_{ef} \).

Kanninen (1974) developed a simple analytical model which extended the energy balance based on linear elastic fracture mechanics to the dynamic situation by considering that energy was dissipated at the crack tip at a rate. It was demonstrated that kinetic energy provides an important contribution to maintaining unstable crack propagation and to determining the crack arrest. The model was restricted to analyse...
rapid crack propagation and arrest in the double cantilever beam (DCB) test. A modified model of closed form solution was developed by Penado (1993) to determine both the compliance and the energy release rate of the adhesively bonded DCB specimen. By assuming that the adhesive behaviour acts as springs in series, the expression for foundation modulus was derived to be applicable to the closed form model. It also added the effects of shear deformation and included orthotropic materials.

Arakawa and Takahashi (1996) proposed another analytical model which evaluates the value of inter-laminar modulus and simplified the solutions for the beam deflection using a beam-to-elastic foundational method. The modulus makes a significant contribution to increasing crack opening displacement, and the strain distribution at the crack tip was also examined and found to be dependent on the inter-laminar modulus as well as the specimen thickness. Hitchings et al. (1996) presented a finite element model with embedded delamination and position variables which indicate the location of delamination front elements. It was capable of simulating the stable delamination propagation of an initial arbitrarily shaped delamination in a laminated composite.

Composite structures are subjected to mixed mode loading in most realistic situations. Song and Waas (1995) developed a nonlinear elastic foundation model to predict the mixed-mode delamination failure in laminated composites. Two types of nonlinear elastic foundations were used to measure mode I and mode II components of the material failure at the crack tip. Both mode I and mode II components are represented by spring foundation in tension and shear which have no interactions between any pair of springs. Each spring element has a non-uniform strain distribution that is assumed to be an approximation to the two-dimensional linear elastic asymptotic solution of the strain field ahead of the crack tip and appropriate nonlinear constitutive laws. A failure criterion for the spring foundation was used to predict the onset of crack propagation as shown below (Johnson and Mangalgiri, 1987).

\[
\left(\frac{G_I}{G_{Ic}}\right)^m + \left(\frac{G_{II}}{G_{IIc}}\right)^n = 1
\]

(2-4)

where \(m = n = 1\). The fracture toughness \(G_{Ic}, G_{IIc}\) are obtained from Double-Cantilever-Beam test (for pure mode I) and End-Loaded-Split test (for pure mode II).
The quantities $G_I$ and $G_{II}$ refer to the energy per unit area absorbed by the tension and the shear spring foundations.

Benzeggagh and Kenane (1996) proposed the characterisation of crack initiation and growth based on a strain energy release rate concept. The Benzeggagh-Kenane fracture criterion is particularly useful when the critical fracture energies during deformation purely along the first and the second shear directions are the same. It is given by the following relationship

\[
G_{Tc} = G_{Ic} + (G_{IIc} - G_{Ic}) \left( \frac{G_{II}}{G_{T}} \right)^m
\]  

(2-5)

where $G_T = G_I + G_{II}$, and $G_{Ic}, G_{IIc}, G_{Tc}$ are mode I, mode II and total critical strain energy release rate. $m$ is a material parameter to be specified.

2.3.2 Cohesive/Interface element Model

The interfacial damage is often localized in a thin, resin-rich layer between two adjacent layers of different fibre orientation angles. The interlaminar damage in laminated composites is therefore normally assumed to occur when the interlaminar stresses satisfy a certain interactive mixed mode criterion. Various cohesive/interface element models have been proposed to simulate the initiation and growth of the inter-laminar damage. In general, cohesive/interface element model assumes a suitable constitutive relationship between the stresses/strains acting on an interface and the displacement discontinuities or separations across the interface. A stress-based criterion is usually used for damage initiation and energy-based criterion for damage propagation.

Cui and Wisnom (1993) introduced a combined stress-based and fracture-mechanics-based interface model using spring elements for predicting delamination in composites. A rigid perfectly plastic behaviour was employed for the spring. The spring deforms elastically first. After the spring force reaches a certain value, further spring displacement takes place at a constant force until the spring is assumed to fail (Fig. 2.2).

Petrossian and Wisnom (1998) also introduced an interface element which was designed to have a smooth transition between linear elastic and plastic behaviour to compete
with the sudden discontinuous stiffness degradation in some interface elements used by other researchers when the stress reaches the critical value.

![Schematic force/displacement relation for springs. (Cui & Wisnom, 1992)](image)

**Figure 2.2: Schematic force/displacement relation for springs. (Cui & Wisnom, 1992)**

![Traction-relative displacement curves employed in various models in the literature. (Zou et al., 2003)](image)

**Figure 2.3: Traction-relative displacement curves employed in various models in the literature. (Zou et al., 2003)**

Different traction-separation relations used to model the fracture process were developed by Tvegaard and Hutchinson (1992), Xu and Needleman (1994), and Reedy et al. (1997), including the elastic-perfectly plastic, bilinear, exponential, trapezoidal constitutive relations as shown in Fig. 2.3. One of the advantages of these models is their easy implementation into finite element code by embedding interface elements. However, the above models are only applicable to single-mode fracture. The interaction between fracture modes in the mixed-mode problems is not considered.
Mi et al. (1997) employed a damage parameter which is expressed in terms of the relative displacement. The scalar damage was then introduced into the interfacial constitutive relationship to produce either a linear or quadratic relationship for the mix-mode fracture. A modified constitutive relationship was proposed by Alfano and Crisfield (2001) which is used for mixed-mode interaction. Both linear and the quadratic and even generalized ellipse criterion have been made available. Alfano (2006) also investigated the influence of the shape of the interfacial law such as bilinear, linear-parabolic, exponential and trapezoidal, respectively, on the fracture prediction. For a typical double cantilever beam test, the trapezoidal law turned out to give the worst results both in terms of numerical stability and in terms of convergence of the finite-element solution to the exact solution. The exponential law was found to be optimal in terms of the degree of approximation achieved in the problems. The bilinear law achieved the best compromise between computational cost and finite-element approximation.

![Diagram](image)

**Figure 2.4**: Cohesive law representing two distinct mechanisms (Yang & Cox, 2005)

Cox and Yang (2005) conducted a comprehensive review on the advantages of the cohesive zone model over the virtual crack closure techniques and the conventional linear elastic fracture mechanics. Another problem of length scales in bridged cracks has also been examined in their work. They mentioned about one of the physical phenomenon involved in the separation process, which acts at higher displacements and will extend further into the crack wake. In Fig. 2.4, when long-range friction effects are presented, one might expect a law possessing a peak at low crack displacements and a long tail at high crack displacements. Such high crack displacement represents the friction that is transmitted through the layer of resin fragments in the crack wake.
Therefore, the extension of CZM (cohesive zone model) to account for crack wake friction was proposed by including a contact law.

Borg et al. (2001) proposed a discrete cohesive failure model which is not based on a damage formulation. Instead, a maximum load surface was introduced. The adhesive nodal forces increase up to a limit which is related to penalty formulation. Delamination starts to initiate and propagate once the adhesive forces exceed its limit. The maximum load surface shrinks gradually as the dissipated energy increases. When the dissipated energy reaches a value corresponding to the fracture energy, i.e. a fracture criterion is satisfied, the adhesive forces are reduced to zero and crack forms. A similar work was carried out by Zou et al. (2003) who introduced a damage surface combining stress-based and fracture-mechanics-based failure criteria to unify the simulation of delamination initiation and propagation. This model will be addressed in detail and further discussed in Chapter 5.

A thermodynamically consistent damage model was proposed by Turon et al. (2006) for the simulation of progressive delamination in composite under variable-mode ratio. A novel initiation criterion that evolves from the Benzegagh-Kenane criterion has been invented to ensure that the model accounts for changes in the loading mode in a thermodynamically consistent way and avoids restoration of the adhesive state.

Cohesive/interface element model has also been applied to fatigue problems. Harper and Hallett (2010) presented an interface element to analyse delamination propagation under cyclic loading. A new formulation for predicting fatigue damage growth was
established. The cohesive zone was divided into two discrete regions, a static damage zone and a fatigue damage zone. In addition to the interface element’s static damage parameter, a fatigue damage parameter was introduced (Fig. 2.5). The interfacial stress was calculated on the basis of the value of the total damage parameter. A rate of strength degradation and consequent failure of the interface element was defined and linked to the fatigue damage parameter. A damage accumulation law was specified for fatigue damage growth. A three-dimensional interface element was developed and the proposed model was implemented in an explicit finite element code.

2.3.3 Continuum-Damage-Mechanics-based Damage Models

It would be computationally inefficient to accommodate physical intra-laminar damage, such as matrix cracks, in a numerical model at the structural scale to simulate their initiation and development. Damage models for intra-laminar damages in composites have therefore been developed in the context of continuum damage mechanics. Instead of the physical form of the damages/cracks, damage parameters are employed to characterise the extent/density of damage in the material. The effect of the damage on the performance of the material is reflected by its influence on the reduction of material’s properties. These models usually contain a physically-based failure criteria, e.g. maximum strain/stress failure criteria, Hashin-Rotem criterion, Puck’s criterion, etc., which is used to predict damage initiation. An evolution law must be established for the damage growth in order to predict post-failure behaviour. Since the models are developed at the material’s level, they can be easily implemented into finite element code for practical structural applications, no matter it is a laminated structure or other types of structures.

Lade`veze (1992) studied the matrix damage in composites. It was proposed that only the shear and transverse tensile moduli are affected by the damage state while the other independent elastic characteristics remain constant up to a rupture point, resulting in the requirement for only two damage variables for laminates. The shear and transverse damage variables are introduced as follow,

\[ d = 1 - \frac{G_{12}}{G_{12}^0} \]  

(2-6)
where $d$ and $d'$ are the shear and transverse damage variable, respectively. $G_{12}$ and $E_{22}$ denote the shear and transverse modulus at a damaged state while the superscripts indicates undamaged state. These two damage variables combined with the rupture criteria and plasticity behaviour therefore describe the progressive damage in laminates. Since Lade’veze’s damage model provides a sufficient way to represent intra-laminar failure mechanisms, it is the basis of a large number of continuum damage models for composites in the literature.

Another in-plane, anisotropic damage model proposed by Matzenmiller et al. (1995) is notable for its damage growth law which is based on a cumulative distribution function of the Weibull distribution. In addition, the reduction of elastic moduli in Matzenmiller’s model is controlled by five damage variables. The evolution of a damage variable is governed by effective stress components acting on the failure plane. Due to its ‘shell element’ based damage mechanics approach, this model is suited for the implementation in plane-stress problem. Other works (Lade’veze and Dantec, 1992; Johnson, 2001) used similar approaches for progressive damage modelling assuming in-plane stress effects only. Williams et al. (2003) developed a model for impact damage modelling. Their model described in particular the physical significance of the choice of damage parameter, the ease of material characterization, and lay-up dependence of the damage growth in laminated structures. Iannucci et al. (2001) proposed a damage model based on the combination of damage mechanics and energy-based approaches for the composite structures under impact loadings. It is claimed that this type of approach, which was implemented in LS-DYNA explicit finite element analysis code and in which a linear stress/strain relationship is assumed for fibre-related damage mechanisms, is a significantly improved method for predicting impact damage compared to the traditional stress based failure criteria modelling techniques. Iannucci and Ankersen (2006) improved the Iannuci’s model (2001) by adopting a crack band method to ensure a certain dissipation of energy regardless of the refinement of the mesh.

Strain localization is exhibited in the constitutive model where the model is normally expressed in terms of stress-strain relations. The strain softening behaviour in FE
modelling of damage will become mesh dependent of the finite element size. Bažant and Oh (1983) proposed a crack band model for establishing a smeared formulation to avoid strain localization. In smeared formulations, the specific or volumetric energy defined by the area underneath the stress-strain curve is related to the fracture energy of the material, i.e. the fracture energy is smeared over the full volume of the element. As a consequence of this method, a geometrical parameter relative to the element dimensions is introduced into the constitutive law. This geometrical quantity is defined as characteristic length, which has the following relation for square element:

\[ L_c = \frac{\sqrt{A_{IP}}}{\cos \theta} \]  

(2-8)

where \( A_{IP} \) is the area associated with an integration point and \( \theta \) is the angle of the mesh line along which the crack band advances with the crack direction. However, this relation must be limited to \( |\theta| \leq 45^\circ \). For an unknown direction of crack propagation, the average of the above expression is computed by Maimí et al. (2006) for a further development. Another method for evaluating characteristic length consistent with the mesh discretization that takes account of the crack orientation was proposed by Oliver (1989).

Li et al. (1998) proposed a continuum damage model for transverse matrix cracking. A single scalar damage parameter, which is related to crack density, is employed and the relationship between the damage parameter and the degraded properties of a lamina is established. Damage evolution is derived from a damage surface. This model overcomes the shortcomings in Talreja’s model (Talreja, 1985) where the damage variables are associated with individual laminae, so that the overall behaviour of the composites can be determined by one of the laminate theories. Li et al. (2005) later improved this model by redefining the damage growth law from laminate-based to a fully lamina-based continuum formulation. This continuum transverse cracking damage model also includes additional feature of material nonlinearities, e.g. shear nonlinearities and fibre reorientation. Furthermore, it takes account of the residual thermal stress by introducing thermal expansion coefficient into stress-strain relationship which has not been considered in the previous work. Recently, a multi-mode damage model was presented by Almaskari et al. (2009) for predicting the developments and the effects of both intra-laminal and inter-laminal damage modes in laminated structures. The matrix
cracking model is virtually the same as Li’s model (2005) and is implemented into FE code via user-defined material behaviour subroutine. However, no evidence shows that delamination simulation has been carried out in the paper.

Lapczyk and Hurtado (2007) adopted the model proposed by Matzenmeiller et al. (1995) and Hashin’s criteria for damage initiation. Their model also used the crack band model to alleviate the mesh dependency of the solution. The damage evolution law in Lapczyk’s model is a generalization of the approach proposed by Camanho and Dávila (2002) for modelling inter-laminar damage using cohesive elements. This model is further made available in the commercial finite element code ABAQUS. A similar model proposed by Maimí et al. (2007) is designed for the prediction of intra-laminar failure mechanisms under plane stress. The methodology used in Maimí’s model is to use the simplified LaRC04 failure criteria (Pinho et al., 2005) to predict the onset of the intra-laminar failure mechanisms related to the four fracture planes, and an exponential softening law for damage evolution. The effect of ply thickness on the matrix cracking initiation is considered by replacing the unidirectional strengths with the in situ strengths in the failure criteria. The softening laws for two damage models using Bažant’s crack band model are illustrated in Fig. 2.6, where $U^{dis}$ indicates the energy dissipated per unit volume. The main discrepancy between these two softening laws is when the element size is chosen too large, the area under the equivalent stress-displacement curve in Lapczyk’s model, i.e. the energy dissipation by the damage process, will become greater than the fracture toughness. As a consequence of the application of the crack band model, there is a maximum element size that can be used to avoid this situation (Maimí et al., 2006).

![Figure 2.6: Softening laws for damage models using crack band method; (a) Maimí’s model ; (b) Lapczyk’s model. (Schuecker et al., 2010)](image-url)
Recently, more researchers proposed damage models based on CDM and extended existing models further. Donadon et al. (2008) proposed a 3-D failure model based on CDM approach for predicting intra-laminar damage of composite laminates under impact loading. The model enables the control of the energy dissipation associated with each failure mode (matrix cracking in tension/compression, fibre failure in tension/compression, and shear failure) regardless of mesh refinement and fracture plane orientation. It also enables the prediction of both in-plane and out-of-plane shear nonlinearities. Donadon et al. also modified the in-plane shear model proposed by Lade\`veze and Dantec (1992) to incorporate a degradation factor in shear failure mode. Pinho et al. (2006), Faggiani and Falzon (2010) developed a 3D failure model for laminates within an explicit dynamic framework and applied Puck's failure theory. Falzon and Apruzzese (2011) developed a 3D failure model which was implemented in an implicit finite element code.

## 2.4 Interaction between Matrix Cracking and Delamination

Extensive work has been carried out to study the interaction between two main damage modes in composite laminates: delamination and matrix cracking. Jen and Sun (1992) used a fracture-mechanics-based model to simulate the multiplication process of matrix cracks and the initiation and propagation of matrix crack-induced delamination in a cross-ply laminates. The probabilistic effect of crack density on the subsequent changes of transverse strength for multiple matrix cracking was addressed. The results showed the failure progression: matrix cracking first, followed by the delamination, and finally the fibre failure in the $0^\circ$ ply.

Wang et al. (1985) modelled the multiple transverse cracks in cross-ply laminate under uniaxial tension. By comparing the strain energy release rate under different layups, the onset of transverse cracking and free-edge delamination were investigated. The results also showed that the matrix cracking process was influenced greatly by the geometry of the laminate and loading environment. Lammerant and Verpoest (1994) performed an impact test on beam-like composite specimens, both experimentally and numerically. The results showed different damage development patterns in two types of cross-ply laminates. Although the initiation and growth of delamination was predicted, the knowledge of the stress field during the damage development was not sufficient to
predict the whole damage development including the matrix crack initiation. A laminated beam model with interleaves incorporated in the inter-laminar region was proposed by Shi and Yee (1994) to numerically investigate the interaction of shear cracks and delamination. The results demonstrated that severe normal (through-thickness) stress occurred at the tips of vertical intraply cracks which were assumed to already exist before delamination initiation. It also showed that the intraply crack caused a local disturbance in stress distribution near crack tips before delamination propagation. Soft interleaves had no influence on the predictions as no reduction in the stress concentration at the delamination front was found. With or without the interleaves technology, the results revealed that the delamination propagation was dominated by mode II fracture rather than mode I under transverse impact loading.

Takeda and Ogihara (1995) used a modified analysis to predict the effect of delamination growth and transverse crack density by modelling progressive damage in carbon fibre-reinforced cross-ply laminates. By considering the interaction between transverse cracks and delamination, the Young’s modulus reduction and permanent strain induced by the damage development was evaluated. In addition, the effect of delamination in the laminates with thicker 90° plies is found to be significant to the prediction of transverse crack behaviour. The transverse cracking only scenario is proved to be inadequate for predicting further matrix cracking in cross-ply laminates at higher crack densities.

A finite element method making use of interface elements was developed by Wisnom and Chang (2000). They modelled the development of delamination in a cross-ply laminate with a centre crack loaded in tension. The interface element was modelled using non-linear springs between the coincident nodes that lie in the 0° and 90° plies. The initial elastic stiffness of the interfacial springs was based on the characteristics of the resin rich layer. It was assumed that spring yields when yielding stress is reached and breaks when the energy release rate reaches the mode II fracture energy. Delamination initiation and propagation was modelled by the failure of the interfacial springs. The model accurately predicted the development of a narrow triangular delamination zone, and the extent of splitting as a function of the applied tensile stress. This approach allowed appropriate modelling in redistributing the loads around areas of stress concentrations and improved understanding of the complex failure process in notched composites as well as simulation.
Moura and Gonçalves (2004) used interface element to investigate intra-laminar and inter-laminar damage mechanisms in laminates under low velocity impact. Both the experimental and numerical results showed the initial damage was vertical bending crack, and the growth of the vertical crack was associated with delamination propagation. Although the model successfully simulated the delamination induced by bending crack, the crack length is underestimated due to no experimental value of intra-laminar fracture toughness.

Lim and Li (2005) provided a better representation of the mode transition including two typical modes of damage: further transverse cracking or delamination induced by existing transverse cracks. Three types of further transverse cracking were considered: continuous, incremental and doubling process. The energy release rate corresponding to each of damage modes was evaluated. Their results as shown in Fig. 2.7 revealed that the “Doubling process of transverse cracking” curve shows a comparably underestimated prediction. Therefore, the damage mode transition from transverse cracking to delamination initiation at tips of existing transverse cracks is invalid in the case of doubling process by using energy release rates. Since there is no intersection between the energy release rate curve for continuous/incremental transverse cracking and delamination initiation as shown in Fig. 2.7, further transverse cracking would remain as a preferred mode of damage and no delamination would be induced by matrix crack.

Figure 2.7: Energy release rate versus crack density for further transverse cracking and initiation of delamination induced by transverse cracks, respectively. (Lim & Li, 2005)
Hallett et al. (2008) used the virtual crack closure technique (VCCT) to determine the applied load that would cause free-edge delamination. Matrix cracks were introduced first into the model and delamination development was predicted by using interface element. Both VCCT and interface elements have been applied successfully to the case of delaminations arising from a free edge. Analysis showed the fibre-dominated failure is initiated by the delaminations which cause local stress concentrations. Moreover, results also indicated that the exact position of the matrix cracks does have some effects on the stress in fibre direction.

The evolution of transverse crack coupled to interface related damage, i.e. delamination, was investigated by numerous researchers. Zhou et al. (2010) used nonlinear cohesive fracture models to study the initiation and propagation of a transverse intra-ply crack coupled to possible delamination at the ply interface in a [0°/90°/0°] laminate. The evolution of transverse cracks was found to be complex due to the various failure modes involving crack initial growth, crack propagation along the ply in a tunnelling mode, and expansion across the thickness of the ply in a plane strain mode. The couple situation was also revealed by the effect of two failure modes: delamination occurring simultaneously with the transverse cracks; and complete tunnelling propagation of the transverse cracks followed by delamination initiation and propagation. A three dimensional analysis based on cohesive zone models was used to simulate the direct coupling between transverse intra-ply cracking and accompanying delamination. Zhou et al. explained that under two different modes, while mode I fracture toughness is typically smaller than the mode II delamination toughness, the tunnelling-without-delamination occurred frequently than tunnelling-with-delamination. Hence, the delamination process will not develop until the tunnelling phase of the transverse crack growth is completed. However, when the mode II on mode I toughness ratio is relatively small and the cohesive strength normalized against the material stiffness is relatively large, the unstable tunnelling-with-delamination mode would occur, i.e. the tunnelling crack initiated from the stress-free edge and propagated into the laminate accompanied by unstable triangular delamination cracks.

Paris et al. (2010a, 2010b) used boundary element method to study damage development mechanism in cross-ply laminates. A two-dimensional boundary element method model was developed to investigate the transverse cracks in the 90° lamina and delamination between both laminae in a [0/90], laminate. By assuming a generalized
plane strain state, numerical studies were performed on two cracked configurations: one having only a transverse crack and the other having a transverse crack that has reached the interface and deflected, appearing as a delamination crack. All the analysis carried out referred to the growth of the damage in the 90° ply or at the interfaces between 0° and 90° plies. The results indicate that a transverse crack grows under mode I and arrests near the interface but without reaching the interface. When the transverse crack in the 90° ply approaches the interface, the normal stress reaches very large value that will induce fibre/material debonding before a clear delamination is formed. The simulation also showed that the energy release rate for an existing crack at the interface between 0° and 90° plies when the transverse crack has not reached the interface is smaller than those when the transverse crack has reached the interface. This eliminated the possibility of potential interaction mechanism between transverse crack and delamination.

Apart from the two cracked configurations described above, other intermediate situations were also analysed to investigate the connection between the former two cases (Fig. 2.8). The first situation was that a transverse crack grows under mode I but stops near the interface. The second situation was that the transverse crack progresses towards the interface and some delamination-like damage appears as the debonding of fibres in the 90° ply closest to the interface or the debonding along the fibres of the 0° ply or the combination of both. In the third situation, the transverse crack has reached the interface and a clear growth path of delamination was considered under mixed mode condition. To extend the damage, either by the extension of the transverse crack or by the progression of the delamination-like damage, more loads must be applied.

Figure 2.8: Schematic diagrams of the evolution of the damage. (Paris et al, 2010)
2.5 Conclusions

This chapter has presented an overview of the methodologies for modelling damage in fibre-reinforced composites. The main focus of this chapter is to assess and discuss the previous research studies in this field so as to obtain useful knowledge in order to justify the originality of the present research. A further detailed review of different aspects of the present research will be discussed in individual chapters.

The various failure criteria and modelling approaches were discussed within the context of the progressive failure analysis (PFA) framework. The simulation of crack initiation involves a failure criterion, and the simulation of crack propagation contains a strategy for modelling the effective properties of the damaged material. This chapter has reviewed most of the damage modelling techniques for progressive failure analysis and the concept of interface element. It is critical to understand the scales involved in any model to consider the links and interactions between matrix cracking and delamination. For the main objective in the present research, the issues of multiple damage sites, structural details and probabilistic variation in properties is considerably essential.
Chapter 3

Meso-macro-mechanics Modelling of Damage in Cross-Ply Laminates

3.1 Introduction

Matrix cracking and delamination are two main damage modes in composite laminates. The damage mechanism has been analysed for decades since transverse matrix cracking and delamination reduce the effective stiffness and strength of laminates. They also provide easy access for moisture or other leaking fluids. It has been suggested in the literature that the primary damage mode in the laminate is usually transverse matrix cracking when a laminate experiences mechanical loading. After sufficient number of matrix cracks have developed in the laminate, delamination starts to initiate from the tip of matrix cracks where local stress concentrations exist and propagate along the interface in the laminate. In other words, the density of matrix cracks increases until a certain value known as saturation crack density, and the secondary damage mechanism may change to delamination induced by transverse cracks. Fig. 3.1 shows a typical representative volume element containing transverse cracks and induced delaminations.

Many practical applications call for a proportion of the fibres to be orientated along the 0° and 90° directions. Cross-ply laminates are generally constituted of unidirectional fibre plies oriented alternatively at 0° and 90°. For these cross-ply laminates, the basic structure is constituted of a three-layered sequence with 90° plies as inner plies and 0° plies as outer plies. This elementary structure has been analysed extensively in the literature. According to the nature of cross-ply laminates, transverse cracks induce local
stress concentrations at crack tips and can involve inter-laminar crack which propagates as a delamination crack between 0° and 90° plies. The failure mechanisms which are observed for these laminates can then be extended to more complex stacking sequences of layer. However, particular damage modes depend upon the type of loading, the lay-up and stacking sequence of the laminates.

In this chapter, a [0°/90°]m cross-ply laminate which is the most common layup in the literature used to investigate damage phenomena was analysed. The damage models in commercial finite element code ABAQUS were employed to simulate the initiation and growth of damages in the laminate. When the [0°/90°ₘ]ₙ cross-ply laminates are subjected to uniaxial loading, several failure mechanisms may be induced: transverse matrix cracking in 90° plies, delamination at the interfaces between 0° and 90° plies, longitudinal matrix cracking and fibre fracture in 0° plies. Therefore, a sufficient number of cohesive elements were inserted in 90° ply to simulate how transverse matrix cracking initiate and develop. The ultimate (saturation) crack density was examined as well. Cohesive elements were also placed at the interface between 0° and 90° plies to simulate the delaminations. The analysis of damage in 0° plies was carried out by the progressive damage model (PDM) in ABAQUS. Although many other damage models exist in the literature, this PDM is readily available and good for its applicability of predicting fibre damage in laminated composites.

![Figure 3.1: Transversely cracked laminate with delamination arising from crack tips](image)

This study aimed to simulate the entire damage process in the laminate up to the final failure of the laminate under various conditions. The following issues have been investigated in particular:
(a) Damage initiation and development sequence, i.e. which damage mode would appear first and how the damages evolve.

(b) Interactions between damage modes, i.e. one damage mode initiates the other damage mode, e.g. whether delamination will start from matrix crack tips or not and under what kind of conditions.

(c) The formation of a single matrix crack. It is usually assumed in the literature that matrix cracking propagates across the entire thickness of the layer immediately after it initiates in a layer. This scenario was investigated thoroughly in this chapter under different loading conditions.

3.2 Description of damage models available in ABAQUS

3.2.1 Cohesive Model

The concept of cohesive zone model was first introduced by Dugdale (1960) and Barenblatt (1962), and later applied by Hillerborg et al. (1976) to the analysis of concrete cracking, allowing for existing crack growth and initiation of new cracks. The cohesive zone model has also been used by Needleman (1987) to describe the relationships between tractions and interfacial separations. Fig. 3.2 shows the traction-separation relationship under pure mode loading. When the interfacial separation increases, the tractions across the interface rise and reach a maximum value (i.e. the interfacial strength). Damage initiation is related to the maximum traction in the traction-separation response. When the area under the traction-separation curve is equal to the critical fracture energy, the traction is reduced to zero and new crack surfaces are formed. The advantages of cohesive models are their simplicity and the unification of crack initiation and growth within one model. Moreover, cohesive element can also be implemented into an ordinary finite element via an “interface element” or “cohesive element” by using proper link relation. The cohesive elements with high penalty stiffness are used before crack onset to prevent additional deformation (point 1 in Fig. 3.2). The cohesive zone models are powerful due to their ability to predict both initiation and crack propagation.
The appropriate constitutive equation in the formulation of cohesive model is fundamental for an accurate simulation of damage process. The constitutive equation describes the relation between the cohesive traction force and the displacement jump in the interfacial region. The traction-separation behaviour is assumed to be linear elastic before damage initiation. The interface traction $\tau$ in the linear elastic region can be expressed as

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{12} & K_{22} & K_{23} \\ K_{13} & K_{23} & K_{33} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = K\delta$$

where $\delta$ is a vector of separations and $K$ is the elasticity matrix. A local coordinate system is defined such that the first component of $\delta$ refers to the separation along the through-thickness direction, whereas the second and third components refer to the separations along the other two orthogonal directions. This coordinate system corresponds to the conventional definition of the fracture modes where the local 1-direction represents the normal traction and the local 2- and 3-directions represent the two shear tractions. The elastic modulus for traction-separation law should be interpreted as penalty stiffness. Assume that the interface is a very thin continuum layer of thickness $t$ and elastic modulus $E$. When loaded, the nominal stress and strain in the interface layer are $\tau$ and $\varepsilon$, then
\[
K = \frac{\tau}{\delta} = \frac{\tau}{\varepsilon t} = \frac{E}{t}
\]

(3-2)

As the thickness of the interface layer reduces to zero, the stiffness \( K \) goes to infinity. Therefore, this stiffness is often chosen as a penalty parameter (Abaqus v6.11, 2011).

The process of damage initiation begins when the stresses or strains or combination of them satisfy certain damage initiation criteria. Under pure loading conditions (Mode I, II or III), damage initiation occurs when the interfacial normal or shear tractions is equal to their respective interfacial tensile or shear strengths (point 2 in Fig. 3.2), and the stiffness are gradually reduced thereafter. A bilinear cohesive law is usually assumed for each of the three fracture modes. The displacement jumps at damage initiation in each fracture modes are shown as follows:

\[
\delta_1^o = \frac{\tau_1^o}{K} \quad \text{(Mode I)}
\]

(3-3)

\[
\delta_2^o = \frac{\tau_2^o}{K} \quad \text{(Mode II)}
\]

(3-4)

\[
\delta_3^o = \frac{\tau_3^o}{K} \quad \text{(Mode III)}
\]

(3-5)

where \( \tau_1^o, \tau_2^o \) and \( \tau_3^o \) represent the peak value of the nominal stress: \( \tau_1^o = N, \tau_2^o = S, \tau_3^o = T \), where \( N \) is the interfacial normal strength, and \( S \) and \( T \) are the interfacial shear strengths as shown in Fig. 3.2. These relationships hold when the deformation is either purely normal to the interface or purely in the first or the second shear direction.

The area under the traction-separation curves is the work done by the traction forces and represents the energy release rate in different fracture modes.

\[
G_i = \int_0^{\delta_i} \tau_i(\delta)d\delta
\]

(3-6)

\[
G_{ii} = \int_0^{\delta_i^f} \tau_i(\delta)d\delta
\]

(3-7)

\[
\delta_i^f = \frac{2G_{ii}}{\tau_i^o}
\]

(3-8)
where $G_i (i = 1, 2, 3)$ are the energy release rates for the mode I, II and III, respectively, and $G_{IC} (i = 1, II, III)$ are the fracture energies required to cause crack in the normal, the first and the second shear directions, respectively. The area under the traction-separation curves is the respective fracture toughness ($G_{IC}, G_{IIC}$, and $G_{IIIC}$ respectively) and defines the final relative displacement $\delta_i^f (i = 1, 2, 3)$. Crack is predicted when the energy release rate attain the corresponding fracture energies (Eq. (3-7); point 4 in Fig. 3.2). In order to formulate the complete constitutive equation, it is important to consider the unloading behaviour where a softening point unloads towards the origin, as shown in Fig. 3.2. The bilinear (linear elastic-linear softening) constitutive behaviour can be defined as

$$
\tau_i = \begin{cases} 
K \delta_i & \text{when } \delta_i^{max} \leq \delta_i^o \\
(1 - d_i) K \delta_i & \text{when } \delta_i^{o} < \delta_i^{max} < \delta_i^{f} \\
0 & \delta_i^{max} \geq \delta_i^{f} 
\end{cases} \quad (3-9)
$$

$$
d_i = \frac{\delta_i^{o} (\delta_i^{max} - \delta_i^{o})}{\delta_i^{max} (\delta_i^{f} - \delta_i^{o})} , \quad i = 1, 2, 3 ; \quad d_i \in [0, 1] \quad (3-10)
$$

where $d_i$ is a scalar damage variable, $\delta_i^{max}$ is the maximum separation the interface has experienced and the subscript $i$ represents the $i$th damage system. The scalar damage variable $d_i$ initially has a zero value and after damage evolution is modelled, it monotonically evolves from 0 to 1 upon further loading after the initiation of damage. In other words, with $\delta_i^{max} \leq \delta_i^o$ (here, the subscript $o$ means onset), there is no damage and therefore $d_i = 0$. Once $\delta_i^{max} \geq \delta_i^{f}$, the material is fully damaged and $d_i = 1$.

Under pure Mode I, II, or III loading, the initiation of damage can be determined directly by comparing the traction components with their respective variables. For mixed-mode loading conditions, however, damage initiation must be defined to consider the effect of the interaction between traction components. It is assumed that the damage initiation can be predicted using the conventional stress-based failure criterion

$$
f_{initiation} = f(\tau_i) = 1 \quad (3-11)
$$

where $f(\tau_i)$ is a stress based failure function criterion. Nevertheless, the quadratic failure criterion has been proven to be more effective in predicting damage initiation.
Based on this criterion, damage is initiated when the following quadratic interaction function is equal to one:

$$
f_{\text{initiation}} = \left( \frac{\tau_1}{\tau_1^o} \right)^2 + \left( \frac{\tau_2}{\tau_2^o} \right)^2 + \left( \frac{\tau_3}{\tau_3^o} \right)^2 \tag{3-12}$$

The symbol $\langle \cdot \rangle$ in the equations is Macauley bracket operator, defined for every $\omega \in \mathbb{R}$ as $\langle \omega \rangle = (\omega + |\omega|)/2$.

The failure criterion used to predict crack propagation under mixed-mode loading conditions are developed in terms of energy release rates and fracture toughness. Damage evolution can be based on energy or displacement, either the total fracture energy or the post damage-initiation effective displacement at failure will be specified in damage propagation prediction. The fracture-mechanics based failure criterion of crack propagation can be expressed as

$$
f_{\text{propagation}} = f\left(G_i\right) = 1 \tag{3-13}$$

where $f(G_i)$ is a failure function based on energy release rates. The power law criterion is developed in terms of an interaction between the energy release rates (Wu & Reuter Jr., 1965).

$$
f_{\text{propagation}} = \left( \frac{G_I}{G_{Ic}} \right)^a + \left( \frac{G_{II}}{G_{IIc}} \right)^a + \left( \frac{G_{III}}{G_{IIIc}} \right)^a = 1 \tag{3-14}$$

In the expression above, the quantities $G_I$, $G_{II}$, $G_{III}$ refer to the work done by the traction over its conjugate relative displacement in the normal, the first, and the second shear directions, respectively as shown in Eq. (3-6). The value $a$ is frequently chosen to be 1 or 2 when no experimental data is available. The power law criterion states that failure under mix-mode loading conditions is governed by a power law interaction of the energies required to cause failure in normal and two shear modes.

To describe the evolution of damage, an effective displacement is introduced (Camanho & Dávila, 2002). The total mixed-mode relative displacement $\delta_m$ is defined as

$$
\delta_m = \sqrt{\langle \delta_i \rangle^2 + \langle \delta_{II} \rangle^2 + \langle \delta_{III} \rangle^2} = \sqrt{\langle \delta_i \rangle^2 + \delta_{\text{shear}}^2} \tag{3-15}
$$
where $\delta_{\text{shear}} = \left(\delta_2^2 + \delta_3^2\right)^{1/2}$ represents the norm of the vector defining the tangential relative displacements (Fig. 3.3).

ABAQUS uses two measures of mode mix, one based on energies and the other based on tractions. The mode-mix ratios defined in terms of energies and tractions can be different, while the dependence of the damage evolution data on the mode mix can be defined either in tabular or analytical form. If the dependence of the fracture energy on the mode mix is specified by using an analytical form, e.g., power law, the mode-mix ratio is assumed to be defined in terms of energies. For linear damage evolution (Fig. 3.4), the damage variable, $D$, was defined as a function of the effective displacement beyond damage initiation as shown below,
\[
D = \frac{\delta_m^f (\delta_m^{\text{max}} - \delta_m^o)}{\delta_m^{\text{max}} (\delta_m^f - \delta_m^o)}
\tag{3-16}
\]

where \(\delta_m^f = 2G^C/T_m^o\) with \(T_m^o\) as the effective traction at damage initiation. \(\delta_m^o\) is the effective displacement at the onset of damage. \(\delta_m^{\text{max}}\) refers to the maximum value of the effective displacement attained during the loading history and the mixed-mode fracture energy \(G^C\) equals to the energy dissipated due to failure when the specific damage evolution criterion is satisfied (i.e. \(G^C = G_{I} + G_{II} + G_{III}\)). The stress components of the traction-separation model are modified by the damage evolution law as follows

\[
\tau_i = (1 - D)\overline{\tau}_i
\tag{3-17}
\]

where \(\overline{\tau}_i\) \((i = 1, 2, 3)\) are the stress components predicted by the elastic traction-separation behaviour for the current strain without damage to tensile or compressive stiffness. \(D\) has an initial value of 0 and if damage evolution is modelled, it monotonically evolves from 0 to 1 upon further loading after initiation of damage.

Material models exhibiting softening behaviour and stiffness degradation usually lead to severe convergence difficulties in implicit analysis programs. ABAQUS/Standard simulation is one of these programs, therefore, a viscous regularization is used in constitutive equations to overcome some of these convergence difficulties (Abaqus v6.11, 2011). It is implemented as suggested by ABAQUS for its cohesive element and composite damage models. The use of viscous regularization of the constitutive equations makes the tangent stiffness matrix of the softening material to be positive for sufficiently small time increments. The procedure is using viscosity by permitting stresses to be outside the limits set by the traction-separation law. Defining a viscous stiffness degradation variable, \(D_v\), the rate of change in the damage variable is a function of the damping factor \(\mu\)

\[
D_v = \frac{1}{\mu} (D - D_v)
\tag{3-18}
\]

where \(D\) is the degradation variable evaluated in the inviscid backbone model. The response of the damaged material is evaluated using viscous degradation variable. Using viscous regularization with a small value (small compared to the time increment) of the viscosity parameter can improve the rate of convergence in the presence of softening regime without compromising the accuracy of the results.
3.2.2 ABAQUS Progressive Damage Model

ABAQUS offers a built-in progressive damage model enabling user to predict the onset of damage and to model damage evolution for elastic-brittle materials with anisotropic behaviour. The model is primarily intended to be used with fibre-reinforced materials.

In this damage model, the initiation of in-plane failure is detected by the Hashin and Rotem damage initiation criteria (Hashin & Rotem, 1973; Hashin, 1980). Usually Hashin and Rotem criteria are implemented within a two-dimensional classical lamination approach for the point-stress calculations with ply discounting as the material degradation model. In Hashin’s theory, the failure surface is expressed in the effective stress space. These criteria consider four different damage initiation mechanisms: fibre tension, fibre compression, matrix tension, and matrix compression. The initiation criteria have the following general forms:

Fibre tension \( (\hat{\sigma}_{11} \geq 0) \):

\[
F^f_j = \left( \frac{\hat{\sigma}_{11}}{X^f} \right)^2 + \alpha \left( \frac{\hat{\tau}_{12}}{S^f} \right)^2 = 1
\]

Fibre compression \( (\hat{\sigma}_{11} < 0) \):

\[
F^c_j = \left( \frac{\hat{\sigma}_{11}}{X^c} \right)^2 = 1
\]

Matrix tension \( (\hat{\sigma}_{22} \geq 0) \):

\[
F^m_t = \left( \frac{\hat{\sigma}_{22}}{Y^t} \right)^2 + \left( \frac{\hat{\tau}_{12}}{S^t} \right)^2 = 1
\]

Matrix compression \( (\hat{\sigma}_{22} < 0) \):

\[
F^m_c = \left( \frac{\hat{\sigma}_{22}}{2S^t} \right)^2 + \left[ \left( \frac{Y^c}{2S^t} \right)^2 - 1 \right] \frac{\hat{\sigma}_{22}^c}{Y^c} + \left( \frac{\hat{\tau}_{12}}{S^t} \right)^2 = 1
\]

where \( X^T \) and \( X^C \) denote the longitudinal tensile and compressive strength, \( Y^T \) and \( Y^C \) the transverse tensile and compressive strength, \( S^L \) and \( S^T \) the longitudinal and transverse shear strength, and \( \alpha \) is a coefficient that determines the contribution of the shear stress to the fibre tensile criterion. To obtain the model proposed in Hashin and
Rotem (Hashin & Rotem, 1973), \( \alpha \) is set to be zero and \( S^T = Y^c / 2 \). Furthermore, \( \hat{\sigma}_{11}, \hat{\sigma}_{22} \) and \( \hat{\tau}_{12} \) are components of the effective stress tensor that is used to evaluate the initiation criteria and which is computed from (Matzenmiller, et al., 1995)

\[
\hat{\sigma} = M \sigma
\]  
(3-23)

where \( \sigma \) is the nominal stress and \( M \) is the damage operator, which has the diagonal form

\[
M = \begin{bmatrix}
\frac{1}{(1-d_f)} & 0 & 0 \\
0 & \frac{1}{(1-d_m)} & 0 \\
0 & 0 & \frac{1}{(1-d_s)}
\end{bmatrix}
\]  
(3-24)

where \( d_f, d_m, d_s \) are internal (damage) variables that characterize fibre, matrix, and shear damage, which are derived from damage variables \( d_f^c, d_m^c \) and \( d_m^c \), corresponding to the four modes previously discussed, as follows:

\[
d_f = \begin{cases} 
  d_f^c & \text{if } \hat{\sigma}_{11} \geq 0, \\
  d_f^c & \text{if } \hat{\sigma}_{11} < 0,
\end{cases}
\]  
(3-25)

\[
d_m = \begin{cases} 
  d_m^c & \text{if } \hat{\sigma}_{22} \geq 0, \\
  d_m^c & \text{if } \hat{\sigma}_{22} < 0,
\end{cases}
\]  
(3-26)

\[
d_s = 1 - (1-d_f^c)(1-d_m^c)(1-d_m^c)
\]  
(3-27)

and the corresponding stiffness matrix is obtained from

\[
C = \frac{1}{D} \begin{bmatrix}
(1-d_f)E_1 & (1-d_f)(1-d_m)v_{12}E_1 & 0 \\
(1-d_f)(1-d_m)v_{12}E_2 & (1-d_m)E_2 & 0 \\
0 & 0 & D(1-d_s)G_{12}
\end{bmatrix}
\]  
(3-28)

where \( D = 1 - (1-d_f)(1-d_m)v_{12}v_{21} \), \( E_1 \), \( E_2 \), \( G_{12} \) are undamaged material modulus, and \( v_{12}, v_{21} \) are Poisson’s ratios.

Once a damage initiation criterion is satisfied, the degradation of material stiffness is controlled by damage variables that might have values between 0 (undamaged state) and 1 (fully damage state for the mode corresponding to this damage variable). The
evolution law is based on the energy that is dissipated as a result of the damage process, also called the fracture energy. This evolution law is a generalization of the approach proposed by Camanho and Dávila (2002). To alleviate mesh dependency during material softening, ABAQUS introduces a characteristic length into the formulation, so that the constitutive law is expressed as a stress-displacement relation. The damage variable will evolve in such a way so that the stress-displacement behaves as shown in Fig. 3.5 in each of the four failure modes. A linear elastic material behaviour prior to damage initiation is assumed, the negative slope after damage initiation is achieved by evolution of the respective damage variables according to the equations shown below.

![Figure 3.5: Equivalent stress versus equivalent displacement](image)

The equivalent displacements and stresses for each damage modes are defined as follows:

Fibre tension ($\hat{\sigma}_{11} \geq 0$):

$$\delta_{eq} = L_c \langle \varepsilon_{11} \rangle$$  \hspace{1cm} (3-29)

$$\sigma_{eq} = \frac{L_c \langle \sigma_{11} \rangle}{\delta_{eq}^b}$$  \hspace{1cm} (3-30)

Fibre compression ($\hat{\sigma}_{11} < 0$):

$$\delta_{eq} = L_c \langle -\varepsilon_{11} \rangle$$  \hspace{1cm} (3-31)
\[ \sigma_{eq} = \frac{L_c (\mathbf{\sigma}_{11} (-e_{11}))}{\delta_{eq}^{f}} \]  

(3-32)

Matrix tension \((\delta_{22} \geq 0)\):

\[ \delta_{eq} = L_c \sqrt{(-e_{22})^2 + e_{12}^2} \]  

(3-33)

\[ \sigma_{eq} = \frac{L_c \left( (\mathbf{\sigma}_{22} (-e_{22}) + \tau_{12}e_{12}) \right)}{\delta_{eq}^{mt}} \]  

(3-34)

Matrix compression \((\delta_{22} < 0)\):

\[ \delta_{eq} = L_c \sqrt{(-e_{22})^2 + e_{12}^2} \]  

(3-35)

\[ \sigma_{eq} = \frac{L_c \left( (\mathbf{\sigma}_{22} (-e_{22}) + \tau_{12}e_{12}) \right)}{\delta_{eq}^{mc}} \]  

(3-36)

where \(L_c\) is a characteristic length of the element that is based on the element geometry and formulation. For membranes and shells it is a characteristic length in the reference surface, computed as the square root of the area. The equivalent displacement for a failure mode is expressed in terms of the components corresponding to the effective stress components used in the initiation criterion for this failure mode.

After damage initiation (i.e., \(\delta_{eq} \geq \delta_{eq}^0\)) for the behaviour shown in Fig. 3.5, the damage variable for each failure mode \(I\) is given by the following relation:

\[ d_i = \frac{\delta_{1,eq}^f (\delta_{1,eq}^f - \delta_{1,eq}^0)}{\delta_{1,eq}^f (\delta_{1,eq}^f - \delta_{1,eq}^0)} ; \quad I \in \{ft, fc, mt, mc\} ; \]  

(3-37)

\[ d_i \in [0,1] \]

where \(\delta_{1,eq}^0\) is the initial equivalent displacement at which the initiation criterion is satisfied, and \(\delta_{1,eq}^f\) is the equivalent displacement at which the material is completely damaged in this failure mode (i.e., \(d_i = 1\)). It can be seen that if the damage variable is computed from Eq. (3-37), the softening response of the material is linear for uniaxial deformations by using Eq. (3-28). The \(\delta_{1,eq}^f\) in Eq. (3-37) is not known and can be computed from the following relation:
\[
\delta_{I,eq}^f = \frac{2G_{I,c}}{\sigma_{I,eq}^p}
\]  

(3-38)

where \(G_{I,c}\) is the critical energy release rate and is specified for each failure mode, each one corresponding to the area of the triangle OAC shown in Fig. 3.5. \(\sigma_{I,eq}^p\) is the equivalent stress at which an initiation for that mode is met. However, the values of \(\delta_{eq}^p\) for the various modes depend on the elastic stiffness and the strength parameters specified as part of the damage initiation discussed above.

### 3.3 Finite Element Model

The typical cross-ply laminates with \([0_n/90_m/0_n]\) and \([90_m/0_n/90_m]\) lay-up as shown in Fig. 3.6 were analysed. The specimen is 4 mm long which is sufficient to show the damage development sequence for the sake of computational efficiency. Both ends of the 90° plies in the specimen were set as free surfaces to represent the earliest transverse cracking occurred at the weakest sites in the 90° plies. The introduction of these pre-existing cracks into the specimen was to avoid a uniform stress state in the entire laminate and the prediction of matrix cracks everywhere in the 90° plies. The specimen could be considered as a representative volume of a laminate immediately after the occurrence of those earliest matrix cracks at the weakest sites. Loads were applied to the ends of the 0° plies to simulate uniaxial tension, pure bending and combined loading.
A two-dimensional finite element model was employed to model the specimen. The layers were modelled by four-node linear plane stress elements CPS4R. The four-node two-dimensional cohesive elements COH2D4 with zero thickness were implemented in the specimen. Cohesive elements were placed along the interfaces between 0° and 90° plies to predict the initiation and propagation of delamination. Cohesive elements were also placed in the 90° plies to model the initiation and propagation of the transverse matrix cracking. Fig. 3.6 shows the positions of the embedded cohesive elements in the specimen.

Since transverse matrix cracking usually appears throughout the thickness of the 90° ply and discretely in the laminate plane, a number of columns of the cohesive elements were evenly arranged in the length direction of the specimen to simulate this process. The idealised doubling multiplication cracking process shown in Fig. 3.7 was often employed in the literature (Nairn, 1989). Fifteen (15) columns of the predefined cohesive elements were used to check the possibility of this doubling cracking process. The space between two neighbouring columns of cohesive elements is 0.25 mm, meaning that the minimum matrix crack spacing can only be equal or greater than a quarter of a millimetre. However, some authors (Shi, et al., 2014) suggested that a saturated matrix cracking has a crack density of 2 cracks/mm. Therefore, the present arrangement of cohesive elements was sufficient for modelling matrix cracking process.
In ABAQUS there are several continuum elements that can be used to represent good bending behaviour. It is known that linear elements with full integration are too stiff in modelling the simple flexural deformation. Full integration refers to the Gauss integration order required for exact integration of the polynomial of the order being integrated when the element is rectangular (Abaqus v6.11, 2011). Although full-order integration elements can act like rigid-body and represent constant-strain displacement fields precisely, they tend to show the “shear-locking” deficiency which greatly increases the flexural rigidity of the model in bending problems. Therefore, 2-dimensional reduced-integration linear elements, CPS4R, are used to eliminate the “shear-locking” phenomenon. However, multiple reduced-integration elements through thickness are suggested to model the bending response properly. In addition, the enhanced hourglass control option available in ABAQUS is enabled to ensure these elements to provide accurate displacement solutions and overcome mesh instability. This approach uses stiffness coefficients based on the enhanced assumed strain method; no scale factor is needed.

The quadrilateral elements involved in the mesh were 0.01 mm × 0.033 mm and there were 15 elements in the thickness direction of each group of layers. Multiple elements through a ply group thickness could predict crack opening by achieving higher order variation of displacement field than single element. Sufficient number of cohesive elements in the ply thickness was also essential for detecting matrix cracking formation sequence in 90° ply. In general, the cohesive element size used in 90° ply for matrix cracking prediction might not be the same as the one used at the interface between 0° and 90° plies for delamination prediction due to the different failure mechanisms of intra-laminar and inter-laminar cracking.
The majority of the material properties used in this simulation were obtained from the literature (Pinho, et al., 2013; Kaddour, et al., 2013) and are listed in Table 3.1. The damage in the $0^\circ$ plies was predicted by using the progressive damage model presented in the previous section. The four damage modes are controlled by the corresponding damage variables. Therefore, in addition to the six values of strength listed in Table 3.1, four values of critical energy release rate must be provided accordingly. The energy release rate associated with crack opening displacements in the literature is expressed as $G = -\frac{\partial U}{\partial A}$, where $U$ is the strain energy and $A$ the crack area. However, in ABAQUS progressive damage model, there is no identifiable crack but rather an equivalent reduction of stiffness. Thus, the function for crack area and the energy release rate is not suitable for ‘equivalent’ energy release rates presented in the section 3.2.2. Barbero et al. (2013) provided the correlations of the energy release rates operated in ABAQUS progressive damage model, using the E-glass/epoxy laminates to identify the parameters for transverse matrix damage. In the present analysis, the remaining fracture energy for the fibre tensile and compressive damage modes were obtained by following the same procedure as Barbero et al. (2013) for the same Glass/Epoxy material used in present work.

The details of material parameters for the cohesive elements are shown in Table 3.2. As far as we have known that the transverse matrix cracking in the $90^\circ$ plies under tension loading in 2-D analysis is predominantly controlled by Mode-I fracture mechanism and the delamination at the ply interface is predominantly controlled by Mode-II fracture. Therefore, the same values were used for fracture toughness $G_{Ic}$ and $G_{IIc}$ for matrix cracks. $G_{Ic}$ and $G_{IIc}$ used for predicting delamination was obtained from Zou et al. (2001). For the transverse cracks, it was found by some authors (van der Meer & Dávila, 2013) that the input strength (i.e. N, S, T) did not influence the stress level associated with longitudinal crack growth as long as the input strength was chosen higher than the in situ strength. For the simulation done in this thesis, the input strength of transverse cracks for Mode-I was chosen to be higher than the delamination strength for the same mode. In addition, Zou et al. (2002, 2003) proposed the value of interface stiffness used in resin-rich area can be taken $10^5$ to $10^7$ times the value of the strength of the interface per unit length and the results are independent of the choice of the value of $K_i$ ($i = N, S, T$), therefore, the penalty stiffness could be chosen with $K_N = K_S = K_T$.
Table 3.1: Material Properties of Glass/epoxy laminate (Kaddour et al., 2013)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (GPa)</td>
<td>45.6</td>
</tr>
<tr>
<td>$E_2$ (GPa)</td>
<td>16.2</td>
</tr>
<tr>
<td>$G_{12}$ (GPa)</td>
<td>5.5</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>0.278</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>0.4</td>
</tr>
<tr>
<td>$X^T$ (MPa)</td>
<td>1280</td>
</tr>
<tr>
<td>$X^C$ (MPa)</td>
<td>800</td>
</tr>
<tr>
<td>$Y^T$ (MPa)</td>
<td>40</td>
</tr>
<tr>
<td>$Y^C$ (MPa)</td>
<td>145</td>
</tr>
<tr>
<td>$S$ (MPa)</td>
<td>73</td>
</tr>
<tr>
<td>$G_{1c}$ (J/m$^2$) (fibre tensile)</td>
<td>45000</td>
</tr>
<tr>
<td>$G_{1c}$ (J/m$^2$) (fibre kinking)</td>
<td>40000</td>
</tr>
<tr>
<td>$G_{1c}$ (J/m$^2$) (matrix tensile)</td>
<td>1500</td>
</tr>
<tr>
<td>$L$ (segment length; mm)</td>
<td>4</td>
</tr>
<tr>
<td>$t$ (ply thickness; mm)</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Table 3.2: Material Properties for the cohesive elements (Kaddour et al., 2013)

<table>
<thead>
<tr>
<th>Direction/Mode</th>
<th>Mode</th>
<th>I</th>
<th>II, III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Penalty stiffness (N/m$^3$)</td>
<td></td>
<td>$10^{14}$</td>
<td>$10^{14}$</td>
</tr>
<tr>
<td>Interface strength (MPa) (matrix cracking)</td>
<td></td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Interface strength (MPa) (delamination)</td>
<td></td>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td>Fracture energy (J/m$^2$) (matrix cracking)</td>
<td></td>
<td>500</td>
<td>1540$^*$</td>
</tr>
<tr>
<td>Fracture energy (J/m$^2$) (delamination)</td>
<td></td>
<td>500</td>
<td>800</td>
</tr>
</tbody>
</table>

* Input data from other authors (Zou, et al., 2001)
3.4 Results and Discussion

3.4.1 \([0_n/90_m/0_n]\) laminates under uniaxial tension

In this section, multiple transverse cracking and delamination in cross-ply laminates and crack density dependence on the applied strain were analysed. The analyses were performed by using ABAQUS/Explicit commercial finite element code. The load was applied to the specimen in displacement-controlled mode. The loading rate employed in the solution is the longitudinal displacement increments of \(\Delta U_x = 1.87 \times 10^{-3}\) mm, corresponding to the tensile strain increments of \(\Delta \varepsilon_x = 0.047\%\), at the time increments of \(\Delta t = 3.75 \times 10^{-5}\). It is worth mentioning that simulation jobs were aborted by ABAQUS when the specimens suddenly lost its load-carrying capacity in conjunction with final fibre failure.

The cracking multiplication process in \([0_n/90_m/0_n]\) laminate can be seen in the Fig. 3.8. The displacement-controlled tensile loading was applied to 0° ply while both boundaries of 90° ply remained traction-free as mentioned in section 3.3. Fig. 3.8(a) shows the embedded interface elements in the 90° ply and the interfaces between 0° and 90° plies, while there is no formation of matrix crack at the 0.187% applied strain. The “hollow gap” across 90° plies in Fig. 3.8(b) represents the entire row of cohesive elements exceeding the damage criterion and propagating through the thickness of 90° ply. For each matrix crack, it initiates in the middle of layer thickness and then propagates stably to the interface between 90° and 0° plies as shown in Fig. 3.8(b) and (c). It can also be seen that the first matrix crack in the 90° ply is formed in the middle of the layer span when the applied strain almost reaches 0.3%. As the load continues to increase, new matrix cracks appear in the middle span between the end of the specimen and the first matrix crack, demonstrating the idealised doubling multiplication cracking process often employed in the literature (Nairn, 1989). When the strain is further increased to 1.22%, more matrix cracks initiate, but they do not propagate through entire thickness of 90° ply as seen in Fig. 3.8(d). When the distance between two neighbouring cracks is sufficiently close, the crack will affect the stress distribution in the whole area between the neighbouring cracks, resulting in high stress near the interface and low stress in the middle of the 90° layers (thickness direction). The loading capacity of the cracked 90° layers is reduced significantly. The 0° layers take more loads and may fail eventually. The
distance between two fully formed cracks is 1 mm in [0°/90°/0°] cracked laminate and the crack density is assumed to reach the saturation level. However, there is no occurrence of delamination between the 0° and 90° plies, i.e. no delamination is triggered by matrix cracking under uniaxial tensile loading.
After sufficient transverse cracks are formed and load is raised to very high level, the damage initiation of fibre tensile failure can be found in the 0° plies. Fig. 3.9 shows the fibre failure in 0° plies when the damage criterion is satisfied (i.e. DAMAGEFT=1.0). The local stress concentration at the tips of transverse cracks can provoke fibre tensile failure. Although interlaminar delaminations are also predicted at this stage, it is believed that these delaminations are caused by the fibre failure, as the intensity of the stress concentration at the delamination tip is usually not as intensified as that at matrix crack tip.
Figure 3.9: Final failure in [0°/90°/0°] E-glass laminate (1.22% applied strain), upper one is related to the damage in fibre tension and lower one is related to the formation of matrix cracking and delaminations.

Fig. 3.10 shows the average stress-strain relationship of the specimen. The slope of the curve in [0°/90°/0°] laminate starts to reduce at the point where the applied strain is around 0.2%. This is caused by the formation of the first matrix crack in the 90° plies at the mid span of the specimen. A “knee” starts to appear on the stress/strain curve at a strain of approximately 0.4% and lasts for a strain increment of 0.05%. This is accompanied by the doubling multiplication process of the matrix cracking in the [0°/90°/0°] laminate, i.e., the formation of the two matrix cracks seen in Fig. 3.8(c). Because the matrix cracking reaches the saturation level since then, no more new matrix cracking can develop in the specimen and no further reduction to the stiffness of the specimen. Stress-strain relationship is therefore almost linear till the final tensile failure of the 0° plies which results in the sudden load drop on stress-strain curve.
To investigate the effect of layup configuration or thickness of different layers on the damage process, three laminates with lay-ups of \([0_2/90_4/0_2]\), \([0_4/90_8/0_4]\) and \([0/90_8/0]\) were also modelled following the same procedure as \([0_4/90_8/0]\) laminate. The size effect of specimen is obvious. Fig. 3.10 shows that \([0_2/90_4/0_2]\) laminate is stiffer than \([0_4/90_8/0_4]\) laminate. The initial applied strain which causes the first matrix crack in \([0_2/90_4/0_2]\) laminate is also larger than \([0_4/90_8/0_4]\) laminate. Parvizi & Bailey (1978) noted that the deviation from linearity at the “knee” is dependent on the total thickness of the 90° plies in the inner of the laminate; the greater the thickness of these plies, the greater the reduction of the laminate stiffness. As the thickness of 90° layers in the \([0_4/90_8/0_4]\) laminate is greater, the drop is more visible than \([0_2/90_4/0_2]\) laminate. Fibre tensile failure in 0° plies can be captured in both \([0_2/90_4/0_2]\) and \([0_4/90_8/0_4]\) laminates when the damage criterion is satisfied. While the final failure in \([0_2/90_4/0_2]\) laminate occurs at a comparable low applied strain, the corresponding final failure stresses are almost the same. The delaminations developed at the interface when final failure of laminate occurs are monitored in Fig. 3.11(b). The level of delamination at final failure in \([0_2/90_4/0_2]\) laminate is much greater than in the \([0_4/90_8/0_4]\) laminate, with delaminations starting at the crack tips.
Figure 3.11(a): Failure in [0₂/90₄/0₂] E-glass laminate (1.22% applied strain): fibre tensile damage (upper), matrix cracking (lower).

Figure 3.11(b): Final failure in [0₂/90₄/0₂] E-glass laminate (1.3% applied strain): fibre tensile damage (upper), matrix cracking and delaminations (lower).
Fig. 3.12 shows matrix cracking formation process in the \([0_4/90_4/0_4]\) laminate. The cracking multiplication sequence is similar to that in \([0_4/90_n/0_4]\). However, more fully damaged cracks appear in the 90° plies in the present laminate. It also shows in Fig. 3.12 that multiple transverse cracks occur and develop in the middle of the ply thickness. These cracks do not propagate through the entire thickness of 90° plies until the final stage of damage propagation. The distance between two fully formed cracks is 0.5 mm. Therefore, a crack density of 2 cracks/mm is reached at the saturation level in the \([0_4/90_4/0_4]\) laminate before other damage mechanisms take place (e.g. delaminations).
Figure 3.12: Cracking multiplication process in [0_4/90_4/0_4] cross-ply laminate (3x deformation scaling)
The predicted crack densities in [0₄/90₆/0₄] and [0/90₆/0] cross-ply laminates are given in Fig. 3.13, together with the effect of the mesh size of cohesive elements in 90° layers for [0₄/90₆/0₄] laminates. The finer mesh size is 0.033 mm for each cohesive element in 90° layers.

The transverse crack densities in [0₄/90₆/0₄] and [0/90₆/0] get converged as the applied strain is increased gradually. Both laminates have a same 1 mm total thickness of the 90° layers which is close to the 1.2 mm total thickness of the 90° layers in a [0/90₆/0] laminate tested by Highsmith & Reifsnide (1982). The crack densities for these three laminates are quite close. The overall comparison gives an estimate tendency of crack density versus applied strain, while the experimental results might not be reliable nowadays.

![Graph](image)

Figure 3.13: Crack density versus applied strain for [0₄/90₆/0₄] and [0/90₆/0] E-glass laminate.
3.4.2 \([90_n/0_m/90_n]\) laminates under uniaxial tension

In this section, damages in cross-ply laminates with \([90_n/0_m/90_n]\) layups were analysed. The simulation procedure was similar to that described in section 3.4.1 except the boundary conditions. For boundary conditions, all the nodes at the ends where the tensile loading is applied are restricted in the longitudinal direction.

Fig. 3.14 shows the development of transverse matrix cracks in a \([90_4/0_4/90_4]\) laminate. For better demonstration of results in Fig. 3.14, the cohesive elements are chosen to be removal when the cracks are fully formed (i.e. SDEG=1.0). Due to numerical rounding error (i.e. stress distribution is not uniform), the transverse cracks are generated in a staggered manner, similar to the damage pattern observed in the experiments (Noda et al., 2005). It should be noted that the local stress distribution at early step is almost uniform and symmetry as shown in Fig. 3.15. However, bending effect caused by the neighbouring transverse cracks will create compression in the intact 90\(^o\) ply group (Nairn & Hu, 1992). The superposition of compression and net tensile load leads to a local minimum in direct stress as shown in the upper 90\(^o\) ply group in Fig. 3.15. In addition, delaminations are generated from the transverse crack tips at around 0.47\% loading strain. Delaminations are not identical and the shape of the delaminations differs from position to position. The major delaminations are generated on both sides of a matrix crack, while two of them are generated on one side of a matrix crack. It seems that the formation of delamination is dependent on the interval between two existing cracks.

Unlike the \([0_n/90_m/0_n]\) laminates, the outside surfaces of the 90\(^o\) plies in the \([90_4/0_4/90_4]\) laminate is more undulated and the delaminations between 0\(^o\) and 90\(^o\) plies become more pronounced. This is because the unsymmetrical stress distribution within the 90\(^o\) plies since their outside surfaces are free of constraint. It is also found that the final failure of fibre breakage is captured in \([90_4/0_4/90_4]\) laminate as shown in Fig. 3.16 at a loading strain of 1.27\%. The initial failure in \([90_4/0_4/90_4]\) laminate is matrix cracking in the 90\(^o\) plies, the secondary failure is delamination at the interface between 0\(^o\) and 90\(^o\) plies, and final failure of the 0\(^o\) plies in fibre tension.
(a) 0.188% applied strain

(b) 0.234% applied strain

(c) 0.469% applied strain
Figure 3.14: Cracking formation process in [90ᵢ/0ᵢ/90ᵢ] laminate (2x deformation scaling)
Figure 3.15: Local stress distribution in [90/0/90] laminate: upper (0.106% applied strain) and lower (0.234% applied strain).

Figure 3.16: Final failure in [90/0/90] E-glass laminate (1.27% applied strain): fibre tensile damage (upper), matrix cracking and delaminations (lower).
Figure 3.17: Average stress versus strain for [90\textdegree]/0\textdegree/90\textdegree] and [90\textdegree]/0\textdegree/90\textdegree] E-glass laminate.

The average stress and strain relationship shown in Fig. 3.17 indicates the “knee” in the plot which is directly related to the onset of transverse cracks as discussed in previous section. However, the load drop (i.e. stiffness reduction) is different in [90\textdegree]/0\textdegree/90\textdegree] and [90\textdegree]/0\textdegree/90\textdegree] laminates while the damage patterns are similar as shown in Fig. 3.18 at a loading strain of 0.234\%. It can be explained that the thicker 0\textdegree plies carry more tensile loadings in the longitudinal direction than the damaged 90\textdegree plies.
Figure 3.18: Damage patterns in $[90_{\circ}/0_{\circ}]$ and $[90_{\circ}/0_{\circ}/90_{\circ}]$ E-glass laminate at 0.234% applied strain. (3x deformation scaling)

The effect of the thickness of 90° plies on the damage pattern is shown in Fig. 3.19. It is obvious that the thickness of 90° plies affects the damage pattern in $[90_{\circ}/0_{\circ}/90_{\circ}]$ laminates significantly. Delaminations are not apparent in the laminate with a $[90_{\circ}/0_{\circ}/90_{\circ}]$ layup at the applied strain of 0.706%, but visible delamination can be seen in the $[90_{\circ}/0_{\circ}/90_{\circ}]$ laminates where more 90° plies are used, despite that more matrix cracking appears in the $[90_{\circ}/0_{\circ}/90_{\circ}]$ laminate. It is also found that the average length of the delamination zones increases with the thickness increase of the 90° plies. The thicker the 90° ply group, the longer the matrix crack formed in this group and more stress intensity at the matrix crack tip. It is therefore easier for delamination to initiate and propagate at the tips of long matrix crack.
Figure 3.19: Damage patterns in [90/0/90] and [90/0/90] E-glass laminate. (2x deformation scaling)
3.4.3 \([0_n/90_m/0_n]\) and \([90_n/0_m/90_n]\) cross-ply laminates under pure bending

In this section, the damage process in cross-ply laminates subjected to pure bending loading was investigated. The schematic of the specimen can be seen in the Fig. 3.20. Instead of applying moments to the edges of the specimen, prescribed rotations were applied to the boundaries under displacement-controlled mode. The loading rate employed in the solution is the rotational displacement increments of \(\Delta \theta = 0.005\) rad at the time increments of \(\Delta t = 5 \times 10^{-5}\). Kinematic coupling constraints were used to tie each edge to one reference point and the rotation was then applied to the reference points. Additional symmetry constraints, such as equal but opposite horizontal displacements (parallel to the 0\(^\circ\) fibre tensile direction) at the centres of the left and right edges, were also introduced. This helped to eliminate the rigid body movement and facilitated Mode I crack propagation in 90\(^\circ\) plies.

![2-D Finite element models of the bend specimen.](image)

Figure 3.20: 2-D Finite element models of the bend specimen.
Cross-ply laminates with three layups, \([0_\pm 90_\pm 0_\pm]\), \([0_\pm 90_\pm 0_\pm]\) and \([0/90_s/0]\), were analysed to study damage process under pure bending. Fig. 3.21 shows the distributions of damages in the laminates before the final failure caused by fibre breakage in the 0\(^\circ\) plies. It can be seen that there is no formation of transverse cracks in the \([0_\pm 90_\pm 0_\pm]\) laminate which has the least number of 90\(^\circ\) layers among the three laminates, see Fig. 3.21(a). This is an expected prediction as the 90\(^\circ\) layers are located in the middle of the laminate and stress in these layers is at lower level due to the pure bending. When more 90\(^\circ\) layers are added to the laminate to form a layup of \([0_\pm 90_\pm 0_\pm]\), bending causes high stress in the 90\(^\circ\) layer most away from mid plane of the laminate and matrix cracking forms in the part under tension as shown in Fig. 3.21(b). If the number of the 0\(^\circ\) layers in the laminate is reduced, then the constraint preventing cracking opening in the 90\(^\circ\) layers imposed by the 0\(^\circ\) layers is weakened. This makes it easier for matrix cracking to form in the 90\(^\circ\) layers and matrix cracks can be observed in the \([0/90_s/0]\) laminate (Fig. 3.21(c)).

Although transverse cracks may initiate and propagate in the 90\(^\circ\) plies, they fail to extend through the entire thickness of 90\(^\circ\) plies. The crack length is less than half of the total thickness of 90\(^\circ\) plies. This is attributed to the fact that matrix cracking is caused by tension and half of the 90\(^\circ\) plies are under compression due to pure bending loading.

A decrease in the number of 0\(^\circ\) layers give rise to an increase in matrix cracking damage and a decrease in loading capacity. The position where fibre breakage failure first takes place depends on the numbers of the 0\(^\circ\) layers and 90\(^\circ\) layers. The damage of fibre tensile failure tends to occur at the matrix crack tips in the \([0/90_s/0]\) laminate while the \([0_\pm 90_\pm 0_\pm]\) and \([0_\pm 90_\pm 0_\pm]\) laminates show considerable fibre damage in 0\(^\circ\) plies on the tension side, since the in-plane stress is small near the neutral axis compared to locations away from the neutral axis.

For all the layups analysed, there is no delamination predicted in the laminate.
Figure 3.21: Damage in $[0_n/90_m/0_n]$ E-glass laminates under bending. (2x deformation scaling)
For the \([90_n/0_m/90_n]\) type laminates, damage is mainly due to matrix cracking in the 90° layers on the tension side. Several “knees” in moment-rotational displacement curves shown in Fig. 3.22 correspond to the instance when the cohesive elements fail as certain crack growth in the 90° layers. Fig. 3.23 shows similar damage pattern in \([90_2/0_2/90_2]\) and \([90_2/0_4/90_2]\) laminates. Therefore, the effect of 0° layer thickness is not clearly seen at this stage of simulation. However, the effect of the thickness of the 90° layers is obvious. Matrix cracking forms in the \([90_4/0_4/90_4]\) and \([90_4/0_4/90_4]\) laminates with more 90° layers, and more importantly, delaminations initiate and propagate at the tips of matrix cracks under a comparatively low loading level.

It can be seen from Fig. 3.23 that the less 90° layers in the laminate, the higher transverse matrix crack density can be found in the cracked plies on the tension side. The failure sequence begins with matrix cracking in the 90° plies, followed by the delamination at the 0/90 interface. The saturation matrix crack density is then reached in the 90° plies on the tension side. It is interesting to observe that matrix tensile failure occurs in the 0° plies as well (Fig. 3.24).

![Figure 3.22: Moment-rotational displacement response curves for \([90_n/0_m/90_n]\) E-glass laminates.](image)
(a) $[90_2/0]_s$ laminate, rotational displacement: 0.2 rad

(b) $[90_2/0]_s$ laminate, rotational displacement: 0.15 rad

(c) $[90_4/0]_s$ laminate, rotational displacement: 0.13 rad
(d) $[90_2/0_2]$-lamine, rotational displacement: 0.11 rad

Figure 3.23: Damage pattern in $[90_4/90_4]$ E-glass laminates at final stage of simulation. (3x deformation scaling)

(a) $[90_2/0_2]$-lamine, rotational displacement: 0.15 rad

(b) $[90_4/0_4]$-lamine, rotational displacement: 0.11 rad

Figure 3.24: Matrix tensile failure in $[90_2/0_4]$ and $[90_4/90_4]$ E-glass laminates at 0 degree ply group.
3.4.4 $[0_4/90_m]$ cross-ply laminates under combined tension-bending loading

Damage process in cross-ply laminates under combined tension-bending loading was also simulated. A loading ratio of curvature/strain equal to 1 (rad/mm$^2$) was applied to the laminates. Only kinematic coupling constraint is applied to the boundary. Two laminates with $[0_4/90_4/0_4]$ and $[0_4/90_8/0_4]$ layups were modelled in this section and the results are shown in Fig. 3.25.

As mentioned in section 3.4.3, when the specimen is under pure bending deformation, there is neither matrix cracking nor delamination occurring in $[0_4/90_4/0_4]$ and $[0_4/90_8/0_4]$ laminates at certain load step (Fig. 3.21(a)&(b)). However, visible damage modes can be seen when specimens are subjected to this combined tension-bending loading. It can be observed that matrix cracking is the major damage in $90^\circ$ plies, while the transverse normal stress is compressive close to the $0/90$ interface for the pure bending case in Fig. 3.21. Even if the upper plies experience negative strains due to the flexural deformation, they also experience positive strain due to the membrane deformation. The overall strain in the upper plies is still positive. Therefore, this combination is more dominated by the extension for the $90^\circ$ plies in order to induce transverse cracks so as the crack propagations.

The delamination is unlikely to form if the curvature/strain ratio is greater than 1 since the tensile loading is assumed to be the significant factor for delaminations to take place. Delaminations can only be observed in this case at the final stage of simulation accompanied by the final failure of laminate as shown in Fig. 3.26. Final failure is considered to be fibre breakage in the $0^\circ$ plies on the tension side.
Figure 3.25: Damage in [0/90/0] and [0/90/0] E-glass laminates under combined loading.

(2x deformation scaling)

Figure 3.26: Final failure in [0/90/0] E-glass laminate under combined loading (rotational
displacement: 0.032 rad, 0.8% applied strain): fibre tensile damage (upper), matrix cracking and
delaminations (lower).
3.5 Conclusion

The cohesive element method has been employed to simulate initiation and propagation of transverse cracking as well as interfacial delamination in cross-ply laminates under uni-axial and bending loading. Multiple damages in \([0_n/90_m/0_n]\) and \([90_n/0_n/90_m]\) cross-ply laminates were investigated by combining cohesive element method and progressive damage model in commercial FEA software ABAQUS. The following conclusions can be drawn from the simulation results:

(a) The first damage mode is usually the formation and accumulation of transverse matrix cracking until a saturation level, then delamination may initiate at the tips of transverse cracks and propagate along the interface. The final failure is fibre breakage in \(0^\circ\) plies which causes laminates to lose their loading capacities completely.

(b) The initiation and development of transverse matrix crack is layup configuration dependent. The thickness of the \(90^\circ\) plies affects the formation of transverse matrix cracking significantly. Matrix cracking propagates across the entire thickness of the thick layers after its initiation. However, matrix cracking propagates quite stably in thin layers and the layers may be only partly cracked (i.e. crack may not reach the interface) before the final failure of the laminate. If bending deformation is significant, the formation of matrix cracking also depends on the position of the layers in the cross section.

(c) The initiation of delamination at matrix crack tip is also layup configuration dependent. Matrix cracks in the layers at the outer surface of the laminate induce delaminations at their tips. It is hard for delamination to start at the tips of matrix cracks in the inner layers. The thicker the layers, the longer the matrix crack, and the easier to trigger delamination.
Chapter 4

An Interface Element/Cohesive Model Taking Account Of Compressive Through-Thickness Stress and Friction

4.1 Introduction

Interface element and cohesive zone models introduce an interface into the solids where potential crack may appear. The mechanical behaviour of the interface is defined by the relationship between tractions and separations on the interface. The initiation of crack takes place when the tractions satisfy a certain failure criterion. Once the energy dissipation rates (i.e., work done by the tractions over the corresponding separations) meet an energy-based criterion, the tractions are reduced to zero and crack propagates. These models combine the stress-based failure and fracture-mechanics-based failure criteria into one single failure model and can be easily implemented into finite element code to simulate crack initiation and propagation in structures.

The failure criteria employed in interface element and cohesive zone models are interactive mixed mode. However, these models ignore the effect of the through-thickness stress when it is compressive. Previous experiments have shown that through-thickness compression can increase the resistance to mode II failure. Cui et al. (1994) performed tensile tests on cut-ply and dropped-ply specimens to investigate the
delamination initiation and growth in the specimens. The results revealed that through-thickness compression enhances the mode II fracture resistance. The experimental study conducted by DeTeresa et al. (2004) also demonstrated that significant gains can be made in improving inter-laminar shear strength by applying through thickness compression.

Various attempts have been made to consider the effect of through thickness compression stress. Cui et al. (1994) employed variable fracture toughness based on compressive stress. Specifically, the average compressive through-thickness stress over the compressive zone at the delamination tip was first determined and the mode II fracture energy was expressed as a linear function of the average through-thickness stress. Nevertheless, this approach is not practical for progressive delamination/crack growth analysis, since the through-thickness compressive stress is a solution variable. Several researchers (Yen & Caiazzo, 2000; Xiao and Gillespie, 2007) took a different approach by linking the inter-laminar shear strength to an internal friction factor. Based on a friction mechanism, the internal material friction was used to explain the increased shear strength under compression.

Friction in the cracked area is a big factor for the increase of delamination stress or fracture toughness. The effect of frictional force on the accuracy of the measured mode II delamination resistance has been studied numerically or analytically in end-notched flexure (ENF) and 4-point bending test (Fan et al., 2007). It was found that the frictional force between the surfaces of the artificial starting defect could slow down the crack propagation significantly, resulting in higher delamination resistance. Li and Wisnom (1994) considered the friction effect on the delamination stress in cut-ply and dropped-ply composite specimens where higher compressive through-thickness stress exists. By modelling friction on the delaminated interface with a coefficient of 1.0, they found that the failure stress increased by 12% for cut-ply specimen and 27% for dropped-ply case.

Interface element or cohesive zone models have also been modified to include the effect of friction. Chaboche et al. (1997) modelled the contact/friction condition in the interface elements after complete interfacial debonding. Alfano and Sacco (2006) combined interface damage and friction in a cohesive zone model; this model has a representative elementary area (REA) with an undamaged part and a damaged part, and friction occurs only on the damaged part. Guiamatsia and Nguyen (2014) developed a
similar model where Helmholtz energy potential, plastic and frictional energy dissipations are all included.

Li et al. (2008) proposed an interfacial failure model that considers the effect of compressive through-thickness stress in the interface element. One parameter was introduced to relate the through-thickness compression to the increase in inter-laminar strength and mode II fracture energy. This modification results in varying inter-laminar shear strength and mode II fracture energy which depends on the loading of the interface element. However, it is possible that the energy consumed to crack an interface element may not increase monotonically as local unloading may occur throughout the entire damage process from damage initiation to propagation. This is against the fact that energy dissipated cannot be restored into the structure.

In this chapter, the damage model originally developed by Zou et al. (2003) was modified and a two-dimensional interface element model was proposed to include interfacial friction. The main idea was to include the friction in the constitutive relationship of the interface element. The inter-laminar shear stress in this model has two components: the cohesive stress and frictional stress. Damage initiation and damage evolution are controlled by the cohesive stress, whereas the interfacial shear strength and mode II fracture energy are constant material properties. As a result, the friction affects the structure response and subsequently the reduction of cohesive stress as well as the increase of the resistance to interfacial damage.

### 4.2 2-D Interface Model Considering Friction

For the conventional interface model or cohesive zone model, the relationship between the tractions $\tau_i$ and separations $\delta_i$ of the interface can be expressed as follows (Zou et al., 2003)

$$\tau_i = (1 - \omega)k_i\delta_i \quad (i=1,2) \quad (4-1)$$

where $k_i$ are constraints or penalty stiffness of the interface. $\omega$ is a damage parameter of the interface that describes the extent of damage. $\omega=0$ represents the undamaged state, and $\omega=1$ indicates a fully cracked state. The subscript $i$ is defined in a canonical way.
such that \( i = 1 \) indicates the normal through-thickness direction and \( i = 2 \) indicates the shear direction in the interface, in accordance with the conventional definitions of the modes of fracture.

The interfacial damage has no effect on the compressive stiffness of the interface; that is, when the interface is under compression \((\delta_1 < 0)\), then

\[
\tilde{\tau}_i = k_i \delta_i \tag{4-2}
\]

To consider the possible contact and friction, a new set of tractions \( \tau_i \) on the surfaces of the interface is introduced and the above interface model is modified as follows:

\[
\tau_1 = \tilde{\tau}_1 \\
\tau_2 = \tilde{\tau}_2 + m \mu \tau_1 \tag{4-3}
\]

where

\[
m = \begin{cases} 
-1 & \text{if } \delta_1 < 0 \text{ and } d\delta_2 > 0 \\
1 & \text{if } \delta_1 < 0 \text{ and } d\delta_2 < 0 \\
0 & \text{if } \delta_1 > 0 \text{ or } d\delta_2 = 0 
\end{cases} \tag{4-4}
\]

and \( \mu \) is the coefficient of friction. For the sake of simplicity, the same value \( m \) and \( \mu \) are used for internal material friction, friction in damaging and fully damaged area. \( d\delta_2 \) is the increment of the sliding displacement and represents the sliding trend of the interface.

The shear stress acting on the interface comes from the two sources: the cohesive shear stress \( \tilde{\tau}_2 \) due to interface deformation; and frictional shear stress \( \mu \tau_1 \) caused by friction. This modification made to the interfacial constitutive law allows the consideration of possible contact and friction between the cracked surfaces after the complete failure of the interface.

The friction contributes to the shear stress on the interface by increasing its resistance to sliding and affects the magnitude of the cohesive shear stress. However, it does not contribute to the failure of the interface directly. This is truly the case when the interface is failed, but contact still occurs between the interface and friction exists. Therefore, a failure function for interfacial damage initiation is proposed as follows:
\[ f_s = \left( \frac{\tau_1}{\tau_{1c}} \right)^2 + \left( \frac{\tau_2}{\tau_{2c}} \right)^2 \text{ when } \tau_1 > 0 \]  
\[ (4-5a) \]
\[ f_s = \left( \frac{\tau_2}{\tau_{2c}} \right)^2 \text{ when } \tau_1 < 0 \]  
\[ (4-5b) \]

where \( \tau_{1c} \) and \( \tau_{2c} \) are the tensile and shear strength of the interface, respectively. By convention, only the cohesive stresses are included in the above quadratic failure function \( f_s \) which is frequently employed in the literature.

The energy consumed by the friction should be excluded from the fracture energy; therefore, the damage-induced energy dissipation rate is defined as

\[ G_i = \int_0^{\delta_i} \tau_i d\delta_i \]  
\[ (4-6) \]

A fracture-mechanics-based failure criterion for interfacial damage propagation can be expressed as

\[ f_g = \left( \frac{G_i}{G_k} \right)^\alpha + \left( \frac{G_{II}}{G_{II_k}} \right)^\beta \]  
\[ (4-7) \]

where \( G_i \) \((i=1, 2)\) correspond to fracture modes I, II, the arabic numbers replacing their roman counterparts without confusion. \( G_{ic} \) \((i=1, II)\) are the individual fracture energies.

The most common settings are \( \alpha = \beta = 1 \) and \( \alpha = \beta = 2 \), which correspond to linear and quadratic fracture failure criteria, respectively.

The damage initiation and evolution are unified by introducing a combined form of the conventional stress-based failure criterion and fracture-mechanics-based failure criterion.

A damage surface in the \( \tau_i \) and \( G_i \) space is constructed as follows:

\[ F = f_s + f_g - 1 = 0 \]  
\[ (4-8) \]

Before damage initiation, the interface is intact and has a large initial stiffness. Therefore, the interface separation \( \delta_0 \), hence, \( G_i \) and \( f_s \) are all very small and can be neglected. The damage surface is indeed a conventional stress-based criterion for damage initiation. As the damage initiates and develops, the stiffness of the interface degrades and \( f_g \) increases rapidly. This results in a shrinking damage surface in the traction space and
consequently lower tractions are required for further damage as expected. A softening interfacial constitutive relationship is therefore established. When the fracture mechanics failure function reached unity, the cohesive tractions are reduced to zero and free crack surfaces form unless contact between the cracked surfaces re-establishes.

An infinitesimal change of the damage state at the interface as a result of an infinitesimal change of tractions requires the satisfaction of the following equation so that the tractions, separations and damage parameter remain on the damage surface Eq. (4-8) i.e.

\[ dF = df_x + df_z = \sum_{i=1}^{2} \left( \frac{\partial f_x}{\partial \bar{\tau}_i} d\bar{\tau}_i + \frac{\partial f_z}{\partial G_i} dG_i \right) = 0 \quad (4-9) \]

The incremental relationships between cohesive tractions, separations and energy dissipations can be obtained from Eqs. (4-1) and (4-6) as

\[ d\bar{\tau}_i = \sum_{j=1}^{2} \frac{\partial \bar{\tau}_i}{\partial \delta_j} d\delta_j + \frac{\partial \bar{\tau}_i}{\partial \omega} d\omega \quad (4-10) \]

\[ dG_i = \bar{\tau}_i d\delta_i \quad (4-11) \]

Substituting Eqs. (4-10) and (4-11) into (4-9), the incremental damage evolution can be described in terms of incremental separations as

\[ d\omega = \left( \sum_{i=1}^{2} \frac{\partial f_x}{\partial G_i} d\delta_i \right) \left( \sum_{i=1}^{2} \frac{\partial f_z}{\partial G_i} d\delta_i \right) \quad (4-12) \]

From Eq. (4-3), we have the incremental interfacial constitutive law

\[ d\tau_i = \sum_{j=1}^{2} \frac{\partial \tau_i}{\partial \delta_j} d\delta_j + \frac{\partial \tau_i}{\partial \omega} d\omega \quad (4-13) \]

The incremental constitutive relationship between the interfacial tractions and separations for the interfaces, which has taken account of the growth of damage, can then be obtained by substituting Eq. (4-12) into Eq. (4-13) to give

\[ d\tau_i = \sum_{j=1}^{2} \frac{\partial \tau_i}{\partial \delta_j} d\delta_j + \frac{\partial \tau_i}{\partial \omega} \left( \sum_{j=1}^{2} \frac{\partial \tau_i}{\partial G_j} d\delta_j \right) \quad (4-14) \]
When the interfacial stress state is either before damage initiation or under unloading conditions after damage has initiated, i.e. \( F(\tilde{\tau}_i, G_i) < 0 \), there will be no damage growth. That is,
\[
d\omega = 0
\]
and the incremental constitutive relationship becomes
\[
d\tau_i = \sum_{j=1}^{2} \frac{\partial \tau_i}{\partial \delta_j} d\delta_j
\]

The interface model developed above has been implemented into the commercial finite element code ABAQUS via a user subroutine, i.e. User Material for the Cohesive Element in ABAQUS.

### 4.3 Delamination in Specimens with Cut and Dropped Plies

The tension tests conducted by Cui et al. (1994) on cut- and dropped-ply specimens have been simulated here using the model developed in the previous section to predict the delamination failure. The specimens are illustrated in Fig. 4.1. There are two cases of cut-ply specimens (without a gap and with a 2 mm gap) and two cases of dropped-ply specimens with different taper angles (shallow angle of 5.7° and sharp angle of 7.6°). The cut-ply specimens are made of two cut and eight continuous plies, and the dropped-ply specimens are made of two dropped and eight continuous plies. Each ply is 0.125 mm thick. Details of the specimens and experimental set-up and results can be found in (Cui et al., 1994).

![Figure 4.1](image-url)

Figure 4.1: Schematic representation of (a) cut-ply and (b) dropped-ply specimens (not to scale)
4.3.1 Finite element model and material properties

A two-dimensional analysis has been performed to simulate the effect of through-thickness normal stress on Mode II failure. For cut-ply specimen without gap and dropped-ply specimens, there are often matrices inside the gap between the ends of the terminated plies. Since these matrices fractured earlier than the delamination initiation, the matrix regions were ignored in the numerical models.

Due to symmetry, only a quarter of the specimen was modelled. The cut/dropped and continuous plies were made of four-node plane stress elements with reduced integration (CPS4R element type in ABAQUS). The interface elements were placed at the interface between the cut/dropped plies and the continuous plies where the possible delamination would occur. Four-node two-dimensional cohesive elements COH2D4 were used for the interfaces. After mesh sensitivity study, an element size of 0.01 mm long and around 0.01 mm wide was used to mesh both cut/dropped and continuous plies. The interface element was 0.01 mm long. The tensile load was applied in a displacement controlled mode at the end of the model while the symmetrical boundary conditions were applied at the symmetry planes.

The specimens were made of unidirectional glass fibre-epoxy pre-preg (E-glass/913). The ply material properties are given in Table 4.1 and the properties for the interface element are shown in Table 4.2.

<table>
<thead>
<tr>
<th>Table 4.1: Material properties for E-glass/913 ply (Li et al., 2008)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus in the fibre direction $E_1$ (GPa)</td>
</tr>
<tr>
<td>Young’s modulus in the transverse direction $E_2$ (GPa)</td>
</tr>
<tr>
<td>Shear modulus $G_{12}$ (GPa)</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu_{12}$</td>
</tr>
</tbody>
</table>
Table 4.2: Interface material properties for E-glass/913

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile yield stress of adhesive material $\tau_c$ (MPa)</td>
<td>94 (94)</td>
</tr>
<tr>
<td>Shear yield stress of adhesive material $\tau_c$ (MPa)</td>
<td>75 (75)</td>
</tr>
<tr>
<td>Fracture energy $G_{IIc}$ (N/mm)</td>
<td>1.05 (0.87)</td>
</tr>
<tr>
<td>Coefficient of friction $\mu$</td>
<td>0.53 (0.74)</td>
</tr>
<tr>
<td>Interface/Penalty stiffness $k_1, k_2$ (N/mm$^3$)</td>
<td>$10^5$</td>
</tr>
</tbody>
</table>

The reference values in the bracket in Table 4.2 are taken from Li et al. (2008). In the present analysis, modified values for $G_{IIc}$ and $\mu$ were used which were obtained from other relevant sources. Wisnom and Jones (1996) performed experiments to measure the coefficient of friction (COF) in mode II delamination with through-thickness compression for glass-fibre/epoxy composites and found that COF is 0.53. This value is consistent with the existing literature where the COF is assumed to be 0.5. $G_{IIc}$ was extrapolated from the experimental results conducted by Cui et al. (1994) on the cut-ply and dropped-ply specimens (Table 4.3).

Table 4.3: Measured Mode II fracture energy for E-glass/913 (Cui et al., 1994)

<table>
<thead>
<tr>
<th>Specimen type</th>
<th>Cut ply without gap</th>
<th>Cut ply with gap</th>
<th>Dropped ply shallow</th>
<th>Dropped ply sharp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Through-thickness stress (MPa)</td>
<td>36.4</td>
<td>62.2</td>
<td>114.7</td>
<td>127.7</td>
</tr>
<tr>
<td>Fracture energy (J/m$^2$)</td>
<td>1.08</td>
<td>1.21</td>
<td>1.69</td>
<td>1.88</td>
</tr>
</tbody>
</table>

* The average through-thickness stress in Table 4.3 was derived over the distance where the stress is compressive on the interface around the cut or the drop (Cui et al., 1994).
The effect of the compressive through-thickness stress on the fracture energy is obvious as shown in Fig. 4.2. The least squares method was used to predict the trend of the best fit for experimental data. The quadratic curve fits the experimental data perfectly for the four specimens. As the through-thickness stress is compressive, the fracture is in pure mode II. Once the through-thickness stress tends to infinitesimal or zero, its effect on mode II fracture energy diminishes and the quadratic trend curve gives a value of $G_{IIc} = 1.05$ N/mm which is the conventional mode II fracture energy and a constant material property for all the four specimens with same material.

Cui et al. (1994) also presented results for one cut-ply specimen under compression which produced tensile through thickness stress on the interface around the cut. They included this data when the linear relationship between through thickness stress and fracture energy was derived. However, the delamination failure under this condition is a mixed-mode fracture instead of pure mode II. Therefore, it is not wise to include this single data for interpolating $G_{IIc}$ unless there is sufficient data to accurately show the trend of the effect of tensile through thickness stress on the fracture energy under mixed mode fracture, especially in the mode transition zone from pure mode II to mixed mode fracture.

The interface stiffness acts as a penalty parameter which should be reasonably large to
simulate a good bonding between plies before occurrence of any interfacial damage while avoid ill-conditioned numerical problems. A sensitivity study on the selection of the interface stiffness has been carried out and a wide range of magnitudes, from $10^4$ to $10^6$, has been attempted in the FE analysis. No noticeable differences have been found among the magnitudes tested.

4.3.2 Results

The numerical simulation shows that damage initiates and develops at the tip of the socket when the tensile load is increased to a certain level. This is, followed by a stable propagation of delamination for a few millimetres during which the load is continuously increasing. The delamination then propagates to the specimen end unstably, as indicated by a drop (or plateau) on the nominal stress-strain curve (Fig. 4.3). This predicted phenomenon is consistent with the experimental observation described by Cui et al. (1994).

![Figure 4.3: Predicted loading curves for cut ply and dropped ply specimens](image)

The nominal stress in Fig. 4.3 is defined as the ratio between the applied tensile load
and the net sectional area at the middle part of the specimen. The nominal strain is defined as the ratio between the applied tensile displacement and the length of the quarter specimen. According to Cui et al. (1994), the initial peak on the nominal stress–strain curve is termed as the delamination propagation stress. Once delamination starts to propagate in the specimen, the cut/dropped plies gradually lose their load-carrying capacity, resulting in the plateau or slow load drop over a long interval shown on the nominal stress–strain curve. After delamination reaches the end of the specimen, the cut/dropped plies lose their load-carrying capacity completely. However, the continuous plies can continue to take on further load, as indicated by the upwards nominal stress–strain curve in the final part with a reduced rate as expected.

Table 4.4 presents the delamination stress for the four types of specimen from experiments (Cui et al., 1994) and the present prediction. When the interfacial material properties $G_{IC} = 1.05$ N/mm and $\mu = 0.53$ are used, the predictions are in very good agreement with the experimental results. As discussed earlier, these two properties were obtained or extracted directly from the experimental data (Cui et al., 1994). No results fittings were attempted to achieve good prediction for the delamination stress.

Table 4.4: Comparison between measured and predicted delamination stress (MPa)

<table>
<thead>
<tr>
<th></th>
<th>Cut ply no gap</th>
<th>Cut ply with gap</th>
<th>Dropped ply shallow</th>
<th>Dropped ply sharp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>966</td>
<td>1021</td>
<td>1208</td>
<td>1274</td>
</tr>
<tr>
<td>Present model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{IC} = 1.05$ N/mm, $\mu = 0.53$</td>
<td>994 (2.9%)</td>
<td>1006 (−1.5%)</td>
<td>1169 (−3.2%)</td>
<td>1210 (−5.0%)</td>
</tr>
<tr>
<td>Present model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{IC} = 1.05$ N/mm, $\mu = 0$</td>
<td>935 (−3.2%)</td>
<td>939 (−8.0%)</td>
<td>934 (−22.7%)</td>
<td>934 (−26.7%)</td>
</tr>
<tr>
<td>ABAQUS Cohesive</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{IC} = 1.05$ N/mm, $\mu = 0$</td>
<td>938 (−2.9%)</td>
<td>935 (−8.4%)</td>
<td>933 (−22.8%)</td>
<td>934 (−26.7%)</td>
</tr>
<tr>
<td>Present model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{IC} = 0.87$ N/mm, $\mu = 0.74$</td>
<td>940 (−2.7%)</td>
<td>955 (−6.5%)</td>
<td>1169 (−3.2%)</td>
<td>1222 (−4.1%)</td>
</tr>
</tbody>
</table>

* Values in bracket are relative errors to the experimental results.
Numerical simulations have also been performed, assuming no friction in order to evaluate the effect of the friction on delamination stress. Both the present model (setting $\mu=0$) and the cohesive model in ABAQUS which does not have the capacity to consider internal friction employed. The predictions shown in Table 4.4 prove that the present model yields almost same predictions as those from the cohesive model in ABAQUS. Without considering friction, the predicted delamination stress is significantly lower than the experimental results for the dropped ply specimens where higher compressive through thickness stress exists at the drop end. This suggests that friction does enhance the mode II shear strength and fracture resistance.

Simulation has also been performed using the same interface material properties as employed by Li et al. (2008): $G_{IIc} = 0.87$ N/mm and $\mu=0.74$. Good results were achieved once again. However, these interface material properties were determined by Li et al. through parametric study for matching predictions to experimental results.

A typical traction-separation relationship of the interface in the area where the compressive through thickness stress exists is shown in Fig. 4.4. The cohesive shear stress-separation relationship (friction stress excluded) shows the typical bilinear behaviour adopted in most cohesive models and interface element models.

Figure 4.4: Typical history of tractions/stresses on the interface in the sharp dropped ply specimen
As expected, the relationship between friction shear stress and separation is nonlinear. Therefore, the energy dissipation by friction is path-dependent and cannot be predicted upfront. Friction stress still exists after the interface is fully damaged, while crack forms as long as there is trend of further sliding between the surfaces of the cracked interface.

Stress distributions along the interface near the drop in the sharp dropped ply specimen are shown in Fig. 4.5 for two cases: (1) when delamination just starts on the interface at the drop end; and (2) immediately before the unstable delamination propagation starts. It can be seen that before the unstable delamination propagation starts, there is a zone around 1.6mm long on the interface where no cohesive shear stress appears. This indicates that this zone has been fully delaminated. In other words, delamination has a stable propagation for 1.6mm.

Fig. 4.5 also shows that the compressive zone expands as delamination propagates. However, high compressive through thickness stress still occurs near the ply drop end, so does friction shear stress consequently, although the magnitude is reduced gradually as delamination propagates. The zone with high compressive through thickness stress
is around 1mm long which is shorter than 1.6mm, the length for stable delamination propagation. This indicates that the friction between surfaces of the delaminated interface may play an important role in enhancing the interfacial shear strength and fracture resistance.

4.3.3 Effect of friction between the delaminated surfaces

To confirm the above elaboration on effect of the friction between the surfaces of delaminated interface, the finite element model of the sharp dropped ply specimen was modified. Initial delamination was introduced into the interface at the drop end. Contact between the surfaces of the initial delamination was modelled by the surface-to-surface contact available in ABAQUS and the coefficient of friction was set to $\mu_1=0.53$. Three initial delamination lengths were used: $d=0.77\text{mm}$ and $d=1.2\text{mm}$ which represent the length of the compression zone when damage initiates ($\omega\neq 0$) and delamination appears ($\omega=1$) at the ply drop end, respectively; and $d=1.6\text{mm}$, which denotes the maximum length for stable delamination propagation in the original specimen. Two values were employed for the coefficient of friction for the interface element model to ignore the friction on the interface beyond the initial delamination ($\mu_2=0.0$) or to consider the friction ($\mu_2=0.53$). The predicted delamination stresses under these conditions are presented in Table 4.5.

<table>
<thead>
<tr>
<th>$d$ (mm)</th>
<th>Prediction ($\mu_1=0.53, \mu_2=0.0$)</th>
<th>Prediction ($\mu_1=0.53, \mu_2=0.53$)</th>
<th>Experimental result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d=0$</td>
<td>934</td>
<td>1210</td>
<td>1274</td>
</tr>
<tr>
<td>$d=0.77$</td>
<td>(−26.7%)</td>
<td>(−5.0%)</td>
<td></td>
</tr>
<tr>
<td>$d=1.2$</td>
<td>(−10.4%)</td>
<td>(−4.7%)</td>
<td></td>
</tr>
<tr>
<td>$d=1.6$</td>
<td>(−9.0%)</td>
<td>(−4.6%)</td>
<td></td>
</tr>
</tbody>
</table>

* Values in bracket are relative errors to the experimental results
The results in Table 4.5 show that delamination stress increases as the length of the initial delamination becomes longer. When only the friction in the initial delamination is considered ($\mu_2=0.0$), the predicted delamination stress is in good agreement with the experimental value. This proves that friction on the delaminated interface contributes significantly to the enhancement of interfacial strength and fracture resistance, which is not the case where there is no friction ($d=0$). The simulation results from the case $\mu_2=0.53$ demonstrate that the friction on the undamaged and damaging interface further increases the fracture resistance and makes the prediction more accurate. The present interface element model can simulate both the contact and the friction before and after the interface is delaminated. The treatment to employ same coefficient of friction on undamaged, damaging and fully damaged interface yields significant difference.

Initial delaminations longer than 1.6mm were also analysed. The results show that longer delamination would not further increase the delamination stress; instead, it decreases the delamination stress. Because the zone with high compressive through thickness stress and therefore large friction is limited to 1mm, the delamination tip of a much longer initial delamination is situated far away from the compression zone and the effect of friction on the delamination tip is therefore very limited.
4.4 Summary

The influence of through-thickness compressive stress on the Mode II fracture resistance has been investigated numerically in this chapter. The study reveals that friction, in particular the friction between the fractured surfaces around the crack tip, is responsible for the increased resistance to Mode II fracture. To simulate progressive interfacial damage propagation accurately, interface element model or cohesive zone model must take into account the friction at the cracking and cracked surfaces. However, models available in the literature do not have this capacity. Simply increasing the Mode II fracture energy cannot consider the effect of friction between the damaged surfaces after they are formed.

The newly developed two-dimensional interface element model incorporates the friction into its interfacial constitutive law. Friction does not change the interfacial shear strength and Mode II fracture energy, both of which remain constant material properties. Instead, the deformation of the interface and the magnitude of the cohesive shear stress are affected due to the existence of friction on the interface, resulting in increased resistance to Mode II fracture when there is compressive through thickness stress on the interface. For progressive damage modelling, new damaged/cracked surfaces are formed continuously. The present model can simulate the friction on the interface whether it is damaging or newly damaged.

This model needs further modification before it can be widely applied to three-dimensional problems. Different coefficient of friction on the undamaged, damaging and fully damaged interface should be introduced to make the model rigorous, in particular in the damaging area.
Chapter 5

Continuum Damage Models for Predicting Failure in Composite Laminates

5.1 Introduction

Modelling schemes based on the use of two continuum damage models have been adopted to represent distributed intra-laminar damage such as matrix cracking and fibre fracture, and inter-laminar damage developing in laminates, i.e. delamination. The continuum damage mechanics (CDM) has been employed by many researchers in literature and its application to impact damage modelling has shown to be efficient. The advantage of the CDM approach is its adaptability to stress-based or/and strain-based failure criteria for predicting damage initiation and easy combination with fracture mechanics approach for damage propagation.

The objective of the present work was to evaluate the capability of the approaches based on the use of two continuum damage models to correctly capture not just planar extent but also the through-thickness distribution of damage induced by impact in laminated composite structures.
5.2 Description of Damage Models

5.2.1 Inter-laminar Damage Model

Delamination is acknowledged as one of the typical destructive interfacial damage modes in laminated composite. The effects of delamination have been considered in the context of continuum damage mechanics. As a result, an inter-laminar damage model which is based on the work conducted by Zou et al. (2003) was obtained and further improved and extended to include 3D effects.

The model introduces an interface into the structure, in the same way as cohesive zone model and interface element model in the literature. The interfacial tractions $\tau_i$, i.e. interlaminar direct and shear stresses, in the undelaminated region are expressed as

$$\tau_i = k_i^0 \delta_i \quad (i = 1, 2, 3) \quad (5-1)$$

where $\delta_i$ are relative displacement components across the interface and $k_i^0$ are constraints or stiffness of the interface. A local co-ordinate system is defined such that subscript 1 indicates the through-thickness direction, and 2 and 3 are the other two orthogonal directions in the interface plane where potential delamination is monitored, corresponding to the conventional definitions of the fracture modes. It is worth to mention that the interface stiffness is often considered as penalty parameter, which should be large enough to provide a reasonable connection between two neighbouring layers before delamination initiation but small enough to prevent ill-conditioning problems in the finite element analysis (Zou et al., 2002). Therefore, a reasonable choice for the interface stiffness is as follows:

$$k_i^0 = k S_{ic} \quad k = 10^7 \sim 10^8 \text{ mm}^{-1} \quad (5-2)$$

where $S_{ic} \ (i = 1, 2, 3)$ are the interfacial tensile and shear strengths respectively.
Since damage may initiate and develop on the interface, a scalar damage parameter $\omega$ ($\omega \in [0,1]$) is employed to describe the extent of damage, $\omega = 0$ indicating an undamaged state and $\omega = 1$ representing a fully delaminated state, as illustrated in Fig. 5.1. The effective stiffness of the interface decreases gradually as the damage grows. Thus, the interfacial tractions in Eq. (5-1) are expressed as functions of the damage parameter, i.e.

$$
\tau_1 = (1-\omega)k_1^0 \langle \delta_1 \rangle + k_1^0 \langle -\delta_1 \rangle \\
\tau_2 = (1-\omega)k_2^0 \delta_2 \\
\tau_3 = (1-\omega)k_3^0 \delta_3
$$

(5-3)

where the symbol $\langle \rangle$ in the equations is Macauley bracket operator. When the normal relative displacement $\delta_1 < 0$, $\tau_1$ is the contact stress resulting from the interpenetration. The stiffness of the interface in normal direction will regain its initial value and it will not degrade.

For an intact interface, delamination initiates when the inter-laminar stress or a combination of them reach a limit. The damage initiation can be predicted by the conventional stress-based failure criterion, i.e.

$$
f_s(\tau_i) - 1 = 0
$$

(5-4)
where \( f_s \) is a stress-based failure criterion frequently adopted in the literature:

\[
f_s = \left( \frac{\max (\tau, 0)}{S_{1c}} \right)^2 + \left( \frac{\tau_2}{S_{2c}} \right)^2 + \left( \frac{\tau_3}{S_{3c}} \right)^2
\]  

(5-5)

For an existing delamination, a fracture mechanics approach has been proved successful in dealing with its propagation. The failure criterion for delamination propagation in terms of energy release rates can be expressed as

\[
f_g (G_i) - 1 = 0
\]  

(5-6)

where \( f_g \) is available in the literature for various considerations under circumstances, e.g.

\[
f_g = \frac{G_i}{G_{ic}} + \frac{G_{II}}{G_{Iic}} + \frac{G_{III}}{G_{IIIC}}
\]  

(5-7)

or

\[
f_g = \left( \frac{G_{I}}{G_{IC}} \right)^2 + \left( \frac{G_{II}}{G_{IIC}} \right)^2 + \left( \frac{G_{III}}{G_{IIIC}} \right)^2
\]  

(5-8)

where \( G_{ic} (i = I, II, III) \) are the critical values of the individual energy release rates \( G_i (i = I, II, III) \) of the material, evaluated as the integration of \( \delta_i \) from

\[
G_i = \int_0^{s_i} \tau_i d\delta_i
\]  

(5-9)

The damage process in the context of continuum damage mechanics involves two aspects, the initiation and the growth of the damage. The concept of a damage surface is introduced for establishing damage evolution laws in continuum damage mechanics. Like the concept of yield surface in classical plasticity theory, a damage surface is constructed in the space of traction \( \tau \) and energy release rates \( G_i \) as follows

\[
F (\tau, G_i) = f_s (\tau) - \left[ 1 - f_g (G_i) \right] = 0
\]  

(5-10)

where \( f_s (\tau) \) and \( f_g (G_i) \) take the same forms as in Eq. (5-5) and (5-7) or (5-8) associated with the stress-based and fracture-mechanics-based failure criteria, respectively.

The damage surface combines the stress-based and fracture-mechanic-based failure criteria. Since the initial stiffness of the interface is chosen to be very large to simulate a
perfect bond between the constituent layers, relative displacements $\delta_i$ are negligible in the early stage of loading, hence, $G_i$ and $f_g$ are all very small and can be neglected. Damage initiates at an intact interface only if the stress-based criterion $f_s(\tau_i) = 1$ is satisfied. As the damage initiates and develops, stiffness of the interface degrades, resulting in quick increment in $\delta_i$, $G_i$ and $f_g$. Formulation of completely damaged surfaces requires that the fracture-based criterion $f_g = 1$ is also satisfied, leading to zero tractions at the interface.

When the damage surface is exceeded, i.e. $F(\tau_i, G_i) > 0$, then damage initiates and develops. An infinitesimal change of the damage state at the interface as a result of an infinitesimal change of tractions requires the satisfaction of the following equation so that the tractions, relative displacements and damage parameter remain on the damage surface, Eq. (5-10), i.e.

$$\begin{align*}
dF &= \sum_{i=1}^{3} \left( \frac{\partial F}{\partial \tau_i} d\tau_i + \frac{\partial F}{\partial G_i} dG_i \right) = 0 \quad (5-11)
\end{align*}$$

When $\delta_i > 0$, the incremental interfacial constitutive law and damage evolution law can be derived in terms of incremental relative displacements by making use of Eq. (5-3), (5-9) and (5-11), i.e.

$$\begin{align*}
d\tau_i &= (1 - \omega)k_i^0 d\delta_i - k_i^0 \delta_i d\omega \\
d\omega &= \sum_{i=1}^{3} C_i d\delta_i \\
\end{align*} \quad (5-12)$$

and

$$\begin{align*}
C_i &= \left[ \frac{\partial F}{\partial \tau_i} (1 - \omega)k_i^0 + \frac{\partial F}{\partial G_i} \tau_i \right] / \sum_{j=1}^{3} \frac{\partial F}{\partial \tau_j} k_j^0 \delta_j \\
\end{align*} \quad (5-13)$$

where

$$\begin{align*}
K &= \begin{bmatrix}
(1 - \omega)k_1^0 - k_1^0 \delta_1 C_1 & -k_1^0 \delta_1 C_2 & -k_1^0 \delta_1 C_3 \\
-k_2^0 \delta_2 C_1 & (1 - \omega)k_2^0 - k_2^0 \delta_2 C_2 & -k_2^0 \delta_2 C_3 \\
-k_3^0 \delta_3 C_1 & -k_3^0 \delta_3 C_2 & (1 - \omega)k_3^0 - k_3^0 \delta_3 C_3
\end{bmatrix} \quad (5-15)
\end{align*}$$

If Eq. (5-11) is written in a matrix form of $\{d\tau\} = [K]\{d\delta\}$, it can be seen that the incremental stiffness matrix $[K]$ is asymmetric as $k_i^0 \delta_i C_j \neq k_j^0 \delta_j C_i$, i.e.

$$\begin{align*}
K &= \begin{bmatrix}
(1 - \omega)k_1^0 - k_1^0 \delta_1 C_1 & -k_1^0 \delta_1 C_2 & -k_1^0 \delta_1 C_3 \\
-k_2^0 \delta_2 C_1 & (1 - \omega)k_2^0 - k_2^0 \delta_2 C_2 & -k_2^0 \delta_2 C_3 \\
-k_3^0 \delta_3 C_1 & -k_3^0 \delta_3 C_2 & (1 - \omega)k_3^0 - k_3^0 \delta_3 C_3
\end{bmatrix}
\end{align*} \quad (5-15)$$
When the damage surface is not exceeded, i.e. \( F(\tau_i, G_i) < 0 \), there will be no damage development, thus \( d\omega = 0 \) and the incremental constitutive relationship becomes

\[
d\tau_i = (1 - \omega)k_i^0 d\delta_i
\]  

(5-16)

It is often assumed that when \( \delta_i < 0 \), the normal stress \( \tau_i \) and energy release rate \( G_i \) have no contribution to damage evolution. Therefore, mode I \( G_i \) is suppressed correspondingly in this case. The incremental interfacial constitutive law becomes

\[
d\tau_i = k_i^0 d\delta_i
\]

\[
d\tau_i = (1 - \omega)k_i^0 d\delta_i - k_i^0 \sum_{j=2}^{3} C_i d\delta_j \ , \ (i = 2, 3)
\]  

(5-17)

where

\[
C_1 = \frac{\partial F}{\partial \tau_i} k_i^0 \left/ \sum_{j=2}^{3} \frac{\partial F}{\partial \tau_j} k_j^0 \delta_j \right. = 0
\]

\[
C_i = \left[ \frac{\partial F}{\partial \tau_i} (1 - \omega)k_i^0 + \frac{\partial F}{\partial G_i} \tau_i \right] \left/ \sum_{j=2}^{3} \frac{\partial F}{\partial \tau_j} k_j^0 \delta_j \right. \ , \ (i = 2, 3)
\]  

(5-18)

The incremental damage evolution law is still expressed in the form of Eq. (5-13) but using the damage evolution rates \( C_i \) in Eq. (5-18).

If damage surface is expressed in the forms in Eq. (5-10), a typical bilinear traction/relative displacement curve for the single mode case and the corresponding damage evolution curve can be obtained, as shown in Fig. 5.2. As the relative displacement increases from zero, the traction increases rapidly. Damage starts to develop once the stress-based criterion reaches to unity. As relative displacement continues to increase, traction decreases while damage develops. The damage parameter reaches unity once the fracture-mechanics-based criterion meets unity. The traction is reduced to zero and complete crack surfaces are formed at the final stage. It is worth mentioning that damage develops at a fast rate at the early stage. As damage continues to grow, traction decreases as well as damage evolution rate.
The construction of damage surface represented in Eq. (5-10) can also be established in a generalized form as follows (Zou et al., 2003):

\[
F(\tau_i, G_i) = f_s(\tau_i) - \left[1 - \varphi(f_s(G_i))\right] = 0
\]  
(5-19)

where \(\varphi\) is a monotonically increasing function of \(f_g\) satisfying \(\varphi(0) = 0\) and \(\varphi(1) = 1\), which implies the trend of the damage surface to shrink in traction space as the damage grows. By choosing different forms for \(\varphi\), the shrinkage rate of the damage surface can be controlled and various softening traction versus relative displacement curves obtained. Suppose \(\varphi(f_g) = f_g^n\), then the traction versus relative displacement curves can have the following forms shown in Fig. 5.2 (right). Increasing parameter \(n\) leads to a slow shrinkage rate of the damage surface in the early stage after damage initiation, while the energy dissipation rate will increase in the later damage state. As \(n\) approaches infinity, the elastic-perfectly plastic curve can be expected. These curves are employed in various models found in the literature.

5.2.2 Intra-laminar Damage Models

The intra-laminar damage model is based on a CDM approach originally developed by Kachanov (1987), who considered the full range of material deterioration which can be measured through the decrease of strength, stiffness, toughness and residual life. The understanding of this general concept has been reviewed by Chaboche and Lemaitre...
The CDM based intra-laminar damage model proposed in this section can account for the degradation effects due to different modes of damage, e.g. fibre tensile and compressive failure, matrix tensile and compressive failure. Another feature of the present model is that according to experimental evidence, shear behaviour in composite laminates is markedly nonlinear. In addition, in-plane shear stress-strain relationship is dominated by nonlinearity. The nonlinearity in the in-plane shear behaviour is a combination of nonlinear elasticity and plasticity with strain hardening. Therefore, a nonlinear in-plane shear damage mode was introduced in this model to capture the matrix plasticity, progressive shear damage and irreversible shear strain.

On the basis of Lemaitre’s strain equivalence principle (1985), any deformation behaviour of a damaged material is represented by the constitutive laws of the undamaged material in which the nominal stress is replaced by the effective stress $\tilde{\sigma}$, defined as

$$\tilde{\sigma} = \frac{\sigma}{(1-d)}$$  \hspace{1cm} (5-20)

where $d$ represents the isotropic damage variable introduced at the lamina level to quantify the crack concentration in the fractured cross-sectional area of the RVE. Relating the effective stress and nominal stress in a compact matrix form as

$$\tilde{\sigma} = M\sigma$$  \hspace{1cm} (5-21)

where

$$\sigma = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{bmatrix}, \quad \tilde{\sigma} = \begin{bmatrix} \tilde{\sigma}_{11} \\ \tilde{\sigma}_{22} \\ \tilde{\tau}_{12} \end{bmatrix}, \quad M = \begin{bmatrix} \frac{1}{(1-d_{11})} & 0 & 0 \\ 0 & \frac{1}{(1-d_{22})} & 0 \\ 0 & 0 & \frac{1}{(1-d_{12})} \end{bmatrix}$$

The subscripts 1 and 2 refer to the material local coordinate system, i.e. local fibre longitudinal and transverse direction. Accordingly, the linear elastic law of a damaged material can be expressed in two equivalent forms as

$$\varepsilon = \frac{\sigma}{E} = \frac{\tilde{\sigma}}{E}$$  \hspace{1cm} (5-22)
where $\varepsilon$ is the elastic strain and $\bar{E}$ is the effective Young’s modulus, which is related to the damage variable as follows

$$\bar{E} = (1-d)E$$

(5-23)

The plane stress orthotropic elasticity matrix can be defined in terms of effective stress as

$$\varepsilon = S\bar{\sigma}$$

(5-24)

where

$$\varepsilon = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{bmatrix}, \quad S = \begin{bmatrix} 1/E_{11} & -\nu_{12}/E_{22} & 0 \\ -\nu_{12}/E_{11} & 1/E_{22} & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix}$$

These basic concepts are used to develop a damage model with five damage variables: $d_{11}^{f,t}, d_{11}^{f,c}, d_{22}^{m,t}, d_{22}^{m,c}, d_{12}^{s}$. Each damage variable is associated with the degradation of elastic modulus due to different mechanisms of fracture in local fibre or transverse matrix direction and in-plane shear. For an undamaged material $d = 0$, while $d = 1$ denotes complete failure. To maintain a positive definite constitutive law, the Poisson’s ratios must be degraded in a similar manner to the Young’s modulus whether in the undamaged or the damaged state, e.g.

$$\frac{\nu_{12}(1-d_{11})}{E_{11}(1-d_{11})} = \frac{\nu_{21}(1-d_{22})}{E_{22}(1-d_{22})}$$

(5-25)

5.2.2.1 Longitudinal and Transverse failure modes

Since the fibre and matrix tensile fracture is predominantly caused by the tensile loading in the longitudinal and transverse directions, respectively, the failure initiation due to tensile loading in longitudinal and transverse direction is predicted using a non-interacting strain based failure criterion. Furthermore, the present formulation of fibre and matrix compressive failure is employed in a similar manner to the tensile failure. However, within the framework of a plane stress formulation only the in-plane stresses
are known directly. The treatment of matrix compression failure described by Puck (1998) was often applied in the literature, which is not employed in the present formulation. It is assumed that no cross-coupling between fibre and matrix failure.

The strain-based failure criteria used to detect tensile or compressive damage initiation are defined as

\[ F^t_i = \left( \frac{\varepsilon_{ii}^t}{\varepsilon_{ii}^o} \right)^2 - 1 \geq 0 \]  
(5-26)

\[ F^c_i = \left( \frac{\varepsilon_{ii}^c}{\varepsilon_{ii}^o} \right)^2 - 1 \geq 0 \]  
(5-27)

where \( i = 1 \) applies to longitudinal failure mode and \( i = 2 \) to transverse failure mode. The indexes \( t \) and \( c \) refer to tension and compression behaviours, respectively, and \( \varepsilon_{ii}^o \) correspond to the strain at failure initiation either in tension and compression, which is given by

\[ \varepsilon_{11}^{o,t} = \frac{\sigma_{11}^{o,t}}{C_{11}} \]  
(5-28)

\[ \varepsilon_{11}^{o,c} = \frac{\sigma_{11}^{o,c}}{C_{11}} \]  
(5-29)

\[ \varepsilon_{22}^{o,t} = \frac{\sigma_{22}^{o,t}}{C_{22}} \]  
(5-30)

\[ \varepsilon_{22}^{o,c} = \frac{\sigma_{22}^{o,c}}{C_{22}} \]  
(5-31)

where \( \sigma_{11}^{o,t}, \sigma_{11}^{o,c} \) are the ultimate value of the longitudinal failure stresses and \( \sigma_{22}^{o,t}, \sigma_{22}^{o,c} \) are the transverse failure stresses. \( C_{ij} \) are the components of the stiffness matrix in the undamaged state.
Figure 5.3: Model behaviour for tensile or compressive failure modes (k=t,c) in longitudinal and transverse direction (i=1,2).

Prior to the failure initiation criterion being satisfied, a linear stress-strain behaviour is assumed. After damage has initiated, the stress is linearly degraded to zero as the strain increases as shown in Fig. 5.3. This results in the rise of the damage variable from zero at failure initiation ($\varepsilon_{ii} = \varepsilon_{ii}^0$) to one at final failure ($\varepsilon_{ii} = \varepsilon_{ii}^f$). The bilinear damage evolution equation for respective failure mode and its respective increments can then be expressed as a function of strain as follows:

Fibre tensile failure ($\varepsilon_{11} \geq 0$):

$$d_{11}^{f,t}(\varepsilon_{11}) = \frac{\varepsilon_{11}^{f,t}}{\varepsilon_{11}^{f,t} - \varepsilon_{11}^{o,t}} \left( 1 - \frac{\varepsilon_{11}^{o,t}}{\varepsilon_{11}} \right)$$  \hspace{1cm} (5-32)

$$\Delta d_{11}^{f,t}(\varepsilon_{11}) = \frac{\varepsilon_{11}^{f,t}}{\varepsilon_{11}^{f,t} - \varepsilon_{11}^{o,t}} \left( \frac{\varepsilon_{11}^{o,t}}{\varepsilon_{11}} \right) \Delta \varepsilon_{11}$$  \hspace{1cm} (5-33)

Fibre compressive failure ($\varepsilon_{11} < 0$):

$$d_{11}^{f,c}(\varepsilon_{11}) = \frac{\varepsilon_{11}^{f,c}}{\varepsilon_{11}^{f,c} - \varepsilon_{11}^{o,c}} \left( 1 - \frac{\varepsilon_{11}^{o,c}}{\varepsilon_{11}} \right)$$  \hspace{1cm} (5-34)

$$\Delta d_{11}^{f,c}(\varepsilon_{11}) = \frac{\varepsilon_{11}^{f,c}}{\varepsilon_{11}^{f,c} - \varepsilon_{11}^{o,c}} \left( \frac{\varepsilon_{11}^{o,c}}{\varepsilon_{11}} \right) \Delta \varepsilon_{11}$$  \hspace{1cm} (5-35)

Matrix tensile failure ($\varepsilon_{22} \geq 0$):
\[ d^{m,t}_{22}(\varepsilon_{22}) = \frac{\varepsilon^{f,t}_{22}}{\varepsilon^{f,t}_{22} - \varepsilon^{o,t}_{22}} \left( 1 - \frac{\varepsilon^{o,t}_{22}}{\varepsilon^{f,t}_{22}} \right) \]  

(5-36)

\[ \Delta d^{m,t}_{22}(\varepsilon_{22}) = \frac{\varepsilon^{f,t}_{22}}{\varepsilon^{f,t}_{22} - \varepsilon^{o,t}_{22}} \left( \frac{\varepsilon^{o,t}_{22}}{\varepsilon^{f,t}_{22}} \right) \Delta \varepsilon_{22} \]  

(5-37)

Matrix compressive failure \((\varepsilon_{22} < 0)\):

\[ d^{m,e}_{22}(\varepsilon_{22}) = \frac{\varepsilon^{f,e}_{22}}{\varepsilon^{f,e}_{22} - \varepsilon^{o,e}_{22}} \left( 1 - \frac{\varepsilon^{o,e}_{22}}{\varepsilon^{f,e}_{22}} \right) \]  

(5-38)

\[ \Delta d^{m,e}_{22}(\varepsilon_{22}) = \frac{\varepsilon^{f,e}_{22}}{\varepsilon^{f,e}_{22} - \varepsilon^{o,e}_{22}} \left( \frac{\varepsilon^{o,e}_{22}}{\varepsilon^{f,e}_{22}} \right) \Delta \varepsilon_{22} \]  

(5-39)

\[ d^{f,t}_{11}, d^{f,c}_{11}, d^{m,t}_{22}, d^{m,c}_{22} \in [0,1] \]

where \(\varepsilon^{f,t}_{11}, \varepsilon^{f,c}_{11}, \varepsilon^{f,t}_{22}, \varepsilon^{f,c}_{22}\) correspond to the maximum strain when the stresses are reduced to zero and damage parameter reaches unity in tension and compression, respectively. The magnitude of these strain value defines the total energy dissipated, i.e., the area underneath the stress-strain curve. Thus, the intra-laminar critical energy release rates associated with tensile and compressive failure of fibre and matrix are input as material properties. For a bilinear stress-strain curve, the specific energy dissipated per unit volume can be defined as

\[ g^{k}_{f,t} = \int^{\varepsilon_{22}} \sigma_{\mu}^o d\varepsilon_{\mu} = \frac{1}{2} \sigma_{\mu}^o \varepsilon_{\mu}^{f,k} \]  

(5-40)

where \(i = 1,2\) applies to the longitudinal and transverse failure mode, respectively. The superscript \(k\) refers to the failure type same as the indexes \(t\) and \(c\). For the sake of simplicity, the objectivity of the solution is obtained using this model which assumes that the intra-laminar fracture energy of the material, \(g_f\), is distributed over the entire volume of the element. The fracture energy within a fully failed element can be written in terms of the specific energy by multiplying the specific energy by a geometric quantity defined as characteristic length \((l^*)\). Therefore, a characteristic length of the finite element associated with an integration point is introduced to correlate the respective critical energy release rate, \(G_f\), which is the fracture energy (energy dissipated per unit area) of the fibre or matrix, with the specific energy (energy dissipated per unit volume), given by
\[ g_{f,j}^k = \frac{G_{f,j}^k}{l_{ii}^k} \]  

(5-41)

The failure strains associated with longitudinal and transverse behaviour can be obtained from Eq. (5-40)~Eq. (5-43) as follows

\[ \varepsilon_{ii}^{f,k} = \frac{2G_{f,j}^k}{\sigma_{ii}^{o,k} l_{ii}^{*k}} \]  

(5-42)

where \( G_{f,i,k} \), \( \sigma_{ii}^{o,k} \) and \( l_{ii}^{*k} \) are the intra-laminar fracture energy, strength and characteristic length, respectively. Different methods have been suggested in the literature for computing the characteristic length. However, since the characteristic length calculation is based only on the element geometry without taking into account the orientation of the crack, some mesh sensitivity remains. Because elements with large aspect ratios will have rather different behaviour depending on the direction in which they crack. Therefore, elements with aspect ratios equal or close to one is recommended.

5.2.2.2 Nonlinear shear behaviour

In general, the nonlinear material behaviour may be attributed to two distinct mechanical processes: plasticity and the degradation in stiffness due to progressive damage. The nonlinear stress-strain relationship of a composite material is normally a predominant effect of the matrix when compared to the fibres. Different approaches to describe these phenomena can be found in literature (Khan et al., 1995, Barbero et al., 2002, Li et al., 2005, Donadon et al., 2008). The in-plane shear model described in this section presents some features in common with the approach taken by Donadon et al. (2008). The stress-strain relationship has been divided into three continuous sections: linear elastic, nonlinear plastic and linear softening, as shown in Fig. 5.4.
In the first section, a linear shear stress-strain relationship has been used to represent the recoverable material behaviour and the strain component is also reversible; once all stresses are removed it does not leave any permanent deformation. The shear stress-strain relationship in elastic region, i.e., \( \gamma_{12} \leq \gamma_{12}^y \), is given by

\[
\tau_{12} = G^0 \gamma_{12}
\]

where \( G^0 \) is the initial elastic in-plane shear modulus. A quartic polynomial stress-strain relationship has been used to represent the non-linear behaviour in plastic region \( (\gamma_{12}^y \leq \gamma_{12} \leq \gamma_{12}^p) \) and is given by

\[
\tau_{12}(\gamma_{12}) = c_0 + c_1 \gamma_{12} + c_2 \gamma_{12}^2 + c_3 \gamma_{12}^3 + c_4 \gamma_{12}^4
\]

where \( c_0, c_1, c_2, c_3 \), and \( c_4 \) are coefficients obtained by curve fitting to the experimental shear stress-strain data. For failure analysis, a proper description of the unloading behaviour is of importance. It is assumed that the unloading path is a straight line, which is governed by the elastic modulus and parallel to initial linear loading path, after passing the yield point \( (\gamma_{12}^y, \tau_{12}^y) \). Upon complete unloading, there is a plastic strain left behind permanently. The reloading path follows the unloading path which is governed by the undamaged modulus since the shear modulus is fairly similar to the initial one (Puck, 1998). Even though experimental cyclic loading-unloading shear response shows
a gradual shear modulus reduction before damage initiation (Donadon et al., 2008, Apruzzese et al., 2007), this additional damage mechanism which is associated with matrix micro-cracking has not been taken into account in the present model. The plastic strain keeps on increasing until damage initiation at $\gamma_{12} = \gamma_{12}^0$, then it remains constant which can be obtained by

$$\gamma_{12}^P = \gamma_{12}^o - \frac{\tau_{12}(\gamma_{12}^o)}{G^0}$$  

(5-45)

A strain-based failure criterion is used to detect damage initiation for $\gamma_{12} > \gamma_{12}^o$, as follows

$$F_{ij}^t = \left(\frac{\gamma_{ij}^o}{\gamma_{ij}}\right)^2 - 1 \geq 0, \text{ for } i, j = 1, 2$$  

(5-46)

where $\gamma_{12}^o$ corresponds to the shear strain when the shear stress reaches the shear strength and its magnitude can be determined from the polynomial equation.

In the region of post damage, the response of material is described by a linear softening law and the in-plane shear stress can be represented as

$$\tau_{12}(\gamma_{12}) = (1 - d_{12}^f)G^0(\gamma_{12} - \gamma_{12}^p), \quad \gamma_{12}^p \leq \gamma_{12} \leq \gamma_{12}^f$$  

(5-47)

The damage variable $d_{12}^f$ used in Eq. (5-47) and its correspondent increments are defined as follows

$$d_{12}^t = \frac{\gamma_{12}^i - \gamma_{12}^p}{\gamma_{12}^f - \gamma_{12}^p} \left(1 - \frac{\gamma_{12}^o - \gamma_{12}^p}{\gamma_{12}^f - \gamma_{12}^o}\right), \quad \gamma_{12}^p \leq \gamma_{12} \leq \gamma_{12}^f$$  

(5-48)

$$\Delta d_{12}^t = \frac{\gamma_{12}^f - \gamma_{12}^p}{\gamma_{12}^f - \gamma_{12}^o} \left(\frac{\gamma_{12}^o - \gamma_{12}^p}{\gamma_{12}^f - \gamma_{12}^o}\right) \Delta \gamma_{12}$$  

(5-49)

where $\gamma_{12}^f$ corresponds to the final failure strain at which the material is fully damaged and its magnitude depends on the amount of energy dissipated at the finite element level. However, in-plane shear cracking is dependent on the fibre orientation within the element, therefore, the characteristic length associated with in-plane shear failure has been assumed to be the same as the one defined for fibre or matrix failure. From Eq.
(5-40), the failure strain associated with in-plane shear failure mode within the finite element can be defined as

$$\gamma_{12}^f = \frac{2}{\tau_{12}} \left[ \frac{G_{f,12}^s}{l_{12}^*} - \int \gamma_{12}^s \, \tau_{12} \, d\gamma_{12} - \frac{1}{2} \gamma_{12}^s \tau_{12} \right] + \gamma_{12}^o$$  (5-50)

where $G_{f,12}^s$ is the intra-laminar fracture energy for in-plane shear fracture and $l_{12}^*$ is the characteristic length for in-plane shear failure. The fracture energy $G_{f,12}^s$ associated with matrix shear failure can be assumed equal to the Mode II inter-laminar fracture toughness which can be measured using standard tests. Subsequently, the fracture energy associated with compressive matrix failure can be related to the value of $G_{f,12}^s$ as suggested by Maimí et al. (2007).

5.2.2.3 Incremental constitutive relationship

For plane stress conditions the stress-strain relationship at a ply or lamina level can be defined as

$$\sigma = C_d \varepsilon$$  (5-51)

where $C_d$ is the effective stiffness matrix, which is reduced by three damage variables $d_{11}^f$, $d_{22}^m$, and $d_{12}^s$ during progressive damage as follows

$$C_d = \begin{bmatrix}
    (1-d_{11}^f)C_{11} & (1-d_{12}^m)(1-d_{22}^m)C_{12} & 0 \\
    (1-d_{11}^f)(1-d_{22}^m)C_{21} & (1-d_{22}^m)C_{22} & 0 \\
    0 & 0 & (1-d_{12}^s)C_{66}
\end{bmatrix} = \begin{bmatrix}
    C_{11}^d & C_{12}^d & 0 \\
    C_{21}^d & C_{22}^d & 0 \\
    0 & 0 & C_{66}^d
\end{bmatrix}$$  (5-52)

Generally, conventional solution techniques for nonlinear problems are frequently based on an incremental approach. The constitutive relation described in this section is therefore expressed in terms of an incremental relation to include stress, strain and damage. The incremental form of the stress, strain and damage relation for lamina can be obtained by differentiating Eq. (5-51) as

$$d\sigma = C_d d\varepsilon + \frac{\partial C_d}{\partial d_{11}} \frac{\partial d_{11}}{\partial \varepsilon} d\varepsilon + \frac{\partial C_d}{\partial d_{22}} \frac{\partial d_{22}}{\partial \varepsilon} d\varepsilon + \frac{\partial C_d}{\partial d_{12}} \frac{\partial d_{12}}{\partial \varepsilon} d\varepsilon$$  (5-53)
Eq. (5-53) can be written in an incremental form in terms of damage for the local 1 and 2 directions;

\[
d \sigma_{11} = (1 - d_{11}) C_{111} \delta \varepsilon_{11} + (1 - d_{11}) (1 - d_{22}) C_{222} \delta \varepsilon_{22} - C_{111} \delta \varepsilon_{11} A_{11} d \varepsilon_{11} \\
- (1 - d_{22}) C_{222} \delta \varepsilon_{22} A_{11} d \varepsilon_{11} - (1 - d_{11}) C_{111} \delta \varepsilon_{11} A_{22} d \varepsilon_{22} \\
= \left[ (1 - d_{11}) C_{111} + (-C_{111} \delta \varepsilon_{11} + (1 - d_{22}) C_{222} \delta \varepsilon_{22}) A_{11} \right] d \varepsilon_{11} \\
+ \left[ (1 - d_{11}) (1 - d_{22}) C_{222} + (1 - d_{11}) C_{111} \delta \varepsilon_{22} A_{22} \right] d \varepsilon_{22} \\
\tag{5-53a}
\]

\[
d \sigma_{22} = (1 - d_{22}) (1 - d_{11}) C_{222} \delta \varepsilon_{22} + (1 - d_{22}) C_{222} \delta \varepsilon_{22} - (1 - d_{22}) C_{222} \delta \varepsilon_{22} A_{11} d \varepsilon_{11} \\
- (1 - d_{11}) C_{222} \delta \varepsilon_{22} A_{22} d \varepsilon_{22} - (1 - d_{22}) C_{222} \delta \varepsilon_{22} A_{22} d \varepsilon_{22} \\
= \left[ (1 - d_{22}) (1 - d_{11}) C_{222} + (-C_{222} \delta \varepsilon_{22} + (1 - d_{11}) C_{111} \delta \varepsilon_{11} A_{11} \right] d \varepsilon_{11} \\
+ \left[ (1 - d_{22}) C_{222} + (-C_{222} \delta \varepsilon_{22} - C_{222} \delta \varepsilon_{22} A_{22} \right] d \varepsilon_{22} \\
\tag{5-53b}
\]

where

\[
A_{11} = \frac{\Delta d_{11}}{\Delta \varepsilon_{11}} = \frac{\delta \varepsilon_{01}}{\varepsilon_{11}^*(\varepsilon_{11} - \varepsilon_{01}^*)} \quad \text{(tension)}
\]

\[
A_{22} = \frac{\Delta d_{22}}{\Delta \varepsilon_{22}} = \frac{\delta \varepsilon_{02}}{\varepsilon_{22}^*(\varepsilon_{22} - \varepsilon_{02}^*)} \quad \text{(compression)}
\]

and the incremental shear term as

\[
d \tau_{12} = C_{66} \delta \gamma_{12} \quad \gamma_{12} \leq \gamma_{12}^v \\
\tag{5-53c}
\]

\[
d \tau_{12} = c_{12} \delta \gamma_{12} + 2c_{22} \delta \gamma_{12}^2 d \gamma_{12} + 3c_{33} \delta \gamma_{12}^3 d \gamma_{12} + 4c_{44} \delta \gamma_{12}^4 d \gamma_{12} \quad \gamma_{12}^v \leq \gamma_{12} \leq \gamma_{12}^o \\
\tag{5-53d}
\]

\[
d \tau_{12} = (1 - d_{12}) C_{66} \delta \gamma_{12} - C_{66} (\gamma_{12} - \gamma_{12}^p) A_{12} d \gamma_{12} \quad \gamma_{12}^o \leq \gamma_{12} \leq \gamma_{12}^f \\
\tag{5-53e}
\]

where

\[
A_{12} = \frac{\Delta d_{12}}{\Delta \gamma_{12}} = \frac{(\gamma_{12}^v - \gamma_{12}^o)(\gamma_{12}^o - \gamma_{12}^p)}{(\gamma_{12}^v - \gamma_{12}^o)(\gamma_{12}^o - \gamma_{12}^p)}
\]
If the above equations were written in a matrix form of \( d\sigma = Q^d d\epsilon \), the tangential stiffness matrix \( Q^d \) is asymmetric in general as \( Q^d_{12} \neq Q^d_{21} \):

\[
Q^d = \begin{bmatrix}
Q^d_{11} & Q^d_{12} & 0 \\
Q^d_{21} & Q^d_{22} & 0 \\
0 & 0 & Q^d_{66}
\end{bmatrix}
\]  

(5-54)

where

\[
Q^d_{11} = (1-d_{11})C_{11} + (-C_{11}\varepsilon_{11} - (1-d_{22})C_{12}\varepsilon_{22})A_{11}
\]

\[
Q^d_{12} = (1-d_{11})(1-d_{22})C_{12} - (1-d_{11})C_{12}\varepsilon_{22}A_{22}
\]

\[
Q^d_{21} = (1-d_{22})(1-d_{11})C_{21} - (1-d_{22})C_{21}\varepsilon_{11}A_{11}
\]

\[
Q^d_{22} = (1-d_{22})C_{22} + (- (1-d_{11})C_{21}\varepsilon_{11} - C_{22}\varepsilon_{22})A_{22}
\]

\[
Q^d_{66} = (1-d_{12})C_{66} - C_{66}(\gamma_{12} - \gamma_{22}^d)A_{12}
\]

and

\[
C_{11} = E_{11}/(1-\nu_{12}\nu_{21})
\]

\[
C_{12} = \nu_{12}E_{22}/(1-\nu_{12}\nu_{21})
\]

\[
C_{21} = \nu_{21}E_{11}/(1-\nu_{12}\nu_{21})
\]

\[
C_{22} = E_{22}/(1-\nu_{22}\nu_{21})
\]

\[
C_{66} = G_{12}
\]

When there is only matrix damage propagation, i.e. \( A_{11} = 0, A_{22} > 0 \), Eq. (5-53a) and (5-53b) become:

\[
d\sigma_{11} = C^d_{11}d\varepsilon_{11} + \left[ C^d_{12} - (1-d_{11})C_{12}\varepsilon_{22}A_{22} \right]d\varepsilon_{22}
\]  

(5-53f)

\[
d\sigma_{22} = C^d_{22}d\varepsilon_{11} + \left[ C^d_{21} + (- (1-d_{11})C_{21}\varepsilon_{11} - C_{22}\varepsilon_{22})A_{22} \right]d\varepsilon_{22}
\]  

(5-53g)
When there is only fibre damage propagation, i.e. $A_{11} > 0$, $A_{22} = 0$, Eq. (5-53a) and (5-53b) become:

$$d\sigma_{11} = \left[ C_{11}^d + (-C_{11}\varepsilon_{11} - (1-d_{22})C_{22}\varepsilon_{22})A_{11} \right] d\varepsilon_{11} + C_{12}^d d\varepsilon_{22} \quad (5-53h)$$

$$d\sigma_{22} = \left[ C_{22}^d - (1-d_{22})C_{21}\varepsilon_{11}A_{11} \right] d\varepsilon_{11} + C_{22}^d d\varepsilon_{22} \quad (5-53i)$$

When there is either no further damage propagation or under unloading condition after damage has initiated, i.e. $A_{11} = A_{22} = 0$, Eq. (5-53a) and (5-53b) become:

$$d\sigma_{11} = C_{11}^d d\varepsilon_{11} + C_{12}^d d\varepsilon_{22} \quad (5-53j)$$

$$d\sigma_{22} = C_{22}^d d\varepsilon_{11} + C_{22}^d d\varepsilon_{22} \quad (5-53k)$$

When there is either no further shear damage propagation or under unloading condition after damage has initiated, i.e. $A_{12} = 0$, the incremental shear term become:

$$d\tau_{12} = C_{66}^d \gamma_{12} \, , \, \gamma_{12} \leq \gamma_{12}^\circ \quad (5-53l)$$

$$d\tau_{12} = (1-d_{12}) C_{66}^d \gamma_{12} \, , \, \gamma_{12}^\circ \leq \gamma_{12} \leq \gamma_{12}^\prime \quad (5-53m)$$

From Eq.(5-53j) to (5-53m), the tangent stiffness matrix can be formed as the effective stiffness matrix in an unloading process.
5.2.3 Implementation of Damage Models in Finite Element Code

The development of the constitutive damage models aimed at the analysis of laminated structures. Since Finite Element Method is a primary tool for such analysis, the constitutive laws for the damage models must be implemented into commercial FEM packages to maximise their application. For present analyses, the inter- and intra-laminar continuum damage models described in the above section have been implemented as a user material subroutine UMAT into the implicit finite element code ABAQUS/Standard. The inter-laminar damage model was implemented as a user material for the cohesive element in ABAQUS and the intra-laminar damage model as user material for solid element or structural shell/continuum shell element.

ABAQUS requires the stresses at integration points and the Jacobian matrix of the material (i.e. the incremental constitutive model $\frac{\partial \Delta \sigma}{\partial \Delta \varepsilon}$ for the material) to be computed in the user subroutine and passed to ABAQUS at the end of each iteration. The strain and its increment are input from ABAQUS. Material properties are defined as user material properties and input into the subroutine by ABAQUS. Damage parameters and other variables such as energy release rates are defined as solution dependent state variables. They are passed into the user subroutine, computed and updated in the user subroutine and passed back to ABAQUS at the end of each iteration.

Due to the softening behaviour of the constitutive law adopted in the damage models after damage initiation, significant divergence issue was encountered when solving the nonlinear damage problem. Some numerical treatments have to be considered in the present model to ease the convergence difficulty.

For the inter-laminar damage model, a numerical treatment has been applied during the iterations in each increment, i.e. $f_B$ is taken as a constant and the value of $f_B$ passed in is the one evaluated at the end of the previous increment. To implement this in the formulations of the incremental damage evolution and interfacial constitutive relationship, change is made in the expression of Eq.(5-14) as

$$C_i = \frac{\partial F}{\partial \tau_i} (1 - \omega) k_i^0 \left/ \sum_{j=1}^{3} \frac{\partial F}{\partial \tau_j} k_j^0 \delta_j \right. \quad (5-55)$$
Although such treatment might cause a discrete multi-step change in continuous softening behaviour, the reduction of the size of each increment should be made for preventing unbalance in the accumulation of increments.

Alternatively, viscous regularization can be used as an optional solution for alleviating convergence problems of the iterations. This artificial viscous regularization model causes the tangent stiffness matrix to be positive for sufficiently small time increments even during the softening part of the constitutive law. It is however only active in the case of softening in intra-laminar damage model and is done by means of the rate equation. A viscosity parameter $\eta$ controls the rate at which the regularized damage variables approach the true damage variables. In this technique, the damage variable calculated from the aforementioned damage evolution equation is replaced by their viscous counterparts. To update the regularized damage variables at time $t_0 + \Delta t$, the following equation is discretized in time as

$$
\frac{d\rho^v}{d\rho} \bigg|_{t_0 + \Delta t} = \frac{\Delta t}{\eta + \Delta t} \frac{d\rho}{d\rho} \bigg|_{t_0 + \Delta t} + \frac{\eta_{p}}{\eta_{p} + \Delta t} \frac{d\rho}{d\rho} \bigg|_{t_0}
$$

where $d\rho$ and $d\rho^v$ are the damage variables concerning damage mode $p$ in the non-viscous and the viscous regularization, respectively. The parameter $\eta_{p}$ represent the viscosity coefficients associated with damage mode $p$. Therefore, the Jacobian matrix can be further formulated as

$$
\frac{\partial \Delta \sigma}{\partial \Delta \varepsilon} = C_d + \begin{bmatrix}
\frac{\partial C_d}{\partial d_{11}} \frac{\partial d_{11}}{\partial \varepsilon} + \frac{\partial C_d}{\partial d_{22}} \frac{\partial d_{22}}{\partial \varepsilon} + \frac{\partial C_d}{\partial d_{12}} \frac{\partial d_{12}}{\partial \varepsilon}
\end{bmatrix} \frac{\Delta t}{\eta + \Delta t}
$$

Care must be taken by choosing an appropriate value of $\eta$ since a large value of viscosity might lead to a noticeable delay in the degradation of the stiffness.

The laminar constitutive relation is expressed in terms of strains, stresses and solution dependent state variables (damage parameters, etc.). The user subroutine UMAT which contains the laminar constitutive relation has been written in FORTRAN and can be referred in the Appendices. A flow chart of UMAT is displayed in Fig. 5.5.
Figure 5.5: Flow chart of UMAT user subroutine.
(UMAT1: Intra-laminar damage model; UMAT2: Inter-laminar damage model)
5.3 Validations

5.3.1 Delamination growth in a double cantilever beam

In this section, delamination in a unidirectional fibre-reinforced DCB specimen was modelled as an example of the application of the continuum damage model. The delamination growth in double cantilever beam specimens is widely tested for Mode I or Mode II fracture behaviour of laminated composites and often employed in many literatures. The geometry of the DCB specimen and loading condition are illustrated in Fig. 5.6. The specimen is 100mm long and 30mm wide, with an initial crack length $a = 30$mm. The material properties are given in Table 5.1.

![Figure 5.6: Double cantilever beam specimen](image)

<table>
<thead>
<tr>
<th>Material properties of the DCB specimen (Mi, 1998; Robinson, 1992)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus in the fibre direction $E_1$ (MPa)</td>
</tr>
<tr>
<td>Young’s modulus in the transverse direction $E_2$ (MPa)</td>
</tr>
<tr>
<td>Poison’s ratio $\nu_{12}$</td>
</tr>
<tr>
<td>Shear modulus $G_{12} = G_{13} = G_{23}$ (MPa)</td>
</tr>
<tr>
<td>Fracture energy $G_{Ic}$ (kJ/m$^2$)</td>
</tr>
</tbody>
</table>
The finite element mesh is illustrated in Fig. 5.7. 8-noded continuum shell elements SC8R with reduced integration were employed for the upper and lower sublaminates. To model the delamination at the mid plane of the DCB specimen, 8-noded three-dimensional cohesive elements with zero thickness were employed for the interface layer between the two sublaminates. The cohesive elements share nodes with the adjacent continuum shell elements. Two layers of continuum shell elements were used in the thickness of each sublaminates. Equal element size of 0.25 mm (coarse mesh) and 0.125 mm (refined mesh) were used in the length direction. Due to the symmetry condition, only half of the beam in the width direction was analysed after the application of symmetry boundary conditions.

Figure 5.7: Finite element model for double cantilever beam

The results from the experiment, analytical beam theory and the present finite element analysis are presented in Fig. 5.8 and 5.9. It can be seen that the FE simulation results for load and crack length versus opening displacement obtained from both meshes are in good agreement with the experimental data and analytical beam theory. The effect of mesh size on the results is also presented for comparison. The predicted crack length versus opening displacement curves are nearly identical for two mesh sizes. However, the critical load in load versus opening displacement curves is slightly different between two meshes. The overall trend for both meshes in Fig. 5.8 does not change while refinement in mesh tends to meet the experimental results even closer.
The propagation of the delamination front through the width and length direction can be seen in Fig. 5.10. Delamination at the interface of two sublaminates tends to propagate from the central region in width direction then spread outwards gradually. The variation of the energy release rate along the delamination front has maximum value in the middle. Moreover, the mesh is fine enough to capture a thumbnail shape for the delamination front. Similar finding has been reported in the literature (Davidson, 1990).

Figure 5.8: Load-opening displacement curve
Figure 5.9: Crack length-opening displacement curve

Figure 5.10: Movement of the delamination front in DCB
5.3.2 Multiple delaminations in composite plates due to indentation

In this section, the test conducted by Finn and Springer (1992) was simulated to model multiple delaminations in graphite-epoxy plates. The plates were manufactured into a $[0_4/90_0/0_4]$ cross-ply laminate. The nominal width and thickness of the plates were 76.2mm and 2.28mm, respectively. The plates were inserted in a specially built aluminium fixture which clamped the two opposite, the narrow edges of the plate (Fig. 5.11). The two longitudinal edges of the plate were free (unsupported). The distance between the clamps, $L$, was 101.6mm. The load was applied by a steel rod with a hemispherical head of 12.7mm diameter mounted to the cross-head of a mechanical testing machine.

![Indentation test fixture (Finn & Springer, 1992).](image)

Because of symmetry, only one quarter of the plate was built and analysed as shown in Fig. 5.12. The composite plate was modelled as three sublaminates coincident with the ply-groups and each sublaminates was modelled with continuum shell element SC8R. Two zero-thickness interface layers were created between the $0^\circ$ and $90^\circ$ sublaminates to simulate the possible delamination and were modelled by COH3D8 cohesive elements. The hemispherical indenter was modelled as analytical rigid body and impacts were applied in a displacement control mode. The interaction between the plate and the indenter was simulated by surface-to-surface contact using penalty friction formulation.
Boundary conditions matching the experimental tested specimen were applied by imposing zero displacement constraints to nodes lying along the edges of the support plate. Symmetry boundaries were applied to the symmetry plane.

![FE model of a [0₄/90₄]s laminated quarter plate](image)

Figure 5.12: FE model of a [0₄/90₄]s laminated quarter plate

The properties of the cross-ply laminate are shown in Table 5.2 where the fracture energy required to delaminate a unit area is not available in (Finn & Springer, 1992). The value of $G_{IIc}$ in Table 5.2 was input as the same value of the mode I critical strain energy release rate in Finn and Springer’s analysis.

**Table 5.2: Material Properties of the cross-ply laminate (Finn & Springer, 1992)**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus $E_1$ (MPa)</td>
<td>130340</td>
</tr>
<tr>
<td>Young’s modulus $E_2 = E_3$ (MPa)</td>
<td>9655</td>
</tr>
<tr>
<td>Shear modulus $G_{12} = G_{13}$ (MPa)</td>
<td>5586</td>
</tr>
<tr>
<td>Shear modulus $G_{23}$ (MPa)</td>
<td>4827</td>
</tr>
<tr>
<td>Poison’s ratio $\nu_{12} = \nu_{13}$</td>
<td>0.29</td>
</tr>
<tr>
<td>Poison’s ratio $\nu_{33}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Interlaminar strengths $\tau_{1c} = \tau_{2c} = \tau_{3c}$ (MPa)</td>
<td>50</td>
</tr>
<tr>
<td>Penalty Stiffness $K$ (N/m³)</td>
<td>$10^{14}$</td>
</tr>
<tr>
<td>Fracture energy $G_{IIc}$ (kJ/m²)</td>
<td>0.149</td>
</tr>
</tbody>
</table>
Different mesh size in laminate plane will affect the accuracy of the results. The comparison between mesh refinements from the refined mesh with element size of 0.254 mm to the coarse mesh with element size of 0.508 mm is shown in Fig. 5.13. The predicted delamination shapes from both meshes are identical while the boundary of the delamination area where the prediction was measured in refined mesh shows smooth curve and more ellipse-like shape.

Figure 5.13: Delamination in bottom interface of the quarter plate under incident energy 0.5J: (a) coarse mesh; (b) refined mesh.

The numerical simulation predicted a major delamination at the lower interface and a much smaller one at the upper interface. This prediction achieves agreement with experimental observation. The delamination at the upper interface appeared much earlier than the delamination at lower interface. However, the delamination at the lower interface propagated at a much faster rate than the damage at the upper interface. The area of delamination at upper interface ceased growing further even the incident energy
still increased. The predicted major delamination matches the experimental observation in principle as shown in Fig. 5.14. Delamination appears much longer along the laminate length than in the width, very likely due to the support of the laminate and the loading.

Figure 5.14: Major delamination in bottom interface from simulation results (upper) and experimental observation (lower) (Finn & Springer, 1992).

The delamination lengths and widths were measured and presented in Fig. 5.15 and 5.16. The predicted dimensions of the delamination match the experiment measurement reasonably well.
Figure 5.15: Comparison between measured and predicted delamination length

Figure 5.16: Comparison between measured and predicted delamination width
5.4 Summary

The inter-laminar continuum damage model based on the concept of damage surface has been presented in this chapter with two applications in modelling the delamination in composite beam and laminate. Cohesive elements are used to implement this continuum damage model into the commercial finite element code ABAQUS via a user defined material subroutine routine UMAT. Simulation results shown in two example applications are in a good agreement with experimental observations. It’s applicability to more sophisticated scenarios such as modelling the damage in impact problems was also explored. In addition, practical delaminations are often in a mixed mode state, it is essential that an appropriate mixed mode criterion is employed when simulating multiple delaminations. The numerical instability associated with a characteristic fast growth of delamination in practical delamination problems can be resolved by applying a numerical treatment.

The intra-laminar continuum damage model for modelling progressive failure in laminated composite is also presented in this chapter. The proposed formulations enable the prediction of different failure modes in composites within an energy based framework avoiding mesh dependence problems arising from the use of orthotropic strain-softening constitutive laws. The shear nonlinearities and irreversible strains are taken in account to reflect realistic material behaviour under shear loading. Applications for this intra-laminar damage model will be exhibited to validate the model against theoretical and experimental results available in the next chapter.

It is relatively straightforward to bring the intra-laminar damage model and the inter-laminar damage model together in the sense of their co-existence to simulate different damage modes in composite laminates. Both models are implemented into commercial finite element code ABAQUS via User Subroutine UMAT. The interactions between different damage modes are important because one mode may initiate another, and they are essential in understanding important issues such as leakage from filament-wound pressurized pipe. The interactions between delamination and transverse cracking and their effects on the behaviour of damaged structures will be discussed in the next chapter.
Chapter 6

Modelling Damage in a Filament-Wound Pipe Subjected to Lateral Indentation

6.1 Introduction

Filament-wound glass fibre composite pipes have found wide engineering applications in for example offshore industry due to their high strength, light-weight and resistance to corrosion. However, lateral concentrated loading such as low-velocity impacts arising from objects falling onto the pipe produce local indentation which may cause significant matrix cracking and delamination. These damages affect the structural integrity of the pipes and may cause fluid leakage and reductions in burst strength.

Research work (Li, et al. 1993, 2005, 2006; Curtis, et al. 2000; Zou, et al. 2002) revealed that matrix cracking is the dominant pre-failure damage mode in thin pipes and delamination the main damage mode in the thick composite pipes. Li et al. (1998, 2005) developed a fully lamina-based continuum damage model to model transverse matrix cracking in filament-wound tubes. A fracture-mechanics-based delamination model was proposed by Zou et al. (2002) together with a ply-discount procedure to deal with the intra-laminar damage mechanisms. Almaskari (2009) incorporates Li’s model into finite
element code via ABAQUS user subroutine UMAT together with cohesive elements for modelling delamination. However, the results of the inclusion of cohesive elements have not been addressed in Almaskari’s work.

In this chapter, filament-wound pipes subjected to lateral indentation were analysed using the damage models presented in last chapter. The aim of the work was to investigate the response of the pipes and the initiation and development of both intra- and inter-laminar damage modes in the pipe. Attempt was also made to seek the first leakage path in the pipe after it was damaged. The results were compared to the experimental observations from Reid et al. (2000)

6.2 Modelling transverse cracking damage in a filament-wound pipe subjected to lateral indentation

6.2.1 Model description

This section is dedicated to simulating the in-plane failure modes such as transverse matrix cracking using intra-laminar damage model as described in section 5.2.2. All the experimental results and observations were obtained from the investigations by Curtis et al. (2000) and Li et al. (1993, 2005). Most of the works done were reproduced in this section numerically and most importantly, to give a verification example of the proposed continuum damage model for intra-laminar damage in filament-wound pipe.

The case to be analysed is a 2-cover (4 laminae) ±55° filament-wound pipe with a wall thickness of 1 mm and diameter of 100 mm. It is 500 mm long and was indented radially by a rigid hemisphere of 50 mm diameter at the centre. The pipe rested on a flat steel plate supporting its whole length. For the validation purposes, the experimental data and material properties were taken from Curtis et al. (2000) and Li et al. (2005) in which the pipe was tested. The material properties of the laminate are given in Table 6.1.

The FE model used in the present simulation is shown in Fig. 6.1. 8-noded continuum shell elements SC8R with reduced integration were employed. A stiffness based hourglass control option was used to avoid the formation of anomalous hourglass modes arising from the use of under integrated elements. A uniform mesh with element size of 2 mm x 1 mm was employed. The hemispherical indenter was modelled as
analytical rigid body and impact load was applied through the indenter in the form of a prescribed displacement. The interaction properties and contact conditions between the indenter and central region were defined as frictionless condition. In the experimental test, the pipe was supported on a flat plate. Boundary conditions matching the experimental tested specimen were applied by imposing zero displacement constraints to nodes lying along the bottom edge to eliminate the rigid body movements.

Due to the implicit dynamic nature of the finite element code the simulations were carried out quasi-statically, a numerical treatment, which is introduced in section 5.2.3, has also been applied to the present damage model for predicting matrix cracking damage in the pipes. The occurrence and development of damage usually causes convergence problem if conventional iterative solution procedures are adopted. With the treatment of damage growth, the convergence behaviour was improved.

Figure 6.1: The finite element model of the composite tube
Table 6.1: Material properties of E-glass/epoxy laminate (Li et al., 2005)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Young’s modulus (MPa)</td>
<td>45600</td>
</tr>
<tr>
<td>Transverse Young’s modulus (MPa)</td>
<td>16200</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.278</td>
</tr>
<tr>
<td>Shear modulus (MPa)</td>
<td>5500</td>
</tr>
<tr>
<td>Longitudinal tensile strength (MPa)</td>
<td>1280</td>
</tr>
<tr>
<td>Longitudinal compressive strength (MPa)</td>
<td>525</td>
</tr>
<tr>
<td>Transverse tensile strength (MPa)</td>
<td>40</td>
</tr>
<tr>
<td>Transverse compressive strength (MPa)</td>
<td>145</td>
</tr>
<tr>
<td>Longitudinal shear strength (MPa)</td>
<td>73</td>
</tr>
<tr>
<td>Intra-laminar fibre fracture energy (tensile) (J/m²)</td>
<td>45000*</td>
</tr>
<tr>
<td>Intra-laminar fibre fracture energy (compressive) (J/m²)</td>
<td>40000*</td>
</tr>
<tr>
<td>Intra-laminar matrix fracture energy (tensile) (J/m²)</td>
<td>165*</td>
</tr>
<tr>
<td>Intra-laminar matrix fracture energy (compressive) (J/m²)</td>
<td>1540*</td>
</tr>
<tr>
<td>In-plane shear fracture energy (J/m²)</td>
<td>1540*</td>
</tr>
<tr>
<td>The nonlinear shear stress-strain relationship (MPa)</td>
<td>$0.031 + 7.2584 \times 10^3 \gamma -309.66 \times 10^3 \gamma^2 + 6.281 \times 10^6 \gamma^3 -48.4 \times 10^6 \gamma^4$</td>
</tr>
</tbody>
</table>

* Input data from Table 3.1 and 3.2.

6.2.2 Numerical simulations results

The predicted load-indentation curves are given in Fig. 6.2 together with experimental results obtained from Curtis et al. (2000). A loading curve predicted using a finite element model with linear elastic material properties, i.e. no damage, is also displayed in the figure. In contrast to the linear, elastic behaviour, the present load-indentation curve shows a lower stiffness. The stiffness reduction can be seen as the result of the changes in material properties resulting from the damage and also shear nonlinearity. The prediction shows a good agreement with the experimental results from 0 mm to 10 mm indentation. It can be noticed that the experimental results show a lower stiffness from 10 mm to 20 mm indentation. This is believed to be the effect of fibre rotation due to large deformation which has not been addressed in the present model.
In Li’s experimental observation, the transverse cracks were not visible until the tube was indented to 5 mm. The predicted first transverse cracking damage, i.e. $d_{22}^m = 1$, has taken place at 1.75 mm indentation and in the area located in the innermost lamina right underneath the indenter. It is believed that this discrepancy is caused by the difficulty in observing small cracks in the early stage of damage development. The contour plots of the damage parameter $d_{22}^{mt}$ in each lamina at 10 and 20 mm indentation are shown in Fig. 6.3 and 6.4. The colour code for cracked region is red. It can be seen that the matrix tensile failure develops rapidly at the innermost lamina after initiating at an early stage. The projected area under the indenter is the main cracked zone from lamina 1 to lamina 3. Matrix tensile failure in that area propagates through the thickness direction but doesn’t reach the outermost lamina. It is worthy to mention that the damage in the projected area under the indenter at the outermost lamina is mostly subjected to matrix compressive failure. This failure is caused by the high in-plane axial compression in the area under the indenter. However, the area away from the indentation point in circumferential direction is subjected to matrix tensile failure in the outermost lamina. The matrix tensile failure in the outermost lamina is dominated by circumferential
bending while the damage in the area under the indenter is due to the development of biaxial bending of the pipe wall under the indenter.
Figure 6.3: Distribution of damage parameter in the laminae of the pipe at 10 mm indentation
Lamina 1
(Innermost)

Lamina 2
Transverse crack map traced at a series of indentation levels can be seen in Fig. 6.5. The developed view of the cylindrical surface obtained from the experiment are compared with top views of predicted crack region. The blue colour code indicates uncracked region and the red colour code indicates the cracked region which includes both matrix tensile and compressive failure. Results show that the pattern of the damaged zones is well captured by the present damage model.
Figure 6.5: Crack map obtained at each level of indentation (top view) (a) 5mm (b) 15mm (c) 20mm (d) experimentally measured (Li, et al 2005)
6.3 Modelling transverse cracking and delamination in a filament-wound pipe subjected to lateral indentation

6.3.1 Experimental Setup

The indentation tests shown in Fig. 6.6 were conducted by Reid et al. (2000). Two types of pipe of 500 mm length were tested. The indenter had a spherical surface of 50mm diameter and the pipe under indentation was supported on a rigid flat surface. The specimens employed in the experiment are GRP filament-wound pipes and they all had a simple helical winding pattern consisting of either 4 covers (which results in 8 layers of reinforcement arranged alternately at +55° and -55° to the pipe axis) or 5 covers (10 layers). The geometry of the specimens is listed in Table 6.1. Details on the tests can be found in Reid et al. (2000).

![Figure 6.6: Experiment set-up of filament-wound pipe under static indentation (Reid et al., 2000)](image)

<table>
<thead>
<tr>
<th>Pipe Type</th>
<th>Wall Thickness, t (mm)</th>
<th>Internal Diameter, 2R (mm)</th>
<th>No. of Covers</th>
</tr>
</thead>
<tbody>
<tr>
<td>3410</td>
<td>2.5</td>
<td>105.2</td>
<td>4</td>
</tr>
<tr>
<td>3450</td>
<td>5</td>
<td>105.2</td>
<td>5</td>
</tr>
</tbody>
</table>
6.3.2 Finite Element Model

The FE model is shown in Fig. 6.7. The pipe was modelled by 8-noded shell SC8R with reduced integration. Each cover was therefore modelled with only one threedimensional shell element in the thickness direction, and interface elements were placed between two neighbouring shell elements in different lay-up. To simplify numerical simulations and shorten the computational time, the use of symmetry and symmetric conditions was considered to be efficient. A half-pipe model has been shown to be sufficient for the analysis of indentation of layered pipes by using the rotational symmetry boundary condition (Li & Reid, 1992) which was applied through an option EQUATIONS provided by ABAQUS for this purpose. Moreover, the contact conditions between the indenter and the pipe have been modelled using the CONTACT PAIR option in ABAQUS under a frictionless condition. The hemispherical indenter was modelled as analytical rigid body as mentioned in section 6.2.1. Load was applied in the displacement-control mode to simulate a quasi-static process in the experiment.

Figure 6.7: Finite element mesh of half-pipe model (4-cover)
The mesh was refined around the centre where the load was applied through a spherical indenter. The construction of transitions from a low mesh density zone to a high mesh density zone and vice versa was a significant aspect of finite element mesh generation. The mesh generation options in ABAQUS are not capable of creating such mesh transitions. Therefore, performing transitions using quadrilateral elements only is possible under such transition scheme shown in Fig. 6.7. The element sizes are nearly 7.8 mm x 7 mm, 4 mm x 3.5 mm, 2 mm x 1.8 mm and 1 mm x 0.9 mm from the coarsest to the finest. This scheme has its limitation but no further techniques are required to solve the incompatibility at the junctions of the fine and more coarsely meshed regions.

The material properties of laminate used in present work are mainly the same as those listed in Table 6.1 and the material properties of the cohesive elements are taken from Table 3.2. In addition, the values of inter-laminar critical energy release rate in four-cover and five-cover pipe are listed in Table 6.3.

<table>
<thead>
<tr>
<th>Table 6.3: Inter-laminar critical energy release rate (Zou et al, 2002)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{IIc}$ (four-cover pipe) (kJ/m$^2$)</td>
</tr>
<tr>
<td>$G_{IIc}$ (five-cover pipe) (kJ/m$^2$)</td>
</tr>
</tbody>
</table>
6.4 Numerical Simulations Results

6.4.1 Load versus indentation curve for four-cover pipe

The predicted load-indentation curve for four-cover pipe is given in Fig. 6.8 along with four test results. The prediction has a good agreement with the experimental results. There is a linearly elastic response prior to a local peak, followed by a clear load drop. Further loading results in a gradual increase in the load, but at much smaller rate. The load-drop shown in four test results and numerical simulations represents the characteristic rapid delamination growth as often observed in experiments. The prediction is very close to the test data within the range of the experimental results.

Due to the incremental solution procedure and the damage growing by many elements, a zigzagged curve appears on the prediction. If finer element meshes and small indentation increments were applied, the predicted curves could be improved to show a smoother appearance. However, this method would have been time consuming.

![Figure 6.8: Load-indentation curves for four-cover pipe](image)
6.4.2 Damage predictions for four-cover pipe

Both matrix cracking and delamination are predicted to occur in the pipe due to indentation, with matrix cracking appearing earlier than delamination. The critical loads for the first appearance of matrix cracking and delamination in the four-cover pipe are listed in Table 6.4. The only pronounced difference in the comparison of results between test data and prediction is the critical load for the first appearance of matrix cracking. It is likely that the property of transverse tensile strength used in the present model is smaller than the real value of the material and the neighbouring layers with different winding angle may enhance the strength of each unidirectional layer. It was also difficult to detect the first matrix cracking which is very tiny using naked eye during the test. The effect of in-situ strength is not considered in the present model. However, the predicted initiation of delamination is reasonably close to the test results.

Table 6.4: Critical loads for first matrix cracking and delamination in 4-cover pipe

<table>
<thead>
<tr>
<th></th>
<th>Matrix Cracking</th>
<th>Delamination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction (kN)</td>
<td>0.63</td>
<td>1.58</td>
</tr>
<tr>
<td>Test (kN)</td>
<td>1.3~1.7</td>
<td>1.4~1.8</td>
</tr>
</tbody>
</table>

In four-cover pipe, delamination is initially formed at the outermost interface at the indentation level of 3 mm as shown in Fig. 6.9. For a better viewing, the damaged zone in Fig. 6.9 is circled by the projected region under the indenter and the colour code for delaminated zone is red. In addition, the symmetrical pattern of the delamination shown in the figure is a 180° rotational symmetry about the z-axis passing through the centreline of the indenter.

At the early stages of delamination propagation, the delamination sizes are small and merely have effect on the stiffness. After the delamination initiates and propagates, the occurrence of delamination redistributes the stresses and causes the new delamination initiation at the other interfaces. In four-cover pipe, delaminations are predicted to occur at each interface between layers at the indentation level of 4.31 mm as shown in Fig. 6.9. The recorded load-drop in the load-indentation curve can be justified by the growth of the multiple delaminations in the pipe. The delaminations at different interfaces take on a similar size when multiple delaminations are captured in the first-
half stage of indentation. However, the innermost interface away from the indenter has the major delamination compared to the upper interfaces when the indentation reaches 10 mm. The delamination at the innermost interface propagates faster than the delaminations at the upper interfaces. Similar phenomenon has been observed in the flat plate in section 5.3.2.
Indentation = 3.0 mm, Load = 1.58 kN

Indentation = 4.31 mm, Load = 2.08 kN
Delaminations at individual interfaces through the pipe wall thickness can be seen in Fig. 6.10 at the indentation level of 7.5 mm. The predicted overlays of the outermost and the innermost delaminations are in good agreement with the test observation, as shown in Fig. 6.11. Moreover, the predicted delaminations by the present model take on a rotational symmetry shape which is similar to the observations in the experiment. It is worth noting that a dead zone was identified in the test observations where no delamination is found. It is due to the fact that there is high compressive stress in the through-thickness direction in the pipe just beneath the indenter which enhances the
fracture resistance as discussed in Chapter 3. According to the present simulation, a few interface elements in the region immediately under the indenter do not fail completely, i.e., $\omega < 1$. Therefore, the predictions capture this local phenomenon.

Figure 6.10: Delaminations at different interfaces in the four-cover pipe (indentation=7.5mm)

Figure 6.11: Overlaid delamination shape in four-cover pipe
The predicted relationship between the overlaid delamination area and the indentation energy is shown in Fig. 6.12. An approximately linear relationship between the indentation energy and delamination area can be seen after the rapid delamination growth reflected by the early stage of the increase in delamination area. Some observations were also made by Reid et al. (1996) with a similar distribution.

Figure 6.12: Overlaid delamination area versus incident energy for the pipe
Matrix cracking initiates first in the innermost layer of the pipe where the material is subjected mostly to the transverse tension, therefore, the matrix tensile failure is the early damage found at about 1.2 mm indentation. Although matrix cracking initiates first in the innermost layer of the pipe, it gradually spreads to other layers radially outwards as indentation continues to increase. Furthermore, numerical simulation indicates that the matrix compression failure due to the high compressive stresses in the contact region right under the indenter is also a predominant failure mode taking place in the early stage of indentation.

The distributions of matrix cracking in 4-cover pipe at indentation level of 10 mm are shown in the Fig. 6.13. Both matrix tensile and compressive failures are predicted and presented. Under the indenter, the inner layers seem to experience the severe damage of matrix cracking. It is worth noticing that no matrix compressive failure found in lamina 2 (-55° layer/upper layer of innermost cover) and lamina 4 (-55° layer) at 10 mm indentation, while a greater matrix tensile failure exists. A delamination causes part of the layers to undergo tension. After the pipe is delaminated, tension builds up in the lower layer of each cover and causes matrix tensile failure there as well as in the layers in the outside covers. For the upper layer of two top covers (Lamina 6&8 in Fig. 6.13) is under compression and also subjected to tension due to the local large deformation near the delamination front. The tensile matrix cracking in the upper layer of two top covers gradually extends as the delamination grows. Around the outer layer of fourth/outermost cover (lamina 8), there is an area on the sides of the loading point in circumferential direction which is dominated by the circumferential bending. The damage in this area is matrix tensile failure while similar phenomena has been mentioned earlier in section 6.2.2.

No further detailed comparisons can be made between the predicted distribution of matrix cracking and the test results because no experimental results on this aspect are available in Reid et al. (2000).
Figure 6.13: The distributions of matrix cracking in 4-cover pipe at indentation level of 10 mm.
(left: matrix tensile failure; right: matrix compressive failure)
6.4.3 Leakage path due to indentation in four-cover pipe

A pipe is considered failed when leakage occurs. It is assumed that a leakage path is composed of the matrix cracking and delamination in each layer and interface, respectively. As the internal fluid may come out through the pipe in the wall-thickness direction, each layer and interface should have been damaged. When each layer of the pipe is cracked transversely at the same axial and circumferential position, leakage will take place. Another possible scenario is when each cover has at least one position where transverse matrix cracking has occurred and delaminations exist at all interfaces between the covers. The fluid can find a way throughout the pipe by linking the cracked position and delaminated interface together to create a path. Therefore, it is not necessary for the cracked position in each cover to appear at the same location in the axial direction.

Leakage path requires each layer in the pipe is fully cracked. However, it has been revealed in Chapter 3 that matrix crack may not always appear across the entire thickness of a layer or group of layers of same orientation, especially under bending deformation. In ABAQUS, multiple integrations are employed for the cross section of continuum shell elements. To ensure a fully cracked layer, damage parameter for matrix cracking is checked at all section points per layer which should reach unity ($d_{22}^{m} = 1$).

Fig. 6.14(a) presents the damage status across the thickness of the outermost cover at ten different sets in their surface plane. Set \{B\} is the set of cracked material points in each surface plane (marked in blue colour). Set \{A\} is specified as the intersection of ten sets, which will be (a) cracked material point at the same position across the entire thickness of the outermost cover (marked in red colour around its contour). The middle surface of the shell (fraction=0.0) is defined as the top surface of layer 1 and also the bottom surface of layer 2. The cover is fully cracked, i.e. set \{A\} is not an empty set, as failure can be found at the same position through the entire shell section from the bottom surface of layer 1 (fraction=-1) to the top surface of layer 2 (fraction=1). However, the cover is only partially cracked at the position where \{B\} \{A\}. This is in consistence with the findings in Chapter 3 that matrix cracking initiates in the layer, but may not propagate across the entire layer. Figure 6.14(b) presents the fully cracked position in the outermost cover. By removing the cracked material points across the entire thickness of the cover, one continuum shell element at the cover will be totally
deleted, while other continuum shell elements remain partly visible. It is clear that the hollow rectangular region is the position where all the section points per layer is cracked.

Figure 6.14(a): Damaged elements in the shell section within the outermost cover (indentation =6.3mm).

Figure 6.14(b): Upper: Damaged area at top surface of layer 2 (indentation=6.3mm); Lower: Overlapping area of two layers across the entire thickness of outermost cover (element deletion).
Fig. 6.15 shows the first leakage path formed in the pipe. The contour of cracked region/elements in all the covers and the delaminations at all the interfaces through the pipe wall are also demonstrated. The leakage path between innermost cover to outermost cover is established by overlapping the damaged region in each cover and interface. It is clear that more materials are cracked in the inner covers. There is only one rectangular region/element in the outermost cover showing the single exit for leakage path to pass through the outer surface. It can also be seen that each layer of the pipe is cracked at one same position in the pipe surface plane. Matrix cracks alone can form the leakage path in this case. This does not indicate that delaminations make no contribution to the formation of this leakage path. The existence of delamination makes it easy for more matrix cracking to occur.

The critical indentation for the formation of the first leakage path is found to be 6.3 mm. The predicted critical indentation for the first leakage path is approximately 2 mm lower than the test result (Reid et al., 2000). The error in the input data of intra-laminar fracture energy is convinced to contribute partially to this discrepancy.

In the leakage criterion, matrix tensile failure is assumed to be the primary damage for establishing leakage path. However, the existence of delamination has a much larger effect on leakage path than matrix cracking. Delamination reduces the transverse stiffness of the pipe and cause local deflections which induce matrix tensile failure due to large tensile strains. While the size of delamination is larger/wider enough to cover the cracked surface, the leakage path can be linked seamlessly between two covers.
Figure 6.15: A schematic diagram for first leakage path (indentation=6.3 mm)
6.4.4 Prediction of Indentation on Five-cover Pipe

The predicted load versus indentation curve for five-cover pipe is given in Fig. 6.16 along with the test results. The prediction is close to the test data but with a much higher slope in the further loading. However, the discrepancy in the comparison of results between the prediction and tests in five-cover pipes has yet to be discovered.

The critical loads for the first matrix cracking and delamination in the five-cover pipe are listed in Table 6.5. Results found in five-cover pipe are similar to the four-cover pipe so the discussion can be referred to the previous section. The predicted overlay of multiple delaminations is also in good agreement with the test observation as shown in Fig. 6.17.

<table>
<thead>
<tr>
<th>Matrix Cracking</th>
<th>Delamination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prediction (kN)</td>
<td>Test (kN)</td>
</tr>
<tr>
<td>Prediction (kN)</td>
<td>Test (kN)</td>
</tr>
<tr>
<td>2.62</td>
<td>3.7~4.3</td>
</tr>
<tr>
<td>3.51</td>
<td>3.9~4.6</td>
</tr>
</tbody>
</table>
Figure 6.17: Overlaid delamination shape in five-cover pipe
Matrix cracking and delamination are the two dominant damage modes in filament-wound pipes under lateral loading. To investigate their effects on the behaviour of the pipes, analyses have been performed under different considerations: (a) no damage was considered, (b) matrix cracking alone considered, (c) delamination alone considered and (d) both matrix cracking and delamination considered. The predicted load-indentation curves from these simulations are shown in Fig. 6.18. With damage being suppressed (i.e. no damage considered), the loading curve with linear elastic behaviour shows higher stiffness than the one with only the intra-laminar damage model implemented. As mentioned earlier in section 6.2.2, the discrepancy between these two curves can be seen as the stiffness reduction in material properties resulting from the intra-laminar damage and also shear nonlinearity. On the other hand, delamination causes more stiffness reduction than matrix cracking and the load-indentation curve is close to the experimental results but on the stiffer side. When both damage modes are considered simultaneously in the model, the most significant stiffness reduction can then be seen and the result is closer to the experimental one.

The load drops on the indentation curves in Fig.6.18 are caused by the initiation and growth of delaminations in the pipe. Earliest load-drop is predicted when both matrix cracking and delamination are modelled. This indicates that the inclusion of the matrix cracking in the simulation prompts initiation and propagation of delaminations in the pipe to take place earlier than the case when matrix cracking is excluded from the simulation. Matrix cracking results in redistribution of the stresses and simultaneously affects the initiation and growth of delamination.

The predicted delaminations in the pipe from delamination alone model and combine matrix cracking and delamination simulation immediately after the load-drops are shown in Fig. 6.19. Although the size and shape of delamination under different damage modes are not exactly identical, the delamination size in the delamination alone case is slightly bigger than the case of combined damage modes, the indentation levels for these two cases are much different. Much larger delamination would be seen if results were presented at the same indentation level for the combined damage simulation as the matrix alone simulation. This is a good example showing that the effects of interactions between the different modes of damage.
Figure 6.18: Comparison between models with different damage modes

Delamination alone, Indentation = 5.1 mm
Delamination alone, Indentation = 5.1 mm
MC&Delamination, Indentation = 4.31 mm

MC&Delamination, Indentation = 4.31 mm
A sensitivity study made by varying the value of viscosity has also been carried out. Fig. 6.20 gives the load-indentation results obtained under different values of the viscosity parameter. Results show that viscous regularization has changed the failure strength while it did not markedly affect the overall damage distribution. However, higher value of viscosity parameter might overestimate the failure strength, even though the convergence is noticeably improved and the simulation takes less time to complete. The two curves with lower value of viscosity parameter are almost identical. Thus, a viscosity of 0.001 is a more reasonable choice for present studies.
Figure 6.20: Load versus indentation curves for different values of the viscosity parameter
6.5 Summary

The proposed model described in Chapter 5 is implemented into Finite Element Code via user subroutine UMAT. It has been proved that the intra-laminar and inter-laminar damage models can predict the response of filament-wound pipe subjected to lateral indentation successfully.

The delamination predicted by the inter-laminar model show a good agreement with experimental observations. Load versus displacement curves obtained from the experiments have been predicted fairly well in the present model. The damage distribution of matrix cracking in the laminae of the pipe have been demonstrated and discussed. The model brings the analysis of matrix cracking and delamination together as both exists in practical application of laminated structures. A leakage path is found due to indentation. The findings in Chapter 3 contributed to the establishment of the criterion used in detecting leakage path.

The effect of different damage modes has been addressed. It is found that delamination causes more reduction in the stiffness of the pipe than matrix cracking. The interaction between matrix cracking and delamination brings early occurrence of delamination in the pipe and more matrix cracking.
Chapter 7
Conclusions and Recommendations for Future Research Studies

7.1 Introduction

This chapter summarises the main findings of the study in each of these researches: damage modelling using cohesive/interface elements, development of new inter-laminar and/or intra-laminar damage model, numerical simulation of laminated structures under tension, bending, and quasi-static impact loading. In addition, recommendations for future research studies to improve or extend current works are also provided.

7.2 Conclusions

The conclusions can be extracted from the following research tasks:

7.2.1 Meso-mechanics modelling of damage development in composite structures

i. The results provided in the thesis can be regarded as an extensive body of evidence that the cohesive elements are capable of modelling matrix cracking and delamination in laminated composites and their interactions.

ii. Damage formation process in cross-ply laminates shows some evidences such that matrix cracking is the pre-dominant failure in the structure. Moreover,
whether transverse matrix cracking propagates across the entire ply thickness is affected by the layup configuration and loading conditions. It has also been seen in the numerical simulation that a doubling multiplication process occurs in [0°/90°/0°] cross-ply laminates and a staggered cracking formation process is captured in [90°/0°/90°] cross-ply laminates which is frequently observed in the experiment.

iii. Delaminations are normally induced at the tips of transverse matrix crack in the layers at the outer surface of the laminate and further propagate along the interface. However, delaminations usually do not occur at the tips of matrix cracks in the inner layers due to the constraints imposed by the neighbouring layers oriented in other directions.

iv. Both delamination initiation and saturated matrix crack density depend on various parameters such as fracture toughness, stiffness of the material and thickness of 90° layers.

7.2.2 Development of a new cohesive zone/interface element model

i. A new cohesive zone/interface element model has been developed successfully. Friction is included in the interfacial constitutive law to consider the effect of through-thickness compressive stress on Mode II fracture resistance.

ii. Friction is the main source for increasing the Mode II fracture resistance when there exists high compressive through-thickness stress on the interface. Results show that the proposed model can provide more accurate delamination stress than conventional cohesive zone model.

iii. Friction should be considered in all the regions on the interface no matter it is damaged, newly damaged or being damaged.

7.2.3 Modelling inter- and intra-laminar damage by using new CDM based damage model

i. An inter-laminar damage model has been proposed in the context of continuum damage mechanics. Numerical simulations were carried out to validate the model against experimental results regarding delamination in double-cantilever
beam test and multiple delaminations in composite plate under quasi-static indentation. The locations, shapes and sizes of delaminations were all predicted with satisfactory accuracy.

ii. Models for intra-laminar damage modes, fibre failure in tension/compression, matrix cracking failure in tension/compression and shear failure, have been developed based on continuum damage mechanics and plane stress formulation. The onset of damage is predicted by stress/strain failure criteria. The damage evolution law is based on fracture energy dissipation, which is associated with each failure mode regardless of mesh refinement. The response of the material in longitudinal and transverse direction is assumed to be bilinear, and the response of the material in shear direction is described as linear elastic, nonlinear plastic and linear softening. The model has been proven to be capable of reproducing experimental results with good accuracy in terms of load/indentation responses and extent of matrix cracking damage. Simulation results also show the important role played by the matrix tensile/compressive failure and shear nonlinearity.

iii. Both inter- and intra-laminar damage models have been implemented into commercial finite element code ABAQUS via its user-subroutine UMAT for practical applications. The issue related to numerical implementation, e.g. convergence in the softening regime, is addressed.

7.2.4 Modelling damage in filament-wound pipe under lateral indentation

i. The two continuum damage models have been successfully applied to composite pipes under lateral indentation. The simulation results show an excellent agreement with experimental observations including load/indentation responses and multiple delaminations shape and size.

ii. Two major failure modes, matrix cracking and delamination, show an interactions between each other in the damage initiation as well as propagation. Matrix cracking is found to be the first damage mode occurred in the pipe and its existence leads to early onset of delaminations in the pipe.

iii. An algorithm is developed for detecting leakage path in the pipe formed by matrix cracking in each layers and delamination in each interface. Each single
layer must be cracked across its entire thickness to let fluids go through.

7.3 Recommendation for future research works

Since the research work in this thesis aimed to model the damage in composite structures, it was not possible to cover all aspects of the problem. A number of further research studies can be pursued to improve the understanding in this area:

i. In the present study, the cohesive elements were placed in 90° layers at predefined positions when modelling the formation of matrix cracking. However, matrix cracks may occur at other locations in real practice. Therefore, cohesive elements should be placed at more locations, ideally, between any neighbouring solid elements. Random distribution, e.g. Weibull distribution, of material properties for cohesive elements and solid elements should also be introduced in the model to reflect the real practice. This makes matrix cracking take place at where it should occur, not at the predefined positions.

ii. The 2-D numerical simulations of interaction between matrix cracking and delamination in the cross-ply laminates can be further extended to 3D modelling and other layup configurations.

iii. In the newly developed interface element model, a further modification can be made to consider 3-D effect. Moreover, different coefficient of friction on the undamaged, damaging and fully damaged interface should be introduced to make the model rigorous, in particular in the damaging area.

iv. The interfacial constitutive law for inter-laminar damage model is implemented in an UMAT subroutine for the ABAQUS implicit solver (ABAQUS/Standard). A further development can be made by implementing the constitutive law into FE code through VUMAT subroutines for the ABAQUS explicit solver (ABAQUS/Explicit).

v. With reference to the numerical simulation in filament-wound pipe under lateral indentation, the present model can be further improved by considering the effects of fibre reorientation due to large deformation, i.e. Scissoring effects. The stress-strain relationship for a thermoelastic problem taking account of residual thermal stress can be added to present model for further needs.
vi. By accomplishing the future work listed in iii, the interface element model can further be implemented into FE code in conjunction with present intra-laminar damage model to take account of the effect of compressive through-thickness stress, especially in the prediction of delamination in pipe subjected to lateral indentation.

vii. Due to the time limitation, the analysis was not continued with an imposed internal pressure applied to the indentation-damaged pipe. The estimation of residual leakage pressure and burst pressure is expected to be carried out in the near future.
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evolution of matrix and interface related damage in [0/90]s laminates under
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1176-1183.

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Appendix (UMAT)

SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
1 RPL,DDSDDT,DRPLDE,DRPLDT,
2 STRAN,DSTRAN,TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,CMNAME,
3 NDI,NSHR,NTENS,NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,
4 CELENT,DFGRD0,DFGRD1,NOEL,NPT,LAYER,KSPT,KSTEP,KINC)

C
INCLUDE 'ABA_PARAM.INC'
C
CHARACTER*80 CMNAME
C
DIMENSION STRESS(NTENS),STATEV(NSTATV),
1 DDSDE(NTENS,NTENS),DDSDDT(NTENS),DRPLDE(NTENS),
2 STRAN(NTENS),DSTRAN(NTENS),TIME(2),PREDEF(1),DPRED(1),
3 PROPS(NPROPS),COORDS(3),DROT(3,3),DFGRD0(3,3),DFGRD1(3,3)
C
IF(CMNAME(1:4).EQ.'MAT1') THEN
    CALL UMAT_MAT1(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
1 RPL,DDSDDT,DRPLDE,DRPLDT,
2 STRAN,DSTRAN,TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,CMNAME,
3 NDI,NSHR,NTENS,NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,
4 CELENT,DFGRD0,DFGRD1,NOEL,NPT,LAYER,KSPT,KSTEP,KINC)
END IF
IF(CMNAME(1:4).EQ.'MAT2') THEN
    CALL UMAT_MAT2(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
1 RPL,DDSDDT,DRPLDE,DRPLDT,
2 STRAN,DSTRAN,TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,CMNAME,
3 NDI,NSHR,NTENS,NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,
4 CELENT,DFGRD0,DFGRD1,NOEL,NPT,LAYER,KSPT,KSTEP,KINC)
END IF
C
RETURN
END
C
*** constitutive material models for MAT1 (Laminates)
C
SUBROUTINE UMAT_MAT1(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
1 RPL,DDSDDT,DRPLDE,DRPLDT,
2 STRAN,DSTRAN,TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,CMNAME,
3 NDI,NSHR,NTENS,NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,
4 CELENT,DFGRD0,DFGRD1,NOEL,NPT,LAYER,KSPT,KSTEP,KINC)

INCLUDE 'ABA_PARAM.INC'

C

CHARACTER*80 MAT1

C

DIMENSION STRESS(NTENS),STATEV(NSTATV),
1 DDSDDE(NTENS,NTENS),DDSDDT(NTENS),DRPLDE(NTENS),
2 STRAN(NTENS),DSTRAN(NTENS),TIME(2),PREDEF(1),DPRED(1),
3 PROPS(NPROPS),COORDS(3),DROT(3,3),DFGRD0(3,3),DFGRD1(3,3)

DIMENSION EC(3),Q(3,3),QD(3,3),SM(3)

DIMENSION DQDDM(2,2),DQDDF(2,2)

DIMENSION ATEMP1(2),ATEMP2(2)

C

E11=PROPS(1) ! E1
E22=PROPS(2) ! E2
V12=PROPS(3) ! Nu12
G12=PROPS(4) ! G12

C

Xt=PROPS(5)  ! longitudinal tensile strength
Xc=PROPS(6)  ! longitudinal compressive strength
Yt=PROPS(7)  ! transverse tensile strength
Yc=PROPS(8)  ! transverse compressive strength
XY=PROPS(9)  ! maximum shear strain

C

GAMMAY=PROPS(10)  ! shear strain at yield point
A1=PROPS(11)
B1=PROPS(12)
C1=PROPS(13)
D1=PROPS(14)
E1=PROPS(15)
Gcft=PROPS(16)  ! fracture energy in fibre tensile dir
Gfcf=PROPS(17)  ! fracture energy in fibre compressive dir
Gcmt=PROPS(18)  ! fracture energy in matrix tensile dir
Gcmc=PROPS(19)  ! fracture energy in matrix compressive dir
ETA=PROPS(20)  ! viscosity for regularization
DF=STATEV(1)
DM=STATEV(2)
DS=STATEV(3)
DFVOLD=STATEV(4)
DMVOLD=STATEV(5)
DSVOLD=STATEV(6)
DL=STATEV(7)
DF1=STATEV(8)
DF2=STATEV(9)
DM1=STATEV(10)
DM2=STATEV(11)
DS1=STATEV(12)
DS2=STATEV(13)
SM(1)=STATEV(14)
SM(2)=STATEV(15)
SM(3)=STATEV(16)
EXXMAX=STATEV(17)
EXXMIN=STATEV(18)
EYYMAX=STATEV(19)
EYYMIN=STATEV(20)
EXYMAX=STATEV(21)
EXYMIN=STATEV(22)

C
DO I=1,3                  ! Calculate the strain
   EC(I)=STRAN(I)+DSTRAN(I)
END DO

C
DO I=1,3
   DO J=1,3
      Q(I,J)=0D0
   END DO
END DO

C=1D0-V12*V12*E22/E11
Q(1,1)=E11/C
Q(2,2)=E22/C
Q(3,3)=G12
Q(1,2)=V12*E22/C
DO I=2,3
DO J=1, I-1
Q(J,J)=Q(J,J)
END DO
END DO

C
E11ot=XT/Q(1,1)
E11oc=Xc/Q(1,1)
E22ot=Yt/Q(2,2)
E22oc=Yc/Q(2,2)
E33ot=XY
E33oc=XY
E33yc=GAMMAY
E33yt=GAMMAY

C
St=A1+B1*E33ot+C1*E33ot**2+D1*E33ot**3+E1*E33ot**4
Sc=-A1+B1*E33oc-C1*E33oc**2+D1*E33oc**3-E1*E33oc**4

C
E33pc=E33oc-Sc/G12
E33pt=E33ot-St/G12

C
E11ft=Gcft/CELENT/Xt*2D0
E11fc=Gcfc/CELENT/Xc*2D0
E22ft=Gcmt/CELENT/Yt*2D0
E22fc=Gcmc/CELENT/Yc*2D0
E33ft=(Gcmc/CELENT-2.102D+06-0.116545275D+06)/St*2D0+E33ot
E33fc=(Gcmc/CELENT-2.102D+06-0.116545275D+06)/Sc*2D0+E33oc

C
DDFDE=0D0
DDMDE=0D0
DDSDE=0D0

C
CALL KDAMAGE(EC,EXXMAX,EXXMIN,EYMAX,EYMIN,EXYMAX,EXYMIN,E11ot,E11oc,E22ot,E22oc,E33ot,E33oc,E11ft,E11fc,
2 E22ft,E22fc,E33ft,E33fc,E33pc,E33pt,DF1,DF2,DM1,
3 DM2,DS1,DS2,DDFDE1,DDFDE2,DDMDE1,DDMDE2,DDSDE1,DDSDE2,
4 DDFDE,DDMDE,DDSDE,DF,DM,DS,DL)

C

C VISCOUS REGULARIZATION

C

184
DFV = ETA/(ETA + DTIME) * DFW + DTIME/(ETA + DTIME) * DF
DMV = ETA/(ETA + DTIME) * DMV + DTIME/(ETA + DTIME) * DM
DSV = ETA/(ETA + DTIME) * DSV + DTIME/(ETA + DTIME) * DS

CALL KQ(Q, QD, E11, E22, V12, G12,
  1 EC, GAMMAY, B1, C1, D1, E1, EXYMAX, EXYMIN, E33yc,
  2 E33yt, E33pc, E33pt, E33ot, E33oc, E33ft, E33fc, DFV, DMV,
  3 DSV, DDSDE, DTIME, ETA)

CALL KSTRESS(QD, EC, DSTRAN, SM, STRESS, GAMMAY,
  1 E33yc, E33yt, E33pc, E33pt, E33ot, E33oc, E33ft, E33fc, EXYMAX,
  2 EXYMIN, G12, A1, B1, C1, D1, E1, DSV)

DO I=1, 2
  DO J=1, 2
    DQDDM(I, J) = 0D0
    DQDDF(I, J) = 0D0
  END DO
END DO

DQDDF(1, 1) = -Q(1, 1)
DQDDF(1, 2) = -(1D0 - DMV) * Q(1, 2)
DQDDF(2, 1) = DQDDF(1, 2)

DQDDM(1, 2) = -(1D0 - DFV) * Q(1, 2)
DQDDM(2, 1) = DQDDM(1, 2)
DQDDM(2, 2) = -Q(2, 2)

C Update the Jacobian

DO I=1, 2
  ATEMP1(I) = 0D0
  DO J=1, 2
    ATEMP1(I) = ATEMP1(I) + DQDDF(I, J) * EC(J)
  END DO
END DO
DO J=1,2
  ATEMP2(I)=ATEMP2(I)+DQDDM(I,J)*EC(J)
END DO
END DO

C
DDSDDE(1,1)=QD(1,1)+(ATEMP1(1)*DDFDE)*DTIME/(DTIME+ETA)
DDSDDE(1,2)=QD(1,2)+(ATEMP2(1)*DDMDE)*DTIME/(DTIME+ETA)
DDSDDE(1,3)=QD(1,3)
DDSDDE(2,1)=QD(2,1)+(ATEMP1(2)*DDFDE)*DTIME/(DTIME+ETA)
DDSDDE(2,2)=QD(2,2)+(ATEMP2(2)*DDMDE)*DTIME/(DTIME+ETA)
DDSDDE(2,3)=QD(2,3)
DDSDDE(3,1)=QD(3,1)
DDSDDE(3,2)=QD(3,2)
DDSDDE(3,3)=QD(3,3)

C
STATEV(1)=DF
STATEV(2)=DM
STATEV(3)=DS
STATEV(4)=DFV
STATEV(5)=DMV
STATEV(6)=DSV
STATEV(7)=DL
STATEV(8)=DF1
STATEV(9)=DF2
STATEV(10)=DM1
STATEV(11)=DM2
STATEV(12)=DS1
STATEV(13)=DS2
STATEV(14)=STRESS(1)
STATEV(15)=STRESS(2)
STATEV(16)=STRESS(3)

C
IF(EC(1).GT.EXXMAX) THEN
  STATEV(17)=EC(1)
END IF
IF(EC(1).LT.EXXMIN) THEN
  STATEV(18)=EC(1)
END IF

C
IF(EC(2).GT.EYYMAX) THEN
  STATEV(19)=EC(2)
END IF
IF(EC(2).LT.EYYMIN) THEN
  STATEV(20)=EC(2)
END IF
C
IF(EC(3).GT.EXYMAX) THEN
  STATEV(21)=EC(3)
END IF
IF(EC(3).LT.EXYMIN) THEN
  STATEV(22)=EC(3)
END IF
C
RETURN
END
C
C
C *** Calculate local reduced stiffness matrix
C
SUBROUTINE KQ(Q,QD,E11,E22,V12,G12,
  1 EC,GAMMAY,B1,C1,D1,E1,EXYMAX,EXYMIN,E33yc,
  2 E33yt,E33pc,E33pt,E33ot,E33ft,E33fc,DFV,DMV,
  3 DSV,DDSD,DTIME,ETA)
INCLUDE 'ABA_PARAM INC'
DIMENSION Q(3,3),QD(3,3),EC(3)
C
DO I=1,3
  DO J=1,3
    QD(I,J)=Q(I,J)
  END DO
END DO
C
QD(1,1)=Q(1,1)*(1D0-DFV)
QD(1,2)=Q(1,2)*(1D0-DFV)*(1D0-DMV)
QD(2,1)=QD(1,2)
QD(2,2)=Q(2,2)*(1D0-DMV)
QD(3,3)=Q(3,3)*(1D0-DSV)
C
IF(DSV.GE.1D0) THEN
  QD(3,3)=0D0
  RETURN
END IF

C

IF(EC(3).LT.EXYMIN.AND.EC(3).GT.E33fc) THEN
  QD(3,3)=Q(3,3)*(1D0-DSV)+(-Q(3,3)*(|EC(3)|-E33pc)*DDSDE)*DTIME/(DTIME+ETA)
END IF

C

IF(EC(3).LT.EXYMIN.AND.EC(3).GT.E33oc) THEN
  QD(3,3)=B1+2D0*C1*EC(3)+3D0*D1*EC(3)**2+4D0*E1*EC(3)**3
END IF

C

IF(EC(3).LT.EXYMAX.AND.EC(3).GT.E33ot) THEN
  QD(3,3)=B1+2D0*C1*EC(3)+3D0*D1*EC(3)**2+4D0*E1*EC(3)**3
END IF

C

RETURN
END

C

C *** Calculate the Stress
C

SUBROUTINE KSTRESS(QD,EC,DSTRAN,SM,STRESS,GAMMAY,
  1 E33yc,E33yt,E33pc,E33pt,E33oc,E33ot,E33fc,EXYMAX,
  2 EXYMIN,G12,A1,B1,C1,D1,E1,DSV)
  INCLUDE 'ABA_PARAM.INC'
  DIMENSION QD(3,3),EC(3),DSTRAN(3),SM(3),STRESS(3)

  DO I=1,2
    STRESS(I)=0D0
    DO J=1,2
      STRESS(I)=STRESS(I)+QD(I,J)*EC(J)
    END DO
  END DO
END

C
IF(DSV.GE.1D0) THEN
STRESS(3)=0D0
RETURN
END IF
C
STRESS(3)=SM(3)+QD(3,3)*DSTRAN(3)
C
IF(EC(3).LT.EXYMIN.AND.EC(3).GT.E33fc) THEN
STRESS(3)=G12*(1D0-DSV)*(EC(3)-E33pc)
END IF
C
IF(EC(3).LT.EXYMIN.AND.EC(3).GE.E33oc) THEN
STRESS(3)=A1+B1*EC(3)**2+D1*EC(3)**3-E1*EC(3)**4
END IF
C
IF(EC(3).GT.EXYMAX.AND.EC(3).GT.EXYMAX.AND.DF1.LT.1D0) THEN
IF(EC(1).GT.E11ot.AND.EC(1).GT.EXXMAX.AND.DF1.LT.1D0) THEN
DF1=E11ft*(EC(1)-E11ot)/EC(1)/(E11ft-E11ot)
END IF
SUBROUTINE KDAMAGE(EC,EXXMAX,EXXMIN,EYYMAX,EYYMIN,EXYMAX,EXYMIN,E11ot,E11oc,E22ot,E22oc,E33ot,E33oc,E11ft,E11fc,
2 E22ft,E22fc,E33ft,E33fc,E33pc,E33pt,DF1,DF2,DM1,
3 DM2,DS1,DS2,DDFDE1,DDFDE2,DDMDE1,DDMDE2,DDSDE1,DDSDE2,
4 DDFDE,DDMDE,DDSDE,DF,DM,DS,DL)
INCLUDE 'ABA_PARAM.INC'
DIMENSION EC(3)
C
IF(EC(1).GT.E11ot.AND.EC(1).GT.EXXMAX.AND.DF1.LT.1D0) THEN
DF1=E11ft*(EC(1)-E11ot)/EC(1)/(E11ft-E11ot)
DDFDE1 = E11f*E11ot/EC(1)**2D0/(E11f-E11ot)
END IF
IF(EC(1).LT.E11oc.AND.EC(1).LT.EXXMIN.AND.DF2.LT.1D0) THEN
  DF2 = E11f*EC(1)E11oc/EC(1)/(E11f-E11oc)
  DDFDE2 = E11f*E11oc/EC(1)**2D0/(E11f-E11oc)
END IF
END IF

C
IF(DF1.GT.1D0) THEN
  DF1 = 1D0
  DDFDE1 = 0D0
END IF
IF(DF2.GT.1D0) THEN
  DF2 = 1D0
  DDFDE2 = 0D0
END IF
IF(DM1.GT.1D0) THEN
  DM1 = 1D0
  DDMDE1 = 0D0
END IF
IF(DM2.GT.1D0) THEN
  DM2 = 1D0
  DDMDE2 = 0D0
END IF
C
IF(DF1.GT.1D0) THEN
  DF1 = 1D0
  DDFDE1 = 0D0
END IF
IF(DF2.GT.1D0) THEN
  DF2 = 1D0
  DDFDE2 = 0D0
END IF
IF(DM1.GT.1D0) THEN
  DM1 = 1D0
  DDMDE1 = 0D0
END IF
IF(DM2.GT.1D0) THEN
  DM2 = 1D0
  DDMDE2 = 0D0
END IF

C
IF(DF1.GT.1D0) THEN
  DF1 = 1D0
  DDFDE1 = 0D0
END IF
IF(DF2.GT.1D0) THEN
  DF2 = 1D0
  DDFDE2 = 0D0
END IF
IF(DM1.GT.1D0) THEN
  DM1 = 1D0
  DDMDE1 = 0D0
END IF
IF(DM2.GT.1D0) THEN
  DM2 = 1D0
  DDMDE2 = 0D0
END IF

C
IF(DF1.GT.1D0) THEN
  DF1 = 1D0
  DDFDE1 = 0D0
END IF
IF(DF2.GT.1D0) THEN
  DF2 = 1D0
  DDFDE2 = 0D0
END IF
IF(DM1.GT.1D0) THEN
  DM1 = 1D0
  DDMDE1 = 0D0
END IF
IF(DM2.GT.1D0) THEN
  DM2 = 1D0
  DDMDE2 = 0D0
END IF

C
IF(DF1.GT.1D0) THEN
  DF1 = 1D0
  DDFDE1 = 0D0
END IF
IF(DF2.GT.1D0) THEN
  DF2 = 1D0
  DDFDE2 = 0D0
END IF
IF(DM1.GT.1D0) THEN
  DM1 = 1D0
  DDMDE1 = 0D0
END IF
IF(DM2.GT.1D0) THEN
  DM2 = 1D0
  DDMDE2 = 0D0
END IF

C
IF(DF1.GT.1D0) THEN
  DF1 = 1D0
  DDFDE1 = 0D0
END IF
IF(DF2.GT.1D0) THEN
  DF2 = 1D0
  DDFDE2 = 0D0
END IF
IF(DM1.GT.1D0) THEN
  DM1 = 1D0
  DDMDE1 = 0D0
END IF
IF(DM2.GT.1D0) THEN
  DM2 = 1D0
  DDMDE2 = 0D0
END IF

C
IF(DF1.GT.1D0) THEN
  DF1 = 1D0
  DDFDE1 = 0D0
END IF
IF(DF2.GT.1D0) THEN
  DF2 = 1D0
  DDFDE2 = 0D0
END IF
IF(DM1.GT.1D0) THEN
  DM1 = 1D0
  DDMDE1 = 0D0
END IF
IF(DM2.GT.1D0) THEN
  DM2 = 1D0
  DDMDE2 = 0D0
END IF

C
IF(DF1.GT.1D0) THEN
  DF1 = 1D0
  DDFDE1 = 0D0
END IF
IF(DF2.GT.1D0) THEN
  DF2 = 1D0
  DDFDE2 = 0D0
END IF
IF(DM1.GT.1D0) THEN
  DM1 = 1D0
  DDMDE1 = 0D0
END IF
IF(DM2.GT.1D0) THEN
  DM2 = 1D0
  DDMDE2 = 0D0
END IF
DM2=1D0
DDMDE2=0D0
END IF
IF(DS1.GT.1D0) THEN
DS1=1D0
DDSDE1=0D0
END IF
IF(DS2.GT.1D0) THEN
DS2=1D0
DDSDE2=0D0
END IF
C
IF(DF1.GT.DF) THEN
DF=DF1
DDFDE=DDFDE1
END IF
IF(DF2.GT.DF) THEN
DF=DF2
DDFDE=DDFDE2
END IF
IF(DM1.GT.DM) THEN
DM=DM1
DDMDE=DDMDE1
END IF
IF(DM2.GT.DM) THEN
DM=DM2
DDMDE=DDMDE2
END IF
IF(DS1.GT.DS) THEN
DS=DS1
DDSDE=DDSDE1
END IF
IF(DS2.GT.DS) THEN
DS=DS2
DDSDE=DDSDE2
END IF
C
DL=DM+DS-DS*DM
RETURN
**User-Subroutine for cohesive elements**

```fortran
SUBROUTINE UMAT_MAT2(STRESS, STATEV, DDSDDE, SSE, SPD, SCD,
  RPL, DDSDDT, DRPLDE, DRPLDT,
  STRAN, DSTRAN, TIME, DTIME, TEMP, DTEMP, PREDEF, DPRED, MAT2,
  NDI, NSHR, NTENS, NSTATV, PROPS, NPROPS, COORDS, DROT, PNEWDT,
  CELENT, DFGRD0, DFGRD1, NOEL, NPT, LAYER, KSTEP, KINC)

INCLUDE 'ABA_PARAM.INC'

CHARACTER*80 MAT2

DIMENSION STRESS(NTENS), STATEV(NSTATV),
  DDSDDE(NTENS, NTENS), DDSDDT(NTENS), DRPLDE(NTENS),
  STRAN(NTENS), DSTRAN(NTENS), TIME(2), PREDEF(1), DPRED(1),
  PROPS(NPROPS), COORDS(3), DROT(3, 3), DFGRD0(3, 3), DFGRD1(3, 3)

DIMENSION Ek(3), St(3), Gc(3), S(3), G(3), TDSP(3), DDSP(3),
  CC(3, 3), S0(3), FS(3), SS(3), SW(3)

Ek(1) = PROPS(1)
Ek(2) = PROPS(2)
Ek(3) = PROPS(3)
St(1) = PROPS(4)
St(2) = PROPS(5)
St(3) = PROPS(6)
Gc(1) = PROPS(7)
Gc(2) = PROPS(8)
Gc(3) = PROPS(9)

S0(1) = STRESS(1)
S0(2) = STRESS(2)
S0(3) = STRESS(3)

DDSP(1) = DSTRAN(1)
```
DDSP(2)=DSTRAN(2)
DDSP(3)=DSTRAN(3)

C
TDSP(1)=DDSP(1)+STRAN(1)
TDSP(2)=DDSP(2)+STRAN(2)
TDSP(3)=DDSP(3)+STRAN(3)

C
G(1)=STATEV(23)
G(2)=STATEV(24)
G(3)=STATEV(25)
OMEGA=STATEV(26)

C

C *** Damage parameter & incremental constitutive law
C

C
Eg=G(1)+G(2)+G(3)
IF(Eg.LE.1D-12) Eg=1D-12
CALL KZERO(3,3,CC)
CALL KZERO(3,1,S)

C
IF(OMEGA.GE.1D0) THEN
IF(TDSP(1).LT.0D0) CC(1,1)=Ek(1)
IF(TDSP(1).LT.0D0) S(1)=Ek(1)*TDSP(1)
GO TO 800
ENDIF

C
OMEG=1D0-OMEGA
DO 100 I=1,50
DO 10 J=1,3
CC(J,J)=OMEG*Ek(J)
IF(TDSP(1).LE.0D0) CC(1,1)=Ek(1)
S(J)=CC(J,J)*TDSP(J)
SS(J)=S(J)/St(J)
10 CONTINUE
IF(TDSP(1).LE.0D0) SS(1)=0D0
Sg=SS(1)*SS(1)+SS(2)*SS(2)+SS(3)*SS(3)
F=Sg+Eg-1D0
IF(I.EQ.1.AND.F.LE.1D-06) GO TO 200
DO 40 J=1,3
   FS(J)=2D0*SS(J)/St(J)
40 CONTINUE
   SW(1)=Ek(1)*TDSP(1)
   SW(2)=Ek(2)*TDSP(2)
   SW(3)=Ek(3)*TDSP(3)
   IF(TDSP(1).LT.0D0) SW(1)=0D0
   FW=FS(1)*SW(1)+FS(2)*SW(2)+FS(3)*SW(3)
   DMEG=-F/FW
   OMEGA=OMEGA+DMEG
100 CONTINUE

C
   IF(DABS(DMEG).GT.1D-06) STOP
C
   DD=FS(1)*SW(1)+FS(2)*SW(2)+FS(3)*SW(3)
   D1=OMEGA*Ek(1)*FS(1)/DD
   D2=OMEGA*Ek(2)*FS(2)/DD
   D3=OMEGA*Ek(3)*FS(3)/DD
   CC(1,1)=-SW(1)*D1+CC(1,1)
   CC(1,2)=-SW(1)*D2
   CC(1,3)=-SW(1)*D3
   CC(2,1)=-SW(2)*D1
   CC(2,2)=-SW(2)*D2+CC(2,2)
   CC(2,3)=-SW(2)*D3
   CC(3,1)=-SW(3)*D1
   CC(3,2)=-SW(3)*D2
   CC(3,3)=-SW(3)*D3+CC(3,3)
   OMEGA=1D0-OMEGA
C
200 IF(OMEGA.GT.0D0) THEN
   DO 300 I=1,3
      DG=0.5D0*(S0(I)+S(I))*DDSP(I)
      G(I)=G(I)+DG/Gc(I)
      IF(G(I).LT.0D0) G(I)=0D0
   300 CONTINUE
   ENDIF
   Eg=G(1)+G(2)+G(3)
   IF(Eg.GE.0.999D0) OMEGA=1.001D0
C
   IF(OMEGA.GE.1D0) THEN
CALL KZERO(3,3,CC)
CALL KZERO(3,1,S)
IF(TDSP(1).LT.0D0) CC(1,1)=Ek(1)
IF(TDSP(1).LT.0D0) S(1)=Ek(1)*TDSP(1)
ENDIF

800 DDSDDE(1,1)=CC(1,1)
DDSDDE(1,2)=CC(1,2)
DDSDDE(1,3)=CC(1,3)
DDSDDE(2,1)=CC(2,1)
DDSDDE(2,2)=CC(2,2)
DDSDDE(2,3)=CC(2,3)
DDSDDE(3,1)=CC(3,1)
DDSDDE(3,2)=CC(3,2)
DDSDDE(3,3)=CC(3,3)
STRESS(1)=S(1)
STRESS(2)=S(2)
STRESS(3)=S(3)
STATEV(23)=G(1)
STATEV(24)=G(2)
STATEV(25)=G(3)
STATEV(26)=OMEGA
RETURN
END

C *** Set a matrix equal to zero
C
SUBROUTINE KZERO(M,N,A)
INCLUDE 'ABA_PARAM.INC'
DIMENSION A(M,N)
DO 10 I=1,M
DO 10 J=1,N
A(I,J)=0D0
10 CONTINUE
RETURN
END