Monte Carlo Simulations of an Anthropomorphic Phantom for Molecular Radiotherapy (MRT) with GATE.

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Abstract

A method for simulating an anthropomorphic phantom and radioisotope distribution using a Monte Carlo based code is presented. A voxel-based simulation of a Jaszczak phantom was performed and the results compared to both a previously validated geometric-based simulation and experimental data. The method was then extended to create a voxel-based simulation of an anthropomorphic phantom which was validated using an equivalent data set. Good agreement is observed between the voxel-based and geometric-based Jaszczak phantom simulations, but it is noted that for complete agreement, it is crucial that the source distribution is fully aligned to the phantom geometry. With regards to the anthropomorphic phantom simulation, an excellent agreement is observed, which suggests that the activity distribution has been accurately aligned with the phantom. The method proposed can be extended to any arbitrary geometry, with the condition that the activity distribution has been accurately aligned to the phantom geometry.
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1.0 Introduction

Molecular Radiotherapy, or MRT, is the delivery of radiation to malignant tissue by the interaction of an administered radiopharmaceutical with targeted molecular receptors [1]. Single Photon Emission Computed Tomography, or SPECT, is a nuclear imaging technique that is employed to acquire a set of 2D gamma distributions of the administered radionuclide tracer within the body, by using one or more gamma cameras. These distributions can then be reconstructed, using tomographical reconstruction techniques [2, 3], to create 3D images of the functional information. It is becoming increasingly important to determine the exact dose administered to a patient, but unfortunately the detection of scattered gamma quanta leads to a degradation of the contrast in reconstructed SPECT images and hinders dosimetry quantification [2].

In recent years, Monte Carlo simulations have been particularly successful in modelling the SPECT imaging process yielding unique information that is not acquired through experimentation [4]. This information includes the ability to track particles from their point of origin to their final destination and any interactions they might experience. The Nuclear Physics Group at the University of Manchester, in conjunction with the nuclear medicine group at the Christie Hospital, have employed Monte Carlo techniques to address the problem of scatter and to develop more accurate scatter correction techniques than those currently available.

Defining the SPECT scanning process using a geometric-based technique allows the user to generate individual components of the geometry and source distribution by a series of mathematically defined volumes. This technique has been previously validated against experimental data, for defining a Jaszczak phantom, but there are limitations associated with the technique, such as its extension to complex geometries and activity distributions which are mathematically indefinable.

As a result of this, a voxel-based approach has been proposed, which divides each volume into a series of 3D discrete volumes (voxels), and by using this technique, more complex geometries, such as anthropomorphic phantoms, can be simulated. The aim of this MSc. Project is to create a voxel-based Jaszczak phantom simulation using the Monte Carlo code Geant4 Application for Tomographic Emission, GATE, and then extend the method to create
a voxel-based anthropomorphic simulation. Validation of the voxel-based Jaszczak model will first be achieved by comparing simulated results against validated geometric-based results and experimental data. The extended anthropomorphic simulation will be validated against experimental data.
2.0 Theory

This section discusses the relevant diagnostic radiological theory required to understand the SPECT imaging process and to interpret the results and the output from the simulation. The theory described includes: the SPECT scanner components, data acquisition and reconstruction techniques associated with each of the SPECT and X-ray/CT scanners, interactions of radiation with matter, the radioactive decay processes associated with the radiopharmaceutical utilised and the Monte-Carlo simulation process.

2.1 The SPECT Scanner

2.1.1 SPECT Scanner Hardware.

The SPECT/CT scanner presented in Figure 1, is comprised of two gamma cameras mounted on a rotating gantry, in a configuration such that the detectors may revolve 360° (each detector rotates 180°) about the patient, or phantom. The detectors acquire a 2D gamma distribution of an administered radiopharmaceutical from multiple angles, only accepting photons which are within a predefined energy range, or energy window. This configuration allows the scanner to acquire a set of 2-D images called projections and thereafter, reconstruct these images into 3D images [3]. This is the process of tomography, which is defined as sectional imaging using a penetrating wave [3, 5], and is the basis for SPECT/CT imaging.

Each gamma camera consists of 4 main components, which include the amplifying electronics, photomultiplier tubes (PMTs), a scintillation crystal and a collimator. An image of a typical SPECT/CT scanner, which is a combined SPECT and x-ray/CT scanner, and schematic of a gamma camera are presented in Figure 1; each element of the scanner is described below.
Figure 1  An Infinia GE Hawkeye 4 SPECT/CT scanner (left) [6]. Labels indicate the different components of the scanner including the two rotating gamma cameras and the x-ray/CT scanner. (Right) A schematic of a gamma camera, including the photo-multiplier tubes (PMTs), scintillation crystal and the collimator (adapted from [5]).

2.1.1.1 The Collimator

Analogous to the lens in one's glasses, which is used for correcting vision and focussing the light onto the retina. The collimator is used as a means of forming an image of the source on the scintillating crystal. This provides a one-to-one correspondence between every point of scintillating crystal and a point in the source. It essentially limits the direction which the photons could travel to interact with the scintillation crystal, by absorbing, the photons which are not travelling parallel to the hole. This is because without knowing the direction of travel, the photon could have originated from any point within the source. Ultimately, without the collimator, the gamma ray's direction of origin is unknown and an image cannot be formed [5]; this is because every point in the image corresponds to a projection of a line through a source, producing a 2D image (again like a camera) from a 3D source [3].
A schematic of a gamma camera with a pin-hole collimator indicating how photons, their trajectories have been coloured in red, would be detected by the camera. The collimator normally consists of a lot more holes per unit area, but for this explanation the dimensions of the holes have been exaggerated. Note that only the photons travelling in a direction parallel to the hole would be detected by the scintillator crystal, those which are travelling in a direction at an angle to the hole are absorbed within the collimator. Adapted from Ref [5].

Collimators, of the type used in Nuclear Imaging Departments, are normally sheets of lead, or tungsten, with a large number of holes drilled through them. These holes are normally around 3 millimetres in diameter and extend the whole thickness of the collimator [3], as seen in Figure 2 and Figure 3. This means that it is unlikely that a gamma ray will make it through the collimator, unless its direction of travel is parallel to the hole; gamma rays travelling at an angle to the hole will be photoabsorbed by the septa (the solid material between holes, see Figure 2 [3, 5].

Different types of collimators are used to image photons of certain energies, whilst also yielding different spatial resolutions and sensitivities. The clinician must use the correct collimator for the typical energy of the photons emitted from the radioactive source being used, whilst also considering the compromise between sensitivity and spatial resolution [5]. Examples of the different classes of collimators include single hole (or pin hole) collimators, which are useful for imaging small superficial organs and diverging/converging types for
whole body scans [5]. The SPECT scanner, at the Christie Hospital, and also the scanner modelled in GATE uses Parallel Hole collimators, an example of which can be seen in Figure 3. Further analysis here, will only consider parallel-hole collimators due to their relevance to the project.

![Figure 3 A section of a lead parallel-hole collimator, notice that the holes run parallel to each other and are all of a set diameter [7].](image)

### 2.1.1.1 Parallel Hole Collimators

In considering the suitability of a collimator, one must consider the septal penetration, the sensitivity and the spatial resolution associated with the parallel-hole collimator.

**Septal Penetration**

Septal penetration is a particularly important characteristic to take into account, when considering the type of parallel hole collimator to use. An ideal collimator would absorb all gamma rays impinging upon the septa. This filters any photons that had interacted with the collimator such that they do not interact with the scintillator crystal and lead to degradation of the overall image [3]. This trait is dependent upon the energy of the gamma-rays being imaged, collimators imaging higher energy gamma rays, for example, need to be thicker, or comprise of a material with a larger atomic number, Z. This ensures that there would be a greater probability of absorption due to the photoelectric effect cross sectional probability [5].
Interactions of this kind and the attenuation of gamma ray energies will be discussed further in section 2.4.

For photons of any given energy, the minimum thickness of the septa, T, such that photons will not penetrate the septa and be detected in the wrong hole, is related to path length through the septum W, hole diameter D and hole length, L, by comparing similar triangles:

$$\frac{T}{W} = \frac{D}{X} \quad (1)$$

From Figure 4, which is a schematic of a collimator coupled to the scintillator crystal, the path length in the hole, X and W can be expressed algebraically:

Path Length in the hole, X,  $$L \approx 2X + W, \quad (2)$$

Rearranging for X,  $$X \approx \frac{1}{2} (L - W) \quad (3)$$

This result is only approximately correct, because the hole length is normally much greater than D. Given this, Figure 4, is not a good representation because it is not to scale.

Figure 4 A diagram of the collimator and scintillator crystal geometry illustrating septal penetration. Ideally, W is large enough to absorb the gamma rays before the scattered photons are absorbed in the crystal, causing degradation to the overall image. [5].
Given equations (1), (2) and (3), and algebraic manipulation, the minimum septal thickness can be expressed as:

\[ T = \frac{2DW}{(L-W)} \]  

(4)

**Spatial Resolution**

The spatial resolution of the scanner is dependent upon the spatial resolution of the collimator. The spatial resolution is defined as the smallest possible distance that two objects might still be defined as separate, or resolvable [3]. From Figure 4, it is possible to define the spatial resolution, \( R \), in terms of the geometric variables: hole length, \( L \), diameter of the hole, \( D \), and a new variable, the distance of the source from the collimator itself, \( Z \):

\[ R = \frac{D(L+Z)}{L} \]  

(5)

From equation (5), the spatial resolution of the collimator can be improved by increasing the hole length, \( L \), or by increasing the number of holes per area of the collimator, such that \( D \) is small. Most importantly of all, the distance of the source to the collimator, will also improve the resolution and this is the most easily optimised by the clinician.

**2.1.1.2 The Scintillator Crystal**

A scintillator crystal is a material that exhibits the property of luminescence and this normally occurs as a result of the crystal absorbing ionizing radiation. The scintillation crystal utilised in SPECT scanner configurations is typically made from thallium-doped sodium iodide and the purpose of the crystal is to convert the energy of the gamma rays into a form that PMTs can detect; this is normally in the visible range, around 500 nm [5].

Scintillation occurs when an incident photon donates either all, or a fraction, of its energy, by the photoelectric effect, or by Compton scattering, inducing the emission of secondary electrons which ionise nearby atoms. Each of the resultant ionised atoms emits an electron, which are excited into the conduction band, from the valence band, thus causing the production of an electron-hole pairs [3]. The emission of scintillation light occurs when electron-hole pairs drift through the crystal and combine with activation centres, emitting scintillation light by radiative transfers [3].
Doping the crystal with thallium, introduces activation centres which trap excited electrons, allowing them to recombine with the vacant holes in the valence band. This provides important advantages in comparison to non-doped crystals, the first being that it increases the number of radiative transfers in relation to non-radiative, thus increasing the light conversion efficiency [3]. Secondly, the impurity shifts the energy levels in the crystal, ensuring that photons emitted in the scintillation process are not of energies that would allow absorption by the crystal [3]. Lastly, emitted photons have a wavelength which matches reasonably to the optimum response to the PMT- 350 – 500 nm, with a peak of 410nm, for an optimum wavelength.

Critical factors that characterise the performance of the scintillating crystal include [3]:

- The scintillator light output should follow proportionally to the energy of the absorbed gamma ray, so the crystal makes a good energy discriminating detector.

- Absorption characteristics of the PMT should match reasonably to the optimum response of the photomultiplier tube. Optical photons produced in scintillation should match as closely as possible to 410nm, for PMTs.

- High efficiency for stopping gamma rays. A optimal scintillator crystal will be made of a material with a high atomic number, Z. A large Z implies high density, and a larger probability of photoelectric absorption.

- Higher probability of photoelectric absorption than Compton scattering.

- Immediate light emission, or fluorescence, rather than phosphorescence, which is the delayed emission of light; briefer light pulses ensure that the detector is effective with higher count rates.

These characteristics are all found in NaI(Tl) which makes it the ideal scintillation crystal for SPECT scanner [3].
2.1.1.3 Photomultiplier Tubes

The purpose of the photomultiplier tube, or PMT, is to convert the optical photons produced in the scintillation crystal into photoelectrons, and amplify the signal using a series of dynodes, to ultimately produce a small charge pulse and output signal [3]. Figure 5, below, is a schematic of a photomultiplier tube and demonstrates the operation of the photomultiplier tube. The five main components of the photomultiplier tube will be discussed here, these components include: the vacuum tube, photocathode, a series of dynodes, anode, and a series of electronics for signal collection and processing [3].

Figure 5 A schematic of a photomultiplier tube and the processes the light photon undergoes to create an output signal, for data collection [8].

The purpose of the photocathode is to absorb any incident optical photons and emit a photoelectron by the photoelectric effect, which is normally converted with a quantum efficiency of 25%. After the photocathode, because of the high potential difference between the anode and cathode, the photoelectrons are accelerated, towards a series of dynodes. Each dynode is held at a greater positive voltage than the last, approximately 100 Volts more, with the first being kept at a voltage of 300 Volts. The premise is that the photoelectrons are accelerated, giving each electron more energy, at each dynode stage. This means that the electron will have sufficient energy to produce multiple electrons by secondary emission on collision with a dynode; this results in gains of around 105 to 106, after 10 stages of dynodes.
These electrons are then collected at the anode, and the charge pulse converted into an output signal by preamplifiers.

2.1.1.4 Preamplifiers and Pulse Arithmetic

Not only do preamplifiers convert the charge pulse into an output signal, or a determinable voltage pulse, they also match the impedances of the photomultiplier tubes with the remainder of the electronics, which are of lower impedance. Gamma cameras contain an array of photomultiplier tubes, and the thousands of optical photons emitted from the scintillation crystal are collected by multiple PMTs at one time. This means that depending on the source position, one particular PMT will receive most of the scintillation light, in comparison to other PMTs which are further away from the source. This will mean that the PMTs closer will produce a stronger signal than those further away. One method might be to use the PMT with the strongest signal as a means of determining the location of the source of light photons, but this procedure produces low resolution images.

![Figure 6](image.png)

*Figure 6 A diagram of scintillation light entering the PMTs, because the event is closer to PMT 3, it has the strongest pre-amp signal. The figure has been adapted from [3].*

It is, however possible to improve event localisation, this is done by determining the centroid of the distribution of signals. The event location, X, occurs in the scintillation crystal and optical photons are emitted in all directions as seen in Figure 6, which are detected in the relevant PMTs at relative positions X₁, X₂, X₃ and X₄. Each preamplifier, or preamp, converts the detected charge pulses into voltages V₁, V₂, V₃, and V₄. It is obvious that the PMT closest
to the interaction will produce the strongest signal and corresponding voltage. In a method analogous to calculating the centre of gravity, from multiple large bodies in space, quantifying the interaction location is the sum of the products of PMT positions and their relative signal; all divided by the total sum of the PMT signals. The relevant formulae have been included in equations 6 and 7:

\[ E = V_1 + V_2 + V_3 + V_4 \]  \hspace{1cm} (6)

\[ X = \frac{X_1V_1 + X_4V_2 + X_4V_3 + X_4V_4}{E} \]  \hspace{1cm} (7)

where \( E \) is the total sum of the PMT signals and \( X \) is the location of the interaction of the incident photon with the scintillator crystal.

2.1.2 SPECT Data Acquisition and Reconstruction

SPECT cameras record spatial and energy information of events it detects, that fall within predefined energy windows. This in turn provides information on the functional and therefore the physiological activity of the part of the body being imaged [5]. For the Infinia GE Hawkeye 4 SPECT/CT scanner, which is the SPECT camera used experimentally and also simulated, the two detectors rotate about a supine patient and acquire the functional 2D gamma distributions, or 2D projections in the XZ plane, of the 3D \(^{177}\)Lu distribution for a predefined number of positions. More details will follow as to the number of positions the detectors will acquire this data, the angle of rotation and the time spent in each position, in section 3.1 [5].

In more detail, in conjunction with a collimator, the SPECT camera provides a compilation of 2D images comprising of multiple profiles which are meant to represent a 1D projection of the radioactivity in a single slice of a patient [5]. This means that each 3D image can be split up into 2D sections, each with a 1D profile of the activity as seen in Figure 7. Every point in a 1D profile can be thought of a linear sum, along the line of view from the source to the detector, of the activity elements, as defined by the collimator [5].
Given that multiple angular samples are acquired, it is then possible to reconstruct the 2D gamma distributions that provide a representation of the $^{177}$Lu distribution contained within the body into 3D source distributions. The techniques utilised for reconstruction include filtered back projection techniques, or more recently, ordered subset expectation maximisation algorithms (OSEM). Further details can be found in section 4.0 of The Physics of Medical Imaging, ref. [5], and information regarding the OSEM algorithm is found in ref. [9].

### 2.1.3 Scatter Correction Techniques: Triple Energy Window.

In most clinical situations, scatter can contribute as much as 40% to the overall photopeak energy window counts and can lead to unfavourable, aforementioned, effects in reconstructed images and dosimetry measurements [2]. The general consensus, in laboratories and clinical situations, has been to adopt an easily implementable approach to scatter correction within SPECT/CT systems, such as dual or Triple Energy Window technique (DEW & TEW, respectively).

In this report, the TEW is used, which employs the use of three energy windows, a main energy window, $C_m$, centred on a photopeak energy, and two further subwindows, $C_l$ and $C_u$, positioned either side of the main window. Counts are then recorded in every pixel and every planar image [10]. The scattered photon count is then estimated from the counts summed, in the 2 subwindows, and this is then subtracted from the photopeak energy window [021]. The positioning of the energy windows used in this method is illustrated below in Figure 8 and the determination of the width and positioning of these energy windows is described in depth by

---

*Figure 7 presents the association between the image profile and the projection of radioactivity in a single position.*
Using this method, an approximation for the number of scattered photons, $S_p$, and therefore the number of primary photons $C_p$ can be expressed in terms of counts in the upper and lower energy windows, $C_u$ and $C_l$ respectively, and the widths of the photopeak and the upper and lower energy windows $W_p$, $W_u$, and $W_l$:

$$S_p = \left( \frac{C_l}{W_l} + \frac{C_u}{W_u} \right) \times \left( \frac{W_p}{2} \right)$$  \hspace{1cm} (8)

The number of primary photons can be expressed with one more variable, the number of counts detected in the main photopeak energy window $C_m$:

$$C_p = C_m - S_p$$  \hspace{1cm} (9)

The TEW method not only accounts for scatter from photopeak emission, but also for any downscatter from the EM2 emission [10,11]. The TEW technique is used in the process of reconstructing the projections, which feature in sections 4.1 and 4.2. With respect to this project, the triple energy window approach was applied to the two photopeaks associated with two gamma rays of energy 113 keV and 208 keV, which are emitted as a result of a $^{177}$Lu nucleus decaying. These radioactive processes are detailed in section 2.3.1.2.
2.2 The X-ray/CT Scanner

The SPECT/CT scanner is comprised of a secondary imaging acquisition technique, x-ray computed tomography or x-ray/CT. The x-ray/CT scanner is probably the most widely known imaging modality, as it fundamentally utilises an imaging technique which a large proportion of people have undergone at some stage of their life. X-rays are widely used in the dental procedure to check for decay within the tooth and low energy x-rays are also utilised in mammography- a means of checking for lesions in the breast. Their main purpose is to provide anatomical information, whereas a SPECT scan can only provide functional information. In this project, the x-ray/CT scanner is not simulated using Monte Carlo techniques, but the data it acquires is used in the methods proposed in section 3.0.

2.2.1 The Scanning Hardware

The x-rays are generated using an x-ray tube; this is a vacuum tube with a heated filament acting as a cathode, producing thermionic electrons which are accelerated towards a tungsten anode, which converts the electron energy into x-ray photons, as shown in Figure 9.

![Figure 9](image)

*Figure 9 A schematic of an x-ray tube for generating x-ray photons. Obtained from ref [12].*

The x-ray tube, as already stated, is a glass evacuated chamber, encased in a lead box with a potential difference of 30 to 100 kV applied between the anode and cathode; this voltage controls the thermionic electron velocity and therefore the kinetic energy of generated x-rays. There are two specifications that make the anode an ideal x-ray emitter:

1) The choice of target material
2) the design for the anode (commonly a rotating design).
2.2.2 CT Data Acquisition and Reconstruction

A radiographic image is the distribution of the transmitted photons through a patient, recorded by a detector. The x-ray photons that are emitted from an x-ray emitter could do one of three things, they could pass through the patient without interacting and be detected, they could interact via the photoelectric effect and thus be completely absorbed within the patient, or they could Compton scatter and then be detected [5]. The third scenario provides the detector with very little useful information and in fact creates a background signal which degrades the image quality, whereas the photons in the latter scenario provide information on the probability of passing through the patient without interaction. This probability is dependent upon the sum of the x-ray attenuating properties of the tissues the photon propagates through [5]. The resultant image is therefore a two dimensional projection of the 3D distribution of the x-ray attenuation properties of all the materials contained within the patient.

In the case of x-ray computed tomography, a planar slice of the body is defined, as in Figure 10 and the x-ray emitter/s rotate about the patient and detector, in one of the arrangements as presented in Figure 11. X-rays are passed through that slice in directions that are parallel to and contained within the plane of the slice. A CT image can be thought as though a slice of the body had been removed and then radiographically imaged, but by passing x-rays perpendicular to the slice of the removed piece of body [5]. Depending on the size of the body being imaged this process is repeated a number of times, each acquiring a two-dimensional projection of the x-ray attenuation properties [5]. These projections are then reconstructed into a 3D image using reconstruction techniques to produce a CT volume; an example of which is shown in Figure 12.
Figure 10 provides a visual representation of how the x-ray source rotates about the patient, whilst the patient bed moves the patient through the beam of x-rays, pausing in multiple positions whilst the scanner radiologically images a slice of the body [13].

Figure 11 The possible arrangements of the x-ray emitter, patient (as the circle) and the detector/s inside the x-ray/CT scanner. (c) and (d) correspond to the most complicated CT scanner designs and also the most recently implemented. [14].
Figure 12. Two slices of a CT volume of two different positions within the phantom, demonstrating the tomographic CT acquisition.

Every pixel in a CT image, for example in Figure 12, is assigned a Hounsfield unit value, which is directly related to the linear attenuation coefficient of the tissue/s that the x-rays have traversed through. The relationship between the linear attenuation coefficient and HU is based on the definition below [14, 15]:-

“The Hounsfield Unit (HU) scale is a linear transformation of the linear attenuation coefficient measurement into one in which the radiodensity of distilled water (at standard pressure and temperature) is defined as zero HU, while the radiodensity of air at STP is defined as -1000 HU.” [14]

For a generic material with a linear attenuation coefficient \( \mu \), the HU value, based on the prior definition is therefore given by [5]:

\[
HU = \frac{\mu_x - \mu_{\text{water}}}{\mu_{\text{water}} - \mu_{\text{air}}} \times 1000 \quad \text{Equation 1}
\]

Where \( \mu_x \) is the measured linear attenuation coefficient of the material being imaged and \( \mu_{\text{water}} \) and \( \mu_{\text{air}} \) are the linear attenuation coefficients for water and air respectively [5].
Table 1 presents the Hounsfield unit value that any given pixel, in a CT image, would be assigned based on the linear attenuation characteristics of that material; notice that some materials have very similar HU values. During procedures monitoring the flow of blood through the heart, or other organs, it can be particularly hard to differentiate between the blood and organ tissue in the reconstructed CT images. In this case, it is normal for the clinician to administer a radiocontrast agent before the CT scan, to a patient, which essentially increases the density of the blood, thus increasing the linear attenuation coefficient and therefore the HU value of the pixels, in the image containing blood. This in turn means that the blood-tissue contrast is enhanced [5, 16]. This is observed in Figure 13, below.

![Figure 13 A CT image of arteries and veins containing radiocontrast, notice that the contrast between the veins or arteries against the muscle is good, allowing the clinician or doctor to distinguish between them [17].](image-url)
2.3 Radionuclides for Imaging.

Nuclear medicine utilises radiopharmaceuticals, to measure the biochemical and physiological processes of the body, by measuring the emitted radiation, from the unique process by which the radioisotope decays [5]. It is therefore crucial that a detailed understanding of the processes that the radiopharmaceutical will undergo, during its lifetime, are understood from a standpoint of generating the Monte Carlo simulation of the SPECT imaging process and understanding the results.

The physical characteristics that make a particular radioisotope desired for its use in nuclear imaging are [5]:

- Appropriate half life
- Radioactive decay by photon emission
- A suitable photon energy such that it can penetrate the body.
- Photons of low enough energy such that upon collision with the collimator, they will be absorbed within the septa (see section 2.1.1.1)

2.3.1 Radioactive Decay

The radioactivity $Q$, of a radioisotope of a given number of nuclei, $N$:

$$ Q = -\lambda N = \frac{dN}{dt} $$

(11)

where $\lambda$ is the decay constant which is defined as the rate of decay of the isotope. Solving equation 11 gives [5]:

$$ N = N_0 \exp \left( -\lambda t \right) $$

(12)

where $N$ is the number of nuclei remaining at a given time, $t$. Given equation 12, the physical half-life of a radioisotope, $T_p$, can be defined. This is the time taken for half of the total nuclei to decay [5]:

$$ T_p = \frac{(Ln 2)}{\lambda} $$

(13)
The effective half-life, $T_E$, of a radiopharmaceutical is however different and is dependent on both the physical half life $T_p$ and the biological half-life, $T_E$, by:

$$\frac{1}{T_E} = \frac{1}{T_B} + \frac{1}{T_P} \tag{14}$$

The biological half-life is essentially the time taken for the number of nuclei to reduce by half due to the body’s normal functions, one being excretion. Matching the effective half-life to the time required for the imaging process is crucial [5].

2.3.1.1 Modes of Radioactive decay

SPECT primarily detects gamma rays, therefore it is integral that the radiopharmaceutical being imaged decays via a process which emits gamma-rays [5]. For this study, Lutetium-177, $^{177}$Lu, is imaged and simulated, which decays by two types of radioactive decay which are $\beta^-$ decay and $\gamma$-decay [18]. The specific details regarding a $^{177}$Lu nuclear disintegration are detailed below:

2.3.1.1.1 Beta-decay

This mode of decay is associated with neutron-rich radionuclides and involves the emission of a $\beta^-$ particle, or an electron, which results in the transformation of a neutron into a proton [5]. Due to this conversion, the atomic number of the radionuclide changes, but its atomic mass stays the same. The energies of the $\beta^-$ particles emitted as a result of the decay are not of a single discrete energy, but a continuum. The energy released from the disintegration is always discrete and this means that another particle must be emitted in the process, which is an electron anti-neutrino [5], $\bar{\nu}_e$. A generic equation states:

$$\frac{A}{Z}X \rightarrow Z+1^A_{+1}X' + \beta^- + \bar{\nu}_e \tag{15}$$

where $\frac{A}{Z}X$, is the parent nuclide, $^Z_{-1}X'$ symbolises the resultant daughter nuclide, given the change in atomic number and the remaining two particles are the electron and electron anti-neutrino [5]. Beta decay can be followed with a gamma-ray emission, if the daughter is produced in an excited state.
2.3.1.1.2 Gamma-decay

Gamma-emission normally follows a previous decay, such as the previously explained beta decay [5], with no change to the atomic structure of the parent nuclide and a decay to a lower state of excitation.

\[ \frac{A}{Z}X^* \rightarrow \frac{A}{Z}X + \gamma, \quad (16) \]

where \( \frac{A}{Z}X^* \) refers to the parent nuclide in an isomeric state and \( \frac{A}{Z}X \) details the resultant nuclide after gamma emission.

2.3.1.2 The nuclear disintegration of Lutetium-177.

A decay scheme has been provided below, detailing the transitions possible from the nuclear disintegration of a \(^{177}\text{Lu}\) atom. The transitions of interest and of greatest intensity are those where typically two gamma rays of 113 keV and 208 keV are emitted. While \(^{177}\text{Lu}\) is a beta emitter, this decay mode is utilised as a therapy treatment and the gamma rays are used for the SPECT imaging aspect.
Figure 14. A decay scheme detailing the transitions possible from the nuclear disintegration of a \( {}^{177}\text{Lu} \) atom via beta-emission and then accompanied by a secondary gamma-emission. Due to selection rules, the transitions of interest and of greatest intensity are those where typically two gamma rays of 113 keV and 208 keV are emitted. \[18\].

It is evident from the nuclear decay scheme, that the most-likely decay mode of Lutetium-177 is to decay via beta-emission to the ground state of \( {}^{177}\text{Hf} \), 79.4% of the time. The other possibility is that the radionuclide decays via beta emission into one of three excited states of \( {}^{177}\text{Hf} \), which are seen in Figure 14. Due to selection rules, the transition leading to the emission of a gamma ray of 112.9 keV is an M1 transition and the 208.4 keV is an E1 transition, with the E1 transition being favoured between the two. The concept of the selection rules is not used in this thesis and the reader is referred to reference [19] for more information.

For imaging \( {}^{177}\text{Lu} \), the SPECT camera is focussed on 2 energy windows, deemed EM1 and EM2 in this thesis, which are centred about the 112.9 keV and 208.4 keV photopeaks respectively. Upper and lower energy windows are also assigned to each photopeak,
implementing the TEW method as described by section 2.1.3.
2.4 Interactions of Radiation with Matter.

To understand the imaging processes associated with SPECT/CT and to be able to interpret results from the Monte Carlo simulation, it is necessary to understand the interactions of radiation with matter, at the energies associated with diagnostic radiology. The subsequent section will detail the corresponding physics, associated with each interaction and also its relevance to the dual imaging modality.

2.4.1 Interactions generating X-rays

The interactions detailed here lead to the generation of x-rays, which are most likely to occur within the x-ray/CT scanner, as detailed in section 2.2, and also as a result of beta decay.

2.4.1.1 Bremsstrahlung Emission

Bremsstrahlung, in German, translates to braking radiation and occurs as a result of high energy electrons interacting with the atomic nuclei in an anode target, or a patient [20]. The most common Bremsstrahlung interaction occurs as an electron passes very close to a positively charged target nucleus, which attracts the negatively charged electron, leading to deceleration. This change in velocity, of the incident electron, is dependent on the distance of approach to the nucleus [5], and translates to a loss of kinetic energy for the electron; an x-ray photon is subsequently emitted with a kinetic energy of that same magnitude. This scenario is presented in Figure 15(a).

2.4.1.2 Inner-shell Ionisation

Figure 15(b), which is the second instance that leads to the emission of x-ray photons, is named inner shell ionisation, which occurs when an incident electron interacts with an inner-shell atomic electron, leading to its displacement and ionisation of the atom. Thereafter, an electron from an outer-shell fills the void in the deficient inner shell and an x-ray photon with characteristic energy defined by the electronic shells is emitted from the target atom. The energies of the x-ray photons are a function of the energy levels of the target atom and thus are specific to the target material. For example, tungsten characteristic Kα and Kβ x-rays occur at 58.5 keV and 66.7 keV respectively, this can be seen on Figure 16.
Figure 15 Interactions (a) and (b) present two interactions that lead to the emission of x-ray photons. In interaction (a), Bremsstrahlung, the incident electron passes particularly close to the atomic nucleus and because of electrostatic attraction the kinetic energy of the electron is reduced. In interaction (b) inner-shell ionisation, the incident photon interacts with an inner-shell electron, knocking it out of its orbit. Outer-shell electrons can occupy the hole, dropping down from their outer orbital shell. Both interactions lead to x-ray emission [5, 16].

Combining the two interactions, a bremsstrahlung spectrum contributes a continuum of energies and inner-shell ionisation produces x-rays with discrete energies, which leads to the overall x-ray spectrum observed in Figure 16. The energies of the tungsten x-rays, produced by inner-shell ionisation, were previously stated in section 2.4.1.2, and contribute the sharp $K_\alpha$ and $K_\beta$ peaks in the overall tungsten x-ray spectrum.

Figure 16 A typical x-ray spectrum, of the x-rays detected from thermionic electrons interacting with a tungsten target by bremsstrahlung and inner-shell ionisation. [21].
2.4.2 Interactions of x-rays and gamma-rays in matter.

Ideally, the emitted photons would not interact with anything before they are detected [2], but unfortunately this is not the case and it is highly likely that the radiation will be scattered perhaps multiple times before it is either absorbed within a material, or detected by the scanner. Interactions are integral to detecting the emitted photons and scattering causes a degradation of the reconstructed SPECT images. Therefore an understanding of how photons interact with the SPECT detector and patient is crucial for creating an accurate Monte Carlo simulation of the imaging process.

2.4.2.1 Compton Scattering

Also known as inelastic scattering, Compton scattering is the dominant interaction of x-rays and gamma rays in diagnostic radiology for photons greater than 26 keV, for scattering in soft tissue. This process normally occurs between an incident photon and valence electrons, as illustrated in Figure 17, causing the atom to eject one of these electrons upon collision [22]. The scattered photon is reduced in energy, relative to that of the incident photon. Given that energy and momentum are conserved, it can be said that the energy of the incident photon, $E_i$, is the sum of the kinetic energy of the ejected electron, $E_e$, and the scattered photon $E_{SC}$ [22]:

![Figure 17A simple representation of a photon undergoing Compton scattering from a collision with a valence electron in an arbitrary atom. This figure was taken from Ref [22].](image)

\[ E_i = E_{SC} + E_e \]  \hspace{1cm} (17)

Compton scattering leads to the ionisation of the atom and the energy of the incident photon apportioned between the ejected electron and scattered photon as stipulated in equation 17. The energy of the scattered photon is determined from the incident photon energy, \( E_{SC} \), and the scattering angle, \( \theta \), of the scattered photon, from the incident trajectory [22]. Using energy and momentum conservation laws [2, 22], it can be shown that:

\[ E_{SC} = \frac{E_i}{1 + \frac{E_i}{511 \text{ keV}}(1 - \cos\theta)} \]  \hspace{1cm} (18)

It can be seen from equation (18) that the greatest transfer of energy, from the incident photon to the ejected electron, occurs when the photon is scattered 180°, or backscattered [2]. With regards to the maximal energy transfer to the scattered photon this occurs at 0° scattering, which can be computed as 511 keV from equation 18. In contrast to the scattered photon, which might undergo additional interactions in the medium, such as further Compton scattering events or even photoelectric interactions, the Compton electron is normally absorbed near the scattering site [2].

One condition must be sustained before a Compton interaction might occur, this is that the incident photon energy is at least greater than the binding energy of an electron- this indicates that the interaction cross-section increases as incident photon energy increases [22]. Another factor, indicating an increased probability of Compton interaction is electron density, in that per unit volume, the probability of Compton interactions is proportional to the electron density of the material [22].

The effect that Compton scattering has on a $^{99m}$Tc spectrum, which similarly to $^{177m}$Lu is used for nuclear imaging, is shown in Figure 18. When the number of scattered photons is equal to zero, \( N_{SC} = 0 \), the spectrum is a distinct photopeak centred about 140 keV, but photons might undergo multiple Compton interactions, or \( M_{SC} > 0 \). In this case, depending on the number of Compton interactions and their deflection angles, the photon would be counted or allocated to lower energy bins of the histogram [2]. This distorts the photopeak, imparting an almost hump-like shape onto the peak. Figure 18 illustrates how various orders of Compton scatter leads to the overall distorted shape of the photopeak [2].
Figure 18 These spectra are particularly indicative of the effects that Compton scatter can have on energy spectra. $N_{SC} = 0$ is the spectrum of events not Compton scattered at all. $N_{SC} = 1$ is the spectrum of events Compton scattered once and $N_{SC} = 2$ is the histogram of even ts scattered twice and so on. Diagram taken from ref [2].

2.4.2.2 Photoelectric Effect

In this mechanism, a photon of incident energy $E_i$ transfers all of its energy to an atomic electron, as illustrated in Figure 19. A bound electron will be ejected if the energy of the incident photon is at least equal to, or greater than, the binding energy of the electron to the atom; the incident photon is completely absorbed [22]. The electron is then ejected from the atom, with any remaining energy in the form of kinetic energy. The equation relating the binding energy, $W$, initial energy of the photon $E_i$ Planck’s constant, $h$, and frequency, $f$, is [22]:

$$E_i = hf - W$$  \hspace{1cm} (19)
The effective cross-section is dependent on the atomic number, Z, and energy of the incident photon, E [5, 22]:

\[ \text{Effective Cross section } \propto \frac{Z^3}{E^3} \quad (20) \]

From equation (20), it can be seen that the lower energy photons are more likely to be photoelectrically absorbed by the tissue and contribute to the ionising radiation absorbed by the patient [5, 22]; the z dependency also indicates that materials with a larger atomic number are more likely to interact by the photoelectric effect.

2.4.2.3 The Overall Effect.

All of the aforementioned processes involve attenuation, since all three interactions reduce the overall intensity of the beam of photons. The attenuation effects of photons vary from material to material and overall, the attenuation effects of a particular material can be characterised by its linear attenuation coefficient, \( \mu \), which is the total linear attenuation coefficient. This coefficient describes the total probability of a photon being attenuated by either the photoelectric effect, or Compton scattering, but \( \mu \) can also be broken down into its own individual constituents, which is expressed in terms of the linear scattering coefficient.
\( \mu_{SC} \) and the linear absorption coefficient \( \mu_{AB} \) \[22\]:

\[
\mu = \mu_{AB} + \mu_{SC} \tag{21}
\]

where \( \mu_{AB} \) and \( \mu_{SC} \) are the contributions to the linear attenuation coefficient from the absorption of energy by the photoelectric effect and the energy loss associated with the Compton interaction, respectively \[22\]. Each material has a unique value of linear attenuation coefficient and the combined effects of the two aforementioned interactions also leads to the value of \( \mu \) being dependent on photon energy, tissue density and atomic number, \( Z \).

Figure 20. Presents the photon interaction cross-section in water, taken from Ref. \[5\].

Figure 20 is a particularly important plot, as it details which of the interactions is dominant over 0-150keV energy range. From the plot, it is observed that the relative importance of the photoelectric effect lessens as the energy of the photon increases and conversely the effect of Compton scattering on the mass attenuation coefficient increases, as the energy increases \[5, 22\]. Putting this into perspective, it is Compton scattering that mainly affects photons in soft tissue, but when the energy of the photons reduces to around 60 keV, which might occur due to Compton interactions, the photoelectric effect dominates.\[2\].
2.5 Monte Carlo Simulations

Monte Carlo Methods use the random sampling of numbers to statistically evaluate the likelihood of a multitude of outcomes of a complicated system, given that the probability distributions of certain processes are known [23].

2.5.1 Geant4

Geant4 is a Monte Carlo Code written by CERN to “simulate the passage of particles through matter” as detailed in ref [24]. The Geant4 kernel encompasses the tracking of particles, describing geometry, material definitions, physics processes and detector response amongst others.

Geant4 generates a particle of known type, energy and momentum from probability distributions [23]. A random number is then generated and with the interaction cross-section of this particle, the distance it will travel, d, before an interaction is based upon equation (22).

\[ d = \frac{1}{\mu \ln (1-R)} \]  

(22)

where \( \mu \) is the linear attenuation coefficient of the material as defined in section 2.4.2.3, with units of inverse length and R is the assigned random number of between 0 and 1 [23]. Given this interaction, the energy of the particle is modified accordingly and the process is repeated until either the energy of the particle reaches a set lower limit, or it is detected. Secondary particles are also modelled in this manner [23].

The information recorded from the simulation is defined by the user and how the data is recorded, in the output, is defined in the digitiser module of GATE.

2.5.2 GATE

GATE provides medical imaging functionality to the Geant4 Monte Carlo Code, by providing the user with an interface to define SPECT detector geometries, source distributions, interaction mechanisms and SPECT detector electronics from a series of in-built libraries. It is this software that was used to create GATE simulations for modelling a voxel-based Jaszczak and anthropomorphic phantom [4, 65].
3.0 Methodology

The methods required to generate and run a SPECT scanner GATE simulation with a voxel-based anthropomorphic phantom and corresponding source distribution are detailed in this section. Also detailed are the methods required to analyse the output from the simulation in ROOT, a data analysis package. An image processing software package, called IMAGEJ, is also referenced multiple times throughout the subsequent sections, this software is used to manipulate images specific to medical imaging, like CT volumes and projections.

3.1 GATE simulation Architecture

The architecture of the GATE simulation first needs to be detailed, so it can be established what specific information is required to run a GATE simulation. From Figure 21, it can be seen that the GATE SPECT simulation requires of a number of inputs which include [4]:-

- SPECT scanner geometry
- Phantom geometry
- Materials database
- Source information
- Physics processes.
- Digitizer information.
- Tomographic acquisition information.
3.1.1 SPECT Scanner and Phantom Geometries

The SPECT scanner geometry input details the dimensions and the materials associated with each component of the SPECT scanner, and the phantom geometry input provides GATE with the dimensions and materials associated with the phantom. Both of these inputs require a materials database, which provides GATE with material information such as the type of materials and their elemental composition and density, amongst others [4]. They both also require the user to define which volumes are “sensitive detectors”, which ultimately tells Geant4 which volumes to track the photons through. [4].
3.1.2 Physics Processes
Defines the interactions that the particles being simulated might undergo, these could include low energy processes, bremsstrahlung, photoelectric effect, Compton scattering amongst others [4].

3.1.3 Source Information
GATE requires source distribution information such as the energy of the gamma rays and the typical Bremsstrahlung distribution of the beta particles associated with the nuclear disintegration of $^{177}$Lu. It also requires the activity of the radiopharmaceutical being imaged, which is to represent the dose that is administered to a patient. Finally, GATE requires the location within the phantom that the desired radioactivity of the radioisotope is allocated to; this is a representation of the lesion and ultimately the distribution of the source of the gamma rays throughout the phantom [4].

3.1.4 The Digitizer Module
The use of the digitizer in GATE is to simulate the behaviour of the detectors and the signal processing chain. GATE uses Geant4 to mimic the interactions of particles and matter, by generating particles and transporting them through the materials defined in the simulation [4]. In more detail, a particle of known type, momentum and energy is generated by Geant4, the trajectory and individual interactions that the particle undergo are recorded as ‘steps’ and if a step was to be recorded in a sensitive detector, the interaction information, including the energy deposited, momentum and the ID of the sensitive detector are stored as ‘hits’. This process is repeated a number of times, forming a trajectory of the particle [4]. If the particle were to interact with the scintillator crystal, as defined in section 2.1.1.2, the digitizer module processes this signal and stores the entire information as a ‘single’. Both the ‘hits’ and ‘singles’ are stored in what is known as the ‘TTree’ in the output ROOT file [4].

3.1.5 Tomographic Acquisition Information.
The information pertaining to the total time for acquisition, the number of positions of the detector heads and the time for data acquisition in each position [4] are required to tell Geant4 when to track photons through the sensitive detectors.
3.2 The SPECT Tomographic GATE Simulation

A simulation of the infinia GE Hawkeye SPECT camera, physics processes and the tomographic process has previously been created with the specifications detailed in Table 2. The information regarding the geometric-based phantom and source distribution has not been included because the subsequent proposed methods detail the addition of the voxel-based equivalent components to the GATE simulation.

*Table 2 contains information used by GATE to create a SPECT scan simulation. Also included is the information pertaining to the decay process associated with Lutetium-177.*

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<th>Geometric GATE Simulation</th>
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<td><strong>SPECT Detector head</strong></td>
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**Gamma/x-ray emission**

Minimum energy 0.0546 MeV
Maximum energy 0.32133 MeV

**X-ray Energies detailed by the histogram:**

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**Output Information**

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<tr>
<td>Number of pixels</td>
<td>128 x 128</td>
</tr>
<tr>
<td>Pixel size</td>
<td>0.924mm x 0.924mm</td>
</tr>
</tbody>
</table>
3.3 The Method for Creating a Voxel-Based Phantom and corresponding voxel based source distribution

3.3.1 The Jaszczak and Anthropomorphic Phantom

A phantom, for applications in nuclear imaging, is a cylinder that serves as a representation of a human torso. Two types of phantom were utilised, in the project, the first termed a “Jasczack” phantom is a very simple phantom, which is filled with water, and three smaller cylinders contained within the body, known as inserts. A CT slice of this phantom and its inserts is presented in Figure 22(b) and the inserts have been labelled one to three, which will be defined as its volumes of interest for the subsequent analysis. These inserts, represent lesions and may be filled with a known activity of a particular radiopharmaceutical to simulate the process undertaken in a hospital scan. The second phantom, labelled an anthropomorph (or human-like) phantom, is a more accurate and more detailed representation of a human torso. Its volumes of interest are the lungs, liver and a liver insert and similar to the Jaszczak phantom, can be filled with a quantity of a radiopharmaceutical. This phantom has been presented in Figure 23 and its volumes of interest visually are presented in Figure 47.

![Figure 22](image.png)

*Figure 22 A photograph of a typical Jaszczak phantom with six inserts discerned in (a). Image (b) is a CT image of the phantom with three cylindrical inserts coloured blue, red and green which are inserts 1, 2 and 3, respectively. These are the volumes of interest for this phantom. Even though a six-insert phantom has been presented, the logic is still the same.*
Phantoms are employed, in nuclear studies to evaluate performance of the scanner or a technique proposed for lesion detection, image reconstruction and collimation design; they are also used in the day-to-day calibration of the SPECT scanner in a hospital.

3.3.2 Voxelisation Techniques in GATE

The voxelisation of an arbitrary geometry means to divide a 3D structure into smaller discrete 3D volumes. An example of this is shown in Figure 24:

![Image of cube before and after voxelisation](image)

**Figure 24** The voxelisation of a simple cube structure into discrete 3D volumes is presented above, with image (a) being the cube before voxelisation and (b) the same cube after voxelisation [26].

GATE can voxelise volumes in different ways, but for this GATE simulation, the “compressed phantom matrix” method is used. As geometries become more complicated, the time taken to voxelise geometries increases and the computational processing power also
increases and this is due to the fact that Geant4 will have to track particles through each individual voxel. The compressed phantom matrix method groups voxels which are assigned the same material to make one larger voxel, meaning that Geant4 will have to voxelise less volumes and track photons through less created voxels [4].

3.3.3 CT Acquisition
For generating voxel-based arbitrary geometries and source distributions, two special x-ray/CT acquisitions of the phantom are required.

3.3.3.1 The Phantom-CT Stack
A CT scan of the phantom must be obtained, but with each separate volumes of interest filled with purified water. The acquired CT stack was used to provide GATE with the phantom geometry and the materials it is comprised of based on the unique information the CT image details, which is the HU value of each pixel.

3.3.3.2 The Source-CT stack
Defining three cylindrical insert-like sources of radioactivity for the Jaszczak phantom is simple, but to define a more complicated anthropomorphic source distribution is extremely difficult. For this case, a second CT scan of the phantom is required so that the geometry of a source distribution can be automatically generated. This is achieved by filling each volume of interest with a differing quantity of radiocontrast to alter the density and therefore range of HU values associated with its containing fluid. This means that each volume of interest will be identifiable by their unique range of HU values. Given that the density of the fluids contained in multiple volumes is being changed with the radiocontrast agent, this CT scan cannot be used to define the materials contained within the phantom. This is because GATE would assign densities to the water in those volumes which were too high and the attenuation of photons would ultimately increase in these volumes.

Figure 25, presents two identical slices of the Jaszczak phantom source CT stack. The first slice, image (a), is a normal view of the CT slice and in image (b) the CT slice has been presented such that the pixel values are represented by colours ranging from red to white, with red meaning least dense and white being most dense. This CT image essentially shows that the volumes of interest, that is to say the water and the liquid in the three inserts, are of different densities thus making their regions of HU values unique. Table 3 details the
concentrations of iodine radiocontrast administered to the volumes of interest in the both the Jaszczak and anthropomorphic phantoms to acquire source-CT volumes.

Table 3 Details the volumes of interest for the Jaszczak and Anthropomorphic phantom and the concentrations of iodine radiocontrast used to acquire the source-CT stack.

<table>
<thead>
<tr>
<th>Volume of Interest</th>
<th>Concentration of Radiocontrast Agent [g/cm$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaszczak Phantom</td>
<td></td>
</tr>
<tr>
<td>Insert 1</td>
<td>1.67E-03</td>
</tr>
<tr>
<td>Insert 2</td>
<td>3.35E-03</td>
</tr>
<tr>
<td>Insert 3</td>
<td>6.96E-03</td>
</tr>
<tr>
<td>Water</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>Anthropomorphic Phantom</td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>Liver Insert</td>
<td>6.29E-03</td>
</tr>
<tr>
<td>Liver</td>
<td>2.14E+02</td>
</tr>
</tbody>
</table>
Figure 25 presents two identical slices of the source CT stack the first, image (a), is a normal view of the CT slice and in image (b) the CT slice has been presented such that the pixel values are represented as colours ranging from purple to white, where white signifies most dense, yellow moderate density and purple/red least dense.

3.3.4 Generating a voxel-based phantom Input.

The voxel-based phantom input utilises the CT scan of the phantom and from a material database, allocates materials to each voxel in the CT image depending on its HU values. The materials database relates HU values to materials and each material has a density, atomic number and proton number also assigned to it. Geant4 requires these pieces of information to track the propagation of photons or the attenuation of photons through the phantom. To create a materials database, GATE requires a Schneider table and a density tolerance. The information required for each of these components is detailed, but how GATE produces a resultant materials database is not required. The reader can find more information on how GATE uses both of these components to automatically create the materials database in ref [4].

3.3.4.1 Information required for the Schneider Table

- **Material composition of the phantom**

  This includes determining the materials associated with the inserts, the body of the phantom and the material that the phantom is filled with. The material composition of each of these components are plexiglass, for the first two components and water, respectively. If other objects are visible in the phantom-CT stack, their composition should also be known. These objects could include the patient bed, or a device for keeping the phantom level such as a bean bag or support.
• *The elemental composition of each of these materials.*

The fractional contribution of each element to the overall composition of the material.

• *The regions of HU values for each of these materials.*

This was achieved by iteratively “thresholding” the image on a region of HU values in an image processing software, IMAGEJ. This means that voxels, in the CT stack, that fall within the specified range of HU values are highlighted visually in the software. By iteratively changing this region to capture all the voxels associated with each material, in every CT slice, the regions for each volume are determined.

This process is difficult because some materials might have similar HU values and this is because they have similar densities. In the scenario where this does occur and two materials are indistinguishable by their regions of HU values, judgement can be used to determine which material is more important, that is to say if one of these materials contributes very little to the attenuation effects of the gamma-rays, it could be defined as a “dummy material” in the Schneider table. This ultimately means that its density is set to a nominal value, thus excluding the attenuating effects of the material and therefore voxel.

Figure 26 presents a GATE visualisation of the voxelised anthropomorphic geometry, showing the results of the “thresholding” process in IMAGEJ. It can be seen that the water, plexiglass and spinal insert have been created as separate voxel-based materials as a result of this process. The same process described above was implemented for the Jaszczak phantom CT image.
A test was carried out, to show that the number of voxels captured in the region of HU values for water, corresponded to the actual volume of water in the phantom. This test first shows that the method is working correctly, but also because the water volume is the greatest volume of attenuating material, it shows that the correct volume of water has at least been captured in the defined material definition. This was carried out by using IMAGEJ to determine the number of voxels in the range of HU values defined for water. This number was then converted to a volume (using the dimension of a voxel) to determine the volume of water captured in the definition. This was then compared to the real volume of water in the phantom and the results of this test are detailed in Table 4 for the Jaszczak and anthropomorphic phantom.

Table 4 details the number of voxels defined as water in the HU region definition and the actual volume of water in the phantom.

<table>
<thead>
<tr>
<th>Phantom</th>
<th>Volume of water captured in HU region definition</th>
<th>Actual Volume of water in phantom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaszczak Phantom</td>
<td>7246 cm$^3$</td>
<td>7300 cm$^3$</td>
</tr>
<tr>
<td>Anthropomorphic Phantom</td>
<td>8364 cm$^3$</td>
<td>7968 cm$^3$</td>
</tr>
</tbody>
</table>
Figure 26 Presents three images showing the successful thresholding of CT images to define regions of HU values for plexiglass in (a), water in (b) and the spinal insert in (c).
3.3.4.2 Defining the Density Tolerance [g/cm³]
Depending on the region of HU values defined for a particular material within the Schneider table, the material could feature as multiple definitions in the materials database. Expanding on this, water might not have a uniform density throughout the phantom. Depending on the range of densities observed for this material, and thus the range of HU values in the CT stack, it will be defined as separate materials. The number of definitions could be limitless, which would slow down the computation time, so with a density tolerance, a single material would only be defined for a range of HU values that correspond to a range of densities differing less than the tolerance value.

3.3.4.3 The materials Database and Optimisation.
Table 5 details the information generated as a result of creating the Schneider table and defining the density tolerance. It is this information that GATE uses, in conjunction with the phantom-CT stack, to create a voxel-based phantom geometry.

Setting the density of a material, in the database, to a nominal value essentially excludes the voxel from the simulation. Table 5 and Table 6 detail the information acquired as a result of implementing the procedure for generating a voxel-based phantom input; the resultant materials database contains the following information. Notice in Table 5, the multiple instances of “Dummy Material”, each with a minimal density of 1E-25 g/cm³. Any voxels in the CT image with a HU value within its HU region range will be assigned this density.

Table 5 The materials defined in the Jaszczak materials database, generated from the Schneider Table and the predefined density tolerance, with the lower and upper HU values defining the overall region of HU values and corresponding densities.

<table>
<thead>
<tr>
<th>Material</th>
<th>Lower HU value</th>
<th>Upper HU value</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air_0</td>
<td>-68</td>
<td>126</td>
<td>1E-22 kg/m³</td>
</tr>
<tr>
<td>Air_1</td>
<td>126</td>
<td>157</td>
<td>99.3348 mg/cm³</td>
</tr>
<tr>
<td>PVC-Fabric_2</td>
<td>157</td>
<td>250</td>
<td>165.964 mg/cm³</td>
</tr>
<tr>
<td>PVC-Fabric_3</td>
<td>250</td>
<td>343.3</td>
<td>265.964 mg/cm³</td>
</tr>
<tr>
<td>PVC-Fabric_4</td>
<td>343.3</td>
<td>432</td>
<td>363.603 mg/cm³</td>
</tr>
<tr>
<td>DummyMaterial_5</td>
<td>432</td>
<td>515.7</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_6</td>
<td>515.7</td>
<td>599.3</td>
<td>1.25E-25 g/cm³</td>
</tr>
</tbody>
</table>
Table 6 The materials defined in the anthropomorphic materials database, generated from the Schneider Table and the predefined density tolerance, with the lower and upper HU values defining the overall region of HU values and corresponding densities.

<table>
<thead>
<tr>
<th>Material</th>
<th>Lower HU value</th>
<th>Upper HU value</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>DummyMaterial_7</td>
<td>599.3</td>
<td>683</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_8</td>
<td>683</td>
<td>766.6</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_9</td>
<td>766.6</td>
<td>850.3</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_10</td>
<td>850.3</td>
<td>933.9</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_11</td>
<td>933.9</td>
<td>967</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>Water_12</td>
<td>967</td>
<td>1012</td>
<td>1.06886 g/cm³</td>
</tr>
<tr>
<td>Plexiglass_13</td>
<td>1012</td>
<td>1121.4</td>
<td>1.13079 g/cm³</td>
</tr>
<tr>
<td>Plexiglass_14</td>
<td>1121.4</td>
<td>1209</td>
<td>1.21996 g/cm³</td>
</tr>
<tr>
<td>BeanBag_15</td>
<td>1209</td>
<td>1210</td>
<td>1.26759 g/cm³</td>
</tr>
<tr>
<td>Material</td>
<td>Lower HU value</td>
<td>Upper HU value</td>
<td>Density</td>
</tr>
<tr>
<td>----------------------</td>
<td>----------------</td>
<td>----------------</td>
<td>------------------</td>
</tr>
<tr>
<td>DummyMaterial_14</td>
<td>1300</td>
<td>1375.2</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_15</td>
<td>1375.2</td>
<td>1450.4</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_16</td>
<td>1450.4</td>
<td>1525.6</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_17</td>
<td>1525.6</td>
<td>1600.8</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_18</td>
<td>1600.8</td>
<td>1676</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_19</td>
<td>1676</td>
<td>1751.2</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_20</td>
<td>1751.2</td>
<td>1826.4</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>DummyMaterial_21</td>
<td>1826.4</td>
<td>1839</td>
<td>1.25E-25 g/cm³</td>
</tr>
<tr>
<td>Beanbag_Spine_22</td>
<td>1839</td>
<td>1840</td>
<td>2.0823 g/cm³</td>
</tr>
<tr>
<td>Beanbag_Spine_23</td>
<td>1840</td>
<td>2000</td>
<td>2.0823 g/cm³</td>
</tr>
</tbody>
</table>
3.3.5 Generating the GATE Source Distribution Input

Essentially, this voxel-based GATE input automatically generates a source distribution from regions of HU values associated with volumes of interest in the phantom. These regions are acquired from a source-specific CT scan of the phantom and from these regions, the input allocates a quantity of radioactivity to each of the voxels.

3.3.5.1 The Information required for the Voxel-Based Source Distribution Input

The source distribution input requires information regarding:

- The particles emitted and their relevant energies or energy distributions from the nuclear disintegration of $^{177}$Lu (this information is defined in Table 2)
- Regions of HU values for each volume of interest in the source-specific CT stack.
- The activity of the radiopharmaceutical per voxel for each region of HU values (this corresponds to each volume of interest).

3.3.5.2 Defining the radioactivity of the $^{177}$Lu source distribution

The source distribution input to GATE requires an input of its own, detailing the regions of HU values for each volume of interest and the activity per voxel, in Becquerels; this input will be named the activity input. The steps for acquiring this specific information and the reasons for doing so are included in subsequent steps one to three:

1) Identifying the regions of HU values for each Volume of Interest in the source-CT

The volumes of interest in the Jaszczak phantom for example are the three inserts and the water in the main body of the phantom. Figure 27(a) and (b) present two images of the same slice of the CT stack where the thresholds, placed on the HU values, have been captured for inserts one and three. To complete this process, the regions of HU values for the water and insert two would be acquired. To optimise this process, the CT stack was cropped to ensure that voxels associated with components, such as the bed, were not included as a part of the source distribution.
Figure 27 Presents two images of the same slice of the CT stack where the thresholds, placed on the HU values, have been selected for inserts one and three, in (a) and (b) respectively. To complete this process, the regions of HU values for the water and insert two were acquired.

2) Determining the number of voxels in each of these regions of HU values.

Step 1 ensured that the source distribution could be defined based on individual regions of HU values. The radioactivity in these volumes is assumed to be distributed uniformly across them and this means that per region of HU values, the number of voxels needs to be calculated. This can either be obtained through data analysis in ROOT, or by simply counting the number of voxels associated with each of the regions.

3) Defining an initial source radioactivity and source distribution.

Due to the complexity of the nuclear disintegration of \(^{177}\)Lu, two activity inputs are required to detail the two possible decay modes. The first details the activity associated with the gamma-emission component and the second for the activity associated with beta-decay. The process is the same for both, but both pieces of information must be stipulated to correctly define the activity of the radioisotope to be simulated.

To define a simple 3-insert source distribution for a Jaszczak phantom, the three possible regions of HU values and three corresponding activity-per-voxels must be stipulated in the input. The activity-per-voxel is calculated by taking the total activity to be simulated in each insert, and divided by the number of voxels associated with the region of HU values for that insert. The process of calculating the number of
voxels associated with a given region of HU values, and thus insert, was explained in step 2. Step 2 implemented for both the gamma-decay contribution activity input and also the beta-decay contribution activity input files.

From steps 1 to 3, the activity inputs for both the gamma emission and beta decay are used by the GATE input to assign an activity per voxel, to voxels that fall within the regions of HU values in the source-CT stack. This process leads to a three insert source distribution being automatically defined and positioned at the location of the inserts of the phantom.

3.3.5.3 The Activity simulated in the Jaszczak and Anthropomorphic Phantom GATE simulations

Presented in Table 7 is the information obtained as a result of implementing steps 1 to 3 in the above section; it also details the radioactivity that was simulated for the voxel-based Jaszczak phantom simulation and the anthropomorphic simulation. The activities in each of the inserts for the Jaszczak phantom are a 1:1 match to that experimentally scanned and geometrically simulated. This will ensure that a comparison of the results can be compared to the experimental and geometric-based data sets. The anthropomorphic phantom simulated activity is only 1/26.47 of the total activity imaged by the SPECT scanner, this corresponds to ~38MBq and the reason for not simulating 1006MBq was because of time constraints towards the end of the project. Even though this is not a 1:1 match, results can be scaled up by a factor of 26.47 to ensure that a comparison of the results available from the experimental dataset is possible. Please note, that only one set of results was scaled-up and in this case there was always a positive number of counts. Statistically speaking this is discouraged, but there was never at any point, a zero number of counts.
Table 7 The data required to create the two activity inputs discussed in steps 1-3. These activity inputs were used to simulate 45.27 MBq of $^{177}$Lu for the Jaszczak phantom and 38 MBq for the Anthropomorphic phantom.

<table>
<thead>
<tr>
<th>Volume of Interest</th>
<th>Lower HU Range</th>
<th>Upper HU Range</th>
<th>Number of Voxels in Region</th>
<th>Beta Total Activity</th>
<th>Beta Activity per voxel [Bq]</th>
<th>Gamma Total Activity</th>
<th>Gamma Activity per Voxel [Bq]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jaszczak Phantom [45.27 MBq]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>935</td>
<td>1021</td>
<td>4.68E+04</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Insert1</td>
<td>1205</td>
<td>1260</td>
<td>1.05E+02</td>
<td>255587326</td>
<td>146213</td>
<td>60318609</td>
<td>34506</td>
</tr>
<tr>
<td>Insert2</td>
<td>1494</td>
<td>1543</td>
<td>1.23E+02</td>
<td>253275481</td>
<td>123687</td>
<td>59773013</td>
<td>29190</td>
</tr>
<tr>
<td>insert3</td>
<td>1445</td>
<td>1488</td>
<td>1.05E+02</td>
<td>244798715</td>
<td>140041</td>
<td>57772497</td>
<td>33050</td>
</tr>
<tr>
<td>Anthropomorphic Phantom [38MBq]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Air</td>
<td>0</td>
<td>39</td>
<td>3.23E+04</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Water</td>
<td>970</td>
<td>1061</td>
<td>1.18E+05</td>
<td>24669020</td>
<td>1940</td>
<td>5821889</td>
<td>460</td>
</tr>
<tr>
<td>Liver Insert</td>
<td>1294</td>
<td>1384</td>
<td>2.21E+02</td>
<td>1541904</td>
<td>64640</td>
<td>363889</td>
<td>15260</td>
</tr>
<tr>
<td>Liver</td>
<td>1153</td>
<td>1300</td>
<td>1.59E+04</td>
<td>11828334</td>
<td>6900</td>
<td>2791487</td>
<td>1630</td>
</tr>
</tbody>
</table>

3.4 The position of the phantom and source CT stacks in the GATE world

Unfortunately, GATE uses two different origins for defining phantom geometries and source distributions so this meant that as a final step in creating the simulation, the two CT stacks, described in section 3.3.3, required aligning. This was to ensure two things:-

1) The activity distribution was positioned within the inserts of the phantom-CT stack, thus ensuring that the emitted particles propagate through the correct and same quantities of attenuating materials before being detected.

2) For the validation of the Jaszczak phantom GATE simulation only: the activity distribution was positioned in the same position with respect to the detector heads, as the geometric-based source distribution and therefore also the experimental source distribution. Please note that, by aligning the two CT stacks, the voxel-based source distribution is automatically positioned in the same location, with respect to the scanner detector, as the geometric-based and experimental datasets.
3.5 Methodology used for Data Analysis in ROOT

3.5.1 The output of the GATE simulation

The output of the simulation was written to ROOT, the data analysis software, which recorded the hits and singles in the TTree, as stipulated in section 3.1. Each piece of information, defined as a “ROOT Object”, is unique to the Monte Carlo simulation and the objects relevant to the analysis of the results in sections 4.0 and 5.0 are described below:

Table 8 details the unique information obtained from the Monte Carlo Simulations relevant to understanding the results presented in section 4.0.

<table>
<thead>
<tr>
<th>ROOT Object</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ComptonVolName</td>
<td>Details the number of Compton interactions experienced in each volume that is defined as a sensitive detector. The number of events not scattered is also detailed.</td>
</tr>
<tr>
<td>SourcePosX, Y or Z</td>
<td>The position that a detected photon originated from. This can be determined as an X, Y and Z position. Can be plot as a combination of two co-ordinates. Indicative of the location of the source distribution in the XY, XZ and ZY planes.</td>
</tr>
<tr>
<td>ComptonPhantom</td>
<td>The number of events scattered once, twice, thrice and so on, by the Compton interaction before being detected.</td>
</tr>
<tr>
<td>headID</td>
<td>Events detected by detector heads 1 or 2.</td>
</tr>
<tr>
<td>runID</td>
<td>This concept is illustrated in section 3.5.1.</td>
</tr>
<tr>
<td>Energy</td>
<td>Plots a histogram of the energies of all photons detected.</td>
</tr>
<tr>
<td>sourceID</td>
<td>Depending on the source distribution there could be a number of sources defined, constituting the radioactivity distribution. In the GATE simulation there are three sources of radioactivity corresponding to inserts 1, 2 and 3 of the Jaszczak phantom.</td>
</tr>
</tbody>
</table>

3.5.1.1 An explanation of runID

For the geometric-based GATE simulation, each detector rotates through an angle of 6°, 30 times, pausing for data acquisition for 40 seconds in each position. This data acquisition process has been characterised using runID, which is a unique parameter to Geant4 and relates the positions of the detector/s as they rotate around the phantom to a range of numbers, or IDs.

RunID = 1 corresponds to detector 1 (top head) beginning at the top (above the patient/phantom) and rotating through 180° to below the patient bed. The detector pauses 30
times during this process to acquire the gamma distribution at each point and a runID is assigned to each of the thirty data acquisitions (runID = 1 to runID = 30). Figure 28 explains this process, with respect to detector head 1. The same process occurs for detector 2, except that its starting position is from below the patient bed and the runIDs assigned to its 30 data acquisitions begin at runID = 31 to runID = 60.

![Diagram of detector heads and patient](image)

*Figure 28 presents a simplified illustration of the rotation of the detector head 1, around the phantom, and the starting position of detector head 2. A cross section of the patient lying on the bed is included in black and the positions of the detectors are signified by the coloured square brackets. The rotation of detector 1 is explained in this figure and the same principle can be applied to detector 2, but its initial position is from below the patient. As detector head 1 rotates about the phantom it pauses and acquires the gamma distribution from thirty positions. Five positions have been included for detector 1: the first, third, fifth, fifteenth, twenty-fifth to explain the premise. One position has been included for detector two, which is its initial position, or (1) for the yellow. Detector 1 does not acquire data in this position given the fact that detector 2 has already acquired the gamma distribution from this angle.*

It is also normal, for the scanner to acquire data from one single position, for each detector head, as opposed to rotating to different positions as presented in Figure 28. For example the detector heads would be kept stationary in their initial positions (position (1)). This is called a PHA scan and features as a component of the Jaszczak results in section 4.1.
3.5.2 Using the Objects for analysing the results.

3.5.2.1 Proving Alignment of the voxel-based source distribution to the geometric-based source distribution

Plotting sourcePosX against sourcePosY, on the same plot, will show how well the two source distributions have been aligned to each other in the XY plane. Similarly, by plotting the objects in the XZ and ZY planes, allows validation across each plane.

3.5.2.2 Energy Spectra

3.5.2.2.1 Total Energy Spectrum

Plotting the object “energy” produces a total energy spectrum. This plots a histogram of the energies of all the events which were detected by the two detectors. Using ROOT, the number of events can be acquired through the use of an “Integral” command, which sums the energies in each energy bin of the histogram to determine the total number of counts.

3.5.2.2.2 Individual Insert Energy Spectra

Similarly to the total energy spectrum, by plotting the objects “energy” gated by “sourceID” this will produce histograms of the energies of the events detected from each of the sources of radioactivity. An integral can be used to determine the number of events detected from each individual source of radiation.

3.5.2.3 Scattering Properties

3.5.2.3.1 Location of Compton Scattered events

Plotting the object “ComptonVolName” yields, for events that have been detected, the sensitive detector location of where the events were Compton scattered or not Compton scattered at all. These results specifically signify the attenuating properties of each of the component volumes and infer any incorrect HU region definitions. The information from the individual volumes can be summed depending on the location of scattering within the component volumes, these being the detector, phantom and table; events not scattered can also be presented in this information.

3.5.2.3.2 Total Energy Spectrum with constituent spectra.

Objects “energy” gated by a varying “ComptonPhantom” value from 0, 1, 2, 3, 4, >4 and >0 and plot on the same histogram. This demonstrates how each spectrum presented is a constituent of the total energy spectrum and that the total energy spectrum is comprised of events that have been Compton scattered a varying number of times. These results are also
indicative of the attenuation properties of the defined materials. An integral command can be used to determine the number of events that have been Compton scattered by varying degrees for both the geometric-based and voxel-based datasets.

### 3.5.2.3.3 Number of Counts versus RunID

The number of counts from each of the sixty projections of the 3D reconstructed images is acquired from IMAGEJ as stipulated in section 3.6. An equivalent analysis can be implemented in GATE, by plotting “runID” gated with “headID”. This details the number of events detected by detectors 1 and 2 for each position of the detector heads as they traverse through their 180° rotations; an explanation of runID was detailed in section 3.5.1.1. By taking these objects and gating them on each of the individual source locations and IDs, the number of counts for each of the sixty runIDs is produced for each of the three sources of radiation.

### 3.6 Projections Analysis

#### 3.6.1 The number of counts as a function of projection.

The SPECT camera GATE simulation acquires 2D gamma distributions of the radioactivity defined in section 3.3.5.3. The projections, as explained in section 2.1.2, detail the spatial and energy information of the events detected from the source distribution, IMAGEJ can be used to extract the number of counts detected, separately for each of the EM1 (113keV) and EM2 (208keV) energy windows, and for every projection. For each energy window the number of counts can therefore be plotted as a function of projection number. These results are indicative of the attenuation characteristics of the materials that have been defined in the voxel-based model and because the number of counts are as a function of detector position about the phantom any erroneous data may be pinpointed to a location in the phantom.

#### 3.6.2 Regions of Interest Analysis

With regards to the analysis of the actual projections, a visual comparison is required for the scope of this thesis, but further, more detailed analyses could be applied by using IMAGEJ to define regions of interest (ROI) for each of the three sources and analyse the intensity of each of these regions.
4.0 Results

4.1 Jaszczak Phantom Results

The following results presented in this section allow the validation of the voxel-based GATE Jaszczak phantom simulation against the geometric-based and experimental datasets. To validate the new voxel-based simulation, the following analysis was performed:

1) Visualisations of the voxel-based Jaszczak phantom geometry in GATE.
2) Source Distribution location.
3) Energy Spectra (Total and Individual Insert).
4) A comparison of the scattering properties of the voxel-based and geometry-based models.
5) Projections and Reconstructed Data.

4.1.1 Experimental Set-up

The voxel-based Jaszczak phantom geometry and source distribution inputs were created in GATE, using the methods discussed in section 3.3, with an activity distribution which simulated a total of 45.27MBq of Lutetium-177, distributed as stipulated in section 3.3.5.3, across the three inserts. The SPECT Tomographic GATE simulation input (information detailed in section 3.2) simulated this radioactivity for a total scan time of 1200 seconds. To validate the voxel-based technique, the activity modelled was the same as that simulated for the validation of the geometric-based phantom model, and also the same as that scanned on the infinia GE Hawkeye SPECT Scanner at the Christie hospital. The results were then analysed using ROOT, utilising the methods in section 3.5.

4.1.2 Geometry of the Voxel-Based Jaszczak Phantom.

A visualisation of the voxel-based geometry in GATE provides an indication of the location of the phantom in the GATE simulated world and it also provides a preliminary outlook of how well the phantom has been constructed on a material and voxel basis from the method proposed in section 3.3.4. The reader is referred to Table 5 for the materials assigned to the Jaszczak phantom and Table 9 for how GATE represents the materials of the phantom as colours.

Figure 29(a) is a visualisation, in GATE, of the voxel-based phantom geometry with all materials visible. Figure 29(b) is a visualisation of the same phantom, but it has been modelled using the geometric-based technique. With reference to the allocation of materials
to the phantom geometry, it appears as though the vast majority of the voxels on its outer exterior have been allocated plexiglass material definitions. Bean bag voxels have been allocated to below the phantom, towards the back, and this is correct because the bean bag was used to keep the phantom from rotating and also keep it level. It is also noted that voxels have been allocated to below the phantom, in the region of the patient bed, in the CT presented in Figure 30(b). This was because this region in the phantom-CT volume (see section 3.3.3.1 for reference) was not cropped out and was originally going to be modelled. Due to optimisations, as described in sections 3.3.4.3 and 3.3.4.1, it was not possible to do so and the geometric-based patient bed was used.

*Table 9 Details the materials and their representative colours used to visualise the simulation in GATE.*

<table>
<thead>
<tr>
<th>Colour</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>Plexiglass</td>
</tr>
<tr>
<td>Blue</td>
<td>Water</td>
</tr>
<tr>
<td>Red</td>
<td>Bean bag (Denser Plastic)</td>
</tr>
</tbody>
</table>

Presented in Figure 30 are two CT images and a visualisation of the voxel-based phantom for comparison. Figure 30(a), is a slice taken from the “Source-CT”, which was used for the creation of the source component of the model and Figure 30(b) is a CT image used for the modelling of the phantom component of the voxel-based model. Finally image (c) is a visualisation of the voxelised phantom. Analysis of the two CT images in this figure shows a clear rotation of the phantom body, in the CT image. This was due to the phantom being supported on a beanbag and a possible shift in its position as the bed was mechanically moved between the SPECT heads.
Figure 29 (a) A GATE visualisation of the Jaszczak phantom using the voxel-based method and the SPECT scanner and bed defined using the geometric-based method. The phantom and scanner has been visualised with the entire array of materials visible. Image (b) presents the same visualisation, but with a geometric-based Jaszczak phantom between the detector heads. The visualisation technique is different in that (a) is a wire frame visualisation to allow the dimensions of the CT volume to become visible (outline of the CT volume), whereas (b) is simply a solid object visualisation.
4.1.3 The positioning of the Source Distribution.

The origin of the detected events from the voxel-based source distribution was compared to the previously validated geometric-based source distribution using the method described in section 3.5.2.1. What is particularly important is the alignment of the source distribution to the phantom inserts, because the location of the origin of the source events, with respect to the phantom and scanner, provide results that are unique to this geometric set-up. This will ensure that the photons propagate through the same quantities of attenuating material.

Figure 31, plots (a), (b) and (c), show the distribution of the events detected, in planes ZY, XZ and XY planes respectively. The coloured data corresponds to events from the voxel-based dataset and the black dataset corresponds to events from the geometric-based dataset. From all of these plots, it is clear that a voxel-based 3-insert gamma distribution has been modelled, but the required alignment of the two CT volumes, as stipulated in section 3.4, has not been successful. This is shown by the fact that the voxel-based source distribution has not been positioned in the same location as the geometric-based and experimental distributions. The source distribution has also not been positioned within the inserts of the phantom-CT, which will be discussed subsequently. The misalignment to the validated source distribution position is clearly shown in Figure 31(c), the events originating from inserts two and three, in the XY plane, do not originate from the same position as their geometry-based counterparts, but the events originating from insert one appear to originate from a very similar position, meaning that this source has been aligned to the correct position.

Table 10 presents a quantified displacement, in each of the X, Y and Z directions, of each of the voxel-based insert distributions with respect to the geometric based insert distributions in increments of voxels. Insert one is the best aligned, with the insert aligned perfectly in the X and Z axis, but the position in the Y axis is off by -1 voxel. Insert 3 is aligned the worst overall, insert 2 is the second worst aligned and both displacements suggest that they are located closer to the phantom plexiglass outerbody.

Notice that the phantom in Figure 30(a) is rotated, GATE cannot correct for a rotation in a source-CT image and this means that upon defining a source distribution, as described in section 3.3.5.2, and aligning the CT image as defined in section 3.4, the source distribution would have been positioned with a rotation within the phantom geometry. This rotation must have occurred due to the placement of the phantom on a bean bag and a possible shift in its position as the bed was mechanically moved between the SPECT detectors, for acquiring the
Please note that in each of (a), (b) and (c), there rogue voxels that have been assigned a quantity of activity based upon their HU values in the source-CT. The reasoning for this is explained by the presence of an image artefact in the CT images and unfortunately these HU values have been captured as a part of capturing the regions of HU values during the method defined in section 3.3.5.2. Fortunately, the number of counts detected in these voxels only represents around 0.002 ± 0.0003% of the total number of detected counts, in the voxel-based simulation, so its effects are negligible.
Figure 30 Three images of the Jaszczak phantom in the XY-plane (a) is a CT image of the three inserts, from the Source-CT stack. (b) is again a CT image of the three inserts taken from the Phantom-CT stack and (c) is a visualisation of the Voxel-based Jaszczak phantom from the front of the phantom. It was not possible to visualise from within the phantom, to capture the voxelised inserts, in this case.
Three plots presenting the distribution of the source events in the ZY, XZ and XY plane respectively. The plots, ultimately demonstrate how well the sources in the voxel-based model have been aligned to the position of the inserts in the experimental data.

Table 10  The delta X, Y and Z positions of each of the voxel-based inserts with respect to each of the geometric-based inserts [1 voxel = 4.418 x 4.418 x 4.418 mm]

<table>
<thead>
<tr>
<th>Insert ID</th>
<th>ΔX [voxels]</th>
<th>ΔY [voxels]</th>
<th>ΔZ [voxels]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insert 1</td>
<td>0 ±1</td>
<td>-1 ±1</td>
<td>0 ±1</td>
</tr>
<tr>
<td>Insert 2</td>
<td>-3 ±1</td>
<td>0 ±1</td>
<td>+2 ±1</td>
</tr>
<tr>
<td>Insert 3</td>
<td>-2 ±1</td>
<td>-2 ±1</td>
<td>+2 ±1</td>
</tr>
</tbody>
</table>
4.1.4 Energy Spectra

4.1.4.1 PHA Energy Spectra

Pulse Height Analysis, or PHA, energy spectra, for both the voxel-based and geometric based phantoms are presented in Figure 32 and Figure 33 respectively. Both data sets have been plotted with the experimental PHA data as a comparison; please note, that for an ideal validation, both datasets should be identical. Both the geometric-based and voxel-based simulations detect more photons overall and detect more x-rays; this is particularly observed for the voxel-based total energy spectrum. In Figure 33, the intensity of the low energy photopeak (x-ray peak) is particularly large in contrast to the same peak in the PHA spectrum. This indicates that either more Compton scattering of the 113 keV gamma rays or more x-rays being produced in the voxel-based phantom simulation. Most importantly, what is also observed, is that gamma-rays and x-rays associated with the nuclear disintegration of $^{177}$Lu have been detected with energies described in section 2.3.1.2.

A quantified analysis could be applied to the subsequent presented spectra, to assess the intensities and widths of the photopeaks, and compare them against the Jaszczak energy spectra. This has not been implemented due to insufficient time.
Figure 32 The energy spectrum from a PHA scan simulation using the geometric-based phantom model in red and the experimental PHA energy spectrum in black. The geometric-based dataset was scaled to the experimental dataset.

Figure 33 The energy spectrum from a PHA scan simulation using the voxel-based phantom model in red and the experimental PHA energy spectrum in blue. The voxel-based data was scaled to the experimental dataset.
4.1.4.2 Individual Source Energy Spectra

Figure 34 contains three spectra, each detailing the energy of events detected from the three inserts of the geometric-based Jaszczak phantom model. The black spectrum presents events which originated from insert 1, the red spectrum presents the energy of events originating from insert 2 and the green spectrum details the events originating from insert 3. Also included is a CT-image of the phantom and its inserts which have been coloured, according to the legend, to aid the identification of the three inserts. An identical plot, for the voxel-based model has been presented, in Figure 35, notice in this plot that more events are detected from inserts 2 and 3 in comparison to insert 1. The converse can be said for the geometric-based spectra in Figure 34, where there are more events detected for insert 1 than the other two inserts.

Please note that from this point, only results that were obtained as a result of a correct alignment (insert one) are going to be presented, because results including the other two inserts have previously only provided results which are invalid.
Figure 34 Three energy spectra of events originating from inserts one, two and three, obtained using the geometric-based phantom model.

Figure 35 Three energy spectra of events originating from inserts one, two and three, obtained using the voxel-based phantom model.
4.1.5 Scattering Results

Section 4.1.5 provides different types of results pertaining to the Compton scattering of photons in both of the models being compared. These results include the degree of Compton scattering, the location of scattered photons and the number of counts in each 2D projection as a function of head position. These are acquired by using the ROOT methods outlined in section 3.5.2.3. By comparing the voxel-based results against the geometric-based equivalents, it is possible to determine if the methods proposed in section 3.3.4, have been successful in defining accurate material attenuation characteristics for materials within the voxel-based phantom geometry. This is because the analysis of Compton scattering results yields valuable information pertaining to material characteristics such as density, atomic number and the linear attenuation coefficient, all of which are defined in section 2.4.2.3.

4.1.5.1 Energy Spectra with varying degrees of scatter

Figure 36 and Figure 37 present the simulated total energy spectra and the energy spectra for events with varying degrees of scatter for the geometry-based and voxel-based models respectively, by plotting the ROOT objects as outlined in section 3.5.2.3.2. By comparing each constituent spectrum, between the techniques, it is possible to determine how the scattering of photons in the voxel-based phantom differs to the scattering in the geometric-based model. Please see the legend for the colouring of the aforementioned spectra. Table 11 details the ratio of events scattered by degree of Compton scattering to the total number of events detected. A good agreement is observed between the two datasets, but there seems to be a larger number of events Compton scattered more than four times and Compton scattered in total; this is also reflected in Table 11.
Figure 36 Energy spectra acquired from simulations using the geometry-based method with varying degrees of Compton scattering. A total energy spectrum (coloured red) is included for reference and the other spectra have been coloured according to the legend included on the histogram.

Figure 37 Energy spectra acquired from simulations using the voxel-based method with varying degrees of Compton scattering. A total energy spectrum (coloured red) is included for reference and the other spectra have been coloured according to the legend included on the histogram.
Table 11 details the number of events detected, varying with the number of Compton interactions that the events have undergone.

<table>
<thead>
<tr>
<th>Degree of Compton Scatter</th>
<th>Ratio to Total Number of Events (Geometry-based)</th>
<th>Ratio to Total Number of Events (Voxel-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Events</td>
<td>2.049E+06 ± 0.001E+06</td>
<td>2.412E+06 ± 0.002E+06</td>
</tr>
<tr>
<td>0</td>
<td>0.31 ± 0.04</td>
<td>0.24 ± 0.03</td>
</tr>
<tr>
<td>1</td>
<td>0.24 ± 0.03</td>
<td>0.23 ± 0.03</td>
</tr>
<tr>
<td>2</td>
<td>0.17 ± 0.03</td>
<td>0.18 ± 0.03</td>
</tr>
<tr>
<td>3</td>
<td>0.11 ± 0.02</td>
<td>0.12 ± 0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.07 ± 0.02</td>
<td>0.09 ± 0.02</td>
</tr>
<tr>
<td>&gt;4</td>
<td>0.11 ± 0.02</td>
<td>0.16 ± 0.03</td>
</tr>
<tr>
<td>&gt;0</td>
<td>0.69 ± 0.06</td>
<td>0.76 ± 0.07</td>
</tr>
</tbody>
</table>

4.1.5.2 Location of Scattered Events

The bar charts in Figure 38, detail the percentage and the location of events that have been Compton scattered in the main volumes of GATE simulated world. The method outlined in section 3.5.2.3.1 has been followed and the events summed for the detectors, the phantom, the patient bed (also known as the patient table) and the bar charts also detail the number of events not scattered. Ideally, between the voxel-based and geometric-based techniques, the same number of events would be scattered, or not scattered at all, in each of the component volumes. This would imply that the materials, defined in the two types of phantom, had the same attenuating characteristics. This however is not the case, there are more photons scattered in the voxel-based phantom volume than the geometry-based phantom equivalent volume and therefore less photons unscattered overall.
Figure 38 A comparison of the percentage of scattered events in the main components of both the Voxel-Based (a) and Geometric-Based Simulation (b).
4.1.5.3 Number of Counts versus Projection Number

The number of counts detected as a function of detector position, has only been carried out for insert 1, because between the three inserts, its source distribution was the only one aligned correctly to the phantom-CT.

4.1.5.4 Number of Events versus Projection Number- Gated by Insert 1 Events

The number of events versus projection number for events originating from insert 1 has been included in the results to illustrate that given a correct alignment of the CT volumes, the results show very good agreement. Figure 39(a) and Figure 39(b) detail the number of events that originated from the insert 1 source distribution and detected by head 1, for each of the 30 runIDs (30 positions) for the geometry-based and voxel-based simulations, respectively. Figure 39(c) and Figure 39(d) detail the same information, but for head 2.

A comparison of Figure 39(a) and (b) indicates an excellent agreement, with the general shape of the two histograms being very similar, but with the main difference being that the voxel-based detectors detect more events in every projection. The initial number of events detected by the voxel-based detector 1 is around 20,000 but for the geometry-based detector, the initial number of events is around 19,000. For projections 20-30 in the voxel-based dataset, when detector 1 traverses below the patient bed, the number of events reduces to ~6500 per projection, whereas for these same projections but in the geometry-based simulation 5500 are detected. For Figure 39(c) and (d), the same trend is observed, in that head 2 of the voxel-based simulation detects more photons overall in all of the projections.
Figure 39 A histogram of the number of counts in each projection, of events originating from Insert 1, and detected by detector 1 in plots (a) and (b) and detector 2 in (c) and (d). Plots (a) and (c) have been acquired from the geometric-based simulation and (b) and (d) from the voxel-based simulation.
4.1.6 Projections and Reconstructed Data

4.1.6.1 Projections

The results presented in this section are particularly important, as they confirm that GATE has accurately modelled a SPECT scanner which has acquired 2D gamma distributions of a voxel-based source from sixty positions about a voxel-based phantom, as described in section 2.1.2. The sixty projections, acquired from the simulation are primarily presented and then the reconstructed 3D data follows subsequently.

4.1.6.1.1 EM1 Window Projections

Figure 40(a) and Figure 40(b) present the sixty projections for the experimental and voxel-based datasets respectively. Note that projections 1 to 30 are events detected by detector 1 and projections 31 to 60 are events detected by detector 2. From a visual comparison of both the voxel-based and experimental projections, both figures appear to be similar, with each projection showing three sources of gamma radiation and also each projection showing the same perspective of the sources as the detector rotates about the phantom. The main difference observed is that two sources in projection 1 of the voxel-based dataset appear to be particularly close together and shifted in the x-direction, this can explained by the rotation in the phantom during the CT scan, as previously stipulated in section 4.1.3.

4.1.6.1.2 EM2 Window Projections.

In an identical manner to section 4.1.6.1, the projections presented in Figure 41 are the 2D gamma distributions of the EM2 window events. Again, the same perspective of the three sources can be seen in each projection for both the voxel-based and experimental data. The same shift in the gamma events in the ZY plane is also observed in the voxel-based projections.
Figure 40 Presents the Sixty 2D gamma distributions of the EM1 window events in the XZ plane from the experimental dataset in (a) and the voxel-based dataset in (b).
Figure 41 Presents the Sixty 2D gamma distributions of the EM2 window events in the XZ plane from the experimental dataset in (a) and the voxel-based dataset in (b).
4.1.6.2 Reconstructed Data

As discussed in section 2.1.2, the projections are manipulated by using a tomographic reconstruction algorithm to combine the 2D gamma distributions and form the 3D reconstructed images shown in Figure 42 and Figure 43. Figure 42 shows the reconstructed gamma distributions for the EM1 data and Figure 43 presents the EM2 dataset. Both figures present the reconstructed datasets, alone, in images (a) and (b) and then the reconstructed images (a) and (b) are overlaid onto the phantom-CT slice to produce images (c) and (d). Both the EM1 and EM2 reconstructions for the voxel-based gamma distributions, in Figure 42(a) and Figure 43(a) resemble the experimental EM1 and EM2 gamma distributions in Figure 42(b) and Figure 43(b) respectively.
Figure 42 The reconstructed 3D images yielded from the EM1 projections after a reconstruction algorithm had been applied to the 2D gamma distributions for both the voxel-based and the experimental data sets in (a) and (b) respectively. Also included are the same two datasets overlaid on a phantom-CT slice to provide a reference of the location that the source distribution was positioned.
Figure 43 The reconstructed 3D images yielded from the EM2 projections after a reconstruction algorithm had been applied to the 2D gamma distributions for both the voxel-based and the experimental data sets in (a) and (b) respectively. Also included are the same two datasets overlaid on a phantom-CT slice to provide a reference of the location that the source distribution was positioned.
4.2 Anthropomorphic Results

The final component of this MSc thesis was to extend the voxel-based methods outlined in section 3.0, to simulate a voxel-based anthropomorphic phantom geometry in GATE. The same results as those presented for the Jaszczak phantom, which are detailed below, have been generated to prove that this method is valid for its extension to arbitrary geometries. The results will also be compared to the Jaszczak phantom results, where valid, to provide insights:-

1) Visualisations of the voxel-based anthropomorphic phantom geometry in GATE.
2) Energy Spectra (Total and Individual Insert).
3) The scattering properties of the phantom geometry assigned materials.
4) Projections and Reconstructed Data.

4.2.1 Experimental Set-up

The voxel-based anthropomorphic phantom geometry and source distribution inputs were created in GATE, using the methods proposed in section 3.3, with an activity distribution which simulated a total of 38 MBq of Lutetium-177, distributed as stipulated in section 3.3.5.3, across the water, liver and the liver insert. These inputs were then combined with the SPECT Tomographic GATE simulation input (information detailed in section 3.2) and simulated for a total scan time of 1200 seconds. Due to time constraints, only 26.47% of the total radioactivity proposed in section 3.3.5.3 was simulated. Given the limited results for comparison, conclusions on the scientific accuracy of the model must be drawn from the Jaszczak voxel-based model results, where applicable, and the experimental data.

4.2.2 Geometry of the anthropomorphic phantom

A visualisation of the voxel-based geometry in GATE provides an indication of the location of the phantom in the GATE simulated world and it also provides a preliminary outlook of how well the phantom has been constructed on a material and voxel basis from the method proposed in section 3.3.4. The reader is referred to Table 6 for the materials assigned to the anthropomorphic phantom, for this analysis and Table 12 for how GATE represents the materials of the phantom as colours.

In Figure 44 there are three images presenting the visualisation of the voxel-based anthropomorphic phantom geometry- Figure 44(a) viewed from the front, Figure 44(b) from the side and the third, Figure 44(c), at polar and azimuthal angle of 30° and 60°, respectively.
In these images, the model has been visualised such that all materials are visible. Firstly, it can be seen that the methods outlined in 3.3.4 have been successful in constructing the phantom as a series of small 3D discrete volumes and when comparing Figure 44 to Figure 23(b) it appears to be a good representation of the phantom.

With reference to the allocation of materials from Table 6 to the phantom geometry, in Figure 44(a), (b) and (c), it appears as though the majority of the voxels on the outer exterior of the phantom have been correctly allocated plexiglass characteristics, but there do appear to be a large amount of water voxels. A comparison of the volume of water actually captured in the region of HU values (for the method see section 3.3.4.1) showed that there was too great a volume of water defined, which was 8364 cm$^3$, the actual volume of water in this phantom was 7968 cm$^3$. It is also noted that materials have been allocated to voxels below the phantom, in the region of the patient bed. This was because this region in the phantom-CT volume (see section 3.3.3.1 for reference) was not cropped out and was originally going to be modelled. Due to optimisations, as described in sections 3.3.4.3 and 3.3.4.1 it was not possible to do so and the geometric-based patient bed was used.
Figure 44 Three visualisations of the Anthropomorphic phantom, viewed from three different angles and in this case all materials have been visualised. Note that the two detector heads and patient bed, of the SPECT scanner is also visible, defined by the geometric-based method.
4.2.4 Source-CT and Phantom-CT Stacks

4.2.4.1 Source-CT Stack

Figure 45(a) and Figure 45(b), present two slices of the source-CT volume, acquired using the CT acquisition method outlined in section 3.3.3.2 and the first of the two, Figure 45(a), presents the slice which corresponds to a position through the liver and liver insert. Notice the differing contrasts, which correspond to volumes of differing density and therefore HU value, between the liver insert, liver volume and the water. Figure 45(b) presents the slice corresponding to a position through the lungs, near the top of the phantom, and the varying HU values in the lungs due to pockets of air. These differing contrasts in the liver, liver insert and water, in combination with Table 7, indicate that the radiocontrast agent, inserted into these volumes of interest, as stipulated in Table 3 was an adequate method to determine regions of HU values to allow the methods in section 3.3.5 to be implemented.

![CT Slice](image)

**Figure 45** Two slices from the source-CT stack, the first, in (a), depicts the liver and its insert, the second, (b), is the slice containing the lungs. The spinal insert at the bottom of the phantom and the patient bed are also visible in both CT slices. Note in (a), the varying contrast of each phantom constituent volume, particularly the difference in contrast between the insert, liver volume and water.
4.2.4.2 Phantom-CT Stack

Figure 46 (a) and Figure 46(b) presents the CT stack acquired as a result of implementing the CT acquisition method stated in section 3.3.3.1. The volumes of interest in the CT images look identical in contrast, which is correct given the fact that the iodine radiocontrast was only inserted into the source-CT stack volumes. These volumes were filled with a purified water, which meant that all the voxels that corresponded to water had the same HU value and therefore the same contrast in the CT image. The theory regarding HU values and CT data acquisition is explained in section 2.2.2. This process was intentional to allow the materials, defined in Table 6, to be successfully defined.

(a)  
(b)  

Figure 46 Two slices from the phantom-CT stack, the first, in (a), depicts the volumes of interest for the anthropomorphic phantom which are liver, liver insert and water volume. The second, image (b), is the slice containing the lungs. The spinal insert at the bottom of the phantom and the patient bed are also visible in both CT slices.
4.2.5 Total Energy Spectrum

Presented below, in Figure 47, is a histogram of all the events detected from the anthropomorphic GATE simulation. This is generated by implementing the method outlined in section 3.5.2.2.1. This energy spectrum demonstrates that the gamma-rays and x-rays associated with the nuclear disintegration of $^{177}$Lu have been correctly defined, with energies described in section 2.3.1.2. A simple comparison against the previously validated Jaszczał phantom total energy spectrum (Figure 36) indicates that the low energy photopeak appears to be intense, indicating either lots of scattering of the 113 keV gamma rays or an abundance of generated x-rays. A quantified analysis could be applied to this spectrum, to assess the intensities and widths of the photopeaks, and compare them against the Jaszczał energy spectra.

![Graph showing energy spectrum](image)

*Figure 47 A histogram of the events detected from the 38MBq GATE simulation of the anthropomorphic phantom. Photopeaks are observed corresponding to gamma ray energies of 113 keV and 208 keV, an x-ray photopeak can also be seen corresponding to the 55.8 keV x-ray.*
4.2.6 Scattering Results

4.2.6.1 Energy Spectra with varying degrees of scatter

Figure 48 presents the simulated total energy spectrum and the simulated energy spectra for events with varying degrees of scatter. These energy spectra have been plot by implementing the methodology outlined in section 3.5.2.3.2 and given that there is no geometric-based equivalent dataset for this phantom a comparison analysis, as was presented for the Jaszczak phantom in Table 11, is not possible. Table 13, details the ratio of events scattered by degree of Compton scattering to the total number of events detected. The largest ratios are associated with no Compton interactions and one Compton scattered events, with 0.271±0.04 and 0.23±0.04 respectively. Also, take note of the unusually high ratio of events Compton scattered more than four times, at 0.14±0.03, this ratio is nearly as high as Compton scattered twice, which is 0.17±0.03, and greater than the ratio associated with Compton scattered three times. This is slightly unusual and might signify incorrect material attenuation characteristics.

Figure 48 Energy spectra acquired from simulations using the voxel-based method with varying degrees of Compton scattering. A total energy spectrum (coloured red) is included for reference.
Table 13 details the number of events detected, varying with the number of Compton interactions that the events have undergone for the anthropomorphic GATE simulation

<table>
<thead>
<tr>
<th>Degree of Compton Scatter</th>
<th>Ratio to Total Number of Events (Voxel-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Events</td>
<td>1.364E+06 ± 0.001E+06</td>
</tr>
<tr>
<td>0</td>
<td>0.27 ± 0.04</td>
</tr>
<tr>
<td>1</td>
<td>0.23 ± 0.04</td>
</tr>
<tr>
<td>2</td>
<td>0.17 ± 0.03</td>
</tr>
<tr>
<td>3</td>
<td>0.11 ± 0.03</td>
</tr>
<tr>
<td>4</td>
<td>0.08 ± 0.02</td>
</tr>
<tr>
<td>&gt;4</td>
<td>0.14 ± 0.03</td>
</tr>
<tr>
<td>&gt;0</td>
<td>0.73 ± 0.07</td>
</tr>
</tbody>
</table>
4.2.6.2 Location of Scattered Events

The bar chart presented in Figure 49 details the percentage and the location of events that have been Compton scattered in the main volumes of GATE simulated world. The method outlined in section 3.5.2.3.1 has been followed and the events summed for the detectors, the phantom, the patient bed (also known as the patient table); the bar charts also detail the number of events not scattered. The greatest number of events were scattered in the phantom, with 65.74% of the total number of events being scattered in this volume and around 27% of the total are not scattered at all. With respect to the scattering volumes, the patient bed scatters the second most photons, with 3.84% of the total number of photons detected and the table follows closely with 3.32% of the total number.

![Bar chart showing percentage of scattered events in each component](chart.png)

**Figure 49** The percentage of scattered events in each of the main components of the voxel-based anthropomorphic phantom model.
4.2.6.3 Number of Counts vs. Projection Number Plot

Figure 50 presents a plot of the number of counts detected per projection, for both the EM1 (113keV) and EM2 (208 keV) energy windows using the method stipulated in section 3.6.1. The blue and yellow triangular points correspond to the EM1 and EM2 experimental data, respectively, and the green and red square points correspond to the EM1 and EM2 equivalent voxel-based simulated data. Please note, that the voxel-based data has been scaled up by a factor of 26.47, because the actual anthropomorphic phantom was imaged on the SPECT scanner at the Christie with ~1006 MBq.

This figure is the most important plot in the entire results as it provides indications on the alignment of the source distribution and also attenuation characteristics of the materials for every point in the phantom geometry. The excellent agreement, in both the EM1 and EM2 datasets, observed in this figure suggests that the CT volumes have been accurately aligned and also the attenuation characteristics have been defined accurately. The only difference observed is in the EM1 data, where there appear to be more events detected between projections 21 and 38, which indicates that there might be an issue with the geometric-based bed.
Figure 50 A histogram of the number of counts in each projection for the EM1 and EM2 energy windows. This has been presented for the voxel-based phantom simulation (scaled up by 26.47) and the experimental scan data. See legend for more information.
4.2.7 Projections and Reconstructed Data.

4.2.7.1 Projections

The results presented in this section are to confirm that the GATE Monte Carlo Simulation has modelled a SPECT scanner which has acquired 2D gamma distributions of a voxel-based source from sixty positions about a voxel-based anthropomorphic phantom, as described in section 2.1.2. The sixty projections, acquired from the simulation are primarily presented and then the reconstructed 3D data, follows subsequently.

4.2.7.1.1 EM1 Window Projections

Presented in this section are sixty projections which each contain the contribution of the EM1 window events to the overall 2D gamma distribution, for the experimental dataset in Figure 51(a) and the voxel-based dataset in Figure 51(b). From a visual comparison of both the voxel-based and experimental projections, both Figure 51(a) and Figure 51(b) show a good similarity, with each projection showing the same perspective of the three sources of gamma radiation, which appear to be located in the water, the liver and the liver insert. There also appears to be more events detected from the projections in Figure 51(a), the experimental projections, this observation is made because the difference in the liver to background (water) radiation appears more pronounced. The main difference between the two sets of projections arises due to the difference in the activity simulated in the voxel-based anthropomorphic GATE simulation to that scanned experimentally. Further research could be carried out with a region of interest analysis, which would provide further information regarding the intensities of each volume as stipulated in section 3.6.2.

4.2.7.1.2 EM2 Window Projections.

In an identical manner to section 4.2.7.1.1, the projections presented in this section are the 2D gamma distributions of the EM2 window events. Again, the same perspective of the three sources of gamma radiation can be seen in each projection for both the voxel-based and experimental data.
Figure 51 Presents the Sixty 2D gamma distributions of the EM1 window events in the XZ plane from the experimental dataset in (a) and the voxel-based dataset in (b).
Figure 52: Presents the Sixty 2D gamma distributions of the EM2 window events in the XZ plane from the experimental dataset in (a) and the voxel-based dataset in (b).
4.2.7.2 Reconstructed Data

As discussed in section 2.1.2, the projections are manipulated by using a tomographic reconstruction algorithm to combine the 2D gamma distributions and form the 3D reconstructed images shown in Figure 53 and Figure 54. Figure 53 shows the reconstructed gamma distributions for the EM1 data and Figure 54 presents the EM2 dataset. Both figures present the reconstructed datasets, alone, in images (a) and (b) and then the reconstructed images (a) and (b) are overlaid onto the phantom-CT slice to produce images (c) and (d). Both the EM1 and EM2 reconstructions for the voxel-based gamma distributions, in Figure 53(a) and Figure 54 (a) resemble the experimental EM1 and EM2 gamma distributions in Figure 53 (b) and Figure 54 (b) respectively.

![VOXEL-BASED](image1) ![VOXEL-BASED OVERLAID ON CT](image2)

![EXPERIMENTAL](image3) ![EXPERIMENTAL OVERLAID ON CT](image4)

Figure 53 The reconstructed 3D images yielded from the EM1 projections after a reconstruction algorithm had been applied to the 2D gamma distributions for both the voxel-based and the geometric data sets in (a) and (b) respectively. Also included are the same two datasets overlaid on a phantom-CT slice to provide a reference of the location that the source distribution was positioned.
The reconstructed 3D images yielded from the EM2 projections after a reconstruction algorithm had been applied to the 2D gamma distributions for both the voxel-based and the experimental data sets in (a) and (b) respectively. Also included are the same two datasets overlaid on a phantom-CT slice to provide a reference of the location that the source distribution was positioned.
5.0 Discussion

5.1 Discussion of the Voxel-Based Jaszczak Phantom Results.
In this section, the results which have provided significant insights will be discussed, assessing how well the methods, defined in section 3.3, for creating a SPECT scanner GATE simulation of a voxel-based phantom and source distribution have performed.

5.1.1 Visualisation of the Voxel-Based Jaszczak Phantom Geometry
From the visualisations of the SPECT scanner GATE simulation, Figure 29(a) shows that a voxel-based Jaszczak phantom geometry has been automatically generated using the method outlined in section 3.3.4 and positioned in a similar position, between the heads, as its geometric counterpart in Figure 29(b). When comparing this figure to Figure 29(b), it also appears as though the geometry is a more accurate representation of the phantom.

Analysing the materials assigned to the outershell of the phantom, it appears as though most of the voxels have been assigned plexiglass definitions, but there do appear to be some erroneous water voxels allocated on the outer exterior. Voxels assigned water and plexiglass characteristics should also not feature in a position that is not associated with the main body of the phantom. This however is not the case and incorrectly defined voxels feature below the phantom, in the voxels which should be defined as the materials associated with the patient bed. These voxels were intentionally retained, in the aim of optimising the regions of HU values for each material and their inclusion ensured that the entire volume of water and plexiglass had been captured during this procedure. This however is a flaw with this method, if the HU values for two or more materials are close, or overlap, it is very difficult to pin point a unique specific range of HU values for each material. One solution for this might be to manipulate the HU value, for the voxel, in IMAGEJ to a value which does not cause conflicting definitions. As a whole though, when considering the volume of water captured in the HU region defined for water (see Table 4), it seems as though this method works, because the volume captured was moderately accurate.

5.1.2 The Positioning of the source Distribution.
Figure 31(a), (b) and (c) firstly illustrate that a voxel-based three-insert source distribution has been automatically created from the generation of the GATE source distribution input defined in section 3.3.5. Unfortunately, these results (in conjunction with Table 10) indicate
that the voxel-based source distribution has been aligned poorly to the geometric-based equivalent and the only component of the voxel-based source distribution to have been aligned correctly, is the source of radioactivity associated with insert 1. It is seen multiple times throughout the results that the most important condition in correctly modelling the voxel-based Jaszczak phantom and corresponding source distribution is to ensure that the sources of radioactivity from the source-CT are positioned within the phantom-CT inserts.

One solution lies in the method proposed in section 3.3 in that it requires the acquisition of two separate CT stacks of the phantom. This provides limitations, because if there is a rotation in the phantom in either the source-CT or phantom-CT volume, it would be impossible to align the CTs in GATE. As was explained in the previous section, the HU values of the voxels corresponding to volumes of interest could be manipulated in an image processing software, to change their HU values to a unique range, therefore eliminating the need to acquire a CT scan of the phantom with the iodine radiocontrast (section 3.3.3.2). This would mean that only one CT image of the phantom would be required, and this could then be duplicated and aligned.

5.1.3 Scattering Results

5.1.3.1 Energy spectra and Location of Scatter Insights

The first component of the scattering results, which is provided in Figure 36 and Figure 37 present the simulated total energy spectra and their constituent spectra. As was explained in section 3.5.3.2.2 these spectra, in conjunction with Table 11, provide information on how much Compton scattering occurs in the phantom. A visual comparison of both of these figures indicates that there are some subtle differences between the two datasets, most notably that there are more events detected in the voxel-based simulation, an extra 17.5% in fact. It is also noted that in the voxel-based phantom simulation, there are a greater number of events Compton scattered at least once, and also more low energy events which are Compton scattered more-than four times before being detected.

Analysing the location of all scattered events in Figure 38(a) and (b), there is a greater percentage of the total number of events scattered in the voxel-based phantom than that of the geometric-based phantom. Given these Compton scattering trends and the location of the scattering, it is most likely that the materials in the voxel-based phantom have been defined with inaccurate attenuating characteristics. This means that the method proposed in section 3.3.4.1 has not accurately defined the materials within the phantom. The fact that there are
more events detected suggests that there is less effective attenuating material occupying the voxels, possibly indicating that materials have too low a density defined in the materials database (Table 5) and therefore too low a linear attenuation coefficient. If the material attenuating characteristics were defined with too high an attenuating coefficient, less photons would be detected and these photons would not scatter at all. This therefore means that the converse must be true, because more photons are detected and more photons are scattered.

It was noted in the energy spectrum discussion that there are a greater number of lower energy photons being detected, and this is explained by the scattering results already discussed. If there are more photons being Compton scattered multiple times, and still being detected, this implies that the photons are not reduced in energy sufficiently to be absorbed by the photoelectric effect but still of sufficient energy to be interacting by the Compton interaction. More Compton scattering means multiple losses of energy as they collide with atomic electrons and therefore more lower energy photons.

As previously discussed in section 4.1.2, visualisation of the components of the model, the geometric-based mattress was not included because its inclusion would result in a collision of the detector head with the table. Looking at Figure 38 and comparing the number of events scattered in table, (which includes the mattress in the geometry-based simulation) between the two techniques, only 1.05% of events are scattered in this volume, as opposed to the geometric-based equivalent where nearly 6% of the events are scattered. It is highly probable that non-inclusion of the mattress is accountable for this effect, but to include the entire table in the simulation, the cropping of the phantom-CT image in IMAGEJ, would be required such that the detector heads, defined in the tomographic input, would not collide with the CT volume.

5.1.3.2 Insights from the Number of Counts versus Projection Number Plot

With a correct alignment of a source distribution to its corresponding insert position in the phantom-CT, for example the insert 1 source of activity as previously mentioned, the number of events detected in each projection originating from the insert 1 source (Figure 39) agrees very well to the number detected in the geometric-based equivalent. This is because the events, originating from insert 1, propagate through the same quantity of attenuating material in both simulations. The only difference observed is that slightly more events are detected in each projection in the voxel-based GATE simulation than the geometric-based equivalent, which could be explained by the previous explanation regarding incorrect attenuating
material characteristics.

Given a misalignment of the CT volumes, a completely different trend was observed in the number of events being detected in each projection. This is because the sources of activity associated with inserts 2 and 3 were positioned too close to the side of the phantom, thus meaning that the photons emitted from these sources had less attenuating material to propagate through (in the phantom), before being detected. This also meant that there was less chance of the photons being scattered and/or absorbed within the water or plexiglass and it lead to more photons being detected overall from the voxel-based source distribution.

5.1.5 Projection and Reconstruction Results
The projections presented in Figure 40(a) and Figure 41(a), when compared to Figure 40(b) and Figure 41(b) of section 4.1.6, show that combining the voxel-based inputs created in section 3.3 with the SPECT camera simulation (information detailed in Table 2 and referred to in section 3.2) validly simulates a voxel-based Jaszczak phantom geometry and source distribution. These figures also show that the two source distributions are very similar, indicating that the method for defining the source distribution (section 3.3.5) is also valid. The only difference observed is that the distributions are not positioned in the same location and they are shifted to the right in each of the sixty projections, which corresponds to a shift in the negative X direction. This is again as a result of the misalignment of the two CT volumes, due to the rotation of the phantom during the CT scan acquisition process (section 3.3.3.2).

5.1.6 The Suitability of the voxel-based method in creating a Jaszczak phantom simulation
Given the above discussion, it can be summarised that the voxel-based method has successfully modelled a Jaszczak phantom, but only in the event that the source distribution has been aligned to the phantom geometry, which implies that the alignment of the CT volumes is crucial. A resolution has been proposed to modify the method in section 5.1.2, to ensure that a misalignment would not occur in the future. The only issue that remains unresolved is why the attenuation characteristics assigned to the phantom geometry are inaccurate. For clarity in this issue, it needs to be determined if the method itself is affecting the results, or if it is another factor.
5.2 Discussion of Anthropomorphic Voxel-Based Phantom Results.

As was with the previous discussion for the voxel-based Jaszczak phantom, the same components of the results will be discussed to assess how well the methods defined in section 3.3 have been applied to create a simulation of a much more complicated phantom geometry and source distribution. The only results not under discussion in this section are those presented in section 4.1.3, the position of its source distribution, because of its complex geometry.

5.2.1 Visualisation of the Voxel-Based Anthropomorphic Phantom Geometry

As with the discussion for the Jaszczak phantom, the visualisation of the GATE simulation in Figure 44 indicates that the method defined in section 3.3.4 has successfully constructed an anthropomorphic phantom from a series of voxels. When comparing this to Figure 23, two photographs of an anthropomorphic phantom, it also looks as though the phantom geometry has been modelled accurately, because even small features such as plastic screws and the lid have been modelled.

Analysing the materials assigned to the phantom, it appears as though most of the voxels have been assigned plexiglass material definitions, but there do appear to be quite a lot of water voxels assigned to the outer exterior. This indicates that the regions of HU values, for water, in Table 6, are inaccurate and the method proposed in defining these regions (section 3.3.4.1) is not sufficient. The volume of water captured in this region was 8364 cm$^3$ and this is perhaps marginally too large when compared to the actual volume of water which was 7958 cm$^3$. There are also quite a large number of water voxels assigned to the patient bed area of the geometry, which explains the greater volume of water overall in the voxel-based simulation. It was thought that the patient bed could be retained in the CT volume and voxelised, but upon determining the regions of HU values for each material, it was more difficult to define a unique range of HU values associated with the patient bed. Overall, it was more difficult to uniquely define regions for all of the materials, than it was during the implementation of the methods for the Jaszczak phantom. This could however be rectified by cropping the patient bed out of the phantom-CT volume entirely and changing the HU values of the voxels, as previously explained in the Jaszczak phantom discussion.

5.2.3 Scattering Results.

Analysis of Figure 48 and Table 13, provide information on how much Compton scattering
occurs in the phantom. Similarly to the Jaszczak phantom results, the ratio of events scattered at least four times is suspiciously large, at 0.14 ±0.03, when compared to the ratio for 2 Compton interactions, which is 0.17 ±0.03. This might indicate that the attenuation characteristics of the defined materials are marginally too low, because this was the trend seen in the Jaszczak phantom.

To investigate this possibility, the analysis of Figure 50, which presents the number of counts detected per projection, is required. As was with the discussion for the Jaszczak phantom, this plot is particularly important as it is indicative of the attenuating affects of the materials and also the alignment of the CT volumes, so will provide answers to the above uncertainties. Firstly, for both the voxel-based simulation and the experimental dataset, the EM1 and EM2 datasets agree very closely, unlike the Jaszczak phantom simulation, which indicates an accurate phantom-CT and source-CT alignment. This means that the photons originating from the correctly aligned source distribution propagate through the same materials and the same quantities of attenuating materials as the experimental photons. Given that the trends fit so closely, it seems as though the attenuating characteristics of the materials defined in the materials database (method and significance outlined in section 3.3.4) have been defined accurately and that the method works.

The only difference observed is for the EM1 data, where there are a greater number of 113keV photons being detected in projections 15 to 45, which correspond to the detector positions beneath the bed. A possible reason for this is that the full patient bed is needed for the voxel-based analysis; this includes the mattress and compressed mattress volumes, rather than only the carbon-fibre component.

5.1.6 The Suitability of the voxel-based method to the Jaszczak phantom Simulation

Given the anthropomorphic phantom simulation discussion and results, it can be summarised that the voxel-based method has again successfully modelled a geometrically complex phantom. In this case each of the methods proposed in section 3.3 has been successfully implemented, which was not the case for the voxel-based Jaszczak phantom simulation. With the results scaled-up, the number of counts per projection, Figure 50, fits very well, this plot implies that the alignment of the source distribution to the phantom geometry, has been
accurate, but it also indicates that the attenuation characteristics of the materials at each point in the material is mostly accurate.
6.0 Conclusions

The Jaszczak phantom results infer that the methods proposed have successfully modelled this phantom using voxel-based techniques in GATE. This conclusion is only valid with the condition that the two CT volumes, required for the definition of the geometry and source distribution, have been aligned accurately. A possible solution to this problem is to only acquire one CT image of the phantom, duplicate it and then manipulate the HU values associated with each volume of interest to a unique range in the source-CT volume. This eradicates the need for 2 acquisitions and also the iodine radioconstrast. The final issue with the results from this simulation was the attenuation characteristics of the materials assigned to the phantom geometry. These appeared to provide insufficient attenuation and more Compton scattering. For clarity in this issue, it needs to be determined if the method itself is affecting the results, or if it is another factor. Therefore a further investigation is required to ensure that this issue is avoided in the future.

Overall, for the anthropomorphic phantom simulation, it can be summarised that the voxel-based method can be applied to any arbitrary geometry. This can be stated because each of the methods proposed in section 3.3 has implemented successfully, which was not the case for the voxel-based Jaszczak phantom simulation. With the results scaled-up, Figure 50, the number of counts per projection fits very well. This plot implies that the alignment of the source distribution to the phantom geometry, has been accurate, but it also indicates that the attenuation characteristics of the materials at each point in the material is mostly accurate. The only inaccuracy is observed when the detectors traverse below the patient bed, because more counts are observed in these positions. Given this, perhaps a full-scale activity simulation could be implemented, with a complete patient bed inserted. This would confirm that the patient bed was to blame for the slight differences in results.

Overall, given the ambiguity surrounding the method proposed for generating attenuation characteristics of phantom materials, it is advised that further investigation is required to determine the factor that is causing uncertainties in the method proposed in section 3.3.4. Only with the further research proposed above and the matter resolved could simulations be generated for increasingly more complex geometries, with the ultimate aim of generating a patient geometry and source distribution.
7.0 References


