Essays on Macroeconomic Models with Nominal Rigidities and Imperfections in the Goods and Credit Markets

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Abstract

In recent years the New Keynesian framework has become widely used to identify the relationship between monetary policy, inflation, the business cycle and welfare. Most commonly in these models inertia in prices are introduced only through the aggregate supply side which generates a short run non-neutrality of money. This thesis begins with an investigation into the impact of sticky prices on the macroeconomic equilibrium through aggregate demand. We show that in models of price stickiness among differentiated goods aggregate consumption deviates from the conventional Euler equation due to relative price distortions. This has some non-negligible implications: there are additional inflation effects, which enter through aggregate demand, that lower the response of the marginal cost and dampen responses of inflation and output; products’ price elasticity of demand affects equilibrium output and inflation dynamics independently of supply factors; monetary policy responses are smoother than in the conventional new Keynesian models, particularly the more competitive are the products markets.

In chapter 2 we continue with an investigation into the impact that the aforementioned channel has on welfare and monetary policy under various regimes. Specifically, we compare our results with the benchmark New Keynesian model with a cost channel for alternative levels of competition in the goods market. When the central bank is assumed to follow a Taylor rule we find, contrary to the standard New Keynesian literature, that welfare losses ultimately fall as the goods market becomes more competitive. Furthermore, there are additional adverse implications for welfare coming through an exaggerated stabilisation bias associated with discretionary policy in our model version. A move to optimal commitment implies significant additional gains compared to the standard literature by; eliminating this amplified stabilisation bias and; reducing further the fall in output gap and inflation fluctuations at the time of shock.

The final part of this thesis develops a Generalised Taylor economy to include a financial market. This finance sector is characterised by savings contracts to households and loan contracts to firms, both of which are differentiated by the duration for which their interest rate remains fixed. Additionally, a time varying external finance premium on loan rates is introduced through an endogenous probability of firm default. Using break-even conditions we show that the fixed markup on loan rates is dependent on, the expected default risk over the lifetime of the contract, and, spillovers from the unexpected losses of current "locked in" financial contracts that must be accounted for in the zero profit condition of the commercial bank. Our results indicate that inertia in loan and savings rates dampens the responses of monetary policy and the business cycle whilst generating a procyclical loan rate spread. In contrast, risk of default amplifies the business cycle and delivers a countercyclical loan rate spread. The overall impact of these two channels on the direction and magnitude of loan rate spreads, spillovers to new contracts and the dynamics of the business cycle, are shown depend on the type of shock hitting the economy.
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Chapter 1

Introduction

"Human wisdom is the aggregate of all human experience, constantly accumulating, selecting and reorganizing its own materials."
- Joseph Story (1779 - 1845)

The current direction and approach to a considerable amount of macroeconomic research can be traced back to a seminal critique by Robert Lucas (1976). Lucas demonstrated that large scale macroeconometric models, based on highly aggregated data, are not policy invariant in so much as they would not adapt to alterations in the conduct of monetary policy.

Perhaps the most powerful outcome of this criticism was that many macroeconomists have become convinced that microfoundations are an essential ingredient for modelling the aggregate economy. By specifying preferences, technology and institutions which govern individual behaviour, we can not only predict what individuals will do, but also how they account for policy changes. The Dynamic Stochastic General Equilibrium (DSGE) methodology works by aggregating all these individual decisions and interactions between hypothetical, rational agents to help explain aggregate economic phenomena and evaluate economic policy.

The origins of DSGE modelling came through the Real Business Cycle (RBC) model of Kydland and Prescott (1982), who showed that macroeconomic fluctuations are, to a great extent, accounted for by real shocks. Associated with New Classical macroeconomics, and given the assumptions of flexible prices and wages, they saw the business cycle as an efficient response to alterations to the economic
environment so had no role for fiscal or monetary stabilisation policy. In contrast, during the same period New Keynesian models introduced nominal rigidities coupled with monopolistic competition and used these assumptions to explain why monetary policy is able to affect employment and production in the short run.\footnote{See for example Mankiw (1985) and Mankiw and Romer (1991).}

Combining the New Classical and New Keynesian theories formed a paradigm known as the New Neoclassical Synthesis (NNS) or alternatively New Keynesian DSGE models.\footnote{NNS models were first introduced by Goodfriend and King (1997). See Woodford (2009) for an overview of NNS models. See Walsh (2009, ch8) for why these two definitions are equivalent.} Central to NNS models is the view that the economy is a dynamic general equilibrium system that can deviate from an efficient allocation of resources in the short run. These New Keynesian features were first implemented into a DSGE model by Rotemberg and Woodford (1997). By introducing market failures into the microfounded RBC framework, most commonly imperfect competition coupled with a nominal rigidity in price or wage setting, the economy may struggle to obtain full employment. This leads to the short run non-neutrality of money and accordingly, a welfare improving input through monetary or fiscal policy may lead to more efficient outcomes, without being subject to the Lucas critique.\footnote{This is to the extent that the economy is sufficiently stable so that the log-linearised equilibrium remains a good approximation. However, based on sticky prices (imparticular Calvo pricing) and other rigidities, the synthesis does not embrace the complete neutrality of money proposed by the new classical economists.}

More recent New Keynesian DSGE models have introduced insightful microeconomic ingredients and captured additional channels of aggregate variables. By developing innovative research in this area economists have been able to provide a quantitative understanding of economic phenomena and help economic policy facilitate welfare maximising outcomes to society.\footnote{See, for example; the sticky wage and labour market imperfection literature of Cecg, Henderson and Levin (2000), Pissarides (2000) and Blanchard and Gali (2008); the optimal policy models of Schmitt-Grohe and Uribe (2004, 2006), and Adam and Billi (2006) and; the financial market imperfection models of Bernanke, Gertler and Gilchrist (1999), Iacoviello (2005), Cúrdia and Woodford (2010), Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).} This remainder of thesis addresses three research agendas in the New Keynesian DSGE framework. Chapter 2 investigates the impact that aggregation of individual product demands under sticky prices
has on aggregate demand and macroeconomic dynamics. Chapter 3 builds on this result with an investigation into implication it has for household welfare and optimal monetary policy. Finally chapter 4 also focuses on the importance of aggregation but this time with an application macroeconomic effects of risky, fixed term loan contracts in the credit market.

Much of the New Keynesian DSGE literature introduces price, and in some cases wage, inertia through a Calvo (1983) process. In this setting, firms face a constant probability of having a price resetting opportunity in given period and so they must bear in mind that their prices will remain fixed for longer than they would like. Imperfect competition is most commonly introduced in these models through aggregate consumption consisting of a basket of differentiated goods where firms have a degree of market power in producing a particular good from this basket. Despite this emphasis on imperfect competition, the price elasticity of demand which governs the market power of firms, seems to play a very limited role for inflation and output dynamics in the baseline New Keynesian DSGE model. The reason for this lack of emphasis on product-market structure stems from least three conventional assumptions made by much of the New Keynesian literature. First, much of the research in this area assumes, largely for modelling simplicity, that the production of firms is characterised by constant returns to scale. This implies that firms’ marginal product of inputs, such as labour and capital, and hence marginal cost, are independent of individual product demands and hence of any relative price effects. Second, distortions arising from the monopolistic power of firms in the product markets are often assumed away through the presence of a lump sum subsidy that perfectly offsets the price mark-up. Third, given no additional market distortions are introduced, the New Keynesian literature retains the neo-classical assumption of the RBC model that the behaviour of aggregate consumption is described entirely by the Euler equation. This implies that aggregate consumption, and in the baseline model aggregate demand, is constructed independently of any relative price
distortions that may exist in the economy due to price stickiness. Price stickiness is introduced conventionally only through the aggregate supply side. As a result, aggregate consumption, remains free of relative price distortions and only deviates from the flexible price, neo-classical level of aggregate consumption through deviations in aggregate inflation dynamics that arise through distortions in aggregate supply.

This last assumption is the main focus of the following chapter. For a framework we employ the cost-channel economy of Ravenna and Walsh (2006). Whilst the cost channel is not central to our results, it does generate an additional channel through which interest rates and thus aggregate consumption can be affected. Additionally, the cost channel is a feature of all the models presented in this thesis with it generating a trade off for the policy maker in chapter 3, and providing the foundations of an enriched credit market in chapter 4.

In Chapter 2 we derive aggregate consumption by aggregating across the actual demands of all individual consumption goods within the household’s typical consumption basket in any given period. In doing so we assume away the symmetric price equilibrium condition that eliminate relative price disturbances; instead we retain the assumption at any given time, aggregate prices will be split into those that can adjust and those that cannot, a la Calvo (1983). Indeed, in actual practice aggregate consumption indices are not constructed assuming symmetric prices, but are based on the aggregation of actual demands of consumer goods, collected in a specific point in time and weighted appropriately within a ‘typical’ consumption basket. We show that when price setters consider the sum of actual consumption demands (rather than the Euler equation) as given in their price setting, then additional relative price distortions affect marginal cost, the output gap and inflation.

Specifically, households are able to switch their products within the consumption

\footnote{This shown analytically through the way in which the cost channel adds an additional mechanism through which our interest rate rule (which itself is influenced by the additional inflation dynamics from) impacts the New Keyensian Phillips Curve and the macroeconomic equilibrium.}
basket, a process which is exaggerated when there is more price dispersion and greater similarities between products. Since firms are aware of this household behaviour, they also know that their product demands are more sensitive to a given price change, especially in product markets characterised by a higher price elasticity of demand. For example, following a shock where it is optimal to reduce relative prices, firms’ with a price-setting opportunity are able to capture more of their fixed price competitors’ product demands, both today and in future periods, for a given change to their relative price. As a result, the optimal adjustment of their relative price is reduced.

In our macroeconomic equilibrium the price elasticity of demand is shown to affect equilibrium output and inflation through aggregate consumption and thus aggregate demand. With firms making less of a price adjustment, this channel is shown to reduce the short run inflation-output trade off, implying smoother responses of the output gap, inflation and interest rates, than those implied by the conventional Euler equation. Furthermore, it is shown, that accounting for relative price distortions in aggregate consumption dampens the short-run dynamics of the output-gap, inflation and interest rates, following economic shocks. Given that this additional channel is increasing with the price elasticity of demand, the more competitive is the goods market, the smoother are the response of these variables in relation to those implied by conventional aggregate consumption.

The model depicted in chapter 2 provides a quantitative illustration of the channels through which additional relative price effects in aggregate consumption impact aggregate variables. Whilst this type of presentation helps improve our understanding of the transmission mechanism through which shocks impact the economy, apart from the benefits of increasing competition to smooth the business cycle, it makes few recommendations for an altruistic policy maker. To do this, one must first establish where and how the potential welfare losses to society may arise and then calculate to what extent and under what conditions alternative policy regimes may
deliver beneficial outcomes. In chapter 3 we analyse the impact that the additional relative price distortions from aggregate consumption, introduced in chapter 2, have on the welfare of households and the conduct of optimal monetary policy.

Following the seminal work of Rotemberg and Woodford (1999), in the optimal monetary policy literature, welfare losses to households are shown to arise endogenously from deviations in output and inflation from their efficient levels. Output fluctuations away from the natural level are costly since the optimal consumption leisure trade-off is distorted. Accordingly, the marginal rate of substitution and the marginal product of labour which determine the economy’s aggregate inefficiency and the extent to which a given deviation reduces the welfare of agents. A given movement in inflation is costly in these models due to the dispersion of prices of the differentiated goods. Because of this dispersion, households buy more of a relatively cheaper good and less of the relatively more expensive goods. However, because of diminishing marginal utility, the utility gains from consuming more of a cheaper good are outweighed by the utility losses in from consuming less of the more expensive good. Additionally, since increasing the price elasticity of demand will exaggerate these movements, a more competitive economy has a greater welfare loss for a given inflation deviation.

The loss function derived in chapter 3 retain all of the above effects. However, the variances of both inflation and the output gap that enter when the additional price dispersion effects that are detailed in chapter 2 are accounted for, are lower and decreasing with the price elasticity of demand. The intuition is that when product demands are more elastic, the potential gains or losses realised by the firm increase for a given relative price change. Accordingly, when firms have a price resetting opportunity they make less of a price adjustment to achieve the optimal price in their profit maximisation. Additionally, the output gap deviations are mitigated in a more competitive product market since households, by switching their products, are able to move closer towards their optimal allocation of aggregate consumption.
Thus, for an equivalent set of shocks, the variances of inflation and the output gap fall when this additional channel is incorporated, especially in more competitive product markets.

When the central bank is assumed to follow a typical Taylor rule, welfare losses following economic shocks are shown to ultimately decrease as the economy becomes more competitive. For the reason that, in our model the positive welfare effect that increasing competition has on reducing the variance of inflation and the output gap dominates the adverse effects of increased asymmetric product demands in the welfare function. This is in contrast to the benchmark case of Ravenna and Walsh (2006) where the variance of inflation and the output gap are unaffected by the price elasticity of demand so that welfare losses will always increase as goods become closer substitutes.

Chapter 3 continues by evaluating to what extent a discretionary or commitment policy, where in both cases the policy maker acts optimally within the constraints of the regime, delivers superior welfare outcomes. In a large part due to the seminal contribution of Kydland and Prescott (1977) and Barro and Gordon (1983) monetary economists can quantitatively analyse the potential welfare gains of moving the conduct of monetary policy from a discretionary regime to a policy plan, namely optimal commitment.

It is the existence of a cost channel that generates a meaningful trade-off between inflation and output stabilisation in these models. For example, a rise in the interest rate to close the output gap must be limited since the increase in the cost of borrowing will induce inflation. In a standard New Keynesian model with a cost channel the adverse implication of discretionary policy is that, by not committing to future actions the policy maker can only use today's output gap to control inflation. A consequence of this outcome is that the policy maker attempts to stabilise the output gap by more than a time consistent policy calls for. In our model this adverse effect is exaggerated as it becomes optimal today to stabilise the output gap.
by even more since a larger interest rate, which causes greater inflation, will reduce further the output gap through our additional channel.

By committing to a policy plan the policy maker is able to influence the expectations of agents in future periods and minimise overall welfare losses further by spreading them over many periods. The advantages of optimal commitment when relative price effects in aggregate consumption are accounted for are twofold. Not only does it eliminate the additional adverse effect of discretion outlined above, but since expectations of future deflation will reduce the product demands of firms adjusting prices today, their price rise and therefore aggregate inflation will increase by less. A consequence of this effect is that the central bank can reduce deviations in inflation and the output gap by even more today at the smaller cost of greater deviations in later periods.

 Whilst chapters 2 and 3 will investigate the macroeconomic impact that price inertia has in generating heterogeneous product demands, chapter 4 explores the role of heterogeneity in the banking sector. To do this, we retain the cost channel from chapters 2 and 3 and develop it to include a range of fixed loan rates and introduce a time varying risk premium that depends upon risk of firm defaults.

 In chapter 4 inertia in prices, wages and interest rates follow the Generalised Taylor contracts of Dixon and Kara (2010, 2011). This is in contrast to the approach to modelling nominal inertia in chapters 2 and 3 which are based on the staggered price adjustment of Calvo (1983). Instead Dixon and Kara build on the model of Taylor (1980) who introduced a single price or wage contract that is fixed for a predetermined number of periods and hence the duration is known \(ex \ ante\). The Generalised Taylor methodology extends the Taylor (1980) model by allowing inertia to be driven by a number of overlapping contracts that are fixed for various durations.\(^6\)

\(^6\)Unlike the random probability of adjustment in Calvo (1983).

\(^7\)Dixon and Kara (2010, 2011), using this approach find an increase in the persistence of output following monetary shocks, an outcome which is initiated by the spillover effect between sectors. This provide a key link between the microeconometric data on prices of Taylor (1993), Bils and
Our financial sector is modeled in such a way that there are potentially many branches of a commercial bank who receive deposits from households and offer loans to firms. Each branch specialises in offering a contract of a particular duration with a cost borrowing and savings rate that remain unchanged until its maturity. The understandable consequence of this is that movements in the policy rate may not be immediately transferred to the real sector, or alternatively, that there is a incomplete short-term pass-through of money market interest rates. One of the advantages of using a range of Taylor contracts is that contract lengths are known by agents and so can be developed according to their anticipated cost.

As has been observable from the recent financial crisis, it is not only inertia in the cost of borrowing that can cause movements in loan rates to differ from deviations in the policy rate. Clearly an understanding of the banking sector and the interactions and risks that they face is paramount in determining the transmission of monetary policy from the central bank to the wider economy. Indeed, a direct consequence of this huge, adverse economic shock has been a wealth of interest and developments in the credit market imperfections and macrofinance literature. In particular, a great deal of research builds upon the contribution of Bernanke, Gertler and Gilchrist (1999) and the role that endogenous firm defaults have on amplification of the business cycle through a countercyclical *external finance premium*. Specifically, we follow Agénor Bratsiotis and Pfajfar (2011) where the expected income from lending to firms is equal to the cost of borrowing these funds from households, so that the commercial bank breaks even each period. Since there is a default risk, the loan rate will be a time varying markup over the deposit rate which depends on the level of collateral, and the probability of default.

Significantly the break even condition for the commercial bank is complicated by a set up where there will always be a proportion of contracts that cannot be adjusted. This raises the possibility that losses incurred from a fixed financial contract may be Klenow (2004), Laurent et al (2004), Le Bihan and Matheron (2011) to the persistence of output observed in aggregate data.
greater than the losses that were anticipated when the loan rate was signed. Since the commercial bank cannot adjust a loan rate mid contract these losses must be accounted for with an additional markup on the contracts up for renegotiation.

Thus a financial contract up for renegotiation will be driven by three elements; the anticipated costs of the firm, which stem from the expected levels of labour demand, productivity, wage costs and borrowing costs over the lifetime of the contract; the expected default risk and default losses associated with the firm and; unexpected default losses which are not accounted for in preset contracts that are passed on to new contracts to maintain the commercial bank’s zero profit condition.

Our results share features with both the incomplete pass through and the external finance premium literature. The inertia in loan rates implies a procyclical interest rate spread and a smoother response of aggregate variables and the business cycle. In contrast, default risk generates a countercyclical external finance premium which amplifies the business cycle. Overall, the dynamics of the spread between the policy rate and the loan rate depend upon the type of shock. Following monetary policy shocks the procyclical incomplete pass through effect dominates the countercyclical external finance premium so that initially the policy rate to loan rate spread is procyclical. In contrast, following technology shocks, and to a greater extent credit shocks, this spread is countercyclical since the change in default risk dominates the inertia in deposit rates. Finally, following adverse credit shocks a significant proportion of losses are passed on to new loan contracts. This amplifies the fall in output, rise in inflation and loan rate spread for many periods.
Chapter 2

Sticky Prices and Relative Price Distortions in Aggregate Consumption

2.1 Introduction

Despite the early emphasis on the role of relative price distortions among differentiated consumption goods in models of sticky prices, such distortions are eliminated at the aggregate macro level in much of the New Keynesian literature. Similarly, the price elasticity of product demand is often missing in the structural parameters of the new Keynesian Phillips curve, while the degree of market competitiveness seems to play little or no role for inflation and output dynamics.¹ This is in great contrast to some earlier microfounded new Keynesian models where relative price distortions and the price elasticity of demand were shown to be some key determinants of the behaviour of aggregate prices and output, (see Ball, Mankiw and Romer (1989), Dixon and Rankin (1994), Rotemberg and Woodford (1997), Ascari (2000)). This can be explained by a combination of, at least, three conventional assumptions made in much of the new Keynesian literature. First, much of the literature assumes, largely for modelling simplicity, that the production of firms is characterised by constant returns to scale. This implies that the firms’ marginal product of inputs (i.e.

¹See for example the seminal papers by Clarida, Galí and Gertler (1999), or Ravenna and Walsh (2006) for a model with a cost-channel.
labour and capital) and hence marginal cost, are independent of individual product demands and hence of relative price effects. Second, the bulk of this literature also assumes the presence of a lump sum subsidy that perfectly offsets the price mark-up and so eliminates any distortions arising from the monopolistic power of firms in the goods market. Third, and the assumption we relax in this chapter, the New Keynesian DSGE literature retains the neo-classical aggregate consumption, where the latter is described by the conventional Euler equation.² This implies that, just as in the flexible price neo-classical literature, aggregate consumption is constructed independently of any relative price distortions that may exist in the economy due to price stickiness. As a result, the behaviour of aggregate consumption in the conventional New Keynesian literature only deviates from the flexible price neo-classical aggregate consumption in the process of aggregate inflation, that is conventionally introduced only through aggregate supply.

This last assumption is the main focus of this chapter. We retain the first two assumptions, (i.e. of constant returns and the conventional lump sum subsidy that offsets the price mark-up), only to keep our results comparable to those of the bulk of the literature, and we focus on the implications that price stickiness among differentiated consumption goods has on the behaviour of aggregate consumption. The central assumption is that households care about their intertemporal consumption, as described by the standard Euler equation, but they also care about intratemporal relative prices, which can be distorted under price stickiness. The motivation of this is that in actual practice aggregate consumption indices are not constructed assuming symmetric prices, but are based on the aggregation of actual demands of consumer goods, collected in some specific point in time and weighted appropriately within a ‘typical’ consumption basket and therefore are subject to intratemporal changes in relative prices. We show that when price setters take the aggregation

²Note this is true in all New Neoclassical Synthesis (NNS) models, which add monopolistic competition and nominal stickiness to the real business cycle paradigm, (see also the discussion in Canzoneri, Cumby and Diba, 2007a).
of actual consumption demands (rather than the Euler equation) as given in their price setting, then relative price distortions affect marginal cost, the output gap and inflation and this has some non-trivial implications for macroeconomic dynamics and monetary policy.

First we show that at the macroeconomic equilibrium aggregate consumption deviates from the conventional Euler equation by some inflation ‘adjustment’ that accounts for the intratemporal effects of relative prices. The extent of this deviation, is shown to be determined by the degree of stickiness in price setting and the degree of competitiveness in the consumption goods market (the price elasticity of demand). In general, in any given point in time, aggregate consumption is shown to be affected by both intertemporal consumption choices and intratemporal relative price effects and it is only at the symmetric price equilibrium, where all prices are the same, that aggregate consumption behaves according to the neo-classical Euler equation.

Second, the price elasticity of demand is shown to affect the output-inflation trade off through aggregate consumption and thus through aggregate demand, hence it is independent of supply side factors, such as returns to scale, or imperfections in the labor or other markets.³

Third, relative price effects in aggregate consumption are shown to imply smoother responses of the output gap, inflation and interest rates than those implied by the conventional Euler equation-based model. This result is supported by studies that provide evidence that actual aggregate consumption data suggest that interest rates and the output gap exhibit much less volatility than that implied by the Euler equation, (see Canzoneri, Cumby and Diba (2007a), and Fuhrer and Rudebusch (2004)). In particular, in a model with a cost-channel, as in this chapter, relative price distortions are shown to affect equilibrium inflation and output through three channels:

³So far the literature shows that, in the absence of labor or other market imperfections, the price elasticity of demand affects equilibrium inflation and output only when the assumption of constant returns to scale is relaxed, since either decreasing returns (i.e Galí, Gertler, and López-Salido (2001), Sbordone (2002), Galí (2008), Damjanovic and Nolan (2010)) or increasing returns (Aoki (2001), Xiao (2008), Huang and Meng (2009)), allow relative prices to affect real marginal cost and hence output and inflation.
(a) directly through aggregate consumption and aggregate demand; (b) through the way aggregate consumption affects real wages and real marginal cost and thus price setting and (c) through monetary policy and the cost-channel. Interest rates respond to the output gap and inflation, but they also affect directly aggregate consumption and marginal cost through the cost of borrowing. It is shown, that accounting for relative price distortions in aggregate consumption moderates the impulse responses of the output-gap, inflation and interest rates. The more competitive is the consumption goods market the smoother become the impulse responses of these variables, in relation to those implied by the conventional Euler equation. This has also monetary policy implications, as the more competitive are consumption are the goods market, the smoother become the response of interest rates, in relation to that implied by the Euler-based model.

Finally, the determinacy of our model is largely consistent with those of the standard cost-channel models. (i.e. Surico (2008), Llosa and Tuesta (2009)). The main difference is that the determinacy region in our model suggests that monetary policy can afford to be relatively more aggressive with inflation, and less concerned with stabilizing the output gap, as the goods market tends to be more competitive; although within our theoretical framework this becomes evident only at very strong responses of monetary policy.

The following section, section 2.2, explains the setup of our model and establishes the macroeconomic equilibrium under sticky prices. Section 2.3 examines the regions of determinacy according to a Taylor policy rule. Section 2.4 evaluates the impulse response functions to taste, productivity, monetary and fiscal shocks, while section 2.5 concludes.
2.2 The Model

We employ the conventional cost-channel New Keynesian model of Ravenna and Walsh (2006), where the private sector consists a infinitely-lived representative household, a continuum of firms, \( f \in (0,1) \) and a financial intermediary, whereas the public sector consists of a government and a central bank. The representative household owns both firms and the financial intermediary, provide labour to firms and consumes a basket of all goods produced in the economy.

2.2.1 Households

Households maximise their expected lifetime utility,

\[
E_0 \sum_{s=0}^{\infty} \beta^s \left( \frac{\xi_{t+s} C_{t+s}^{1-\sigma}}{1-\sigma} - \frac{\chi N_{t+s}^{1+\gamma}}{1+\gamma} \right),
\]

where \( \beta \in (0,1) \) is a subjective discount factor; \( C_t \) is aggregate consumption in period \( t \); \( \xi_t \) is a taste shock, \( N_t \) is working hours supplied to firms; \( \sigma, \gamma, \chi > 0 \), and \( E_t \) is the expectation operator conditional on the information available at \( t \).

At the beginning of period \( t \), households enter with money \( M_t \) and gross nominal interest payments on their deposits \( R_t D_t \), where \( R_t = (1 + i_t) \); they receive nominal wage payments \( W_t N_t \) and pay lump-sum (nominal) taxes, \( T_t \). At the end of period \( t \) households enter the goods market and spend on consumption with their additional wealth allocated to money and deposits; they also receive total profits, \( \Pi_t \), from all firms and intermediary; their budget constraint is,

\[
P_tC_t + M_{t+1} + D_t = W_t N_t + M_t + R_tD_t + \Pi_t - T_t.
\]

Maximizing (2.1) subject to (2.2), yields the following first-order conditions,

\[
\xi_t C_t^{-\sigma} = \beta E_t \xi_{t+1} C_{t+1}^{-\sigma} \left( \frac{R_t P_t}{P_{t+1}} \right),
\]

(2.3)
\[
W_t/P_t = \frac{\chi N_t^{\gamma}}{\xi_t C_t^{\sigma}} 
\]
(2.4)
\[
P_tC_t = M_t + W_tN_t - D_t. 
\]
(2.5)

Equations (2.3)-(2.5) describe the conventional Euler equation, the labour supply and the demand for real money balances and deposits respectively.

### 2.2.2 Government Consumption and the Resource Constraint

As in Ravenna and Walsh (2006) we assume that the government purchases, \( \tilde{G}_t \), are proportional to output, \( \tilde{G}_t = (1 - g_t)\tilde{Y}_t \); where \( g_t \) is stochastic and bounded between zero and one. Note that as with all other agents, the government treats aggregate private consumption as given; thus government purchases are assumed to be proportional to actual output as shown below, (rather the conventional output determined by the Euler equation). In the steady state, \( G = (1 - g)Y \), and so government consumption is the same as in Ravenna and Walsh (2006). Equilibrium in the goods market requires that what is produced is consumed by all households and the government, i.e. \( Y_t = C_t + G_t \). Since government purchases are proportional to output, \( G_t = (1 - g_t)Y_t \), the aggregate resource constraint (in the RW2006 case) becomes,

\[
Y_t = C_t + (1 - g_t)Y_t \quad \text{or} \quad C_t = g_tY_t. 
\]
(2.6)

### 2.2.3 Financial Intermediaries

Following Ravenna and Walsh (2006) we assume that financial intermediaries receive an additional cash injection from the monetary authority, \( X_t \), these funds are lent at the nominal rate \( R_t \). This cash injection can be expressed as, \( X_t = (M_{t+1} - M_t) = (G_{t+1} - 1)M_t \), where \( G_{t+1} \) denotes the growth rate of money from \( t \) to \( t + 1 \). Finally in equilibrium the demand for loans requires that \( W_tN_t = D_t + X_t \). Intermediaries

\[\text{Note that this specification of government expenditure based on Ravenna and Walsh (2006) is only used so as to keep our results comparable to their study. It should be emphasized that this specification is independent to our main results.}\]
operate costlessly in a competitive environment, so that profits in the intermediary 
industry are $R_t (D_t + X_t) - R_t D_t = R_t X_t = \Pi_t^b$.

### 2.2.4 Wholesale Firms

A continuum of perfectly competitive wholesale producers bundle all intermediate 
goods into the production of a final composite good. The bundling technology is,

$$C_t = \left( \int_0^1 C_t(f)^{(\theta-1)/\theta} df \right)^{\theta/(\theta-1)}, \quad \theta > 1. \tag{2.7}$$

The parameter $\theta$ governs the price elasticity of demand for each individual good $j$ in the consumption basket. A higher $\theta$ implies that goods are closer substitutes and hence individual firms have less market power. We assume that the final good produced is used entirely for consumption. Given the price of each good, $P_t(f)$, the household’s decision problem is,

$$\min \int_0^1 P_t(f) C_t(f) \quad \text{subject to} \quad C_t \geq \left( \int_0^1 C_t(f)^{(\theta-1)/\theta} df \right)^{\theta/(\theta-1)}.$$

This yields the following demand for consumption of good $f$,

$$C_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\theta} C_t, \tag{2.8}$$

where the average price index (free of price stickiness) is,

$$P_t = \left( \int_0^1 P_t(f)^{1-\theta} df \right)^{1/(1-\theta)}. \tag{2.9}$$

### 2.2.5 Intermediate Firms

The economy consists of many imperfectly competitive firms, $f \in (0,1)$, each en-
gaging in the production of an intermediate good $Y_t(f)$, which is used towards the 
production of the composite final good, $Y_t$. Each intermediate goods firm $f$, faces
a demand for its consumption good, \( C_t(f) \), as determined in 2.8, it hires labour \( N_t(f) \) and produces output based on the following linear production technology with constant returns to scale,

\[
Y_t(f) = A_t N_t(f),
\]  

(2.10)

where \( A_t \) is a common technology shock. Firms pay workers the market determined wage rate \( W_t \). The demand for loans is

\[
L_t(f) = W_t N_t(f).
\]  

(2.11)

All profits are distributed to households, hence in the beginning of each period production costs must be fully covered by external finance; hence \( TC_t = R_t W_t N_t(f) \), and cost minimization results in the corresponding real marginal cost,

\[
mc_t(f) = mc_t = \frac{R_t w_t}{A_t},
\]  

(2.12)

where \( w_t = W_t / P_t \) is the real wage.

### 2.2.6 The Flexible-Price Equilibrium

Our flexible price equilibrium, where \( P_t(f) = P_t \), is the same as in all related literature (see Ravenna and Walsh, 2006). In log-linearized terms, from (2.12), \(^5\)

\[
\hat{w}_t = \hat{A}_t - \hat{R}_t.
\]  

(2.13)

From equations (2.4), and (2.10) and 2.6, we obtain,

\[
\hat{w}_t = (\gamma + \sigma) \hat{Y}_t + \sigma \hat{g}_t - \gamma \hat{A}_t - \hat{\xi}_t.
\]  

(2.14)

\(^5\)See Appendix 2.A.1 for derivation
Hence, the flexible-price equilibrium output can be written as,

$$\hat{Y}_t^f = \frac{1}{\gamma + \sigma} \left[ (1 + \gamma) \hat{A}_t + \hat{\xi}_t - \sigma \hat{\gamma}_t - \hat{R}_t^f \right].$$  \hspace{1cm} (2.15)$$

Note that as in Ravenna and Walsh (2006), we later also assume, \( \hat{R}_t^f = 0 \).

### 2.2.7 Sticky Prices, Price Setting and Aggregate Consumption

In this section we consider the behaviour of price setting and aggregate consumption in the presence of nominal price stickiness. We impose firm level price rigidity using the common Calvo (1983) price setting assumption, where firms face a constant and common probability \( (1 - \omega) \) of having a price setting opportunity. Firms that do not reset prices keep their prices fixed as in the last period. As with the rest of the literature, the average price index (2.9) under this sticky price setup is,

$$\hat{P}_t = \frac{P_t}{P_{t-1}} = \int_0^1 P_t^*(f)^{1-\theta} df,$$

\hspace{1cm} (2.16)$$

where, \( P_t^*(f) \) is the optimal price level at time \( t \); \( q_t(f) = \frac{P_t(f)}{P_t} \) and \( \pi_t = \frac{P_t}{P_{t-1}} \).

Therefore, in log-linearized terms, around steady state inflation \( (\pi = 1) \), the relative price chosen by all firms adjusting their price at time \( t \) is,\(^6\)

$$\hat{q}_t = \int_0^1 \hat{q}_t(f) df = \frac{\omega}{(1 - \omega)} \hat{\pi}_t.$$

\hspace{1cm} (2.17)$$

**Conventional Case: ‘Euler’ Aggregate Consumption**

In the conventional approach, price setters treat aggregate consumption as driven by the Euler equation, that is free of relative prices distortions. Aggregate consumption

\(^6\)See in the Appendix 2.A.3
and inflation are estimated in the following way.

Stage 1: Firms observe the behaviour of the consumption demand for their own product, (based on eq. 2.8), and of the aggregate consumption, as given by the Euler equation, (eq. 2.3), for time, \( t + k \),

\[
C_{t+k}(f) = \left( \frac{P_{t+k}(f)}{P_{t+k}} \right)^{-\theta} C_{t+k}, \tag{2.18}
\]

where, from the Euler equation,

\[
C_{t+k} = E_t \left( \frac{\beta C_{t+k}^{-\sigma} P_{t+k} \xi_{t+1+k}}{P_{t+1+k}/P_{t+k} \xi_{t+k}} \right)^{-1/\sigma}. \tag{2.19}
\]

The behaviour of the Euler equation determines how consumers want to allocate their aggregate consumption over time, in general. This intertemporal choice is independent of individual consumption demands and hence of changes in relative prices over time.

Stage 2: In maximizing expected discounted real profits, the conventional literature assumes that firms set prices by taking as given how their relative price will be affected by their own reset price probability in any given period \( t + k \), (i.e. \( \omega^k \)), and subject to equations (2.18) and (2.19), above. The firm’s profit maximization problem is,

\[
\max_{P_t(f)} \sum_{k=0}^{\infty} \omega^k \Lambda_{t,t+k} \left( 1 + \tau \right) \left( \frac{P_t(f)}{P_{t+k}} \right)^{1-\theta} - mc_{t+k} \left( \frac{P_t(f)}{P_{t+k}} \right)^{-\theta} C_{t+k} \tag{2.20}
\]

where, \( \Lambda_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right) \), and,

\[
m_{c_{t+k}} = \frac{R_{t+k} w_{t+k} [C_{t+k}]}{A_{t+k}} \tag{2.21}
\]
where from (2.4), (2.10) and 2.6, the real wage is,

\[\begin{align*}
w_{t+k} &= \frac{\chi A_{t+k}^{-\gamma} C_{t+k}^{\gamma+\sigma} g_{t}^{-\gamma}}{\xi_{t+k}}.\end{align*}\] (2.22)

Using, the conventional lump subsidy that eliminates monopolistic distortions, i.e. $(\theta - 1)(1 + \tau)/\theta = 1$, and employing the definitions $q_{t}(f) = \frac{P_{t}(f)}{P_{t}}$ and $\pi_{t+k} = \frac{P_{t+k}}{P_{t}}$, the first order condition, in log-linearized terms, is

\[E_{t} \sum_{k=0}^{\infty} (\omega \beta)^{k} \left[ \hat{g}_{t}(f) - \sum_{l=1}^{k} \hat{R}_{t+l} - \hat{A}_{t+k} \right] = 0.\] (2.23)

Substituting the log-linearized, $\hat{m}_{t+k}$ and $\hat{w}_{t+k}$ (based on 2.21 and 2.22), into 2.23 and using the resource constraint, $\hat{C}_{t+k} = \hat{Y}_{t+k} + \hat{g}_{t}$, (see 2.6) we derive the standard NKPC,

\[\hat{\pi}_{t} = \beta E_{t} \hat{\pi}_{t+1} + \kappa (\gamma + \sigma) \hat{x}_{t} + \kappa (\hat{R}_{t} - \hat{R}_{t}^{f})\] (2.24)

where, $\hat{x}_{t} = \hat{Y}_{t} - \hat{Y}_{t}^{f}$, is the output gap from the flexible-price output level, $\hat{Y}_{t}^{f}$, that is defined in (2.15); $\hat{Y}_{t} = \hat{C}_{t} - \hat{g}_{t}$ and $\hat{C}_{t}$ is the conventional log-linearised Euler equation,

\[\hat{C}_{t} = E_{t} \hat{C}_{t+1} - \frac{1}{\sigma} \left[ \hat{R}_{t} - E_{t} \pi_{t+1} + \left( \hat{\xi}_{t} - E_{t} \hat{\xi}_{t+1} \right) \right].\] (2.25)

From the definition of $\hat{x}_{t}$ and equations (2.24), and (2.25), it is evident that in deriving this NKPC, price setters treat the dynamic behaviour of aggregate consumption as driven purely by the intertemporal consumption allocation of households, (i.e. as described by the Euler equation), that is independent of relative price effects.

\footnote{See Appendix 2.A.2}
**Aggregation of Actual Consumption Demands: Accounting for Relative Price Effects**

In this section, we use the fact that in actual practice aggregate consumption indices are constructed based on the aggregation of actual demands of consumer goods, collected in some specific point in time and weighted appropriately within a ‘typical’ consumption basket. In this case, as we show below, the behaviour of aggregate consumption is determined by both the intertemporal consumption allocation of households, (i.e. as described by the Euler equation), and the dynamics of relative prices over time, since here all relative prices are observed in firm’s price setting, but at any given period $t$ only a fraction $(1 - \omega)$ of firms will be able to adjust prices. Using this approach, aggregate consumption and inflation can be estimated in the following way.

**Stage 1:** Firms observe the behaviour of consumption demand of their own product, (based on eq. 2.8), but also of the aggregate consumption as given by the aggregation of all consumption demands, (i.e. based on eq. 2.7 and 2.8), for time, $t + k$,

$$C_{t+k}(f) = \left( \frac{P_t(f)}{P_{t+k}} \right)^{-\theta} \hat{C}_{t+k}, \quad (2.26)$$

subject to

$$\hat{C}_{t+k} = \left( \int_0^1 \left( \frac{P_t(f)}{P_{t+k}} \right)^{-\theta} df \right)^{(\theta-1)/\theta} \left( \frac{P_t}{P_{t+k}} \right)^{\theta/(\theta-1)} \quad (2.27)$$

where as before from the *Euler* equation,

$$C_{t+k} = E_t \left( \frac{\beta C_{t+k}^{-\sigma} R_{t+k} \xi_{t+1+k}}{P_{t+1+k}/P_{t+k} \xi_{t+k}} \right)^{\sigma},$$

**Stage 2:** Firms set prices by taking as given how their relative price will be affected by their own reset price probability in any given period $t + k$, (i.e. $\omega^k$) as before, but now subject to equations (2.26) and (2.27), where the latter is subject to relative price distortions as it is given by the aggregation of all actual consumption demands.

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Intuitively, in selecting its optimal price, each firm here recognises that having a fixed probability $\omega$ of not being able to reset its price, for say $t+k$ periods, its relative price will be affected by both the intertemporal consumption dynamics, (as given by the Euler equation), and the dynamics of relative prices across its competitors. This is because at any point in time, only a fraction $(1-\omega)$ of firms will be able to adjust prices to changing economic conditions and thus the aggregate relative price of all firms that can respond to a stochastic shock in any period $t$, is $\frac{\omega}{1-\omega} \bar{\pi}_t$ (see eq. 2.17). Since all firms setting price internalize these relative price effects into their optimal prices, at the aggregate level, relative price distortions cannot be eliminated within any single period, and therefore their effect will spill over to marginal cost and inflation.

The firm’s profit maximization problem here becomes,$^8$

$$\max_{P_t(f)} \mathbb{E}_t \sum_{k=0}^{\infty} \omega^k \bar{\Lambda}_{t,t+k} \left( (1+\tau) \left( \frac{P_t(f)}{P_{t+k}} \right)^{1-\theta} - mc_{t+k} \left[ \frac{P_t(f)}{P_{t+k}} \right]^{-\theta} \right) \dot{C}_{t+k}$$

(2.28)

where $\bar{\Lambda}_{t,t+k} = \beta^k \left( \frac{\dot{C}_{t+k}}{C_t} \right)$ and,$^9$

$$mc_{t+k} = \frac{R_{t+k} w_{t+k} [\dot{C}_{t+k}]}{A_{t+k}};$$

(2.29)

---

$^8$At this stage in deriving the optimal price of firms setting prices at time $t$, there are two assumptions one can make: (a) Price setters treat aggregate consumption $\dot{C}_{t+k}$ as given, that is with relative price effects affecting the dynamics of consumption, as shown above. (b) Large price setter recognise their own effect on aggregate $\dot{C}_{t+k}$ and account for that, (i.e. in the sense of the atomistic price/wage setter literature). In this case the price elasticity of demand, $\theta$, will also be affected. In this paper we adopt the standard case, (a), in that we assume many small firms, that just take these relative price distortions as given. This means that at the aggregate price level the optimal relative price of all firms setting price at time $t$ will generate a non-negligible relative price effect with respect to those that cannot set prices. Since our competition uses a Dixit -Stiglitz aggregator with a continuum of firms, relative price distortions are taken as given by each firm.

$^9$Note that the marginal utility of consumption between period $t$ and $t+k$, and thus the discount factor, must be consistent with the actual aggregate consumption considered in the model. As we show below, the households also adjust their Euler equation to account for relative price effects, and thus, at equilibrium, both households and firms treat $\dot{C}_t$ as the aggregate consumption in the economy.
where from (2.4), (2.10) and 2.6, the real wage is,\(^{10}\)

\[ w_{t+k} = \frac{\chi A_{t+k}^{-\gamma} \tilde{C}_{t+k}^\gamma \tilde{g}_{t}^{-\gamma}}{\xi_{t+k}} \]  

(2.30)

where \(mc_{t+k}\), and \(w_{t+k}\) are real marginal cost and real wages respectively. Using, as in the conventional case above, the lump subsidy that eliminates monopolistic distortions and employing the definitions \(q_t(f) = \frac{P_t(f)}{P_t}\) and \(\pi_{t+k} = \frac{P_{t+k}}{P_t}\), the first order condition, in log-linearized terms, is

\[ E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{q}_t(f) - \sum_{l=1}^{k} \hat{\pi}_{t+l} - \hat{R}_{t+k} - \hat{\alpha}_{t+k}[\hat{C}_{t+k}] + \hat{A}_{t+k} \right] = 0. \]  

(2.31)

Following standard log-linearization techniques we show that in any period, \(t+k\), the dynamics of consumption will be described by both the intertemporal consumption allocation over time (as described by the Euler equation), but also by the dynamics of relative prices due to the price asynchronization among firms. In log-linearised terms, the dynamics of aggregate consumption in time, \(t+k\), is,\(^{11}\)

\[ \hat{C}_{t+k} = \hat{C}_{t+k} + \hat{A}_{t+k}, \]  

(2.32)

where \(\hat{C}_{t+k}\) is based on the conventional Euler equation (2.25) above, and \(\hat{A}_{t+k}\) is the deviation from the Euler equation,

\[ \hat{A}_{t+k} = -\theta \left( \frac{\hat{q}_t - \sum_{l=1}^{k} \hat{\pi}_{t+l}}{(1 - \omega)} \hat{\pi}_t - \sum_{l=1}^{k} \hat{\pi}_{t+l} \right). \]  

(2.33)

In the absence of changing economic conditions (or shocks), where prices are the same, \(\int_0^1 P_t(f)df \approx P_{t+k}\) or when \(\omega = 0\) (i.e. relaxing price stickiness), then \(\hat{A}_{t+k} = 0\) and \(\hat{C}_{t+k} = \hat{C}_{t+k}\), and thus we are back to the conventional case.\(^{12}\)

\(^{10}\)Note that for equilibrium consistency here we use \(\tilde{C}_t = g_t\tilde{Y}_t\), (see eq. 2.42 below).

\(^{11}\)See Appendix 2.A.4

\(^{12}\)Note that some recent literature that is concerned with the steady state effects of price dispersion, (i.e. due to a positive inflation trend, or partially indexed prices to past inflation), interprets
Substituting the log-linearized, \( \tilde{m}c_{t+k} \) and \( \tilde{w}_{t+k} \) (based on (2.29) and (2.30)), into (2.31), and using the appropriate resource constraint, \( \tilde{C}_t = \tilde{Y}_t + \tilde{g}_t \) we can derive the ‘adjusted’ NKPC,

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\gamma + \sigma) \hat{x}_t + \kappa (\hat{R}_t - \hat{R}_f^f)
\]

(2.34)

where, \( \kappa = \frac{(1-\omega)(1-\omega \beta)}{\omega} \); \( \hat{x}_t = \tilde{Y}_t - \tilde{Y}_f^f \) is the output gap with relative price effects, and \( \hat{R}_t - \hat{R}_f^f \) is the nominal interest rate gap, both expressed as deviations from their flexible price levels. Therefore, based on the standard forward-looking price-setting Calvo model, with constant returns to scale, the difference between (2.34) and the conventional NKPC, (2.24), is the output gap, that now accounts for the relative price effect, \( \hat{\Delta}_{t+k} \). We discuss this in more detail in eq. (2.38) below.

**Stage 3**: We estimate the equilibrium aggregate consumption \( \tilde{C}_t \) in period \( t \).

Having established the dynamic behaviour of aggregate consumption up to any period \( t + k \), (see 2.32), we need to solve for the equilibrium level of aggregate consumption, at any time \( t \). In effect in this final stage we find the ‘adjusted’ Euler equation, that is consistent with all households and firms at the macroeconomic equilibrium, when relative price effects are accounted for in prices.

From (2.33) we know that the behavior of actual aggregate consumption, \( \tilde{C}_{t+k} \), deviates from the conventional case, \( \tilde{C}_{t+k} \), by some additional inflation dynamics \( \hat{\Delta}_{t+k} \). Therefore to find the latter at time \( t \), we take the difference (at any period \( t \)) between the conventional NKPC and the relative price-adjusted NKPC, both

\[
\Delta_t = \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{-\theta} df, \text{ as a measure of the degree of price dispersion, and shows that a first-order log-linear approximation around zero-inflation steady state (i.e. } \pi = 1 \text{), where the symmetric price approximation, } \int_0^1 \tilde{P}_t(f) df \approx \tilde{P}_t \text{ is used, implies } \hat{\Delta}_t \approx 0 \text{, (see Ravenna and Walsh, 2006 and Ascari, Castelnovo and Rossi, 2011, Damjanovic and Nolan 2010). We should stress that this effect does not hold in this model, because here price setters internalise these relative price effects into their price setting, and since only a fraction } (1 - \omega) \text{ of price setters can adjust prices at any given period } t, \text{ these effect are not eliminated at the aggregate level. In this model, } \hat{\Delta}_t \approx 0, \text{ will only hold if there was no price stickiness } \omega = 0, \text{ or if price did not change over time, i.e. } \int_0^1 P_t(f) df \approx P_t, \text{ In both of these case the behaviour of aggregate consumption would be, } \tilde{C}_{t+k} = C_{t+k}. \]

\(^{13}\)See Appendix 2.A.5

36
expressed in terms of $C_t$, to obtain,\textsuperscript{14}

\[
\hat{C}_t = \hat{C}_t + \hat{\Delta}_t, \tag{2.35}
\]

where, $\hat{\Delta}_t$ is,

\[
\hat{\Delta}_t = -\frac{\theta}{\kappa} [\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}]. \tag{2.36}
\]

Eq. (2.35) is the ‘adjusted’ aggregate consumption, as given in any period $t$, when price setters consider actual aggregate consumptions across the economy, that is, when the relative price effects of all differentiated consumption goods are considered within the asynchronous sticky price setting assumed by Calvo-price contracts. Within the forward-looking price-setting Calvo model, the deviation, $\hat{\Delta}_t$, is determined by size of the structural parameters $\omega$ and $\theta$, and the way that relative price distortions in aggregate consumption causes inflation in period $t$ to deviate from expected inflation in period $t + 1$.

\textbf{2.2.8 The Output Gap}

To find the full output gap, $\hat{x}_t = \hat{Y}_t - \hat{Y}_t^f$, we use $\hat{C}_t = \hat{Y}_t + \hat{g}_t$, and write equation (2.35) in terms of aggregate output,

\[
\hat{Y}_t = \hat{Y}_t + \hat{\Delta}_t. \tag{2.37}
\]

Substituting, $\hat{x}_t = \hat{Y}_t - \hat{Y}_t^f$ and $\hat{x}_t = \hat{Y}_t - \hat{Y}_t^f$, into (2.37) we obtain,

\[
\hat{x}_t = \hat{x}_t + \hat{\Delta}_t, \tag{2.38}
\]

\textsuperscript{14}See Appendix 2.A.6
where \( \hat{x}_t \) is the conventional dynamic IS equation,\(^{15}\)

\[
\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} \left[ \hat{R}_t - E_t \hat{\pi}_{t+1} \right] + \mu_t, \tag{2.39}
\]

and,

\[
\mu_t = \left( \frac{1 + \gamma}{\gamma + \sigma} \right) [E_t \hat{\Delta}_{t+1} - \hat{\Delta}_t] + \left( \frac{\gamma}{\gamma + \sigma} \right) [E_t \hat{\delta}_{t+1} - \hat{\delta}_t] - \frac{\gamma}{\sigma (\gamma + \sigma)} [E_t \hat{\xi}_{t+1} - \hat{\xi}_t].
\]

In the absence of price stickiness, \( \omega = 0 \), \( \hat{\Delta}_t = 0 \), and \( \hat{C}_t = \hat{C}_t \) and similarly, \( \hat{x}_t = \hat{x}_t \). However, in the presence of price stickiness, the output gap deviates from the conventional Euler based output-gap, \( \hat{x}_t \), by \( \hat{\Delta}_t \) as shown in (2.36). The more competitive is the consumption goods market, (i.e. the higher is \( \theta \)) the more sensitive becomes the response of price changes and thus inflation to relative price distortions, \( \hat{\Delta}_t \). Intuitively, a higher \( \theta \) implies that goods become closer substitutes, hence given an innovation that raises (lowers) inflation at time \( t \), the higher is \( \theta \), the larger will be the relative price distortions (i.e. \( \hat{\Delta}_t \)), arising from those firms that can adjust prices. Hence, as we demonstrate below, at the aggregate level and for any given \( \omega \), the presence of \( \hat{\Delta}_t \) moderates the impulse responses of the inflation rate, the output gap and interest rates and this effect is stronger the higher is the value of \( \theta \).

### 2.2.9 Monetary Policy and the Macro Equilibrium

In this section we complete our macroeconomic model by assuming that the central bank implements monetary policy through the familiar Taylor rule used widely in the literature. In terms of deviations from its steady state, and assuming \( \hat{R}_t^f = 0 \), the policy rate is,

\[
\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left( \phi_x \hat{\pi}_t + \phi_x \hat{x}_t \right) + \hat{\nu}_t. \tag{2.40}
\]

\(^{15}\)See Appendix 2.A.7
where $\phi_\pi > 0$ and $\phi_x > 0$ are policy weights and $\rho_R$ defines the degree of smoothing in the interest rate.\textsuperscript{16}

Equilibrium in the goods market requires that what is produced is consumed by households and the government, hence the following resource constraint holds,\textsuperscript{17}

\begin{align*}
\tilde{Y}_t &= \tilde{C}_t + (1 - g_t)\tilde{Y}_t, & (2.41) \\
\tilde{C}_t &= g_t\tilde{Y}_t & (2.42)
\end{align*}

Equations (2.34),(2.38), (2.39) and (2.40) together with (2.41), define our macroeconomic equilibrium. Equilibrium in the goods market requires that $\tilde{Y}_t = \tilde{C}_t + \tilde{G}_t$, where $\tilde{G}_t$ are government purchases. We follow Ravenna and Walsh (2006) and assume that the government purchases are proportional to output $\tilde{G}_t = (1 - g_t)\tilde{Y}_t$.\textsuperscript{18} The aggregate resource constraint then takes the form $\tilde{Y}_t = \tilde{C}_t + (1 - g_t)\tilde{Y}_t$ or $\tilde{C}_t = g_t\tilde{Y}_t$.

In terms of log deviations from steady state, $\tilde{C}_t = \tilde{g}_t + \tilde{Y}_t$.

## 2.3 Determinacy

In this section we provide regions of determinacy of the Taylor rule parameters $\phi_\pi$ and $\phi_x$ under different levels of goods market competition and with a cost channel.\textsuperscript{19}

Our basic parameter values are given in Table 2.1 (see Galí and Gertler 1999 and Ravenna and Walsh, 2006).

To capture the implications of the degree of competition among differentiated goods for determinacy in our model, we examine determinacy under $\theta = 3$ and $\theta = 10$, but also under the standard cost channel model of Ravenna and Walsh.

---

\textsuperscript{16}Note that as expected the policy rule responds always to the actual output gap in the model, (i.e. $\tilde{x}_t$ here rather than the output gap according to the Euler equation $\tilde{x}_t$).

\textsuperscript{17}Note that our aggregate resource constraint already incorporates the price dispersion related inefficiencies.

\textsuperscript{18}Note that government purchases are assumed to be proportional to actual output rather euler output in the standard case. This way we are assuming the the government behaves in the same optimal fashion of consumers that we have outlined in this paper.

\textsuperscript{19}All simulations carried out in MATLAB.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Inverse of the Frisch elasticity of labour supply</td>
<td>1.0</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Intertemporal elasticity of substitution in consumption</td>
<td>1.5</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Probability of not adjusting prices</td>
<td>0.75</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution in demand</td>
<td>3, 10</td>
</tr>
</tbody>
</table>

Table 2.1: Baseline Parameter Values

(2006) which we refer to as $R&W^{20}$. Assuming that $\rho = 0^{21}$, we have the following conditions for determinacy$^{22}$,

$$\phi_x + \frac{(1 - \beta - \kappa)}{(\sigma + \gamma) \kappa} \phi_x > 1$$

(2.43)

and

$$\phi_x (1 + \beta + \kappa + 2\theta \sigma) + \kappa(\sigma + \gamma) - \phi_x \kappa(\sigma - \gamma) + 2(1 + \beta)(\sigma + \gamma \theta \sigma + \theta \sigma^2) > 0$$

(2.44)

Condition (2.43) can be interpreted as a version of the long-run Taylor principle that guarantees determinacy which measures the impact of interest rate on inflation through the cost channel. In line with Surico (2008), Llosa and Tuesta (2009), and Woodford (2003) each percentage point of permanently higher inflation implies a $\frac{1 - \beta - \kappa}{(\sigma + \gamma) \kappa}$ permanent change in the output gap.$^{23}$ The regions of determinacy with a cost channel, indicated in equation (2.43), are displayed by the lower bound in Figure (2.1); since this condition is independent of $\theta$ the region of determinacy is identical to R&W (Ravenna and Walsh, 2006).

---

$^{20}$For the simulations, the R&W model provides identical results to assuming $\theta = 0$ in our macroeconomic equilibrium.

$^{21}$When $\rho > 0$ we show through simulation that the constraint in 2.43 remains the same and the constraint 2.44 is weakened.

$^{22}$See Appendix 2.A.8 for details.

$^{23}$When there is no cost-channel, the output gap is higher following an increment in steady state inflation, whereas when a cost channel is present, it leads to a permanent reduction in the output gap.
Figure 2.1: Region of Determinacy

The upper bound on inflation is given by condition (2.44). Our economic interpretations for the most part follow, Surico (2008) and Llosa and Tuesta (2009), for a given consumption elasticity, an increase in the real interest rate boosts labour supply and reduces real wages. Higher values of the labour supply elasticity, given by a lower $\gamma$ imply a smaller decline in real wages for a given change in the interest rate. If the response to the nominal interest rate is too great, then the cost of higher lending rates outweighs the benefit of lower real wages and firms will raise prices making inflation self-fulfilling. However, unlike Surico (2008) and Llosa and Tuesta (2009), the responses of inflation to a given shock is also affected by the relative price effects, through $\theta^{24}$. By setting $\theta$ to zero in (2.44) we obtain an identical result to Llosa and Tuesta (2009), where given a null response to the output gap $\phi_\pi$ must be less than 145.$^{25}$ In our case if, $\theta = 3$, $\phi_\pi$ must be less than 1187.41 and if $\theta = 10$, $\phi_\pi$ must be less than 3621.78.$^{26}$ Although in the content of our setup it

---

$^{24}$See the following section
$^{25}$Not shown in Figure 2.1, because of its high value.
$^{26}$Through simulation we know that when $\rho = 0.85$, equation 2.43 is identical and constraint given in equation 2.44 is weaken. For example given a null response to the output gap $\phi_\pi$ must
is empirically implausible that these values can become binding in actual monetary policy, there are some clear theoretically implications of this: for any given value of \( \phi_x \), and \( \phi_x \), and for \( \sigma > \gamma \), (as assumed in the literature), the constraint given by (2.44) binds at a higher level as the goods market becomes more competitive (i.e. for higher values of \( \theta \)). This implies that monetary policy can afford to be relatively more aggressive with inflation and less concerned with stabilising output, as markets tend to be more competitive.

### 2.4 Impulse Response Functions

Figures (2.2 – 2.5) below, provide the impulse response functions for taste, productivity, monetary policy and fiscal policy shocks respectively. In this section we simulate our model for two different values of the elasticity of substitution in product demand (\( \theta = 3 \) and \( \theta = 10 \)) and compare it to Ravenna and Walsh (2006) that is based on the conventional aggregate consumption (shown as Euler (R&W)).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_A )</td>
<td>Persistence of productivity shock</td>
<td>0.85</td>
</tr>
<tr>
<td>( \rho_\xi )</td>
<td>Persistence of taste shock</td>
<td>0.85</td>
</tr>
<tr>
<td>( \rho_v )</td>
<td>Persistence of monetary policy shock</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_R )</td>
<td>Persistence of the interest rate</td>
<td>0.85</td>
</tr>
<tr>
<td>( \rho_g )</td>
<td>Persistence of fiscal policy shock</td>
<td>0.85</td>
</tr>
<tr>
<td>( \phi_\pi )</td>
<td>Interest rate response to inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>( \phi_x )</td>
<td>Interest rate response to the output gap</td>
<td>0.125</td>
</tr>
<tr>
<td>( \hat{A}_t )</td>
<td>Size of quarterly productivity shock</td>
<td>1</td>
</tr>
<tr>
<td>( \hat{\xi}_t )</td>
<td>Size of quarterly taste shock</td>
<td>1</td>
</tr>
<tr>
<td>( \hat{g}_t )</td>
<td>Size of quarterly fiscal policy shock</td>
<td>1</td>
</tr>
<tr>
<td>( \hat{\nu}_t )</td>
<td>Size of quarterly monetary policy shock</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 2.2: Parameter Values of Shocks and Policy Weights

For the impulse responses we use the baseline parameter values in Table 2.1 and the impulse response values in Table 2.2. Our interest rate smoothing value, follows be less than 14,644 when \( \theta = 3 \), and less than 44,678 when \( \theta = 10 \).

\(^{27}\)See Appendix 2.A.8 for details of derivation of impulse response functions
a value between the 0.8 used in some papers, (see Amano, Ambler and Rebei, 2007) and the 0.9 used in much of the literature (see Rudebusch 2002). Our monetary policy baseline parameter values, $\phi_x = 1.5$ and $\phi_x = 0.125$ are standard in the literature.\footnote{Note that the policy response to inflation and the output gap are roughly in keeping with observed variations in the Federal Funds rate over the Greenspan era, and also satisfy our determinacy conditions. The monetary policy shock is 25 basis points which corresponds to a 1% annualised nominal rate on impact. Conversion to quarterly rates also requires that the output gap coefficient, 0.5 here, be divided by 4.}

### 2.4.1 Taste Shock

Figure (2.2) shows the responses to a taste shock, with relative price distortion effects in consumption (i.e. with two different values, $\theta = 3, 10$) and under the conventional Euler equation (i.e. Euler (R&W)).\footnote{The case of R&W, is effectively the same as eliminating $\theta$ in the macroeconomic equilibrium. Thus, setting $\theta = 0$, in our model collapses to R&W (2006).} A positive taste shock increases the demand for consumption and output but it also pushes up the output gap and inflation, together resulting in higher interest rates. The presence of relative price distortions in aggregate consumption, dampens the responses of these variables, in relation to the conventional case, Euler (R&W). As a result interest rate responses and consumption are also substantially smoother than that implied by the conventional case. This effect is stronger the higher is the competition among differentiated goods (i.e. the higher is $\theta$), because, as we explained earlier, as the substitutability of products increases the change in prices (a fall here) are moderated and this also moderates the fall in the output gap and thus the response of interest rates and consumption.
Figure 2.2: Impulse Responses to a Taste Shock
2.4.2 Productivity Shock

Figure (2.3) displays the impulse response functions to a positive productivity shock. The productivity shock results in a fall in the inflation rate and the output gap and a rise in the level of output and consumption; the fall in inflation and the output gap causes the nominal policy rate also to fall. However, the presence of relative price distortions in consumption are shown to dampen the fall in the output gap. This also moderates the drop in inflation rate, thus implying a smoother interest rate response, in relation to that implied by the conventional Euler model and hence the standard cost channel model (R&W 2006). Positive productivity shocks are thus shown to result in a relatively higher response of consumption. Intuitively, as we explained above, as the goods market competition increases, firms who are able to raise prices following the taste shock, will lose share of their market for a given price increase, as a result the more competitive are the goods markets the more dampened become the responses of the output gap and inflation and so is the interest rate response and its effects on consumption. Note the small hump in consumption in the Euler (R&W) is due to the exogenous interest smoothing in the interest rate policy rule. The sluggishness that the latter causes, in combination with the lack of relative price distortion that tend to smooth out the output gap and dampen the interest rate response, cause the interest rate to overreact first downwards and then upwards resulting in this hump response.\footnote{\textsuperscript{30}}

\footnote{\textsuperscript{30}For specific interest rate smoothing and AR(1) parameter values in the shock process we can generate a hump in both cases.}
Figure 2.3: Impulse Responses to a Productivity Shock
2.4.3 Monetary Policy Shock

Figure (2.4) displays the impulse responses of annualised 1% increase in the interest rate. A policy shock reduces inflation and the output gap but also consumption. The presence of relative price distortions in aggregate consumption here dampens again significantly the fall in inflation as the latter exhibits more persistence and this makes interest rate policy more effective as shown below. This effect is amplified here through the cost-channel, which plays a key role for the behaviour of consumption. The presence of the cost-channel in combination with the AR(1) process of the policy rate, generate a hump-shaped response of consumption. The more competitive is the goods market, the more effective is monetary policy in reducing inflation, and this implies a smaller 'crowding out' effect on private consumption, in relation to the conventional Euler (R&W) case.

---

31 Note that in the absence of fiscal policy (a stochastic fiscal shock here), a monetary policy shock implies identical responses for the output gap, output and consumption.
32 This hump is independent of habit persistence or real wage stickiness or other assumptions employed in the literature.
Figure 2.4: Impulse Responses to a Monetary Policy Shock
Fiscal Policy Shock

Figure (2.5) displays a 1% increase in $g_t$, which represents a decrease in government spending. As a result of fiscal contracting, output and the output gap fall which generates a fall in both prices and the nominal interest rate. This fall in both prices and the savings rate leads to an increase in aggregate consumption. In our model, since agents are able to switch their products optimally, consumption rises by more and therefore the fall in aggregate output is reduced by less. In effect, here the fiscal policy contraction reduces interest rates less and ‘crowds in’ more private consumption when relative price distortions are accounted for. For reasons explained above, these effects are stronger the higher are the relative price frictions among consumption goods and the more competitive is the goods market.\textsuperscript{33}

\textsuperscript{33}Note that as explained above, the small hump in consumption in the Euler (R&W) is due to the sluggishness that the exogenous interest smoothing causes, in combination with the lack of relative price distortion that cause the interest rate to overreact first downwards and then upwards.
Figure 2.5: Impulse Responses to a Fiscal Policy Shock
2.5 Concluding Remarks

The conventional Euler equation has come under much criticism recently for failing to match the dynamic behaviour of actual aggregate consumption data, which exhibits more stickiness that is implied by the Euler equation (see, Fuhrer and Rudebusch, 2004, and Carroll, Slacalek and Sommer, 2011). Similarly, Canzoneri, Cumby and Diba (2007a) show that the interest rates implied by the conventional Euler equation are much more volatile than actual interest rates, as set by the Fed. Could this partly be explained by the fact that the neo-classical Euler equation is mainly concerned with the intertemporal consumption choices of households, while neglecting the intratemporal implications of relative price dynamics in the presence of price stickiness?

This chapter shows that in the presence of price stickiness among differentiated consumption goods, both intertemporal consumption choices and intratemporal relative price effects should affect the behaviour of aggregate consumption. Using a standard forward-looking Calvo-type price-setting model, we show that aggregate consumption, as described by the conventional Euler equation, is ‘adjusted’ by the extent to which, in any given period, relative price distortions among differentiated consumption goods affect, through marginal cost, the inflation dynamics between current and future periods. This ‘adjustment’ works to moderate the impulse responses of the output gap and interest rates.

The model can be extended to account for different types of price setting. An obvious extension here would be to include some price-stickiness to past inflation, that would endogenously introduce persistence in the behaviour of aggregate consumption. Such an addition could not only reduce the volatility of the output gap and interest rates, but also add more persistence in the behaviour of aggregate consumption, without the need to employ additional ad-hoc features, such as different types of consumption habits.
2.A Appendix 2

2.A.1 Flexible price equilibrium output ($\hat{Y}_t^f$)

Here we employ, $P_t(f) = P_t$, and $X_t = X_t$. Given eq. (2.8), that implies a price mark-up of $\frac{(\theta-1)}{\theta}$, and $mc_t(f) = mc_t = \frac{R_t}{A_t}$, our flexible real price implies $\frac{\theta}{(\theta-1)} = mc_t$. In log-linearized terms,

$$\hat{w}_t^f = \hat{A}_t - \hat{R}_t^f. \tag{2.45}$$

From the wage equation, (2.4), $W_t/P_t = \frac{\chi N_t^\gamma}{\xi C_t^{\gamma - \sigma}}$, and the production function, $Y_t(f) = A_t N_t(f)$, we obtain

$$W_t = w_t = \frac{\chi N_t^\gamma}{\xi C_t^{\gamma - \sigma}} = \frac{\chi A_t^{-\gamma} Y_t^\gamma C_t^\sigma}{\xi}. \tag{2.46}$$

From the resource constraint, $Y_t = \frac{C_t}{g_t}$ or $C_t = g_t Y_t$,

$$w_t^f = \frac{\chi A_t^{-\gamma} Y_t^\gamma (g_t Y_t)^\sigma}{\xi}. \tag{2.47}$$

Denoting flexible price output as, $\hat{Y}_t^f$, and log-linearizing around the steady state,

$$\hat{w}_t^f = (\gamma + \sigma) \hat{Y}_t^f + \sigma \hat{g}_t - \gamma \hat{A}_t - \hat{\xi}_t. \tag{2.46}$$

and using (2.45) and 2.46 we obtain,

$$\hat{Y}_t^f = \frac{1}{\gamma + \sigma} \left[ (1 + \gamma) \hat{A}_t + \hat{\xi}_t - \sigma \hat{g}_t - \hat{R}_t^f \right] \tag{2.47}$$
2.A.2 The Euler equation (Derivation of $\hat{C}_t$)

$$
\xi_t C_t^{-\sigma} = \beta E_t \xi_{t+1} C_{t+1}^{-\sigma} \left( \frac{R_t P_t}{P_{t+1}} \right)
$$

$$
\xi(e^{\hat{\xi}_t}) C (e^{\hat{C}_t})^{-\sigma} = \beta E_t \left[ \xi \left( e^{\hat{\xi}_{t+1}} \right) C \left( e^{\hat{C}_{t+1}} \right)^{-\sigma} \right] \left( e^{\hat{R}_t} \right) - E_t \pi_{t+1}
$$

$$
\xi(C(1 + \hat{\xi}_t - \sigma \hat{C}_t)) = \beta C \xi R \left( 1 + E_t \hat{\xi}_{t+1} - \sigma E_t \hat{C}_{t+1} + \hat{R}_t - E_t \pi_{t+1} \right)
$$

$$
\hat{\xi}_t - \sigma \hat{C}_t = E_t \hat{\xi}_{t+1} - \sigma E_t \hat{C}_{t+1} + \hat{R}_t - E_t \pi_{t+1}
$$

$$
\hat{C}_t = E_t \hat{C}_{t+1} + \frac{1}{\sigma} \left( \hat{\xi}_t - E_t \hat{\xi}_{t+1} \right) - \frac{1}{\sigma} \left( \hat{R}_t + E_t \pi_{t+1} \right)
$$

$$
\hat{C}_t = E_t \hat{C}_{t+1} + \frac{1}{\sigma} \left( \hat{\xi}_t - E_t \hat{\xi}_{t+1} \right) - \frac{1}{\sigma} \left( \hat{R}_t + E_t \pi_{t+1} \right)
$$

2.A.3 Aggregate relative price ($\hat{q}_t$)

From the average price implied by the Calvo-type price stickiness,

$$
P_t^{1-\theta} = \omega P_{t-1}^{1-\theta} + (1 - \omega) \int_0^1 P_{t-1}^{*} (f)^{1-\theta} df
$$

where $P_{f,t}^{*}$ is the optimal price level for firms. Using $\frac{P_{t-1}^*}{P_t} = \frac{1}{\pi_t}$, we are able to define aggregate relative prices in terms of inflation and the probability of price adjustment

$$
1 = \omega \left( \frac{1}{\pi_t} \right)^{1-\theta} + (1 - \omega) \int_0^1 q_t(f)^{1-\theta} df
$$

Log-linearising we obtain,

$$
1 = \omega \left( e^{\hat{\pi}_t} \right)^{\theta-1} + (1 - \omega) \left( e^{\int_0^1 \hat{q}_t(f) df} \right)^{1-\theta}
$$

$$
1 = 1 + \omega (\theta - 1) \hat{\pi}_t + (1 - \omega) (1 - \theta) \int_0^1 \hat{q}_t(f) df
$$
\[
\omega \tilde{\pi}_t = (1 - \omega) \int_0^1 \hat{q}_t(f) df
\]
\[
\hat{q}_t = \int_0^1 \hat{q}_t(f) df = \frac{\omega}{1 - \omega} \tilde{\pi}_t.
\] (2.49)

2.A.4 Aggregate Consumption with Relative Price Effects
(Derivation of \(\hat{C}_{t+k}\))

We require to find \(\hat{C}_{t+k}\) within the dynamic optimisation problem of the firm. From eq (2.27),
\[
\hat{C}_{t+k} = \left( \int_0^1 \left( \frac{P_t(f)}{P_{t+k}} \right)^{-\theta} df \ C_{t+k} \right)^{(\theta-1)/\theta} \theta/(\theta-1)
\] (2.50)

Defining \(q_t(f) = \frac{P_t(f)}{P_t}\) and \(\pi_{t+k} = \frac{P_{t+k}}{P_t}\) as inflation between period \(t\) and \(t + k\) we can write,
\[
\hat{C}_{t+k}^{(\theta-1)/\theta} = \left( \int_0^1 \left( \frac{q_t(f)}{\sum_{l=1}^k \pi_{t+l}} \right)^{-\theta} df \ C_{t+k} \right)^{(\theta-1)/\theta} \theta/(\theta-1)
\]
\[
\hat{C}_{t+k}^{\theta/(\theta-1)} = C_{t+k}^{\theta/(\theta-1)} \left( \int_0^1 \left( q_t(f) \right)^{-\theta} df \right) ^{(\theta-1)/\theta} \left( \sum_{l=1}^k \pi_{t+l} \right)^{\theta/(\theta-1)}
\]

Log-linearising around a steady state where \(q(f) = q = \pi = 1\),
\[
C^{(\theta-1)/\theta} \left( e^{\hat{C}_{t+k}} \right)^{\theta-1} = C^{(\theta-1)/\theta} \left( e^{\hat{C}_{t+k}} \right)^{\theta-1} e^{\left( \int_0^1 \hat{q}_t(f) df \right) - \theta(\theta-1)/\theta} \left( \sum_{l=1}^k \hat{\pi}_{t+l} \right)^{\theta/(\theta-1)}
\]
\[
\left( 1 + \frac{\theta - 1}{\theta} \hat{\pi}_{t+k} \right) = \left( 1 + \frac{\theta - 1}{\theta} \hat{\pi}_{t+k} \right) - \frac{\theta (\theta - 1)}{\theta} \int_0^1 \hat{q}_t(f) df + \frac{\theta (\theta - 1)}{\theta} \sum_{l=1}^k \hat{\pi}_{t+l}
\]
\[
\hat{C}_{t+k} = \hat{C}_{t+k} - \theta \int_0^1 \hat{q}_t(f) df + \theta \sum_{l=1}^k \hat{\pi}_{t+l},
\]
\[
\hat{C}_{t+k} = \hat{C}_{t+k} - \theta \left( \int_0^1 \hat{q}_t(f) df - \sum_{l=1}^k \hat{\pi}_{t+l} \right)
\]
and using 2.49,

\[ \hat{C}_{t+k} = \hat{C}_{t+k} - \theta \left( \frac{\omega}{1 - \omega} \tilde{\pi}_t - \sum_{l=1}^{k} \hat{\pi}_{t+l} \right) \]

or

\[ \hat{C}_{t+k} = \hat{C}_{t+k} + \Delta_{t+k} \]  

(2.51)

where

\[ \Delta_{t+k} = -\theta \left( \frac{\omega}{1 - \omega} \tilde{\pi}_t - \sum_{l=1}^{k} \hat{\pi}_{t+l} \right) \]

2.A.5 The NKPC with relative price effects (Derivation of eq. 2.34)

From equations, 2.28, 2.29 and 2.30 in the text we have

\[ \max_{P_t(f)} E_t \sum_{k=0}^{\infty} \omega^k \hat{\lambda}_{t+k} \left( 1 + \tau \right) \left( \frac{P_{t+k}(f)}{P_{t+k}} \right)^{1-\theta} - mc_{t+k} \hat{C}_{t+k} \left( \frac{P_{t+k}(f)}{P_{t+k}} \right)^{-\theta} \hat{C}_{t+k} \]

(2.52)

where \( \hat{\lambda}_{t+k} = \beta^k \left( \frac{\hat{C}_{t+k}}{\hat{C}_t} \right) \) and

\[ mc_{t+k} = \frac{R_{t+k}w_{t+k}[\hat{C}_{t+k}]}{\hat{A}_{t+k}} \]  

(2.53)

\[ w_{t+k} = \frac{\chi A_{t+k}^{-\gamma} \hat{C}_{t+k}^{(\gamma+\sigma)} g_t^{-\gamma}}{\xi_t} \]  

(2.54)

Substituting 2.53 into 2.52,

\[ \max_{P_t(f)} E_t \sum_{k=0}^{\infty} \omega^k \hat{\lambda}_{t+k} \left( 1 + \tau \right) \left( \frac{P_{t+k}(f)}{P_{t+k}} \right)^{1-\theta} - R_{t+k}w_{t+k}[\hat{C}_{t+k}] \left( \frac{P_{t+k}(f)}{P_{t+k}} \right)^{-\theta} \hat{C}_{t+k} \]
and assuming that each price setter is not too large so as to perceive an effect on aggregate consumption, \( \partial \tilde{C}_{t+k} / \partial P_t(f) = 0 \), the first order condition is,

\[
E_t \sum_{k=0}^{\infty} \omega^k \tilde{\Lambda}_{t,t+k} \left[ \frac{(1 + \tau)(1 - \theta)}{\theta} \left( \frac{P_t(f)}{P_{t+k}} \right) + \frac{R_{t+k} w_{t+k}[\tilde{C}_{t+k}]}{A_{t+k}} \right] = 0 \quad (2.55)
\]

Using the conventional lump tax that eliminates monopolistic distortions, \( \left( \frac{\theta - 1}{\theta} \right)(1 + \tau) = 1 \), and employing the definition \( q_t(f) = \frac{P_t(f)}{P_t} \) and \( \pi_{t+k} = \frac{P_{t+k}}{P_t} \) we can write equation (2.55), as,

\[
E_t \sum_{k=0}^{\infty} \omega^k \tilde{\Lambda}_{t,t+k} \left[ \frac{q_t(f)}{\sum_{l=1}^{k} \pi_{t+l}} - \frac{R_{t+k} w_{t+k}[\tilde{C}_{t+k}]}{A_{t+k}} \right] = 0. \quad (2.56)
\]

Log-linearization of eq. (2.56):

Using our definition of \( \tilde{\Lambda}_{t,t+k} = \beta^k \left( \frac{\hat{C}_{t+k}}{C_t} \right) \) we can rewrite (2.56) as,

\[
E_t \sum_{k=0}^{\infty} \omega^k \beta^k \left( \frac{\hat{C}_{t+k}}{\hat{C}_t} \right) \left[ \frac{q_t(f)}{\sum_{l=1}^{k} \pi_{t+l}} - \frac{R_{t+k} w_{t+k}[\tilde{C}_{t+k}]}{A_{t+k}} \right] = 0
\]

can be written in terms of deviations around its steady state

\[
E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \frac{\tilde{q}_t(f)}{\sum_{l=1}^{k} \hat{\pi}_{t+l}} + \left( \hat{\tilde{C}}_{t+k} - \hat{\tilde{C}}_t \right) - \hat{\tilde{R}}_{t+k} + \hat{\tilde{A}}_{t+k} - \hat{\tilde{w}}_{t+k} - \left( \hat{\tilde{C}}_{t+k} - \hat{\tilde{C}}_t \right) \right] = 0
\]

\[
E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \tilde{q}_t(f) - \sum_{l=1}^{k} \hat{\pi}_{t+l} + \left( \tilde{\hat{C}}_{t+k} - \hat{\tilde{C}}_t \right) - \tilde{\hat{R}}_{t+k} + \tilde{\hat{A}}_{t+k} - \tilde{\hat{w}}_{t+k} + \left( \tilde{\hat{C}}_{t+k} - \hat{\tilde{C}}_t \right) \right] = 0 \quad (2.57)
\]

The real wage,
From equations (2.4),

\[
\frac{W_{t+k}}{P_{t+k}} = w_{t+k} = \frac{\chi N_{t+k}^\gamma}{\xi_{t+k} \tilde{C}_{t+k}^{-\sigma}}
\]

Using the production function, (2.10) the real wage is,

\[
w_{t+k} = \frac{\chi A_{t+k}^{-\gamma} \tilde{Y}_{t+k}^\gamma \tilde{C}_{t+k}^\sigma}{\xi_{t+k}}
\]

Using the production function and the resource constraint, \( \tilde{Y}_t = \frac{\tilde{C}_t}{g_t} \),

\[
w_{t+k} = \frac{\chi A_{t+k}^{-\gamma} \tilde{Y}_{t+k}^\gamma \tilde{C}_{t+k}^\sigma}{\xi_{t+k}}
\] (2.58)

Log-linearization of (2.58):

This can be written in terms of deviations around a steady state as

\[ w(e^{\tilde{\omega}_{t+k}}) = \chi A(e^{\hat{A}_{t+k}})^{-\gamma} \xi(e^{\hat{\delta}_{t+k}})^{-1} g(e^{\hat{g}_{t+k}})^{-\gamma} \tilde{C}^{(\gamma+\sigma)}(e^{\hat{\tilde{C}}_{t+k}})^{(\gamma+\sigma)} \]

\[ \hat{w}_{t+k} = (\sigma + \gamma) \tilde{C}_{t+k} - \gamma \hat{g}_{t+k} - \gamma \hat{A}_{t+k} - \hat{\xi}_{t+k}, \] (2.59)

or using (2.51) real wage deviations become

\[ \hat{w}_{t+k} = (\gamma + \sigma) \left( \tilde{C}_{t+k} + \Delta_{t+k} \right) - \gamma \hat{g}_{t+k} - \gamma \hat{A}_{t+k} - \hat{\xi}_{t+k}. \]

where, \( \Delta_{t+k} = -\theta \left( \hat{q}_t - \sum_{l=1}^k \hat{\pi}_{t+l} \right) \).

Flexible-price real wage

From the resource constraint \( \hat{C}_{t+k} = \hat{Y}_{t+k} + \hat{g}_{t+k} \) we can rewrite equation (2.51) as,

\[ \hat{w}_{t+k} = (\sigma + \gamma) \left( \hat{Y}_{t+k} + \hat{g}_{t+k} \right) - \gamma \hat{g}_{t+k} - \gamma \hat{A}_{t+k} - \hat{\xi}_{t+k} \]

\[ = (\sigma + \gamma) \hat{Y}_{t+k} + \sigma \hat{g}_{t+k} - \gamma \hat{A}_{t+k} - \hat{\xi}_{t+k} \]
Given flexible price, $k = 0$ and $\hat{Y}_t^f = \hat{Y}_t^f$ thus,

$$\hat{w}_t = (\sigma + \gamma) \hat{Y}_t^f + \sigma \hat{g}_t - \gamma \hat{A}_t - \hat{\xi}_t$$

which is the same as 2.46.

Derivation of the NKPC expressed in terms of ‘adjusted’ output gap $\hat{C}_{t+k}$

Substitute 2.59 into (2.57) we obtain

$$E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{q}_t(f) - \sum_{l=1}^{k} \hat{\pi}_{t+l} - \hat{R}_{t+k} - \hat{w}_{t+k}[\hat{C}_{t+k}] + \hat{A}_{t+k} \right] = 0. $$

$$E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{q}_t(f) - \sum_{l=1}^{k} \hat{\pi}_{t+l} - \hat{R}_{t+k} - (\sigma + \gamma) \hat{C}_{t+k} - \gamma \hat{g}_{t+k} + (1 + \gamma) \hat{A}_{t+k} + \hat{\xi}_{t+k} \right] = 0.$$  \hfill (2.60)

Aggregating overall all firms setting prices at time $t$

$$\left( \frac{1}{1 - \omega \beta} \right) \hat{q}_t = E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \sum_{l=1}^{k} \hat{\pi}_{t+l} + \hat{R}_{t+k} + \right. $$

$$(\gamma + \sigma) \hat{C}_{t+k} - (1 + \gamma) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right] = 0$$

Using $E_t \sum_{k=0}^{\infty} (\omega \beta)^k \sum_{l=1}^{k} \hat{\pi}_{t+l} = \sum_{k=1}^{\infty} \frac{(\omega \beta)^k}{1 - \omega \beta} E_t \hat{\pi}_{t+k}$, and $\hat{q}_t = \frac{\omega}{(1-\omega)} \hat{\pi}_t$ we can write,

$$\left( \frac{1}{1 - \omega \beta} \right) \frac{\omega}{1 - \omega} \hat{\pi}_t - \sum_{k=1}^{\infty} \frac{(\omega \beta)^k}{1 - \omega \beta} E_t \hat{\pi}_{t+l}$$

$$-E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{R}_{t+k} + (\gamma + \sigma) \hat{C}_{t+k} - (1 + \gamma) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right] = 0$$

$$\hat{\pi}_t = \frac{(1-\omega)}{\omega} \sum_{k=1}^{\infty} (\omega \beta)^k E_t \hat{\pi}_{t+k}$$

$$+ \frac{(1-\omega)(1-\omega \beta)}{\omega} E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{R}_{t+k} + (\gamma + \sigma) \hat{C}_{t+k} - (\gamma + 1) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right]$$

58
\[
\hat{\pi}_t = \frac{(1 - \omega) \omega \beta}{\omega} E_t \hat{\pi}_{t+1} + \frac{(1 - \omega)(1 - \omega \beta)}{\omega} \left[ \hat{R}_t + (\gamma + \sigma) \hat{C}_t - (\gamma + 1) \hat{A}_t - \hat{\xi}_t - \gamma \hat{g}_t \right] + \frac{(1 - \omega)}{\omega} \sum_{k=2}^{\infty} (\omega \beta)^k E_t \hat{\pi}_{t+k} + \frac{(1 - \omega)(1 - \omega \beta)}{\omega} E_t \sum_{k=1}^{\infty} (\omega \beta)^k \left[ \hat{R}_{t+k} + (\gamma + \sigma) \hat{C}_{t+k} - (\gamma + 1) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right]
\]

and using into the above equation the following forward equation,

\[
\omega \beta E_t \hat{\pi}_{t+1} = \frac{(1 - \omega)}{\omega} \sum_{k=2}^{\infty} (\omega \beta)^k E_t \hat{\pi}_{t+k} + \frac{(1 - \omega)(1 - \omega \beta)}{\omega} E_t \sum_{k=1}^{\infty} (\omega \beta)^k \left[ \hat{R}_{t+k} + (\gamma + \sigma) \hat{C}_{t+k} - (\gamma + 1) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right]
\]

we obtain,

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \left[ \hat{R}_t + (\gamma + \sigma) \hat{C}_t - \gamma \hat{g}_t - (\gamma + 1) \hat{A}_t - \hat{\xi}_t \right] \quad (2.61)
\]

where, \(\kappa = \frac{(1 - \omega)(1 - \omega \beta)}{\omega}\). or using \( \hat{\xi}_t = \hat{Y}_t + \hat{\gamma}_t \)

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \left[ \hat{R}_t + (\gamma + \sigma) \hat{Y}_t + \sigma \hat{g}_{t+k} - (\gamma + 1) \hat{A}_t - \hat{\xi}_t \right].
\]

Using the definition of flexible output (2.47) we can define the NKPC above in terms of the output gap that account for relative price distortions, \( \left( \hat{Y}_t - Y_t^f \right) \),

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \left[ (\gamma + \sigma) \left( \hat{Y}_t - Y_t^f \right) + \hat{R}_t - R_t^f \right]. \quad (2.62)
\]

where, the appropriate resource constraint is, \( \hat{Y}_t + \hat{g}_t = \hat{C}_t \). From this it is easy to see that setting \( \Delta_{t+k} = 0 \), equation (2.62) is reduced to (2.24), the conventional NKPC.
2.A.6 Aggregate Consumption at time $t$, $\hat{C}_t, = \hat{C}_t + \hat{\Delta}_t$, (Derivation of eqs 2.35 and 2.36)

Derivation of the ‘adjusted’ NKPC, expressed in terms of the conventional Euler ($\hat{C}_{t+k}$)

Substitute (2.59) and (2.51) into (2.57),

$$E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{q}_t(f) - \sum_{l=1}^{k} \hat{\pi}_{t+l} - \hat{R}_{t+k} - \hat{\omega}_{t+k} + \hat{A}_{t+k} \right] = 0.$$

$$E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{q}_t(f) - \sum_{l=1}^{k} \hat{\pi}_{t+l} - \hat{R}_{t+k} - (\gamma + \sigma) \hat{C}_{t+k} \right.$$

$$\left. - \left( \gamma + \sigma \right) \left( \hat{C}_{t+k} + \hat{\Delta}_{t+k} \right) - \gamma \hat{g}_{t+k} - \gamma \hat{A}_{t+k} - \hat{\xi}_{t+k} + \hat{\Delta}_{t+k} \right] = 0.$$

Using, $\hat{\Delta}_{t+k} = -\theta \left( \frac{\omega \pi_t}{1-\omega} - \sum_{l=1}^{k} \hat{\pi}_{t+l} \right)$, we have,

$$E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \left( \hat{q}_t(f) - \sum_{l=1}^{k} \hat{\pi}_{t+l} \right) - \hat{R}_{t+k} - (\gamma + \sigma) \hat{C}_{t+k} \right.$$

$$\left. + (\gamma + \sigma) \theta \left( \hat{q}_t - \sum_{l=1}^{k} \hat{\pi}_{t+l} \right) + (1 + \gamma) \hat{A}_{t+k} + \hat{\xi}_{t+k} + \gamma \hat{g}_{t+k} \right] = 0$$

and aggregating overall all firms,

$$\hat{q}_t \left( \frac{1 + (\gamma + \sigma) \theta)}{1 - \omega \beta} \right) = \left( 1 + (\gamma + \sigma) \theta \right) E_t \sum_{k=0}^{\infty} (\omega \beta)^k \sum_{l=1}^{k} \hat{\pi}_{t+l}$$

$$+ E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{R}_{t+k} + (\gamma + \sigma) \hat{C}_{t+k} - (1 + \gamma) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right]$$

or,

$$\hat{q}_t = (1 - \omega \beta) E_t \sum_{k=0}^{\infty} (\omega \beta)^k \sum_{l=1}^{k} \hat{\pi}_{t+l}$$

$$+ \frac{(1 - \omega \beta)}{(1 + (\gamma + \sigma) \theta)} E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{R}_{t+k} + (\gamma + \sigma) \hat{C}_{t+k} - (1 + \gamma) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right]$$
and using $E_t \sum_{k=0}^{\infty} (\omega \beta)^k \sum_{t=1}^{k} \hat{\pi}_{t+t} = E_t \sum_{k=1}^{\infty} \frac{(\omega \beta)^k}{1-\omega \beta} \hat{\pi}_{t+k}$, and $\hat{q}_t = \frac{\omega}{(1-\omega)} \hat{\pi}_t$ we can write,

$$\hat{\pi}_t = \frac{(1-\omega)}{\omega} \sum_{k=1}^{\infty} (\omega \beta)^k E_t \hat{\pi}_{t+k}$$  \hspace{1cm} (2.63)

$$+ \frac{(1-\omega)(1-\omega \beta)}{\omega(1+(\gamma+\sigma)\theta)} E_t \sum_{k=0}^{\infty} (\omega \beta)^k \left[ \hat{R}_{t+k} + (\gamma+\sigma) \hat{C}_{t+k} - (1+\gamma) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right]$$

or,

$$\hat{\pi}_t = \frac{(1-\omega)}{\omega} (\omega \beta) E_t \hat{\pi}_{t+1} + \frac{(1-\omega)}{\omega} \sum_{k=2}^{\infty} (\omega \beta)^k E_t \hat{\pi}_{t+k}$$  \hspace{1cm} (2.64)

$$+ \frac{(1-\omega)(1-\omega \beta)}{\omega(1+(\gamma+\sigma)\theta)} E_t \sum_{k=1}^{\infty} (\omega \beta)^k \left[ \hat{R}_{t+k} + (\gamma+\sigma) \hat{C}_{t+k} - (1+\gamma) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right]$$

From 2.63 we can write

$$ (\omega \beta) \hat{\pi}_{t+1} = \frac{(1-\omega)}{\omega} \sum_{k=2}^{\infty} (\omega \beta)^k E_t \hat{\pi}_{t+k}$$  \hspace{1cm} (2.65)

$$+ \frac{(1-\omega)(1-\omega \beta)}{\omega(1+(\gamma+\sigma)\theta)} E_t \sum_{k=1}^{\infty} (\omega \beta)^k \left[ \hat{R}_{t+k} + (\gamma+\sigma) \hat{C}_{t+k} - (1+\gamma) \hat{A}_{t+k} - \hat{\xi}_{t+k} - \gamma \hat{g}_{t+k} \right]$$

and substituting 2.65 into 2.64 we obtain,

$$\hat{\pi}_t = \frac{(1-\omega)}{\omega} (\omega \beta) E_t \hat{\pi}_{t+1} + (\omega \beta) E_t \hat{\pi}_{t+1}$$

$$+ \frac{(1-\omega)(1-\omega \beta)}{\omega(1+(\gamma+\sigma)\theta)} \left[ \hat{R}_t + (\gamma+\sigma) \hat{C}_t - (1+\gamma) \hat{A}_t - \hat{\xi}_t - \gamma \hat{g}_t \right]$$

or

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\kappa}{(1+(\gamma+\sigma)\theta)} \left[ \hat{R}_t + (\gamma+\sigma) \hat{C}_t - (\gamma+1) \hat{A}_t - \gamma \hat{g}_{t+k} - \hat{\xi}_t \right]$$  \hspace{1cm} (2.66)
where $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$. Equation 2.66 is the relative price NKPC expressed in terms of the conventional Euler consumption, $\hat{C}_t$.

From equations (2.61) and (2.66) we can solve for $\hat{C}_t$ and $\hat{C}_t$, respectively:

$$\hat{C}_t = \frac{1}{\kappa (\gamma + \sigma)} [\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}] - \frac{1}{(\gamma + \sigma)} \left( \hat{R}_t - (\gamma + 1) \hat{A}_t - \gamma \hat{g}_t - \hat{\xi}_t \right)$$ (2.67) 

and

$$\hat{\bar{C}}_t = \frac{(1 + (\gamma + \sigma)\theta)}{\kappa (\gamma + \sigma)} [\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}] - \frac{1}{(\gamma + \sigma)} \left( \hat{R}_t - (\gamma + 1) \hat{A}_t - \gamma \hat{g}_t - \hat{\xi}_t \right)$$ (2.68)

Finding the difference, $\hat{C}_t - \hat{C}_t$ we obtain,

$$\hat{C}_t = \hat{C}_t - \frac{\theta}{\kappa} [\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}]$$. (2.69)

### 2.A.7 The output gap, $\hat{x}_t = \hat{Y}_t - \hat{Y}_t^f$, (Derivation of 2.38)

The full output gap is:

$$\hat{x}_t = \hat{Y}_t - \hat{Y}_t^f$$ (2.70)

Use $\hat{C}_t = \hat{Y}_t + \hat{g}_t$, and write equation (2.35) in terms of aggregate output,

$$\hat{Y}_t = \hat{Y}_t + \hat{\Delta}_t$$ (2.71)

Substituting, $\hat{x}_t = \hat{Y}_t - \hat{Y}_t^f$ and $\hat{x}_t = \hat{Y}_t - \hat{Y}_t^f$, into (2.71) we obtain,

$$\hat{x}_t = \hat{x}_t + \hat{\Delta}_t$$ (2.72)

Derivation of $\hat{x}_t = \hat{Y}_t - \hat{Y}_t^f$
We know \( \hat{Y}_t = \hat{C}_t - \hat{g}_t \) hence from 2.48 we can write

\[
\hat{x}_t = \hat{C}_t - \hat{g}_t - \hat{Y}_t
\]

\[
\hat{g}_t + \hat{x}_t + \hat{Y}_f = E_t \hat{x}_{t+1} + \hat{Y}_{t+1} + E_t \hat{g}_{t+1} - \frac{1}{\sigma} \left[ \hat{R}_t - E_t \hat{\pi}_{t+1} + \left( \hat{\xi}_t - E_t \hat{\xi}_{t+1} \right) \right]
\]

Substituting in (2.47) and rearranging

\[
\hat{g}_t + \hat{x}_t + \frac{1}{(\sigma + \gamma)} \left[ (1 + \gamma) \hat{A}_t - \sigma \hat{g}_t + \hat{\xi}_t - \hat{R}_t \right] = E_t \hat{x}_{t+1} + E_t \hat{g}_{t+1} + \frac{1}{(\sigma + \gamma)} \left[ (1 + \gamma) E_t \hat{\xi}_{t+1} - E_t \hat{\xi}_{t+1} \right]
\]

\[
\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} \left[ \hat{R}_t - E_t \hat{\pi}_{t+1} \right] + \frac{1}{\gamma + \sigma} \left[ (1 + \gamma) E_t \hat{A}_{t+1} + \hat{\xi}_t - \hat{\xi}_{t+1} \right] - \frac{1}{\gamma + \sigma} \left[ (1 + \gamma) \hat{A}_t + \hat{g}_{t+1} + \hat{\xi}_t - \hat{R}_t \right] - \frac{1}{\sigma} \left[ E_t \hat{\xi}_{t+1} - \hat{\xi}_t \right]
\]

And using, \( \hat{R}_t' = \hat{R}_{t+1} = 0 \),

\[
\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} \left[ \hat{R}_t - E_t \hat{\pi}_{t+1} \right] + \frac{1 + \gamma}{\gamma + \sigma} \left[ E_t \hat{A}_{t+1} - \hat{A}_t \right] + \frac{\gamma}{\gamma + \sigma} \left[ E_t \hat{g}_{t+1} - \hat{g}_t \right] - \frac{\gamma}{\sigma (\gamma + \sigma)} \left[ E_t \hat{\xi}_{t+1} - \hat{\xi}_t \right]
\]

Which can be simplified to give

\[
\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} \left[ \hat{R}_t - E_t \hat{\pi}_{t+1} \right] + \mu_t, \quad (2.73)
\]
where,

$$
\mu_t = \left(\frac{1+\gamma}{\gamma+\sigma}\right) \left[ E_t \hat{A}_{t+1} - \hat{A}_t \right] + \left(\frac{\gamma}{\gamma+\sigma}\right) \left[ E_t \hat{g}_{t+1} - \hat{g}_t \right] - \frac{\gamma}{\sigma(\gamma+\sigma)} \left[ E_t \hat{\xi}_{t+1} - \hat{\xi}_t \right]
$$

### 2.A.8 Determinacy

Equation (2.73) can be rewritten as,

$$
\hat{x}_t - \frac{1}{\sigma} E_t \pi_{t+1} = E_t \hat{x}_{t+1} - \frac{1}{\sigma} \hat{R}_t 
$$

$$
+ \left(\frac{1}{\gamma+\sigma}\right) \left[ (1+\gamma) \left( E_t \hat{A}_{t+1} - \hat{A}_t \right) + \gamma (E_t \hat{g}_{t+1} - \hat{g}_t) - \frac{\gamma}{\sigma} \left( E_t \hat{\xi}_{t+1} - \hat{\xi}_t \right) \right]
$$

Combining this with our output gap, equation (2.72), we can write:

$$
E_t \hat{\xi}_{t+1} + \frac{\theta}{\kappa} \hat{x}_{t+1} - \frac{\theta}{\kappa} \beta E_t \hat{\pi}_{t+2} + \frac{1}{\sigma} E_t \hat{\pi}_{t+1}
$$

$$
+ \left(\frac{1+\gamma}{\gamma+\sigma}\right) E_t \hat{A}_{t+1} - \left(\frac{1}{\gamma+\sigma}\right) \gamma E_t \hat{\xi}_{t+1} + \left(\frac{\gamma}{\gamma+\sigma}\right) E_t \hat{g}_{t+1}
$$

$$
= \frac{1}{\sigma} \hat{R}_t + \hat{x}_t + \frac{\theta}{\kappa} \hat{\pi}_t - \frac{\theta}{\kappa} \beta E_t \hat{\pi}_{t+1}
$$

$$
+ \left(\frac{1}{\gamma+\sigma}\right) \hat{A}_t - \left(\frac{1}{\gamma+\sigma}\right) \gamma \hat{\xi}_t + \left(\frac{\gamma}{\gamma+\sigma}\right) E_t \hat{g}_t
$$

Recall our Phillips curve (2.34) yields

$$
\beta E_t \hat{\pi}_{t+1} = -\kappa(\sigma + \gamma) \hat{x}_t + \hat{\pi}_t - \kappa \hat{R}_t
$$

Iterating forward one period we have,

$$
\beta E_t \hat{\pi}_{t+2} = -\kappa(\sigma + \gamma) \hat{x}_{t+1} + \hat{\pi}_{t+1} - \kappa \hat{R}_{t+1}
$$
substituting (2.77) into (2.75) we are able to define our dynamic IS equation over two periods

\[
E_t \hat{x}_{t+1} + \frac{\theta}{\kappa} \hat{\pi}_{t+1} - \frac{\theta}{\kappa} \left[ -\kappa(\sigma + \gamma) \hat{x}_{t+1} + \hat{\pi}_{t+1} - \kappa \hat{R}_{t+1} \right] + \frac{1}{\sigma} E_t \hat{\pi}_{t+1} \\
+ \left( \frac{1 + \gamma}{\gamma + \sigma} \right) E_t \hat{A}_{t+1} - \left( \frac{1}{\gamma + \sigma} \right) \frac{\gamma}{\sigma} E_t \hat{\xi}_{t+1} + \left( \frac{\gamma}{\gamma + \sigma} \right) E_t \hat{g}_{t+1} \\
= \frac{1}{\sigma} \hat{R}_t + \hat{x}_t + \frac{\theta}{\kappa} \hat{\pi}_t - \frac{\theta}{\kappa} \beta E_t \hat{\pi}_{t+1} + \left( \frac{1}{\gamma + \sigma} \right) \hat{A}_t - \left( \frac{1}{\gamma + \sigma} \right) \frac{\gamma}{\sigma} \hat{\xi}_t + \left( \frac{\gamma}{\gamma + \sigma} \right) E_t \hat{g}_t \\
\]

Equations (2.76) and (2.78) define our aggregate supply and aggregate demand respectively.

In state space form

We can define our system as,

\[
A_0 E_t X_{t+1} = A_1 X_t + B_0 \xi_{t+1} \\
\]

where \( X_t = \begin{bmatrix} w_t \\ y_t \end{bmatrix} \) with \( w_t = (\hat{A}_t, \hat{\xi}_t, \hat{\pi}_t)' \) and \( y_t = (\hat{x}_t, \hat{\pi}_t)' \).
We assume that the shocks follow an independent AR(1) processes given by

\[
\begin{pmatrix}
\hat{A}_{t+1} \\
\hat{\xi}_{t+1} \\
\hat{g}_{t+1} \\
\hat{v}_{t+1}
\end{pmatrix}
= \begin{pmatrix}
\rho_A & 0 & 0 & 0 \\
0 & \rho_\xi & 0 & 0 \\
0 & 0 & \rho_g & 0 \\
0 & 0 & 0 & \rho_v
\end{pmatrix}
\begin{pmatrix}
\hat{A}_t \\
\hat{\xi}_t \\
\hat{g}_t \\
\hat{v}_t
\end{pmatrix}
+ \begin{pmatrix}
\nu_A & 0 & 0 & 0 \\
0 & \nu_\xi & 0 & 0 \\
0 & 0 & \nu_g & 0 \\
0 & 0 & 0 & \nu_v
\end{pmatrix}
\begin{pmatrix}
\varepsilon^A_{t+1} \\
\varepsilon^\xi_{t+1} \\
\varepsilon^g_{t+1} \\
\varepsilon^v_{t+1}
\end{pmatrix}
\]

Combining (2.34) (2.38) (2.39) and (2.40) we can write our DSGE model in generalized state space form as,
\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 \\
\end{pmatrix}
\begin{pmatrix}
(\frac{1+\gamma}{1+\sigma}) & -\left(\frac{1}{1+\sigma}\right) & \frac{\gamma}{\sigma} & 0 & -\theta & 1 + \theta(\sigma + \gamma) & \frac{\sigma}{\sigma + \gamma} + \frac{1}{\sigma} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\dot{A}_{t+1} \\
\dot{\xi}_{t+1} \\
\dot{g}_{t+1} \\
E_t \bar{v}_{t+1} \\
E_t R_{t+1} \\
E_t \bar{x}_{t+1} \\
E_t \bar{\pi}_{t+1} \\
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\rho \dot{A} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \rho \dot{\xi} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho \dot{g} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho \dot{v} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho_R & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\rho \ddot{A} \\
\rho \ddot{\xi} \\
\rho \ddot{g} \\
\rho \ddot{v} \\
\rho_R \ddot{R} \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
v \dot{A} & 0 & 0 & 0 \\
0 & v \dot{\xi} & 0 & 0 \\
0 & 0 & v \dot{g} & 0 \\
0 & 0 & v \dot{v} & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{t+1}^A \\
\varepsilon_{t+1}^\xi \\
\varepsilon_{t+1}^g \\
\varepsilon_{t+1}^v \\
\end{pmatrix}
\]

\[\begin{pmatrix}
\gamma \\
\dot{\gamma} \\
\dot{\theta} \\
\dot{\sigma} \\
\end{pmatrix}
\]
To establish our regions of determinacy we first rewrite (2.79) in terms of the forward and backward looking variables

\[ E_t y_{t+1} = \Omega_1 y_t + \ell w_t \]  

(2.81)

where \( E_{t+1} w_{t+1} = \Omega_2 w_{t+1} + B_0 \theta_{t+1} \). Following Blanchard and Kahn (1980), equation (2.81) has a unique stable solution for the output gap, inflation and the interest rate if and only if the number of eigenvalues outside the unit circle in \( \Omega_1 \) is equal to the number of forward looking variables, in our case two. Thus, by examining all possible combinations of \( \phi_x \) and \( \phi_\pi \) and the influence they have on the size of the eigenvalues, we can map the influence that different policy coefficients have on determinacy.

Setting \( \rho_R \) to zero. From the matrices \( A_o \) and \( A_1 \) we can define the respective, forward looking control elements

\[
H_o = \begin{pmatrix}
1 + \theta(\sigma + \gamma + \phi_x) & \frac{\theta}{\nu} \beta + \frac{1}{\sigma} + \theta \phi_\pi \\
0 & \beta
\end{pmatrix}
\]

and

\[
H_1 = \begin{pmatrix}
\frac{1}{\sigma} \phi_x + 1 & \frac{1}{\sigma} \phi_\pi + \frac{\theta}{\nu} \\
-\kappa(\sigma + \gamma + \phi_x) & 1 - \kappa \phi_\pi
\end{pmatrix}
\]

So that in Blanchard and Khan (1980) conditions are satisfied in our system so long as \( \det \Omega_1 - tr \Omega_1 > -1 \) and \( \det \Omega_1 + tr \Omega_1 > -1 \).

The determinate and trace of \( \Omega_1 \) is given as

\[
\det \Omega_1 = \frac{\phi_x + \kappa \gamma \phi_\pi + \theta \sigma \phi_x + \sigma (1 + \gamma \theta + \sigma \theta)}{\beta \sigma (1 + \phi_x \theta + \gamma \theta + \sigma \theta)}
\]
and

$$tr \Omega_1 = \frac{\phi_x (\beta + \kappa + \sigma \theta + \beta \sigma \theta) + \sigma (1 + \beta + \kappa - \kappa \phi_x + \sigma \theta + \beta \sigma \theta) + \gamma (\kappa + (1 + \beta) \theta \sigma)}{\beta \sigma (1 + \theta (\phi_x + \sigma + \gamma))}$$

Thus when $\rho = 0$, we have the following necessary and sufficient conditions for determinacy

$$\phi_x + \frac{(1 - \beta - \kappa)}{(\sigma + \gamma)}\frac{\kappa \phi_x}{\kappa} > 1$$

and

$$\phi_x (1 + \beta + \kappa + 2\theta \sigma) + \kappa (\gamma + \sigma) + \phi_x (\kappa (\gamma - \sigma)) + 2(1 + \beta) (\sigma + \gamma \theta \sigma + \theta \sigma^2) > 0$$
Chapter 3

Optimal Monetary Policy with a Cost Channel and Relative Price Distortions in Aggregate Consumption

3.1 Introduction

Since the seminal work of Kydland and Prescott (1977) and Barro and Gordon (1983) monetary economists have recognised the potential welfare gains of moving the conduct of monetary policy from a discretionary regime to a policy plan, namely optimal commitment. Furthermore, given the prominence of the New Keynesian framework for monetary analysis, optimal policy has more recently been discussed in this microfounded setting\(^1\).

This Chapter will analyse the impact that the additional relative price distortions from aggregate consumption, introduced in the previous chapter, have on the welfare of households and the conduct of optimal monetary policy. Additionally, we consider these issues under various levels on goods market competition by adjusting the price elasticity of demand accordingly. In this setting we evaluate the performance of policy according to the Taylor rule considered in chapter 2 as well as optimal policy under commitment and discretion.

\(^1\)See for example Clarida, Gali and Gertler (1999), Woodford (2003).
When considering the appropriate objectives of the central bank we refer to a policy objective function that can be interpreted as an approximation of the utility of the representative household. This follows of the seminal work of Rotemberg and Woodford (1999)\(^2\) where a welfare-based-criterion relies upon a second order approximation to the utility losses experienced by the representative consumer as a consequence of deviating away from the efficient allocation. Specifically, for a given movement in output away from its efficient level, the marginal rate of substitution and the marginal product of labour determine the economies aggregate inefficiency and the extent to which this reduces the welfare of agents. Inflation is costly in these models due to the dispersion of prices of the differentiated goods. Because of this dispersion, households buy more of a relatively cheaper good and less of the relatively more expensive goods. As mentioned in the previous chapter, a consequence of diminishing marginal utility is that the increase in utility from consumption of a cheaper good is less than the loss in utility from consumption of a more expensive good. Similarly the dispersion on the production side is costly. Since increasing the price elasticity of demand will exaggerate these movements, a more competitive economy has a greater welfare loss for a given inflation deviation.

In this chapter we retain all of the above effects and, as we will see, welfare losses are shown to increase monotonically with respect to the price elasticity of substitution among consumption goods, (see Ravenna and Walsh 2006, Galf, 2008, Canzoneri, Cumby and Diba 2007b); this comes from the standard second-order effect shown in the literature and outlined above. However, the variances of both inflation and the output gap that enter in our case are lower and decreasing with the price elasticity of demand. The intuition is that when product demands are more elastic, the expected gains or losses realised by the firm increase for a given price change. Accordingly, when firms have a price resetting opportunity they make less of a price adjustment to achieve the optimal price in their profit maximisation, since

\(^2\)See Woodford (2003) chapter 6 for a detailed discussion of welfare-based evaluations of policy rules
a smaller price adjustment is required to shift demands to the optimal allocation\(^3\). Additionally, the output gap variance falls in a more competitive product market since households, by switching their products, are able to move closer towards their optimal allocation of aggregate consumption given by the natural rate.

We first assume that the central bank follows a typical Taylor rule and compare our model with the benchmark case of Ravenna and Walsh (2006). Welfare losses following economic shocks are shown to ultimately decrease as the economy becomes more competitive. For the reason that, in our model the positive welfare effect that increasing competition has on reducing the variance of inflation and the output gap dominates the adverse effects of increased asymmetric product demands in the welfare function. This is in contrast to the benchmark case where the variance of inflation and the output gap are unaffected by the price elasticity of demand so that welfare losses will always increase as goods become closer substitutes.

We continue by considering optimal monetary policy when the policy maker faces a trade off between inflation and output gap stabilisation. This compromise is generated by the inclusion of the cost channel and follows Ravenna and Walsh (2006). For example, a rise in the interest rate to close the output gap must be limited since the increase in the cost of borrowing will manufacture inflation. We first consider an environment where the policy maker makes an optimal decision each period, since they are unable to control agents future expectations, namely optimal discretionary policy. In a standard New Keynesian model the adverse implication of discretionary policy is that, by not committing to future actions the policy maker can only use todays output gap to control inflation. A consequence of this outcome is that the policy maker attempts to stabilise the output gap by more than a time consistent policy calls for, in what is known as the stabilization bias associated with discretionary policy. In our model version this effect is exaggerated as it becomes optimal today to stabilise the output gap by even more since a larger interest rate, which

\(^3\)See Chapter 2 for details
causes greater inflation, will reduce further the output gap through our additional channel.

By committing to a policy plan the policy maker is able to influence the expectations of agents in future periods and can minimise total welfare losses by committing to an optimal future path for the output gap and inflation. With the additional relative price effects in aggregate consumption accounted for this effect is exaggerated; not only does it eliminate the additional adverse effect of discretion outlined above, but since expectations of, for example, future deflation\footnote{Given that we have been pushed above our zero inflation steady state inflation following the shock.} act to reduce the product demands of firms adjusting prices today, their price rise and therefore aggregate inflation will increase by less. A consequence of this effect is that the central bank can reduce deviations in inflation and the output gap by even more today at the smaller cost of greater deviations in later periods, as a result we observe much smoother, more persistent "humpshaped" responses of the output gap\footnote{Alternatively, habit persistence can be introduced to achieve a smoother more hump shaped output gap response, see for example, Amato and Laubach (2004) or Leith, Moldovan and Rossi (2012).}.

The following section, section 2.2, establishes our economic environment derived from chapter 2 whilst the welfare objectives of the central bank are outlined in section 3.3. Given these objectives we outline optimal policy and simulated impulse responses under; discretion in section 3.5; and commitment in section 3.6. The comparison of welfare losses under commitment and discretion will be investigated in section 3.7, while 3.8 provides some concluding remarks.

\section{3.2 The Economic Environment}

In this section we present the log-linearised model economy. The framework can be summarised as a typical New Keynesian model derived with Calvo (1983) type price stickiness with the additional characteristics of a cost channel (See Ravenna and
Walsh, 2006) and an additional inflation channel on the standard IS equation. This channel is derived from the dynamic aggregation of product demands in consumption which was outlined in chapter 2.

### 3.2.1 Baseline Parameter Values

We consider our model with the following fixed parameter values:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.75</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>0.85</td>
</tr>
<tr>
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<tr>
<td>$\phi_t$</td>
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</tr>
<tr>
<td>$\sigma_t^2$</td>
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</tr>
<tr>
<td>$\sigma_e^2$</td>
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</tr>
<tr>
<td>$\sigma_y^2$</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_g^2$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.1: Parameter Values and Notation

### 3.2.2 The New Keynesian Phillips Curve

Our New Keynesian Phillips Curve is defined as:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\gamma + \sigma) \hat{x}_t + \kappa \hat{R}_t,$$  \hspace{1cm} (3.1)

where, $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$ is the slope of the Phillips curve; $\hat{\pi}_t$ is inflation; $\hat{x}_t = \hat{Y}_t - \hat{Y}_t^f$, is the output gap and $\hat{R}_t$ is the nominal interest rate gap or cost channel, expressed as a deviation from their flexible price levels.

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3.2.3 The Output Gap

Our output gap, with additional price dispersion from aggregate consumption is given as

\[ \hat{x}_t = \hat{x}_t - \frac{\theta}{\kappa} [\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}] . \]  

(3.2)

where, \( \hat{x}_t \) is the conventional dynamic IS equation,

\[ \hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} [\hat{R}_t - E_t \hat{\pi}_{t+1}] + \mu_t . \]  

(3.3)

and

\[ \mu_t = \left( \frac{1 + \gamma}{\gamma + \sigma} \right) [E_t \hat{A}_{t+1} - \hat{A}_t] + \left( \frac{\gamma}{\gamma + \sigma} \right) [E_t \hat{g}_{t+1} - \hat{g}_t] - \frac{\gamma}{\sigma[(\gamma + \sigma)]} [E_t \hat{\xi}_{t+1} - \hat{\xi}_t] \]

is our disturbance terms made up of deviations in technology \( \hat{A}_t \), the proportion of government purchases \( \hat{g}_t \), and tastes \( \hat{\xi}_t \).

3.2.4 Monetary Policy

We assume the following Taylor rule

\[ \hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) (\phi \hat{\pi}_t + \phi \hat{\pi}_t) + \nu_t \]  

(3.4)

where \( \nu_t \) is a monetary policy shock.

\[ ^6 \text{Here a taste shock effects the utility from consumption which therefore alters the marginal utility of consumption.} \]
3.2.5 Shock Process

We assume that all shocks follow a first order autoregressive process,

\[
\begin{align*}
\hat{A}_t &= \rho_a \hat{A}_{t-1} + \varepsilon^a_t, \\
\hat{\xi}_t &= \rho_{\xi} \hat{\xi}_{t-1} + \varepsilon^\xi_t, \\
\hat{g}_t &= \rho_g \hat{g}_{t-1} + \varepsilon^g_t, \\
\hat{v}_t &= \rho_v \hat{v}_{t-1} + \varepsilon^v_t
\end{align*}
\]

where \(\varepsilon^a_t, \varepsilon^g_t, \varepsilon^\xi_t, \varepsilon^v_t\) are all independent identically distributed shocks with respective variances \(\sigma^2_a, \sigma^2_g, \sigma^2_\xi, \sigma^2_v\).

3.3 Policy Objectives

Given the specification and economic environment that led to our macroeconomic equilibrium, we now consider what are the appropriate objectives for the central bank. Following the log-linear second order approximation outlined by Rotemberg and Woodford (1999), the central bank is assumed to minimise a policy objective function that accounts for the welfare losses of households following economic shocks. The focus of this welfare function will be to highlight losses that stem from the presence of price rigidity, specifically the welfare losses encountered through a move from the flexible to the sticky price equilibrium.

For the central bank to determine the course of action that minimises welfare losses to the household we must first define the household preferences and the products they consume\(^7\). Households choose to maximise their expected present discounted value of utility given by

\[
V_t = E_t \sum_{i=0}^{\infty} \left[ \frac{\xi_{t+i}C_{t+i}}{1 - \sigma} - \eta \frac{N_t^{1+\gamma}}{1+\gamma} \right]
\]

\(^7\)These two definitions must be the equivalent to the equations of the model that generated our macroeconomic equilibrium in Chapter 2.
In addition our consumption is defined as a continuum of differentiated goods with a composite consumption good.

\[ C_t = \left[ \int_0^1 c_t(i) \frac{\theta + 1}{\sigma} di \right]^{\frac{\sigma}{\sigma - 1}} \]

where \( \eta_{1 + \gamma}^{N_{t+i}} \) is the household's disutility of production and \( \xi_{t+i} \), \( C_{t+i} \) and \( N_{t+i} \) represent a taste parameter, aggregate consumption and labour hours respectively. Following Erceg, Henderson and Levin (2000), Woodford (2003) and Ravenna and Walsh (2006) we obtain our policy objective function by taking a second order Taylor approximation of the utility function (equation 3.5). In doing so we define the welfare loss per period as:

\[ W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{\theta}{\kappa} \right) \tilde{x}_t^2 + (\sigma + \gamma) \tilde{x}_t^2 \right] \quad (3.6) \]

As in the models of Khan, King and Wolman (2003) and Benigo and Woodford (2005) the fiscal variable \( g_t \) generates a wedge between the output gap and what Ravenna and Walsh refer to as the welfare gap. The welfare gap \( \tilde{x}_t^e \) is defined as

\[ \tilde{x}_t^e = \tilde{x}_t - \left( \frac{1}{\sigma + \gamma} \left( \tilde{R}_t + \tilde{\gamma}_t \right) \right) - z^* \quad (3.7) \]

where

\[ z^* = \frac{\tilde{g}_t \Phi \tilde{R} - 1}{\tilde{g}_t \Phi \tilde{R}(\sigma + \gamma)} \]

The differences between the welfare gap and the output gap, depend on monopolistic competition via the markup \( \Phi \), and the monetary distortion generated by the non-zero interest rate \( \tilde{R} \). Since the focus of this chapter is on stabilisation policies, we follow Ravenna and Walsh (2006) and assume that these average efficiency distortions are.

\[ \text{See Appendix 3.A.1 for derivation.} \]

\[ \text{Note that we use tilde notation to define our aggregate output gap with the additional price dispersion. Also note that indirectly this price dispersion enters into labour. This notation is used from the start because we know this to be the observed level of consumption and labour.} \]
tions are eliminated so that \( z^* = 0 \). We also follow Ravenna and Walsh (2006) in assuming an interest rate peg, so that \( \hat{R}_t^f = 0^{10} \).

The second term in equation (3.7) arises because government purchases increase with output, yet households do not account for these changes in deciding on labour supply and consumption decisions. A result of this is that even when the distortions associated with \( z^* \) are accounted for, \( \hat{g}_t \) creates a wedge between the efficient output gap and the welfare gap even if the output gap is fully closed. Thus it may be optimal to offset deviations in \( \hat{g}_t \) by allowing \( \hat{x}_t \) to fluctuate, even if this leads to inflation deviations. Accounting for \( z^* = \hat{R}_t^f = 0 \) we have the following welfare loss function

\[
W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{\theta}{\kappa} \right) \hat{\pi}_t^2 + (\sigma + \gamma) \left( \hat{x}_t - \frac{1}{(\gamma + \sigma)} \hat{g}_t \right)^2 \right] 
\]

which can be written in terms of average welfare losses each period as,

\[
L = \frac{1}{2} \left[ \left( \frac{\theta}{\kappa} \right) var(\hat{\pi}_t) + (\sigma + \gamma) var(\hat{x}_t) + \frac{1}{(\gamma + \sigma)} var(\hat{g}_t) - 2cov(\hat{g}_t, \hat{x}_t) \right] 
\]

Our loss function is identical to Ravenna and Walsh (2006). Significantly, our additional effects do not effect the structure of equation (3.8). This is because it is only the aggregate inefficiencies in the economy that relate the gap between the marginal rate of substitution and the marginal product of labour. Thus, although our effects will impact the size of the distortion through the impact that relative price distortions have on the variances of both the output gap and inflation, they do not effect the weight one places on welfare losses due to this distortion.

Overall, there are two conflicting welfare effects with respect to the competitiveness in the goods markets. First there is the direct effect shown in the literature where welfare losses are shown to increase monotonically with respect to the price elasticity of substitution among consumption goods (see eq. (3.9) and Gali 2008).

\(^{10}\)Since our focus is on stabilisation policies in terms of deviations from the flexible level there is no reason for this flexible rate to fluctuate anyway.
The second effect, which is an innovation here, comes endogenously from the way that relative price distortions affect inflation and the output gap. As we have shown earlier, the more competitive are the goods markets the lower is the variance of the output gap and inflation, intuitively implying smaller welfare losses in time of shocks or monetary policy intervention.

3.4 Welfare Losses Under a Taylor Rule

3.4.1 Taste and Technology Shocks

Figures 3.1 - 3.8, show separately the welfare losses associated with the output gap and inflation as a result of taste, technology, monetary policy and fiscal shocks, respectively.\(^\text{11}\) This is examined over a continuum of \(\theta\) values ranging between 1 and 15 and for both our model and the baseline model of Ravenna and Walsh (2006).\(^\text{12}\) In all cases the Taylor rule is given by equation 3.4.

\(^{11}\)These are welfare losses associated with a 1% change in the (annualised) interest rate, a 1% quarterly taste shock and a 1% quarterly technology shock.

\(^{12}\)The Ravenna and Walsh (2006) case is equivalent to setting \(x_t = \hat{x}_t\) in equation (3.2) so that

\[
\frac{\hat{\pi}_t}{\pi} [\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}] = 0
\]
Figure 3.1: Welfare Losses Following a Taste Shock

Figure 3.2: Welfare Losses Following a Taste Shock (Ravenna and Walsh)
Figure 3.3: Welfare Losses Following a Technology Shock
Figure 3.4: Welfare Losses Following a Technology Shock (Ravenna and Walsh)

Figure 3.5: Welfare Losses Following a Monetary Policy Shock
Figures 3.1-3.6 define the welfare losses associated with a 1% change in the (annualised) interested rate\textsuperscript{13}, a 1% quarterly taste shock and a 1% quarterly technology shock. To analyse the welfare implications we first consider the standard case of Ravenna and Walsh (2006) given in Figures 3.2, 3.4 and 3.6. Equation (3.9) define the welfare loss each a period as a function of the variances of inflation and the output gap\textsuperscript{14}. Welfare losses associated with the output gap are increasing with the levels of $\sigma$ and $\gamma$ since they both amplify the size of the gap between the marginal rate of substitution and the marginal product of labour. On the other hand, increases in $\theta$ amplify the welfare losses associated with a given variance in inflation. A more competitive economy is associated with larger swings in product demands since the substitutability of goods is higher, thus for a given movement in inflation this result is amplified the higher is the price elasticity of demand. Since

\textsuperscript{13}Recall that as in chapter 2 this corresponds with a 0.25 quarterly policy shock.

\textsuperscript{14}Note that when we consider loss in welfare we are defining the loss compared to the flexible price equilibrium, as a result we do not consider the effect that levels of $\theta$ have on the mark up (distortions and welfare analysis unrelated to sticky prices).
the macroeconomic equilibrium in the baseline model of Ravenna and Walsh (2006), is not influenced by the level the price elasticity of demand, the variances of inflation and the output gap will not be a function of this elasticity. As a result, welfare losses (in comparison with the flexible price equilibrium) will always be greater in more competitive economies following all shocks. Additionally, the relative weight on inflation increases as the product markets become more competitive. Finally, since the price elasticity of demand effects neither the weight nor the variance of the output gap, the output gap losses do not change for alternative competition levels in the goods market.

Figures 3.1, 3.3 and 3.5 display the results of our model version. Our loss function is identical to Ravenna and Walsh (2006), indeed we too find that household behavior which is characterised by the switching of product demand weights, to generate a greater welfare loss for a given variance of inflation, which is increasing in the level of \( \theta \). However, we also get a decrease in the variance of both inflation and the output gap for economies with a higher \( \theta \) following an identical set of economic shocks. Intuitively, firms who have a price resetting opportunity also account for this type of consumer behavior and its effects on the demand for their product in their profit maximisation. With this knowledge, firms make less of a movement in prices to achieve their optimal level of production, a channel which is increasing in the competitiveness of the goods market. Additionally, since households are able to alter their relative product weights (which is again determined by the extent to which similar alternatives are available) they are able to increase their aggregate consumption through switching products\(^{15} \). This behaviour, which depends on the trade off between the price of a product and its similarity to other products, mitigates the utility losses in consumption caused by price dispersion. Its overall effect is to push aggregate consumption, and therefore output, towards the flexible price equilibrium or optimal allocation thus reducing the absolute size of the output gap.

\(^{15}\text{When compared with the symmetric product demands case of the Euler equation,}\)
As a result the variance of both the output gap and inflation are inversely related to the levels of $\theta$.

Our results indicate that any increases in $\theta$ reduces output gap losses since the switching process outlined above allows households to alter their optimal consumption towards the optimal flexible price level thus reducing output gap variance. Since the weight on the output variance is not affected by the level of $\theta$, falls in the variance of the output gap will always generate a fall in losses.

Overall the effects of an increase in $\theta$ on inflation losses are less obvious. This is because it will increase the welfare losses for a given price change but also reduces that very given price change by reducing the variance. Our results indicate (for all shocks) as theta increases from $\theta = 1$, initially welfare losses from inflation increase (in that increases in the weight on losses is greater than the decrease in the variance). However, from around $\theta = 3^{16}$ onwards each incremental increase leads to an overall fall in welfare losses from inflation and the initial effect in reversed$^{17}$.

With the losses from inflation and the output gap combined, increases in competition in the goods market, in general, lead to a reduction in welfare losses. A result which is in stark contrast to Ravenna and Walsh (2006) and the bulk of the related literature that utilises the Rotemberg and Woodford (1999) welfare function.

More recently, Soderberg (2011) obtains similar results, in this case however welfare is decreasing with respect to increases in the fraction of product demands that remains from the previous period$^{18}$.

---

$^{16}$Given that most economies are more competitive than $\theta = 3$ we can infer that in general increasing competition, increases welfare.

$^{17}$Note that increasing persistence on the shocks, (and in the case of the policy shocks also increasing the persistence of the policy rule) leads to an exaggeration of this effect. That is the direction of welfare losses begin to turn at higher values of theta. Also note that taste and technology shocks have these increases for a longer period of time than policy shocks.

$^{18}$In this paper, Soderberg considers a Calvo (1993) type setup in the goods market where a fraction of customers in market each period are unable to alter their consumption demands through habit persistence.
3.4.2 Fiscal Distortions

When analysing welfare losses following fiscal distortions we must also consider the variance of the government expenditure and the covariance between the output gap and the government expenditure parameter (the final two terms in equation (3.9)). This is because fiscal shocks effects $g_t$ and also the welfare gap directly in equation (3.8).

![Figure 3.7: Welfare Losses Following a Fiscal Policy Shock](image-url)
Figures 3.7 and 3.8 display the welfare losses from fiscal distortions both with and without our additional relative price distortions. Here we only present the welfare losses that can be altered through monetary policy. In both cases there are also a fixed amount of losses coming through the $\text{var}(\hat{g}_t)$ term. This term is only dependent on the shock process and the disturbance term $\mu_t$ thus it is not dependent on either $\theta$ or the price dispersion that arises through aggregate consumption. For a 1% fiscal shock $\text{var}(\hat{g}_t)$ are equal to 0.72 which is the lower bound on the horizontal axis.

Our results indicate that, as with the other shocks, inflation losses are, in general, increasing with $\theta$ and the output losses constant in the Ravenna and Walsh (2006) version. The response of inflation and the output gap also follow a line of reasoning identical to the shocks previously analysed. However, there is an additional channel which reduces welfare losses as $\theta$ increases in our model and which is constant with respect to theta in the baseline. This comes through the covariance between $\hat{x}_t$ and $\hat{g}_t$.\textsuperscript{19} Consider a rise in government spending, $(\hat{g}_t$ falls), maintaining a zero output gap causes a rise in the welfare gap so that a movement in inflation will decrease

\textsuperscript{19}Specifically covariance losses $= -2(\sigma + \gamma)\text{cov}(\hat{g}_t, \hat{x}_t)$ in equation (3.9).
less than is optimal. The actual level of output falls by less than the natural level causing a fall in the output gap. This reinforces the negative realisation of \( \hat{g} \) further and thus increases the variability of consumption and welfare losses. However in our model the output gap falls by less as \( \theta \) increase, hence the losses associated with the covariance between \( \hat{x}_t \) and \( \hat{g}_t \) also fall.

### 3.4.3 Welfare Losses: The Degree of Price Rigidity (\( \omega \))

Canzoneri, Cumby and Diba (2007b) show that decreasing the degree of price rigidity, \( \omega \), reduces welfare losses and that, in general, there is an increasing cost associated with the degree of nominal rigidity.\(^20\) Figures, 3.9, 3.10 and 3.11 examine the welfare losses associated with different levels of price rigidity under taste, technology and monetary policy shocks, respectively.\(^21\) It shown that in the case of R&W (2006), that is based on the Euler aggregate consumption, increasing the degree of price rigidity increases welfare losses, (i.e. up to around \( \omega = 0.8 \) for inflation, and \( \omega \approx 1 \) for the output gap losses). This result is consistent with that in Canzoneri, Cumby and Diba (2007b). Largely, this is also true for our model, but in general accounting for all relative prices in price setting seems to exhibit a hump-shaped relationship between welfare losses and the degree of price rigidity, where the peak of the hump is determined by the competitiveness in the goods markets. As shown below, raising \( \theta \) from \( \theta = 3 \) to \( \theta = 10 \) shifts the hump from around 0.6 to 0.5. Intuitively, this is because as the degree of rigidity reaches a substantial level, (i.e. over \( \omega = 0.5 \)), the welfare gains from inflation adjustments start offsetting the overall welfare losses from both inflation and the output gap. As price setting becomes more sensitive to relative price distortions, the more competitive is the consumption

\(^{20}\)Note that Canzoneri, Cumby and Diba (2007b), examine the costs of both price and wage rigidities. In their model price dispersion effects enter aggregate output and consumption through the second order approximation effect in deriving welfare losses, and not endogenously, through price setting, via the the Phillips curve. In this paper both of these effects are present.

\(^{21}\)Following fiscal shocks (\( \hat{g}_t \)), welfare losses are again lower when relative price are accounted for, but they do not share as interesting differences as the the rest of the shocks, mainly due to the way they are modelled in R&W (2006), so we do not report them here, but they are available upon request by the authors.
Figure 3.9: Welfare Losses and Price Rigidity: A Taste Shock

goods market (i.e., the higher is $\theta$) the earlier will welfare losses start falling along the $\omega$-axis.
Figure 3.10: Welfare Losses and Price Rigidity: A Technology Shock
Figure 3.11: Welfare Losses and Price Rigidity: A Monetary Policy Shock
3.5 Discretionary Equilibrium

We now turn to consider optimal policy under discretion. Under discretion the central bank optimises each period without being able to make any commitment to future actions beyond time $t$. Consequently, the decisions of the central bank at time $t$ cannot influence the private sector’s expectations of future aggregate variables. Following Ravenna and Walsh (2006), a policy problem is generated for the central bank through the presence of the cost channel. For example, increasing the interest rate to close the output gap generates inflation through the cost channel. The problem for the central bank is to select the path for $R_t$ and the implied paths for $\hat{x}_t$ and $\hat{\pi}_t$ to minimise

$$W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \hat{\pi}_t^2 + \lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{\pi}_t \right)^2 \right]$$

(3.10)

where $\lambda = \frac{\kappa(\gamma + \sigma)}{\sigma}$, subject to the Phillips curve

$$\hat{\pi}_t = \kappa (\gamma + \sigma) \hat{x}_t + \kappa \delta \hat{R}_t + \beta E_t \hat{\pi}_{t+1}$$

and the output gap

$$\hat{x}_t = \bar{x}_t - \frac{\theta}{\kappa} [\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}]$$

where

$$\hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} \left[ \hat{R}_t - E_t \hat{\pi}_{t+1} \right] + \mu_t$$

22The weights on $\left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{\pi}_t \right)^2$ and $\hat{\pi}^2$ are directly proportional to those used to determine welfare losses in the previous section. As a result, the optimal weight between the output gap and inflation will be identical regardless of which function is used.

23The $\delta$ term allows the cost channel to be switched off by turning it to zero. When the cost channel is included this term is 1.
and

\[
\mu_t = \left( \frac{1 + \gamma}{\gamma + \sigma} \right) [E_t \hat{A}_{t+1} - \hat{A}_t] + \left( \frac{\gamma}{\gamma + \sigma} \right) [E_t \hat{g}_{t+1} - \hat{g}_t] - \frac{\gamma}{\sigma [\gamma + \sigma]} [E_t \hat{\xi}_{t+1} - \hat{\xi}_t]
\]

The optimally condition for the above problem is given by\textsuperscript{24}

\[
\hat{\pi}_t = -\frac{(1 + \theta \delta \sigma)}{\kappa (\gamma + \sigma (1 - \delta))} \lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right)
\]  

(3.11)

Equation (3.11) indicates that we have an additional effect that only exists with the cost channel and our model version, the \( \theta \delta \sigma \) term. Whilst the cost channel already demonstrates that optimal discretionary policy requires a greater inflation variability for a given output variability, since the cost channel reduces the size of the denominator, our result exaggerates this movement by increasing the size of the numerator.

In the baseline cost channel model an increase in \( \hat{R}_t \) decreases \( \hat{x}_t \) but the direct impact of the rise of the nominal interest rate partly offsets the deflationary impact of tighter monetary policy. When the cost channel is not present the relationship is equivalent to the standard New Keynesian literature where it becomes optimal to completely shut off inflation, as in the Lagrangian \( \Psi_t = 0 \). That is, only the output gap effects inflation in the Phillips curve so it becomes optimal to adjust \( \hat{R}_t \) to completely close \( \hat{x}_t \) which will also cause inflation deviations to diminish.

Typically, in the New Keynesian literature\textsuperscript{26}, the price elasticity of demand \( \theta \) only enters through \( \lambda \) which suggests in economies with a more competitive goods market, inflation will move less for a given movement in the output gap. Intuitively, for higher levels of \( \theta \) a given price movement generates larger swings in product demands and losses in welfare, hence it becomes optimal to weigh more highly

\textsuperscript{24}See Appendix 3.A.2 for derivation.

\textsuperscript{25}See, for example Woodford (2003)

\textsuperscript{26}A cost push shock is usually added to the Phillips curve when the cost channel is not used to create a policy tradeoff.
inflation stabilisation relative to output gap swings.

When the additional price dispersion in aggregate consumption is accounted for we retain the above effect, however we also have an additional channel which relates negatively the output gap with inflation. Since firms price resetting behaviour accounts for the above effect, the variance of inflation and the output gap will fall as $\theta$ increases. Furthermore, households can utilise the non symmetric prices and the substitutability of products to reduce the deviation in the output gap. Thus, a rise in inflation helps to generate an additional fall in the output gap. As in the baseline model of Ravenna and Walsh (2006), optimal policy must consider the adverse effect that interest adjustments have on inflation when the central bank responds to a movement in the output gap. Additionally, our model also considers the beneficial effects that these inflation movements have on complementing the initial interest rate movement in closing the output gap.\footnote{As we will see this is only benificial in the context of a one period optimal policy maker who is not able to control agents expectations of aggregate variables. Not in terms of total future welfare losses.}

One of the benefits of this approach is that the increased weight on the output gap is much more in line with the monetary policy literature\footnote{When compared with the Ravena and Walsh (2006) paper}. Consider $\tilde{\lambda}$ where we include the additional effect $(1 + \theta \delta \sigma)$ which increases the weight on the output gap in our model,

$$
\tilde{\lambda} = \lambda (1 + \theta \delta \sigma) = \frac{\kappa (\gamma + \sigma)(1 + \theta \delta \sigma)}{\theta}.
$$

(3.12)

In the baseline model of Ravenna and Walsh (2006) where $\sigma = 1.5, \kappa = 0.0858333, \gamma = 1$ and $\theta = 11$ then $\tilde{\lambda} = 0.0195$. As the authors point out generally much larger values are used (typically $\tilde{\lambda} = 0.25$)\footnote{See for example McCallum and Nelson (2000), Jensen (2002), and Walsh (2003) who all set $\tilde{\lambda} = 0.25$. or thereabouts}. However, in their model it is not possible to obtain this value of $\tilde{\lambda}$ for empirically plausible parameter values for example \textit{ceteris paribus} either $\sigma = 31$ or $\gamma = 30.5$. In contrast, in our model a combination of $\sigma = 1.19$ and $\gamma = 1$ achieves this result which lies in the range of empirically
plausible parameter space.

3.5.1 Determinacy Under Discretion

We now analyse under what conditions (3.11) delivers a determinate equilibrium. Significantly our model lies in a critical region for determinacy for a range empirically plausible parameter values. For example, ceteris paribus when \( \theta = 3, \gamma > 1.62 \), when \( \theta = 6, \gamma > 1.47 \), and when \( \theta = 10, \gamma < 1.42 \). Intuitively, for plausible parameter values the one period optimal weight on output gap stabilisation may be so much that the increase in the interest rate causes the cost channel to make inflation self fulfilling. In the case where the optimal discretionary point is indeterminate, the policy maker chooses to weigh output gap stabilisation at the highest determinate point.

3.5.2 Simulations Under Discretion

To allow for direct comparison we adjust \( \gamma \), the intertemporal elasticity of substitution in the labour market to a value which is determinate for all values of \( \theta \). Since the lowest value we simulate is \( \theta = 3 \) we set \( \gamma = 1.62 \). (As opposed to \( \gamma = 1 \), the parameter value used by Ravenna and Walsh (2006).

---

30 Determinacy conditions are obtained through Dynare.
31 See Surico (2008) and Llosa and Tuesta (2009) for detailed analysis of determinacy conditions with a cost channel.
32 Alternatively when \( \gamma = 1 \) ceteris paribus \( \theta = 3, \sigma < 0.79 \), when \( \theta = 6, \sigma < 0.95 \), and when \( \theta = 10, \sigma < 1.03 \)
33 This fits with the parameter space of the literature.
\[ \beta = 0.99 \] Discount rate 
\[ \sigma = 1.5 \] Intertemporal elasticity of substitution in consumption 
\[ \omega = 0.75 \] Proportion of firms unable to alter their price in each period 
\[ \gamma = 1.62 \] Intertemporal elasticity of substitution in the labour market 
\[ \rho_a = 0.85 \] Persistence of productivity shocks 
\[ \rho_z = 0.85 \] Persistence of taste shocks 
\[ \rho_r = 0.85 \] Persistence of the policy rate 
\[ \sigma^2_a = 1 \] Variance of technology shocks 
\[ \sigma^2_z = 1 \] Variance of taste shocks 
\[ \sigma^2_u = 1 \] Variance of fiscal shocks

Table 3.2: Alternative Parameter Values

Figure 3.12: Impulse Responses to a Taste Shock Under Discretion
Figure 3.13: The Impulse Responses to a Technology Shock Under Discretion

The impulse response functions following positive taste and technology shocks are shown in figures 3.12 and 3.13. In all of the above model versions a positive technology shock raises the current output gap relative to future output gaps so that the interest rate must fall to raise current consumption relative to future consumption. The nominal interest rate fall generates a fall in inflation. The trade-off comes from how much the central bank chooses keep inflation down relative to letting the output gap rise\textsuperscript{34}. We also consider how different levels of competition in product markets effect both our model and the baseline model of Ravenna and Walsh (2006) by considering a move from $\theta = 3$ to $\theta = 10$. From (3.12) in the R&W case the output gap weight is reduced from $\lambda = 0.0715$ to $\lambda = 0.0215$ a factor of 3.3 whereas in our model $\tilde{\lambda} = 0.393$ to $\tilde{\lambda} = 0.343$, a factor of 1.15. In both cases it becomes optimal to allow for the output gap to deviate by more than inflation. In the baseline environment a higher value of $\theta$ is only more costly due to the in-

\textsuperscript{34}Note that the responses of taste shocks are the opposite to technology shocks.
creased welfare losses from inflation, thus it becomes optimal to increase the focus on inflation deviations and we see a large fall in the relative weight on the output gap. In our model, whilst the $\theta$ increases also increases the welfare losses for a given inflation movement, the inflation movement itself has the effect of reducing the output gap. As a result, the relative weight on the output gap is reduced by less. It is for this reason that we also see (both through our simulations and the weights on $\lambda$ and $\tilde{\lambda}$) a higher weight on output gap deviations in our model compared to the R&W case. Consequently, *ceteris paribus* we see more of a movement in inflation following a technology or taste shocks in our model. Furthermore, the magnitude of this difference is increasing with $\theta$.

In our model, $\theta$ also operates through our output gap (equation 3.2). Thus the increase in competition will result in both a reduction in the output gap and inflation since the impact of the shock is mitigated in a more competitive goods market\textsuperscript{35}. This effect also permits the nominal interest to have a smoother response in our model and as $\theta$ increases this channel is exaggerated.

\textsuperscript{35}For a discussion see chapter 3.
Figure 3.14 shows the impact of a fiscal shock (a rise in $g_t$). Whilst inflation behaves in the same manner as the taste and technology shocks, the relative responses of the output gap appear in the opposite order. Recall that fiscal shocks impact the welfare function directly as well as through the exogenous process $\mu_t$.

Consider a rise in $\hat{g}_t$ (a fall in government spending), maintaining a zero output gap so that inflation remains at zero causes a fall in the welfare gap, $\tilde{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t$. To limit the fall in the welfare gap the output gap must rise, and optimal policy trades off some inflation increases. As a result, the fiscal shock decreases the flexible-price level of output by more than the actual level causing the output gap to rise.

With the additional price dispersion in aggregate consumption, optimal policy allows for greater swings in inflation for a given movement in the welfare gap. Indeed, the welfare gap behaves analogously to the output gap following taste and technology shocks. To achieve this level of welfare gap, policy is much more aggressive, highlighted by the large fall in the interest rate in our model and in both models as theta increases. Furthermore, in our model for a unit increase in $\theta$, since
inflation has more of an impact on closing the output gap, the interest rate change must be of greater magnitude to achieve the desired level of output gap. Recall that in the case of taste and technology shocks inflation helped to close the output gap allowing for a smoother interest rate response. In contrast, in this case, where it is optimal to increase the output gap response to shut off the welfare gap, the interest rate must counteract this effect and therefore becomes stronger and more prolonged.

Significantly, since the central bank is unable to influence the expectations of agents they may not necessarily achieve what is an optimal rule. This is known as the stabilisation bias associated with discretionary policy. Indeed, as mentioned in the baseline model of Ravenna and Walsh (2006) it is theoretically possible that discretionary policy can lead to indeterminacy which is equivalent to infinite losses. In our model the additional channel adds a new consideration for the time inconsistency problem under discretion. When the policy maker considers 

\[ \hat{x}_t = \hat{x}_t - \frac{\beta}{\pi_t} [\pi_t - \beta E_t \hat{\pi}_{t+1}] \],

by only reacting to \( \pi_t \) they ignore the agents expectations of \( \beta E_t \hat{\pi}_{t+1} \). Because they are not able to influence future inflation the central bank will put too much weight on the output gap and welfare losses from inflation over the lifetime of the shock will be suboptimally high. Thus, there is even greater rationale in the context of our model to consider the potential gains from a time consistent policy plan by investigating optimal policy under commitment.

### 3.6 Optimal Commitment

We now consider an environment where the central bank is able to commit, with full credibility, to a policy plan. Under optimal commitment the central bank can manipulate both \( E_t \hat{x}_{t+i} \) and \( E_t \hat{\pi}_{t+i} \) to control losses. Specifically, policy makers are assumed to be able to choose a path for inflation and the output gap that minimise the objective function over all states of nature, current and future. Thus, the central bank minimises the welfare losses in equation (3.10) for time \( t \) and also for future
periods $t + i$. This yields the following first order conditions\textsuperscript{36}

$$\lambda \left( \hat{x}_{t+i} - \frac{1}{\sigma + \gamma} \hat{g}_{t+i} \right) = -\kappa \gamma \Phi_{t+i} - \beta^{-1} \sigma \kappa \Phi_{t+i-1} = 0 \quad (3.13)$$

$$\hat{\pi}_{t+i} = (1 + \theta \sigma) \Phi_{t+i} - (1 + \beta^{-1} \kappa + \theta \sigma (\beta^{-1} + 1)) \Phi_{t+i-1} + \theta \sigma \beta^{-1} \Phi_{t+i-2} \quad (3.14)$$

Svensson and Woodford (1999) have described a policy that implements equations (3.13) and (3.14) for all $t$ as the timeless pre-commitment policy. Notice that for the initial period (setting $i = 0$ in the above equation), and given that we start from the steady state where there are no deviations in any endogenous backward looking variables ($\hat{X}_{t-k} = 0$, for and positive $k$), our model reduces to (3.11), which is the discretionary equilibrium of the previous section. Consequently, can define (3.13) and (3.14) for any period $t$ as, (3.15) and (3.16)

$$\hat{x}_t = -\frac{\kappa \gamma}{\lambda} \Phi_t - \frac{\beta^{-1} \sigma \kappa}{\lambda} \Phi_{t-1} + \frac{1}{\sigma + \gamma} \hat{g}_t \quad (3.15)$$

$$\Phi_t = \frac{\hat{\pi}_t + (1 + \beta^{-1} \kappa + \theta \sigma (\beta^{-1} + 1)) \Phi_{t-1} - \theta \beta^{-1} \sigma \Phi_{t-2}}{(1 + \theta \sigma)} \quad (3.16)$$

Equation (3.15) is identical in both our model and the Ravenna and Walsh (2006) case. However, the $\Phi_t$ term given in (3.16) displays much more endogenous persistence in inflation and output.

\subsection{3.6.1 Determinacy Under Optimal Commitment}

Following Surico (2008) and Llosa and Tuesta (2009), the issues of determinacy found under discretion, do not apply in this case and optimal commitment guarantees determinacy (shown through simulation). Intuitively, since the policy maker is able to commit to a policy plan and control expectations for all $t + i$ it would never be optimal to design such a plan that results in an explosive economy.

\textsuperscript{36}See Appendix 3.A.3 for details of derivation.
3.6.2 Simulations Under Optimal Commitment

Our simulations for optimal policy under commitment can be achieved by jointly solving (3.15) and (3.16), which govern the optimal relationship between inflation and the output gap, together with our macroeconomic equilibrium (equations 3.1-3.3).

![Figure 3.15: The Impulse Responses to a Taste Shock Under Optimal Commitment](image)

Figures 3.15 and 3.16 display the impulse responses to both taste and technology shocks under optimal commitment. All responses depict a degree of intrinsic or endogenous persistence, however in our model this level of persistence is higher, especially with respect to the output gap. There is also a much more of a "humped shape" output gap response with the impulse responses taking much more time to reach their peak. Finally, the change from the discretionary equilibrium in our model, relative to the R&W baseline, depicts a larger reduction in the initial inflation.

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37 As in the decretionary equilibrium the movement of the output gap with respect to taste and technology shocks is analogous to the welfare gap under fiscal distortions.
Figure 3.16: The Impulse Responses to a Technology Shock Under Optimal Commitment

response and a larger fall (in percentage terms) in the initial output gap response\(^{38}\). These effects come at the cost of a larger maximum deviation of the output gap (6 quarters in when \(\Theta = 3\) and 8 quarters in when \(\Theta = 10\)) compared with the initial (and therefore maximum) response of the output gap under discretion\(^{39}\).

The intuition is as follows: Optimal discretionary policy requires that the central bank only considers the additional effects that inflation has on the output gap today. In \(\hat{x}_t = \hat{x}_t - \theta \hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}\), only \(\hat{\pi}_t\) need be considered and a small increase in inflation deviations has a large impact on reducing today’s output.

Under optimal commitment the impact on future inflation terms on the output gap can also be influenced by the policy maker. Consider the positive taste shock, under optimal commitment; the central bank can now create expectations of deflation at \(t + 1\), so that the reduction in output need not be as large today. In our version of the model this effect is exaggerated as firms know their product demands

\(^{38}\)See figures 3.12 and 3.13 for comparison.

\(^{39}\)Leith, Moldovan and Rossi (2012) also find a smoother more persistent response to output in their case as habit persistence is increased.
in the future will also suffer because of deflation if they set a too higher price today and they are unable to adjust, as a result they will not set such a high price today.

As in the discretionary equilibrium case, the fiscal shock depicted below in figure 3.17, shows the welfare gap to present the same relationship with inflation as the output gap did for taste and technology shocks. Beyond this the overall result of moving from discretion to commitment follow the same line of reasoning as taste and technology shocks.

Figure 3.17: The Impulse Responses to a Fiscal Shock Under Optimal Commitment
Figure 3.18: Comparison of Commitment and Discretion Following a Technology Shock When There is no Exogenous Persistence

Figure 3.18 compares the welfare losses following a technology shock when $\rho_a = 0^{40}$. We can clearly see endogenous or intrinsic persistence of inflation and the output gap active in all models under commitment. By maintaining a persistent deviation in inflation and the output gap the central bank manages to increase the output-inflation trade off when the shock is realised. Furthermore, in our model version the magnitude of this endogenous persistence is exacerbated. For a positive technology shock, the presence of persistent inflation in future periods allows firms to deflate their price by less today since they know that their product demands will increase when the aggregate price level increases. This process is increasing with the price elasticity of demand, $\theta$.

Regardless of whether or not shocks are serially correlated all the models show that under discretionary policy, in an attempt to minimise losses today, the output gap stabilises immediately by more than optimal policy under commitment requires. This stabilisation bias associated with discretionary policy is exaggerated in our

\footnote{The move of the impulse responses from $\rho = 0.85$ to $\rho = 0$ are analogous for taste and fiscal policy shock so are not displayed here.}
model version since the immediate benefits of allowing inflation to deviate further act to close the output gap by an even greater amount. As a result, the relative gains from a move to optimal commitment are greatest in our model. This can be seen more clearly by comparing the total welfare losses under the two regimes, which will be considered in the next section.

3.7 Comparison of Welfare Losses under Optimal Commitment and Discretion

We can provide some quantitative evidence for the intuition regarding the extra welfare gains from moving from discretion to optimal commitment that were hinted towards in the previous section. It is well documented that moving from commitment to discretion leads to welfare gains since under commitment the central bank can control expectations and thus minimise losses over the lifetime of the shock\(^{41}\). Figures 3.19 - 3.21 display the welfare losses under discretion and commitment for alternative values of \(\theta\) and in our model and the baseline of Ravenna and Walsh (2006).

\(^{41}\)See Woodford (2003) and Gali (2008) for a discussion.
Figure 3.19: Various Welfare Losses Following a Taste Shock

Figure 3.20: Various Welfare Losses Following a Technology Shock
Figure 3.21: Various Welfare Losses Following a Taste Shock
The welfare losses under commitment and discretion reenforce the results from sections 3.5 and 3.6. Moving from discretion to optimal commitment eliminates the stabilisation bias associated with optimal discretionary policy. Furthermore, this effect is exaggerated in our model version for two reasons; (a) Discretionary policy places an even greater emphasis on output gap stabilisation since in a single period inflation helps reduce output gap variations; (b) In our model and as $\theta$ increases and under optimal commitment, firms swings in their product demands in the future is much more elastic for a given price adjustment. Thus, the threat of future deflation (inflation) limits the extent to which firms can hold (choose to reduce) their reset price. As a result there is less of a variation in both the output gap and inflation. Furthermore, because this effect is increasing in $\theta$ and in the benchmark model the cost of inflation is increasing with $\theta$, the losses from the output gap, and in total, fall in our model as $\theta$ increases (whereas they rise in the Ravenna and Walsh case).

Under fiscal policy shocks the welfare results outlined above remain, however there are also gains from allowing the output gap to deviate and additional losses from the distortion that the shock itself has on aggregate consumption. Intuitively, allowing output gap variance to rise limits the fall in the welfare gap, $\hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t$ following a fall in government spending, thus the $\text{cov}(\hat{g}_t, \hat{x}_t)$ reflects negative losses (or welfare gains) in the in the loss function 3.9. Indeed, in our model the welfare gap and inflation are shown to behave in a similar fashion to taste and technology shock for the reasons outlined above.

### 3.8 Concluding Remarks

This chapter has investigated the implications that additional relative price effects in aggregate consumption, that were introduced in chapter 2, have on the welfare losses to society and the conduct of optimal monetary policy. Analogous to a typical welfare loss function (see Woodford 2003), increases in the price elasticity of
demand, amplify the welfare losses associated with a given variance of inflation as they generate larger swings in product demands. However in this, and the previous, chapter the variance of inflation is also reduced as the price elasticity of demand increases. This is because firms relative market share is much more volatile and hence less of a price alteration is made when they have a price setting opportunity.

Assuming the central bank follows a typical Taylor rule, welfare losses following economic shocks are shown to ultimately decrease as the economy becomes more competitive as by switching the relative weights on their product demands, households are able to move their aggregate consumption towards the optimal, efficient allocation. This is opposed to the standard literature, where in a typical New Keynesian model with constant returns to scale, welfare losses from inflation (and in total) increase as the product markets move towards perfect competition.

This chapter reinforces the seminal result of Rotemberg and Woodford (1999) that there are additional gains, to be made through a move from optimal discretion to optimal commitment. Furthermore, these gains are exaggerated in more competitive economies. There are two reasons why a credible policy plan delivers an even more beneficial result. Firstly, our additional channel on the output gap means that under discretionary policy the central bank places an even greater, ultimately suboptimal, emphasis on current output stabilisation. Moving to a policy plan eliminates this result as the central bank can now control future inflation expectations. Secondly, as firms know their product demands in the future will swing because of future price changes, they make smaller adjustments to their price today. A consequence of this effect is that the central bank can reduce deviations in inflation and the output gap by even more today at the smaller cost of greater deviations in later periods. As a result we see much smoother, more persistent responses of the output gap and inflation when the central bank is able to commit to a time consistent policy plan.
3.A Appendix 3

3.A.1

To derive an approximation of the representatives utility it is first necessary to introduce some additional notation. For any variable $X_t$, let $\bar{X}$ be its steady state value, $X_t^*$ be its efficient level (if this is relevant), let $\bar{X}_t = X_t - \bar{X}$ be the deviation of $X_t$ around a steady state and let $\bar{X}_t = \log(X_t/\bar{X})$ be the log deviation of $X_t$ around a steady-state value. Using a second order Taylor approximation, the variables $\bar{X}_t$ and $\bar{X}_t$ can be related as

$$
\frac{X_t}{\bar{X}} = 1 + \log \left( \frac{X_t}{\bar{X}} \right) + \frac{1}{2} \left[ \log \left( \frac{X_t}{\bar{X}} \right) \right]^2 = 1 + \bar{X}_t + \frac{1}{2} \bar{X}_t^2 \quad (3.17)
$$

Since we can write $\bar{X} = X_t - \bar{X}$, it follows that $\bar{X} \approx \bar{X} \left( \bar{X}_t + \frac{1}{2} \bar{X}_t^2 \right)$

Employing this notation, we are able to develop a second order approximation of utility

We start by focusing on the first term on the right hand side of (3.5) This can be approximated around a steady state as

$$
U \left( \bar{C}_t, \xi_t \right) \approx U \left( \bar{C}, 1 \right) + U_c \left( \bar{C}, 1 \right) \bar{C}_t + \frac{1}{2} U_{cc} \bar{C}_t^2 + U_{\xi} \left( \bar{C}, 1 \right) \xi_t + \frac{1}{2} U_{\xi \xi} \xi_t^2 + U_{c \xi} \xi_t \bar{C}_t \quad (3.18)
$$

As Ravenna and Walsh (2006) do, we consider the government to purchase of individual goods in the same proportions as households; $G_t = (1 - g_t) \bar{Y}_t$. The aggregate resource constraint $\bar{Y}_t = \bar{C}_t + G_t = \bar{C}_t + (1 - g_t) \bar{Y}_t$ or

$$
\bar{C}_t = g_t \bar{Y}_t \quad (3.19)
$$

\footnote{Since our output is driven by the demands of the economy, our price dispersion from aggregate consumption is transformed into price dispersion in aggregate output. Hence we retain the tilde notation.}
Using (3.17) and ignoring the terms greater than second order

\[ \tilde{C}_t \approx \hat{g}Y \left( \hat{g}_t + \tilde{Y}_t + \frac{1}{2} \left( \hat{g}_t + \tilde{Y}_t \right)^2 \right) \]

Which given our utility specification in (3.5) generates the following utility specification

\[
U \left( \tilde{C}_t, \xi_t \right) \approx U \left( \tilde{C}, 1 \right) + U_c \left( \tilde{C}, 1 \right) \hat{g}Y \left( \hat{g}_t + \tilde{Y}_t + \frac{1}{2} \left( \hat{g}_t + \tilde{Y}_t \right)^2 \right) \\
- \frac{1}{2} \sigma U_c \left( \tilde{C}, 1 \right) \hat{g}Y \left[ \hat{g}_t + \tilde{Y}_t + \frac{1}{2} \left( \hat{g}_t + \tilde{Y}_t \right)^2 \right]^2 \\
+ U_\xi(\tilde{C}, 1) \xi_s + \frac{1}{2} U_\xi \xi_s^2 + U_c \left( \tilde{C} \right) \hat{g}Y \xi_s \left( \hat{g}_t + \tilde{Y}_t + \frac{1}{2} \left( \hat{g}_t + \tilde{Y}_t \right)^2 \right)
\]

Since they are very small we ignore terms of order \( X^i \) for \( i > 2 \),

\[
U \left( \tilde{C}_t, \xi_t \right) \approx U \left( \tilde{C}, 1 \right) + U_c \left( \tilde{C}, 1 \right) \hat{g}Y \left( \left( 1 + \xi_s \right) \left( \hat{g}_t + \tilde{Y}_t \right) + \frac{1}{2} \left(1 - \sigma\right) \tilde{Y}_t^2 \right) \\
+ U_\xi(\tilde{C}, 1) \xi_s + \frac{1}{2} U_\xi \xi_s^2
\]

(3.19)

This concludes the utility from consumption section.

We can now move on to obtain an approximation for the disutility from work.

The second order Taylor expansion for \( V(N_t) \) is

\[
V(N_t) \approx V(\tilde{N}) + V_N(\tilde{N}) \tilde{N}_t + \frac{1}{2} V_{NN}(\tilde{N}) \tilde{N}_t^2
\]

(3.20)

Where our aggregate employment is given by

\[
\tilde{N}_t = \int_0^1 \tilde{n}_t(i) di
\]

For employment at firm \( i \),

\[
\tilde{n}_t(i) \approx \bar{n} \left[ \tilde{n}_t(i) + \frac{1}{2} \tilde{n}_t(i)^2 \right]
\]

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Each firm has production technology given by

$$\hat{n}_t(i) = \hat{y}_t(i) - \hat{A}_t.$$  

Thus

$$\bar{N}_t = \int_0^1 \hat{n}_t(i) di = \bar{n} \left\lceil \hat{n}_t(i) + \frac{1}{2} \hat{n}_t(i)^2 \right\rceil$$

$$= \bar{y} \left\lceil \int_0^1 \hat{y}_t(i) di - \hat{A}_t + \frac{1}{2} \int_0^1 \left( \hat{y}_t(i) - \hat{A}_t \right)^2 di \right\rceil$$

$$+ \frac{1}{2} V_{NN}(\bar{N}) \bar{y}^2 \left\lceil \int_0^1 \hat{y}_t(i) di - \hat{A}_t \right\rceil^2$$

Substituting this into 3.20

$$V(N_t) \approx V(\bar{N}) + V_N(\bar{N}) \bar{y} \left\lceil \int_0^1 \hat{y}_t(i) di - \hat{A}_t + \frac{1}{2} \int_0^1 \left( \hat{y}_t(i) - \hat{A}_t \right)^2 di \right\rceil$$

$$+ \frac{1}{2} V_{NN}(\bar{N}) \bar{y}^2 \left\lceil \int_0^1 \hat{y}_t(i) di - \hat{A}_t \right\rceil^2$$

(3.21)

Given the demand function that each firm faces, aggregate output $\tilde{Y}_t$ is defined as

$$\tilde{Y}_t = \int_0^1 \hat{y}_t(i)^{\frac{\theta - 1}{\theta}} di.$$  

This implies

$$\tilde{Y}_t = \int_0^1 \hat{y}_t(i) di + \frac{1}{2} \left( \frac{\theta - 1}{\theta} \right) var(\hat{y}_t(i))$$

Hence we have,

$$\left( \int_0^1 \hat{y}_t(i) di \right)^2 = \left( \tilde{Y}_t - \frac{1}{2} \left( \frac{\theta - 1}{\theta} \right) var(\hat{y}_t(i)) \right)^2 \approx \tilde{Y}_t^2$$

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Additionally not that

\[ \int_0^1 \left( \int_0^1 \hat{y}_t(i) \, di \right)^2 \, di = \left( \int_0^1 \hat{y}_t(i) \, di \right)^2 + \text{var}_i \hat{y}_t(i). \]

Therefore,

\[ \int_0^1 \hat{y}_t(i)^2 \, di \approx \hat{Y}_t^2 + \text{var}_i \hat{y}_t(i) \]

In addition,

\[ \hat{A}_t \int_0^1 \hat{y}_t(i) \, di \approx \hat{A}_t \hat{Y}_t - \frac{1}{2} \hat{A}_t \left( \frac{\theta - 1}{\theta} \right) \text{var}_i \hat{y}_t(i) \approx \hat{A}_t \hat{Y}_t \quad (3.22) \]

Using these results, (3.21) becomes

\[ V(N_t) \approx V(\bar{N}) + V_N(\bar{N}) \bar{y} \left[ \hat{Y}_t - \hat{A}_t - \frac{1}{2} \left( \frac{\theta - 1}{\theta} \right) \text{var}_i \hat{y}_t(i) \right] \]
\[ + V_N(\bar{N}) \bar{y} \left[ \frac{1}{2} \left( \hat{Y}_t^2 + \text{var}_i \hat{y}_t(i) \right) - \hat{A}_t \hat{Y}_t + \frac{1}{2} \hat{A}_t^2 di \right] \]
\[ + \frac{1}{2} V_{NN}(\bar{N}) \bar{y}^2 \left( \hat{Y}_t - \hat{A}_t \right)^2 \]

Combining terms and using the utility function (3.5)

\[ V(N_t) \approx V(\bar{N}) + V_N(\bar{N}) \bar{y} \left[ \hat{Y}_t - \hat{A}_t + \frac{1}{2} \left( \frac{1}{\theta} \right) \text{var}_i \hat{y}_t(i) + \frac{1}{2} (1 + \gamma) \left( \hat{Y}_t - \hat{A}_t \right)^2 \right] \]

(3.24)

Combining equations (3.21) and (3.24),
The steady state labour market equilibrium condition becomes

$$\frac{V_N(\tilde{\bar{N}}) \tilde{g}}{U_c} = \bar{\omega} = \frac{1}{\Psi R}.$$ 

We define $\Xi$ such that

$$1 - \Xi \equiv \frac{1}{g\Phi R}$$

then $V_N(\tilde{\bar{N}}) \tilde{g}$ can be written as $U_c(\tilde{\bar{C}}) \tilde{g}(1 - \Xi)$. As Ravenna and Walsh (2006) do, we assume that the term $\Xi$ is so small that terms such as $(g\Phi R)^{-1} \tilde{Y}^2 = (1 - \Xi)\tilde{Y}^2$ simply become $\tilde{Y}^2_t$. With this assumption we can now rewrite equation (3.25) as

$$U \left( \tilde{C}_t, \xi_t \right) - V(N_t)$$

$$\approx\quad U \left( \tilde{C}, 1 \right) - V(\bar{N})$$

$$+ U_c(\tilde{\bar{C}}) \tilde{g} \tilde{Y} \left( \left( 1 + \tilde{\xi}_t \right) \left( \tilde{g}_t + \tilde{\bar{Y}}_t \right) + \frac{1}{2} \left( 1 - \sigma \right) \tilde{\bar{Y}}^2_t \right)$$

$$+ U_c(\tilde{\bar{C}}) \tilde{g} \tilde{Y} \left( \left( 1 + \tilde{\xi}_t \right) \left( \tilde{g}_t + \tilde{\bar{Y}}_t \right) + \frac{1}{2} \left( 1 - \sigma \right) \tilde{\bar{Y}}^2_t \right)$$

$$- U_c(\tilde{\bar{C}}) \tilde{g} \tilde{Y} \left( \tilde{Y}_t - \hat{A}_t + \frac{1}{2} \left( 1 + \gamma \right) \left( \tilde{Y}_t - \hat{A}_t \right)^2 \right)$$

$$- U_c(\tilde{\bar{C}}) \tilde{g} \tilde{Y} \left( \tilde{Y}_t - \hat{A}_t + \frac{1}{2} \left( 1 + \gamma \right) \left( \tilde{Y}_t - \hat{A}_t \right)^2 \right)$$

Collecting terms we have,
\[ U(C_t, \xi_t) - V(N_t) \approx U(C, 1) - V(\bar{N}) + U_c(C, 1) \bar{Y}\left[3\hat{Y}_t + \hat{\xi}_t\right] \]
\[ + U_c(C, 1) \bar{g}_t \bar{Y}\left(\frac{1}{2}(1 - \sigma)\left(\hat{g}_t + \bar{Y}_t\right)^2 - \frac{1}{2}(1 + \gamma)\left(\hat{Y}_t - \hat{A}_t\right)^2\right) \]
\[ - \frac{1}{2}U_c(C, 1) \bar{Y}\left(\frac{1}{\theta}\right) \text{var}\hat{g}_t(i) \]
\[ + U_c(C, 1) \hat{g}_t + \frac{1}{2}U_c\xi\hat{\xi}_t^2 + U_c\bar{g}(C, 1) \bar{Y}\left[\left(1 + \hat{\xi}_t\right)\hat{g}_t + (1 - \Xi)\hat{A}_t\right] \] (3.26)

Define

\[ \hat{Z}_t = \frac{(1 + \gamma)\hat{A}_t + \hat{\xi}_t}{\sigma + \gamma} \]

and

\[ z^* = \frac{\Xi}{\sigma + \gamma} \]

Then the utility approximation can be written as

\[ U(\hat{g}_t, \hat{Y}_t, \hat{\xi}_t) - V(N_t) \approx U(\bar{Y}, 1) - V(\bar{N}) - \frac{1}{2}(1 - \sigma)U_c(\bar{Y}) \bar{Y}\left(\hat{Y}_t - \hat{Z}_t - z^*\right)^2 \]
\[ - \frac{1}{2}U_c(\bar{Y}) \bar{Y}\left(\frac{1}{\theta}\right) \text{var}\hat{g}_t(i) + \text{tip} \]

where \(\text{tip}\) denotes terms independent of policy and is given by

\[ \text{tip} = U_c(\bar{g})(C, 1) \bar{Y}\left[\left(1 + \hat{\xi}_t\right)\hat{g}_t + (1 - \Xi)\hat{A}_t\right] + \frac{1}{2}U_c\xi\hat{\xi}_t^2 \]
\[ + U_c(C, 1) \hat{\xi}_t - \hat{Z}_t - \frac{1}{2}U_c(C, 1) \bar{Y}\hat{Z}_t^2 \]

Recall our flexible price equilibrium

\[ \hat{Y}_t' = \left(\frac{1 + \gamma}{\gamma + \sigma}\right)\hat{A}_t - \frac{1}{(\gamma + \sigma)}\left[\hat{R}_t' - \hat{\xi}_t\right] - \frac{\sigma}{(\gamma + \sigma)}\hat{g}_t \]
\( \hat{Z}_t \) can be rewritten as

\[
\hat{Z}_t = \hat{Y}_t^f + \frac{1}{(\gamma + \sigma)} \left( \hat{R}_t^f + \hat{g}_t \right)
\]

With the assumed utility function,

\[
\log y_t(i) = \log \hat{Y}_t = \theta (\log p_t(i) - \log P_t)
\]

so that

\[
\text{var}_i \log y_t(i) = \theta^2 \text{var}_i \log p_t(i)
\]

Note the price adjustment mechanism involves a randomly chosen fraction \(1 - \omega\) of all firms acting optimally adjusting price each period. If we define \( \bar{P}_t \equiv E_t \log p_t(i) \) then

\[
\Delta_t \approx \omega \Delta_{t-1} + \left( \frac{\omega}{1 - \omega} \right) \pi_t^2
\]

**Proof:**

This Proof follows Woodford (2003)

Our prices are staggered in the discrete time Calvo model where a fraction \(\omega\) of prices remain unchanged each period. We let price dispersion at any point in time be measured by

\[
\Delta_t \equiv \text{var}_i \log p_t(i) \tag{3.27}
\]

The Calvo pricing model implies that the disturbance in prices \(\{p_t(i)\}\) consists of \(\omega\) times the distribution of prices in the previous period \(t - 1\), plus an atom size \((1 - \omega)\) at the optimal price \(p_t^*\) selected at time \(t\) by all firms. Defining,

\[
\bar{P}_t \equiv E_t \log p_t(i)
\]

one observes from this recursive characterisation of the distribution of prices at time
that

\[
\bar{P}_t - \bar{P}_{t-1} = E_i \left[ \log p_t(i) - \bar{P}_{t-1} \right] \\
= \omega E_i \left[ \log p_t(i) - \bar{P}_{t-1} \right] + (1 - \omega) \left( \log p_t^* - \bar{P}_{t-1} \right) \\
= (1 - \omega) \left( \log p_t^* - \bar{P}_{t-1} \right).
\]

Along the same line of reasoning the dispersion measure \( \Delta_t \) yields

\[
\Delta_t = \text{var}_i \left[ \log p_t(i) - \bar{P}_{t-1} \right] \\
= E_i \left\{ \left[ \log p_t(i) - \bar{P}_{t-1} \right]^2 \right\} - \left( E_i \log p_t(i) - \bar{P}_{t-1} \right)^2 \\
= \omega E_i \left\{ \left[ \log p_t(i) - \bar{P}_{t-1} \right]^2 \right\} + (1 - \omega) \left( \log p_t^* - \bar{P}_{t-1} \right)^2 - (\bar{P}_t - \bar{P}_{t-1})^2 \\
= \omega \Delta_{t-1} + \frac{\omega}{1 - \omega} (\bar{P}_t - \bar{P}_{t-1})^2 \tag{3.28}
\]

Finally, \( \bar{P}_t \) may be related to the Dixit - Stiglitz price the following log linear approximation

\[
\bar{P}_t = \log P_t + \mathcal{O} \left( \| \Delta_{t-1}^{\frac{1}{2}}, \phi, \tilde{\xi} \| \right) \tag{3.29}
\]

Note that in our aggregate supply relation (Phillips Curve), small policies through \( \phi \), in which the long run level of \( \hat{Y}_t \) remain near zero, must also be policies in which the inflation rate remains near zero. As a result the equilibrium inflation process must satisfy a bound of order \( \|, \phi, \tilde{\xi} \| \), together with the bound on the degree of preexisting price dispersion \( \Delta_{t-1} \). Note also that in our model the extent to which this result in reduced and diminishes as \( \theta \to \infty \) as the level of price dispersion and associated policy response diminishes.
Finally combining (3.29) and (3.28) yields\(^{43}\)

\[
\Delta_t \approx \omega \Delta_{t-1} + \left( \frac{\omega}{1-\omega} \right)\pi_t^2
\]

where we assume that \(\tilde{\sigma} \left( \| \Delta_{t-1}^2, \phi, \xi \| \right) = 0.\)

**This completes the proof.**

Furthermore, if \(\Delta_{t-1}\) is the initial degree of price dispersion, then

\[
\sum_{t=0}^{\infty} \beta^t \Delta_t = \left[ \frac{\omega}{(1-\omega)(1-\omega^2)} \right] \sum_{t=0}^{\infty} \beta^t \pi_t^2 + tip
\]

Combining this with (3.26), we can approximate the present discounted value of the Utility of the representative household as

\[
\sum_{t=0}^{\infty} \beta^t U_t \approx \tilde{U} - \sum_{t=0}^{\infty} \beta^t L^t
\]

where the associated losses from welfare are given as the final term, specifically.

\[
\sum_{t=0}^{\infty} \beta^t L^t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{\theta}{\kappa} \right) \hat{\pi}_t^2 + (\sigma + \gamma) \left( \hat{\gamma}_t + \hat{\phi}_t \right) \right]^2
\]

\[
\hat{Z}_t = \hat{Y}_t^f + \frac{1}{\gamma + \sigma} \hat{R}_t^f - \frac{\Xi}{\sigma + \gamma} \quad \text{and} \quad \kappa = \frac{(1-\omega)(1-\omega^2)}{\omega}
\]

which assuming that the central bank follow an interest rate peg (under flexible prices the interest rate does not deviate from its steady state level) and \(\frac{\Xi}{\sigma + \gamma}\) is so small it is ignored, With our fiscal distortions, \(\hat{g}_t\) included we have the following welfare function\(^{44}\),

\[
W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{\theta}{\kappa} \right) \hat{\pi}_t^2 + (\sigma + \gamma) \left( \hat{\gamma}_t + \frac{1}{(\gamma + \sigma)} \hat{g}_t \right) \right]^2
\]

\(^{43}\)Note that this is the same under our case as the standard case. (It is derived from the price evolution equation).

\(^{44}\)Note that this is identical to Ravenna and Walsh (2006).
or alternatively utility function of the central bank

\[ U = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \frac{\theta}{\kappa} \right) \hat{\pi}_t^2 + (\sigma + \gamma) \left( \hat{x}_t - \frac{1}{\gamma + \sigma} \hat{g}_t \right)^2 \right] \]

3.A.2

The problem for the central bank is to choose a path for \( \hat{R}_t \), and the implied paths for \( x_t \) and \( \hat{\pi}_t \), to minimise,

\[ U = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \hat{\pi}_t^2 + \lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right)^2 \right] \]

where \( \lambda = \frac{\kappa(\gamma + \sigma)}{\theta} \), subject to following constraints; the Phillips curve

\[ \hat{\pi}_t = \kappa (\gamma + \sigma) \hat{x}_t + \kappa \delta \hat{R}_t + \beta E_t \hat{\pi}_{t+1} \]

and the output gap

\[ \hat{x}_t = \hat{x}_t - \frac{\theta}{\kappa} [\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1}] \]

where

\[ \hat{x}_t = E_t \hat{x}_{t+1} - \frac{1}{\sigma} [\hat{R}_t - E_t \hat{\pi}_{t+1}] + \mu_t \]

and

\[ \mu_t = \left( \frac{1 + \gamma}{\gamma + \sigma} \right) \left[ E_t \hat{A}_{t+1} - \hat{A}_t \right] + \left( \frac{\gamma}{\gamma + \sigma} \right) \left[ E_t \hat{g}_{t+1} - \hat{g}_t \right] - \frac{\gamma}{\sigma (\gamma + \sigma)} \left[ E_t \hat{\xi}_{t+1} - \hat{\xi}_t \right] \]
Setting up the Lagrangian we have,

\[
-\frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t \left\{ \hat{\pi}^2_t + \lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right)^2 \right. \\
+ \Phi_t \left[ \hat{\pi}_t - \kappa (\gamma + \sigma) \hat{x}_t - \kappa \delta \hat{R}_t - \beta E_t \hat{\pi}_{t+1} \right] \\
+ \Psi_t \left[ \hat{x}_t - E_t \hat{x}_{t+1} + \frac{1}{\sigma} \left[ \hat{R}_t - E_t \hat{\pi}_{t+1} \right] - \mu_t \right] \\
+ \Upsilon_t \left[ \hat{x}_t - \hat{x}_t + \frac{\theta}{\kappa} \left[ \hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} \right] \right) \\
\]

Let $\Psi_t$, $\Phi_t$ and $\Upsilon_t$ be the Lagrangian multipliers associated with each of these constraints.

\[
\frac{\partial W}{\partial \hat{x}_t} = -\lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right) - \kappa (\gamma + \sigma) \Phi_t + \Upsilon_t = 0 \quad (3.30)
\]

\[
\frac{\partial W}{\partial x_t} = \Psi_t - \Upsilon_t = 0 \quad (3.31)
\]

\[
\frac{\partial W}{\partial \hat{\pi}_t} = -\hat{\pi}_t + \Phi_t + \frac{\theta}{\kappa} \Upsilon_t = 0 \quad (3.32)
\]

\[
\frac{\partial W}{\partial \hat{R}_t} = -\delta \kappa \Phi_t + \left( \frac{1}{\sigma} \right) \Psi_t = 0 \quad (3.33)
\]

From (3.31) and (3.33), $\Upsilon_t = \Psi_t = \delta \sigma \kappa \Phi_t$. Substituting this into (3.32) and (3.30) we have

\[
-\hat{\pi}_t + \Phi_t + \frac{\theta}{\kappa} \delta \sigma \kappa \Phi_t = 0
\]

\[
-\hat{\pi}_t + (1 + \theta \delta \sigma) \Phi_t = 0
\]

\[
\Phi_t = \frac{1}{(1 + \theta \delta \sigma)} \hat{\pi}_t
\]

into (3.30),

\[
-\lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right) + (\delta \sigma \kappa - \kappa (\gamma + \sigma)) \Phi_t = 0
\]
Finally,

\[-\lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right) + \frac{(\delta \sigma \kappa - \kappa (\gamma + \sigma))}{(1 + \theta \delta \sigma)} \hat{\pi}_t = 0\]

\[\frac{(\delta \sigma \kappa - \kappa (\gamma + \sigma))}{(1 + \theta \delta \sigma)} \hat{\pi}_t = \lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right)\]

\[\hat{\pi}_t = -\frac{(1 + \theta \delta \sigma)}{\kappa (\gamma + (\sigma(1 - \delta))) \lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right)\]

3.A.3

Setting up the Lagrangian,

\[-\frac{1}{2} E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\hat{x}_t^2}{\sigma} + \lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right)^2 \right\}

+ \Phi_t \left[ \hat{x}_t - \kappa (\gamma + \sigma) \hat{x}_t - \kappa \delta \hat{R}_t - \beta E_t \hat{\pi}_{t+1} \right]

+ \Psi_t \left[ \hat{x}_t - E_t \hat{x}_{t+1} + \frac{1}{\sigma} \left[ \hat{R}_t - E_t \hat{\pi}_{t+1} \right] - \mu_t \right]

+ \Gamma_t \left[ \hat{x}_t - \hat{x}_t + \frac{\theta}{\kappa} \left[ \hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} \right] \right]\]

Let \( \Psi_t, \Phi_t \) and \( \Gamma_t \) be the Lagrangian multipliers associated with each of these constraints. In period \( t \),

\[\frac{\partial W}{\partial \hat{x}_t} = -\lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right) - \kappa (\gamma + \sigma) \Phi_t + \kappa = 0 \quad (3.34)\]

\[\frac{\partial W}{\partial x_t} = \Psi_t - \Gamma_t = 0 \quad (3.35)\]

\[\frac{\partial W}{\partial \pi_t} = -\hat{\pi}_t + \Phi_t + \frac{\theta}{\kappa} \Gamma_t = 0 \quad (3.36)\]

\[\frac{\partial W}{\partial \hat{R}_t} = -\delta \kappa \Phi_t + \left( \frac{1}{\sigma} \right) \Psi_t = 0 \quad (3.37)\]
From (3.37) and (3.35), $\Psi_t = \delta \kappa \Phi_t$ substituting this into (3.36) we have

\[-\hat{\pi}_t + \Phi_t + \frac{\theta}{\kappa} \delta \kappa \Phi_t = 0\]

\[-\hat{\pi}_t + (1 + \theta \delta \sigma) \Phi_t = 0\]

\[\Phi_t = \frac{1}{(1 + \theta \delta \sigma)} \hat{\pi}_t\]

which substituted into (3.34) yields,

\[- \left( \frac{\hat{x}_t - \frac{1}{\sigma + \gamma} \hat{y}_t}{\sigma + \gamma} \right) + (\delta \kappa - \kappa (\gamma + \sigma)) \Phi_t = 0\]

Finally,

\[- \left( \frac{\hat{x}_t - \frac{1}{\sigma + \gamma} \hat{y}_t}{\sigma + \gamma} \right) + \frac{(\delta \kappa - \kappa (\gamma + \sigma))}{(1 + \theta \delta \sigma)} \hat{\pi}_t = 0\]

\[\frac{(\delta \kappa - \kappa (\gamma + \sigma))}{(1 + \theta \delta \sigma)} \hat{\pi}_t = \lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{y}_t \right)\]

\[\hat{\pi}_t = - \frac{(1 + \theta \delta \sigma)}{\kappa (\gamma + \sigma (1 - \delta))} \lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{y}_t \right)\]  \hspace{1cm} (3.38)

Under optimal commitment we must also consider the optimal responses of periods $t + i > t$ for $i > 0$.

\[\frac{\partial W}{\partial \hat{x}_t} = -\lambda \left( \frac{\hat{x}_{t+i} - \frac{1}{\sigma + \gamma} \hat{y}_{t+i}}{\sigma + \gamma} \right) - \kappa (\gamma + \sigma) \Phi_{t+i} + \Psi_{t+i} = 0\]  \hspace{1cm} (3.39)

\[\frac{\partial W}{\partial x_t} = \Psi_{t+i} - \beta^{-1} \Psi_{t+i-1} - \Psi_{t+i} = 0\]  \hspace{1cm} (3.40)

\[\frac{\partial W}{\partial \hat{\pi}_t} = -\hat{\pi}_{t+i} + (\Phi_{t+i} - \Phi_{t+i-1}) - \frac{1}{\kappa} \beta^{-1} \Psi_{t+i-1} + \frac{\theta}{\kappa} (\Psi_{t+i} - \Psi_{t+i-1}) = 0\]  \hspace{1cm} (3.41)
\[
\frac{\partial W}{\partial R_t} = -\delta \kappa \Phi_t + \left( \frac{1}{\sigma} \right) \Psi_t = 0 \quad (3.42)
\]

we first substitute (3.42) \((\Psi_t = \delta \sigma \kappa \Phi_{t+i})\) into (3.41) and (3.40)

\[
\begin{align*}
\frac{\partial W}{\partial \hat{\pi}_t} &= -\hat{\pi}_{t+i} + (\Phi_{t+i} - \Phi_{t+i-1}) - \delta \kappa \beta^{-1} \Phi_{t+i-1} + \frac{\theta}{\kappa} (\Upsilon_{t+i} - \Upsilon_{t+i-1}) = 0 \\
\hat{\pi}_{t+i} &= (\Phi_{t+i} - \Phi_{t+i-1}) - \delta \kappa \beta^{-1} \Phi_{t+i-1} + \frac{\theta}{\kappa} (\Upsilon_{t+i} - \Upsilon_{t+i-1}) \quad (3.43)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial W}{\partial \hat{x}_t} &= \delta \sigma \kappa \Phi_{t+i} - \beta^{-1} \delta \sigma \kappa \Phi_{t+i-1} - \Upsilon_{t+i} = 0 \\
\Upsilon_{t+i} &= \delta \sigma \kappa \Phi_{t+i} - \beta^{-1} \delta \sigma \kappa \Phi_{t+i-1} \quad (3.44)
\end{align*}
\]

Finally we substitute (3.44) into (3.39) and (3.43) (as \(\delta = 1\) with the cost channel).

\[
\begin{align*}
\lambda \left( \hat{x}_{t+i} - \frac{1}{\sigma + \gamma} \hat{g}_{t+i} \right) &= -\kappa (\gamma + \sigma) \Phi_{t+i} + \sigma \kappa \Phi_{t+i} - \beta^{-1} \sigma \kappa \Phi_{t+i-1} = 0 \\
\lambda \left( \hat{x}_{t+i} - \frac{1}{\sigma + \gamma} \hat{g}_{t+i} \right) &= -\kappa \gamma \Phi_{t+i} - \beta^{-1} \sigma \kappa \Phi_{t+i-1} = 0 \quad (3.45)
\end{align*}
\]

\[
\begin{align*}
\hat{\pi}_{t+i} &= (\Phi_{t+i} - \Phi_{t+i-1}) - \kappa \Phi_{t+i-1} \\
&\quad + \frac{\theta}{\kappa} (\sigma \kappa \Phi_{t+i} - \beta^{-1} \sigma \kappa \Phi_{t+i-1} - \sigma \kappa \Phi_{t+i-1} - \beta^{-1} \sigma \kappa \Phi_{t+i-2}) \\
\hat{\pi}_{t+i} &= \Phi_{t+i} - (1 + \kappa) \Phi_{t+i-1} + \theta (\sigma \Phi_{t+i} - \beta^{-1} \sigma \Phi_{t+i-1} - \sigma \Phi_{t+i-1} + \beta^{-1} \sigma \Phi_{t+i-2}) \\
\hat{\pi}_{t+i} &= (1 + \theta \sigma) \Phi_{t+i} - (1 + \beta^{-1} \kappa + \theta \sigma (\beta^{-1} + 1)) \Phi_{t+i-1} + \theta \sigma \beta^{-1} \Phi_{t+i-2} \quad (3.46)
\end{align*}
\]

We can define (3.45) and (3.46) for any period \(t\) as, (3.47) and (3.48)

\[
\lambda \left( \hat{x}_t - \frac{1}{\sigma + \gamma} \hat{g}_t \right) = -\kappa \gamma \Phi_t - \beta^{-1} \sigma \kappa \Phi_{t-1} = 0
\]

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\[
\hat{x}_t = -\frac{\kappa \gamma}{\lambda} \Phi_t - \frac{\beta^{-1} \sigma \kappa}{\lambda} \Phi_{t-1} + \frac{1}{\sigma + \gamma} \hat{y}_t
\]  
(3.47)

\[
\hat{\pi}_{t+i} = (1 + \theta \sigma) \Phi_{t+i} - (1 + \beta^{-1} \kappa + \theta \sigma (\beta^{-1} + 1)) \Phi_{t+i-1} + \theta \sigma \beta^{-1} \Phi_{t+i-2}
\]

\[
\hat{\pi}_t = (1 + \theta \sigma) \Phi_t - (1 + \beta^{-1} \kappa + \theta \sigma (\beta^{-1} + 1)) \Phi_{t-1} + \theta \beta^{-1} \sigma \Phi_{t-2}
\]

\[
\Phi_t = \frac{\hat{\pi}_t + (1 + \beta^{-1} \kappa + \theta \sigma (\beta^{-1} + 1)) \Phi_{t-1} - \theta \beta^{-1} \sigma \Phi_{t-2}}{(1 + \theta \sigma)}
\]  
(3.48)
Chapter 4

Varying Term Loan Contracts, Spillovers and Macroeconomic Dynamics

4.1 Introduction

The approach to modelling nominal inertia that generated the short run real effects in Chapter 2, and a welfare problem for the central bank in chapter 3, were based on the staggered price adjustment of Calvo (1983). In this setting, firms face a constant probability of price adjustment and do not know the length of contract fixation \textit{ex ante}. In the other core strand of the literature, nominal rigidities are introduced due to Taylor (1980) where a single price or wage contract is fixed for a predetermined number of periods and hence the duration is known \textit{ex ante}.

More recently, Dixon and Kara (2010, 2011) have proposed a Generalised Taylor Economy (GTE). They allow for an economy with many different sectors that vary in terms of the duration for which the nominal contracts remain fixed\textsuperscript{1}. Their results indicate an increase in the persistence of output following monetary shocks, an outcome which is initiated by the spillover effect between sectors. These findings also provide a key link between the microeconometric data on prices of Taylor (1993), Bils and Klenow (2004), Laurent and Le Bihan (2004) and Le Bihan and Matheron

\textsuperscript{1}Also see Dixon and Le Bihan (2012), Carvalho (2006), Kara(2010), Mash (2004), Sheedy (2007) and Wolman (1999) for other papers that recognise the significance of heterogeneity on output and inflation dynamics
(2011) with the persistence of output observed in aggregate data\(^2\).

This chapter extends the contract mechanism proposed by Dixon and Kara to also incorporate imperfect credit markets, with loan and deposit contracts able to vary in terms of their interest rate fixation. Specifically, our financial sector is modelled with a commercial bank which is divided into many individual branches who receive deposits from households and offer loans to firms. Each branch specialises in offering a fixed term contract of a particular duration with a cost borrowing and savings rate that remain unchanged during the contract life.\(^3\) The consequence of this is that movements in the policy rate may not be immediately transferred to the real sector, but instead there may be an incomplete short-term pass-through of monetary policy to the other interest rates. This concept is widely supported by empirical evidence. For example, Sander (2002) Kleimeier (2002), Hofmann and Mizen (2004), De Bondt (2005) and Scharler (2008) document that a significant proportion of loan rates are sluggish in their response to interest rate movements made both in the money market and at central banks\(^4\). Additionally, Mojon (2001) finds an incomplete short-term pass-through for deposit rates, notably for savings deposits. To reinforce these results, in section 4.2 we provide a simple overview of UK loan and deposit rates between January 2004 and January 2012, as well as a categorisation and breakdown of the proportion of loans that are on a fixed rate for a particular duration. The incomplete pass through of interest rates has implications for monetary policy since it limits the core policy tool of the central bank\(^5\). As a result, it is important to model the macroeconomic implications of this phenomenon from a theoretical perspective. To introduce loan rate inertia, Güntner (2011), Hülsewig,

\(^2\)Both Chari, Kehoe and McGrattan (2000) and Ascari (2000) find that nominal rigidities cannot explain the level of persistence found in aggregate data.

\(^3\)So, there is one branch, for example, offering one-year loan contracts, and other branches offering longer period contracts. Decision for pricing each different duration loan contracts are made independently by each branch, however as we discuss later the commercial bank as a whole can also distribute losses from riskier contracts across all branches.

\(^4\)For a review see De Bont (2005).

\(^5\)See Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) for recent macroeconomic models that capture unconventional monetary policy instruments that have been used in the wake of the financial crisis of 2008.
Mayer, et al. (2009) employ a Calvo (1983) type of stickiness in loan rates, whereas Gerali, Neri, Sessa and Signoretti (2010) introduce quadratic adjustment costs to prevent some firms from altering their borrowing costs each period. Inertia on the demand side is introduced by Christiano, Motto and Rostagno (2009) who allow for a proportion of deposits to be fixed each period.

In this chapter, GTE contracts are shown to drive interest rate inertia. The relative size and duration of contracts offered by each branch, is taken to match the relative sector weights and durations, given from our UK data, in section 4.2. Just as the micro data on prices provided an empirical justification for the duration of contracts and the relative size of sectors in Dixon and Kara (2010, 2011) we use this evidence, and an equivalent approach, to structure our financial sector. One of the advantage of using a range of Taylor (rather than Calvo or quadratic adjustment costs) contracts is that contract lengths are known by the financial intermediary. Furthermore, knowledge of contract length allows financial intermediaries to pass on unexpected losses or profits to new contracts, through an adjustment to the interest rate premium, since expected returns can be calculated.

It is not only inertia in the cost of borrowing that causes movements in loan rates to deviate from the policy rate. A considerable amount of empirical evidence indicates that the dynamics of the cost of borrowing are crucially determined by an observed countercyclical finance premium, that varies for loan contracts of different risk, (see for example, De Grave (2008), Nolan and Thoenissen (2009), Aliag-Diazand Olivero (2011)). Indeed, over the past two decades there has been significant advances in research into the formation and influence that financial risk has on the business cycle. Moreover, given the scale of the recent financial crisis and the subsequent economic recession, one can appreciate the importance of accurately modelling the linkages between financial markets and the real economy, especially the impact that private credit has on the transmission of economic shocks (see for example Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Gilchrist
(2004), Faia and Monacelli (2007), De Fiore and Tristani (2009), Christiano, Motto and Rostagno (2010), Carlstrom, Fuerst and Paustian (2010), Curdia and Woodford (2010), Gertler and Kiyotaki (2010), Pesaran and Xu (2011), and Agénor, Bratsiotis and Pfajfar (2012). In these papers credit market imperfections are introduced due agency costs, asymmetric information or collateral constraints which act as a financial accelerator and lead to an amplification of the business cycle.

The model introduced in this chapter is closely related to the theoretical literature that generates these credit frictions from the assumption that firms are involved in risky production (see Bernanke, Gertler and Gilchrist (1999), Faia and Monacelli (2007), De Fiore and Tristani, (2009), Christiano, Motto and Rostagno (2010), Pesaran and Xu (2011) and Agénor Bratsiotis and Pfajfar (2012). Specifically, we follow Agénor, Bratsiotis and Pfajfar (2011) by assuming that in equilibrium the expected income from lending to firms, given their positive probability of default, is equal to the cost of borrowing these funds from households so that the commercial bank breaks even each period. Since there is risk of default the loan rate will be a time varying markup over the deposit rate or, alternatively, an external finance premium which depends on the level of collateral, and the probability of default.

The break even condition for the bank must account for a proportion of contracts that cannot be adjusted since we allow for the possibility that each contract can be agreed at either a fixed rate or flexible rate, with potentially many fixed rate sectors. For example, following an adverse shock the losses incurred from a fixed financial contract may be greater than the losses that were anticipated when the loan rate was set. Since a branch of the commercial bank cannot adjust a loan rate mid contract these losses must be accounted for with an additional markup on the contracts up for renegotiation. Significantly, this feature is not present in the other sticky rate models that assume imperfect competition,⁶ where there is no interaction or

spillovers between sectors.

To summarise a financial contract up for renegotiation will be driven by three factors; (a) the anticipated working capital costs of the firm, which stem from the expected levels of labour demand, productivity, wage costs and borrowing costs over the lifetime of the contract; (b) the expected default risk and default losses associated with each firm’s idiosyncratic risk (as introduced by Agénor, Bratsiotis and Pfajfar (2012)) which affects the level of collateral the bank is expected to receive and; (c) the unexpected default losses which are not accounted for by preset contracts that are passed on to new contracts to maintain the commercial bank’s zero profit condition, given that fixed contracts of different durations imply a spillover of losses from one branch to another.

Within this framework it is possible to examine the transmission and dynamics of loan rates, specifically the loan rate risk weight or spread. In particular we focus on the spillover effects that may exist between sectors and their impact on aggregate variables. With this established, the response of aggregate variables as well as the transmission of monetary policy according to a Taylor rule will be examined. In line with the incomplete pass through literature, the impulse responses of output, inflation and all interest rates are shown to be smoother and more persistent when a proportion of interest rates are unable to react following an economic shock. Additionally, in line with the external finance premium literature, we observe a countercyclical external finance premium or spread between the loan and deposit rates. When both the incomplete-pass through and external finance premium channels are combined we observe an ambiguous result with respect to the dynamics of the spread between the policy rate and the loan rate. Following shocks to the policy rate, aggregate technology and the collateral the bank receives from firms, inertia in deposit rates generates initially a procyclical spread between the policy rate and deposit rates. For monetary policy shocks that raise the policy and deposit rates, inertia in fixed loan contracts dominates the countercyclical external
finance premium so that initially the policy rate to loan rate spread is procyclical. In contrast, following technology shocks, and to a greater extent credit shocks, this spread is countercyclical since the change in default risk dominates the inertia in deposit rates. Finally, following adverse credit shocks a significant proportion of losses are passed on to new loan contracts. This effect amplifies the fall in output, rise in inflation and loan rate spread for a number of periods.

The remainder of this Chapter will be structured in the following way. The next section will present some stylised facts on loans to private sector non-financial firms between 2004 and 2012. In Section 4.3 we introduce our model economy. Section 4.5 provides some simulated results whilst section 4.6 briefly concludes.

### 4.2 Stylised Facts on Loans to Private Sector Non-Financial Firms

In this section we present some stylised facts on the dynamics of loan rates to private non-financial firms to motivate the GTE approach to inertia in the financial sector. The monthly data set used begins from 2004 and is provided by the Bank of England. Within this period we are able to capture the dynamics of loan rates following the fall 450 basis points fall in base rate that began in September 2008.

#### 4.2.1 Outstanding Loans

Figures (4.1-4.3) display information on the monthly average of all loans currently on the market to private non financial companies. The data in these figures is disaggregated in terms of length of fixation: Figure 4.1 displays the loan rates offered to UK private sector non financial companies; Figure 4.2 indicates the proportion of outstanding loans split between flexible and fixed rates; Finally, figure 4.3 indicates the distribution of fixed rate contracts according to length of fixation.

Figure 4.1 displays the set of average loan rates to private sector non finan-
Figure 4.1: UK Loan Rates to Private Sector Non Financial Firms

cial firms in the UK between January 2004 and January 2012. These rates are
categorised by the length of time for which they remain fixed. There is clearly
an asymmetric change when base rate falls 450 basis points. After this point all
markups are higher and there is greater persistence in the rates with a longer inter-
est rate fixation. As a result, we see the spread between fixed and flexible rates also
increases. Given that there has also been a huge amount of quantitative easing since
the end of 2008 which should in theory bring these rates back towards the central
bank rate, making the above even more of an interesting result.
From figure 4.2 we can calculate the average proportion of fixed and flexible loans in the UK over the sample period as 42.3% and 57.7% respectively. Interestingly there also appears to be a move towards move fixed contracts recently\(^7\). Combining these sector sizes with the variation in countercyclical spreads from figure 4.1, it is clear that both fixed and flexible loan contracts have a role to play in determining the aggregate cost of borrowing and the transmission of monetary policy though this channel. In the context of our model it is the average contract length that is important since this will determine the contract lengths in our Generalised Taylor Economy (GTE).

\(^{7}\)This may be due to the increased liquidity issues at commercial banks who need to assure credit through fixed maturities.
Figure 4.3: Breakdown of Proportions of Fixed Rate Contracts Based on Initial Fixation

Figure 4.3 provides a breakdown of loans with a fixed rate. Over this time period there is an average of 75.2% loans that are one year or less, 9.1% are 1-5 year long and 15.8% are fixed for more than 5 years. The proportion of total loans a year or less is large (89.5%) are either flexible or fixed for a year or less whilst the remaining loans have a much longer interest rate fixation.

4.2.2 New Loans

Figures 4.4 and 4.5 display information on the monthly average of all new loans currently in the market to private non financial firms. This differs from figures 4.2 and 4.3 which display the proportion of loans that are currently on the market. As we will see, the relationship between new and outstanding loans will help us to ascertain a more specific length of interest rate fixation for each group of contracts.
In figure 4.4 the average over the displayed period is 62.55% fixed and 37.45% flexible.
4.2.3 Calculating the Relative Length of Contracts.

Since we have information on the breakdown of total existing loans in the economy and total new loans, we can calculate the relative contract lengths for each loan type as

\[
\text{Relative contract length} = \frac{\text{Proportion of current loans in category}}{\text{Proportion of new loans in category}}.
\]

Intuitively, longer contracts will have more current loans relative to new contracts since new contracts will be issued less often\(^8\). The results are displayed in table 4.1

---

\(^8\)Ideally we would wish to have more precise data on the length of contract (for example in line with the Bils Klenow (2004) dataset on prices). Naturally using the relative length of contract assumes that current contracts are not amended. However, we do get a more specific idea of where the average contract lies in the ranges we have.
Table 4.1 tells us that if the contracts of less than a year last for 82% of a year then the remaining contracts last for 1.98 years and 4.26 years respectively. If instead we assume that one year loans are fixed in terms of rate and maturity for 1 year\(^9\) we have the 1-5 year loans at 2.22 years the loans in the initial fixation between one and five years and five year plus loans category with a loan fixed for 5.202 years\(^{10}\). Overall the weighted average for the fixed sector is,

\[
91.7 \times 1 + (2.2 \times 4.6) + (5.2 \times 3.7) = 121
\]

or 1.21 years. Finally, to calculate the average inertia of the interest rate of loan contracts we must also refer to the size of the fixed relative to the flexible sector. Weighted with the total amount of fixed contracts in the economy the average duration of fixed maturity is 42.3*1.2=0.5, a maturity of around six months or 2 quarters, (Huelsewig, Mayer and Wollmershaeuser (2006) find this to be 1.5 quarters for Germany whilst Gerali et al (2010) find similar results for the Euro area).

### 4.2.4 Deposit Rates

Next we focus on the deposit rates faced by households.

\(^{9}\)It is likely that the majority of the rates that are fixed to private non financial firms are fixed for one year. However we can alter this start point, so long as the relative contract lengths remain unchanged.

\(^{10}\)Since 1.98/0.82=2.22 and 4.26/0.82=5.202
Figure 4.6: UK Household Deposit Rates

Figure 4.6 shows the quarterly percentage of UK financial institutions’ deposit rates offered to households (in sterling millions) not seasonally adjusted. The above graph indicates that deposits (as with loans) behave more sluggishly for contracts which are fixed for a longer duration. Furthermore, deposits have also fallen much less than the base rate overall but more than loans following the base rate cuts at the end of 2008 / beginning of 2009. As a result, the cost of funds from deposits have not become as cheap for commercial banks as the policy rate may suggest. Thus, contracts that continue to lend at a high fixed rate *ceteris paribus* are not necessarily more profit since they continue to borrow at the higher household savings rate.

4.3 The Model

4.3.1 Notation and Institutions

We introduce the following notation in this model.
<table>
<thead>
<tr>
<th>Notation</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>Time period</td>
</tr>
<tr>
<td>$f$</td>
<td>Firm</td>
</tr>
<tr>
<td>$h$</td>
<td>Household</td>
</tr>
<tr>
<td>$i$</td>
<td>Sector $i$ where contracts are held for $i$ periods</td>
</tr>
<tr>
<td>$T$</td>
<td>Longest duration in sector $i$</td>
</tr>
<tr>
<td>$j$</td>
<td>Cohort</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>Relative size of sector $i$</td>
</tr>
<tr>
<td>$X_t$</td>
<td>Reset variable $X$ at time $t$</td>
</tr>
<tr>
<td>$X_{ijf,t}$</td>
<td>Value of Variable $X$ for firm $f$ in cohort $j$ in sector $i$ at time $t$</td>
</tr>
<tr>
<td>$b$</td>
<td>Commercial bank</td>
</tr>
<tr>
<td>$b_i$</td>
<td>Branch of commercial bank that lends to sector $i$</td>
</tr>
<tr>
<td>$s$</td>
<td>Number of periods since resetting variable</td>
</tr>
<tr>
<td>$X_t$</td>
<td>Deviation of variable $X$ from its steady state value at time $t$.</td>
</tr>
</tbody>
</table>

Table 4.2: Notation

Our model economy consists of a private sector with five types of agents: a representative infinitely lived household, a continuum of firms $f \in (0, 1)$, a commercial bank, the individual branches of the commercial bank and a central bank. The representative household that owns both firms and commercial banks, provide labour to firms and consumes a basket of all goods produced in the economy.

### 4.3.2 Firms

In this section we describe the standard optimal behavior of firms\textsuperscript{11}. We assume that each firm produces a single differentiated good, $y_f$ which are combined to produce a final composite good $Y$ each period.

Output is defined by the following composite good,

$$ Y_t = \left( \int_0^1 y_{f,t}^{(\theta - 1)/\theta} \, df \right)^{\theta/(\theta - 1)} \quad \theta > 1 $$

(4.1)

where the parameter $\theta$ governs the price elasticity of demand for individual goods.

We assume that the final good produced is used entirely for consumption. The

\textsuperscript{11}At this point all firms are unconstrained by nominal rigidities and are therefore able to act optimally
demand for the output of firm $f$ is given by

$$y_{f,t} = \left( \frac{P_{f,t}}{P_t} \right)^{-\theta} Y_t$$  \hspace{1cm} (4.2)

where

$$P_t = \left( \int_0^1 P_{f,t}^{-\theta} \, df \right)^{1/(1-\theta)}.$$  \hspace{1cm} (4.3)

Production is based on firm specific labour with the constant returns to scale production function given as

$$y_{f,t} = Z_{f,t} N_{f,t}$$  \hspace{1cm} (4.4)

where $Z_{f,t}$ is firm specific productivity$^{12}$. From the production function of firms we can define our labour demand as

$$N_{f,t}^d = \frac{y_{f,t}}{Z_{f,t}}$$  \hspace{1cm} (4.5)

Firms select, $P_{f,t}$, $y_{f,t}$ and $N_{f,t}^d$ to maximise profits subject to (4.2) and (4.3). Furthermore, we assume that the firm must borrow from the financial intermediary at the gross nominal loan rate $(1 + r_{f,t}^L)^{14}$.

The problem for the firm is to

$$\max \frac{P_{f,t}}{P_t} y_{f,t} - (1 + r_{f,t}^L) \frac{W_{f,t}}{P_t} \frac{y_{f,t}}{Z_{f,t}}$$

subject to

$$y_{f,t} = \left( \frac{P_{f,t}}{P_t} \right)^{-\theta} Y_t$$

---

$^{12}$Production $Z_{f,t}$ will include an aggregate component and an idiosyncratic component. This will be outlined later in the paper.

$^{13}$For simplicity we assume a constant returns to scale production function.

$^{14}$See the cost channel literature of Ravenna and Walsh (2006) and the models outlined in chapters 2 and 3 for more details of loan rates entering into firms marginal costs.
This yields the following first order conditions, and solution under flexible prices\textsuperscript{15},

\[ P_{f,t} = \frac{\theta}{(\theta - 1)} (1 + r_{f,t}^L) W_{f,t} Z_{f,t}^{-1} \]  

(4.6)

Since we assume that there is constant returns and that production comes from only labour and technology, optimal prices are a constant markup \( \frac{\theta}{(\theta - 1)} \) over the wage, nominal loan rate and inverse of technology (which together define our marginal cost). Using our above definition of prices (4.6) and our demand function (4.2) we have

\[ y_{f,t} = \frac{\theta}{(\theta - 1)} Z_{f,t}^{\theta}(1 + r_{f,t}^L)^{-\theta} \left( \frac{W_{f,t}}{P_t} \right)^{-\theta} Y_t \]  

(4.7)

which from our labour demand (taken from the inverse of the production function (4.2)) is

\[ N_{ft}^d = \frac{\theta}{(\theta - 1)} Z_{ft}^{\theta-1}(1 + r_{f,t}^L)^{-\theta} \left( \frac{W_{f,t}}{P_t} \right)^{-\theta} Y_t \]  

(4.8)

Product demands and employment depend upon the levels of productivity, the cost of borrowing, the real wage and the total output in the economy.

### 4.3.3 Financial Intermediation and the Structure of Loan Contracts

We employ a version of the GT wage-contract model of Dixon an Kara’s (2010, 2011) in a model that focuses on credit markets with loan contracts of different duration taken by firms. We assume that in this model firms make loans from banks in order to finance their working capital, namely wage payment to workers. When a firm has the opportunity to renegotiate and fix for a finite period their financial contract, they simultaneously reset their wage contracts; as a result loan contracts are fixed for the same duration as wage contracts\textsuperscript{16}. This has two advantages: first, the timing

\textsuperscript{15}See Appendix 4.A.1 for derivation

\textsuperscript{16}This is except for the flexible loan contracts with are assumed to only have the opportunity to alter their wages every 5 periods. This allows us to have a level of wage stickiness consistent with the related literature.
of contracts is much easier to observe and thus follows the structure of Dixon and Kara (2010, 2011); second, the firm is able to identify the expected nominal cost for each period, a feature not available with Calvo (1983) contracts where contract duration is unknown at the micro level.

We consider a representative bank $b$ consisting of many different branches $i...T$, each specialising in a specific duration of loan contracts (i.e. from flexible loan to the longest fixed loan contracts $T$). The representative bank is risk-neutral and makes zero profit each period\textsuperscript{17}. To determine the loan rate and maintain zero profit the bank in each period requires each of the branches $b_i$ to work in collaboration with the whole bank $b$. We assume that the size and number of branches $b_i$ directly matches the equivalent market share of firms who take the contract (in terms of contract length) in the goods market. Essentially, the size and duration of the contract offered by each branch has arisen from the demands in the goods market, which in turn is reflected in the demand for labour and hence loan contracts.

Given the idiosyncratic nature of each firm, the problem for each branch $b_i$ of our representative bank is to calculate the expected risk and losses from default. They then apply an appropriate risk premium to the loan rate which cover these anticipated costs. In the steady state this is enough to ensure that each branch, and therefore the bank as a whole, makes zero profit.

However, if there are alterations to the economic conditions the zero profit condition for each branch may be challenged. Following aggregate shocks the actual level of default may differ from the level expected when an individual contract was agreed. As a result, it is possible that branches may make a profit or loss from cohorts with a predetermined loan rate markup. The problem for the (whole) bank $b$ is to make sure that the aggregate return on all contracts is at the break even level. Since predetermined contracts cannot be altered\textsuperscript{18}, there is an additional premium

\textsuperscript{17}We assume that the bank has access to the central bank funds which can only be used to fund liquidity issues. These liquidity issues may arise when current losses are guaranteed by additional markups on new fixed term contracts. (See spillover effects section)

\textsuperscript{18}Note that only new contracts can have their loan rate adjusted.
(or bursary\textsuperscript{19}) that is added to new contracts to maintain the zero profit condition and cover the above variation. This additional premium, which has a steady state value of zero, is split between branches according to the relative weights of each sector and passed on to new contracts. When a firm defaults we assume the financial intermediary incurs losses, after this the firm is taken over and the agreed terms of the contract remain in place. Alternatively we could think of this as a bailout by the bank. This assumption guarantees that cohort sizes do not fluctuate following deviations in default levels\textsuperscript{20}.

In particular, each firm $f$ in sector $i$ borrows from a branch $b_i$ of the financial intermediary for a fixed number of periods $i$, to finance all of their labour costs, $W_{ijf,t}N_{ijf,t}$. The timing of this decision is taken along with the rest of their cohort $j$, thus we assume that each branch, $b_i$ agrees a new contract with one cohort $j$ in sector $i$ each period. The loan rate is fixed for the life of the contract and each period the firm borrows the level required to meet their labour costs at this predetermined rate. These labour costs are made up of the wage, which is also fixed for the life of the contract and labour which is driven by the product demands the firm faces each period. We define the number of sectors in the whole economy as simply $T$, with banking sectors $i = 1...T$. Within each of these sectors there is a standard Taylor process with overlapping contacts of a specified length. The market share of the $T$ sectors with uniform loan rates (where prices $p_f$ are equal for all $f \in [0, 1]$) are given by $\alpha_i$ with $\sum_{i=1}^{T} \alpha_i = 1$, where the $T$ vector $(\alpha_i)^T_{i=1}$ being denoted by $\alpha$, where $\alpha \in \Delta^{T-1}$. Since, the sole commonality within a sector is the length of the contract, we can, without loss of generality, suppose that in each sector $i$ there are $i$-period contracts, so that the longest contracts are $T$.

Within each sector $i$, each firm is matched with a firm-specific union: there are $j$ equal sized cohorts $j = 1...i$ of unions and firms. Each cohort sets their wage and

\textsuperscript{19}A bursary represents a reduction in new loan rates when the losses from preset contracts are less than anticipated.

\textsuperscript{20}This assumption is essential since allowing for fluctuations in sector sizes in this setup will leave the commercial bank in disequilibrium between deposits and loans.
loan rate, which remains fixed for \( i \) periods: one cohort moves each period\(^{21} \). The log linearised price in sector \( i \) is the average over the prices set in each cohort \( j = 1, 2, \ldots, i \),

\[
\hat{P}_i = \frac{1}{i} \sum_{j=1}^{i} \hat{p}_{ij}
\]

Similarly, the average price in the whole economy is the weighted aggregation of all sectors

\[
\hat{P} = \sum_{i=1}^{T} \hat{p}_i
\]

which can be written in terms of cohort prices as

\[
\hat{P} = \sum_{i=1}^{T} \sum_{j=1}^{i} \alpha_i \hat{p}_{ij}
\]

Since we assume that there are no sector or cohort specific aspects of technology or preferences the sole commonalty within a sector is the length of the contract and the sole commonalty within the cohort is the timing of the contract. We extend the baseline model so that when firms have the opportunity to set their wage they are able to reset their loan rate. Thus, the timing decision outlined above not only applies to the resetting of wages but also the loan rate.

**The Balance Sheet of the Commercial Bank and the Zero Profit Condition**

Following the portfolio allocation decisions of households, the commercial bank receives deposits. Each branch of the commercial bank provides credit to firms by paying interest on household deposits and savings. The length and size of the savings contract directly matches the loan contract at each branch. Specifically, each period the savings deposits made by households at time \( j \) which are fixed for \( i \) pe-

\(^{21}\) For details of the decomposition of the price index see Appendix 4.A.2 or ECB working paper 489 (Dixon and Kara, 2005).
periods, match the loans for firms in sector $i$ cohort $j$, $D_{ij,t} = L_{ijt}$ and if both $r^D_{ij,t}$ and $r^L_{ij,t}$ are predetermined at time $t$ they will remain fixed for $i$ periods. The assumption is essentially that households agree to a fixed rate on a proportion of their savings. The actual element of deposits that go to this form of savings is driven by the credit demands of the firm. A consequence of this is that total deposits in each sector equal total sector loans, $D_{i,t} = L_{i,t}$ and aggregate deposits equal aggregate loans, $D_t = L_t$.

The zero profit condition in the deposit market requires that the return on deposits is equal to central bank rate, $r^D_t = r^{CB}_t$ at the time of contract renegotiation.

We assume that each household invests in both savings deposits, where the interest rate is fixed for more than one period ($i > 1$), and time deposits where the interest rate is fixed for only one period ($i = 1$). Both types of deposits are offered at central bank rate, however savings deposits ($i > 1$) remain fixed at that rate for $i$ periods. Aggregating over sector $i$ at time $t$ we have the following definitions for the total return on sectoral deposits and loans.

$$\frac{(1 + r^D_t) D_i}{i} = \sum_{j=1}^{i} (1 + \tilde{r}^D_{ij,t+1-j}) D_{i,t} \quad (4.9)$$

$$\frac{(1 + r^L_t) L_i}{i} = \sum_{j=1}^{i} (1 + \tilde{r}^L_{ij,t+1-j}) L_{i,t} \quad (4.10)$$

For example, for the branch that delivers 1 year contract at time $t$ where $i = 4$,

$$\frac{(1 + r^L_4) L_4}{i} = \frac{1}{4} \left[ (1 + \tilde{r}^L_L) L_{4,t} + (1 + \tilde{r}^L_{t-1}) L_{4,t} + (1 + \tilde{r}^L_{t-2}) L_{4,t} + (1 + \tilde{r}^L_{t-3}) L_{4,t} \right] \quad (4.11)$$

is going to be made up of loans to four cohorts. Each cohort has a cost of borrowing $\tilde{r}^L_{ij,t+1-j}$, where we define $\tilde{r}^L_{ij,t}$ as the rest loan rate in sector $i$, at time $t$. Since $3^2$ out of the 4 cohorts have a previously set loan rate the aggregate rate in the sector

\[\text{Note that since only one cohort resets in sector } i \text{ we do not need the } j \text{ subscript.}\]
is going to be a function of previously determined rates, specifically,

\[
    r^L_{i,t} = \frac{1}{i} \sum_{j=1}^{i} \tilde{r}^L_{ij,t+1-j}
\]

\[
    r^L_{4t} = \frac{1}{4} \sum_{j=1}^{4} \tilde{r}^L_{4,t+1-j}
\]

\[
    = \frac{1}{4} \left[ \tilde{r}^L_{4,t} + \tilde{r}^L_{4,t-1} + \tilde{r}^L_{4,t-2} + \tilde{r}^L_{4,t-3} \right]
\]

We use the tilde notation to define a reset level for any variable. Aggregating
over all sectors recalling that \( \alpha_i \) defines the relative size of the sector. We have

\[
(1 + r^D_t)D_t = \sum_{i=1}^{T} \alpha_i (1 + r^D_{it})D_{it} = \sum_{i=1}^{T} \sum_{j=1}^{i} \frac{\alpha_i}{i} (1 + \tilde{r}^D_{ij,t+1-j})D_{it} \tag{4.12}
\]

\[
(1 + r^L_t)L_t = \sum_{i=1}^{T} \alpha_i (1 + r^L_{it})L_{it} = \sum_{i=1}^{T} \sum_{j=1}^{i} \frac{\alpha_i}{i} (1 + \tilde{r}^L_{ij,t+1-j})L_{it} \tag{4.13}
\]

which can be rewritten as

\[
    r^L_t = \frac{\alpha_i}{i} \sum_{i=1}^{T} \sum_{j=1}^{i} \tilde{r}^L_{ij,t+1-j} \tag{4.14}
\]

\[
    r^D_t = \frac{\alpha_i}{i} \sum_{i=1}^{T} \sum_{j=1}^{i} \tilde{r}^D_{ij,t+1-j} \tag{4.15}
\]

The problem for each branch of the bank is to calculate a markup that leaves a contract that is expected to break even. If the bank has no outstanding liabilities or assets incurred from the end of the last period then they simply set the loan rate markup (the risk premium) to cover expected losses throughout the contract. If we only have one period loans \((i = 1, \alpha_i = 1)\) then our cut off point in \((4.23)\) is known and the branch \(b_i\) always makes zero profit. However, since some of our contracts are predetermined (when \(i > 1\)) then they are not necessarily correct for
future economic states beyond the reset date, so unexpected losses or gains may be incurred during the lifetime of the contract. It is the responsibility of the whole bank $b$ to cover any additional liabilities (assets) with an increase (reduction) in the interest rate premium. This rebalancing will be financed through markup premiums split evenly over all sectors.

We define $\Psi_t$ as the proportion of total unaccounted losses that the bank realises in period $t$. Later in this section we can calculate endogenously the value of $\Psi_t$. When $\Psi_t \neq 0$, branch $b_i$ delivering a contract to cohort $j$ in sector $i$ will have $\alpha_i \Psi_t L_t$ costs (profits) to cover by increasing (reducing) the loan rate markup$^{23}$. This additional levy (bursary) is imposed on them by the commercial bank. We can define zero profit condition for the bank in period $t$. The balance sheet of the commercial bank is given as, $L_t = D_t$ and the zero profit condition$^{24}$ requires that

$$
\sum_{i=1}^{T} \sum_{j=1}^{i} (1 + r_{t+1-j}^{L}) \frac{\alpha_i}{i} L_{i,t} - \sum_{i=1}^{T} \sum_{j=1}^{i} (1 + r_{t+1-j}^{D}) \frac{\alpha_i}{i} D_{i,t}
$$

$$
\left[ \sum_{i=2}^{T} \sum_{j=2}^{i} \rho_{i,t+1-j}^{L} \frac{\alpha_i}{i} L_{i,t} \right] - \sum_{i=1}^{T} \frac{\alpha_i}{i} \rho_{i,t}^{b} L_{i,t}
$$

$$
- \sum_{i=1}^{T} \frac{\alpha_i}{i} \Psi_t L_{i,t} - \xi_t = 0
$$

where

$$
\xi_t = \left( \sum_{i=2}^{T} ((i - 1) \alpha_i / i) E_t L_{i,t+i-1} - L_t^{CB} \right) \Psi_t = 0
$$

defines the proportion of unexpected losses that are accounted for by a markup premium that is set today on fixed rate contracts but not fully repaid until the contracts expire. Additionally, $\hat{\rho}_{i,t}^{b}$ defines the reset markup at time $t$ to cover the

$^{23}$Recall that $\alpha_i$ is the relative size of sector $i$.

$^{24}$Note that some of the additional losses (or gains) will be accounted for with a markup on new contracts, since these contracts are fixed for a number of period the zero profit condition could deviate (very slightly) for a number of periods. However these losses will eventually be accounted for once all of the contracts have been repaid. In the meantime central bank liquidity borrowing covers this very issue.
expected losses from default, \( \hat{\rho}^L_{i,t} \) is the final loan rate markup in sector \( i \). We assume that \( \xi_t \) is funded by borrowing from the central bank\(^{25,26} \), \( L_t^{CB} \). Each reset loan will be fixed for \( i \) periods. Essentially equation 4.16 reads:

\[
\text{All loans} - \text{All Deposits}
\]

- Markup on predetermined loans
- Markup on new loans at branch level
- Spillovers added to today’s new loans

\[ = 0. \]

Furthermore, since \( D_t = L_t \) and

\[
\left( \sum_{i=2}^{T} \sum_{j=2}^{i} \hat{\rho}^L_{i,t+1-j} \frac{\alpha_i}{i} L_{i,t} + \sum_{i=1}^{T} \frac{\alpha_i}{i} \tilde{p}_{i,t}^{Lb} L_{i,t} \right) = \left[ \sum_{i=2}^{T} \sum_{j=2}^{i} \hat{\rho}^L_{i,t+1-j} \frac{\alpha_i}{i} + \sum_{i=1}^{T} \frac{\alpha_i}{i} \tilde{p}_{i,t}^{Lb} \right] L_t 
\]

(see description below), the 4.16 condition can be reduced to

\[
r^L_t L_t = r^D_t L_t + \sum_{i=2}^{T} \sum_{j=2}^{i} \hat{\rho}^L_{i,t+1-j} \frac{\alpha_i}{i} L_{i,t} + \sum_{i=1}^{T} \frac{\alpha_i}{i} \tilde{p}_{i,t}^{Lb} L_{i,t} + \Psi_t L_t \sum_{i=1}^{T} \frac{\alpha_i}{i} \tag{4.18}
\]

\[
r^L_t = r^D_t + \sum_{i=2}^{T} \sum_{j=2}^{i} \hat{\rho}^L_{i,t+1-j} \frac{\alpha_i}{i} + \sum_{i=1}^{T} \frac{\alpha_i}{i} \tilde{p}_{i,t}^{Lb} + \Psi_t \sum_{i=1}^{T} \frac{\alpha_i}{i} \tag{4.19}
\]

Note in (4.19) as in Agénor, Bratsiotis and Pfajfar (2011) the loan rate is a markup over deposits which depends on default risk. However, there are some key differences

- The term on the left-hand side is the loan rate, although \( L_t \) is an aggregation of all loans made today the cohort specific loan rates were determined some finite period ago when the loan contract was renegotiated, an aggregation of

\(^{25}\text{Note that a very small fraction of the borrowing (saving) must come from the central bank loans as additions (reductions) to future loans will not be covered by deposits, } \Psi_t \sum_{i=2}^{T} (i-1) \alpha_i / i \text{ as a result the bank must raise finance from the central bank to cover these additions. See section 4.3.3 for details.}\)

\(^{26}\text{Note that } \sum_{i=2}^{T} (i-1) \alpha_i / i + \sum_{i=1}^{T} \alpha_i / i = 1 \text{ so that all unexpected losses (in time) will be accounted for.}\)
these rates define \( r^L_t \).

- The first term on the right hand side defines the deposits which mirror loan rates in terms of their cohort specific timing and volumes.

- The second term on the right hand captures the proportion of loans that have a predetermined markup. Note that in the summation both \( i \) and \( j \) begin from 2. This is because all flexible loans and cohorts who reset in the other sectors can reset their contracts. For example, in a one year contract sector the loan rate markups set 3, 6 and 9 months ago would be included here.

- The third term defines the markup determined at the branch level on all new contracts. This markup covers the current and expected default risks of the firm.

- Since terms two and three on the right hand side in 4.18 cover all loans the sum of \( L_{i,t} \) becomes \( L_t \) and we can reduce 4.18 to 4.19.

- The final term captures the unexpected losses (gains) from previously agreed contracts (see the second term). Total losses will equal \( \Psi_t L_t \). We will see that this corrects for the aggregation of all the \( \rho^L_{t,i,t-j} \) that are misspecified by the bank for today. The aggregate markup, \( \rho^L_t \) is an aggregation of all previously set markups. In the steady state and all contracts were correctly specified this term is zero. As a result, the model remains balanced and Woodford’s (1999) timeless perspective is realised. These losses must be covered with new loans placed on the market in period \( t \). Since we assume that some of the loans (where \( i > 1 \)) are financed with an agreed markup on future loans, not all of the losses will be accounted for through an addition to today’s markup (see \( \xi_t \)).

For example if our economy is determined as 70% flexible (\( \alpha_1 = 0.7, i = 1 \)) and 30% 3 period loans (\( \alpha_3 = 0.3, i = 3 \)) then \( \Psi_t \sum_{i=1}^{T} \frac{\alpha_i}{i} = \Psi_t (0.7 + 0.1) = 0.8 \Psi_t \) and \( \sum_{i=2}^{T} (i - 1) \alpha_i / i = 0.2 \). In this situation the markup on the three period
loans is fixed for two more periods which is where the additional 20% of losses come from.\footnote{See spillover effects section}

**Financial Intermediation at the Branch level**

Following Agénor, Bratsiotis and Pfajfar (2011) we assume that total productivity of firm $f$ is subject to an idiosyncratic productivity shock each period.

$$Z_{ijf,t} = A_t \varepsilon_{ijf,t} \quad (4.20)$$

where $Z_{ijf,t}$ denotes the level of technology faced by firm $f$ who is situated in sector $i$, cohort $j$.\footnote{Kara (2010) investigates sector specific technology shocks. In our paper aggregate technology is identical over all sectors. It is only the firm specific idiosyncratic element that differs at the firm (but not sector) level.} The level of technology that the financial intermediary considers when developing the contract is given as a function of the minimum level of technology plus the additional idiosyncratic element. In equation (4.20) deviations in $A_t$ come through shocks to aggregate technology whereas $\varepsilon_{ijf,t}$ is an idiosyncratic uniform productivity shock, distributed over the constant interval $(\varepsilon, \bar{\varepsilon})$.

We first consider the flexible sector ($i, j = 1$) who’s wages and loan rates can be adjusted freely each period\footnote{Since we work in discrete time the minimum fixation is 1 period.} and thus all agents only have to consider the observable economic conditions today. Each period the firms in this sector will borrow from the commercial bank to meet their labour costs. These will be repaid at the nominal rate $r_{ij,t}$ at the end of the period. We let $L_{ijf,t}$ denote the nominal amount of borrowing by firm $f$ who is situated in sector $i$, cohort $j$,

$$L^R_{ijf,t} = W^R_{ijt} N_{ijf,t} \quad (4.21)$$

where $L^R_{ijf,t} \equiv L_{ijf,t} / P_t$ defines loans in real terms. In the case of default the revenue that the representative branch of our bank is able to seize is a fraction of of the
firms output, \( \chi_t \) where \( \chi \in (0, 1) \) is the steady state fraction and \( \chi_t = \chi \xi_{xt} \). In the steady state \( \xi_{xt} = 1 \), we introduce the variable \( \xi_{xt} \) to allow for a credit shock\(^{30}\). This allows use to directly alter the amount of assets the bank is able to seize in the case of a default. By allowing for credit shocks we are able to directly shock the balance sheet of the commercial bank in a manner similar to the origins of the 2008 global financial crisis.

As a result, a firm will choose to default if

\[
\chi_t Y_{ij,t} < (1 + r_{ij,t}^L) L_{ijf,t}^R. \tag{4.22}
\]

where \( r_{ij,t}^L \) defines the final loan rate offered to the firm. In equation (4.22) the left hand side is the firms actual repayment in the case of default and the right hand side is the firms contractual repayment, expressed in real terms. We denote \( \xi_{ijf,t}^M \) as the highest value of the realised contract specific productivity shock below which firms will lie in negative equity. Above this point firms will make profit which is distributed to households each period. Using our definition of productivity given by (4.20) we have

\[
\chi_t (A_t \xi_{ijf,t}^M) N_{ijf,t} = (1 + r_{ij,t}^L) L_{ijf,t}^R.
\]

Furthermore, from our definition of loans given in (4.21), holding with equality, this expression can be written as\(^{31}\)

\[
\xi_{ij,t}^M = \frac{(1 + r_{ij,t}^L) W_{ij,t}^R}{\chi_t A_t}. \tag{4.23}
\]

**The Break Even Condition** We define \( E_t S_{ij,t} \) as the expected income from lending over the lifetime of the contract for branch \( b_i \). As was the case in the previous subsection, to begin with we only consider the flexible sector \( (i,j=1) \) where the

---

\(^{30}\)See for example Pesaran and Xu (2011) for credit shocks in a macroeconomic model with a financial market defined by break even conditions.

\(^{31}\)Since all firms in the same cohort face the same wage, aggregate technology and loan rate we can drop the \( f \) subscript.
state of the economy for the life of the contract is known\textsuperscript{32}. The concern for the branch is to develop a risk premium over the savings costs that covers the expected losses associated with the contract. Assuming that the commercial has zero debts passed on from the previous period we have that the income from lending must be equal to the cost of financing the loan for the duration of the contract.

\[
\tilde{S}_{ij,t} = (1 + \tilde{r}^D_{ij,t}) L^R_{ijf,t}
\]  

Equation (4.24) states that the contract breaks even.

Each branch of our commercial bank ties into an obligation to pay savers the interest rate \( r^C_{t} \) for the lifetime of the contract\textsuperscript{33}. In the event of a default we know that the representative bank seizes a fraction \( \chi_t \) of average output throughout the contract. Thus, the expected income from the loans in real terms is given by,\textsuperscript{34}

\[
\tilde{S}^R_{ij,t} = \left[ \int_{\varepsilon^M_{ij,t}}^{\varepsilon^S_{ij,t}} (1 + \tilde{r}^{Lb}_{ij,t}) L^R_{ijf,t} f(\varepsilon_{ijf,t}) d\varepsilon_{ijf,t} + \int_{\varepsilon^M_{ij,t}}^{\varepsilon^S_{ij,t}} \chi_t Y_{ijf,t} f(\varepsilon_{f,t}) d\varepsilon_{ijf,t} \right] (4.25)
\]

where \( \varepsilon^M_{ij,t} \) is the cutoff value of \( \varepsilon_{f,t} \) (see 4.23) which is the same for all firms in cohort \( j \) and \( f(\varepsilon_{f,t}) d\varepsilon_{f,t} \) is the density function of \( \varepsilon_{f,t} \). We can rewrite the above equation as

\[
\tilde{S}^R_{ij,t} = L^R_{ijf,t} \left( 1 + \tilde{r}^{Lb}_{ij,t} \right) - \int_{\varepsilon^M_{ij,t}}^{\varepsilon^S_{ij,t}} \left[ (1 + \tilde{r}^{Lb}_{ijf,t}) L^R_{ijf,t} - \chi_t Y_{ijf,t} \right] f(\varepsilon_{j,t}) d\varepsilon_{ijf,t} (4.26)
\]

substituting in \( \chi_t (A_t \varepsilon^M_{ij,t} N_{ijf,t}) = (1 + \tilde{r}^{Lb}_{ij,t}) L^R_{ijf,t} \) and \( Y_{ijf,t} = N_{ijf,t} A_t \varepsilon_{ij,t} \), we

\textsuperscript{32} As a result we can drop the expectations operator.

\textsuperscript{33} In the same way savers are obliged to provide finance at this rate.

\textsuperscript{34} Note that the cutoff point \( \varepsilon^M_{ij,t} \) can alter during the contract if economic conditions are expected to change.
obtain

$$\tilde{S}_{ij,t}^R = L_{ij,t}^R (1 + \tilde{r}_{ij,t}^L) - \int_{\xi}^M (\varepsilon_{ij,t}^M - \varepsilon_{ij,t}) \chi_t N_{ij,t} A_t \right] f(\varepsilon_{j,t})d\varepsilon_{j,t}$$

Since the expected value of $\varepsilon_{ij,t+s}$ is the same for all firms in and sector or cohort we can drop the subscript $f$.

$$\tilde{S}_{ij,t}^R = L_{ij,t}^R (1 + \tilde{r}_{ij,t}^L) - \int_{\xi}^M (\varepsilon_{ij,t}^M - \varepsilon_t) \chi_t N_{ij,t} A_t \right] f(\varepsilon_{j,t})d\varepsilon_{j,t} \quad (4.27)$$

To maintain zero profit at the branch level, the income from lending must be equal to the agreed savings rate offered to households by this branch $\tilde{S}_{ij,t}^R = L_{ij,t+s}^R (1 + \tilde{r}_{ij,t}^D)$. Since we have the potential in the next section to finance for multiple periods this is the saving rate and will not necessarily be the current policy rate.\textsuperscript{35}

$$L_{ij,t}^R (1 + \tilde{r}_{ij,t}^D) = L_{ij,t}^R (1 + \tilde{r}_{ij,t}^L) \quad (4.28)$$

Dividing by $L_{ij,t}^R$

$$(1 + \tilde{r}_{ij,t}^D) L_{ij,t}^R = (1 + \tilde{r}_{ij,t}^L) L_{ij,t}^R - \int_{\xi}^M \left( (\varepsilon_{ij,t}^M - \varepsilon_t) \chi_t N_{ij,t} A_t \right] f(\varepsilon_{j,t})d\varepsilon_{j,t}$$

and then canceling yields,

$$\tilde{r}_{ij,t}^D = \tilde{r}_{ij,t}^L - \frac{E_t \int_{\xi}^M \left( (\varepsilon_{ij,t}^M - \varepsilon_t) \chi_t N_{ij,t} A_t \right] f(\varepsilon_{j,t})d\varepsilon_{j,t+s}}{L_{ij,t}^R}$$

\textsuperscript{35}For the flexible rate case the savings rate will always equal the policy rate.
or

\[ \frac{\tilde{v}_{ij,t} - \tilde{v}_t}{r_{ij,t} - r_t} = \int_{\xi}^{\varepsilon M_{ij,t}} \frac{[\varepsilon_{ij,t} - \varepsilon_t] \chi_i A_t}{iW_{ij,t}^R} f(\varepsilon_{j,t}) d\varepsilon_{j,t} = \frac{\rho_{ij,t}^{LR}}{\bar{\rho}_{ij,t}} \]  

(4.29)

where \( \rho_{ij,t}^{LR} \) is the loan rate markup at the branch level.

We assume that the idiosyncratic productivity shock \( \varepsilon_t \) follows a uniform distribution over the interval \( (\xi, \bar{\xi}) \). Its probability density is therefore \( \frac{1}{\bar{\xi} - \xi} \) and its mean \( \mu_\varepsilon = (\xi + \bar{\xi})/2 \). Under these assumptions if the markup is fixed for the contract duration then

\[ \rho_{ij,t}^{LR} = \frac{(\bar{\xi} - \xi) \chi_i A_i \Phi_{ij,t}^2}{2 \frac{W_{ij,t}^R}{W_{ij,t}}} \]  

(4.30)

where steady state default is given as

\[ \Phi = \left( \frac{\varepsilon M - \xi}{\bar{\xi} - \xi} \right) = \left( \frac{\mu_\varepsilon}{\nu_{p\lambda}} - \frac{\bar{\xi}}{\xi - \bar{\xi}} \right) \]

and the actual level of default as,

\[ \Phi_t = \int_{\xi}^{\varepsilon M_{ij,t}} f(\varepsilon_{j,t}) d\varepsilon_{j,t} = \left( \frac{\varepsilon M_{ij,t} - \bar{\varepsilon}}{(\bar{\xi} - \xi)} \right) \]  

(4.31)

where \( \Phi_t \in (0, 1) \) is the aggregate probability of default, which is known at time \( t \); \( \varepsilon_{ij,t} \) is given in (4.23). Here using, \( \varepsilon M = \frac{(1+i^L)W_R}{\chi A} \) and \( mC = \frac{(1+i^L)W_R}{\mu_\varepsilon} = \frac{1}{\nu_{p\lambda}} \), hence \( \varepsilon M = \frac{(1+i^L)W_R \mu_\varepsilon}{\chi A} = \frac{\mu_\varepsilon}{\nu_{p\lambda}} \) so that the steady state level of default for all sectors is given as

\[ \bar{\rho}^{LR} = \rho_{ij,t}^{LR} = \frac{(\bar{\xi} - \xi) \chi_i A_i \Phi_{ij,t}^2}{2 \frac{W_{ij,t}^R}{W_{ij,t}}} \]

\[ = \frac{(\bar{\xi} - \xi) \chi_i A_i \Phi_{ij,t}^2}{2 \frac{W_{ij,t}^R}{W_{ij,t}}} \]  

(4.32)
Financial Intermediation at the Branch Level When \( i > 1 \). We now develop the loan rate at the branch \( b_i \) level when \( i > 1 \), i.e. when loan contracts are fixed and longer than one period. Recall it is the responsibility of the branch to calculate the expected losses and design an appropriate risk markup. To do this they must first calculate the expected amount of borrowing which is going to depend upon the expected wages, expected loan rates and expected technology\(^{36}\). In period \( t \) when the contract is signed the bank can estimate a risk premium given over the predetermined savings rate (which is fixed at \( r_{t}^{CB} \) for \( i \) periods) which will be expected to break even over the life of the contract. For any time \( s \) periods after the contract was signed we can estimate an appropriate markup to be.

\[
E_{t}p_{i,t+s}^{lr} = E_{t}\left(\bar{\varepsilon} - \tilde{\varepsilon}\right)\chi_{t+s}A_{t+s}\Phi_{t+s}^{2}W_{i,t+s}^{R}W_{ij,t+s}^{R}.
\]

That is, we can estimate what the expected conditions of the economy will be for any period during the life of the contract. Since the problem for the branch is the same each period, it is only the values of the variables that differ. As a result, the branch \( b_i \) can design a contract that is expected to break even by summing all of the risk premia over the lifetime of the contract

\[
E_{t}\sum_{s=0}^{i-1}p_{i,t+s}^{lr} = E_{t}\left(\bar{\varepsilon} - \tilde{\varepsilon}\right)\sum_{s=0}^{i-1}\chi_{t+s}A_{t+s}\Phi_{t+s}^{2}W_{i,t+s}^{R}W_{ij,t+s}^{R}.
\]

where \( W_{ij,t+s}^{R} \)\(^{37}\) is the reset real wage. Since the loan rate is fixed for the life of the contract the branch designs the markup in such a way that the contract breaks even. Accordingly, the rate offered each period will be given as an average of all the risk premia over the lifetime of the contract so that the rate is fixed for the lifetime of the contract\(^{38}\). As a result the reset markup is given by

\(^{36}\)It is only one period loans for which the whole amount of borrowing is known since this is realised today.

\(^{37}\)Note that the real wage, \( W_{ij,t+s}^{R} \) can deviate over the lifetime of the contract as expected deviations in prices will cause movements. This is as opposed to the fixed nominal wage.

\(^{38}\)Note that this the bank may incur losses during some periods of the contract, however these losses are exactly counteracted with equivalent gains over the contract life. This is corrects through
\[
\tilde{p}_{ij,t}^{L_r} = E_t \frac{1}{i} \sum_{s=0}^{i-1} p_{ij,t+s}^{L_r} \\
= E_t (\bar{\xi} - \xi) \gamma \frac{1}{i^2} \sum_{s=0}^{i-1} \chi_{t+s} A_{t+s} \Phi_{t+s}^2 W_{ij,t+s}^R
\]

where
\[
\Phi_{t+s} = \left( \frac{\varepsilon_{ij,t+s}^M - \xi}{(\bar{\xi} - \xi)} \right)
\]

and
\[
\varepsilon_{ij,t+s}^M = \frac{(1 + r_{ij,t}^L) W_{ij,t+s}^R}{\chi_{t+s} A_{t+s}}
\]

note that \(\tilde{r}_{ij,t}^L\) will be the reset interest rate which was fixed at time \(t\) in the same way as the wage. Equation (4.32) is the reset risk weight at the branch level which is expected to break even over the contract. It is calculated in an identical way to (4.30) to given us an appropriate risk weight each period which are averaged to provide a contract that is expected to break even. However, just because it is expected to break even does not mean it actually will. We address this issue in the next section.

**Spillover Effects**

In this section we will consider how the bank \(b\) maintains the zero profit condition. At the start of period \(t\) the commercial bank recognises that predetermined contracts from previous periods may not currently be optimal,\(^39\)

\[
\tilde{p}_{ij,t-j}^L L_{ijf,t}^R \neq \tilde{p}_{ij,t-j}^L L_{ijf,t}^R
\]

\[
\tilde{p}_{ij,t-j}^L \neq \tilde{p}_{ij,t}^L
\]

the \(\xi\) term in the zero profit condition.

\(^39\)Note that since it is only the cut-off point which determines the default risk not the amount of loans

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Here \( \hat{\mu}_{ij,t-j}^{L} \) is a hypothetical "ideal" markup offered to the cohort that would exactly cover default losses of cohort \( j \) in period \( t \), if it was offered in period \( t - j \). Each period the commercial bank is able to calculate what markup should be offered for every contract. By summing all of these hypothetical markups and subtracting from this all of the actual markups the commercial bank can calculate the losses or profits that were unexpected. The bank is able to calculate the optimal markup for each current contract, \( \hat{\mu}_{ij,t}^{L} \) which will exactly cover the default losses. Using the same method as the previous section.

\[
\hat{\mu}_{ij,t-j}^{L} = \mu_{ij} \chi \frac{A_t \Phi_{t-j}^{L}}{W_{ij,t-j}^{R}}
\]

where
\[
\Phi_{t-j} = \int_{0}^{\infty} \frac{f(\xi_{j,t})}{\xi} d\xi_{j,t+s} = \left( \frac{\xi_{j,t-j}^{M} - \xi}{\xi - \xi^{M}} \right).
\]

and
\[
\xi_{j,t-j}^{M} = \frac{(1 + \hat{r}_{ij,t-j}^{Lb})}{\chi A_t} W_{ij,t-j}^{R}
\]

We can infer how shocks can cause an alteration in default risk in (4.33). Changes to the policy rate do not (directly) alter the risk of default on the contract. This is because the borrowing costs, \( \hat{r}^{D}_{ij,t} \) remain identical since they too are fixed for the life of the contract. Although nominal wages are fixed for the lifetime of the contract the real wage may deviate through changes in the price level, finally alterations to aggregate technology, \( A_t \) and the amount the bank is able to seize in the case of default \( \chi_t \), can cause the original contract to be misspecified and thus for spillovers to exist.

We can now determine the difference between the expected and actual default levels as

\[
\Psi_t = \sum_{i=1}^{T} \sum_{j=0}^{i-1} \frac{\alpha_{i}}{i} \left( \hat{\mu}_{ij,t-j}^{L} - \hat{\mu}_{i,t-j}^{Lb} \right)
\]

where \( \Psi_t \) is the additional markup which needed to cover todays bank losses.
The Commercial Bank and the Reset Loan Rate

Given that the bank may incur losses or gains due to the fixed nature of contracts (see 4.37) their financing costs will alter for this periods loans. Specifically, the commercial bank will need to recoup $\Psi_t L_t^R$ with $\alpha_i \Psi_t L_t^R$ coming from branch $b_i$. Essentially, this assumption means that the relative size of the branch directly determines the proportion of losses which they must recover.

This debt (profit) must be imposed (released) to firms so that the commercial banks zero profit condition is satisfied in (4.16). The only way that the commercial bank can meet this condition is to pass $\Psi_t$ onto contracts up for renewal. Since only one cohort in each sector renegotiates each period all of these additional funds will come through the contracts offered in period $t$. Furthermore, because the commercial bank can borrow for this purpose from the central bank, these funds can come from future profits. Specifically, if the branch needs to raise $\alpha_i \Psi_t L_t^R$, then the finance costs are $\sum_{s=0}^{i-1} (1 + (1 + \delta r^c)) \alpha_i \Psi_t L_t^R$. For example, a flexible one-period contract is simply $\alpha_i \Psi_t L_t^R$, for a 2 period contract $(1 + r_t^{CB}) \frac{\alpha_i}{2} \Psi_t L_t^R$, a 4 period contract is $(1 + \frac{6r_t^{CB}}{4}) \alpha_i \Psi_t L_t^R$, and a 16 period contract, $(1 + 7.5r_t^{CB}) \alpha_i \Psi_t L_t^R$. Thus, as a rule we have the cost of external finance as $(1 + \frac{i(i-1)}{2} r_t^{CB}) = \Delta^{CB}$. Also note that the amount a branch needs to recoup can be written as

$$\alpha_i \Psi_t L_t^R = \Psi_t E_t \sum_{s=0}^{i-1} L_{t,t+s}^R \frac{\alpha_i L_t^R}{\Psi_t} \sum_{s=0}^{i-1} L_{t,t+s}^R$$

If we now return to the financing of the contract (equation 4.19) the branch is now

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40 See equation 4.17 in the zero profit condition of the commercial bank for details.

41 Specifically, in the first period the cost of finance is zero, the borrowing for the second period is $r$, for the third period it is $r + r$, Ultimately, for any contract total financing costs are $(1+r+2r+...+(T-1)r)$.

42 We can assume that either the commercial bank is offered central bank finance at the current market rates, expected or the actual rate in future periods. Since the result is of second order importance it is only the steady state interest rate that effects the log linearised macroeconomic equilibrium.
expected to raise \((1 + \tilde{\tau}_{i,t}^L) \sum_{s=0}^{i-1} E_t L_{i,t+s}\), where

\[
(1 + \tilde{\tau}_{i,t}^L) \frac{1}{i} \sum_{s=0}^{i-1} E_t L_{i,t+s}^R = (1 + \tilde{\tau}_{i,t}^{lb}) \frac{1}{i} \sum_{s=0}^{i-1} E_t L_{i,t+s}^R + (1 + \frac{(i - 1)}{2} r_t^{CB}) \Psi_t \frac{1}{i} E_t \sum_{s=0}^{i-1} L_{i,t+s} \frac{\alpha_i L_t^R}{\frac{1}{i} E_t \sum_{s=0}^{i-1} L_{i,t+s}^R}
\]

or

\[
(1 + \tilde{\tau}_{i,t}^L) = (1 + r_t^{CB} + \tilde{\rho}_{i,t}^{lb} + \Delta_{CB} \left( \frac{\alpha_i L_t^R}{\frac{1}{i} E_t \sum_{s=0}^{i-1} L_{i,t+s}^R} \right) \Psi_t)
\]

Since we do not look at shocks on shocks, the expected value of future loans will be equal to the actual\(^{43}\)

\[
\frac{1}{i} E_t \sum_{s=0}^{i-1} L_{i,t+s}^R = \frac{1}{i} \sum_{s=0}^{i-1} L_{i,t+s}
\]

If not the difference would be passed on to new contracts in the same fashion as above. Simplifying we have

\[
\tilde{\tau}_{i,t}^L = r_t^{CB} + \tilde{\rho}_{i,t}^{lb} + \Delta_{CB} \Delta_{L,t} \Psi_t
\]

where

\[
\Delta_{L,t} = \left( \frac{\alpha_i L_t^R}{\frac{1}{i} E_t \sum_{s=0}^{i-1} L_{i,t+s}^R} \right)
\]

The term \(\Delta_{L,t}\) acts as an additional multiplier or dampener in the non-linear model.\(^{44}\)

\[
\Delta_{CB} = \left(1 + \frac{(i - 1)}{2} r_t^{CB}\right)
\]

\[
\tilde{\rho}_{i,t}^{lb} = \frac{(\tilde{\varepsilon} - \varepsilon) E_t \sum_{s=0}^{i-1} \chi_{t+s} A_{t+s} \Phi_{i+s}^2}{i W_{i,j,t}^R}
\]

\(^{43}\)This is because once the shock has occurred the expected loans afterwards will equal the actual.

\(^{44}\)Intuitively, if future loan demand falls then to cover today’s unaccounted losses the premium agreed upon today must increase by a greater amount. In the linearised version (to first order) this term is always 1, so drops out.
and total spillovers are the weighted average of spillovers passed on to all sectors
given by equation (4.37) or alternatively in terms of the aggregate ideal markup and
the aggregate actual markup at the branch level

\[ \Psi_t = \gamma_t^L - \bar{\gamma}_t^{Lb} \]

where \( \gamma_t^L \) is the aggregate ideal markup and \( \bar{\gamma}_t^{Lb} \) is the aggregate actual markup at
the branch level.

### 4.3.4 Households, Unions and Contracts.

Households \( h \in [0; 1] \) have preferences defined over consumption, and labour. The
expected life-time utility function takes the form

\[ U_h = E_t \sum_{s=0}^{\infty} \beta^t \left[ \frac{C_{ht+s}^{1-\sigma}}{1-\sigma} - \eta N_{ht,t+s}^{1+\eta} \right] \]  \hspace{1cm} (4.38)

where \( C_{ht}, \frac{M_{ht}}{P_t}, 1 - N_{ht} \) are household h’s consumption, end of period real money
balances, hours worked, and leisure respectively,

Each household belongs to a particular employment union within their sector,
given that each household \( h \) is twinned with a firm \( f, (f = h) \).

The households budget constraint is given by\(^\text{45}\)

\[ P_tC_{ht} + M_{ht} + D_{ht} \leq M_{ht-1} + (1 + r_D^{t-1})D_{ht-1} \]

\[ + W_{ht} N_{ht} + \Pi_{h,ft} + \tau_t \]  \hspace{1cm} (4.39)

where \( D_t \) is deposits, \( r_D^t \) is the interest rate on deposits outlined in the previous
section, \( M_{ht} \) denotes money holdings at the end of period \( t \), \( W_{ht} \) is the nominal wage,
\( \Pi_{ht} \) is the profits distributed by firms and \( W_{ht} N_{ht} \) is the labour income. Additionally,

\(^{45}\)Note we follow Ravenna and Walsh (2006) by assuming the presence of both real money
balances and deposits in the households budget constraint.
recall that deposits are fixed in terms of size and duration to match the demand and contract duration in the loan market. Finally, \( \tau_i \) is a nominal lump-sum transfer from the government. The households optimisation breaks down into two parts. First, there is the choice of consumption, to maximise expected lifetime utility in (4.38) given the budget constraint (4.39). The first order conditions derived from the consumer’s problem are as follows.

\[
L = E_t \sum_{s=0}^{\infty} \beta^t \left[ C_{ht+s}^{1-\sigma} - \eta N_{t+s}^{1+\eta} \frac{N_t^{1+\eta}}{1+\eta} \right] +
\]

\[
E_t \sum_{s=0}^{\infty} \beta^t \lambda_t \left[ \frac{M_{ht-1}}{P_t} + \frac{D_{ht}}{P_t} + \frac{W_{ht} N_{ht}}{P_t} + \Pi_{ht} \right]
\]

\[
+ T_t - C_{ht} - \frac{M_{ht}}{P_t} + (1 + i_{t-1}) \frac{D_{ht}}{P_t}
\]

From (4.38) and (4.39) we obtain the Euler equation.

\[
C_t^{1-\sigma} = \beta(1 + r_t^D) E_t \left( \frac{P_t}{P_{t+1}} \right) C_{t+1}^{1-\sigma} \quad (4.40)
\]

As with Dixon and Kara (2010) have removed the \( h \) subscript as consumption is identical across all households. Note that the deposit rate in not the same as central bank rate as it contains savings rates agreed in previous periods.\(^{46}\)

### 4.3.5 The Reset Wage

The reset wage is for household \( h \) in sector \( i \) is chosen to maximise lifetime utility given labour demand and the additional constraint that the nominal wage will be fixed for \( T_i \) periods in which the aggregate output and price level are given. For our wage and price setting we follow the GTE contract setting of Dixon and Kara (2010). From the unions point of view we can collect together all of the terms in

\(^{46}\) See section 4.3.3
(4.8) where they treat $K_t$ as a constant,

$$K_t = \frac{\theta}{(\theta - 1)} Z_t^{\theta - 1} (1 + r_t')^{-\theta} P_t^0 Y_t$$  \hspace{1cm} (4.41)$$

Note that in terms of our model the households also treat (4.41) as exogenous, that is individual household decisions have no control of technology, loan rates, aggregate prices and aggregate output. This yields the following reset wage$^{47}$,

$$\bar{W}_{h,t} = \frac{\theta}{\theta - 1} \left[ \frac{E_t \sum_{s=0}^{i-1} \beta^s \left[ \eta_D R_{t+s}^{1+\eta} \right]^1}{E_t \sum_{s=0}^{i-1} \beta^s \left[ \frac{C_{h,t+s}}{P_{t+s}} K_{t+s} \right]} \right]$$  \hspace{1cm} (4.42)$$

Like Dixon and Kara (2010) equation (4.42) shows that the optimal wage is constant markup over the ratio of the marginal utilities of leisure and marginal utilities from consumption within the finite contract duration.

### 4.3.6 The Reset Price

We now return to consider optimal prices in an economic environment where a proportion of firms are unable to change their prices each period. We follow Dixon and Kara (2010) and allow for heterogeneity in the price contracts$^{48}$.

The problem for the firm in cohort $j$ in sector $i$ is to maximise their profits by selection the optimal reset price $\tilde{P}_{ij,t}$

$$\max \sum_{i=0}^{i-1} \beta^i \left[ \tilde{P}_{ij,t} - mc_{t+i} P_{t+i} \right] Y_{ij,t+i}$$  \hspace{1cm} s.t \hspace{0.5cm} y_{ij,t} = \left( \frac{P_{ij,t}}{P_t} \right)^{\theta} Y_t$$

$^{47}$See Appendix 4.A.3 for derivation

$^{48}$In this paper wages are determined in this way but the contract structure is identical.
This yields the following solution for prices\(^49\),

\[
\tilde{P}_{ij,t} = \frac{\sum_{i=0}^{i-1} \beta^i \left[ \frac{\theta}{(\theta-1)} R_{ij,t+i} W_{ij,t+i} Z_{i+1}^{-1} \right]}{\sum_{i=0}^{t-1} \beta^i}
\]

which from our definition of flexible prices given in (4.6) can be written as

\[
\tilde{P}_{ij,t} = \frac{\sum_{i=0}^{i-1} \beta^i P_{ij,t+i}^{*}}{\sum_{i=0}^{t-1} \beta^i}
\]

where \(P_{ij,t+i}^{*}\) is the optimal price at time \(t + i\).

### 4.4 General Equilibrium

#### 4.4.1 The Complete linearised System

Below we define the complete log linearised system\(^50\).

**Aggregate Demand**  We have a typical IS equation except for the deposit rate, which contains elements of inertia, enters rather than the central bank rate

\[
\hat{Y}_t = E_t Y_{t+1} - \frac{1}{\sigma} \left( \hat{r}^D_t - E_t \hat{\pi}_{t+1} \right).
\]

Our deposit rate is an aggregation of all deposits in the economy

\[
\hat{r}^D_t = \sum_{i=1}^{T} \alpha^i \hat{r}^D_{it}
\]

\[
\hat{r}^D_{i,t} = \frac{1}{i} \sum_{j=1}^{i} \hat{r}^D_{t+1-j}
\]

**Aggregate Supply**

\(^{49}\)See Appendix 4.A.4 for details of derivation

\(^{50}\)See Appendix 4.A.5 for details of derivation
Wages  Cohort reset wages

\[ \hat{W}_{ij,t} = \frac{1}{\sum_{s=0}^{i-1} \beta^s} \left[ E_t \sum_{s=0}^{i-1} \beta^s \left[ \delta_a \hat{Y}_{t+s} + \delta_b \hat{Z}_{t+s} + \hat{P}_{t+s} - \delta_c \hat{r}^L_{ij,t} \right] \right] \]

where (above for flexible sector the loan rate is going to be the expected flexible rate)

\[ \delta_a = \frac{(\eta + \sigma)}{(\theta \eta + 1)}, \quad \delta_b = \frac{\eta (\theta - 1)}{(\theta \eta + 1)}, \quad \delta_c = \frac{\eta \theta}{\theta \eta + 1} \]

Aggregate sector wages

\[ \hat{W}_{it} = \frac{1}{i} \sum_{j=1}^{i} \hat{W}_{i,t+1-j} \]

Prices  Flexible Prices

\[ \hat{P}^*_{i} = \hat{W}_{i,t} + \hat{r}^L_{i,t} - \hat{Z}_t \]

(4.43)

The linearised reset price for each cohort in each sector

\[ \hat{P}_{i,t} = \sum_{i=0}^{i-1} \beta^i \hat{P}^*_{ij,t+i} / \sum_{i=0}^{i-1} \beta^i \]

Sector prices

\[ \hat{P}_{it} = \frac{1}{i} \sum_{j=1}^{i} \hat{P}_{i,t+1-j} \]

Aggregate prices

\[ \hat{P}_t = \sum_{i=1}^{T} \alpha_i \hat{P}_{i,t} \]

Loan Rates  Sector loan rate

\[ \hat{r}^L_{it} = \frac{1}{i} \sum_{j=1}^{i} \hat{r}^L_{ij,t+1-j} \]
Aggregate loan rate

\[ \hat{r}_{ij,t}^L = \sum_{i=1}^{T} \alpha_i \hat{r}_{it} \]

\[
\hat{r}_{ij,t}^L = \frac{1}{(1 + i)^L} \left\{ (1 + \hat{r}^R_t) + \rho^L \left[ 2\hat{s}_1 \hat{r}_{ij,t}^L + \hat{s}_2 \left( \hat{W}_{ij,t}^R - \hat{a}_{ij,t} - \hat{c}_{ij,t} \right) \right] + \Delta^C B \hat{s}_2 \left[ (\hat{a}_t - \hat{A}_t) + (\hat{c}_t - \hat{\chi}_t) + (1 - \alpha_1) \left( \hat{W}_t - \hat{P}_t - \hat{W}_t^R \right) \right] \right\} \tag{4.44}
\]

where \( \Delta^C B = (1 + (i-1)^C B) \), \( \hat{s}_1 = \left( \frac{\epsilon^M}{\epsilon^M - \frac{\epsilon^P}{2}} \right) \), \( \hat{s}_2 = \left( \frac{\epsilon^M + \epsilon^P}{\epsilon^M - \frac{\epsilon^P}{2}} \right) \).

Equation (4.44) defines the linearised reset loan rate. It is clear that elements of this loan rate are functions of the expectations of default for those who currently resetting, this is the markup at the branch level, additionally there are spillover effects which arise from aggregate default levels. The first line of this equation defines as a loans as a markup over the current central bank rate. The loan rate markup is increasing in terms of the fixed cost of borrowing and expected real wage costs, yet it is decreasing in terms of the expected level of productivity and level of that is collateral pledged to the commercial bank\(^{51}\). The second line corrects for the unaccounted losses that the commercial bank incurs at the aggregate level, if for example technology or the collateral placed to the commercial bank unexpectedly fall \((\hat{a}_t > \hat{A}_t)\) or \((\hat{c}_t > \hat{\chi}_t)\) then the contracts up for renegotiation must rise to correct for the losses on the preset contracts. If however, there is unexpected inflation then the real wage will fall which reduces default risk on the preset contracts which leads to the spillover effect reducing the current loan rate spread on new contracts. Finally, the cost of financing spillovers increases as the length of contract increases through \( \Delta^C B \).

When \( i, j = 1 \) we have the aggregate (not reset) wage in the flexible sector agents

\(^{51}\)The results are in line with Agenor Bratsiotis and Pfafar (2012)
consider the secotral (rather than reset) real wage.

\[
\tilde{r}_{ij,t}^L = \frac{1}{(1+i^L)} \left\{ (1+i) \tilde{r}_{t}^C + \rho^L \left[ 2 \varsigma_1 \tilde{r}_{ij,t}^L + \varsigma_2 (\tilde{W}_{i,t} - \tilde{a}_{ij,t} - \tilde{c}_{ij,t}) \right] \right. \\
+ \Delta^C \varsigma_2 \left[ (\tilde{a}_t - \tilde{A}_t) + (\tilde{c}_t - \tilde{\chi}_t) + (1 - \alpha_1) \left( \tilde{W}_t - \tilde{P}_t - \tilde{W}_t^R \right) \right] \right) \right\} 
\] (4.45)

In the two reset loan rate equations (4.44 and 4.45) above there are the terms \( \tilde{a}_t, \tilde{c}_t \) and \( \tilde{W}_t^R \), these terms define the anticipated levels of technology, collateral and real wages for \( t \) at the time when their contract was renegotiated. We can define these values as,

**Technology used in Loan Rates**  \( \text{Reset technology} \)

\[
\tilde{a}_{ij,t} = \frac{1}{i} \sum_{s=0}^{i-1} \tilde{A}_{t+s}
\]

Sector technology

\[
\tilde{a}_i,t = \frac{1}{i} \sum_{j=0}^{i-1} \left( \tilde{a}_{ij,t+1-j} \right)
\]

Aggregate technology

\[
\tilde{a}_t = \sum_{i=1}^{T} \alpha_i \tilde{a}_{i,t}
\]

**Collateral used in Loan Rates**  \( \text{Reset collateral} \)

\[
\tilde{c}_{ij,t} = \frac{1}{i} \sum_{s=0}^{i-1} \tilde{\chi}_{t+s}
\]

Sector collateral

\[
\tilde{c}_i,t = \frac{1}{i} \sum_{j=0}^{i-1} \left( \tilde{c}_{ij,t+1-j} \right)
\]

Aggregate
\[
\hat{c}_t = \sum_{i=1}^{T} \alpha_i \hat{c}_i
\]

**Real wages used in loan rates**  
Reset real wages

\[
\tilde{W}_{ij,t}^{R} = \tilde{W}_{ij,t}^{R} - \frac{1}{T} \sum_{s=0}^{i-1} \tilde{P}_{t+s}
\]

Sector real wages

\[
\tilde{W}_{i,t}^{R} = \frac{1}{T} \sum_{j=0}^{i-1} \left( \tilde{W}_{ij,t+1-j}^{R} \right)
\]

Aggregate real wages

\[
\tilde{W}_{t}^{R} = \sum_{i=1}^{T} \alpha_i \tilde{W}_{i,t}^{R}
\]

**Monetary Policy and Shocks**  
We assume a typical Taylor Rule

\[
\hat{r}_{t}^{CB} = \psi \hat{r}_{t-1}^{CB} + (1 - \psi) \left[ \psi_y \hat{Y}_t + \psi_{\pi} \hat{\pi}_t \right] + \hat{\epsilon}_{t}^{mp}
\]

with the following monetary policy shock

\[
\hat{\epsilon}_{t}^{mp} = \hat{s}_t^{mp}
\]

and AR(1) supply and credit shock

\[
\hat{A}_t = \rho^A \hat{A}_{t-1} + \hat{\zeta}_t^A
\]

\[
\hat{\chi}_t = \rho^\chi \hat{\chi}_{t-1} + \hat{\zeta}_t^\chi
\]

where \( \hat{s}_t^{mp}, \hat{\zeta}_t^A, \) and \( \hat{\zeta}_t^\chi \) are monetary policy, technology and credit shocks
Aggregate Default and Loans  Finally we have the following aggregate default and aggregate loans\textsuperscript{52}

\[ \hat{\Phi}_t = \left( \frac{e^M}{\bar{e} - \bar{e}} \right) \left( \hat{r}_t + \hat{W}_t^R - \hat{A}_t - \hat{\chi}_t \right) \]

\[ \hat{L}_t = (\theta - 1) \hat{Z}_t - \theta \hat{R}_t^d - (\theta - 1) \hat{W}_t^R + \hat{Y}_t \]

4.5  Simulations

4.5.1  Parameter Values

The key parameters in the model are given in the table below

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>Inverse of the elasticity of substitution in consumption</td>
</tr>
<tr>
<td>$\eta$</td>
<td>4.5</td>
<td>Inverse of the intertemporal labour supply elasticity</td>
</tr>
<tr>
<td>$(\bar{e}, \bar{\bar{e}})$</td>
<td>(1, 1.306)</td>
<td>Range of idiosyncratic technology element</td>
</tr>
<tr>
<td>$\theta$</td>
<td>11</td>
<td>Price elasticity of demand</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.97</td>
<td>Steady state level of output pledged to the bank</td>
</tr>
<tr>
<td>$\psi_{\pi}$</td>
<td>1.5</td>
<td>Policy weight on inflation</td>
</tr>
<tr>
<td>$\psi_{\mu}$</td>
<td>0.5/4</td>
<td>Policy weight on output</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.5</td>
<td>Persistence of the Taylor rule</td>
</tr>
<tr>
<td>$\rho^A$</td>
<td>0.7</td>
<td>Persistence of technology shocks</td>
</tr>
<tr>
<td>$\rho^c$</td>
<td>0.7</td>
<td>Persistence of credit shocks</td>
</tr>
<tr>
<td>$\rho^r$</td>
<td>1%</td>
<td>Steady state interest rate spread</td>
</tr>
</tbody>
</table>

Table 4.3: Parameter Values

In line with the stylised facts outlined at the start of this chapter we have the following distribution of loan rate stickiness. There are 58% of loans which are flexible\textsuperscript{53}, 31.5% are fixed for a year, 3.78% are fixed for two years and 6.72% are

\textsuperscript{52}Note that it is the disaggregated versions that enter into the equations above, however these equations will be useful for defining the behaviour of aggregate default and loans.

\textsuperscript{53}Since $t$ is one quarter these are modelled as 3 months inertia.
fixed for four years. This generates and aggregate loan rate inertia of 3.2 quarters. Our wages are reset at the same time as the loan rates in all sectors apart for the flexible sector who fixes their wages for five periods. Admittedly, this is a shortcut but it allows us to generate an aggregate wage contract of 5.5 quarters which is more in-line with empirical studies, whilst at the same time retain the financial contract length that is in line with our micro evidence.

We follow the study of Taylor (1993) to define price inertia. In this model Taylor calibrates the US economy as having contract lengths $T=(1,2,3,4,5,6,7,8)$, with sector shares being $\alpha_1 = 0.07$, $\alpha_2 = 0.19$, $\alpha_3 = 0.23$, $\alpha_4 = 0.21$, $\alpha_5 = 0.15$, $\alpha_6 = 0.08$, $\alpha_7 = 0.04$, $\alpha_8 = 0.03$, the average contract length is 3.6 periods.

**4.5.2 Policy Shock**

Figure 4.7 displays the response of a number of variables to a 1% increase in the central bank rate. When loan and deposit interest rates are flexible, monetary shocks impact the cost of borrowing through very similar channels to Agénor Bratsiotis and Pfajfar (2012). These are: (a) through the standard cost channel of monetary policy (see Ravenna and Walsh, 2006) and (b) through the endogenous effect that a change the central bank rate has on default risk. In this example, the fall in demand resulting from a higher central bank rate, reduces output, demand for loans and wage inflation. Furthermore, in line with the price puzzle, the two above effects act to raise the initial inflation response which is reversed in later periods by the relative response of the demand channel. Finally, like Agénor Bratsiotis and Pfajfar (2011) we observe countercyclical external finance premium shown through the spread between deposits and loan rates.

When we account for inertia in the loan and deposit rates two additional channels affect the cost of borrowing following monetary policy shocks: (c) A proportion

Note that we use four rather than five year contracts for our longest length. This does not alter our qualitative results and reflects the observation that a higher proportion of long term contracts will be altered or hedged in reality.
of loans will be a predetermined markup over the deposit rate that was agreed before the realisation of the shock and; (d) Any unaccounted losses or gains from previous contracts are transferred to new contracts. This involves aggregating all unaccounted losses from all sectors and distributing them to new contracts according to the relative size of the sector.

Following policy shocks point (c) has a significant effect on the persistence and dynamics of the model: Since deposit rates are more sluggish than central bank rate (initially $r_D - r_{CB} < 0$), output responds by less initially and then displays more persistence\(^5\). Furthermore, because the cost of borrowing is a markup over deposits, even though the risk of default and the $r_L - r_D$ spread increases, the $r_L - r_{CB}$ spread initially falls. In later periods the persistence of output and default lead the $r_L - r_D$ spread to dominate, causing the loan to central bank rate spread

\(^5\)This is in line with the findings of Mojon (2001) who investigates the deposit rate pass through of the Euro area to be only 11% on savings deposits and 65% on time deposits after three months.
Figure 4.8: Impulse Responses to an Increase in the Policy Rate With and Without Spillover Effects
to move from procyclical to countercyclical.

In general, the inclusion of intertia in the interest rates of firms and households leads to a more of a cushion following monetary policy shocks but with much greater persistence of market rates and aggregate variables. This result is in line with results of Gerali, Neri, Sessa and Signoretti (2010) and Hulsewig, Mayer and Wollmerhaeuser (2009) who also find that inertia in loan rates following monetary policy shocks moderates the impact on consumption and mitigates the strength of the cost channel. Indeed, it is only the real wage which is amplified in this version. Because of wage inertia when unions reset their wage they consider the dynamics of output and the cost of borrowing over the life of the contract. With both of these variables more persistent wages fall by more when they are reset.

The final graph in figure 4.8 depicts the effects of point (d). Here we compare the results with and without spillover effects to the new contracts. Essentially this involves comparing our model with a model where $\Psi_t = 0$ every period. Importantly, borrowing costs and nominal wages are fixed on the preset contracts. Additionally, increase in prices cause the real wage to fall slightly. As a result, the default rate falls on these contracts so that the wage spillovers are positive and the gains are passed on to those who reset. However, this channel is weak and has no significant impact on either aggregate loan rates, output or inflation.

4.5.3 Technology Shock

Figures 4.9 and 4.10 display the Impulse responses to a 1% negative shock to technology. The fall in productivity initially reduces output, raises inflation and nominal wages, whilst reducing the demand for loans. Technology shocks retain the same channels (a-d) as were outlined in policy shocks above but with channel (d) additionally containing spillovers from misspecified technology on predetermined contracts. Like policy shocks, we observe the smoother more persistent responses of loan rates and aggregate variables when the $r^D$ and $r^L$ rates contain inertia. In this version
Figure 4.9: Impulse Responses to a Negative Shock to Technology With and Without Inertia in Loan and Deposit Rates
we still observe the sluggish behaviour of deposits so that initially $r^D - r^{CB}$ falls, however since the movement in central bank rate is less than following a policy shock this spread has a reduced impact. We also retain a countercyclical ($r^L - r^D$) spread. When we examine a combination of these two channels, we find the fall in productivity coupled with higher borrowing costs causes both the ($r^L - r^D$) and to a muted extent ($r^L - r^{CB}$) spread to be countercyclical.

The spillovers from old to new contracts come through a combination of the unaccounted real wage losses we observed with policy shocks as well as additional unaccounted technology losses. Because both the fall in technology and the rise in the interest rate cause prices to rise, the real wage spillovers have more of an impact than they did following a policy shock. However, this effect is dominated by the increased default from lower productivity so that the rate on new contracts must increase.
The final graph in figure 4.10 indicates that when the spillover effects are included they amplify the \( r^L - r^D \) spread on new contracts and therefore the \( r^L - r^{CB} \) spread so that the reset lending rate reacts by more. However, this amplification has little impact on output and inflation so has a limited contribution to the central banks reaction according to a Taylor rule. This result also highlights the weakness of the cost channel in driving the output and inflation responses following technology shocks. This is because the main deviations in prices and wages come from the impact of the movement in productivity rather than the secondary effect it has on increasing the cost of borrowing.

### 4.5.4 Credit Shock

Figures 4.11 and 4.12 display the impulse responses to a 1% fall in the amount of collateral that the bank is able to seize following a default. This essentially directly shocks the balance sheet of the bank.\(^{56}\) Changes to creditworthiness impact our economy through the cost channel by directly shocking the risk of default. Indeed, without a cost channel and external borrowing there would be no place for this type of credit shock in this model.

As was the case with policy shocks, a negative credit shock leads to an increase in inflation, this causes the interest rate to rise so that output falls. Furthermore, the model displays more smoothing and persistence when there is inertia in \( r^L \) and \( r^D \) and the interest rate pass through is incomplete. The main difference from policy shocks come through the impact on the spread. The \( r^L - r^D \) spread and the \( r^L - r^{CB} \) spread are clearly countercyclical and display more amplification than the case of both technology and policy shocks, (indeed it is the countercyclical spread that is entirely driving the movements in prices and output). This is because, the downward pressure on the real wage is much weaker so that the cost channel effect dominates the incomplete deposit rate pass through the \( r^D - r^{CB} \) spread.

\(^{56}\) See for example, Christiano, Motto, and Rostagno (2010), Gertler and Karadi (2011) and Pesaran and Xu (2011) for examples of Credit shocks in DSGE models.
As a result, the effects of points (c) and (d) displayed in figure 4.12 are much stronger and the spillover effect work to both amplify the loan rate spread and the movement of aggregate variables. In line with the 'Bank funding Channel' of Christiano Motto, and Rostagno (2010) the bank must raise additional funds from an increase in the loan rate, when their balance sheet has be adversely effected. Indeed, the credit shocks may go some way in explaining the greater persistence and amplification from credit shocks that have been observed in the recent financial crisis (See IMF world economic outlook (April 2009)) although we leave this for future empirical work.
Figure 4.12: Impulse Responses to a Negative Shock to Collateral With and Without Spillover Effects
4.6 Concluding Remarks

Evidence from UK lending has indicated that, since the onset of the financial crisis, the cost of lending to firms has remained high whilst the policy rate has fallen to record lows. This chapter has introduced a financial sector into the Generalised Taylor Economy of Dixon and Kara (2010, 2011) to capture two channels which may generate these loan rate dynamics. Firstly, we allow for loans to firms and the savings of households to be on a fixed rate for a range of predetermined durations. Additionally, we introduce default risk at the firm level through idiosyncratic productivity shocks. These losses encountered from default risks are covered by a perfectly competitive commercial bank who breaks even each period by charging a premium over the savings rate.

The combination of legally bound fixed financial contracts and state dependent default risk generates a problem for our commercial bank since changes to the aggregate state of the economy may lead to the originally agreed contract being misspecified. To overcome this issue, new contracts up for renegotiation must make up for any unaccounted losses or gains through an adjustment to their risk premium. Thus, a financial contract up for renegotiation will be driven by three elements; the anticipated costs of the firm, the expected default risk and unexpected default losses which are not accounted for in the preset contracts.

Our results indicate that the impulse response of output, inflation and all interest rates are smoother and more persistent when a proportion of savings and loan rates are unable to react following an economic shock. This generates a procyclical spread between the central bank rate and the deposit rate. Additionally, the presence of default risk generates a countercyclical external finance premium or spread between the loan and deposit rates. For a central bank concerned with the transmission of the policy rate to the loan rate it is dynamics of the sum these two spreads, and the impact that has on aggregate variables which will ultimately be of interest. Following monetary policy shocks the procyclical spread dominates so that initially the policy
rate to loan rate spread is also procyclical. In contrast, following technology shocks, and to a greater extent credit shocks, this spread is countercyclical since the change in default risk dominates the inertia in deposit rates. Finally, following adverse credit shocks a significant proportion of losses are passed on to new loan contracts. This effect amplifies the fall in output, rise in inflation and loan rate spread for a number of periods.

The implications for monetary policy suggest that the direction and persistence of credit spreads, and their effect on aggregate variables, depends on the type of economic shock. In particular following adverse credit shocks the countercyclical spread and spillovers of losses to new contracts is greater than the aftermath of adverse technology and policy shocks.

4.A  Appendix 4

4.A.1

The problem for the firm is to

$$\max \frac{P_{f,t}}{P_t} y_{f,t} - (1 + r_{f,t}^L) \frac{W_{f,t}}{P_t} z_{f,t}$$

subject to

$$y_{f,t} = \left( \frac{P_{f,t}}{P_t} \right)^{-\theta} Y_t$$

This yields the following first order conditions, and solution for prices,

$$\Omega = \left( \frac{P_{f,t}}{P_t} \right)^{1-\theta} Y_t - (1 + r_{f,t}^L) \frac{W_{f,t}}{P_t} z_{f,t} \left( \frac{P_{f,t}}{P_t} \right)^{-\theta} Y_t$$
By defining each sector as a sub interval of the unit interval we are able to define the cumulative share of sectors $k = 1...i$.

$$\tilde{\alpha}_i = \sum_{k=1}^{i} \alpha_k$$

where $\tilde{\alpha}_0 = 0$ and $\tilde{\alpha}_N = 1$. The interval for sector $i$ is then $[\tilde{\alpha}_{i-1}, \tilde{\alpha}_i]$. We assume that there are $B_{n,i}$ unions in each sector. that is each firm and bank is matched with. There is one union per a firm, which is attached to the fixed contracts in that firm. Thus, they are partitioned into cohort interval within the interval $[\tilde{\alpha}_{i-1}, \tilde{\alpha}_i]$. We define the share of each cohort within the sector as $\lambda_{ij}$ so that $\sum_{j=1}^{B_{n,i}} \lambda_{ij} = 1$ with the $B_{n,i}$ - vector $\lambda_i \in \Delta^{B_{n,i}-1}$. The cumulative share $\tilde{\lambda}_{ij}$ is defined analogously to $\tilde{\alpha}_k$. We can therefore define the interval of contract setting unions corresponding to cohort $j$ in sector $i$ as

$$[\tilde{\alpha}_{i-1} + \tilde{\lambda}_{ij-1}\tilde{\alpha}_i, \tilde{\alpha}_{i-1} + \tilde{\lambda}_{ij}\tilde{\alpha}_i]$$

if cohorts are symmetric $\lambda_{ij} = B_{n,i}^{-1}$ and $\tilde{\lambda}_{ij} = jB_{n,i}^{-1}$

The sectors are differentiated by integer contract length $T_i \in \mathbb{Z}_{++}$ which is the same for all banks and firms within a sector. The timing of the contract setting process with the sector is summarised by an $B_n - 1$ tuple of integers $\{T_{ij}\}_{j=2}$ which
specifies when the age setting cycle cohort $j$ moves. The cycle begins when cohort 1 moves first (period 1): this defines the beginning of the cycle, so that $1 \leq T_{ij} \leq B_i$ by convention we assume that the $j$’s are ordered so that $T_{ij}$ is strictly increasing. (see Dixon and Kara for more discussion)

In order to characterise our economy with non uniform contract setting we need to specify the calendar date $t_i$ when each interest rate setting process starts for each contract length $T_i$. In a uniform interest rate setting case these start dates are irrelevant as each period is equivalent in all sectors.

We can therefore characterise the contract setting process in a $GTE$ (Generalised Taylor Economy) by $(T, \alpha) \in Z_+ \times \Delta^{B-1}$, which give the contract lengths and sizes of the $B_n$ sectors, and $(B_ni, \lambda_i, t_i) \in Z_+ \times \Delta^{N_i-1} \times Z_+$ which describes the number and relative size of the cohorts in each sector $i$, and the timing / synchronisation of cohorts in that sector:

$$GTE := \{ (T, \alpha), \{B_i, \lambda_i, t_i\}_{i=1}^N \}$$

In the case where each sector has a uniform interest rate setting process, we have a uniform $GTE$ which is more simply parametrised by $(T, \alpha)$ as $(B_i, \lambda_i) = (T_i, T_i^{-1})$ and $t_i$ is equivalent in each period and therefore irrelevant.

The general interest rate index $P$ can be defined in terms of sectors or subintervals $[\tilde{\alpha}_{i-1}, \tilde{\alpha}_i]$ for each sector $i$.

$$P = \left[ \sum_{i=1}^{T} \int_{\tilde{\alpha}_{i-1}}^{\tilde{\alpha}_i} p(f)^{1-\epsilon} df \right]^{1-\epsilon}$$

This can be broken down further into intervals for each cohort, where we note that all firms in the same cohort have the same price $p(f) = p_{ij}$ for $b \in [\tilde{\alpha}_{i-1} + \tilde{\lambda}_{ij-1} \alpha_i \tilde{\alpha}_{i-1} + \tilde{\lambda}_{ij} \alpha_i]$.

\(^{57}\)In fact all that is required is the start date of one cycle since then the start date of all cycles is given.
We can log-linearise the price equations around the steady state. Whilst Dixon and Kara assume that all firms with the same wage set the same price we assume that all firms that set the same wage set the same price receive the same market determined interest rate, and demand the same labour. This can be written in log-linearisation terms around a steady state (where we assume $P = 1$):

$$
P = \left[ \sum_{i=1}^{T} \sum_{i=1}^{i} \int_{\tilde{\alpha}_{i-1} + \tilde{\lambda}_{ij} \alpha_{i}}^{\tilde{\alpha}_{i} + \tilde{\lambda}_{ij} \alpha_{i}} p(f)^{1-\epsilon} \, db \right]^{\frac{1}{1-\epsilon}} \tag{4.46}
$$

(Note that $B_{i}$ can be replaced with $i$ since the only difference between sectors is the contract length).

**4.3.4**

The Lagrangian is given by,

$$
L = E_{t} \sum_{s=0}^{i-1} \beta^{t} \left[ C_{h,t+s}^{1-\sigma} + \frac{\gamma}{1-\beta} \left( \frac{M_{h,t+s}}{P_{t+s}} \right)^{1-b} - \eta D \frac{N_{t+s}^{1+\eta}}{1+\eta} \right] + 
E_{t} \sum_{s=0}^{i-1} \beta^{t} \lambda_{t} \left[ \frac{M_{h,t-1}}{P_{t}} + \frac{B_{ht}}{P_{t}} + \frac{W_{ht}N_{ht}}{P_{t}} + \Pi_{ht} + T_{t} - C_{ht} - \frac{M_{ht}}{P_{t}} - \frac{B_{ht}}{P_{t}} \right]
$$

where

$$N_{ht+s} = W_{h,t+1}^{-\theta} K_{t+s} \tag{4.48}$$

$$
L = E_{t} \sum_{s=0}^{i-1} \beta^{t} \left[ C_{h,t+s}^{1-\sigma} + \frac{\gamma}{1-\beta} \left( \frac{M_{h,t+s}}{P_{t+s}} \right)^{1-b} - \eta D \frac{\left(W_{h,t}^{-\theta} K_{t+s}\right)^{1+\eta}}{1+\eta} \right] + 
E_{t} \sum_{s=0}^{i-1} \beta^{t} \lambda_{t} \left[ \frac{M_{h,t-1}}{P_{t}} + \frac{B_{ht}}{P_{t}} + \frac{W_{h,t+1}^{-\theta} K_{t+s}}{P_{t}} + \Pi_{ht} + T_{t} - C_{ht} - \frac{M_{ht}}{P_{t}} - \frac{B_{ht}}{P_{t}} \right]
$$
\[ L = \sum_{s=0}^{i-1} \beta^t \left[ \frac{C_{ht+s}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left( \frac{M_{ht+s}}{P_{t+s}} \right)^{1-b} - \eta_D \frac{K_{t+s}^{1+\eta}}{1+\eta} W_{h,t}^{-\theta(1+\eta)} \right] + \]
\[ \sum_{s=0}^{i-1} \beta^t \lambda_t \left[ \frac{M_{ht-1}}{P_t} + \frac{B_{ht}}{P_t} + W_{h,t}^{1-\theta} \frac{K_{t+s}}{P_{t+s}} + \Pi_{ht} + T_t - C_{ht} - \frac{M_{ht}}{P_t} - \frac{B_{ht}}{P_t} \right] \]

\[ \frac{\partial L}{\partial W_{t+s}} = \sum_{s=0}^{i-1} \beta^t \left[ \lambda_{t+s} (1-\theta) W_{ht}^{-\theta} \frac{K_{t+s}}{P_{t+s}} + \theta W_{h,t}^{-\theta(1+\eta)-1} \eta_D K_{t+s}^{1+\eta} \right] = 0 \]

sub in the Lagrangian multiplier

\[ \frac{\partial L}{\partial W_{t+s}} = \sum_{s=0}^{i-1} \beta^t \left[ C_{h,t+s}^{1-\sigma} (1-\theta) W_{ht}^{-\theta} \frac{K_{t+s}}{P_{t+s}} + \theta W_{h,t}^{-\theta(1+\eta)-1} \eta_D K_{t+s}^{1+\eta} \right] = 0 \]

divide by \((1-\theta)\)

\[ \sum_{s=0}^{i-1} \beta^t \left[ C_{h,t+s}^{1-\sigma} W_{ht}^{-\theta} \frac{K_{t+s}}{P_{t+s}} - \frac{\theta}{\theta-1} W_{h,t}^{-\theta(1+\eta)-1} \eta_D K_{t+s}^{1+\eta} \right] = 0 \]

multiply by \(W_{h,t}^{-\theta}\) and rearranging we have,

\[ \sum_{s=0}^{i-1} \beta^t \left[ C_{h,t+s}^{1-\sigma} \frac{K_{t+s}}{P_{t+s}} - \frac{\theta}{\theta-1} W_{h,t}^{-\theta(1+\eta)-1} \eta_D K_{t+s}^{1+\eta} \right] = 0 \]

\[ \sum_{s=0}^{i-1} \beta^t \left[ C_{h,t+s}^{1-\sigma} \frac{K_{t+s}}{P_{t+s}} \right] = W_{h,t}^{-\theta(1+\eta)-1} \frac{\theta}{\theta-1} \sum_{s=0}^{i-1} \beta^t \left[ \eta_D K_{t+s}^{1+\eta} \right] \]

\[ \sum_{s=0}^{i-1} \beta^t \left[ C_{h,t+s}^{1-\sigma} \frac{K_{t+s}}{P_{t+s}} \right] W_{h,t}^{\theta(1+\eta)+1} = \frac{\theta}{\theta-1} \sum_{s=0}^{i-1} \beta^t \left[ \eta_D K_{t+s}^{1+\eta} \right] \]

\[ \beta C^{1-\sigma} P^{-1} K W = \frac{\theta}{\theta-1} \eta_D \beta K^{1+\eta} \]

\[ W_{h,t}^{\theta(1+\eta)+1} = \frac{\theta}{\theta-1} \left[ \frac{E_t \sum_{s=0}^{i-1} \beta^s \left[ \eta_D K_{t+s}^{1+\eta} \right]}{E_t \sum_{s=0}^{i-1} \beta^s \left[ C_{h,t+s}^{1-\sigma} \frac{K_{t+s}}{P_{t+s}} \right]} \right] \]

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\[ W_{h,t} = \frac{\theta}{\theta - 1} \left[ \frac{\sum_{s=0}^{i-1} \beta^s E_t \left[ \eta_D R_{i+1}^{1+\eta} \right]}{E_t \sum_{s=0}^{i-1} \beta^s \left[ \frac{C_{h,t+s}^{1+\eta}}{P_{t+s} K_{t+s}} \right]} \right]^{\frac{1}{\theta - 1}} \]

### 4.A.4

The problem for the firm in cohort \( j \) in sector \( i \) is to maximise their profits by selection the optimal reset price \( \tilde{P}_{ij,t} \)

\[
\max_{\tilde{P}_{ij,t}} \sum_{i=0}^{i-1} \beta^i \left[ \tilde{P}_{ij,t} - m c_{t+i} P_{t+i} \right] Y_{ij,t+i}
\]

s.t \( Y_{ij,t} = \left( \frac{P_{ij,t}}{P_t} \right)^{-\theta} Y_t \)

This yields the following first order conditions, and solution for prices,

\[
\Omega = \sum_{i=0}^{i-1} \beta^i \left[ \left( \frac{\tilde{P}_{ij,t}}{P_{t+i}} \right)^{1-\theta} - m c_{t+i} \left( \frac{\tilde{P}_{ij,t}}{P_{t+i}} \right)^{-\theta} \right] Y_{t+i}
\]

\[
\frac{\partial \Omega}{\partial \tilde{P}_{ij,t}} = \sum_{i=0}^{i-1} \beta^i \left[ (1 - \theta) \left( \frac{\tilde{P}_{ij,t}}{P_{t+i}} \right)^{-\theta} P_{t+i}^{-1} Y_{t+i} - \theta m c_{t+i} \left( \frac{\tilde{P}_{ij,t}}{P_{t+i}} \right)^{-\theta - 1} \right] = 0
\]

\[
\sum_{i=0}^{i-1} \beta^i \left[ \left( \frac{\tilde{P}_{ij,t}}{P_{t+i}} \right)^{-\theta} P_{t+i}^{-1} Y_{t+i} \right] = \frac{\theta}{(\theta - 1)} \sum_{i=0}^{i-1} \beta^i \left[ m c_{t+i} \left( \frac{\tilde{P}_{ij,t}}{P_{t+i}} \right)^{-\theta - 1} P_{t+i}^{-1} Y_{t+i} \right]
\]

\[
\tilde{P}_{ij,t}^{-\theta} \sum_{i=0}^{i-1} \beta^i \left[ P_{t+i}^{\theta-1} Y_{t+i} \right] = \tilde{P}_{ij,t}^{-\theta - 1} \frac{\theta}{(\theta - 1)} \sum_{i=0}^{i-1} \beta^i \left[ m c_{t+i} P_{t+i}^{\theta} Y_{t+i} \right]
\]
substitute in our marginal cost

\[ \tilde{P}_{ij,t}^{-\theta} \sum_{i=0}^{i-1} \beta^i [P_{t+i}^{\theta-1}Y_{t+i}] = \tilde{P}_{ij,t}^{-\theta-1} \frac{\theta}{(\theta-1)} \sum_{i=0}^{i-1} \beta^i \left[ R_{ij,t+i}^l \frac{W_{ij,t+i}}{P_{t+i}^l} Z_{t+i}^{-1} P_{t+i}^\theta Y_{t+i} \right] \]

\[ \tilde{P}_{ij,t}^{-\theta} \sum_{i=0}^{i-1} \beta^i \left[ P_{t+i}^{\theta-1}Y_{t+i} \right] = \tilde{P}_{ij,t}^{-\theta-1} \frac{\theta}{(\theta-1)} \sum_{i=0}^{i-1} \beta^i \left[ R_{ij,t+i}^l W_{ij,t+i} Z_{t+i}^{-1} P_{t+i}^{\theta-1} Y_{t+i} \right] \]

\[ \tilde{P}_{ij,t}^{\theta} \sum_{i=0}^{i-1} \beta^i = \tilde{P}_{ij,t}^{\theta-1} \frac{\theta}{(\theta-1)} \sum_{i=0}^{i-1} \beta^i \left[ R_{ij,t+i}^l W_{ij,t+i} Z_{t+i}^{-1} \right] \]

\[ \tilde{P}_{ij,t} = \frac{\sum_{i=0}^{i-1} \beta^i \left[ R_{ij,t+i}^l W_{ij,t+i} Z_{t+i}^{-1} \right]}{\sum_{i=0}^{i-1} \beta^i} \]

which from our definition of flexible prices given in (4.6) can be written as

\[ \tilde{P}_{ij,t} = \frac{\sum_{i=0}^{i-1} \beta^i P_{ij,t+i}^s}{\sum_{i=0}^{i-1} \beta^i} \]

where \( P_{ij,t+i}^s \) is the optimal price at time \( t+i \).

4.A.5

Log linearised Aggregate Demand

Log-linearising equation (4.40) from the household’s first order conditions,

\[ (1 - \sigma \dot{C}_t) = \beta E_t \left[ (1 + r^D)(1 + 1 + \ddot{r}_t^D)(1 + \dot{P}_t)(1 - \dot{P}_{t+1})(1 - \sigma \dot{C}_{t+1}) \right] \]

or,

\[ (1 - \sigma \dot{C}_t) = \beta (1 + r^D) E_t \left[ 1 - \dot{P}_{t+1} + \dot{P}_t - \sigma \dot{C}_{t+1} + 1 + \ddot{r}_t^D \right] \]

Defining \( E_t \dot{\pi}_t^{P} = E_t \dot{P}_{t+1} - \dot{P}_t \) as the expected log deviation of inflation from its steady state value (assuming \( \pi_t^{P,T} = 0 \)) then the above equation reduces to,
\[
(1 - \sigma \hat{C}_t) = \beta(1 + r^D) - \beta(1 + r^D)E_t\hat{\pi}_{t+1}^P - \beta(1 + r^D)\sigma E_t\hat{C}_{t+1} + \beta(1 + r^D)E_t(1 + r_t^D)
\]

Using the steady state value of the deposit rate and re-arranging,

\[
\hat{C}_t = E_t\hat{C}_{t+1} - \frac{1}{\sigma}[E_t\hat{r}_t^D - E_t\hat{\pi}_{t+1}^P]
\]

Substituting the market clearing condition for the goods market and noting that households know the actual deposit rate (refinance rate) when making their consumption decision, the above equation can be written as,

\[
\hat{Y}_t = E_t\hat{Y}_{t+1} - \frac{1}{\sigma} [\hat{r}_t^D - E_t\hat{\pi}_{t+1}^P]
\]

where our deposit rate is given by

\[
\hat{r}_{i,t}^D = \frac{1}{i} \sum_{j=1}^{i} r_{t+1-i}^{CB}
\]

\[
\hat{r}_t^D = \sum_{i=1}^{T} \alpha_i \hat{r}_{it}^D
\]

**Log linearised wage equation**

We can rewrite our wage equation (4.42) as

\[
\left[ E_t \sum_{s=0}^{i-1} \beta^s \left[ \frac{C_{h,t+s}^{\theta-\sigma}}{P_{t+s}} K_{t+s} \right] \right] W_{h,t}^{\theta_{\eta+1}} = \frac{\theta}{\theta - 1} E_t \sum_{s=0}^{i-1} \beta^s \left[ \eta_D K_{ij,t+s}^{1+\eta} \right]
\]

\[
E_t \sum_{s=0}^{i-1} \beta^s \left[ C_{h,t+s}^{\theta-\sigma} P_{t+s}^{-1} K_{t+s} \right] W_{h,t}^{\theta_{\eta+1}} = \frac{\theta}{\theta - 1} E_t \sum_{s=0}^{i-1} \beta^s \left[ (1 + \eta)\eta_D K_{ij,t+s} \right]
\]
log linearising we have

\[ \sum_{s=0}^{i-1} \beta^s \left[ C^{-\sigma} P^{-1} KW (1 - \sigma \hat{C}_{h,t+s} - \hat{P}_{t+s} + \hat{K}_{t+s} + (\theta \eta + 1) \hat{W}_{h,t} \right] \]

\[ = \frac{\theta}{\theta - 1} E_t \sum_{s=0}^{i-1} \beta^s \eta_D K^{1 + \eta} \left[ (1 + \eta) \hat{K}_{ij,t+s} \right] \]

In the steady state note that

\[ C^{-\sigma} P^{-1} KW = \frac{\theta}{\theta - 1} \eta_D K^{1 + \eta} \]

As a result we have,

\[ \sum_{s=0}^{i-1} \beta^s \left[ (1 - \sigma \hat{C}_{h,t+s} - \hat{P}_{t+s} + \hat{K}_{t+s} + (\theta \eta + 1) \hat{W}_{h,t} \right] = \sum_{s=0}^{i-1} \beta^s \left[ 1 + (1 + \eta) \hat{K}_{ij,t+s} \right] \]

if we minus \( \sum_{s=0}^{i-1} \beta^s \ast 1 \) from both sides we have

\[ \sum_{s=0}^{i-1} \beta^s \left[ -\sigma \hat{C}_{h,t+s} - \hat{P}_{t+s} + \hat{K}_{t+s} + (\theta \eta + 1) \hat{W}_{h,t} \right] = E_t \sum_{s=0}^{i-1} \beta^s \left[ (1 + \eta) \hat{K}_{ij,t+s} \right] \]

rearranging

\[ \sum_{s=0}^{i-1} \beta^s (\theta \eta + 1) \hat{W}_{h,t} = E_t \sum_{s=0}^{i-1} \beta^s \left[ (1 + \eta) \hat{K}_{ij,t+s} + \sigma \hat{C}_{h,t+s} + \hat{P}_{t+s} - \hat{K}_{ij,t+s} \right] \]

\[ \sum_{s=0}^{i-1} \beta^s (\theta \eta + 1) \hat{W}_{h,t} = E_t \sum_{s=0}^{i-1} \beta^s \left[ \eta \hat{K}_{ij,t+s} + \sigma \hat{C}_{h,t+s} + \hat{P}_{t+s} \right] \]

substituting in our definition of \( K \) Recall that

\[ K = \frac{\theta}{(\theta - 1)} Z^{(\theta - 1)} P^{\theta} Y \]
\[ K(1 + \hat{K}_{ij,t}) = \frac{\theta}{(\theta - 1)} Z^{\theta - 1}_j r^{L(\theta - \theta)} P^\theta Y(1 + (\theta - 1) \hat{Z}_{j,t} - \theta \hat{r}_{ij,t} + \theta \hat{P}_t + \hat{Y}_t) \]

\[ \hat{K}_{ij,t} = (\theta - 1) \hat{Z}_{j,t} - \theta \hat{r}_{ij,t} + \theta \hat{P}_t + Y_t \]

plugging in to our wage equation

\[
\sum_{s=0}^{i-1} \beta^s (\theta \eta + 1) \hat{W}_{h,t} \\
= E_t \sum_{s=0}^{i-1} \beta^s \left[ \eta \left( \theta - 1 \right) \hat{Z}_{j,t+s} - \theta \hat{r}_{ij,t} + \theta \hat{P}_{t+s} + \hat{Y}_{t+s} \right] + \sigma \hat{C}_{h,t+s} + \hat{P}_{t+s} \]

using \( Y_t = C_t \) note that this assumption assumes that we set \( \chi = 1 \).

\[
\sum_{s=0}^{i-1} \beta^s (\theta \eta + 1) \hat{W}_{h,t} \\
= E_t \sum_{s=0}^{i-1} \beta^s \left[ \eta \left( \theta - 1 \right) \hat{Z}_{j,t+s} - \theta \hat{r}_{ij,t} + \theta \hat{P}_{t+s} + \hat{Y}_{t+s} \right] + \sigma \hat{C}_{h,t+s} + \hat{P}_{t+s} \]

\[
(\theta \eta + 1) \hat{W}_{h,t} \\
= \frac{1}{\sum_{s=0}^{i-1} \beta^s} \left[ E_t \sum_{s=0}^{i-1} \beta^s \left[ \eta \left( \theta - 1 \right) \hat{Z}_{j,t+s} - \eta \theta \hat{r}_{ij,t} + (\eta \theta + 1) \hat{P}_{t+s} + (\eta + \sigma) \hat{Y}_{t+s} \right] \right] \]

Since we do not know our location on the idiosyncratic element of production,
\( E_t \hat{Z}_{j,t+s} = E_t \hat{Z}_{t,s} \)

\[ \hat{W}_{ij,t} = \frac{1}{\sum_{s=0}^{i-1} \beta^s} \left[ E_t \sum_{s=0}^{i-1} \beta^s \left[ \frac{\eta \left( \theta - 1 \right) \hat{Z}_{t+s} - \eta \theta \hat{r}_{ij,t} + (\eta \theta + 1) \hat{P}_{t+s} + (\eta + \sigma) \hat{Y}_{t+s}}{(\theta \eta + 1)} \right] \right] \]

or

\[ \hat{W}_{h,t} = \frac{1}{\sum_{s=0}^{i-1} \beta^s} \left[ E_t \sum_{s=0}^{i-1} \beta^s \left[ \delta_a \hat{Y}_{t+s} + \delta_h \hat{Z}_{t,s} + \hat{P}_{t+s} - \delta \hat{r}_{ij,t} \right] \right] \]

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where

$$\delta_a = \frac{(\eta + \sigma)}{(\theta \eta + 1)}, \quad \delta_b = \frac{\eta (\theta - 1)}{(\theta \eta + 1)}, \quad \delta_c = \frac{\eta \theta}{\theta \eta + 1}$$

Finally, we can define the aggregate wage in the sector as

$$\hat{W}_{it} = \frac{1}{i} \sum_{j=1}^{i} \hat{W}_{i,t+1-j}$$

The wage definition in real terms

$$\hat{W}^R_{ht} = \frac{1}{\sum_{s=0}^{i-1} \beta_s^*} \left[ E_t \sum_{s=0}^{i-1} \beta_s^* \left[ \delta_a \hat{Y}_{t+s} + \delta_b \hat{Z}_{t+s} - \delta_c \hat{r}_{ij,t} \right] \right]$$

and

$$\hat{W}^R_{it} = \frac{1}{i} \sum_{j=1}^{i} \hat{W}^R_{i,t+1-j}$$

Log linearised Prices

Log-linearising (4.6), prices in each sector we have

$$P_i^* = \frac{\theta}{(\theta - 1)} (1 + \hat{r}_i^L) W_{i,t} Z_t$$

$$(1 + \hat{P}_i^*) P = \frac{\theta}{(\theta - 1)} (1 + \hat{r}_i^L) W_{i,t} Z_{i,t}^{-1} (1 + \hat{r}_i^L + \hat{W}_{i,t} - \hat{Z}_t)$$

$$\hat{P}_{i,t} = \hat{r}_i^L + \hat{W}_{i,t} - \hat{Z}_t$$

$$\hat{P}_{i,t} = \hat{W}_{i,t} + \hat{r}_{i,t}^L - \hat{Z}_t$$

where is the $\hat{Z}_t^{-1}$is the average productivity across the economy. The linearised reset price for each cohort in each sector is given as,

$$\hat{P}_{i,t} = \frac{\sum_{i=0}^{i-1} \beta_i \hat{P}_{i,t+i}}{\sum_{i=0}^{i-1} \beta_i}$$

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The sector prices are therefore

\[ \hat{P}_{it} = \frac{1}{i} \sum_{j=1}^{i} \hat{P}_{i,t+1-j} \]

Aggregate prices are given as

\[ \hat{P}_t = \sum_{i=1}^{T} \alpha_i \hat{P}_{i,t} \]

Aggregate inflation is given as

\[ \pi_t = \hat{P}_t - \hat{P}_{t-1} \]

**Log linearised loan rates**

define the reset loan rate and sector and aggregate rates

will need the phi, which will depend on the aggregate markup and ideal markup

\[ \hat{r}_{it}^L = \frac{1}{i} \sum_{j=1}^{i} \hat{r}_{i,t+1-j} \]

\[ \hat{r}_t^L = \sum_{i=1}^{T} \alpha_i \hat{r}_{it} \]

**Log linearised reset loan rate**

The lending rate equation is,

\[ \hat{r}_{ij,t} = r_{t}^{CB} + \hat{\rho}_{ij,t} + \frac{\alpha_i L_i^R}{\frac{1}{\tau} E_t \sum_{s=0}^{i-1} L_{i,t+s}^R} \left( 1 + \frac{(i - 1)}{2} r_{t}^{CB} \right) \left( \hat{\rho}_t - \hat{\rho}_t \right) \]

which in the steady state is

\[ \hat{r}^L = r^{CB} + \rho \]

since

\[ \frac{\alpha_i L_i^R}{\frac{1}{\tau} E_t \sum_{s=0}^{i-1} L_{i,+s}^R} = 1 \]

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and \( E_t \rho_t^{Lb} = E_t \tilde{\rho}_t \).

Log linearising,

\[
(1 + r^L) \left( 1 + 1 + r_{ij,t}^L \right) = (1 + r^{CB}) \left( 1 + 1 + r_t^{CB} \right) + \rho(1 + \tilde{\rho}_t^{Lb})
\]

\[
+ \rho(1 + \alpha_i \hat{L}_t^R - \frac{1}{i} E_t \sum_{s=0}^{i-1} \hat{L}_{i,s,t+s}^R + \hat{\rho}_t^L) - \rho(1 + \alpha_i \hat{L}_t^R - \frac{1}{i} E_t \sum_{s=0}^{i-1} \hat{L}_{i,s,t+s}^R + \hat{\rho}_t) \\
+ \rho \frac{(i - 1)}{2} r^{CB} (1 + \alpha_i \hat{L}_t^R - \frac{1}{i} E_t \sum_{s=0}^{i-1} \hat{L}_{i,s,t+s}^R + \hat{\rho}_t + \hat{r}_t^{CB}) \\
- \rho \frac{(i - 1)}{2} r^{CB} (1 + \alpha_i \hat{L}_t^R - \frac{1}{i} E_t \sum_{s=0}^{i-1} \hat{L}_{i,s,t+s}^R + \hat{\rho}_t + \hat{r}_t^{CB}) \\
\]

\[
(1 + r^L) \left( 1 + 1 + r_{ij,t}^L \right) = (1 + r) \left( 1 + 1 + r_t^D \right) + \rho(1 + \tilde{\rho}_t^{Lb}) \\
+ \rho(1 + \hat{\rho}_t^{Lb}) - \rho(1 + \hat{\rho}_t) \\
+ \rho \frac{(i - 1)}{2} r^{CB} (1 + \hat{\rho}_t^{Lb}) \\
- \rho \frac{(i - 1)}{2} r^{CB} (1 + \hat{\rho}_t)
\]

\[
(1 + r^L) + (1 + r^L) \left( 1 + r_{ij,t}^L \right) = (1 + r) + (1 + r) \left( 1 + r_t^D \right) \\
\hat{\rho}_{ij,t}^{Lb} + \rho(1 + \frac{(i - 1)}{2} r^{CB}) \hat{\rho}_t^{Lb} - \rho(1 + \frac{(i - 1)}{2} r^{CB}) \hat{\rho}_t
\]

\[
(1 + r^L) \tilde{r}_{ij,t}^L = (1 + r) \tilde{r}_t^D \\
\rho \left[ \hat{\rho}_{ij,t}^{Lb} + \frac{(i - 1)}{2} r^{CB} \left( \hat{\rho}_t^{Lb} - \hat{\rho}_t \right) \right]
\]

\[
\tilde{r}_{ij,t}^L = \frac{1}{(1 + r^L)} \left\{ (1 + r) r_t^{CB} + \rho \left[ \hat{\rho}_{ij,t}^{Lb} + \frac{(i - 1)}{2} r^{CB} \left( \hat{\rho}_t^{Lb} - \hat{\rho}_t \right) \right] \right\}
\]

recall that \( \hat{\rho}_t^{Lb} \) is the aggregate of all markups made at the branch level and \( \hat{\rho}_t \) is
the "ideal" aggregate markup that would be given to all firms if there was no loan rate stickiness (but there is wage stickiness).

**log linearised reset markup at the branch level**

Real the loan rate markup is

\[
\tilde{\rho}^{Lb}_{i,t} = \frac{(\bar{\varepsilon} - \bar{\varepsilon})}{2i} E_t \sum_{s=0}^{i-1} \chi_{t+s} A_{t+s} \Phi_{ij,t+s}^2 \sum_{s=0}^{i-1} W_{ij,t+s}
\]

or

\[
\tilde{\rho}^{Lr}_{i,t} = \frac{(\bar{\varepsilon} - \bar{\varepsilon})}{2i} E_t \sum_{s=0}^{i-1} \chi_{t+s} A_{t+s} \Phi_{ij,t+s}^2 W_{ij,t+s}^{R(-1)}
\]

Since all firms are identical, in the steady state this is given as

\[
\tilde{\rho}^{Lr}_i = \frac{(\bar{\varepsilon} - \bar{\varepsilon})}{2i} E_t \sum_{s=0}^{i-1} \chi_{0+s} A_{0+s} \Phi_{ij,0+s}^2 W_{ij,0+s}^{R(-1)}
\]

\[
\rho^{Lr} = p^{Lr} = \frac{(\bar{\varepsilon} - \bar{\varepsilon})}{2} \frac{\chi A \Phi^2}{W^R}
\]

as we will see it also holds that \(\rho^{Lr} = p^L\).

Log-linearising we have
Log linearised Probability of Default

Now we turn to log-linearise $\Phi_{ij,t+s} = \left( \frac{\hat{\varepsilon}_{ij,t+s} - \varepsilon}{\hat{\varepsilon} - \varepsilon} \right)$, or $\Phi_{ij,t+s} = \left( \frac{\varepsilon_{ij,t+s}}{\varepsilon - \hat{\varepsilon}} \right) - \frac{\varepsilon}{\hat{\varepsilon} - \varepsilon}$

$$\Phi \left( 1 + \hat{\Phi}_{ij,t+s} \right) = \left( \frac{1}{\varepsilon - \hat{\varepsilon}} \right) \left( \varepsilon M \right) \left( 1 + \hat{\varepsilon}_{ij,t+s} M \right) - \frac{\varepsilon}{\hat{\varepsilon} - \varepsilon}$$

or,

$$\Phi \Phi_{ij,t+s} = \left( \frac{1}{\varepsilon - \hat{\varepsilon}} \right) \left( \varepsilon M \right) \hat{\varepsilon}_{ij,t+s} M$$

using that in steady state $\Phi_\ast = \left( \frac{\varepsilon M}{\hat{\varepsilon} - \varepsilon} \right)$ we obtain,

$$\Phi \hat{\Phi}_{ij,t+s} = \left( \frac{1}{\varepsilon - \hat{\varepsilon}} \right) \left( \varepsilon M \right) \hat{\varepsilon}_{ij,t+s} M$$

using again the steady state value $\Phi_\ast = \left( \frac{\varepsilon M - \varepsilon}{\hat{\varepsilon} - \varepsilon} \right)$ we obtain,

$$\hat{\Phi}_{ij,t+s} = \left( \frac{1}{\varepsilon - \hat{\varepsilon}} \right) \left( \varepsilon M \right) \hat{\varepsilon}_{ij,t+s} M$$

or,
\[ \hat{\Phi}_{ij,t+s} = \left( \frac{\varepsilon^M}{\varepsilon^M - \varepsilon} \right) \hat{\varepsilon}_{ij,t+s} \]

We now turn to log-linearise

\[ E_t \varepsilon^M_{ij,t+s} = \frac{(1 + \hat{r}_{ij,t}^L) W^R_{ij,t+s}}{E_t \hat{A}_{t+s} + E_t \hat{\chi}_{t+s}} \]

\[ \varepsilon^M (1 + \hat{\varepsilon}_{ij,t+s}^M) = \frac{(1 + \hat{r}_t^L) W^R}{\chi A} \left( 1 + 1 + \hat{\rho}_{ij,t}^L \right) \left( 1 + \hat{W}_{ij,t+s}^R \right) \left( 1 - \hat{A}_{t+s} \right) + (1 - \hat{\chi}_{t+s}) \]

using that in steady state \( \varepsilon^M = \frac{(1+r^L)W^R}{\chi A} \) we obtain,

\[ \hat{\varepsilon}^M_{ij,t+s} = \hat{r}_{ij,t}^L + \hat{W}_{ij,t+s}^R - \hat{A}_{t+s} - \hat{\chi}_{t+s} \]

using \( \hat{\Phi}_t = \left( \frac{\varepsilon^M}{\varepsilon - \varepsilon} \right) \hat{\varepsilon}^M_t \) then the probability of default at time \( t + s \) is

\[ \hat{\Phi}_{ij,t+s} = \left( \frac{\varepsilon^M_t}{\varepsilon^M - \varepsilon} \right) \left( \hat{r}_{ij,t} + \hat{W}_{ij,t+s}^R - \hat{A}_{t+s} - \hat{\chi}_{t+s} \right) \quad (4.49) \]

Combining this with our linearised risk premium,
\[ \hat{p}^L_{ij,t} = \frac{1}{\bar{\kappa}} \sum_{s=0}^{i-1} \left[ \hat{A}_{t+s} + 2\hat{\Phi}_{t+s} + \hat{\chi}_{t+s} - \hat{W}^R_{ij,t+s} \right] \]

\[ \hat{p}^R_{ij,t} = \frac{1}{\bar{\kappa}} \sum_{s=0}^{i-1} \left[ \hat{A}_{t+s} + \hat{\chi}_{t+s} - \hat{W}^R_{ij,t+s} + 2 \left( \frac{\varepsilon_M}{\varepsilon_M - \varepsilon} \right) \left( \hat{r}^L_{ij,t} + \hat{W}^R_{ij,t+s} - \hat{A}_{t+s} - \hat{\chi}_{t+s} \right) \right] \]

\[ \hat{p}^L_{ij,t} = 2 \left( \frac{\varepsilon_M}{\varepsilon_M - \varepsilon} \right) \hat{r}^L_{ij,t} - \frac{1}{\bar{\kappa}} \sum_{s=0}^{i-1} \left[ 2 \left( \frac{\varepsilon_M}{\varepsilon_M - \varepsilon} \right) \hat{A}_{t+s} - \hat{A}_{t+s} + 2 \left( \frac{\varepsilon_M}{\varepsilon_M - \varepsilon} \right) \hat{\chi}_{t+s} \right] \]

\[ \hat{p}^R_{ij,t} = 2 \left( \frac{\varepsilon_M}{\varepsilon_M - \varepsilon} \right) \hat{r}^L_{ij,t} - \frac{1}{\bar{\kappa}} \sum_{s=0}^{i-1} \left[ 2 \left( \frac{\varepsilon_M}{\varepsilon_M - \varepsilon} \right) \hat{A}_{t+s} - \hat{A}_{t+s} + \left( \frac{2\varepsilon_M}{\varepsilon_M - \varepsilon} \right) \hat{\chi}_{t+s} \right] \]

\[ \hat{\rho}^R_{ij,t} = 2\hat{\varsigma}_1 \hat{\varsigma}_2 + \hat{\varsigma}_2 \hat{\varsigma}_2 \hat{\varsigma}_3 - \hat{\varsigma}_2 \hat{\varsigma}_3 - \hat{\varsigma}_2 \hat{\varsigma}_3 \]

where \( \varsigma_1 = \left( \frac{\varepsilon_M}{\varepsilon_M - \varepsilon} \right) \), \( \varsigma_2 = \left( \frac{\varepsilon_M + \varepsilon}{\varepsilon_M - \varepsilon} \right) \), and \( \hat{\varsigma}_{ij,t} = \frac{1}{\bar{\kappa}} \sum_{s=0}^{i-1} \hat{A}_{t+s} \), \( \hat{\varsigma}_{ij,t} = \frac{1}{\bar{\kappa}} \sum_{s=0}^{i-1} \hat{\chi}_{t+s} \)

and \( \hat{W}^R_{ij,t} = \hat{W}_{ij,t} - \frac{1}{\bar{\kappa}} \sum_{s=0}^{i-1} \hat{p}_{t+s} \) are the reset level of technology, collateral and real wages that the branch considers.

We can now consider the flexible sector the loan rate markup to firm \( f \) in the flexible sector will have a wage that is fixed for a finite duration.

\[ \hat{\rho}^L_{ij,t} = 2\hat{\varsigma}_1 \hat{\varsigma}_2 + \hat{\varsigma}_2 \hat{\varsigma}_2 \hat{\varsigma}_3 - \hat{\varsigma}_2 \hat{\varsigma}_3 - \hat{\varsigma}_2 \hat{\varsigma}_3 \]

We can now look at the \( \Psi \) term. Recall,
\[ \rho^{Lb}_t = \sum_{i=1}^{i} \alpha_i \hat{\rho}^{Lb}_{i,t} \]

\[ \rho_{st} = \frac{1}{i} \sum_{j=1}^{i} \hat{\rho}^{Lb}_{ij,t+1-j} \]

and

\[ \tilde{\rho}_t = \sum_{i=1}^{i} \alpha_i \tilde{\rho}_{i,t} \]

\[ \tilde{\rho}_{st} = \frac{1}{i} \sum_{j=0}^{i-1} \tilde{\rho}^{Lb}_{ij,t+1-j} \]

the linearised versions are

\[ \tilde{\rho}^{Lb}_t = \sum_{i=1}^{i} \alpha_i \tilde{\rho}^{Lb}_{i,t} \]

\[ \tilde{\rho}_{st} = \frac{1}{i} \sum_{j=0}^{i-1} \tilde{\rho}^{Lb}_{ij,t+1-j} \]

and

\[ \tilde{\hat{\rho}}^{L}_{t} = \sum_{i=1}^{i} \alpha_i \tilde{\hat{\rho}}^{Lb}_{i,t} \]

\[ \tilde{\hat{\rho}}_{st} = \frac{1}{i} \sum_{j=0}^{i-1} \tilde{\hat{\rho}}^{Lb}_{ij,t+1-j} \]

we can write

\[
\tilde{\hat{\rho}}^{L}_{ij,t+1-j} - \tilde{\rho}^{Lb}_{ij,t+1-j} = \left[ 2\xi_1 \tilde{\hat{\rho}}^{L}_{t+1-j} + \xi_2 \left( \tilde{W}_{t+1-j} - P_t \right) - \xi_2 \tilde{\hat{A}}_t - \xi_2 \tilde{X}_t \right] - \left[ 2\xi_1 \tilde{\hat{\rho}}^{L}_{t+1-j} + \xi_2 \tilde{\hat{W}}\tilde{R}_{ij,t+1-j} - \xi_2 \tilde{\hat{\theta}}_{ij,t+1-j} \tilde{\eta}_2 - \tilde{\hat{\theta}}_{ij,t+1-j} \right] \\
= \xi_2 \left( \tilde{W}_{t+1-j} - P_t \right) - \xi_2 \tilde{\hat{W}}\tilde{R}_{ij,t+1-j} - \xi_2 \tilde{\hat{A}}_t + \xi_2 \tilde{\hat{\theta}}_{ij,t+1-j} - \xi_2 \tilde{\hat{X}}_t + \xi_2 \tilde{\hat{c}}_{ij,t+1-j}
\]
Thus at the sector level

\[
\widehat{\rho}_t - \widehat{\rho}_t^{Rb} = \varsigma_2 \left( \frac{1}{i} \sum_{j=0}^{i-1} \widehat{W}_{ij,t+1-j} - \varsigma_2 P_t \right) - \varsigma_2 \left( \frac{1}{i} \sum_{j=0}^{i-1} \widehat{W}_{ij,t+1-j}^R + \frac{1}{i} \sum_{j=0}^{i-1} \varsigma_2 \left( \widehat{a}_{ij,t+1-j} - \varsigma_2 \hat{A}_t \right) \right)
\]

\[
-\varsigma_2 \frac{1}{i} \sum_{j=0}^{i-1} \left( \hat{A}_t \right) + \frac{1}{i} \sum_{j=0}^{i-1} \varsigma_2 \left( \widehat{c}_{ij,t+1-j} \right) - \varsigma_2 \frac{1}{i} \sum_{j=0}^{i-1} \left( \hat{\chi}_t \right)
\]

\[
= \left( \frac{1}{i} \sum_{j=0}^{i-1} \widehat{W}_{ij,t+1-j} \right) + \varsigma_2 P_t - \varsigma_2 \frac{1}{i} \sum_{j=0}^{i-1} \left( \widehat{W}_{ij,t+1-j}^R \right) + \frac{1}{i} \sum_{j=0}^{i-1} \varsigma_2 \left( \widehat{a}_{ij,t+1-j} \right) - \varsigma_2 \hat{A}_t
\]

\[
+ \frac{1}{i} \sum_{j=0}^{i-1} \varsigma_2 \left( \widehat{c}_{ij,t+1-j} \right) - \varsigma_2 \hat{\chi}_t
\]

let \( \hat{a}_{i,t} = \frac{1}{i} \sum_{j=0}^{i-1} \left( \widehat{a}_{ij,t+1-j} \right), \hat{c}_{i,t} = \frac{1}{i} \sum_{j=0}^{i-1} \left( \widehat{c}_{ij,t+1-j} \right) \) and \( \widehat{W}_{i,t} = \frac{1}{i} \sum_{j=0}^{i-1} \widehat{W}_{ij,t+1-j} \)

additionally we now account for the fixed wage flexible rate sector.

\[
\widehat{\rho}_t - \widehat{\rho}_t^{Rb} = \varsigma_2 \left( \frac{1}{i} \sum_{j=0}^{i-1} \sum_{t=1}^{T} \alpha_i \widehat{W}_{ij,t+1-j} - P_t \right) - \varsigma_2 \alpha_1 \left( \widehat{W}_{i,t} - P_t \right)
\]

\[
-\varsigma_2 \frac{1}{i} \sum_{j=0}^{i-1} \sum_{t=1}^{T} \alpha_i \widehat{W}_{ij,t+1-j}^R + \varsigma_2 \alpha_1 \widehat{W}_{i,t}^R + \sum_{t=1}^{T} \varsigma_2 \alpha_i \hat{a}_{i,t} - \varsigma_2 \sum_{t=1}^{T} \alpha_i \hat{A}_t
\]

\[
+ \sum_{t=1}^{T} \varsigma_2 \alpha_i \hat{c}_{i,t} - \varsigma_2 \sum_{t=1}^{T} \alpha_i \hat{\chi}_t
\]

\[
= \varsigma_2 (1 - \alpha_1) \left( \widehat{W}_{i,t} - P_t - \widehat{W}_{i,t}^R \right) + \sum_{t=1}^{T} \varsigma_2 \hat{a}_{i,t} - \varsigma_2 \hat{A}_t + \sum_{t=1}^{T} \varsigma_2 \hat{c}_{i,t} - \varsigma_2 \hat{\chi}_t
\]

\[
= \varsigma_2 (1 - \alpha_1) \left( \widehat{W}_{i,t} - P_t - \widehat{W}_{i,t}^R \right) + \varsigma_2 \left( \hat{a}_t - \hat{A}_t \right) + \varsigma_2 \left( \hat{c}_t - \hat{\chi}_t \right)
\]

where \( \hat{a}_t = \sum_{i=1}^{T} \alpha_i \hat{a}_i, \hat{c}_t = \sum_{i=1}^{T} \alpha_i \hat{c}_i \) and \( \widehat{W}_{i,t}^R = \sum_{i=1}^{T} \alpha_i \widehat{W}_{i,t}^R. \) Also \( \varsigma_2 \left( \hat{a}_t - \hat{A}_t \right) \) and \( \varsigma_2 \left( \hat{c}_t - \hat{\chi}_t \right) \) is the deviation in technology and collateral considered when firms take up the contract to the actual state of technology today and \( \varsigma_2 \left( \widehat{W}_{i,t} - \widehat{P}_t - \widehat{W}_{i,t}^R \right) \) is the deviation between the expected real wage and the actual real wage. Note that the spillovers on the wage term are \((1 - \alpha_1)\) since wages may deviate in the flexible interest rate sector \(\alpha_1\) but the loan rate is able to adjusted optimally.

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As a result the interest rate premium becomes

\[
\tilde{r}_{ij,t}^L = \frac{1}{(1+i^L)} \left\{ (1+i)\tilde{r}_{i}^{CB} + \rho^L \left[ 2\zeta_1\tilde{r}_{ij,t}^L + \zeta_2 (\tilde{W}_{ij,t} - \tilde{a}_{ij,t} - \tilde{c}_{ij,t}) \right. \right.
\]
\[
+ \Delta^C \zeta_2 \left[ (\tilde{a}_t - \tilde{A}_t) + (\tilde{c}_t - \chi_t) + (1 - \alpha_1) \left( \tilde{W}_t - \tilde{P}_t - \tilde{W}_t^R \right) \right]\right\}
\]

where \( \Delta^C = (1 + \frac{(i-1)}{2})r^{CB} \).
Chapter 5

Overall Conclusions

At the heart of modern monetary theory lie groups of rational representative agents making intertemporal decisions. Capturing and aggregating their optimal behaviour and interactions with one another, in the presence of real and nominal rigidities, is imperative for the construction of meaningful quantitative models with policy relevance. We have contributed to this dynamic and evolving literature through an investigation into the role of some specific frictions in the goods and credit markets.

This thesis begins by examining the impact that a combination of price stickiness and the switching behaviour of consumers to cheaper products has on variation in the relative weights of the consumption basket and the impact this behaviour has on the dynamics of aggregate variables. Much of the New Keynesian literature places great emphasis on both price stickiness, in particular the staggered price setting due to Calvo (1983), and product differentiation in the goods market. As a result, how we model the interaction between these two assumptions as well as the impact they have on macroeconomic variables is important for our understanding of the transmission mechanism and conclusions of these models, which will be of interest to macroeconomists in academia and policy makers at central banks.

Chapter 2 addresses the above research question and lays the foundations of our innovation by building a model where the price stickiness, usually only assumed to
directly affect the supply side of the economy, influences the product demands of firms and therefore aggregate consumption. As is well known, the price elasticity of demand dictates the extent to which product demands will fluctuate for a given relative price alteration. We show that the consequence of these dynamics in the presence of price stickiness is that relative price distortions among differentiated consumption goods may not be eliminated between two points in time. Since firms understand this type of switching behaviour, and that it is more prevalent in product markets characterised by similar consumption alternatives, they account for this in their profit maximisation.

Aggregating across all agents we show that following economic shocks consumption will deviate from that described by the Euler equation by some additional inflation dynamics. This additional channel works to dampen the responses of inflation and the output gap the more competitive is the goods market. The model also suggests that in more competitive economies the response of interest rates are smoother as increased competition improves the inflation-output trade-off.

Chapter 3 addresses the question of how monetary policy should be conducted under various policy regimes in light of the additional inflation dynamics shown in chapter 2. We address this issue since the implications of the aforementioned channel on household welfare and, in turn, optimal monetary policy have non-negligible implications for the policy decision of central bankers.

As is standard in the optimal policy literature, household welfare is shown to be costly due to deviations in inflation and output away from their efficient levels. It is the price elasticity of demand, that amplifies the welfare losses associated with a given inflation variance as it generate larger swings in product demands. However due to our additional inflation channel outlined in chapter 2, the variance of inflation itself is also reduced as the price elasticity of demand increases.

Assuming the central bank follows a typical Taylor rule, welfare losses following economic shocks are shown to ultimately decrease as the economy becomes
more competitive. This result, shown in chapter 3, is contrary to the standard new Keynesian literature where instead a move towards a more competitive economy is detrimental for welfare. Our results also indicate that there are additional gains, compared to the baseline version of Ravenna and Walsh (2006), to be made by moving from optimal discretion to optimal commitment. Since these additional gains arise from the introduction of our inflation channel, they are also exaggerated in more competitive economies. Although it is known that optimal commitment delivers a superior outcome throughout the literature, there are two reasons why commitment to a credible policy plan marks an even more improved result in our model version. Firstly, in the discretionary equilibrium the central bank places a greater, ultimately suboptimal, emphasis on current output stabilisation. By moving to a policy plan this outcome is eliminated as the central bank is able to control future inflation expectations to minimise losses over many periods. Secondly, as firms know their product demands are more susceptible to future price changes, they make smaller adjustments to their price today. Consequently, the central bank can reduce the variance of inflation and the output gap by even more today at the smaller cost of a greater variance in later periods. As a result our impulse response functions display smoother and more persistent responses for the output gap and inflation when we have a central bank in place who can convince society that they are committed to a time consistent policy plan.

Whilst chapter 4 shares features with the previous two chapters the focus is more in keeping with the credit market imperfections literature with an emphasis on the role of frictions in the cost of borrowing for firms. Given that central banks are unable to directly set either loan or deposit rates, understanding the dynamics of the transmission mechanism of interest rates to firms and households is an important issue for macroeconomists. Chapter 4 focuses on the interaction between time-varying risk of default and fixed term interest rate contacts and examines the impact that these frictions in the financial market have on the wider economy. Given that
during the recent financial crisis we have observed an increase in firm defaults as well as a sharp fall in interest rates, the cost of financial contracts varied greatly according to when they were last re-negotiated. Consequently, this area of research is both topical and of value for monetary policy makers and economists in this research field.

By introducing a financial sector into the Generalised Taylor Economy of Dixon and Kara (2010, 2011) we allow for loans to firms and the saving of households to be on a fixed rate for a range of predetermined durations. The credit market is further enriched with the introduction of default risk at the firm level which is driven by idiosyncratic productivity shocks. Like much of the literature in this area our commercial bank accounts for this risk with an appropriate premium over the savings rate.

The combination of legally bound fixed financial contracts and state dependent default risk generates the potential for some contracts to be misspriced. To overcome this issue, new contracts up for renegotiation must make up for any otherwise unaccounted losses or gains through an adjustment to their risk premium. As a result our reset financial contracts, must account for these unexpected losses as well as the anticipated costs of the firm and the expected default risk over the contract term.

Like much of the incomplete interest rate pass through literature our results indicate that the impulse response of output, inflation and all of the interest rates are smoother and more persistent when a proportion of savings and loan rates are unable to react following an economic shock. This generates a procyclical spread between the central bank rate and the deposit rate. In contrast, risk of default generates a countercyclical external finance premium between the loan and deposit rates. We show the aggregate of these two spreads and therefore the behaviour of key variables to differ according to the type of economic shock. In particular, following adverse credit shocks which add a disturbance to the amount of collateral
the bank is able to recover, the countercyclical spread and spillovers of losses to new contracts is greater than the aftermath of adverse technology or monetary policy shocks.

As we have outlined above, the approaches adopted and the results depicted in this thesis have implications for our understanding of the channels, and manner with which, certain nominal rigidities impact the new Keynesian framework. Not only has this raised relevant insights for economic policy at central banks, but it has also opened up potential avenues for future research. The main theme of this thesis outlined in chapters 2 and 3 could be extended to analyse the role of our price dispersion effects in aggregate consumption when the economy is structured in an alternative way. An obvious extension would be to investigate how the presence of diminishing returns in production, which cause the price elasticity of demand to enter aggregate supply, interact with the same elasticity which we find to enter aggregate demand. Another approach would be to extend the model to include alternative product market structures, for example the role of a large firm who’s individual, not collective, price setting is able to influence others. Finally, given the recent emphasis of papers indicating the adverse effects of price dispersion on welfare in a trend inflation environment it would be very interesting to see how our macroeconomic equilibrium and welfare effects differ when there is a positive steady state level of inflation.

Chapter 4 also raises a number of interesting policy questions and avenues of future analysis, especially regarding the optimal behaviour of the policymaker. Given the range of frictions and potential policy levers that arise from the setup of this framework, a number of interesting research questions regarding the welfare implications and the conduct of optimal monetary policy arise. For example; What are the welfare implications of credit spreads? Should, and under what circumstances, policymakers target a rate different to the policy rate such as the loan rate or reset loan rate? Do, and if so under what conditions, alternative approaches such
as macroprudential policies and liquidity injections into the financial sector, have a place in the toolkit of policy makers?
Bibliography


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