Study of Homopolar DC Generator

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Nomenclature

\((\hat{a}_x, \hat{a}_y, \hat{a}_z)\) Unit vector in Cartesian coordinate system

\((\hat{a}_r, \hat{a}_\theta, \hat{a}_z)\) Unit vector in cylindrical coordinate system

\(\hat{a}_n\) Unit vector perpendicular to surface

\(A\) Magnetic vector potential

\((A_x, A_y, A_z)\) Components of magnetic vector potential in Cartesian coordinate system

\(B\) Magnetic flux density

\(|B|\) Magnitude of magnetic flux density

\(B_t\) Tangential magnetic flux density

\(B_n\) Normal magnetic flux density

\((B_x, B_y, B_z)\) Components of magnetic flux density in Cartesian coordinate system

\((B_r, B_\theta, B_z)\) Components of magnetic flux density in cylindrical coordinate system

\(e, \varepsilon\) Induced Voltage

\(E\) Electric field intensity

\(F\) Force

\(g\) Gravity

\(H\) Magnet field intensity

\(I\) Current

\(J\) Current density

\(K\) Moment of inertia

\(m\) Mass

\(R, r\) Radius

\(R_B\) Brush resistance

\(R_o\) Brush body resistance

\(R_f\) Surface film resistance

\(R_c\) Constriction resistance

\(R_{\text{Load}}\) Load resistance

\(R_{\text{Total}}\) Total resistance
Note:

- Bold symbols used in this dissertation represent vector quantities.
Abstract

The aerospace and marine sectors are currently using or actively considering the use of DC networks for electrical distribution. This has several advantages: higher VA rating per unit volume of cable and ease of generator connections to the network. In these systems the generators are almost exclusively ac generator (permanent magnet or wound field synchronous) that are linked to the dc network via an electric converter that transforms the ac generator output voltage to the dc rail voltage.

The main objective of this project is to develop a Homopolar DC Generator (HDG) that is capable of generating pure DC voltage and could therefore remove the need for an electric converter and ease connection issues to a dc electrical distribution network. The project aim is to design, build and test a small technology demonstrator, as well as electromagnetic modeling validation.

In Chapter 1, the initial generator concepts proposed to fulfill the aforementioned requirements of DC generator are presented, as well as an obscurity in electromagnetic induction law faced at the beginning of this project. Also the advantages, disadvantages and different applications of Homopolar DC Generators are covered in Chapter 1. In Chapter 2, Faraday's law of induction and the ways of using it properly are discussed using some example. The preliminary design calculations to construct the prototype HDG are presented in Chapter 3. Also the prototype construction and assembly procedure are discussed in this chapter. In this project, magnetostatics and current flow Finite Element (FE) simulations were used to assess the prototype HDG. In Chapter 4, the results of 2D and 3D-FE simulation are presented; furthermore the limitations of the FE simulations to assess the HDG performance are included. In Chapter 5, the results of the practical tests are demonstrated and assessed, as well as comparison between some of the results obtained practically and those obtained using FE-modeling. Using sliding contacts in the HDG is obligatory so some definitions corresponding to electrical contact resistances are given in Chapter 5. Final chapter is conclusions including the results assessments, future works to design, simulation and construction of the HDG.

**Keywords:** Homopolar DC Generator, 3D-Finite Element, Magnetostatics modeling, Current Flow Analysis
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Finally, I wish to recall the name and memorial of my grandfather, Mr. Ahmad Fahimi Pour whom I missed during the writing of my dissertation.
CHAPTER 1

INTRODUCTION

1.1 Aims and objectives
In the aerospace and marine sectors, two types of power distribution system are utilized; namely, AC systems and DC systems. There are two methods to supply DC networks by means of rotating electrical generators; one technique is to utilize AC generators like synchronous generators; the output voltage of the AC generator must be rectified first, the rectified voltage can then be connected to the DC network. The existence of an AC/DC converter between the AC generator and the DC network is inevitable. The other technique is to use conventional DC generators; in this type of generator, the voltage induced in its armature winding is actually AC, but the mechanical commutator rectifies the armature voltage to produce DC. If the output voltage and current of a DC generator exactly fulfills the requirement of a DC network, it is possible to connect its output directly to the DC network. If the output of the DC generator does not match to the network, a DC/DC converter is needed to regulate the DC voltage.

The primary objective of this project is to design and construct a novel DC generator, capable of generating DC voltage without using any sort of electronic or mechanical commutator to supply a DC network directly. This thesis makes the following contributions to knowledge:

1. Analysis of homopolar generator topologies
2. Design, analysis and test of a homopolar generator specially when it is used as a DC generator with continuous output current
3. Development of 2D and 3D finite element models for homopolar generators
4. Construction of a prototype homopolar generator and experimental investigation
5. Assessment of the effect of armature reaction using finite element models

1.2 Initial DC generator concept
To build an innovative DC generator with a specification as described in the previous section, the generator concept shown in Figure 1.1 was initially developed. The rotor of this concept, shown in Figure 1.1(A), is an assembly of two solid cylinders made of
electrical steel and placed at either side of a cylindrical magnet. The magnet is axially magnetised; a positive magnetic pole is formed therefore on one end of the rotor and the negative magnetic pole on the other end. Industrial magnets in term of their mechanical properties are fragile, for this reason a torque tube made of non-magnetic steel should be mounted around the magnet. The torque tube connects the two solid rotor cylinders together; a cut-away view of this tube and its location on the rotor is shown in Figure 1.1(A). The steel torque tube on the rotor has two duties: firstly it works as a torque tube, ensuring the magnet itself is not subjected to any torque; and secondly it provides containment for the magnet. Stainless steel was chosen for the tube because of its non-magnetic properties. At either end of the rotor, two shafts of stainless steel were attached.

The stator of this generator concept is shown in Figure 1.1(B). One part of the stator is a hollow cylinder made of a ferromagnetic material, typically electrical steel same as the material used for the rotor poles.

In the inside of the stator, two separate copper tubes are attached, as shown in Figure 1.1(B). The lengths of the copper tubes correspond to the axial length of the rotor poles. At each end of the copper tubes, several pairs of electrical connections are attached. These connections create the electrical output from the generator. The number and thickness of these connections depends on the magnitude of current that the copper tubes carry.

In Figure 1.1(C) and (D), a cut-away view and a full view of the proposed generator are illustrated.

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\* Ferromagnetic Materials: Materials with very high magnetic permeability.
To illustrate the magnetic flux paths in the concept generator, the structure of the generator is redrawn in two dimensions and illustrated in Figure 1.2. The red lines in Figure 1.2 show the path of magnetic flux originated from permanent magnet through the stator, rotor and two airgaps. As can be seen from Figure 1.2, constant and uniform magnetic flux cut the copper tubes.

Initially it was expected that a voltage would be induced in the copper tubes when the rotor starts to rotate, because there is a relative motion between the magnetic flux and the copper tubes.
The flux cutting rule could be used to calculate the magnitude of the voltage induced in the copper tubes:

\[ e = L \cdot B \times v \]  

(1-1)

Each term of Equation (1-1) is described as follow:

\( e \) - Induced Voltage  
\( B \) - Magnetic flux density cut a conductor  
\( L \) - Length of the conductor

In an electrical machine, \( v \) in Eq (1-1) is always considered to be the relative velocity between the magnetic flux and the conductor, and it does not matter if the conductor remains stationary and the magnetic flux moves or vice-versa.

For example, in a conventional DC generator, the field windings are on the stator so the magnetic flux is stationary and the conductors on the rotor move and cut the magnetic flux. To calculate the induced voltage in the conductors of the DC generator using Eq (1-1), \( v \) is the velocity of rotor conductors.

In a synchronous generator for example the field windings are on the rotor and the conductors are on the stator, so in this case the magnetic flux moves and the conductors are stationary. To calculate the induced voltage in the conductors in this example, \( v \) is the velocity of magnetic flux.

To calculate the voltage induced in the copper tubes of the generator proposed in Figure 1.2, Eq (1-1) was applied, the velocity of the rotor or in other words the velocity of magnetic flux was used for \( v \), and the axial length of the copper tube was \( L \), and magnetic flux originating from the magnet and cutting the copper tube was \( B \).

The two black arrows in Figure 1.2 show the polarity of expected generated voltage, \( e \), in the copper tubes.

Replacing the linear velocity with the rotational rotor velocity in Eq (1-1), it is possible to obtain the induced voltage as a function of the rotational velocity:

\[ v = \omega_r \times R \]  

(1-2)

\( \omega_r \) - Rotational velocity of the rotor  
\( R \) - Radius of the rotor
The induced voltage as a function of rotational speed is equal to:

\[ e = L.B \times (\mathbf{\omega} \times \mathbf{R}) \]  

(1-3)

Two additional features of this generator concept were:

- The electrical outputs of the copper cans on the stator would be connected in series to double the output voltage of the generator.
- There is the possibility of using a high speed prime mover, such as a gas turbine, due to the simple and robust construction of the rotor of the generator. The high speed would also increase the voltage output of the generator.

One drawback of this first concept was the small cross-sectional area of the magnet which would reduce the airgap magnetic flux density and the generator voltage as a consequence.

This problem was eliminated by increasing the diameter of the magnet up to the diameter of the rotor poles as illustrated in Figure 1.3(A). This would increase the magnet costs. Voltage regulation in the generator was also improved by modifying the stator as illustrated in Figure 1.3(B), and adding a single solenoid field winding in such a way that the rotor can freely rotate inside this winding.

Unfortunately although the initial design concept appears attractive, the Faraday paradox relating to homopolar DC generators indicates that the proposed generator in Figure 1.1, Figure 1.2 and Figure 1.3 does no generate any voltage. It also demonstrates
that any homopolar DC generator without the use of sliding contacts is impossible. The proposed generator concept shown in Figure 1.3 therefore was modified to the topology shown in Figure 1.4.

In Figure 1.4(A), the two copper tubes on the original concept were combined into a single copper tube and mounted onto the rotor surface. Three brush assemblies, each one consisting of six brushes, were then mounted on the stator in such a way that contacts were made with the two ends and the middle of the rotor. The magnitude and polarity of the voltage induced in the copper tube is explained in CHAPTERS 4 and 5. It is also possible to remove the rotor magnet and use the stator coils to provide the full field. The rotor magnet was removed therefore and a single solid cylinder covered by the copper tube was used as a rotor, shown in Figure 1.4 (B). The rotor construction remains simple and robust but the high speed operation is now limited by the sliding electrical contact.
1.3 Literature review on some previous Homopolar DC Machines (HDG)

"DC homopolar machines are based on Faraday's disk machine demonstrated in 1883" [1]; which indicates that the first Homopolar DC Machine was invented and constructed by Michael Faraday. In the next chapter, electromagnetic induction and the operating principles of the Homopolar DC Generator have been explained. Any Homopolar DC Machine can operate as a motor or generator.

Homopolar DC machines have the following advantages and disadvantages:

Advantages of Homopolar DC Machines

- The only true DC generator/motor
  For example, a conventional DC generator is an AC generator in reality, because the induced voltage in the armature winding of a conventional DC generator is AC and a mechanical commutator or electronic commutator is used to produce a DC output.
- Simple and robust rotor and stator
- Control of these machines is much easier than AC machines and conventional DC generators
- High torque at low speed is achievable if used as motor because the torque is not function of velocity
- "It is intrinsically quiet since there are no AC magnet fields present to create acoustical noise." [2]

Disadvantages of Homopolar DC generators

- Using sliding contact is unavoidable; the brushes should carry high current and should have very small contact resistance∗.
- Commonly produce very low DC voltages

The armatures of Homopolar DC machines have a very low electric resistance, thus if they are used as a generator, the generator can produce very high currents but a low output DC voltage. Similarly Homopolar DC motors require a high current, low voltage DC supply.

High current, low voltage armatures can be advantageous or disadvantageous depending on the application.

∗Contact resistance is explained in CHAPTER 5.
The following applications have been proposed for Homopolar DC Machines:

- Homopolar DC generator for pipe line welding (used as a pulsed output)[3]
- Ship propulsion [2], [4]…
- Homopolar tachogenerators, to measure speed (The output of generator is open circuit) [5]
- Using HDG to supply an electrolyser to electrolyze water and generate hydrogen[6]
- DC generator (continuous output)
- In the patent [7], using a HDG as a pulsed power source to weld the bars and end rings of the cage rotor of an electric motor together was proposed.
- In [8], HDG was proposed, which was wind-powered, as a high current, low voltage source to supply water electrolysis unit producing hydrogen and oxygen.
  Both hydrogen and oxygen could then be used as an energy source.

In some applications, Homopolar DC generators were used because of their high current and low voltage output. Today with the advancement of power electronics, it is possible to design and construct a power supply to produce high current and low voltage output. The design and construction of such a power supply was presented in [9]; the output of this power supply was 6 [V] and 20 [kA].

The following differences are evident in the Homopolar DC machines built to date:

In terms of the magnetic circuit:

- Air cored
- Ferromagnetic cored

In terms of field winding:

- Super-conductor (Both low temperature and high temperature are applicable)
- Copper winding

In terms of armature (input/output):

- Pulse
- Continuous

In terms of the sliding contacts:

- Liquid sliding contact
- Solid contact (brush)
Homopolar DC machines may be designed and built in a variety of sizes, powers and shapes. In the following pages, some of the Homopolar DC Machines (HDM) constructed and designed previously are discussed.

In [10] published at 1958, the design and test of a 10 [kW] HDG is presented. Mercury was used as the current collector in this machine. The output range of the machine was 10 to 16 [kA] at 1-0.625 [V].

In [11], the HDG was studied analytically to observe its capability as a pulse generator. The configuration illustrated in Figure 1.5 was used in the study and "a transient expression for the field flux" and its time constant', "an expression for the flux density due to eddy currents" and "an expression for the transient current in the magnetizing coils" were all obtained.

In [12] by the same author of [11], "the effect of a sudden short circuit" on a HDG was also studied analytically. In this paper, the equivalent circuit of the HDG was also presented. In [12] as in [11], the HDG configuration shown in Figure 1.5 was used in the study.

![Homopolar DC Generator as a pulse Generator](image)

**Figure 1.5** Homopolar DC Generator as a pulse Generator [11], © 1968 IEEE

In [5], the operating principle of a Homopolar DC Generator (HDG) was used to design a tachometer; in this application, the output of the generator was open circuit, and the generated voltage was linearly proportional to the speed. In [5], the configurations shown in Figure 1.6 were considered in the design of this type of tachometer; all of these configurations could also be used in the design and construction of general Homopolar DC machines.

*The permission grant of figures used in the literature review is presented in Appendix (III)*
In [13], the fabrication and test of a 50 [kW] homopolar dc motor was presented. The field windings in this motor were made of superconducting material. The aim of this project was propulsion for an ice-breaker ship; two requirements were stated: "high torque at zero speed" and "fast change in shaft speed and direction". The tested machine was rated at 51[kW], 5.77[V], 10 [kA] and its efficiency was 92%; liquid metal brushes were used in this motor. The simple configuration of this machine is shown in Figure 1.7.
In [14], some HDG topologies to be used as pulsed power generators were compared. In this type of generator, initially the output of the machine is open circuit, and by means of the prime mover the speed of the rotor is increased to the required value. Then for a short period of time the output of the generator is connected to the load, so that the kinetic energy stored in the generator rotor is converted to electrical energy. This electrical energy is converted to the load in the form of high current and low voltages.

One of the issues considered in the design of the HDG was "generate more power and store more energy per unit mass" [14]. Some of the topologies compared in [14] are shown in Figure 1.8. All of these configurations can also be used to construct a HDM with continuous input and output. In the configuration shown in Figure 1.8 (A), "most of magnetic circuit is rotated." The field distribution shown in Figure 1.8 (B) is similar to that found in a Faraday disc. Figure 1.8 (C) illustrates another field distribution of a HDG which consists of two counter-rotating rotors; these machines have an advantage which is explained on page (29). Figure 1.8 (C) shows a configuration of a HDG known as a drum type. More details about power density and the advantages and disadvantages of each configuration used as a pulse generator can be obtained in [14].

![Figure 1.8](image)
In [15], the design of an iron core HDG was presented. This machine was designed to be used as a pulse generator. Copper-graphite brushes were specified for this machine. In the design of this machine, it was assumed that the rotational speed of the rotor reaches 12500 \( [\text{RPM}] \), and at this speed the brushes make contact with the rotor by means of a pneumatic actuator. The machine generated up to 1.5 \([\text{MA}]\) and 60 \([\text{V}]\). This machine is shown in Figure 1.9.

![Figure 1.9](image)

"A 10-MJ compact homopolar generator" [15], © 1986 IEEE

In [16], the design of a 40 \([\text{MW}]\) generator was presented. This machine was designed to provide 40 \([\text{MW}]\) with continuous output current for 5 seconds only. The operating speed range of this machine was assumed to be in the range 0 to 7000 \( [\text{RPM}] \). This machine is shown in Figure 1.10. The maximum output current of this machine was 2 \([\text{MA}]\).

![Figure 1.10](image)

Fig. 1. Schematic representation of a 40 megawatt homopolar generator, with gas turbine prime mover.

"A 40 Megawatt homopolar generator" [16], © 1986 IEEE
In [17], construction and test of a high voltage pulsed homopolar DC generator was presented. This generator was air-cored; its output voltage and current were $500[V]$ and $500[kA]$, respectively. In this machine in order to increase the output voltage, a superconducting field winding was used to increase the average magnetic flux density to $5[T]$. This machine was designed and constructed to be used as a pulsed power supply. The machine is shown in Figure 1.11.

![High voltage homopolar generator](image1.png)

**Figure 1.11** High voltage homopolar generator [17], ©1986 IEEE

In [18] and the companion paper [19], the electrical, mechanical and thermal designs of HDG were presented. The application of the HDGs proposed in [18] and [19] was pulse power generation. These machines were also designed to become self excited. In these papers three different configurations were proposed; simple drawings of these configurations are shown in Figure 1.12.

![Self-excited HDG](image2.png)

(A) Drum type  
(B) Contra-rotating disk-type

**Figure 1.12** Self-excited HDG [18], ©1989 IEEE
Figure 1.12(A) shows a drum-type HDG which is air-cored. According to [18], the advantage of designing machine with an air-core was to avoid reducing the magnetic flux density through magnetic saturation. Higher armature emf therefore can be obtained. The rotor of this machine consisted of "a high strength isotropic metal rotor body and copper alloy conducting sleeve [18]". One of drawbacks of the configuration shown in Figure 1.12 (A) was that when the rotor reached full speed and the output of the rotor was suddenly connected to the load, a high current passes through the rotor producing high electromagnetic torques. This torque decelerated the rotor with a high reaction torque transferred to the stator. As a result the structure of the stator had to be capable of withstanding this reaction torque. In the HDG proposed and shown in Figure 1.12(B), the generator had two rotors which rotated in the opposite direction at identical speed. Therefore when the rotors reached the specified speed and the output of the generator was suddenly connected to the load, the stator experienced two equal and opposite reaction torques. The net reaction torque therefore on the stator was zero; Figure 1.13(A) shows a detailed drawing of the HDG shown in Figure 1.12(A). Figure 1.13(B) illustrates another counter-rotating HDG proposed in [18] and [19]; this HDG was very similar to the configuration shown in Figure 1.12(B); the only difference was that this HDG had two auxiliary windings at each end. The magnetic flux distribution in each of these three configurations is shown in Figure 1.14.
In [20], two machines were designed and constructed using superconducting materials; one of these machines was a Homopolar DC motor. This machine was constructed and designed to be used with high temperature superconducting (HTSC) material for the field windings. In [20], it was stated that "the motor will be the test bed for evaluating new HTSC wire coils as they are developed". This machine is shown in Figure 1.15.
In [21], the design of a Homopolar DC machine used as a pulsed generator was presented. The structure of this machine is shown in Figure 1.16. This machine stored the energy mechanically and was capable of generating \(895 \text{ kA} \) at a maximum of 460 [V] "for several seconds and recharge in less than a minute".

In [22], the design and testing of a 300 [kW] Homopolar Generator was described; the schematic drawing of this machine is shown in Figure 1.17. The field winding of this generator was made from a superconductor, and the brushes used were silver graphite. The nominal velocity of this generator was 1300 [RPM] and its output was 230 [V], 1305 [A]. The weight of the rotor was 2.5 tons, and the overall length and outer diameter of the machine were 2.7 [m] and 1.05 [m] respectively.
The most recent designs of Homopolar DC machines for industrial applications have been undertaken by General Atomics Company (GA). These machines were designed and constructed to be used for ship propulsion. According to GA [23], Homopolar DC motors have the following advantages over AC motors:

- Significantly quieter, smaller, and lighter than AC motors
- More efficient than the AC motor systems
- Control is more straightforward and simpler than the AC motor systems
- Suited to simpler and less costly ship electrical distribution architectures

GA constructed and tested 300 [kW] and 3.7 [MW] motors. Both of these machines were prototypes and were used for test. A 36.5 [MW] at 120 [RPM] Homopolar motor was also under development by GA [23]. In [4], some of the characteristics of the 3.7 [MW] motor were presented. The nominal speed of this machine was 500 [RPM], and it was air-cored. The field winding of this machine was superconducting. The average magnetic flux density in this machine was 2 [Tesla], and it had an efficiency of 97%, an input voltage of 145[V] and an input current of 26 [kA]. The weight of this machine was 11.4 tons. The concept model of this machine is shown in Figure 1.18.

![Figure 1.18 Homopolar motor for ship propulsion developed by GA [4], © 2002 IEEE](image)

In [24], the design and fabrication of two field windings made of high temperature superconductor for the GA’s 3.7 [MW] machine was presented. Each field winding
produces $2 \times 10^6$ [A. turns]. The operating current of the field windings was 154 [A]. The maximum magnetic flux density produced by these field windings was 6 [T].

In [25], a homopolar micro motor was built and tested; the rotor of this micro motor was made of mercury and it reached an angular velocity of 100 [RPM] when supplied with 0.5 [V] and 14 [mA]. The volume of this micro motor was $4 \times 4 \times 5$ [mm$^3$].

A schematic and cross section view of this motor is shown in Figure 1.19. In this configuration, the layer made of silicon nitride acts as an insulator, and the doped silicon layers act as the conductor.

Two types of electrical contact have commonly been used in Homopolar DC machines: a liquid sliding contact and a solid brush contact. Each one has advantages and disadvantages.

In [26], an acoustic emission transducer was used in a Homopolar DC motor to monitor the brushes performance such as wear and friction. This type of monitoring can also determine a suitable maintenance strategy. In this paper, it was also shown that it was possible to monitor the condition of the rotor. The type of brush used in this study was a copper wire which formed a sliding contact with the copper rotor.

In homopolar DC machines, brushes are exposed to the magnetic flux and this can affect the performance of the brushes. In [27], the performance of fiber brushes was studied in the presence of magnetic flux.
In [28], different types of liquid sliding contacts were compared and the advantages and disadvantages of each were studied. In [28], it was stated "In contrast to solid sliding electric contacts, liquid metals provide uniform coverage to a slip ring and therefore have very low electrical contact losses and are essentially wear free."

One of the materials used in the study was NaK. This liquid material is a type of electrical contacts made of sodium and potassium, but it can oxidize from exposure to oxygen and water. Another material studied in this paper was gallium indium tin. In this study the power dissipation of both of these liquid electrical contacts in the presence of a magnetic field and rotational speed were examined.

In [29], the design, construction and use of an apparatus to study the performance of liquid metal sliding electrical contacts in the presence of large magnetic fields was presented. The maximum current to be passed through the liquid sliding contact in this device was 180 [kA] at a rotational speed of 180 [RPM]. In this test rig, a superconducting magnetic winding was used to create a magnetic field up to 2 [T]. The aim of this test rig was to study the performance of liquid metal electrical contacts in a Homopolar DC Machine. One drawback of a liquid contact is that "the contacts are subject to hydrodynamic instabilities which can cause the liquid to leave the electric region and therefore not function." [29]

In [29], a concept Homopolar motor was described and is shown in Figure 1.20. In this concept, the field coils were made of superconducting wire and the current collectors were liquid metal. Due to the presence of the high magnetic field created by the superconducting field coils, the performance of the liquid metal current collectors in the presence of this high magnetic field was the focus of this paper.

![Figure 1.20 Concept Homopolar DC Motor presented in [29] © 2010 IEEE](image-url)
The majority of the papers published to date concentrate mainly on using a HDG as a pulse generator or ship propulsion. In this project, the prototype was developed to be used as a generator with continuous output current.

1.4 Structure of the dissertation

The current chapter is followed by five chapters. From CHAPTER 2 to CHAPTER 6 the following material is presented, respectively:

- **CHAPTER 2 HOMOPOLAR DC GENERATOR (HDG), OPERATION, PRINCIPLE AND APPLICATIONS**

  Faraday's law of induction is explained in addition to its limitations to solve some types of problem, and the methods that can be employed. In CHAPTER 2, it can be seen clearly why the first concept to construct a DC generator was unrealisable.

- **CHAPTER 3 PRELIMINARY DESIGN, CONSTRUCTION AND ASSEMBLY OF THE PROTOTYPE HDG**

  The procedure to design and construct the prototype HDG are presented in CHAPTER 3, including all parts and materials used to construct the prototype, as well as the assembly procedure.

- **CHAPTER 4 FINITE ELEMENT MODELS**

  In CHAPTER 4, two and three dimensional Finite Element simulations of the constructed prototype are described, as well as the limitations of the Finite Element in analysing the HDG.

- **CHAPTER 5 EXPERIMENTAL INVESTIGATION**

  In CHAPTER 5, test procedures for the experimental investigation of the prototype are presented, along with results obtained from experimental analysis. The results are compared with the Finite Element simulations.
CHAPTER 6 CONCLUSIONS

In CHAPTER 6, all the results obtained during the project are concluded and future research is highlighted in the field of design and simulation of HDGs.
CHAPTER 2
HOMOPOLAR DC GENERATOR (HDG), OPERATION, PRINCIPLE AND APPLICATIONS

This chapter discusses the basic physics to explain clearly the reasons why the generator concepts illustrated in Figure 1.1 and 1.3 do not generate any voltage and also explain why creating a HDG without any sliding electrical contacts is not possible.

2.1 Faraday's law of induction
Imagine an electric circuit (C) bounds a surface S, as shown in Figure 2.1. Magnetic flux lines with magnetic flux density (B) pass through C. The induced voltage in the circuit (C) according to Faraday's law of induction, Eq (2-1), is equal to the negative rate of change of magnetic flux \( \phi \) through C. If we define vector area \( S \) corresponding to surface S, the hatched area in Figure 2.1, and bounded by contour C, the magnetic flux is equal to the scalar product of \( B \) and \( S \).

\[
e = -\frac{d\phi}{dt} = -\frac{d(B.S)}{dt}
\]  

(2-1)

![Figure 2.1 Surface S bounded by contour C (Circuit C)](image)

To induce a voltage in circuit (C), the magnetic flux \( \phi \), in Eq (2-1) has to change relative to time in three ways:

- \( B \) varies with time but \( S \) does not.
- \( S \) varies with time but \( B \) does not.
- Both \( B \) and \( S \) vary with time.

To clarify how Eq (2-1) may be used to calculate the voltage induced in circuit (C) and also to illustrate the limitation of Eq (2-1), three different cases in which \( B \) remains
unchanged relative to time are presented here; (In the subsequent examples, Eq (2-1) is used without the minus sign because this simply indicates the direction of the induced voltage.) in the most electromagnetics references, some of these cases are used to explain electromagnetic induction. The material presented in this section was extracted from [30-34].

**Case 1**
Consider two stationary conducting rails (R1 and R2) in parallel. R1 and R2 are connected to each other at one end by a stationary conducting bar perpendicular to both R1 and R2, as shown in Figure 2.2(A).

A conducting bar, perpendicular to R1 and R2 slides along R1 and R2 with a constant velocity along the positive x-axis. The velocity of the moving bar is $v$.

The whole structure is situated in a constant and stationary magnetic field, $\mathbf{B}$, given by Eq (2-2), as shown in Figure 2.2.

To measure the voltage induced in the circuit formed by the rails R1 and R2 and the sliding rail, a voltmeter with a high internal resistance is mounted into one of the rails.

$$\mathbf{B} = B \hat{a}_y$$  \hspace{1cm} (2-2)

**Figure 2.2** A moving bar slides over R1 and R2
Using Eq (2-1) to calculate the voltage induced in the circuit PQRS, an imaginary surface bounded by PQRS, the hatched surface in Figure 2.2(B), should be chosen:

\[ e = \frac{d\phi}{dt} = \frac{d(BS_{PQRS})}{dt} \]  

(2-3)

\( S_{PQRS} \) is the vector area of the PQRS plane, given by:

\[ S_{PQRS} = L.x\hat{a}_y \]  

(2-4)

In Eq (2-4), \( x \) is the distance of the moving bar from the stationary bar PS. Substituting Eq (2-2) and Eq (2-4) into Eq (2-3), we get:

\[ e = \frac{d(B\hat{a}_y.Lx\hat{a}_y)}{dt} = B.L \frac{dx}{dt} \]  

(2-5)

The width of the PQRS plane, \( L \), and the magnetic flux density \( B \) are not functions of time—they are constant. The length of the PQRS plane, \( x \), is however a function of time. The velocity of the moving bar (\( v \)) is equal to the time derivative of \( x \):

\[ v = \frac{dx}{dt} \]  

(2-6)

By substituting Eq (2-6) into Eq (2-5), the magnitude of the voltage induced (\( e \)) in the circuit PQRS produce the classic equation:

\[ e = B.L.v \]  

(2-7)

**Case 2**

Let us assume there is a non-magnetic and conducting disc, illustrated in Figure 2.3(A), rotating with a constant angular velocity. Let us further assume that a constant magnet flux is applied externally and parallel to the axis of rotation. The rotating disc therefore cuts the magnetic flux. A conducting and non-magnetic shaft is also attached to the disc. To measure the induced voltage, a voltmeter with a high internal resistance is connected between the shaft and the rim of the disc using two brush contacts. To determine the voltage induced in the disc, an imaginary plane, as shown in Figure 2.3(B), and defined by the vector area given by Eq (2-9), is created. This plane, which is perpendicular to the disc, is bounded by the circuit PQRS. The circuit PQRS is stationary and not moving with respect to the rotating disc.

*In Cartesian coordinate system, \((x, y, z)\) the following relations exist between base vectors:

\[ \hat{a}_x.\hat{a}_y = \hat{a}_y.\hat{a}_x = 0, \quad \hat{a}_x.\hat{a}_z = 1 \]

\[ \hat{a}_y.\hat{a}_z = \hat{a}_z.\hat{a}_y = 0, \quad \hat{a}_y.\hat{a}_x = 1 \]

\[ \hat{a}_z.\hat{a}_x = \hat{a}_x.\hat{a}_z = 0, \quad \hat{a}_z.\hat{a}_y = 1 \]
In the interests of simplicity, the voltmeter and brushes are not shown in Figure 2-3(B), but it should be noted that the voltmeter and brushes are integral elements of the circuit PQRS.

\[ \mathbf{B} = B \hat{a}_y \]  
(2-8)

\[ S_{PQRS} = S_{PQRS} \hat{a}_z \]  
(2-9)

Substituting Eq (2-8) and Eq (2-9) into Eq (2-1), we get:

\[ e = \frac{d \phi}{dt} = \frac{d(B \cdot \hat{a}_z S_{PQRS} \cdot \hat{a}_z)}{dt} = 0 \]  
(2-10)

According to Eq (2-10), the voltage induced in the circuit PQRS or in other words the voltage induced in the rotating disc is equal to zero, because no flux passes through the plane PQRS. However if we perform an experiment to measure the voltage induced in the rotating disc, it is found that the voltmeter shows a non-zero voltage. It is evident from Case 2 that Eq (2-1) is either utilised incorrectly or is not capable of calculating the voltage induced in the rotating disc.
Case 3

This case is very similar to Case 2. In this case, a very long non-magnetic conducting bar is moving with constant velocity along the positive x-axis. Two brushes are placed at either side of the bar and the bar cuts a constant magnetic flux given by Eq (2-11). The magnetic flux is applied externally and perpendicular to the movement of the bar. A voltmeter with a high internal resistance is used to measure the voltage, as shown in Figure 2.4(A).

To calculate the induced voltage, like Case 1 and Case 2, an imaginary surface bounded by the contour PQRS is used; this surface can be defined by the vector area given in Eq (2-12).
\[ B = B \hat{a}_y \]  \hspace{2cm} (2-11)

\[ S_{PQRS} = S_{PQRS} \hat{a}_x \]  \hspace{2cm} (2-12)

In Eq (2-12), \( S_{PQRS} \) is the area of the surface bounded by the circuit PQRS; this surface is depicted by the hatched area in Figure 2.4(B). The voltmeter and brushes are part of the circuit PQRS circuit, but again for simplicity, the voltmeter and brushes are not shown in Figure 2.4(B). The circuit PQRS is stationary and not moving with respect to the moving bar.

Substituting Eq (2-11) and (2-12) into Eq (2-1) gives:

\[ e = \frac{d\phi}{dt} = \frac{d}{dt}(B \hat{a}_y, S_{PQRS} \hat{a}_x) = 0 \]  \hspace{2cm} (2-13)

Again according to Eq (2-13), the voltage induced in the circuit PQRS or in other words, the voltage in the moving bar is equal to zero, because no flux passes through the plane PQRS. However if we create an experiment to measure the voltage induced in the moving bar experimentally, we would find the voltmeter shows a non-zero voltage.

**Figure 2.4** Non-magnetic conducting bar moves with constant velocity through a constant magnetic field
In Case 3, as in Case 2, Eq (2-1) (Faraday's law) produces an incorrect result. However, let us use the integral form of Faraday's law to determine the voltage induced in Cases 1, 2 and 3.

Eq (2-14) represents the integral form of Faraday's law; it is possible to determine Eq (2-14) from Eq (2-1).

\[ e = \oint E \cdot dl = -\oint \frac{\partial B}{\partial t} dS + \oint (v_i \times B) \cdot dl \]  

(2-14)

Based on Eq (2-14), the voltage induced in any closed circuit (C) is equal to the line integral of the electric field intensity (E).

The right side of Eq (2.14) has two terms, the voltage defined by the first term is known as the transformer voltage (emf) and the voltage defined by the second term is known as motional voltage (emf).

In Eq (2-14), S is the surface bounded by the circuit (C) as shown in Figure (2-1), \( v_i \) is the velocity of the path of integration \( dl \), and \( B \) is the magnetic flux density of magnetic flux through the circuit (C).

In Case 1, 2 and 3, \( B \) is not a function of time and is constant, thus:

\[ \frac{\partial B}{\partial t} = 0 \]  

(2-15)

By substituting Eq (2-15) into Eq (2-14), we get:

\[ e = \oint (v_i \times B) \cdot dl \]  

(2-16)

**Case 1**

By using Eq (2-16) for Case 1, Eq (2-17) is obtained:

\[ e = \oint E \cdot dl = \oint_{PQRS} (v_i \times B) \cdot dl \]

\[ = \oint_{PQ} (v_i \times B) \cdot dl + \oint_{QR} (v_i \times B) \cdot dl + \oint_{RS} (v_i \times B) \cdot dl + \oint_{SP} (v_i \times B) \cdot dl \]  

(2-17)

The velocity of the lines, \( PQ \), \( QR \) and \( SQ \) are equal to zero, (lines PQ, QR, RS and SP make up circuit PQRS), so:

\[ e = \oint_{RS} (v_i \times B) \cdot dl \]  

(2-18)

*emf: abbreviation of electromotive force
By using \( v = v_1 \hat{a}_1 \) for \( v_1 \), and Eq (2-2) for \( B \), the voltage induced in the circuit PQRS is:

\[
e = BLv
\]

(2-19)

So, the result is exactly same as the result calculated in Eq (2-7)

**Case 2 and 3**

In both Cases 2 and 3, if we use Eq (2-16), the velocity of the path of integration, namely \( PQ, QR, RS \) and \( SP \) is zero; therefore using Eq (2-16), the result will again be zero as obtained for Cases 2 and 3 in Eq (2-10) and Eq (2-13) respectively.

If we look at the circuit PQRS, or in other words the path of integration used to calculate the voltage induced in Cases 1, 2 and 3 using Eq (2-18), a key difference exists between the paths of integration.

In Case 1, \( dl \) lies on RS and the material of \( dl \) does not change by the motion of the moving bar, or in other words there is no relative motion between \( dl \) and the materials on which \( dl \) lies.

In both Cases 2 and 3, with the rotation of the disc in Case 2 and the movement of the bar in Case 3, the material of \( dl \) on RS changes, or in other words there is a relative motion between the path of integration \( dl \) and the materials on which \( dl \) lies.

In [1] and [2], two definitions exist that describe the reason why using Eq (2-1) and Eq (2-14) gives wrong answer. G.W.Carter [30] states:

"The equation \( \mathcal{E} = -\frac{d\Lambda}{dt} \) * always gives the induced e.m.f. correctly, provided the flux-linkage is evaluated for a circuit so chosen that at no point are the particles of the material moving across it."

Feynman [31] says:

"It [Flux rule- Eq (2-1)] must be applied to a circuit in which the material of the circuit remains the same. When the material of the circuit is changing, we must return to the basic laws. The correct physics is always given by the two basic laws:

\[
F = q(E + v \times B),
\]

\[
\nabla \times E = -\frac{\partial B}{\partial t}
\]

"*

* This equation is same as Equation (2-1). \( \mathcal{E} \) is voltage induced in a circuit C, \( \Lambda \) is flux linkage equal to the number of turns of C multiplied to magnetic flux \( \phi \) pass through circuit C.
Both of these statements show that the path of integration PQRS and the plane bounded by PQRS are chosen incorrectly in Case 2 and 3. The correct selection for the path of integration for Case 2 and 3 are shown in Figure 2.5 and Figure 2.6, respectively.

For Case 2 (Figure 2.5), PQRR'SP should be chosen as the correct circuit or in other words, the path of integration; this path bounds the hatched surface shown in Figure 2.5. Lines PQ, QR and SP are elements of circuit PQRR'SP and they are stationary. Lines RR' and R'S are also part of the circuit PQRR'SP circuit and they move with the rotating disc.

It is possible to divide the hatched surface to two sections, namely PQRSP given by vector area $A_{PQRSP}$ and SRR'S given by vector area $A_{SRR'S}$.

Eq (2-1) can now be used to calculate the voltage induced in circuit PQRR'SP:

$$
e = \frac{d\phi}{dt} = \frac{d(B.A_{PQRSP})}{dt} = \frac{d(B.A_{PQRSP})}{dt} + \frac{d(B.A_{SRR'})}{dt} \quad (2-20)$$

The right side of Eq (2-20) has two terms, the first term is equal to zero because no flux passes through the PQRSP plane; we get therefore:

$$
e = \frac{d(B.A_{SRR'})}{dt} = \frac{d(B\hat{y}.A_{SRR'}\hat{y})}{dt} = B \frac{d(A_{SRR'})}{dt} \quad (2-21)$$

In Figure 2.5, the rotation of SR' relative to SR is determined by the angle $\theta$, so if SR' rotates $2\pi$ radians relative to SR, the area swept by SR' is equal to $\pi b^2$ ($b$ is the radius of the disc).
of the rotating disc). If SR' rotates by $\theta$ radians relative to the SR, the area swept by SR' is equal to area of SRR'S, $A_{SRR'S}$, that is:

$$A_{SRR'S} = \frac{\pi b^2}{2\pi} \times \theta$$  \hspace{1cm} (2-22)

The angular velocity of the disc is:

$$\omega_r = \frac{d\theta}{dt}$$  \hspace{1cm} (2-23)

Substituting Eq (2-22) and Eq (2-23) into Eq (2-21), the voltage induced in the circuit PQRR'SP is:

$$e = \frac{B\omega b^2}{2}$$  \hspace{1cm} (2-24)

The induced emf defined in Eq (2-24) represents the value obtained experimentally. One can also calculate the voltage induced in the circuit PQRR'SP using the integral form of Faraday's law Eq (2-14):

$$e = \oint (\mathbf{v} \times \mathbf{B}).d\mathbf{l} = \oint (\mathbf{v}_{PQ} \times \mathbf{B}).d\mathbf{l} + \oint (\mathbf{v}_{QR} \times \mathbf{B}).d\mathbf{l} + \oint (\mathbf{v}_{RS} \times \mathbf{B}).d\mathbf{l} + \oint (\mathbf{v}_{SP} \times \mathbf{B}).d\mathbf{l}$$

PQ, QR and SP are stationary, so $\mathbf{v}_{PQ}$, $\mathbf{v}_{QR}$ and $\mathbf{v}_{SP}$ in Eq (2-25) are equal to zero. $(\mathbf{V}_{RS} \times \mathbf{B})$ is perpendicular to $d\mathbf{l}$ in the fifth term of Eq (2-25), hence this term is also equal to zero. Eq (2-25) is therefore reduced to:

$$e = \oint (\mathbf{v}_{RS} \times \mathbf{B}).d\mathbf{l} = \oint [(r\omega_0 \hat{a}_\theta) \times B\hat{a}_z].d\mathbf{l} = B\omega_0 \int_b^0 rdr = B\omega_0 \frac{b^2}{2}$$  \hspace{1cm} (2-26)

Therefore:

$$e = \frac{B\omega b^2}{2}$$  \hspace{1cm} (2-27)

The result obtained in Eq (2-27) is identical to the result obtained in Eq (2-24). (Note: Eq (2-27) shows absolute value of the induced voltage, the negative sign is neglected.)
Constant and external magnetic field \( \mathbf{B} = B \hat{z} \)

non-magnetic conducting bar moving along x-axis with constant speed \( \mathbf{x} = v \hat{t} \)

**Figure 2.6** Correct selection of integration path in Case 3

For Case 3 in Figure 2.6, PQRR'S'SP is defined as the circuit or the path of integration; this path bounds the hatched surface shown in Figure 2.6. Lines PQ, QR and SP are elements of the circuit PQRR'S'SP and they are stationary; lines RR' and R'S' and S'S are also part of the circuit PQRR'S'SP circuit and these move with the moving bar.

It is possible to divide the hatched surface into two sections, namely PQRSP given by the vector surface \( \mathbf{A}_{PQRSP} \) and RR'S'SR given by the vector surface \( \mathbf{A}_{RR'S'SR} \).

Using Eq (2-1) to calculate the voltage induced in circuit PQRR'S'SP:

\[
e = \frac{d\phi}{dt} = \frac{d(B \cdot \mathbf{A}_{PQRSP})}{dt} + \frac{d(B \cdot \mathbf{A}_{RR'S'SR})}{dt} \tag{2-28}
\]

The right side of Eq (2-28) has two terms, the first term is equal to zero because no flux passes through PQRSP plane, therefore:

\[
e = \frac{d(B \cdot \mathbf{A}_{RR'S'SR})}{dt} = \frac{d(B \hat{z} \cdot A_{RR'S'SR} \hat{z})}{dt} = B \frac{d(A_{RR'S'SR})}{dt} \tag{2-29}
\]

In Eq (2-29), \( A_{RR'S'SR} \) is equal to:

\[
A_{RR'S'SR} = L x \tag{2-30}
\]

The velocity of the moving bar (\( v \)) is:

\[
v = \frac{dx}{dt} \tag{2-31}
\]

Substituting Eq (2-30) and (2-31) into Eq (2-29) we get:

\[
e = BLv \tag{2-32}
\]

Again this is the value of induced emf found experimentally. One can also again calculate the voltage induced in the circuit PQRR'S'SP using integral form of the Faraday's law:
\[ e = \oint (v \times B) \, dl = \oint (v_{PQ} \times B) \, dl + \oint (v_{QR} \times B) \, dl + \oint (v_{RR} \times B) \, dl + \oint (v_{RS} \times B) \, dl + \oint (v_{SP} \times B) \, dl \]

\[ + \oint (v_{SS} \times B) \, dl + \oint (v_{SS} \times B) \, dl + \oint (v_{SP} \times B) \, dl \]

PQ, QR and SP are stationary, so \( v_{PQ} \), \( v_{QR} \) and \( v_{SP} \) in Eq (2-33) are zero, also \( (v_{RR} \times B) \) and \( (v_{SS} \times B) \) are perpendicular to \( dl \) in the third and fifth terms respectively, hence these terms are also equal to zero. Eq (2-33) reduces to:

\[ e = \int (v_{RS} \times B) \, dl = \int (v_{RS} \cdot \hat{a}_z) \times (B \hat{a}_z) \, (\hat{a}_z \, dz) \]  

\[ e = B L v \]  

The final result obtained in Eq (2-35) is identical to the result obtained using Eq (2-32).

### 2.2 Calculating induced voltage with relative motion

In Section 2.1, we have seen that using Faraday's law of induction in both the derivative and integral forms can lead to some ambiguity particularly when there is a relative motion between a segment of path of integration and the material (the material is always the conductor) on which the segment lies. In Case 1 presented in Section 2.1, the segment of the path of integration that lies on the moving bar, moves with the moving bar, so there is no relative motion. In Case 2, when the path of integration is considered to be stationary, Faraday's law gives the wrong answer, but if a segment of the path of integration lies on the moving disc, moving with the moving disc, Faraday's law gives the correct answer. This condition occurs in Case 3. In Eq (2-14), \( v \) is the velocity of \( dl \) that lies on the conductor (Note: the conductor is the moving bar in Case 1, the rotating disc in Case 2 and the moving bar in Case 3) and moves with the conductor, so that no relative motion exists between \( dl \) and the conductor.

To solve problems in which a segment of the path of integration lies on a conductor that has a relative motion to it, the following method can be used to calculate induced voltage in a conductor*:

Imagine there exists a magnetic field and an electric field in space; the magnetic field is given by the magnetic flux density \( (B) \) and the electric field by the electric field intensity \( (E) \); when a sample electric charge moves through these fields with velocity \( (v_q) \) relative to an observer in the stationary reference frame, the observer in the

---

* The calculation of induced voltage when there is no relative motion (Farday's law) and when there is relative motion, are explained clearly in [32] with proof and examples;
stationary reference frame sees a force on the electric charge, \( q \), as follow:

\[
F = q(E + \mathbf{v}_q \times \mathbf{B})
\]  

(2-36)

This is known as the classical Lorenz Force. If we imagine another observer who moves with the electric charge \( q \), the moving observer detects a force \((F')\) on \( q \) as a result of electric field only, because with respect to the moving observer, \( q \) is stationary and no magnetic force therefore exists on \( q \).

The electric field observed by the moving observer is given by the electric field intensity \((E')\):

\[
F' = qE'
\]  

(2-37)

If the velocity of the moving charge \((q)\) becomes significantly less than the velocity of light, \( F \) and \( F' \) should be identical, therefore:

\[
E' = E + qv_q \times \mathbf{B}
\]  

(2-38)

Also

\[
\nabla \times E = -\frac{\partial \mathbf{B}}{\partial t}
\]  

(2-39)

Substituting Eq (2-38) into Eq (2-39), we get:

\[
\nabla \times E' = -\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v}_q \times \mathbf{B}
\]  

(2-40)

Applying stockes' theorem* to the above equation, we get:

\[
e = \oint_C \mathbf{E'} \cdot d\mathbf{l} = -\oint_{C'} \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{t} + \oint_C \mathbf{v}_q \times \mathbf{B} \cdot d\mathbf{l}
\]  

(2-41)

This equation looks like Eq (2-14), but in Eq (2-14) we use the velocity of the segment of the path of integration lying on the conductor and moving with the conductor, but here \( \mathbf{v}_q \) "is the velocity of the segment of the conductor at the point corresponding to \( d\mathbf{l} \) relative to some given observer"[32]

To clarify Eq (2-41) we can apply it to Cases 1, 2 and 3; in all three cases \( \frac{\partial \mathbf{B}}{\partial t} \) is equal to zero because \( \mathbf{B} \) is not function of time.

---

* Stokes's theorem: For a vector function like \( \mathbf{A} \), the following relation exists:

\[
\int_s (\nabla \cdot \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\mathbf{l}
\]

In the above, \( C \) is the contour of surface \( S \).
In Case 1, the velocity of the path of integration, $v_l$, and the velocity of the conductor on which the path of integration lies, $v_q$, are identical for an observer in the stationary reference frame, so the voltage calculated by Eq (2-14) is exactly the same as the voltage calculated using Eq (2-41).

In Case 2, when we consider the path of integration, PQRS in Figure 2.3(B), it is stationary relative to the observer in the stationary reference frame, using Equation (2-14), the wrong result is obtained, but Eq (2-41) provides the correct answer if we use PQRS in Figure 2.3(B) as the path of integration, therefore:

$$e = \int_{RS} v_q \times B \cdot dl = BLv = \frac{B\omega b^2}{2}$$

(2-42)

It should be emphasized that RS is stationary and in Eq (2-42), $v_q$ is the linear velocity of the disc.

Case 3 is the same as Case 2; when we use PQRS in Figure 2.4(B) as the path of integration in Eq (2-41), we get:

$$e = \int_{RS} v_q \times B \cdot dl = BLv$$

(2-43)

In Eq (2-43), $v_q$ is the linear velocity of the moving bar and it should be emphasized that RS is stationary. So in Case 2, when the path of integration is stationary relative to the observer in the stationary reference frame, using Eq (2-41) results in the correct result.

2-3 A special problem

In Case 3, we consider that the source of the magnetic flux and the voltmeter (observer) are stationary and the non-magnetic conducting bar moves with constant velocity in a constant and stationary magnetic flux; what happens if the bar remains stationary and the source of magnetic field moves or alternatively the bar and the source of magnetic field move and the voltmeter remain stationary and so on. Based on the movement of each element (the non-magnetic conducting bar, voltmeter and the source of magnetic flux) eight states may be defined. The procedure used to calculate the voltage induced in each state is explained here. Firstly the pass of integration should be determined and then using Equation (2-41) the voltage induced in each state can be determined. It should be noted that $\frac{\partial B}{\partial t}$ in this example is equal to zero, so the first term of Eq (2-41) is equal to zero. PQRS is selected as the path of integration and it is assumed that this
path is stationary relative to the observer (voltmeter) for all the states. The structure and path of integration are shown in Figure 2.7.

**Figure 2-7** Long non-magnetic conducting bar, voltmeter and constant and external magnetic field
**State 1:**
Bar \( \Rightarrow \) Stationary
Voltmeter (Observer) \( \Rightarrow \) Stationary
Source of magnetic field \( \Rightarrow \) Stationary

**Induced voltage**
In this state, the full structure is stationary, so no voltage should be induced.

**State 2:**
Bar \( \Rightarrow \) Moving (With velocity \( \mathbf{v} = v\mathbf{\hat{a}}_x \))
Voltmeter (Observer) \( \Rightarrow \) Stationary
Source of magnetic field \( \Rightarrow \) Stationary

**Induced voltage**
The bar moves with velocity \( \mathbf{v} \) relative to the observer (voltmeter) in the stationary reference frame, so there is a relative motion between \( d\mathbf{l} \) and the conductor (bar) on which \( d\mathbf{l} \) lies, therefore \( \mathbf{v}_q \) is equal to \( \mathbf{v} \), and:

\[
e = \oint_C \mathbf{v}_q \times \mathbf{B} \, d\mathbf{l} = vBL
\]

**State 3:**
Bar \( \Rightarrow \) Stationary
Voltmeter (Observer) \( \Rightarrow \) Stationary
Source of magnetic field \( \Rightarrow \) Moving (With velocity \( \mathbf{v} = v\mathbf{\hat{a}}_x \))

**Induced voltage**
Firstly, is should be noted that observer in both the stationary reference frame and in the moving reference frame witness the same magnetic flux \( (\mathbf{B}) \). In this state, there is no relative motion between \( d\mathbf{l} \) and the conductor (bar) on which \( d\mathbf{l} \) lies, so \( \mathbf{v}_q \) is equal to zero:

\[
e = \oint_C \mathbf{v}_q \times \mathbf{B} \, d\mathbf{l} = 0
\]

**State 4:**
Bar \( \Rightarrow \) Stationary
Voltmeter (Observer) \( \Rightarrow \) Moving (With velocity \( \mathbf{v} = v\mathbf{\hat{a}}_x \))
Source of magnetic field \( \Rightarrow \) Moving (With velocity \( \mathbf{v} = v\mathbf{\hat{a}}_x \))
Induced voltage

In this state, there is relative motion between $d\mathbf{l}$ and the conductor (bar) on which $d\mathbf{l}$ lies, for a moving observer (voltmeter) it appears that the bar moves with velocity $-\mathbf{v}$, so $\mathbf{v}_q$ is equal to $(-\mathbf{v})$, and:

$$e = \oint \mathbf{v}_q \times \mathbf{B} \cdot d\mathbf{l} = \oint \mathbf{v} \times \mathbf{B} \cdot d\mathbf{l} = -\mathbf{vBl}$$

**State 5:**

- Bar $\Rightarrow$ Moving (With velocity $\mathbf{v} = v\hat{a}_x$)
- Voltmeter (Observer) $\Rightarrow$ Stationary
- Source of magnetic field $\Rightarrow$ Moving (With velocity $\mathbf{v} = v\hat{a}_x$)

**Induced voltage**

In this state if we imagine another observer moving with the bar, the observer measures the same magnetic field as an observer (voltmeter) who is stationary, so moving the source of the magnetic field has no effect on the induced voltage. Furthermore there is a relative motion between $d\mathbf{l}$ and the conductor (bar) on which $d\mathbf{l}$ lies, so $\mathbf{v}_q$ is equal to $\mathbf{v}$, therefore:

$$e = \oint \mathbf{v}_q \times \mathbf{B} \cdot d\mathbf{l} = \oint \mathbf{v} \times \mathbf{B} \cdot d\mathbf{l} = \mathbf{vBL}$$

**State 6:**

- Bar $\Rightarrow$ Stationary
- Voltmeter (Observer) $\Rightarrow$ Moving (With velocity $\mathbf{v} = v\hat{a}_x$)
- Source of magnetic field $\Rightarrow$ Stationary

**Induced voltage**

Firstly for an observer moving with the voltmeter and an observer in the stationary reference frame, the magnitude of the flux density ($\mathbf{B}$) is constant, so motion of the source of the magnetic field has no effect on the induced voltage. In this state, there is a relative motion between $d\mathbf{l}$ and the material (bar) on which $d\mathbf{l}$ lies, and for an observer who measures the voltage, it appears that the bar is moving with velocity $-\mathbf{v}$, so $\mathbf{v}_q$ is equal to $-\mathbf{v}$, therefore:

$$e = \oint \mathbf{v}_q \times \mathbf{B} \cdot d\mathbf{l} = \oint (-\mathbf{v}) \times \mathbf{B} \cdot d\mathbf{l} = -\mathbf{vBL}$$
**State 7:**
Bar  ➔ Moving (With velocity $\mathbf{v} = v\hat{a}$)
Voltmeter (Observer)  ➔ Moving (With velocity $\mathbf{v} = v\hat{a}$)
Source of magnetic field  ➔ Stationary

**Induced voltage**
In this state, there is no relative motion between the path of integration $d\mathbf{l}$ and the material on which the path lies, so $\mathbf{v}_q$ is equal to zero:

$$e = \oint_c \mathbf{v}_q \times \mathbf{B} \cdot d\mathbf{l} = 0$$

**State 8:**
Bar  ➔ Moving (With velocity $\mathbf{v} = v\hat{a}$)
Voltmeter (Observer)  ➔ Moving (With velocity $\mathbf{v} = v\hat{a}$)
Source of magnetic field  ➔ Moving (With velocity $\mathbf{v} = v\hat{a}$)

**Induced voltage**
For the observer moving with the voltmeter and an observer in the stationary reference frame the magnitude of the flux density $B$ is constant, so motion of the source of the magnetic field has no effect on induction of voltage. Also in this state there is no relative motion between the path of integration $d\mathbf{l}$ and the material on which $d\mathbf{l}$ lies, so $\mathbf{v}_q$ is equal to zero:

$$e = \oint_c \mathbf{v}_q \times \mathbf{B} \cdot d\mathbf{l} = 0$$

All the results obtained from state 1 to 8 are summarised in Table 2.1. Based on Table 2.1, the following points can be deduced:

- The movement of the source of $\mathbf{B}$ has no effect on the induced voltage.
- To generate voltage, there must be a relative motion between the bar and the voltmeter.

These statements clarify the reasons why the homopolar concept proposed in Figure 1.1 and Figure 1.3 do not generate any voltage. The conditions present in this machine are exactly the same as State 3 in this section.
From Table 2.1, it is clear that creating a HDG without any sliding contacts is not feasible.

In [4], the system, shown in Figure 2.7 (consisting of a non-magnetic conducting bar, voltmeter and external magnetic field), used to study electromagnetic induction provides a table similar to Table 2.1. More detail is added to Table 2.1 to clarify the induced voltage observed by the voltmeter.

<table>
<thead>
<tr>
<th>State</th>
<th>Bar (magnet)</th>
<th>Observer (Voltmeter)</th>
<th>The induced voltage measured by the voltmeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>v</td>
<td>0</td>
<td>vBL</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>v</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>v</td>
<td>-vBL</td>
</tr>
<tr>
<td>5</td>
<td>v</td>
<td>v</td>
<td>vBL</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>-vBL</td>
</tr>
<tr>
<td>7</td>
<td>v</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>v</td>
<td>v</td>
<td>0</td>
</tr>
</tbody>
</table>

If the system shown in Figure 2.7, is replaced by that shown in Figure 2.3, and examining various states of rotation of the disc, voltmeter and the source of magnetic field, the same result is obtained for the disc, voltmeter and the source of magnetic field, with $L$ in Table 2.1 replaced by the radius of the disc and $v$ by angular velocity.

2-4 Conclusion

In the previous section, Eq (2-41) is used to study electromagnetic induction. In [36] Weber's electrodynamics is used to study analytically electromagnetic induction in systems consisting of a disc, a permanent magnet as the source of magnetic flux, a copper disc and a galvanometer and based on the motion of each element (disc, magnet and galvanometer). The induced voltages measured by the galvanometer are determined. A table similar to Table 2.1 was produced. Other methods have also been proposed to study electromagnetic induction, for example in [37], "it is shown that the use of $A$ [Magnetic vector-potential] to describe the magnetic effects of the source currents help to provide a clearer view of induction". In [37], this technique is applied.
to electromagnetic machines such as those described in the classification Case 1 and Case 2. In the previous sections, the rotating disc in Case 2 and the moving bar in Case 3 were the electric conductor but made of a non-magnetic material, and the source of magnetic flux was applied externally. If the rotating disc and the moving bar in Cases 2 and 3, respectively, were made of a magnetic material such as permanent magnet for example, then the information in Table 2.1 is still valid, but their operating principles are completely different. These can only be explained using the Special Theory of Relativity. [38-41]

The information presented in this chapter relating to electromagnetic induction in homopolar dc machines is presented historically in [42].

In [43], a method was proposed to explain electromagnetic induction in a rotating cylinder magnet, which is a homopolar DC generator, the proposed method was to use "the forces acting on conduction electrons inside the magnet, rather than in terms of flux linkage and the cutting of lines of force."
CHAPTER 3

PRELIMINARY DESIGN, CONSTRUCTION AND ASSEMBLY OF THE PROTOTYPE HDG

In this chapter, the design and construction of the prototype Homopolar DC Generator is described. The preliminary design calculations are presented, as well as materials issues related to the construction. The prototype concept used in this project is shown in Figure 3.1.

Two key points should be noted here:

- Some of the design calculations were modified during the construction of the prototype machine. This was done in order to use low-cost and readily available materials
- The experimental test values for this prototype were lower than the rated design. The HDG operates at low voltage and high current, therefore a test voltage was selected that kept the output currents to acceptable level in the test rig.

![Prototype HDG](image)

**Figure 3.1 Prototype HDG**

3.1 Preliminary design of the prototype HDG

The preliminary design of the HDG is based on the magnetic circuit illustrated in Figure 3.2. The following specification was assumed for the prototype.

- Output voltage: 1.5 ~ 2.0 [V]
- Output current: approx 200 [A]
• Length of airgap, $l_g : 0.6 \text{ [mm]}$

• Thickness of the copper tube, $l_{cu} : 0.6 \text{ [mm]}$

• Magnetic flux density in the airgap, $B_g : 1 \text{ [T]}$

• Maximum Magnetic flux density in the rotor, $B_{rotor} : 2 \text{ [T]}$

• The maximum linear velocity of the rotor, $v : 25 \text{ [m/sec]}$ (Restricted by the maximum working velocity of the brush contacts)

![Magnetic circuit of the prototype HDG](image)

The voltage generated under each stator pole in the rotor copper can is:

$$e = B_g \cdot L \cdot v$$  \hspace{1cm} (3-1)

The axial pole length, $L$, was chosen as $75 \text{[mm]}$, and the design output voltage, $e$, therefore would be $1.875 \text{[V]}$. Thus:

• The length of stator pole $L, 75 \text{[mm]}$

If $l_g$ and $l_{cu}$ are initially ignored, Eq (3-2) may be used to relate the rotor flux density to the airgap flux density:

$$A_{rotor} \cdot B_{rotor} = A_{gap} \cdot B_{gap}$$  \hspace{1cm} (3-2)

Where $A_{rotor}$ and $A_{gap}$ are the cross-sectional area of the airgap and rotor respectively.

Thus:

$$(\pi \cdot R^2) \cdot B_{rotor} = (2\pi R L) \cdot B_{gap}$$  \hspace{1cm} (3-3)
Based on the values assumed for $B_{\text{rotor}}$ and $B_{\text{gap}}$, the radius of the rotor $R$ should be equal to $L(75[\text{mm}])$, however allowing for the copper can thickness on the rotor surface the inner radius was set to $76.2[\text{mm}]$.

To insert a shaft with radius $S$ inside the rotor, leads to a reduction of $A_{\text{rotor}}, B_{\text{rotor}}$ however remains close to the design values. $S$ is shown in Figure 3.3.

To determine $C$, shown in Figure 3.3, again ignoring $l_g$, $l_{cu}$ and $S$, we get:

$$A_{\text{rotor}} B_{\text{rotor}} = A_{\text{stator}} B_{\text{stator}} \quad (3-4)$$

In Eq (3-4), $B_{\text{stator}}$ denotes the magnetic flux density through the surface $A_{\text{stator}}$ between the stator poles. The value of $B_{\text{stator}}$ was constrained to lie in the range 1 to 2 Tesla.

Based on Eq (3-4), we get:

$$A_{\text{rotor}} B_{\text{rotor}} = \pi (R + B + C)^2 - \pi (R + B)) B_{\text{stator}} \quad (3-5)$$

In Eq (3-5), $B$ and $M$ are determined by the space needed for the centre brushes, brush holders and the space required for the field windings. (See Figure 3.1)

In Figure 3.3, the rotor overhang, $N$, is determined by the space required for end brushes and brush holders.

![Figure 3.3 Two dimensional layout of prototype HDG](image)

![Figure 3.4 The magnetic flux path in the prototype HDG](image)
The prototype HDG has two field windings, as shown in Figure 3.4: the number of turns and current rating of each winding is calculated by using Ampere's law:

\[ \oint H \cdot dl = 2nI_f \]  

(3-6)

Where:
- \( H \) - Magnetic flux strength \([A/m]\)
- \( n \) - Number of turns of each field winding
- \( I_f \) - Current rating of each field winding \([A]\)

Applying Eq (3-6) to the magnetic flux path shown in Figure 3.4, we get:

\[ \frac{B_m l_m}{\mu_m \mu_0} + \frac{B_{\text{airgap}} 2(l_{\text{gap}} + l_{\text{ca}})}{\mu_0} = 2nI_f \]  

(3-7)

Where:
- \( B_m \) - Magnetic flux density in the electrical steel parts of the HDG magnetic circuit
- \( l_m \) - Length of the electrical steel
- \( \mu_m \) - Magnetic permeability of the material used to construct the stator and rotor

The LHS of Eq (3-7) has two terms: the first term corresponds to the magnetic flux within the stator and rotor of the HDG, and the second term corresponds to the magnetic flux through the two airgaps and the copper can. The solution of Eq (3-7) manually is difficult due to the non-linear B-H characteristics of the electrical steel material used in the stator and rotor, as well as edge effects around the poles of the stator. A Finite Element solution therefore was used to check the magnetic flux in each part of the prototype after estimating the design dimensions using Eq (3-7).

![Figure 3.5](Image)

*Figure 3.5* The path of electric current in the brushes and the copper can
In Figure 3.5, the three brush sets are in contact with the copper can as shown. The direction of the induced voltage \( (e) \) in the copper can is also shown. The middle set of brushes, shown in Figure 3.1, are all at the same polarity as are both sets of brushes at each end of the rotor. The middle brush set therefore form one output connect whilst the other two end brush sets are connected together to form the other output. It should be noted that the number of brushes used at each end of the HDG and the number of brushes used in the middle of the HDG are identical. The number of brushes chosen for each axial rotor position in the prototype design was six. Ideally a large number of brushes should be used to maintain a uniform axial current flow in the rotor copper can. A large number of brushes however introduce complexity and cost. The design compromise here to the selection of six brushes per set.

In Figure 3.5, the path of the current in the copper can is shown. Each central brush carries twice the current compared to each end brush. The cross section of the middle brushes therefore should be twice the cross section of the end brushes.

The alternative option would be to have twice the number of brushes in the centre set: i.e. in this case 12. However for simplicity the prototype design retained the same number (6). The generator current rating therefore was determined by the current loading on the central brushes.

If the total number of the brushes in the centre brush set is \( n \), and \( I_{\text{total}} \) denotes the total output current of HDG, the current in each central brush is equal to \( (I_{\text{total}} / n) \) and the current in each end brush is equal to \( (I_{\text{total}} / 2n) \).

The copper can on the rotor of the HDG has three functions:

- The magnetic flux of each stator pole induces the main output voltage in it.
- The brushes slide on the copper can surface and due to its high conductivity; this provides a lower contact resistance.
- It acts as a heat sink to remove the heat generated by the following three sources:

  1. Electrical losses, due to the electrical resistance of the copper can itself
  2. Mechanical friction at the interface of brushes and the copper can
  3. Electrical losses due to the contact resistance between the brushes and the copper can
If it is assumed that the copper can is electrically insulated from the iron rotor, the output current, $I_{\text{Total}}$, passes through the copper as shown in Figure 3.5. For uniform 2D axial current flow, the current density, $J_{\text{cu}}$, in the copper can may be calculated as:

$$J_{\text{cu}} = \frac{I_{\text{Total}}}{2A_{\text{cu}}}$$  \hspace{1cm} (3-8)

In Eq (3-8), $A_{\text{cu}}$ is the cross section of the copper can:

$$A_{\text{cu}} = \pi[(R + l_{\text{cu}})^2 - R^2]$$  \hspace{1cm} (3-9)

Where:

- $R$ - Radius of the core of the rotor, shown in Figure 3.3
- $l_{\text{cu}}$ - Thickness of the copper can, shown in Figure 3.3

### 3.2 Complete prototype design

In Figure 3.6, an exploded view of the prototype HDG is shown and in Figure 3.7, cutaway isometric views from the right and left are illustrated. In Figure 3.8, three photos of the completed prototype are also included.
**Figure 3.6** Exploded view of prototype HDG

- **A1, A2** HDG's end housing
- **B1, B2** Ball bearing
- **C1, C2** Side brush assembly
- **D1, D2** Stator iron
- **E** Rotor
- **F** Middle brush assembly (On stator)
- **G1, G2** Field winding (On stator)
Figure 3.7 Cutaway views of prototype HDG

- **H** Clamping bolts
- **I1, I2** Mounting Feet
- **K** Nut
- **J** The surface of rig frame
Figure 3.8 Photograph of the prototype generator

(A) Photograph of the prototype generator showing:
- End housing
- Field winding wires
- The terminals of the central brushes (The prototype has 6 central brushes, in this photo, only 3 terminals of central brushes can be seen)

(B) Photograph of the prototype generator showing:
- Shaft
- Mounting foot
- The terminals of end brushes

(C) Photograph of the prototype generator showing:
- Shaft
- The terminals of the end brushes
The photograph shown in Figure 3.9 was taken after removing one end housing and half of the stator back iron. In Figure 3.9, a black trace can be seen at one end of the rotor copper can; this trace is caused by the sliding contact of the end set of brushes.

Figure 3.9 Inside the prototype

In the following figures, each component in the prototype is illustrated in detail.

**A1, A2 HDG's end housing**

Figure 3.10 HDG's end housing, Cutaway view (Left) and prototype (Right)
The prototype generator consists of two end housings, shown in Figure 3.10; each one should be made from a non-ferromagnetic material, so aluminium was used for these. At the centre of the end housing, a central recess houses the shaft bearing, and six holes are drilled for the wire connections.

**C1, C2 Side brush assembly**

A) Cutaway view of C1, C2  
B) The real photography of C1 and C2  
C) Brush holder, P2  
D) Brush, P3  
E) The fixture of the brush holder, P5

**Figure 3.11 End Brush assembly**

**P1:** The frame of the end brush assembly  
The thickness was 6 [mm] and it was made of PVC, isolating the brushes electrically from each other and from the stator.  
**P2:** Brush holder (Dimensions in Appendix (I))  
Six brush holders were mounted on the frame, P1; the angle between each adjacent brush was 60 degrees; the brush holders were made of brass.  
**P3:** Brush  
The brush type was copper graphite. The brushes have the following characteristics (The data sheet for the brushes is included in Appendix (II)):

- Type: BRUSH-RC87  
- Specific electric resistance: 0.10 [$\mu \Omega m$]  
- Maximum current density: 22 [$A/cm^2$] (continuous mode)
- Maximum linear velocity: 25 [m/sec]
- Cross section: 8×20 [mm²]
- Length of brush: 40 [mm]

Based on the maximum current density and the cross section of each brush used in the prototype, the maximum current that each brush could carry in the continuous mode was 35.2 [A]; as explained previously, the current rating is determined by the capacity of the central set of brushes. The current rating of the prototype generator is therefore (6×35.2 [A]) or 211.2 [A]

Each brush was fitted with two insulated flexible wires onto a single terminal. A diagonal slot was cut into each brush face to remove brush dust.

**P4: Spring**
All the brush holders were fitted with one constant-force spring.

**P5: The fixture of the brush holder.**
The brush holders slide over the fixtures and then fastened to the frame using two screws.

**D1, D2 The stator magnetic circuit**

![Figure 3.12 Cutaway view D1 and D2 (The dimensions are given in Figure 3.16)](image)

The stator is made of two identical electrical steel parts as shown in Figure 3.12. D1 and D2 were machined from Mild Steel (Grade: EN3)

On D1, six radial slots were machined to provide channels for the wires of the central brush set. The slots were rectangular (3[mm]×12[mm]) and are illustrated in red in Figure 3.12.
E Rotor

![Component of the rotor](image1)

![Copper can on the rotor](image2)

**Figure 3.13** Rotor (The dimensions are given in Figures 3.16 and 3.17)

**Q1: Copper can**
The thickness of the copper can was chosen to be $0.6\,[mm]$, so according to Eq (3-9), $A_{cu}$ is equal to $287.26\,[mm^2]$. The rated current density ($J_{cu}$) therefore in the copper can is equal to $0.367\,[A/mm^2]$. In reality the steel rotor also has some conductivity, so a small amount of current may flow in the steel rotor if there is a good electrical contact between the copper and the steel. The current density in the copper can therefore may be lower than the current density calculate here. However it is possible to calculate the current through the copper can and the steel rotor more precisely using a Finite Element model. To mount the copper can on the steel rotor, a copper tube was heated up, then slide over the steel rotor and cooled. The outer surface of the copper can was then machined to the finished thickness required.

**Q2: Rotor Back Iron**
This part was a simple cylinder made of Mild steel (Grade: EN 1A), with a shaft hole machined out.

**Q3: Shaft**
This was made from a non-magnetic stainless steel.
**R1: Frame**
The frame was made of PVC, insulating the brushes from the stator and from each other.

**R2: Brush holder**
The size and type of brush holders used in the central brush assembly were the same as the brush holders used in the end brush assemblies.

**R3: Brush**
The brushes used in the central brush assembly were the same as the brushes used in the end brush assembly in terms of size and type.

**R4: Spring**
Same size and type used in the end brush assemblies

**R5: The fixture of the brush holder**

**R6: Screw**
Two field windings were used in the prototype machine. Each field winding was wound onto a PVC bobbin. The wire used for each field winding has the following specification:

- Polyester enameled copper wire
- Length of wire: 400 [m] approximately
- Resistance: 0.0156 [Ω/m]
- Wire diameter: 1.18 [mm]
- Rated current: 3 [A]
- Number of turns: 620
- Bobbin window area: 55×40 [mm²]

3.3 Two-dimensional design drawings of the prototype

In Figure 3.16, a 2D layout of the prototype is shown, as well as the dimensions of all the main parts. In Figure 3.17, the brush placement of one brush assembly in contact with the rotor is illustrated. In Figure 3.18, the placement of three brushes that are parallel to each other on each brush set is shown. This is used in the next chapter to calculate the electric potential between the two brushes. Finally in Figure 3.19, the dimensions of the field winding bobbin are illustrated.
Figure 3.16 2D drawing of the prototype (Unit [mm])
Figure 3.17 End elevation of the rotor brushes (Unit [mm])
Figure 3.18 Plan view of the rotor and placement of the brush sets (Unit [mm])
Figure 3.19 Side elevation of the field winding bobbin (Unit [mm])
3.4 Prototype assembly

After assembly of the two field windings and the central brush assembly, they were bonded together using PVC glue, as shown in Figure 3.20. The dimensions of the components shown in Figure 3.20(B) were such that it fits between D1 and D2 shown in Figure 3.12. The values of M and B shown in Figure 3.3 were determined by the dimensions of this component.

![Field windings and central brush assembly](image)

(A) Before assembly  
(B) After assembly

Figure 3.20 Field windings and central brush assembly

The next step in the prototype assembly was to connect the end brush assemblies to the end housing as shown in Figure 3.21. After screwing the frame of the brush assembly to the end housing, the brushes were placed in the brush holders, as shown in Figure 3.21(A). The brushes were then pushed inside the brush holders and the brush wire connections brought out through the end housing and the brush wires were then taped to the end housing as shown in Figure 3.21(B). The end housing was then ready for mounting, Figure 3.21(C), onto the stator back iron.
A) The body of the end brush assembly screwed to the end housing, the brushes placed in the brush holders

B) The brush wires taped to the end housing

C) Brushes inside the brush holders

**Figure 3.21** Preparation of end housing for final assembly

The same procedure as illustrated in Figure 3.21 was used on the brushes in the central brush assembly, as shown in Figure 3.22. After placing the field winding bobbin in the stator, the brushes of the central assembly were inserted inside the brush holders and the brush wires taped to the stator.
Preparing the stator for rotor insertion, the wires of the central brush assembly were taped to the stator. The rotor was then located into the stator as shown in Figure 3.23, and the end housings were attached to the stator. Finally, all the tapes used to keep the brushes in the brush holders were removed.

The complete assembly of the final prototype homopolar generator is shown in Figure 3.24, ready for mounting on the laboratory test bed.
CHAPTER 4
FINITE ELEMENT MODELS

To understand a detailed electromagnetic analysis of the performance of the prototype, a Finite Element (FE) simulation was used. Initially a simple magnetostatic simulation was performed to assess the prototype. In this chapter both 2D-FE and 3D-FE models are used in the magnetostatic simulations, and their results compared directly.

As explained in Chapters 1 and 2, using a sliding electrical contact in a HDG is unavoidable. This introduces difficulties therefore for the FE models, and this is discussed in more detail in this chapter. To study the distribution of the current in the motor, a current flow analysis was used. Due to the 3-dimensional distribution of the current in the rotor, a 2D-FE model is not possible; therefore a 3D-FE model therefore was developed to simulate the current flow in the rotor, brushes, etc.

4.1 Finite Element Software
In this project, all the two-dimensional simulations were carried out using FEMM [44] (Version 4.2) which was designed for magnetostatics, current flow and heat problems. OPERA-3D was utilised for all the three-dimensional simulations. Vector Field (OPERA-3D, Version 13) [45-46] is commercial software designed for static and dynamic electromagnetic simulations. In practice, OPERA-3D is a set of analysis programs for a diverse range of simulations; some of these were tried on the prototype HDG but difficulties were encountered. The difficulties are discussed in this chapter. The limitations of each program that was tried in this project is explained briefly.

- **TOSCA**
  This analysis program can be used for magnetostatics, current flow and electrostatic problems. In this project TOSCA was utilised for the magnetostatics and the current flow.

- **ELECTRA**
  This analysis program can be used for steady state and transient analyses. ELEKTRA may also be used for eddy current analysis due to motion of conductor in a static magnetic field; for example an eddy current brake comprising a conducting disc that rotates and cuts static magnetic flux. In this example using ELEKTRA it is possible to assign a constant velocity to the disc and observe eddy current produced in it.
• CARMEN
This analysis program can be used to analyse transient electromagnetic fields. For example, simulating a synchronous generator with a rotating rotor coupled to a mechanical load. It is mainly used for dynamic simulations when there is relative velocity between the stator and rotor.

Initially it was assumed that CARMEN could be used to undertake the electromagnetic analysis of the first generator concept illustrated in Figures 1.1 and 1.3. These were brushless machines, but after understanding that it was not possible to construct a HDG without any sliding contacts, CARMEN was no longer applicable. CARMEN could not be used to model the HDG concept, because it has brushes.

4.2 Finite Element models of the HDG
The FE models were used to determine the magnitude of the voltage induced in the rotor copper can when it rotates inside the static magnetic field produced by the field windings. Initially a model as shown in Figure 4.1 was used in ELEKTRA to observe the voltage induced in the rotor. The software however could not solve the model indicating a convergence error. This probably occurred because the software attempted to solve the eddy current distribution in the rotor. In the model however, there is not any eddy current, so a convergence problem is encountered; in order to increase the speed of the simulation, only a 10 degree slice of the model was used. An alternative 2D-FE software package (FLUX-2D) was used and the model defined as an axisymmetric problem. However another limitation was encountered; when a model is defined as axisymmetric, it is not possible to define rotational velocity to the rotor, i.e. rotation around the axis of the symmetry. It is only possible to define movement along the axis of symmetry.
Magnetostatic simulation using a stationary rotor can however provide the following information:

- The distribution of the magnetic flux in the airgaps, in the stator and in the rotor of the prototype.
- Due to the nonlinear characteristics of mild steel used in stator and rotor, employing a magnetostatic simulation it was easier to calculate the ampere-turns of the field windings needed to set up the design values of the magnetic flux in the airgaps.
- By knowing the magnetic flux cutting the copper can, the voltage induced in the copper can may be calculated.

If the radial slots for brush connections on the stator of the prototype are neglected, a magnetostatic analysis in two-dimensions is possible using cylindrical symmetry. The model can then be defined as an axisymmetric model in the 2D-FE software.

To observe the distribution of the current in the copper can and the rotor iron, a separate solution using TOSCA Current Flow was used. In section 4.5, this will explained in more detail.

**4.3 Two-Dimensional magnetostatic simulations using FEMM**

For a magnetostatic simulation, two-dimensional layout of the prototype was created in the Preprocessor of FEMM, as shown in Figure 4.2. Appropriate materials are then assigned to each part of the layout.
The radial slots for the brush wires are small and can be ignored. The prototype therefore has cylindrical symmetry and can be defined as an axisymmetric problem. As shown in Figure 4.2, mild steel is assigned for both the stator and the rotor of the prototype; the same B-H curved was used for both stator and rotor. The B-H curve of any ferromagnetic material may be obtained by practical test. In some handbooks however the B-H curve of some materials are detailed. It is worth noting here that in the construction of the prototype, the grade of mild steel used for the stator was different from that used for the rotor. Two different B-H curves would be needed therefore in the model. In the FE model however the same B-H curve was used for the stator and the rotor, because B-H curves corresponding to each grade of mild steel could not be found in the reference material.

In [47], a B-H curve for mild steel was detailed; however the grade was not given. This B-H data was used in the model shown in Table 4.1 and plotted in Figure 4.3.
Table 4.1 Data extracted from B-H curve presented in [47] for mild steel

<table>
<thead>
<tr>
<th>B [T]</th>
<th>H [A/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.19817</td>
<td>111.11</td>
</tr>
<tr>
<td>0.50092</td>
<td>166.67</td>
</tr>
<tr>
<td>0.79817</td>
<td>222.22</td>
</tr>
<tr>
<td>0.99633</td>
<td>333.33</td>
</tr>
<tr>
<td>1.1394</td>
<td>583.33</td>
</tr>
<tr>
<td>1.2495</td>
<td>888.89</td>
</tr>
<tr>
<td>1.3431</td>
<td>1277.8</td>
</tr>
<tr>
<td>1.3872</td>
<td>1500</td>
</tr>
<tr>
<td>1.4477</td>
<td>1916.7</td>
</tr>
<tr>
<td>1.5468</td>
<td>2777.8</td>
</tr>
<tr>
<td>1.6349</td>
<td>3750</td>
</tr>
<tr>
<td>1.6789</td>
<td>4583.3</td>
</tr>
<tr>
<td>1.7229</td>
<td>5694.4</td>
</tr>
<tr>
<td>1.7505</td>
<td>6750</td>
</tr>
<tr>
<td>1.767</td>
<td>7694.4</td>
</tr>
<tr>
<td>1.8</td>
<td>9027.8</td>
</tr>
</tbody>
</table>

Figure 4.3 B-H curve of mild steel
As seen in Figure 4.2, two square areas in the machines are labeled coil; these two blocks correspond to the two field windings in the prototype. Each field coil in the model has the following properties:

- Number of turn: 1
- Cross section area, $A_{\text{coil}}$: $40 \times 45 \text{ [mm}^2\text{]}$
- Current density, $J_{\text{coil}}$ [A/mm$^2$]

$A_{\text{coil}}$ in the model corresponds to the window area of the field winding bobbin. It should be noted that the total cross section area of the field winding coil is less than $A_{\text{coil}}$ due to the filling factor.

As explained in the previous chapter, the current rating of the wire used in the prototype was $3 \text{ [A]}$; in the magnetostatic simulation, the magnetic flux in stator, rotor and airgap was obtained for a field winding current, $I_f$, equal to 0.5, 1.0, 1.5, 2.0, 2.5, and $3.0 \text{ [A]}$. It should be emphasised that $I_f$ is the current in each field winding, with each field winding is supplied separately.

$J_{\text{coil}}$ in the model may be calculated as:

$$J_{\text{coil}} = \frac{nI_f}{A_{\text{coil}}} \quad (4-1)$$

$n$ in Eq (4-1) is the number of turns of each field winding in the prototype. ($n$ is equal to 620)

In Table 4.2, $J_{\text{coil}}$ is listed for different values of $I_f$.

**Table 4.2** Calculation of current density, $J_{\text{coil}}$, in each field winding

<table>
<thead>
<tr>
<th>Current in each the field winding</th>
<th>$nI_f$ [ATurns]</th>
<th>Current density $J_{\text{coil}}$ [A/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_f$ [A]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>310</td>
<td>0.172</td>
</tr>
<tr>
<td>1.0</td>
<td>620</td>
<td>0.344</td>
</tr>
<tr>
<td>1.5</td>
<td>930</td>
<td>0.516</td>
</tr>
<tr>
<td>2.0</td>
<td>1240</td>
<td>0.688</td>
</tr>
<tr>
<td>2.5</td>
<td>1550</td>
<td>0.861</td>
</tr>
<tr>
<td>3.0</td>
<td>1860</td>
<td>1.033</td>
</tr>
</tbody>
</table>
In Figure 4.2, there is also a rectangular region denoted air; this region corresponds to the shaft which was made of non-magnetic stainless steel.

The outer boundary of the model was placed along way from the airgap, so that the field in the airgap is not influenced by the outer boundary; a fixed potential (flux boundary) \( A = 0 \), was selected as the outer boundary as shown in Figure 4.2, where \( A \) is magnetic potential.

A copper can was also placed on the surface of the rotor, but due to its small radial thickness it is difficult to see in Figure 4.2.

Figure 4.4 and Figure 4.6 illustrate magnetic flux lines and current density of the coils for different field currents, and Figure 4.5 and 4.7 show the corresponding magnetic flux density in the model.

As it can be seen from Figure 4.7, when \( I_f \) is 3 [A], the rotor core is magnetically saturated; the magnetic flux density in the rotor core is about 2 [T] which is equal to the design flux level \( B_{\text{Rotor}} \).
Figure 4.4 Magnetic flux lines in the model and current density ($J_{coil}$) of the field winding for $I_f$ equals to 0.5, 1.0 and 1.5 [A]
Figure 4.5 Magnetic Flux density $|B|$ in the model for $I_f$ equals to 0.5, 1.0 and 1.5 [A]
Figure 4.6 Magnetic flux lines in the model and current density ($J_{\text{coll}}$) of the field winding for $I_f$ equals to 2.0, 2.5 and 3.0 [A].
Figure 4.7 Magnetic Flux density ($|B|$) in the model for $I_f$ equals to 2.0, 2.5 and 3.0 [A]
To determine the generated voltage due to rotation of the rotor, the magnetic flux linking the copper can is required. In the FE package (FEMM), a contour is created along the surface of the copper can, as shown in Figure 4.8.

![Figure 4.8 Line along interface of brushes and copper can (This figure is just for explanation and it is not to scale)](image)

In Figure 4.9, the magnitude of magnetic flux density ($|B|$) along the chosen contour is plotted. Figure 4.9 illustrate that by increasing $I_f$, $|B|$ at interface of the brushes and the copper can increases nonlinearly which is due to the B-H characteristics of the stator and rotor mild steel cores. $|B|$ can be divided into two components, $B_n$ - the normal magnetic flux cutting the copper can, and $B_t$ - the magnetic flux tangential to the copper can. $B_n$ produces the induced motor voltage. To calculate the voltage induced in the copper can therefore, normal magnetic flux was used and plotted in Figure 4.11. The magnitude of the magnetic flux density ($B_t$) tangential to the copper can plotted in the Figure 4.10. Fringing field off the poles can be seen in Figures 4.9, 4.10 and 4.11.
Figure 4.9 Magnitude of magnetic flux density ($|B|$) on a contour along interface of brushes and copper can
Figure 4.10 Magnitude of magnetic flux density ($B_t$) tangential to copper can on a contour along interface of brushes and copper can
Figure 4.11 Magnitude of magnetic flux density ($B_n$) normal to copper can on a contour along interface of brushes and copper can
In both Figures 4.10 and Figure 4.11, similar to Figure 4.9, increasing $I_f$ results in $B_i$ and $B_n$ increasing nonlinearly.

![Figure 4.12 Division of contour at interface of the brushes and the copper can](image)

To calculate the voltage induced in the copper can, using Eq (1-1) can be problematic, because the distribution of the magnetic flux on the copper can surface is completely uniform but it is not completely constant.

If there are $n$ points on the contour along the interface of the copper can and the brushes, it is possible to discretise the contour into $(n-1)$ identical segments, each segment of length $\Delta l$, as shown in Figure 4.12.

The following procedure was used to calculate the electric potential at each point on the contour:

1. The flux density between nodes $N_{n-1}$ and $N_n$ is denoted, $B_n$, and is constant due to the discretised nature of the FE simulation. The induced voltage ($e$) along $\Delta l$ between nodes $N_{n-1}$ and $N_n$ therefore is:

   $$e = B_n v_{Linear} \Delta l$$

   \hspace{1cm} (4-2)

2. If the linear velocity of the rotor is $v_{Linear}$, and the electric potential $V_1$ at point $N_1$ is assumed to be zero ($V_1 = 0$), the electric potential, $V_n$ at point $N_n$ can be calculated as follows:

   $V_1 = 0$

   $V_2 = V_1 + B_2 \Delta l v_{Linear}$

   $V_3 = V_2 + B_3 \Delta l v_{Linear}$

   $V_4 = V_3 + B_4 \Delta l v_{Linear}$

   .

   .

   .

   $V_{N-1} = V_{N-2} + B_{N-2} \Delta l v_{Linear}$

   $V_N = V_{N-1} + B_N \Delta l v_{Linear}$
The number of nodes \((n)\) should be large to ensure a good representation of the voltage distribution; 7201 nodes were used. The length of the contour at the interface of the copper can and the brushes was \(360\, [mm]\), therefore the segment length \((\Delta l)\) was \(0.05\, [mm]\).

For an estimated rotational velocity \((\omega_r)\) of the rotor of \(3274\, [RPM]\), the linear velocity of the copper can is:

\[
v_{\text{linear}} = (R + l_{cu})\omega_r = 26.33107 \, [m/sec]
\]  
(4-3)

The magnitude of \(R\) and \(l_{cu}\) are given in Chapter 3. The above procedure for calculating the electric potential at each point was used for different values of \(I_f\); the electric potential distribution is plotted in Figure 4.13.

Increasing \(I_f\), the electric potential distribution increases nonlinearly due to the nonlinear steel B-H characteristic.
Figure 4.13 Electric potential at the contour along interface of brushes and copper can, angular velocity 3274 [RPM]
The contact points of the brushes with the copper can are illustrated in Figure 4.14. Using the results obtained in Figure 4.13, it is possible to calculate the electric voltage between the end brush set and the central brush set. These were validated in the experimental investigation.

In Figure 4.14, the first point (POINT 1) and the last point (POINT 7201) on the contour along interface of brushes and the copper can are shown. POINT 3601 is placed at the centre of the central brushes, and POINT 181 is placed at the leading edge of the end brush contact with the copper can.

![Figure 4.14 Identification of the key point on the contour (The figure is not to scale)](image)

The magnitude of the electric potential for POINTS 3601 and POINT 181 at various magnitudes of $I_f$ are summarised in Table 4.3. The induced voltage ($e$), i.e. the voltage that appears between the central and end brushes, for various values of $I_f$ are also presented in Table 4.3.

<table>
<thead>
<tr>
<th>$I_f$ [A]</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>POINT 181 [mV]</td>
<td>3.20</td>
<td>6.23</td>
<td>7.95</td>
<td>8.85</td>
<td>9.28</td>
<td>9.75</td>
</tr>
<tr>
<td>POINT 3601 [V]</td>
<td>0.602</td>
<td>1.17</td>
<td>1.50</td>
<td>1.67</td>
<td>1.76</td>
<td>1.85</td>
</tr>
<tr>
<td>Electric Voltage ($e$) [V]</td>
<td>0.598</td>
<td>1.17</td>
<td>1.49</td>
<td>1.67</td>
<td>1.75</td>
<td>1.84</td>
</tr>
</tbody>
</table>
4.4 Three-Dimensional magnetostatic model using OPERA-3D

TOSCA-Magnetostatic analysis program is a part of the OPERA-3D for magnetostatic analysis of 3D-model. In the 2D-FE model, the radial slots on the stator for the wire output connections of the central brush set were ignored. If these slots are included, the complete model may be divided into 6 identical segments as shown in Figure 4.15. A 3D-FE simulation of one segment is enough to observe the magnetic flux density in the whole model.

To perform magnetostatic simulations in OPERA-3D, initially the whole model of the prototype should be plotted in the Modeler section of the software; OPERA-3D Modeler defines the model, boundary conditions, mesh, etc. Figure 4.16(A) illustrates parts of the model plotted in Modeler. For clarity part of the stator is not shown. Figure 4.16(B) shows the complete model. Appropriate materials were defined for all parts of the model similar to the 2D-simulation.

Six radial slots were placed in the model, equivalent to the six slots in the stator for the central brush wires. In Figure 4.16(B) it is difficult to see these slots due to their small dimension. The final model is shown in Figure 4.16(C).

Using the complete model for 3D-simulations increases the time of simulation significantly, so the boundary conditions are used to reduce the size of the model and computational solution times.

Once the model symmetry has been assigned, the complete model shown in Figure 4.16 is reduced to the model illustrated in Figure 4.17(A). The model is then meshed. The mesh of the model is depicted in Figure 4.17(B). The number of mesh elements was 3113196.

Figure 4.15 Dividing the model into 6 identical segments
(A) Inside the model, some parts of the stator hidden

(B) Complete model

(C) Air around the model

**Figure 4.16** Drawing the model for 3D-simulation
The final step is to define the boundary conditions. The outer surface of the model was chosen to be a tangential magnetic boundary as shown in Figure 4.18. In 3D-FE, tangential magnetic boundary means that $A_x, A_y,$ and $A_z$ are zero. ($A_x, A_y,$ and $A_z$ are components of the magnetic potential in 3D-coordinate system.)
Each field winding was selected as a Biot-Savart current source as shown in Figure 4.19, and the current density defined for each winding according to Table 4.2 used for the 2D-simulation.
The models were then run for various magnitudes of $I_f$. 

Using the OPERA-3D post-processor, the magnitude of the magnetic flux density $|B|$ when $I_f$ is equal to 2.5 [A] for example, was obtained and illustrated in Figure 4.20(A).

A cylindrical surface was created at the interface of the copper can and the brushes, and the magnitude of the magnetic flux $|B|$ plotted on this cylindrical surface, as shown in Figure 4.20(B).

Figure 4.20 (B) illustrates clearly a uniform magnetic flux cutting the copper can. The model was defined such that the symmetry axis of the model coincides with the cylindrical coordinate system. The cylindrical coordinate system is defined as $(r, \theta, z)$; in the postprocessor, it is possible to view $B_r$, $B_\theta$, and $B_z$ inside the model in the cylindrical coordinate system. Figure 4.21(A) shows $B_r$ inside the model. $B_r$ is equivalent to $B_n$ in the 2D-FE simulations discussed in the previous section.

As explained in Section 4.3, to calculate the voltage induced in HDG, the normal magnetic flux cutting the copper can is needed. Therefore $B_r$ cutting the copper can was used to calculate the voltage induced. In Figure 4.21(B), $B_r$ is plotted on the cylindrical surface placed at interface of the brushes and copper can.

In Figure 4.22, $B_z$ both inside the model and on the cylindrical surface at interface of the brushes and copper can are plotted; $B_z$ in the 3D-simulations is identical to $B_r$ in the 2D-simulations presented in Section 4.3.
A) Magnitude of $|B|$ (Tesla) in the model

B) Magnitude of $|B|$ (Tesla) plotted on the cylinder surface at interface of the brushes and the copper can

Figure 4.20 Distribution of the magnetic flux ($|B|$) (Tesla) in the model, $I_j = 2.5[A]$
A) Magnitude of $B_z$ (Tesla) at the model

B) Magnitude of $B_z$ plotted on the cylinder surface at the interface of the brushes and the copper can

Figure 4.21 Distribution of the magnetic flux $B_z$ (Tesla) in the model, $I_f = 2.5[A]$
A) Magnitude of $B_z$ (Tesla) at the model

B) Magnitude of $B_z$ plotted on the cylinder surface at interface of the brushes and the copper can

**Figure 4.22** Distribution of the magnetic flux $B_z$ (Tesla) in the model, $I_j = 2.5[A]$
In the postprocessor, it is possible to view half or even the whole model. In Figure 4.23, the distribution of the magnetic flux density $|B|$ in one half of the model is shown from two different views; in Figure 4.23(B), the distribution of the magnetic flux density around the slots on the stator can be seen. The 3D-simulations were repeated for different values of $I_f$. Table 4.2 was used for the magnitude of current densities in the field windings in the 3D-simulations. In Figure 4.24, the distribution of the magnetic flux density $|B|$ in the model for various magnitudes of $I_f$ are illustrated. The procedure used to calculate the induced voltage in the previous 2D-FE simulations, can be repeated for the 3D-FE simulations. In Figures 4.25 and 4.26, $|B|$ and $B_n$ obtained from the 2D-FE and 3D-FE respectively are placed side by side for comparison. A close agreement between the 2D and 3D simulations was obtained. In Figures 4.25 and 4.26, $|B|$ and $B_n$ for $I_f$ equal to $3[A]$ is not presented. Convergence of the 3D-model was not achieved at this current level, however it is possible to increase the number of elements in the mesh and solve this convergence problem.
Figure 4.23 Distribution of the magnetic flux $|B|$ (Tesla) in one half of the model, $I_f = 2.5[A]$
Figure 4.24 Distribution of the magnetic flux $|\mathbf{B}|$ (Tesla) in the model for various magnitudes of $I_f$.
The magnitude of magnetic flux density [Tesla]

Figure 4.25 Comparison of $|\beta|$ obtained by 2D-FE and 3D-FE simulation
Figure 4.26 Comparison of $B_o$ obtained by 2D-FE and 3D-FE simulation
4.5 Finite Element simulation of current in the rotor of HDG

Before explaining the finite element simulation of the current distribution in the HDG rotor, an issue relating to the brush contact should be noted. Due to existence of the brushes in the HDG Velocity Skin Effect (VSE) occurs in the brushes. "Because of the velocity skin effect, the current distribution in sliding contacts is uneven and mainly concentrated in the rear of the brushes." [48]

The performance and the life of the brushes may be affected by VSE. In [49] measurement of VSE was presented, and in [48] the Homopolar Disk generator was analyzed considering VSE; in this project however VSE is not considered.

As explained previously, the dynamic simulation of the HDG is not possible, so to determine the distribution of the current in the rotor, a current flow analysis has to be done using TOSCA.

Another difficulty in the current flow analysis is the existence of the contact resistance between the brushes and copper can which is also not possible to include in the FE-simulations.

In Figure 4.27, the model used for the current flow analysis is illustrated. This model consists of the rotor core, which has electrical conductivity, the copper can, brushes, wires and some small wire resistance acting as the load. The aim of the current flow simulation is to inject current into the wire connections and determine the current distribution in the rotor. In the version of OPERA used in the project, a current boundary condition used to inject current into the model, was not available.* Therefore to inject the current into the model, small resistance loads were added to the wires at the end brush sets as shown in Figure 4.27. In this figure the electrical load coloured red.

By adjusting the electrical conductivity to each load in the model, the load resistance can be determined. In the model, the resistance of the load was set to 10 [Ω]; the wires were assumed to be copper and the conductivity was set to 5.8E7 [S/m]. The brushes were also assumed to be copper. The conductivity of the rotor core (mild steel) was set to 1.0E7 [S/m] obtained from [50].The load resistances were chosen to dominate the resistance of the wires, brushes and rotor. By applying appropriate voltage boundary condition to the cross sections of the end brush sets therefore controlled the magnitude of the current in the wires.

* In the newer version of the software, Version 15, current boundary condition is added.
In the model for example if the current in the wires of the end brushes was required to be 20 \( A \), and the current in the central brushes therefore 40 \( A \), a voltage of 200 \( V \) was required on each cross section of the end wires and a voltage of 0 \( V \) on each cross section of the central brushes, as shown in Figure 4.28.

![Figure 4.27 The model for current flow analysis](image)

The mesh of the model is shown in Figure 4.29.
Figure 4.28 Assignment of the voltage boundary condition to the wire cross section

Figure 4.29 The meshed model for current flow analysis
In Figure 4.30, the current distribution in the model is shown. To observe the current distribution inside the rotor, a surface was placed between the end brushes and the central brushes inside the rotor. In Figure 4.31(A), the model is shown and the placement of the surface is included in Figure 4.31(B). The current passing through this surface was 120 [A], equivalent to 20 [A] through each of six brushes at the end of the rotor. The cross-sectional area of the wire was 20\(mm^2\), so the wire current density shown in Figure 4.30 corresponds to 1 [A/mm\(^2\)] for a current of 20 [A]. In Figure 4.32, the distribution of the current density on the surface defined in Figure 4.31(B) in the form of a histogram. In Figure 4.31(B), an orange line is shown; the current density along this line is plotted in Figure 4.33. Figure 4.32 and Figure 4.33 illustrate the current density in the copper can and the rotor core which is useful for the design of the rotor and may help to choose the correct thickness of the copper can.

In Figures 4.32 and 4.33, the current density in the shaft is zero, as expected, due to the selection of Air for the shaft in the model; also it is evident that the current density in the copper can is more than the current density in the rotor core.
Figure 4.30 Current density in the model (Unit: A/mm²)

(A) The complete model

(B) The model with wires, brushes and loads removed
(A) The complete model

(B) Planer surface inside the model between the end and central brushes with a line defined

Figure 4.31 The full model for current flow analysis
Figure 4.32 Histogram showing distribution of the rotor current (Unit: $A/mm^2$)

Figure 4.33 Current distribution along the line shown in Figure 4.31 (B) (Unit: $A/mm^2$)
To determine the current passing through the copper can, another surface similar to that shown in Figure 4.31(B) was defined, but with a radius equal to the outer radius of the rotor core. The current passing through this surface therefore corresponds to the current passing through the rotor core. A histogram of the current passing through this surface is shown in Figure 4.34.

![Figure 4.34](image)

**Figure 4.34** Histogram showing distribution of the current in the rotor core [A/mm²]

Integrating the current density of Figure 4.34 shows that 54.74013 [A] passes through the mild steel rotor core, and therefore 65.25987 [A] should pass through the copper can. The current in the rotor core is clearly significant and this is due to the relative large cross-sectional area of the core in comparison to the copper can area. It also assumes a perfect electrical contact between the rotor core and the copper can.
The same procedure can be applied to the current distribution with only one end brush and one central brush in the model as shown in Figure 4.35. The current density in the model with two brushes is also shown in Figures 4.36 and 4.37. Similar to Figure 4.31(B), it is possible to define a surface in the model to illustrate the current distribution with only two brushes. A histogram of the current passing through this surface is also shown in Figure 4.38. If a line is used from the middle of the side brush to the middle of the central brush, the maximum current density shown in Figure 4.38 on the rotor occurs at the centre of this line in the rotor as expected. The full 3D current flow model therefore allows the current distribution for any combination of brush inputs and outputs to be determined in the rotor can and rotor core.
Figure 4.35 Current flow model with only two brushes

Figure 4.36 Current density with only two brushes \([\text{A/mm}^2]\)
Figure 4.37 Current density with only two brushes, the model with wires, brushes and loads removed \( [A/mm^2] \)

Figure 4.38 Histogram showing distribution of the current in the rotor with only two brushes \( [A/mm^2] \)
4.6 The effects of the magnetic field produced by the armature current

In the HDG studied in this dissertation, when current passes through the rotor, it produces a magnetic field. This magnetic field may lead to magnetic saturation in the machine magnetic core due to the interaction with the magnetic flux originating from the field windings. In Figure 4.39, four regions, \( A, A', B \) and \( B' \) are specified in the HDG.

![Figure 4.39 Four regions specified in the HDG](image)

The cross-section of region \( A \) is illustrated in Figure 4.40(A); here it is assumed that the entire current passes through the copper can only and also the current is uniformly distributed in the copper can. This current generates a magnetic field which is denoted by \( B_\theta \). In Figure 4.40(A), it is assumed that the direction of the current is axial; therefore the direction of the magnetic flux is counter-clockwise. From Section 4.3, the direction and magnitude of the magnetic field from the field winding is known; the direction of magnetic field from the field windings in the stator is shown by \( B_r \) in Figure 4.40(A). The two magnetic fluxes exist in the stator, \( B_r \) and \( B_\theta \), and the magnitude of the total magnetic flux density, \( |B| \), in the stator is:

\[
|B| = \sqrt{(B_r)^2 + (B_\theta)^2} \leq 1.8 \ [Tesla]
\]

\( |B| \) should be less than 1.8 \([Tesla]\), otherwise the stator can become saturated magnetically. The aims of this section are:

- To determine the magnitude of \( B_\theta \), and therefore calculate \( |B| \)
- If \( |B| \) exceeds 1.8 \([Tesla]\), a slit with appropriate width \( (w_{slit}) \) should be placed in the stator as shown in Figure 4.41(A), to reduce the flux density levels and reduce the likelihood of saturation.
The cross-section presented in Figure 4.40(A) is also applicable to region A’. The only difference between region A and A’ is the direction of \( B_r \) and \( B_r' \). The direction of the magnetic fluxes shown in Figure 4.40(A) should be reversed in region A’.

In Figure 4.40(B), the direction of the magnetic flux originating from the field winding \( B_z \), and the magnetic flux produced by the current passing through the copper can, \( B_r' \), are shown for region B in Figure 4.39. Figure 4.40(B) is also applicable to region B’ if the direction of \( B_z \) and \( B_r' \) are reversed. \( B_z \) and \( B_r' \) are perpendicular to each other, as shown in Figure 4.40(B) and (C). In regions B and B’, the magnitude of stator magnetic flux, \(|B|\) can be calculated by Eq (4-5).

\[
|B| = \sqrt{(B_z)^2 + (B_r')^2} \leq 1.8 \text{ [Tesla]}
\]  

\(|B|\) in the regions B and B’ should be less than 1.8 [Tesla], to prevent saturation. As before if \(|B|\) exceeds 1.8 [T], a slit can be added whose width is \( w_{slit} \), as shown in Figure 4.41(B). The magnitude of \( B_z \) and \( B_r \) from the field windings are determined from the magnetostatic 2D-FE simulation presented in Section 4.3. In this section the magnitude of \( B_r' \) is first obtained using a magnetostatic 2D-FE simulation for regions A and B, then \(|B|\) is calculated for the both regions. This can be repeated when there is a slit of width \( w_{slit} \) in the stator. The simulations have been done for two slits with different width, 0.5 and 1.0 [mm].
A) Region A

B) Region B

C) Direction of the resultant magnetic flux in region B

Figure 4.40 Cross-section view of the HDG
Figures 4.42 and 4.43 show the models defined in the preprocessor of the FE software. The dimensions of these models are based on the dimensions of the prototype presented in Chapter 3. The current density corresponding to the maximum current rating of the HDG was assigned to the copper can. According to Chapter 3, the maximum current rating of the prototype is 211.2 [A]; half of this current passes through one half of the copper can, and the other half through the other half of the can. The cross-section area of the copper can is 288.39 [mm$^2$], therefore the current density in the copper can is
0.3661 \( [A/mm^2] \). A small air region was defined around the model as shown in the Figures 4.42 and 4.43.

**Figure 4.42** The models defined in the FE preprocessor for regions A
Figure 4.43 The models defined in the FE preprocessor for regions B

In Figures 4.44-4.47, the magnetic flux densities and magnetic flux lines are presented for regions A and B with and without the slits of width 0.5 and 1.0 [mm].
Figure 4.44 Magnetic flux density for region A

A) Slit-None

B) Slit- 0.5 [mm]

C) Slit- 1.0 [mm]
Figure 4.45 Magnetic flux lines for region A
Figure 4.46 Magnetic flux density for region \( B \)
Figure 4.47 Magnetic flux lines for region \( B \)
The red lines shown in Figures 4.45 and 4.47 are contour inserted by the postprocessor of the FE-software. The magnitude of the magnetic flux density along this contour is shown in Figure 4.48 for the current passing through the HDG in regions A and B. The starting point of this contour is point G and the end point is point H as shown in Figures 4.45 and 4.47. In Figure 4.48 the points G and H are specified. In the models with a slit, the contour passes through the middle of the slit. The magnitude of the magnetic flux density on this line is plotted and shown in Figures 4.48 for the both regions, A and B.

Figure 4.48 The magnitude of the magnetic flux density on a contour for the model with and without with 0.5 and 1.0 [mm] slits
The values of $B_r$ and $B_z$, respectively, in regions A and B as expected are not constant. The maximum values of these quantities, when $I_f$ is equal to 3.0 [A], have been extracted from Figure 4.7. The maximum of $B_r$ and $B_z$ are about 0.8 [T] and 1.1[T] respectively. These values are given in Table 4.4.

**Table 4.4 Magnetic flux density in regions A and B ($I_f = 3.0$ [A])**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Region A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>0.8</td>
<td>-</td>
<td>0.7996</td>
<td>1.1310</td>
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<tr>
<td>0.5 [mm]</td>
<td>0.8</td>
<td>-</td>
<td>0.2657</td>
<td>0.8429</td>
</tr>
<tr>
<td>1.0 [mm]</td>
<td>0.8</td>
<td>-</td>
<td>0.1896</td>
<td>0.8221</td>
</tr>
<tr>
<td>Region B</td>
<td></td>
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<td></td>
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<td>-</td>
<td>1.1</td>
<td>0.2028</td>
<td>1.1185</td>
</tr>
<tr>
<td>0.5 [mm]</td>
<td>-</td>
<td>1.1</td>
<td>0.1198</td>
<td>1.1065</td>
</tr>
<tr>
<td>1.0 [mm]</td>
<td>-</td>
<td>1.1</td>
<td>0.0884</td>
<td>1.1035</td>
</tr>
</tbody>
</table>

From the results presented in Table 4.4, we can conclude:

**In Region A**

- When there is no slit, the value of $|B|$ is less than 1.8 [Tesla], therefore adding slit in the HDG is not required.

- By adding a slit (0.5 [mm]), the value of $B_r^c$ drops to approximately 30% of the value of $B_r^c$ in the model without slit, and therefore $|B|$ is reduced.

- By adding a slit (1.0 [mm]), the value of $B_r^c$ drops to approximately 25% of the value of $B_r^c$ in the model without slit. Therefore the impact of the magnetic field from the current passing through the copper can be considered to be negligible.

**In Region B**

- When there is no slit in the model, the value of $B_r^c$ is much smaller than the value of $B_z$, and $|B|$ is less than 1.8 [Tesla]. Therefore adding a slit in the model is not required.
• By adding a slit (0.5 \text{ [mm]}), the value of $B_r^c$ drops by approximately 50\% of the value of $B_r^c$ in the model without the slit.

• By adding a slit (1.0 \text{ [mm]}), the value of $B_r^c$ drops by approximately 50\% of the value of $B_r^c$ in the model without slit.

4.7 Conclusion
This chapter introduced the FE-software models used in this project. The results of the two and three dimensional magnetostatic simulations are presented and compared. Good agreement was obtained between the 2D and 3D magnetostatic simulations. The voltage induced in the HDG was also obtained by the algorithm explained at the end of Section 4.3. In Section 4.5, the current distribution in the rotor of the HDG was analysed using 3D-FE (current flow analysis); this simulation is helpful in choosing the thickness of the copper can more accurately. It can also help to select the number and placement of the brushes in the machine to produce more uniform current distribution in the copper can. At the end of chapter, the effect of the magnetic field from the current passing through the copper can on the saturation levels in the magnetic core of the HDG was studied. The effect of adding slits to reduce the armature reaction in the stator was also studied using the 2D-FE simulations. The results obtained from these simulations could be used in the future design and optimisation of Homopolar DC machines.
CHAPTER 5
EXPERIMENTAL INVESTIGATION

In this chapter, the experimental investigation of the prototype HDG is presented and the results assessed. The results were also validated against the Finite Element model simulations.

5.1 Measurement of the generated voltages
The objective of this test is to measure the HDG generated voltage when the outputs of the generator are open-circuit. The HDG was coupled to a DC motor which acted as the prime mover. The test rig is illustrated in Figure 5.1. A separate DC power supply was used to supply the field windings of the HDG.

As mentioned in the previous chapters, the HDG had eighteen electrical outputs with the two end brush sets of the generator having the same polarity and the central brush set having the opposite polarity. Figure 5.2 shows the output electrical connections. The experimental test was accomplished in two different ways:

1. The current in the field windings ($I_f$) was kept constant, and the generated voltages ($e$) measured for various magnitudes of the rotational velocity ($\omega$).
2. $\omega$ was kept constant, and $e$ was measured for various magnitudes of $I_f$. 
5.1.1 $I_f$ constant, varying $\omega_r$

In this test, $I_f$ was initially fixed at 0.5[A] and by means of the DC motor $\omega_r$ was changed from 98 [RPM] to 3274[RPM]. At the different values of $\omega_r$, $e$ was measured using a DC voltmeter. The procedure was then repeated to measure $e$ for $I_f$ in the range 1, 1.5, 2, 2.5 and 3[A]. The measured values of $e$ against $I_f$ and $\omega_r$ are presented in Table 5.1. The maximum linear working velocity of the brushes was 25 [m/sec], this equated to a maximum rotational velocity of 3274[RPM]. The results presented in Table 5.1 are illustrated in Figure 5.3. As can be seen from Figure 5.3, if $I_f$ is held constant and $\omega_r$ varies from 98 [RPM] to 3274[RPM], the induced emf, $e$, changes linearly. It is also clear from Figure 5.3 that for a given rotational velocity, $e$ increases nonlinearly for increasing $I_f$. This nonlinearity variation can be related to the nonlinear electromagnetic properties of the ferromagnetic materials used in the stator and rotor. According to Table 6.1, the maximum value of $e$ is obtained when $I_f$ and $\omega_r$ are equal to 3 [A] and 3274[RPM], respectively.
Table 5.1 Measured values of $e$ [V] with $I_f$ constant and $\omega$ varying

<table>
<thead>
<tr>
<th>Field Winding Current, $I_f$ [A]</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
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<td>248</td>
<td>0.049</td>
<td>0.09</td>
<td>0.112</td>
<td>0.123</td>
<td>0.13</td>
<td>0.135</td>
</tr>
<tr>
<td>500</td>
<td>0.098</td>
<td>0.181</td>
<td>0.227</td>
<td>0.248</td>
<td>0.261</td>
<td>0.272</td>
</tr>
<tr>
<td>751</td>
<td>0.148</td>
<td>0.272</td>
<td>0.34</td>
<td>0.373</td>
<td>0.393</td>
<td>0.408</td>
</tr>
<tr>
<td>1004</td>
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</tr>
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<td>1255</td>
<td>0.247</td>
<td>0.454</td>
<td>0.568</td>
<td>0.623</td>
<td>0.656</td>
<td>0.681</td>
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<td>1507</td>
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<td>0.545</td>
<td>0.681</td>
<td>0.748</td>
<td>0.787</td>
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<td>1759</td>
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<td>0.636</td>
<td>0.796</td>
<td>0.873</td>
<td>0.919</td>
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<td>2012</td>
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<td>0.727</td>
<td>0.91</td>
<td>0.998</td>
<td>1.05</td>
<td>1.091</td>
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<td>2264</td>
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<td>1.183</td>
<td>1.228</td>
</tr>
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<td>2516</td>
<td>0.496</td>
<td>0.91</td>
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<td>1.249</td>
<td>1.315</td>
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<td>2766</td>
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<td>1.183</td>
<td>1.481</td>
<td>1.624</td>
<td>1.71</td>
<td>1.775</td>
</tr>
</tbody>
</table>

Figure 5.3 Measured values of $e$ [V] with $I_f$ is kept constant and $\omega$, varying
5.1.2 \( \omega_r \) constant, \( I_f \) varying

In this test, \( \omega_r \) was kept constant and the field current \( I_f \) varied. The following test procedure, shown in the flowchart of Figure 5.4, was used to measure the values of \( e \) for a rotational speed of 248 [RPM].

![Flowchart](image)

**Figure 5.4** Flowchart illustrating the procedure to measure \( e \) with \( \omega_r \) constant
This test was accomplished at rotational speeds of 248, 500, 751, 1004, 1507 and 2012 [RPM]; the recorded values of $e$ are tabulated in Table 5.2 as a function of $\omega_r$ and $I_f$.

The term *increase* in Table 5.2 means that $I_f$ was increasing from zero during this test and the term *decrease* means that $I_f$ was decreasing from 3.25 [A].

The results of the measurements presented in Table 5.2 are plotted and shown in Figure 5.5.

From Table 5.2 and Figure 5.5, two prominent issues are clear:

1. When $\omega_r$ is kept constant and $I_f$ is increased from zero to 3.25 [A], $e$ increases nonlinearly because of the nonlinear properties of the ferromagnetic materials used in the stator and rotor.

2. For given values of $I_f$ and $\omega_r$, $e$ has two different values corresponding to an increasing or decreasing current. This is due to the magnetic hysteresis effect in the stator and rotor cores.

It is also worth noting that when $I_f$ is equal to zero and $\omega_r$ is non-zero, small values for $e$ may be measured due to the remnant magnetic flux in the stator and rotor cores. These are not included in Table 5.2 and Figure 5.5.
Table 5-2 The measured values of $e$ when $\omega_f$ is kept constant and $I_f$ is changed (Unit of $e$: [V])

<table>
<thead>
<tr>
<th>Field Winding Current, [A]</th>
<th>248</th>
<th>248</th>
<th>500</th>
<th>500</th>
<th>751</th>
<th>751</th>
<th>1004</th>
<th>1004</th>
<th>1507</th>
<th>1507</th>
<th>2012</th>
<th>2012</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Increase</td>
<td>Decrease</td>
<td>Increase</td>
<td>Decrease</td>
<td>Increase</td>
<td>Decrease</td>
<td>Increase</td>
<td>Decrease</td>
<td>Increase</td>
<td>Decrease</td>
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<td>0.066</td>
<td>0.077</td>
<td>0.098</td>
<td>0.102</td>
<td>0.131</td>
<td>0.158</td>
<td>0.194</td>
<td>0.211</td>
<td>0.263</td>
</tr>
<tr>
<td>0.5</td>
<td>0.048</td>
<td>0.057</td>
<td>0.097</td>
<td>0.116</td>
<td>0.147</td>
<td>0.174</td>
<td>0.198</td>
<td>0.231</td>
<td>0.297</td>
<td>0.348</td>
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<td>0.467</td>
</tr>
<tr>
<td>0.75</td>
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<td>0.079</td>
<td>0.143</td>
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<td>0.215</td>
<td>0.243</td>
<td>0.287</td>
<td>0.324</td>
<td>0.432</td>
<td>0.486</td>
<td>0.578</td>
<td>0.647</td>
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<td>0.199</td>
<td>0.273</td>
<td>0.3</td>
<td>0.37</td>
<td>0.398</td>
<td>0.546</td>
<td>0.596</td>
<td>0.733</td>
<td>0.799</td>
</tr>
<tr>
<td>1.25</td>
<td>0.103</td>
<td>0.109</td>
<td>0.209</td>
<td>0.222</td>
<td>0.314</td>
<td>0.334</td>
<td>0.419</td>
<td>0.445</td>
<td>0.631</td>
<td>0.669</td>
<td>0.841</td>
<td>0.895</td>
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<tr>
<td>1.5</td>
<td>0.112</td>
<td>0.116</td>
<td>0.227</td>
<td>0.235</td>
<td>0.34</td>
<td>0.354</td>
<td>0.455</td>
<td>0.472</td>
<td>0.682</td>
<td>0.71</td>
<td>0.918</td>
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</tr>
<tr>
<td>1.75</td>
<td>0.118</td>
<td>0.121</td>
<td>0.239</td>
<td>0.245</td>
<td>0.359</td>
<td>0.368</td>
<td>0.479</td>
<td>0.491</td>
<td>0.719</td>
<td>0.737</td>
<td>0.959</td>
<td>0.985</td>
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<tr>
<td>2</td>
<td>0.122</td>
<td>0.125</td>
<td>0.248</td>
<td>0.252</td>
<td>0.372</td>
<td>0.379</td>
<td>0.497</td>
<td>0.505</td>
<td>0.746</td>
<td>0.759</td>
<td>0.995</td>
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</tr>
<tr>
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<td>0.255</td>
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<tr>
<td>2.5</td>
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<td>0.261</td>
<td>0.263</td>
<td>0.392</td>
<td>0.395</td>
<td>0.523</td>
<td>0.527</td>
<td>0.785</td>
<td>0.792</td>
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<td>1.057</td>
</tr>
<tr>
<td>2.75</td>
<td>0.132</td>
<td>0.132</td>
<td>0.266</td>
<td>0.268</td>
<td>0.4</td>
<td>0.402</td>
<td>0.534</td>
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<td>0.801</td>
<td>0.806</td>
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<td>1.075</td>
</tr>
<tr>
<td>3</td>
<td>0.134</td>
<td>0.134</td>
<td>0.271</td>
<td>0.272</td>
<td>0.407</td>
<td>0.408</td>
<td>0.543</td>
<td>0.545</td>
<td>0.815</td>
<td>0.817</td>
<td>1.088</td>
<td>1.091</td>
</tr>
<tr>
<td>3.25</td>
<td>0.136</td>
<td>0.136</td>
<td>0.275</td>
<td>0.275</td>
<td>0.413</td>
<td>0.413</td>
<td>0.552</td>
<td>0.552</td>
<td>0.828</td>
<td>0.828</td>
<td>1.105</td>
<td>1.105</td>
</tr>
</tbody>
</table>
5.2 Comparison $e$ obtained from practical tests and from FE simulations

In Chapter 4, the values of $e$ were calculated using FE model for different values of $I_f$ (0.5, 1.0, 1.5, 2.0, 2.5 and 3 [A]) and $\omega_r$ equal to 3274 [RPM]. There are presented in Table 4.3. Comparison with the results obtained from the practical tests at 3274 [RPM] are summarised in Table 5.1.

Table 5.3 Comparing the values of $e$ obtained from experimental investigations and from the FE model ($\omega_r = 3274$ [RPM])

<table>
<thead>
<tr>
<th>$I_f$ [A]</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Voltage $e$ [V]</td>
<td>0.598</td>
<td>1.17</td>
<td>1.49</td>
<td>1.67</td>
<td>1.75</td>
<td>1.84</td>
</tr>
<tr>
<td>Calculated voltage by Finite Element [V]</td>
<td>0.645</td>
<td>1.183</td>
<td>1.481</td>
<td>1.624</td>
<td>1.710</td>
<td>1.775</td>
</tr>
<tr>
<td>Error [%]</td>
<td>7.22%</td>
<td>1.23%</td>
<td>-0.76%</td>
<td>-2.5%</td>
<td>-2.13%</td>
<td>-3.55%</td>
</tr>
</tbody>
</table>

The results shown in Table 5.3 and Figure 5.6, illustrate good agreement between the measured and calculated values of $e$; minor errors may be caused by the uncertainty associated with the B-H curve used in the FE simulation.
5.3 current connections

To measure the output current of the prototype HDG, three busbars were constructed. The busbars were effectively loops made of plain copper braid wire of rating 100 [A], as shown in Figure 5.7. The approximate cross section of the copper braid wire used was (20.0×1.5 [mm]). At one point on these busbar loops, the wire was extended to form the output connection. The busbar were finally covered with PVC tape for insulation. For the central brush assembly, two pieces of copper braid wire were utilised in parallel to increase the rating of the busbar because the current in the central brush set was twice the current in the end brush sets. The final configuration of the busbars and the output wires can be seen in Figure 5.7. To connect the brushes to the busbar, small sections of the busbar insulation was removed.
The final connections and arrangements of the busbars and brushes are shown in Figure 5.8. The rating of each end busbar was equal to 100 [A] and the rating of the central busbar 200 [A].

![Figure 5.8 Connections and arrangements of busbars and brushes](image)

Schematic drawings of the brushes and busbars' connections are illustrated in Figure 5.9, along with the corresponding terminal to each busbar. Terminal A and Terminal B belong to the end brush assemblies with the same polarity, and were connected together and denoted Terminal N. Terminal M belongs to the central brush assembly. The external load could be connected between Terminal M and N.

![Figure 5.9 Schematic drawing of the connections of the brushes and busbars](image)

In Figure 5.9, the end brushes are denoted B1, B2, ..., B6 and A1, A2, ..., A6; the central brushes are denoted M1, M2, ..., M6.
In Figure 5.10(A), the full electrical circuit of the prototype HDG is illustrated. The electrical resistance of the rotor, busbar and the output wires was assumed to be negligible. $R_{\text{Brush}}$ denotes the resistance of each brush (there are 18 brushes in HDG) and the resistance of all brushes was assumed to be identical. In Figure 5.10(A), two DC voltage sources are shown corresponding to the voltage, $e$, induced on one side of the rotor. The equivalent circuit of the electrical set up of Figure 5.10(A) is shown in Figure 5.10(B). The total output current ($I_{\text{Total}}$) of the prototype HDG is:

$$I_{\text{Total}} = \frac{e}{R_{\text{Brush}} + R_{\text{Load}}} = \frac{e}{R_{\text{Total}}}$$  \hspace{1cm} (5-1)

In Eq (5-1), $R_{\text{Load}}$ is the total resistance of the electrical load connected between Terminal M and N. In Figure 5.10(A), $I_A$ and $I_B$ are the total current passing through the brushes of end A and B, respectively. $I_A$ and $I_B$ are assumed to be equal. One point should be emphasised here, the equivalent circuit shown in Figure 5.10(B), applies to steady state conditions. For transient operation, the inductance of the wires, busbars, and rotor would need to be included.
A) Electrical schematic of the HDG

\[ I_B = \frac{I_{\text{Total}}}{2} \quad I_A = \frac{I_{\text{Total}}}{2} \]

B) Steady-state equivalent circuit of the HDG

**Figure 5.10** Electrical circuit of the HDG
5.4 Introduction to electrical contacts

The contact resistance of the brushes on the rotor copper can is explained briefly here. The aim of this section is not to cover the detailed physics of electrical contacts but provide a summary of only some very basic concepts and definitions; more comprehensive details may be found in [51-53]. These contact resistance terms and definitions will be used in later parts of this chapter.

The electrical resistance of a simple cuboid, shown in Figure 5.11, whose cross section area, length and resistivity are \( A \), \( L \) and \( \rho \) respectively, if an electric potential exists between surfaces, S1 and S2, is simply:

\[
R = \rho \frac{L}{A}
\]

Figure 5.11 Resistance of a body

In Figure 5.12, two materials A and B are placed together so that they are in mechanical and electrical contact and have an electric potential applied across them. If we look at the interface of two objects microscopically, the interface is uneven with random contacts point along the surface.

Figure 5.12 Mechanical contact of two conducting materials
The random material contact at the interface leads to an electric resistance known as contact resistance.

If an electric potential is applied across the two materials like A and B, as shown in Figure 5.12, an electric current flows through the interface. The region voltage drop across A and B is not only due to body resistance but also the interfacial resistance or contact resistance.

Sometimes the contact resistance is negligible but sometimes it can have a significant effect on the voltage drop, especially in cases dealing with low voltages. Calculation of the contact resistance is a difficult task because many parameters are involved.

The contact resistance comprises two diverse resistances, namely the constriction resistance and the surface film resistance. To understand these, we should consider at the apparent contact area $A_a$; there is mechanical contact for only a portion of $A_a$, thus the real contact area, $A_r$ is smaller than the apparent contact area.

The real mechanical contact area is divided into two parts, one part that has an electrical contact known as the contact spot or a-spot, and one part containing insulation, due to oxidization for example. A sample contact area is shown in Figure 5.13.

In some parts of the interface, the insulating material is thick, so no current passes through these points. In some parts however the insulating material is thicker which leads to semi-conducting regions, the resistance of these is known as the film resistance.

In an ideal contact that is smooth, as shown in Figure 5.14(A), current flow exists over all of the contact area. In a contact that has unevenness however; current flow only exists at a-spots, as shown in Figure 5.14(B). This restriction to the current path is known as the restriction resistance.

![Figure 5.13 Sample contact area](image-url)
From [51], "the electrical resistance of the brushes ($R_B$) consists of three parts,
\[ R_B = R_O + R_F + R_C \]
being the sum of the resistance of the brush body ($R_O$), the surface film resistance ($R_F$) and the constriction resistance ($R_C$)."

As explained in Chapter 3, Copper Graphite brushes were used in this project due to its low contact resistance. In the data sheet for the brushes, presented in Appendix (II), the voltage drop across a pair of brushes as a function of current passing through the area of the brushes is given as a function of the slip ring velocity, the brush pressure, temperature and the type of slip ring. The contact resistance of each brush is functions of the rotor can material, the pressure of the spring of the brush holder on the spring, the rotor surface velocity, etc.
5.5 Current measurements on the prototype HDG

The aims of the tests presented in this section were to study the output current of the prototype HDG. Due to the low voltage output of the HDG, the electrical loads connected to HDG require very low resistance values. In the experimental tests therefore, the output busbars were simply short-circuited as shown in Figure 5.15. The electrical load was effectively the resistance of the current busbar.

![Figure 5.15 Short circuit output of the HDG and the current transducers (CT) to measure the output currents](image)

To measure the output currents, current transducers (CTs) were used; the schematic circuit of the connections and the placement of the CTs are shown in Figure 5.16.

![Figure 5.16 Position of the current transducers are shown by the orange arrows](image)

One sample of the oscilloscope output is shown in Figure 5.17 and is explained in detail. The oscilloscope output includes the maximum, minimum, peak to peak and mean values of each signal.
Figure 5.17 Sample output of the oscilloscope
Three cases summarised in Table 5.4, were used to study the current output from the prototype HDG. For these cases, the generated voltage \((e)\) with the output open circuited was extracted from Table 5.1.

The difference between these three cases is that the value of \(e\) for each is approximately constant, whilst \(I_f\) and \(\omega_r\) are different, so the influence of \(I_f\) and \(\omega_r\) on the output currents can be assessed.

<table>
<thead>
<tr>
<th>Case</th>
<th>Rotational velocity, (\omega_r) [RPM]</th>
<th>Current of field windings, (I_f) [A]</th>
<th>Generated voltage at open circuit, (e) [V]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>751</td>
<td>2</td>
<td>0.373</td>
</tr>
<tr>
<td>B</td>
<td>1004</td>
<td>1</td>
<td>0.363</td>
</tr>
<tr>
<td>C</td>
<td>1759</td>
<td>0.5</td>
<td>0.347</td>
</tr>
</tbody>
</table>

To measure the values of \(I_A\), \(I_B\) and \(I_{Total}\) shown in Figure 5.16, \(I_f\) and \(\omega_r\) were first set for Case A; CTs were mounted as shown in Figure 5.16. Figure 5.18 illustrates the outputs of \(I_A\), \(I_B\) and \(I_{Total}\) obtained from the test for Case A. As seen in Figure 5.18(A), the mean value of \(I_{Total}\) was 98.70 [A]. The induced emf \(e\), in Case A was 0.373 [V]; \(R_{Total}\) is therefore equal to \(3.78\Omega\).

This effective load resistance would correspond to the brushes only if the resistance of the busbars, wires and the rotor core are neglected. Figure 5.18(A) also illustrates that there is some high frequency noise/ripple in the current signal, the peak to peak value in the total current is estimated to be 17.5[A], this noise/ripple is due to the contact resistance of the brushes. In Figure 5.18(A), the mean values of \(I_A\) and \(I_B\) are estimated to be 43.29[A] and 60.72[A] respectively*. The current \(I_A\) and \(I_B\) should ideally be identical but the difference are due to sensitivity of the electrical load to the brush contact resistance. In Figure 5.18, \(I_A\) and \(I_B\) appears to be negative- this simply means the reference direction for these currents is opposite to that shown In Figure 5.16.

In Figure 5.18(A), the summation of the mean values of \(I_A\) and \(I_B\) is 104.01 [A] which is about 5 [A] more than the mean measured value of \(I_{Total}\); this is due to error measurement in the CTs.

*In the oscilloscope outputs shown in this section, the unit of some output currents are V; each 10 [mV] is equal to 1 [A].
Figure 5.18(A) illustrates one time sample of the output currents for Case A. During the tests, it was observed that the mean values of $I_A$ and $I_B$ and $I_{Total}$ do not remain strictly constant. For example, Figure 5.18(B) and 5.18(C) are samples at two other time instants. This again confirms the sensitivity of the load resistance to the brush contact with the rotor. In Table 5.5, $R_{Total}$ is calculated using the mean value of $I_{Total}$ and $e$ for the results depicted in Figures 5.18(A), (B) and (C).

<table>
<thead>
<tr>
<th></th>
<th>Mean value of $I_{Total}$ [A]</th>
<th>$R_{Total}$ [mΩ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 5.18 (A)</td>
<td>98.70</td>
<td>3.78</td>
</tr>
<tr>
<td>Figure 5.18(B)</td>
<td>135.06</td>
<td>2.76</td>
</tr>
<tr>
<td>Figure 5.18 (C)</td>
<td>102.58</td>
<td>3.63</td>
</tr>
</tbody>
</table>

During the tests, it was observed that any slight movement of the busbars and/or the brush wires lead to changes in the output current. These current variations are due to the contact resistances between the brushes and the rotor. In Figure 5.18(C), it is clear that $I_A$ and $I_B$ are identical as expected. In Figures 5.18(A) and 5.18(B), $I_A$ and $I_B$ are not absolutely identical.
(A)

(I_{Total} (C2))

(I_{B} (C1))

(I_{A} (C3))

(B)

(The continue of figure in the next page)
The test described for Case A was repeated for Cases B and C. Figures 5.19(A) and 5.19(B) illustrate the values of $I_A$, $I_B$ and $I_{\text{Total}}$ at two time instants for Case B; the values of $I_A$, $I_B$ and $I_{\text{Total}}$ show similar descriptions to those found for Case A.
Figure 5.19 Values of $I_A$, $I_B$ and $I_{\text{Total}}$ in Case B ($e = 0.363[V], \omega_r = 1004[RPM], I_j = I[A]$)
In Figure 5.20, a single time sample of the results obtained for $I_A$, $I_B$ and $I_{Total}$ in Case C are shown; Again as in Cases A and B, there are variations in the mean values of $I_A$, $I_B$ and $I_{Total}$.

According to Figure 5.20, the mean value of $I_{Total}$ is 139.27 [A], and $e$ for Case C is 0.347 [V], therefore the effective value of $R_{Total}$ is equal therefore to $2.49E-03$ [Ω].

The rotor speed was held constant (1004 [RPM]) and the field winding current switched on or off whilst recording the output currents $I_A$, $I_B$ and $I_{Total}$. Figure 5.21(A) and (B) illustrate the current responses for Case B. The exponential rise and decay of the output currents is due to the time constant of the field windings.

It was noticed during the test that when $I_f$ was flowing in the field windings, and the prime mover (DC motor) was turned-on, occasionally large oscillations were seen in the output currents of the prototype HDG. After a short period of time however, these
oscillations would disappear. This was thought to be caused by mechanical vibrations transferred to the electrical contacts resulting in variations in the contact resistance; two samples of this situation are depicted in Figures 5.22 (A) and (B); the values of $I_f$ and $\omega$ in these two figures were the value used in Cases B and C.
Figure 5.21 Values of $I_A$, $I_B$ and $I_{total}$ during (A) turn-on and (B) turn-off the current in the field winding for Case B ($e = 0.363[V]$, $\omega_f = 1004[RPM]$, $I_f = 1.0[A]$)
Figure 5.22 Oscillatory current outputs of the HDG during the run up of the prime mover (DC motor)
As already mentioned, any movement of the wire of the brushes and busbars lead to significant variations in the output currents due to the change in the contact resistances. The short length wire to the brushes in particular meant that any small movement caused the spring pressure on the brushes to change so the contact resistances also varied. To reduce the problems, the length of the wires to the end brushes was extended, as shown in Figure 5.23.

In Figures 5.24, 5.25 and 5.26, the values of $I_A$, $I_B$ and $I_{Total}$ were again recorded with the extended wiring shown in Figure 5.23 (B). The variations in the output currents HDG were reduced but some variation was still present. All the results presented in the following are for modified wiring.
Figure 5.23 Extending the wires of the end brushes

Figure 5.24 Values of $I_A$, $I_B$ and $I_{Total}$ in Case A for the new wiring

\[ (e = 0.373[V], \omega_f = 75[\text{RPM}], I_f = 2[A]) \]
Figure 5.25 Values of $I_A$, $I_B$ and $I_{Total}$ in Case B with the new wiring

\[ (e = 0.363[V], \omega_r = 1004[RPM], I_f = 1.0[A]) \]

Figure 5.26 Values of $I_A$, $I_B$ and $I_{Total}$ in Case C with the new winding

\[ (e = 0.347[V], \omega_r = 1759[RPM], I_f = 0.5[A]) \]
18 brushes in total were used in the prototype HDG, therefore measuring the current in all the brushes together was not practical. The current therefore in only two brushes were measured. In Figure 5.27, the placement of the CTs and the measured values of $I_{M1}$, $I_{M2}$ and $I_{Total}$ are illustrated; $I_{M1}$ and $I_{M2}$ are the currents in Brush M1 and Brush M2, respectively. In the sample shown in Figure 5.27(B), $I_{Total}$ is equal to 89.24 [A], the current therefore in each individual brush in the central set should be equal to 14.87[A]. Measurement shows that $I_{M1}$ and $I_{M2}$ were equal to 7.68[A] and 14.38[A], respectively; the current in Brush M1 therefore is less than the expected current value.

Figure 5.27 to 5.33 illustrate the current measurements of different brushes along with the total current for each measurement. These were all undertaken for Case A. Table 5.6 below summarises the measured current results.

| Table 5.6 Measured brush current at the central brush set (Unit [A]) |
|-------------------------|-------------------|-----------------|-----------------|-----------------|-----------------|---------------|-----------------|
|                         | $I_{M1}$ | $I_{M2}$ | $I_{M3}$ | $I_{M4}$ | $I_{M5}$ | $I_{M6}$ | $I_{Total}$ | $I_{Total}$ |
| Figure 5.27(B)         | 7.68     | 14.38    | ×       | ×       | ×       | ×       | 89.24        | 14.87         |
| Figure 5.28(B)         | ×        | ×        | 22.65   | 12.14   | ×       | ×       | 83.85        | 13.97         |
| Figure 5.28(C)         | ×        | ×        | 22.53   | 11.52   | ×       | ×       | 84.85        | 14.14         |
| Figure 5.29(B)         | ×        | ×        | ×       | ×       | 34.52   | ×       | 10.04        | 89.08         | 14.84         |
|                         | $I_{A1}$ | $I_{A2}$ | $I_{A3}$ | $I_{A4}$ | $I_{A5}$ | $I_{A6}$ | $I_{Total}$ | $I_{Total}$ |
| Figure 5.30(B)         | 4.88     | ×        | ×       | ×       | ×       | ×       | 92.78        | 7.73          |
| Figure 5.31(B)         | ×        | ×        | ×       | ×       | ×       | ×       | 7.71         | 95.03         | 7.91          |
|                         | $I_{B1}$ | $I_{B2}$ | $I_{B3}$ | $I_{B4}$ | $I_{B5}$ | $I_{B6}$ | $I_{Total}$ | $I_{Total}$ |
| Figure 5.32(B)         | 9.34     | ×        | ×       | ×       | ×       | ×       | 89.35        | 7.44          |
| Figure 5.33(B)         | ×        | ×        | ×       | ×       | ×       | 12.28   | 101.91       | 8.49          |

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It is clear from Table 5.6 that there is a considerable variation in the measured current values at each brush and from one test to another. Figure 5.28 highlights this problem by illustrating the current measurements at two different times, Figures 5.28(B) and (C). There is significant noise and some deviation in the current measurements and these make it very difficult to provide accurate current measurements.

Figure 5.30-5.33 illustrates similar results for brush at each end of the machine, (A) and (B). Figure 5.31(C) includes an enlarged section illustrating the distortion on the current signal.

There was clearly considerable variation in the output current due to issues associated with the brush resistance. The difficulty is compounded by the basic problem that due to very low induced emf's, the load impedance also needs to be very low to produce reasonable current levels. Because of this load requirement, the experimental load was dominated by the brush contact resistance which was the source of the problems highlighted. Correlation with the 3D Finite Element current flow models was not directly possible because the 3D software was limited to current injection rather than determining the current produced by the induced emf and the load. The current FE model would allow the voltage distribution in the rotor copper can to be determined but this could not be measured on the rotating rotor because of the uncertainty associated with brush voltage drops.

The key conclusions that can be drawn from the experimental current measurement are:

- The prototype HDG produces an output DC current of level consistent with the design estimates.
- The output DC current increases with increasing stator field current. The stator field winding therefore demonstrates the intended modulation of the output.
- The output DC current increases with increasing speed. The maximum rotor speed of the proposed design will likely be limited by the rotor surface velocity at the brushes.
A) The placement of the current transducers to measure $I_{M_1}$, $I_{M_2}$ and $I_{Total}$

B) $I_{M_1}$, $I_{M_2}$ and $I_{Total}$

Figure 5.27 Values of $I_{M_1}$, $I_{M_2}$ and $I_{Total}$ in Case A

($e = 0.373 [V]$, $\omega_b = 751 [RPM]$, $I_f = 2 [A]$)
A) Placement of the current transducers to measure $I_{M3}$, $I_{M4}$ and $I_{Total}$

B) $I_{M3}$, $I_{M4}$ and $I_{Total}$

(The continue of Figure in the next page)
Figure 5.28 Values of $I_{M3}$, $I_{M4}$ and $I_{Total}$ in Case A

$(e = 0.373[V], \omega_f = 751[RPM], I_f = 2[A])$
A) Placement of the current transducers to measure $I_{M5}$, $I_{M6}$ and $I_{Total}$

B) $I_{M5}$, $I_{M6}$ and $I_{Total}$

Figure 5.29 Values of $I_{M5}$, $I_{M6}$ and $I_{Total}$ in Case A

($e = 0.373[V], \omega = 751[RPM], I_f = 2[A]$)
A) Placement of the current transducers to measure $I_{A1}$ and $I_{Total}$

Figure 5.30 Values of $I_{A1}$ and $I_{Total}$ in Case A

$v = 0.373[V], \omega_r = 75[RPM], I_f = 2[A]$
A) Placement of the current transducers to measure $I_{A6}$ and $I_{Total}$

B) $I_{A6}$ and $I_{Total}$

(The continue of Figure in the next page)
C) Zoom of $I_{A6}$

**Figure 5.31** Values of $I_{A6}$ and $I_{Total}$ in Case A

($e = 0.373[V], \omega = 751[RPM], I_f = 2[A]$)
A) Placement of the current transducers to measure $I_{B1}$ and $I_{Total}$

![Diagram of current transducers placement]

B) $I_{B1}$ and $I_{Total}$

Figure 5.32 Values of $I_{B1}$ and $I_{Total}$ in Case A

$(e = 0.373[V], \omega_r = 75[RPM], I_f = 2[A])$
A) Placement of the current transducers to measure $I_{B6}$ and $I_{Total}$

<table>
<thead>
<tr>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
<th>$A_6$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$M_4$</th>
<th>$M_5$</th>
<th>$M_6$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
<th>$B_6$</th>
</tr>
</thead>
</table>

B) $I_{B6}$ and $I_{Total}$

**Figure 5.33** Values of $I_{B6}$ and $I_{Total}$ in Case A

\[(e = 0.373[V], \omega_e = 751[RPM], I_f = 2[A])\]

For the results illustrated in Figure 5.34, firstly $I_f$ was set to 1 $[A]$, then the rotor speed was set to 1001 $[RPM]$. The supply to the prime mover was then turned-on and the values of $I_A$, $I_B$ and $I_{Total}$ are recorded and shown in Figure 5.34 (A). Figure 5.34 (B) shows the output current of the HDG when the DC motor is turned-off. It is clear as expected, that the output currents in Figure 5.34 are direct functions of the rotor speed.
A) Turning-on the prime mover (DC Motor)

B) Turning-off the prime mover (DC Motor)

Figure 5.34 Values of $I_A$, $I_B$ and $I_{Total}$ during (A) turn-on and (B) turn-off the prime mover (DC motor) for Case B ($e = 0.363[V], \omega_i = 1004[RPM], I_f = 1.0[A]$)
5.6 Testing prototype in the motoring mode

If the prototype is supplied with DC current, it works in the motoring mode. To test the prototype in the motoring mode, the DC motor used in the tests in the previous section should be mechanically detached from the prototype HDG, and then the field windings connected to a DC power supply. Terminals M and N shown in Figure 5.9 are then connected to the output terminals of a separate variable DC power supply which is capable of providing high current. The tests show that when $I_f$ was equal to 3[A], the minimum current required for the motor to rotate was about 155 [A]. Initially, 180 [A] was injected into the motor with $I_f$ set to 3 [A], the velocity of the rotor was then increased gradually and slowly, up to a rotational velocity of 3000 [RPM]. To measure the torque of the motor, a torque arm was constructed; the exploded view of the torque arm is shown in Figure 5.35 (A). The torque arm was fixed to the shaft of the motor with clamps, as illustrated in Figure 5.35(B). Weights could be added to the torque arm as shown in Figure 5.35 to determine the torque developed when the rotor is stationary. The rotor current was measured using a shunt resistor as shown in Figure 5.35(C).

(A) The exploded view of the torque arm

(B) The torque arm

(The continue of Figure in the next page)
The following procedure was used:

1. Record the mass \( m_1 \) on the scale.
2. Supply the field windings of the motor.
3. Inject current, \( I_{\text{Total}} \), into the rotor using the high current DC power supply.
4. Record the mass \( m_2 \) on the scale after inserting the current into the motor.
5. Calculate the torque \( T_x \)

\[
T_x = (m_1 g - m_2 g) \times l \tag{5-3}
\]

Where \( l \) is the length of the torque arm shown in Figure 5.36

The torque generated can also be approximated as follows:

1. Calculate \( F_{\text{Motor}} \) which is the electromagnetic force on the rotor:

\[
F_{\text{Motor}} = I_{\text{Total}} \times l_p \times B_g \tag{5-4}
\]

where:

\( l_p \) - Length of each pole of the motor (0.075 [m])

Figure 5.35 Measuring the torque by the torque rod

(C) Test rig to measure the torque
$B_g$ - Average magnetic flux density cutting the copper can (approximately 0.8 [T] when $I_f$ is 3[A])

2. $T_{Motor}$ can be approximated as:

$$T_{Motor} = F_{Motor} \times R$$

where $R$ is the radius of the rotor.

![Figure 5.36](image-url) Calculation of torque

Two points should be noted:

1. $m_s$ should be large enough to assure the torque arm and chain remain stationary when $I_{Total}$ is injected into the rotor.
2. The direction of the current in the field windings and $I_{Total}$ are such that the torque acts in the anti-clockwise direction.

The mass ($m_s$) measured on the scale therefore should be less than the mass ($m_t$) before injecting $I_{Total}$.

To measure torque, it was difficult to get any consistency in the measurements using the above procedure, because torque was very small. The torque was estimated using the following values and Eqs 5.5 and 5.6:

- $l_p$ -0.075[m]
- $B_g$ -0.8 [T] (When $I_f$ is 3 [A])
- $I_{Total}$ -300 [A]
- $l$ - 0.5 [m]
- $R$ -0.0762[m]
Assuming that all the current \( I_{\text{Total}} \) passed through the copper can, \( F_{\text{Motor}} \) and \( T_{\text{Motor}} \) were estimated to be 18 [\( N \)] and 1.3716 [\( N.m \)], respectively.

From Eq (5-3), it is possible to determine \( \Delta m \), (\( \Delta m = m_1 - m_2 \)):

\[
\Delta m = \frac{T_A}{l \times g} = 0.279 \text{ [Kg]}
\]  
(5-6)

The change in the measured mass therefore was only 279 [gr].

In Section 4.5, using the FE model it was shown that when 240 [A] is injected into the rotor, approximately half of the total current passed through the copper can, and the rest passed through the rotor core. The calculated torque above should be higher than the real value of the torque may be measured practically; also friction between the brushes and rotor reduces the output torque of the motor.

The above procedure was applied several times to measure \( \Delta m \), but different values of \( \Delta m \) were recorded and the measurement errors were significant. Part of the difficulty associated with the torque measurements was the stiction created by the rotor brushes; this appeared to be significant and was thought to be largely responsible for the measurement errors. One option was to increase the torque by increasing the rotor current, however this was limited by the current rating of brushes and busbars; another option was to calculate the torque using a speed-time curve for an acceleration and a deceleration test.

The moment inertia of the rotor was calculated using the dimensions and densities of the rotor components: shaft, rotor core and the copper can. To calculate the moment of inertia of each component, Eqs (5-7) and (5-8) were used. Eq (5-7) and Eq (5-8) represent the moment inertia of a solid cylinder and hollow cylinder respectively, around the z-axis as shown in Figure 5-37.
After calculating the component moments of inertia, Eq (5-9) was used to determine the total rotor moment of inertia, $K_{Rotor}$.

*Generally "I" and "J" symbols are used for moment of inertia, but in this dissertation these symbols are used for current and current density, so $K$ is used for moment of inertia.*
\[ K_{\text{Rotor}} = K_{\text{Shaft}} + K_{\text{Rotor-core}} + K_{\text{Copper-can}} \] (5-9)

- \( K_{\text{Shaft}} \)  Shaft moment of inertia
- \( K_{\text{Rotor-core}} \)  Rotor-core moment of inertia
- \( K_{\text{Copper-can}} \)  Copper-can moment of inertia

In Table 5.7, the dimensions, volumes and densities of the rotor components are given. The mass and moment of inertia of each rotor component are presented in Table 5.8, along with the total rotor mass and \( K_{\text{Rotor}} \).

In Table 5.7, a range of densities is presented for stainless steel. The density of stainless steel depends on its grade; the grade of the shaft made of stainless steel however was unknown, so the minimum and maximum values of density were used to calculate the shaft moment of inertia. In Table 5.8, it is clear that the shaft inertia is negligible compared to the inertia of the rotor-core and the copper can. The variation of the density of stainless steel also applies to mild steel; in this case the density of the mild steel was assumed to be 7850 [Kg / m³].

Table 5.7 Dimensions, volumes and densities of the rotor components

<table>
<thead>
<tr>
<th>Material</th>
<th>( r ) [mm]</th>
<th>( r_i ) [mm]</th>
<th>( r_o ) [mm]</th>
<th>( l ) [mm]</th>
<th>Volume [m³]</th>
<th>Density, [Kg / m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft</td>
<td></td>
<td>-</td>
<td>-</td>
<td>612</td>
<td>1.922\times10^{-4}</td>
<td>7480-8000</td>
</tr>
<tr>
<td>Rotor core</td>
<td></td>
<td>10</td>
<td>76.2</td>
<td>360</td>
<td>6.453\times10^{-3}</td>
<td>7850</td>
</tr>
<tr>
<td>Copper can</td>
<td></td>
<td>76.2</td>
<td>76.8</td>
<td>360</td>
<td>1.038\times10^{-4}</td>
<td>8940</td>
</tr>
</tbody>
</table>

Table 5.8 Masses and moments of inertia of the rotor components

<table>
<thead>
<tr>
<th>Material</th>
<th>Mass [Kg]</th>
<th>Moment of inertia [Kg.m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft</td>
<td>1.437-1.537</td>
<td>7.185\times10^{-5} - 7.685\times10^{-5}</td>
</tr>
<tr>
<td>Rotor core</td>
<td>50.656</td>
<td>0.1496</td>
</tr>
<tr>
<td>Copper can</td>
<td>0.928</td>
<td>5.430\times10^{-3}</td>
</tr>
<tr>
<td><strong>Rotor</strong></td>
<td>Mass=53.073 [Kg]</td>
<td>( K_{\text{Rotor}} = 0.155 ) [Kg.m²]</td>
</tr>
</tbody>
</table>
To record and measure the rotor speed, a high precision encoder was attached to the rotor shaft. The output of encoder was recorded using the LABVIEW data acquisition system. Using this system the rotor speed as a function of time was recorded. To obtain a set of speed-time curves corresponding to different acceleration and deceleration rates, the field winding current $I_f$ was initially set at a required value, then the required rotor current $I_{Total}$ was injected into the rotor using a DC power supply. When the rotational speed of the rotor reached 2500 [RPM], the power supply was turned off. The speed-time curves during acceleration and deceleration for various values of $I_f$ and $I_{Total}$ were obtained using this procedure.

The rotor speed-time curves were recorded for a rotor current $I_{Total}$ equal to 200 [A], and for various values of $I_f$ (2, 2.5 and 3 [A]). These curves are shown in Figure 5.38. Figure 5.39 shows the curves for a fixed field current $I_f$ equal to 3.0 [A], and for various values of rotor current $I_{Total}$ (200, 250, 300, 350, 400 and 450 [A]).

In Figures 5.38 and 5.39, a trend line was added on the graph corresponding to acceleration, and another one added corresponding to deceleration. For each trend line, the line equation was obtained, $x$ and $y$ corresponds respectively to time and rotational speed of the rotor. The gradients of these trend lines correspond to angular acceleration and deceleration of the rotor.

In Figure 5.38(A), noise on the encoder or data acquisition device is evident; this noise however did not have any effect on the determination of the gradient of the trend line corresponding to rotor acceleration.
A) $I_f = 2.0 \, [A]$, $I_{Total} = 200 \, [A]$

B) $I_f = 2.5 \, [A]$, $I_{Total} = 200 \, [A]$
C) $I_f = 3.0 \, [A]$, $I_{Total} = 200 \, [A]$

Figure 5.38 (A)-(C) Acceleration-deceleration curves when $I_{Total}$ is equal to 200 [A], and for various values of $I_f$ (2, 2.5 and 3 [A])
A) $I_f = 3.0 \, [A], \quad I_{total} = 200 \, [A]$

B) $I_f = 3.0 \, [A], \quad I_{total} = 250 \, [A]$
C) \( I_f = 3.0 \) [A], \( I_{Total} = 300 \) [A]

D) \( I_f = 3.0 \) [A], \( I_{Total} = 350 \) [A]
From Newton’s law, the product of the rotor inertia and its angular acceleration is equal to the total rotor torque as given in Eq (5-10).

$$\sum_i T_i = K.\alpha$$  \hspace{1cm} (5-10)
$T_r$  Rotor torque
$K$  Rotor inertia
$\alpha$  Rotor angular acceleration

Eq (5-10) can be used for the rotor of HDG during acceleration and deceleration interval, as follows:

**Acceleration:**
$$T_{LF} - T_{\text{friction}} = K_{\text{Rotor}} \cdot \alpha_{\text{Acc}}$$  \hspace{1cm} (5-11)

**Deceleration:**
$$-T_{\text{friction}} = K_{\text{Rotor}} \cdot \alpha_{\text{Dec}}$$  \hspace{1cm} (5-12)

where:

- $T_{LF}$  Torque developed by the rotor (Lorentz force)
- $T_{\text{friction}}$  Torque from friction between the brushes and the copper can, and bearing friction
- $K_{\text{Rotor}}$  Rotor inertia
- $\alpha_{\text{Acc}}$  Rotor angular acceleration
- $\alpha_{\text{Dec}}$  Rotor angular deceleration

It is clear from Figures 5.38 and 5.39, that both the acceleration and deceleration are closely linear. From Eq (5-12), this implies that the friction torque is constant as the speed reduces. Normally one would expect the friction torque to vary slowly with rotor speed, however it is assumed here to be constant. From Eq (5-11) this then also implies the torque developed by the rotor, $T_{LF}$, is also constant.

Using the values in Table 5.8, and the value of $\alpha_{\text{Dec}}$ from speed-time curves, $T_{\text{friction}}$ was calculated from Eq (5-12) for various values of $I_f$ and $I_{\text{Total}}$. Then using $\alpha_{\text{Acc}}$ for various values of $I_f$ and $I_{\text{Total}}$, and $T_{\text{friction}}$ attained from Eq (5-12), $T_{LF}$ was determined for various values of $I_f$ and $I_{\text{Total}}$. The figures for $\alpha_{\text{Acc}}, \alpha_{\text{Dec}}, T_{\text{friction}}$, and $T_{LF}$ are given in Table 5.9.

This Table also includes the values of $T_{LF}$ calculated using the finite element model. In Figure 4.11, the magnitude of the magnetic flux density ($B_n$) normal to the copper can
is illustrated along a contour on the interface of the brushes and the copper can using 7201 points. The values of $B_n$ along one pole of the HDG were extracted from this figure. The placement of the contour is illustrated in Figure 5.40. The values of $B_n$ determined on this contour are shown in Figure 5.41, when $I_f$ is equal to 2.0, 2.5, and 3.0 [A].

![Contour line along the interface between the brushes and the copper can for one pole of the HDG](image)

**Figure 5.40** Contour line along the interface between the brushes and the copper can for one pole of the HDG

![Magnitude of magnetic flux density ($B_n$) normal to the contour along the interface between the brushes and the copper can for one pole of the HDG ($I_f$ equal to 2.0, 2.5 and 3.0 [A])](image)

**Figure 5.41** Magnitude of magnetic flux density ($B_n$) normal to the contour along the interface between the brushes and the copper can for one pole of the HDG ($I_f$ equal to 2.0, 2.5 and 3.0 [A])

To calculate the torque on the rotor, it was assumed that the can current $I_{Total}$ flows uniformly through the copper can, with the current split into half flowing inwards to the central brush set from the brush set at each end of the rotor. The full length of the contour line is shown in Figure 5.42; this line was divided into 7200 segments of
identical length ($\Delta l$). It was assumed that the magnetic flux passing through any segment between points $N_i$ and $N_{i+1}$ was $B_{n(i+1)}$.

![Diagram](attachment:image.png)

**Figure 5.42** The division of the contour to 7201 points

The torque produced on segment $(i)$ between points $N_i$ and $N_{i+1}$ therefore is:

\[ T_i = F_i \cdot r_{\text{mean}} = \frac{l_{\text{Total}}}{2} \cdot \Delta l \cdot B_{i+1} \]  \hspace{1cm} (5-13)

In Eq (5-13), $F_i$ is Lorentz force on segment $(i)$, and $r_{\text{mean}}$ is the mean radius:

\[ r_{\text{mean}} = \frac{r_i + r_o}{2} = 76.5 \ [mm] \]  \hspace{1cm} (5-14)

$r_i$ and $r_o$ are respectively the inner and outer radius of the copper can. Their values are given in Table 5.7. The total torque on the rotor can be calculated by summing the torque on each segment:

\[ T_{LF} = 2 \sum_{i=2}^{3601} T_i = 2 \sum_{i=2}^{3601} \frac{l_{\text{Total}}}{2} \Delta l \cdot B_i \cdot r_{\text{mean}} = l_{\text{Total}} \sum_{i=2}^{3601} \Delta l \cdot r_{\text{mean}} \cdot B_i \]  \hspace{1cm} (5-15)

In Eq (5-15), $\Delta l$ is equal to 0.05 [mm], the values of $l_{\text{Total}}$ and $r_{\text{mean}}$ are known, so to determine $T_{LF}$, the values of $\sum B_i$ are required. These are obtained from the 2D-FE simulation.
By substituting Eq (5-15) into Eq (5-14), we get:

\[
\begin{aligned}
\text{For } I_f = 2.0 [A], & \quad T_{LF} = (4863.15 \times 10^{-6}) I_{Total} \quad [T] \\
\text{For } I_f = 2.5 [A], & \quad T_{LF} = (5101.31 \times 10^{-6}) I_{Total} \quad [T] \\
\text{For } I_f = 3.0 [A], & \quad T_{LF} = (5368.52 \times 10^{-6}) I_{Total} \quad [T]
\end{aligned}
\] (5-16)

By substituting \( I_{Total} \) into Eq (5-16), \( T_{LF} \) for any value of \( I_f \) and \( I_{Total} \) can be calculated. The values are given in Table 5.9. These were also compared with the results obtained directly from the rotor speed-time curves. A good correlation was obtained between the experimental results and those obtained by using the 2D-FE and the method described here. The comparison is illustrated in Figure 5.43 and 5.44. From the results in Table 5.9, one point should be noted:

- To calculate \( T_{LF} \) using the finite element model, it was assumed that \( I_{Total} \) passes through the copper can only. Comparison between the results obtained from the experimental tests and those obtained using the FE model, shows that the assumption was valid.
Table 5.9 The values of angular acceleration, deceleration, and the rotor torque for various $I_f$ and $I_{Total}$

<table>
<thead>
<tr>
<th>$I_f$</th>
<th>$I_{Total}$ [RPM / sec]</th>
<th>$\alpha_{Acc}$ [RPM / sec]</th>
<th>$\alpha_{Dec}$ [rad / sec^2]</th>
<th>$\alpha_{Acc}$ [rad / sec^2]</th>
<th>$\alpha_{Dec}$ [rad / sec^2]</th>
<th>$J_{Rotor} \cdot \alpha_{Acc}$ [N.m]</th>
<th>$T_{\text{friction}}$ (Measured) [N.m]</th>
<th>$T_{LF}$ (Measured) [N.m]</th>
<th>$T_{LF}$ (Calculated) [N.m]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>200</td>
<td>6.294</td>
<td>-50.736</td>
<td>0.659</td>
<td>-5.313</td>
<td>0.102</td>
<td>-0.824</td>
<td>0.926</td>
<td>0.973</td>
<td>4.826</td>
</tr>
<tr>
<td>2.5</td>
<td>200</td>
<td>9.438</td>
<td>-50.477</td>
<td>0.988</td>
<td>-5.286</td>
<td>0.153</td>
<td>-0.819</td>
<td>0.973</td>
<td>1.020</td>
<td>4.680</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>11.038</td>
<td>-50.801</td>
<td>1.156</td>
<td>-5.320</td>
<td>0.179</td>
<td>-0.825</td>
<td>1.004</td>
<td>1.074</td>
<td>6.996</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>29.793</td>
<td>-49.082</td>
<td>3.120</td>
<td>-5.140</td>
<td>0.484</td>
<td>-0.797</td>
<td>1.280</td>
<td>1.342</td>
<td>4.610</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>43.429</td>
<td>-49.333</td>
<td>4.548</td>
<td>-5.166</td>
<td>0.705</td>
<td>-0.801</td>
<td>1.506</td>
<td>1.611</td>
<td>6.512</td>
</tr>
<tr>
<td>3</td>
<td>350</td>
<td>59.64</td>
<td>-49.845</td>
<td>6.245</td>
<td>-5.220</td>
<td>0.968</td>
<td>-0.809</td>
<td>1.777</td>
<td>1.879</td>
<td>5.422</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>75.889</td>
<td>-49.567</td>
<td>7.947</td>
<td>-5.191</td>
<td>1.232</td>
<td>-0.805</td>
<td>2.036</td>
<td>2.147</td>
<td>5.172</td>
</tr>
<tr>
<td>3</td>
<td>450</td>
<td>95.16</td>
<td>-49.143</td>
<td>9.965</td>
<td>-5.146</td>
<td>1.545</td>
<td>-0.798</td>
<td>2.342</td>
<td>2.416</td>
<td>3.045</td>
</tr>
</tbody>
</table>
Figure 5.43 Rotor torque ($T_{LF}$), for $I_{Total} = 200$ [A], and for various values of $I_f$ (2.0, 2.5 and 3.0 [A])

Figure 5.44 Rotor torque ($T_{LF}$), for $I_f = 3$ [A], and for various values of $I_{Total}$ (200, 250, 300, 350, 400 and 450 [A])

5.7 Conclusion

The results obtained from the measurement of the voltage induced in the HDG were presented. These results have been compared with those obtained using 2D finite element model. The comparison of the results shows a good agreement. The electrical contact has been briefly discussed and the results of the current measurements given. The sensitivity of the contact resistance on the output current was shown. The results of torque measurements obtained from acceleration and deceleration tests have been presented. These results are compared with the calculated values using the finite element model and again the correlation is very good, validating the torques predicted by the model.
CHAPTER 6
CONCLUSION

The main contributions of this dissertation are the study, design, simulation, construction and test of a Homopolar DC generator. In this chapter, a short review of the results obtained in the previous chapters is presented and along with a discussion of revisions and future work.

As stated in Chapter 1, the first aim of this project was to construct a DC generator producing DC voltage without using an electronic or mechanical commutator. The Initial concept generator shown in Figure 1.1 was proposed. Also in Chapter 1, it was demonstrated that due to the fundamental physics of electromagnetic induction, the first concept generator would not work. Conventional material on electromagnetic induction is not enough to explain why the initial concept generator would not work.

The comprehensive explanation of Unipolar Induction can only be found in books including the explanation of Relativity Theory. In Chapter 2, a brief explanation of the physics of electromagnetic induction and the operating principle of HDGs was presented. The configuration of the prototype shown in Figure 1.4(B) fulfills the requirements of this project, but has sliding electrical contacts that require conventional brushes. The function of the HDG brushes is not the same as those used with a mechanical commutator. In practice, the brushes and commutators used in a conventional DC generator behave like a rectifier while the HDG itself is a true DC generator. The reason why brushes or sliding contacts must be used in the HDG can be explained using the results presented in Table 2.1. The last part of Chapter 1 provides an overview on construction of the prototype HDG as well as the various applications of a HDG. In Chapter 2, the physics of electromagnetic induction is presented. This section provides an explanation as to why the initial concept generator does not work. In Chapter 3, the preliminary design, detailed design procedure and prototype assembly are presented. In Chapter 4, the limitations of the FE-models due to the existence of the electrical contacts are discussed; the results of the 2D and 3D-FE magnetostatics modeling are shown and compared. The procedure to calculate the distribution of electric potential in the copper can is also presented and using these results, the voltage appearing between the end brush set and the central brush set can be determined.
chapter concludes with the results of the 3D-current flow modeling as well as the investigation of the effects of the magnetic field produced by the armature current. The results of four sets of experimental tests undertaken on the prototype HDG are presented in Chapter 5:

1. **Measurement of the induced voltages**
   The generated voltage on the prototype HDG was measured using two different methods. In the first method, the field winding current was kept constant and for different values of angular velocity, the generated voltages were measured. In the second method, the rotational velocity of the HDG’s rotor was kept constant and by varying the field winding current, the induced voltages were measured. The maximum voltage generated by the prototype HDG in this project was $1.775 \, [V]$ when $I_f$ and $\omega_l$ were $3 \, [A]$ and $3274 \, [RPM]$ respectively.

2. **Comparison of the results obtained from the modeling and the experimental tests**
   The results obtained from the open circuit tests were compared with the results obtained from the Finite Element simulations. Good agreement between the practical tests and the numerical simulations was evident.

3. **Measurement of the prototype HDG output currents**
   The HDG output currents were measured for different values of $I_f$ and $\omega_l$. Due to the very low internal resistance of the prototype HDG, two different variations in the output currents were observed. A variation of the output current was evident as well as a variation of the mean values. Both of these effects were due to the contact resistance. An accurate measurement of the output currents was very difficult, and in the results presented some uncertainty exists.

4. **Motoring mode of operation**
   In the last section of Chapter 5, the results of testing the prototype in the motoring mode of operation are presented. In this chapter, an attempt to measure the torque using a simple torque arm was also presented; this was unsuccessful due to the small output torque. The torque was subsequently measured indirectly using a speed-time curve.
The following comments are worth noting relating to future work on the construction and analysis of HFG's:

- The brush holders should be re-designed; as demonstrated in Chapter 3, placing and removing the prototype HDG brushes were a difficult task.

- Before constructing and designing a HDG, selecting proper brushes with a low contact resistance is very important. In this project, copper graphite brushes were used. There are metal graphite brushes however made of silver which has more than 90 percent silver; these kinds of brushes have a very low contact resistance. Silver graphite brushes are very suitable for use in a HDG; in this project however the high price of silver graphite brushes meant that copper graphite brushes were used.

- The effect of temperature on the contact resistance should be studied. The variation in the mean values of the output currents may be as a result of the changing temperature of the copper can and the brushes due to friction between the brushes and the copper can. This may cause a changing contact resistance.

- A further literature review and study on skin velocity effect is necessary because it has a direct effect on the healthy performance of the brushes, as well as brush life.

- In order to increase the generated voltage, two methods could be examined:
  
  1. Increasing the magnetic flux density cutting the copper can. Possible use of a superconductor for the field winding, removing the magnetic core, or in other words using an air core. This would increase the output voltage.
  
  2. It might be possible to increase the output voltage using several layers of copper cans insulated from each other. This is equivalent to increasing the length of conductor.

- Increasing the number and size of the brushes, and increasing the busbar ratings, it might be possible to increase the prototype HDG output current.

- The prototype HDG is not fully optimized; a full optimisation study would be beneficial.

- Analysis of the HDG using other electromagnetic techniques such as the hybrid finite element boundary element method [54] might also help to predict the full performance.
REFERENCES


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CONSTANT FORCE SPRING BRUSH HOLDERS

| Part Number | t | a | r | C | E | f | g | i | m | n | o | p |
|-------------|---|---|---|---|---|---|---|---|---|---|---|---|---|
| RTR620      | 8 | 20| 40| 14| 24| M6| 10.5| 32| 44| 71| 17| 4  |   |
Appendix (II) [56]
Appendix (III)
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