RANS Modelling for Compressible Turbulent Flows
Involving Shock Wave Boundary Layer Interactions

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Abstract
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RANS Modelling for Compressible Turbulent Flows Involving Shock Wave Boundary Layer Interactions
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The main objective of the thesis is to provide a detailed assessment of the performance of four types of Low Reynolds Number (LRN) Eddy Viscosity Models (EVM), widely used for industrial purposes, on flows featuring SWBLI, using experimental and direct numerical simulation data. Within this framework the two-equation linear $k-\varepsilon$ of Launder and Sharma (1974) (LS), the two-equation linear $k-\omega$ SST, the four-equation linear $\phi-f$ of Laurence et al. (2004) (PHIF) and the non-linear $k-\varepsilon$ scheme of Craft et al. (1996b,1999) (CLSa,b) have been selected for testing. As initial test cases supersonic 2D compression ramps and impinging shocks of different angles and Reynolds numbers of the incoming boundary layer have been selected. Additional test cases are then considered, including normal shock/isotropic turbulence interaction and an axisymmetric transonic bump, in order to examine the predictions of the selected models on a range of Mach numbers and shock structures.

For the purposes of this study the PHIF and CLSa,b models have been implemented in the open source CFD package OpenFOAM. Some results from validation studies of these models are presented, and some explorations are reported of certain modelled source terms in the $\varepsilon$-equation of the PHIF and CLSb models in compressible flows. Finally, before considering the main applications of the study, an examination is made of the performance of different solvers and numerical methods available in OpenFOAM for handling compressible flows with shocks.

The performance of the above models, is analysed with comparisons of wall-quantities (skin-friction and wall-pressure), velocity profiles and profiles of turbulent quantities (turbulent kinetic energy and Reynolds stresses) in locations throughout the SWBLI zones. All the selected models demonstrate a broadly consistent performance over the considered flow configurations, with the CLSb scheme generally giving some improvements in predictions over the other models. The role of Reynolds stress anisotropy in giving a better representation of the evolution of the boundary layer in these flows is discussed through the performance of the CLSb model. It is concluded that some of the main deficiencies of the selected models is the overestimation of the dissipation rate levels in the non-equilibrium regions of the flow and the underestimation of the amplification of Reynolds stress anisotropy, especially within the recirculation bubble of the flows. Additionally, the analysis of the performance of the considered EVM’s in a normal shock/isotropic turbulence interaction illustrates some drawbacks of the EVM formulation similar to the ones observed in normally-strained incompressible flows. Finally, a hybrid Detached Eddy Simulation (DES) approach is incorporated for the prediction of the transonic buffet around a wing.
Declaration

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Nomenclature

Latin Letters

\( a_{ij} \) - Anisotropic stress tensor

\( C_f \) - Skin-friction coefficient, \((=\tau_w/\frac{1}{2}\rho\infty U_\infty^2)\)

\( C_p \) - Specific heat coefficient for constant volume

\( C_v \) - Specific heat coefficient for constant volume

\( c \) - Speed of sound or chord of bump

\( \mathbf{d} \) - Vector between the centroids P and N

\( \mathbf{d}_{fN} \) - Vector between the centre of the face f and the centroid N

\( d_{ij}' \) - Turbulent diffusion rate in the Reynolds stress model

\( d_{ij}'' \) - Viscous diffusion rate in the Reynolds stress equation

\( E \) - Total energy in the instantaneous or Favre averaged N-S equations

\( e \) - Specific internal energy

\( F_f \) - Volumetric flux
Nomenclature

\( f \) - Auxiliary variable in the elliptic relaxation model

\( G_\delta \) - Filter function

\( H \) - Total enthalpy in the Favre averaged N-S equations

\( h \) - Specific enthalpy

\( \mathbf{I} \) - Identity tensor

\( k \) - Turbulent kinetic energy, Reynolds or Favre averaged

\( k_{SGS} \) - Sub-grid scale kinetic energy

\( L_m \) - Length scale in the elliptic relaxation model

\( l \) - Turbulent length-scale

\( l_o \) - Characteristic length of the energy containing large scales

\( M_t \) - Turbulent Mach number

\( P \) - Mean pressure

\( P_k \) - Production rate in the Reynolds or Favre averaged the \( k \) equation

\( p \) - Pressure

\( p' \) - Fluctuating pressure in terms of Reynolds averaging

\( P_{ij} \) - Production rate in the Reynolds stress equation

\( Pr_l \) - Laminar Prandtl number

\( Pr_t \) - Turbulent Prandtl number

\( \mathbf{q} \) - Heat-flux vector
Nomenclature

$q_i$ - ith component of the heat-flux vector

$q_{li}$ - ith component of the laminar heat-flux vector

$q_{tu}$ - ith component of the turbulent heat-flux vector

$R$ - Perfect gas constant

$r$ - Ratio of successive gradients for flux-limited schemes

$Re$ - Reynolds number

$Re_t$ - Turbulent Reynolds number

$S$ - Strain invariant

$S_f$ - Face area vector

$S_{ij}$ - Reynolds or Favre averaged strain-rate tensor

$s_{ij}$ - Strain-rate tensor

$S_\phi$ - Source term

$T$ - Temperature

$T_m$ - Time-scale in the elliptic relaxation model

$t$ - Time

$u$ - Velocity vector

$U_i$ - ith component of mean velocity

$u_i$ - ith velocity component (u,v,w)

$u'_i$ - ith component of fluctuating velocity in terms of Reynolds averaging
Nomenclature

\(u_o\) - Characteristic velocity of the energy containing large scales

\(u_r\) - Friction velocity

\(\overline{u_i'u_j'}\) - Reynolds stress tensor in terms of Reynolds averaging

\(\overline{\widetilde{u}_i'\widetilde{u}_j'}\) - Reynolds stress tensor in terms of Favre averaging

\(V_p\) - Volume of control volume

\(w_f\) - Weighting factor

\(x\) - Location vector

\(x\) - Coordinate direction, or direction aligned with the wall

\(x_I\) - Inviscid Impingement point

\(x_i\) - Coordinate direction \((x,y,z)\)

\(x_p\) - Location vector of the centroid of a CV

\(x_s\) - Separation point

\(x_r\) - Reattachment point

\(y\) - Coordinate direction or wall normal distance

\(y^+\) - Non-dimensional wall distance, \(= \frac{u_\tau y}{\nu}\)

Greek Letters

\(\Gamma_\phi\) - Diffusivity

\(\gamma\) - Specific heat ratio or blending factor
\( \Delta \) - Width of filter

\( \delta \) - Boundary layer thickness

\( \delta_{ij} \) - Kronecker’s delta

\( \varepsilon \) - Dissipation rate in the Reynolds or Favre averaged \( k \) equation

\( \varepsilon_{ij} \) - Viscous dissipation rate in the Reynolds stress equation

\( \varepsilon_{iso} \) - Isotropic eddy dissipation

\( \eta \) - Kolmogorov length scale

\( \theta \) - Momentum thickness, \( \int_0^\delta \frac{\bar{u}}{\rho_\infty} \frac{\bar{u}}{\bar{u}_\infty} \left( 1 - \frac{\bar{u}}{\bar{u}_\infty} \right) dy \)

\( \kappa \) - Von Karman constant

\( \lambda \) - Thermal conductivity

\( \mu \) - Dynamic viscosity

\( \mu_t \) - Dynamic eddy viscosity

\( \nu \) - Kinematic viscosity

\( \nu_{SGS} \) - Sub-grid scale viscosity

\( \nu_t \) - Kinematic eddy viscosity

\( \rho \) - Density

\( \sigma_{ij}, \sigma \) - Stress tensor

\( \tau_\eta \) - Kolmogorov time-scale

\( \tau_w \) - Wall shear stress
Nomenclature

$\phi$ - Normalised wall normal Reynolds stress in the elliptic relaxation model or transported variable in the general transport equation

$\phi_{ij}$ - Pressure-strain term in the Reynolds stress equation

$\Psi$ - Flux limiter

$\Omega$ - Vorticity invariant

$\Omega_{ij}$ - Vorticity tensor

$\omega \equiv \varepsilon/k$

Subscript

f - Face value

P,N - Value at the centroids P,N

w - Value at the wall

0 - Value at the location of the incoming boundary layer

$\infty$ - Freestream value

Superscripts

imd - Intermediate value

init - Initial value

n - New time level
Nomenclature

- $o$ - Old time level
- $T$ - Transpose
- $'$ - Fluctuating part in terms of Reynolds averaging
- $''$ - Fluctuating part in terms of Favre averaging
- $\overline{}$ - Reynolds averaged value
- $\tilde{}$ - Favre averaged value

Acronyms

- APG - Adverse Pressure Gradient
- BD - Blended Differencing
- BSL - Base Line (model)
- CD - Central Differencing
- EVM - Eddy Viscosity Model
- CFD - Computational Fluid Dynamics
- CFL - Courant Friedrichs Lewy (condition)
- CLS - Craft Launder Suga (model)
- CPU - Central Processing Unit
- CV - Control Volume
- DNS - Direct Numerical Simulation
**Nomenclature**

FS - Flux Splitting (scheme)

GGDH - Generalised Gradient Diffusion Hypothesis

HO - High Order

JL - Jones Launder (model)

KNP - Kurganov Noelle Petrova (scheme)

KT - Kurganov Tadmor (scheme)

LES - Large Eddy Simulation

LRN - Low Reynolds Number

LRR - Launder Reece Rodi (model)

LS - Launder Sharma (model)

MLH - Mixing Length Hypothesis

NLEVM - Non-Linear Eddy Viscosity Model

N-S - Navier-Stokes

PHIF - $\phi - f$ (model)

PISO - Pressure-Implicit solution by Splitting of Operators

RANS - Reynolds Averaged Navier-Stokes

RHS - Right Hand Side

RSTM - Reynolds Stress Transport Model

SSG - Speziale Sarkar Gatski (model)
Nomenclature

SST - Shear Stress Transport (model)

SWBLI - Shock Wave Boundary Layer Interaction

TCL - Two Component Limit (model)

TVD - Total Variation Diminishing

UD - Upwind Differencing
Chapter 1

Introduction

In a range of applications in the aerospace and energy industries Shock Wave Boundary Layer Interaction (SWBLI) is critical for the effective aerodynamic and thermodynamic design. Some examples of such applications are transonic airfoils and turbines, supersonic intakes of jet engines, rocket nozzles, jet thrust vectoring, where SWBLI’s affect the performance of such machines. One example of the effects of SWBLI is the boundary layer separation usually induced by the Adverse Pressure Gradient (APG) imposed on the flow by shocks, which can lead to an increase of the internal machine losses and the rise of drag force for blades and wings. Additionally, shocks increase turbulent mixing with direct effects on the heat transfer rates, which can lead to thermal fatigue of structures. A further feature is the shock unsteadiness coming from interactions with boundary layers, which can result in pressure fluctuations affecting the aerelasticity and structural fatigue of machines. Therefore, the effective prediction of the flow physics associated with SWBLI is of high importance for the performance of high speed air vehicles and internal engines.

Computational Fluid Dynamics (CFD) is an economical tool for the designers in
industry. However, the turbulent flows typically encountered in the applications mentioned above mean that it is important to employ appropriate turbulence models. The high Reynolds number of many of such applications means that industrial simulations generally employ the Reynolds Averaged Navier-Stokes (RANS) equations, with the need to use suitable models for the turbulent Reynolds stresses. A wide variety of RANS turbulence models is available in literature, ranging from zero-equation Eddy Viscosity Models (EVM) to full Reynolds Stress Transport Models (RSTM). A well established class of RANS models in industry is the one/two equation EVM’s, due to their relatively low computational resource requirements, their numerical stability and their relatively acceptable levels of accuracy in a range of flows.

Many of the simpler RANS models have been developed principally based on assumptions associated with simple flows (e.g. isotropic turbulence, channel flows). Consequently, appropriate experimental data on flows through more complex geometries are necessary for the validation of such RANS models in flow configurations closer to the ones encountered in industrial applications. In the case of flows involving SWBLI, a variety of experimental databases are described in, for example, Settles and Dodson (1994) and Roy and Blottner (2006) including studies on for e.g. compression ramps, impinging shocks and three dimensional fins flows. A number of previous studies [see for example Wilcox (1990), Zha and Knight (1996), Rizzetta (1998), Liou et al. (2000), Gerolymos et al.(2004), Brown (2013)] have tested a range of RANS models ranging from one-equation EVM’s to RSTM’s, demonstrating different levels of accuracy among the models in predicting features such as the size of recirculation bubbles, the shock location and the wall heat transfer. The precise conclusions that can be drawn on the evaluation of the performance of the RANS models in the above studies can be restricted by the accuracy of the experimental
measurements, especially of near wall quantities and turbulent quantities. For example the Reynolds stresses measured by Kuntz et al. (1987) and Smits and Muck (1987) in a compression ramp case with nominally the same flow conditions differ by a factor of $\approx 4$. Another parameter that introduces uncertainty in the experimental measurements is the effect of the side-walls used in the experimental set up on SWBLI’s (Reda and Murphy, 1973), and the possibility of consequent three-dimensional features. Recent Direct Numerical Simulation (DNS) studies (e.g. Wu and Martin, 2007; Pirozzoli and Grasso, 2006) on compression ramp and impinging shock configurations provide data, including Reynolds stress profiles and turbulent kinetic energy budgets, appropriate for an elaborate examination of the performance of RANS models in predicting SWBLI. These can, in principle, avoid some of the above limitations of the experimental measurements. However, accurate such simulations do require significant computational resources, and specialised numerical methods and are restricted to fairly low-Reynolds-number cases. As a result, as will be seen later, there can also be issues in using some of these data sets for detailed comparisons. Furthermore, the industrial needs today focus on simulation of the unsteady shock-wave boundary-layer interaction (SWBLI) in aerodynamics, in respect of the new generation of aircraft design, including instability development in cruise speed. In this context, hybrid-RANS-LES methods have been developed in the last decade and they start to be currently used in the aeronautics industry (cf European research programs FLOMANIA, DESIDER, ATAAC, UFAST, TFAST. For these reasons, the development and validation of reliable RANS modelling is crucial, in order to enable the hybrid RANS-LES modelling to accurately predict the near-wall region and the amplification of instabilities related with the compressibility effects, such as the buffet in transonic speeds and in general, the unsteady SWBLI in transonic and supersonic speeds. In this context, the present thesis will examine the ability of adapted URANS
methodology in capturing the SWBLI instability as well as a recent hybrid RANS-LES method, the Delayed Detached Eddy Simulation, DDES, for the prediction of the buffet around a wing.

1.1 Study Objectives

The objective of this project is to assess the performance of four types of Low Reynolds Number (LRN) EVM’s, widely used for industrial purposes, including both linear and non-linear varieties, on flows featuring SWBLI, using experimental and DNS data. Within this framework the two-equation linear $k - \varepsilon$ of Launder and Sharma (1974) LS, the two-equation linear $k - \omega$ SST, the four-equation linear $\phi - f$ of Laurence et al. (2004) and the non-linear $k - \varepsilon$ scheme of Craft et al. (1996b, 1999) (CLS$a,b$) have been selected for testing. One reason for the choice of these schemes is to explore the efficiency and accuracy of the different methods they employ to handle near-wall turbulence. Additionally, the use of the CLS model will give evidence of the effect of the resolution of Reynolds stress anisotropy on predicting SWBLIs. Supersonic 2D compression ramps and impinging shocks of different angles and Reynolds numbers of the incoming boundary layer have been selected as initial test cases. In this way the selected turbulence models are tested in predicting flow features, such as recirculation regions of different size, coming from the interaction of boundary layers with oblique and reflected shocks of different strength. The sensitivity of the models to Reynolds number effects (Ringuette et al., 2008) is also investigated. Additional test cases are then considered, including axisymmetric transonic bump and normal shock/isotropic turbulence interaction, in order to examine the predictions of the selected models on a range of Mach numbers and shock structures. Furthermore, the objective of the present thesis focuses on an adaptation of the RANS methodology to the unsteady
SWBLI simulation and to the capturing of the related predominant instability frequencies, as well as the investigation of the hybrid RANS-LES methodology in the context of DDES, by using efficient URANS approaches in the near-wall region, in order to predict the transonic buffet, the SWBLI unsteadiness, the near-wake vortex motion and the aerodynamic unsteady forces at high Reynolds numbers.

1.2 Thesis Outline

The thesis is organised as follows. In Chapter 2 some basic types of SWBLI’s are described. In Chapter 3 some basic concepts of turbulence and some different methods for the simulation of turbulence are presented. In Chapter 4 some of the basic types of RANS turbulence models are summarized beginning with their incompressible formulation and some basic features associated with turbulence modelling for compressible flows are also discussed. In Chapter 5 some of the basic types of the Delayed Detached-Eddy Simulation modelling are presented. In Chapter 6 the equations of the turbulence models selected for the investigations of this study are described. Some results from validation studies of the PHIF and CLS models, which have been implemented in OpenFOAM software during this project, are also presented. Furthermore, the source terms introduced in the $\varepsilon$-equations of the PHIF and CLS models to improve their predictions in the considered flow configurations are discussed. In Chapter 7 the finite volume method for discretizing the N-S equations is described and a variety of convection schemes for resolving flow discontinuities are presented. Furthermore, different types of solvers for handling compressible flows available in OpenFOAM are presented. The performance of a variety of solvers and numerical methods, described in Chapter 7, in a shock tube case is discussed in Chapter 8. In Chapters 9 to 12 the performance of the employed RANS models in the 2D
compression ramp, 2D impinging shock, axisymmetric transonic bump and normal shock/isotropic turbulence interaction cases is analysed. In Chapter 13 the three-dimensional calculation for the V2C laminar transonic airfoil using the DDES-SST model is presented. The main conclusions of the thesis are summarised in Chapter 14 and some suggestions for future work are also included.
Chapter 2

Shock Wave Boundary Layer Interactions

A range of different types of SWBLI’s are described in Mundell and Mabey (1986) and Dolling (2001), including SWBLI’s for example around transonic airfoils, supersonic compression ramps and reflected shocks.

2.1 Normal Shock/Isotropic Turbulence Interaction

One of the simplest cases commonly examined is the interaction of a normal shock with isotropic and homogeneous turbulence. This configuration has been examined in terms of Direct Numerical Simulation (DNS) by Lee et al. (1997) and Larsson and Lele (2009) for different shock strengths and turbulent Mach numbers of the incoming turbulence. This type of interaction results in the amplification of turbulent kinetic energy downstream the shock and the anisotropy of Reynolds stresses. These features
are enhanced with increasing shock strength and turbulent Mach number. Finally, both Kolmogorov and Taylor scales are decreased.

2.2 Transonic Flow Around Airfoils

Typical industrial applications involve the interaction of shock waves with turbulent boundary layers, which can result in flow separation, modification of the shock structure and shock unsteadiness. A flow configuration that has gained a lot of attention is the transonic flow around airfoils. Mundell and Mabey (1986) proposed a classification of SWBLI’s in transonic flows based on pressure measurements around a NACA section. For constant freestream Mach number and variable angle of attack they suggested three types of transonic SWBLI’s, shown in Figure 2.1. In the case of a low angle of attack, denoted as Type 1 in Figure 2.1, the formation of a weak shock wave results in a thickened attached boundary layer. Upstream of the interaction of Type 1, denoted as region 1 in Figure 2.1, pressure fluctuation levels are insignificant. Near the shock, in region 2, small-scale unsteadiness of low-frequency can be measured, which disappear further downstream in region 3. For an intermediate angle of attack, denoted as Type 2 interaction in Figure 2.1, the shock wave is strong enough to cause flow separation and a recirculation region is formulated at the shock foot. This type of interaction is characterized by large-scale unsteadiness of low frequency near the shock location, in region 2. Motions of high frequency may exist in region 3, whilst insignificant levels of unsteadiness may exist upstream of the shock, in region 1, and downstream of the separation bubble, in regions 4 and 5. For a high angle of attack, Type 3 interaction in Figure 2.1, the strong shock wave can cause the formulation of a separation bubble ranging from the shock foot to the trailing edge. This type of interaction is characterized by low-frequency large-scale unsteadiness over a large
area downstream of the flow separation. Near the trailing edge, in region 2A, high-frequency oscillations may be detected produced by the separated shear layer, whilst upstream of the shock, in region 1, pressure fluctuations are of low level. It should be mentioned that these types of interactions have been encountered for a constant angle of attack and variable freestream Mach number.

![Types of shock wave/boundary layer interactions](from Mundell and Mabey (1986).

The flow around a transonic airfoil can be highly unstable and this can result in a low-frequency self-sustained oscillation of the shock wave. This phenomenon is called transonic buffet and can happen for specific combinations of angles of attack and freestream Mach numbers. Experiments for the investigation of transonic buffet have been conducted by McDevitt et al. (1976), for a symmetric circular-arc airfoil measuring buffet frequencies \( \approx 190 \) Hz, by Lee et al. (1989), for the BGK No. 1 and WTEA II supercritical airfoils measuring buffet frequencies between 50 and 80 Hz, and by Jacquin et al. (2005, 2009), for the OAT15 supercritical airfoil detecting buffet frequencies \( \approx 70 \) Hz.
2.3 Compression Ramp

Another test case that has attracted a lot of attention is the supersonic compression ramp. This flow configuration has been studied experimentally by Settles et al. (1976) for ramp angles of 8, 16, 20 and 24°, freestream Mach number of 2.85 and Reynolds number based on the momentum thickness of the incoming boundary layer (Re$\theta$) of 1.7x10$^6$.

For the 8° compression ramp the flow remains attached and a shock is formulated at the corner from the deflection of the boundary layer at the ramp. For the 16° compression ramp the flow is under incipient separation conditions. For 20 and 24° ramp angles the adverse pressure gradient imposed by the shock is sufficient to cause flow separation and a recirculation region is developed. In this case, the flow is deflected by the separation bubble and a compression fan is developed intruding the boundary layer in the region of the flow separation. At the reattachment point an additional compression fan arises and a $\lambda$-shock is formulated, as shown in Figure 2.2.

Figure 2.2: Sketch of the compression ramp flow pattern.
Additionally, the velocity profiles downstream of the interaction are decelerated, with the inner region of the boundary layer being decelerated more than the outer part. Finally, the boundary layer gradually recovers to its equilibrium state. Ardonceau (1984) and Kuntz et al. (1987) measured streamwise and vertical turbulent intensities and Reynolds shear stress profiles using Laser Doppler Velocimetry (LDV) techniques for a range of ramp angles concluding that Reynolds stresses are amplified due to the interaction, a feature that is enhanced with increasing ramp angle. The unsteadiness of the shock for the 24° compression ramp has been studied experimentally by Dolling et al. (1983), Ringuette et al. (2009) and in terms of DNS by Wu and Martin (2007). These studies indicate that the shock oscillates at a low-frequency of $O(10^3)$ Hz.

### 2.4 Impinging Shock

A flow configuration that has been also studied extensively is the interaction of an impinging shock wave with a turbulent boundary layer. In this case an oblique shock, produced by an external source, impinges on a flat plate and reflects imposing an adverse pressure gradient on an incoming turbulent boundary layer. This case has been studied experimentally by Green (1970) at Mach 2.5, $Re_\theta$ of $4 \times 10^5$ and for deflection angles varying from 2° to 10.5°. Measurements included surface-pressure distributions, pitot surveys and flow visualizations using surface oil, schlieren, and shadowgraph techniques. For deflection angles up to 5°, the shock is weak, the boundary layer thickens and remains attached and the shock reflection is essentially inviscid. For higher deflection angles, the adverse pressure gradient imposed by the shock is sufficient enough to cause flow separation well upstream the nominal impingement point. In this case a separation bubble is formulated the flow is turned at the sep-
aration point, where a compression fan is developed intruding the boundary layer. Further away from the wall, the separation compression waves coalesce to form the principal reflected shock. At the maximum height of the separation bubble, where the incident shock meets the sonic line, the flow is turned towards the wall through an expansion fan and reattaches. Near the reattachment point a sequence of compression waves realigns the flow with the wall. Finally, downstream of the reattachment of the flow the boundary layer relaxes to its equilibrium state. A sketch of the flow pattern (taken from Delery and Marvin (1986)) is shown in Figure 2.3, where the separation and reattachment points are denoted by S and R respectively.

![Figure 2.3: Sketch of the impinging shock wave/turbulent boundary layer interaction flow pattern (from Delery and Marvin (1986)).](image)

Deleuze (1995) and Laurent (1996) investigated experimentally the incident reflected shock test case at Mach 2.3, $Re_\theta$ of 4808 and deflection angles varying from 7 to 9.5°, using hot-wire anemometry, laser Doppler velocimetry and particle image velocimetry methods. The experimental data showed a different amplification of the shear and normal Reynolds stresses in the interaction zone. Under these conditions a separation bubble is formulated. The reflected shock experiences a large scale low-frequency oscillation of $O(10^3)$ Hz. The spatial displacement of the shock foot is of
the order of the incoming boundary layer thickness. The experiments indicate that higher frequencies of $O(10^4)$ Hz are dominant within the interaction zone. The low-frequency oscillation of the reflected shock has been also studied experimentally by Dupont et al. (2004) and Dussage et al. (2006) and in terms of DNS by Pirozzoli and Grasso (2006) and Priebe et al. (2009) reaching similar conclusions.
Chapter 3

Turbulence Theory and Turbulence Modelling

3.1 Turbulence Theory

One of the phenomena encountered in SWBLI is turbulence. The main characteristic of a turbulent flow field is the irregular, unsteady and three dimensional velocity field. Reynolds (1883) defined a non-dimensional number that can be used to indicate when transition to turbulence will typically occur. The Reynolds number is defined as \( Re = \frac{U_c L_c}{\nu} \), where \( U_c \) and \( L_c \) are characteristic velocity and length scales of the flow. Above a certain Reynolds number, called critical, the small perturbations of the flowfield amplify and lead to the breakdown of the initially laminar flow to turbulence. Richardson (1922) introduced the idea that the turbulent flowfield can be seen as a superposition of vortical modes, called eddies, of different scales. He proposed that the large scale eddies are destabilized and break down to smaller eddies forming a cascade process of eddy-breakdown. Each eddy has a characteristic lengthscale \( l \), velocity scale
$u(l)$ and a time scale $\tau(l) = l/u(l)$. In this process energy transfer happens in the direction from the larger eddies to the smaller ones. Finally, the energy is dissipated at the smallest scales eddies to internal energy. Kolmogorov (1941) quantified the energy cascade of Richardson, making some assumptions. The first assumption he made was that as the energy is transferred in the cascade, the anisotropy of the larger scales is gradually lost and eventually the small eddies become statistically isotropic. The smallest range of scales is called the Kolmogorov dissipative range and is considered universal. The scales of this range come from the fact that since they are dominated by viscous effects, they characterised by the dissipation rate $\varepsilon$ and the kinematic molecular viscosity $\nu$, thus dimensional analysis gives,

\[
\eta = (\nu^3/\varepsilon)^{1/4} \quad (3.1)
\]

\[
u_\eta = (\varepsilon \nu)^{1/4} \quad (3.2)
\]

\[
\tau_\eta = (\nu/\varepsilon)^{1/2} \quad (3.3)
\]

where $\eta$, $u_\eta$ and $\tau_\eta$ are the length, velocity and time Kolmogorov scales. He additionally assumed that for sufficiently high Reynolds numbers there is a range of scales, where the break-up process is mainly inviscid. These scales are called the inertial scales and they transfer energy to the smaller eddies at a constant rate $\varepsilon$. This rate is expressed as

\[
\varepsilon \sim \frac{u_o^3}{l_o} \quad (3.4)
\]

where $u_o$ and $l_o$ are the characteristic velocity and size of the energy containing large scale modes. The second hypothesis is based on the assumption that $l_o$ is related to the size and geometry of the flow, and this then leads to the ratio $\eta/l_o$ scaling according to $Re^{-3/4}$, indicating that the Kolmogorov scales decrease as Re increases.
The conclusions drawn by the Kolmogorov theory are valid for flows with distinct inertial range, meaning that the Reynolds number is sufficiently high. Additionally, the flow should be free from solid boundaries, in order for the size $l_0$ of the large scales to be independent of the viscous effects. Examples of such flows are free shear flows.

In the case of wall-bounded flows the behaviour of the turbulent structures is different. Near the wall the flow quantities are dependent on the kinematic viscosity $\nu$ and the shear stress at the wall $\tau_w$. Again dimensional analysis gives the friction velocity scale $u_\tau$ and length scale $\delta_\nu$ defined for the near wall region as,

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$  \hspace{1cm} (3.5) \\
$$\delta_\nu = \frac{\nu}{u_\tau}$$  \hspace{1cm} (3.6)

For the examination of the different regions near the wall $u_\tau$ and $\delta_\nu$ are used to non-dimensionalise the different quantities, resulting in variables often referred to as being expressed in wall units. In this case, these quantities are expressed in wall units. The non-dimensional wall distance and velocity used for this characterization are

$$y^+ = \frac{y}{\delta_\nu} = \frac{u_\tau y}{\nu}$$  \hspace{1cm} (3.7) \\
$$u^+ = \frac{U}{u_\tau}$$  \hspace{1cm} (3.8)

The region where the viscous effects are influential is called the viscous sublayer. In the very near-wall laminar layer the velocity profile follows the law of the wall $u^+ = y^+$ and is considered universal. It extends from the wall to around $y^+ < 5$, and the physical thickness therefore decreases as the Reynolds number increases. In that region molecular dissipation is dominant, and thus this region is characterised
by the the Kolmogorov scales. The region extending to around \( y^+ \) of around 20-30 is called buffer layer, where both viscous and turbulence effects need to be accounted for. Outside the viscous sublayer, for \( y^+ > 30 \) and \( y/\delta < 0.3 \), the velocity profiles for simple boundary layers follow the log-law of the wall, initially expressed by von Karman (1930), who assumed local equilibrium of the turbulence field in that region. Local equilibrium is here taken to mean that the local production and dissipation rates of turbulent kinetic energy are equal. The log-law of the wall can be expressed as

\[
u^+ = \frac{1}{\kappa} \ln y^+ + B
\]

with the usually adopted values of \( \kappa = 0.41 \) (the von Karman constant) and \( B = 5.2 \).

The existence of such a log-law region in plane channel flow was demonstrated by the DNS data of Kim et al. (1987). Further away from the wall the eddies become independent of the viscosity and their size is determined by the geometry of the flow, as opposed to wall distance, and so the mean velocity here will depart from the log-law.

### 3.2 Turbulence Modelling

The equations that describe the physics of a continuum medium are the Navier-Stokes (N-S) equations. The differential formulation of these in tensor form \((i = 1, 2, 3)\) is:

Conservation of the mass:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0
\]
Conservation of the momentum:

\[ \frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} \quad (3.11) \]

Conservation of energy:

\[ \frac{\partial}{\partial t} \rho (e + \frac{1}{2} u_i u_i) + \frac{\partial}{\partial x_j} \left[ \rho u_j (h + \frac{1}{2} u_i u_i) \right] = \frac{\partial}{\partial x_j} (\sigma_{ij} u_i) - \frac{\partial q_j}{\partial x_j} \quad (3.12) \]

where \( \rho \) is the density, \( u_i \) is the \( i \)th component of the velocity vector, \( x_i \) is the \( i \)th component of the position vector, \( p \) is the pressure, \( e \) is the specific internal energy, and \( h = e - p/\rho \) is the specific enthalpy. Additionally, \( \sigma_{ij} \) is the stress tensor, which is of second order and symmetric, and \( q_j \) \( j \)th component of the heat-flux vector. For a Newtonian fluid the stress tensor relation is defined as \( \sigma_{ij} = \mu s_{ij} \), where \( s_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \) is the strain-rate tensor and \( \mu \) the dynamic viscosity.

In the case of a calorically perfect fluid, the equation of state is \( p = \rho RT \), where \( T \) is the temperature and \( R \) the perfect gas constant. Additionally, \( e = C_v T \) and \( h = C_p T \), where \( C_v \) and \( C_p \) are the specific heat coefficients for constant volume and pressure respectively. Finally, the heat-flux vector based on Fourier’s law is defined as \( q_j = -\frac{\mu}{Pr_l} \frac{\partial h}{\partial x_j} \), where \( Pr_l = \frac{C_p \mu}{\lambda} \) is the molecular Prandtl number and \( \lambda \) the thermal conductivity.

### 3.2.1 Direct Numerical Simulation (DNS)

This method resolves all the turbulent scales in time and space. Therefore, the N-S equations are solved directly without any modelling treatment and the only source of error comes from the method used for the numerical solution of the equations. The size of the computational box, \( H \), required in such a computation is determined by
the largest energy containing scales, and the mesh increment $DX$ by the Kolmogorov dissipative scales. For isotropic turbulence $H$ should be at least eight integral length scales $L_{11}$ (Pope, 2000). In order for the smallest scales of turbulence, $\eta$, to be resolved the mesh increment should follow $DX/\eta \approx 2.1$ (Pope, 2000). Therefore, the number of grid points along a specified direction of the mesh should be proportional to $Re_L^{3/4}$, where $L = \frac{k^{3/2}}{\varepsilon}$ is the turbulent length scale. Since turbulence is three dimensional the total amount of computational nodes required is proportional to $Re_L^{9/4}$. Additionally, the Courant number sets a limitation on the time step for specific mesh spacing. This means that the number of time steps is also proportional to $Re_L^{3/4}$. As a result the computational cost for DNS scales with $Re_L^3$, a fact that makes it currently impractical for industrial applications. DNS is used for research purposes and for developing reliable databases of low Reynolds number flows for the validation of the industrial RANS codes and models.

### 3.2.2 Large Eddy Simulation (LES)

This method solves for the time dependent flowfield, but resolving only the large energy containing turbulent scales. In this case the flow variables are decomposed in the filtered field, denoted with brackets, and the residual field, denoted with star, as

$$
\mathbf{u} = \langle \mathbf{u}(\mathbf{x}, t) \rangle + \mathbf{u}^*(\mathbf{x}, t)
$$

(3.13)

where symbols in bold refer to vector quantities. The separation of the scales is achieved with the use of a low-pass filter, which is defined as the convolution product

$$
\langle \mathbf{u}(\mathbf{x}, t) \rangle = \int G_\Delta(\mathbf{r}, \mathbf{x})\mathbf{u}(\mathbf{x} - \mathbf{r}, t) d\mathbf{r}
$$

(3.14)
where $G_\Delta$ is the filter function and $\Delta$ is the width of the filter, which is typically associated with the mesh size. It should be noted that the filtering operation on the residual field is non zero. The filtered N-S equations (in the incompressible form) are

$$\frac{\partial \langle u_i \rangle}{\partial t} = 0$$ (3.15)

$$\frac{\partial \langle u_i \rangle}{\partial x_i} + \frac{\partial \langle u_i u_j \rangle}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial \nu \langle s_{ij} \rangle}{\partial x_j}$$ (3.16)

The filtering of the non-linear term introduces the so-called residual-stress tensor, which is defined as

$$\tau_{ij}^R = \langle u_i u_j \rangle - \langle u_i \rangle \langle u_j \rangle$$ (3.17)

Now decomposing the $\tau_{ij}^R$ into its isotropic and the anisotropic part, the anisotropic part is written as

$$\tau_{ij}^{R\alpha} = \tau_{ij}^R - \frac{2}{3} k_r \delta_{ij}$$ (3.18)

where $k_r = \frac{1}{2} \tau_{ii}^R$ is the residual stress tensor. Therefore, the only term that needs to be modelled for the closure of the equations is $\tau_{ij}^{R\alpha}$. Smagorinsky (1963) proposed the Sub-Grid Scale (SGS) model, assuming that $\tau_{ij}^{R\alpha}$ acts in the same manner as the viscous stress tensor. In this case, $\tau_{ij}^{R\alpha}$ is expressed as

$$\tau_{ij}^{R\alpha} = -\nu_{SGS} \langle s_{ij} \rangle$$ (3.19)

where $\nu_{SGS}$ is the sub-grid viscosity, $\langle s_{ij} \rangle$ is the filtered strain-rate tensor and $k_{SGS}$ the sub-grid scale energy. Based on the mixing-length-hypothesis, which is introduced in the following section, the $\nu_{SGS}$ is expressed as

$$\nu_{SGS} = (C_s \Delta)^2 \sqrt{\langle s_{ij} \rangle \langle s_{ij} \rangle / 2}$$ (3.20)
where \( C_S \) is the Smagorinsky coefficient. It should be noted that LES is still too expensive for everyday engineering calculations, but can be used in higher Reynolds number flows than DNS, and is employed as a research tool in a number of industries.

### 3.2.3 Reynolds Averaged Navier Stokes (RANS)

This method solves only for the mean flow quantities and thus all the details of turbulence have to be modelled. Using the Reynolds decomposition the velocity field can be expressed as

\[
\mathbf{u}(x, t) = \overline{\mathbf{u}}(x) + u'(x, t)
\]  

where \( \overline{\mathbf{u}} \) is the time-averaged or mean component, denoted with overbar and defined as

\[
\overline{u}_i(x) = \lim_{T \to \infty} \frac{1}{T} \int_{t}^{t+T} u_i(x, t) dt
\]

and \( u'_i(x, t) \) the fluctuating part, denoted with single prime. In the case that a flow field has a characteristic frequency \( \tau \), like in the case of transonic buffet, ensemble averaging can be defined as

\[
\overline{u}_i(x, t) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N} u_i(x, t + \tau)
\]

Performing the averaging operation on the N-S equations, the non-linear term introduces an additional second order tensor, consisting of the so-called Reynolds stresses, which need appropriate modelling. In the case of a RANS simulation, none of the scales of turbulence structures are resolved. Therefore, the computational cost, which is associated with the mesh spacing and the time step size, can be sufficiently low for industrial applications. However, the accuracy of RANS is typically limited by the...
modelling introduced for the Reynolds stresses.

3.2.4 Hybrid RANS-LES Modelling

Hybrid RANS-LES modelling attempts to use computationally affordable RANS approaches in regions of attached boundary layers, where the accuracy of their predictions is established, and scale resolving LES modelling in regions of detached boundary layers, where turbulent field is far from equilibrium. A categorization of different types of hybrid methods, including zonal and non-zonal approaches, is presented in Mocket (2009). Zonal hybrid methods require the determination of RANS and LES regions by the user. In non-zonal methods the transition between the RANS and LES regions is defined by the model and for this reason this kind of methods is attractive for complex industrial flows.


Chapter 4

RANS Turbulence Modelling

4.1 Introduction

In this section some of the basic types of RANS turbulence models are summarized beginning with their incompressible formulation, since most have been developed principally for such flows. Some basic features associated with turbulence modelling for compressible flows are also discussed.

In incompressible flows, the density is assumed as constant and the continuity equation becomes $\frac{\partial u_i}{\partial x_i} = 0$. By decomposing the velocity and pressure fields into their mean and fluctuating components,

$$ u_i = U_i + u_i', \quad p = P + p' $$

(4.1)

and by substituting them into the continuity and momentum equation and applying
Reynolds averaging, the RANS equations become,

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (4.2)$$

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} [\nu S_{ij} - \overline{u'_i u'_j}] \quad (4.3)$$

where overline denotes Reynolds averaging, $U_i = \overline{u_i}$, $P = \overline{p}$ and $S_{ij} = \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}$ is the mean strain-rate tensor in this case. Therefore, an additional term $\overline{u'_i u'_j}$ is introduced in the momentum equation coming from the averaging of the non-linear term. From Equation 4.3 one can conclude that this term acts as an additional stress, and $\overline{u'_i u'_j}$ is therefore called Reynolds-stress tensor, which is of second order and symmetric. Thus, six new unknowns are introduced to the equations, the Reynolds normal stresses $\overline{u'^2}, \overline{v'^2}, \overline{w'^2}$ and the Reynolds shear stresses $\overline{u'v'}, \overline{v'w'}, \overline{w'u'}$ and consequently models are needed for these to close the set of equations.

Early studies, based on simple shear flows, assumed that Reynolds shear acts in an analogous fashion to the viscous one, suggesting

$$\overline{u'v'} = -\nu_t \frac{\partial U}{\partial y} \quad (4.4)$$

where $\nu_t$ is the turbulent viscosity. A generalization of Equation 4.4 is

$$\overline{u'_i u'_j} = -\nu_t S_{ij} + \frac{2}{3} k \delta_{ij} \quad (4.5)$$

where $k = \frac{1}{2} \overline{u'_i u'_i}$ the mean turbulent kinetic energy per unit mass. This form is known as Boussinesq’s hypothesis. The term $\frac{2}{3} k \delta_{ij}$ (\(\delta_{ij}\) is the Kronecker’s delta) is included in order to ensure the correct trace of the Reynolds stress tensor.

To complete the model, a suitable expression is needed for the turbulent viscosity.
\( \nu_t \). Models of the form of Equation 4.5 are often referred to generically as linear EVM’s, and some models for \( \nu_t \) are outlined below.

### 4.2 Zero-Equation EVM

In Zero-Equation EVM’s the turbulent viscosity is prescribed algebraically. Prandtl (1925) proposed that turbulent viscosity could be expressed via a turbulent velocity-scale \( u_m \) and a turbulent length-scale \( l_m \) as

\[
\nu_t = u_m l_m \tag{4.6}
\]

Prandtl, focusing on a simple shear flow, where only the Reynolds-shear stress appears in the momentum equation, suggested the turbulent velocity-scale should scale according to the mean-shear rate, and based on dimensional analysis he proposed \( u_m = l_m |\frac{\partial U}{\partial y}| \), which results in

\[
\nu_t = l_m^2 |\frac{\partial U}{\partial y}| \tag{4.7}
\]

The expression of eddy viscosity in Equation 4.7 is known as the Mixing-Length-Hypothesis (MLH) and \( l_m \) is known as the mixing-length. The prescription of the mixing-length is flow dependent and takes different expressions for different free shear flows, e.g. far wake, mixing layer and jet flows (Rodi, 1993), demonstrating lack of universality. For the application of the MLH model to wall-bounded flows the mixing-length is typically taken as \( l_m = ky \) in the fully turbulent near wall region, in order for the log-layer law to be satisfied. Furthermore, Van Driest (1956) introduced a damping function to account for viscous damping effects very close to the wall. The
resulting mixing-length is

\[ l_m = ky(1 - \exp(-y^*/26)) \tag{4.8} \]

Variations of MLH model have been proposed by Smagorinsky (1963), Cebici and Smith (1974) and Baldwin and Lomax (1978), referred to in Speziale (1995). Zero-Equation models often give satisfactory results for simple two-dimensional shear flows, although they frequently show a weaker performance in more complex flows, such as separated boundary layers, partly due to the need for an empirical length-scale prescription. Additional drawbacks of this type of model are the fact that, unphysically, turbulent viscosity vanishes in the absence of mean-strain rates, e.g. in the middle of a channel flow, and the difficulty in prescribing wall-distance in complex geometries. A discussion on the performance of this type of models is included in Rodi (1993) and Versteeg and Malalasekera (1995).

### 4.3 One-Equation EVM

In One-Equation models transport effects are typically introduced in the calculation of turbulent velocity-scale. Prandtl (1945) and Kolmogorov (1942) proposed that an alternative choice for the turbulent velocity-scale is \( k^{1/2} \), in which case turbulent viscosity becomes

\[ \nu_t = k^{1/2}l \tag{4.9} \]
where \( l \) is a turbulent length-scale. The transport equation for the turbulent kinetic energy is

\[
\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = -\frac{u_i' u_j'}{2} \frac{\partial U_i}{\partial x_j} - \varepsilon - \frac{\partial}{\partial x_i} \left[ \frac{1}{3} u_i' u_j' u_j' + \frac{u_i' p'}{\rho} - \nu \frac{\partial k}{\partial x_i} \right]
\]  

(4.10)

which, with some modelling input, can be solved to obtain \( k \). The first term on the Right-Hand-Side (RHS) of Equation 4.10 is referred to as the production rate of \( k \), usually denoted as \( P_k \). This term exists in the mean kinetic energy equation with the opposite sign and the physical interpretation of it is that it transfers kinetic energy from the mean flow to the turbulence scales. The \( \varepsilon \) quantity, which is defined as

\[ \varepsilon = \nu \frac{\partial u_i'}{\partial x_j} \frac{\partial u_j'}{\partial x_i}, \]

is the dissipation rate of \( k \) per unit mass and it is associated with a viscous procedure dissipating the turbulent kinetic energy into internal energy. The last term on the RHS of the \( k \)-equation is related with diffusive processes. The turbulent diffusion, \(-\frac{1}{2} u_i' u_j' u_j'\), is usually assumed to act in analogy to molecular diffusion. The pressure diffusion term, \(-\frac{u_i' p'}{\rho}\), is usually assumed negligible, which is valid for simple shear flows as discussed in Mansour et al. (1988), though it is significant in the near-wall region within the recirculation region of a separated boundary layer as discussed in Na and Moin (1998) and Le et al. (1997). The typical modelled transport equation for \( k \) is

\[
\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = -\frac{u_i' u_j'}{2} \frac{\partial U_i}{\partial x_j} - \varepsilon + \frac{\partial}{\partial x_i} \left[ \left( \nu + \frac{\nu_i}{\sigma_k} \right) \frac{\partial k}{\partial x_i} \right]
\]

(4.11)

Turbulent dissipation can be expressed based on dimensional analysis as \( \varepsilon \sim k^{2/3}/l \). The turbulent length-scale \( l \) is commonly prescribed in a similar way as in Zero-Equation models and the model constant, \( \sigma_k \), is usually assumed unity. Rodi (1993) noted that under local-equilibrium conditions, where \( P_k \) balances \( \varepsilon \), One-Equation models reduce to Zero-Equation ones, indicating that MLH models can be regarded
as models that prescribe $k$ algebraically based on the local-equilibrium assumption. Therefore, in One-Equation models transport properties are introduced in the calculation of $k$ and thus of $\mu_t$, due to the presence of the diffusion and convection terms in the $k$-equation. In this way the predicted $\mu_t$ is not necessarily zero in the absence of mean-strain rates and this anomaly of MLH models is removed. Other types of One-Equation models solve a transport equation for the eddy viscosity, such as that proposed by Baldwin and Barth (1990) and Spalart and Almaras (1992). However, the need for an empirical prescription of the turbulent length-scale has limited the performance of this type of models and their predictions in complex flows are generally not superior compared to Zero-Equation models.

4.4 Two-Equation EVM

In Two-Equation models a second transport equation is incorporated for the calculation of the turbulent length-scale. The choice of the additional variable is not unique and the most widely validated models of this type solve either for the dissipation rate $\varepsilon$ or for the dissipation rate per unit turbulence kinetic energy $\omega = \varepsilon/k$.

In the case of the $\varepsilon$-based models the turbulent length-scale is defined as $l \sim k^{3/2}/\varepsilon$ and turbulent viscosity as $\nu_t = C_\mu k^2/\varepsilon$. The $k-\varepsilon$ model often referred to as standard is the one referred in Launder and Spalding (1974), in which the modelled $\varepsilon$-equation is expressed as

$$\frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_j} = C_{\varepsilon 1} \frac{\varepsilon}{k} P_k - C_{\varepsilon 2} \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j} \left[ (\nu + \nu_t/\sigma_\varepsilon) \frac{\partial \varepsilon}{\partial x_j} \right]$$

(4.12)

Equation 4.12 is an empirical equation for $\varepsilon$ and it is based on the assumption that the processes that govern the conservation equations of fluid motion should apply
to the evolution of $\varepsilon$. The exact equation of $\varepsilon$ is complex including correlations difficult to measure experimentally. The closure coefficients, $C_\mu$, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$ and $\sigma_\varepsilon$ are typically taken as constants and are tuned to a range of flows. More specifically, $C_\mu$ is calibrated for the local-equilibrium region of a simple boundary layer and $C_{\varepsilon 2}$ is tuned for decaying homogeneous isotropic turbulence. Additionally, $C_{\varepsilon 1}$ and $\sigma_\varepsilon$ are defined such that the log-law of an equilibrium boundary layer should be satisfied and the model return an appropriate spreading rate of free shear flows, e.g. plane jet or mixing layer.

The model as given above would overestimate the Reynolds shear stress in the viscosity affected near-wall region of a boundary layer and this is usually attributed to the modelling of $\varepsilon$, which is modelled in an empirical way, and to the constant value of $C_\mu$, which is essentially reduced in the viscous sublayer as shown in the DNS studies of Kim et al. (1987) and Spalart (1988). A common practice to take into account near-wall behaviour of turbulent quantities is the application of damping functions. Jones and Launder (1972) (JL) proposed a Low-Reynolds-Number (LRN) $k-\varepsilon$ model, where damping functions for the eddy viscosity, $f_\mu$, and for the sink term in the $\varepsilon$-equation, $f_2$, were introduced. In this model both $f_\mu$ and $f_2$ are functions of the turbulent Reynolds number, defined as $R_\kappa = k^2/(\nu \varepsilon)$. Additionally, the JL model solved this second transport equation for a slightly modified variable, $\varepsilon_{\text{iso}}$, called isotropic eddy dissipation, which is defined as $\varepsilon_{\text{iso}} = \varepsilon - D_\varepsilon$. The $D_\varepsilon$ term was designed to ensure that $\varepsilon_{\text{iso}}$ should go to zero at the wall, providing a more convenient and robust boundary condition. Furthermore, the JL model incorporates an additional source term in the $\varepsilon$-equation, usually denoted as $P_{\varepsilon 3}$, which is designed to increase the level of dissipation in the near-wall region.

The near-wall performance of a range of LRN EVM’s has been discussed in Patel
et al. (1985) and Speziale et al. (1992), concluding that none of the examined models, including the $k - \varepsilon$ of Launder and Sharma (1974) (LS), the $k - \varepsilon$ of Chien (1982) and the $k - \omega$ of Wilcox (1988), satisfy the immediate near-wall limiting behaviour of turbulent quantities. Since some of these early models were proposed, DNS studies, e.g. Kim et al. (1987) and Spalart (1988), have provided data that can be used to evaluate all the terms in the exact $\varepsilon$-equation. Various modifications have been proposed for the formulation of the damping functions, $f_\mu$ and $f_2$, and the $P_{\varepsilon 3}$ term, e.g. Nagano and Tagawa (1990), Shih and Mansour (1990), So et al.(1991), and Yang and Shih (1993) (YS), which generally capture the near-wall behaviour of turbulent quantities for the flows they have been calibrated for. However, most of these “improved” LRN models depend on the wall-distance (unlike LS, YS), which make them difficult to apply in complex geometries. Moreover, Sarkar and So (1997) examined a variety of LRN models and concluded that among the examined models those dependent on wall-distance predicted unrealistic behaviour of the friction coefficient in a backstep flow, despite their good near-wall performance in simple equilibrium shear flows.

An alternative way to handle near-wall behaviour of turbulent scales without the incorporation of the above algebraic form of damping functions is the elliptic relaxation model proposed by Durbin (1991). Durbin, in the $\bar{\nu}^2 - f$ model, employed $\bar{\nu}^2$ (which behaves as $y^4$ near the wall) as a turbulent velocity scale instead of k (behaves as $y^2$) to account for the near-wall damping of $\nu_t$ (behaves as $y^3$ near the wall). Therefore, he redefined the turbulent viscosity as $\nu_t = C_\mu \bar{\nu}^2 T_m$ and based on second moment closures he introduced an additional transport equation for $\bar{\nu}^2$,

$$\frac{\partial \bar{\nu}^2}{\partial t} + U_i \frac{\partial \bar{\nu}^2}{\partial x_i} = k f - \bar{\nu}^2 \frac{\varepsilon}{k} + \frac{\partial}{\partial x_i} \left[ (\nu + \nu_t/\sigma_k) \frac{\partial \bar{\nu}^2}{\partial x_i} \right]$$ (4.13)

where the auxiliary variable $f$ is associated with the modelling of the pressure-velocity
correlation of the pressure-strain term $\Phi_{ij}$ and the anisotropic dissipation. Durbin (1991) proposed a Poisson equation for the evolution of $\Phi$, of the form.

$$L_m^2 \frac{\partial^2 \Phi}{\partial x_j^2} - \Phi = \frac{1}{T} (C_1 - 1) \left[ \frac{\overline{u^2}}{k} - \frac{2}{3} \right] - C_2 \frac{P_k}{k}$$

(4.14)

In the local-equilibrium region, the pressure-velocity term in the $\Phi$-equation is modelled using the model of Launder et al. (1975) (LRR). In the near-wall non-equilibrium region, Durbin arrived at the so-called elliptic relaxation model (the Laplacian term in the $\Phi$-equation) by assuming exponential decay of the two-point correlation of the velocity-pressure gradient correlation in the pressure-strain term. The $\overline{u^2} - \Phi$ model has been calibrated for a channel flow and predicts accurate near-wall behaviour of turbulent quantities. In the case of segregated solvers, numerical instabilities may arise due to the boundary conditions for $\Phi$. Modifications to overcome these numerical issues have been proposed by Lien and Durbin (1996), Laurence et al. (2004) and Billard and Laurence (2012). The $\overline{u^2} - \Phi$ model is a four-equation model, though it is included in this sections because it offers an alternative near-wall treatment for the $k - \varepsilon$ model, and still falls within the class of linear EVM’s.

In the case of $\omega$-based models, the eddy-viscosity is usually defined as $\nu_t = k/\omega$, the dissipation rate as $\varepsilon = \beta^* \omega k$ and the commonly used transport equation for $\omega$ is the one proposed by Wilcox (1998),

$$\frac{\partial \omega}{\partial t} + U_i \frac{\partial \omega}{\partial x_i} = a \frac{\omega}{k} P_k - \beta \omega^2 + \frac{\partial}{\partial x_i} \left[ (\nu + \sigma \nu_t) \frac{\partial \omega}{\partial x_i} \right]$$

(4.15)

which is an empirical equation constructed in a similar way as the $\varepsilon$-equation. The closure coefficients, $a$, $\beta$, $\beta^*$, $\sigma$ and $\sigma^*$ (in expression of the diffusion term in $k$-equation) are typically assumed constant and are calibrated for simple flow configurations. More
specifically, a value for the ratio $\beta^*/\beta$ can be derived by applying the model to decaying homogeneous, isotropic turbulence. The values of $a$ and $\beta^*$ are selected in order the law of the wall to be satisfied. Finally, $\sigma$ and $\sigma^*$ can be determined with analysis of the defect layer and the viscous sublayer. The $\omega$-equation resolves the turbulent scales up to the wall by applying appropriate wall-boundary conditions for $\omega$, Wilcox (1988). The $k - \omega$ model exhibits good performance in two-dimensional equilibrium boundary layers, though it is found to be overly sensitive to free-stream turbulence conditions. To overcome this issue Menter (1994) proposed the Base Line (BSL) model, which switches between the $\omega$ model (near the wall), and the $\varepsilon$ model (away from the wall). He achieved this in the BSL model by introducing a blending function, designed to switch the equation from the standard $\omega$ form, to an equivalent $\varepsilon$ form, as the distance from the wall increased.

The various types of linear EVM’s discussed in this section have good performance in equilibrium boundary layers, though can exhibit a weaker performance in the case of non-equilibrium flows, where, for example, the flow field experiences rapid changes of mean strain rates. A range of modifications have been proposed in the literature, designed to tackle some of the drawbacks demonstrated by the models in specific flow configurations, and some of these are outlined here. Yap (1987) introduced an additional source term in the $\varepsilon$-equation, the so-called Yap-correction, to reduce the overexcessive turbulent length-scales that the typical form of $\varepsilon$-equation often produces around reattachment and stagnation regions. Menter (1994), in the Shear Stress Transport (SST) model, introduced a limiter in the eddy viscosity of the BSL model, to avoid overexcessive $P_k$ levels that linear EVM’s usually produce in Adverse Pressure Gradient (APG) boundary layers and recirculating flows, as discussed in Menter (1992). Durbin (1993) incorporated a functional form of $C_{\varepsilon 1}$ dependent on $P_k/\varepsilon$ to
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improve the predictions of the $\overline{v^2} - f$ model in APG boundary layers. Additionally, Menter (1993) proposed a limiter for $P_k$ and Durbin (1996) proposed a timescale limiter for $k/\varepsilon$ to reduce the overexcessive $P_k$ levels around stagnation regions that linear EVM’s typically produce as discussed in Craft et al. (1993). Some additional concepts accounting for non-equilibrium effects within the linear EVM framework are the multiple timescale models, e.g. of Hanjalic and Launder (1980). Another approach is the $\text{Cas}$ model of Revell et al. (2006), which solves for an additional transport equation for $\text{Cas} = -\frac{a_{ij} S_{ij}}{\|S\|}$ in order to take into account the misalignment of the stress and strain-rate tensors encountered in non-equilibrium flows. Additionally, Bourget et al. (2008) developed a tensorial eddy viscosity model, named as anisotropic Organised Eddy Simulation (OES), by projecting the turbulence anisotropy tensor onto the strain rate principal matrices in order to take into account the effect of stress-strain misalignment in the three directions.

Despite the fact that the above mentioned adaptations improve considerably the performance of EVM’s in the flow configurations mentioned above, linear EVM’s still usually demonstrate weak performance in flows involving stream-line curvature effects, swirling flows and secondary flows in non-circular ducts. This is commonly attributed to the linear dependence of the Reynolds stresses on mean-strain rates and to the fact that linear EVM’s do not generally predict the correct anisotropy of the Reynolds normal stresses. Non-linear EVM’s, and the more advance Reynolds Stress Transport Models, described later, attempt to tackle these issues.
4.5 Non-Linear EVM

Non-Linear EVM’s (NLEVM) include a non-linear stress-strain relation, which includes products of the mean rate of strain $S_{ij}$ and vorticity $\Omega_{ij} \left( = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right) \right)$ tensors, to account for the anisotropy of the Reynolds-normal stresses. Pope (1975), based on the Cayley-Hamilton theorem derived a general formulation for the stress-strain relation including up to fifth-order products of the strain and vorticity tensors,

$$a_{ij} \equiv \frac{\bar{u}_i' u_j'}{k} - \frac{2}{3} \delta_{ij} = \sum_{\lambda=1}^{10} G^\lambda T^\lambda_{ij} \quad (4.16)$$

where,

- $T^1_{ij} = S_{ij}$
- $T^2_{ij} = S_{ik} \Omega_{kj} - \Omega_{ik} S_{kj}$
- $T^3_{ij} = S_{ik} S_{kj} - \frac{1}{3} S_{ik} S_{kl} \delta_{ij}$
- $T^4_{ij} = \Omega_{ik} \Omega_{kj} - \frac{1}{3} \Omega_{ik} \Omega_{kl} \delta_{ij}$
- $T^5_{ij} = \Omega_{il} S_{lm} S_{mj} - S_{il} S_{lm} \Omega_{mj}$
- $T^6_{ij} = \Omega_{il} \Omega_{lm} S_{mj} - S_{il} \Omega_{lm} \Omega_{mj} - \frac{2}{3} S_{lm} \Omega_{mn} \Omega_{nl} \delta_{ij}$
- $T^7_{ij} = \Omega_{ik} S_{kl} \Omega_{lm} \Omega_{mj} - \Omega_{ik} \Omega_{kl} S_{lm} \Omega_{mj}$
- $T^8_{ij} = S_{ik} \Omega_{kl} S_{lm} S_{mj} - S_{ik} S_{kl} \Omega_{lm} S_{mj}$
- $T^9_{ij} = \Omega_{ik} \Omega_{kl} S_{lm} S_{mj} + S_{ik} S_{kl} \Omega_{lm} \Omega_{mj} - \frac{2}{3} S_{kl} S_{lm} \Omega_{mn} \Omega_{nk} \delta_{ij}$
- $T^{10}_{ij} = \Omega_{ik} S_{kl} S_{lm} \Omega_{mn} \Omega_{nj} - \Omega_{ik} \Omega_{kl} S_{lm} S_{mj} \Omega_{nj}$

where the coefficients $G^\lambda$ can be functions of $k$, $\varepsilon$ and the invariants: $S_{ik} S_{ki}$, $\Omega_{ik} \Omega_{ki}$, $S_{ik} S_{ki} S_{li}$, $\Omega_{ik} \Omega_{kl} S_{li}$, $\Omega_{ik} \Omega_{kl} S_{lm} S_{mi}$. Models that include up to quadratic products
of mean velocity gradients have been proposed by Speziale (1987), Nisizima and Yoshizawa (1987)(NY), Myong and Kashagi (1990)(MK) and Shih et al. (1993). Among these only the NY and MK models include low-Re terms and the others should be used in conjunction with wall functions. Quadratic NLEVM’s have been shown to give improved performance over linear forms in capturing secondary flows, such as those found in square ducts, though they do not capture effects on the Reynolds-shear stress in flows that involve stream-line curvature and swirl. To account for these effects Craft et al. (1996b) proposed a cubic stress-strain relation including low-Re terms. The CLS model was tuned to homogeneous shear flow, straight and curved channel flow, impinging flow, and a rotating pipe flow. In addition to the cubic tensorial form of the stress-strain relation, the model also included a functional form of $C_\mu$ (dependent on the local mean strain rate). In a later work Craft et al. (1999) proposed a revised version of this $C_\mu$ function (and the corresponding $f_\mu$ damping term) to improve the stability of the model without seriously affecting its predictive capability.

### 4.6 Reynolds Stress Transport Models

The Reynolds Stress Transport Models (RSTM) solve a separate transport equation for each individual Reynolds stress. The stress transport equation has the form ,

$$\frac{\partial u'_i u'_j}{\partial t} + U_k \frac{\partial u'_i u'_j}{\partial x_k} = P_{ij} - \varepsilon_{ij} + \phi_{ij} + d^i_{ij} + d^t_{ij} \quad (4.17)$$

where

- $P_{ij} = -(u'_i u'_k \frac{\partial U_j}{\partial x_k} + u'_j u'_k \frac{\partial U_i}{\partial x_k})$ is the generation rate.
\[ \varepsilon_{ij} = -2\nu \frac{\partial u_i'}{\partial x_k} \frac{\partial u_j'}{\partial x_k} \] is the viscous dissipation rate.

- \[ \phi_{ij} = \frac{\rho}{p} \left( \frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right) \] is called the pressure-strain term.

- \[ d_{ij}^v = \nu \frac{\partial^2 u_i' u_j'}{\partial x_k \partial x_k} \] is the viscous diffusion rate.

- \[ d_{ij}^t = -\frac{\partial}{\partial x_k} \left[ u_i' u_j' u_k + p' u_i' / \rho \delta_{jk} + p' u_j' / \rho \delta_{ik} \right] \] is the turbulent diffusion rate.

In the case of a RSTM, the terms that need modelling are \( \varepsilon_{ij}, \phi_{ij} \) and \( d_{ij}^t \). The dissipation rate is typically modelled as \( \varepsilon_{ij} = (2/3)\varepsilon \delta_{ij} \), where the assumption of isotropy of the small dissipative scales in high Reynolds number flows is made. Therefore, an additional transport equation for \( \varepsilon \) (or for \( \omega \)) is usually incorporated, similar to the one used in the \( k-\varepsilon \) (or \( k-\omega \)) model. The turbulent diffusion term can be approximated with the Generalised Gradient Diffusion Hypothesis (GGDH) of Daly and Harlow (1970) as

\[
\frac{\partial}{\partial x_k} \left[ \frac{k}{\varepsilon} u_k' \frac{\partial u_i' u_j'}{\partial x_l} \right] \quad (4.18)
\]

which introduces characteristics of anisotropy in the “diffusivity” term, \( c_s \frac{k}{\varepsilon} u_k' u_l' \), and typically has improved performance over simple gradient diffusion models when the Reynolds stresses are predicted accurately. The role that the pressure-strain plays is to redistribute the energy between the Reynolds normal stresses, without contributing directly to the turbulent kinetic energy levels. It is typically decomposed into two parts

\[
\phi_{ij} = \phi_{ij1} + \phi_{ij2} \quad (4.19)
\]

The \( \phi_{ij1} \) term is linked with interactions between fluctuating quantities and is usually called the return-to-isotropy or slow term. A simple model, based on decaying non-isotropic turbulence in the absence of mean strain rates, has been proposed by Rotta
\[ \phi_{ij1} = -c_{\phi1} \varepsilon a_{ij} \]  
(4.20)

where \( a_{ij} = \frac{\overline{u_i' u_j'}}{k} - \frac{2}{3} \delta_{ij} \) is the normalised anisotropic stress tensor.

The \( \phi_{ij2} \) term, usually called the mean-strain or rapid term, is associated with interactions between fluctuating quantities and mean strain rates. A simple model for \( \phi_{ij2} \) is the isotropisation-of-production model

\[ \phi_{ij2} = -c_{\phi2}(P_{ij} - \frac{1}{3}P_{kk}\delta_{ij}) \]  
(4.21)

A variety of models for the pressure-strain term have been proposed, with some popular examples being the high Reynolds number LRR model of Launder, Reece and Rodi (1975) (linear in \( a_{ij} \)), the high-Reynolds number SSG model of Speziale, Sarkar and Gatski (1991) (quadratic in \( a_{ij} \)) and the low-Reynolds number Two Component Limit (TCL) model of Craft (1998) (cubic in \( a_{ij} \)). Gibson and Launder (1978) (GL) introduced corrections, proposed by Shir (1973), in the formulation of the \( \phi_{ij1} \) and \( \phi_{ij2} \) of the LRR model to account for near-wall proximity effects. These additional terms are referred to as wall-reflection corrections and depend on the wall-normal unit vectors and wall-distance. Hanjalic et al. (1997) proposed a LRN RSTM by sensitizing the model-coefficients of the pressure term in the GL model using the so-called flatness parameters \( A \) and \( E \). The flatness parameters are defined as

\[ A = 1 - \frac{9}{8}(A_2 - A_3) \] and \[ E = 1 - \frac{9}{8}(E_2 - E_3) \],

where \( A_2, A_3, E_2 \) and \( E_3 \) are the second and third invariants of the anisotropy of the Reynolds stress \( (a_{ij}) \) and the dissipation \( (\varepsilon_{ij}) \) tensors respectively. The TCL model in addition to the flatness parameters incorporates the so-called normalized length scale gradients, \( d_i \) and \( d_i^A \), to sensitize the model to near-wall effects and avoid using the wall-distance and wall-normal unit
vectors. The normalized length scale gradients are defined as \( d_i = \frac{N_i}{0.5 + \sqrt{N_k N_{ik}}} \) and \( d_i^A = \frac{N_i^A}{0.5 + \sqrt{N_k^A N_{ik}^A}} \), where \( N_i = \frac{\partial (k^{(3/2)/\varepsilon})}{\partial x_i} \) and \( N_i^A = \frac{\partial (k^{(3/2)/\varepsilon})}{\partial x_i} \). The anisotropy of \( \varepsilon_{ij} \) in the near-wall region is typically modelled in an algebraic way to avoid additional transport equations and a general formulation commonly adopted is

\[
\varepsilon_{ij} = (1 - f_s)^2 \delta_{ij} \varepsilon + f_s \varepsilon_{ij}^* \tag{4.22}
\]

where \( f_s \) is a blending function used for the gradual transition from anisotropic to isotropic state and \( \varepsilon_{ij}^* \) is an algebraic expression for the anisotropy of \( \varepsilon_{ij} \) typically dependent on the Reynolds stress tensor. A number of expressions for \( f_s \) and \( \varepsilon_{ij}^* \) have been proposed, for example Hanjalic and Launder (1976), Launder and Tselepidakis (1993), Hanjalic and Jakirlic (1993), Craft and Launder (1996b) and Jakirlic and Hanjalic (2002).

### 4.7 Turbulence Modelling for Compressible Flows

In compressible flows, density variations due to pressure variations can be significant, and consequently these enter into the conservation equations when considering their averaged forms. In this case, it is convenient to use Favre, or mass, averaging, introduced by Favre (1965), to avoid the introduction of a number of additional terms in the equations. The Favre-average of a variable, \( \tilde{f} \), is defined as \( \bar{\rho} \tilde{f} = \bar{\rho} \tilde{f} \) (overbar denotes Reynolds averaging) and the instantaneous value of it is decomposed as \( f = \tilde{f} + f'' \) (double prime denotes the fluctuating part with respect to Favre averaging). The Favre-averaged mean conservation equations and turbulent kinetic energy equation in the form described in Huang et al. (1995) are,
Conservation of the mass

\[ \frac{\partial \bar{p}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{p} \bar{u}_i) = 0 \] (4.23)

Conservation of the momentum

\[ \frac{\partial}{\partial t}(\bar{p} \bar{u}_i) + \frac{\partial}{\partial x_j}(\bar{p} \bar{u}_i \bar{u}_j) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(\bar{\sigma}_{ij} - \rho \bar{u}_i \bar{u}_j') \] (4.24)

Conservation of energy

\[ \frac{\partial}{\partial t}(\bar{p}E) + \frac{\partial}{\partial x_j}(\bar{p} \bar{u}_j H) = \frac{\partial}{\partial x_j} \left[ u_i \bar{\sigma}_{ij} - \bar{u}_i \rho \bar{u}_j' u_j' \right] + \frac{\partial}{\partial x_j} \left[ -q_{ij} - q_{ij} + \bar{\sigma}_{ij} u_i' - \rho \bar{u}_j' \frac{1}{2} u_i' u_j' \right] \] (4.25)

Turbulent kinetic energy equation

\[ \frac{\partial}{\partial t}(\bar{p}k) + \frac{\partial}{\partial x_j}(\bar{p} \bar{u}_j k) = -\bar{p} \bar{u}_j' u_j' \frac{\partial \bar{u}_i}{\partial x_j} - \bar{\sigma}_{ij}' \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial}{\partial x_j} (\bar{\sigma}_{ij}' u_i') - \frac{\partial}{\partial x_j} \left[ \frac{1}{2} \bar{p} \bar{u}_j' u_j' + p' u_j' \right] + p' \frac{\partial u_i'}{\partial x_i} + u_i' \left[ \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} - \frac{\partial \bar{p}}{\partial x_i} \right] \] (4.26)

Equation of state

\[ \bar{p} = \bar{p} R \bar{T} \] (4.27)

where \( E = \dot{e} + \frac{1}{2} \bar{u}_i \bar{u}_i + k \) is the total energy, \( H = \dot{e} + \bar{p}/\rho + \frac{1}{2} \bar{u}_i \bar{u}_i + k \) the total enthalpy and \( k = \frac{1}{2} \frac{\rho \bar{u}_i' u_i'}{\bar{p}} \) is the turbulent kinetic energy. Turbulent kinetic energy \( k \) is often neglected in the calculation of \( E \) and \( H \), which is usually a good approximation up to the supersonic regime (see Wilcox, 1993). The heat-flux vector is taken as

\[ q_{ij} = -\frac{C_{nu}}{Pr_t} \frac{\partial \bar{T}}{\partial x_i}, \] where \( Pr_t = \frac{C_{nu}}{\lambda} \) the molecular Prandtl number. In the case of an
EVM, the Reynolds-stress tensor $-\rho u_i'' u_j''$ is modelled by generalizing the Boussinesq approximation for compressible flows, and taking $-\rho u_i'' u_j'' = \mu_t S_{ij} - \frac{2}{3} \rho k \delta_{ij}$, where $S_{ij} = \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial \tilde{u}_k}{\partial x_k}$ is the Favre-averaged strain-rate tensor. Additionally, $Q_{ij} = \rho w_j' h''$ is the turbulent heat-flux vector and is modelled in analogy to the molecular one as $q_{ij} = -\frac{\mu_c}{Pr_t} \frac{\partial T}{\partial x_j}$. A constant value of 0.9 is assumed for the turbulent Prandtl number $Pr_t$ in the case of a boundary layer and 0.5 for free shear layers (see Wilcox, 1993). Molecular diffusion, $\sigma_{ij}' u_i''$, and turbulent transport, $\rho u_j' u_i'' u_i''$, can often be neglected in the mean-energy equation, particularly for flows in the supersonic regime (see Wilcox, 1993). In the k-equation these terms are commonly modelled using the simple gradient approximation

$$\sigma_{ij}' u_i'' - \rho u_j'' \frac{1}{2} u_i'' u_i'' = \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_i}$$ \hspace{1cm} (4.28)

The pressure diffusion term, involving $p' u_j''$, in the k-equation is commonly neglected (or assumed to be included in the rest of the modelled diffusion terms), as it is in incompressible flows. The pressure-dilatation, $p' \frac{\partial u_i''}{\partial x_i}$, and pressure-work, $u_i'' \frac{\partial p}{\partial x_i}$, terms are typically ignored and this is a valid approximation at least in the case of a supersonic turbulent boundary layer (Guarini et al., 1999), a supersonic channel flow (Huang et al., 1995) and an impinging shock flow (Pirozzoli and Bernardini, 2011). Zeman (1990) and Sarkar et al. (1991) proposed models for the dilatation dissipation, which is associated with the compressibility effects on dissipation $\sigma_{ij} \frac{\partial u_i''}{\partial x_j}$. Both models are functions of turbulence Mach number $M_t(= \sqrt{2k/\tau}$, where $\tau$ is the mean speed of sound) and were tuned by considering compressible isotropic decaying turbulence. Wilcox (1992) showed that the Sarkar/Zeman compressibility corrections improve the performance of linear EVM’s in predicting the spreading rate of compressible mixing layers, though they have a negative impact in predicting the friction
coefficient of a compressible turbulent boundary layer, and proposed a modification of the Sarkar/Zeman models to extent their applicability in wall-bounded flows. An investigation of the performance of these compressibility models in a Mach 5 impinging shock flow is given in Brown (2013), which shows that the predictions of the size of the separation bubble are not improved. Additionally, DNS studies on supersonic channel flows (Coleman et al., 1995) and a compressible annular mixing layer (Freund et al., 2000) have shown that the dilatation-dissipation is negligibly small comparing to the solenoidal part of the dissipation, which makes the physical reasoning of the proposed compressibility models debatable. Therefore, the straightforward generalization of the incompressible models for dissipation is usually adopted for compressible flows, without adding extra modelled source or sink terms.

4.8 Summary

In this section some basic type of RANS turbulence models, including linear and non-linear EVM’s and Reynolds stress transport models, have been discussed. Additionally, some aspects associated with turbulence modelling for compressible flows are also included.
Chapter 5

Hybrid RANS-LES Modelling

5.1 Introduction

In this section some basic variances of a popular non-zonal hybrid RANS-LES approach, namely the Delayed Detached Simulation, are summarized.

5.2 Detached Eddy Simulation (DES)

A popular zonal hybrid method is the Detached Eddy Simulation (DES), which is based on the transport equations of a selected RANS model. The switching between RANS and LES mode is controlled by a modified turbulence length scale defined as

\[ l_{DES} = \min(l_{RANS}, l_{LES}) \]  \hspace{1cm} (5.1)

where \( l_{RANS} \) is the turbulence length scale defined by the selected RANS model and \( l_{LES} \) is the LES length scale typically dependent on local grid spacing. The first version of DES has been proposed by Spalart et al. (1997) based on the one-equation
model of Spalart and Allmaras (1994) (SA) and is commonly referred to as DES97. The turbulence length scale in DES97 model is defined as

$$l_{DES} = \min(d, C_{DES} \Delta_{max})$$  \hspace{1cm} (5.2)

where $d$ is the wall-distance in the destruction term of the SA model, $C_{DES}$ is a model constant in the definition of $l_{LES}$ and $\Delta_{max} = \max(\Delta_i, \Delta_j, \Delta_k)$ is the larger local grid spacing. Within the DES context, grid spacing parallel to the wall should be significantly larger than in the wall-normal direction within the attached regions of the flowfield in order the DES model to act in RANS mode. Additionally, in detached regions of the flowfield a mesh should be fine enough and nearly isotropic in order the DES model to act as a Smagorinsky type of model. The DES approach has been extended for the SST model by Travin et al. (2000) and Strelets (2001), in which case turbulence length scale in the dissipation term of $k$-equation is modified as

$$l_{DES} = \min(\sqrt{k/(\beta^* \omega)}, C_{DES} \Delta_{max})$$  \hspace{1cm} (5.3)

### 5.3 Delayed Detached Eddy Simulation (DDES)

There are occasions that a mesh can be fine enough to activate inappropriately the LES mode in regions of attached boundary layers, for example in the case of the presence of shock waves or complex geometric features. In these regions, which are referred to as gray areas (Spalart et al., 2006), the mesh may be not sufficiently fine for pure LES resulting in the underestimation of the modelled Reynolds stresses. This problematic behaviour, known as Modeled-Stress Depletion (MSD), can have a negative effect on the predictions of the flow separation, as highlighted in Menter and
Kuntz (2004). A general modification for the elimination of the MSD applicable to any EVM has been proposed by Spalart et al. (2006). They redefined $l_{DES}$ as

$$l_{DES} = l_{RANS} - f_d \max(0, l_{RANS} - l_{LES})$$

(5.4)

where $f_d = 1 - \tanh[(8r_d)^3]$ is a function used to delay the activation of the LES mode in ambiguous regions of a grid and $r_d = \frac{\nu + \nu^*}{S \cdot \kappa^* d^2}$ a function that is one in the log-layer and tends to zero at the edge of the boundary layer. This formulation eliminates the possible gray areas in a grid and is known as Delayed DES (DDES). Different DDES formulations have been proposed by Menter and Kuntz (2004) (based on the SST model), Spalart et al. (2006) (based on the SA model) and Bourguet et al. (2008) (based on the OES model).

### 5.4 Improved Delayed Detached Eddy Simulation (IDDES)

It should be mentioned that for a grid resolution and time step size suitable for LES the above mentioned DES approaches can be used as a Wall Model LES (WMLES), as demonstrated by Nikitin et al. (2000). In this case RANS equations are used to model near-wall turbulent scales and LES is enabled to resolve the energy containing eddies at sufficient distance from the wall in order to avoid the very fine grids that pure LES would otherwise require for the resolution of near-wall anisotropic turbulent motions. Nikitin et al. (2000), showed in a channel flow case that the modeled part of the log-layer predicted from RANS and the resolved part of it from LES are not in agreement. To solve this problematic behaviour Shur et al. (2008) redefined the
sub-grid filter dependent on the local grid spacing and wall-distance and proposed a WMLES based on DDES equations. Additionally, Shur et al. (2006) proposed a unified model that blends a DES and a WMLES resulting in a type of model referred to as Improved DDES (IDDES).

5.5 Summary

In this section some basic variances of a popular non-zonal hybrid RANS-LES approach, namely the Delayed Detached Simulation, have been discussed.
Chapter 6

Turbulence Models Incorporated

6.1 Introduction

In this section the Favre-averaged transport equations of the turbulence models selected for testing are described. Four types of LRN EVM’s, widely used for industrial purposes, have been selected in order to demonstrate the efficiency of some basic alternative approaches available in literature (see Chapter 4) in predicting SWBLI’s. Within this framework the $k$-$\varepsilon$ of Launder and Sharma (1974) (LS), the $k$-$\omega$ SST of Menter (1994), the $v^2$-$f$ variant of Laurence et al. (2004) (PHIF) and the non-linear $k$-$\varepsilon$ scheme of Craft et al. (1996b,1999) (CLSa,b) have been selected for testing. The LS model employs damping functions dependent on $R_t$ for the resolution of near-wall turbulent length scales, and in the present study the Yap (1987) correction has been included in the LS model, to control predicted turbulent lengthscales around separation and reattachment points. The SST solves an $\omega$-equation for the resolution of near-wall turbulence and switches to the $\varepsilon$-equation in the outer part of the boundary layer. Additionally, SST employs a limiter in the eddy viscosity to avoid
overexcessive $P_k$ levels in APG boundary layers and recirculating flows. The four-equation PHIF model solves an elliptic relaxation equation for the near wall damping of turbulence. The CLSa,b models, based on the $k-\varepsilon$ equations of LS, include a cubic algebraic expression for the stress-strain relation. Additionally, the CLS models employ functional forms of $C_\mu$ dependent on the local mean strain rates. Therefore, the utilization of these models will demonstrate the efficiency of the different methods of handling near-wall turbulence, that these models incorporate, in SWBLI cases. Additionally, the use of the CLS model will give evidence of the effect of the resolution of Reynolds stress anisotropy in predicting SWBLIs. Finally, in this chapter the high Reynolds number RSTM of Launder, Reece and Rodi (1975) (LRR) is described. The LRR model is utilized only to compare the predictions of a RSTM and the selected EVM's in a normally-strained flow, namely the case of a 1D normal shock/isotropic turbulence interaction, where it will be seen that EVM's generally show a weak performance

It should be highlighted that the PHIF and CLSa,b models have been implemented during this project in the open source CFD package OpenFOAM v1.7.1 and some results from validation studies are included in this chapter, to confirm the accuracy of these implementations. Furthermore, the source terms introduced as compressibility corrections in the $\varepsilon$-equations in the PHIF and CLS models are discussed.

Finally, the equations of the SST based DDES approach, which is used for the analysis of the three-dimensional flow around the V2C supercritical airfoil, are presented
6.2 $k-\varepsilon$ model of Launder and Sharma (1974)

Launder and Sharma (1974) (LS) proposed a LRN $k-\varepsilon$ model, in which the transport equations for $k$ and $\varepsilon$ are defined as

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho \tilde{u}_j k}{\partial x_j} = -\rho u_i' u_j' \frac{\partial \tilde{u}_i}{\partial x_j} - \rho \varepsilon + \frac{\partial}{\partial x_j} \left[ (\mu + \mu_t) \frac{\partial k}{\partial x_j} \right]$$

(6.1)

$$\frac{\partial \rho \varepsilon_{iso}}{\partial t} + \frac{\partial \rho \tilde{u}_j \varepsilon_{iso}}{\partial x_j} = C_{\varepsilon 1} \frac{\varepsilon_{iso}}{k} \left( -\rho u_i' u_j' \frac{\partial \tilde{u}_i}{\partial x_j} \right) - C_{\varepsilon 2} \rho f_{\mu} \frac{\varepsilon_{iso}^2}{k} + \frac{\partial}{\partial x_j} \left[ (\mu + \mu_t) \frac{\partial \varepsilon_{iso}}{\partial x_j} \right] + P_{\varepsilon 3} + S_{\varepsilon}$$

(6.2)

where

$$\varepsilon_{iso} \equiv \varepsilon - D_\varepsilon$$

(6.3)

$$\mu_t = \bar{\rho} C_{\mu f} f_{\mu} \frac{k^2}{\varepsilon_{iso}}$$

(6.4)

$$-\rho u_i' u_j' = \mu_t S_{ij} - \frac{2}{3} \rho k \delta_{ij}$$

(6.5)

where $S_{ij} = \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} \frac{\partial \tilde{u}_k}{\partial x_k}$. The quantity $\varepsilon_{iso}$ is referred to as the isotropic part of the dissipation and goes to zero at the wall, while the term $D_\varepsilon = 2\mu \left( \frac{\partial k^{1/2}}{\partial x_j} \right)^2$ provides a non-zero value of the dissipation rate near the wall. The quantity $P_{\varepsilon 3} = 2 \frac{\mu \mu_t}{\bar{\rho}} \left( \frac{\partial \tilde{U}_k}{\partial x_i \partial x_j} \right)^2$ is designed to increase the level of dissipation in the near-wall region.

The proposed damping functions are,

$$f_{\mu} = \exp(-3.4/(1 + R_t/50)^2)$$

(6.6)
\[ f_2 = 1.0 - 0.3 \exp(-R_t^2) \]  \hspace{1cm} (6.7)

where \( R_t = \frac{R}{\mu \varepsilon} \) is the turbulent Reynolds number defined using \( \varepsilon_{iso} \) in this case. The source term \( S_\varepsilon \) in the dissipation equation is the so-called Yap correction

\[ S_\varepsilon = \max \left[ 0.83 \left( \frac{k^{3/2}}{2.5 \varepsilon_{iso}} - 1 \right) \left( \frac{k^{3/2}}{2.5 \varepsilon_{iso}} \right)^2 \frac{\varepsilon^2}{k}, 0 \right] \]  \hspace{1cm} (6.8)

and it is employed to reduce the predicted turbulent lengthscale around reattachment and impingement points. The model coefficients are shown in Table 6.1.

<table>
<thead>
<tr>
<th>( C_\mu )</th>
<th>( \sigma_k )</th>
<th>( \sigma_\varepsilon )</th>
<th>( C_{\varepsilon 1} )</th>
<th>( C_{\varepsilon 2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>1.0</td>
<td>1.3</td>
<td>1.44</td>
<td>1.92</td>
</tr>
</tbody>
</table>

The boundary values at the wall of the turbulent variables are \( k = 0 \) and \( \varepsilon_{iso} = 0 \).

### 6.3 \( k - \omega \) SST Model of Menter (1994)

The Shear Stress Transport model of Menter (1994) (SST) switches between the \( k - \omega \) and \( k - \varepsilon \) models. The transport equations for \( k \) and \( \omega \) are defined as,

\[ \frac{\partial \rho k}{\partial t} + \frac{\partial (\rho u_j k)}{\partial x_j} = -\rho u_i u_j \frac{\partial u_i}{\partial x_j} - \beta \rho \omega k + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_\varepsilon) \frac{\partial k}{\partial x_j} \right] \]  \hspace{1cm} (6.9)

\[ \frac{\partial \rho \omega}{\partial t} + \frac{\partial (\rho u_j \omega)}{\partial x_j} = \frac{\gamma P}{\mu_t} \left( -\rho u_i u_j \frac{\partial u_i}{\partial x_j} \right) - \beta \rho \omega^2 + \frac{\partial}{\partial x_j} \left[ (\mu + \sigma_\varepsilon) \frac{\partial \omega}{\partial x_j} \right] 
+ 2(1-F) \frac{\rho \sigma_\omega}{\rho} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} \]  \hspace{1cm} (6.10)
where

\[ \mu_t = \frac{\bar{\rho}k_{\text{max}}}{\max(\omega; F_2\|S\|/a_1)} \]  \hfill (6.11)

where \( \|S\| = \sqrt{S_{ij}S_{ij}/2} \) is the magnitude of the mean-strain rate tensor. (The implementation of this model in OpenFOAM v1.7.1, which is the CFD package used for this study, includes the quantity \( \sqrt{\frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)^2} \) instead of \( \|S\| \)). The timescale limiter in the eddy viscosity formulation was introduced to reduce the overprediction of \( P_k \) in APG boundary layers and separated flows. The auxiliary function \( F_2 \) is defined as,

\[ F_2 = \tanh \left[ \max \left( 2 \frac{\sqrt{k}}{0.09\omega y} ; \frac{500\mu}{\bar{\rho}g^2\omega} \right)^2 \right] \]  \hfill (6.12)

where \( y \) is the wall-distance. The final term in the above \( \omega \)-equation arises from the formal transformation of the \( \varepsilon \) equation model to be written in terms of \( \omega \). The model coefficients, \( \phi(\sigma_k, \sigma_\omega, \beta, \gamma) \), are defined as

\[ \phi = F_1\phi_1 + (1 - F_1)\phi_2 \]  \hfill (6.13)

so that these switch between the values of the constants of the \( k - \omega \) model, with subscript 1, and those of the \( k - \varepsilon \), with subscript 2. The auxiliary function \( F_1 \) is the so-called blending function and is defined as

\[ F_1 = \tanh \left( \text{arg}g_1^4 \right) \]  \hfill (6.14)

\[ \text{arg}g_1 = \min \left[ \max \left( \frac{\sqrt{k}}{0.09\omega y} ; \frac{500\mu}{\bar{\rho}g^2\omega} ; \frac{4\rho\sigma_\omega k}{C_{D_{k\omega}}y^2} \right) \right] \]  \hfill (6.15)

\[ C_{D_{k\omega}} = \max \left( \frac{2\rho\sigma_\omega}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, 10^{-20} \right) \]  \hfill (6.16)

The model coefficients are given in Table 6.2.
Table 6.2: Model coefficients of the SST model

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>$\beta^*$</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31</td>
<td>0.09</td>
<td>0.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma_{k1}$</th>
<th>$\sigma_{\omega1}$</th>
<th>$\beta_1$</th>
<th>$\gamma_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>0.5</td>
<td>0.075</td>
<td>$\frac{\sigma_{k1}^2}{\sigma_{\omega1}^2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma_{k2}$</th>
<th>$\sigma_{\omega2}$</th>
<th>$\beta_2$</th>
<th>$\gamma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.856</td>
<td>0.0828</td>
<td>$\frac{\sigma_{k2}^2}{\sigma_{\omega2}^2}$</td>
</tr>
</tbody>
</table>

The boundary values at the wall of the turbulent variables are $k = 0$ and $\omega = \frac{6\mu}{\rho \beta_1 y_1^2}$, where $y_1$ is the wall-distance of the first computational node away from the wall.

### 6.4 $\phi - f$ Model of Laurence et al. (2004)

Laurence et al. (2004) proposed an elliptic relaxation $\overline{\nu^2} - f$ model with improved numerical behaviour compared to the original formulation of this model proposed by Durbin (1991). The transport equations for $k$ and $\varepsilon$ are given by

\[
\frac{\partial \rho k}{\partial t} + \frac{\partial \rho \tilde{u}_j k}{\partial x_j} = -\rho u_i' u_j'' \frac{\partial \tilde{u}_i}{\partial x_j} - \overline{\nu^2} + \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] \tag{6.17}
\]

\[
\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial \rho \tilde{u}_j \varepsilon}{\partial x_j} = C_{\varepsilon 1} \left( -\frac{\rho u_i' u_j'' \frac{\partial u_i}{\partial x_j}}{\overline{T_m}} - C_{\varepsilon 2} \overline{\nu^2} \right) + \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] \tag{6.18}
\]

In this model, $\overline{\nu^2}$ is substituted with $\phi = \overline{\nu^2}/k$ and $f$ with $\bar{f} = f + \frac{2\mu}{\rho \sigma_k} \frac{\partial \phi}{\partial x_j}$. The transport equation for $\phi$ and the Poisson equation for $f$ are

\[
\frac{\partial \rho \phi}{\partial t} + \frac{\partial \rho \tilde{u}_j \phi}{\partial x_j} = \overline{\nu^2} \bar{f} + \rho u_i' u_j'' \frac{\partial \tilde{u}_i}{\partial x_j} k + 2 \frac{\mu_t}{\sigma_k} \frac{\partial \phi}{\partial x_j} \frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial \phi}{\partial x_j} \right] \tag{6.19}
\]

\[
T_m^2 \frac{\partial^2 \bar{f}}{\partial x_j^2} - \bar{f} = \frac{1}{T_m} (C_1 - 1) \left( \phi - \frac{2}{3} \right) + C_2 \frac{\rho u_i' u_j'' \frac{\partial u_i}{\partial x_j}}{k^2} - 2 \frac{\mu_t}{\rho k} \frac{\partial \phi}{\partial x_j} \frac{\partial k}{\partial x_j} - \frac{\mu}{\rho} \frac{\partial^2 \phi}{\partial x_j^2} \tag{6.20}
\]
where the turbulent viscosity is defined as

$$\mu_t = \bar{p}C_\mu\phi kT_m$$

(6.21)

The lower limit on the timescale $T_m = \max \left[ \frac{k}{\varepsilon}, 6\sqrt{\frac{\mu}{\varepsilon}} \right]$, set in terms of the Kolmogorov timescale, is to avoid numerical singularities near the wall. Similarly, the turbulent length-scale is defined as $L_m = C_L\max \left[ \frac{k^{3/2}}{\varepsilon}, C_\eta \left( \frac{\mu}{\varepsilon} \right)^{3/4} \right]$. The model coefficients are given in Table 6.3.

<table>
<thead>
<tr>
<th>$C_\mu$</th>
<th>$\sigma_k$</th>
<th>$\sigma_\varepsilon$</th>
<th>$C_{\varepsilon 1}$</th>
<th>$C_{\varepsilon 2}$</th>
<th>$C_L$</th>
<th>$C_\eta$</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.22</td>
<td>1.0</td>
<td>1.3</td>
<td>1.4(1 + 0.05$\sqrt{1/\phi}$)</td>
<td>1.85</td>
<td>0.25</td>
<td>1.10</td>
<td>1.4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The boundary values at the wall of the turbulent variables are $k = 0$, $\varepsilon \to \frac{2\nu k}{y^2}$, $\phi = 0$, $\bar{f} = 0$. The zero value of $\bar{f}$ at the wall improves the numerical stability of the model when the system of the equations is solved in a segregated manner.

### 6.4.1 Validation

The implementation of the PHIF model in OpenFOAM v1.7.1 has been validated in the channel flow with $Re_\tau = 395$ of Kim et al. (1987) and the 2D asymmetric plane diffuser of Obi et al. (1993) and Buice and Eaton (1997), shown in Figure 6.1. For these cases results have been produced by Laurence et al. (2004) with the open-source CFD package Code_Saturne (Archambeau et al., 2004), in which the published PHIF form was developed. The mesh for the asymmetric diffuser, shown in Figure 6.2, extends from $x/H = -11$ to $x/H = 60$, where $H = 1$ m is the inlet channel height, has size 292x92 control volumes and the non-dimensional wall-distance of the near-wall node is around $y^+ \leq 1$. The same mesh has been used for the Code_Saturne
calculations. The results from the OpenFOAM implementation are consistent with those from Code_Saturne, as shown in Figures 6.3 and 6.4.

Figure 6.1: Geometry for the 2D asymmetric plane diffuser.

Figure 6.2: Grid for the 2D asymmetric plane diffuser.
Figure 6.3: Channel $Re_\tau = 395$: Validation of the implementation of the PHIF model in OpenFOAM v1.7.1. Profiles of $U^+$, $k^+$, $\varepsilon^+$, $\phi$ and $f^+$ from OpenFOAM (red) and Code_Saturne (blue).
Figure 6.4: 2D Asymmetric Plane Diffuser: Validation of the implementation of the PHIF model in OpenFOAM v1.7.1. Profiles of $U/U_b$ and $\overline{u'v'}/U_b^2$ from OpenFOAM (red) and Code_Saturne (blue).
6.4.2 Modification

It should be highlighted that initial calculations for a 24° compression ramp, studied by Bookey et al. (2005), and a 8° impinging shock, studied by Pirozzoli and Grasso (2006), showed that the PHIF scheme overestimates the extent of the recirculation region of these flows significantly. To improve the predictions of the PHIF model a source term coming from the production of dissipation, \( \frac{2}{3} C_{\mu} \rho k \frac{\partial \bar{u}_i \bar{u}_k}{\partial x_k} \), has been introduced on the RHS of the \( \varepsilon \)-equation. This dilatation term is used to modify turbulence length scales in regions close to the separation point, which are surrounded by compression waves in the flows under investigation. More specifically, this source term acts to increase the predicted Reynolds shear stress close to the separation point delaying the flow separation. Predictions of the PHIF scheme including this modification for compression ramp and impinging shock cases are presented in Chapters 9 and 10 indicating promising performance of the model. Therefore, the PHIF model including this modification has been incorporated for the main calculations of this project and hereafter this version of the model is denoted as PHIF.
6.5 Cubic Non-Linear $k-\varepsilon$ Model of Craft et al. (1996b), (1999)

Craft et al. (1996b) (CLSa) proposed a cubic stress-strain relation, the formulation of which is:

$$a_{ij} = -\frac{\mu_t}{\overline{p}k} S_{ij} + c_1 \frac{\mu_t}{\overline{p}^2\varepsilon_{iso}} (S_{ik}S_{kj} - \frac{1}{3}S_{kl}S_{kl}\delta_{ij})$$

$$+ c_2 \frac{\mu_t}{\overline{p}^2\varepsilon_{iso}}(\Omega_{ik}S_{kj} + \Omega_{jk}S_{ki})$$

$$+ c_3 \frac{\mu_t}{\overline{p}^2\varepsilon_{iso}}(\Omega_{ik}\Omega_{jk} - \frac{1}{3}\Omega_{ik}\Omega_{lk}\delta_{ij})$$

$$+ c_4 \frac{\mu_t}{\overline{p}^2\varepsilon_{iso}}(S_{kl}\Omega_{ij} + S_{kj}\Omega_{li})S_{kl}$$

$$+ c_5 \frac{\mu_t}{\overline{p}^2\varepsilon_{iso}}(\Omega_{il}\Omega_{jm}S_{mj} + S_{il}\Omega_{im}\Omega_{mj} - \frac{2}{3}S_{im}\Omega_{mn}\Omega_{nl}\delta_{ij})$$

$$+ c_6 \frac{\mu_t}{\overline{p}^2\varepsilon_{iso}}S_{ij}S_{kl}S_{kl} + c_7 \frac{\mu_t}{\overline{p}^2\varepsilon_{iso}} S_{ij}\Omega_{kl}\Omega_{kl}$$

(6.22)

where $S_{ij} \equiv \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}\frac{\partial \bar{u}_k}{\partial x_k}$ is the mean-strain rate tensor and $\Omega_{ij} \equiv \left(\frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \bar{u}_j}{\partial x_i}\right)$ the mean-vorticity tensor. Additionally, $S \equiv \frac{k}{\varepsilon_{iso}} \sqrt{S_{ij}S_{ij}/2}$ and $\Omega \equiv \frac{k}{\varepsilon_{iso}} \sqrt{\Omega_{ij}\Omega_{ij}/2}$ are the strain and vorticity invariants respectively. In this model the $k$ and $\varepsilon$ equations are the same as those in the LS model, as is the form adopted for the turbulent viscosity (albeit with different forms of $C_\mu$ and $f_\mu$). The model terms and coefficients are given in Table 6.4. The terms and coefficients not present in Table 6.4 are the same as those in the LS model.

Craft et al. (1999) (CLSb) proposed a revised $C_\mu$, designed to be slightly less dependent on the local mean strain rate in regions away from the wall, to improve the stability of the original CLS model in the separated shear layers. The form
Table 6.4: Model terms and coefficients of the CLS model

<table>
<thead>
<tr>
<th></th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.1</td>
<td>0.1</td>
<td>0.26</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>(-10C^2_{\mu} )</td>
<td>0</td>
<td>(-5C^2_{\mu} )</td>
</tr>
<tr>
<td>( c_7 )</td>
<td>( C_{\mu} )</td>
<td>( f_{\mu} )</td>
<td></td>
</tr>
<tr>
<td>( 5C^2_{\mu} )</td>
<td>( \frac{0.3[1-\exp(-0.36\exp(0.75\eta))]}{1+0.35\eta} )</td>
<td>( 1 - \exp\left[-\left(\frac{R_t}{300}\right)^{1/2} - \left(\frac{R_t}{400}\right)^2\right])</td>
<td></td>
</tr>
<tr>
<td>( \eta )</td>
<td>( E )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| \( max(S,\Omega) \) | 0.0022 \( \frac{\rho_kS_{ij}^2}{\varepsilon_{iso}} \left(\frac{\partial \tilde{u}_i}{\partial x_j}\right)^2 \) for \( R_t \leq 250 \)

The boundary values at the wall of the turbulent variables are \( k = 0 \) and \( \varepsilon_{iso} = 0 \).

\[ C_{\mu} = \min \left[ 0.09, \frac{1.2}{1 + 3.5\eta + f_{RS}} \right] \] (6.23)

where \( f_{RS} = 0.235 (\max(0, \eta - 3.33))^2 \left[ \exp(-R_t/400) + \sqrt{S_I^2} \right] \) and \( S_I \) is the third invariant of the strain rate tensor in non-dimensional form \( S_I = S_{ij}S_{jk}S_{ki}/(S_{nl}S_{nl}/2)^{3/2} \).

6.5.1 Validation

The implementation of the CLSa model in OpenFOAM v1.7.1 has been validated in the channel flow with \( Re = 5600 \) of Kim et al. (1987) and the curved channel of Ellis and Joubert (1974). The results from the CLS implementation in OpenFOAM, shown in Figures 6.5 and 6.6, are consistent with the ones reported in Craft et al. (1996).
Figure 6.5: Channel $Re = 5600$: Validation of the implementation of the CLSa model in OpenFOAM v1.7.1 (DNS; circle, CLSa; line).

Figure 6.6: Curved Channel: Validation of the implementation of the CLSa model in OpenFOAM v1.7.1 (experiment; circle, CLSa; line).
6.5.2 Modification

It should be highlighted that initial calculations for a 24° compression ramp, studied by Bookey et al. (2005), and a 8° impinging shock, studied by Pirozzoli and Grasso (2006), showed that both versions of the CLS model overestimate the extent of the recirculation region significantly (Asproulias et al. 2013), with the CLS.b model giving improved predictions. To improve the predictions of the CLS.b model a source term coming from the production of dissipation, \( \frac{4}{3}C_{\varepsilon 1} \bar{p} \varepsilon_{iso} \frac{\partial \tilde{u}_k}{\partial x_k} \), has been introduced on the RHS of the \( \varepsilon \)-equation. This dilatation term is used to modify turbulence length scales in regions close to the separation point, which are surrounded by compression waves in the flows under investigation. More specifically, this source term acts to increase the predicted Reynolds shear stress close to the separation point delaying the flow separation. Predictions of the CLS.b scheme including this modification for compression ramp and impinging shock cases are presented in Chapters 9 and 10 indicating promising performance of the model. Therefore, the CLS.b model including this modification has been used for the main calculations of this project and hereafter this version of the model is denoted as CLS.

6.6 Reynolds Stress Transport Model of Launder, Reece and Rodi (1975)

The model of Launder, Reece and Rodi (1975) (LRR) is a high Reynolds number RSTM, the transport equation for the Reynolds stresses of which is

\[
\frac{\partial \bar{p} u_i'' u_j''}{\partial t} + \frac{\partial \bar{p} u_k'' u_i'' u_j''}{\partial x_k} = P_{ij} - (2/3)\bar{p} \delta_{ij} + \phi_{ij} + \frac{\partial}{\partial x_k} [(\mu \delta_{lk} + c_s \bar{u}_k'' u_l'') \frac{\partial u_i'' u_j''}{\partial x_k}]
\] (6.24)
where \( P_{ij} = -\bar{p}(\bar{u}_i' \bar{u}_k' \frac{\partial \bar{u}_j}{\partial x_k} + \bar{u}_j'' \bar{u}_k'' \frac{\partial \bar{u}_i}{\partial x_k}) \). The proposed pressure-strain term is

\[
\phi_{ij} = -c_1 \bar{p} \frac{\varepsilon}{k} (u_i'' u_j' - \frac{2}{3} k \delta_{ij}) - c_2 (P_{ij} - \frac{1}{3} P_{kk} \delta_{ij})
\] (6.25)

The transport equation for \( \varepsilon \) is

\[
\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial (\rho \bar{u}_i \varepsilon)}{\partial x_i} = c_{\varepsilon 1} \frac{\varepsilon}{k} P_k - c_{\varepsilon 2} \bar{p} \frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_l} \left[ (\mu \delta_{kl} + \sigma_s \bar{p} u_k'' u_l'') \frac{\partial \varepsilon}{\partial x_k} \right]
\] (6.26)

The constants of the model are given in Table 6.5.

Table 6.5: Model coefficients of the LRR model

<table>
<thead>
<tr>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_s )</th>
<th>( c_{\varepsilon 1} )</th>
<th>( c_{\varepsilon 2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>0.6</td>
<td>0.22</td>
<td>1.44</td>
<td>1.92</td>
</tr>
</tbody>
</table>

6.7 Delayed Detached-Eddy Simulation Based on the SST Model

Menter and Kuntz (2004) proposed a Delayed Detached-Eddy Simulation based on the SST model by modifying the turbulence length scale in the dissipation term of the SST model as

\[
l_{\text{SST-DDES}} = d - f_d \max(0, \sqrt{k}/(\beta^* \omega) - C_{\text{DES}} \Delta_{\max})
\] (6.27)

where \( f_d = 1 - \tanh[(8 r_d)^3] \) is a function used to delay the activation of the LES mode in ambiguous regions of a grid and the quantity \( r_d \)

\[
r_d = \frac{\nu_t + \nu}{S \kappa^2 d^2}
\] (6.28)
is a function that is one in the log-layer and tends to zero at the edge of the boundary layer. $\Delta_{\text{max}}$ is the maximum local grid spacing and $C_{DES}$ is locally computed as

$$C_{DES} = F_1 C_{k-\omega} + (1 - F_1 C_{k-\varepsilon})$$  

where $C_{k-\omega} = 0.78$ and $C_{k-\varepsilon} = 0.61$, which have been calibrated by Travin et al. (2000) for homogeneous isotropic turbulence. The quantities $F_1$ and $\beta^*$ are the same as in the SST model.

### 6.8 Summary

In this section the turbulence models, namely LS, SST, PHIF, CLSa,b and LRR, selected for the investigations of the current study have been described. Additionally, some results from validation studies of the PHIF and CLS implementations in OpenFOAM v1.7.1 have been presented. Finally, the additional source terms introduced in the $\varepsilon$-equations of the PHIF and CLS models to improve their performance in the examined flow configurations are given.
Chapter 7

Numerical Modelling

7.1 Introduction

In this section the formulation of the finite volume method for discretizing the N-
S equations is discussed and the different types of compressible solvers available in
OpenFOAM are presented. The vector notation is used here to write the N-S equa-
tions in the form:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot [u \rho] = 0 \]  
\[ \frac{\partial (\rho u)}{\partial t} + \nabla \cdot [u(\rho u)] + \nabla p - \nabla \cdot \tau = 0 \]  
\[ \frac{\partial (\rho E)}{\partial t} + \nabla \cdot [u(\rho E + p)] - \nabla \cdot (\tau \cdot u) + \nabla \cdot q = 0 \]

where the stress tensor is expressed as \( \tau = \mu [\nabla u + (\nabla u)^T - \frac{2}{3} \nabla \cdot u I] \), the total
specific energy as \( E = e + \frac{1}{2}|u|^2 \), the heat flux vector as \( q = -\lambda \nabla T \) and \( I \) is the
identity tensor. The magnitude of the velocity vector is expressed as \( |u| = (u \cdot u)^{\frac{1}{2}} \).
7.2 Finite Volume Method

According to the finite volume method, the computational domain is divided into a finite number of discrete regions, which are called Control Volumes (CV). A typical CV, shown in Figure 7.1, has a centroid P (defined by \( \int_{V_P} (x - x_P) dV = 0 \)) and volume \( V_P \) and is constructed with a set of flat faces. The area vector \( S_f \) of a face \( f \) of a CV passes through the centre of the face, has magnitude equal to the area of the face surface and points along the direction normal to the face pointing out of the CV. In Figure 7.1, a neighboring CV with centroid N is shown. The vector \( d \) connects the centroids P and N and the vector \( d_{fN} \) connects the centre of the face \( f \) and the centroid N. In the colocated grid arrangement (Rhie and Chow, 1982), all the dependent variables are stored at the same location, which is the centroid of a CV.

Figure 7.1: Control Volume.
The finite volume method requires that the conservation equations are satisfied over the CV in their integral form. The transport equations included in the N-S equations can be expressed in the general form as

\[
\frac{\partial \phi}{\partial t} + \nabla \cdot [u\phi] - \nabla \cdot (\Gamma \phi \nabla \phi) = S_\phi(\phi) \tag{7.4}
\]

where \( \phi = \phi(x, t) \) is the transported variable and \( \Gamma \phi \) is the diffusivity. The equation to be discretized is obtained by applying temporal and spatial integration on Equation 5.4

\[
\int_t^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_{V_p} \phi \, dV + \int_{S_p} \nabla \cdot (u\phi - \Gamma \phi \nabla \phi) \cdot dS \right] dt = \int_{V_p} S_\phi(\phi) \, dV \tag{7.5}
\]

and using the Gauss theorem \( \int_{V_p} \nabla \cdot A \, dV = \int_{S_p} A \cdot dS \)

\[
\int_t^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_{V_p} \phi \, dV + \int_{S_p} (u\phi - \Gamma \phi \nabla \phi) \cdot dS \right] dt = \int_{V_p} S_\phi(\phi) \, dV \tag{7.6}
\]

where \( S_p \) is the closed surface that bounds the CV.

### 7.3 Numerical Discretization

#### 7.3.1 Unsteady Term

The unsteady term is evaluated as

\[
\int_t^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_{V_p} \phi(x, t) \, dV \right] dt = \int_{V_p} \phi(x, t + \Delta t) \, dV - \int_{V_p} \phi(x, t) \, dV \tag{7.7}
\]
and the volume integral is approximated as

\[ \int_{V_P} \phi dV \approx \phi_P V_P \quad (7.8) \]

where \( \phi_P = \phi(x_P) \). This approximation is obtained assuming linear variation in space of \( \phi \) in a CV, which leads to accuracy of second order as indicated by Taylor series expansion. Assuming that \( V_P \) is constant in time, the unsteady term is thus approximated as

\[ \int_{t}^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_{V_P} \phi(x,t) dV \right] dt = (\phi^n_P - \phi^o_P)V_P \quad (7.9) \]

where \( n \) and \( o \) denote the new and the old time levels, respectively. The order of accuracy of the time discretization depends on the approximation of the time integral for the remaining terms, which can also have an impact on the solution stability. Fully implicit methods, such as the first order accurate Euler implicit method, take all the dependent variables in these terms to be evaluated at the new time level. Fully explicit methods, such as the first order accurate Euler explicit method, evaluate all the dependent variables at the old time level. Semi-implicit methods, such as the second order accurate Crank-Nicholson method, consider a combination of the values of the dependent variables at both the old and new time levels. The temporal approximations employed in the present work will be discussed later, when detailing the particular solvers. In the paragraphs below, the spatial discretization of the convection, diffusion and source terms is discussed.
7.3.2 Convection Term

Taking into consideration that a CV is constructed with flat faces, the convection term is approximated as

$$\int_{S_p} (u\phi) \cdot dS \approx \sum_f S_f \cdot u_f \phi_f = \sum_f F_f \phi_f$$

(7.10)

where the subscript \( f \) denotes value of a variable in the middle of a face, \( \sum_f \) denotes summation over the faces of a CV and \( F_f = S_f \cdot u_f \) is the volumetric flux through a face \( f \). This approximation leads to accuracy of second order because the assumption of the linear variation of \( \phi \) on a face is made. To evaluate this term the volumetric flux \( F_f \) and the face value of \( \phi \) are required, which are obtained by interpolating the values of \( u \) and \( \phi \) at the centroids of neighboring CV’s. For unstructured meshes it is practical to base this interpolation on the values from the two neighboring CV’s that the face belongs to. The interpolation employed depends on the convection discretisation scheme. One of the simplest second order accurate differencing schemes is Central Differencing (CD), which assumes linear variation of \( \phi \) between the centroids of P and N,

$$\phi_f = w_f \phi_P + (1 - w_f) \phi_N$$

(7.11)

where the weighting factor is \( w_f = |S_f \cdot d_{fN}|/|S_f \cdot d| \). The accuracy of the CD scheme is of second order, although it can introduce unbounded solutions with unphysical oscillations in cases where convection plays a dominant role (Hirsch, 1991).

The Upwind Differencing (UD) scheme assumes that the value of \( \phi \) is constant in the CV and the interpolation is determined by the direction of the flow and thus
from the sign of the volumetric flux:

\[ \phi_f = \phi_P \quad \text{for} \quad F_f \geq 0 \]  
(7.12)

\[ \phi_f = \phi_N \quad \text{for} \quad F_f < 0 \]  
(7.13)

The UD scheme ensures the boundedness of the solution (Patankar, 1981), though the accuracy of the scheme is of first order and numerical diffusion is introduced in the solution.

The Blended Differencing (BD) combines linearly the UD and CD schemes as

\[ \phi_f = [(1 - \gamma)\max(\text{sgn}(F_f), 0) + \gamma w_f]\phi_P \]
\[ + [(1 - \gamma)\min(\text{sgn}(F_f), 0) + \gamma (1 - w_f)]\phi_N \]  
(7.14)

The BD scheme is a compromise between boundedness and accuracy of the solution, though none of these characteristics is guaranteed because the behaviour of the scheme depends on the a-priori definition of the blending factor \( \gamma \).

Another class of schemes that attempt to achieve a compromise between boundedness and accuracy of the solution is the flux-limited schemes, which combine non-linearly the first-order “diffusive” bounded scheme (UD) and a “limited” higher-order unbounded scheme (HO). Following Sweby (1984), the face value \( \phi_f \) using a flux-limited scheme is expressed as

\[ \phi_f = (\phi_f)_{UD} + \Psi(r)[(\phi_f)_{HO} - (\phi_f)_{UD}] \]  
(7.15)

where \( (\phi_f)_{HO} \) is the face value of \( \phi \) from a high-order scheme, \( \Psi(r) \) is the flux-limiter function and \( r \) is the ratio of successive gradients of \( \phi \). In the case of unstructured
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meshes $r$ can be written as (Greenshields et al., 2010),

$$ r = 2 \frac{d \cdot (\nabla \phi)_P}{(\nabla_d \phi)_f} - 1 $$ \hfill (7.16)

where $(\nabla \phi)_P(= \frac{1}{V_P} \sum_f S_f \phi_f)$ is the gradient calculated at the centroid $P$ of the CV
and $(\nabla_d \phi)_f(= \phi_N - \phi_P)$ is the face gradient in the direction of the area vector $S_f$, multiplied by $|d|$. For unstructured meshes it is practical to use the CD scheme as the HO one and in that case a flux-limited scheme is written as

$$ \phi_f = [1 - \Psi(1 - w_f)]\phi_P + \Psi(1 - w_f)\phi_N $$ \hfill (7.17)

The flux-limiters are designed to provide boundedness in the solution by satisfying particular boundedness criteria. One of them is the Total Variation Diminishing (TVD) criterion which was introduced by Harten (1983). The total variation, $TV(\phi^n)$, is defined as

$$ TV(\phi^n) = \sum_f | \phi^n_N - \phi^n_P | $$ \hfill (7.18)

and in order for this to be bounded for every timestep, one needs

$$ TV(\phi^n) \leq TV(\phi^o) $$ \hfill (7.19)

In order for the convection scheme to satisfy the TVD condition, the flux-limiter should lie in the Sweby’s diagram (Sweby 1984), which imposes limitations to the behaviour of the flux-limiter function,

$$ 0 \leq \Psi(r) \leq min(2r, 2) \quad \text{for} \quad r \geq 0 $$ \hfill (7.20)

$$ \Psi(r) = 0 \quad \text{for} \quad r < 0 $$ \hfill (7.21)
Some of the commonly used flux-limiters that lie in the Sweby’s diagram are:

- the limiter of van Leer (1974)

$$\Psi(r) = \frac{r + |r|}{1 + r}$$ (7.22)

- the MUSCL limiter of van Leer (1977)

$$\Psi(r) = max[0, min(2r, \frac{r + 1}{2}, 2)]$$ (7.23)

- the Superbee limiter of Roe (1983)

$$\Psi(r) = max[0, min(2r, 1), min(r, 2)]$$ (7.24)

- the Minmod limiter of Roe (1986)

$$\Psi(r) = max[0, min(r, 1)]$$ (7.25)

A extended discussion on TVD schemes and on other classes of schemes that satisfy alternative boundedness criteria is include in Waterson and Deconinck (2007).

Another class of second-order non-oscillatory schemes is the so-called central or Flux-Splitting (FS) schemes. In this case, the face interpolation method is split into two directions, one inward and one outward of the face. The discretization procedure of the Kurganov and Tadmor (2001) (KT) and the Kurganov, Noelle and Petrova (2001)(KNP) FS schemes for unstructured grids within the colocated, finite volume framework is described in Greenshields et al. (2010). In this case the convection term
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is discretized as

\[ (F_f \phi_f)_{FS} = \alpha F_{f+} \phi_{f+} + (1 - \alpha) F_{f-} \phi_{f-} + \omega_f (\phi_{f-} - \phi_{f+}) \]  

(7.26)

where the $f+$ direction corresponds to the $+S_f$ direction and the $f-$ direction corresponds to the $-S_f$ direction. The third term in Equation 7.24 introduces additional numerical diffusion and is used only for the convection term of the momentum equation, which is part of a substantive derivative. The diffusive volumetric flux is

\[ \omega_f = \text{amax}(\psi_{f+}, \psi_{f-}) \quad \text{KT method} \]

\[ \omega_f = a(1 - a)(\psi_{f+} + \psi_{f-}) \quad \text{KNP method} \]

The weighting factor is

\[ a = 1/2 \quad \text{KT method} \]

\[ a = \frac{\psi_{f+}}{\psi_{f+} + \psi_{f-}} \quad \text{KNP method} \]

The volumetric fluxes based on the local speeds of propagation are

\[ \psi_{f+} = \text{max}(c_{f+} |S_f| + \phi_{f+}, c_{f-} |S_f| + \phi_{f-}, 0) \]  

(7.27)

\[ \psi_{f-} = \text{max}(c_{f+} |S_f| - \phi_{f+}, c_{f-} |S_f| - \phi_{f-}, 0) \]  

(7.28)

where $c_{f\pm} = \sqrt{\gamma RT_{f\pm}}$ are the speeds of sound at the face, outward and inward of the CV. A TVD flux-limited scheme can be used for the $f+$ and the $f-$ face interpolations.
7.3.3 Diffusion Term

In an analogous way the diffusion term is approximated as,

$$\int_{S_p} (\Gamma_\phi \nabla \phi) \cdot dS \approx \sum_f \Gamma_\phi f S_f \cdot (\nabla \phi)_f$$

(7.29)

where $\Gamma_\phi$ is the diffusivity and $(\nabla \phi)_f$ is the face gradient of $\phi$. In the case of an orthogonal grid the face gradient expressed in the direction of the area vector $S_f$ is approximated as,

$$S_f \cdot (\nabla \phi)_f = |S_f| \frac{\phi_N - \phi_P}{|d|}$$

(7.30)

This approximation leads to accuracy of second-order because of the assumption of linear variation. In the case of a non-orthogonal grid the area vector, $S_f$, and the vector connecting the centroids of two neighboring CV’s, $d$, are not parallel and the second-order accuracy of the discretization scheme can be violated. A further correction to Equation 7.28 can be applied by decomposing the product $S_f \cdot (\nabla \phi)_f$ into two parts as

$$S_f \cdot (\nabla \phi)_f = D \cdot (\nabla \phi)_f + k \cdot (\nabla \phi)_f$$

(7.31)

where $D$ is chosen to be parallel with $d$ and is associated with the orthogonal contribution and $k$ is a vector satisfying the condition $S_f = D + k$ and is associated with the non-orthogonal contribution. The area vector can be decomposed in various ways, discussed in Jasak (1996), and one of them is the minimum-correction approach which defines the vector parallel to $d$ as

$$D = \frac{S_f \cdot d}{d \cdot d} d$$

(7.32)
In this way the contribution from $\phi_P$ and $\phi_N$ decreases as the non-orthogonality increases. An alternative decomposition is the orthogonal-correction approach, where

$$D = \frac{d}{d} | S_f |$$  \hspace{1cm} (7.33)

In this way the contribution from $\phi_P$ and $\phi_N$ is the same as in the case of an orthogonal mesh and does not depended on the non-orthogonality. Another method is the over-relaxed approach, where

$$D = \frac{d}{d} | S_f |^2$$  \hspace{1cm} (7.34)

With this method the contribution of $\phi_P$ and $\phi_N$ increases as the non-orthogonality increases. The diffusivity $\Gamma$ is usually interpolated with the CD scheme.

### 7.3.4 Source Terms

A general source term $S_\phi(\phi)$ is approximated as

$$\int_{V_P} S_\phi dV \approx S_{\phi_P} V_P$$  \hspace{1cm} (7.35)

where $S_\phi$ is a general non-linear function of $\phi$. To aid stability this can often be linearized, according to Patankar (1981). The source term can then be split into two parts and written as

$$S_\phi = S_c + S_p \phi$$  \hspace{1cm} (7.36)

and the volume integral as,

$$\int_{V_P} S_\phi dV = S_c V_P + S_p \phi_P V_P$$  \hspace{1cm} (7.37)
where $S_c$ and $S_p$ can also be functions of $\phi$. $S_p$ should be less than zero and $\phi_P$ should be treated as implicitly as possible in the case of implicit methods in order for this term to contribute to the diagonal dominance of the system matrix. Diagonal dominance is strongly related to the convergence rate and stability of the solution when an iterative solver is used.

### 7.3.5 Pressure-Gradient Term

The gradient of pressure in the momentum equation is discretized as

$$\int_{V_p} \nabla p dV = \int_{S_p} p dS \approx \sum_f S_f p_f$$  \hfill (7.38)

In the case of a FS interpolation the face values of pressure are approximated as

$$(p_f)_{FS} = \alpha p_{f+} + (1 - \alpha) p_{f-}$$  \hfill (7.39)

and a TVD flux-limited scheme can be used for the $f_+$ and the $f_-$ face interpolations.

Combining all the above treatments, the discretized transport equation for one CV results in an algebraic equation of the form,

$$a_P \phi_P^n + \sum_f a_N \phi_N^n = R_P$$  \hfill (7.40)

The unknown implicit terms are included on the right hand side of Equation 7.34 and the known explicit terms on the left hand side. Thus, in the discretized domain the discretization of one conservation equation results in an algebraic system of equations of the form,

$$[A][\phi] = [B]$$  \hfill (7.41)
The system matrix $A$ is a sparse matrix and has in its main diagonal the coefficients $a_P$. The coefficients $a_N$ comprise the remaining non-zero elements of $A$, the number of which depends on the number of the faces of the CV and the selected discretization scheme, which determines the computational molecule.

In the case of an implicit method the N-S equations are coupled and all the equations should, in principle, be solved simultaneously. This results in a system matrix that needs substantial CPU and memory usage. In order to overcome this issue the equations can be solved in a segregated manner. In this case each conservation equation is solved separately by treating implicitly only the transported variable in the equation and the rest explicitly. The decoupling of the equations in this manner introduces a lag in the solution, which could have significant effect in unsteady flows. Such lag effects in the solution are usually eliminated by performing additional iterations within the time loop [e.g. Issa (1986), Dermirdžić et al. (1993)]. Alternatively, these lag effects can be neglected, but a significantly reduced time-step is then often necessary.

### 7.4 Solvers for Compressible Flows in OpenFOAM

For this study the open source Computational Fluid Dynamics software package OpenFOAM v1.7.1 has been employed. The OpenFOAM software is based on colocated, polyhedral, finite volume numerics. A variety of solvers for compressible flows are implemented in OpenFOAM including both density-based and pressure-based ones. In the case of a density-based algorithm, the conservation equations are solved for the conserved variables $[\rho, (\rho u), (\rho E)]$, density is obtained from continuity equation and pressure from an equation of state. In the case of a pressure-based algorithm,
the conservation equations are solved for the primitive variables $[\rho, u, e]$, density is obtained from an equation of state and pressure from a pressure-velocity coupling equation. Some representative results of them are,

### 7.4.1 rhoCentralFoam

The rhoCentralFoam solver is a segregated, density-based algorithm and the convection terms are discretized with the flux-splitting KT or KNP schemes. The algorithm is:

- Initialize all the dependent variables

- Start of time loop

- Solve continuity equation for $\rho^n$:

$$V_P \frac{\rho^n_P - \rho^n_o}{\Delta t} - \sum_f (F_f \rho_f)_F S = 0 \quad (7.42)$$

- Solve momentum equations, excluding the diffusion terms, for an intermediate estimate of the momentum $(\rho u)^{imd}$:

$$V_P \frac{(\rho u)^{imd}_P - (\rho u)^{imd}_o}{\Delta t} + \sum_f [F_f(\rho u)_f]_FS + \sum_f S_f(p_f)_FS = 0 \quad (7.43)$$

- Update $u^{imd} = [(\rho u)^{imd}/\rho^n]$

- Solve momentum equations for $u^n$ including the diffusion term:
\[
\begin{aligned}
V_P \frac{\rho_P^n u_P^o - \rho_P^o u_P^n}{\Delta t} - V_P \frac{\rho_P^n u_P^{imd} - \rho_P^o u_P^o}{\Delta t} - \sum_f \mu_f^o S_f \cdot (\nabla u)_f^n \\
- \sum_f \mu_f^o S_f \cdot [(\nabla u)^T - \frac{2}{3} \nabla \cdot u]_f^n = 0 \quad (7.44)
\end{aligned}
\]

- Update \((\rho u)^n = \rho^n u^n\)

- Solve energy equation for \((\rho E)^{imd}\) excluding the heat flux:

\[
\begin{aligned}
V_P \frac{(\rho E)_P^{imd} - (\rho E)^o_P}{\Delta t} + \sum_f [F_f (\rho E)_{FS}^o]_f + \sum_f (F_f \rho_f)_{FS}^o - \sum_f \mu_f^o S_f \cdot (\nabla u)_f^n \cdot (u_f)_{FS}^o \\
- \sum_f \mu_f^o S_f \cdot [(\nabla u)^T - \frac{2}{3} \nabla \cdot u]_f^n \cdot (u_f)_{FS}^o = 0 \quad (7.45)
\end{aligned}
\]

- Update \(e^{imd} = (\rho E)^{imd} / \rho^n - 0.5(u^n \cdot u^n), T^{imd}(e^{imd}), \mu^{imd}(T^{imd})\) and \(\lambda^{imd}(\mu^{imd})\)

- Solve the energy equation for \(e^n\) including the heat flux:

\[
\begin{aligned}
V_P \frac{\rho_P^n e_P^o - \rho_P^o e_P^n}{\Delta t} - V_P \frac{\rho_P^n e_P^{imd} - \rho_P^o e_P^o}{\Delta t} - \sum_f \lambda_f^{imd} S_f \cdot (\nabla T)_f^{imd} = 0 \quad (7.46)
\end{aligned}
\]

- Update \(T^n(e^n), \mu^n(T^n), (\rho E)^n = \rho^n [e^n + 0.5(u^n \cdot u^n)]\) and \(p^n = \rho^n / (RT^n)\)

- End of time loop

### 7.4.2 rhoSonicFoam

The rhoSonicFoam solver is a segregated, density-based algorithm and a flux-limited scheme can be used for the discretization of the convection terms. This solver is only for inviscid calculations. The algorithm is:

- Initialize all the dependent variables
- Start of time loop

- Compute the volumetric flux \( F^o_j = S_f \cdot (\rho u)^o_j / \rho^o_f \)

- Solve continuity equation for \( \rho^n \):

\[
V_P \frac{\rho^n_P - \rho^n_o}{\Delta t} + \sum_f F^o_f \rho^n_f = 0 \tag{7.47}
\]

- Update pressure from equation of state \( p^{upd} = \rho^n R T^o \)

- Solve momentum equation for \((\rho u)^n\):

\[
V_P \frac{(\rho u)^n_P - (\rho u)^n_o}{\Delta t} + \sum_f F^o_f (\rho u)^n_f = - \sum_f S_f p^{upd}_f \tag{7.48}
\]

- Update velocity \( u^n = (\rho u)^n / \rho^n \) and volumetric flux \( F^n_j = S_f \cdot (\rho u)^n_j / \rho^n_j \)

- Solve energy equation for \((\rho E)^n\):

\[
V_P \frac{(\rho E)^n_P - (\rho E)^n_o}{\Delta t} + \sum_f F^o_f (\rho E)^n_f = - \sum_f F^n_f p^{upd}_f \tag{7.49}
\]

- Update temperature \( T^n = [(\rho E)^n / \rho^n - 0.5(\rho u)^n / \rho^n]^2] / C_v \)

- End of time loop

**7.4.3 sonicFoam**

The sonicFoam solver is a segregated, pressure-based solver and a flux-limited scheme can be used for the discretization of the convection terms. The algorithm is:

- Initialize all the dependent variables and the mass flux \( M^o_f = S_f \cdot (u^o \rho^o)_f \)
-Start of time loop

-The continuity equation is solved for an initial approximation of density $\rho^{\text{init}}$:

$$ V_P \frac{\rho_P^{\text{init}} - \rho_P^o}{\Delta t} + \sum_j M_f^o = 0 \quad (7.50) $$

-Solve momentum equation for an initial approximation of velocity $\mathbf{u}^{\text{init}}$:

$$ V_P \frac{\mathbf{u}_P^{\text{init}} - \mathbf{u}_P^o}{\Delta t} + \sum_j M_j^o \mathbf{u}_j^{\text{init}} - \sum_j \mu_j^o \mathbf{S}_j \cdot (\nabla \mathbf{u})_j^{\text{init}} $$

$$ - \sum_j \mu_j^o \mathbf{S}_j \cdot [(\nabla \mathbf{u})^T - \frac{2}{3} \nabla \cdot \mathbf{u}]_j^o $$

$$ - \sum_j \mu_j^o \mathbf{S}_j \cdot \{ (\nabla \mathbf{u})_j^T - \frac{2}{3} \nabla \cdot \mathbf{u} \} = \sum_j M_j^o \rho_j^o $$

(7.51)

-Solve energy equation for internal energy $e^n$ excluding the third term in Equation 7.3:

$$ V_P \frac{e_P^{\text{init}} - e_P^o}{\Delta t} + \sum_j M_j^o e_j^o - \sum_j (\lambda_j^o/C_v) \mathbf{S}_j \cdot (\nabla e)_j^o $$

$$ = - \sum_j (M_j^o/\rho_j^o) p_j^o $$

(7.52)

-Update $T^n$ using $e^n$, $\mu^n$ using $T^n$ and $\lambda^n$ using $\mu^n$

At this stage the updated velocity and density fields will not satisfy the continuity. The correction of the velocity field, and determination of the pressure field, is achieved with a prediction-correction iterative procedure called the PISO loop.

-Start of PISO loop:

1. Update density $\rho^n$ from the equation of state

2. The velocity field is expressed in a semi-discretized form as $\mathbf{u} = \frac{1}{a_P} \mathbf{H}(\mathbf{u}) - \frac{1}{a_P} \nabla p$. As an initial step the velocity field without any influence of pressure $\mathbf{u}^* = \frac{1}{a_P} \mathbf{H}(\mathbf{u})$ is calculated, using $\mathbf{u}^{\text{init}}$ to construct the $\mathbf{H}(\mathbf{u})$ operator in the first PISO loop.
3. Compute the volumetric fluxes multiplied by $\frac{1}{RT}$ as $F_j^* = \frac{1}{RT_j} u_j^* \cdot S_j$.

4. A Poisson equation is then obtained for pressure by substituting $u = u^* - \frac{1}{a_p} \nabla p$ in the continuity equation, assuming that the corrected velocity $u$ should satisfy continuity:

$$V_P \frac{1}{RT_P} \rho^n_P - \frac{1}{RT_P} \rho^0_P \Delta t + \sum_j F_j^* p_f^* - \sum_j (\rho_f^0 / a_p) S_j \cdot (\nabla p)_j^n = 0 \quad (7.53)$$

This is solved for the updated pressure, $p^n$.

5. Correct the velocity field using the corrected pressure $u = u - \frac{1}{a_p} \nabla p^n$. The discretized pressure gradient $\nabla p$, which is calculated at the centroids of the CV’s, is $\sum_j S_j p_f$.

-Repeat the PISO loop until the defined tolerance is reached.

At stage 4 of the PISO loop, the face gradient of pressure is calculated using the centroid values of $p$. At stage 5, the pressure gradient evaluated at the centroid of the CV is obtained using the face values of $p$. This method is described in Jasak (1994) as being ”in the spirit” of Rhie and Chow (1982) interpolation and provides oscillation-free solutions.

-Update density from the equation of state, using the corrected pressure from the last PISO loop.

-End of time loop
7.5 Summary

In this section the finite volume method for discretizing the N-S equations has been discussed. Additionally, a variety of convection schemes, including TVD flux-limited and flux-splitting ones, have been described. Finally, some basic algorithms, including density-based and pressure-based ones, for compressible flows available in OpenFOAM have been presented.
Chapter 8

Initial Shock Tube Tests

8.1 Introduction

In this section a variety of different numerical methods for resolving compressible flows including discontinuities available in OpenFOAM v1.7.1 are compared in order to select an appropriate numerical set up for the main calculations of this project. Therefore, the algorithms described in Chapter 7 along with a variety of convection schemes, including TVD flux-limited and flux-splitting schemes, are compared in a shock tube case.

8.2 Case Description

The shock tube test case is typically used for testing the accuracy of numerical methods since three discontinuities evolve simultaneously, namely an expansion fan, a contact discontinuity and a shock wave, introducing various steep gradient areas in the solution. Here the one-dimensional inviscid shock tube of Sod (1978) is consid-
Initial Shock Tube Tests

The gas is initially at rest and a diaphragm divides the shock tube in two flow regions of different pressure and temperature. When the diaphragm is removed a shock wave and a contact discontinuity propagate towards the low-pressure region and an expansion wave towards the high-pressure region of the flow. Since, these discontinuities evolve with different speeds the flowfield is separated into regions with uniform conditions.

8.3 Case Set Up

A shock tube, 15m long, is considered and the initial conditions of this case are shown in Figure 8.1. A thermally and calorically perfect gas was used as working medium, in which case the equation of state is $p = \rho RT$. For air the gas constant is taken as $R = 287.06 \, m^2 s^{-2} K^{-1}$, the specific heat coefficient for constant pressure as $C_p = 1004.06 \, J/(kgK)$ and the laminar Prandtl number as $P_l = 0.7$. A coarse one-dimensional uniform orthogonal mesh of 150 cells was used in order to demonstrate the accuracy of the different numerical set ups. A timestep of size 0.002ms was used to give time independent solutions.

![Figure 8.1: Initial conditions for the shock tube case.](image-url)
8.4 Results

Figure 8.2 displays a comparison of a range of TVD flux-limited schemes using the rhoSonicFoam algorithm. The numerical results for $\rho$ are compared with the analytical solution (Anderson, 2003) at time 7ms. The Minmod limiter gives the least oscillatory solution. Figure 8.2 displays also a comparison of the different algorithms. The density-based algorithms resolve more accurately the flow discontinuities and the KT flux-splitting scheme integrated in rhoCentralFoam solver gives a highly non-oscillatory solution. Therefore, the rhoCentralFoam solver was utilized for the rest of the calculations of this study. The flux-splitting KT scheme was used for the discretization of the convection terms in the continuity, momentum and energy equations. The Van-Leer limiter was used for the construction of the KT scheme. The UD scheme was used for the convection terms in turbulence equations. It should be mentioned that the performance of the pressure-based sonicFoam solver and the density-based rhoCentralFoam solver in a compression ramp case has been tested in Asproulias et al. 2011, indicating that a pressure-based solver can have a negative effect on the prediction of shock location in highly compressible flows.
Figure 8.2: Comparison of a range of TVD flux-limited convection schemes in OpenFOAM (top). Comparison of a variety of algorithms for compressible flows in OpenFoam (bottom).
8.5 Summary

A variety of numerical methods available in OpenFOAM v1.7.1 were tested in a shock tube case in order to determine the most appropriate numerical set up for the main calculations of this project. The density-based rhoCentralFoam solver, which incorporates the flux-splitting KT scheme for the discretization of the convection terms in N-S equations, is more efficient in resolving flow discontinuities and has been selected for the rest of the calculations.
Chapter 9

2D Compression Ramp

9.1 Case Description

The two-dimensional compression ramp is a test case widely used for validation of RANS models on predicting SWBLI’s. In this particular project, experimental measurements for an incoming boundary layer at $Re_{\theta} = 2400$, ramp angle of 24° and Mach number of 2.9, reported by Bookey et al. (2005) and Ringuette et al. (2009) have been used, with corresponding DNS data from Wu and Martin (2007). This case is denoted here as the low Reynolds number compression ramp. Additional measurements for $Re_{\theta} \approx 80000$, Mach number of 2.9 and ramp angles of 16, 20 and 24° are available in Settles and Dodson (1994). This case is referred to here as the high Reynolds number compression ramp.

In the compression ramp flow, the deflection of the boundary layer on the inclined wall results in a shock wave, which imposes an adverse pressure gradient on the incoming turbulent boundary layer. For the 16° ramp the flow is under incipient separation conditions. For the higher ramp angles the shock is sufficiently strong to
cause flow separation and a recirculation region. Two compression fans arise, one in the separation region and another in the reattachment region, forming a $\lambda$-shock.

### 9.2 Case Set Up

A thermally and calorically perfect gas was used as working medium, in which case the equation of state is $p = \rho RT$. For air the gas constant is taken as $R = 287.06 \text{ m}^2\text{s}^{-2}\text{K}^{-1}$, the specific heat coefficient for constant pressure as $C_p = 1004.06 \text{ J/(kgK)}$ and the laminar Prandtl number as $Pr_l = 0.7$. The turbulent Prandtl number is taken as $Pr_t = 0.9$. The dynamic viscosity is estimated using a Sutherland's law, $\mu = 1.458 \times 10^{-6} T^{3/2}/(T + 110.3)$.

The DNS flow conditions, which were used for the calculations in the low Reynolds number case, are given in Table 9.1. Additionally, the experimental flow conditions used for the calculations in the high Reynolds number cases are shown in Table 9.2.

| Table 9.1: DNS flow conditions for the low Reynolds number compression ramp reported in Wu and Martin (2007). |
|---|---|---|---|---|---|---|
| $M_\infty$ | $Re_{\theta_0}$ | $\theta_0,\text{mm}$ | $\delta_0,\text{mm}$ | $U_\infty$ | $\rho_\infty,\text{kg/m}^3$ | $T_\infty,\text{K}$ |
| 2.9 | 2300 | 0.38 | 6.4 | 609.1 | 0.077 | 107.1 |
| | | | | | | 307 |

| Table 9.2: Experimental flow conditions for the high Reynolds number compression ramps reported in Settles and Dodson (1994). |
|---|---|---|---|---|---|---|
| $M_\infty$ | $Re_{\theta_0}$ | $\theta_0,\text{cm}$ | $\delta_0,\text{cm}$ | $U_\infty,\text{m/s}$ | $p_\infty,\text{N/m}^2$ | $T_\infty,\text{K}$ |
| 16° | 2.85 | 81900 | 0.13 | 2.6 | 576 | 2.29e+04 | 102.1 |
| 20° | 2.85 | 81900 | 0.13 | 2.5 | 562 | 2.32e+04 | 98.3 |
| 24° | 2.84 | 75600 | 0.12 | 2.3 | 569 | 2.36e+04 | 100.3 |
| | | | | | | 276 |

Structured grids were used, uniform in the streamwise direction and refined normal to the wall, with the non-dimensional wall-distance of the near-wall node being around
$y_w^+ \leq 0.6$. The size of the computational domains used and a sample grid are shown in Figure 9.1.

At the inlet suitable profiles to match the DNS and experimental characteristics of the incoming boundary layer (given in Tables 9.1 and 9.2) were applied, and these were taken from a separate flat plate boundary layer simulation with 1.5% freestream turbulence intensity and a ratio of $\mu_t/\mu = 10$. At the wall no slip conditions were applied, with zero gradient for the pressure, and temperature was fixed to the DNS and experimental values given in Tables 9.1 and 9.2. At the top and outlet boundaries zero gradient conditions were applied for all variables. A total number of (150x70) CV’s were used for the low Reynolds number case and (250x150) for the high Reynolds number cases. Figure 9.2 and 9.3 display grid-independence studies indicating the used meshes give grid-independent solutions. In all cases the calculations were initialised with the corresponding inlet profiles. The time step size was defined by the maximum CFL number, chosen as 0.3.

### 9.3 Results

Figure 9.4 displays iso-lines of mean pressure superimposed on contours of mean streamwise velocity for the low Reynolds compression ramp. All the models broadly capture the flow features. The CLS model predicts a kink in the pressure contour patterns as ones moves away from the wall downstream of the reattachment of the flow, which is not predicted by the other models. Figure 9.5 displays wall-quantities for the low Reynolds number case. The LS, SST and CLS models predict quite accurately the wall-pressure variation, whilst PHIF predicts an upstream shift of the wall-pressure rise. The CLS model gives a quite accurate estimation of the size of
the separation bubble as indicated by the skin-friction variation, whilst the other models overestimate it. Downstream of the flow reattachment, all the models predict a steeper variation of skin-friction comparing to the one suggested by DNS.

Figure 9.6 displays comparisons of mean velocity profiles for the low Reynolds number compression ramp at five stations through the SWBLI zone. At $x/\delta = -8$ the incoming boundary layer imposed for the RANS calculations is in good agreement with DNS. At $x/\delta = -1.9$ the DNS $C_f$ results show the flow has separated, although the velocity profile suggests only a rather weak backflow. The PHIF scheme predicts a thicker boundary layer, due to the earlier separation, whilst the other models predict a boundary layer thickness comparable to the DNS one. The CLS model gives a better estimation of the backflow velocity, whilst the other models overestimate it. Downstream of the separation bubble, at $x/\delta = 4.2$ and 6.1, the boundary layer relaxes towards its equilibrium state. The DNS velocity profile is of lower gradient in the inner part of the boundary layer than in the outer, a feature that is broadly captured only by the CLS scheme. Figure 9.7 displays comparisons of Reynolds shear stress profiles for the low Reynolds number compression ramp. The DNS data are rather spiky, which could be possibly associated with insufficient averaging. Therefore, only qualitative comparisons can be made between the results from DNS and RANS calculations. At $x/\delta = 1$, where the DNS flow is very close to the reattachment point, all the models predict a considerably weaker amplification of the $\tilde{u}'\tilde{v}'$ levels. Further downstream in the relaxation zone, at $x/\delta = 4.2$, all the models still underestimate $\tilde{u}'\tilde{v}'$. At the other stations, $x/\delta = -8$, -1.9 and 6.1, the models give better estimations of Reynolds shear stress. The DNS results indicate an enhancement of the Reynolds stress anisotropy in the SWBLI zone and a relaxation of it to its equilibrium levels downstream of the flow reattachment, as shown in Figure 9.8.
The streamwise and wall-normal Reynolds stress components predicted by the LS model are quite isotropic, as expected from a linear EVM. The CLS models give better estimations of the Reynolds stress anisotropy, due to the quadratic terms in the non-linear stress-strain relation, although the amplification of it is underestimated. Tests using the CLS model without the non-linear stress-strain relation show that the resolution of Reynolds stress anisotropy affects the mean velocity mainly within the recirculation region of the flow, where mean normal-strain rates are significant. The incorporation of the non-linear stress-strain relation improves the prediction of $C_f$ within the separation bubble, as shown in Figure 9.5. Therefore, a better representation of the Reynolds stress anisotropy could possibly improve the predictions of the models within the SWBLI zone. Additionally, all the models predict a weaker amplification of turbulent kinetic energy than shown in the DNS data within the separation bubble, as shown in Figure 9.9. In general, the EVM’s tested here underestimate the effect of the SWBLI on turbulent quantities.

Figure 9.10 displays iso-lines of mean pressure superimposed on contours of mean streamwise velocity for the high Reynolds number compression ramps predicted by the CLS model. For increasing shock strength, as the compression angle is increased, the thickening of the boundary layer due to the SWBLI is enhanced and the size of the separation bubble increases. Figure 9.11 displays wall-quantities for the high Reynolds number compression ramps. For the 16° compression ramp the experimental flow is under incipient separation conditions and all the models predict only a weak flow separation. For the 20 and 24° compression ramps all the models predict a separation bubble size comparable to the experimental one as indicated by the skin-friction variation, although they do not fully capture the correct shock location in all cases as indicated by the wall-pressure variation. Downstream of the reattachment of
the flow most of the models broadly capture the recovery of $C_f$, although the PHIF scheme returns a large initial overshoot of skin-friction.

Figure 9.1: Computational domains and a sample grid for the two-dimensional compression ramp case. LR denotes the low Reynolds number case and HR the high Reynolds number case.
Figure 9.2: Grid independency study for the low Reynolds number $24^\circ$ compression ramp case. Results are from LS (solid line) and PHIF (dashed line).
Figure 9.3: Grid independency study for the high Reynolds number 24° compression ramp case. Results are from LS.
Figure 9.4: Iso-lines of mean pressure superimposed on contours of mean streamwise velocity for the low Reynolds number compression ramp. Predictions are from LS (top left), SST (top right), PHIF (bottom left) and CLS (bottom right). $x_s$ and $x_r$ denote the mean separation and reattachment points respectively, whilst the CLS plot also shows the locations (in terms of $x/\delta_0$) where subsequent comparisons of profiles with the data of Wu and Martin (2007) will be made.
Figure 9.5: Mean wall-pressure (top) and skin-friction (bottom) distribution for the low Reynolds number compression ramp. Experiment (circle), DNS (square), LS (black), SST (blue), PHIF (orange), CLS (red) and linear CLS (green).
Figure 9.6: Comparison of mean velocity profiles for the low Reynolds number compression ramp at five streamwise locations (indicated as lines on Figure 9.4). DNS (square), LS (black), SST (blue), PHIF (orange) and CLS (red).
2D Compression Ramp

Figure 9.7: Comparison of Reynolds shear stress ($-\overline{uv''}$) profiles for the low Reynolds compression ramp at five streamwise locations (indicated as lines on Figure 9.4). DNS (square), LS (black), SST (blue), PHIF (orange) and CLS (red).
Figure 9.8: Comparison of Reynolds normal stresses ($\overline{u''u''}$; solid line), ($\overline{v''v''}$; dashed line) profiles for the low Reynolds compression ramp at five streamwise locations (indicated as lines on Figure 9.4). DNS($\overline{u''u''}$; square, $\overline{v''v''}$; triangle), LS (black) and CLS (red).
Figure 9.9: Comparison of turbulent kinetic energy ($k \equiv \overline{u_i'u_i}/2$) profiles for the low Reynolds compression ramp at five streamwise locations (indicated as lines on Figure 9.4). DNS (square), LS (black), SST (blue), PHIF (orange) and CLS (red).
Figure 9.10: Iso-lines of mean pressure superimposed on contours of mean streamwise velocity for the high Reynolds number compression ramps. Predictions are for the 16° (top left), for the 20° (top right) and for the 24° (bottom left) compression ramps from CLS. $x_s$ and $x_r$ denote the mean separation and reattachment points respectively.
Figure 9.11: Mean wall-pressure (left) and skin-friction (right) distribution for the high Reynolds compression ramps. Experiment (circle), LS (black), SST (blue), PHIF (orange) and CLS (red). Each row corresponds to a different case; 16° (top), 20° (middle) and 24° (bottom).
9.4 Summary

The LS, SST, PHIF and CLS models have been tested in supersonic 2D compression ramps of different angles and Reynolds numbers of the incoming boundary layer.

The main conclusions drawn for the low Reynolds number $24^\circ$ compression ramp case are:

- The CLS model predicts a kink in the pressure contour patterns as ones moves away from the wall downstream of the reattachment of the flow, which is not predicted by the other models.

- The LS, SST and CLS models predict quite accurately the wall-pressure variation, whilst the PHIF scheme predicts an upstream shift of the wall-pressure rise.

- The CLS model predicts quite accurately the size of the separation bubble, whilst the other models overestimate it.

- The CLS model is the only model to predict the correct qualitative behaviour of the boundary layer through the SWBLI zone.

- All the models underestimate the Reynolds stress and turbulent kinetic energy amplification in the region close to the reattachment of the flow.

- The CLS model, which as expected is the only one to predict qualitatively correct normal stress anisotropy, underestimates the amplified levels of anisotropy between the Reynolds normal stresses through the SWBLI zone.

The main conclusions drawn for the high Reynolds number $16$, $20$ and $24^\circ$ compression ramp cases are:
• For all the ramp angles all the models predict a size of separation bubble comparable to the experimental one, with LS giving the best approximation for the separation point of the flow.

• For all the ramp angles, most of the models broadly capture the recovery of $C_f$ downstream of the reattachment of the flow, although the PHIF scheme returns a large initial overshoot of skin-friction.

• The LS model has been also examined in the high Reynolds number compression ramp cases for example by Gerolymos et al. (2004) and Sinha et al. (2005). In both these previous studies, the LS model predicts an initial overshoot of skin-friction downstream of the flow reattachment, unlike in the present study. This could be attributed to the effect of the Yap correction, which has not been included in these previous studies. The size of the separation bubble and the wall-pressure rise predicted in these previous studies are close to the ones predicted in the present study.

• The SST model has been also examined in the high Reynolds number compression ramp cases for example by Liou et al. (2000) and Oliver et al. (2007). In both these previous studies, the SST model overpredicts the separation bubble size significantly, unlike in the present study. This could be attributed to the formulation of the eddy viscosity limiter used in these previous studies, which is defined using the mean-vorticity tensor. In the present study the eddy viscosity limiter is defined using the mean-strain rate tensor.
Chapter 10

2D Impinging Shock

10.1 Case Description

Another test case typically used for validation of RANS models on predicting SWBLI’s is the two-dimensional impinging shock. Experimental data for shock turning angle of 8°, $Re_\theta = 5350$ and Mach number of 2.25 have been reported by Deleuze (1995), Laurent (1996) and DNS for $Re_\theta = 3725$ by Pirozzoli and Grasso (2006). This case is denoted here as the low Reynolds number impinging shock (despite the slight difference in Reynolds number between the DNS and experimental cases). Further experimental data for $Re_\theta = 47000$, shock turning angles of 7, 10 and 13° and Mach number of 2.9 have been reported by Reda and Murphy (1973), Murthy and Rose (1978) and Modaress and Johnson (1979). This case is referred to here as the high Reynolds number case.

In this test case an oblique shock, produced by an external source, impinges on a flat plate and reflects, imposing an adverse pressure gradient on the incoming turbulent boundary layer. For the 7° shock the flow remains attached. For the rest of
the cases the shock is sufficiently strong to cause flow separation and a recirculation region is developed. In those cases, the flow turns at the separation point and a compression fan is developed intruding into the boundary layer. Further away from the wall, the separation compression waves coalesce to form the principal reflected shock. Additionally, at the maximum height of the separation bubble the flow is turned towards the wall through an expansion fan and reattaches. Finally, near the reattachment point a sequence of compression waves realigns the flow with the wall.

### 10.2 Case Set Up

The working medium and the thermodynamic properties were the same as in the compression ramp case. The DNS flow conditions, which were used for the calculations in the low Reynolds number impinging shock, are given in Table 10.1. Additionally, the experimental flow conditions used for the calculations in the high Reynolds number impinging shocks are shown in Table 10.2.

Table 10.1: DNS flow conditions for the low Reynolds number impinging shock reported in Pirozzoli and Grasso (2006).

<table>
<thead>
<tr>
<th>$M_{\infty}$</th>
<th>$Re_{\theta_0}$</th>
<th>$\theta_0$,mm</th>
<th>$\theta_0$,mm</th>
<th>$p_{\infty},N/m^2$</th>
<th>$T_{\infty},K$</th>
<th>$T_w,K$</th>
</tr>
</thead>
<tbody>
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<td>2.25</td>
<td>3725</td>
<td>0.147</td>
<td>2</td>
<td>23813</td>
<td>169.4</td>
<td>322.2</td>
</tr>
</tbody>
</table>

Table 10.2: Experimental flow conditions for the high Reynolds number impinging shocks reported in Reda and Murphy (1973).

<table>
<thead>
<tr>
<th>$M_{\infty}$</th>
<th>$Re_{\theta_0}$</th>
<th>$\theta_0$,cm</th>
<th>$\delta_0$,cm</th>
<th>$p_{\infty},N/m^2$</th>
<th>$T_{\infty},K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>47000</td>
<td>0.082</td>
<td>1.69</td>
<td>689010</td>
<td>291</td>
</tr>
</tbody>
</table>

Structured grids were used, uniform in the streamwise direction and refined normal to the wall, with the non-dimensional wall-distance of the near-wall node being around
The size of the computational domains used and sample grids are shown in Figure 10.1.

At the lower part of the inlet boundary (shown in Figure 10.1) suitable profiles to match the DNS and experimental characteristics of the incoming boundary layer (given in Tables 10.1 and 10.2) were applied, and these were taken from a separate flat plate boundary layer simulation with 1.5% freestream turbulence intensity and a ratio of $\mu_t/\mu = 10$. At the upper part of the inlet boundary the shock-relations (Anderson, 2003) were applied to introduce the incident shocks in the computational domain. At the wall no-slip conditions were applied, with zero gradient for the pressure. For the low Reynolds number case temperature was fixed to the DNS value given in Table 10.1. For the high Reynolds number cases the experimental wall-temperature is not reported and a fixed value of 271 K, which is the one used by Gerolymos et al. (2004), was applied. At the top and outlet boundaries zero gradient conditions were applied for all variables. A total number of (150x78) CV’s were used for the low Reynolds number case and (400x203) for the high Reynolds number cases. Figure 10.2 and 10.3 display grid-independence studies indicating the used meshes give grid-independent solutions. In all cases the calculations were initialised with the corresponding inlet profiles. The time step size was defined by the maximum CFL number, chosen as 0.3.

### 10.3 Results

Figure 10.4 displays iso-lines of mean pressure superimposed on contours of mean streamwise velocity for the low Reynolds number impinging shock. All the models broadly capture the flow features. Downstream of the separation bubble the DNS shows a kink in the pressure contour patterns as ones moves away from the wall, a
feature that is broadly captured only by the CLS model. The CLS predicts a similar pressure variation in the case of the compression ramp, though DNS data are not available to make comparisons. The amplification of turbulent kinetic energy due to the compression of the flow and the thickened shear layer through the interaction zone is shown in Figure 10.5. The DNS suggests an initial amplification of $k$ in the region of the separation point which continues to nearly half the streamwise length of the recirculation region of the flow (the mean separation point of the DNS flow is at 8.549 and the mean reattachment point is at 8.626 on the axis of the DNS results). This flow feature is broadly captured only by the CLS model, whilst the other models predict continuous amplification of $k$ within the separation bubble. Tests have shown that this growth pattern is captured by the CLS model due to the non-linear tensorial elements in the stress-strain relation. Within the separation bubble, mean normal strain rates are significant, as well as shearing, and consequently the Reynolds normal stresses here also affect the generation rate of $k$. Since the CLS model is the only one tested here which captures, at least qualitatively, the Reynolds normal stresses, as well as the Reynolds shear stress in the SWBLI region, as shown in Figure 10.6, this explains its improved predictions of $k$. Additionally, CLS is the only model to capture the qualitative manner the Reynolds shear stress is transported through the SWBLI zone, as shown in Figure 10.6.

Figure 10.7 displays wall quantities for the low Reynolds number case. The LS model predicts a downstream shift of the shock structure, as indicated by the wall-pressure variation. The other models predict shock locations closer to that suggested from the experiment and the DNS, though the pressure rise is smoother, associated with the larger separation bubble these models predict, as indicated by the skin-friction variation. The LS model gives a better estimation of the separation point of
the flow than the other models, whilst the reattachment point is better predicted by CLS. The SST model overestimates the magnitude of $C_f$ in the recirculation region of the flow, whilst the other models give better estimations. Downstream of the SWBLI most of the models broadly capture the recovery of $C_f$, although the PHIF scheme returns a large initial overshoot of skin-friction.

Figure 10.8 displays comparisons of mean velocity profiles for the low Reynolds number impinging shock at six stations (indicated as lines on a separate graph in Figure 10.8). At a station upstream of the SWBLI (probe1) the incoming boundary layers used for the RANS calculations are in good agreement with the DNS one. At a station downstream of the separation point (probe 4), LS predicts a boundary layer thickness comparable to the DNS one. All the other models predict a thicker boundary layer, due to the earlier separation. At a station further downstream within the separation bubble (probe 6) LS and SST predict quite accurately the boundary layer thickness, whilst PHIF and CLS models overestimate it. The SST model overpredicts the maximum backflow velocity, whilst the other models give better estimations. The DNS velocity profile at this location is of lower gradient in the inner part of the boundary layer than in the outer, a characteristic captured only by the CLS model, due to the non-linear tensorial elements in the stress-strain relation indicated by relative tests. Downstream of the flow reattachment (probes 9, 10 and 13) the boundary layer relaxes to its equilibrium state. The DNS suggests a boundary layer of lower gradient in the inner part of the boundary layer than in the outer, as shown at probe 10. This feature is broadly captured only by the CLS model, which could be linked with the predicted development of the boundary layer in the recirculation region of the flow.

Figure 10.9 displays a comparison of k profiles predicted by the models at six
stations for the low Reynolds number impinging shock. At probe 4 all the models predict amplification of $k$. At this station CLS and PHIF models predict higher peak value of $k$, which could be associated with the steeper velocity profiles across the boundary layer predicted by these models. Further downstream in the recirculation region at probe 6 LS, SST and PHIF predict higher $k$ levels in comparison to the predictions at probe 4. On the other hand, CLS predicts similar levels of $k$, which is in agreement with the conclusions drawn from the results in Figure 10.5. Downstream of the flow reattachment $k$ recovers to its equilibrium value. In the recovery region the PHIF model gives higher levels of $k$ near the wall than the other models.

Figure 10.10 displays a comparison of Reynolds shear stress profiles predicted by the models at six stations. At probe 4 all the models predict amplification of $-\tilde{u}''\tilde{v}''$, with PHIF giving a higher peak value than the other models. At probe 6, LS, SST and PHIF predict amplification of $-\tilde{u}''\tilde{v}''$ comparing to the predictions at probe 6. On the other hand, the Reynolds shear stress levels predicted by CLS are similar to the ones predicted at probe 4, which is consistent with the conclusions drawn from the results in Figure 10.6. The lower levels of $-\tilde{u}''\tilde{v}''$ predicted by CLS in this region could be linked with the improved response of the boundary layer predicted by CLS at probe 6. In the recovery zone PHIF predicts higher levels of Reynolds shear stress in the near wall region in comparison to the predictions of the other models, which could be linked with the considerable overprediction of $C_f$ by PHIF downstream of the flow reattachment. This, in combination with the higher near-wall $k$ levels predicted by PHIF in this region, leads to the conclusion that near-wall turbulent scales are considerably overestimated by PHIF downstream of the flow reattachment.

A comparison of turbulent kinetic energy generation rates, $P_k$, and turbulent diffusion, $T_k$, profiles for the low Reynolds number case are displayed in Figures 10.11
and 10.12 respectively. At a station downstream of the separation point (probe 4) the models underestimate considerably $P_k$ levels. Further downstream, but still within the separation bubble (probe 6), the near-wall peak value of $P_k$ is overestimated by LS, SST and PHIF. The CLS model predicts better the peak value of $P_k$, though the location of the maximum is somewhat further away from the wall than in the DNS data, due to the earlier separation. In the recovery zone SST gives better estimations of near-wall peak values of $P_k$, whilst the other models overestimate it. The PHIF scheme overestimates considerably the near-wall $P_k$ levels in this region of the flow, which is consistent with the higher levels of near-wall Reynolds shear stress predicted by PHIF in this region. The turbulent diffusion term predicted by the models is not generally in good agreement with the DNS, especially in the recirculation region of the flow. The peak values of $T_k$ in most locations are not well captured by the models. Though, it should be noted that turbulent diffusion is generally fairly small and often does not have a huge impact on overall behaviour. Figure 10.13 displays a comparison of dissipation rate profiles for the low Reynolds number case at five stations upstream and downstream of the recirculation region. At probe 1 the boundary layer is in equilibrium and PHIF is the only model to predict quite accurately the asymptotic near-wall behaviour of $\varepsilon$, which is associated with the formulation of the $\varepsilon$-equation incorporated in this model. The other models predict lower levels of $\varepsilon$ in the near-wall region. At probe 3 the DNS shows that the boundary layer is starting to feel the effects of the adverse pressure gradient from the compression fan upstream of the separation point. The LS model, which is the only model to predict attached flow at this station, overestimates the amplified peak value of $\varepsilon$. At stations in the relaxation zone only qualitative comparisons can be made, since the quantities are normalised with mean local wall units. All the models overestimate the levels of normalized dissipation rate in this region, even though the friction velocity predicted by them is
somewhat higher than in the DNS, as indicated by the skin-friction variation (Figure 10.7). Therefore, the levels of dissipation rate are overpredicted by the models in the non-equilibrium region of the flow upstream and downstream of the separation bubble, where the flow is subject to fast changes of mean strain rates. It should be noted the equations that determine the turbulence length scale, the modelling of which includes a lot of empiricism, reproduce reasonably well the shape of the $\varepsilon$ profile through the SWBLI zone.

Figure 10.14 displays iso-lines of mean pressure superimposed on contours of mean streamwise velocity for the high Reynolds number impinging shocks predicted by the CLS scheme. For increasing shock strength (as the shock angle increases) the thickening of the boundary layer due to the SWBLI is enhanced, the structure of the reflected shock is shifted further upstream of the inviscid impingement point $x_I$ and the size of the separation bubble increases in the cases where the flow is separated. The CLS model predicts a pressure kink downstream of the SWBLI, whilst LS (included in the same figure, for the $13^\circ$ case) does not predict such a flow feature. The predictions of SST and PHIF are similar to LS. Figure 10.15 displays wall-quantities for the high Reynolds number impinging shocks. For the $7^\circ$ shock the wall-pressure variation predicted by all the models is in good agreement with the experimental one. The SST is the only model to predict a weak separation of the flow, whilst the experimental flow and the flow predicted by the other models are attached. In the case of the $20^\circ$ impinging shock, all the models predict reasonably well the shock location, as indicated by the wall-pressure variation, and the size of the separation bubble, as indicated by the skin-friction variation. In the case of the $13^\circ$ impinging shock, the CLS model predicts better the wall-pressure rise, whilst LS and SST models predict a downstream shift of it. The CLS model gives better
estimations of the negative values of $C_f$ in the region of the separation bubble, which can be attributed to the effect of the non-linear stress-strain relation.

Figure 10.16 displays a comparison of the velocity profiles at seven stations (the locations of which are indicated as lines on Figure 10.14). The performance of the models is similar to the that noted for the low Reynolds number case. The CLS gives improved predictions of the streamwise Reynolds stress levels, as shown in Figure 10.17, though underestimating them in some locations, especially in the separation bubble.

![Figure 10.1: Computational domains and a sample grid for the two-dimensional impinging shock case. LR denotes the low Reynolds number case and HR the high Reynolds number case.](image-url)
Figure 10.2: Grid independency study for the low Reynolds number impinging shock case. Results are from LS (solid line) and PHIF (dashed line).
Figure 10.3: Grid independency study for the 13° high Reynolds number impinging shock case. Results are from LS.
Figure 10.4: Iso-lines of mean pressure superimposed on contours of mean streamwise velocity. Predictions are from DNS (top left), LS (middle left), SST (middle right), PHIF (bottom left) and CLS (bottom right) for the low Reynolds number impinging shock. $x_s$ and $x_r$ are the mean separation and reattachment points respectively.
Figure 10.5: Iso-lines of turbulent kinetic energy \( (k \equiv \frac{u''u''}{2}) \). Predictions are from DNS (top left), LS (middle left), SST (middle right), PHIF (bottom left) and CLS (bottom right) for the low Reynolds number impinging shock. \( x_s \) and \( x_r \) are the mean separation and reattachment points respectively.
Figure 10.6: Iso-lines of Reynolds shear stress ($-\overline{u'v''}$). Predictions are from DNS (top left), LS (middle left), SST (middle right), PHIF (bottom left) and CLS (bottom right) for the low Reynolds number impinging shock.
Figure 10.7: Mean wall-pressure (top) and skin-friction (bottom) distribution for the low Reynolds number impinging shock case. Experiment (circle), DNS (square), LS (black), SST (blue), PHIF (orange) and CLS (red).
Figure 10.8: Comparison of mean velocity profiles for the low Reynolds number impinging shock at six streamwise locations. DNS (square), LS (black), SST (blue), PHIF (orange) and CLS (red). At the top of the figure, the locations of the probes (defined in Pirozzoli and Grasso 2006) for data comparison are shown. $x_s$ and $x_r$ are the mean separation and reattachment points respectively. $x_I$ is the inviscid impingement point. Iso-lines of mean pressure superimposed on contours of mean streamwise velocity predicted by LS are shown.
Figure 10.9: Comparison of turbulent kinetic energy \( (k = u''_i u''_i / 2) \) profiles for the low Reynolds number impinging shock at six streamwise locations (indicated as lines on Figure 10.8). LS (black), SST (blue), PHIF (orange) and CLS (red).
Figure 10.10: Comparison of Reynolds shear stress ($-\bar{u}''v''$) profiles for the low Reynolds number impinging shock at six streamwise locations (indicated as lines on Figure 10.8). LS (black), SST (blue), PHIF (orange) and CLS (red).
Figure 10.11: Comparison of turbulent kinetic energy production term \( P_k = -\bar{\rho}u_i'\bar{u}_j'\frac{\partial u_i}{\partial x_j} \) profiles for the low Reynolds number impinging shock at six streamwise locations (indicated as lines on Figure 10.8). DNS (square), LS (solid black), SST (solid blue), PHIF (solid orange) and CLS (solid red). Comparison of dissipation rate \( D_k = -\sigma_{ij}\frac{\partial u_i}{\partial x_j} \); dashed line) profiles predicted by the models is shown.
Figure 10.12: Comparison of turbulent diffusion term \( T_k = -\frac{\partial}{\partial x_j} \left[ \frac{1}{2} \bar{\rho} \bar{u}_i'' \bar{u}_j'' \right] \) profiles for the low Reynolds number impinging shock at six streamwise locations (indicated as lines on Figure 10.8). DNS (square), LS (solid black), SST (solid blue), PHIF (solid orange) and CLS (solid red). Comparisons of viscous diffusion \( V_k = \frac{\partial}{\partial x_i} \left( \sigma_{ij} \bar{u}_j'' \right) \) (dashed line) and convection \( C_k = -\frac{\partial}{\partial x_i} \left( \bar{p} \bar{u}_j'' \right) \) (dotted line) terms profiles between the models is shown.
Figure 10.13: Comparison of turbulent dissipation rate ($\varepsilon^+ = \nu \bar{\rho} \varepsilon / (\bar{\rho} \bar{u}_t^4)$) profiles for the low Reynolds number impinging shock at five streamwise locations (indicated as lines on Figure 10.8). DNS (square), LS (black), SST (blue), PHIF (orange) and CLS (red).
Figure 10.14: Iso-lines of mean pressure superimposed on contours of mean streamwise velocity for the high Reynolds number impinging shocks. Predictions are for the 7° impinging shock from CLS (top left), for the 10° impinging shock from CLS (top right), for the 13° impinging shock from LS (bottom left) and for the 13° impinging shock from CLS (bottom right). Locations (defined in Modaress and Johnson 1979) for data comparison for the 13° high Reynolds number impinging shock is shown (bottom right), where the numbers are non-dimensional locations \((x - x_I)/\delta_0\).
Figure 10.15: Mean wall-pressure (left) and skin-friction (right) distribution for the high Reynolds number impinging shocks. Experiment (circle), LS (black), SST (blue), PHIF (orange) and CLS (red). Each row corresponds to a different case; 7° (top), 10° (middle) and 13° (bottom).
Figure 10.16: Comparison of mean velocity ($\bar{u}$; calculations, $\bar{u}$; experiment) profiles for the 13° high Reynolds number impinging shock at seven streamwise locations (indicated as lines on Figure 10.14). Experiment (circle), LS (black), SST (blue) and CLS (red).
Figure 10.17: Comparison of Reynolds streamwise stress $\langle u''u'' \rangle$; calculations $\langle u'\bar{u}' \rangle$; experiment) profiles for the 13° high Reynolds number impinging shock at seven streamwise locations (indicated as lines on Figure 10.14). Experiment (circle), LS (black) and CLS (red).
10.4 Initial Investigations for Capturing Shock Unsteadiness

It should be mentioned that none of the incorporated models could predict the low-frequency oscillation of the reflected shock. Figure 10.18 shows the variation of the $C_\mu$ functional form incorporated by the CLS model throughout the interaction zone for the low Reynolds number impinging shock case. The CLS model predicts $C_\mu$ values in the range of 0.02-0.05 in regions of high shear within the interaction zone. Values of $C_\mu$ around 0.02-0.05 have been identified by Hoarau et al. (2005) in non-equilibrium regions of the unsteady flow around a NACA0012 airfoil. In order to examine the effect of $C_\mu$ variation in predicting shock unsteadiness a constant value of 0.04 has been imposed using the LS model. Figure 10.19 displays the Power Spectral Density (PSD) of pressure predicted by LS with constant $C_\mu$ value of 0.04 for the low Reynolds number $8^\circ$ impinging shock case at four stations throughout the interaction zone. At all stations high energetic frequencies arise of $O(10^5)$ Hz, which are one order of magnitude higher than the characteristic frequencies of wall-pressure signals measured by Dupont et al. (2005) for an $8^\circ$ impinging shock case and $Re_\theta = 4500$. 
Figure 10.18: Contours of $C_\mu$ predicted by the CLS model superimposed on iso-lines of mean pressure for the low Reynolds number impinging shock case.
Figure 10.19: Power spectral density of pressure predicted by LS with constant $C_\mu$ value of 0.04 for the low Reynolds number 8° impinging shock case at four stations. Inside the mixing layer within the interaction zone (top left and top right), on the wall within the recirculation region (bottom left) and on the wall downstream of the flow reattachment (bottom right).
Figure 10.20: Power spectral density of wall-pressure at the middle of the recirculation bubble (left) and at the end of the interaction (right) measured experimentally by Dupont et al. (2004) for an $8^\circ$ impinging shock case.
10.5 Summary

The LS, SST, PHIF and CLS models have been tested in supersonic 2D impinging shocks of different angles and Reynolds numbers of the incoming boundary layer.

The main conclusions drawn for the low Reynolds number $8^\circ$ impinging shock case are:

- Downstream of the separation bubble the DNS shows a kink in the pressure contour patterns as ones moves away from the wall, a feature that is broadly captured only by the CLS model.

- The CLS is the only model to capture the qualitative manner in which the Reynolds shear stress and turbulent kinetic energy is transported through the SWBLI zone.

- The SST, PHIF and CLS models predict shock locations closer to the one suggested by the experiment and the DNS, whilst LS predicts a downstream shift of the shock structure.

- The LS gives a better estimation of the separation point of the flow than the other models, whilst the reattachment point is better predicted by the CLS model.

- Downstream of the reattachment of the flow most of the models broadly capture the recovery of $C_f$, although the PHIF scheme returns a large initial overshoot of skin-friction.

- The CLS is the only model to predict the qualitative behaviour of the boundary layer through the SWBLI zone.
• All the models overpredict the levels of dissipation rate in the non-equilibrium region of the flow upstream and downstream of the separation bubble.

• The better representation of basic features of this flow configuration by the CLS model is attributed to the at least qualitative resolution of the Reynolds stress anisotropy, especially within the region of the separation bubble, where mean normal strain rates become significant.

• None of the incorporated models could predict the low-frequency unsteadiness of the shock.

• High energetic frequencies of pressure signals of $O(10^5)$ Hz are predicted using a $C_\mu$ value of 0.04 with LS.

The main conclusions drawn for the high Reynolds number 7, 10 and 13° impinging shock cases are (LS, SST and CLS have been tested in these cases):

• For the 7° shock the wall-pressure variation predicted by all the models is in good agreement with the experimental one. The SST is the only model to predict a weak separation of the flow, whilst the experimental flow and the flow predicted by the other models are attached.

• For the 20° shock all the models predict reasonably well the shock location and the size of the separation bubble.

• For the 13° shock the CLS model predicts better the wall-pressure rise, whilst LS and SST models predict a downstream shift of it. Additionally, the CLS scheme gives improved predictions of the streamwise Reynolds stress levels, though underestimating them in some locations, especially in the separation bubble.
• The LS model has been also examined in the high Reynolds number 13° impinging shock case by Gerolymos et al. (2004), in which case the LS model predicts a somewhat steeper variation of skin-friction downstream of the flow reattachment. This could be attributed to the effect of Yap-correction, which has not been included in this previous study. The size of the separation bubble and the wall-pressure rise predicted in this previous study is close to the ones predicted in the present study.

• The SST model has been also examined in the high Reynolds number impinging shock cases by Liou et al. (2000). In this previous study, the SST model overpredicts the separation bubble size significantly, unlike in the present study. This could be attributed to the formulation of the eddy viscosity limiter used in this previous study, which is defined using the mean-vorticity tensor. In the present study the eddy viscosity limiter is defined using the mean-strain rate tensor.
Chapter 11

Axisymmetric Transonic Bump

11.1 Case Description

The axisymmetric transonic bump is another case involving SWBLI and commonly used for validation of turbulence models. Experimental measurements have been reported by Bachalo and Johnson (1986) for a variety of freestream Mach numbers. For this study the experimental flow with Mach number of 0.875 and $Re/m$ of $13.6 \times 10^6$ has been considered, with the total pressure being $p_t = 95000$ Pa and temperature $T_t = 302$ K, as reported in Loyau et al. (1998).

The experimental model consists of a circular cylinder of 0.152 m diameter with its axis parallel to the flow direction. An annular circular-arc bump, of chord $c = 20.32$ cm and 1.91 cm thick, is affixed to the cylinder. A circular arc of radius 18.3 cm, which is tangent to the cylinder 3.3 cm upstream and to the bump 2.2 cm downstream of the intersection of the arc of the bump with the cylinder, joins the cylinder with the leading edge of the bump. In the experiment the incident boundary layer upstream of the bump was reported to be $\approx 1$ cm thick.
In this case the initially subsonic flow is decelerated as the leading edge of the bump is approached, and the adverse pressure gradient imposed on the flow causes the thickening of the incoming boundary layer. Downstream of the leading edge of the bump the flow is accelerated and becomes supersonic. As the trailing edge of the bump is approached the flow is diverted towards the cylinder and a shock wave is formed. The adverse pressure gradient imposed on the flow by the shock, in combination with the deceleration of the flow at the trailing edge of the bump, causes flow separation and a recirculation region is developed.

11.2 Case Set Up

Here the axisymmetric flow is simulated by taking a wedge of 1° angle from the experimental geometry and applying symmetry conditions at the side planes of the computational domain used, shown in Figure 11.1. A structured mesh, with a total of 237x150x1 was used, with the non-dimensional wall-distance of the near-wall node being around $y^+ \leq 0.7$. A 2D plane of the mesh is shown in Figure 11.1. At the inlet a boundary layer of 2 mm thickness, taken from a separate flat plate boundary layer simulation with 1.5% freestream turbulence intensity and a ratio of $\mu_t/\mu = 10$, was applied to match the experimental boundary layer thickness of $\approx 1$ cm upstream the leading edge of the bump. At the wall no slip conditions were applied, with zero gradient for the pressure. The wall is assumed adiabatic since the wall temperature is not reported in the literature.
11.3 Results

Figure 11.2 displays iso-lines of mean pressure superimposed on contours of mean Mach number, indicating that the basic flow features are predicted by the models. Both models underestimate the size of the separation bubble with CLS giving better estimations, indicated by the locations of the separation and reattachment points shown in Table 11.1. Figure 11.3 displays the wall-pressure distribution. The CLS model captures more accurately the shock location, whilst LS predicts a downstream shift of it. Additionally, CLS shows improved predictions in the plateau region of pressure within the region of the separation bubble. Figure 11.4 displays a comparison of mean velocity profiles at seven stations (indicated as lines on Figure 11.2). At $x/c = -0.25$ the incident boundary layer upstream of the bump predicted by the models is in good agreement with the experimental one. Both CLS and LS give generally good predictions of the velocity profiles. Only at $x/c = 1$, upstream of the flow reattachment, both models considerably underestimate the maximum backflow velocity. Figure 11.5 displays a comparison of Reynolds shear stress profiles predicted by the models at seven stations. The CLS model shows improved predictions of the peak values of Reynolds shear stress compared to the estimations from the LS scheme, though underestimates them around the flow reattachment point, as indicated in the profiles at $x/c = 1, 1.25$. 
Figure 11.1: Computational domain (top) and grid (bottom) for the axisymmetric transonic bump.
Figure 11.2: Iso-lines of mean pressure superimposed on contours of mean Mach number for the axisymmetric transonic bump. Predictions are from CLS. Locations (defined in Bachalo and Johnson 1986) for data comparison, where the numbers are non-dimensional locations $x/c$. 
Table 11.1: Separation and reattachment points (axisymmetric transonic bump).

<table>
<thead>
<tr>
<th></th>
<th>$x_s/c$</th>
<th>$x_r/c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP</td>
<td>0.7</td>
<td>1.10</td>
</tr>
<tr>
<td>LS</td>
<td>0.888</td>
<td>1.061</td>
</tr>
<tr>
<td>CLS</td>
<td>0.832</td>
<td>1.083</td>
</tr>
</tbody>
</table>

Figure 11.3: Mean wall-pressure distribution for the axisymmetric transonic bump. Experiment (circle), LS (black) and CLS (red)
Figure 11.4: Comparison of mean velocity profiles for the axisymmetric transonic bump at seven streamwise locations (indicated as lines on Figure 11.2). Experiment (circle), LS (black) and CLS (red).
Figure 11.5: Comparison of Reynolds shear stress ($-\overline{u''v''}$) profiles for the axisymmetric transonic bump at seven streamwise locations (indicated as lines on Figure 11.2). Experiment (circle), LS (black) and CLS (red).
11.4 Summary

The LS and CLS models have been tested in an axisymmetric transonic bump case.

The main conclusions drawn are:

- Both models underestimate the size of the separation bubble, with CLS giving the better prediction.

- The CLS model captures more accurately the shock location, whilst the LS scheme predicts a downstream shift of it. Additionally, the CLS model gives better predictions than the LS in the plateau region of pressure within the region of the separation bubble.

- Both CLS and LS models give generally good predictions of the mean velocity profiles. However, upstream of the flow reattachment both models considerably underestimate the maximum backflow velocity.

- The CLS model leads to improved predictions of the peak values of Reynolds shear stress compared to the estimations from the LS scheme, though underestimates them at locations around the flow reattachment point.

- The LS model including the Yap-correction and the first version of the CLS model (Craft et al., 1996) have been also studied by Loyau et al. (1998). In these computations the LS model predicts a downstream shift of the wall-pressure rise and overestimates the wall-pressure in the plateau region. Similar predictions have been obtained with the current calculations, though Loyau et al. predict a somewhat more downstream location of the shock. The first version of the CLS model predicts quite accurately the wall-pressure and the size of the separation bubble in Loyau et al.’s computations, whilst the second version of the CLS
model including the modification introduced in this project underestimates the size of the recirculation region, as indicated by the current calculations
Chapter 12

1D Normal Shock/Isotropic Turbulence Interaction

12.1 Case Description

The simplest flow geometry studied here is the interaction of isotropic turbulence with an one-dimensional normal shock, a case for which a number of DNS studies have been reported. In this particular project, DNS results for a normal shock of Mach number of 1.29, turbulent Mach number, $Ma_t(=\sqrt{2k/\tau})$, of 0.14 and Reynolds number based on longitudinal Taylor microscale, $Re_\lambda(=\upsilon_{rms}\lambda/\nu)$, of 19.1 reported by Mahesh et al. (1997) have been considered.

In this flow an initially isotopic turbulent field interacts with an one-dimensional planar shock. The turbulent field undergoes compression within the region of the shock, resulting in amplified turbulent kinetic energy levels. Additionally, the normal shock distorts the turbulent field from its isotropic state and as a result the Reynolds normal stresses become anisotropic within the region of the shock. Downstream of
the shock, turbulent kinetic energy and Reynolds stress anisotropy levels relax to
their isotropic state due to turbulent dissipation.

12.2 Case Set Up

A structured grid was used, extending from $x/L = 0$ to 6, where $L$ is an arbitrary
length scale determined such that the Reynolds number based on the reference quan-
tities, $Re_{ref} = \overline{\sigma}L/\nu$, where $\overline{\sigma}$ is the mean speed of sound upstream of the shock
location, was 750. A steady shock was imposed at $x/L = 2$ and only the transport
equations of the selected RANS models were solved in this case. For the profiles
of mean velocity, pressure and temperature across the shock the Rankine-Hugoniot
relations were used for upstream Mach number 1.29:

$$V(x) = V^u + (V^d - V^u)\text{tanh}\left(\frac{x - x^s}{b_s}\right)$$

(12.1)

where $V$ is one of velocity, pressure and temperature, the superscript $u$ denotes a value
upstream of the shock, the superscript $d$ denotes a value downstream of the shock, $x^s$ is
the location of the shock and $b_s$ represent the shock thickness parameter. The value of
$b_s$ was determined by the thickness of the shock, defined as

$$\delta_s = |\tilde{u}^u - \tilde{u}^d|/|\partial \tilde{u}/\partial x|_{\text{max}}.$$

For weak shocks (Shapiro, 1953) $\delta_s$ is estimated as,

$$\delta_s \approx \frac{4}{\gamma + 1} \left(\frac{4}{3} + \frac{\gamma - 1}{Pr}\right) \frac{1}{Re_{ref}(Ma^* - 1)}$$

(12.2)

where

$$Ma^* = \frac{\frac{\gamma + 1}{2} Ma^u_2}{1 + \frac{\gamma - 1}{2} Ma^u_2}$$

(12.3)
At the inlet, the level of turbulent kinetic energy is obtained from $M_t$ and the level of dissipation rate is evaluated for isotropic turbulence as $\varepsilon = \frac{15}{4} \mu u_o^2 / k_o$, where $u_o$ is the $u_{\text{rms}} = \sqrt{\frac{2}{3} k}$ and $k_o$ is the most energetic wave number obtained from $Re_\lambda = 2 \frac{u_o}{\nu k_o}$.

### 12.3 Results

Figure 12.1 displays the evolution of turbulent kinetic energy across the shock predicted by the models and DNS. The levels of turbulent kinetic energy from DNS are very high at the shock location, $x/L = 2$. Some of this apparent turbulence might be due to the fact that the shock location was reported to exhibit some unsteady motion in the DNS, and when applying a RANS averaging procedure this motion would contribute to the “turbulence” part. In the present RANS calculations the shock location was imposed, and thus meaningful comparisons should mainly be made in the region downstream of the shock. The LRR model gives a good approximation of the $k$ amplification compared to the DNS. The constant $C_\mu$ in the eddy viscosity formulation of the linear $k - \varepsilon$ scheme leads to an excessive amplification of $k$. On the other hand, when the functional form of $C_\mu$ from the CLS model is incorporated in the linear $k - \varepsilon$ scheme the generation rates of $k$ driven from the mean normal strain rates imposed by the shock are reduced and the levels of $k$-amplification are comparable to the DNS.

Figure 12.2 displays a comparison of the evolution of the Reynolds normal stresses. DNS gives very high levels of $\overline{u'u''}$ in the region of the shock for the same reasons as noted above. The LRR model captures quite accurately the anisotropy between the streamwise and normal Reynolds stresses downstream of the shock. In this case the mean normal strain rates imposed by the shock drive the generation rate of $\overline{u'u''}$, leading to the amplification of it. The amplification of $\overline{v'v''}$, the generation rate
of which is zero through the interaction, is driven by the action of the pressure-strain term, which redistributes the energy from the streamwise Reynolds stress to the shock-parallel one, and the agreement between DNS and the LRR predictions indicates a good performance of the modelled $\phi_{ij}$ adopted in LRR. The linear $k - \varepsilon$ scheme predicts a rather amplified $\overline{u''u''}$ within the shock region compared to the LRR one, leading to excessive $P_k$ levels and explaining the overestimated amplification of $k$. Additionally, the linear $k - \varepsilon$ scheme gives unphysically negative values of $\overline{v''v''}$ at the shock location. This indicates that the linear eddy-viscosity formulation gives erroneous Reynolds normal stresses when compression is imposed on the flow.

Downstream of the shock, the linear $k - \varepsilon$ model predicts immediate Reynolds stress isotropy because the Reynolds stresses depend algebraically on the mean local strain rates. The CLS model does predict realizable Reynolds stresses, though for the same reasons as with the linear scheme they are not correctly transported downstream of the shock. The overall good performance of the LRR model indicates that compressibility effects have only a minor effect in the modelling of such an interaction, since the LRR model was originally developed for incompressible flows.
Figure 12.1: Comparison of turbulent kinetic energy for the 1D normal shock/isotropic turbulence interaction case. Linear $k - \varepsilon$ with constant $C_\mu$ (black), linear $k - \varepsilon$ with the functional form of $C_\mu$ in the CLS model (red), LRR (green). $k^*$ is the normalised turbulent kinetic energy with the value immediately upstream of the shock.

Figure 12.2: Comparison of Reynolds normal stresses ($\overline{u'u''}$; RANS:solid line, DNS:square), ($\overline{v'v''}$; RANS:dashed line, DNS:circle) for the 1D normal shock/isotropic turbulence interaction case. Linear $k - \varepsilon$ (black), CLS (red), LRR (green). Reynolds stresses are normalised Reynolds with their values immediately upstream of the shock.
12.4 Summary

The linear $k - \varepsilon$, the linear $k - \varepsilon$ with the functional $C_\mu$ formulation from the CLS model, the CLS and the LRR models have been tested in a 1D normal shock/isotropic turbulence interaction.

The main conclusions drawn are:

- The linear $k - \varepsilon$ model with constant $C_\mu$ overestimates the amplification of turbulent kinetic energy due to the interaction with the shock. It also gives erroneous predictions of the Reynolds normal stresses, giving unphysically negative values of $\tilde{\nu}'\tilde{\nu}''$ at the shock location.

- The linear $k - \varepsilon$ model with the functional $C_\mu$ formulation from the CLS model predicts quite accurately the amplification of turbulent kinetic energy.

- The CLS scheme predicts realizable Reynolds stresses though they are not correctly transported downstream of the shock.

- Downstream of the shock, the EVM’s predict immediate Reynolds stress isotropy because the Reynolds stresses depend algebraically on the mean local strain rates.

- The LRR model predicts quite accurately the amplification of turbulent kinetic energy and the levels of Reynolds stress anisotropy.
Chapter 13

V2C Laminar Transonic Airfoil

13.1 Case Description

The V2C profile has been developed by Dassault Aviation in the framework of TFAST (Transition Location Effect on Shock Wave Boundary Layer Interaction) European project. The main objective for the design of the V2C profile is to provide laminar flow on the upper surface from the leading edge to the shock wave up to buffet onset. Dassault has examined the performance of this geometry numerically for chord-length \( c \) of 0.25 m, for a range of angles of attack \( \alpha \) and for freestream mach numbers of 0.7 and 0.75, which correspond to Reynolds numbers based on the chord of 3.245x10^6 and 3.378x10^6 respectively. Computations for the estimation of the transition location using a three-dimensional compressible boundary-layer code, described in Courty et al. (1993), have shown that the boundary layer remains laminar up to the shock location for angles of attack ranging from 1° to 7°. Computations for \( Ma_\infty = 0.7 \) using a two-layer \( k - \varepsilon \) model, indicate flow separation for angles of attack between 6° and 7°. Additional analysis has been performed by Grossi (2014) using the Navier-
Stokes Multi Block (NSMB) code, described in Vos et al. (1998), and incorporating RANS and DDES approaches. Two-dimensional calculations using the SST model indicate that for $Ma_{\infty} = 0.7$ the flow separates for angles of attack higher than $4^\circ$ and the critical angle of attack for transonic buffet onset is approximately $5.5^\circ$. For Mach number 0.75 no transonic buffet has been detected for angles of attack ranging from $1^\circ$ to $7^\circ$. Grossi (2014) also studied the effect of the transition location for $Ma_{\infty} = 0.7$ and $a = 4^\circ, 7^\circ$ concluding that the delay of transition to turbulence causes a downstream displacement of the shock-separation bubble structure resulting into a rise of the lift force. An initial study of the 3D unsteady flow dynamics of the V2C profile for $Ma_{\infty} = 0.7$ and $a = 5.5^\circ$ has been performed by Grossi (2014) using the SST-based DDES model and the SST model. The main conclusions drawn is that the incorporated DDES approach predicts lift-coefficient oscillations of higher frequency and amplitude than those predicted by two-dimensional computations with the SST model. Unfortunately, the lift coefficient obtained from Grossi’s DDES calculation is not periodical, which could possibly indicate that the flow is still in transient phase.

13.2 Case Set Up

Within the current project the DDES-SST calculation of the V2C profile for $Ma_{\infty} = 0.7$ and $a = 5.5^\circ$ performed by Grossi is extended for one buffet cycle and compared with predictions from a two-dimensional calculation using the SST model. The numerical approach used in this study is the same as in Grossi (2014). More specifically, the incorporated code is the Navier-Stokes Multi Block (NSMB) code, which is a density-based finite volume solver. The third-order TVD of Roe upwind scheme using the van-Leer flux limiter has been selected for the spatial discretization of the convection terms. Implicit time integration has been performed using a second-order
backward scheme in the context of the dual time-stepping approach. A physical time step size of 0.1 $\mu$s, similar to the one selected by Grossi et al. (2013) for the study of the OAT15A airfoil, has been used. At each time step a typical number of 30 inner iterations were necessary to ensure convergence based on the criterion of $1 \times 10^{-3}$ tolerance of the norm of density.

The computational domain is extended 0.33$c$ in the spanwise direction, following a procedure similar to the one described in Deck (2005), and the far field is located 80 chords from the airfoil. A two-dimensional planar grid, a zoom of which is shown in Figure 13.1, has been copied equally spaced in the spanwise direction. The planar grid has a C-H topology, with the non-dimensional wall-distance of the near-wall node being $y^+ \leq 0.55$, and has size of 163,584 cells. A grid-dependency study of the 2D planar grid has been performed by Grossi (2014) for the steady flow at $Ma_\infty = 0.7$ and $\alpha = 4^\circ$ using the SST model. A number of 59 cells in the spanwise direction has been selected in order the spanwise spacing to have nearly the same size as the streamwise spacing. The grid used has size of about 9.65 million cells.

At the far-field boundary total pressure is set to $P_o = 10^5 Pa$, total temperature is fixed to $T_o = 290 K$, Mach number is 0.7 and Reynolds number based on the chord-length is $3.245 \times 10^6$. The upstream turbulence intensity is 0.08%. At the wall impermeability and no-slip conditions are imposed and in the spanwise direction periodic boundary conditions are applied.

### 13.3 Results

Figure 13.2 shows the time history and the power spectral density of the lift coefficient ($C_L$). The two-dimensional URANS calculation with SST predicts oscillation of the
lift force at a frequency \(\approx 82\) Hz, as suggested from the PSD, whilst the DDES-SST predicts a higher lift oscillation of \(\approx 105\) Hz. Additionally, DDES-SST gives lift fluctuations of larger amplitude than SST indicating a wider range of the shock-wave motion. It should be noted that DDES-SST still predicts buffet of variable frequency.

Figure 13.3 demonstrates a series of snapshots of instantaneous isosurfaces of vorticity colored with the mach number as a function of the non-dimensional time \(t^* = \frac{t U_\infty}{c}\), where \(t^* = 0\) is an instant of maximum lift. As the shock wave moves towards the leading edge, at \(t^* = 0\) and 0.82, the separated flow field is extended all over the upper surface of the airfoil. Such an extensive flow separation results in the mixing of the turbulent structures in the wake downstream of the shock with those near the trailing edge. As the shock wave moves away from the leading edge, at \(t^* = 1.64, 2.46, 3.28\) and 4.10, the separated region starts shrinking and the number of the vortical structures decreases. Finally, alternate vortex shedding occurs at the trailing edge, as the shock wave moves towards its most downstream location, at \(t^* = 4.92\) and 5.74. The present DDES simulation shows the ability of the method in capturing the spanwise large-wavelength undulation of the main von Karman vortices downstream of the separation and the creation of smaller-scale vortex structures with a rich statistical content in the shear regions past the separation and in the wake. It will be noticed that these structures are considerably attenuated in the phases of the shock motion where the separation decreases. At this stage, there are not yet experimental results in the TFAST European program in which this test-case is investigated and therefore a direct comparison is not possible. However, from a qualitative point of view, the vortex dynamics and the forces amplitudes are similar to the dynamics obtained in previous studies in the context of the ATAAC European program, concerning the OAT15A airfoil by means of DDES.
Figure 13.1: Two-dimensional planar grid for the V2C laminar transonic airfoil.
Figure 13.2: Time histories (top) and spectra (bottom) of the lift coefficient for the V2C airfoil. DDES-SST (red) and URANS-SST (black).
Figure 13.3: Instantaneous vorticity magnitude isosurfaces by DDES-SST simulation, for \( \text{Wc}/U_\infty = 10 \) colored with the Mach number for the V2C airfoil. The non-dimensional time is \( t^* = tU_\infty/c \), where \( t^* = 0 \) is an instant of maximum lift, and \( c \) is the chord-length.
13.4 Summary

The three-dimensional unsteady simulation for the V2C laminar transonic airfoil has been extended for one buffet cycle using the DDES-SST model and starting from the final solution of the calculation performed by Grossi (2014).

The main conclusions drawn are:

- The DDES-SST model predicts lift-coefficient oscillation of slightly higher frequency than the SST model. Furthermore, the DDES predicts different classes of eddies and the secondary instability of large wavelength arising in a number of DNS, LES, URANS and Hybrid method simulations, in accordance with the onset of the secondary instability and spanwise irregularities (dislocations) with previous DNS studies (Braza, Faghani and Persillon, JFM 2001).

- The lift-coefficient predicted by the DDES-SST is still not periodical.
Chapter 14

Conclusions and Suggested Future Work

14.1 Conclusions

This thesis has described the evaluation of the performance of four types of EVM’s, widely used for industrial purposes, including both linear and non-linear varieties, in predicting flows involving SWBLI’s using experimental and DNS data. More specifically, the LS, SST, PHIF and CLS models have initially been tested in supersonic 2D compression ramps and impinging shocks of different angle and Reynolds numbers of the incoming boundary layers. The employed models demonstrated a broadly consistent behaviour over these cases and the main conclusions drawn are:

- The CLS model predicts a kink in the pressure contour patterns as ones moves away from the wall, downstream of the reattachment of the flows, which is not predicted by the other models. This flow feature is suggested by DNS data available in the impinging shock case.
Conclusions and Suggested Future Work

- All the models predict the size of the separation bubble broadly comparable to the experimental and DNS ones, although in a number of cases the size is somewhat overestimated.

- The LS model predicts quite accurately the separation point in most cases, whilst the other models typically predict an earlier flow separation.

- Downstream of the reattachment of the flows most of the models broadly capture the recovery of $C_f$, although the PHIF scheme returns a large initial overshoot of skin-friction.

- The CLS model captures the qualitative behaviour of the turbulent kinetic energy and Reynolds stresses through the SWBLI zone of the impinging shock case better than the other models.

- In the compression ramp and impinging shock cases the DNS velocity profile at locations downstream of the reattachment of the flow is of lower gradient in the inner part of the boundary layer than in the outer, a feature broadly captured only by the CLS model.

- The better representation of the flow patterns by the CLS model is associated with the better, at least qualitative, prediction of the Reynolds stress anisotropy, especially within the recirculation region of these flows, where mean normal strain rates become significant. However, the CLS model underestimates the amplified levels of Reynolds stress anisotropy through the SWBLI zone, when these are compared with DNS data in the compression ramp case.

- All the models overpredict the levels of dissipation rate in the non-equilibrium region of the flow upstream and downstream of the separation bubble in the impinging shock case.
- The evaluation of the eddy-diffusion coefficient by means of the CLS model indicates that in the regions of high shear of an impinging shock, this coefficient takes values close to unsteady flow predictions by the Cas model (A. Revell et al, Conference BBVIV 2006) as well as with the OES, organised Eddy Simulation modelling (Y. Hoarau et al, BBVIV06) for unsteady separated flows. This fact motivated our study the behaviour of the LS RANS modelling towards a first step of URANS-OES by employing the same order of magnitude of the eddy-diffusion coefficient in the impinging shock case.

- The LS model modified in the sense of a preliminary URANS-OES mode predicted quite well the existence of a predominant unsteadiness in the impinging shock case and a spectral form similar to the experiments of the research group led by J.P. Dussauge - Marseille, who initiated this test-case within the UFAST European project.

Additionally, the performance of the LS and CLS models has been evaluated in an axisymmetric transonic bump case. The main conclusions drawn are:

- Both models underestimate the size of the separation bubble, with CLS giving a better prediction.

- The CLS model captures more accurately the shock location, whilst LS predicts a downstream shift of it.

- Both CLS and LS give generally good predictions of the velocity profiles. The main discrepancy is at a location upstream of the flow reattachment, where both models considerably underestimate the maximum backflow velocity.

- The CLS model shows improved predictions of the peak values of Reynolds
shear stress compared to the estimations from LS, though underestimates them at locations around the flow reattachment point.

Moreover, the linear $k - \varepsilon$, the linear $k - \varepsilon$ with the functional $C_\mu$ formulation in the CLS model, the CLS and the LRR models have been tested in a 1D normal shock/isotropic turbulence interaction. The main conclusions drawn are:

- The linear $k - \varepsilon$ with constant $C_\mu$ overestimates the amplification of turbulent kinetic energy. Additionally, it predicts the Reynolds normal stresses erroneously, giving unphysically negative values of $\tilde{v}'v''$ at the shock location.

- The linear $k - \varepsilon$ with the functional $C_\mu$ formulation of CLS predicts quite accurately the amplification of turbulent kinetic energy.

- The CLS scheme predicts realizable Reynolds stresses, though they are not correctly transported downstream of the shock.

- Downstream of the shock, the EVM’s predict an immediate return of the normal Reynolds stresses to an isotropic state, because the Reynolds stresses depend algebraically on the mean local strain rates.

- The LRR model predicts quite accurately the amplification of turbulent kinetic energy and the levels of Reynolds stress anisotropy.

Finally, the hybrid RANS-LES method DDES using the URANS-SST model in the near-wall region is proven efficient for the prediction of the onset of the buffet instability and of its predominant frequency, which is found slightly higher than previous URANS -SST approaches. The statistical content of the present DDES approach is found in qualitative agreement with previous simulations (ATAAC - EU program for the transonic buffet around a supercritical airfoil, the OAT15A). Therefore, the
Conclusions and Suggested Future Work

The present thesis offers a detailed study of RANS models that can contribute in future studies to improvement of current hybrid RANS-LES methods. In particular, the improvement obtained by the compressibility modification in the CLS model derived from the present thesis, together with the fact that this class of modelling provides values of the eddy-diffusion coefficient in accordance with the non-equilibrium turbulence effects related to coherent flow unsteadiness, open a significant issue in further improvement of hybrid RANS-LES methods.

14.2 Suggested Future Work

The results of this thesis have contributed to the understanding of the performance of a range of types of EVM’s in predicting flows involving SWBLI’s. Nevertheless, some further explorations could be useful in confirming and extending the objectives of this particular study. It is believed that at least the following work would be desirable:

- In the compression ramp case a DNS study providing sufficiently averaged Reynolds stresses and including turbulent kinetic energy budgets would be beneficial for more precise conclusions on the performance of turbulence models.

- The resolution of Reynolds stress anisotropy levels is essential for an improved prediction of the flow features within the SWBLI region, as indicated by the results from the CLS model. However, the CLS model underestimates the Reynolds stress anisotropy and thus testing of turbulence models that predict more accurately the normal Reynolds stresses would be desirable. Within the EVM framework one option would be the three-equation cubic non-linear $k - \varepsilon$ of Craft et al. (1997). The three-equation NLEVM incorporates the tensorial elements in the stress-strain relation in the CLS model sensitized to the second
stress anisotropy invariant $A_2 = a_{ij} a_{ij}$, resolving more accurately the Reynolds stress anisotropy near the wall and in the buffer layer of a boundary layer. The investigation of the performance of Reynolds stress transport models would be also desirable.

- One of the deficiencies of the selected models is the overestimation of the dissipation rate levels in the non-equilibrium region of the flow, as indicated from the results in the impinging shock case. All the selected models assume constant energy transfer rate between the large energy containing scales, the inertial scales and the dissipative scales by considering a single time-scale. This assumption, which is valid under equilibrium conditions, may be violated in flows subject to rapid changes of mean strain rates. Therefore, the examination of two-time-scale concepts, for example the model of Hanjalic and Launder (1980), which attempt to account for the delay in response from different regions of the turbulence spectrum in non-equilibrium flows, would be desirable.

- Another flow characteristic not captured by the EVM’s employed here is the misalignment between the stress and strain tensor in non-equilibrium flows (Revell et al., 2006). Therefore, the testing of the three-equation stress-strain lag EVM of Revell et al. (2006), which solves a transport equation for $Cas = -\frac{a_{ij} S_{ij}}{\left\| S \right\|}$ besides the turbulent kinetic energy and the length determining equations, would be also desirable.
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