Measurement of the Mass Difference between Top and Anti-top Quarks in Top Pair Events from $pp$ Collisions at $\sqrt{s} = 7$ TeV

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Abstract

A measurement of the mass difference between top and anti-top quarks produced in \( pp \) collisions at \( \sqrt{s} = 7 \) TeV with the ATLAS detector at the LHC is presented. The analysis uses the full 2011 data sample, corresponding to an integrated luminosity of 4.7 fb\(^{-1}\). An event-by-event mass difference is calculated for every event consistent with \( t\bar{t} \) production and decay in the semi-leptonic channel. A likelihood fit to the full double \( b \)-tagged data set yields a measured value of \( \Delta(m) = m_t - m_{\bar{t}} = 0.67 \pm 0.61 \) (stat.) \( \pm 0.19 \) (syst.) GeV, consistent with the Standard Model, and more generally, the CPT Theorem, prediction of no mass difference.
Declaration

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Most of all I would like to thank my parents and my sister for their loving support, without whom, this achievement would never have been possible.
Chapter 1

Introduction

The Standard Model (SM) has been very successful in describing the fundamental particles and their interactions. Within the SM, symmetries play a fundamental role, including specific transformations of Charge conjugation (C), Parity (P) and Time reversal (T). The combination of the three is known as the CPT symmetry. The symmetry is described by the CPT Theorem \[1\], which states that any local quantum field theory, invariant under Lorentz transformations, conserves CPT. The CPT symmetry implies that particles and anti-particles have the same mass and lifetime. A mass difference between a particle and its anti-particle would be evidence of CPT violation. CP violation and T violation in weak interactions have been observed, but CPT violation has not been observed. An observation of CPT violation will have a large impact on the history of elementary particle physics.

The top quark is an ideal candidate in the search for CPT violation in the quark sector. It has a mass, almost a factor of 40 times larger than the bottom quark and is close to the electroweak symmetry breaking scale. It is the only quark to decay before it hadronises, due to its short lifetime ($\sim 10^{-25} \text{ s}$). This is different from the other quarks, which form hadronic bound states before decaying. The states form because the time scale for hadronisation is orders of magnitude less than for electroweak decay. Since the top quark does not form bound states, there is an opportunity to study free quarks, and subsequently, make a precise measurement of the difference between the top and the anti-top masses.

The top quark was discovered in 1995, by the two Tevatron experiments, CDF and D0, at Fermilab. The Tevatron used proton anti-proton collisions at $\sqrt{s} =$
1.8 TeV. An interesting fact in the announcement of the discovery of the top, was its large mass, which was measured to be around 178 GeV [2] (using Run I data). The latest precision measurement of the top quark mass is $173.18 \pm 0.94$ GeV [3], assuming that the top and the anti-top have the same mass.

For the last few years, the possible violation of CPT in the top quark sector has been explored. The first mass difference measurement between the top and anti-top quarks was made by the D0 collaboration, with a $\Delta(m) = 0.8 \pm 1.8 \pm 0.5$ GeV [4], in agreement with the Standard Model value. The CDF collaboration measured $\Delta(m) = -3.3 \pm 1.4 \pm 1.0$ GeV [5], which is approximately 2 standard deviation away from the predicted value of zero.

The latest measurement of the mass difference by the CMS collaboration is $\Delta(m) = -0.44 \pm 0.46 \pm 0.27$ GeV [6]. This measurement is consistent with the Standard Model and is currently the highest precision measurement of the mass difference.

This thesis presents the first mass difference measurement in the top quark sector made by the ATLAS collaboration.

The structure of this thesis is as follows. Chapter 2 describes the theoretical background involved in the analysis. Chapter 3 discusses the LHC accelerator and the ATLAS detector itself, singling out key detector components and features. The discussion on object selection and the signal events being analysed will be covered in Chapter 4. Chapter 5 covers the data and Monte Carlo (MC) samples used in the analysis. A section will be dedicated to the generation of the signal MC, which was one of the major challenges in this analysis. The MC section will also include the background contributions. The main crux of the analysis will be discussed in Chapter 6. A detailed explanation of the systematic studies will also be included in this chapter. The final two chapters will discuss the results and the conclusions.
Chapter 2

Top Quark Theory

2.1 The Standard Model

The Standard Model (SM) is a theory of elementary particles and forces. The SM contains twelve fermions, four spin-1 gauge bosons and one spin-0 boson. The spin-0 boson is known as the Higgs boson, and discovery of its existence, at a mass of 125 GeV, is currently being verified at CERN. The twelve fermions consist of three quark and two lepton families as shown in Figure 2.1. The “up” type quarks (up, charm and top) have a charge of $+\frac{2}{3}$ and the “down” type quarks (down, strange, bottom) have a charge of $-\frac{1}{3}$. Each fermion has an associated anti-fermion (complex conjugate).

The strong interaction is felt by quarks, and is mediated through massless gluons. Quarks can bind with one another because they have colour charge. Quarks come in three colours; red, blue and green. A combination of all three colours creates a colourless particle, such as a baryon (proton for instance), which is a quark triplet state. A quark doublet is known as a meson, which contains a quark anti-quark pair and is colour neutral. Gluons exist in eight independent colour states. The model which describes the interactions of coloured particles through the exchange of gluons is called Quantum Chromodynamics (QCD). The strong interactions conserve C,P and CP-symmetry

The Electromagnetic (EM) interaction is mediated by the photon. Charged particles interact through the exchange of the photon. Photons are massless and uncharged. The model which describes this interact is known as Quantum Electrodynamics (QED). C, P and CP-symmetry is conserved in electromagnetic interactions.
The short range weak interaction is mediated through the $W^\pm$ and the $Z$ gauge bosons. Flavoured particles interact weakly though the exchange of one of these weak bosons. However, at high energies, the weak and the EM forces are indistinguishable, and a combined theory to describe both, known as Electroweak Theory (EWT) is formed. Weak interactions allow quarks to change flavour, by unit charge $e$. It is only in the weak interactions, in which C, P and CP-symmetry is violated.

The strong, weak and electromagnetic interactions are described by $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetries. The subscripts $C$, $L$ and $Y$ represent the colour, weak and hypercharge symmetries respectively. $SU(3)_C$ transformations describe the strong interaction, and $SU(2)_L \otimes U(1)_Y$ transformation describe the electroweak interaction.

**Electroweak Symmetry Breaking**

The EWT describes the interactions of particles via weak and electromagnetic forces. The underlying symmetry of the EWT is described by the gauge transformation, $SU(2)_L \otimes U(1)_Y$. Under this symmetry, the weak gauge bosons are massless. The electroweak symmetry must be broken to allow the weak gauge
bosons to acquire mass, whilst still requiring the Lagrangian to remain invariant under gauge transformation [8][9][10]. Starting with the classical Lagrangian density [11]

\[ \mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi) \]  

(2.1)

\[ \phi = \frac{\sqrt{2}}{2} [\phi_1 + i\phi_2] \]  

(2.2)

\[ V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4, \]  

(2.3)

where \( \phi \) represents a complex field and \( V(\phi) \) the potential energy, \( \mu^2 < 0 \) and \( \lambda > 0 \) are real. This Lagrangian (2.1) is invariant under the global U(1) transformation. \( \phi(x) \) is a constant in this vacuum state, if the state is invariant under Lorentz transformations. There are two possibilities for the existence of the vacuum state, depending on the \( \mu^2 \), as shown in Figure 2.2 If \( \mu^2 > 0 \), one sees the minimum for the potential at \( \phi = 0 \). If \( \mu^2 < 0 \), the minimum energy corresponds to a minimum in a ring in the complex plane

\[ V(\phi_{\text{min}}) = \sqrt{-\frac{\mu^2}{2\lambda} \exp i\theta}, 0 \leq \theta < 2\pi. \]  

(2.4)

The Lagrangian is invariant under rotations in the complex plane of \( \phi \). Setting \( \theta \) to zero gives

\[ V(\phi_{\text{min}}) = \sqrt{-\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}. \]  

(2.5)
Deviation from the minimum can be described by two real fields, $\eta$ and $\xi$ defined through
\[
\phi = \sqrt{\frac{1}{2}} [v + \xi + i\eta].
\]  
(2.6)

Rewriting the Lagrangian in terms of $\eta$ and $\xi$ gives
\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} \xi \partial_{\mu} \xi - \lambda v^2 \xi^2 + \frac{1}{2} \partial_{\mu} \eta \partial_{\mu} \eta
\]  
(2.7)

where $\eta$ and $\xi$ are two real Klein-Gordon fields. Quantising these fields, the Lagrangian describes two different spin-0 particle fields. The $\xi$ bosons will have mass
\[
m_{\xi} = v \sqrt{2\lambda}
\]  
(2.8)
arising from the $\xi^2$ term while the $\eta$ bosons will be massless. The masslessness of the $\eta$ bosons are a direct consequence of the minimum being degenerate. The remaining terms can be treated as interactions among the $\xi$ and $\eta$ particles through perturbation theory.

**The Higgs Mechanism**

The Higgs mechanism describes how elementary particles are given mass, through spontaneous symmetry breaking.

Continuing from Section 2.1, replacing the normal derivative in Equation 2.1 with the covariant derivative
\[
D_{\mu} = \partial_{\mu} + iqA_{\mu}
\]  
(2.9)

and adding the Lagrangian of the free gauge field $A_{\mu}$, gives
\[
\mathcal{L} = D_{\mu} \phi^* D_{\mu} \phi - V(\phi) - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}.
\]  
(2.10)

This Lagrangian is invariant under $U(1)$ gauge transformations. Using the negative $\mu^2$ requirement, expressing the Lagrangian in terms of $\xi$ and $\eta$ and following the steps discussed in [11], one gets the following Lagrangian density (2.11) and vacuum expectation value (2.12)
\[
\mathcal{L} = \partial_{\mu} \sigma \partial_{\mu} \sigma - \lambda v^2 \sigma^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} g^2 v^2 A_{\mu} A^\mu + \text{higher order terms}
\]  
(2.11)
The top has the largest Yukawa coupling, $y_t \sim 1$, among SM fermions. The Yukawa interaction describes the coupling between the Higgs field and the massless quark and lepton fields. The quarks and leptons acquire a mass proportional to the vacuum expectation energy of the Higgs field. The mass of the top is at the electroweak scale, $m_t \sim \frac{v}{\sqrt{2}}$, and is considered to be naturally related to the electroweak symmetry breaking (EWSB). This makes precision top quark measurements important in Higgs physics.

\[ v = \frac{2m_W}{g} = 246 \text{ GeV}. \quad (2.12) \]

\[ 2.2 \text{ CPT Theorem} \]

Symmetries play a vital role within the Standard Model (SM). Three key symmetries are Charge conjugation (C), Parity (P) and Time reversal (T). Charge conjugation replaces particles with their anti-particles, while preserving helicity,

\[ t^-_L \rightarrow t^+_L. \quad (2.13) \]

Parity transformation inverts the spatial coordinates and reverses the helicity,

\[ r \rightarrow -r \]
\[ t^-_L \rightarrow t^-_R. \quad (2.14) \]

Time reversal reverses the time direction, preserving helicity,

\[ \tau \rightarrow -\tau \]
\[ t^-_L \rightarrow t^+_L. \quad (2.15) \]

The conservation of the combination of all three is guaranteed by the CPT Theorem which states that any local theory, which is Lorentz invariant, and has a Hermitian Hamiltonian, conserves CPT [13][14]. This theory implies that any inequality between a particles mass compared with its anti-particle counterpart, or differences in the lifetime, violates CPT conservation.

C and P are both conserved in the strong and electromagnetic interactions, but not in weak interactions. CP violation also occurs in weak interactions. It is violated in the neutral Kaon systems (which shows a favour to decay into matter
as opposed to anti-matter) and in $B$ and $D$ mesons. However, the combination of CPT has not yet been observed. Any violation in CPT automatically leads to the breaking of Lorentz symmetry [15] and new physics. A mass difference between a particle and anti-particle would be a model independent investigation into CPT violation.

### 2.3 Top Quark Production and Decays

The top quark has the largest mass of any observed fundamental particle. The current leading measurements of the mass are made by the two Tevatron experiments, CDF and D0, with an average top mass of [3],

$$m_{\text{top}} = 173.18 \pm 0.94 \text{ GeV}.$$  \hfill (2.16)

Figure 2.3 shows the current top mass measurements made by the LHC and Tevatron.

The predicted Standard Model $t\bar{t}$ cross-section for $pp$ collisions at a centre-of-mass energy of $\sqrt{s} = 7$ TeV is

$$\sigma_{t\bar{t}}^{\text{approx, NNLO}} = 167^{+17}_{-18} \text{ pb},$$  \hfill (2.17)
for a top quark mass of 172.5 GeV as obtained from approximate NNLO QCD calculations [17].

The units of the cross-section in this thesis will be in barns. A barn is defined as $10^{-28}$ m$^2$. This production cross-section is a factor of almost 25 higher than that at the Tevatron. With the huge number of events being produced, the LHC will be a “top” factory and properties of the top quark, like the CPT symmetry for example, can be studied with higher precision. Figure 2.4 shows the leading order Feynman diagrams for top pair production at the LHC. The dominant production process is gluon-gluon fusion, with quark anti-quark production having a smaller contribution. This is opposite to the Tevatron proton anti-proton collider, in which the quark anti-quark production is dominant.

Figure 2.5 shows the top quark pair branching ratios. There are three decay channels in which top pairs are studied. The type of decay depends upon the manner in which the two $W$ bosons in the final state decay. If both, one or neither of the $W$ bosons decays into two light jets, then the decay is considered to be hadronic, semi-leptonic or di-leptonic respectively. The hadronic channel has both $W$ bosons decay into light jets giving a total of six high $p_T$ jets in the final
state, two of which are $b$-jets. The branching ratio for this decay is 46% of all top pair events. However, due to the large number of jets in the final state, the background contribution from QCD multi-jets is very large, making this a difficult channel for this analysis. The QCD multi-jet background comprise of a pair of lighter quarks or gluons in the final state, instead of a top quark pair, which emit gluons that hadronise to form jets. Therefore the jets in the background event originate from gluon radiation whereas the jets in the top pair decay are predominately caused by the hadronisation of quarks. The di-leptonic channel has both $W$ bosons decay into charged lepton and neutrino pairs. There are only two $b$-jets in the final state. The background contribution to this channel is low. However, due to two neutrinos in the final state, it is difficult to reconstruct the top system as the neutrinos pass through the detector undetected, leaving a large amount of missing energy. The semi-leptonic decay channel, shown in Figure 2.6 is the case in which one $W$ boson decays into light jet pairs and the other into a charged lepton neutrino pair. The branching ratio for this decay is just under 45% of all top pair events. This is considered the golden channel for top physics because the background contribution to this channel is far less then for the hadronic channel, but one can fully reconstruct the top system (unlike the di-lepton channel).

There have been three measurements of the top anti-top quark mass difference from D0, CDF and CMS. The experiments all use different strategies to measure
the mass difference.

D0 uses the matrix element (ME) method, in the semi-leptonic decay channel to measure the mass difference. Each event is given a probability to be observed, as a function of the top and anti-top quark masses. The probability to observe an event is split into signal and background contributions. The probabilities are combined to form a likelihood function from which the mass difference and top mass parameters are extracted. The likelihood function is maximised for each event pair. The mass difference is extracted by projecting the likelihood function onto the mass difference axis, and taking the mean mass difference that maximises the function. The mass difference at D0, was measured to be $\Delta(m) = 0.8 \pm 1.8 \pm 0.5$ GeV. A detailed description of the method is given in [4].

CDF measures an event-by-event mass difference, by constructing templates for the top and anti-top quark pairs for signal and background. A kinematic mass fitter is used to reconstruct the mass difference. Probability density functions are constructed for the signal and background contributions, which are then fit to data. After 3000 pseudoexperiments, the mass difference is extracted and measured to be $\Delta(m) = -3.3 \pm 1.4 \pm 1.0$ GeV. This is explained in further detail in [5].

At CMS, the top and anti-top mass are reconstructed separately, using the Ideogram method, in the hadronic component of the semi-leptonic decay events. The mass difference is measured as the difference between the reconstructed top and anti-top mass (not done on an event-by-event basis like the previous two experiments). The mass difference was measured to be $\Delta(m) = -0.44 \pm 0.46 \pm 0.27$ GeV. Further details are found in [6].
The techniques that CDF use in their mass difference measurement are used in the ATLAS analysis.
Chapter 3

LHC and ATLAS

3.1 LHC

The Large Hadron Collider, (LHC), is a large particle collider built on the Franco-Swiss border. It is built and operated in collaboration with over 10,000 scientists and engineers from over 100 countries coming from hundreds of universities and laboratories. The LHC utilises a pre-existing tunnel, that was previously used for the Large Electron Positron (LEP) collider which stopped running in 2000. The LHC accelerates protons or heavy ions in the beam pipe and collides them at high energies. The collider itself is large in size with a circumference of just under 27 km. It consists of two beam pipes, which contain beams traveling in opposite directions, and collides them at four designated collision points. At each collision point a detector is placed. There are four major experiments on the LHC; ATLAS, CMS, LHCb and ALICE.

ATLAS (A Toroidal LHC Apparatus) and CMS (Compact Muon Solenoid) are the two large multi-purpose experiments which are designed to probe for beyond the Standard Model (BSM) physics, to search for the existence of the Standard Model Higgs, and to make precision SM measurements. LHCb (Large Hadron Collider Beauty) is designed to explore $b$-quark physics and to understand the reasons for a matter-dominated universe (CP Violation). ALICE (A Large Ion Collider Experiment) is a detector designed to study heavy ion collisions, which will take advantage of the heavy ion runs at the LHC, to investigate the properties of matter as they reach extreme temperatures (those similar to that at the early stage of the universe).

The LHC is designed to run at a centre of mass energy of $\sqrt{s} = 14$ TeV and luminosity of $10^{-34}$ cm$^{-2}$s$^{-1}$. At this luminosity, there are an expected 40 million
bunch crossing per second (40 MHz) with bunch spacings of 25 ns.

The LHC was started with a centre of mass energy of 900 GeV and then planned to move onto 7 TeV per beam at a later date. The reason the LHC is not running at its design $\sqrt{s}$ is due to an unforeseen incident [20] in which a large number of magnets were damaged. A faulty electrical connection between two accelerator magnets caused mechanical damage and the release of helium from the magnet cold mass to the tunnel. This release of helium caused the beam vacuum to degrade and the pressure to rise. The spring-loaded relief discs were unable to cope with the relatively large increase in pressure (0.15 MPa) causing a separation of the central sub-sector from its neighboring one. The repair of the magnets and further analysis of the situation caused a year in delay and the decision was taken to reduce the running energy down to 3.5 TeV per beam. In March 2010, the first collisions at a centre of mass energy of 7 TeV occurred at the LHC. In 2012, the LHC operated at an energy of 4 TeV per beam, with a bunch spacing of 50 ns.

There are many stages that take place to get protons to travel around the LHC ring at 3.5 TeV per beam and to be ready for collisions as shown in Figure 3.1. Initially the protons are obtained by stripping electrons from hydrogen atoms, leaving only the protons remaining. These protons are then accelerated into a series of systems to reach the desired beam energy.
Once the protons are obtained, they are accelerated in the first system, the Linear Accelerator (LINAC2). The LINAC2 accelerates the protons to an energy of 50 MeV such that they are ready for injection to the Proton Synchrotron Booster (PSB), where the protons are accelerated to an energy of 1.4 GeV. The Proton Synchrotron (PS) takes the protons from the PSB and accelerates them to an energy of 26 GeV, at which point they are ready for injection in the Super Proton Synchrotron (SPS). The SPS accelerates the protons to 450 GeV. At this energy, the protons are then finally injected into the main LHC ring, where the proton bunches are accumulated and accelerated to the final energy of 3.5 TeV, with a bunch spacing of 50 ns. The protons are injected as two counter-rotating beams into the LHC main ring, and are collided at the four designated collisions points.

The number of events produced at the LHC is given by

\[ N_{\text{events}} = L \sigma_{\text{event}}, \]  

(3.1)

where \( \sigma_{\text{event}} \) is the cross-section for the event in question, and \( L \) is the machine luminosity. The luminosity is further defined as

\[ L = f \frac{n_1 n_2}{4 \pi \varepsilon \beta^*}, \]  

(3.2)

where \( n_1 \) and \( n_2 \) are the numbers of particles per bunch, \( f \) is the collision frequency, \( \varepsilon \) is the transverse emittance and \( \beta^* \) is the amplitude function.

There are several ways to increase the luminosity according to Equation 3.2. Increasing the frequency and the number of particles per bunch or reducing the emittance and the amplitude function increases the luminosity. During the 2011 data taking period, the number of proton per bunch reached \( 10^{11} \) and the peak luminosity reached \( 10^{33} \text{ cm}^{-2}\text{s}^{-1} \).

### 3.2 ATLAS

The ATLAS detector is a multipurpose detector designed to search for new physics and to take precision measurements of the SM. The detector is 45 m long, 25 m high and weighs almost 7000 tonnes. It is located at point 1 on the LHC ring, approximately 100 m below the ground. A full technical design report can be read in [22].

Figure 3.2 shows the layout of the ATLAS detector. The inner detector is in the innermost region of ATLAS. It comprises of three sub-detectors; the pixel
detector and the semi-conductor and transition radiation trackers. These detectors are designed to track charged particles and make precision momentum and vertex measurements. The inner detector is surrounded by the electromagnetic and hadronic calorimeters. These calorimeters measure the energies of charged and neutral particles from the showers of particles. The final part of the ATLAS detector is the muon spectrometer, which is designed to measure high transverse momentum muon tracks. The major considerations in the detector layout and composition of ATLAS are given below:

- Due to the high energy and luminosity conditions at the LHC, the detectors require fast, radiation-hard electronics and sensor elements.

- High granularity detectors are needed to handle the particle flux from collision runs and reduce the effects of overlapping events. These overlapping events come from pileup and underlying events. Pileup comes from the extra proton-proton interactions that occur in the bunches, and the underlying event (UE) occurs from multiple parton-parton interactions, where the remnants of two interacting protons further interact.

- Large acceptance in pseudorapidity (Equation 3.4) is needed, with almost full azimuthal coverage ($\phi$) throughout the detector.

- It is vital to have a good charged-particle momentum resolution and reconstruction efficiency on muons, electrons, charged hadrons and also to identify heavy meson decays. For example, secondary vertices from $B$-meson decays are vital in identifying $b$-quarks.

- Good coverage in the electromagnetic calorimeter and full coverage in the hadronic calorimeter are needed for accurate jet and missing transverse energy ($E_{T}^{miss}$) measurements. This is vital in semi-leptonic top decays as there is missing energy in the form of neutrinos in the final state as well as four jets. Good energy resolution is also required.

- Muon identification and resolution over a wide momenta range is required from the muon spectrometer. The ability to identify muon tracks of high transverse momentum ($p_T$) is important.

- The trigger must be highly efficient in triggering high $p_T$ objects with sufficient background rejection.
3.3 ATLAS Coordinate System

The ATLAS experiment uses a right-handed coordinate system, in which the beam direction defines the $z$-axis, and the $x - y$ plane is transverse to the beam. The origin of the coordinate system is offset from the interaction point (IP), where the collisions occur. The positive $x$-axis is from the IP to the centre of the LHC ring and the positive $y$-axis is pointing upwards from the IP. The azimuthal angle, $\phi$, is measured around the beam axis from the range $-\pi$ to $+\pi$, where $\phi = 0$ points to the $x$-axis and the polar angle, $\theta$, is the angle from the beam axis. However, in the analysis, the variable pseudorapidity, $\eta$, is used to describe the angle of a particle relative to the beam pipe. It is calculated using $\theta$, in Equation 3.3.

$$\eta = -\ln \tan\left(\frac{\theta}{2}\right). \tag{3.3}$$

The pseudorapidity can also be defined in terms of the momentum of a particle

$$\eta = \frac{1}{2} \ln \left(\frac{|p| + p_z}{|p| - p_z}\right). \tag{3.4}$$
The variable $\Delta R$, is the distance in the $\eta$-$\phi$ space, and is defined as

$$\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2}. \quad (3.5)$$

Some variables such as the transverse momentum, $p_T$ and transverse energy, $E_T$ which are key in this analysis are defined in Equation 3.6 and Equation 3.7 respectively

$$p_T = |p| \sin \theta \quad (3.6)$$
$$E_T = E \sin \theta. \quad (3.7)$$

For this thesis, the natural units notation is used; $c = \hbar = 1$. Thus a mass unit GeV/$c^2$ will be GeV.

### 3.4 ATLAS Detector

#### 3.4.1 Inner Detector

The inner detector (ID) is used to make high-precision measurements of momentum and vertex location for charged particles. The ID measures the momentum of charged particles using the curvature of tracks. It is contained within an envelope of length $\pm 7024$ mm and radius 1150 mm, shown in Figure 3.3. It is comprised of the pixel detector and semi-conductor tracker (SCT) used in conjunction with the straw tubes of the transition radiation tracker (TRT). The precision tracking occurs in the central region, $|\eta| < 2.5$, which is the coverage of the ID. Each of the three detector sections are further split into three sub-detectors; the barrel and two end-caps. The barrel region is located around the beam pipe, with the IP at the centre, and the end-caps are located either side of the IP.

The pixel detector provides high granularity and high precision set of measurements as close to the IP as possible. It is used to determine the impact parameters of particles which are useful in identifying metastable particles like the $B$-mesons. The pixel detector contributes typically three measurements for each track that passes through it. There are three pixel layers in the barrel, and three pixel discs in each end-cap. The barrel has 1744 pixel modules (each module is 10 cm²), with 46080 readout channels per module. Each pixel is $50 \times 400 \, \mu m^2$ with an intrinsic accuracy of $10 \, \mu m$ in $R - \phi$ and $115 \, \mu m$ in $z$. This is the same for the barrel region and the end-cap region. The first layer of the pixel detector,
Figure 3.3: The barrel region of the inner detector, traversed by a charged track [22].
the $b$-layer, is very powerful in determining the primary vertex, calculating the impact parameter of particles and locating the secondary vertices.

The SCT consists of eight strip layers, which contribute four measurements for each track that passes through it. Each silicon microstrip tracker consists of 4088 two-sided module and over six million implanted readout strips. In the barrel region, these eight layers provide precision points in the $R - \phi$ and $z$ coordinates. The intrinsic accuracy of the SCT is 17 $\mu$m in the $R - \phi$ region, and 580 $\mu$m in the $z$ region. This is the same for the barrel and end-cap regions.

The pixel and SCT detectors are both based on silicon technology. When a charged particle enters the silicon, electron-hole pairs are created due to the excitation of a valence band electron to the conduction band. An applied electric field allows the charges to be collected on the surface of the silicon.

The TRT is different to the other sub-detectors of the ID. Instead of being comprised of a semi-conducting material, it consists of a series of straw tubes which can operate at the expected high rates due to their small diameter (4 mm) and the isolated sense wires within individual gas volumes. There are many very thin foils and fibres of plastic embedded between the straw tubes. When a highly ionising charged particle passes through the boundary between the gas inside the tubes (carbon dioxide) and the plastic, soft X-rays are emitted called transition radiation. The radiated energy is proportional to the Lorentz factor, $\gamma$ of the incident particle. For a given measured momentum, the transition radiation can be used to infer the particle mass. The TRT information is predominantly used to distinguish electrons from charged pions, since heavier charged particles such as charged pions produce less transition radiation.

With an average of 36 hits per track, the TRT provides continuous tracking to provide improved momentum resolution and pattern recognition.

The ID is immersed in a 2 T superconducting solenoid, which bends the charged particles. The momentum is calculated by measuring the curvature of these charged tracks, defined by the hits in the ID. If one considers a charged particle, with momentum $p$, passing through a region of length $l$, in a magnetic field, $B$, the momentum $p$, is defined as:

$$p = \frac{l^2QB}{8s},$$

(3.8)

where $s$ is the sagitta, which is the distance from the centre of an arc to the centre of its base, shown in Figure 3.3. The systematic alignment uncertainties in the ID are unlikely to improve beyond the 1 $\mu$m level (which is 0.1% of the
Figure 3.4: The sagitta, where $r$ is the radius of curvature, $s$ is the sagitta and $l$ is half the length of the chord spanning the base of the arc.

A charged particle typically leaves three pixel hits, eight SCT layer hits, and around 36 TRT hits. Pattern recognition software is used to reconstruct the tracks. From Monte Carlo studies, the resolution on the inverse momentum, $1/p_T$, of a track is expected to be

$$\sigma_{1/p_T} = 0.34 \text{ TeV}^{-1} \left(1 + \frac{44 \text{ GeV}}{p_T}\right),$$

where $p_T$ is expressed in units of GeV. The first term represents the intrinsic resolution at infinite momentum, and the second term the multiple scattering component, which is small for high $p_T$ tracks. This inverse resolution was found for the barrel region ($0.25 < |\eta| < 0.50$). Figure 3.5 shows the relative transverse momentum as a function of $|\eta|$, without any beam constraints and assuming the effects of misalignment to be negligible.

### 3.4.2 Electromagnetic and Hadronic Calorimeters

The calorimeters measure the energies of electrons, photons and jets of particles from the hadronisation of partons. There are two calorimeters in the ATLAS detector; the electromagnetic calorimeter and the hadronic calorimeter, shown in Figure 3.6. They consist of metal plates, known as absorbers, and sensing elements. The interaction with the absorbers transforms the incident energy into a shower of particles which are then detected by the sensing equipment. They are designed to measure the energies of the electromagnetic and hadronic showers that are initiated by particle contact with the absorbers. By containing the shower, the calorimeters aim to limit the amount of punch-through in the muon
Figure 3.5: Relative transverse momentum resolution as function of $|\eta|$ for $p_T = 1$ GeV (open circles), 5 GeV (full triangles) and 100 GeV (full squares) in Monte Carlo simulation [22].

Figure 3.6: Cut-away view of the ATLAS calorimeter system [22].
system. Punch-through is defined as the shower particles that leave the dense material of the calorimeters and penetrate surrounding detector systems. Therefore a large amount of material is required to perform this task. The calorimeters are also vital for the $E_T^{\text{miss}}$ measurement.

**Electromagnetic Calorimeter**

The electromagnetic calorimeter (ECal) is a lead-liquid argon (Pb-LAr) detector with accordion-shaped electrodes and lead absorber plates. The showers of particles in the liquid argon (LAr), liberate electrons which are then collected and recorded. Immersed in the argon, is copper, which acts like an electrode used to make measurements of particles which pass through. As an electron passes through calorimeter, it interacts producing shower electrons, positrons and photons. Several layers are passed through before it eventually stops. The shower of particles then pass through the LAr and ionises the atoms, creating more negatively charged electrons, and positively charged ions. The negative charge is attracted towards copper electrodes, where it is measured.

The ECal is comprised of three sub-detectors; the barrel region ($|\eta| < 1.475$) and two end caps ($1.375 < |\eta| < 2.5$), each housed in their own cryostat. The calorimeter must be sufficiently large to contain the EM showers that are produced. In the barrel region, the ECal is 22-30 radiation lengths ($X_0$) in size, whereas in the end-cap region, it is larger, 24-33 $X_0$.

In the barrel region of the calorimeter, the response and energy resolution were studied as a function of the energy, in the range 10 to 245 GeV, at an $\eta$ of 0.687. The segmentation of the barrel calorimeter is given in Figure 3.7. There are three distinct layers; strip towers in the first sampling layer (S1), square towers in the second layer (S2) and the trigger towers in the final sampling layer (S3). S1 comprises of layers of strips of size $\Delta \phi \times \Delta \eta = 0.100 \times 0.003$ in $\eta - \phi$ space. S2 has the finest granularity, $\Delta \phi \times \Delta \eta = 0.025 \times 0.025$, and is where the majority of the energy is deposited. S3 layer is the outermost part of the ECal. It is comprised of larger cells than S2 ($\Delta \phi \times \Delta \eta = 0.025 \times 0.050$) and is used to catch the high energy shower tails. An additional layer of LAr before the ECal, known as the pre-sampler, is used to correct for the energy lost by electrons and photons in dead material in front of the calorimeter. The resolution on the energy of the calorimeter is given by Equation (3.10)

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E(\text{GeV})}} \oplus b, \quad (3.10)$$
Figure 3.7: The barrel electromagnetic calorimeter [22].
where $a$ is the stochastic term, and $b$ is a constant term describing non-linearities in the calorimeter response. Test-beam studies of barrel modules found the sampling term to be $10% \cdot \sqrt{\text{GeV}}$ and the constant term to be $0.17\%$. This corresponds to an uncertainty of $3.2\%$ for 10 GeV electrons and $1\%$ for 100 GeV electrons as seen in Figure 3.8. The resolution varies as a function of pseudo-rapidity, as the calorimeter depth varies along with changes in thickness to the upstream material.

### Hadronic Calorimeter

The hadronic calorimeter (HCal) is located outside the ECal. It is used to measure the energies of hadrons, such as neutrons, protons and mesons. The HCal is comprised of large arrays of interleaved steel and scintillator sheets called tiles. A scintillator is a material which radiates light when a charged particle passes through it. When a particle such as a proton passes through the steel, it induces a shower. These particle showers enter the plastic scintillators, causing light to be emitted. The intensity of the emitted light is measured and the result is converted into an electric current, which is then analysed. The HCal needs to be sufficiently large to encompass the hadronic showers that are produced in the interactions.
with the steel absorbers. It is built to a total thickness of 11 interaction lengths \( \lambda \), at \( \eta = 0 \).

The HCal is split into three sub-detectors; the barrel and two end-caps. The barrel covers a range \( |\eta| < 1.0 \) and two extended barrel regions cover the range \( 0.8 < |\eta| < 1.7 \). The hadronic end-caps (HEC) are comprised of two independent wheels per end-cap. They reduce the drop in material density at the transition between the end-cap and the forward calorimeter (around \( |\eta| < 3.1 \)). The HEC extends out to \( |\eta| < 3.2 \) and is comprised of copper plates for the absorber material.

The HCal is calculated using Equation 3.10. Test-beam studies of the tile calorimeter using charged pions found the sampling term to be 56.4\% and the constant term to be 5.5\%, for \( \eta = 0.35 \). This again can vary with \( \eta \), largely due to the varying effective depth of the calorimeter material.

The final component of the calorimeters is the forward calorimeters (FCal). The FCal is designed to reduce the neutron albedo, which is the probability of a neutron entering into a region through a surface, returning back through the same surface. Therefore, the front face of the FCal is recessed by 1.2 m with respect to the ECal front face. This calorimeter is of a higher density design then the other parts of the calorimeter to compensate for this recess. It is around 10 \( \lambda \) deep and made up of three components. The first is made of copper (EM measurements) and the next two of tungsten (for hadronic interactions). The LAr gaps in the FCal are much smaller then in the ECal and HCal. This is done to reduce the collection time of the signal and to reduce ion build up issues. The FCal is integrated into the end-cap cyrostat and provides benefits in terms of uniformity of the calorimetric coverage as well as reduced radiation background levels in the muon spectrometer.

3.4.3 Muon Detector

The muon spectrometer (MS), shown in Figure 3.9, is the outermost part of the ATLAS detector. It is designed to measure the momentum and charge sign of muons within a region of \( |\eta| < 2.7 \). Large toroidal magnets provide the magnetic field required to bend the muons for the momentum measurement.

Within the range \( |\eta| < 1.4 \), the magnetic bending is provided by the large barrel toroid and within the range \( 1.6 < |\eta| < 2.7 \), the tracks are bent by two smaller end-cap magnets, which are inserted into both ends of the barrel toroids. The magnetic field in the transition region between the two \( \eta \) ranges is provided.
Figure 3.9: Cut-away view of the ATLAS muon spectrometer [22].
by a combination of the two magnetic systems.

Precision measurements of the momentum are made by the monitored drift tube chambers (MDTs). These cover the full pseudorapidity range of \(|\eta| < 2.7\). However, in the innermost part of the end-cap (EC), the MDTs are restricted to be within a range of \(|\eta| < 2\). The MDTs consist of between three and eight layers of drift tubes, which achieve an average position resolution of 80 \(\mu\text{m}\) per tube and 35 \(\mu\text{m}\) per chamber.

The cathode-strip chambers (CSCs) are used in the innermost tracking layer of the forward region \((2 < |\eta| < 2.7)\). The position resolution of the chamber is 40 \(\mu\text{m}\) in the bending plane and about 5 mm in the transverse plane. To achieve this resolution, the location of the MDT wires and CSC strips need to be known to a precision of 30 \(\mu\text{m}\).

Two additional chambers are required; the resistive plate chambers (RPCs), located in the barrel region \((|\eta| < 1.05)\) and the thin gap chambers (TGCs), located in the end-cap region \((1.05 < |\eta| < 2.4)\). Collectively, these are known as the trigger chambers, and both measure the coordinates of the track in the bending plane \((\eta)\), and in the non-bending plane \((\phi)\).

The MS was designed to withstand the radiation in the experimental hall. To account for this radiation, the components of the MS were tested to tolerate at least five times the radiation levels predicted in simulation.

The muon reconstruction can work in two modes; the standalone mode in which the momentum is inferred solely from the tracks in the MS, or the combined mode, in which the tracks in the MS are matched to those in the ID. This second mode allows a precise determination of the muon momentum and rejects fake muons.

### 3.4.4 Magnetic System

The ATLAS detector has two magnetic systems, which are cooled by liquid helium:

- The solenoid system, which is designed to provide a magnetic field of 2 T axial field for the ID. The magnets are found on the boundary between the ID and the ECal, and are used to bend the tracks of charged particles in the ID. These magnets are aligned on the beam axis and also minimise the radiative thickness in front of the barrel ECal to \(\sim 0.66 X_0\). The solenoid is kept at a temperature of 4.5 K. In case of a quench, the temperature of
the magnet increases to a safe limit of 120 K and a day is required to cool back down to 4.5 K.

- The toroid system, which consists of a barrel toroid, producing 0.5 T magnetic field and two end-cap toroids, producing a field of 1 T. The toroids are found outside the MS, and are used to bend the tracks of muons in the MS. The toroidal magnet system consists of eight barrel toroids housed in separate cryostats and two end-cap cryostats housing eight coils each. The EC coils systems are rotated by 22.5° with respect to the barrel toroids in order to provide radial overlap and to optimise the bending power in the interface regions of both coil systems.
3.4.5 Trigger System

The design luminosity of the LHC is $10^{34}$ cm$^{-2}$s$^{-1}$, which corresponds to a bunch crossing rate of 40 MHz. There is only a maximum amount of data that can be stored from collision, 300 Mbs$^{-1}$, which corresponds to an event rate of 200 Hz. The event rate of 200Hz is limited by the offline processing capacity. In 2012, the offline processing capacity was raised to almost 400Hz, by having a delayed processing of some fraction of the data which won’t be available till after the run stops. In order to achieve this challenging goal, ATLAS has a three level trigger system, shown in Figure 3.10, designed to reduce the event rate to a manageable level. The level 1 trigger consists of hardware while the subsequent two triggers, level 2 and event filter are software based. The combination of the level 2 and event filter is known as the high level trigger (HLT). Each subsequent level refines the decisions made in the previous levels.

Level 1

The level 1 (L1) is completely hardware based, designed to handle an output rate of up to 100 kHz. The L1 trigger has $2.5 \mu s$ latency time to accept or reject each event. At this timescale, only the reduced granularity of the calorimeters and the muon spectrometer is used, no ID information can be used.

The calorimeter selection utilises the L1 calorimeter trigger (L1Calo). This trigger is based on information from the ECal and HCal, in both the barrel and EC regions. The L1Calo can identify electrons, photons, jets and taus (decaying to hadrons) as well as large $E_T^{miss}$. Isolation can be required for the electron, photon and tau triggers to reject the calorimeter clusters which are not well separated from surrounding energy deposits.

The muon selection is governed by the L1 muon trigger. This is based on signals in the muon trigger chambers; RPCs in the barrel region and the TGCs in the EC region. The trigger searches for patterns of hits consistent with high-$p_T$ muons originating from the interaction region.

The central trigger processor (CTP) makes the overall L1 decision based on only the multiplicity of trigger objects. The information on the location of the trigger objects is kept in the muon and calorimeter trigger processors. Trigger menus, which can be programmed with up to 256 distinct items, are used to select events of interest. If the trigger menu requirements are passed, then the L1 trigger defines one or more regions of interest (RoIs), which are the geographical coordinates in $\eta$ and $\phi$. The full detector readout information within the RoIs at L1 is sent to L2.
For many of the triggers, the event rate at high luminosity exceeds the bandwidth available, so only a fraction of the events can be stored. This pre-scaling of the triggers is vital at high luminosities and is pre-defined for a given luminosity.

**High Level Trigger**

The L2 trigger is seeded from the L1 RoI information. It consists of fast algorithms which are run on dedicated farms of computers. The high level trigger, HLT, algorithms use full granularity and precision of calorimeter and muon chamber data, as well as data from inner detector to refine trigger selections. The L2 processing time is of the order of 40 ms per event, and the rate can be reduced to the order of 3.5 kHz.

The algorithms look for features within the RoIs such as ID tracks, calorimeter clusters or muon spectrometer tracks.

The event filter (EF) uses algorithms used in the offline reconstruction to further reduce the event rate to 200 Hz, with an event processing time of around a few seconds per event. The EF performs a complete reconstruction of the event using the full detector information and uses this for a selection directly based on the offline reconstruction and analysis algorithms.
Chapter 4

Object Reconstruction

This analysis determines the mass difference between top and the anti-top quarks. The semi-leptonic decay channel has six objects in the final state; lepton, neutrino and four partons which are expected to be two $b$-jets and two light jets. The definition of these objects will be described and discussed in this chapter.

4.1 Electrons

Electrons are charged particles that leave hits in the ID and ideally deposit all of their energy in the electromagnetic calorimeter (ECal). Only electrons that have a transverse energy, $E_T$, greater than 25 GeV are used in the analysis. The definition of the $E_T$ used is given in Equation 4.1

$$E_T = \frac{E_{\text{cluster}}}{\cosh(\eta_{\text{track}})},$$ (4.1)

where $E_{\text{cluster}}$ is defined as the collection of cells in the calorimeter in which the electron deposits its energy and $\eta_{\text{track}}$ is defined as the $\eta$ of the inner detector electron track. The electron candidates must be found within a range $|\eta| < 2.47$. Electron reconstruction algorithms use a combination of high energy deposits in a collection of neighbouring cells in the second layer of the ECal, known as clusters, with hits in the ID. These hits are used to reconstruct tracks, which are extrapolated to the ECal. For each cluster, the closest matching track in $\Delta R$ is associated with the cluster to form an electron candidate. An additional $E_T/p_T$ balancing requirement is needed as a further discriminant. The matching of the ID tracks to the calorimeter clusters helps remove fake electron candidates coming from $\pi^0$ decays, and increases electron reconstruction purity. Additionally, the electron candidates can be identified using the transition radi-
A series of EM cluster corrections need to be made to correct for biases that arise in the measurement. The corrections are applied for leakage, calibrations and dead material. The cluster $\eta$ and $\phi$ position dependent energy corrections are applied, by correcting for the calibration in the strip and middle layers. The energy modulation in both $\eta$ and $\phi$ is corrected for, since the measured energy varies depending upon the position of the particle impact in the calorimeter.

Hadronic activity can lead to ID tracks and energy deposits in the electromagnetic calorimeter (hadronic leakage). A series of selection criteria have been developed to reject this background known as the “loose”, “medium” and “tight” cuts, which are described in detail in [24].

The first level of cuts concentrate on a loose definition of an electron. A cut on the hadronic leakage (when the EM showers deposit a small amount of energy in the HCal) is also applied. The hadronic leakage is defined as the ratio of $E_T$ in the first layer of the HCal to the $E_T$ of the EM cluster. The second layer of the ECal is where the EM showers tend to deposit most of their energy in. The lateral shower width and the ratio in $\eta$ of cell energies in 3x7 (in which electrons deposit most of their energy) window versus 7x7 window cells are also taken into account.

The second level of cuts, called the medium definition of an electron, builds upon the loose criteria, but with more stringent cuts. In addition, the first layer of the ECal is used, which utilises the total shower width information along with the ratio of the difference between two leading energy deposits, with the sum of the energy. These cuts help reject jets with high energy pions and wide showers, which could fake electrons. The medium selection is the first instance in which track quality cuts are applied. At least one hit in the pixel detector and at least seven in the SCT are required. The transverse impact parameter, $d_0$, has a cut of $d_0 < 5$ mm. The $\Delta \eta$ between the matching track and cluster must be below 0.01.

The loose and medium selection cuts are further improved by applying the tight selection requirements, which this analysis uses. The tight electron selection criteria requires:

- Longitudinal shower containment and shower shape used for coarse rejection of hadronic background

- Track quality cuts applied based on number of pixel (hits in the b-layer $\geq 1$), SCT ($\geq 7$) and TRT hits
• Transverse impact parameter requirement of $d_0 < 1\, \text{mm}$
• Matching between cluster and track; $\Delta \eta < 0.005$ and $\Delta \phi < 0.02$
• Ratio of the cluster energy to the track momentum, $E_T/p_T$

There are regions of the ECal which are un-instrumented. The boundary between the barrel and end-cap region requires a cut between $1.37 < |\eta| < 1.52$, to remove such events.

Two types of isolation cuts can be applied to suppress fake electrons from jets and electrons from heavy-flavour decays; calorimeter based and tracking based isolation [24]. The calorimeter isolation cut is based on the reconstructed energy in a cone of radius $R$ around an electron candidate, where the energy of the electron is excluded. A cone of radius 0.2 in $\eta - \phi$ space is chosen as a compromise between allowing for more energy in the cone, in case of misidentified jets, and reducing the effect of energy deposits from pileup events. Tracking based isolation takes the summed scalar $p_T$ of tracks in a cone size of 0.3. This method has the advantage of track quality criteria applied to reject tracks from secondary vertices.

When applying cuts on the isolation variables, the dependence of the isolation variables on $p_T$ and $|\eta|$ is taken into account by separately optimising the cuts for different regions in $p_T$ and $|\eta|$. The optimisation is done in such a way that the efficiency for isolated electrons, with respect to the pre-selection, is constant in all ranges of $p_T$ and $|\eta|$. This allows one to retain 99%, 98%, 95%, or 90% efficiency for isolated electrons. This analysis uses the optimisation in which 90% efficiency for isolated electrons is retained for both track based and calorimeter based isolation. The EM energy scale [25] accounts correctly for the energy deposited in the calorimeter by the showers. It is determined through in-situ calibration of constraining the di-electron invariant mass distribution. This is constrained to the well-known $Z$ boson shape obtained from LEP measurements [26]. The EM scale is known to a high precision level; of the order of 0.5% in the central region.

An electron is removed from the event if it is found within a $\Delta R < 0.4$ of a jet.

### 4.2 Muons

Muons are charged particles which leave tracks in the ID and the MS, with little energy deposited in the calorimeters. The MS has a coverage of $|\eta| < 2.7$, and can be used to reconstruct the muon tracks to measure the $p_T$. However, multiple scattering effects, particularly for low $p_T$ muons, can give a poor resolution on
the measured momentum. A combination of muons tracks from the MS and ID are used to improve the stand-alone measured momentum. The muon track in the MS is projected backwards towards the interaction point. Matched ID and MS tracks are combined and any muon candidate without a matching ID track, is rejected. The MUID algorithm [27] is used to perform the muon reconstruction. The energy correction is a function of the track $p_T$ and $\eta$, and reflects the non-Gaussian energy loss of a muon traversing the beampipe, ID and calorimeters before reaching the MS. A tight requirement on the muon selection ensures that only high-quality muons are reconstructed:

- Track quality cuts applied, including at least one pixel, six SCT hits and hits in the TRT.
- Muon $p_T > 20$ GeV
- Pseudorapidity $|\eta| < 2.5$

To reduce fake muons, an isolation cut is also applied to the muon events. The $E_T$ in a cone in $\eta - \phi$ space of radius 0.2 is required to be less than 4 GeV in the calorimeter. In addition, the scalar sum of the transverse momentum for additional tracks inside a cone of $R = 0.3$ is required to be less than 2.5 GeV. Muons within a $\Delta R < 0.4$ to a jet are removed from the event, where only jets with $p_T > 25$ GeV are considered. The optimisation of the isolation cuts and the efficiencies are given in the ATLAS internal note [28]. Combining the calorimeter and track based efficiencies, the efficiency of the isolation cut for muons is $\sim 75\%$ [28].

An absolute isolation requirement is used for the muons, but a relative isolation is used for electrons. The higher the energy of an electron, the more energy it will emit during flight. The muon, however, emits a relatively low amount of energy during flight. A relative isolation cut is therefore required to account for the emission for the electron, whereas for muons, an absolute isolation cut is suitable.

4.3 Jets

Proton-proton collisions produce partons in the final state. These partons fragment before they can be directly detected, forming showers of particles that can be measured in the detector. The particle showers can be grouped together to form jets. This procedure is shown schematically in Figure 4.1. The kinematic
properties of these jets ($p_T, \eta, \phi$) can be associated with the properties of the hard scatter partons that produced them. A jet-finding algorithm is used to cluster particles together that are likely to have originated from the same partons. The energy in these jets have to be calibrated, as detector effects need to be accounted for.

There are numerous jet algorithms available for an experimentalist to use, but in this analysis, the \textit{anti}−$k_t$ algorithm \cite{30} is used to reconstruct jets. Only jets that lie within a radius 0.4 are used. The \textit{anti}−$k_t$ jet algorithm is infrared and collinear safe. Infrared safe algorithms ensure that the emission of a soft gluon between two jets does not affect the jet. Collinear safe means that splitting a parton into two collinear ones, does not change what one identifies as the jet during clustering. The jet shape is not influenced by soft radiation, giving rise to circular jets. The algorithm is based on existing sequential recombination algorithms like the $k_t$ \cite{31} and Cambridge-Aachen algorithms \cite{32}.

A distance $d_{ij}$ is introduced as the distance between particles (or pseudo-jets as the algorithm progresses) $i$ and $j$ and a distance $d_{iB}$ between the particle, $i$, and the beam (B). The clustering begins by comparing the distances $d_{ij}$ and $d_{iB}$. If the distance $d_{ij}$ is smaller than $d_{iB}$, then the particles $i$ and $j$ are combined. However, if $d_{ij}$ is larger then $d_{iB}$, then particle $i$ is called a jet, removed from the list of particles to loop over, and the whole process is repeated until no particles are left. The definitions of the distances are given in Equation 4.2 and 4.3

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta_{ij}^2}{R^2}$$

(4.2)
where $\Delta^2_{ij} = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$, $k_{ti}$ is the particle transverse momentum, $y_i$ the rapidity, $\phi_i$ the azimuthal angle of particle $i$, $R$ defines the “radius” of the resultant jet, and $p$, is used to govern the relative power of the energy versus geometrical ($\Delta_{ij}$) scales. Jets with an $R$ of 0.4 are used in this analysis. The parameter $p$ is important as its value determines the style of jet algorithm used. For $p = 1$, the $k_T$ algorithm is obtained, $p = 0$ returns the Cambridge-Aachen algorithm, and $p = -1$, returns the anti-$k_t$ algorithm. Setting $p$ to -1, one can see that the equations in 4.2 and 4.3 now go as the inverse of the momentum. The softer particles will be separated by larger distances, which means the soft particles are less likely to cluster with themselves, but more with harder particles. If a hard particle has no particles near it within a radius, $2R$, then it will be classed as a jet, with a perfect conical shape.

Topo-cluster jets are used in this analysis. These clusters are built from topologically connected calorimeter cells that contain a signal that is significantly above the noise level. The EM+JES calibrations are applied to these jets, which need to be applied before the jets are used in the pre-selection for the analysis. The calibration consists of a variety of steps starting from the measured calorimeter energy at the electromagnetic (EM) energy scale (discussed in Section 4.1). Next, the jet energy scale (JES) correction is applied. The JES aims to correct for detector effects. It includes pileup correction (which corrects for the expected energy offset caused by pileup interactions), vertex corrections (which corrects the momentum of the clusters to point from the primary vertex, not the origin of the coordinate system) and energy correction (which corrects to the particle level).

The jets are calibrated from the raw electromagnetic scale to the particle level. Jets are required to have $p_T > 25$ GeV within $|\eta| < 2.5$. An additional cut on the jet vertex fraction (JVF) is used. The JVF is defined (after associating tracks with jets by requiring $\Delta R < 0.4$ ) as the scalar sum of $p_T$ of all associated tracks from the primary vertex divided by the total $p_T$ associated with that jet from all vertices.

The JVF method selects jets that originated from the hard scatter interaction by using the tracking and vertex information. A requirement of $|JVF| < 0.75$ removes the jets that have come from pileup interactions. If a jet is the closest jet to an electron candidate and the corresponding distance $\Delta R$ between the jet
and the electron calorimeter position is less than 0.2, the jet is removed from consideration in order to avoid double-counting of electrons as jets.

4.4 B-jets

The semi-leptonic top decay channel has four quarks in the final state, two of which are measured as $b$-jets. These jets can be distinguished from jets that were produced from light quarks. A secondary vertex is produced, as shown in Figure 4.2, from the decay of, for example, the neutral $B^0$-meson (formed from the hadronisation of the $b$-quark at the hard scatter). This meson travels $\sim 500 \mu m$ before it decays. Having the ability to locate and measure the secondary vertex, allows tagging of subsequent jets produced, as a $b$-jet. There are a variety of $b$-tagging algorithms available [33], but in this analysis, the MV1 tagger is used. The MV1 tagger is itself a combination of a variety of different $b$-tagging algorithms; IP3D, SV1 and JetFitterCombNN. The jet $p_T$ and jet $\eta$ are also required when using this tagger.

The IP3D tagger is an impact parameter based algorithm, which uses a likelihood ratio technique, in which input variables are compared to pre-defined reference probability distributions for $b$-jets and light jets, obtained from Monte Carlo simulation [33]. The ratio of the two probabilities defines a weight factor which is used to discriminate between light and $b$-jets. This tagger uses a combination of likelihood ratios for both the transverse impact parameter significance, $d_0/\sigma_{d_0}$ and longitudinal impact parameter significance, $z_0/\sigma_{z_0}$.

The secondary vertex tagger (SV1) is used to help discriminate between light and $b$-jets. It uses the vertex formed by decay products of the $B$-mesons, building all two-track pairs that form good vertices and using tracks only associated to jets and far enough from primary vertex. Tracks must have a $p_T > 400$ MeV, $|d_0| < 3.5$ mm, and a requirement of at least one hit in the pixel detector in order to pass selection cuts. Figure 4.2 shows schematically the secondary vertex and impact parameters. The decay length significance, $L_{3D}/\sigma_{L_{3D}}$ can be used as a discriminating variable between light jets and $b$-jets. This is based on the SV0 tagger [35], which was used with the 2010 ATLAS data. To increase the discriminating power, the SV1 tagger takes advantage of three properties of the vertex; the invariant mass of all tracks associated to the vertex, the ratio of the sum of the energies of the tracks in the vertex to the sum of the energies of all tracks in the jet, and the number of two-track vertices. These variables are combined using a likelihood ratio technique and by taking the ratio with
a predefined reference probability distributions for light and $b$-jets, a tagging discrimination weight can be obtained \[28, 33\].

The last algorithm used in the MV1 $b$-tag tagger is the JetFitterCombNN. This exploits the topology of the weak $b$- and $c$-hadron decay in jets. A Kalman filter is used to find the common line on which the primary vertex and the $b$- and $c$-vertices lie, as well as their position on this line, giving the approximate flight path of the $b$-hadron. The discrimination between $b$-, $c$- and light jets is based on a likelihood using the masses, momenta, flight-length significances, and track multiplicities of the reconstructed vertices as inputs. Further information on this algorithm is given in \[36\].

The MV1 tagger takes the weights from the three aforementioned taggers, as well as the $p_T$ and $\eta$ of the jet as an input to a neural network, to determine a single discriminant variable. An operating point corresponding to a 70\% $b$-tag efficiency in $t\bar{t}$ events is used for this analysis. The $b$-tagging efficiencies and mis-tag probabilities are measured in data and are discussed in further detail in \[37, 38\]. Jet $p_T$ dependent scale factors are applied to simulation to match the efficiencies measured in data. The uncertainties in scale factors applied to $b$-jets ranges from 8\% to 20\% depending on jet $p_T$. 

Figure 4.2: *Secondary vertex and transverse impact parameter* \[34\].
4.5 Missing Transverse Momentum

During proton-proton collisions, a significant portion of energy in the $z$-direction, is not detected in ATLAS. The total initial/final momentum is zero in the transverse direction. An imbalance of energy in the transverse plane, therefore signifies the presence of weakly or non-interacting particles (like the neutrino that is present in the final state of the semi-leptonic top decay). The missing transverse energy ($E^\text{miss}_T$) is calculated as the transverse vector sum of all clusters in the calorimeters and the muons reconstructed in the muon spectrometer, and is discussed in detail in [39].

The $E^\text{miss}_T$ contribution to the MET calculation is reconstructed from cells, in jets with $p_T > 20$ GeV (with EM+JES (Section 4.3) calibration applied), from electrons with $p_T > 10$ GeV (with default electron calibration), and photons with $p_T > 10$ GeV (with EM scale calibration, in Section 4.1).

4.6 Scale factors

Scale factors are weights that have been applied to Monte Carlo models in order to improve their description of various efficiencies obtained from data. Lepton scale factors are applied to account for efficiencies in reconstruction, identification, trigger, isolation, and systematic uncertainties. The muon reconstruction and trigger efficiencies are estimated by the Muon Combined Performance (MCP) group [40] using the tag-and-probe method in $Z \rightarrow \mu\mu$ events in both data and MC simulation [41]. The trigger efficiency is parameterised as a function of the muon $\eta$ and $\phi$ [28][41]. For low $p_T$ ranges, $J/\psi \rightarrow \mu\mu$ samples are used [42]. The muon momentum scale offset and resolution smearing factors are calculated by the MCP, and use the weighted average of the ID and MS smeared components. These scale factors are calculated as a function of the data taking period, and are typically within 1% of unity.

The electron reconstruction and trigger efficiencies are measured with the tag-and-probe method using $Z \rightarrow ee$ samples [25]. The electron selection efficiency is derived from the combined measurements using $Z \rightarrow ee$ and $W \rightarrow e\nu$ samples. These samples cover similar $E_T$ and $\eta$ ranges. For the low $E_T$ ranges, $J/\psi \rightarrow ee$ samples are used. However, the available statistics after trigger requirements are limited for the $J/\psi$ samples. The scale factors are evaluated separately in different data periods to correct the MC simulation to match the data. Isolation scale factors on pileup effects are evaluated by measuring the scale factors (integrated over $\eta$ and $E_T$) as a function of the reconstructed number of vertices [28]. These
are provided by the egamma working group [43].

The lepton reconstruction and trigger scale factor corrections are relatively small and are typically of order one [28] [44].

Scale factors are applied to correct the efficiency of tagging a jet originating from $b$-quark, $c$-quark and the mis-tagging rate with which a light flavour jet (gluon, $u$, $d$, $s$-quark) is mistakenly tagged [28] [37] [45]. These scale factors are $p_T$-dependent and used to optimise the description of the data by MC simulation. Additional factors are applied to correct for the rates of $W$+ light flavour ($u,d$ and $s$ type quarks) jets and $W$+ heavy flavour ($c$ and $b$ type quarks) jets between data and MC predictions. The scale factors by which the $b$-tagging efficiency and mis-tagging rates in simulation have to be adjusted to be compared to data are both of order one [33].

These correction factors have been centrally derived by the egamma, muon, flavour tagging, and $W$+jets performance groups respectively. Each of the scale factors applied, have an associated systematic uncertainty. It is these uncertainties that are varied when calculating the systematic uncertainty on the mass difference. These uncertainties are defined in Table 6.5 [28]. Centrally produced tools have been used to apply these scale factors to this analysis and a complete list of packages used are given in Appendix [13].
Chapter 5

Data and Monte Carlo Samples

A series of Monte Carlo (MC) samples were generated and data samples were collected to use in this analysis. The data samples were split into the muon and electron decay channels, and were obtained from the events recorded in the 2011 proton-proton collisions. MC samples were used to replicate the signal and background processes seen in data.

This chapter concentrates on the production process of the MC samples, the various background and systematic study samples used and ends with the discussion of the data samples and the associated triggers. The creation of the signal samples was one of the major challenges in this analysis, so this section will be described in detail in this chapter.

5.1 Monte Carlo Samples

Monte Carlo techniques are numerical methods in which integrals are estimated by random evaluations of the integrand. In particle physics terms, this refers to a probability density function which can be converted into a simulation of a physical process (a cross-section for example). An MC generator is the name given to event generators which employ the MC technique.

Parton distribution functions (PDFs) are a vital part of calculating cross-sections for SM processes. The PDFs of the form, $f(x, Q^2)$ gives the probability of finding a parton $i$ (quark or gluon) carrying a fraction $x$ of the proton momentum, at four-momentum transfer $Q^2$. The PDFs are extracted by fits to inelastic scattering data. The deep inelastic scattering (DIS) results from HERA and fixed target lepton-nucleon experiments, as well as results from Tevatron and LHC have been used to extract the PDFs.
For this analysis, a variety of different MC samples were used to represent both signal and background events. All MC samples used in this analysis have been generated at $\sqrt{s} = 7$ TeV. For an MC sample to be used in an analysis, there is a series of steps and procedures that must be undertaken. A schematic of this is shown in Figure 5.1.

Event generation is the first step. This involves generating the tree level diagram first, and then applying processes, such as those shown in Figure 5.2. A variety of different generators can be used in this first step. The choice of MC generator depends on the physics process under study. In this analysis, mc@nlo\cite{47,48}, AcerMc\cite{49}, Powheg\cite{50} and Pythia\cite{51} are used. For the parton shower model, either Herwig\cite{52} or Pythia are used.

The following stage is simulation. The Geant4 toolkit\cite{54} is used to describe how generated particles interact with the material in the ATLAS detector. The energy deposited by the particles is converted to signals which the simulated detector sees. This is the most CPU time consuming of all the steps. Here to reduce CPU time, frozen showers\cite{55} are used. Events previously simulated with GEANT4, are stored in libraries and used as a substitute to EM particles that fall below a threshold of 1 GeV. The output of the simulation stage is the G4 Hits file. The next stage is digitisation.

Digitisation refers to the conversion of the GEANT4 simulated hits in active volumes of the detector, to raw data objects (RDOs). These RDOs are the input files to the reconstruction step. Within the digitisation stage, the propagation of charge to the readout electrodes (the measuring of the detector interaction with the particles) and the electronic simulation occurs. Further variables, such as the contribution of the minimum bias, cavern background, beam halo and beam gas interactions can also be added in this step.

One of the final steps in this chain is the reconstruction stage. At this stage, both MC samples and the data undergo the same steps. Outputs known as ESDs (event summary data) and AODs (analysis object data) are used in the analyses. The ESDs contain a detailed output of reconstruction, whereas the AODs contain enough information about the event to support all the typical usage patterns of a physics analysis. For this particular analysis, AODs are preferred to the ESDs due to storage reasons, as AODs are smaller in size.

This analysis utilises further reduced files called ntuples. These files contain analysis specific physics objects, which are useful for the final analysis. The size of these files are greatly reduced as a large fraction of the information stored in AODs that is not required, can be removed.
Figure 5.1: Schematic of the full Monte Carlo production chain from event generation, to the production of Analysis Object Data (AODs). The boxes in blue are the steps in the MC production, and the eclipses represent the output of these steps. HepMC \cite{46} is the storage container class which is used to store events from generation. The G4 hits and G4 digits are the \textit{Geant4} output from simulation and digitisation steps. The reconstruction step results in Event Summary Data (ESDs) and AODs. The green box represents the frozen shower sequence in which the usually long simulation process is made faster. The yellow eclipse represents the stage of the production chain at which the real data files are used. The final step, analysis, represents an output smaller than that of the AODs, known as ntuples, which are suitable for specific analysis use.
5.1.1 Signal Sample

In the SM, the CPT symmetry is conserved, and therefore particles and antiparticles have the same mass. This is reflected in the MC samples, in which particles and anti-particles are generated to have the same mass. For this analysis, it is required that one is able to generate the top and anti-top quarks at different masses in order to measure the mass difference.

For the generation of events with a non-zero mass difference, a modification has to be made to an existing MC generator. PYTHIA 6.425 [51] was modified by Stephen Mrenna [56] to allow for the generation of the signal samples. The leading order (LO) PYTHIA was chosen since the modifications in next-to-leading order (NLO) mc@nlo and POWHEG would be difficult. The MRST [57][58] LO PDF was used in the signal sample generation along with the AMBT2B and AUET2B tunes [59] for minimum bias and the underlying event. The process for the SM $t\bar{t}$ pair production is replaced by a new process, $t,\bar{t}^{*}$, where $\bar{t}^{*}$ is set to match the anti-top quark with a different mass using the Supersymmetric Les Houches Accord (SLHA) [60] file system. The SLHA is a unique set of conventions for supersymmetric extensions of the SM which together with generic file structures, provides a universal interface between spectrum calculation programs, decay packages and high energy physics event generators. In the SLHA file, the
branching ratio of the newly defined anti-top quark decays and width are set. The top quark width is calculated internally within Pythia. However, the newly defined anti-top quark has a default width of zero. Therefore the width of an anti-top quark at a given signal generation mass needs to be calculated, and then using this value, one must manually set the anti-top width within the SLHA file. Once all these variables are set, $\bar{t}^*$ is set to be the anti-top quark. This is where the modification is made to Pythia in the two routines, Pyofsh.F and Pyscat.F, to allow for the substitution of the anti-top quark for the newly created particle.

To generate events with this modification of Pythia, a couple of changes need to be made in a job options (JO) file. A JO is a python script which contains the important settings for the Pythia routines used in the event generation. It is this JO that is run to generate the events. A routine to allow for the use of the SLHA is added, as well as a lepton filter. This lepton filter helps in the generation of only di-leptonic and semi-leptonic events by filtering out the all hadronic decay events. The lepton filter has two requirements; at least one lepton in the final state (electron and muon only) with a minimum $p_T$ of 5 GeV and within the range $|\eta| < 2.8$.

Using this modified Pythia, 15 signal samples were generated, with a mass difference of $\pm 15$ GeV, $\pm 10$ GeV, $\pm 5$ GeV, $\pm 3$ GeV, $\pm 1$ GeV, $\pm 0.6$ GeV, $\pm 0.3$ GeV and 0 GeV. The mass difference is always defined as $m_t - m_{\bar{t}}$ and the average of the top and anti-top mass for all the signal samples is set to be 172.5 GeV. These samples range in size from 150,000 events for the large mass difference samples ($\pm 15, \pm 10, \pm 5$ GeV) to 400,000 events for the remaining samples. All the samples were fully simulated using Geant4 as discussed in Section 5.1.

Figure 5.3 shows a selection of mass difference distributions, at truth level for the different signal sample. Truth level here refers to the true mass difference, at the generator level before detector simulation and reconstruction were added.

### 5.1.2 Control and Background Samples

This section will concentrate on the control and background samples used in the analysis. A full summary of these samples are found in Tables 5.1 and 5.2. In these two tables, $\sigma$ represents the cross-section multiplied by the generator filter efficiency [61].

SM $t\bar{t}$ was generated using the NLO MC mc@nlo version 4.01. For this sample, Herwig version 6.520 was used as the parton shower (PS) model and hadronisation. Jimmy version 4.31 [62] was used to model the underlying event
Figure 5.3: Truth level mass difference distributions generated with a range of input values in simulation.

(UE). The NLO CT10 PDF \[63\] is used in this generation. This sample has the hadronic channel filtered out. In the mc@nlo events, the full NLO effects are given by using both negative and positive event weights. This sample is high in statistics (15 million events generated) and simulated using GEANT4. The cross-section used here was 79.01 pb, with a $K$-factor of 1.146. The $K$ factor is defined as:

$$K = \frac{\sigma_{N^{i+1}LO}}{\sigma_{N^iLO}}$$  \hspace{1cm} (5.1)$$

where $i = 0,1,2$ etc. For mc@NLO, $i$ is equal to 1. The $K$ factor is thus defined as the ratio of the approximate next-to-next-to-leading order (NNLO) cross-section to the NLO.

Further $t\bar{t}$ samples were created using the NLO Powheg 1.0 generator. Two sets of samples were generated here. The first sample used the HERWIG PS, whereas the second sample used PYTHIA for the PS. Both samples use CTEQ6L \[64\] LO PDFs. With these two samples, the effect of changing the PS can be investigated. The major difference between HERWIG and PYTHIA here is the way in which the parton shower evolves. For PYTHIA, the showers are transverse mo-
Figure 5.4: Parton shower evolution for angular ordered (left) and transverse momentum ordered (right) where $t_1 > t_2 > t_3$ \cite{53}.

momentum ($p_T$) ordered, whereas for HERWIG, they are angular ordered. This is shown in Figure 5.4. For a $p_T$-ordered PS model, the shower starts at a momentum of $t$, and then continues with emissions with decreasing $t$ until one hits the cutoff scale ($\Lambda_{QCD}$). For angular ordered PS model, on average, emissions have decreasing angles with respect to the emitters. The hadronic channel has been filtered out for this sample.

The LO AcerMc samples with different levels of initial state radiation (ISR) and final state radiation (FSR) were generated. ISR is the radiation that is emitted from the incoming partons before the hard scatter, whereas FSR is the radiation that is emitted from the outgoing partons from the hard scatter. ISR requires backward evolution, which starts at the hard scatter, and traces back. This sample uses the MRST LO PDFs. These samples are used to calculate the ISR/FSR contribution to the systematic uncertainty.

For the single top backgrounds, samples were generated in the three production channels; $Wt^-, t^-$- and $s$-channel, and in the three decay channels; electron, muon and tau. The $s$-channel samples were generated using Mc@NLO and the NLO CT10 PDF, whereas the $Wt^-$- and $t^-$-channel samples were generated using AcerMc and MRST LO PDFs.

The $W+jets$ samples were generated using Alpgen version 2.13 \cite{65}, with HERWIG used for the PS and Jimmy for the UE. These samples are normalised to the production cross-sections. With these samples used in analysis, heavy flavour
composition of the $W$+jets, have been calculated using data-driven methods. These samples are generated for the electron, muon and tau channels, and also for light and heavy jet flavours. Here a light flavoured jet is defined as a jet from a up, down, strange type quark or gluon jets, and a heavy jet flavour is one from a charm or bottom type quark. For the $W$+light jets, samples ranging from $W^+(\text{lepton}+\text{neutrino})+0$ parton final states up to $W^+(\text{lepton}+\text{neutrino})+5$ additional partons have been generated.

The $W$+heavy flavour samples include $W+bb$, $W+cc$, and $W+c$. Samples ranging from $W^+(\text{heavy flavour})+0$ parton final states up to $W^+(\text{heavy flavour})+\text{multiple additional jets}$ have been generated.

The $W^+(\text{lepton}+\text{neutrino})+4$ jets sample, and the $W+bb+2\text{jet}$ samples mimic the semi-leptonic decay signal, and are expected to dominate in the background contribution. A double $b$-tag requirement can reduce the $W$+light jets contribution.

These $W$+jets samples are used purely for the shape, and the normalisations themselves are calculated using data-driven methods. These methods typically provide a normalisation of the $W$+jets background, but still rely on the shape from the MC. The charge asymmetry method is used to estimate an overall normalisation on $W$+jets backgrounds. At the LHC, $W^+$ production has a higher cross section than $W^-$ production because of the PDFs. Since the ratio, $r_{MC}$ of the two production cross-sections is well predicted in the MC, the scale factors with their uncertainty are extracted for $W$+jets samples. The flavour-specific scale factors are also used. The uncertainties on $W$+jets estimates are 26 % for the electron channel and 28 % for the muon channel. A detailed description of the $W$+jets background estimation can be found in [66].

In the $Z$+jets samples, the cross-sections are normalised to the NNLO $Z$ boson production cross-section [66]. ALPGEN is again used to create this sample, with additional jets generated.

Three di-boson samples have been generated using HERWIG for $WW$, $WZ$ and $ZZ$ events. A lepton filter of one lepton with $p_T > 10$ GeV within $|\eta| < 2.8$ is required for these samples.

The final background sample used in the analysis is the QCD multi-jet sample. Unlike for the other backgrounds, this sample is not generated from simulated MC. Instead, a data-driven method, described in [67], is used to simulate the shape and normalisation. The uncertainty on the QCD multi-jets background is taken to be 50 %. The reconstructed mass distribution is obtained from a control region in the data. In the control region, leptons are required to be semi-isolated,
such that the transverse momentum of the inner detector tracks in a cone of radius $\Delta R = 0.3$ must be in the region of $0.1 < \frac{\sum P_t^{\Delta R=0.3}}{P_t(e, \mu)} < 0.3$. If an anti-isolation cut is used, where the large fraction of the QCD multi-jets in which the opening angles between the lepton and jets are very small, a large bias can result in the reconstruction of the top mass. In addition, leptons are required to have large impact parameters ($0.2 \text{mm} < |d_0| < 2 \text{ mm}$) and impact parameter significances ($|d_0|/\sigma_{d_0} > 3$).
<table>
<thead>
<tr>
<th>Process</th>
<th>$\sigma$ [pb]</th>
<th>K-Factor</th>
<th>Generator</th>
<th>$N_{MC}$</th>
</tr>
</thead>
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<td>150k</td>
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<td></td>
<td>Pythia</td>
<td>400k</td>
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<td>Pythia</td>
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<td></td>
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<td>300k</td>
</tr>
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</tr>
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<td>400k</td>
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<td>15M</td>
</tr>
<tr>
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<td>1.462</td>
<td>AcerMc+Pythia</td>
<td>12M</td>
</tr>
<tr>
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<td>1.469</td>
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<td>1.131</td>
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<td>10M</td>
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</table>

Table 5.1: Monte Carlo samples used for the signal events and to assess systematic uncertainties are shown with the cross-section including generator efficiency (branching ratios) and K-factors.
<table>
<thead>
<tr>
<th>Process</th>
<th>$\sigma$ [pb]</th>
<th>K-Factor</th>
<th>Generator</th>
<th>$N_{MC}$</th>
</tr>
</thead>
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<tr>
<td>Single top $Wt$ all decays</td>
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<td>1.079</td>
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</tr>
<tr>
<td>Single top $t$-chan (e)</td>
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<td>0.865</td>
<td>ACERMc+PYTHIA</td>
<td>507k</td>
</tr>
<tr>
<td>Single top $t$-chan ($\mu$)</td>
<td>8.06</td>
<td>0.865</td>
<td>ACERMc+PYTHIA</td>
<td>1,097k</td>
</tr>
<tr>
<td>Single top $t$-chan ($\tau$)</td>
<td>8.05</td>
<td>0.866</td>
<td>ACERMc+PYTHIA</td>
<td>254k</td>
</tr>
<tr>
<td>Single top $s$-chan (inclusive)</td>
<td>0.47</td>
<td>1.064</td>
<td>MC@NLO+HERWIG</td>
<td>253k</td>
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<tr>
<td>$W \to e\nu + 0$ parton</td>
<td>6.930</td>
<td>1.196</td>
<td>ALPGEN+HERWIG</td>
<td>1,600k</td>
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<tr>
<td>$W \to e\nu + 1$ parton</td>
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<td>1.196</td>
<td>ALPGEN+HERWIG</td>
<td>1,314k</td>
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<tr>
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<td>1.196</td>
<td>ALPGEN+HERWIG</td>
<td>2,045k</td>
</tr>
<tr>
<td>$W \to e\nu + 3$ parton</td>
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<td>1.196</td>
<td>ALPGEN+HERWIG</td>
<td>564k</td>
</tr>
<tr>
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<td>1.196</td>
<td>ALPGEN+HERWIG</td>
<td>142k</td>
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<td>1.196</td>
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<td>40k</td>
</tr>
<tr>
<td>$W \to \mu\nu + 0$ parton</td>
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<td>ALPGEN+HERWIG</td>
<td>3,463k</td>
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<td>1.195</td>
<td>ALPGEN+HERWIG</td>
<td>4,997k</td>
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<td>ALPGEN+HERWIG</td>
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<td>255k</td>
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<td>70k</td>
</tr>
<tr>
<td>$W + b\bar{b} + 0$ parton</td>
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<td>1.200</td>
<td>ALPGEN+HERWIG</td>
<td>475k</td>
</tr>
<tr>
<td>$W + b\bar{b} + 1$ partons</td>
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<td>1.200</td>
<td>ALPGEN+HERWIG</td>
<td>505k</td>
</tr>
<tr>
<td>$W + b\bar{b} + 2$ partons</td>
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<td>1.200</td>
<td>ALPGEN+HERWIG</td>
<td>175k</td>
</tr>
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<td>1.200</td>
<td>ALPGEN+HERWIG</td>
<td>70k</td>
</tr>
<tr>
<td>$W + c\bar{c} + 0$ parton</td>
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<td>1.20</td>
<td>ALPGEN+HERWIG</td>
<td>1275k</td>
</tr>
<tr>
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<td>1.20</td>
<td>ALPGEN+HERWIG</td>
<td>1050k</td>
</tr>
<tr>
<td>$W + c\bar{c} + 2$ partons</td>
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<td>1.20</td>
<td>ALPGEN+HERWIG</td>
<td>525k</td>
</tr>
<tr>
<td>$W + c\bar{c} + 3$ partons</td>
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<td>1.20</td>
<td>ALPGEN+HERWIG</td>
<td>170k</td>
</tr>
<tr>
<td>$W + c + 0$ parton</td>
<td>644.4</td>
<td>1.52</td>
<td>ALPGEN+HERWIG</td>
<td>6499k</td>
</tr>
<tr>
<td>$W + c + 1$ partons</td>
<td>205.0</td>
<td>1.52</td>
<td>ALPGEN+HERWIG</td>
<td>2070k</td>
</tr>
<tr>
<td>$W + c + 2$ partons</td>
<td>50.8</td>
<td>1.52</td>
<td>ALPGEN+HERWIG</td>
<td>520k</td>
</tr>
<tr>
<td>$W + c + 3$ partons</td>
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<td>1.52</td>
<td>ALPGEN+HERWIG</td>
<td>115k</td>
</tr>
<tr>
<td>$W + c + 4$ partons</td>
<td>2.8</td>
<td>1.52</td>
<td>ALPGEN+HERWIG</td>
<td>30k</td>
</tr>
<tr>
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<td>11.5003</td>
<td>1.48</td>
<td>HERWIG</td>
<td>2.5M</td>
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<tr>
<td>$WZ$</td>
<td>3.4641</td>
<td>1.30</td>
<td>HERWIG</td>
<td>1M</td>
</tr>
<tr>
<td>$ZZ$</td>
<td>0.9722</td>
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<td>HERWIG</td>
<td>250k</td>
</tr>
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<td>$Z(e)bb + 0$ parton</td>
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<td>1.25</td>
<td>ALPGEN+HERWIG</td>
<td>150k</td>
</tr>
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<td>$Z(e)bb + 1$ parton</td>
<td>2.48</td>
<td>1.25</td>
<td>ALPGEN+HERWIG</td>
<td>100k</td>
</tr>
<tr>
<td>$Z(e)bb + 2$ parton</td>
<td>0.89</td>
<td>1.25</td>
<td>ALPGEN+HERWIG</td>
<td>40k</td>
</tr>
<tr>
<td>$Z(e)bb + 3$ parton</td>
<td>0.39</td>
<td>1.25</td>
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<td>10k</td>
</tr>
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<td>1.25</td>
<td>ALPGEN+HERWIG</td>
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<td>$Z(\mu)bb + 1$ parton</td>
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<td>1.25</td>
<td>ALPGEN+HERWIG</td>
<td>10k</td>
</tr>
</tbody>
</table>

Table 5.2: Background Monte Carlo samples are shown with the cross-section including generator efficiency (branching ratios) and K-factors.
5.2 Trigger

With an instantaneous luminosity of the order $10^{33}$ cm$^{-2}$s$^{-1}$ being achieved at the LHC, a good trigger strategy is needed in order to efficiently record events. Two single lepton triggers are used, one for the electrons and one for the muons.

ATLAS uses a three level trigger system, as described in Section 3.4.5, to select events of interest. The L1 is a hardware trigger, while the L2 and EF are software-based triggers commonly called the HLT. Semi-leptonic $t\bar{t}$ events are characterised by a high-$p_T$ charged lepton, which is used at the trigger level to discriminate against the dominant QCD multi-jet processes. The triggers and the data running periods (B-M) are shown in Table 5.3. The data periods represent a period of time in which there was stable detector conditions. The period only changes when important changes for the data quality are made. The electron triggers EF$_{e20}$ medium, EF$_{e22}$ medium and EF$_{e22vh}$ medium1 have increasing $p_T$ threshold requirements to deal with the increase in the luminosity. The EF$_{e20}$ medium trigger for data taking periods B-H, EF$_{e22}$ medium trigger for periods I-K, EF$_{e22vh}$ medium1 trigger for periods L-M were used. The vh component on the EF$_{e22vh}$ trigger represent the addition of an $\eta$ threshold and hadronic leakage cut required at L1.

At L1 the muon trigger is based on information from the muon spectrometer trigger system. At the HLT, muon tracks are reconstructed using the trigger hits from the precision MDT chambers. These tracks were then matched to inner detector tracks to form combined muons. For the data taking periods B-H, EF$_{mu18}$ trigger is used, and EF$_{mu18}$ medium for periods I-M.

The mu18 trigger is seeded by L1$_{mu10}$ and mu18 medium is seeded by L1$_{mu11}$, which are both hardware based trigger that selects muon candidates above 10 GeV. The difference between the two triggers are the L1$_{mu11}$ trigger requires at least three muon stations (inner, middle and outer part of the muon spectrometer) to fire, whereas L1$_{mu10}$ requires only two. The efficiency for the L1$_{mu11}$ trigger is 6% lower when compared with the L1$_{mu10}$ trigger [68].
<table>
<thead>
<tr>
<th>Data period</th>
<th>Muon Trigger</th>
<th>Electron Trigger</th>
<th>Run Range</th>
<th>Luminosity [pb⁻¹]</th>
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<td>B-D</td>
<td>mu_18</td>
<td>e20_medium</td>
<td>177986-180481</td>
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<td>E-H</td>
<td>mu_18</td>
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<td>180614-184169</td>
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<td>186516-186755</td>
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<td>mu_18_medium</td>
<td>e22_medium</td>
<td>186873-187815</td>
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</tr>
<tr>
<td>L-M</td>
<td>mu_18_medium</td>
<td>e22vh_medium</td>
<td>188902-191933</td>
<td>2432</td>
</tr>
</tbody>
</table>

Table 5.3: Triggers used for the different data taking periods.
5.3 Data

This analysis uses 7 TeV proton-proton collision data recorded in 2011 (period B-M) by the ATLAS experiment. Events triggered by single electron and muon triggers (Section 5.2) and filtered using a Good Runs List (GRL) are used. When collision data is collected, one must know which datasets are suitable for physics analysis. Data quality (DQ) information is required to define suitable datasets. A dedicated list of runs and luminosity blocks (LB) are collectively known as the GRL. ATLAS data quality revolves around the concept of “defects”. A defect is defined as a problem with a detector sub-system that affects the quality of the data acquired. There is a defect for each class of problem, which need to be set if there are any issues for a given LB (which is a period of 120 s of data recording). Intolerable defects mean the associated luminosity blocks must be removed before use in analyses.

These events require stable beams in the LHC and pass the ATLAS DQ requirements for all detector systems critical to $E_{T}^{miss}$ determination and muon, electron, and jet identification.

The total data collected in 2011 used for this analysis is 4.7 fb$^{-1}$. The breakdown for each period is given in Table 5.3.
Chapter 6

Analysis

This analysis presents a measurement of the mass difference between top and anti-top quarks using the full 2011 ATLAS collected data. The search is made in the semi-leptonic top decay channel in which one of the tops decays hadronically, and the other leptonically. Hadronic decay refers to the $W$ boson from a top decay, subsequently decaying into two light jets, and leptonic decay refers to the other $W$ boson from the top decay subsequently decaying into a charged lepton (electron, muon) and neutrino. In the final state, there are two $b$-jets present, two light-jets, one charged lepton and large missing momentum that comes from a neutrino.

With this event structure in place, a selection criteria needs to be devised in order to pick events with higher signal efficiency and reduced background contributions.

This chapter concentrates on the experimental method and the measurement of the mass difference. It begins with the search criteria for selecting semi-leptonic $t\bar{t}$ events, then moving onto reconstructing the top system using the kinematic fitter. An optimisation study was undertaken to improve the signal reconstruction and remove background contributions. Finally, the last section describes experimental techniques such as parameterisation, maximum likelihood fits and sensitivity studies.

6.1 Event Selection

In order to select semi-leptonic $t\bar{t}$ events, a common set of selections have been developed and applied in the semi-leptonic analyses on ATLAS. This strategy allows for the sharing of many of the same common studies of systematic uncertainties and tools. The selection cuts are given below:
1. Require the event to pass the GRL and to pass the relevant single charged lepton trigger,

2. Require events to have a primary vertex with \( N_{\text{tracks}} > 4 \) to remove non-collision backgrounds,

3. Exactly one electron with \( E_T > 25 \) GeV and \( |\eta| < 2.47 \), excluding the crack region \( (1.37 < |\eta| < 1.52) \), or exactly one muon with \( p_T > 20 \) GeV and \( |\eta| < 2.5 \),

4. Require the selected charged lepton to match to the triggered object,

5. Remove events where an electron shares an inner detector track with a non-isolated muon,

6. Require at least four jets with \( E_T > 25 \) GeV and \( |\eta| < 2.5 \),

7. Require a missing transverse energy of \( E_{\text{miss}}^T > 30 \) GeV for the electron channel, or \( E_{\text{miss}}^T > 20 \) GeV for the muon channel,

8. Transverse \( W \) mass is required to be \( M_t(W) > 30 \) GeV for the electron channel or \( (E_{\text{miss}}^T + M_t(W)) > 60 \) GeV for the muon channel,

9. At least two jets to be \( b \)-tagged (with a \( b \)-tagging MV1 weight larger than 0.601713).

The event selection is chosen in this manner to increase the event yield and reduce background contribution to the signal. The same event selection is used for data as well as MC simulation. The first cut selects events that pass the GRL and trigger requirements. This cut is applied only to data, by selecting run periods that are included in the GRL, and the events that are triggered by the high-\( p_T \), single charged lepton triggers. The GRL was designed for the 2011 data taking period, and the specific package code and GRL used in this analysis are given in Appendix B. The muon and egamma streams were used for the muon and electron channels respectively. In MC samples, the simulated trigger is included offline, and the events are passed through this. The primary vertex cut removes the non-collision background contributions that arise from cosmic rays and beam gas events. These events have low number of associated tracks, so a requirement of greater than four tracks reduces this contribution. Cuts three to nine deal with objects found in the final state; one charged lepton, four jets, and \( E_{\text{miss}}^T \). Exactly one charged lepton is needed and any events where the electron and muon
share the same ID track are removed. A cut on the $E_T^{miss}$ and the requirement of greater than four jets in the event are also needed. The $M_t(W)$ cut is defined as the transverse mass of the charged lepton and $E_T^{miss}$ present in the event. The final cut aims to significantly reduce the background contribution by requiring at least two tagged $b$-jets in the final state.

Tables 6.1 and 6.2 show the comparison of the data and MC predictions at each stage of the event selection. Many of the selection cuts have been combined to simplify the tables. The trigger column represents cuts one and two and the one electron/muon column represents cuts three, four and five. The four jet column represents cut six and $E_T^{miss}$ represents cuts seven and eight. The double $b$-tag column, represents cut 9. A good agreement is observed between the data and MC simulation predictions. The background contribution for both the electron and muon channel before the $\chi^2$ cut is $\sim15\%$. However, by applying a $\chi^2$ cut, the background contribution is reduced to only $\sim10\%$. This final cut on $\chi^2$ will be described in further detail in Section 6.2. The number of muon events that passes all selection criteria is $\sim40\%$ larger then the number of electrons that pass the event selection. Looking at Tables 6.1 and 6.2 one can see that more muons are retained at the $E_T^{miss}$ stage, then electrons. At this stage, cut eight differs between the two channels as the muon uses a combination of $E_T^{miss}$ and $M_t(W)$ cuts, whereas the electron selection requires both cuts be applied sequentially (i.e. first the $E_T^{miss}$ then the $M_t(W)$).
<table>
<thead>
<tr>
<th>Process electron channel</th>
<th>no cut $\times 10^5$</th>
<th>trigger $\times 10^5$</th>
<th>1 electron $\times 10^4$</th>
<th>4 jets $\times 10^4$</th>
<th>$E_T^{\text{miss}}$ $\times 10^2$</th>
<th>2 $b$-tag</th>
<th>$\chi^2$</th>
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<tbody>
<tr>
<td>Data</td>
<td>164</td>
<td>1278</td>
<td>1926</td>
<td>77287</td>
<td>48190</td>
<td>9752</td>
<td>4941</td>
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<tr>
<td>SM $t\bar{t}$, not all hadronic</td>
<td>427</td>
<td>113</td>
<td>612</td>
<td>26818</td>
<td>21395</td>
<td>8380</td>
<td>4501</td>
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<td>6244</td>
<td>1145</td>
<td>903</td>
<td>231</td>
<td>97</td>
</tr>
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<td>21077</td>
<td>13040</td>
<td>956</td>
<td>744</td>
<td>261</td>
<td>64</td>
</tr>
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<td>1373</td>
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<td>31</td>
<td>14</td>
<td>5</td>
</tr>
<tr>
<td>$Wb\bar{b}$ + jets</td>
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<td>1097</td>
<td>67382</td>
<td>1437</td>
<td>1058</td>
<td>174</td>
<td>38</td>
</tr>
<tr>
<td>$Wc\bar{c}$ + jets</td>
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<td>2985</td>
<td>1839</td>
<td>1784</td>
<td>1313</td>
<td>70</td>
<td>12</td>
</tr>
<tr>
<td>$Wc$ + jets</td>
<td>4502</td>
<td>5568</td>
<td>5752</td>
<td>3828</td>
<td>2596</td>
<td>67</td>
<td>18</td>
</tr>
<tr>
<td>$W$ + jets</td>
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<td>2057</td>
<td>28325</td>
<td>20571</td>
<td>288</td>
<td>76</td>
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<tr>
<td>$Zb\bar{b}$ + jets</td>
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<td>46606</td>
<td>25639</td>
<td>4150</td>
<td>342</td>
<td>54</td>
<td>14</td>
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<tr>
<td>Dibosons</td>
<td>1122</td>
<td>36449</td>
<td>20600</td>
<td>454</td>
<td>276</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>QCD($e$)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>233</td>
</tr>
<tr>
<td>Total SM background</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9781</td>
</tr>
</tbody>
</table>

Table 6.1: Cut flow table for signals and the main SM backgrounds in the electron channel. The integrated luminosity is 4.7 fb$^{-1}$. 
<table>
<thead>
<tr>
<th>Process</th>
<th>no cut</th>
<th>trigger</th>
<th>1 muon</th>
<th>4 jets</th>
<th>$E_{T}^{\text{miss}}$</th>
<th>2 b-tag</th>
<th>$\chi^2$</th>
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</tr>
<tr>
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<td>$1217 \times 10^2$</td>
<td>$81495$</td>
<td>$35139$</td>
<td>$32110$</td>
<td>$13972$</td>
<td>$7709$</td>
</tr>
<tr>
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<td>$74185$</td>
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<td>$1319$</td>
<td>$384$</td>
<td>$170$</td>
</tr>
<tr>
<td>Single top, $t$-channel</td>
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<td>$22235$</td>
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<td>$118$</td>
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<td>$9$</td>
</tr>
<tr>
<td>$Wb +$ jets</td>
<td>$6512 \times 10^2$</td>
<td>$1683 \times 10^2$</td>
<td>$1079 \times 10^2$</td>
<td>$2220$</td>
<td>$1982$</td>
<td>$342$</td>
<td>$94$</td>
</tr>
<tr>
<td>$Wc +$ jets</td>
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<td>$1374 \times 10^3$</td>
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<td>$3209 \times 10^4$</td>
<td>$51526$</td>
<td>$45687$</td>
<td>$547$</td>
<td>$137$</td>
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<tr>
<td>Dibosons</td>
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<tr>
<td>Total SM background</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$16584$</td>
</tr>
</tbody>
</table>

Table 6.2: Cut flow table for signals and the main SM backgrounds in the muon channel. The integrated luminosity is 4.7 fb$^{-1}$.
6.2 Kinematic Fitter

In order to reconstruct the invariant masses of the top and anti-top quark for the selected $t\bar{t}$ candidates, a kinematic mass fitter is used. The kinematic fitter fully reconstructs $t\bar{t}$ events from the reconstructed charged lepton, four jets and $E_T^{miss}$. This requires the correct assignment of the jets from the top quark decays to the four original partons. The two jets from the $W$ boson do not need to be distinguished, giving $4! \over 2!$ jet permutations. Applying a requirement of 0,1 or 2 $b$-tags significantly reduces the jet permutations available to 12, 6 and 2 respectively. This analysis selects events with two $b$-tags applied, therefore there are only two possible jet configurations. The correct jet assignment for the two possible jet configurations is 70% before the $\chi^2$ cut and 73% after the $\chi^2$ cut. This relatively high efficiency is down to the double $b$-tag requirement, which significantly reduced the possible jet combinations.

In order to improve the efficiency of the kinematic mass fitter, a fifth jet is included as one of the four jets and can be swapped with one of last two leading jets. If a better efficiency is obtained, this new jet combination is used. This addition is motivated by the possibility of a high $p_T$ jet replacing one of the leading four jets from the top decays.

The reconstruction of the leptonically decaying $W$ boson requires the knowledge of the neutrino momentum. The transverse momentum of the neutrino, $p_T(\nu)$, is inferred from $E_T^{miss}$. In the fitter, the mass of the leptonically decaying $W$ boson is constrained to be 80.4 GeV [71], allowing the longitudinal momentum of the neutrino, $p_L(\nu)$, to be expressed in terms of two measurable quantities; $p_T(\nu)$ and the charged lepton momentum $p_l$. The resulting quadratic equation in $p_L(\nu)$ provides two solutions for each permutation of jet assignment. This is discussed in further detail in Appendix A.

For each combination, an additional top-specific correction was applied to each jet to convert the measured jet energy to an average partonic level energy. The $p_T$-dependent top-specific correction and their resolution functions for $b$-jets and light jets were derived using MC@NLO $t\bar{t}$ events separately. Events were selected where all four leading jets were matched to the truth partons within $\Delta R < 0.2$. The true parton $p_T$, $p_T^{parton}$, was then compared with the reconstructed jet $p_T$, $p_T^{jet}$, and the correction was derived as a function of the jet $p_T$. This method additionally provides an estimate of the parton-jet $p_T$ resolution $\sigma_{jet}$ as a function of $p_T$. The top-specific correction and resolution functions are shown
The response function, $R$, is defined as
\[ R = \frac{(p^\text{parton}_T - p^\text{jet}_T)}{p^\text{jet}_T}. \] (6.1)

The correction is applied by multiplying $(1+R)$ to the jet $p_T$. Further details on the derivation of this method is given in [18].

The kinematic mass fitter has a significant role to play in the analysis. It selects the combination of jets for which it is most likely that the jet assignment is correct and it leads to an improved resolution of the mass distributions of top and anti-top quarks. In the fitting process, MINUIT [72] was used to minimise the following $\chi^2$ function:

\[
\chi^2 = \frac{(p^\text{jets,fit}_{T} - p^\text{jets,meas}_{T})^2}{\sigma^2_{\text{jets}}} + \frac{(p^\text{fit}_{T} - p^\text{meas}_{T})^2}{\sigma^2_{T}} + \sum_{j=x,y} \frac{(p^\text{SEJ,fit}_{j} - p^\text{SEJ,meas}_{j})^2}{\sigma^2_{\text{SEJ}}} + \frac{(M_{jj} - M_{W})^2}{\sigma^2_{W}} + \frac{(M_{l\nu} - M_{W})^2}{\sigma^2_{W}} + \frac{(M^\text{meas}_{bl\nu} - (M_{top} + \frac{\Delta M^\text{fit}}{2}))^2}{\sigma^2_{\text{top}}} + \frac{(M^\text{meas}_{bjj} - (M_{top} - \frac{\Delta M^\text{fit}}{2}))^2}{\sigma^2_{\text{top}}} \] (6.2)

\[
\Delta M^\text{fit} = M^\text{fit}_{bl\nu} - M^\text{fit}_{bjj} \] (6.3)

\[
\Delta^\text{reco} = q \times \Delta M^\text{fit}. \] (6.4)
The first and second terms in Equation \ref{eq:6.2} allows the measured values of the leading four jets, $p_T^{4 \text{jets,meas}}$, and the charged lepton, $p_T^{l \text{meas}}$, to vary within the experimental uncertainty, $\sigma_{4 \text{jets}}$ and $\sigma_{l}$. The jet resolution, $\sigma_{4 \text{jets}}$ is defined as

$$\sigma_{b \text{jet}} = \exp \left( (-1.94-0.022 p_T^{j \text{et}} + 7.77) \frac{p_T}{E_T} + 0.083 \right)$$

for the $b$-jets and

$$\sigma_{l \text{jet}} = \exp \left( (-2.24-0.023 p_T^{j \text{et}} + 14.33) \frac{p_T}{E_T} + 0.063 \right)$$

for the light flavour jets. A detailed derivation of these resolutions are given in \cite{[18]}. The fitted jets and charged lepton are denoted by $p_T^{4 \text{jets,fit}}$ and $p_T^{l \text{fit}}$. The sum of extra jet energy in the third term, $SEJ$, is defined as a quantity absorbing all jet $E_T$ not associated with the primary charged lepton or the four leading jets, and is used to correct $E_T^{\text{miss}}$. The fourth and fifth terms are the $W$ boson mass constraints, where $M_{jj}$ and $M_{l\nu}$ represent the hadronic and leptonic $W$ boson masses. The $W$ boson mass is constrained to $M_W = 80.42$ GeV \cite{[71]} within the $W$ boson width, $\sigma_W$, which is set to its measured value \cite{[71]}. The final two terms contain the measured hadronic top mass, $M_{tjj}^{\text{meas}}$ and the leptonic top mass, $M_{l\nu}^{\text{meas}}$. The average value of the top quark and anti-top quark masses in the final two terms, $\Delta M_{\text{fit}}$, is constrained to $M_{t\bar{t}} = 172.5$ GeV \cite{[71]} because the precision measurements on the top quark mass from the Tevatron experiments were obtained assuming that $m_t = m_{\bar{t}}$. The top quark width, $\sigma_t$, is set to the value predicted from theory \cite{[71]}. From the fit, a mass difference in the reconstructed invariant masses between top and anti-top quarks is obtained. $\Delta_m^{\text{reco}}$ is calculated from the product of the lepton charge, $q$, and the difference between the fitted leptonic, $M_{l\nu}^{\text{fit}}$ and fitted hadronic, $M_{tjj}^{\text{fit}}$ top masses, as seen in Equations \ref{eq:6.3} and \ref{eq:6.4}. To find the mass difference between the top and the anti-top, one must find a way to distinguish between the two quarks. One way to do this is to use the charge of the lepton from the semi-leptonic decay of the tops. From this, one can conclude that if a negatively charged lepton is seen in the final state, it has come from a leptonically decaying anti-top. The opposite is true for a positive lepton in the final state.

In the fitting process, the measured momenta are permitted to vary within their experimental resolutions from the top specific corrections while the $\eta$ and $\phi$ of final state particles are fixed. It is this re-scaling of measured values that facilitates the improvement of the $\Delta_m^{\text{reco}}$ resolution. The effect of the re-scalings are propagated to the $E_T^{\text{miss}}$. The $SEJ$ energy is allowed to vary within an un-
certainty of 25% for energy above 20 GeV and 40% for lower energy. These uncertainties were taken by extrapolating the jet transfer function uncertainties, discussed in [18], back to lower energies. The SEJ resolution is therefore estimated to be $0.25\sqrt{SEJ}$. In practice, the contribution to the total $\chi^2$ from the charged lepton and SEJ terms is small in comparison to that from the four leading jets.

In each event, only the combination with the lowest $\chi^2$ value is used for this analysis. The lowest $\chi^2$ value is required to be less than 10. The optimisation of this cut is discussed in Section 6.3. If the best combination does not pass these cuts, the event is rejected.

Figure 6.2 plots shows how the $\Delta^{\text{reco}}_{m}$ distribution is improved if the fitted energy of the jets and charged lepton, with $\chi^2 < 10$ is used. The long tails observed in the distribution is due to increased combinatorial backgrounds. A similar improvement can be observed in the reconstructed leptonic and hadronic top mass distributions, shown in Figure 6.3.

Figure 6.2: Comparison of the $\Delta_{m}^{\text{reco}}$ distributions before/after the fitter with $\chi^2 < 10$ for MC@NLO $t\bar{t}$ events at $\Delta_{m}^{\text{truth}} = 0$. The unfitted $\Delta_{m}^{\text{reco}}$ distribution has the top-specific correction applied.

The mis-identification rate of the lepton charge could play a key part in this analysis. At ATLAS, the mis-identification probability of electrons is $\sim 10^{-3}$ [73]. Considering the number of electrons found in data is under 5000, this implies that fewer than five electrons may be mis-identified. The mis-identification for muon events is smaller, $\sim 10^{-4}$ [74]. There are just under 9000 muons found in data, which results in less then one muon on average, may be mis-identified. For
Figure 6.3: Comparison of the reconstructed top mass distributions before/after the fitter with $\chi^2 < 10$ for MC@NLO $t\bar{t}$ events at $\Delta_m^{\text{truth}} = 0$. The leptonic top and anti-top quarks ($M_{t\bar{t}}^\text{fit}$) is shown to the left and the hadronic top and anti-top quarks ($M_{b\bar{b}}^\text{fit}$) to the right. The unfitted mass distributions have the top-specific correction applied.

this analysis, the mis-identification of charged leptons plays a small part in the mass difference measurement and its contribution to the systematic uncertainty is discussed in Section 6.7.2.

6.3 Optimisation of the Kinematic Fitter

The $\chi^2$ value returned by the kinematic fitter can be used to reject background and also to remove poorly reconstructed signal events. A simple optimisation selects the best choice of $\chi^2$ to cut on. This optimisation was performed by Jahred Adelman [75] and is discussed in further detail in [76]. For the optimisation study, the background is ignored since its contribution is small when requiring two $b$-tags in each event. The optimisation uses several other features of the $\chi^2$:

- A tighter (looser) $\chi^2$ cut will allow fewer (more) signal events to pass the selection. In the absence of any other changes, the fewer $t\bar{t}$ events that pass the cut, the larger the statistical uncertainty in the measurement.

- A tighter (looser) cut will shrink (widen) the $\Delta^{\text{reco}}_m$ distribution. This is because a tighter cut rejects poorly reconstructed events. In the absence of any other changes, a measurement using a narrower distribution will have a smaller statistical uncertainty.

- As the tighter (looser) $\chi^2$ rejects more (fewer) poorly reconstructed signal events, the $\Delta^{\text{reco}}_m$ distribution will shift towards the (away from) true $\Delta^{\text{truth}}_m$ value.
A simple metric of the form \(6.7\) can be defined based on the three bullet points above:

\[
\sqrt{N} \cdot \frac{1}{\sigma_{\text{core}}},
\]

where \(\sigma_{\text{core}}\) is the RMS of the nominal \((\Delta m^{\text{reco}})\) distribution and \(N\) is taken as the integral of the expected number of signal events at a given \(\chi^2\), as seen in Figure 6.4. The parameter \(m\) is defined as the gradient of a linear fit, which is obtained at each \(\chi^2\) value by fitting a Gaussian to each signal sample within a \(\pm 75\) GeV window on the \(\Delta m^{\text{reco}}\) (similar to those seen in Figure 6.14). For a given \(\chi^2\) cut, the mean of the Gaussians from each of the signal samples is fitted with a linear function to obtain \(m\) at that \(\chi^2\). The \(m\) parameter as a function of the \(\chi^2\) is shown in Figure 6.5\textsuperscript{76}. As expected, \(m\) decreases as the \(\chi^2\) increases. If \(m\) was equal to zero, then there would be no sensitivity to the mass difference.

The form of the metric is motivated by the three bullet points and thus has three parameters. The \(\sqrt{N}\) term, which accounts for the first bullet point, allows the metric to scale with the statistics of the samples used in the optimisation. The \(\frac{1}{\sigma_{\text{core}}}\) term is accounted for by looking at the core of the \(\Delta m^{\text{reco}}\) distributions from the signal samples. The \(m\) term is accounted for by seeing how much the core of the \(\Delta m^{\text{reco}}\) distribution changes with \(\Delta m^{\text{truth}}\) when the \(\chi^2\) is changed.
Figure 6.5: Gradient parameter, $m$, from fits to the core of the mass difference distributions [75][76].

The resulting optimisation values are shown in Figure 6.6. The $\chi^2$ is optimised at a cut of 12-13. These optimisation studies do not include any background events, so a tighter cut is chosen at a $\chi^2 < 10$ (since the optimisation values are rather flat in $\chi^2$ in that region).

### 6.4 Comparison of Kinematic Distributions

In this section, a number of kinematic distributions have been examined to check the agreement between the data and MC simulation predictions. The $t\bar{t}$ pairs generated with MC@NLO was used as the signal sample. Good agreements in the total number of events between data and the predicted events from MC simulation are shown for both electron and muon channels, within the systematic
Figure 6.6: $\chi^2$ optimisation results. The optimisation values are calculated using Equation 6.7. [75][76].

uncertainty. All systematics uncertainties listed in Table 6.5 are included in the uncertainty band, assuming no correlation among all systematics. The systematic uncertainty is discussed in Section 6.7. The luminosity uncertainty is not included in the systematic uncertainty band.

Figure 6.7 shows the comparison between data and the predicted backgrounds for the charged lepton $p_T$ and $\eta$, and the $E_{\text{miss}}^T$. The charged lepton $p_T$ peaks between 30-40 GeV in data, and is well modeled by the signal and background simulation. The $E_{\text{miss}}^T$ shows larger background contributions at low $p_T$. The distributions show the charged lepton kinematics and the $E_{\text{miss}}^T$ is modeled well in all $p_T$ and $\eta$ ranges. The transition region from the barrel to the end-cap is also described well by the MC simulation.

Figure 6.8 shows the leptonic and hadronic $b$-jet $p_T$ and $\eta$ distributions. The leptonic $b$-jet $p_T$ distribution shows an excess in data around the peak region of $\sim 60$ GeV, but this is covered by the systematic uncertainty band. The majority of the background events lie in the low $p_T$ region.
The light jet $p_T$ and $\eta$ are shown in Figure 6.9. Here, a small excess is seen in the peak region of the $p_T$ distributions. This excess lies within the systematic uncertainty band, so one can conclude there is good agreement between data and MC prediction.

The reconstructed top, anti-top and di-jet masses are shown in Figure 6.10. The mass distributions in the MC simulation models the data well. The same level of agreement is seen in Figure 6.11, which shows the mass difference between the top and anti-top quarks.

Figure 6.12 shows the $\chi^2$ distribution in the electron and muon channel. The response to the fitter in the data and MC simulation case is consistent with one another.

There is good agreement between data and MC predictions between all the key kinematic variables presented. The modeling of the background and signal is consistent with the data within the systematic uncertainty. Tables 6.1 and 6.2 reflects this good agreement between data and MC simulation, after the $\chi^2$ cut is applied.
Figure 6.7: Comparison of the data and predicted backgrounds of the charged lepton $p_T$, $\eta$ and the missing transverse energy for the electron channel (left column) and muon channel (right column). The signal and background components of the MC simulation prediction have been normalised to 4.7 fb$^{-1}$ of data.
Figure 6.8: Comparison of the data and predicted backgrounds for the leptonic and hadronic b-jet $p_T$ and $\eta$ for the electron channel (left column) and muon channel (right column). No top-specific correction or JES SFs were applied to the jet $p_T$. The signal and background components of the MC simulation prediction have been normalised to $4.7 \text{ fb}^{-1}$ of data.
Figure 6.9: Comparison of the data and predicted backgrounds for the two light jets $p_T$ and $\eta$, for the electron channel (left column) and muon channel (right column). No top-specific correction or JES SFs were applied to the jet $p_T$. The signal and background components of the MC simulation prediction have been normalised to 4.7 fb$^{-1}$ of data.
Figure 6.10: Comparison of the data and predicted backgrounds for the di-jet mass and the reconstructed masses of the top and anti-top quarks, for the electron channel (left column) and muon channel (right column). The signal and background components of the MC simulation prediction have been normalised to $4.7 \, \text{fb}^{-1}$ of data.
Figure 6.11: Comparison of the $\Delta^{\text{reco}}$ distribution in data and the predicted backgrounds for the electron channel (left) and muon channel (right). The signal and background components of the MC simulation prediction have been normalised to $4.7 \text{ fb}^{-1}$ of data.

Figure 6.12: Comparison of the $\chi^2$ distribution in data and the predicted backgrounds for the electron channel (left) and muon channel (right). The signal and background components of the MC simulation prediction have been normalised to $4.7 \text{ fb}^{-1}$ of data.
6.5 Parameterisation

The template method is used in this analysis to determine the mass difference between the top and anti-top quarks, in data. The signal templates are derived using the signal Monte Carlo samples in Section 5.1.1. There is only one template for the background, which is taken as the sum of the various background processes discussed in Section 5.1.2. In order to extract the mass difference, $\Delta(m)$, the $\Delta_{\text{reco}}$ distributions for all signal MC samples need to be parameterised as a function of the true mass difference.

First, each of the signal $\Delta_{\text{reco}}$ distributions are fitted to two Gaussians of the form
\[
F(x) = A \cdot \frac{\exp\left(-\frac{(x-\bar{x}_1)^2}{\sigma_1^2}\right)}{\sigma_1} + B \cdot (1 - \text{frac}) \cdot \frac{\exp\left(-\frac{(x-\bar{x}_2)^2}{\sigma_2^2}\right)}{\sigma_2},
\]
where $A$ and $B$ are the normalisation constants for the Gaussians, $\bar{x}_1$ and $\sigma_1$ are the mean and width of the wide Gaussian and $\bar{x}_2$ and $\sigma_2$ are the mean and width of the narrow Gaussian. The narrow Gaussian corresponds to the correct jet-parton assignment, and the wider Gaussian takes into account the incorrect jet-parton pairings.

Figure 6.13 shows the double Gaussian fit to the nominal sample for $\Delta^{\text{truth}} = 0$ GeV sample. The fit performs very well, with a $\chi^2$ per degree of freedom of only 0.56. Figure 6.14 shows the double Gaussian fits to the remaining signal samples. Table 6.3 gives the list of parameters from the double Gaussian fit to the signal templates. These templates were created in the range $-180$ GeV to $180$ GeV, using 100 bins.

The means of the two Gaussians for all different mass samples are well parameterised by a linear function of the input $\Delta^{\text{truth}}$ as shown in Figure 6.15. The widths of the two Gaussians are described by a quadratic function. The relative weight or fraction, frac, which is defined as the contribution to the double Gaussian fit from the wider Gaussian, is also fitted to a quadratic function. Before deciding on using quadratic fits to $\sigma_1$, $\sigma_2$ and frac, a small study was undertaken in which the fit provided by a constant term was compared to that provided by a quadratic function. The $\chi^2$ per degree of freedom for the quadratic fits to $\sigma_1$, $\sigma_2$ and frac were 1.35, 1.93 and 2.59 respectively. The $\chi^2$ per degree of freedom for a constant fit to $\sigma_1$, $\sigma_2$ and frac were 1.57, 1.93 and 2.93 respectively. Quadratic functions were chosen to fit to the aforementioned parameters because they provided a better fit than the constant term. Figure 6.15 shows how the parameters of the two Gaussian fits vary as a function of the input top mass.
The entire ensemble of MC samples is fit to a functional form, including variations with input $\Delta_m^{\text{truth}}$. Figure 6.16 shows parameterised functions for different mass difference values.

The gradient of the two Gaussian means shown in Figure 6.15 are well constrained by the samples with a large $\Delta_m^{\text{truth}}$. But in the low mass region, (within $\Delta_m^{\text{truth}} = \pm 1$ GeV), large fluctuations appear due to low statistics of the signal samples (400K generated events). Since high statistics PYTHIA samples are not available, two high statistics systematic samples (ISR more and less) are used as a cross-check on the parameterisation, as these two systematic samples show a negligible difference in the mass difference with respect to the nominal signal sample ($\Delta_m^{\text{truth}} = 0$ GeV). Figure 6.17 shows the average of the ISR more and less samples agree very well with the parameterised curves at $\Delta_m^{\text{truth}} = 0$ GeV.

The background samples have no dependence on the mass difference, but must still be parameterised to smooth out statistical fluctuations. A single combined background template is used in which all backgrounds are merged together according to their respective weights and expected number of events. The combined background is fit to a single Gaussian. A small study was undertaken in which an alternate fit function was considered for the background template. The results of this study and the associated systematic uncertainty are discussed in
Table 6.3: A Table of parameters from the double Gaussian signal fit in Equation 6.8. Parameters \( \bar{x}_1 \), \( \bar{x}_2 \), \( \sigma_1 \) and \( \sigma_2 \) are given in GeV.

Section 6.7.14 Figure 6.18 shows the background parameterisations for the total background, QCD, single top and W+jets respectively.

6.6 Maximum Likelihood Fit and Sensitivity

6.6.1 Likelihood Fit

In order to extract the mass difference \( \Delta(m) \) from the data, one needs to fit to the \( \Delta_{m}^{\text{reco}} \) of the data using the parameterised template distributions. An extended maximum likelihood fit is used to extract the mass difference, \( \Delta(m) \), the number of signal events \( (n_s) \), and the number of background events \( (n_b) \). For a given data \( D \), with \( N \) events, the likelihood, \( L \), is defined by:

\[
L(D|n_s, n_b, \Delta(m)) = q(N, n_s + n_b) \times \sum_{i=1}^{N} \frac{n_s p_s(\Delta_{m}^{\text{reco},i} | \Delta(m)) + n_b p_b(\Delta_{m}^{\text{reco},i})}{n_s + n_b} (6.9)
\]

where \( q(N, n_s + n_b) \) is the Poisson probability to observe \( N \) events given \( n_s + n_b \) expected events and the sum over \( i \) is over the \( N \) reconstructed events. \( p_s \) and \( p_i \) are PDFs for the signal and background respectively.

The likelihood is maximised over all three parameters \( (n_s, n_b, \Delta(m)) \), where \( \Delta(m) \) is the quantity of interest.
6.6.2 Sensitivity Studies

Pseudoexperiments (PEs) are used in order to check the sensitivity to the mass difference for 4.7 fb$^{-1}$ of data in this analysis.

First, the fit machinery must be tested. Figure 6.19 shows the output of PEs, in the case where events were drawn from the parameterisation itself. The mean of the pull distribution is very close to being flat and consistent with zero, indicating that there is no bias in the fit machinery. The pull is defined as

$$\text{Pull} = \frac{\Delta_m^{\text{fit}} - \Delta_m^{\text{truth}}}{\sigma_{\Delta_m^{\text{fit}}}},$$ (6.10)

where $\Delta_m^{\text{fit}}$ is the mass difference returned from the fit to the distribution drawn from the parameterisation, $\Delta_m^{\text{truth}}$ is the mass difference at generation level and $\sigma_{\Delta_m^{\text{fit}}}$ is the uncertainty on the fitted value. Any small, residual bias is taken as a systematic uncertainty. The pull width is consistent with one, indicating the likelihood is Gaussian and that there is no need to inflate the returned errors from our fits. The expected statistical error is $\sim 700$ MeV. The expected statistical error drops at larger mass differences, as the narrow Gaussian gets larger weight at larger mass differences (see Section 6.5). Figure 6.20 shows pull values for the number of events. The returned pull RMS for the number of background events in Figure 6.20(d), has a poor $\chi^2$ per degree of freedom. This maybe due to fitting for low statistics process. Figure 6.21 shows the distributions of returned error from PEs from the nominal SM sample with $\Delta_m^{\text{truth}} = 0$ GeV.

Once the fit machinery was tested, PEs using the MC simulation were undertaken. In each PE and fit, the number of signal and background events are fluctuated around their expected values. In order to smooth out statistics in the pseudoexperiments, events are drawn from histograms created from the simulated samples. The histograms have 1000 bins and have a range [-180,180] GeV. $N$ events are randomly drawn from the histogram (using a dedicated ROOT random function). Two sets of histograms are obtained, one for the signal, which is filled with the expected number of signal events, and a second histogram, filled with the expected number of background events. Figure 6.22 is an example of the signal histogram, from which $N$ signal events are drawn. Note that the histograms drawn are derived from the simulated data, not from the parameterised function, so this method directly tests the goodness-of-fit of the global parameterisation.
6.6.3 Validation

The current signal samples used in the analysis have relatively low statistics (only 400K events generated). With this in mind, a study was taken to investigate the robustness of the parameterisation used, and to ensure that the parameterisation is sensitive enough to be able to make sub-GeV level precision measurement.

Two methods of investigation can be deployed. The first involves using the ISR more/less sample discussed in Section 6.5. The second method is to reweight the \textsc{mc@nlo} sample (15 Million generated events with full \textsc{Geant4} simulation) to the \textsc{Pythia} signal samples, in order to create high statistic signal samples at various values of $\Delta_{m}^{\text{truth}}$. Using these newly reweighted samples, the sensitivity of the parameterisation can be tested.

To begin, the top and anti-top masses for \textsc{mc@nlo} at truth level were individually reweighted to those of the \textsc{Pythia} samples for each mass difference signal sample. Another possible method would be to reweight the mass difference at truth level. However for this case, the top and anti-top mass at truth level need to be correlated in order to preserve the average top mass to be 172.5 GeV requirement needed for the kinematic fitter. Since the top and anti-top masses were not correlated, the first method was employed. Once the reweighting factors are found, the combination of both the top and the anti-top weights can be applied, on an event-by-event basis, to the reconstructed mass difference.

For the study, seven samples were reweighted; $\pm 1$, $\pm 0.6$, $\pm 0.3$, and the 0 GeV sample. Using these reweighted samples, 2000 pseudoexperiments were run using the parameterisation of which the results are shown in Table 6.4.

The results in the table are consistent with the input mass values within 50 MeV, with an offset of 175 MeV. Figure 6.23 shows the difference between the reconstructed and the truth mass difference as a function of the truth mass difference. The offset of 175 MeV is mostly due to the difference between \textsc{Pythia} and \textsc{mc@nlo}, plus a possible small bias by the fitter. This offset is subtracted from the final $\Delta(m)$ measurement.

The measurement value from the fit is calibrated to the next to leading order \textsc{mc@nlo} sample, with an additional calibration uncertainty of 50 MeV.
<table>
<thead>
<tr>
<th>Sample</th>
<th>$\Delta(m)$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC@NLO default sample</td>
<td>0.181</td>
</tr>
<tr>
<td>MC@NLO reweighted to PYTHIA : +0.3 GeV</td>
<td>0.458</td>
</tr>
<tr>
<td>MC@NLO reweighted to PYTHIA : −0.3 GeV</td>
<td>−0.118</td>
</tr>
<tr>
<td>MC@NLO reweighted to PYTHIA : +0.6 GeV</td>
<td>0.733</td>
</tr>
<tr>
<td>MC@NLO reweighted to PYTHIA : −0.6 GeV</td>
<td>−0.430</td>
</tr>
<tr>
<td>MC@NLO reweighted to PYTHIA : +1 GeV</td>
<td>1.19</td>
</tr>
<tr>
<td>MC@NLO reweighted to PYTHIA : −1 GeV</td>
<td>−0.799</td>
</tr>
</tbody>
</table>

Table 6.4: Pseudoexperiment results for the seven reweighted MC@NLO samples.
Figure 6.14: Parameterisation of the signal only templates in order of smallest (most negative) $\Delta_m^{\text{truth}}$ to largest $\Delta_m^{\text{truth}}$ in simulation. The red function is the wide Gaussian and the blue distribution is the sum of the two.
Figure 6.15: Parameters for the signal fits. Shown are the mean (a) and width (b) of the wide Gaussian ($\bar{x}_1$ and $\sigma_1$), the mean (c) and width (d) of the narrow Gaussian ($\bar{x}_2$ and $\sigma_2$), and the weight/normalisation (e) of the wide Gaussian ($\text{frac}$).
Figure 6.16: Parameterised signal templates for the different input $\Delta_m^{\text{truth}}$, drawn from the global parameterised fit.
Figure 6.17: Parameters for the signal fits with the ISR More, ISR Less and ISR average samples overlaid. Shown are the mean (a) and width (b) of the wide Gaussian ($\bar{x}_1$ and $\sigma_1$), the mean (c) and width (d) of the narrow Gaussian ($\bar{x}_2$ and $\sigma_2$), and the weight/normalisation (e) of the wide Gaussian ($\text{frac}$).
Figure 6.18: Parameterised background templates. Shown are the total background (a), QCD (b), single top (c) and W+jets (d) samples. The background contributions have been normalised to the expected number of events from 4.7 fb\(^{-1}\) of data.

Figure 6.19: Results from pseudoexperiments. Shown are the pull mean (a) and the pull width (b) of the mass difference distributions.
Figure 6.20: Results from pseudoexperiments. Shown are the pull mean for the number of signal events (a), pull mean for the number of background events (b), pull RMS for the number of signal events (c), and pull RMS for the number of background events (d).

Figure 6.21: Distributions of returned errors on the extracted $\Delta(m)$ from pseudoexperiments using $\Delta^{\text{truth}}_m = 0$ GeV sample (a) and the mean of the error returned for the different signal samples (b).
Figure 6.22: Histogram from which signal events are drawn for the pseudoexperiments. This histogram contains $t\bar{t}$ pairs generated with MC@NLO.

Figure 6.23: The difference between the reconstructed and the truth level mass difference as a function of the truth mass difference.
6.7 Systematic Uncertainties

Several sources of systematic uncertainties can arise when measuring $\Delta(m)$. One must consider which systematic effects, which the $\Delta(m)$ measurement would be sensitive to. This analysis is particularly sensitive to any asymmetry between the top and the anti-top quarks. Therefore, any systematic effect that can bring about an asymmetry in the top and anti-top quark’s system, is expected to contribute a large amount to the total systematic uncertainty.

Uncertainties in the difference in jet energy response between $b$ and $\bar{b}$-jets ($b\bar{b}$ asymmetry) and $c$ and $\bar{c}$-jets ($c\bar{c}$ asymmetry), parton shower model, MC generator, and the level of ISR and FSR, are expected to have a relatively large contribution to the systematic uncertainty.

However, uncertainties which are applied to both the top and the anti-top quarks equally, are expected to have a relatively small contribution to the systematic uncertainty. This is due to the fact that any changes applied equally to the top and anti-top will be negated when the mass difference is taken. Systematic effects that are expected to have a smaller contribution to the total systematic uncertainty include uncertainties in the charged lepton identification, jet energy scale (JES), $b$-jet energy scale (BJES), the jet energy resolution (JER), and the identification and mis-identification of the $b$-jets.

6.7.1 Procedure to Evaluate Systematic Uncertainty

The $\Delta_{\text{reco}}$ from the NLO mc@nlo $t\bar{t}$ events with the background $\Delta_{\text{reco}}$ distribution is considered the reference distribution in the systematic uncertainty evaluation. This combination of signal and background events is taken as the nominal simulation sample. As stated in Section 6.6.2, the expected number of signal and background events are drawn from the signal and background histograms at random. The likelihood fit to each set of distributions is considered one pseudoexperiment (PE). This process is repeated 2000 times, which gives 2000 PEs in total. Since the likelihood function is a three-parameter fit, one of which is the mass difference, the 2000 PEs will give a distribution of different mass difference values. The average value for the mass difference is taken as $\Delta(m)$.

This value of $\Delta(m)$ is considered the nominal and central value for the systematic uncertainty. From this central value, the majority of the systematic uncertainties will be evaluated. The systematic uncertainties for the analysis are evaluated by applying the various uncertainties and scale factors to the respec-
tive kinematic variables, re-running the full analysis and re-deriving the $\Delta_m^{\text{reco}}$ distributions. Using these new distributions, 2000 PEs are run, using the method described in section 6.6.2 and comparing the $\Delta(m)$ result to that of the nominal case (the SM $t\bar{t}$ MC@NLO sample with 15 million fully GEANT4 simulated events). The absolute difference between the two results is taken as the systematic uncertainty in most cases. There are some cases in which two samples are generated, and the difference between the two is the systematic uncertainty; the ISR more/less and the parton shower model samples are an example of this case. This will be further explained in the subsequent sections.

6.7.2 List of Systematic Uncertainties

There are numerous effects that can affect the $\Delta(m)$ measurement. Many include the uncertainties on the objects used in the reconstruction of the top and anti-top quark system. All the effects that contribute to the systematic uncertainty are given in Table 6.5. This table shows the $\pm 1\sigma$ or one standard deviation, variation used as the uncertainty. The difference in the shape between the nominal sample and sample used for systematic uncertainty is measured. For the case of the ISR/FSR, the half difference between the two is taken as the systematic uncertainty. For the rest of the uncertainties, the full difference between the systematic sample and the nominal sample discussed in Section 6.7.1 is taken as the systematic uncertainty.
<table>
<thead>
<tr>
<th>Systematic</th>
<th>Definition ±1σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet energy scale</td>
<td>up to 4.6%</td>
</tr>
<tr>
<td>$b$-jet energy scale</td>
<td>1~2.5%*</td>
</tr>
<tr>
<td>Jet energy resolution</td>
<td>up to 16%</td>
</tr>
<tr>
<td>$b$/$\bar{b}$ asymmetry in jet energy</td>
<td>0.13%*</td>
</tr>
<tr>
<td>$c$/$\bar{c}$ asymmetry in jet energy</td>
<td>0.1%</td>
</tr>
<tr>
<td>$b$-tagging efficiency</td>
<td>5 $\sim$ 17%*</td>
</tr>
<tr>
<td>Mis $b$-tagging efficiency</td>
<td>12 $\sim$ 21%*</td>
</tr>
<tr>
<td>Electron energy scale</td>
<td>0.5 $\sim$ 2.4%</td>
</tr>
<tr>
<td>Muon momentum scale</td>
<td>up to 1%</td>
</tr>
<tr>
<td>Muon reconstruction &amp; identification</td>
<td>1%</td>
</tr>
<tr>
<td>Electron trigger &amp; identification</td>
<td>up to 3.5%</td>
</tr>
<tr>
<td>Muon trigger &amp; identification</td>
<td>1%</td>
</tr>
<tr>
<td>Parton shower</td>
<td><strong>POWHEG + HERWIG vs POWHEG + PYTHIA</strong></td>
</tr>
<tr>
<td>MC generator</td>
<td><strong>MC@NLO+HERWIG vs POWHEG + HERWIG</strong></td>
</tr>
<tr>
<td>ISR/FSR</td>
<td>ACER: (More - Less)/2</td>
</tr>
<tr>
<td>top mass</td>
<td>$\delta m_t = \pm 1.5$ GeV</td>
</tr>
<tr>
<td>Non-$t\bar{t}$ backgrounds normalisation</td>
<td>±30%</td>
</tr>
<tr>
<td>Non-$t\bar{t}$ backgrounds shape</td>
<td>$W+$jets, QCD</td>
</tr>
<tr>
<td>Template parameterisation</td>
<td>±4%</td>
</tr>
</tbody>
</table>

Table 6.5: Definitions of ±1σ for each systematic uncertainty. The sources marked with * are dependent upon kinematic variables defined in bins of $p_T$ and $\eta$ upon kinematic variables. For the non-$t\bar{t}$ backgrounds shape systematic uncertainty, the $W+$jets contribution was replaced by the QCD contribution.
6.7.3 Jet Energy Scale

The jet energy scale (JES) is a detector level correction of the jet energy response in the calorimeters, back to that at particle level. The JES uncertainty is the uncertainty on this correction factor. The JES systematic uncertainty is evaluated by varying the jet $p_T$ and $\eta$ by $\pm \sigma$. The uncertainty of the jet is calculated for a given $p_T$ and $\eta$. A detailed description of all the contributions to the JES uncertainty for the EM+JES scheme used in this analysis is given in [77]. A brief description of some of these are given below:

- **Uncertainty in the JES calibration**
  
  After the jet calibration, there are still slight deviations from unity seen at low $p_T$ when comparing the MC simulated jet to its truth matched jet. Any deviations from unity in the jet energy or jet $p_T$ implies that even though the JES correction has been applied, the calorimeter jet kinematics are not being fully matched to the truth jet (non-closure). The non-closure is mainly due to the application of the same correction factor for jet energy and jet $p_T$. The systematic uncertainty due to the non-closure of the JES calibration is taken as the larger deviation of the response in either jet energy or $p_T$ of the jet from unity.

- **Calorimeter response uncertainty**
  
  The uncertainty in the single particle response in the calorimeters also contributes to the overall JES uncertainty. In simulation, the energy deposits in the calorimeter can be traced back to the particles generated in the collisions. The individual particles constituting the jet, and the response uncertainty associated with them, can be used to give the uncertainty in the calorimeter response.

- **Uncertainty in the detector simulation**
  
  The topo-clusters used to build jets are constructed based on a signal-to-noise ratio of energy. A difference between the simulated noise and the real noise in data can lead to differences in the cluster shapes and result in fake topo-clusters. For data, the noise can change over time, but in the simulation, the noise is fixed, depending upon which simulation data sets are used. This can bias the jet calibration and reconstruction. Therefore, the topo-clusters in MC simulation are reconstructed using the calorimeter noise measured from data [77]. The difference between the default MC sim-
ulation noise threshold and a sample in which the noise thresholds have been modified is used to estimate the uncertainty on the jet energy measurement.

- **MC generator modeling uncertainty**
  The modeling of the additional energy deposit in the calorimeter from multiple parton-parton interactions, and the fragmentation (parton showering and hadronisation) models is obtained by using a variety of different MC samples. The variation between the samples is used to calculate the systematic uncertainty contribution.

- **In-situ intercalibrations using di-jet topologies**
  The response of jets in the calorimeters depends upon the direction of the jets and the calorimeter layout. To ensure that a uniform response is obtained for jets, a calibration is required. The relative uncertainty is obtained by comparing a well calibrated central jet with one from the forward region in events with only two jets in it. The JES uncertainty in the central region ($|\eta| < 0.8$) is calculated using the single particle response and systematic variations of the MC simulation. This uncertainty is transferred to the forward region using results from the di-jet balancing method [78][79][80] (also known as the $\eta$-intercalibration method). These uncertainties contribute to the final JES uncertainty.

- **Uncertainty in pileup collisions**
  The pileup corrections are obtained by investigating the jet response with respect to the $p_T$ of the track jets, as a function of the number of primary vertices. Track jets are defined as jets built from charged particle tracks originating from the primary hard scattering vertex. This uncertainty is added separately to the estimate of the total JES uncertainty.

  The total JES uncertainty is expected to have a small effect on the total systematic uncertainty. Since the JES uncertainty is applied to all jets, the effect is negated when the mass difference is taken between the top and antitop quarks. In a top mass measurement, for example, this would be one of the more dominant sources of the systematic uncertainty. The total contribution to the systematic uncertainty from the JES uncertainty is $\pm$43 MeV. Figure 6.24 shows the reconstructed mass difference for the nominal, and JES $\pm\sigma$ case, using unfitted and fitted jets. One sees the improvement in the mass resolution by using the fitted jets.
6.7.4 B-jet Energy Scale

The JES uncertainty is calculated under the assumption that jets were either quark or gluon initiated, making no separate consideration for $b$-quark initiated jets. The response between $b$-jets and light jets in the calorimeters is different, which leads to a further uncertainty to be calculated for the $b$-jets, known as the $b$-jet energy scale (BJES) uncertainty. The $b$-jets have harder fragmentation and have a larger semi-leptonic decay fraction than the light jets.

For this analysis, the truth information for the $b$-jets needs to be known, so that uncertainties in the BJES are applied only to the $b$-jets. The uncertainty on the $b$-jet $p_T$ varies from the 1% level up to 2.5% depending on the $\eta$ and $p_T$ of the $b$-jets in question. The contribution to the total systematic uncertainty from the BJES uncertainty is $\pm$50 MeV. Figure 6.25 shows the reconstructed mass difference for the $\pm$ and nominal case.

6.7.5 Jet Energy Resolution

The measurement of the jet energy resolution (JER) is quite important in many key physics measurements, like QCD and $E_T^{\text{miss}}$ measurements. The JER is based on data/MC simulation studies using two in-situ techniques; di-jet balancing and bi-sector techniques $^{[78][79][80]}$, which are 100% correlated point by point. It is calibrated to the EM+JES scheme for $\text{anti} - k_t$ jets, with an $R$ of 0.4. The JER can be measured with truth matching at the generator level, or by direct measurement from di-jet events. The MC simulations need to match the JER seen in data, therefore they are corrected to data using the results derived from the in-situ measurements.

In the central region ($|y| < 0.8$), the data collected allows for the validation
of the results for $p_T$ between 30 and 1000 GeV. The coverage has been split into six regions in rapidity. The uncertainty in the $|y| < 2.8$ region is $p_T$ and rapidity dependent. However in the forward region ($|y| > 2.8$) the uncertainty is assigned a constant value.

The JER uncertainty is expected to contribute little to the total systematic uncertainty, as these corrections are applied to all jets, and when the mass difference is taken, its effect is nullified. The contribution to the total systematics from the JER uncertainty is ±33 MeV.

### 6.7.6 Asymmetry in B-jet and C-jet Energy Scale

The uncertainties of the $b$-jet modeling and potential differences between $b$-jet and $\bar{b}$-jet $p_T$ responses are considered. Figure 6.26 shows the jet $p_T$ responses of the $b$ and $\bar{b}$ jets with respect to its parton $p_T$ in simulation. A Gaussian fit is applied, as seen in Figure 6.26, in order to find the peak response for the $b$ and $\bar{b}$-jets. The peak response for $b$ jets is seen to be 0.27% higher than for $\bar{b}$-jets. Half the difference is taken as the uncertainty of the different responses of $b$ and $\bar{b}$-jets in addition to the absolute $b$-jet energy uncertainties recommended by the Top Working group on ATLAS. For this systematic effect, the $b$-jet $p_T$ is reduced by 0.14%, but at the same time the $\bar{b}$-jet $p_T$ is increased by the same amount to conserve the average top mass used in the kinematic fitter. The $b\bar{b}$ asymmetry has a ±80 MeV contribution to the total systematic uncertainty.

In addition, the peak response for $c$-jets is seen to be 0.1% higher than for $\bar{c}$-jets. Using the same procedure as with $b\bar{b}$ asymmetry, the contribution to the
Figure 6.26: Comparisons of the jet $p_T$ responses of the $b$-jets (left) and $\bar{b}$-jets (right) with respect to the parton $p_T$ in simulation. The mean parameter from the Gaussian fit for the $b$-jets is 0.27% larger than for the $\bar{b}$-jets.

Figure 6.27: Comparisons of the $\Delta_m^{\text{reco}}$ distributions for the $b\bar{b}$ (left) and $c\bar{c}$ (right) asymmetry in simulation.
systematic uncertainty for the $c\bar{c}$ asymmetry is $\pm 13$ MeV.

Figure 6.27 shows a comparison of the $\Delta_{m}^{\text{reco}}$ distributions for the $b\bar{b}$ and $c\bar{c}$ asymmetry. The effect on the overall shape of the $\Delta_{m}^{\text{reco}}$ distribution is small.

### 6.7.7 Asymmetry in the Light Jet Energy Scale

The single pion response in the calorimeter is not the same for $\pi^+$ and $\pi^-$, as seen in Figure 6.28. $R(\pi^+)$ and $R(\pi^-)$ are defined as the average response ($E/p$) for $\pi^+$ and $\pi^-$ in a given $\eta$ region. At low $p_T$, the MC simulation and data show a difference in response [81]. This difference is in agreement with the CMS study discussed in [82], wherein this difference is assigned to charge-exchange. The difference may also be related to the “Barkas correction” described in [71]. The Monte Carlo simulation is consistent with the data above 1 GeV. The difference in pion response could potentially manifest itself as an asymmetry in the di-jets from $W^+$ and $W^-$. The ratio of the reconstructed/truth $p_T$ is shown in Figure 6.29 for a combination of $u$, $d$ and $s$-jets and $\bar{u}$, $\bar{d}$ and $\bar{s}$-jets. No asymmetry is seen in the MC simulation and they are consistent within 0.1%.

This is explained by the multiplicity ratio of $\pi^+$ and $\pi^-$ particles, which is of order one from the $u$ and $\bar{u}$-jets (same for the $d$-jets) in QCD. Any asymmetry between $W^+$ and $W^-$ would be further reduced due to the fact that light quark jets from the $W$ boson consist of quark and anti-quark jets. Also, there is the additional $W$ mass constraint, which further reduces any asymmetry that might manifest itself.

Figure 6.30 shows the light quark jet response for the two jets from the hadronic $W$-boson decay. The response, $R_{LJ1}$ and $R_{LJ2}$ is defined as

$$\frac{(p_{T_{\text{jet,fit}}} - p_{T_{\text{jet,meas}}})}{p_{T_{\text{jet,meas}}}}, \quad (6.11)$$

where $p_{T_{\text{jet,fit}}}$ is the fitted light jet $p_T$ and $p_{T_{\text{jet,meas}}}$ is the measured light jet $p_T$. The distributions are centred on zero and show a good agreement between data and the MC simulation. Although data and MC simulation are different within statistical uncertainty, the distributions are very similar and the effect is minimal on the systematic uncertainty.

### 6.7.8 B-tagging and Mis-tagging Efficiency

The tagging of $b$-jets is very important in this analysis as having a double $b$-tag requirement significantly reduces the background contribution. The uncertainty
Figure 6.28: Pion response in the calorimeter for $\pi^+\pi^-$ for $|\eta| < 0.6$ on the left and $0.6 < |\eta| < 1.1$ on the right. The available energy is defined as the pions total energy and should be distinguished from the measured energy.

Figure 6.29: Comparisons of the light quark jet $p_T$ responses for the combined $u,d$ and $s$ jets (left) and combined $\bar{u},\bar{d}$ and $\bar{s}$ (right) with respect to the parton $p_T$ in simulation. The mean parameter from the Gaussian fit for the $u,d$ and $s$-jets is 0.1% larger than for the $\bar{u},\bar{d}$ and $\bar{s}$-jets.
Figure 6.30: Data and MC comparison of the jet response, $R_{LJ1}$ and $R_{LJ2}$, for each of the two light jets from the hadronic $W$ boson decay.

in the $b$-tagging and mis-tagging of $b$-jets is important as this can affect the final mass difference shape if a jet is wrongly tagged as a $b$-jet. The mis-tag rates have been measured using two complementary methods, using the 2011 data from the ATLAS detector. The first method uses the invariant mass spectrum of tracks associated with the secondary vertices to separate light and heavy-flavour jets and the second method is based on the rate at which secondary vertices with negative decay length, or tracks with negative impact parameters are present in data. Both methods are discussed in detail in [37][38]. The $b$, $c$- and mis-tagging efficiencies are applied with the appropriate $p_T$ and $\eta$ dependent scale factors. The efficiencies of the $b$-tag and mis-tag in data have been matched to the MC simulation with an associated uncertainty. The efficiencies are varied in the systematic uncertainty calculations by $\pm \sigma$ giving a relatively large contribution to the total systematic uncertainty. The $b$-tagging systematic uncertainty contribution is $\pm 83$ MeV and the mis-tag contribution is $\pm 50$ MeV.

6.7.9 Charged Lepton Reconstruction and Trigger Efficiency

The muon and electron channels both have associated uncertainties in their reconstruction and trigger efficiency. The scale factors on the trigger, reconstruction and standard ID efficiencies and associated uncertainties are found in tools
recommended by the respective charged lepton working groups\[40\][43]. The momentum and energy of the charged leptons were changed by the uncertainty for each of these cases, which varied from around 1% for the muon uncertainties, up to 3.5% for the electron uncertainties\[28\]. Due to the $W$ mass constraint in place, these effects are expected to be relatively small, and contribute only a small amount to the total systematic uncertainty. The total reconstruction and trigger systematic uncertainty contribution is ±23 MeV for electrons and ±36 MeV for muons. The energy scale and resolution shifts for the charged leptons are ignored for this analysis.

### 6.7.10 Parton Shower Model

The uncertainty associated with the different parton shower models PYTHIA and HERWIG, when interfaced with POWHEG is also calculated. The comparison of the $\Delta_{\text{m reco}}$ distributions for both cases is shown in Figure 6.31. The full difference between the mass difference from the pseudoexperiments for the two cases is taken. HERWIG uses an angular ordered parton shower system, whereas PYTHIA uses a $p_T$ ordered system (Figure 5.4).

Table 6.6 shows the $b$-jet and the $\bar{b}$-jet response for Mc@NLO and POWHEG for different pileup and simulation conditions. Both mc11b and mc11c samples used provide an accurate description of the 2011 data taking period, and only differ in the pileup description. The pileup description used for the mc11b samples comes from PYTHIA8\[83\] and for the mc11c samples, the pileup description comes from Fortran PYTHIA \[51\][61]. HERWIG shows a $b\bar{b}$ asymmetry at the 0.32% level, whereas PYTHIA does not show any asymmetry. This is important in the calculation of the parton shower systematic effect since a difference due to the parton shower models between PYTHIA and HERWIG also shows up as an effect on the $b\bar{b}$ asymmetry. Taking the full PS and a full $b\bar{b}$ asymmetry effects can double-count the systematic uncertainty. It is evident from this table that the $b\bar{b}$ asymmetry depends upon which parton shower model we use (PYTHIA or HERWIG) as opposed to the degree of simulation applied (full simulation and fast simulation). The systematic uncertainty contribution is therefore split into two different effects. The first measures the $b$-jet and $\bar{b}$-jet energy response to the detector (discussed in Section 6.7.6) and the second measures the effect of the parton shower and hadronisation of the $b$ and $\bar{b}$-quark, including different colour flow effects. The contribution to the systematic uncertainty from the parton shower model itself, gives a systematic uncertainty contribution of ±50 MeV.
### Table 6.6: Asymmetry in the response of the $b$ and $\bar{b}$ jets for MC@NLO and POWHEG Monte Carlo generators (full simulation and fast simulation).

<table>
<thead>
<tr>
<th>Sample</th>
<th>Simulation Type</th>
<th>$b$ asymmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC@NLO (mc11c)</td>
<td>Full Simulation</td>
<td>0.27 ± 0.1%</td>
</tr>
<tr>
<td>MC@NLO (mc11c)</td>
<td>Fast Simulation</td>
<td>0.25 ± 0.1%</td>
</tr>
<tr>
<td>MC@NLO (mc11b)</td>
<td>Fast Simulation</td>
<td>0.17 ± 0.1%</td>
</tr>
<tr>
<td>POWHEG+HERWIG (mc11b)</td>
<td>Fast Simulation</td>
<td>0.32 ± 0.1%</td>
</tr>
<tr>
<td>POWHEG+PYTHIA (mc11b)</td>
<td>Fast Simulation</td>
<td>0.07 ± 0.1%</td>
</tr>
<tr>
<td>POWHEG+PYTHIA (mc11c)</td>
<td>Full Simulation</td>
<td>0.04 ± 0.1%</td>
</tr>
</tbody>
</table>

6.7.11 Monte Carlo Generator

The uncertainty due to the choice of MC generator is evaluated by comparing the results using the MC@NLO and POWHEG generators. Both are NLO MC generators, with HERWIG used as the parton shower model. These samples were both run under fast simulation, using ATLFAST II [84]. Fast simulation reduces the time taken in simulation of the particle interactions with the detector by running full simulation for the inner detector interactions and muon tracks, but fast simulation for the calorimeters.

This systematic effect was taken as the full difference between these two samples and was found to be relatively small, contributing ±42 MeV to the final systematic uncertainty.

6.7.12 Initial and Final State Radiation

The contribution to the systematic uncertainty when the ISR/FSR showering of the LO ACERMC MC generator was varied, was investigated. The showering involves additional radiation from the initial and final state partons affecting the final $\Delta(m)$ measurement. Hard FSR emissions can take considerable energy away from the jets, which will give a lower jet energy measurement. ISR emissions can create high $p_T$ jets which could be included as part of the $t\bar{t}$ system. It is important to check the effect of this on this analysis as there are four jets in the final state. The ACERMC samples used contain both more and less parton shower activity, which were recommended by the top working group at ATLAS. A comparison of the $\Delta m_{\text{reco}}$ distribution for more/less ISR/FSR modeling is shown in Figure 6.31. In this distribution, one can clearly see the worsening resolution, when the ISR/FSR activity is increased. Half the difference between the ISR more
Figure 6.31: Comparison of the $\Delta^{\text{reco}}_{m}$ distributions for ACERMC with ISR more/less (left), and POWHEG interfaced with PYTHIA and HERWIG (right).

and ISR less samples is taken as the systematic uncertainty. This was found to have a ±45 MeV contribution to the total systematic uncertainty.

6.7.13 Top Mass Uncertainty

An investigation into the effect on the measured $\Delta(m)$ if a real mass difference existed in the single top channel was undertaken. The most dominant channel, the $Wt$ channel, was used.

An artificial mass difference is created by combining single top events from MC samples in which the top quark is generated at 170 GeV and the anti-top quark at 175 GeV. The effect on $\Delta(m)$ using this sample was ±20 MeV effect.

Changing the average mass of the top and anti-top in the kinematic fitter was also investigated. In total, three average mass values were chosen; 171, 172.5 (default) and 174 GeV, so the variation in the measured mass difference was taken to be ±1.5 GeV. Re-running the pseudoexperiments for each of these cases resulted in a systematic uncertainty contribution of ±40 MeV.

6.7.14 Normalisation and Shape Uncertainty on the Non-$t\bar{t}$ Background

The magnitude of the normalisation uncertainty on the non-$t\bar{t}$ background has been estimated to be 26% (28%) in the electron (muon) channel for the $W+$jets background. The uncertainty on the data-driven QCD measurement was taken to be 50%. The same uncertainty for $Z+$jets were used for the $W+$jets backgrounds. For the normalisation of the single top and di-boson backgrounds, the uncertainty is taken to be 15 %. Combined, these estimates gave an overall nor-
malisation uncertainty of 30% on the backgrounds. The background contribution to the PEs was changed by \( \pm 30\% \). This effect contributed \( \pm 36 \) MeV to the total systematic uncertainty. The non-\( t\bar{t} \) shape systematic uncertainty was calculated by replacing the \( W + \text{jets} \) background with QCD shape. The result was found to have a \( \pm 37 \) MeV effect.

As mentioned in Section 6.5, a study was undertaken to find the contribution to the systematic uncertainty when an alternative function was fitted to the background template. A double Gaussian function was used, and a PE-by-PE difference between the default fit (single Gaussian) and the new fit (double Gaussian) was taken. The systematic uncertainty when the alternative fit was used was \( \pm 22 \) MeV. This contribution was added in quadrature to the non-\( t\bar{t} \) shape systematic uncertainty, to give a total contribution of \( \pm 43 \) MeV.

### 6.7.15 Summary of Systematic Study

A summary of the systematic uncertainty on the \( \Delta(m) \) measurement is shown in Table 6.7. The systematic uncertainty was calculated to be \( \pm 0.19 \) GeV. The systematic uncertainty, coupled with the statistical uncertainty of \( \pm 0.61 \) GeV are the total uncertainty on the \( \Delta(m) \) measurement.
<table>
<thead>
<tr>
<th>Systematic</th>
<th>$\Delta(m)$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet energy scale</td>
<td>0.043</td>
</tr>
<tr>
<td>b-jet energy scale</td>
<td>0.050</td>
</tr>
<tr>
<td>Jet energy resolution</td>
<td>0.033</td>
</tr>
<tr>
<td>$b/\bar{b}$ asymmetry in jet energy</td>
<td>0.08</td>
</tr>
<tr>
<td>$c/\bar{c}$ asymmetry in jet energy</td>
<td>0.013</td>
</tr>
<tr>
<td>$b$-tagging</td>
<td>0.083</td>
</tr>
<tr>
<td>mis $b$-tagging</td>
<td>0.050</td>
</tr>
<tr>
<td>Electron reconstruction &amp; identification</td>
<td>0.023</td>
</tr>
<tr>
<td>Muon reconstruction &amp; identification</td>
<td>0.036</td>
</tr>
<tr>
<td>Asymmetry in Lepton Energy Scale</td>
<td>$&lt; 0.010$</td>
</tr>
<tr>
<td>Parton Shower</td>
<td>0.056</td>
</tr>
<tr>
<td>MC generator</td>
<td>0.042</td>
</tr>
<tr>
<td>ISR/FSR</td>
<td>0.045</td>
</tr>
<tr>
<td>Top Mass</td>
<td>0.040</td>
</tr>
<tr>
<td>Non-$t\bar{t}$ normalization</td>
<td>0.036</td>
</tr>
<tr>
<td>Non-$t\bar{t}$ shape</td>
<td>0.043</td>
</tr>
<tr>
<td>Calibration Method</td>
<td>0.050</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.193</strong></td>
</tr>
</tbody>
</table>

Table 6.7: A summary of the contribution to the total systematic uncertainty on $\Delta(m)$ measurement.
Chapter 7

Results

7.1 Results on the Mass Difference

The results from the likelihood fit to the $\Delta_{\text{reco}}^m$ distribution for 4.7 fb$^{-1}$ of data, yields a $\Delta(m)$ of 0.84 GeV with an associated statistical uncertainty of 0.61 GeV.

The $\Delta_{\text{reco}}^m$ is shown in Figure 7.1. Subtracting the final calibration offset of 175 MeV from the result of the likelihood fit gives a final measurement of

$$\Delta(m) = m_t - m_\bar{t} = 0.67 \pm 0.61 \text{ (stat.)} \pm 0.19 \text{ (syst.)} \text{ GeV}, \quad (7.1)$$

which is consistent with the Standard Model prediction of no mass difference.

The scan of the profile of the likelihood is shown in Figure 7.2. The red line at 0.5 shows the error on the measurement.

7.2 Cross Checks

A few cross checks were made to check the consistency of the measurement. One thing that was noticed was the low background contribution in the final distribution. This is due to the parameterisation method used. The parameterisation was undertaken using a leading order (LO) MC generator. A next-to-leading order (NLO) MC generator describes the data much better (the central region is better described with NLO instead of LO MC generator). The parameterisation predicts more events in the tail then in the central region. Thus, in the fitting, the parameterisation pulls events from the background to compensate for the tail regions. This explains why the background contribution seems so low in Figure 7.1.

To test this hypothesis, a check was run using a background constraint.
Figure 7.1: $\Delta^{\text{reco}}_m$ distribution using 4.7 fb$^{-1}$ of data.

Figure 7.2: Scan of the profile likelihood with associated error shown in red.
the background contribution is set to be within ±30% of the expected background. When comparing the $\Delta_m^{\text{reco}}$ from data with and without the background constraint, the difference was at the level of 3 MeV. This shows the effect of the background contribution on the final measurement of $\Delta(m)$ is negligible.

The next cross check was to measure $\Delta(m)$ in the two decay channels, electron and muon, separately in data and MC simulation. The difference in $\Delta(m)$ between the electron and muon channel in the MC simulation was of the order of ±200 MeV. In data, the results are consistent with this value within statistical error. The $\Delta m$ in the electron channel is $0.31\pm0.02$ (stat) GeV, and in the muon channel is $1.1\pm0.76$ (stat) GeV.

The final cross check undertaken was to measure $\Delta(m)$ as a function of the data taking period. The data collected in 2011 by ATLAS is split into 11 data taking periods, ranging from period B to M. For the cross check, these data periods were combined into three groups; B-H, I-K and L-M. Figure 7.3 shows the $\Delta(m)$ is consistent with our measured value within the associated statistical uncertainty. This distribution does not include the calibration offset applied in the final measurement.
Chapter 8

Conclusions

The CPT symmetry is a fundamental part of elementary particle physics. It is a symmetry in which particles and anti-particles have the same mass and lifetime. Any difference in mass between a particle and its anti-particle would lead to the breaking of Lorentz symmetry and would make a profound impact on particle physics by opening a window for new physics. The large mass of the top quark, which is at the electroweak symmetry breaking scale, and the large production cross-section at the LHC make it a perfect candidate to measure the mass difference between a quark and its anti-quark.

A measurement of the mass difference between the top and anti-top quarks has been performed using the semi-leptonic decay channel. The full ATLAS 2011 data set, corresponding to an integrated luminosity of 4.7 fb$^{-1}$ of $pp$ collisions at $\sqrt{s} = 7$ TeV, was used in the measurement. A mass difference of $\Delta(m) = m_t - m_{\bar{t}} = 0.67 \pm 0.61 \, \text{(stat)} \pm 0.19 \, \text{(syst)}$ GeV is measured. This result is consistent with the Standard Model and CPT Theorem prediction of no mass difference.

The semi-leptonic decay channel was used in the analysis, which contains four-partons, measured as two $b$-jets and two light jets, one charged lepton, and a large amount of missing momentum. The electron and muon decay channels are used in this study. The decay channel makes full use of the ATLAS detector; $E_T^{\text{miss}}$ and jet measurements from the calorimeters, electrons from the inner detector tracks and showers deposited in the electromagnetic calorimeters, muons from inner detector and muon spectrometer tracks and $b$-tagging using the secondary vertex information from $b$-layer in the pixel detector.

A host of leading order and next-to-leading order Monte Carlo samples were used to generate the signal and background events. Data-driven techniques were used to represent the QCD contribution to the background. Event selections were
optimised to pick events with higher signal efficiency and reduced background contributions. Of the selected events, a kinematic fitter is used to reconstruct the top and anti-top masses. The mass difference between the invariant masses of the top and anti-top was chosen as the variable to measure the mass difference of the top and anti-top quarks.

A modified PYTHIA Monte Carlo generator was used to generate 15 signal samples, with mass differences ranging from $\Delta_m^{\text{truth}}$ of -15 GeV to +15 GeV. An extended maximum likelihood fit, derived from signal and background templates, is used to extract mass difference, $\Delta(m)$ from data. Pseudoexperiments were run, and the statistical and systematic uncertainties were extracted. The expected statistical uncertainty was calculated to be 0.61 GeV.

In this analysis, many of the systematic uncertainties are reduced due to the mass difference being taken between the top and anti-top quarks. Systematic effects that are based on an asymmetry in the top anti-top system have larger contributions to the systematic uncertainty, whereas those in which the uncertainty is applied to both the top and anti-top quark system (JES, JER) have a smaller contribution. The overall systematic uncertainty on the $\Delta(m)$ measurement was calculated to be 0.19 GeV.

The largest contribution to the uncertainty on $\Delta(m)$ in this analysis comes from the statistical uncertainty. To reduce this uncertainty, the full 2012 dataset, corresponding to an integrated luminosity of over 21 $\text{fb}^{-1}$ (at $\sqrt{s} = 8$ TeV) can be used. This analysis would also benefit from high statistics signal MC simulation samples. Samples in which 400K events were generated, were used in the signal parameterisation, but larger samples (3-15M generated events) would help improve the parameterisation.
Bibliography


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Appendix A

Neutrino Momentum Solution

The neutrino is a very weakly interacting particle. It passes straight through the detector without leaving tracks or depositing energy. Its presence is inferred by the amount of missing energy in the detector.

Taking the neutrino mass to be negligible, the total $E_T^{miss}$ gives the transverse momentum of the neutrino $P_{\nu,T}$. However, to reconstruct the leptonic $W$ boson mass, the full neutrino momentum is required. Since the centre-of-mass energy of the colliding parton system is unknown, the longitudinal component of the neutrino momentum $P_{\nu,z}$ is also unknown. If one assigns all the missing $E_T$ to the neutrino, one can express $P_{\nu,z}$ in terms of the two measurable quantities; $P_{\nu,T}$ and the charged lepton momentum, $P_l$. The four-vectors of the lepton and the neutrino are constrained to be exactly equal to the $W$ boson mass, which is 80.42 GeV.

$$M_{W}^2 = (P_l + P_{\nu})^2,$$ (A.1)

where $P_l$ and $P_{\nu}$ are the four-momentum of the charged lepton and neutrino respectively. Expanding Equation [A.1] gives

$$M_{W}^2 = P_l^2 + P_{\nu}^2 + 2P_l \cdot P_{\nu}.$$ (A.2)

Using the mass shell conditions, $P_{\nu}^2 = 0$ and $P_l^2 = M_l^2$, Equation [A.2] simplifies to

$$M_{W}^2 = M_l^2 + 2(E_l E_{\nu} - P_l \cdot P_{\nu}),$$ (A.3)
where

\[ E_\nu = \frac{1}{2E_l} (M_W^2 - M_l^2 + 2P_l.P_\nu). \] (A.4)

One can change Equation (A.4) to the form \( E_\nu = A + BP_{\nu,z} \), where

\[ A = \frac{1}{2E_l} \left( M_W^2 - M_l^2 \right) + P_{l,x}.P_{\nu,x} + P_{l,y}.P_{\nu,y}, \] (A.5)

and

\[ B = \frac{P_{l,z}}{E_T}. \] (A.6)

Since the \( z \)-axis has been defined as the same direction as the beamline, the longitudinal and transverse momentum are defined as

\[ P_L = P_z, \] (A.7)

\[ P_T = \left( P_{\nu,x}^2 + P_{\nu,y}^2 \right)^{\frac{1}{2}}. \] (A.8)

The neutrino energy is expressed as

\[ E_\nu = \left( P_{\nu,x}^2 + P_{\nu,y}^2 + P_{\nu,z}^2 \right)^{\frac{1}{2}} \] (A.9)

and the quadratic formula for the longitudinal momentum of the neutrino is then

\[ (B^2 - 1)P_{\nu,L}^2 + 2ABP_{\nu,L} + A^2 - P_{\nu,T}^2 = 0. \] (A.10)

Therefore, there are two possible solutions for \( P_{\nu,L} \) for every event. The \( P_{\nu,L} \) solution which gives the lowest \( \chi^2 \) is used.
Appendix B

Scale Factor Packages

A complete list of the tags of all the packages used in this analysis are given. This list includes scale factors and uncertainty packages:

- ApplyJetCalibration-00-01-00
- GoodRunsLists-00-00-97
- JetEffiProvider-00-00-02
- JetTagAlgorithms-00-00-01
- JetUncertainties-00-03-05-branch
- MissingETUtility-00-02-11
- MuonEfficiency Corrections-01-01-00
- MuonIsolationCorrection-00-08
- MuonMomentumCorrections-00-05-03
- PileupReweighting-00-02-01
- TopD3PDAnalysis-00-00-61
- TopD3PDCorrections-00-00-35
- TopD3PDSelection-00-02-37
- TopElectronSFUtils-00-00-13
- TopGoodRunsList-00-00-01
- TopJetUtils-00-00-03
The specific GRL used in this analysis is: \textit{data11.7TeV.periodAllYear.DetStatus-v36-pro10.CoolRunQuery - 00 - 04 - 08.Top_allchannels.xml}.