Optimisation of Plate/Plate-Fin Heat Exchanger Design

A thesis submitted to The University of Manchester for the degree of Doctor of Philosophy in the Faculty of Engineering and Physical Sciences

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Optimisation of Plate/Plate-fin Heat Exchanger Design

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Abstract-PhD Thesis

With increasing global energy consumption, stringent environmental protection legislation and safety regulations in industrialised nations, energy saving has been put under high priority. One of the most efficient ways of energy reduction is through heat transfer enhancement for additional heat recovery. Applying compact heat exchanger is one of the main strategies of heat transfer enhancement. However, the application of compact heat exchangers is prohibited by the lack of design methodology. Therefore, the aim of this research is to tackle the problem of developing optimisation methodologies of plate/plate-fin heat exchanger design.

A mathematical model of plate-fin heat exchanger design is proposed to consider fin type selection with detailed geometry and imposed constraints simultaneously. The concept of mix-and-match fin type combinations is put forward to include all possible fin type combinations in a heat exchanger. The mixed integer nonlinear programming (MINLP) model can be converted to a nonlinear programming (NLP) model by employing continuous heat transfer and pressure drop correlations and considering the basic fin geometric parameters as continuous variables. The whole optimisation is based on volumetric minimisation or capital cost minimisation and completed by CONOPT solver in GAMS. Case studies are carried out to demonstrate the effectiveness and benefits of the new proposed methodology.

For plate heat exchangers, the design methodology is developed on the basis of plate-fin heat exchanger methodology, and takes phase change, plate pattern selection, flow arrangement and pressure drop constraints simultaneously. The phase change problem is tackled by dividing the whole process into several subsections and considering constant physical properties in each subsection. The performances of various flow arrangements are evaluated by correction factors of logarithmic mean temperature difference. For two-phase conditions, the heat transfer and pressure drop performance are predicted by continuous two-phase Nusselt number and Fanning friction factor correlations to avoid the MINLP problem. The optimisation is solved by CONOPT solver as well. The feasibility and accuracy of the new proposed methodology is examined by case studies.
Declaration

No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Kunpeng Guo
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Dedicated to My Father
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To the beloved parents and my wife for untiring love, thoughtfulness and financial support;

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Chapter 1 Introduction

Global demand for energy has risen steadily in the last 20 years in step with industrial development and population growth, which is shown in Table 1 (BP, 2004, 2015). Coupled with stringent environmental protection legislation and safety regulations in industrialised nations, energy saving has been paid more and more attention. One of the main strategies for energy saving is to recover the heat already generated in the process as efficiently and economically as possible. Heat recovery not only saves on primary energy, but also contributes to environmental protection due to reduced emissions. One way to improve heat recovery performance is through heat transfer enhancement. Nowadays, a significant number of thermal engineering researchers are seeking new heat transfer enhancement methods. Thermal reuse requires efficient and economically designed heat exchangers. The most efficient and widely accepted heat exchanger is compact heat exchangers (Kays and London, 1984).

<table>
<thead>
<tr>
<th>Energy Consumption</th>
<th>Million tonnes oil equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil</td>
<td>3204.4</td>
</tr>
<tr>
<td>Nature Gas</td>
<td>1876.7</td>
</tr>
<tr>
<td>Coal</td>
<td>2185.5</td>
</tr>
<tr>
<td>Nuclear</td>
<td>504.0</td>
</tr>
<tr>
<td>Hydroelectricity</td>
<td>539.5</td>
</tr>
<tr>
<td>Renewable Energy</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>8310.1</td>
</tr>
</tbody>
</table>

Compact heat exchangers are a class of heat exchangers that incorporate a large amount of heat transfer surface area per unit volume. The typical and commonly used compact heat
exchangers include plate heat exchangers and plate-fin heat exchangers (Kays and London, 1984). These compact heat exchangers are characterised by:

- Small minimum approach temperature: Compared with 10-20 °C in the shell and tube heat exchanger, the minimum approach temperature in the compact heat exchangers can be as low as 2-5 °C, which contributes to more heat recovery duties and less utility requirements.

- Large heat transfer surface area: the heat transfer surface area of compact heat exchangers can reach 1000 m²/m³, which is twice as large as that of shell and tube heat exchangers.

- High heat transfer coefficient: Due to small hydraulic diameter or inserted fins, the fluid turbulence is enlarged and the local convective heat transfer coefficient is enhanced correspondingly.

- Possibility of handling several streams in one unit: this unique structure gives an opportunity of reducing the number of heat exchangers and reduces the capital cost of heat exchangers.

Based on the above distinguished advantages, compact heat exchangers are widely used in various energy-intensive process industries, such as refrigeration, petrochemical plants, refineries and natural-gas processing (Hesselgreave, 2001).

### 1.1 Challenges for application of plate/plate-fin heat exchangers

The application of plate and plate-fin heat exchangers falls far behind shell and tube heat exchangers. The major challenges are the lack of generalised heat transfer and pressure drop correlations and design optimisation methodologies.

#### 1.1.1 Plate-fin heat exchangers

Plate-fin heat exchangers consist of a series of fin surfaces sandwiched between parting sheets and stacked together. The introduced fin, work as a secondary surface, can increase efficient heat transfer area and transfer heat to fluid through parting sheets, enlarge fluid turbulence and enhance the local convective heat transfer coefficient to increase the efficiency of heat transfer (Shah, 2003). According to Kays and London (1984), there are
approximately sixty standardised fin parts (plain fin, louvered fin, offset strip fin and wavy fin) with specific heat transfer and pressure drop performance. A major challenge in designing optimal multi-stream plate fin heat exchangers is the lack of fin selection method and the method how to address the discrete design problem caused by standardised fin types.

In the past decades, many publications proposed various design methodologies of multi-stream plate-fin heat exchangers. Shah (1982) and Cowell (1990) analysed individual fins in various ways: volume vs. power consumption, frontal area vs. power consumption, area goodness. But these qualitative analysis can only give a general guide on selection of fin types. Lee and Zhu (1999) proposed two new concepts: identical-fin concept and Z-Y graph to select optimum fin types to select fin types. This time-consuming screening method uses the same fin type for all streams in the early design stage and ignores the benefits of mix and match fin types, which may not reach the smaller plate fin heat exchanger size. Picon-Nunez et al. (2002) rearranged heat balance equations, proposed a new term “Volume Performance Index” and plotted a graph of VPI vs Reynolds number to select appropriate fin types. This trial and error design methodology is based on assumed Reynolds number and neglects the effect of imposed design constraints on fin selection, which will deviate from the real design. Recently, Peng, et al. (2008) and Yousefi, et al. (2012) took basic fin geometries as design variables and integrated into a cross flow plate fin heat exchanger design model to optimise total weight and total heat transfer area respectively. Discrete fin geometry variables result in a very complicated mixed-integer nonlinear programming (MINLP) optimisation design problem. Although Yousefi, et al. (2013) later developed a proposed variant of harmony search algorithm for design optimisation to solve these discrete variables, the method still cannot guarantee the global optimum design solution. In summary, few general optimisation methodologies of multi-stream plate-fin heat exchanger design include fin selectin and pressure drop consideration simultaneously and can find the optimum design solution promptly.

1.1.2 Plate heat exchangers

Plate heat exchangers, including gasket plate heat exchangers and welded plate heat exchangers, use metal plates to transfer heat between two fluids (Wang, 2007). Like plate-fin heat exchangers, various plate patterns have their specific heat transfer and pressure
drop performance. The large number of plates give a freedom of flow arrangement to achieve the required heat duty within the specific pressure drop. Currently, the challenges of plate heat exchanger application are unavailable industrial owned heat transfer and pressure drop performance data and the lack of general design optimisation methodology which can include selection of plate pattern, determination of flow arrangement and consider pressure drop simultaneously. Additionally, few design methodology addressed phase change problem, which is common in the heat transfer process.

The traditional design methodologies proposed by Cooper (1983) and Shah (1988), use either $\varepsilon$-NTU (number of heat transfer units) or LMTD (logarithmic mean temperature difference) thermal-hydraulic models and employ time-consuming trial and error methods to test different plate patterns with detailed geometries in order to consider pressure drop simultaneously, which is related to plate geometry and considered as constraints in the design. Wang and Sunden (2003) employed the correlations of Nusselt number and friction factor, which are functions of Reynolds number and chevron angle, to avoid time-consuming trial iterations. In this way, all possible plate patterns can be included and optimised by maximising the utilisation of pressure drop or minimising the total cost for with or without pressure drop consideration respectively in the optimal design methodology for plate heat exchangers. Both methodologies neglect the effect of flow arrangement on heat transfer and pressure drop performance. Still, the discrete optimisation problem caused by standardised plate patterns increases degree of difficulty in finding a global optimum design solution. To overcome the limitations of representing the problem as a mixed nonlinear programming (MINLP) problem, Gut and Pinto (2004) proposed a screening method to eliminate infeasible and sub-optimal solutions. In the optimisation design methodology, the detailed configuration is determined by minimising the total heat transfer area. Similarly, Picon-Nenuz and Martinez (2007) employed $\varepsilon$-NTU method and evaluated the temperature correction factor to determine flow arrangements. Najafi H. and Najafi D. (2010) developed a design optimisation methodology with multiply objectives of minimising pressure drop and maximising heat transfer coefficient, solved by genetic algorithm, to obtain the optimum plate heat exchanger configurations. These methodologies optimised flow arrangements but neglected the plate pattern selection. Arsenyeva et al. (2011) proposed a temperature difference matrix to include all possible flow arrangement by considering a
plate heat exchanger as a system of one pass blocks of plates. In the optimisation procedure, the total heat transfer area is considered as the objective function to find the optimum plate heat exchanger configuration. Based on Gut and Pinto’s model, Mota et al. (2014) proposed a system of normalised ordinary differential equations and wrote in the matrix form to include all possible flow arrangements. In these two methods, the plate pattern selection is completed by enumeration and the effect of chevron angle on heat transfer and pressure drop performance is neglected as well.

1.1.3 Optimisation
The plate/plate fin heat exchanger design includes integer variables determining fin type or plate pattern selection and flow arrangement options. Coupled with nonlinear heat transfer and pressure drop performance, the overall design problem is an MINLP problem. In the mid 1970’s, the MINLP problem was converted to MIP problem and solved by SCICONIC, which linked Special-Ordered-Set variables to represent low dimensional nonlinear terms by a piecewise linear approximation (Beale, 1980). In the mid 1980’s, Grossmann (1986) developed DICOPT, a general purpose algorithm for convex MINLP based on the outer approximation method, to find the global optimum solution. Since then, the MINLP problem was also converted to a LP or NLP problem (Grossmann, 1992; Leyffer, 2001). Peng, et al. (2008) and Yousefi, et al. (2012) used GA algorithm and improved harmony algorithm respectively to find the global optimum solution of plate fin heat exchanger. Guo and Liu (2014) employed genetic and Monte Carlo algorithm to optimise plate fin heat exchangers.

1.2 Objectives of this thesis
As mentioned in Section 1.1, the major challenge of plate/plate-fin heat exchanger application is the lack of general optimisation methodology of plate/plate-fin heat exchanger design. Therefore, this thesis proposes an optimisation approach for the plate-fin heat exchanger and plate heat exchanger design to find the optimum plate/plate-fin heat exchanger configurations. The aims of this thesis include:

a) Propose an optimisation methodology of plate-fin heat exchanger design. The proposed methodology includes:

- Develop a thermal-hydraulic design model of plate-fin heat exchangers.
• Develop a fin type selection approach and take account of heat transfer and pressure drop simultaneously.

• Address the discrete design problem caused by standardised fin types and their unique heat transfer and pressure drop performance.

• Address multi-stream plate heat exchanger design problem.

• Use of an optimisation algorithm that overcomes the local optima solutions.

b) Propose an optimisation methodology of plate heat exchanger design. The proposed methodology includes:

• Develop a thermal-hydraulic design model of plate heat exchangers.

• Develop a plate pattern approach and take account of heat transfer and pressure drop performance simultaneously.

• Develop an approach of flow arrangement determination to find the optimum flow arrangement

• Address the discrete design problem caused by standardised plate pattern and flow arrangement.

• Address multi-stream plate heat exchanger design problem.

• Use of an optimisation algorithm to find the global optimum design solution.

1.3 Overview of this thesis

This thesis is presented as a series of papers that have been published or submitted to relevant journals. Each chapter contains one publication. Chapter 2 and chapter 3 proposed design methodologies of plate-fin heat exchangers with identical fin category and mix-and-match fin types respectively. The optimisation of two-stream and multi-stream multi-pass plate heat exchanger with or without phase change are represented in chapter 4 and 5.

Chapter 2 presents an optimisation methodology of fin selection and thermal design of counter-current plate-fin heat exchangers, which is published in Applied Thermal Engineering.
Chapter 3 presents design optimisation of multi-stream plate-fin heat exchangers with multiple fin types, which is submitted to Applied Thermal Engineering for publication.

In chapter 4, the optimisation of two-stream multi-pass plate heat exchanger design is presented and submitted to Energy for publication.

Design optimisation of multi-stream multi-pass plate heat exchangers with phase change is presented in Chapter 5 and submitted to Energy for publication.

The thesis ends with a conclusions and suggestions for future work chapter that synthesises the work and their significant in the heat exchanger design field.
Chapter 2
Publication 1: Optimisation of Fin Selection and Thermal Design of Counter-current Plate-fin Heat Exchangers

Optimisation of Fin Selection and Thermal Design of Counter-current Plate-fin Heat Exchangers

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Abstract

A major challenge in designing optimal multi-stream plate-fin heat exchangers is the large number of combination of standardised fin geometries for various fin types to choose from, which adds discrete aspects to an already complicated design problem. In this work, a new design algorithm is proposed to address this issue. By treating basic fin geometries such as plate spacing, fin pitch, fin length, fin thickness as continuous variables for all the fin types, different fin types are characterised based on the work published by different researchers. Then by taking into account thermal hydraulic performance of different fin types, optimal fin types and their corresponding design parameters can be obtained simultaneously by minimising the total volume of heat exchanger. The design parameters can be rounded to the nearest standardised fin parts for a feasible design. A case study with a comparison of published results is carried out to demonstrate the effectiveness of the method.

Key words: Fin selection   Design  Optimisation  Continuous

Nomenclature

Parameters

A  Total heat transfer area of one side, m²
A_c  Free flow area of one side, m²
A_tr  Total heat exchanger front area, m²
b_st  Standardised plate spacing, m
c_st  Standardised fin pitch, m
c_p  Heat capacity, J/Kg °C
C  Capital cost per unit Q/ΔT_LM, £/ (W/K)
d_h  Hydraulic diameter, m
ER  Relative difference
f  Friction factor
f_s  Ratio of secondary surface area to total surface area for heat transfer
h  Heat transfer coefficient, W/m² °C

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Email: nan.zhang@manchester.ac.uk
i  Interest rate
j  Colburn factor
k  Thermal conductivity of fins, W/m °C
L  Flow length on one side, m
m  Mass flow rate, Kg/s
n  Number of holes per unit length
N  Number of layers per stream
ΔP  Pressure drop, Pa
Pr  Prandlt number
Q  Heat duty of heat exchanger, W
R  Fouling resistance, m² °C /W
Re  Reynolds number
St  Stanton number
\(t_{f, st}\)  Standardised fin thickness, m
Δt  Stream temperature difference, °C
ΔT_{LM}  Logarithmic temperature difference, °C
T_{cap, a}  Total annual capital cost, £/a
U  Overall heat transfer coefficient, W/m² °C
V  The total exchanger volume, m³
W  Exchanger width, m
\(x_{st}\)  Standardised fin length, m
xi  Annual factor

**Variables**

b  Plate spacing, m
c  Fin pitch, m
e  Wavy fin amplitude, m
l  Wavy fin wavelength, m
\(t_f\)  Fin thickness, m
x  Fin length, m

**Greek Letters**

\(\alpha\)  Ratio of total transfer area of one side exchanger to total exchanger volume, m²/m³
\(\beta\)  Ratio of total transfer area of one side of exchanger to volume between plates of that side, m²/m³
\(\sigma\)  Ratio of free flow area of one side to total frontal area, m²/m²
\(\mu\)  Viscosity, W/m °C
1 Introduction

A plate-fin heat exchanger is a type of compact heat exchangers that use plates and finned chambers to transfer heat between fluids. The main structure of plate-fin heat exchangers, shown in Figure 1, consists of nozzle (stub pipe), distributor and plate-fin. Typical materials are brazed aluminium, stainless steel and titanium. Figure 2 presents a typical single chamber made up of plates (parting sheet), fins and side bar (edge seal). Fins are introduced in the design of compact heat exchangers because: A) as a secondary surface, fin can transfer heat to fluid streams through parting sheets; B) due to higher thermal conductivity, heat transfer is more efficient; C) fins can enlarge fluid turbulence and enhance the local convective heat transfer coefficient. Typically, there are four categories of fins, namely, plain fin, louver fin, serrated fin (offset strip fin) and wavy fin (herringbone fin) shown in Figure 3.
Compared with normal heat exchangers, plate-fin heat exchangers have several distinctive advantages. Firstly, the minimum temperature approach can be as low as 1°C in plate-fin heat exchangers, which can contribute to more heat recovery and less utility requirement. Secondly, due to higher heat transfer efficient and large surface area per unit volume, a heat exchanger unit can be much smaller and lighter. Furthermore, the possibility of handling several streams in one unit provides the opportunity of reducing the heat exchanger number [4]. Based on these merits, plate-fin heat exchangers have been extensively used in low temperature process systems, such as cryogenics for separation, LNG plants and ethylene plants, etc.

However, there are still several barriers in the application of plate-fin heat exchangers [5]. First of all, fouling limits the application only to clean streams. Secondly, materials allow the
operation condition only at moderate temperature and pressure. Thirdly, the manufacturability and capital cost is another major hurdle. More importantly, there are few general systematic design methods that consider fin selection and thermal design of plate-fin heat exchangers simultaneously [4-12].

From Kays and London [13], there are approximately sixty standardised fin parts. Therefore, the large number of discrete combinations of standardised fin parts and types involved in the thermal design is one major difficulty in optimising the design of plate-fin heat exchangers. Lee [4] proposed two new concepts: identical-fin concept and Z-Y graph, which means that the same fins are used for all streams in the early design stage and a Z-Y graph is developed to select fin types. Picon-Nunez et al. [6-9] put up with a new term “Volume Performance Index (VPI)”, drew a graph of VPI vs Reynolds number and conducted sensitive analysis to select appropriate fins based on assumed Reynolds number. Based on the identical fin concept and assumed Reynolds number, fin selection could not provide sophisticated results. On top of that, it is a time-consuming procedure.

Hao et al. [10-11] designed a cross-flow heat exchanger, taking fin height and fin pitch as variables to minimize the total exchanger weight or annual cost by Genetic Algorithm combined with back propagation neural network. Yousefi [12] optimised cross-flow plate-fin heat exchangers using an ε-NTU model, treating hot flow length, cold flow length, fin height, fin frequency and length, fin thickness and numbers of layers as variables to minimise the total heat transfer area by hybrid Genetic Algorithm with particle swarm solution. He [14] then used a self-adaptive penalty function scheme and employed an improved harmony search algorithm to minimise the total heat transfer area and total pressure drop. Each standardised fin has its corresponding Colburn factor j and friction factor f correlations, which makes the design problem discrete. The optimisation of fin selection cannot converge easily. Therefore, the priority should be given to a general fin selection method, which can select fin types and consider imposed constraints simultaneously.

Current plate-fin heat exchanger design methodologies divide the process streams into several sections or intervals by composite curves and analyse each interval in detail [4-9]. The main design method is based on a thermal-hydraulic model and a ε-NTU model that present the relationship between heat exchanger volume, heat transfer coefficients and pressure drop.

In general, due to the complex discrete nature of plate-fin heat exchanger design caused by a large number of standardised fin parameters, the design of plate-fin heat exchangers still contains a major element of trial and error, which often leads to sub-optimal design configurations. Therefore, the aim of this study is to develop a systematic design methodology for optimal multi-stream plate-fin heat exchangers that takes into account fin selection and pressure drop utilization simultaneously. A case study from Picon [15] is redesigned to verify the feasibility of the new algorithm.
2 Problem Statement

This design methodology is developed based on the following assumptions:

- Steady-state operation
- Single-phase heat transfer
- Constant fluid properties
- Constant heat transfer coefficients
- Common wall temperature
- Counter-current arrangement
- Identical fin category

Physical properties and thermal information (mass flow rate, inlet and outlet temperature and pressure) of process streams, as well as fin physical properties, are given in the design stage. The design objective is to find the optimum design of a plate-fin heat exchanger by minimising the total heat exchanger volume. Total heat exchanger volume, exchanger dimensions (height, width and length), number of layers per streams, heat transfer coefficients and pressure drop should all be taken into consideration simultaneously.

3 Characterisation of different fin types

For a two-stream plate-fin heat exchanger, there are approximately 3600 combinations that need to be screened. The number will be increased exponentially with more streams involved. Without an appropriate fin selection method, the design would be extremely time-consuming especially for multi-stream plate-fin heat exchangers.

Based on the published data, pressure drop, heat transfer performance and design parameters can be expressed as continuous correlations in this model to convert the discrete design problem to a continuous problem. The Colburn factor $j$ and the friction factor $f$ are expressed as a function of Reynolds number and basic fin geometry parameters. Similarly, several characteristic dimensions, hydraulic diameter $d_h$, ratio of transfer area to volume of one side $\beta$ and ratio of secondary surface area to total surface area $f_s$, are described as a function of basic fin geometry parameters.

The Colburn factor $j$ and the Fanning friction factor $f$ for offset strip fin cores were regressed by Manglik [16] as a function of Reynolds number and basic fin geometry, such as, fin pitch $c$, fin height $b$, fin length $x$ and fin thickness $t_f$ shown in Figure 4. Manglik et al. [16] claimed that these correlations predicted the experimental data within 14%, which is much better than that in previous design work.

$$j = 0.6522 \times 10^{-0.5403 \left(\frac{c}{b}\right)^{-0.1541} \left(\frac{t_f}{x}\right)^{0.1499} \left(\frac{t_f}{c}\right)^{-0.0678} \left[1 + 5.269 \times 10^{-5} \times 1.34 \left(\frac{c}{b}\right)^{0.504} \left(\frac{t_f}{x}\right)^{0.450} \left(\frac{t_f}{c}\right)^{-1.055}\right]^{0.1}}$$

(1)
\[ f = 9.6243 \text{Re}^{-0.7422} \left( \frac{c}{b} \right)^{-0.1856} t_f \left( \frac{x}{x} \right)^{0.3053} t_c \left( \frac{x}{c} \right)^{0.2659} \times [1 + 7.669 \times 10^{-8} \text{Re}^{4.429} \left( \frac{c}{b} \right)^{0.920} t_f \left( \frac{x}{x} \right)^{3.767} t_c \left( \frac{x}{c} \right)^{0.236}]^{0.1} \]  \tag{2}

Re is the Reynolds number which is defined as a function of the surface hydraulic diameter:

\[ \text{Re} = \frac{\dot{m}d_h}{\mu A_c} \]  \tag{3}

![Figure 4 Basic offset strip fin geometry](image)

Equation (1) and (2) are valid for \(120 < \text{Re} < 10^4\), \(0.134 < c/b < 0.997\), \(0.012 < t_f/x < 0.048\) and \(0.041 < t_f/c < 0.121\).

For plain fin, Diani et al [17] claimed these correlations correctly predicted with a relative deviation of -0.4% and absolute deviation of 4.3% when the Reynolds number is around 2700 to 10100:

\[ j = 0.233 \text{Re}^{-0.48} \left( \frac{b}{c} \right)^{0.192} t_f \left( \frac{x}{b} \right)^{0.208} \]  \tag{4}

\[ f = 0.029 \text{Re}^{-0.09} \left( \frac{b}{c} \right)^{0.034} t_f \left( \frac{x}{b} \right)^{-0.169} \]  \tag{5}

For louver fin, Davenport [18] derived these correlations and agreed with experimental data within 10% and 12% respectively when the Reynolds number is around 300 to 4000:

\[ j = 0.249 \text{Re}^{-0.42} \left( \frac{x}{b} \right)^{1.1} x^{0.33} \]  \tag{6}

\[ f = 0.494 \text{Re}^{-0.39} \left( \frac{x}{c} \right)^{1.1} \left( \frac{b}{c} \right)^{0.33} \]  \tag{7}

For wavy fin, Dong [19] proved that the mean deviations of these correlations in the range of Reynolds number between 800 and 6500 for j and f factors were 4.4% and 5.1% respectively:
\[ j = 0.0836 \text{Re}^{-0.2309} \left( \frac{c}{b} \right)^{0.1284} \left( \frac{x}{t_f} \right)^{-0.326} \left( \frac{c}{2e} \right)^{-0.153} \]  
\[ f = 1.16 \text{Re}^{-0.309} \left( \frac{c}{b} \right)^{0.3703} \left( \frac{x}{t_f} \right)^{-0.1152} \left( \frac{c}{2e} \right)^{-0.25} \]

To make the model continuous, for each category of fin, hydraulic diameter \( d_h \), secondary surface ratio \( f_s \), and heat transfer surface area per unit volume \( \beta \) can be simply regressed as a function of basic fin geometry parameters. The continuous correlations for four fin categories were listed as below:

For offset strip-fin [20]:

\[
\beta = \frac{2(b - t_f)x + 2(c - t_f)x + 2(b - t_f)t_f + ct_f}{bcx} \]

\[
f_s = \frac{2(b - t_f)x + 2(b - 2t_f)t_f + ct_f}{2(b - t_f)x + 2(c - t_f)x + 2(b - t_f)t_f + ct_f} \]

\[
d_h = \frac{4(c - t_f)(b - t_f)x}{2 \left( (c - t_f)x + (b - t_f)x + t_f(b - t_f) \right) + t_f(c - t_f) - t_f^2} \]

For plain fin [21]:

\[
\beta = \frac{2(b + c)}{bc} \]

\[
f_s = \frac{2b + 5c/3}{2b + 4c} \]

\[
d_h = \frac{2b(c - t_f)}{b + c} \]

For louver fin [20]:

\[
\beta = \frac{2x(c - t_f) + 2bc + 2(\sqrt{c^2 + b^2} - t_f)}{bc} \]

\[
f_s = \frac{2(\sqrt{c^2 + b^2} - t_f)}{2x(c - t_f) + 2bc + 2(\sqrt{c^2 + b^2} - t_f)} \]

\[
d_h = \frac{4bc - t_f(\sqrt{c^2 + b^2} - t_f)}{2x(c - t_f) + 2bc + 2(\sqrt{c^2 + b^2} - t_f)} \]

For wavy fin [21]:

\[
\beta = \frac{2\Psi b + 2c}{bc} \]

\[
f_s = \frac{2\Psi b + c}{2\Psi b + 2c} \]
\[ d_h = \frac{2bc}{b + \varphi_c} \]  

4 Modelling of thermal-hydraulic performance of plate-fin heat exchangers

The thermal-hydraulic model is based on the basic heat transfer design equation for a two-stream heat exchanger [6-9, 20].

\[ Q = UA \Delta T_{LM} \]  

With the inclusion of surface fouling and fin effects, the overall heat transfer coefficient is defined as below [20]:

\[
\frac{1}{UA} = \frac{1}{U_1 A_1} + \frac{1}{U_2 A_2} = \frac{1}{\eta_1 A_1} \left( \frac{1}{h_1} + R_1 \right) + \frac{1}{\eta_2 A_2} \left( \frac{1}{h_2} + R_2 \right) + R_w
\]  

Combining it with the basic equation gives

\[
A_1 = \frac{Q}{\Delta T_{LM}} \left[ \frac{1}{\eta_1} \left( \frac{1}{h_1} + R_1 \right) + \frac{1}{\eta_2} \left( \frac{A_1}{h_2 A_2} + R_2 \right) + R_w \right]
\]  

where \( A_1 \) and \( A_2 \) represent the total heat transfer area of each side; \( h_1 \) and \( h_2 \) are the filmed heat transfer coefficients of each side; \( R_1 \) and \( R_2 \) are the thermal resistance due to fouling in each side. \( R_w \) is the wall thermal resistance.

Due to different fin types and densities for both streams, the heat exchanger volume will be more reasonable as the key indicator of sizing than heat transfer area in plate-fin heat exchanger design. Kays and London [13] set a new parameter \( \alpha \) that relates the total heat transfer area of one side of heat exchanger to the total heat exchanger volume. Based on the definition, the total heat transfer area can be expressed as:

\[
A_1 = \alpha_1 V_T
\]
\[
A_2 = \alpha_2 V_T
\]

where \( \alpha \) can be calculated by the heat exchanger geometry:

\[
\alpha_1 = \beta_1 \left( \frac{b_1}{b_1 + b_2} \right)
\]
\[
\alpha_2 = \beta_2 \left( \frac{b_2}{b_1 + b_2} \right)
\]

where \( \beta \) is the ratio of total heat transfer area of one side to the volume between plates of that side and \( b \) is the plate spacing of each side respectively.

Therefore, the heat exchanger volume can be rearranged as [13]:
\[ V_T = \frac{Q}{\Delta T_{LM}} \left[ \frac{1}{\eta_1 \alpha_1} \left( \frac{1}{h_1} + R_1 \right) + \frac{1}{\eta_2 \alpha_2} \left( \frac{1}{h_2} + R_2 \right) \right] \]  

(29)

The overall surface efficiency \( \eta_1 \) and \( \eta_2 \) can be calculated by [13]:

\[
\eta = 1 + f_s \left\{ \tanh \left[ \frac{(2h)^{1/2} b}{k t_f} \right] \right\} - 1
\]

\[ (30) \]

where \( k \) is the thermal conductivity of fin; \( t_f \) is the fin thickness.

The heat transfer performance is presented as the Colburn factor \( j \) [20],

\[ j = \frac{St}{Pr^{2/3}} \]

\[ (31) \]

where \( Pr \) is the Prandtl number, \( St \) is the Stanton number which are given by [20]:

\[ St = \frac{h A_c}{\dot{m} C_p} \]

\[ (32) \]

\[ Pr = \frac{\mu C_p}{k} \]

\[ (33) \]

Therefore, the heat transfer coefficient can be rearranged as

\[ h = \frac{\dot{m} C_p j}{A_c P r^{2/3}} \]

\[ (34) \]

The similar procedure can be applied to express the pressure drop. The pressure drop across the core of a heat exchanger is [13]:

\[ \Delta P = \frac{2 f L \dot{m}^2}{\rho d_h A_c^2} \]

\[ (35) \]

5 Methodology for optimal fin selection

Based on the above models for different fin categories and the thermal-hydraulic model, the discrete design problem can be converted to a continuous NLP problem. Since friction factor, Colburn factor and design parameters \( (d_n, f_s, \beta) \) can be expressed as a function of Reynolds number \( Re \) and basic fin geometries, plate spacing \( b \), fin pitch \( c \), fin thickness \( t_f \) and fin length \( x \) are taken as continuous design variables in this methodology. The objective function, total exchanger volume can be minimised by the CONOPT solver in GAMS to find the optimum fin types. BARON is another computation system for solving nonconvex optimisation problems to global optimality. BARON solver can represent local optimum and global optimum clearly but only be professional for small and medium problems.

By combining these two models together, heat transfer coefficient and pressure drop can also be related to basic geometry parameters. Therefore, fin selection can be carried out
with consideration of imposed constraints, such as pressure drop and fluid flow types. In other words, there is no need to select the fin type blindly based on assumed Reynolds number and regardless of pressure drop in the early design stage.

Due to standardised fin parts, the optimum fin geometry may not exist. Compared with fixed standardised fin geometries, the “relative difference” ER should be calculated to select the closet fin types. A smaller ER value indicates a better match.

\[
ER = \left| \frac{b - b_{st}}{b_{st}} \right| + \left| \frac{c - c_{st}}{c_{st}} \right| + \left| \frac{x - x_{st}}{x_{st}} \right| + \left| \frac{t_f - t_{f, st}}{t_{f, st}} \right|
\]  

(36)

6 Overall design methodology of plate-fin heat exchanger

The whole general plate-fin heat exchanger design methodology is presented in Figure 4.

Pinch technology [22] is employed to represent process streams directly in a graphical format. The composite curves include hot and cold composite curves, which depict the heat balance of process streams and give the minimum hot and cold utility requirements. Based on assumed constant fluid physical properties, the composite curve is formed by several straight lines. The kink points indicate certain process streams inlet or outlet points. Therefore, the composite curves can be divided into several enthalpy intervals by kink points shown in Figure 5 [4-9].

With the interval decomposition, the flow rate, permissible pressure drop and heat load per stream are fixed in each enthalpy interval. The allowable pressure drop per stream in each interval is distributed linearly according to the fraction of heat load [6].

\[
\Delta P_{i,k} = \Delta P_{i,total} \left( \frac{\Delta H_{i,k}}{\Delta H_{i,total}} \right)
\]  

(37)

where \(i\) is the stream number, \(k\) is the interval number.
Figure 4 Overall optimum design algorithm
In each interval, the entry and exit temperature are fixed. The temperature driving force is uniform for all hot streams and cold streams. Therefore, each hot stream can match any cold stream in the same interval.

In this work, a superstructure based heat exchanger network is employed in the design stage. Each hot stream is split into several streams to match every cold stream. Conversely, each cold stream should be split to match all the hot streams. Figure 6 illustrates a simple superstructure example.

The heat load distribution is calculated by

\[ Q_{i,j,k} = Q_{i,k} \times \frac{Q_{j,k}}{\sum_j Q_{j,k}} \]  

(38)

Every match can be regarded as a two-stream plate-fin heat exchanger \( V_{i,j,k} \). Each interval is a plate-fin heat exchanger network. Consequently, the volume of each interval is the sum of
each match included. The whole plate-fin heat exchanger volume is the sum of volume of each interval involved.

\[ V_k = \sum_i \sum_j V_{i,j,k} \]  

(39)

\[ V = \sum_k V_k = \sum_k \sum_i \sum_j V_{i,j,k} \]  

(40)

As mentioned above, each interval is a two-stream heat exchanger network. In the design stage, once a match is set and optimised, the fin types for these two streams will be determined to ensure the unified pressure drop for the same stream in the same interval. Therefore, different design sequences will give different heat exchanger volumes. Fin type should be determined based on the minimum interval volume.

Based on selected fin types, the heat exchanger volume must be recalculated and used to determine the interval and exchanger dimensions (width \( W \), length \( L \) and height \( H \)) and number of layers per stream.

The frontal area \( A_{fr} \) can be obtained by free flow area \( A_c \) [13]:

\[ \sigma = \frac{A_c}{A_{fr}} = \frac{\alpha d_h}{4} \]  

(41)

The length can be determined by [13]:

\[ L = \frac{V_T}{A_{fr}} \]  

(42)

With the assumed width \( W \), the height of exchanger \( H \) and the number of layers per stream can be deduced by [13]:

\[ H = \frac{A_{fr}}{W} \]  

(43)

\[ N = \frac{A_{fr}}{W(b_1 + b_2)} \]  

(44)

7 Case Study

A four-stream plate-fin heat exchanger [14] is revisited in this work. The process stream information is listed below in Table 1. To test and verify the feasibility of the new design methodology, the minimum temperature approach of 20 °C is assumed the same as the previous work.
Table 1 Process data for case study

<table>
<thead>
<tr>
<th>Stream</th>
<th>T_{s}  (°C)</th>
<th>T_{t}  (°C)</th>
<th>Flow rate (kg/s)</th>
<th>∆P (KPa)</th>
<th>ρ (kg/m³)</th>
<th>C_{p} (J/kg °C)</th>
<th>μ (cP)</th>
<th>k (W/m °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>150</td>
<td>60</td>
<td>25.0</td>
<td>46</td>
<td>700</td>
<td>800</td>
<td>0.3</td>
<td>0.12</td>
</tr>
<tr>
<td>H2</td>
<td>90</td>
<td>60</td>
<td>106.7</td>
<td>60</td>
<td>700</td>
<td>750</td>
<td>0.4</td>
<td>0.12</td>
</tr>
<tr>
<td>C1</td>
<td>20</td>
<td>125</td>
<td>27.7</td>
<td>30</td>
<td>750</td>
<td>900</td>
<td>0.5</td>
<td>0.12</td>
</tr>
<tr>
<td>C2</td>
<td>25</td>
<td>100</td>
<td>37.5</td>
<td>86</td>
<td>750</td>
<td>800</td>
<td>0.5</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 2 Interval decomposition results

<table>
<thead>
<tr>
<th>Interval</th>
<th>T_{H,in} (°C)</th>
<th>T_{H,out} (°C)</th>
<th>T_{C,in} (°C)</th>
<th>T_{C,out} (°C)</th>
<th>∆T_{LM} (°C)</th>
<th>∆H (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65.3</td>
<td>64</td>
<td>20</td>
<td>25</td>
<td>42.13</td>
<td>124.65</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>65.3</td>
<td>25</td>
<td>70</td>
<td>29.00</td>
<td>2417.8</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>90</td>
<td>70</td>
<td>91.8</td>
<td>35.74</td>
<td>1200.0</td>
</tr>
</tbody>
</table>

Table 3 Pressure drop distribution

<table>
<thead>
<tr>
<th>Stream</th>
<th>∆P distribution (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interval 1</td>
</tr>
<tr>
<td>H1</td>
<td>0.72</td>
</tr>
<tr>
<td>H2</td>
<td>2.60</td>
</tr>
<tr>
<td>C1</td>
<td>1.43</td>
</tr>
<tr>
<td>C2</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 5 presents the hot and cold composite curves and shows the interval decomposition in this case. The detail information of each interval is shown in Table 2. The distribution of pressure drop per stream is listed in Table 3.

The optimisation is executed with GAMS version 23.4 on a 2.6GHz 4th Intel Core i5 PC with 8GB memory. The CONPT solver is used in this study and the computation time is several microseconds. There are 9 variables and 8 nonlinear equations in the optimisation program.

1. Heat exchanger volume and fin selection

The identical fin category is assumed in this case study. With highest heat transfer efficiency and largest pressure drop, offset strip fin and louvered fin cannot meet the heat exchanger requirement because of unrealistic heat exchanger size and pressure drop. Therefore, the plain fin is employed in this case in spite of lowest heat transfer efficiency. Take interval 1/H1-C1 match as an example, the optimum fin height, fin thickness and fin pitch for H1 side obtained from GAMS are 6.05mm, 1.20mm and 0.152mm respectively. By calculating ER
value, the most similar fin type is plain-fin 19.86. While the optimum fin height, fin thickness and fin pitch for C1 side are 6.45mm, 1.24mm and 0.152mm respectively. Compared with standardised parts size, plain-fin 19.86 (fin height 6.35mm, fin pitch 1.278mm, fin thickness 0.152mm) is the best choice for C1 side. The same method is employed in each match and interval. To determine the interval dimensions, the width is assumed as 0.75m in the basic and new design. The detail design information is shown below in Tables 4-6.

From Table 4-6, fin types employed (plain fin 15.08 and plain fin 11.94T) in the basic design cannot provide the minimum heat exchanger volume. Because the fin type selection in the basic design is based on the graph of VPI (Volume Performance Index) vs Reynolds number Re and assumed Reynolds number. Once the Reynolds number is calculated, the fin type will be reselected. The iteration will not stop until the final Reynolds number agrees well with the previous Reynolds number within the accepted limit. But in the new methodology, the fin selection and thermal design are optimised simultaneously, which avoid blind selection and improve the precision. Using plain-fin 19.86 saves the block volume by almost 16.7%, 1% and 37.6% respectively. The number of layers in each block changed in different directions correspondingly. Through the verification of this case study, the fin selection can be completed while designing the plate-fin heat exchanger, rather than drawing a graph of special terms vs Reynolds number and selecting fin types based on assumed Reynolds number at early design stage. The fin selection problem can be taken as a continuous problem through the integration of continuous expression of fin parameters in spite of dealing with standardised parts. Moreover, the new design methodology not only provides a better design, but also improves the job efficiency, especially for multi-stream plate-fin heat exchanger design.

Table 4 Interval 1 design details

<table>
<thead>
<tr>
<th>Interval Dimensions</th>
<th>Fin types</th>
<th>Number of layers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volume (m³)</td>
<td>Length (m)</td>
</tr>
<tr>
<td>Basic Case</td>
<td>0.012</td>
<td>0.053</td>
</tr>
<tr>
<td>New Design</td>
<td>0.010</td>
<td>0.170</td>
</tr>
</tbody>
</table>

* PF – plate fin
Table 5 Interval 2 design details

<table>
<thead>
<tr>
<th>Interval Dimensions</th>
<th>Fin types</th>
<th>Number of layers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (m³)</td>
<td>Length (m)</td>
<td>Width (m)</td>
</tr>
<tr>
<td>Basic Case</td>
<td>0.204</td>
<td>0.868</td>
</tr>
<tr>
<td>New Design</td>
<td>0.204</td>
<td>1.008</td>
</tr>
</tbody>
</table>

Table 6 Interval 3 design details

<table>
<thead>
<tr>
<th>Interval Dimensions</th>
<th>Fin types</th>
<th>Number of layers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (m³)</td>
<td>Length (m)</td>
<td>Width (m)</td>
</tr>
<tr>
<td>Basic Case</td>
<td>0.178</td>
<td>0.780</td>
</tr>
<tr>
<td>New Design</td>
<td>0.111</td>
<td>0.530</td>
</tr>
</tbody>
</table>

2. Pressure drop sensitive analysis

Pressure drop is an important constraint, which should be considered in the plate fin heat exchanger design process. In the current design methodology, the allowed pressure drop of critical stream is maximised or fully utilised, while pressure drop values for other streams are calculated consequently. But all pressure drops should be less than allowed maximum pressure drop.

Table 7 lists the allowed maximum pressure drop, pressure drop of each stream in each interval in the basic case and new design.

Table 7 Pressure drop details

<table>
<thead>
<tr>
<th>Interval</th>
<th>Interval 1</th>
<th>Interval 2</th>
<th>Interval 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H1</td>
<td>H2</td>
<td>C1</td>
</tr>
<tr>
<td>Allowed</td>
<td>0.72</td>
<td>2.60</td>
<td>1.43</td>
</tr>
<tr>
<td>Basic Case</td>
<td>0.243</td>
<td>0.48</td>
<td>0.17</td>
</tr>
<tr>
<td>New Design</td>
<td>0.46</td>
<td>0.91</td>
<td>0.322</td>
</tr>
</tbody>
</table>

*All pressure drops are in kPa*
Compared with plain fin 15.08, plain fin 19.86 provides a smaller heat exchanger but gives higher pressure drop. Actually, fin type has an effect on the rate of heat transfer and pressure drop utilization. The large fin density gives a larger heat transfer area, which can enhance heat transfer efficiency and decrease heat exchanger size. But the pressure drop will increase consequently. When pressure drop of critical stream is maximised, the pressure drop of other streams should be under the allowed pressure drop.

To verify the influence of pressure drop on fin selection, a sensitive analysis is studied by changing the critical stream C1 pressure drop in the interval 3 and keeping other pressure drops constant.

![Figure 7 pressure drop sensitive analysis](image)

When the pressure drop of critical stream increases from 6.23kPa to 8.23kPa or larger, the corresponding pressure drop for other streams violate the maximum allowed pressure drop. Thus the lower density fin pitch should be selected at expense of heat exchanger volume. Actually, there is a trade-off between pressure drop and heat exchanger volume. Therefore, fin selection and pressure optimisation for every single stream should be considered simultaneously.

8 Conclusions

A new design methodology for multi-stream plate-fin heat exchanger was proposed to minimise heat exchanger volume. The selection of fin types and imposed constraints can be considered simultaneously in the optimal design process by regressing friction factor f, Colburn factor j and some design parameters (d_h, f_s, β) as a function of basic fin geometries such as fin height b, fin pitch c, fin length x and fin thickness. The discontinuous optimisation problem caused by discrete standardised fin types would be converted to a continuous
problem to overcome computation difficulties. For a large scale multi-stream plate-fin heat exchanger, the design quality could be greatly improved.

However, this design methodology is developed based on constant physical properties and identical fin categories for both sides. In practical, especially in low temperature systems, varied physical properties, induced by phase or temperature change, should be taken into consideration in the design methodology as well as mixing and matching fin categories in the plate-fin heat exchanger. Further research work is needed to tackle these challenges.

References


Chapter 3

Publication 2: Design Optimisation of Multi-stream Plate Fin Heat Exchangers with Multiple Fin Types

(Guo, K., Zhang, N., Smith, R., Design Optimisation of Multi-stream Plate-Fin Heat Exchangers with Multiple Fin Types. Applied Thermal Engineering, submitted)
Design Optimisation of Multi-stream Plate Fin Heat Exchangers with Multiple Fin Types

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Abstract

For multi-stream plate fin heat exchanger design, one needs to consider the optimal combination of fin categories and detailed geometries of fin types. Due to the complexity of the design problem for multi-stream plate fin heat exchangers, a major limitation of existing technology is the lack of a general design methodology that can consider mix-and-match fin type selection and imposed constraints simultaneously to take advantage of different fin characteristics. In this work, a new design method is proposed to consider the possibility of mix-and-match fin types. Multi-stream plate fin heat exchangers with mix-and-match fin types is considered as a network of two-stream plate fin heat exchangers. A mixed integer nonlinear programming (MINLP) model is converted into a nonlinear programming (NLP) model, and solved in combination of enumeration for a few remaining binary variables. The objective is to minimise the total capital cost. Case studies are carried out to validate the effectiveness of new proposed design methodology and demonstrate the benefits of mix-and-match fin type combinations in the overall design methodology.

Key words: plate fin heat exchanger  mix and match  fin types  optimisation

Nomenclature

Parameters

- $A$: Total heat transfer area of one side, $m^2$
- $A_c$: Free flow area of one side, $m^2$
- $A_{fr}$: Total heat exchanger front area, $m^2$
- $c_p$: Heat capacity, J/Kg ºC
- $d_h$: Hydraulic diameter, m
- $ER$: Relative difference
- $f$: Friction factor
- $F_f$: Fixed cost factor

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fs Ratio of secondary surface area to total surface area for heat transfer

FV Variable cost factor

h Heat transfer coefficient, W/m² °C

j Colburn factor

k Thermal conductivity of fins, W/m °C

L Flow length on one side, m

m Mass flow rate, Kg/s

N Number of layers per stream

ΔP Pressure drop, Pa

Pr Prandlt number

Q Heat duty of heat exchanger, W

R Fouling resistance, m² °C /W

Re Reynolds number

St Stanton number

TC Capital cost, $

Δt Stream temperature difference, °C

ΔTLM Logarithmic temperature difference, °C

U Overall heat transfer coefficient, W/m² °C

V Total exchanger volume, m³

W Exchanger width, m

Variables

b Plate spacing, m

c Fin pitch, m

e Wavy fin amplitude, m

l Wavy fin wavelength, m

tf Fin thickness, m

x Fin length, m

Greek Letters

α Ratio of total transfer area of one side exchanger to total exchanger volume, m²/m³

β Ratio of total transfer area of one side of exchanger to volume between plates of that side, m²/m³

σ Ratio of free flow area of one side to total frontal area, m²/m²

μ Viscosity, W/m °C

ρ Density, Kg/m³

η Fin efficiency
1. Introduction

With extremely small minimum approach temperature, large surface area per volume, high heat transfer coefficient and possibility of accommodating several streams (up to 10), multi-stream plate fin heat exchangers, categorized as compact heat exchangers, are widely used in various industrial areas such as chemical, petrochemical, cryogenics and aerospace industries [1]. Plate fin heat exchangers, commonly known as brazed aluminium heat exchangers, consist of a series of fin surfaces sandwiched between the parting sheets and stacked together. The introduced fin, work as a secondary surface, can increase efficient heat transfer area and transfer heat to fluid through parting sheets, enlarge fluid turbulence and enhance the local convective heat transfer coefficient to strength the efficiency of heat transfer [2]. Therefore, compared with conventional shell and tube heat exchangers, plate fin heat exchangers have larger heat transfer load per unit volume. Consequently, the capital cost and the process operating cost will be reduced to a large extent, and the smaller size of plate fin heat exchanger can give more freedom of plant layout [3].

However, why have plate fin heat exchangers not been widely applied in most process industries? In practice, there are some disadvantages of plate fin heat exchangers: they cannot handle high pressure and temperature for material issue, and are normally used for clean and non-corrosive fluids [1]. The lack of general plate fin heat exchanger design methodology that can consider fin type selection and imposed constraints simultaneously also restricts their applications. Even some leading commercial software, such as ASPEN and UniSim, can only simulate plate fin heat exchangers and produce a “first shot” design without optimisation [4, 5].
According to Kays and London [6], there are approximately sixty standardised fin parts (plain fin, louvered fin, offset strip fin and wavy fin) with specific heat transfer and pressure drop performance. A major challenge in designing optimal multi-stream plate fin heat exchangers is the large number of combinations of standardised fin geometries for various fin categories and types to choose from, which adds discrete aspects to an already complicated design problem. Therefore, an appropriate fin selection method is important in the plate fin heat exchanger design. In the past, few methodologies considered all possible fin type combinations in the design procedure to guarantee the optimum solution. Shah [7] and Cowell [8] analysed individual fins in a variety of different ways: volume vs. power consumption, frontal area vs. power consumption, area goodness. But when it comes to the actual design, the selection of another surface can have an effect on the overall performance of an exchanger. Therefore, the performance of fins in combination is important if there is any degree of interaction between the two. Lee and Zhu [9] in 1999 proposed two new concepts: identical-fin concept and Z-Y graph to select optimum fin types. The identical fin indicates that the same fin type is employed for all streams in the early design stage. Apart from time-consuming, this screening method ignores mix and match fin types and may not reach the minimum plate fin heat exchanger size. Picon-Nunez et al. [10] rearranged heat balance equations, proposed a new term “Volume Performance Index” and plotted a graph of VPI vs Reynolds number to select appropriate fin types. But the selection is based on the assumed Reynolds number relying on a time-consuming trial and error procedure. Moreover, this method neglects the effect of imposed design constraints on fin selection. Recently, Peng, et al. [11] and Yousefi, et al. [12] took basic fin geometries as variables and integrated into a cross flow plate fin heat exchanger design.
model to optimise total weight and total heat transfer area respectively. Discrete fin geometry variables result in a very complicated mixed-integer nonlinear programming (MINLP) design problem. To solve these discrete variables, Yousefi, et al. [13] later developed a proposed variant of harmony search algorithm for design optimisation. Guo, et al. [14] set basic fin geometries as continuous variables in plate fin heat exchanger design optimisation, with the final selected fin types to be the closest standardised fin types. Although these design methodologies consider the effect of imposed design constraints on fin selection, there is one common assumption of identical fin category. In other words, only one fin category, such as plain fin or offset strip fin, is employed in the design procedure. However, ignoring the possibility of mix and match fin categories in plate fin heat exchanger design could lead to sub-optimal design solutions.

A major challenge to consider mix and match fin categories in a single plate fin heat exchanger design is the discrete thermal-hydraulic model associated with various standardised fin types in different fin categories. As a matter of fact, the detailed fin geometry is of great importance to heat transfer and pressure drop performance of plate fin heat exchangers. In other words, each standardised fin type has unique heat transfer and pressure drop performance. When including all possible fin type combinations, the overall design problem is an MINLP problem, which increases the difficulty of finding the optimum design solution. In terms of heat transfer coefficient and pressure drop in the thermal-hydraulic model, Picon-Nunez and Robles [15], and Wang and Sunden [16] regressed friction factor and Colburn factor of each fin type only as a function of Reynolds number. This regression neglects the effect of detailed fin geometry on heat transfer coefficient and pressure drop performance, and still cannot avoid the discrete design problem. Yousefi et al. [13] employed published expressions of Colburn factor and friction factor, which are functions of Reynolds number and basic fin geometries, in the design optimisation problem. Sepehr and Hassan [17] and Amin [18] employed genetic algorithm and biogeography-based optimisation algorithm to optimise the heat exchanger configurations. But these designs limit to offset-strip fins and also cannot avoid solving an MINLP problem. Guo et al. [19] integrated published continuous fanning factor and Colburn factor expression of various fin categories into the optimisation design model to select fin types by minimising the heat exchanger volume, which converts the MINLP problem to an NLP problem for easier convergence. But it does not consider mix and match fin types in a single plate fin heat exchanger.

Therefore, in this work, a modified design methodology of multi-stream plate fin heat exchangers is developed to consider mix-and-match fin types in designing a single heat exchanger, while taking fin selection optimisation and pressure drop performance into
consideration simultaneously.

2. Modelling of mix-and-match fin type selection

From Kays and London [7], four fin categories, plain fin, louvered fin, offset strip fin and wavy fin, shown in Figure 2, are commonly employed in plate fin heat exchangers. Each fin category has 10-20 fin types with different detailed fin geometries. With four available fin categories and totally 60 fin type options, a simple two-stream plate fin heat exchanger has 3600 possible combinations. In the past few decades, using same fin category for each heat exchange match is employed in the plate fin heat exchangers design to simplify the design, which cannot guarantee a true optimum result for all cases.

In practice, each fin category has unique heat transfer and pressure drop performance. The heat transfer performance of offset strip fins is increased by a factor of about 5 over plain fins of similar geometry, because offset strip fins increase heat transfer coefficient and heat transfer area through repeated growth and destruction of boundary layers and increase the effective surface area, while plain fins simply increase heat transfer area per volume. But the increase of heat transfer coefficient is at the expense of higher pressure drop for offset strip fins [1]. Without appreciably destructing layers and changing heat transfer area, the heat transfer characteristics of louvered fin and wavy fin lie between those of plain and offset strip fins, and their pressure drop performance ranks in between them as well [1].

Pressure drop is an important reference factor in the fin selection method. Minimising heat exchanger volume in the optimisation design will prefer fin types with high heat transfer coefficient, such as offset strip fin, and give a smaller heat exchanger size. But the pressure drop may not be in the allowed range. If that is the case, the fin type must be switched to other fin types with lower pressure drop.

Therefore, regarding different heat transfer and pressure drop performance, and if there is a big pressure drop difference requirements for two sides, especially in multi-stream heat exchangers, mix-and-match fin types can provide a better design solution. Hence, in this work, mix-and-match fin type selection is introduced to find the optimal plate fin heat exchanger design.
As mentioned before, a simple two-stream plate fin heat exchanger has 3600 possible combinations of fin types with different geometry. For both fin types employed in each combination, they have unique heat transfer and pressure drop performance. Without appropriate algorithms for fin type selection, design of multi-stream plate fin heat exchangers will be a big challenge. To simplify the fin selection problem, Guo, et al. [19] proposed an identical fin category screening method, in which the typical fin geometries of different standardised fin categories and types, such as fin pitch, fin length, fin thickness and fin height, are considered as continuous variables. By employing continuous expressions of Fanning friction factor and Colburn factor in Table 1, which are functions of Reynolds
number and basic fin geometry parameters, and regressing some design parameters as a function of basic geometry parameters, the discrete optimisation problem caused by standardised fin parts can be converted into a continuous problem to avoid solving the MINLP problem directly.

Table 1 Continuous Colburn factor and friction factor expression

<table>
<thead>
<tr>
<th>Fin Type</th>
<th>Colburn Factor Expression</th>
<th>Friction Factor Expression</th>
<th>Re Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain fin [20]</td>
<td>$j = 0.2333Re^{-0.48} \left( \frac{L}{b} \right)^{0.192} \left( \frac{t_f}{b} \right)^{-0.208}$</td>
<td>$f = 0.029Re^{-0.09} \left( \frac{L}{b} \right)^{0.034} \left( \frac{t_f}{b} \right)^{-0.169}$</td>
<td>(2700, 10100)</td>
</tr>
<tr>
<td>Offset strip fin [21]</td>
<td>$j = 0.6522Re^{-0.5403} \left( \frac{L}{b} \right)^{-0.1541} \left( \frac{t_f}{x} \right)^{0.1499} \left( \frac{t_f}{c} \right)^{-0.0678}$</td>
<td>$f = 0.96243Re^{-0.7422} \left( \frac{L}{b} \right)^{-0.1856} \left( \frac{t_f}{x} \right)^{0.3053} \left( \frac{t_f}{c} \right)^{-0.2659}$</td>
<td>(120, 10000)</td>
</tr>
<tr>
<td>Louvered fin [22]</td>
<td>$j = 0.249Re^{-0.42} \left( \frac{L}{b} \right)^{1.1} \left( \frac{c}{b} \right)^{0.33}$</td>
<td>$f = 0.494Re^{-0.39} \left( \frac{L}{c} \right)^{1.1} \left( \frac{b}{c} \right)^{0.33}$</td>
<td>(300, 4000)</td>
</tr>
<tr>
<td>Wavy fin [23]</td>
<td>$j = 0.0836Re^{-0.2309} \left( \frac{L}{b} \right)^{0.1284} \left( \frac{x}{L} \right)^{-0.326} \left( \frac{c}{2e} \right)^{-0.153}$</td>
<td>$f = 1.16Re^{-0.309} \left( \frac{L}{b} \right)^{0.3703} \left( \frac{x}{L} \right)^{-0.1152} \left( \frac{c}{2e} \right)^{-0.25}$</td>
<td>(800, 6500)</td>
</tr>
</tbody>
</table>

Therefore, in this work, the corresponding fin geometrical parameters in each mix-and-match fin type combination is set as continuous variables, which are listed in Table 2. Also, the unique heat transfer and pressure drop performance of different fin types in the same fin category can be expressed generally by continuous Colburn factor and friction factor correlations, which are functions of Reynolds number and typical fin geometrical parameters. With assumptions of continuous basic fin geometry variables and the above continuous Colburn factor and friction factor expressions, one fin category with different detailed fin geometries can be simplified to one fin type option. Therefore, for a two-stream heat exchanger with multiple fin types, 3600 different fin type combinations can be simplified to 16 possible mix-and-match combinations listed in Table 3. In this way, the design problem is simplified to a large extent. To differentiate the remaining sixteen combinations of different fin categories, enumeration can be applied to compare all these possible combination options.
Table 2 Design variables for different fins

<table>
<thead>
<tr>
<th>Fin type</th>
<th>Model Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain fin</td>
<td>Fin height ((b_1, b_2)); Fin pitch ((c_1, c_2)); Fin thickness ((t_{f1}, t_{f2}))</td>
</tr>
<tr>
<td>Serrated fin</td>
<td>Fin height ((b_1, b_2)); Fin pitch ((c_1, c_2)); Fin thickness ((t_{f1}, t_{f2})); Fin offset length ((x_{s1}, x_{s2}))</td>
</tr>
<tr>
<td>Louvered fin</td>
<td>Fin height ((b_1, b_2)); Fin pitch ((c_1, c_2)); Fin thickness ((t_{f1}, t_{f2})); Louver fin pitch ((L_{p1}, L_{p2})); Louver fin cut length ((L_{i1}, L_{i2})); Louver fin height ((L_{h1}, L_{h2}))</td>
</tr>
<tr>
<td>Wavy fin</td>
<td>Fin height ((b_1, b_2)); Fin pitch ((c_1, c_2)); Fin thickness ((t_{f1}, t_{f2})); Wavy fin wave length ((L_{d1}, L_{d2})); Heat exchanger length (L)</td>
</tr>
</tbody>
</table>

Table 3 Possible mix and match combinations

<table>
<thead>
<tr>
<th>Side 1</th>
<th>Side 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain-Plain</td>
<td>Plain-Plain</td>
</tr>
<tr>
<td>Serrated-Plain</td>
<td>Serrated-Plain</td>
</tr>
<tr>
<td>Louvered-Plain</td>
<td>Louvered-Plain</td>
</tr>
<tr>
<td>Wavy-Plain</td>
<td>Wavy-Plain</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In terms of the objective function, Picon-Nenuz, et al. [10] took the plate fin heat exchanger volume rather than heat transfer area as the evaluation criterion of heat exchangers design. In practice, due to material and manufacture feasibility, capital costs of different fin categories are different. Moreover, due to porosity issues, the total weights of different fin categories per unit volume are different as well. Hence, when mix-and-match fin selection are considered in the design methodology, minimising the total capital cost is used as the objective function.

The unit capital cost can be estimated on the basis of various involved fin types capital costs, which are related to heat exchanger weight, heat transfer area and heat exchanger volume.

\[
A_1 = \alpha_1 V \quad A_2 = \alpha_2 V
\]

Where \(\alpha\) is relates the total heat transfer area of one side of heat exchanger to the total heat exchanger volume. 1 and 2 represent one side and the other side of the heat exchanger.

Then the total weight can be determined by:

\[
W_{e1} = \rho \alpha_1 V t_{f_1,1} \quad W_{e2} = \rho \alpha_2 V t_{f_2,2}
\]

where \(\rho\) is fin material density, \(t_i\) is fin thickness.

The total capital cost \(TC\) is calculated as:

\[
TC_1 = F_{f_1} + F_{v_1} \rho \alpha_1 V t_{f_1,1} \quad TC_2 = F_{f_2} + F_{v_2} \rho \alpha_2 V t_{f_2,2}
\]
\[ TC = F_{f,1} + F_{v,1} \rho \alpha_1 V t_{f,1} + F_{f,2} + F_{v,2} \rho \alpha_2 V t_{f,2} \] (4)

where \( F_{f} \) and \( F_{v} \) are fixed cost factor and variable cost factor.

3. Design methodology of two-stream plate fin heat exchangers with multiple fin types

The overall design methodology is based on the assumptions of constant fluid physical properties, constant heat transfer coefficients, common wall temperature and steady state operation. Counter-current flow is set as default flow arrangement in this work, and only single phase heat transfer is considered in this research.

The thermal-hydraulic model employed in this work, adopted from Picon-Nunez [15] and Guo, et al. [19], is summarised as below. The basic heat balance equation for a two-stream heat exchanger is shown as:

\[ Q = U A \Delta T_{LM} \] (5)

where \( Q \) is the heat load, \( U \) is the overall heat transfer coefficient, \( A \) is the total heat transfer area and \( \Delta T_{LM} \) is the logarithmic mean temperature difference.

Including surface fouling and fin effects, the overall heat transfer coefficient is defined as [24]:

\[
\frac{1}{U A} = \frac{1}{U_1 A_1} = \frac{1}{U_2 A_2} = \frac{1}{\eta_1 A_1} \left( \frac{1}{h_1} + R_1 \right) + \frac{1}{\eta_2 A_2} \left( \frac{1}{h_2} + R_2 \right) + R_w
\] (6)

where \( A_1 \) and \( A_2 \) represent the total heat transfer area of each side; \( h_1 \) and \( h_2 \) are the film heat transfer coefficients of each side; \( R_1 \) and \( R_2 \) are the fouling thermal resistance of each side. \( R_w \) is the wall thermal resistance, which can be neglected compared to fouling in this study.

Neglecting the wall thermal resistance and combining equations (5) and (6) gives

\[
A_1 = \frac{Q}{\Delta T_{LM}} \left[ \frac{1}{\eta_1} \left( \frac{1}{h_1} + R_1 \right) + \frac{1}{\eta_2} \left( \frac{A_1}{h_2 A_2} + R_2 \right) \right]
\] (7)

By introducing a parameter \( \alpha \) that relates the total heat transfer area of one side of heat exchanger to the total heat exchanger volume, the heat exchanger volume can be expressed as [8]:

\[
V_T = \frac{Q}{\Delta T_{LM}} \left[ \frac{1}{\eta_1 \alpha_1} \left( \frac{1}{h_1} + R_1 \right) + \frac{1}{\eta_2 \alpha_2} \left( \frac{1}{h_2} + R_2 \right) \right]
\] (8)

The overall surface efficiency \( \eta_1 \) and \( \eta_2 \) of fin types employed in the design can be generally calculated by following equation [6]:
\[ \eta = 1 + f_s \left\{ \tanh \left( \frac{2h}{(k_f t_f) \frac{1}{2}} b \right) \right\} - 1 \]  

(9)

where \( k_f \) is the thermal conductivity of fin; \( t_f \) is the fin thickness.

Based on the definitions of Colburn factor \( j \), Prandlt number \( Pr \) and Stanton number \( St \), the heat transfer coefficient \( h \) can be calculated as below [24]:

\[ j = St \frac{Pr^{2/3}}{} \]  

(10)

\[ St = \frac{hA_c}{\dot{m}C_p} \]  

(11)

\[ Pr = \frac{\mu C_p}{k} \]  

(12)

\[ h = \frac{\dot{m} C_p j}{A_c Pr^{2/3}} \]  

(13)

A similar procedure can be applied to express the pressure drop. The pressure drop across the core of a heat exchanger is [24]:

\[ \Delta P = \frac{2fL\dot{m}^2}{\rho d_h A_c^2} \]  

(14)

As mentioned in Section 2, for optimisation process, basic fin geometry parameters, such as fin pitch \( c \), plate spacing \( b \) and fin thickness \( t_f \), are set as continuous variables to minimise the total capital cost. The optimisation (fin type, layer number etc.) for each mix-and-match combination option is completed by the CONOPT solver in GAMS. To help with the convergence, one random feasible design solution is set as the initial point. Because of standardised fin parts, the fin type with optimum basic fin geometry parameters may not exist. The closest standardised fin type is selected, and the heat exchanger dimensions and cost are recalculated. The selected fin type combination is considered as the local optimum design of current mix-and-match combination option. Afterwards, switch the design calculation to next possible mix and match combination until all possible combinations are examined. By comparing total capital costs of all combinations, the general optimum fin types for both sides with the minimum capital cost and heat exchanger dimensions can be determined. Therefore, the optimisation problem can be converted to multiple continuous NLP problem, which can save calculation workload and easily obtain the optimum design results.

The overall design optimisation methodology for a two stream plate-fin heat exchanger with multiple fin types is summarised in Figure 4.
4. Design methodology of multi-stream plate fin heat exchangers with multiple fin types

Multi-stream plate fin heat exchangers can be considered as a network of two-stream plate fin heat exchangers. With the help of pinch technology [25], the network can be graphically represented by composite curves, which include hot and cold composite curves. Based on assumed constant fluid physical properties, composite curves are formed by several straight lines with several kink points. The kink points indicate certain process stream inlet or outlet points. Therefore, the composite curves can be divided into several enthalpy intervals by kink points shown in Figure 5 [25]. Each enthalpy interval has fixed inlet and outlet temperature, flow rate, permissible pressure drop and heat load.
The allowable pressure drop per stream in each interval is assumed to be distributed linearly on the basis of heat load fraction [10].

\[ \Delta P_{i,k} = \Delta P_{i,total} \left( \frac{\Delta H_{i,k}}{\Delta H_{i,total}} \right) \]  

(15)

where \( i \) is the stream number, \( k \) is the interval number.

As a result of fixed entry and exit temperature in each enthalpy interval, each hot stream can match any cold stream in the same enthalpy interval. Therefore, in this work, a superstructure based heat exchanger network is employed in the design stage. Each hot stream is split into several streams to match every cold stream in the same enthalpy interval. Correspondingly, each cold stream is split to allow a match with all involved hot streams. Figure 6 illustrates a simple heat exchanger superstructure example. In an enthalpy interval, the splitting heat load ratio distribution of a hot stream is based on the heat load fractions of cold streams.

\[ Q_{i,j,k} = Q_{i,k} \times \frac{Q_{j,k}}{\sum_j Q_{j,k}} \]  

(16)
In the enthalpy interval superstructure, every match can be regarded as a two-stream plate-fin heat exchanger with a volume $V_{i,j,k}$ and capital cost $TC_{i,j,k}$. The optimisation methodology of one match can refer to the optimisation design of a two-stream plate fin heat exchanger, explained in Section 3. The whole enthalpy interval is a two-stream plate-fin heat exchanger network with several involved streams. Consequently, the capital cost of each enthalpy interval is the sum of every match included. The capital cost of whole plate-fin heat exchanger is the sum of capital cost of all enthalpy intervals involved.

$$TC_k = \sum_i \sum_j TC_{i,j,k}$$  \(17\)

$$TC = \sum_k TC_k = \sum_k \sum_i \sum_j TC_{i,j,k}$$  \(18\)

In the interval design process, the pressure drop performance in each split should be no larger than the allowed pressure drop in this enthalpy interval. Due to structural features, the maximum allowed pressure drop of both streams cannot be utilised simultaneously. Therefore, the reference stream, the pressure drop of which will be maximised in the optimisation, should be set in the early design stage. In this design, to avoid local optimum design solutions caused by the definition of the reference stream, the design sequence $y$ is introduced. In other words, each stream in one interval can be set as the reference stream to search the general optimum design. Once the reference stream is set, the match included reference stream should be optimised firstly to determine the fin type of the reference stream. In order to ensure the unified pressure drop for the reference stream in the same interval, the fin type for the reference stream will be fixed. The fin types for remaining streams can be determined through optimisation. By comparing the total capital cost of that interval, the optimum heat exchanger network configuration can be determined. The
optimum heat exchanger network design for other enthalpy intervals can be completed in a similar way.

Based on selected fin types, the heat exchanger volume should be recalculated and used to determine the interval exchanger dimensions (width W, length L and height H) and the number of layers per stream.

The frontal area $A_{fr}$ can be obtained by free flow area $A_c$ [6]:

$$\sigma = \frac{A_c}{A_{fr}} = \frac{ad_h}{4} \tag{19}$$

The length $L$ can be determined by [6]:

$$L = \frac{V_T}{A_{fr}} \tag{20}$$

With the assumed width W, the height of exchanger $H$ and the number of layers per stream can be deduced by [6]:

$$H = \frac{A_{fr}}{W} \tag{21}$$

$$N = \frac{A_{fr}}{W(b_1 + b_2)} \tag{22}$$

The overall design optimisation methodology of a multi-stream plate fin heat exchanger is summarised in Figure 7.
Figure 7 Overall optimisation design algorithm of multi-stream plate fin heat exchanger with multiple fin types
5. Case studies

5.1 Case Study 1: the effectiveness of new design model

A two-stream plate-fin heat exchanger from Picon-Nunez [26] is studied in this work to validate the effectiveness of the new design model. The basic process information and physical information and physical properties are listed in Table 4. The thermal conductivity of fin (aluminium) is set as 90 W m$^{-2}$ K$^{-1}$. In order to predict the heat transfer coefficient, the outlet temperature and pressure drop, the fin type Strip-fin 1/10-19.35 and Strip-fin 1/9-24.12 are selected for streams 1 and 2 respectively in this case. The accuracy of the new proposed design methodology is validated by comparing these results of the new proposed model and the published design model as shown in Table 5 [26].

<table>
<thead>
<tr>
<th>Process information and Physical properties for Case Study 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stream</strong></td>
</tr>
<tr>
<td>Mass flow rate (kg s$^{-1}$)</td>
</tr>
<tr>
<td>Allowed pressure drop (Pa)</td>
</tr>
<tr>
<td>Inlet temperature (°C)</td>
</tr>
<tr>
<td>Outlet temperature (°C)</td>
</tr>
<tr>
<td>Density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>Heat capacity (J kg$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>Thermal conductivity (W m$^{-2}$ K$^{-1}$)</td>
</tr>
<tr>
<td>Viscosity (cP)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results comparison of exchanger performance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stream</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Stream 1</td>
</tr>
<tr>
<td>Stream 2</td>
</tr>
</tbody>
</table>

It is obvious from Table 5 that heat transfer coefficients in the new model are slightly higher than those in the published model and the outlet temperature of stream 2 is 3°C higher, because the Colburn factor expression employed in the new model is related to Reynolds number and basic fin geometry parameters as well. Similarly, the pressure drop of stream 2 is higher than the original value, because the friction factor is considered as a function of Reynolds number and basic fin geometry parameters in the new model, other than only the function of Reynolds number. Based on these results, it can be seen that the results
obtained by applying the new proposed methodology are in good agreement with the published thermal-hydraulic model. Therefore, the new proposed thermal-hydraulic model can be employed to find the optimum multi-stream plate fin heat exchanger design.

5.2 Case study 2: the need of considering mix-and-match fin types

The same case is further studied to examine the benefits of mix-and-match fin selection and verify how pressure drop performance affects fin type selection. In this case study, the allowed pressure drop of stream 2 varies from 502 Pa to 40000 Pa, while other process information and physical properties remain the same. Basic fin geometry parameters are set as variables, and the capital cost or volume is optimised to select the optimum fin type. Design results are listed in Table 6. Due to unavailable cost data, the capital cost ratio of offset-strip fin, louvered fin, wavy fin and plain fin is assumed as 10:5:2:1 in this work to examine the important of mix-and-match fin types.

Table 6 Design results comparison for different allowed pressure drop

<table>
<thead>
<tr>
<th>Case</th>
<th>Allowed ΔP for Stream 2 (Pa)</th>
<th>Fin Type (stream 1)</th>
<th>Fin Type (stream 2)</th>
<th>Total Volume (m³)</th>
<th>Total Capital Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>502.6</td>
<td>ST/10-19.74</td>
<td>ST/10-19.74</td>
<td>1.86</td>
<td>20689.7</td>
</tr>
<tr>
<td>2</td>
<td>521.3</td>
<td>PF19.86</td>
<td>PF19.86</td>
<td>2.54</td>
<td>12510.1</td>
</tr>
<tr>
<td>3</td>
<td>7469.7</td>
<td>PF19.86</td>
<td>LF1/4-11.1</td>
<td>1.69</td>
<td>10197.8</td>
</tr>
<tr>
<td>4</td>
<td>39109.1</td>
<td>PF19.86</td>
<td>ST/10-19.74</td>
<td>1.93</td>
<td>11962.9</td>
</tr>
</tbody>
</table>

* PF-plain fin, ST-offset strip fin, LF-louvered fin

Comparing Case 1 with Case 2, the pressure drop performances of both cases are around 500 Pa. In Case 1, strip fin 1/10-19.74 is employed for both streams with a volume of 1.86m³, while plain fin PF19.86 is selected in Case 2 with a volume of 2.54m³. Although the volume of Case 2 is larger than Case 1, the capital cost is much cheaper. Therefore, different objective functions will lead to different design solutions. If the space is priority such as in the aerospace industry, the total heat exchanger volume should be set as the objective function. But when cost is prioritised such as in oil refineries, the objective function should be the total capital cost instead.

In Case 3, the allowed pressure drop of stream 2 is similar to stream 1, Louvered fin LF1/4-11.1 is selected with a smaller volume of 1.69 m³ with a cheaper capital cost. If the pressure drop allowance is increased further to around 40 kPa in Case 4, strip fin ST/10-19.74 is the best choice and the volume and capital cost are even smaller than those of Case 2. It is clear that mix-and-match may give a better design solution when there is very different pressure drop allowance between streams.
Therefore, mix-and-match fin types can provide more flexibility for exploiting the optimum design solutions, especially in the occasion of big difference in pressure drop allowance. Also, it is worth to note that the pressure drop is a key factor for fin selection, and should be taken into consideration simultaneously in the plate fin heat exchanger design.

5.3 Case study 3: a multi-stream plate fin heat exchanger design

A multi-stream plate fin heat exchanger [10, 17] is designed to examine the benefits of mix and match and check the robustness of modified design methodology. The process stream and physical properties are shown in Table 6. The minimum approach temperature remains as the published literature [10, 17] at 20 °C. The width of heat exchanger is set as 0.75 m to calculate each interval dimension.

Table 7 Process data for Case Study 3

<table>
<thead>
<tr>
<th>Stream</th>
<th>$T_s$ (°C)</th>
<th>$T_t$ (°C)</th>
<th>Flow rate (kg/s)</th>
<th>$\Delta P$ (KPa)</th>
<th>$p$ (kg/m$^3$)</th>
<th>$C_p$ (J/kg °C)</th>
<th>$\mu$ (cP)</th>
<th>$k$ (W m$^{-2}$ K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>150</td>
<td>60</td>
<td>25.0</td>
<td>46</td>
<td>700</td>
<td>800</td>
<td>0.3</td>
<td>0.12</td>
</tr>
<tr>
<td>H2</td>
<td>90</td>
<td>60</td>
<td>106.7</td>
<td>60</td>
<td>700</td>
<td>750</td>
<td>0.4</td>
<td>0.12</td>
</tr>
<tr>
<td>C1</td>
<td>20</td>
<td>125</td>
<td>27.7</td>
<td>30</td>
<td>750</td>
<td>900</td>
<td>0.5</td>
<td>0.12</td>
</tr>
<tr>
<td>C2</td>
<td>25</td>
<td>100</td>
<td>37.5</td>
<td>86</td>
<td>750</td>
<td>800</td>
<td>0.5</td>
<td>0.12</td>
</tr>
</tbody>
</table>

According to pinch analysis, process stream is represented graphically by hot and cold composite curves. The whole process is divided into three enthalpy intervals, shown in Figure 4. Table 8 presents the interval decomposition details (inlet and outlet temperature, heat load). Based on linear distribution, the pressure drop allowance of each interval is divided by the fraction of heat load and listed in Table 9.

In this design, basic fin geometry parameters are considered as continuous variables, and the total capital cost is taken as the objective function. Overall, there are 144 variables and 128 nonlinear equations. The CONOPT solver in GAMS version 23.4 is used to optimise the plate fin heat exchanger design. The CPU time of whole design is 68 seconds on a 2.6 GHz 4th Intel Core i5 PC with 8GB memory.

Table 8 Interval decomposition results for Case Study 3

<table>
<thead>
<tr>
<th>Interval</th>
<th>$T_{H,in}$ (°C)</th>
<th>$T_{H,out}$ (°C)</th>
<th>$T_{C,in}$ (°C)</th>
<th>$T_{C,out}$ (°C)</th>
<th>$\Delta T_{LM}$ (°C)</th>
<th>$\Delta H$ (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65.3</td>
<td>64</td>
<td>20</td>
<td>25</td>
<td>42.13</td>
<td>124.65</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>65.3</td>
<td>25</td>
<td>70</td>
<td>29.00</td>
<td>2417.8</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>90</td>
<td>70</td>
<td>91.8</td>
<td>35.74</td>
<td>1200.0</td>
</tr>
</tbody>
</table>
Table 9 Pressure drop distribution for Case Study 3

<table>
<thead>
<tr>
<th>Stream</th>
<th>∆P distribution (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interval 1</td>
</tr>
<tr>
<td>H1</td>
<td>0.72</td>
</tr>
<tr>
<td>H2</td>
<td>2.60</td>
</tr>
<tr>
<td>C1</td>
<td>1.43</td>
</tr>
<tr>
<td>C2</td>
<td>-</td>
</tr>
</tbody>
</table>

The design results for three intervals are shown in Tables 10-12. The basic case is from the published literature. The design results shown in the second line is based on identical fin category and minimising the total heat exchanger volume. The new design results is obtained from minimising the total capital cost, including all possible mix and match combinations. In this case, if offset strip fin and louvered fin are employed, the length of the heat exchanger may not exist because of higher heat transfer coefficient and pressure drop performance. So only wavy fin and plain fin are mixed and matched in this design.

Table 10 Interval 1 design details

<table>
<thead>
<tr>
<th>Interval Dimensions</th>
<th>Fin types</th>
<th>Number of layers</th>
<th>TC ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vol. (m³)</td>
<td>L (m)</td>
<td>W (m)</td>
<td>H (m)</td>
</tr>
<tr>
<td>Base Case</td>
<td>0.012</td>
<td>0.05</td>
<td>0.75</td>
</tr>
<tr>
<td>Guo [18]</td>
<td>0.010</td>
<td>0.17</td>
<td>0.75</td>
</tr>
<tr>
<td>New Design</td>
<td>0.010</td>
<td>0.17</td>
<td>0.75</td>
</tr>
</tbody>
</table>

*PF – plate fin

Table 11 Interval 2 design details

<table>
<thead>
<tr>
<th>Interval Dimensions</th>
<th>Fin types</th>
<th>Number of layers</th>
<th>TC ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (m³)</td>
<td>L (m)</td>
<td>W (m)</td>
<td>H (m)</td>
</tr>
<tr>
<td>Base Case</td>
<td>0.204</td>
<td>0.868</td>
<td>0.75</td>
</tr>
<tr>
<td>Guo [18]</td>
<td>0.204</td>
<td>1.008</td>
<td>0.75</td>
</tr>
<tr>
<td>New Design</td>
<td>0.187</td>
<td>0.819</td>
<td>0.75</td>
</tr>
</tbody>
</table>

*W–wavy fin
Table 12 Interval 3 design details

<table>
<thead>
<tr>
<th>Interval Dimensions</th>
<th>Fin types</th>
<th>Number of layers</th>
<th>TC ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (m$^3$)</td>
<td>L (m)</td>
<td>W (m)</td>
<td>H (m)</td>
</tr>
<tr>
<td>Base Case</td>
<td>0.178</td>
<td>0.78</td>
<td>0.75</td>
</tr>
<tr>
<td>Guo [18]</td>
<td>0.111</td>
<td>0.53</td>
<td>0.75</td>
</tr>
<tr>
<td>New Design</td>
<td>0.102</td>
<td>0.51</td>
<td>0.75</td>
</tr>
</tbody>
</table>

From Tables 10-12, it is clear that the new design results keep consistence with that in Guo’s research work [19] in the first interval, but in the following two intervals, wavy fin W17.8 appears in our optimum design result. Take interval 3 as an example, in the base case, plain fin 15.08 is employed with a biggest heat exchanger volume and expensive capital cost. In the second case, plain fin PF19.86 is used and have a smaller volume and cheaper capital cost, while in the new design, plain fin PF19.86 and wavy fin W17.8 are selected, and both the heat exchanger volume and capital cost are reduced by 42% and 13% respectively. In the base case, the Reynolds number is assumed in the early design stage, and the fin selection method is based on trial and error. In the second case, the identical fin category limits some possible fin type combinations, resulting in sub-optimal design results. In the new design, mix and match is introduced to consider all possible fin type combinations to find the optimum fin type combination for minimum total capital cost. The number of layers in each enthalpy interval changed correspondingly. The capital cost for the whole heat exchanger is reduced by 8%.

6. Conclusions

In this study, a new design algorithm is proposed to optimise the mix-and-match of different fin types in multi-stream plate fin heat exchanger design. Under a heat exchanger network superstructure, fin selection is optimised within pressure drop constraints. The problem is MINLP in nature, with two categories of integer variables, one relating to the selection and combination of different fin types, and the other relating to the choice of discrete fin geometry parameters. The first category of integer variables is overcome using enumeration to compare all the possible options, while the second category is dealt with by treating basic fin geometries of each fin in one interval as continuous variables, and considering thermal hydraulic performance of different fin types as a function of basic fin geometries. Therefore, the design model is converted into several continuous NLP problems. The optimal fin types for the whole enthalpy interval and their corresponding design parameters can be obtained.
by minimising the total interval volume or capital cost. However, this design methodology is developed on the assumption of constant fluid physical properties and single phase. Phase change and various physical properties are other big challenges of plate fin heat exchangers design. Future work is required to address these gaps.

Reference


Chapter 4

Publication 3: Optimisation of Two-stream Multi-pass Plate Heat Exchanger Design

(Guo, K., Zhang, N., Smith, R., Optimisation of Two-stream Multi-pass Plate Heat Exchanger Design. Energy, Submitted)
Optimisation of Two-stream Multi-pass Plate Heat Exchanger Design

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Abstract

With features of compactness, high effectiveness, small minimum temperature approach and cost competitiveness, plate heat exchangers are widely employed in the energy-intensive process industries. A major difficult of applying plate heat exchangers is the lack of a general multi-pass plate heat exchanger design optimisation methodology, which can take plate pattern selection, flow arrangement selection and imposed design constraints into account simultaneously. Various plate patterns with detailed geometry are available to meet various process requirements. Multi-pass arrangement makes fluid undergo the heat transfer process during a longer time and increase the pressure drop. In this work, a new optimisation methodology of two-stream multi-pass plate heat exchanger design is presented to include plate pattern selection and determination of flow arrangement. The flow arrangement is represented by the correction factor of logarithm mean temperature difference and integrated into the thermal-hydraulic model. The mixed-integer nonlinear programming (MINLP) model caused by standardised plate patterns and various flow arrangements is overcome and converted to several nonlinear programming (NLP) problem by continuous expressions of Nusselt number (Nu) and friction factor (f) and enumeration respectively. The basic plate geometric parameters are considered as continuous variables and optimised by capital cost minimisation in GAMS. Case studies are carried out to validate the design model and examine the effectiveness of the proposed design methodology. Results show that new design methodology agrees well with the literature and saves the computation time.

Key Words: Plate pattern multi-pass optimisation plate heat exchanger

Nomenclature

Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Heat transfer area, m²</td>
<td></td>
</tr>
<tr>
<td>Ac</td>
<td>Free flow area, m²</td>
<td></td>
</tr>
<tr>
<td>AT</td>
<td>Total heat transfer area, m²</td>
<td></td>
</tr>
<tr>
<td>cp</td>
<td>Heat capacity, J kg⁻¹ K⁻¹</td>
<td></td>
</tr>
</tbody>
</table>

† Corresponding author: Tel: +44 1613064384; Fax:+44 1612367439; Email: nan.zhang@manchester.ac.uk
CP
Heat Capacity mass flowrate, W

$dh$
Hydraulic diameter, m

$f$
Fanning friction factor

$F$
Logarithm temperature difference correction factor

$h$
Film heat transfer coefficient

$H$
Plate heat exchanger height, m

$k$
Thermal conductivity, W m$^{-1}$ K$^{-1}$

$m$
Mass flow rate, kg s$^{-1}$

$N_C$
Number of channels

$NTU$
Number of heat transfer units

$Nu$
Nusselt number

$P$
Temperature effectiveness

$Pr$
Prandlt number

$Q$
Heat Duty, W

$R$
Heat capacity mass flowrate ratio

$Re$
Reynolds number

$R_w$
Thermal resistance due to heat conduction through the wall, (m$^2$ K)/ W

$R_f$
Thermal resistance due to fouling, m$^2$ K W$^{-1}$

$TC$
Total capital cost, GBP

$U$
Overall heat transfer coefficient, W m$^2$ K$^{-1}$

$W$
Plate width, m

$\Delta P$
Pressure drop, Pa

$\Delta T_{LM}$
Logarithm temperature difference, K

**Variables**

$b$
Plate spacing, m

$d_{port}$
Port diameter, m

$K$
Flow arrangement option

$L_h$
Plate length for heat transfer, m

$L_p$
Plate length for pressure drop, m

$N_p$
Number of Passes

$\beta$
Chevron angle, $^\circ$

**Greek letters**

$\rho$
Density, kg m$^{-3}$

$\mu$
Viscosity, cP

$\phi$
Area enlargement factor
1. Introduction

With increasing global energy consumption, stringent environmental protection legislation and safety regulations in industrialised nations, energy saving has been put under high priority. Energy saving not only saves on primary energy, but also is an important contribution for environmental protection due to reduced emissions. One of the main strategies for energy saving is the reusing of heat already generated in the process as efficiently and economically as possible. Thermal reuse requires efficient and economically designed heat exchangers. Currently the most common used type of heat transfer equipment is the conventional shell-and-tube heat exchanger. This paper will address issues about the design, thermal and hydraulic properties of more efficient plate heat exchangers.

A plate heat exchanger, one of the most efficient heat transfer equipment, is a typical kind of compact heat exchangers that uses metal plates to transfer heat between two fluids [1]. Commonly-used plate heat exchangers include gasketed plate heat exchangers and welded plate heat exchangers, shown in Figure 1 and 2. Like normal compact heat exchangers, plate heat exchangers are characterised by small minimum approach temperature, large heat transfer surface per unit volume, efficient heat transfer due to turbulence and small hydraulic diameter, and possibility of accommodating several streams in one unit, which can increase heat recovery duty, decrease capital cost and give more freedom of plant layout. Various Chevron type plates with detailed geometry (Chevron angle, plate width and length) are commonly employed to meet the process requirement. The stacked plates give a degree of freedom about passes number per stream and overall flow arrangement types (counter-current flow, co-current flow or cross flow). Therefore, the unique structure of gasketed plate heat exchangers facilitates cleaning and permits changes in configuration (changing various plate patterns, adding or removing a few plates, and altering flow arrangements) to meet processes requirements changes. While welded plate heat exchangers can handle extreme temperature, pressure and special design that exceed gasket limitation. Additional, welded plate heat exchangers give high leakage protection and high resistance to tough operating conditions.
The question, which naturally arises, is why plate heat exchangers have not been applied to a large extent for most process industries? In practice, some technical problems prohibit the application. Firstly, due to material issue, plate heat exchangers cannot be used in applications with high pressure (larger than 40 bar) and temperature (higher than 623 K) or in situations where a large difference in pressure between streams for possible plate deformations [1]. Secondly, the great flexibility of plate heat exchangers brings several distinguished advantages, but at the same time it poses considerable complexity when it
comes to designing this type of heat exchangers. And few general design optimisation methodology is open to public. A complete optimisation design methodology of plate heat exchangers should include: 1) determination of various flow arrangement (pass number $N_P$, channel number per pass $N_C$ for all streams); 2) selection of plate patterns (chevron angle, plate spacing, plate length etc.); 3) imposed constraints, such as pressure drop.

Currently, the biggest challenge of plate heat exchanger design is that most essential information required (heat transfer and pressure drop performance data) is industrially owned and not readily available to the public [3]. The traditional design methodologies proposed by Shah [4] and Cooper [5], uses either $\varepsilon$-NTU (number of heat transfer units) or LMTD (logarithmic mean temperature difference) thermal-hydraulic model and employs trial and error to test different plate patterns with detailed geometries in order to consider pressure drop simultaneously, which is related to plate geometry and considered as constraints in the design. But these traditional design methods are time-consuming and also neglect the effect of flow arrangement on heat transfer. To avoid time-consuming trial iterations, Wang and Sunden [6] employed the correlations of heat transfer and pressure drop, which are functions of Reynolds number and chevron angle, to include all possible plate patterns in the optimal design methodology for plate heat exchanger. The optimisation with or without pressure drop specification are completed by maximising the utilisation of pressure drop or minimising the total cost respectively. This methodology applies well when designing single pass heat exchangers, but it doesn’t include the possibility of multi-pass plate heat exchangers, where may give a better design solution. Still, the discrete optimisation problem caused by standardised plate patterns increases freedom of difficulty in finding a global optimum design solution. To overcome the limitations of representing the problem as a mixed nonlinear programming (MINLP) problem, Gut and Pinto [7] proposed a screening method to eliminate infeasible and sub-optimal solutions. In the optimisation design methodology, the detailed configuration, characterised by number of channels, number of passes for two sides, feed connection relative locations, hot fluid location and type of flow in the channels are determined by minimising the total heat transfer area. But the design methodology doesn’t include various plate patterns selection. Picon-Nenuz and Martinez [3] employed $\varepsilon$-NTU method to evaluate the temperature correction factor and determine flow arrangements when applying plate heat exchangers in heat recovery systems. Similarly, this design is based on assumed plate patterns and ignores the plate pattern optimisation. Najafi H. and Najafi D. [8] developed a design optimisation methodology to obtain the optimum plate heat exchanger configuration with multiply objectives of minimising pressure drop and maximising heat transfer coefficient, which are solved by genetic algorithm. But flow arrangement selection is not included in the existing MINLP optimisation problem, which is time-consuming to solve and difficult to find a global optimum design solution. Arsenyeva et al. [9] proposed a temperature difference matrix to include all possible flow arrangement by considering a plate heat exchanger as a system of one pass blocks of plates. In the optimisation procedure, the total heat transfer area is
considered as the objective function to find the optimum plate heat exchanger configuration. But complicated plate pattern optimisation is completed by enumeration. Moreover, the design neglected the effect of chevron angle on heat transfer and pressure drop performance. Therefore, this design method is still time-consuming and cannot guarantee the global optimum solution. Based on Gut and Pinto’s model, Mota et al. [10] proposed a system of normalised ordinary differential equations and wrote in the matrix form to include all possible flow arrangements. The plate pattern selection is addressed by enumeration. For each plate pattern, the optimisation gives one local optimum solution with detailed flow arrangement. By comparing all possible local optima, the global optimum is obtained. But the effect of plate chevron angle on heat transfer and pressure drop performance has not been mentioned in this design.

Therefore, in this paper, an optimal design method for two stream multi-pass plate heat exchangers is presented to solve various flow arrangement selection, plate pattern selection and simultaneously consider pressure drop constraints as well.

2. Problem Statement

The design methodology is developed based on the following assumptions:

- Steady-state heat transfer process
- Constant fluid physical properties
- No phase change in streams
- The number of heat transfer plates is big enough to ignore the effect of end plate
- Heat losses to surroundings are negligible
- Uniform flow distribution between channels
- Constant heat transfer coefficients along the length of the channel.

Based on given physical properties and thermal information (mass flow rate, inlet and outlet temperature, pressure drop) of all process streams, as well as plate physical properties, the design objective is to find the optimum design of a plate heat exchanger with the minimum capital cost. The flow arrangement (the number of passes, the number of channels per pass), plate patterns (plate length, plate width, chevron angle and others), heat exchanger dimensions, heat transfer coefficient and pressure drop should be considered simultaneously.

3. Modelling of two-stream multi-pass plate heat exchangers

The overall mathematical modelling of two-stream multi-pass plate heat exchanger design includes plate patterns, flow arrangements, basic thermal-hydraulic properties, design constraints and the objective function.
3.1 Plate pattern selection

Plate pattern is of great importance to the plate heat exchanger design because different plate geometries will contribute to different heat transfer and pressure drop performance. From Wang [6], a large number of plate corrugation patterns have been developed worldwide to promote heat transfer with minimum pressure drop penalty, such as, washboard, zigzag, chevron, protrusions and depressions, washboard with secondary corrugations and oblique washboard. The chevron plate geometry has proved to be the most efficient design and the most common in use in the past decades [6]. For the chevron-type plate, around 20 plate patterns with different plate sizes and chevron angles are available to choose from. Therefore, the plate pattern selection should be included in the optimisation design methodology. In order to simplify the design procedure, only the chevron type plate is considered in this research work.

The typical structure of the chevron type plate includes: plate length for heat transfer ($L_h$), plate width ($W$), chevron angle ($\beta$), plate spacing ($b$), and port diameter ($d_{port}$), shown in Figure 3.

![Figure 3 Basic geometry of a chevron plate](image)

Due to assemblage issue, the length and width of adjacent plates should be identical. Therefore, the big difference of two adjacent plates is the Chevron angle. Chevron angle is a crucial factor that directly affects heat transfer and pressure drop performance. In modern plate heat exchangers, to improve heat transfer and pressure drop performance, one plate channel is usually sandwiched by two plates with different chevron angles [1], which can form three different channels, shown in Figure 4. L-type channels (Figure 4a) have corrugations with lower chevron angles, which give lower heat transfer coefficient and smaller pressure drop. H-type channels (Figure 4c) have corrugations with larger chevron angles, which form the H channels with higher intensity of heat transfer and higher pressure drop performance. M-type channels (Figure 4b) mix and match large and low chevron angles configurations together and give intermediate heat transfer and pressure drop performance [1]. In other words, if allowed pressure drops of both streams are rather large, the H type
plate channel is recommended to improve heat transfer performance. Conversely, the L type plate channel will be appreciated when both allowed pressure drops are low. But if there is a big pressure drop difference between two streams, M type plate channel is a better choice, which can maximise the utilisation of pressure drop and heat transfer performance. Therefore, chevron angle should be taken as a design variable to find the global optimum design solution.

![Figure 3 Different channel types: (a) L type channel; (b) M type channel; (c) H type channel](image)

A major challenge of plate pattern modelling is a large number of standardized plates with detailed geometries and their unique heat transfer and pressure drop performance. If enumeration is employed to tackle these problems, it will be time-consuming and the optimisation design problem will be an MINLP problem, which is difficult to find a global optimum solution.

From available plate basic geometry data [11], plate size (plate length, plate width, etc.) is limited to a rather small range. Therefore, plate length $L_p$, plate width $W$ and chevron angle $\beta$ for two adjacent plates and other basic geometry parameters can be considered as continuous variables with respective lower and upper bounds in the design model. In this way, the discrete problem caused by standardised plates size can be turned into a continuous problem.

In order to include the heat transfer and pressure drop performance of most chevron type plates, Martin [12, 13] regressed the general Nusselt number and Fanning friction factor as continuous functions of Reynolds number and chevron angle, shown as below:

\[
\frac{1}{\sqrt{f}} = \frac{\cos\beta}{\sqrt{0.045 \tan\beta + 0.09 \sin\beta + f_0 / \cos\beta}} + \frac{1 - \cos\beta}{\sqrt{3.8 f_1}}
\]

(1)

where $f_0 = 16/\text{Re}$ for $\text{Re} < 2000$ and $f_0 = (1.56 \ln \text{Re} - 3.0)^2$ for $\text{Re} \geq 2000$, and $f_1 = (149/\text{Re}) + 0.9625$ for $\text{Re} < 2000$ and $f_1 = 9.75 / \text{Re}^{0.289}$ for $\text{Re} \geq 2000$. The correlation is valid for the chevron angle $\beta$ within $0$ - $80^\circ$.

\[
\text{Nu} = 0.205 \text{Pr}^{\frac{1}{3}} (\frac{\mu_m}{\mu_w})^{\frac{1}{6}} (f \text{Re}^2 \sin 2\beta)^{0.374}
\]

(2)
If this correlation is used for gases, the viscosity correction term \((\mu_m/\mu_w)^{1/6}\) should be omitted. The friction correlation is valid for the chevron angle \(\beta\) within 10 - 80°.

By employing these correlations in the thermal-hydraulic model, the continuous basic geometry variables can represent the heat transfer and pressure drop performance and make the design problem continuous. The MINLP problem in nature for plate pattern selection can be converted into a easily solved NLP problem.

In the plate pattern model, the basic design parameters (hydraulic diameter, free flow area and heat transfer area etc.) can be derived by basic plate geometric parameters. Due to the fact that the influence from amplitude and wavelength can be summarised into the area enlargement factor, the chevron angle becomes the most important parameter for the thermal and hydraulic performance [13]. Normally, the area enlargement factor \(\varphi\) is around the value of 1.22 [6].

The plate length for heat transfer \(L_h\) is different from the plate length for pressure drop \(L_p\), and the relationship of both can be given by:

\[
L_p = L_h + d_{port}
\]

The heat transfer surface area per pass on one stream side of a plate heat exchanger is given by:

\[
A = 2\varphi WL_h N_C
\]

where \(N_C\) is channel number per pass.

The free flow area per pass on one fluid side of a plate heat exchanger is given by:

\[
A_C = bWN_C
\]

In the design, some other design parameters can be related to hydraulic diameter. Because of the irregular sectional area, the hydraulic diameter can be defined as [2]:

\[
d_h = \frac{4 \times \text{Free flow area}}{\text{Wetted Perimeter}} = \frac{4bW}{2(b + \varphi W)} \approx \frac{2b}{\varphi}
\]

### 3.2 Multi-pass Flow Arrangement selection

A plate heat exchanger consists of a large number of plates, which give a freedom of flow arrangement to achieve the required heat load within the specified pressure drop. Apart from changing the number of plates and changing plate patterns, modifying the flow pass
arrangement is another efficient way to meet process requirements [3]. Multi-pass arrangement makes the fluid undergo the heat transfer process during a longer time and a longer distance. Regarding the heat transfer performance, multi-pass heat exchangers will give higher heat transfer coefficient. For pressure drop, as mentioned above, the larger the passes number, the higher the pressure drop. It is another trade-off problem between heat transfer coefficient and pressure drop. Therefore, the flow pass arrangement optimisation is a very important factor in the plate heat exchanger design problem.

The common way to classify the two-stream plate heat exchangers is on the basis of the number of passes for each stream. A detailed classification of different plate heat exchanger configurations on the basis of the number of plates and passes is presented by Kandlikar and Shah [14]. The term “2-4” refers to 2 passes on the fluid stream 1 and 4 passes on the fluid stream 2, shown in Figure 4c. In each pass, the stream may be divided into several channels to transfer heat and then combined to the next pass. The stream is supposed to follow the $N_c \times N_p$ arrangement, which means the stream has $N_p$ passes and $N_c$ channels per pass. Based on the arrangement of gaskets, many combinations of passes are possible on each stream side in a plate heat exchanger. The basic flow arrangements include series flow, circuit flow, and complex flow, as shown in Figure 4.

Figure 4 Basic flow arrangements in plate heat exchanger
The series flow (see Figure 4a) is where the total fluid passes over each plate and changes direction over each subsequent plate. It contains \(2N-1\) thermal plates, where \(N\) is the total pass number. But it is only feasible with small flow rates [1]. The circuit flow (see Figure 4b) is where the fluids are divided into sub-streams that mix before exiting from the plate heat exchanger, which can give a pure counter-current flow arrangement and achieve small temperature approach. Therefore, the circuit flow is widely used with large flow rates [1]. The complex flow (see Figure 4c) is made up of a combination of the series and the circuit flow, which is characterised by more than one pass.

To involve various flow arrangements, the log mean temperature difference correction factor \(F\) is employed in the thermal hydraulic model. The definition is presented as below [15]:

\[
F = \frac{NTU_{\text{counter-current}}}{NTU_{\text{other}}}
\]

(7)

where \(NTU_{\text{counter-current}}\) is the number of heat transfer units when a pure counter-current is employed for the specific duty, while \(NTU_{\text{other}}\) is the number of heat transfer units of actual arrangement in the design.

The number of heat transfer units (NTU) is defined as [15]:

\[
NTU = \frac{UA}{CP}
\]

(8)

where \(U\) is the overall heat transfer coefficient, \(A\) is heat transfer area, and \(CP\) is the heat capacity mass flow rate.

The P-NTU method proposed by Kandlikar and Shah [16] is the most efficient method to present the log mean temperature difference correction factor. \(P\) is temperature effectiveness of individual fluid stream. The basic calculations for counter-flow, parallel flow and complex flow are shown as below [16]:

2-2 with overall parallel flow arrangement:

\[
P_1 = P_{A,1} + P_{B,1} - P_{A,1}P_{B,1}(1 + R_1)
\]

(9)

Where \(P_{A,1}\) and \(P_{B,1}\) are temperature effectiveness of fluid 1 for pass A and B respectively.

2-2 with overall counter-current flow arrangement:

\[
P_1 = \frac{P_{A,1} + P_{B,1} - P_{A,1}P_{B,1}(1 + R_1)}{1 - R_1P_{A,1}P_{B,1}}
\]

(10)

1-2 flow arrangement with fluid 1 divided in two stream:

\[
P_2 = P_{A,2} + P_{B,2} - P_{A,2}P_{B,2}
\]

(11)
The expression for $P_1$ can be obtained from the above equation by utilising the following correlations:

\begin{align*}
P_2 &= R_1 P_1 \\ P_{A,2} &= R_{A,1} P_{A,1} \\ P_{B,2} &= R_{B,1} P_{B,1}
\end{align*}

Combining the above equations, $P_1$ can be expressed as

$$P_1 = \frac{1}{R_1} \left( R_{A,1} P_{A,1} + R_{B,1} P_{B,1} - R_{A,1} R_{B,1} P_{A,1} P_{B,1} \right)$$

$p_{A,1}$, $p_{A,2}$ and $p_{B,1}$, $p_{B,2}$ are the corresponding individual temperature effectiveness of fluids 1 and 2 in passes A and B.

For a parallel flow arrangement, the temperature effectiveness $P$, number of heat transfer units NTU and heat capacity ratio $R$ are related by:

$$P_1 = \frac{1 - \exp \left( -NTU_1 (1 - R_1) \right)}{1 + R_1}$$

For a counter-current flow arrangement, temperature effectiveness can be related to NTU and $R$ in a similar format:

$$P_1 = \frac{1 - \exp \left( -NTU_1 (1 - R_1) \right)}{1 - R_1 \exp \left( -NTU_1 (1 - R_1) \right)}$$

Once NTUother and NTU_counter-current are known, the logarithm mean temperature correction factor $F$ can be obtained. In this work, to simplify the problem, the maximum pass number is set as 4. Therefore, there are at least 16 flow arrangements (1-1, 1-2, 1-3, 1-4, 2-1, 2-2, 2-3, 2-4, 3-1, 3-2, 3-3, 3-4, 4-1, 4-2, 4-3, 4-4), and each flow arrangement can be cross flow, parallel flow or counter-current flow. Obviously, the overall counter-current flow or parallel flow does not always mean individual count-current flow or parallel flow arrangement. The results of different flow arrangement have been published in papers [1, 14, 16] and can be applied directly in the design.

In terms of various flow arrangements, enumeration is employed to compare all possible flow arrangements in the design optimisation problem. Each flow pass arrangement option is considered as a possible complete design. By comparing the objective function values (capital costs) of all possible flow arrangements, select the most efficient flow arrangement as the optimum design solution.
3.3 Thermal-hydraulic model

The basic thermal-hydraulic model for plate heat exchanger is developed on the basis of plate-fin heat exchanger thermal-hydraulic model proposed by Picon [17] and Guo [18].

The plate heat exchanger general design equation can be used to calculate the total area $A_T$ [15]:

$$Q = FU A_T \Delta T_{LM}$$

(18)

where $Q$ is the heat duty, $F$ is temperature difference correction factor and $\Delta T_{LM}$ is logarithmic mean temperature difference.

With the inclusion of surface fouling, the overall heat transfer coefficient $U$ can be defined as below [15]:

$$U = \frac{1}{\frac{1}{h_1} + \frac{1}{h_2} + R_w + R_f}$$

(19)

Combining it with the equation (18) gives:

$$A = \frac{Q}{F \Delta T_{LM}} \left( \frac{1}{\frac{1}{h_1} + \frac{1}{h_2} + R_w + R_f} \right)$$

(20)

where $h_1$ and $h_2$ are the heat transfer coefficients of streams 1 and 2 respectively, $R_w$ is the thermal resistance due to heat conduction through the wall, and $R_f$ is the thermal resistance due to fouling.

For plate heat exchangers, the film heat transfer coefficients are usually calculated on the basis of Nusselt number $Nu$,

$$Nu = \frac{hd_h}{k}$$

(21)

As mentioned above, to eliminate the discrete problem caused by various standardised plates, the continuous empirical correlations of Nusselt number is employed in this model [12]:

$$Nu = f(Re, Pr, \beta)$$

(22)

The details have been demonstrated in Section 3.1. Therefore, the film heat transfer coefficient can be expressed as below:

$$h = \frac{kNu}{d_h} = f(k, d_h, Re, Pr, \beta)$$

(23)
where \( k \) is thermal conductivity of the respective stream and \( d_h \) is hydraulic diameter of plate.

The Reynolds number (Re) is given by [19]:

\[
Re = \frac{md_h}{\rho A_c}
\]

(24)

where \( A_c \) is free flow area, \( \text{m}^2 \); \( \rho \) is fluid density, \( \text{kg/m}^3 \);

The Prandtl number is given by [15]:

\[
Pr = \frac{\mu c_p}{k}
\]

(25)

where \( c_p \) is heat capacity of the fluid, \( \text{J kg}^{-1} \text{K}^{-1} \); \( \mu \) is fluid viscosity, \( \text{cP} \).

As mentioned in Sections 3 and 4, plate pattern and flow arrangement determination will affect the performance of pressure drop. Therefore, the pressure drop and plate patterns selection and flow arrangement determination should be addressed simultaneously. Although it is not possible to fully satisfy the specified pressure drops on both streams because of the standardised configuration and size, the pressure drop should be maximised within the allowed range.

In a plate heat exchanger, the pressure drop from inlet to outlet of one channel included pressure drop \( \Delta P_{\text{channel}} \) due to friction in the channels of the exchanger and pressure drop \( \Delta P_{\text{elevation}} \) due to changes in height [1].

\[
\Delta P_{\text{channel}} = \frac{4f m^2 L_p}{2d_h \rho A_c^2}
\]

(26)

\[
\Delta P_{\text{elevation}} = \pm \rho g H
\]

(27)

where \( f \) is Fanning friction factor, which is a function of Reynolds number and basic plate geometry and shown in the above Section 2, \( L_p \) is plate length for pressure drop, \( \text{m} \). Considering that the height change in a plate exchanger is relatively small, the total pressure drop is equal to the pressure drop \( \Delta P_{\text{channel}} \). With the continuous correlation of friction factor, the pressure drop can be expressed as a continuous function of Reynolds number, chevron angle, plate length, density, mass flow rate, etc. As a result, the discontinuous problem caused by standardised plate pattern can be converted to a continuous problem.

\[
\Delta P = f(Re, \beta, L_p, m, \rho \ldots)
\]

(28)
For multi-pass plate heat exchangers, with the assumption of even flow distribution, the pressure drop of each channel in the same pass has the same value, and the total pressure drop $\Delta P$ should be the pressure drop per pass multiplied by the number of passes $N_p$.

$$\Delta P = N_p \Delta P_{\text{pass}}$$ (29)

### 3.4 Design constraints

In the overall design methodology, the utilisation of pressure drop is usually maximised to increase heat transfer performance, but the pressure drops of both streams should not exceed the maximum allowed values:

$$\Delta P_1 \leq \Delta P_{1,\text{max}}$$ (30)

$$\Delta P_2 \leq \Delta P_{2,\text{max}}$$ (31)

The total heat transfer area of a plate heat exchanger can be given by:

$$A_T = 2(N_C N_P)_{1} \varphi W L_h + 2(N_C N_P)_{2} \varphi W L_h$$ (32)

Once the plate pattern is set, the height of a plate heat exchanger can be determined by:

$$H = b((N_C N_P)_{1} + (N_C N_P)_{2})$$ (33)

### 3.5 Objective function

When it comes to the objective function in the optimisation model, the total capital cost is the common choice, which is related to the total heat transfer area $A_T$ and shown as below [20]:

$$TC = 2070 A_T^{0.85}$$ (34)

The units for capital cost TC and heat exchanger area $A_T$ are GBP and $m^2$, respectively.

### 4 Optimisation of two-stream multi-pass plate heat exchanger design

In the design methodology, the basic plate geometric parameters (plate length $L_p$, plate spacing $b$, plate width $W$, port diameter $D_p$, chevron angles for both sides $\beta_1$, $\beta_2$), passes number $N_p$ and channel number per pass $N_c$ for each stream are considered as variables. These variables are optimised by minimising the total capital cost to find the optimum plate heat exchanger configuration. To avoid discrete problem caused by standardised plate pattern and unique thermal-hydraulic properties, the typical plate geometric parameters are considered, and continuous expression of Nusselt number and friction number is integrated into the design model. In terms of flow arrangements, enumeration is employed to compare and select the optimum flow arrangement. As a result, the MINLP problem is
converted to several NLP problems, which can be easily solved by the CONOPT solver in GAMS.

For certain flow arrangement, a local optimum plate pattern and heat exchanger configurations can be determined by optimising the capital cost. Until comparing all possible flow arrangements, the overall optimum heat exchanger configurations can be obtained with the minimum capital cost.

The overall design optimisation methodology of a two-stream multi-pass plate heat exchanger is shown in Figure 5.

**Figure 5** Overall design optimisation methodology of a multi-pass plate heat exchanger
5 Case Studies

5.1 Case Study 1: the effectiveness of new design model

A case from the published literature [9] is studied to validate the new plate heat exchanger design model. The overall process information of case study is summarised in Table 1. It is required to heat 5 m$^3$h$^{-1}$ of distillery wash fluid from 28 to 90 °C using hot water, which has a temperature of 95 °C and a flow rate 15 m$^3$h$^{-1}$. The allowed pressure drops for the hot and cold streams are both 1.0 bar. The properties of the wash fluid are considered constant as follows: density – 978.4 kg m$^{-3}$; heat capacity – 3.18 kJ kg$^{-1}$ K$^{-1}$; thermal conductivity – 0.66 W m$^{-1}$ K$^{-1}$. Dynamic viscosity at temperatures $t = 25; 60; 90$ °C is taken as $\mu = 19.5; 16.6; 9.0$ cP.

Table 1 Process data for case study

<table>
<thead>
<tr>
<th>Stream</th>
<th>$T_{in}$ (°C)</th>
<th>$T_{out}$ (°C)</th>
<th>$V$ (m$^3$/h)</th>
<th>$\Delta P_{allowed}$ (bar)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$C_p$ (kJ (kg K))</th>
<th>$k$ (W/ (m K))</th>
<th>$\mu$ (cP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stream 1</td>
<td>28</td>
<td>90</td>
<td>5</td>
<td>1</td>
<td>978.4</td>
<td>3.18</td>
<td>0.66</td>
<td>19.5</td>
</tr>
<tr>
<td>Stream 2</td>
<td>95</td>
<td>79</td>
<td>15</td>
<td>1</td>
<td>962.1</td>
<td>4.21</td>
<td>0.68</td>
<td>0.297 (95 °C)</td>
</tr>
</tbody>
</table>

To verify the effectiveness of the new proposed design model, both the published model and new proposed model are simulated to predict the heat transfer coefficient, pressure drop and outlet temperature. By comparing these results, the degree of agreement between two models can be stated clearly. In the validation case, plate pattern M3 with chevron angle 45° is employed for both streams and the outlet temperature of stream 2 is fixed. The result comparison is listed in Table 2.

Table 2 Exchanger performance result comparison

<table>
<thead>
<tr>
<th>Stream</th>
<th>Published Model</th>
<th>New Model</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{out}$ (°C)</td>
<td>$h$ (W/m$^2$K)</td>
<td>$\Delta P$ (bar)</td>
</tr>
<tr>
<td>Stream 1</td>
<td>90</td>
<td>519.2</td>
<td>0.41</td>
</tr>
<tr>
<td>Stream 2</td>
<td>79</td>
<td>557.3</td>
<td>1</td>
</tr>
</tbody>
</table>

It is clear from the above table that heat transfer coefficients in the new model are slightly higher than those in the published model and the outlet temperature of stream 1 is almost the same as that in the literature model. Similarly, the pressure drop of stream 1 is slightly higher than the original value. The main reason for these difference is that the design models are correlation-based. Thus changing or using appropriate correlations can give different prediction. Based on these results, it can be seen that the results obtained by applying the new proposed methodology are in good agreement with the published
thermal-hydraulic model. Therefore, the new proposed thermal-hydraulic model can be employed for optimisation of plate heat exchanger design.

5.2 Case Study 2: Two-stream multi-pass plate exchanger optimisation design

The case studied in model validation is carried out again here to verify the effectiveness and accuracy of new proposed optimisation design methodology. In this case study, plate patterns data in literature [17], shown in Table 3, is employed to facilitate design results comparison.

Table 3 Plate heat exchanger geometrical parameters

<table>
<thead>
<tr>
<th></th>
<th>b (mm)</th>
<th>Lh (mm)</th>
<th>W (mm)</th>
<th>Dport (mm)</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>M3</td>
<td>2.4</td>
<td>320</td>
<td>100</td>
<td>36</td>
<td>30°(T), 45°(F), 60°(S)</td>
</tr>
<tr>
<td>M6</td>
<td>2.0</td>
<td>694</td>
<td>216</td>
<td>50</td>
<td>30°(T), 45°(F), 60°(S)</td>
</tr>
<tr>
<td>M6M</td>
<td>3.0</td>
<td>666</td>
<td>210</td>
<td>50</td>
<td>30°(T), 45°(F), 60°(S)</td>
</tr>
<tr>
<td>M10B</td>
<td>2.5</td>
<td>719</td>
<td>334</td>
<td>100</td>
<td>30°(T), 45°(F), 60°(S)</td>
</tr>
<tr>
<td>M15B</td>
<td>2.5</td>
<td>1381</td>
<td>449</td>
<td>150</td>
<td>30°(T), 45°(F), 60°(S)</td>
</tr>
</tbody>
</table>

There are 7 variables and 6 nonlinear equations in the optimisation program. The optimisation design is executed with GAMS version 23.4 on a 2.6 GHz 4th Intel Core i5 PC with 8GB memory. The CONOPT solver is employed in this study and the CPU time for one optimisation with specific flow pass arrangement is 13 seconds, which is much faster than that in other published methodologies using the same computer [9].

The design details for different flow pass arrangements are listed in Table 4. In the basic design, the optimal solution is obtained for plate pattern M6M, employed in the all design solutions. H, L and M mentioned in the basic design results mean high duty, low duty and medium duty plate. The specific Chevron angle is not given in detail. While in the new design, apart from M6M, both M10B and M6 are also selected as the optimum solution for specific flow pass arrangements. T, F and S stand for plates with chevron angle 30°, 45°, 60°. For each specific flow arrangement, subject to certain constraints, local optimal configurations are found. For all flow arrangements considered, by plotting a graph of capital cost (area) vs. flow pass arrangement, the overall optimum design can be obtained easily.
Figure 6 Graphical representation of design results for different flow arrangement

It is obvious from Figure 6 that design results agree very well with those in the published literature [9]. However, compare with several hours computation time in the published literature, only 130 seconds is needed for whole heat exchanger design in this case. Therefore, for a multi-stream multi-pass plate heat exchanger, this optimisation methodology can significantly reduce the computation time.

In the new design, the overall optimum design solution is based on the flow pass arrangement 2-4, which means two passes for stream 1 and 4 passes for stream 2. The total heat transfer area and capital cost are 5.04 m$^2$ and 8185.33 GBP respectively. For stream 1, the optimum plate pattern is M6M with Chevron angle of 30°, the optimum flow pass arrangement is 2 passes with 9 channels each pass. For stream 2, the optimum plate pattern is also M6M with Chevron angle of 60°, the optimum flow pass arrangement is 4 passes with 4 channels for 2 passes and 5 channels for other 2 passes. The optimum design result agrees well with the published literature.

The regressed Nusselt number and friction factor correlations employed in the base case are based on experimental results, which are different from the numerical modelling correlations employed in this new model. Due to the above reasons, some slightly different design results appear in the flow arrangements 2-1, 3-1, 4-1 and 1-4. But the trend is almost the same.

In the design methodology, the allowed pressure drop of stream 1 is fully utilised, while the pressure drop is calculated automatically but should be no larger than the allowed maximum value. Actually, the pressure drop, plate pattern selection and flow pass arrangement are interrelated and interacting on each other. In other words, if the number of passes is large, the pressure drop will be high. Correspondingly, the design will
automatically select plate pattern with lower Chevron angle to avoid exceeding the allowed maximum pressure drop. Conversely, if the higher Chevron angle plate is selected, the smaller number of passes will be preferable. Therefore, the pressure drop constraints should be taken into account simultaneously in the design optimisation.
### Table 4 Design results for different flow pass arrangements

<table>
<thead>
<tr>
<th>Flow Arrangement</th>
<th>Basic Design</th>
<th>New Design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Plate pattern and $N_c$</td>
<td>Area (m²)</td>
</tr>
<tr>
<td>1-1</td>
<td>$1 \times 28H/1 \times 27H$ (M6M)</td>
<td>7.56</td>
</tr>
<tr>
<td>2-1</td>
<td>$2 \times 58H/1 \times 116H$ (M6M)</td>
<td>32.34</td>
</tr>
<tr>
<td>3-1</td>
<td>$3 \times 26H/1 \times 78H$ (M6M)</td>
<td>21.70</td>
</tr>
<tr>
<td>4-1</td>
<td>$4 \times 23H/1 \times 91H$ (M6M)</td>
<td>25.48</td>
</tr>
<tr>
<td>1-2</td>
<td>$1 \times 35H/2 \times 18H$ (M6M)</td>
<td>9.80</td>
</tr>
<tr>
<td>2-2</td>
<td>$2 \times (6H+6M)/2 \times (6H+6M)$ (M6M)</td>
<td>6.58</td>
</tr>
<tr>
<td>3-2</td>
<td>$3 \times 10M/2 \times 15M$ (M6M)</td>
<td>8.26</td>
</tr>
<tr>
<td>4-2</td>
<td>$4 \times 8L/1 \times 15L+1 \times 16L$ (M6M)</td>
<td>8.68</td>
</tr>
<tr>
<td>1-3</td>
<td>$1 \times 21H/2 \times 7H+1 \times 8H$ (M6M)</td>
<td>5.88</td>
</tr>
<tr>
<td>2-3</td>
<td>$2 \times 12M/3 \times 8M$ (M6M)</td>
<td>6.58</td>
</tr>
<tr>
<td>3-3</td>
<td>$3 \times (4M+3L)/3 \times (4M+3L)$ (M6M)</td>
<td>5.74</td>
</tr>
<tr>
<td>4-3</td>
<td>$3 \times 6L+1 \times 7L/3 \times 8L$ (M6M)</td>
<td>6.72</td>
</tr>
<tr>
<td>1-4</td>
<td>$1 \times 31M/1 \times 7M+3 \times 8M$ (M6M)</td>
<td>8.54</td>
</tr>
<tr>
<td>2-4</td>
<td>$2 \times 9M/1 \times 4M+3 \times 5M$ (M6M)</td>
<td>5.04</td>
</tr>
<tr>
<td>3-4</td>
<td>$3 \times 8L/1 \times 5L+3 \times 6L$ (M6M)</td>
<td>6.44</td>
</tr>
<tr>
<td>4-4</td>
<td>$3 \times 6L+1 \times 7L/4 \times 6L$ (M6M)</td>
<td>6.72</td>
</tr>
</tbody>
</table>

* T: chevron angle $\beta = 30^\circ$; F: chevron angle $\beta = 45^\circ$; S: chevron angle $\beta = 60^\circ$
6. Conclusions

A new design optimisation methodology for two-stream multi-pass plate heat exchangers is developed. The basic plate geometrical parameters, such as plate spacing, plate width, plate length, chevron angle, are set as continuous variables for minimisation of heat exchanger capital cost. By employing continuous expressions of Nusselt number and friction factor, the plate pattern selection and pressure drop constraints can be taken into consideration simultaneously. And with enumeration, the optimum flow pass arrangement can be determined by comparing all capital costs. On the basis of these, the MINLP design problem can be converted to several NLP problems for easy convergence and short computation time.

However, the two-stream multi-pass plate heat exchanger optimisation design methodology in this work is proposed within assumptions of constant fluid physical properties and no phase change in the process. Further research work should tackle phase changes and also extend the methodology to multi-stream and multi-pass plate heat exchangers.

Reference


Chapter 5

Publication 4: Design Optimisation of Multi-stream Multi-pass Plate Heat Exchangers with Phase Change

Design Optimisation of Multi-stream Multi-pass Plate Heat Exchangers with Phase Change

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Abstract

For multi-stream multi-pass plate heat exchanger design with phase change, one needs to consider the plate pattern selection, flow arrangement determination and varied heat transfer coefficient and pressure drop caused by phase change simultaneously. Major difficulties of multi-stream multi-pass plate heat exchanger design with phase change arise from limited general heat transfer and pressure drop correlations for phase change, complicated plate pattern selection and flow arrangement determination. In this work, published continuous correlations of Nusselt number and friction factor for boiling and condensation are employed to avoid solving discrete optimisation problem caused by boiling and condensation are employed to avoid solving discrete optimisation problem caused by standardised plate patterns. And a multi-stream plate heat exchanger with phase change is considered as a network of two-stream plate heat exchangers. The basic plate geometric parameters and flow arrangement configurations are optimised to minimise the total capital cost. Case studies are conducted to verify the effectiveness of the proposed methodology.

Key words: multi-stream phase change optimisation plate heat exchanger

Nomenclature

Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Heat transfer area, m²</td>
</tr>
<tr>
<td>Ac</td>
<td>Free flow area, m²</td>
</tr>
<tr>
<td>AT</td>
<td>Total heat transfer area, m²</td>
</tr>
<tr>
<td>Bd</td>
<td>Bond number</td>
</tr>
<tr>
<td>Bo</td>
<td>Boiling number</td>
</tr>
<tr>
<td>cp</td>
<td>Heat capacity, J/(kg K)</td>
</tr>
<tr>
<td>CP</td>
<td>Heat Capacity mass flowrate, W</td>
</tr>
<tr>
<td>d_h</td>
<td>Hydraulic diameter, m</td>
</tr>
<tr>
<td>f</td>
<td>Fanning friction factor</td>
</tr>
<tr>
<td>F</td>
<td>Logarithm temperature difference correction factor</td>
</tr>
</tbody>
</table>

‡ Corresponding author: Tel: +44 1613064384; Fax:+44 1612367439; Email: nan.zhang@manchester.ac.uk
\( g \)  Gravity constant
\( G \)  Mass flux
\( G_e \)  Geometric parameter
\( h \)  Film heat transfer coefficient
\( H \)  Plate heat exchanger height, m
\( k \)  Thermal conductivity, W/ (m K)
\( m \)  Mass flow rate, kg/s
\( N_C \)  Number of channels
\( \text{NTU} \)  Number of heat transfer units
\( \text{Nu} \)  Nusselt number
\( P \)  Temperature effectiveness
\( \text{Pr} \)  Prandtl number
\( Q \)  Heat Duty, W
\( R \)  Heat capacity mass flowrate ratio
\( \text{Re} \)  Reynolds number
\( R_w \)  Thermal resistance due to heat conduction through the wall, (m² K)/ W
\( R_f \)  Thermal resistance due to fouling, (m² K)/ W
\( \text{TC} \)  Total capital cost, GBP
\( U \)  Overall heat transfer coefficient, W/ (m² K)
\( \text{We} \)  Weber number
\( x \)  Vapour quality
\( z \)  Multi-component factor
\( \Delta P \)  Pressure drop, Pa
\( \Delta T_{LM} \)  Logarithm temperature difference, K

**Variables**
\( b \)  Plate spacing, m
\( d_{\text{port}} \)  Port diameter, m
\( L_h \)  Plate length, m
\( L_p \)  Plate length, m
\( N_p \)  Number of Passes
\( W \)  Plate width, m
\( \beta \)  Chevron angle, °

**Greek letters**
\( \rho \)  Density, kg/m³
**ρ** Density ratio

**μ** Viscosity, cP

**B** Chevron angle ratio

**φ** Area enlargement factor

**σ** Surface tension force

**Subscript**

1 Stream 1

2 Stream 2

A Pass number A

B Pass number B

tp Two-phase

eq Equivalent

l liquid

v vapour

## 1. Introduction

Plate heat exchangers (PHEs), a type of compact heat exchangers, consist of thin, rectangular, pressed steel plates that are stacked together, such that hot and cold streams alternate through the inter-plate passages [1]. With small approach temperature and large heat transfer surface area per unit volume, plate heat exchangers can enlarge the heat recovery duty and save utility and capital cost. Compared with shell and tube heat exchangers, plate heat exchangers have a higher convective heat transfer coefficient, encouraged by the large heat transfer area, small flow cross section or hydraulic diameter, flow disruption and reattachment of boundary layers, and the generation of swirl, vortex and helical secondary flows [2]. Commonly-used plate heat exchangers include gasket plate heat exchanger and welded plate heat exchangers. In the gasket plate heat exchangers, shown in Figure 1, the flow arrangement is governed by gaskets, which allows the sealing of each plate and generates alternative flow channels for two fluids. This unique structure facilitates cleaning and permits changes in configuration (changing plate patterns, adding or removing a few plates, and altering flow arrangements) to meet processes requirements. Therefore, they are widely used in the chemical, pharmaceutical, food and beverage industries. In the welded plate heat exchangers, shown in Figure 2, all plates are welded together. The operating temperature range can be between -50 °C and 350 °C, while the range of operating pressure can vary from vacuum to 40 bars. The major applications for this type of plate heat exchangers are oil cooling, process chemicals, expensive fluids and refrigeration [1].
For single phase applications, the heat transfer coefficient changes along the flow length of an exchanger only as a result of changes in the physical properties of the fluid. Single phase flow inside plate heat exchangers has been widely studied and the design optimisation methodology of plate heat exchangers has been well developed [1]. As mentioned above, plate heat exchangers are increasingly used for many evaporation and condensation applications, such as steam generators, petrochemical reboilers, oil cooler and refrigeration systems [3]. For phase change duties, however, the heat transfer coefficient can also change very significantly because of the changes in the vapour quality (local vapour phase fraction). Furthermore, the changes of local vapour quality can also result in different local two-phase...
flow patterns within the exchanger flow passages, causing further impact on the local thermal-hydraulics \[4\]. But most research work until now only proposed prediction methods for limited plates and fluids, instead of general correlations for predicting the heat transfer and pressure drop performance. Additionally, less attention has been given to two-phase plate heat exchanger design methodology. Therefore, addressing phase change problems and integrating plate pattern selection, flow arrangement determination and imposed constraints simultaneously in the overall design methodology are major challenges in evaporation and condensation applications with plate heat exchangers.

In terms of two-phase heat transfer and pressure drop performance, Yan and Lin \[5, 6\] investigated flow boiling and condensing of R-134a in a single channel plate heat exchanger and reported correlations for the local heat transfer coefficient and pressure drop performance. Hsieh and Lin \[7\] proposed boiling heat transfer coefficient and pressure drop correlations from the experimental data for R-410A in a plate heat exchanger with chevron angle of 60 °C. These regressed correlations are based on specific plate patterns. Han, Lee and Kim \[8, 9\] varied mass flow rate, evaporating (condensation) temperature, vapour quality, heat flux and plate pattern and regressed correlations for heat transfer and pressure drop performance with refrigerants R-22 and R-410A. Amalfi et al. \[10\] proposed boiling heat transfer and pressure drop performance prediction correlations on the basis of published experimental database.

When it comes to design methodology, few published papers focus on plate heat exchanger design with multi-stream and phase change. Najafi \[11\], Arsenyeva \[12\] and Wang \[13\] proposed various novel design methodologies to address plate pattern selection, flow arrangement selection and optimisation problem for two-stream plate heat exchanger. Picon-Nunez \[14\] proposed plate heat exchanger design methodology and also addressed multi-stream problem. But all these design methodologies haven’t considered the phase change problem. Qiao et al. \[15\] developed a new model for the analysis of plate heat exchangers with multi-stream and multi-pass configurations, which divides an entire heat exchanger into multiple slices in the direction of flow, and the wall temperatures are assumed to be constant. But this model excludes the effect of plate pattern and flow arrangement. Therefore, the aim of this paper is propose a new design optimisation methodology for multi-stream multi-pass plate heat exchangers with phase change.

2. Thermal-hydraulic modelling of two-phase systems

For the design of evaporators, boilers and condensers, it is necessary to have Nusselt number and friction number correlations that can predict the heat transfer and pressure drop performance for fluids undergoing phase change, for which boiling and condensation have somewhat different characteristics. Therefore, different correlations needs to be developed for boiling and condensation respectively.
2.1 Boiling correlations for pure components

The evaporation heat transfer can be typically divided into: (i) a nucleate boiling dominated regime (where heat transfer is associated with bubble nucleation and growth), (ii) a two-phase convective boiling dominated regime (where heat is transferred by convection through a layer of liquid from the wall to the liquid-vapour interface), (iii) a transition regime where both nucleate and two-phase convective boiling contribute [16]. However, evaporation in confined corrugated channels of plate heat exchangers belongs mainly in the convective dominated regime as the heat fluxes in such applications are low as narrow temperature approaches between the evaporating fluid and the heating fluid are the norm, and getting even smaller for energetic reasons [17]. During the evaporation process the refrigerant vapour quality grows and thus the specific volume grows, and consequently the fluid velocity rises. The larger velocity promotes more shear between the liquid and vapour phases and provides higher turbulence. Thus the convective heat transfer coefficient is enhanced and the associated frictional pressure drop grows as well. Heat transfer mechanisms in plate heat exchangers are in fact a complex function of the flow regime, plate geometry, fluid properties, heat flux and many other parameters. Amalfi, Vakili-Farahani and Thome [10] collected all published two-phase heat transfer experiment data and regressed new prediction correlations for heat transfer and pressure drop performance on the basis of a comprehensive database. These correlations are suitable for most plate patterns, most fluid media and different operating conditions.

The prediction of the two-phase Nusselt number should be performed according to the value of Bond number $Bd$, which is a dimensionless number measuring the importance of surface tension forces compared to body forces [10].

$$Bd = \frac{\Delta \rho g L^2}{\sigma}$$  \hspace{1cm} (1)

When the bond number is less than 4, the two-phase Nusselt number is found to associate with Weber number $We_m$, Bond number, boiling number $Bo$, density ratio (liquid/vapour) $\rho^*$ and Chevron angle as follows [10]:

$$Nu_{tp} = 982\beta^{1.101} We_m^{0.315} Bo^{0.320} \rho^{*-0.224}$$  \hspace{1cm} (2)

where reduced Chevron angle $\beta^*$ is the ratio of Chevron angle and maximum Chevron angle, Weber number is a dimensionless number that is often useful in analysing thin film flows and the formation of droplet and bubbles [10], boiling number is a dimensionless number that is to evaluate the effect of heat flux, mass flux and latent heat on the heat transfer rate [10], and density ratio is the ratio of liquid density and vapour density.

$$\beta^* = \frac{\beta}{\beta_{max}}$$  \hspace{1cm} (3)

$$We_m = \frac{G^2 d_h}{\rho_m \sigma}$$  \hspace{1cm} (4)
When the bond number $Bd$ is larger than or equal to 4, the homogeneous Weber number $We_m$ is replaced by vapour Reynolds number and liquid only Reynolds number [10].

$$Nu_{tp} = 18.495\beta^{0.248}Re_v^{0.135}Re_l^{0.351}Bd^{0.235}Bo^{0.198}\rho^{*-0.223}$$ (7)

The two-phase Fanning friction factor $f_{tp}$ is also related to the homogeneous Weber number, Bond number, density ratio (liquid/vapour) and Chevron angle [10]:

$$f_{tp} = 15.698(2.15\beta^{9.993} + 0.955)We_m^{-0.475}Bd^{0.255}\rho^{*-0.571}$$ (8)

The friction factor increases with Chevron angle because of the longer fluid path, but decreases with the Weber number. The reason may be explained by the fact that with an increasing vapour quality, the liquid film thickness close to the wall becomes thinner, and as a consequence lower shear stress can be observed. Furthermore, the friction factor rises when the Bond number grows because of the late transition from laminar to turbulent flow regime, which provides higher shear stress. But on the other hand, it decreases with an increase of the density ratio [4].

### 2.2 Condensing correlations for pure components

Analogous to the boiling process, the condensing process can be divided into two different regimes: drop-wise and film-wise condensation. In drop wise condensation, vapour condenses in drops that grow as they are further condensed and their coalescence and roll over the surface. In film-wise condensation, an annular liquid film is formed on the cooled surface [18]. In the heat exchanger design, the film-wise condensation is accepted [18].

The condensation heat transfer coefficient is correlated as a function of the Reynolds number and Prandtl number. Han, Lee and Kim proposed new correlations to include the effect of mass flux, condensation temperature and vapour quality [9].

$$Nu = Ge_1Re_{eq}^{Ge_2}Pr^{1/3}$$ (9)

where the coefficients $Ge_1$ and $Ge_2$ are functions of the heat exchanger geometric parameters [9]:

$$Ge_1 = 11.22\left(p_{co}/d_h\right)^{-2.83}\left(\frac{\pi}{2} - \beta\right)^{-4.5}$$ (10)

$$Ge_2 = 0.35\left(p_{co}/d_h\right)^{0.23}\left(\frac{\pi}{2} - \beta\right)^{1.48}$$ (11)

Similarly, the condensation friction factor $f_{tp}$ is regressed in the same form, as an only
function of Reynolds number [9]:

\[ f_{tp} = Ge_3 Re_{eq}^{Ge_4} \]  

(12)

where \( Ge_3 \) and \( Ge_4 \) are also dimensionless geometric parameters that include the corrugation pitch, hydraulic diameter and Chevron angle [9].

\[ Ge_3 = 3521.1 \left( \frac{p_{co}}{d_h} \right)^{4.17} \left( \frac{\pi}{2} - \beta \right)^{-7.75} \]  

(13)

\[ Ge_4 = -1.024 \left( \frac{p_{co}}{d_h} \right)^{0.0925} \left( \frac{\pi}{2} - \beta \right)^{-1.3} \]  

(14)

And \( Re_{eq} \) is the equivalent Reynolds number, and \( Ge_{eq} \) is the equivalent mass flux [9]:

\[ Re_{eq} = \frac{Ge_{eq} d_h}{\mu} \]  

(15)

\[ Ge_{eq} = G_c [1 - x + x(\rho_l/\rho_v)^{0.5}] \]  

(16)

\[ G_c = \frac{m}{N_c b W} \]  

(17)

where \( x \) is vapour quality, \( b \) is plate spacing and \( W \) is plate width.

### 2.3 Correlations for multi-component streams

All the above boiling and condensing heat transfer and pressure drop correlations are developed for single-component streams. But multicomponent are more common in practice, especially in refrigeration systems and refineries. Bell and Ghaly [19] proposed a correction factor \( Z \) to account for the effect of multi-components on the boiling and condensation heat transfer performance. The correction factor \( Z \) is defined as follows [19]:

\[ Z = x C_{p,v} \left( \frac{dh}{dT} \right)^{-1} \]  

(18)

where \( dh/dT \) is the slope of the boiling or condensing enthalpy-temperature curve. The corrected boiling and condensation heat transfer coefficient for multi-component streams are calculated by [19]:

\[ h_{b,m} = \frac{1}{\frac{1}{h_b} + \frac{Z}{h_v}} \]  

(19)

\[ h_{c,m} = \frac{1}{\frac{1}{h_c} + \frac{Z}{h_v}} \]  

(20)

where \( h_v \) is the vapour fraction single phase heat transfer area.
3. Modelling of multi-stream plate heat exchangers

Multi-stream plate heat exchangers give a possibility of transferring heat among several streams in one unit, which can maximise the heat recovery duty and save the heat exchanger capital cost. As a multi-stream plate heat exchanger can be considered as a network of two-stream plate heat exchangers, the design method developed by Guo, et al [20] for multi-stream plate fin heat exchangers is adopted in this work. With the help of pinch analysis, the network can be graphically represented by composite curves. Based on inlet and outlet stream information or kink points, multi-stream plate heat exchangers with no phase change are divided into several enthalpy intervals. But for multi-stream plate heat exchangers with phase change, the composite curves are no longer straight, caused by various physical properties. For a multi-stream plate heat exchanger with phase change, physical properties (density, vapour quality, viscosity, heat capacity, etc.) of involved fluids will vary with temperature. Such changes to the physical properties affect the heat transfer and pressure drop performance, which will make the heat exchanger design more challenging. Therefore, how to address these changes is the key issue of plate heat exchanger design.

In this work, by dividing the whole process into several sections, the heat transfer coefficient and pressure drop can be calculated based on the average vapour fraction of the section. The original enthalpy intervals can still be divided based on the inlet and outlet information. And each enthalpy interval with phase change can be further divided into several subsections to improve the accuracy of fluid physical properties. In terms of subsections distribution, the temperature range of the streams in each subsection can be set to the same. The smaller the temperature range, the more accurate and complicated the design.

The allowed pressure drop per stream in each enthalpy is proportional to its corresponding heat duty:

$$\Delta P_{i,k(z)} = \Delta P_{total} \left( \frac{\Delta H_{i,k(z)}}{\Delta H_{total}} \right)$$

where $i$ is the hot or cold stream number, $k$ is the interval number and $z$ subsection number.

In each enthalpy interval or subsection, the inlet and outlet temperature, flow rate, allowed pressure drop and heat duty are fixed. Therefore, each hot stream can match any cold stream in the same enthalpy interval or subsection. A superstructure based plate heat exchanger network is employed in this work. The heat load distribution of each possible match is based on corresponding stream heat load fraction:

$$Q_{i,j,k} = Q_{i,k} \times \frac{Q_{j,k}}{\sum_j Q_{j,k}}$$

where $j$ is the cold or hot stream number.
In this way, each match can be regarded as a two-stream plate heat exchanger, and each subsection in an enthalpy interval can be considered as a two-stream plate heat exchanger network. The total capital cost of each interval is the sum of capital cost of individual two-stream plate heat exchangers.

Due to the small temperature range in each subsection, the dimension of heat exchanger will be very small if one subsection is considered as a separate heat exchanger. Therefore, in this study, each enthalpy interval is considered as a separate heat exchanger with specific dimensions instead of each subsection. In other words, except Chevron angle, the plate pattern (plate length, plate width and port diameter) employed should be uniform to allow the fluid pass through ports smoothly. Each subsection can be considered as a prior subsection sequence to determine the plate pattern. This kind of plate pattern is applied in the other subsections in the same enthalpy interval, and the whole interval capital cost is calculated. Similar to multi-stream plate fin heat exchanger design [20], design sequence y is introduced to determine the reference subsection. By comparing the total capital cost, the optimum plate heat exchanger configuration can be derived.

4. Overall modelling of multi-stream plate heat exchangers with phase change

The overall modelling of multi-stream plate heat exchangers with phase change includes the above mentioned multi-stream model and the phase change model to address multi-stream and phase change design problems. Also, it contains the basic thermal hydraulic model (including design constraints), the plate pattern model and flow arrangement model to tackle the selection of plate patterns and determination of flow arrangements, which are explained in detail as follows.

4.1 Basic thermal-hydraulic model

The plate heat exchanger heat transfer area \( A_T \) is calculated by the general design equation [21]:

\[
Q = FU_A T \Delta T_{LM}
\]  

(23)

where \( Q \) is the heat duty, \( F \) is temperature difference correction factor and \( \Delta T_{LM} \) is logarithmic mean temperature difference.

With the inclusion of surface fouling, the overall heat transfer coefficient \( U \) can be defined as below [21]:

\[
U = \frac{1}{\frac{1}{h_1} + \frac{1}{h_2} + R_w + R_f}
\]  

(24)

Combining it with equation (23) gives:
\[ A_T = \frac{Q}{F \Delta T_{LM}} \left( \frac{1}{h_1} + \frac{1}{h_2} + R_w + R_f \right) \]  

(25)

where \( h_1 \) and \( h_2 \) are the heat transfer coefficients of streams 1 and 2 respectively, \( R_w \) is the thermal resistance due to heat conduction through the wall, and \( R_f \) is the thermal resistance due to fouling.

For plate heat exchangers, the film heat transfer coefficients are usually related to Nusselt number \( Nu \) [21],

\[ Nu = \frac{hd_h}{k} \]  

(26)

To eliminate the discrete problem caused by various standardised plates, the continuous empirical correlations of Nusselt number for single phase and two-phase are employed in this model. The Nusselt number correlation for single phase is a function of Reynolds number, Prandtl number and Chevron angle [1], shown as follows:

\[ Nu = 0.205Pr^{\frac{1}{3}} \left( \frac{\mu_m}{\mu_w} \right)^{\frac{1}{3}} (f Re^2 \sin 2\beta)^{0.374} \]  

(27)

While Nusselt number correlations for two phases (boiling and condensing) have been demonstrated in the thermal-hydraulic model for two-phase sections. Therefore, no matter if there is phase change or not, the film heat transfer coefficient can be expressed as a continuous function of physical properties and geometric parameters:

\[ h = \frac{kNu}{d_h} = f(k, d_h, Re, Pr, \beta, etc.) \]  

(28)

where \( k \) is thermal conductivity of the respective stream and \( d_h \) is hydraulic diameter of plate.

The basic definition of Reynolds number (\( Re \)) is given by [22]:

\[ Re = \frac{md_h}{\rho A_c} \]  

(29)

where \( A_c \) is free flow area, \( m^2 \); \( \rho \) is fluid density, \( \text{kg/m}^3 \), the Reynolds number for liquid phase, vapour phase or two phases can be obtained based on the basic definition.

The Prandtl number is given by [21]:

\[ Pr = \frac{\mu c_p}{k} \]  

(30)

where \( c_p \) is heat capacity of the fluid, \( \text{J kg}^{-1} \text{K}^{-1} \); \( \mu \) is fluid viscosity, \( \text{cP} \).
In the plate heat exchanger, the pressure drop from inlet to outlet of one channel includes pressure drop $\Delta P_{\text{channel}}$ (due to friction in the channels of the exchanger) and pressure drop $\Delta P_{\text{elevation}}$ (due to changes in height) [4].

$$\Delta P_{\text{channel}} = \frac{4fm^2L_p}{2d_h\rho A_c^2}$$  \hspace{1cm} (31)

$$\Delta P_{\text{elevation}} = \pm \rho gH$$  \hspace{1cm} (32)

where $f$ is Fanning friction factor, which is a function of Reynolds number and basic plate geometry shown in Section 2, and $L_p$ is plate length for pressure drop.

Pressure drops in two-phase flows also include acceleration (or deceleration) part, shown as follows [4]:

$$\Delta P_{\text{acc}} = G^2 \Delta x \left( \frac{1}{\rho_v} - \frac{1}{\rho_l} \right)$$  \hspace{1cm} (33)

For prediction of frictional pressure drop in a two-phase flow, the two-phase Fanning friction factor $f_{tp}$, shown in Section 2, is employed instead of single phase friction factor. Two-phase friction multiplier, which is a function of Lockhart-Martinelli parameter and flow regime depended constant, is widely applied [23]. But it is difficult to find general correlations for most cases.

Considering that the height change in a plate exchanger is relatively small and the relatively small acceleration pressure drop, the total pressure drop for two-phase is equal to the pressure drop $\Delta P_{\text{channel}}$. With the continuous correlation of friction factor, the pressure drop can also be expressed as a continuous function of Reynolds number, Chevron angle, plate length, density, mass flow rate, etc. As a result, the discontinuous problem caused by standardised plate pattern can be converted to a continuous problem.

$$\Delta P = f(Re, \beta, L_p, m, \rho, We_m \text{ etc.})$$  \hspace{1cm} (34)

For multi-pass plate heat exchangers, with the assumption of even flow distribution, the pressure drop of each channel in the same pass has the same value and the total pressure drop $\Delta P$ should be the pressure drop per pass multiplied by the number of passes $N_p$.

$$\Delta P = N_p \Delta P_{\text{pass}}$$  \hspace{1cm} (35)

In the overall design methodology, the utilisation of pressure drop is usually maximised. But the pressure drops of both streams must not exceed the maximum allowed values:

$$\Delta P_1 \leq \Delta P_{1,\text{max}}$$  \hspace{1cm} (36)

$$\Delta P_2 \leq \Delta P_{2,\text{max}}$$  \hspace{1cm} (37)
The total heat transfer area of a plate heat exchanger can be given by:

\[ A_T = 2(N_C N_P)_1 \varphi W L_h + 2(N_C N_P)_2 \varphi W L_h \]  

(38)

Once the plate pattern is set, the height of a plate heat exchanger can be determined by:

\[ H = b((N_C N_P)_1 + (N_C N_P)_2) \]  

(39)

4.2 Modelling of plate patterns

The Chevron plate geometry has proven to be the most efficient design and the most common in use in the past decades [24]. For the Chevron-type plate, around 20 plate patterns with different plate sizes and Chevron angles are available to choose from. Therefore, plate pattern selection should be considered in the design optimisation. The typical structure of the Chevron type plate includes: plate length for heat transfer \( L_h \), plate width \( W \), Chevron angle \( \beta \), plate spacing \( b \), and port diameter \( d_{\text{port}} \), shown in Figure 3.

![Figure 3 Basic geometry of a chevron plate](image)

The plate length for heat transfer \( L_h \) is different from the plate length for pressure drop \( L_p \), and the relationship of both can be given by:

\[ L_p = L_h + d_{\text{port}} \]  

(40)

The heat transfer surface area per pass on one stream side of a plate heat exchanger is given by:

\[ A = 2\varphi WL_h N_C \]  

(41)

where \( N_C \) is channel number per pass.

The free flow area per pass on one fluid side of a plate heat exchanger is given by:

\[ A_C = bWN_C \]  

(42)

In the design, some other design parameters can be related to hydraulic diameter. Because
of the irregular sectional area, the hydraulic diameter can be defined as [2]:

$$d_h = \frac{4 \times \text{Free flow area}}{\text{Wetted Perimeter}} = \frac{4bW}{2(b + \varphi W)} \approx \frac{2b}{\varphi}$$  \hspace{1cm} (43)

In the design process, plate length \(L_p\), plate spacing \(b\), plate width \(W\), port diameter \(D_p\), Chevron angles for both sides \(\beta_1, \beta_2\) are considered as continuous variables to avoid discrete standardised plate patterns. The unique heat transfer and pressure drop performance of different standardised plates can be expressed by continuous Nusselt number and Fanning friction number correlations, shown in Section 2. In this way, plate pattern selection can be integrated into the design model.

### 4.3 Modelling of flow arrangements

The stacked plates in plate heat exchangers provide a freedom of flow arrangement to achieve the required heat load within the specified pressure drop. Multi-pass heat exchangers will give higher heat transfer coefficient, at the expenses of higher pressure drop. Therefore, the flow pass arrangement determination should be included in the design methodology. In this work, passes number \(N_p\) and channel numbers per pass \(N_c\) for both streams are set as variables. But the pass number and channel number are integer variables. To avoid introducing a discrete optimisation problem and involve various flow arrangements, the correction factor \(F\) of log mean temperature difference is employed and integrated into the thermal-hydraulic model. The definition is presented as below [25]:

$$F = \frac{NTU_{\text{counter-current}}}{NTU_{\text{other}}}$$  \hspace{1cm} (44)

where \(NTU_{\text{counter-current}}\) is the number of heat transfer units when a pure counter-current is employed for the specific duty, while \(NTU_{\text{other}}\) is the number of heat transfer units of actual arrangement in the design.

The P-NTU method proposed by Kandlikar and Shah [26] is employed to calculate the correction factor of log mean temperature difference. To simplify the design problem, the maximum pass number is set as 4. Therefore, there are at least 16 flow arrangements (1-1, 1-2, 1-3, 1-4, 2-1, 2-2, 2-3, 2-4, 3-1, 3-2, 3-3, 3-4, 4-1, 4-2, 4-3, 4-4), and each flow arrangement can be cross flow, parallel flow or counter-current flow. Obviously, the overall counter-current flow or parallel flow does not always mean individual count-current flow or parallel flow arrangement. Enumeration is employed to compare all possible flow arrangements. Each flow pass arrangement option is considered as a possible design. By comparing the objective function values (capital costs) of all possible flow arrangements, select the most efficient flow arrangement as the optimum design solution.
5 Optimisation of multi-stream plate heat exchanger design with phase change

In the optimisation model, plate length $L_p$, plate spacing $b$, plate width $W$, port diameter $D_p$, Chevron angles $\beta$, passes number $N_p$ and channel number per pass $N_c$ for each stream are considered as variables. Minimising the total capital cost is set as the objective function to find the optimum plate heat exchanger configuration. By considering the typical plate geometric parameters as continuous variables, integrating continuous expression of Nusselt number and friction number for single phase and two-phase conditions, and employing enumeration to select optimum flow arrangement, the MINLP optimisation problem is converted to multiple NLP problems, which can be easily solved by the CONOPT solver in GAMS.

When it comes to the objective function in the optimisation methodology, the total capital cost is the common choice, which is related to the total heat transfer area and shown as below [27]:

$$TC = 2070A_T^{0.85}$$  \hspace{1cm} (45)

The units for capital cost $TC$ and heat exchanger area $A_T$ are GBP and m$^2$, respectively.

In one subsection with specific design sequence $y$, the capital cost is compared to determine the local optimum plate pattern and flow arrangement of each match. By comparing different design sequence $y$ in the certain subsection, the local optimum plate pattern and flow arrangement of each subsection can be determined. After finishing all design sequences $Z_y$ and subsections, the overall optimum plate patterns and flow arrangements can be determined by comparing the overall enthalpy interval capital cost. Finally, by adding up all enthalpy intervals, the whole heat exchanger configurations and dimensions can be obtained.

The overall design methodology of multi-stream multi-pass plate heat exchangers with phase change is shown in Figure 4.
Figure 4 Overall optimisation methodology of plate heat exchanger design
6. Case studies

6.1 Case Study 1: the effectiveness of new design model

A heat transfer case between water and R22 from the published literature [15] is conducted to verify the effectiveness of the design model. The overall process information of the case study is summarised in Table 1. The thermal conductivity of plate is set as 22.5 W m K$^{-1}$. In order to predict the heat transfer coefficient, the outlet temperature and pressure drop, 1 pass-1 pass, 17 channels per pass and counter-current flow are assumed in this case. The cross sectional area per channel is 0.00063 m$^2$. The accuracy of the new proposed design methodology is validated by comparing these results of the new proposed model and the published design model.

Table 1 Process information for Case Study 1

<table>
<thead>
<tr>
<th>Stream</th>
<th>Mass flow Kg s$^{-1}$</th>
<th>$T_{in}$ °C</th>
<th>$T_{out}$ °C</th>
<th>$P_{in}$ kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.5</td>
<td>25.0</td>
<td>21</td>
<td>150</td>
</tr>
<tr>
<td>R22</td>
<td>0.05</td>
<td>8.5</td>
<td>10.2</td>
<td>950</td>
</tr>
</tbody>
</table>

Table 2 Results comparison of exchanger performance

<table>
<thead>
<tr>
<th>Stream</th>
<th>$T_{out}$ °C</th>
<th>h (W/m$^2$ K)</th>
<th>$\Delta P$ (Pa)</th>
<th>$T_{out}$ °C</th>
<th>h (W/m$^2$ K)</th>
<th>$\Delta P$ (Pa)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stream 1</td>
<td>21</td>
<td>301.4</td>
<td>21.2</td>
<td>21</td>
<td>309.7</td>
<td>21.2</td>
<td>0</td>
</tr>
<tr>
<td>Stream 2</td>
<td>10.2</td>
<td>356.1</td>
<td>46.8</td>
<td>10.7</td>
<td>361.4</td>
<td>47.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

It is obvious from Table 2 that heat transfer coefficients in the new model are slightly higher than those in the published model and the outlet temperature of stream 2 is 0.5 °C higher, because the phase change condition is considered in the new model and the Nusselt number is related to plate geometric parameter. Similarly, the pressure drop of stream 2 is almost the same as the original value. It can be seen that the simulation results obtained by applying the new proposed model are in good agreement with the published thermal-hydraulic model, which provides a solid basis for optimising multi-stream multi-pass plate heat exchanger design with phase change.

6.2 Case Study 2: multi-stream multi-pass plate heat exchanger design with phase change

In this case study, the impact of the proposed methodologies is examined for the optimisation design of multi-stream multi-pass plate heat exchangers with phase change. Due to unavailable reference cases, the process data employed in this case is generated from a sample file of Aspen HYSYS 8.2. The minimum approach temperature is 5°C and the overall process data are shown in Table 3. The plate data and thermal-hydraulic behaviour of plates
According to pinch analysis, the stream population per enthalpy interval can be described in Figure 4. Each enthalpy can be considered as a separate heat exchanger.

According to pinch analysis, the stream population per enthalpy interval can be described in Figure 4. Each enthalpy can be considered as a separate heat exchanger.

As mentioned in Section 3, each interval is divided into several subsections, in each of which the physical properties of the streams involved are considered constant. The heat transfer coefficients and pressure drops are obtained based on the average density, heat capacity and vapour quality. The number of subsections of each enthalpy interval is determined based on the variations of the physical properties and condensing enthalpy-temperature curves of hot stream in this case. In this case, the enthalpy interval is evenly divided into four subsections to simplify the design process. The pressure drop is also distributed according to the fraction of heat load.

In this design, basic plate geometric parameters are considered as continuous variables, and the total capital cost is taken as the objective function. Take interval 2 as an example, there are 14 variables and 10 nonlinear equations. The CONOPT solver in GAMS version 23.4 is employed to optimise the plate heat exchanger design. The computation time of an interval is 245 seconds on a 2.6 GHz 4th Intel Core i5 PC with 8GB memory.
Table 4 Overall design results for interval 1

<table>
<thead>
<tr>
<th>Stream</th>
<th>h [W m² K⁻¹]</th>
<th>ΔP [kPa]</th>
<th>Plate pattern</th>
<th>Flow arrangement</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>552.7</td>
<td>11.5</td>
<td>M6M(60°)</td>
<td>2×28-2×28</td>
</tr>
<tr>
<td>H2</td>
<td>412.8</td>
<td>0.85</td>
<td>M6M(30°)</td>
<td>1×22-2×11</td>
</tr>
<tr>
<td>C1</td>
<td>702.3</td>
<td>11.12</td>
<td>M6M(60°)</td>
<td>-</td>
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</table>

Table 5 Overall design results for interval 2

<table>
<thead>
<tr>
<th>Stream</th>
<th>h [W m² K⁻¹]</th>
<th>ΔP [kPa]</th>
<th>Plate pattern</th>
<th>Flow arrangement</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>342.5</td>
<td>5.58</td>
<td>M6M(60°)</td>
<td>2×30-1×60</td>
</tr>
<tr>
<td>H2</td>
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<td>M6M(30°)</td>
<td>2×11-1×22</td>
</tr>
<tr>
<td>C2</td>
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<td>-</td>
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</tbody>
</table>

Table 6 Overall design results for interval 3

<table>
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<tr>
<th>Stream</th>
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<th>ΔP [kPa]</th>
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<th>Flow arrangement</th>
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</thead>
<tbody>
<tr>
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<tr>
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<td>4×25-2×25</td>
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<td>2.67</td>
<td>M10B(60°)</td>
<td>-</td>
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</table>

The design details for all three enthalpy intervals are listed in Tables 4, 5 and 6. For the plate pattern, M6M and M10B are selected as the optimum plate patterns in this case. For H1-C1 in interval 1, the optimum flow arrangement is 2-2 on the basis of M6M plates with 60°C. The plate pattern and flow arrangement selection is determined by comparing the capital cost of different design sequencing and subsections. To meet the pressure drop requirement and maximise the heat transfer coefficients, the Chevron angles are optimised for each stream. H, L and M plate channels, which is make up of large Chevron angle, small Chevron angle and mix and match Chevron angle plates respectively, giving higher, lower or medium heat transfer and pressure drop performance. Also, the larger the pass number, the higher the pressure drop and heat transfer coefficient. The final flow arrangements and plate patterns are determined based on the total capital cost. Although the methodology is applied successfully in academic view, the accuracy of the proposed methodology should be verified in practice.

7. Conclusions

A new design optimisation methodology for multi-stream multi-pass plate heat exchangers is developed and successfully applied. In the optimisation, the basic plate geometric parameters, such as plate length, plate width and Chevron angle, and numbers of passes and channels per pass are considered as design variables. By employing the continuous two-phase Nusselt number and friction factor expressions, varied heat transfer coefficient and pressure drop caused by phase change can be addressed and plate pattern selection can be completed simultaneously with pressure drop consideration. The discrete flow arrangement
selection problem can be overcome by using enumeration and comparing the total capital cost of heat exchanger. Eventually, the MINLP multi-stream multi-pass plate heat exchanger design optimisation problem can be converted to a semi-continuous problem, which can be solved efficiently.

Reference


Chapter 6 Conclusions and Future Work

6.1 Conclusions

This work focuses on developing systematic design methods for plate and plate fin heat exchangers.

For plate-fin heat exchangers, an optimisation methodology of multi-stream multiple fin types plate-fin heat exchanger design is developed to obtain the optimum plate heat exchanger configuration. A multi-stream plate-fin heat exchanger is addressed by considering them as a network of two-stream plate-fin heat exchangers. The mixed integer nonlinear linear programming (MINLP) problem caused by standardised fin types and their unique heat transfer and pressure drop performances is converted to a nonlinear linear programming (NLP) problem by employing continuous Colburn factor and Fanning factor correlations and considering basic fin geometric parameters (fin spacing, fin pitch, thickness, etc.) as continuous design variables. Mix-and-match fin types is also taken into account to include all possible fin type combinations and avoid obtaining the local optimum design solutions. In this way, the large number of fin type combinations can reduce to few combinations, which are solved by enumeration. The optimum plate-fin heat exchanger configuration is found by comparing the total capital cost. Compared with published methods, the new design methodology can give a better design solution and save computation time.

An optimisation methodology of multi-stream multi-pass plate heat exchanger design is also proposed in this work. The thermal-hydraulic model of plate-fin heat exchangers is modified and employed in the plate heat exchanger design. The MINLP problem caused by standardised plate patterns and various flow arrangements can be converted to an NLP problem by relating Nusselt number and friction number as continuous functions of Reynolds number and basic plate geometric parameters, considering basic plate geometric parameters as continuous variables and introducing logarithmic mean temperature difference correction factor. Each flow arrangement will give a local optimum plate heat
exchanger configuration. By comparing capital costs of all local optimum design solutions, the overall optimum design solution can be determined. In this way, plate pattern selection and determination of flow arrangement can be completed simultaneously. A multi-stream plate heat exchanger is considered as a network of two-stream plate heat exchangers as well. The benefit of plate heat exchanger application in crude preheat trains is examined from academic views.

The commonly conducted phase change is also addressed in the plate heat exchanger design. The multipliers of Nusselt number and friction factor for two-phase condition are added to correct the heat transfer and pressure drop performance. Additionally, the composite curves are divided further into several subsections and physical properties of each subsection is considered as constants to simplify the design.

### 6.2 Future work

Although the present work presented new design methodologies for plate/plate-fin heat exchangers, assumptions and simplifications were made that limit the applicability of the design models. Therefore, further research area should address the following problems:

Up to now, the optimisation methodologies of plate-fin heat exchanger design for single phase have been developed based on constant physical properties. There are still many aspects could be explored to improve and expand the content in the thesis. Once the two-phase Nusselt number and friction factor correlations for different fin types are available, the proposed optimisation methodology of plate heat exchanger design with phase change can be transferred to the plate-fin heat exchanger design.

Moreover, flow arrangement in plate-fin heat exchangers is another factor affects heat transfer and pressure drop performance. Therefore, flow arrangement selection should be considered and integrated into the plate-fin heat exchanger design model.

In terms of plate/plate-fin heat exchangers, the current design methodologies are correlation-based, which means the effectiveness of new proposed design methodology depends on the accuracy of Nusselt number and friction factor correlations. Therefore, find more suitable correlations for thermal-hydraulic calculations to improve the effectiveness of the design model.
Find better ways to test the accuracy of the proposed plate heat exchanger with phase change model. Since no real operation date was available for comparison, the case study in this work is completed without validation and can be set as a benchmark for further study.

With regards to optimisation, the global optimum design result cannot be easily to find. Therefore, more efficient optimisation solver or tools could be found and employed to improve the accuracy and increase the efficiency.

Besides, the rating methodology of plate/plate-fin heat exchanger with or without phase change should be developed to evaluating the given heat exchanger performance. The eventual objective is to integrate these plate/plate-fin heat exchanger models into the overall heat exchanger network or processes, like refrigeration cycle and crude preheat trains, and optimise the whole heat exchanger network or process.
Reference


## Appendix A: Fin Types with Detailed Geometry

<table>
<thead>
<tr>
<th>Surface</th>
<th>Plate Spacing ($10^{-3}$ m)</th>
<th>Hydraulic Diameter ($10^{-3}$ m)</th>
<th>Fin Thickness ($10^{-3}$ m)</th>
<th>Heat Transfer Area/volume between plates $\beta$</th>
<th>Fin Area/Total Area f</th>
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<td>Fin Area/Total Area $f$</td>
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Appendix B: GAMS Code Sample

* 20-06-2014 Kunpeng Guo Multi-stream Plate-fin Heat Exchanger Design

* Plain-Strip Mix and Match Model

* include pressure drop

option decimals=6;

Positive variables

\[ yita1, yita2, alpha1, alpha2, h1, h2, vt, fs1, fs2, b1, b2, c1, c2, \]
\[ x2, beta1, beta2, alphas1, alphas2, segama1, segama2, gama1, gama2, canshu2, dh1, \]
\[ dh2, Ac1, Ac2, kh1, kh2, kp1, kp2, deltap2, \]
\[ zh1, zh2, zp1, zp2, ml1, ml2; \]

variables

\[ z; \]

parameters

\[ \text{deltat} \quad \text{LMTD log mean temperature difference} /23/ \]
\[ \text{q} \quad \text{heat transfered} /10949001/ \]
\[ \text{m1} \quad \text{flow rate of stream 1} /49/ \]
\[ \text{m2} \quad \text{flow rate of stream 2} /49/ \]
\[ \text{k1} \quad \text{thermal conductivity of fin 1} /90/ \]
\[ \text{k2} \quad \text{thermal conductivity of fin 2} /90/ \]
\[ \text{kk1} \quad \text{thermal conductivity of stream 1} /0.078/ \]
\[ \text{kk2} \quad \text{thermal conductivity of stream 2} /0.0789/ \]
\[ \text{cp1} \quad \text{heat capacity of stream 1} /1059/ \]
\[ \text{cp2} \quad \text{heat capacity of stream 2} /1059/ \]
\[ \text{miu1} \quad \text{viscosity of stream 1} /0.0000509/ \]
\[ \text{miu2} \quad \text{viscosity of stream 2} /0.0000509/ \]
\[ \text{ro1} \quad \text{density of stream 1} /0.55/ \]
ro2  density of stream 2      /9.63/
deltap1  pressure drop of side 1        /8800/
tf1 /0.000051/  
tf2 /0.000051/  
pr1       prantl number 1        ;
pr2       prantl number 2        ;
pr1=miu1*cp1/kk1; pr2=miu2*cp2/kk2 ;
equations
  segama1equation  segama1 calculation equation  
  segama2equation  segama2 calculation equation  
  gama1equation  gama1 calculation equation  
  gama2equation  gama2 calculation equation  
  canshu2equation  canshu2 calculation equation  
  yitaequation1  yita1 calculation equation  
  yitaequation2  yita2 calculation equation  
  ml1equation  ml1 calculation  
  ml2equation  ml2 calculation  
  alpha1equation  alpha1 calculation  
  alpha2equation  alpha2 calculation  
  beta1equation  beta1 calculation  
  beta2equation  beta2 calculation  
  fs1equation  fs1 calculation  
  fs2equation  fs2 calculation  
  dh1equation  hydraulic diameter 1 calculation  
  dh2equation  hydraulic diameter 2 calculation  
  h1equation  heat transfer coefficient 1 calculation  
  h2equation  heat transfer coefficient 2 calculation  

zh1equation  zh1 calculation
zh2equation  zh2 calculation
zp1equation  zp1 calculation
zp2equation  zp2 calculation
kp1equation  kp1 calculation
kp2equation  kp2 calculation
kh1equation  kh1 calculation
kh2equation  kh2 calculation
vtequation  vt calculation
ac1equation  ac1 calculation
ac2equation  ac2 calculation
deltap2equation  deltap2 calculation
obj  objective function;
segama1equation..  b1*segama1=e=2*c1;
segama2equation..  b2*segama2=e=c2;
gama1equation..  b1*gama1=e=tf1;
gama2equation..  x2*gama2=e=tf2;
canshu2equation..  c2*canshu2=e=tf2;
yitaequation1..  yita1*ml1=e=ml1+fs1*(tanh(ml1)-ml1);
yitaequation2..  yita2*ml2=e=ml2+fs2*(tanh(ml2)-ml2);
ml1equation..  sqr(ml1)*(k1*tf1)=e=(2*h1)*sqr(b1/2);
ml2equation..  sqr(ml2)*(k2*tf2)=e=(2*h2)*sqr(b2/2);
alpha1equation..  alpha1*(b1+b2)=e=beta1*b1;
alpha2equation..  alpha2*(b1+b2)=e=beta2*b2;
beta1equation..  beta1*c1*b1=e=2*(b1+c1);
beta2equation..  beta2*(b2*c2*x2)=e=(2*(b2-tf2)*x2+2*(c2-tf2)*x2+2*(b2-tf2)*tf2+c2*tf2);
fs1equation..  fs1*(2*b1+4*c1)=e=2*b1+5*c1/3;
fs2equation..  fs2*(2*(b2-tf2))*x2+2*(c2-tf2)*x2+2*(b2-tf2)*tf2+c2*tf2)=e=2*(b2-tf2)*x2+2*(b2-2*tf2)*tf2+c2*tf2;

dh1equation..  dh1*(c1+b1)=e=2*(c1-tf1)*b1;

dh2equation..  dh2*2*((c2-tf2)*x2+(b2-tf2)*x2+tf2*(b2-tf2))+tf2*(c2-tf2)-

power(tf2,2))=e=4*(c2-tf2)*(b2-tf2)*x2;

h1equation..  h1*(kp1*zp1*vt)**(0.52/2.91)=e=kh1*zh1*(deltap1**((0.52/2.91));

h2equation..  h2*(kp2*zp2*vt)**((0.4597/2.2578)=e=kh2*zh2*(deltap2**((0.4597/2.2578));

zh1equation..  zh1=e=(segama1**0.192)*(gama1**(-0.208));

zh2equation..  zh2=e=(segama2**(-0.1541))*(gama2**0.1499)*(canshu2**(-0.0678));

zp1equation..  zp1=e=(segama1**0.034)*(gama1**(-0.169));

zp2equation..  zp2=e=(segama2**(-0.1856))*(gama2**0.3053)*(canshu2**(-0.2659));

kp1equation..  kp1*(2*ro1*(dh1**0.09))=e=0.029*(m1**1.91)*(miu1**0.09)*alpha1;

kp2equation..  kp2*(2*ro2*(dh2**0.742))=e=9.6243*(m2**1.2578)*(miu2**0.7422)*alpha2;

kh1equation..  kh1*(dh1**0.48)*(pr1**(2/3))=e=0.233*(m1**0.52)*(miu1**0.48)*cp1;

kh2equation..  kh2*(dh2**0.5403)*(pr2**(2/3))=e=0.6522*(m2**0.4597)*(miu2**0.5403)*cp2;

vtequation..  vt*deltat*alpha1*alpha2*yita1*yita2*h1*h2=e=q*(yita1*h1*alpha1+yita2*h2*alpha2);

ac1equation..  Ac1*((kp1*zp1*vt)**(-1/2.91))=e=deltap1**(-1/2.91);

ac2equation..  Ac2*b1*beta1*dh1=e=Ac1*b2*beta2*dh2;

deltap2equation.. deltap2=e=kp2*zp2*vt*((1/Ac2)**(-1/2.2578));

obj.   z=e=vt;

* set fin geometric variables bound

  b1.lo=0.0012;b2.lo=0.0012;

  c1.lo=0.0009;c2.lo=0.0009;

  x2.lo=0.0024;

  b1.up=0.02;b2.up=0.0123;

  c1.up=0.0022;c2.up=0.0022;

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* set initial value
b1.l=0.0106; b2.l=0.00129;
c1.l=0.001684; c2.l=0.001287;
x2.l=0.00254;
segama1.l=0.3178; segama2.l=0.9976;
gama1.l=0.01434; gama2.l=0.02;
canshu2.l=0.0198;
dh1.l=0.002644; dh2.l=0.001226;
fs1.l=0.859345; fs2.l=0.507711;
alpha1.l=1227; alpha2.l=328.41;
beta1.l=1376.3; beta2.l=3027.05;
zh1.l=1.94; zh2.l=0.693;
zp1.l=1.9706; zp2.l=0.7158;
kh1.l=358.66; kh2.l=1045.42;
kp1.l=38364; kp2.l=2068.2;
h1.l=394.2; h2.l=1045.42;
yita1.l=0.7178; yita2.l=0.97;
ml1.l=1.2724; ml2.l=0.4353;
vt.l=2.8;
Ac1.l=2.983; Ac2.l=0.37;
deltap2.l=39109;

Model plainstrip /all/;
option nlp=conopt;
*option nlp=BARON;
Solve plainstrip using nlp minimizing z;
display z.l,vt.l,h1.l,h2.l,c1.l,c2.l,b1.l,b2.l,x2.l,deltap2.l;