Fracture under Combined Primary and Secondary Stresses

A thesis submitted to The University of Manchester for the degree of Doctor in the Faculty of Engineering and Physical Sciences

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Peter Michael James
Supervisor: Professor Andrew Sherry
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Abstract

Fracture under Combined Primary and Secondary Stresses

Peter Michael James, The University of Manchester, January 2013

Degree Title: Doctor of Philosophy

Components found within many industries contain crack like defects. The work detailed here considers such a component under the combined influence of primary and secondary stresses; where primary stresses contribute to plastic collapse and secondary stresses are redistributed under plastic deformation. A number of approaches are available to detail the combined loading on the crack tip parameter $J$, or $K_I$, which is used to assess proximity to failure from crack extension. However, these approaches are recognised to be conservative and can lead to the unnecessary replacement of components, stricter surveillance and inspection regulations, and further costs associated with downtime.

The aim of the work presented is to investigate these conservatisms and develop a further approach to quantify the interaction of primary and secondary stresses on fracture. A large matrix of cracked body finite element analyses of a circumferentially cracked cylinder has been performed under a range of loadings. This is then used to detail the interaction of primary and secondary stresses on fracture by providing a function to describe a scaling term, $g$, that multiplies the secondary crack driving force contribution. This term has been shown to be relatively independent on the magnitude of secondary stresses and is also dependent on the material stress strain relation. This relation for $g$ has also been shown to be compatible with the R6 defect assessment procedures $V$ factor approach, through the $V_g$ plasticity interaction term, that provides a scaling term to the secondary contribution in R6.

A review of experiments considering combined loading has indicated that the number of tests that cover a range of primary stress induced plasticity levels is limited. Further experiments were therefore considered within this research to provide added experimental fracture toughness data by which to compare the R6 $V$ factor and $V_g$ approaches. These experiments introduced a compressive pre-load to the ends of three-point bend specimens so that a tensile residual stress resulted on unloading. A crack was introduced and the specimens tested at one of three temperatures so that changes in the materials fracture toughness with temperature ensured different levels of plasticity at failure; so that crack growth occurred over three sets of load normalised to the load for plastic collapse. Tests were also conducted that did not include the residual stress so that the effect of residual stress could be shown under different levels of plastic redistribution. The $V_g$ Approach and the existing Complex R6 $V$ Approach have then been applied to all available experimental data for validation. The results show that both approaches conservatively predict the failure of all tests and that the $V_g$ Approach can reduce the level of conservatism.
Declaration

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1. Introduction

1.1. Preface

Within industry, including the nuclear power generation industry, there are components which contain welds operating at high temperatures and undergoing rapid changes in temperature during service. Therefore, the effects of secondary (e.g. thermal loads) and primary (e.g. pressure) stresses need to be considered in developing a complete understanding of structural response. It is also likely that defects within the structure exist, either undetected from the start of life or formed during service, and these are more likely to be found in regions of high stress [1]. Understanding the behaviour of cracks within these structures is therefore an area of great importance. A range of defect assessment codes exist to quantify the integrity of structures under combined primary and secondary stresses; where the most advanced of these, with respect to the inclusion of secondary stresses, are the EDF Energy R5 and R6 defect assessment procedures [2], [3].

1.2. Primary and Secondary Stresses

The concept of primary and secondary stresses was first defined within the American Society of Mechanical Engineers (ASME) design code [1] in the 1960’s. The distinction between primary and secondary stresses, as defined below, was made to allow simplified elastic calculations. The approach was also adopted in the original R6 [3] procedure.

The main distinction between primary and secondary stresses is that a secondary stress is displacement controlled, arising from a structural or internal strain mismatch, and does not contribute to plastic collapse. As such, secondary stresses are globally self-equilibrating, although not necessarily at the flaw’s location [4]. Conversely, primary stresses are those that do contribute to plastic collapse and result from an externally applied load.

A simple comparison of the nature and causes of primary and secondary stresses can be seen in Table 1.
Table 1 - Table summarising some of the basic phenomenological differences between primary and secondary stresses

<table>
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<tr>
<th>Causes of Stress</th>
<th>Primary Stress</th>
<th>Secondary Stress</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Pressure, End Load, Gravity, Bending Moment, Torsion Moment</td>
<td>Temperature Gradient, Weld Residual Stresses, Fit-up Stresses, Local Deformation</td>
</tr>
<tr>
<td>Load Type</td>
<td>Load Controlled</td>
<td>Displacement Controlled</td>
</tr>
<tr>
<td>Effect of Material Yielding</td>
<td>Enhances the localised damage and stress does not reduce</td>
<td>Relieves the displacement and reduces the stress</td>
</tr>
<tr>
<td>Contributes to Plastic Collapse?</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Effect on Structure</td>
<td>Externally applied and deforms entire structure</td>
<td>Self-balancing in entire structure, but not necessarily locally, and generally will only cause localised deformation</td>
</tr>
<tr>
<td>Range of Effect</td>
<td>Over entire structure</td>
<td>Long range secondary* (i.e. global temperature gradient)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Short range secondary (i.e. weld residual stress)</td>
</tr>
</tbody>
</table>

* The case of a long range thermal may induce elastic follow-up and be considered to share features of both primary and secondary stresses (see Section 2.8)

1.3. Typical Examples of Secondary Stresses

Thermal stresses are normally considered as secondary stresses and arise from a temperature difference through the material. One example of this is a localised through wall temperature difference which, if the component is prevented from bending (either
fixed at either end or geometrically constrained as in a cylinder), will induce a through wall bending stress as illustrated in Figure 1a. Another example is a difference along the length of a component as illustrated in Figure 1b. A third example may be a uniform temperature change over a component that contains two materials of different thermal expansion coefficients (which may occur at a bimetallic weld or austenitic cladding on a ferritic pressure vessel).

Figure 1 – Examples of a cylinder with a central fully circumferential crack of length a with (a) through thickness temperature distribution giving through wall bending and (b) longitudinal temperature variation giving rise to global bending

Residual stresses, which are also considered secondary stresses, are those that typically arise during fabrication or repair of a component, normally from welding processes. A Post Weld Heat Treatment (PWHT) can be applied to the weld to reduce, but not remove, the residual stress. However, this process can be very difficult because of the components location, the expense, and, depending on the material, can cause localised distortions or sensitise\(^1\) the material [5].

It is critically important that secondary stresses must be considered in the assessment of many components used in the power generation industry as well as in other areas such as

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\(^1\) At PWHT temperatures the material, for example Grade 304 stainless steel, forms chromium carbides at grain boundaries that denudes the adjacent material of chromium in solid solution. This increases the susceptibility of the material to intergranular corrosion [116].
in oil and gas pipelines. Examples of components that include severe secondary stresses in the power generation industry are stream tubes and cooler pipes. These components are subjected to a high through wall temperature difference and contain a large number of welds, with some experiencing high magnitude residual stress due to a lack of PWHT. Fabrication methods are a further source of residual stress, along with pipe mis-fit. A typical component in EDF Energy’s Advanced Gas Cooled Reactor (AGR) reheater outlet penetration illustrated in Figure 2 [6], which shows the location of a defect detected near a weld. This component includes a number of features which add to the levels of secondary stress to be considered such as: (a) weld residual stresses, (b) global bending arising from a temperature change, (c) a localised mismatch and (d) residual stresses in the forged section. The component was also subjected to a moderate internal pressure. In this component, a defect formed on the notched section and was not detected prior to a steam leak.

![Diagram of AGR re-heater outlet penetration component](Figure 2 - Simplified geometry of the AGR re-heater outlet penetration component [6])

1.4. Aim of Project Investigation

In order to include secondary stresses in a defect assessment it is necessary to adopt generic approaches so that the range of components, materials and stress types contained in Table 1 can be accounted for. Approaches should be relatively straightforward to implement so that a large number of conditions and postulated defects can be considered such that confidence can be gained from sensitivity studies. As a consequence, the methods contained within defect assessment procedures are generally designed to contain
significant levels of conservatism as quantified by experimental results, finite element analyses and nuclear plant experience [4], [7]. This conservatism can drive the replacement of components that may be suitable for service; unnecessarily exposing personnel to irradiated environments and incurring additional costs and downtime for the plant operators. A review of the published literature (Section 2) shows that the simplified defect assessment methods do not build on a complete understanding of how primary and secondary stresses interact in the presence of plasticity and this lack of understanding and consequent lack of sophistication results in unnecessarily severe conservatisms.

The objective of this research was therefore to develop new approaches to improve the assessment of defects in engineering components that contain secondary stresses. It is anticipated that some outcomes of this work will improve on the existing guidance for such assessments that already exist in the R5 and R6 procedures [2], [3].

1.5. Structure of Thesis

Chapter 2 provides a review of the relevant published literature considering fracture, the reference stress methodology and the interaction of primary and secondary stresses. Section 3 outlines a more detailed project aim with reference to the current state of knowledge reviewed in Section 2. Section 4 presents work performed to develop a new plasticity interaction parameter, $V_g$, that quantifies the interaction of primary and secondary stresses under combined loading. An experimental programme is then presented in Section 5 which was designed to provide new fracture toughness data under combined loading over a range of plastic conditions. In Section 6 the results from this experimental programme, as well as other experiments identified in Section 2, are then used to assess the $V_g$ approach. Sections 7 and 8 provide the discussion and conclusions respectively that have been taken from the work performed. Finally Section 9 provides some recommendations for further work.

Appendix 1 contains a compendium of known experiments undertaken world-wide pertaining to the interaction of primary and secondary stresses, of which a summary is provided in Section 2. Appendix 2 details an investigation to consider welded cylinders to cover gaps in the discussion.
2. Literature Review

2.1. Introduction

The review presented within this section reviews the relevant published literature to fracture, particularly to the conditions where primary and secondary stresses are combined. The review progresses from the concept of elastic fracture mechanics, through to elastic plastic fracture and constraint modified fracture when assessing crack like defects under any applied stresses. Simplified approaches for fracture, predominantly for primary loading alone, are then presented based upon the reference stress concept. The use of the elastic follow-up factor is then discussed when comparing primary and secondary stresses before considering different approaches for including combined primary and secondary stresses in a simplified fracture assessment. Finally a short review of relevant experiments, which consider both primary and secondary stresses in fracture mechanics tests, is then provided before a short overview of the approaches used to measure residual stresses.

2.2. Examples of Failure with a Secondary Stresses Contribution

The need for inclusion of secondary stresses in component assessments is not immediately obvious since such secondary stresses do not contribute to plastic collapse\(^2\), and, as such, are not always considered when designing a component or structure. However, experience has shown that there are a number of reasons for acknowledging the influence of secondary stresses on structural integrity because such stresses;

1. act near an initiation site of cracks,
2. are the driving force for crack growth,
3. introduce significant plasticity and high strains,
4. cause significant deformation locally,
5. act near a local stress raising feature,
6. are associated with degraded material properties for example in the Heat Affected Zone (HAZ) of a weld.

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\(^2\) neglecting a potential contribution to ratcheting effects under cyclic loading
The primary cause of many failures attributed, at least in part, to secondary stresses is from the presence of weld residual stresses. Historical examples of this include the cases of the Silver Bridge suspension bridge [8–10] and the John Thompson pressure vessel [11]. The rapid failure of the Silver Bridge could be traced back to residual “fit-up” stresses, a residual stress resulting from forming the structure, in the suspension rope eyes that provided a site for micro-crack initiation that grew to a critical size through fatigue crack growth. Conversely the John Thompson pressure vessel was found to have failed in the HAZ region with high residual stress levels as the vessel was not correctly stress relieved, causing failure within the initial pressure test.

2.3. Linear Elastic Fracture Mechanics

The fracture of a material from sharp, crack like defects, is generally characterised by single parameters such as $K$ or $J$; where $K$ is the stress intensity factor (SIF) and $J$ is the energy associated with opening a crack by a unit area. The normal unit for $K$ is MPa.m$^{1/2}$ whereas $J$ is kJ/m$^2$. These terms provide a measure of the increase in localised crack tip stress from the presence of the crack.

The basic concepts of linear elastic SIFs are not discussed here. It is however noted that in defect assessments the Mode 1 linear elastic stress intensity factors, $K_I$, can be obtained from SIF solutions contained within procedures (such as R6 [3]) which have been developed over many years. How these are subsequently used within elastic-plastic fracture assessments, by way of estimating the elastic-plastic SIF, $K_J$, is detailed further below; where $K_J$ is the value of $K$ defined from the value of $J$.

Linear Elastic Fracture Mechanics (LEFM) is appropriate for the assessment of a component without any significant contribution from plasticity. These conditions are normally met when the temperature at which the component or test specimen is assessed ensures that large scale (or gross) plasticity does not develop before the crack extends. Normally, this is when the materials response is brittle in nature. Under these conditions the plastic zone is so small as to have a negligible effect and the material response is proportional to the load.
Under elastic conditions, the stress intensity factor, $K_I$, provides a measure of the stress field ahead of a crack like defect under an applied load. Under elastic conditions the stress field ahead of a crack follows an inverse $\sqrt{r}$ relationship, where $r$ is the radial distance ahead of the crack tip. The amplitude of this stress field is proportional to the applied load. It is these relations that are captured by the linear elastic stress intensity factor.

Under elastic conditions the energy release per unit crack growth, $J_e$, is found to equate to the Griffith energy term $G$, which was the first term used to assess energy release in brittle materials (glass) [12]. When an elastic material is assumed, a relationship between $K_I, J_e$ and $G$ is found as:

$$
G = J_e = \frac{K_I^2}{E'}
$$

where $E$ is Young’s modulus and $\nu$ is Poisson’s ratio. In assessments of component integrity, the crack driving force terms are compared to a resistive term characterising the materials “toughness” (e.g. $K_{mat}, J_e$), nominally termed the fracture toughness. The measurement of these resistive terms is generally obtained by fracture tests performed at the temperature of interest in compact tension or bend specimens to ensure a lower bound measure of toughness (see Section 2.6).

### 2.4. Energy Release Term under Crack Growth, $J$, and the Hutchinson, Rice and Rosengren Stress Field

When extending fracture to consider a crack like defect that lies outside the validity of LEFM but has not reached the plastic collapse limit\(^3\), it is necessary to consider $J$ defined under elastic-plastic conditions. This section considers the relationship between $J$ and the Hutchinson, Rice and Rosengren (HRR) stress field.

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\(^3\) where plastic collapse is defined at the point where plastic strain has extended such that either the component has deformed beyond the design tolerance of the component or the structure can no longer sustain any more load without excessive deformation or wall thinning.
In 1968 Hutchinson [13], Rice and Rosengren [14] (HRR) developed mathematical solutions for the stress fields at a crack tip undergoing non-linear elastic deformation. The assumption of non-linear elastic behaviour is valid in the analysis of elastic-plastic materials providing no unloading occurs. Further, as noted by Anderson [15] the deformation theory of plasticity, which relates total strain to stresses in a material, is equivalent to non-linear elasticity. Equation 2 defines the elastic strain energy contained at a singularity, from the presence of a sharp crack, as a function of the distance from the crack tip, $r$. This was extended to use $\eta$ as the scaling parameter as shown in Equation 3.

$$\frac{1}{2} \sigma_{ij} \varepsilon_{ij} = \left( \frac{K_f^2}{E'} \right) r^{-1} f$$  \hspace{1cm} \text{Equation 2}$$

$$\sigma_{ij} \varepsilon_{ij} = J r^{-1} f$$  \hspace{1cm} \text{Equation 3}$$

where $\sigma_{ij}$ and $\varepsilon_{ij}$ are the stress, normally defined in MPa, and strain at position $r$ and $f$ is a dimensionless function defined by the polar angle, $\theta$, (centred on the crack tip) and material. It was found for a power law material (i.e. one where the plastic strain can be related to the stress raised to a power, $n$, as defined by Shih [16]) that the definition of the stress field close to the crack tip is given by Equation 4 and the corresponding strain field by Equation 5.

$$\sigma_{ij} = \sigma_y \left( \frac{J}{\alpha l_n \sigma_y \varepsilon_y r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(\theta, n)$$  \hspace{1cm} \text{Equation 4}$$

$$\varepsilon_{ij} = \alpha \varepsilon_y \left( \frac{J}{\alpha l_n \sigma_y \varepsilon_y r} \right)^{\frac{n}{n+1}} \tilde{\varepsilon}_{ij}(\theta, n)$$  \hspace{1cm} \text{Equation 5}$$

where $\alpha$ is the scaling term used in the power law relationship, $l_n$ is a dimensionless integration parameter, $\sigma_y$ and $\varepsilon_y$ are the stress and strain at yield, and $\tilde{\sigma}_{ij}$
and $\tilde{\epsilon}_{ij}$ are dimensionless functions of $\theta$ and $n$. Within these equations it is also noted that, by setting the strain hardening index $n$ to unity, the $\sqrt{R}$ relationship for elastic conditions can be recovered. This means that $J_e$ is realised from this definition of $J$ when $n$ is set to perfectly elastic conditions. Under elastic-plastic conditions (i.e. $n > unity$) it is generally assumed that the elastic plastic stress intensity factor, $K_f$, can be defined from the energy release rate, $J$, in a similar means as $K_t$ is defined from $J_e$ (i.e. $K_f = \sqrt{E'J}$).

A comparative plot of the evolution of $K_t$ and $K_f$ as a function of applied load, $P$, normalised by the elastic perfectly plastic limit load for that structure, $P_L$, is provided below (data from [17]). Note that this ratio of applied load to a material and structural limit for that load type is termed $L_r$ ($L_r = P/P_L$). This term provides a useful measure of the applied forces and is commonly adopted in fracture assessment codes [3], [18]. As $K_f$ is defined from the elastic-plastic stress field, the use of $K_f$ should include all plasticity and redistribution effects under loading. Within the example shown in Figure 3 it can be seen that $K_t$ remains linear over all loads whereas $K_f$ is the same as $K_t$ at lower loads, starts to increase as the load is increased beyond $L_r = 0.5$ before deviating significantly at higher loads (i.e. $L_r > 1.0$). This increase accounts for the additional damage being sustained at the crack tip by local plastic deformation.

Figure 3 – Example $K_t$ versus $K_f$ for a circumferentially cracked pipe made from 316L stainless steel under internal pressure [17]
2.5. \( J \)-Contour Integral

The evaluation of \( J \) can also be found through analyses of the stress strain condition in the material close to the crack tip. This sub-section details how this is achieved from the \( J \)-contour integral, which is normally obtained via finite element analyses.

\( J \) can be found from the energy associated with the local crack tip region as described [19] as:

\[
J = \int_{\Gamma} \left[ W_s \, dy - T_i \left( \frac{\partial u_i}{\partial x} \right) \, ds \right]
\]

\[ W_s = \int_{0}^{\epsilon_{ij}} \sigma_{ij} \, d\epsilon_{ij} \tag{Equation 6} \]

where \( T_i \) is a term describing the traction\(^4 \), \( u_i \) describes the displacement vectors and \( s \) is the arc-length of the contour integral path, \( \Gamma \). The path of the integral is taken as an arbitrary anticlockwise path that must enclose the crack tip (at least to the free surfaces), although it is common to use uniform circular rings surrounding the crack (Figure 4). The form of the integral means that it is almost always estimated by computational means, often as part of finite element analyses. It has been found that for most elastic and elastic plastic materials the estimation of \( J \) is independent of the path chosen and, as such, it is commonly referred to as a path-independent integral.

\[ T \rightarrow \text{force per unit area} \]

\[ u \rightarrow \text{displacement vectors} \]

\[ s \rightarrow \text{arc-length} \]

\[ \Gamma \rightarrow \text{path} \]

\[ W_s \rightarrow \text{work} \]

\[ \epsilon_{ij} \rightarrow \text{strain energy} \]

\[ \sigma_{ij} \rightarrow \text{stress tensor} \]

\[ \frac{\partial u_i}{\partial x} \rightarrow \text{partial derivative} \]

\[ \int \rightarrow \text{integral} \]

\[ ds \rightarrow \text{arc-length differential} \]

\[ \text{Equation 6} \]

\[ \text{Figure 4} \]

\[ \text{Illustration of conventional} \ J \text{-contour integral} \]

It was noted in [20], [21] and summarised in [22] that the estimation of \( J \) becomes dependent on the contour used (i.e. path dependent) for cases which include pre-straining or non-proportional loading; where non-proportional loading arise in cases where a loading

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\(^4\) Traction is the force per unit area on a surface (3D) or a line (2D). The force should include all normal and shear components of stress contributing to the force.
history or complex thermal or residual strains exists such that the stress and strain state can
not be related to the uniaxial stress-strain curve. It is also noted that this estimation of \( J \)
often becomes path dependent in the presence of secondary stresses, even when the
stresses at the crack tip remain proportional. Recently there have been two independent
post-processor programmes developed that can take account of this effect using the
detailed results from finite element analyses of cracked structures (J-MOD [23] and JEDI
[24], [25]) which can be used alongside the finite element programme ABAQUS [26]:
where JEDI stands for the J-Equivalent Domain Integral. Both JEDI and J-MOD include
more refined definitions of the contour integral allowing for secondary stresses and non-
proportional effects. Within these two post-processors the definition of \( J \) incorporates the
non-proportionality by an additional term that account for the influence of prior work on
the material. In JEDI this is achieved through the \( W^{p0} \) term in [25]:
\[
J = \int_{\Gamma} \left[ W^m \delta_{ik} - \sigma_{ij} \left( \frac{\partial u_j}{\partial x_k} \right) \right] n_i \eta_k ds
\]
\[
W^m = W - W^{p0}
\]
where \( \eta \) is a unit vector defining a local coordinate system at a position on the
contour path with the normal \( n \), \( W \) is the stress-work (or strain energy) density and \( W^{p0} \) is
the prior plastic work density present before the crack is inserted. As the path forms a
closed section about the fracture process zone the evaluation of \( J \) can be re-written over an
area, \( A \), enclosed by the contour \( C \) as Equation 8, and implemented in JEDI as in Equation
9 (Figure 5).
\[
\int_{C} \left[ W^m \delta_{ik} - \sigma_{ij} \left( \frac{\partial u_j}{\partial x_k} \right) \right] m_i q_k ds
\]
\[
J = \int_{A} \left\{ \left( \sigma_{ij} \frac{\partial u_j}{\partial x_k} - W^m \delta_{ik} \right) \frac{\partial q_k}{\partial x_i} - (M_k - E_k) q_k \right\}
\]
\[
M_k = \frac{\partial W^m}{\partial x_k} - \sigma_{ij} \frac{\partial (\varepsilon_{ij}^p + \varepsilon_{ij}^{p0})}{\partial x_k}
\]
\[
E_k = \sigma_{ij} \frac{\partial (\varepsilon_{ij}^l + \varepsilon_{ij}^{p0})}{\partial x_k}
\]
where \( q \) is defined to range from 0 at \( C \) to \( \eta \) at the innermost contour (although the form of \( q \) is irrelevant), \( \varepsilon_{ij}^{p0} \) is the pre-strain, \( \varepsilon_{ij}^t \) is the thermal strain, \( \varepsilon_{ij}^e \) and \( \varepsilon_{ij}^p \) are the elastic and plastic strains and \( x_k \) is the position.

![Diagram of J-Integral based on area as implemented in JEDI](image)

Figure 5 – \( J \)-Integral based on area as implemented in JEDI

The form of the Lie expression used in J-MOD [23] is of a similar form:

\[
J = \int_A \left\{ \left( \sigma_{ij} \frac{\partial u_j}{\partial x_k} - W^m \delta_{ik} \right) \frac{\partial q_k}{\partial x_i} \right. \\
\left. - \left( \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1} - \frac{\partial W}{\partial x_1} \right) q \right\} dA
\]

Equation 10

These two expressions have been shown to provide contour independent results [23], [25], [27] for cases that include non-proportional loading and residual stresses. The expression contained within ABAQUS allows for thermal strains in the assessment but not for non-proportional loading. As both expressions reduce back to the nominal contour integral formulation when proportional loading is present they are applicable to a wide range of loading conditions, including primary, residual and thermal secondary stresses.

As inferred above, strictly the definition of \( J \) means that the contour path must encompass the fracture process zone. This fracture process zone can be interpreted as the plastic zone which would invalidate \( J \) under net-section yielding. However, it is relatively common to adopt \( J \) estimates in procedures such as R6 until plastic collapse is predicted.
2.6. Two Parameter Fracture Mechanics

The measured fracture toughness, i.e. the value of $J$ or $K_f$ at failure measured within fracture tests, is dependent on the geometry and crack depths used in the measurement. Illustrations of this can be seen in Figure 6 for the observed fracture toughness when altering the crack depth, $a$, normalised by the specimen thickness, $W$. It can be seen that the shallower the crack the higher the apparent fracture toughness observed.

![Figure 6](image)

**Figure 6 – Illustration showing change in fracture toughness for a three point bend specimen with crack depths [28]; data taken from Kirk et al [29]**

An explanation for this can be made when considering the expansion of the elastic crack tip field within a cylindrical coordinate system [30].

\[ \sigma_{ij} = A_{ij}r^{-\frac{1}{2}} + B_{ij}r^0 + Cr^{\frac{1}{2}} + \ldots \]  \hspace{1cm} \text{Equation 11}

where $A, B, C$ are scaling factors. It has already been noted, for an elastic material, that an elastic $K$ controlled stress field follows an $\sqrt{r}$ relationship. Within the relationship provided by the HRR stress field [13], [14] it is assumed that only the first term needs to be considered; which is the basic premise adopted in most LEFM calculations. However, the effect of including the second term, where $B_{ij}$ is generally referred to as $T$ (or $T'$ stress), allows the observed change in fracture toughness to be accounted for. This is illustrated in Figure 7 where the critical value of $J$ at cleavage failure, $J_c$, as a function of crack depth...
normalised by the specimen thickness, \( a/w \), has the same observed trend as when plotted as a function of the \( T \) stress normalised by the materials yield stress, \( T/\sigma_0 \).

![Graph showing the change in apparent fracture toughness for different depth edge cracked bend specimens plotted against (a) crack depth and (b) elastic \( T \) stress; taken from [31]; data taken from Betegón and Hancock [32]](image)

Under this two parameter, elastic, definition the stress field is given by Equation 12 below.

\[
\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_i(\theta) + T \delta_{ii} \delta_{ij} \quad \text{Equation 12}
\]

where \( \delta_{ij} \) is the Kronecker delta. In plane strain, the effect of this \( T \) stress is to shift the opening-mode stress field negatively, thus reducing the local crack opening stress, \( \sigma_{22} \), field as a function of the distance ahead of the crack tip as illustrated in Figure 8; where in the figure the opening stress has been normalised by the yield stress, \( \sigma_y \), and the distance ahead of the crack has been normalised as a function of \( J \) and yield stress. In the figure a modified boundary layer analysis, which ensures small scale yielding, has been compared to the HRR stress field predictions. The change in \( T \) shown has been achieved by modifying the stress level applied parallel to the crack.
As in Figure 8 it is sometimes more appropriate to define the $T$ stress normalised by the yield stress, which, as the $T$ stress can only be applied under elastic conditions,

$$
\frac{T}{\sigma_y} = \beta_T \frac{L_r}{\sigma_y} \rightarrow \frac{T}{L_r \sigma_y} = \beta_T
$$

Equation 13

where $\beta_T$ is the normalised $T$ stress. The value of $\beta_T$ should remain approximately constant for a given geometry and crack size as the any increase in $T$ is balanced by an increase in $L_r$ under elastic conditions. An illustration of how the material fracture toughness, expressed as $K_{mat}$, changes with $\beta_T$ can be seen in Figure 9 for a three-point bend specimen (3PB), and a centre cracked plate in tension (CCT) with two normalised crack depths under different loading conditions. Also shown in Figure 9 is a lower bound curve to the data demonstrating how the apparent fracture toughness is seen to increase as the $T$ stress decreases. The effect seen under changing values of $T$ stress has been termed constraint.
Figure 9 – Change in fracture toughness for a BS4360 43A mild steel plate at -50 °C with different geometries and specimen conditions shown in [33]

Where the $T$ stress is zero or positive there is little effect on the crack-tip stress field, Figure 8, and hence on the fracture toughness measured under normal testing conditions, i.e. from deeply cracked bend or compact tension, CT, specimens, which are high constraint. However, when the $T$ stress is negative such that the stress field ahead of the crack is lower than the HRR stress field, which will then show an increase in the apparent fracture toughness, this condition is referred to as low constraint. Typically, deeply cracked specimens that develop a bending stress under loading generate high constraint as the stress field is close to (or above) that for the HRR stress field close to the crack tip whereas specimens that are under tensile loading generate low constraint as the stress field is below the HRR stress field near to the crack tip. This can also be considered in how far the plastic zone extends for the specimen in question as the HRR stress field is only for small scale yielding (i.e. the plastic zone is fully contained in the material): in a bending stress field the plastic zone is constrained to a smaller area at the crack tip whereas a tensile stress field allows the plastic zone to develop across the specimen moving away from small scale yielding conditions. An illustration of constraint levels in normal laboratory specimens, including single edge crack in tension, SECT, double edge crack in tension, DECT, centre crack in tension, CCT, as well as SECB and CT specimens, was provided in Ainsworth [34] and is repeated in Figure 10 with cartoons of the specimens.
Figure 10 – Change in normalised $T$ stress against normalised crack depth shown for a range of specimen geometries [34]

Under elastic plastic conditions the $T$ stress is no longer applicable. To assess constraint in these conditions the first term in the stress field expansion is controlled by $J$, as described in the HRR stress field, and all remaining terms in the elastic-plastic stress field expansion can be defined by $Q$, as introduced by O’Dowd and Shih [35], [36]. The $Q$ term provides a measure of the difference between the actual stress field, commonly determined from finite element analyses, and the HRR stress field. As such a definition for $Q$ was provided as:
\[ Q = \frac{\sigma_{ij} - (\sigma_{ij})_{HRR}}{\sigma_y} \]  \hspace{1cm} \text{Equation 14}

It is normal to consider the definition of \( Q \) at a normalised distance, \( \bar{r} = r\sigma_y/J \), between 1 and 5 from the crack tip, which is the micro-structurally significant range over which the stresses and strains control fracture [28]. However, it is common practice to use a distance \( \bar{r} = 2 \).

This more detailed estimate of \( Q \) which allows for full elastic-plastic material properties can also be normalised to be nominally load independent by:

\[ \beta_Q = \frac{Q}{L_r} \]  \hspace{1cm} \text{Equation 15}

Constraint can also be linked to ratio of the hydrostatic stress, which is the first invariant of the stress tensor and does not cause plastic flow, and the von Mises stress, as considered by Thaulow [37]. Under elastic-plastic conditions the plastic zone at a crack tip will therefore develop differently with different levels of constraint; as noted by changing from plane stress to plane strain conditions.

Ainsworth [34] showed how the lower bound curve to the constraint modified fracture toughness, i.e. to similar data sets as in Figure 9, can be described by the following equation.

\[ K_{mat}^c = K_{mat}(1 + \alpha(-\beta L_r)^k), \beta \leq 0 \]  \hspace{1cm} \text{Equation 16}

where \( \beta \) can be defined in terms of either \( T \) or \( Q \) and it is important to differentiate between \( K_{mat}^c \), which is a constraint corrected fracture toughness, and \( K_{mat} \) which is the fracture toughness under high constraint conditions. The \( \alpha \) and \( k \) terms are material constants. Figure 9 includes this estimate of \( K_{mat}^c \), as the lower bound fit to the experimental data where \( \alpha = 2.15 \) and \( k = 2 \) are used to define the curve. When loading
a cracked specimen failure can be predicted at the point where the loading line crosses the failure line (or failure locus) defined. This can be seen in Figure 11.

![Figure 11 – Figure showing failure locus of a specimen under loading][38]

2.7. Reference Stress and Fracture

The reference stress is a very powerful concept that defines a characteristic parameter that can help to quantify the behaviour of a stressed body. The concept allows a range of factors (including applied stresses magnitude and type, geometry and material properties) to be captured within a single parameter that can be simply extended to allow for fracture. This section provides a review of the origins of a reference stress before progressing to how this has been developed for use in fracture assessments.

2.7.1. Origins of the Reference Stress Concept for Creep

The concept of a reference stress was first introduced in the early 1960s. Anderson et al [39] showed that the use of uniquely defined material properties is not required for the assessment of beams undergoing bending. This work showed the independence of the deformation on the creep material properties through the consideration of a simple creep law applied to a range of simply supported beams with uniformly applied loads. It was noted that the required material property is a function of the “characteristic stress” which is only dependent on geometrical values and the applied load.
Anderson et al stated that “at a representative stress which can be calculated for many simple systems it is possible to make good estimations of creep deformation which do not require a detailed knowledge of the creep behaviour for the whole stress range”. They further stated that “the fact that one particular stress can be found to play such a useful part in determining bounds for the displacement may have general implications which could lead to simplifications in the creep analysis of more complex structures and creep laws”.

Marriott and Leckie [40] showed how observations of the stress field within structures undergoing creep can be used to simplify calculations. Creep rate and deformation calculations were applied to three differing geometries; a rectangular beam section under a constant moment, a thick cylinder subject to a constant internal pressure and a uniform disc spinning at a constant speed. The rectangular beam calculations demonstrated a unique position through the structure at which the stress was observed to remain constant at all times: this was termed the “skeletal point”.

Similar trends were observed in calculations relating to the thick cylinder, where the hoop, radial and axial stresses were found to remain almost constant at the skeletal point. This therefore demonstrated that the stress at the skeletal point was constant for a range of geometries. The spinning disc calculations showed similar results.

Some consideration was also given to the behaviour of the skeletal point under variable loading. As this stress is constant over time, the stress at the skeletal point was considered to be directly and instantaneously proportional to the load applied. If the stress at the skeletal point follows the loading history, then the stress distributions and creep behaviour must do the same, opening the way for ‘simplified’ methods to be developed to assess the stress and strain conditions under both static and variable loads. The characteristic stress defined by Anderson et al [39] and the stress at the skeletal point were found to correspond and have collectively been termed as the reference stress.

Goodall [41] indicated that “reference stresses could be defined for structures subjected to fluctuating load, as well as those under constant load”. Therefore, the essence of a

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3 Here Anderson is referring to the “characteristic stress”
reference stress is to select a reference stress that “will minimise the error for two values of stress index, $n$, either side of the expected real value of $n$”. This can be extended to extreme values of $n$ corresponding to unity and infinity. The reference stress can then be expressed as:

$$\sigma_{ref} = \frac{P}{P_L} \sigma_y = L_r \sigma_y$$  \hspace{1cm}  \text{Equation 17}$$

A strong analogy between plasticity and creep strains exists so that results under creep deformation such as the concept of a skeletal point have also been observed in elastic-plastic materials. Recently Wang [42] has considered the numerical estimation of a reference stress for a range of cracked bodies both by obtaining the skeletal point and via Equation 17, which used both upper and lower bound estimates of $P_L$. It was found that the difference between the two methods was just 5% in the extreme case but in most cases was less than 3%.

### 2.7.2. Development of a Reference Stress for Fracture

This section describes how the reference stress concept has been used to provide crack driving force estimates. This is then progressed to show how this can be used to define a failure assessment curve, or failure assessment diagram (FAD), which can be used to convert between elastic and elastic-plastic estimates of crack driving force.

### 2.7.3. J Estimation

In 1984 Ainsworth [43] showed how an estimation of $J$ can be made for a crack-like defect in a material without any prior knowledge of the material strain hardening behaviour. This was achieved by use of the reference stress as defined by Equation 17, itself dependent on the limit load of the structure. Ainsworth started by assuming a strain hardening material that has a Ramberg Osgood relation between the stress, $\sigma$, and strain, $\varepsilon$, such as [44]:

$$\sigma = \sigma_y + \left( \frac{\sigma}{\epsilon} \right)^n$$
\[ \varepsilon = \frac{\sigma}{E} + \alpha \varepsilon_0 (\sigma/\sigma_0)^n \]  \hspace{1cm} \text{Equation 18}

where \( \alpha, \sigma_0, \varepsilon_0 \) and \( n \) are material constants that define a fit to the level of plastic strain, normalising stress, normalising strain and the coefficient of strain hardening (note that commonly \( \sigma_0 \) and \( \varepsilon_0 \) are defined at yield and \( \alpha \) set to unity). From the GE-EPRI \( J \) estimation scheme [45], based on elastic-plastic finite element analyses, it had been suggested that \( J \) can be estimated for a material obeying this stress-strain relation by:

\[ J = \frac{K_i^2}{E} + \alpha \sigma_0 \varepsilon_0 c h_1 (P/P_0)^{n+1} \]  \hspace{1cm} \text{Equation 19}

where \( h_1 \) is a non-dimensional function for different geometries and loading, \( P_0 \) is a nominal normalising load and \( c \) is a characteristic length (e.g. un-cracked ligament). By adopting an arbitrary reference stress for \( P/P_0 \) as shown in Equation 17, \( J \) becomes:

\[ J = \frac{K_i^2 (\alpha_c)}{E} + c h_1 \sigma_{ref} (\varepsilon_{ref} - \sigma_{ref} \varepsilon_0/\sigma_0) \]  \hspace{1cm} \text{Equation 20}

where \( \varepsilon_{ref} \) is the reference strain, defined from the reference stress and the materials stress-strain curve. Equation 19 can be applied for any definition of \( P_0 \) but modifying \( P_0 \) also modifies \( h_1 \) and it was shown that:

\[ \frac{h_1}{P_0^{n+1}} = \text{constant} \]  \hspace{1cm} \text{Equation 21}

The value of \( h_1 \) for a range of the strain hardening component was shown (see Figure 12) for a number of different normalising loads \( P_0 \) when assessing a compact tension specimen in plane strain conditions. Also shown in the figure are the limit load solutions by Kumar and Shih and by Ewing (see [43]). It was found that under certain conditions \( h_1 \) could be independent of \( n \). The normalising load required to create \( h_1 \) independent of \( n \) was within 5% of the limit load of the cracked geometry, provided by Ewing so that it can be assumed
that \( P_0 = P_L \). The use of the limit load as the normalising parameter also meant that the \( J \) estimation can easily be represented in terms of the reference stress.

![Figure 12 – Change in \( h_1 \) for different normalising loads, \( P_0 \) [43]](image)

The use of the reference stress estimation of \( J \) was shown to be within 5% of the values predicted from finite element analyses, for almost all structural geometries\(^6\) considered. By using the definition of the reference stress above, it is shown in [43] how the estimation of \( J \) therefore becomes:

\[
J = \sigma_{\text{ref}} \varepsilon_{\text{ref}} \left( \frac{K}{\sigma_{\text{ref}}} \right)^2 \left( \frac{E}{E'} \right)
\]

*Equation 22*

Validation of the reference stress method was considered by Miller [46] who compared the reference stress method to the GE-EPRI methodology [45]. The GE-EPRI report based the estimation of \( J \) upon the summation of the elastic, small-scale yielding and plastic response as in Equation 19. However, the report provided a range of finite element results for the variables in Equation 19 to solve for \( J \). Miller suggests that this meant that the cumulative error was estimated to be bounded within 15% of the true solution [46], whereas the error associated within a reference stress approximation was of the order of 5% from the true

---

\(^6\) These geometries included CT, Centre Cracked Tension (CCT), Double Edge Notch Tension (DENT) and single edge notch BEND (BEND) under plane stress or strain conditions with a crack depth of \( a/w \) of 0.5 in most cases.
solution. It was however noted that non-conservative $J$ estimates using the reference stress method could be obtained. These were mainly confined to shallow cracks in a tensile stress field in a material with a high strain hardening.

The accuracy of the reference stress methodology was also estimated in a validation paper associated with R6 [47] where both experimental and finite element estimates of $J$ for common specimen types were compared. Milne [47] concluded that the reference stress method is accurate and that any potential non-conservatisms would be avoided by conservative estimates of the input variables used in the calculations such as material properties or fracture toughness.

More recently (between 2001 and 2004) Kim [48–51] reported the results from a range of finite element analyses for a range of cracked cylinders. Comparisons to the R6 reference stress methodology and an alternate definition of the reference stress were made which showed that both methods provided conservative results. The alternate estimate of the reference stress was defined through a modification to the limit load solution used. The improved reference stress method defined by Kim could therefore be considered an improved limit load solution and the increase in accuracy only reflects the requirement of accurate limit load estimates for estimating $J$.

### 2.7.4. Failure Assessment Diagrams

This section outlines how the reference stress concept can be used to define a failure assessment diagram (FAD), specifically the Option 2 FAD, before considering the Option 1 and 3 FADs. The Option 2 FAD allows an estimate of $J$ or $K_f$ that accounts for a materials hardening properties, as defined from the stress-strain curve, and has some consideration of the geometry in the definition of $L_r$.

The concept of a FAD is very important for simplified assessments of fracture, such as those presented in R6, as it provides a relationship between $K_f$ and $K_f$ as a function of $L_r$. The FAD is shown as $f(L_r)$ in Equation 23. This therefore provides an assessment of $K_f$ based on the LEFM parameter $K_f$, and an estimate of the levels of plasticity in the material defined through $f(L_r)$.
\[ K_j = K_i / f(L_r) \]  \hspace{1cm} \text{Equation 23}

There are a number of options for defining the FAD but, perhaps, the most important is the Option 2 FAD. This can be defined by combining Equation 1, Equation 22 and Equation 23 whilst recognising that \( G = J_e \) so that:

\[ f_2(L_r) = \left( \frac{\varepsilon_{ref}E}{\sigma_{ref}} \right)^{\frac{-1}{2}} \]  \hspace{1cm} \text{Equation 24}

It was noted by Ainsworth [43] that this form of failure assessment curve is appropriate for elastic behaviour and gross yielding but not under small-scale yielding. Therefore a small-scale yielding correction was introduced as:

\[ f_2(L_r) = \left( \frac{\varepsilon_{ref}E}{\sigma_{ref}} + \frac{0.5L_r^2}{1 + L_r^2} \right)^{\frac{-1}{2}} \]  \hspace{1cm} \text{Equation 25}

\[ f_2(L_r) = 0 \quad \text{if} \quad L_r > L_r^{max} \]

where \( L_r^{max} \) is a limiting value of \( L_r \) to conservatively predict plastic collapse.

This was further modified as outlined in [47] to allow the denominator of the second term to reflect the first term as shown in Equation 26. This allowed the failure assessment curve to be derived directly from the material stress-strain curve. Whilst the derived failure assessment curve is thus material specific, it is not specific to a particular geometry. Examples of Option 2 FADs can be seen in Figure 13.

\[ f_2(L_r) = \left( \frac{\varepsilon_{ref}E}{\sigma_{ref}} + \frac{0.5L_r^2}{\varepsilon_{ref}E/\sigma_{ref}} \right)^{\frac{-1}{2}} \]  \hspace{1cm} \text{Equation 26}

\[ f_2(L_r) = 0 \quad \text{if} \quad L_r > L_r^{max} \]
Figure 13 – Option 2 FAD for (1) EN40B, (2) A533B, (3) C-Mn Steel, (4) Austenitic Steel, (5) Elastic Perfectly Plastic Material and (6) Option 1 Curve [47]

The Option 2 FAD defined above is simply obtained by the materials stress and strain relation. If this information is not available then a simplified, sometimes conservative approach, termed Option 1 can be used. However, if detailed finite element analyses of the geometry and material conditions of interest can be used, an Option 3 FAD can be defined, from Equation 23, which should be the most accurate method of all three. Therefore, the accuracy of the FADs can be related to increasing rank, as is the complexity of use.

A basic definition of the FAD (Option 1) is provided, in R6 [2] as in Equation 27, which has been obtained by a lower bound curve fit to failure assessment curves derived for a range of steels and geometries [47], [52].

\[
f_1(L_T) = (1 + 0.5L_T^2)^{-\frac{1}{2}}(0.3 + 0.7e^{-0.6L_T^6})
\]

\[
f_1(L_T) = 0 \quad \text{if} \quad L_T > L_T^{\text{max}}
\]

Equation 27

The Option 1 FAD, also shown in Figure 13, is independent of the component material and geometry and is conservative in the majority of cases. It is noted that for high strain hardening materials, the use of an Option 1 curve may provide inadequate predictions, and Option 2 or 3 FADs should be used.

An estimate of the Option 3 FAD can be provided from finite element analyses via Equation 28
The FAD is commonly used as a bounding curve to predict the failure of a component when plotted as $K_i/K_{mat}$ as a function of $L_r$. However, despite the initial concept that the FAD provides a 2-parameter assessment approach for fracture, the FAD approach has become more powerful as a $J$ (or $K_J$) estimation scheme (i.e. R6 Issue 4 [3]) whereby the FAD is used to provide the plasticity correction to an elastic assessment.

It is also possible to adapt the FAD to consider 2-parameter, or $J - Q$, fracture mechanics. As $K_i/K_{mat}$ is used to define the vertical axis of the FAD, constraint can be incorporated to redefine $K_{mat}$ in the FAD so that a constraint corrected FAD results, $f_c$ as:

$$f_c = f(L_r)(1 + \alpha(-\beta L_r)^k)$$

**Equation 29**

where $\alpha$ and $k$ are material constants and $\beta$ defines the level of constraint, as defined for Equation 16. An illustration of $f_c$ for a number of values of $\alpha$ with $k = 2$ can be seen in Figure 14.

![Figure 14 – Copy of figure showing change in constraint corrected FAD for three different values of $\alpha$, illustration taken from [34]](image-url)
This approach of modifying the failure assessment curve was extended by Sherry et al [33] to provide look-up tables of $\alpha$ and $k$, which were obtained from a matrix of finite element analyses that used the Beremin\(^7\) [53] local approach model to predict cleavage fracture probabilities for different material properties. The failure points in Figure 9 were assessed using this prediction of modified fracture toughness and is repeated in Figure 15 which shows that the Beremin approach allows largely conservative predictions for this dataset with a model parameter, $m$, of 10 [33], [38]. Here, $m$ defines the shape parameter within the Weibull statistical model which quantifies the scatter associated with the fracture toughness data.

**Figure 15** – Copy of figure showing Figure 9 reassessed using the constraint corrected fracture toughness, $K^C_{\text{mat}}$, using a constraint corrected failure assessment curve. Illustration taken from [33]

\(^7\) The Beremin Model for cleavage fracture describes the statistical likelihood of fracture, based on the model parameter, $m$, which nominally relates to the size distribution of the initiating brittle carbides, and a critical stress, $\sigma_c$, which need calibrating alongside a reference volume, $V_0$, with fracture data. However, once calibrated it is assumed that the model can be used as long as the material does not change.
2.8. Elastic Follow-Up

To be able to consider fracture under combined stresses it is important to distinguish between primary and secondary stress. One means by which this can be achieved is by considering the effect the applied stresses has on the elastic follow-up of the structure.

There are many factors, such as geometry, physical distance of the stress and stress type that can influence elastic follow-up. For primary stresses the remote stress is not seen to diminish with distance away from the defect and can be well characterised despite plasticity in the defective region. Therefore, a primary stress will always act at the defective region regardless of the location where the load is applied and, additionally, will enhance deformation. Primary stresses therefore are considered to have a large elastic follow-up. However, for stresses localised to a small region, such as weld residual stresses, the action of elastic follow-up is negligible as, if a crack was placed far enough away from the stress, the crack would not experience the stress. Further, this means that such a local stress will maintain equilibrium globally if the stress is removed, allowing for stress relaxation. Therefore, under large levels of plasticity the displacement controlled residual stresses are able to redistribute and relax prior to plastic collapse. Such cases are said to have a low elastic follow-up as the local stress is not enhanced from a remote stress. As the original definition of primary stresses were those that contributed to plastic collapse, and secondary stresses as those which do not, the concept of elastic follow-up can be used as an alternate means to define primary and secondary stresses.

Elastic follow-up can be considered a measure of how the structure responds, elastically, to the changes in stiffness under plastic deformation (or crack growth). There are, however, a number of different interpretations that can be made which are detailed in [54]. It may be considered that elastic follow-up is a measure of the level of load at which the least stressed parts of the structure act, as elastically loaded springs, on the remaining structure. Smith [54] considered the phenomenon of elastic follow-up as resulting from when highly stressed parts exhibit a change in general compliance through any mechanism to alter the stiffness of the component (i.e. creep, plasticity or crack growth). Further to these, R5 considers that elastic follow-up [2] is described as “a consequence of the change in kinematics between fully elastic and inelastic behaviour”.
Historically elastic follow-up was first considered with respect to the relaxation of bolted joints under creep conditions. The investigation focused on the residual stress present in the bolts after being tightened and the rate at which this stress reduced due to creep deformation. The role of elastic follow-up in creep was further explored in references [55–57] and its role in stress classification, although the approach to define elastic follow-up differed between papers. There is, however, only a limited experimental base for comparison and little agreement on the analytical approaches to provide consistent estimates. Available experiments performed at high temperature creep conditions have been considered by Smith [54]. However Smith considered that the available experiments only concern the relaxation of residual stress and the effect of elastic follow-up, whereas none consider the effect of combined loading with applied stresses. This led a number of investigations [54] to consider the evaluation of elastic follow-up for structures containing an initial misfit. These experiments focused on the use of multi-bar structures surrounding a central column of, commonly, a second material. An example of such a structure is shown in Figure 16. The experiments showed that the behaviour of the central bar can be explained by the relative stiffness of the central bar and the surrounding structure.

![Figure 16 – Experimental configuration of multi-bar structures adopted in [54]](image)

Schematically elastic follow-up can be considered by use of elastic-perfectly plastic material (with reference to the multi-bar structure). Such a condition is shown in Figure 17, which is adapted from [54], where a three-bar structure, with the central bar accommodating a mis-fit in length, is loaded so that the structure undergoes yielding. If the elastic stress in the central bar, point A, is converted to the elastic-plastic state the final stress strain position, point B, C or D, is affected by the level of elastic-follow-up in the structure. Under pure load controlled conditions the resultant stress will run parallel to the
strain axis towards point C, and hence, will not meet the yielding line. This therefore means that, under load controlled conditions the material will deform indefinitely. Under pure displacement control conditions no additional strain is required to bring the elastic point at A to the final point D. However, under conditions where a level of elastic follow-up somewhere between these extremes is considered, the final point may be point B. In such cases the surrounding structure further deforms the structure above the displacement controlled case, but not as much as that observed under the load controlled case. The numerical definition of elastic follow-up is related to the slope of the line AB, designated R in the figure below.

Figure 17 – Schematic illustration of elastic follow-up for an elastic perfectly-plastic material demonstrating load control (A to C), displacement controlled (A to D) and moderate elastic follow-up (A to B) [54]

A measure of elastic follow-up, generally termed Z when calculated, is provided by a measure of the slope of R as:

\[ Z = \frac{\varepsilon_f - \varepsilon_e}{\varepsilon_i - \varepsilon_e} \]

Equation 30
where $\varepsilon_f$ is the final strain, $\varepsilon_e$ is the elastic strain at the final stress state and $\varepsilon_i$ is the elastic strain for the load applied. Under such conditions the displacement controlled stresses have an elastic follow-up of unity and load controlled stresses have an infinite elastic follow-up factor. Therefore, it is generally considered [2] that the use of elastic follow-up factors provides a means to distinguish between load and displacement controlled stresses.

In a low temperature fracture assessment, in the presence of a residual stress field and additional primary loads, which are assumed to be purely load controlled, the level of elastic follow-up in the residual stress field can be found by incrementally applying and removing the primary load to further plastically deform the structure. After each additional level of plasticity the reduction of stress and positive increment of strain can be used to estimate the elastic follow-up for the structure [54]. An adapted illustration of this approach for a multi-bar structure, of a steel bar surrounded by aluminium bars, can be seen in Figure 18 with the values of $\varepsilon_f$, $\varepsilon_e$ and $\varepsilon_i$ noted, where the initial and elastic strains are now modified by the materials strain hardening response when compared to the elastic perfectly-plastic material in Figure 17. In Figure 18 the applied secondary stress is shown in red and the effect of the primary load is shown in blue through load-upload cycles. Here it can be seen that the secondary stress is being removed through plastic redistribution but not at a slope indicating the secondary stress is purely displacement controlled.

![Figure 18](image_url)

**Figure 18 – Estimate of elastic follow-up from an elastic-plastic experiment including an initial residual stress field (adapted from [54])**
Therefore, it can be observed that the use of elastic follow-up provides a means to characterise a stress which, although resulting from displacements internal to the material, such as a thermal or a weld residual stress, may share some characteristics with a primary load such that the secondary stress provides additional plastic strain. It is, however, noted that even if a secondary load exhibits a large elastic follow-up it is still considered a secondary load as it still does not contribute to the plastic collapse of a structure. Nonetheless, R6 [3] recommends that any structure that exhibits elastic follow-up the secondary stress should, conservatively, be considered as an additional primary load.

Furthermore, this approach within R6 [3] is very conservative; especially under low or moderate levels of elastic follow-up. It may be considered useful to demonstrate the effective level of elastic follow-up within a structure and be able to use this within an assessment so that an improved understanding of the redistribution of secondary stresses can be obtained. However, currently there is little understanding of how this can be considered in an assessment.

2.9. Estimations of Elastic-Plastic Crack Driving Force for Combined Primary and Secondary Stresses

2.9.1. Introduction

The assessment of defects inevitably needs to consider the combined influence of primary and secondary stresses, when present. Under circumstances of such combined loading the interaction results in levels of plasticity, and so $K_f$ or $J$ which are greater than would be achieved by the simple addition of the stresses. Ainsworth [58] noted that for combined loading the following features are maintained:

- under pure thermal loading, plasticity may cause $J$ to differ from $J_e$;
- thermal loading creates a plastic zone at the crack tip which at low mechanical loads causes $J/J_e$ to rise more rapidly than under primary loads alone, which can be seen in Figure 19;
• at larger mechanical loads the behaviour is dominated by the approach to plastic collapse: the secondary stress reduces due to the removal of the displacement mismatch causing it, and so the primary stress remains dominant, which can also be seen in Figure 19.

This section considers in detail the generalised treatment of combined loading in R6. Some reference is also made to other international procedures. The TAGSI (UK Technical Advisory Group on the Structural Integrity of Nuclear Plant) report [7] also notes that the inclusion of secondary stresses is vital to a valid assessment, but the degree of conservatism inherent to these procedures can be significant. The following sections describe various methods for estimating elastic-plastic crack driving force for combined primary and secondary stresses.

![Figure 19 – Example of the influence of secondary stresses upon J compared to J resulting from mechanical (primary) loading alone [58]](image)

### 2.9.2. International methods of dealing with primary and secondary stress interaction

The majority of international defect assessment procedures use either the methods contained in the R6 defect assessment procedure [3] or the American Society of Mechanical Engineers (ASME) approach [1]. Procedures that use the former include BS7910 and API 579. Those that use the latter include some aspects of RCC-MR and Japanese codes. Table 1 summarises various methods to account for the interaction
between primary and secondary stresses, noting their advantages, limitations and any deviations from R6 or ASME procedures. Where the equations of the R6 type methods are shown in Table 1 the method is discussed further in the section detailed.

The ASME approach contains several appendices that addresses elastic and ductile materials and situations of more severe consequence [7]. Appendix H in ASME follows a similar approach to R6. Appendix C allows for defects in both austenitic and ferritic piping. In this appendix, a process is followed to select the failure mode. If failure is by plastic collapse, secondary stresses are neglected; if failure is within the elastic regime then secondary elastic SIFs are included. However, if the failure is between these, then an elastic-plastic approach is recommended, but not necessarily including secondary stresses. One feature of the ASME approach which is widely quoted is that Appendix A allows the assessor to ignore secondary stresses if the material is “on the upper shelf” i.e. behaves in a fully ductile manner [7]. The reasoning behind this is that, on the upper shelf, the component is likely to fail under extensive plasticity, hence reducing or even removing the contribution of secondary stress. However there is a condition that states that “residual stresses should be considered”, but it is not made clear how this should be undertaken. It is, however, allowed that an R6 approach be adopted if the interaction of primary and secondary stresses needs to be considered.

One potential area of interest is the API-579 [59] approach which follows the R6 methodology in nearly all aspects except for defining the conditions where the complex approach should be used over the simple upper bound estimate. When defining this limitation the secondary reference stress is directly estimated from a limit load solution. This is different from the R6 method where the magnitude of the secondary reference stress is estimated from the secondary elastic-plastic SIF by $K_i^S/(K_i^P/L_r)$. Finally a further approach is adopted within the French nuclear code RSE-M that fits a number of finite element solutions for specific geometries and loading types [60]. This allows for a very accurate approach but is limited in the number of scenarios within which it can be used. Interestingly, the approach used also allows for the reduction in the elastic plastic secondary SIF, $K_i^S$, compared to the elastic SIF, $K_i^E$, which is not allowed elsewhere without detailed finite element analyses. The approach also includes an interaction term based on multiplying the elastic-plastic SIF as in the R6 $V$ Factor approach, but is not then enhanced by the failure assessment curve.
It is clear that many of these international methods defer to those contained in R6 for more complex elastic-plastic conditions. It is predominantly the background to these R6 methods that are discussed in more detail through the remainder of this Chapter.
### Table 1 – Summary of Methods to Account for Primary and Secondary Stress Interaction in Fracture

<table>
<thead>
<tr>
<th>Method</th>
<th>Equation*</th>
<th>Benefits</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>R6 ρ Factor Simple</td>
<td>$\rho = \rho_i$ $\rho = 4\rho_i(1.05 - L_i)$ $\rho = 0$</td>
<td>• Simple to Use&lt;br&gt;• Conservative ($\rho_i$ is an upper bound profile for $\rho$ described further below)</td>
<td>• Potentially Over Conservative&lt;br&gt;• Limited to small $\sigma^S$ (where $K_i^S / (K_i^P / L_i) &lt; 4$)</td>
</tr>
<tr>
<td>(Section 2.9.3)</td>
<td>$K_r = \frac{K_i^P}{K_{mat}} + \frac{K_j^S}{K_{mat}} + \rho$</td>
<td>• Can be used with all $\sigma^S$&lt;br&gt;• Conservative&lt;br&gt;• Used extensively in Industry</td>
<td>• Can be very conservative&lt;br&gt;• Relies on look-up tables for $\psi$ and $\phi$&lt;br&gt;• Can be complex to use&lt;br&gt;• Based on upper bound deformation theorem</td>
</tr>
<tr>
<td>R6 ρ Factor Complex</td>
<td>$\rho = \psi + \phi \left(1 - \frac{K_i^S}{K_j^S}\right)$</td>
<td>• Similar to R6 V Factor&lt;br&gt;• More accurate&lt;br&gt;• Derived from Time Dependent method (links fracture and creep)</td>
<td>• V factor defined from $\phi$ and $\psi$ so it has the same limitations as R6 Complex $\rho$ Factor method</td>
</tr>
<tr>
<td>(Section 2.9.4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R6 V Factor Complex</td>
<td>$K_r = \frac{K_i^P + VK_j^S}{K_{mat}}$</td>
<td>• Similar to the Simplified Method and R6 V Approach&lt;br&gt;• Allows a new elastic plastic interaction term, $g$, to be defined&lt;br&gt;• Allows primary and secondary reference lengths to be defined independently</td>
<td>• The $g$ term needs to be defined&lt;br&gt;• Limited validation&lt;br&gt;• Not widely known</td>
</tr>
<tr>
<td>(Section 2.9.5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simplified Method</td>
<td>$K_j = \frac{K_i^P}{f(L_i)} + K_j^S$</td>
<td>• Similar to Simplified Method and R6 V Approach&lt;br&gt;• Allows a new elastic plastic interaction term, $g$, to be defined&lt;br&gt;• Allows primary and secondary reference lengths to be defined independently</td>
<td>• The $g$ term needs to be defined&lt;br&gt;• Limited validation&lt;br&gt;• Not widely known</td>
</tr>
<tr>
<td>(Section 2.9.6)</td>
<td></td>
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<tr>
<td>Average a-bar approach</td>
<td>$K_j = \frac{K_i^P}{f(L_i)} + gK_j^S$</td>
<td>• Similar to the Simplified Method and R6 V Approach&lt;br&gt;• Allows a new elastic plastic interaction term, $g$, to be defined&lt;br&gt;• Allows primary and secondary reference lengths to be defined independently</td>
<td>• Might miss cases where $\sigma^S$ is important&lt;br&gt;• Advice is not clear&lt;br&gt;• Safety factors might cause excessive conservatism</td>
</tr>
<tr>
<td>(Section 2.9.7)</td>
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<tr>
<td>ASME</td>
<td>Appendix A – Potentially Ignore $\sigma^S$&lt;br&gt;Appendix C – Determines failure mode before including $\sigma^S$ in brittle (elastic) assessment&lt;br&gt;Appendix H – Follows an R6 style approach</td>
<td>• Can make assessment a lot simpler&lt;br&gt;• Determines if $\sigma^S$ needs to be included</td>
<td>• Might miss cases where $\sigma^S$ is important&lt;br&gt;• Advice is not clear&lt;br&gt;• Safety factors might cause excessive conservatism</td>
</tr>
</tbody>
</table>
| **RSE-M** | $K_J = \frac{K_P}{f(L_c)} + k^* K^*_f$ | • Based upon finite element analyses of cylinders specifically for use in PWR NPP  
• The $k^*$ term can reduce the secondary contribution significantly  
• $J$-estimation scheme based on reference stress concept as in R6 | • Not validated beyond finite element analyses  
• Specific to cylinders with internal cracks  
• Only used for thermal transient secondary stresses |
<table>
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</thead>
<tbody>
<tr>
<td>API 579</td>
<td>Follow similar approach as R6</td>
<td>• Uses direct estimate of $\sigma^S_{ref}$</td>
<td>Difference not validated [7]</td>
</tr>
<tr>
<td>BS7910</td>
<td>Follow similar approach as R6 (even if some of the terminology is altered) with some historical differences</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>RCC-M</td>
<td>Follow similar approach as ASME but allows for improvements in SIF estimation (i.e. Irwin correction) [7]</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Japanese</td>
<td>Follow similar approach as ASME</td>
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</tr>
</tbody>
</table>

* note that all terms are defined in the main text
2.9.3.  Simplified $\rho$ factor

Ainsworth [58] originally outlined the generalised procedure, applicable to any component for which a value of $K$ can be defined. This procedure aimed to treat the total combined stress in the same manner as that used for primary stress acting alone. This was defined through using a reference stress, $\sigma_{ref}$, which provides the same total crack driving force, when used as a primary stress, as that from the combined primary and secondary stresses.

Ainsworth considered that it is generally necessary to include thermal and residual stresses in an analysis and that, elastically, this is through the summation of the separate stress intensity factors. However, once plastic yielding occurs, the reference stress concept can be used to combine the primary and secondary contributions. When detailing the approach to the reference stress estimate of $J$ or $K_j$ within [58], Ainsworth starts with an estimate of $J$ under primary loads as:

$$ J = \left[ \frac{\sigma_{ref}^p}{f(\sigma_{ref}/\sigma_y)} \right]^2 \frac{\pi \bar{a}}{E} $$

Equation 31

where $\sigma_{ref}^p$ is the primary reference stress and $\bar{a}$ is a reference length, or reference crack length, term defined by the stress field and crack size. Under the assumption that the combined reference stress, $\sigma_{ref}$, is treated the same as the primary reference stress;

$$ J = \left[ \frac{\sigma_{ref}}{f(\sigma_{ref}/\sigma_y)} \right]^2 \frac{\pi \bar{a}}{E} $$

Equation 32

By making the further assumption that, for secondary stresses, $J \approx J_e$, and by taking Equation 32 to the limit where the primary stress contribution is negligible, the secondary reference stress was defined as in Equation 33.
To obtain the reference stress, an estimate of the equivalent mechanical load is obtained from an interpretation of a deformation-bounding theorem. Figure 20 illustrates how the deformation of a plate under large primary tensile and secondary bending stresses, where the primary stress is given in blue and the secondary stress is provided in red (where the arrows suggest the magnitude of the stress and the dashed lines the stresses deformation if considered as a primary stress), is compared to the overall deformation from the combined stresses, shown by the surround. The left figure shows that a relatively large secondary stress provides only a small deformation (shown by the strain resulting from the secondary bending stress) to the structure. This shows that the combined effect of the primary and secondary stress cannot simply be summated. Under the same primary load and an additional primary load, \( R \), instead of the secondary stress, is shown in the right hand figure to provide the same overall deformation. The figure therefore shows how a smaller primary bending stress can be applied to the structure to provide the same overall deformation as the higher secondary stress, i.e. \( \sigma_b^S \neq \sigma_b^R \), which is a result of the redistribution of secondary stresses on interaction with primary stresses.

\[
\frac{\sigma_{ref}^S}{f(\sigma_{ref}^S / \sigma_y)} = \frac{K_i^S}{\sqrt{\pi a}}
\]

Equation 33

Figure 20 – Example of deformation bounding theorem as used by Ainsworth where a secondary bending displacement, \( \sigma_b^S \), on the left is represented by a lower primary stress, \( \sigma_b^R \), as applied on the right.
Ainsworth shows that the mechanical load that is equivalent to the secondary stress, \( R \), can be used to provide an estimate of the reference stress from:

\[
\varepsilon(\sigma_{\text{ref}}) = \varepsilon(\sigma_{\text{ref}}^p + \sigma_{\text{ref}}^R) + \left[ \sigma_{\text{ref}}^S + \sigma_{\text{ref}}^R \right] \frac{\varepsilon(\sigma_{\text{ref}}^S) - \varepsilon(\sigma_{\text{ref}}^R)}{2\sigma_{\text{ref}}^R} \tag{34}
\]

where \( \sigma_{\text{ref}}^R = R \sigma_{\text{ref}}^P / P \).

Ainsworth [58] noted that the value of \( R \) is best chosen to minimise the combined reference stress, to make \( \sigma_{\text{ref}}^R = 0 \) when no secondary stress is present, to make \( \sigma_{\text{ref}} = \sigma_{\text{ref}}^S \) when no primary load is present and to make \( \sigma_{\text{ref}} = \sigma_{\text{ref}}^S + \sigma_{\text{ref}}^P \) under elastic conditions. For a general elastic-plastic stress-strain curve, the reference strain for combined loading is generally greater than the sum of the individual primary and secondary induced strains (\( \varepsilon_{\text{ref}} > \varepsilon_{\text{ref}}^P + \varepsilon_{\text{ref}}^S \)). This means that the combined \( J \) cannot be estimated by summing the primary and secondary contributions.

Ainsworth subsequently used an Option 1 FAD for \( f(L_r) \) to define a combined reference stress over a range of primary and secondary reference stresses. He then used a matrix of \( \sigma_{\text{ref}}^P \) and \( \sigma_{\text{ref}}^S \) to gain \( \sigma_{\text{ref}} \) from which an estimate of \( J \) can be obtained. This method is interpreted as a correction to the elastic-plastic crack driving force, \( K_f \), relative to that for mechanical loads alone, by means of a parameter \( \psi \).
The derived relationship of the reference stress, $\sigma_{ref}$, to the primary reference stress, $\sigma_{ref}^p$, can be seen in Figure 21. The primary reference stress can be found from the limit load and the secondary stress through using the linear elastic SIF, which can be calculated from the equation below (note that this equation is Equation 33 with $J \approx J_e$ so that $f(\sigma_{ref}^s/\sigma_y) \approx 1$):

$$K_i^S = \sigma_{ref}^S \sqrt{\pi \bar{a}}$$

Equation 35

where $\bar{a}$ is defined in such a way that it relates the reference stress and SIF solutions. For a given geometry the reference length $\bar{a}$ is defined to be the same for both the primary and secondary stresses. This means that $\bar{a}$ under secondary stresses can be found from:

$$\sqrt{\pi \bar{a}} = \frac{K_i^P}{\sigma_{ref}^P}$$

Equation 36

Incorporating the correction factor $\psi$ into the definition of $K_f$ results in the following;
\begin{equation}
K_j = \frac{K_i^P + K_i^S}{f(L_r) - \psi}
\end{equation}

Equation 37

The \( \psi \) function is defined as in Equation 38 below and illustrated as a function of \( L_r \) in Figure 22.

\begin{equation}
\psi = f\left(\frac{\sigma_{ref}}{\sigma_y}\right) - \frac{\sigma_{ref}}{f\left(\frac{\sigma_{ref}}{\sigma_y}\right)}\left[\sigma_{ref}^p + \frac{\sigma_{ref}^s}{f\left(\frac{\sigma_{ref}}{\sigma_y}\right)}\right]
\end{equation}

Equation 38

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{plot.png}
\caption{Original plot of \( \psi \) within the Ainsworth paper [58]}
\end{figure}

where \( \sigma_{ref} \) can be estimated from Figure 21. The dashed line, representing the upper bound to all curves in Figure 22, can be described as in Equation 39 below [58].

\begin{align*}
\psi &= \psi_1 & L_r \leq 0.8 \\
\psi &= 4\psi_1 (1.05 - L_r) & 0.8 < L_r < 1.05 \\
\psi &= 0 & 1.05 \leq L_r
\end{align*}

Equation 39
Note that the conservative expression shown in Equation 39 can still be found in R6 – Section II.6.3 [3] as the R6 Simplified (ρ factor⁸) approach and only for $K_i^S/(K_i^p/L_r) \leq 4$. The factor $\psi_1$ provides an estimate of the magnitude of the peak of $\psi$ as shown in Figure 22 for a given magnitude of secondary stress.

Within R6, $\psi_1$ is either found graphically, or from a curve fit to graphs of $\psi_1$ as a function of $L_r$. A graph of $\psi_1$ plotted as a function of $K_i^S/K_i^pL_r$ within [58] can be found in Figure 23.

![Figure 23 – Original plot of $\psi_1$ within the Ainsworth paper [58]](image)

### 2.9.4. Complex ρ factor method

Equation 39 was found by Hooton and Budden [61] to be overly conservative for low values of primary load and potentially non conservative for high levels of secondary stress acting alone. A further methodology was therefore developed by Hooton and Budden [61] by replacing $K_i^S$ by the effective stress intensity factor, $K_j^S$, resulting from secondary stress in a plastic material. The complex ρ method in R6 originates by defining $K_j^S$ by Equation

⁸ Note that $\psi$ has changed notation to $\rho$ when first inserted to R6
33 (as also noted by Hong-Liang [62] two years later) and re-calculating for \( \rho \) as defined in Equation 37.

\[
\frac{\sigma_{\text{ref}}^S}{f\left(\frac{\sigma_{\text{ref}}^S}{\sigma_y}\right)} = \frac{K_j^S}{\sqrt{\pi a}}
\]

Equation 40

The resulting value for the correction parameter, \( \rho \), is shown below,

\[
\rho = \psi + \phi \left(1 - \frac{K_i^S}{K_j^S}\right)
\]

Equation 41

where \( \psi \) is given in Equation 38 and \( \phi \) is defined in Equation 42 below and is also illustrated as a function of \( L_r \) in Figure 24. It is also noted that an error exists in Reference [61] where the first term in Equation 42 is normalised by \( \sigma_y \), not \( \sigma_{\text{ref}} \).

\[
\phi = \frac{f\left(\frac{\sigma_{\text{ref}}}{\sigma_y}\right)}{\sigma_{\text{ref}}} \frac{\sigma_{\text{ref}}^S}{f\left(\frac{\sigma_{\text{ref}}^S}{\sigma_y}\right)}
\]

Equation 42

The influence of \( \phi \) is to allow \( \rho \) to equal \( \psi \) for low values of secondary stress, where the assumption that \( K_i^S \approx K_j^S \) still holds, and to reduce the influence of \( \rho \) for higher values of \( L_r \), thus allowing the secondary stress to redistribute more completely.

![Figure 24 – Variation of \( \phi \) with \( L_r \) for primary reference stress for various values of secondary reference stress [61]](image)

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The current estimation procedure for combined primary and secondary stresses, as appears in Section II.6.4 of R6 [3] Revision 4, is summarised in Equation 43 below.

\[ K_j = \frac{K_j^p + K_j^s}{f(L_r) - \rho} \]  

Equation 43

Look up tables are provided in R6 to determine \( \psi \) and \( \phi \) as a function of \( K_j^s / (K_j^p / L_r) \).

### 2.9.5. V-Factor Method

R6 [3] incorporates an alternate method for dealing with combined primary and secondary loading known as the “V Factor” [63]. The method was first proposed by Smith [64] where a factor \( V \) multiplies the secondary stress intensity factor as shown in Equation 44.

\[ K_j = \frac{K_j^p + VK_j^s}{f(L_r)} \]  

Equation 44

By defining the ratio of the secondary stress intensity factor including plasticity to that for elastic response only provides a value for \( V \) at zero primary loads;

\[ V_0 = \frac{K_j^s}{K_j^s} \]  

Equation 45

By use of Equation 35, Equation 36, Equation 40, Equation 44 and Equation 45 and by taking \( K_{mat} = K_j \) and \( K_r = f(L_r) = f(\sigma_{ref}^p / \sigma_y) \), the definition of \( V \) is:

\[ V = \left( \frac{\sigma_{ref}}{\sigma_{ref}^s} \right) f(\sigma_{ref}^p / \sigma_y) \left( \frac{\sigma_{ref}^p}{\sigma_{ref}^s} \right) - \left( \frac{\sigma_{ref}^p}{\sigma_{ref}^s} \right) \]  

Equation 46

which can be directly related to \( \psi \) and \( \phi \) as:
\[
\frac{V}{V_0} = \xi = 1 + \frac{\psi}{\phi}
\]

*Equation 47*

where \(\xi\) is the look-up term for the \(V\) factor method in R6. As a result, estimates for \(K_f\) when using either Equation 43 or Equation 44 are nominally the same. The advantages that the \(V\) factor approach has over the (original) \(\rho\) factor approach, as outlined in [65], are:

- The factor \(V\) provides a direct multiplication to the secondary stress contribution so that if a value of less that unity is used, stress relaxation is occurring.
- \(V/V_0\) is “relatively insensitive to the magnitude of secondary stresses suggesting that a simplified procedure for estimating \(V\) could be developed” [65].

The second of these points was taken further to provide a bounding equation for \(V/V_0\) under a range of different secondary stresses and \(L_r\). This bounding curve can be found in R6 and is reproduced below.

\[
\begin{align*}
V &= 1 + 0.2L_r + 0.02\frac{K_1^S}{K_1^P/L_r} (1 + 2L_r) \quad L_r < L_r^* \\
V &= 3.1 - 2L_r \quad L_r^* < L_r < 1.05 \\
V &= 1 \quad L_r > 1.05
\end{align*}
\]

*Equation 48*

where \(L_r^*\) is the point which intercepts the first two curves defined above.

A range of finite element based validation reports that detail the suitability of the current R6 methods (both the \(\rho\) and \(V\) factor) were performed by Goldthorpe [17], [66], [67]. The work was based upon the finite element modelling of a fully external circumferentially cracked cylinder and, in [67], also includes analyses of an edge cracked plate. The work considered the interaction of primary and secondary stresses of different magnitudes and the additional effect of residual stress on the accuracy of the methods contained in R6.
In this work, inferred values of $K_f$ calculated using the recommendation of R6 were compared with values derived from the finite element analyses. Of the conclusions derived within [17], those which apply to the interaction of primary and secondary stresses are interpreted below:

- The maximum level of conservatism of the hand calculations compared to the finite element results were 30%, at $L_r = 0.8$ when residual stresses are not included, and approximately 36% when residual stresses are included.
- Estimates of $K_f$ should only include weld residual stresses when the crack forms after the welding process.
- A strong dependence on the value of $K_f^S$ is found.

The work reported in [17] nominally represents a weld residual stress, but, by the author’s admission, is through a thermal field and is not “necessarily representative of the plastic strain histories adjacent to real welds”. This would therefore not reproduce the same complex strain field observed for weld residual stresses.

Goldthorpe [67] also examined the simplified form of the $V$ factor approach. The report details the investigation of multiple thermal and residual stresses for both an external circumferentially cracked cylinder and a centre cracked plate. The relevant conclusions from the report drawn were:

- The ratio $V/V_0$ peaks between $L_r$ values of 0.8 to 1.2.
- Provided the estimate of $V_0$ is accurate, the results are generally conservative.
- Where severe elastic follow-up occurs, some non-conservatism occurs.

It is noted that the majority of the finite element validation exercises performed are based upon the use of a thermally induced residual stress field. This might be of some concern when considering their applicability to a weld residual stress field as the thermally induces stress field does not contain the complex strain history resulting from the welding process.
2.9.6. Simplified Method

More recently a further method for the interaction of primary and secondary stresses has been obtained [68] from the Time Dependent FAD (TDFAD) [69] approach of R5 [2]. This method was obtained by first considering the TDFAD [69]:

\[
K_r = \sqrt{\frac{K_i^p \left( \frac{E \varepsilon_{ref}^p}{\sigma_{ref}^p} \right) + K_i^s + 2K_i^p K_i^s}{\left( \frac{E \varepsilon_{ref}^p}{\sigma_{ref}^p} \right) K_{mat}^C}} 
\]

Equation 49

where \( \sigma_{ref}^p \) is the primary reference stress defined from the isochronous stress strain curve yield stress, \( \varepsilon_{ref} \) is the total strain for \( \sigma_{ref}^p \) accumulated over creep time (not necessarily at the current stress and yield), and \( K_{mat}^C \) is a time dependent material toughness. In the absence of creep effects \( K_{mat}^C = K_{mat} \). Also, as the R6 failure assessment diagram, apart from the minor correction for small scale yielding, can be expressed as

\[
f(L_r) = \left( \frac{E \varepsilon_{ref}^p}{\sigma_{ref}^p} \right)^{-\frac{1}{2}} 
\]

Equation 50

Hence, at failure, when \( K_{mat} = K_f \) and \( K_r = f(L_r) \), the crack driving force is described by Equation 51:

\[
K_f = \sqrt{\left( \frac{K_i^p}{f(L_r)} \right)^2 + K_i^s + 2K_i^p K_i^s} 
\]

Equation 51

Under the assumption that \( K_f^s \approx K_i^s \)
\[ K_j = \sqrt{\left(\frac{K_i^p}{f(L_r)}\right)^2 + K_j^S + 2K_i^p K_j^S} \quad \text{Equation 52} \]

Hence, \( K_j \) can be derived directly without requiring \( \rho \). It is also seen that \( K_j = K_j^S \) in the absence of primary stress and in the absence of secondary stress \( K_j = K_i^p / f(L_r) \). It has been noted [70] (with the same information included in [71]) that this equation can be simplified by taking \( f(L_r) \) into the square root and simplifying so that:

\[ K_j = \frac{K_i^p}{f(L_r)} + K_j^S \quad \text{Equation 53} \]

It is noted that the Simplified Method is very similar to the \( V \) Factor Method in R6 with \( K_j^S = K_i^S V / f(L_r) \).

### 2.9.7. Reference Length

When using the reference stress approach, it is important to consider the means of estimating the reference length. The form of \( \bar{a} \) has been discussed by Piques \textit{et al} [72], Molinié \textit{et al} [73] and Webster and Ainsworth [19], although the nomenclature changes so that \( \pi \bar{a} = R' = L_{ref} \); where \( R' \) is used by Webster and Ainsworth (as in R5 [2]) and \( L_{ref} \) has been adopted by Piques and Molinié. Piques \textit{et al} [72] provided a range of different means to estimate \( \bar{a} \), outlined below in Equation 54, Equation 55 and Equation 56.

\[ \pi \bar{a} = \left(\frac{K}{\sigma_{ref}}\right)^2 \quad \text{Equation 54} \]

\[ \pi \bar{a} = 0.75 \left(\frac{K}{\sigma_{ref}}\right)^2 \quad \text{Equation 55} \]
\[ \pi \bar{a} = \eta(W - a) \quad \text{Equation 56} \]

An additional means to estimate the reference length is possible through use of the GE-EPRI [45] \(J\) estimation scheme where \(\pi \bar{a} = a h_1\) and tabulated values of \(h_1\) exist, that depend on \(n\). All these estimates of reference length refer to primary load only.

Reference [72] notes that the reference length for a deep external crack in a cylinder can be taken as being “equal to the radius of the uncracked ligament” for deep external cracks in a cylinder. It was therefore suggested that the reference length for other geometries could be related to the size of the crack or the uncracked ligament, or a proportion \(\eta\) of the latter, as indicated in Equation 56. An example of this is shown in Figure 25 whereby the resultant value of the reference length (under primary loads alone) is calculated to relate to the crack length at small crack depths and to the length of the uncracked ligament at larger crack depths.

![Figure 25 – Schematic illustration of how \(\bar{a}\) changes with crack depth, \(a\)](image)

A further modification to the simplified approach, as detailed above, has been provided by James et al [71] by considering the reference length term under both primary and secondary stresses. In this approach reference lengths under primary and secondary stresses are considered independently and added together as a weighted average for use in determining the combined elastic plastic SIF. This approach has the useful benefit of not
assuming the reference length is always provided by the primary stress field. This means that for cases where the primary stress is tensile and the secondary stress bending, or the reverse case, the secondary contribution is not over or underestimated.

The approach defined by James et al [71] was achieved by calculating an interpolated average value for $\bar{a}$ from primary stress, $\bar{a}^p$ (as defined in Equation 57), and from secondary stress, $\bar{a}^s$ (as defined within Equation 58). This weighted average term is called the average reference length, $\bar{a}^{ave}$.

\[
K_i^p = \sigma_{ref}^p \sqrt{\pi \bar{a}^p} \tag{Equation 57}
\]

\[
K_i^s = \sigma_{ref}^s \sqrt{\pi \bar{a}^s} \tag{Equation 58}
\]

$K_f$ can therefore be provided by Equation 59.

\[
K_f = \frac{\sigma_{ref} \sqrt{\pi \bar{a}^{ave}}}{f(\sigma_{ref}/\sigma_y)} \tag{Equation 59}
\]

By taking an elastic summation and correcting each component reference length for plasticity, when normalised to the total reference stress, James et al showed that the $\bar{a}^{ave}$ could be described by:

\[
\bar{a}^{ave} = \bar{a}^p \left( \frac{\sigma_{ref}^p f(\sigma_{ref}/\sigma_y)}{\sigma_{ref} f(\sigma_{ref}/\sigma_y)} \right)^2 \\
+ \bar{a}^s \left( \frac{\sigma_{ref}^s f(\sigma_{ref}/\sigma_y) g(\sigma_{ref}^p, \sigma_{ref}^s)}{\sigma_{ref} f(\sigma_{ref}/\sigma_y)} \right)^2 \\
+ \frac{\sigma_{ref}^p \sigma_{ref}^s}{\sigma_{ref}^2} \frac{f^2(\sigma_{ref}/\sigma_y) g(\sigma_{ref}^p, \sigma_{ref}^s)}{f(\sigma_{ref}/\sigma_y)} \sqrt{\bar{a}^p \bar{a}^s} \tag{Equation 60}
\]
Equation 60 provides an $\tilde{a}^{ave}$ that becomes more aligned to $\tilde{a}^S$ when the reference stress approaches the secondary reference stress. At larger values of $\sigma_{ref}^p$, the $\tilde{a}^S$ term is removed through the $g\left(\sigma_{ref}^p, \sigma_{ref}^S\right)$ term. $K_j$ can be defined, by means of Equation 60 and Equation 59, as in Equation 61.

$$K_j = \frac{K_i^p}{f\left(\sigma_{ref}^p/\sigma_y\right)} + K_i^S g\left(\sigma_{ref}^p, \sigma_{ref}^S\right) \quad \text{Equation 61}$$

This result is similar to the form of the “Simplified Method” recently proposed by Hooton [61], but with a term added to account for the effects of plasticity. It is noted that this is very similar in form to the $V$ factor method of the R6 procedure but with $g\left(\sigma_{ref}^p, \sigma_{ref}^S\right)$ replacing $V/f(L_r)$. This approach does allow a means to further consider the plasticity interaction term, $g\left(\sigma_{ref}^p, \sigma_{ref}^S\right)$, so that the potentially excessive conservatisms of R6 can be examined and potentially reduced.

### 2.9.8. Recent Finite Element Analyses Observations

Recently Song [74], [75] has performed a range of finite element analyses with respect to thermally stressed pipes containing circumferential cracks under combined loading. This section reviews this work.

A large range of finite element analyses have been performed which reference:

- Ramberg-Osgood material with three strain hardening coefficients ($n = 5, 10, 20$),
- Two crack lengths ($\theta/\pi = 0.125$ and $1$),
- Two crack depths ($a/t = 0.3$ and $0.5$),
- Three thermal gradients (axial temperature gradient, radial temperature gradient and a sectional temperature gradient). Figures outlining these different temperature gradients can be seen in Figure 26.
Figure 26 – Thermal gradient cases considered in [74], [75] showing (left) Radial Temperature Gradient, (middle) Axial Temperature Gradient and (right) Sectional Temperature Gradient.

The primary loads were applied as membrane tension in most cases and the thermal loads applied to give values of $\beta$ as 0.5, 1.0, 2.0 and 5.0, where $\beta$ has been defined as:

$$\beta = \frac{K^S_f}{K^P_f / L_r}$$

so that, when assuming that $\bar{\alpha}$ is defined under primary loading, $\beta$ provides an estimate of the secondary reference stress normalised by the yield stress. The results indicate that even the most accurate R6 methods, when using the complex $\rho$ factor and an Option 3 FAD, are conservative when predicting failure. It was also shown that an Option 2 FAD provides an improved estimate of the stress redistribution resulting from plasticity, presented as an estimate of $V/V_0$, as shown in Figure 27 (where Eq. 20 in the figures is an Option 2 FAD) for the axial temperature gradient cases. The worst agreement between finite element and R6 Option 2 FAD estimates of $V/V_0$ are seen for the sectional temperature gradient; where the results can be seen in Figure 28.

The work detailed in [74], [75] does represent a large range of finite element analysis data which has shown that the estimates of plasticity induced stress redistribution are insensitive “to the crack geometry (length and depth), loading mode (axial tension and bending), the material strain hardening exponent and the thermal loading type”.

Equation 62
would therefore indicate that a reference stress methodology is suitable for estimating stress redistribution. This is because the reference stress would capture all of these variables, apart from strain hardening, allowing the interaction to be defined by reference stress alone. Song [74], [75] notes the advice in R6 that for short or shallow cracks, it may be necessary to treat long-range thermal stresses as primary as they will show a higher degree of elastic follow-up.

Figure 27 – Example results from [74] demonstrating finite element prediction of $V/V_0$ for the axial temperature gradient

note that Eq. (20) shown in the above figures is the Option 2 FAD.
note that Eq. (20) shown in the above figures is the Option 2 FAD.

Figure 28 – Example Results from [74] demonstrating finite element prediction of $V/V_0$ for the sectional temperature gradient
2.10. Experiments Examining the Effects of Combined Loading on Fracture

This section provides an overview of relevant experiments that have been performed to investigate the influence of combined primary and secondary stresses on fracture (see Table 2). Full details regarding each experiment is provided in Appendix 1. This section also describes a number of methods used to quantify secondary stresses, mostly residual in nature, in fracture tests.

It is noted that the testing undertaken in Sweden by Inspectra Technology AB, funded by the Swedish radiation safety authority, Strål-Säkerhets-Myndigheten, was conducted in parallel with the experiments undertaken as part of this research and detailed in Section 5. Collaboration on the experimental design and provision of material has ensured that the results from both programmes have been made available to create a new pool of experimental data considered within this project [76]. This has, however, meant that the experiments performed in Sweden and those presented later within this thesis are somewhat similar in their approach, although the focus of the two experiments, and therefore specimen designs, was slightly different.
Table 2 – Overview of Experiments Considering Combined Primary and Secondary Stresses

<table>
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<tr>
<th>Experiment</th>
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<th>Geometries</th>
<th>Details of Secondary Stress</th>
<th>Primary Load</th>
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</table>
| Aluminium Plate Tests             | Ainsworth et al [77]             | Centre cracked plate. Two sizes. | (1) Two parallel weld beads with crack in middle region of high tensile stress.  
(2) End tabs cut into plate with centre tab pulled and outer tabs compressed prior to welding to provide fit-up residual stresses.                                                                                       | Tensile membrane stress with crack growth measured. | Two grades of aluminium alloy were used to provide different levels of plasticity at failure. | Used to validate R6. Failure observed over a range of values of $L_r$. |
<p>| A533B Steel Plate Tests           | Ainsworth et al [77]             | Centre cracked plate.        | Weld residual stress from welding two halves of a plate back together. Some samples underwent PWHT.                                                                                                                      | Four Point Bending with crack growth measured. | A533B steel (to provide structurally relevant material) | Used to validate R6. Experiments showed that R6 method is overly conservative. Some potential explanations given. |
| 16MnR Plate Tests                 | Hong-Liang [62]                  | Centre cracked plate.        | Weld residual stress with three different weld / crack combinations to create different residual stress profiles                                                                                                        | Tensile membrane stress                  | 16MnR steel (Chinese specification)                                      | Show that R6 is conservative apart from case with a high tensile secondary stress. Failure observed over a range of values of $L_r$. |
| A533B bend tests                  | Mirzaee-Sisan [78]               | Bend specimen with central semi-circular scallop with crack at base of scalloped notch.                                                                                                                                   | The specimens were pre-compressed at room temperature in the longitudinal direction to establish a plastic residual stress field. The tests were then unloaded, cooled and tested to fracture. | Three point bend test at cooled temperature (to ensure brittle fracture) | A533B steel               | Showed that R6 methods are conservative. Direct finite element estimate is still conservative. |
| A533B steel constraint tests      | Lee et al. [79]                  | Compact-tension and Single-edge notched plate | Residual stress induced by in-plane compressive preload prior to precracking                                                                                                                                           | Bending stress for compact-tension specimen, and tension for single-edge cracked plate. | Heat treated A533B steel | Showed that residual stresses can increase crack-tip constraint.          |
| A4333 M4 C-                       | Kamel [28]                       | C-Ring with                  | One level of pre-compression                                                                                                                                                                                             | Tensile load at AISI 4333 M4.            | Argued that tests are brittle                                          |                                                                          |</p>
<table>
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<tr>
<th>Residual Stress Induction Method</th>
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<tr>
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<td>Shallow crack</td>
<td>To induce a tensile residual stress field on unloading. An opposite level of pre-tension to result in a compressive residual stress field.</td>
<td>Loading pins (note that this induces a bend at the crack)</td>
<td>High yield (1050 MPa) and very low hardening (UTS = 1100 MPa). Also showed strong Bauschinger effect. but elastic summation of K appears to be incorrect. Neutron diffraction used to measure stress field. Used to investigate constraint effects.</td>
</tr>
<tr>
<td>A533B Side-punched residual stress</td>
<td>Bend specimen with side punches</td>
<td>A controlled secondary stress field is inserted by compressing the bend specimen in the thickness direction by use of side punches.</td>
<td>3 point bend test at cooled temperature (to ensure brittle fracture)</td>
<td>A533B steel Showed that crack-tip constraint can be included in the assessment of defects under secondary loads.</td>
</tr>
<tr>
<td>A533B and Weldex 900 Swedish Tests</td>
<td>Bend specimen with central semi-circular scallop with crack at base of scalloped notch.</td>
<td>The specimens were pre-compressed at room temperature in the longitudinal direction to establish a plastic residual stress field. The tests were then unloaded and tested to failure without cooling. A number of different cases (material and crack length) were considered.</td>
<td>3 point bend test at room temperature</td>
<td>Weldex 900 and A533B steel Showed that R6 methods are conservative and the levels of stress redistribution at higher levels of plasticity.</td>
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<td>A508 Spinning Cylinder Tests 1-6</td>
<td>Large scale rotating cylinder</td>
<td>The cylinders were heated (note exact temperatures vary between experiments) and cold water sprayed internally to produce a thermal shock profile through thickness.</td>
<td>High rotational velocity to simulate pressure hoop stress in a cylinder</td>
<td>Forged cylinders heat treated to represent A508 steel Provided validation of R6. One of the only experiments able to include a thermal transient.</td>
</tr>
</tbody>
</table>
2.11. Methods of residual stress measurement

There are a number of experimental approaches to quantify residual stresses within a component or laboratory specimen. A review of the approaches used to measure residual stresses is provided by Withers and Bhadeshia [87]. The measurement approaches considered by Withers and Bhadeshia are separated to the methods below:

1. Mechanical quantification of stress
   a. Curvature methods
   b. Hole Drilling
   c. Compliance / Sectioning
2. Diffraction with incident wavelengths, and
   a. Electron Beam Diffraction
   b. X-Ray Diffraction
   c. Hard X-Rays
   d. Neutron Diffraction
3. “Other” methods

Both the first and second approaches outlined above are discussed further below. The third approach (other methods) is not considered as this includes approaches such as ultrasonic and magnetic approaches that are not conventionally applied to components or laboratory specimens to determine the residual stress. Also included below is a further approach to measure stresses during loading that is based on Digital Image Correlation (DIC).

A further classification to the approach has been provided by Leggatt [88] where the level of sample destruction is characterised to classify the mechanical approaches from (a) to fully destructive approaches, such as for the sectioning approaches, (b) partially destructive approaches, such as the hole drilling, and (c) non-destructive approaches such as neutron diffraction.
2.11.1.1. Mechanical quantification of stress

The mechanical quantification of stress outlined as the first approach type in [87] relies on the sectional removal of material, thus allowing the residual stress component to be relieved at the free surface. A measure of the deformation at the free surface allows the original stress field to be back calculated. Of these approaches the hole drilling approach is the most common. The hole drilling approach can be separated to two techniques; surface-hole drilling and deep hole drilling. The basic theory associated with these two approaches are similar, a hole is drilled into the sample at the location where the stress value(s) is required and the local deformation measured. The surface-hole drilling approach adopts a measure of strain, normally by adopting a strain gauge similar to that shown in Figure 29, on the surface of the sample with a hole drilled at the centre. Elastic relations can then use the strain values measured by the strain gauge to define the principal stresses parallel to the surface.

![Rosetta strain gauge](image)

**Figure 29** – Rosetta strain gauge adopted in hole drilling measures of residual stress [87]

As the stress calculated in this way relies on the strain gauge positioned on the surface of the material it is typically not possible to determine stresses deeper that the diameter of the hole [87]. Other approaches are available where the measured depth can be extended, such as incremental deep-hole drilling [88]. This approach considers the measured deformation
of the hole as a function of depth by an air probe inserted to the hole as the hole is incrementally extended through the wall section. The measure of incremental-hole drilling is more complex than surface-hole drilling, as the stress tensor will include all components of stress whereas at the surface this can be reduced to consider only two in plane principal directions. Nonetheless, the approach still adopts elastic relations to calculate the stress profile.

2.11.1.2. Diffraction based measures of stress

The change in the atomic lattice spacing under stresses, be those residual or not, can be used to calculate the stress present. The lattice spacing can be measured by incident waves of various types including X-rays or neutrons and the diffraction pattern produced. If the incident wavelength, $\lambda$, is known then the Bragg equation can be used to calculate the lattice spacing, $d$, as:

$$
d = \frac{\lambda}{2 \sin \theta}
$$

Equation 63

where $\theta$ is the scattering angle. It is also noted that, as the resulting estimate of $d$ is from an interference pattern any integer multiple of $\lambda$ will produce the same pattern but might miss some of the peaks necessary for accurate measurements. The measure of strain can therefore be calculated from the measure of lattice spacing as:

$$
\varepsilon = \frac{\Delta d}{d_0} = \frac{d - d_0}{d_0}
$$

Equation 64

where $d_0$ is the unstressed lattice spacing. It is therefore necessary to have a good measure from the unstressed material to successfully obtain results from diffraction approaches. This can be achieved from either a specially machined sample where the sample is as stress relieved as possible, as adopted by McCluskey [89], or, if the stress is
localised, by taking a measure close to a free surface away from the stressed region such as Hurlston [81], both when adopting neutron diffraction.

The main difference between the approaches that adopt diffraction is the wavelength of the incident wave. The smaller the wavelength, the more refined the measurement can be, but the lower the penetration depth and the more intricate the sample preparation. The lowest extreme of this is electron diffraction that allows resolutions of up to 10 nm but is coupled with a shallow penetration depth that necessitate thin samples of less than 100 nm thickness.

X-rays and neutron diffraction approaches both can have a spatial resolution similar to the atomic spacing of common materials but X-rays only have a penetration depth of approximately 0.01 mm (unless Hard X-rays are used) whereas neutrons can penetrate up to 300 mm with suitable accuracy. Hard X-rays, also referred to as synchrotron X-Rays, can be used to increase the penetration depth to similar levels as neutron diffraction but can affect the spatial resolution of the measured area and narrow the angular resolution of the diffracted pattern [87] which may lead to loss of accuracy.

A diagram illustrating the different types of measurement processes and their associated penetration and resolution was provided by Withers et al [90], and is repeated in Figure 30. The shaded regions of the diagram also provide a key to approaches that are destructive or partially destructive in nature.

For the purposes of measuring a through thickness stress distribution in a typical nuclear component or test specimen, with a relevant spatial resolution and penetration, the neutron diffraction approach has been considered in a number of more recent investigations [28], [79], [81], [89], [91]. There are two different techniques for using neutron diffraction because of the two different sources of neutrons, those from a reactor or from a spallation source. The first approach allows a uniform, well-characterised continuous neutron beam to be used that allows conventional diffraction approaches to be adopted in which the sample is rotated within the incident beam to determine the various stress components. The second approach, referred to as time-of-flight, is considered for spallation sources as the incident neutron beam contains a number of wavelengths of different energies. As the
high-energy neutrons reach the specimen first a change in diffraction pattern with time can be calibrated to the change in incident wavelength with time and the stress calculated. The first approach allows better accuracy with smaller spatial resolution such as required in a rapidly changing stress field whilst the second approach includes additional information that can increase capture times and also provides a bulk material response [87].

Figure 30 – Diagram illustrating the range of different residual stress measurement techniques by their penetration depth and spatial resolution [90]

Neutron diffraction allows a non-destructive measure of stress field from within a specimen with a resolution suitable for characterising residual stress fields. The approach can, however, be time consuming and requires access to specialised facilities.

2.11.1.3. Digital Image Correlation

The approaches outlined above for quantifying stress fields are generally only applied to cases where the stress field is residual and the component does not see any additional loading. Some approaches, such as neutron diffraction can be applied whilst the specimen is being loaded but the stress measured is still only in a localised region. Another approach
that can allow the development of a strain field to be monitored during loading and hence the stress be quantified, over the surface of structure, is by way of image correlation techniques, commonly referred to as Digital Image Correlation (DIC). The DIC approach measures the strain field evolution on the surface of a specimen by capturing a sequence of images and correlating the displacements to a reference image, normally a case that has not been loaded. The DIC approach provides a full field measure of strain which allows a direct comparison to computational predictions such as finite element analyses [92]. For the approach to be successful the surface of the material should be prepared so that a random pattern of speckles is measured, with the typical scale of the speckled pattern being of a similar scale as the pixels of the image [92] so that the motion can be accurately monitored. As the specimen is loaded the speckled pattern is seen to move and specialist-processing software, such as Vic2D and Vic3D can convert these displacements to a strain.

Wasylyk [93] adopted the approach to monitor the plastic zone development in a number of different sized CT specimens. The wider applicability of the approach is demonstrated by the successful application to monitor the stress field of a growing crack under fatigue loading [94]. In this second case the specimen monitored was a CT specimen with the camera focused on a region of 0.6 by 0.4 mm about the initial crack tip with a pixel resolution of 0.93 by 0.93 µm at a capture rate of 30 Hz. This small region only required a single camera, as the surface of the specimen remained relatively normal to the camera. Nonetheless, the approach can also be extended to capture 3-dimensional motion by adopting two cameras positioned to provide a stereoscopic view of the surface [95], [96] which allows movement in the direction normal to the specimen surface to be captured.

This method of stress measurement can be very accurate, as it is taking a measure of strain directly from the specimen, and it also allows the strain field to be measured over a large area in a way that allows direct comparison to finite element analysis.
3. Aim

3.1. Preface

This section provides an overview of the main project aim and the approaches taken within the project to achieve this aim. This section is separated to further sub-sections which consider the relevant background information from the literature review in Section 2 before providing a detailed overview of the aim of the project.

3.2. Estimates of Stress Redistribution from Plasticity

A number of experimental [77], [78], [97] and analytical results [17], [66], [67], [74], [75] have shown that current approaches to assess the influence of combined primary and secondary stresses on structural integrity are, under some circumstances, overly conservative. This has been further noted by Banahan [4] and by a UK Technical Advisory Group for Structural Integrity (TAGSI) report [7]. This results from simplifications to the influence of plasticity on the local stress state at the crack tip, i.e. stress relaxation and enhancement under plasticity. There is, consequently, an opportunity to further developed these methods to reduce conservatism. It is noted here that the term “stress redistribution” is further used to capture both stress relaxation and enhancement.

The development of a new approach for assessing the influence of the interaction of primary and secondary stresses under combined loading on structural integrity of cracked components is required to both reduce the conservatisms maintained in the current approaches, such as those in R6 [3], and to make the approaches more straight forward to implement. The simplified approach by Hooton [68], extended by James et al [71], allows an easily implemented approach to estimate the crack driving force under combined loading which will be extended further. The \( g \) term defined by James et al when considering the effect of reference length under combined loading will form the basis of a new plasticity interaction term, \( V_g \) (or \( V \) from \( g \)). This therefore allows correlation with the R6 \( V \) term whilst maintaining its origin in the \( g \) term by James et al [71].
The enhanced definition of secondary stress redistribution provided by, $V_g$, can be achieved by examining the results of finite element analyses and validated through subsequent testing against experimental results. The finite element analyses can be used to provide an approach which details the secondary stress interaction without variability in the results. As such finite element analyses allow a reproducible means to quantify how the secondary stress behaves and provides a basis for developing a simplified method to conservatively predict the results.

The first aim from this work is therefore to investigate the plasticity interaction under a range of plasticity conditions so that a new, simplified expression for plasticity interaction under combined loads, for a cracked body, can be provided.

### 3.3. Experimental Verification of Plasticity Interaction Behaviour

In addition to the first aim, it is also important to compare the new approach to experiments to show how a real material might respond compared to the idealised component in the finite element analyses. The plasticity interaction parameter should be applicable to both engineering components and test specimens. Therefore the proposed $V_g$ term must remain conservative for a range of different geometries.

However, the range of experiments reported in the literature, as presented in Section 2, does not contain many experiments that cover a range of $L_r$ values with different specimens. Of those available in the literature, only References [77], [97] consider values of $L_r$ greater than 0.8 where the effect of plastic interaction is likely to be greatest, and, as such, provide the requisite results required to validate any approach to consider plasticity interaction. Therefore, a further aim of this work is to provide a base of experimental data for combined loading configurations at higher levels of plasticity. This was performed in conjunction with Swedish colleagues at Inspectra AB [76] where provision of material, and collaboration at the concept and design stage, ensured close communication and exchange of results.
The second aim of this work is, therefore, to perform experiments to extend the number of fracture toughness results available to validate the approach defined.

To validate the defined approach it is important that all the available experimental data, including that published in the literature and obtained in this work, be assessed. Therefore, the third aim of the work is to use the results of the experiments performed, and those identified in the literature, to validate the approach defined.

### 3.4. Objective of the PhD

The areas of investigation outlined above provide three main aims for the work. These areas allow an approach to be derived, experimentally considered and validated by external, third party data. As a summary, the three aims considered in this work are:

1. The first aim from this work is to investigate the plasticity interaction under a range of plasticity conditions so that a new, simplified expression for plasticity interaction under combined loads, for a cracked body, can be provided.

2. The second aim from this work is to perform experiments to extend the number of experimental results available to validate the new approach.

3. A third aim of the work is to take the results of the experiments performed, and those identified in the literature, to validate the new approach.

The work performed to consider Aim 1 is presented in Section 4 where an analytical, finite element investigation of a fully circumferential externally cracked cylinder under a number of secondary stress levels forms the basis of the new $V_g$ interaction term. Primary loads corresponding to different combinations of internal pressure and end load were incrementally applied to the cylinder to consider how the combined crack driving force develops with applied load.
The work undertaken with respect to the second aim is discussed in Section 5 which includes details of the experimental programme performed using three-point bend specimens containing a mechanically induced residual stress field.

Section 6 provides details of the work performed with respect to the third aim. The validation exercise was performed to validate the approach defined in Section 4. The experiments included in the assessment include those defined in References [28], [77–80], [97] in addition to the new tests described in Section 5 (and the results from the tests performed in Sweden [76]). The results are presented both in terms of the existing detailed R6 approach [3] as well as the new $V_g$ approach.
4. Development of Plasticity Interaction Term, $V_g$, to Account for Redistribution of the Secondary Stress in Combined Primary and Secondary Loading

4.1. Preface

This section describes the finite element analyses and post-processing performed to provide an estimate of $g(\sigma_{ref}^p, \sigma_{ref}^S)$, henceforth simply referred to as $g$, and from that $V_g$ (where $V_g = gf(L_r)$). The assessment of plasticity interaction here adopted the $g$ term of James et al [71], as this allows an easily applied multiplicative factor to assess the effect of secondary stress redistribution in terms of a $J$-estimation approach and can easily be modified to the $V_g$ approach which can then be used in an R6 assessment of a defect under combined loading [3]. Further reasons for adopting the $g$ term of James et al [71] were the new understanding obtained from the derivation of this approach, by considering the reference length under primary and secondary stress, and the author’s involvement in originally defining the $g$ term.

In the assessment, a fully external circumferentially cracked cylinder has been used to provide an estimate of plasticity interaction with respect to a range of secondary and primary stress levels with four different material strain hardenings. The use of the cylinder geometry was chosen to reflect its relevance to pressure vessels and piping.

A range of secondary stresses magnitudes was applied to the cylinder to represent different severities of thermal through-wall bending stress, representative of a thermal shock or heat exchange conditions. These were applied as they represent nominal transients under normal operating conditions and will, again, provide enhanced local deformation in the cracked region compared with other loading conditions, such as weld residual stresses. It is, however, noted that other through-wall thermal transients to those adopted here could have been used, such as those from Song et al [74], [75], but these are more complex and it...
was considered that a simple linear gradient was a suitable initial thermal field to study for development purposes.

4.2. Methodology to Investigate the Plasticity Interaction Term, $g$

The approach considered to investigate $V_g$ was to perform a number of finite element analyses to provide the combined elastic-plastic crack driving force, $K_f$, the primary elastic-plastic crack driving force, $K_f^p$, and the secondary elastic crack driving force, $K_i^s$. These parameters were then used to provide an estimate of $g$ from the finite element results, defined as $g_{FE}$, by:

$$g_{FE} = \frac{K_f - K_f^p}{K_i^s}$$  \hspace{1cm} \text{Equation 65}

Through this work, where an estimate from finite element analyses is adopted this is always referred to as $g_{FE}$, and $g$ defines the fit to $g_{FE}$. The evolution of $g_{FE}$ was calculated as a function increasing primary, $\sigma_{ref}^p$, and secondary, $\sigma_{ref}^s$, reference stresses. As such, the derivation of primary and secondary reference stresses was also derived from the finite element analyses. This allowed for comparisons of $g_{FE}$ versus $L_r$ to be made with regard to different levels of $\sigma_{ref}^s$ so that any trends with applied load and plasticity development could be observed.

The variations of $g_{FE}$ versus $L_r$ were then used to develop trend-lines, providing a simplified method to describe $g_{FE}$. This approach is very dependent on the accuracy of the finite element analyses, particularly on the estimate of primary reference stress; details of the finite element analyses are provided in the next section. This adopted approach is also dependent on the transferability of results from this geometry to others, including other constraint conditions. This aspect is discussed later in Section 7.
4.3. **Finite Element Analyses**

This section provides an overview of the finite element analyses performed to investigate and quantify the $g_{FE}$ term for an externally cracked cylinder. Details of the geometry, mesh and material properties, loading conditions, data extraction and model validation are described before the results are presented.

4.3.1. **Geometry**

In this series of analyses only one model, with two crack depths, was created using ABAQUS, version 6.8 [26]. The model used represented a cylinder with a mean radius, $r_m$, of 5 m, an overall model height of 10 m and a wall thickness, $t$, of 0.5 m. This provides a cylinder with a $r_m/t$ of 10, which is on the limit of thin shell applicability and hence allows the use of either thin or thick shell approximations when validating the mesh and in defining the applied loads. A simple schematic illustration of the geometry adopted is shown in Figure 31.

The geometry was chosen solely to represent the normalised $r_m/t$ ratio of 10 and does not represent a specific component, although, as all dimensions are normalised, it is possible to apply the results to any cylinder that follows the thin shell approximation. The dimensions are, however, a close approximation of those of a PWR pressure vessel. Two crack depths of $a/t = 0.1$ and 0.4 were used. These two crack depths were chosen to represent both a shallow and a deep crack to explore the influence of crack size on $g_{FE}$, or to assess if any constraint effects on $g_{FE}$ could be inferred.
4.3.2. **Meshing Characteristics**

Four finite element meshes were created to model the cylinder: two meshes for the analyses of the two crack depths under elastic conditions and a further two for analyses under elastic-plastic conditions. In all models reduced integration, eight noded, axi-symmetric elements (ABAQUS type CAX8R) were used. Axi-symmetric elements were adopted as they allow a simple two-dimensional mesh to account for the full three-dimensional behaviour of a cylinder. It is noted that this is only possible under conditions that allow for uniform loading around the cylinder, such as the internal pressure and end load used here, as the load applied to the two-dimensional model represents all parts of the cylinder with the same polar radial and axial position (i.e. it is symmetrical about $\theta$ under polar coordinates). The use of quadratic elements was to allow additional mid-side nodes so that deformation can be better modelled, especially at the crack tip location. Reduced
integration elements were adopted to ensure relatively quick analysis times and to prevent “shear locking”\textsuperscript{9} under bending (from the thermal stress field) or under excessive plasticity. An illustration of the mesh is shown in Figure 32.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure32.png}
\caption{Illustration of global mesh for deep cracked cylinder (note that the 2D mesh has been partially rotated about the central axis to demonstrate the modelled geometry)}
\end{figure}

\textsuperscript{9} Shear locking is where full integration elements deform, typically under shear or bending conditions, such that the integration points approach or overlap each-other such that they approach the neutral axis which introduces a singularity in the solution. Therefore, to prevent the singularity, additional motion is restricted, increasing the effective stiffness of the element. Adopting reduced integration elements with one integration point easily solves this.
A focused crack tip mesh was adopted with positions of the mid side nodes of the quadratic elements moved to a quarter element distance under elastic conditions (see below). However, under elastic-plastic and elastic-perfectly plastic conditions the mid side nodes were maintained at their mid-side position. As indicated previously, this modification necessitated two meshes under elastic and elastic-plastic conditions. However, the number and size of elements in both the elastic and elastic-plastic meshes was maintained. An illustration of the crack tip mesh is shown in Figure 33.

![Image](image.png)

**Figure 33 – Close-up of focused crack tip region with illustration of boundary conditions applied to the un-cracked ligament**

The degenerate behaviour of the crack tip nodes was also altered between these two models. In the elastic model the crack tip nodes were set to remain fixed, to ensure a sharp crack under loading with quarter-point nodes to provide the $1/\sqrt{r}$ stress singularity relationship required [98]. This is simply achieved by having one node present at the crack tip and moving the mid side nodes to the quarter point position nearest the crack tip. However, with the elastic-plastic model multiple nodes were coincident at the crack tip and were free to move, to allow crack tip blunting, reflecting the guidance in ABAQUS [26] and Reference [98].

The only geometric boundary condition required was a symmetry condition applied to the uncracked ligament as the model is fixed in the radial direction, and from all rotation, by
the symmetry imposed by the axi-symmetric elements. The act of applying the boundary conditions to the uncracked ligament also has the effect of introducing the crack on the symmetry plane. Therefore the model only represents one half of the cylinder. This means that the energy release term, $J$, requires multiplying by two (note that this is automatic in ABAQUS if the SYMMETRY word is included). This also means that the crack is present throughout the analyses and will experience all loadings applied. The illustration shown in Figure 33 also includes orange triangles denoting the position of the symmetry boundary condition upon the nodes located on the ligament ahead of the crack tip in ABAQUS.

### 4.3.3. Material Properties

The material model used had elastic properties based on those for Type 316L stainless steel with plastic properties represented by an idealised Ramberg Osgood form shown below with a Young’s modulus, $E$, taken as 196,000 MPa, Poisson’s ratio, $\nu$, of 0.296, $\sigma_y$ of 290 MPa and four values of $n$, the strain hardening component: 5, 7.5, 10 and 12.5.

$$
\varepsilon = \frac{\sigma}{E} + 0.002 \left( \frac{\sigma}{\sigma_y} \right)^n
$$

Equation 66

where $\varepsilon$ is the von Mises equivalent strain and $\sigma$ is the von Mises equivalent stress. By adapting the basic properties defined for a common stainless steel used for many piping components in nuclear power plant it is ensured that the materials response is representative of typical components that may be considered. However, the use of four different strain hardening exponents in the Ramberg-Osgood relation for plastic strains may take the results further away from that seen in plant as a typical 316 stainless steel will have a strain hardening index of approximately 6. Nonetheless, adopting a number of different strain hardening terms does allow an improved estimate of the reference stress (see below) for a wider range of materials and also provides a means to investigate the effect of plasticity on $V_\theta$. An illustration showing the four different elastic-plastic stress-strain curves adopted in the analyses is shown in Figure 34.
As the loading was applied consecutively with no unloading it can be assumed that the crack tip stress and strain field follow a proportional response. That is to say that the stress state at a given time is independent of the order in which the loads were applied. Also, as no unloading was considered, a simple isotropic hardening law was used. In all models small strain approximations were assumed.

4.3.4. Application of Primary Stress

The primary load was simulated via a pressure to the element faces located on the end surface of the cylinder and, where a hoop stress was to be induced, to the elements faces on the internal surface (see Figure 35). To ensure the pressure on the end surface correctly transferred as an axial stress, nodes located at the top surface were constrained to maintain equal axial displacements with no rotation. This is analogous to a remote load applied to a long cylinder.
The magnitudes of the applied primary loads were characterised by the $R_6$ $L_r$ parameter. The loads were applied in 0.1 increments of $L_r$ up to a value of 1.6. In the analyses the pressures required for each values of $L_r$ were obtained from $L_r = P/P_L$, where $P$ is the applied load and $P_L$ is the elastic-plastic limit load for each geometry and material combination.

For each loading condition a value of $P_L$ was obtained by elastic-perfectly plastic finite element analyses, i.e. $n = \infty$. A potential range of values of $P_L$ were found depending on the local limit load, which is where the plastic zone extends across the uncracked ligament, and the global limit load, which is defined by the load at which the structure is no longer able to support further stress and the finite element analyses can not converge. However, the value for $P_L$ used in this investigation was not defined from the local or global limit.

Figure 35 – Schematic illustration of application of loads to the cylinder and finite element mesh
load but from its effect on the resulting Option 3 FAD. As such the value of $P_L$ used was that required to provide Option 3 FADs for the same geometry, but different Ramberg Osgood strain hardening coefficients, that crossed at $L_r = 1$, as also adopted by Song [74], [75]. This was to ensure the primary reference stress was equal to the yield stress, i.e. $\sigma_{ref}^p = \sigma_y$, at $L_r = 1$, which is a feature of reference stress approaches [43], and as such provides a more accurate estimate of the primary reference stress that does not require assumptions inherent to local and global limit load solutions. Nonetheless, the value of $P_L$ used was always between the local and global limit load solutions (see Table 3), which helps validate the approach. Handbook limit load solutions from R6 [3] were found to be somewhat conservative compared to the finite element measure, especially when including hoop stress. This was, however, expected as handbook solutions were Tresca stress based and the finite element solution was von Mises stress based. This means that there is a potential discrepancy of $2/\sqrt{3}$ between the results which brings the results closer for the shallow crack and beyond for the deep crack.

**Table 3 – Limit load adopted in the calculation of $g_{FE}$, expressed in terms of applied axial stress, for all cases compared to global and local finite element predictions and R6 predictions**

<table>
<thead>
<tr>
<th>$\sigma_a$</th>
<th>$\sigma_n$</th>
<th>$a/t$</th>
<th>Local Limit Load</th>
<th>Global Limit Load</th>
<th>R6 Estimate</th>
<th>Used in Analyses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>No</td>
<td>0.1</td>
<td>277</td>
<td>290</td>
<td>273</td>
<td>277</td>
</tr>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>0.1</td>
<td>164</td>
<td>168</td>
<td>145* (167)</td>
<td>165</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>0.4</td>
<td>194</td>
<td>203</td>
<td>198</td>
<td>203</td>
</tr>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>0.4</td>
<td>140</td>
<td>160</td>
<td>140* (162)</td>
<td>148</td>
</tr>
</tbody>
</table>

* In these cases the R6 solution is only defined by the Tresca yield criterion for global plastic collapse; and the maximum predicted von Mises stress is provided in brackets by multiplying the R6 estimate by $2/\sqrt{3}$ as defined by Miller [99].

It is also worth noting that in all cases the primary loads were applied until either $L_r$ reached a value of 1.6 or where the finite element analyses was unable to converge. For the lower strain hardening cases ($n = 10, n = 12.5$), as the material behaves closer to an elastic-perfectly-plastic material, it was not always possible to reach higher levels of loading. This is, however, captured within the R6 $L_r^{max}$ term. When considering a strain
hardening material an estimate of the materials ultimate tensile strength, $\sigma_u$, can be made in R6 from Considère’s rule as below [3];

$$\sigma_u = \sigma_y \left( \frac{1}{n} \right)^{\frac{1}{n}} e^{-\frac{1}{n}}$$  \hspace{1cm} \text{Equation 67}

which can then be used to estimate $L_r^{\text{max}}$ by;

$$L_r^{\text{max}} = \frac{(\sigma_y + \sigma_u)}{2\sigma_y}$$  \hspace{1cm} \text{Equation 68}

This leads to predictions of $L_r^{\text{max}}$ shown below in Table 4

<table>
<thead>
<tr>
<th>$\sigma_u/\sigma_y$</th>
<th>$L_r^{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 5$</td>
<td>2.06</td>
</tr>
<tr>
<td>$n = 7.5$</td>
<td>1.53</td>
</tr>
<tr>
<td>$n = 10$</td>
<td>1.34</td>
</tr>
<tr>
<td>$n = 12.5$</td>
<td>1.24</td>
</tr>
</tbody>
</table>

Therefore, as long as the analyses converged for loads beyond $L_r^{\text{max}}$ the analyses was considered valid even if a value of $L_r = 1.6$ could not be attained. It is noted that the value of $L_r^{\text{max}}$ is a conservative estimate of plastic collapse and does not necessarily represent the limiting condition as defined by the finite element analyses. Therefore, all analyses were allowed to continue where possible to accumulate as many additional data points beyond $L_r^{\text{max}}$ for comparison. As such these values of $L_r^{\text{max}}$ are not included in the FADs plotted in this section.

4.3.5. Application of Secondary Stress

A secondary stress field has been applied to the model corresponding to an elastic bending stress in the range $0 < \sigma_{ref}^s/\sigma_y < 1.6$. As secondary loads do not contribute to plastic
collapse it is not possible to determine the secondary reference stress from finite element analyses. Instead, handbook solutions defined for an edge cracked plate, as shown in Figure 36, under primary loads were used to determine the secondary reference stress [99].

\[
\sigma_{ref}^2 = \frac{3\sigma_h^2}{4} + \frac{3Q^2}{(t-a)^2} + \left(\frac{2M}{(t-a)^2} + \left(\frac{2M}{(t-a)^2} + \left(\frac{\sigma_h}{2} - \frac{N}{t-a}\right)^2\right)^2\right)
\]

Equation 69

Figure 36 – Illustration of edge cracked plate geometry assumed in secondary reference stress calculation

where \(\sigma_h\) is the stress acting tangential to the crack (i.e. hoop stress in the circumferentially cracked cylinder), \(Q\) is the mode II shear force per unit length, \(M\) is applied bending moment per unit length and \(N\) is the applied tensile force per unit length. For the cases considered here, i.e. under thermally induced equi-biaxial bending stresses, \(Q\) and \(N\) are zero, an equivalent hoop stress can be obtained by multiplying the maximum bending stress by \(\sqrt{1/3}\) and the bending moment was taken as that for a plate, \(M = \frac{1}{6}t^2\sigma_b\) [99]. The approach adopted to provide the equivalent membrane hoop stress was from the equivalent strain energy approach, a version of which can be found in R6 [3].

In each case the secondary stress was applied via a linear through-thickness thermal gradient to open the crack. These values of temperature were defined within a user-
subroutine to equate to the through thickness values of $\sigma_{r_{ef}}/\sigma_y$ required by evaluating the bending stress, $\sigma_b$. In all cases the crack was present from the start of the analyses.

4.3.6. Data Interpretation

Within the calculation of $g_{FE}$ it was assumed that the value of $J$, estimated by the $J$-contour integral in ABAQUS, and $K_f$ are related by the plane strain relation shown in Equation 70.

$$K_f = \sqrt{\frac{EJ}{1 - \nu^2}}$$

Equation 70

To estimate values of $K_f^p$ and $K_f^p$, finite element analyses were performed with only the primary load applied under elastic and elastic-plastic conditions, respectively. Likewise estimates of $K_f^S$ and $K_f^S$ were obtained from elastic and elastic-plastic finite element analyses, respectively, with only the secondary stress present. Estimates of $K_f$ were obtained with both primary and secondary loads applied in elastic-plastic finite element analyses. In all cases the secondary load was applied before the primary loads. This allows the effect of increasing plasticity on $g_{FE}$ to be examined by incrementally increasing the primary load and monitoring the inbuilt ABAQUS contour integral used to define $J$.

4.3.7. Model Validation

To validate the models (meshes, boundary conditions and application of loads) the predicted radial, $\sigma_r$, and hoop, $\sigma_h$, stresses were compared to analytical solutions for thick cylinders [100]. The location on the finite element model upon which the measures of hoop and radial stresses were taken was near the upper surface of the cylinder, such that the effect of the crack was not included. The solution adopted was for an open cylinder and as such only included radial and hoop components of stress. As the axial load was applied as a pressure on the top surface, no variation through thickness existed and the stress is simply the applied pressure; this does however help confirm that the application of
pressure is correct. The radial and hoop stresses were seen to match the standard solutions to within 0.02% for all cases and the axial stresses matched exactly. To validate the modelling of the crack-tip mesh, comparisons were also made to stress intensity factor solutions from R6 [3] and those predicted in the elastic finite element analyses. In this comparison the derived values of $K_f^p$ were shown to be within 0.2% of handbook solutions [3] for the shallow cracked cylinder and within 1.3% for the deeper cracked cylinder.

A further validation of the approach was made by comparison of the material and geometry specific Option 3 FADs, constructed from $K_f^p$ and $K_f^p$, and the material specific Option 2 FADs of R6 [3]. This comparison can be seen in Figure 37 (A-D). In the figure the solid lines are the Option 3 FADs and the points are the Option 2 FADs. The FAD for the highest strain hardening ($n = 5$) can be seen in black, moving through red, blue and green as the strain hardening decreases through $n = 7.5$ to 10 and 12.5, respectively. Generally, the Option 2 and Option 3 FADs show good agreement, especially at elastic and post-yield levels of plasticity, and for high strain hardening material. The best agreement is at the lower elastic and the fully plastic loads and this is indicative of the use of the reference stress approach adopted in the simplified Option 2 approach. This is because the Option 2 FAD does not allow for geometric constraint differences on yielding, which an Option 3 FAD will do. This geometric constraint will have most effect when plasticity is contained but will be less significant under global yielding, i.e. large scale yielding. Further aspects of the FADs are discussed below. Note that the results for $K_f^p$ used in the FAD estimates are presented in the following section.

### 4.3.8. Results

The finite element estimates of $K_f^p$ and $K_f^p$ as a function of $L_r$ can be seen in Figure 38 (A-D) for the different loading conditions and crack depths. The figures show four coloured lines with the data points shown for the elastic-plastic $K_f^p$ and a dashed black line for the elastic $K_f^p$. Of the $K_f^p$ curves, the black is for $n = 5$, red is for $n = 7.5$, blue is for $n = 10$ and green is for $n = 12.5$.  

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The cases that only include the axial end load can be seen to generate a higher $K_f^p$ at a given value of $L_r$ than the cases that include both axial end load and pressure. This would be expected, as the effect of the pressure, for the same primary reference stress, is to create more stress in the hoop direction so that the applied axial stress is reduced. This therefore means that the resultant values of $K_f^p$ and $K_f^p$ are lower than if the entire load was applied to open the crack.
Figure 37 – Option 2 and Option 3 FADs for: (A) shallow cracked cylinder with end load, (B) shallow cracked cylinder with end load and pressure load, (C) deep cracked cylinder with end load; (D) deep cracked cylinder with end load and pressure load.
It can also be seen that the cases that only include the axial end load are those that did not all converge to a value of $L_r$ of 1.6. For the shallow crack only the finite element analyses for the weakest hardening material case did not converge. The deeper crack analyses were seen to stop at values of $L_r$ closer to the predicted values of $L_r^{\text{max}}$ in Table 4. The cases that include pressure loading are seen to complete through to value of $L_r$ of 1.6. As discussed above the reason for this difference is the reduction is applied load to open the crack whilst inducing more plasticity in the hoop direction.

The values of $K_f^p$ and $K_f^p$ are observed to be higher for the deeper crack than for the shallow crack, as $K_f \propto \sqrt{a}$. It can also be seen, as expected, that $K_f^p$ increases substantially once yielding occurs. Collectively, these trends help add confidence to the results attained.

The variation of the secondary stress intensity factor, $K_f^S$, as a function of $\sigma_{ref}^{S}/\sigma_y$ is shown in Figure 39 (A-D) for the shallow and deeply cracked cylinder cases. The results show a negligible difference with strain hardening exponent; which indicates a low elastic follow-up under secondary loads alone as this would act to enhance the deformation of the cylinder, thus increasing $K_f^S$. As would be expected, the deeper crack has a larger $K_f^S$ (approximately 1.6 times) than the shallow crack. That the elastic results match the elastic-plastic results at low loads, and that thermal stresses are equivalent to primary stresses under elastic loading, confidence can be gained in the validity of the results given the validation applied under primary loads.
Figure 38 – Elastic and elastic-plastic Primary SIFs for: (A) shallow cracked cylinder with end load, (B) shallow cracked cylinder with end load and pressure load, (C) deep cracked cylinder with end load; (D) deep cracked cylinder with end load and pressure.
Figure 39 – Secondary elastic and elastic-plastic stress intensity factor predictions for; shallow cracked cylinder (A) and; deep cracked cylinder (B)

The variation of $K_j$ as a function of $L_r$ for the combined primary and secondary stresses can be seen in Figure 40 (A-D) for the shallow cracked cylinder with an axial end load, Figure 41 (A-D) for the shallow cracked cylinder with both an axial end load and pressure, Figure 42 (A-D) for the deep cracked cylinder with an axial end load and Figure 43 (A-D) for the deeply cracked cylinder with both pressure and an axial end load. In each figure four plots are included which show each of the different strain hardening exponents. Only four of the sixteen different levels of secondary reference stress are included; $\sigma_{ref}^s/\sigma_y = 0$, 0.5, 1.0 and 1.5.
Figure 40 – Combined elastic-plastic SIFs for shallow cracked cylinder with end load for 4 values of strain hardening index: (A) $n = 5$; (B) $n = 7.5$; (C) $n = 10$; and (D) $n = 12.5$. 

(A) – $n = 5$

(B) – $n = 7.5$

(C) – $n = 10$

(D) – $n = 12.5$
Figure 41 – Combined elastic-plastic SIFs for shallow cracked cylinder with pressure and end load for 4 values of strain hardening index: (A) $n = 5$; (B) $n = 7.5$; (C) $n = 10$; and (D) $n = 12.5$
Figure 42 – Combined elastic-plastic SIFs for deep cracked cylinder with end load for 4 values of strain hardening index: (A) $n = 5$; (B) $n = 7.5$; (C) $n = 10$; and (D) $n = 12.5$
Figure 43 – Combined elastic-plastic SIFs for deep cracked cylinder with pressure and end load for 4 values of strain hardening index: (A) $n = 5$; (B) $n = 7.5$; (C) $n = 10$; and (D) $n = 12.5$
The combined stress intensity factor, $K_J$, shown in Figure 40 through to Figure 43 indicates a number of features, many of which are similar to those observed for the primary loading alone.

With regard to the inclusion of the secondary stress, in all cases the initial secondary stress intensity factor can be seen to dominate the crack driving force at low primary loads, i.e. low $L_r$. However, as the primary load ($L_r$) is increased the curves are seen to converge as the secondary stress is redistributed within the structure. Typically the applied primary load at which this convergence starts to occur is near $L_r = 1$ but is slightly earlier for the higher strain hardening cases and slightly after for the low strain hardening cases. Conversely, once the convergence starts, the rate of convergence is greatest for the lower strain hardening.

Nonetheless, the observations for the relative contribution of the secondary stress to the crack driving force can better be considered by calculating $g_{FE}$ for each case using Equation 65. Plots showing this comparison as a function of $L_r$ can be seen in Figure 44 for the shallow cracked cylinder with an axial end load, Figure 45 for the shallow cracked cylinder with pressure and an axial end load, Figure 46 for the deep cracked cylinder with an axial end load and Figure 47 for the deep cracked cylinder with an axial end load and pressure. It can be seen that in all cases the estimate of $g_{FE}$ is largely independent of the magnitude of the secondary reference stress. This is evident as the highest variation in $g_{FE}$ at any given applied primary load, for any given case, is 0.1 and is, in most cases, significantly less. Therefore, the variation of $g_{FE}$ as a function of $L_r$ can be approximated, conservatively, as the maximum, or upper bound, curve for each case considered. An illustration of this maximum $g_{FE}$ can be seen in Figure 48 for the four different cases.
Figure 44 – Estimate of $g_{FE}$ for shallow cracked cylinder with end load for 4 values of strain hardening index: (A) $n = 5$; (B) $n = 7.5$; (C) $n = 10$; and (D) $n = 12.5$
Figure 45 – Estimate of $g_{FE}$ for shallow cracked cylinder with pressure and end load for 4 values of strain hardening index:

(A) $n = 5$; (B) $n = 7.5$; (C) $n = 10$; and (D) $n = 12.5$
Figure 46 – Estimate of $g_{FE}$ for deep cracked cylinder with end load for 4 values of strain hardening index: (A) $n = 5$; (B) $n = 7.5$; (C) $n = 10$; and (D) $n = 12.5$
Figure 47 – Estimate of $g_{FE}$ for deep cracked cylinder with pressure and end load for 4 values of strain hardening index:

(A) $n = 5$; (B) $n = 7.5$; (C) $n = 10$; and (D) $n = 12.5$
Figure 48 – Upper bound variation of $g_{FE}$ for: (A) shallow cracked cylinder with end load; (B) shallow cracked cylinder with pressure and axial end load; (C) deep cracked cylinder with axial end load; and (D) deep cracked cylinder with pressure and axial end load.
4.4. **Simplified Calculation of g**

The results for $g_{FE}$ illustrated in Figure 48 show a number of features:

1) All curves trend to unity at $L_r = 0$.
2) All curves have a skeletal point (i.e. a point where $g$ is independent of the strain hardening exponent) at a specific value of $L_r$ close to 1.2.
3) All curves appear to reduce quickly (sometimes following a peak) at large values of $L_r$.
4) The lower the strain hardening behaviour (larger $n$), the higher the peak value of $g_{FE}$ prior to the skeletal point; and the lower the value of $g_{FE}$ beyond the skeletal point, i.e. at high $L_r$.

The above observations are also characteristics of the Option 2 and Option 3 R6 failure assessment curves. However, some additional features of the plots are not observed for a failure assessment curve under primary loading. These additional points are:

5) the value of $g_{FE}$ rises above unity for intermediate values of $L_r$, whereas the value of $K_r$ for the Option 2 and Option 3 FAD reduces below unity for intermediate values of $L_r$, and
6) the skeletal point for $g_{FE}$ is at $L_r$ of approximately 1.2 for all cases (apart from the case with a short crack and an axial end load and pressure which is at $L_r = 1$), whilst that for $K_r$ for the Option 2 FAD is at $L_r = 1$ [43].

However, the qualitative similarity of the form of the $g_{FE}$ curves as a function of $L_r$ noted in points (1) to (4) to an Option 2 or 3 failure assessment curve suggests that an equation of a similar form to that used to define the Option 2 FAD may be used to represent $g_{FE}$ as a function of $L_r$. Here the use of the Option 2 FAD is used as a starting point because of its dependence on material properties, more specifically the strain-hardening component. It is noted that the R6 Option 2 FAD is given by;
When considering points (5) and (6) above, where the form of \( g_{FE} \) does not conform to the conventional Option 2 FAD, it is possible to make some minor modifications to Equation 71 to account for these. When considering point (5) it is noted that the region where the results are seen to increase above unity is in the region of \( L_r \) between 0.8 and 1.2, i.e. the region that is affected by the small strain correction in the Option 2 FAD. Therefore, the small-scale correction term in the Option 2 FAD may be modified to allow the FAD to increase above unity. When considering point (6) it can also be seen that the value of \( L_r \) where the skeletal point can be found is simply increased by approximately 0.2 to a value of \( L_r = 1.2 \). To provide a simple way to shift the \( L_r \) axis, the primary reference stress can be modified by a scaling parameter.

Thus, the following form of the Option 2 FAD should be able to fit the finite element estimate of \( g_{FE} \) by fitting two parameters, \( A \) and \( B \).

\[
g = \left[ \frac{E \varepsilon_{ref}^{mod}}{\sigma_{ref}^{mod}} + A \left( \frac{\sigma_{ref}^{mod} / \sigma_y}{2E \varepsilon_{ref}^{mod} / \sigma_{ref}^{mod}} \right)^{1/2} \right]^{1/2}
\]

Equation 72

where \( \sigma_{ref}^{mod} \) is a shifted, or modified, reference stress and is defined by \( \sigma_{ref}^{mod} = \sigma_{ref}^p / B \). As discussed above the value for \( B \) is linked to the position of the skeletal point and the \( A \) term provides a measure of the enhancement above unity seen under small scale yielding. Results of applying Equation 72 to fit the finite element data can be seen as the open circles within Figure 49, with the optimum values of \( A \) and \( B \) included, where the solid lines are the upper bound finite element results and the circles are the curve fits of Equation 72.
Figure 49 – Curve-fit of $g_{FE}$ for: (A) Shallow cracked cylinder with end load; (B) shallow cracked cylinder with end load and pressure; (C) deep cracked cylinder with end load; and (D) deep cracked cylinder with end load and pressure
From these fits, over 90% of the points were within 10% of the finite element results, with approximately 40% of the remaining 10% showing a potential non-conservatism. The values of $A$ and $B$ can be defined by Equation 74 and Equation 73, to provide a conservative, upper bound, estimate of $g_{FE}$ for all load cases and work hardening exponents studied.

$$A = \frac{\sigma_{in\ plane}}{1.25\bar{\sigma}_{mises}}$$ \hspace{1cm} \text{Equation 73}

$$B = 1.25$$ \hspace{1cm} \text{Equation 74}

where $\sigma_{in\ plane}$ is the remotely applied primary stress acting to open the crack, i.e. the axial stress in the current analyses, and $\bar{\sigma}_{mises}$ is the von Mises equivalent stress for all remotely applied primary stresses, i.e. the equivalent stress defined from the hoop and axial stress. The comparison of the conservative estimate of $g$ and the finite element analyses can be seen in Figure 50 where, again, the solid lines are the upper bound finite element results and the circles are the curve fits of Equation 72 when adopting Equation 74 and Equation 73 to define $A$ and $B$.

This added conservatism removes all the points showing a potential non-conservatism of more than 10% and means that less than 5% of all points shows a potential non-conservatism, of which less than 1% are more than 5% non-conservative. This brings the approach to a level of conservatism that can be usefully adopted within an assessment procedure. However, it does mean that approximately 30% of cases show a conservatism of more than 10%, with a quarter of these showing a conservatism of more than 100% in terms of $g$, i.e. the value of $g_{FE}$ is half that predicted using Equation 72, Equation 74 and Equation 73.
Figure 50 – Comparison of $g$ and $g_{FE}$ for: (A) Shallow cracked cylinder with end load; (B) shallow cracked cylinder with end load and pressure; (C) deep cracked cylinder with end load; and (D) deep cracked cylinder with end load and pressure.
4.5. Overview of $V_g$ Approach

Within R6 it may be more appropriate to adopt a plasticity correction factor that is equivalent to the $\rho$ or $V$ terms so that assessment points can be plotted on the FAD, rather than adopting a $J$ estimation approach as considered previously. Therefore, the developed $g$ approach can be adapted to provide a different definition of the $V$ term, defined here as $V_g$.

Therefore, the approach that may be adopted within R6 would be very similar to that of the conventional $V$ approach as below:

$$K_r = \frac{K_r^p + V_g K_i^S}{K_{mat}}$$  \hspace{1cm} \text{Equation 75}

where;

$$V_g = \frac{K_i^S}{K_i^s} f(L_r) g$$  \hspace{1cm} \text{Equation 76}

and;

$$g = \left[ \frac{E \varepsilon_{ref}^{mod}}{\sigma_{ref}^{mod}} + A \left( \frac{\sigma_{ref}^{mod}}{\sigma_{ref}} \right)^2 \right]^{-\frac{1}{2}}$$  \hspace{1cm} \text{Equation 77}

$$A = \frac{\sigma_{in\ plane}}{1.25 \sigma_{mises}}$$
$$\sigma_{ref}^{mod} = \sigma_{ref}^p / 1.25$$

In this approach it is noted that the ratio $K_i^S / K_i^S$ has been introduced to consider cases which do not follow $K_i^S \approx K_i^S$, which was observed in the research outlined above. The results shown in Figure 50 (A-D) are repeated in Figure 51 (A-D) after being converted to $V_g$, as are the finite element predictions of $g_{FE}$, shown as $V_{gFE}$, by way of Equation 76, where the solid lines are $V_{gFE}$ and the circles are $V_g$. In the conversion an Option 1 FAD has been assumed.
Figure 51 – Comparison of \( V_g \) and \( V_{gFE} \) for: (A) Shallow cracked cylinder with end load; (B) Shallow cracked cylinder with end load and pressure; (C) Deep cracked cylinder with end load; and (D) Deep cracked cylinder with end load and pressure
4.6. Application of $V_g$ to the Finite Element Predictions of Failure

To assess the accuracy of the fit of $g$ to the finite element values, $g_{FE}$, and hence the accuracy of $V_g$, it is possible to provide an estimate of $K_j$ from Equation 75 and Equation 76, when recognising that at failure $K_r = f(L_r)$ and $K_{mat} = K_j$, as shown below:

$$K_j = K_j^p + gK_j^S$$  \hspace{1cm} \text{Equation 78}

where the $K_j^p$ and $K_j^S$ are defined from finite element analyses and $g$ is defined as in Equation 72 through to Equation 73. This means that the combined elastic-plastic stress intensity factor defined above can be compared to the finite element prediction to see the levels of conservatism of the approach for practical application. It is also possible to use the same data to provide a direct comparison with the detailed R6 $V$ Factor approach.

To provide the comparison the results are presented as the ratio of $K_j$, predicted from the $g$ approach in Equation 78 or the R6 $V$ Factor approach, with $K_j$ defined directly from the finite element analyses. An accurate comparison has a ratio of unity, a conservative answer, i.e. when $K_j$ defined from $g$ or $V$ yields higher value of $K_j$ that the corresponding finite element analysis, is above unity and a non-conservative solution is less than unity. Figure 52 shows the maximum level of conservatism seen for each solution, at each value of $L_r$, from all different magnitudes of secondary stress. Four plots are shown, for the different crack depth and loading cases considered, within which the colour denotes the material strain hardening index; black: $n = 5$; red: $n = 7.5$; blue: $n = 10$; green: $n = 12.5$. The solid curves show the $g$ approach in Equation 78 and the diamonds show the detailed R6 $V$ Factor approach. Figure 53 repeats these plots but adopts the minimum estimate of $K_j$ from both methods, such that the level of non-conservatism can be established.

The results presented in Figure 52 show that the R6 approach can be very conservative, where the maximum level of conservatism seen is over 60% (Figure 52(B)). The maximum conservatism occurs at $L_r \approx 1$ in all cases. This indicates a consistent
conservatism in the approach. The maximum conservatism of the $g$ approach is less than 10% in plots (A), (C) and (D), whereas for plot (B), which is the shallow crack subjected to an axial and hoop stress, it is 20%. It is this case that provided the poorest fit in Figure 50 and would be expected to provide conservative results; though even this conservative estimate is still a 40% improvement relative to the R6 approach.

The results presented in Figure 53 showing the minimum estimate of $K_f$ indicate that the $g$ approach and the R6 approach have very similar levels of potential non-conservatism. The maximum non-conservatism seen for the $g$ approach can be seen to be 3%. The maximum non-conservatism of the R6 approach is 30%, but this is at very high values of $L_r$ for the lowest strain hardening cases, which is beyond the calculated value of $L_r^{max}$ calculated in Table 4. If the estimate of non-conservatism is restricted to “valid” values of $L_r$, then no non-conservatism is seen.
Figure 52 – Maximum values of $K_J$ estimated by $g$ and $R_6$ factor when normalised by $K_{JFE}$ for: (A) Shallow cracked cylinder with end load; (B) Shallow cracked cylinder with end load and pressure; (C) Deep cracked cylinder with end load; and (D) Deep cracked cylinder with end load and pressure.
Figure 53 – Minimum values of $K_J$ estimated by $g$ and R6 $V$ factor when normalised by $K_{JFE}$ for: (A) Shallow cracked cylinder with end load; (B) shallow cracked cylinder with end load and pressure; (C) deep cracked cylinder with end load; and (D) deep cracked cylinder with end load and pressure
4.7. **Summary**

The results presented in this section relate to a fully external circumferentially cracked cylinder, with a deep and shallow crack, with primary loads applied from $\frac{\sigma_{ref}}{\sigma_y} = L_r = 0$ to 1.6 as an axial stress in isolation or in combination with a hoop stress. These primary stresses were applied in combination with a large range of magnitudes of a thermally induced secondary biaxial bending stress ranging from $\frac{\sigma_{ref}^S}{\sigma_y} = 0.1$ to 1.6. The finite element analyses demonstrate correct behaviour compared to existing solutions and expected trends under both primary loads alone and under combined loading.

A new function, $g$, has been defined to describe this redistribution of secondary stresses as primary stress induced plasticity increases. An estimation of $g$ was obtained from numerous finite element analyses, $g_{FE}$, that demonstrated insensitivity to the magnitude of the secondary stress. The value of $g_{FE}$ was found to be dependent upon the materials strain-hardening exponent (see Figure 48).

There was a negligible difference in the finite element prediction of $g$ with the increased application of secondary loading. It is upon this independence that the developed approach is based but it is possible that this is a result of the secondary stress considered having a low level of elastic follow-up; these areas are discussed further in Section 7.

Many important features of $g$, as defined through finite element analyses, are shared with an Option 2 failure assessment curve of the R6 reference stress approach, upon which a function for $g$ has been based. However, one important difference to the Option 2 FAD is that the curve increases above unity at moderate to high levels of plasticity. The need to account for secondary stresses is especially true if it is demonstrated that crack extension initiates before widespread plasticity.

Comparison of the developed $g$ approach to the finite element results and the R6 $V$ factor approach showed that the existing R6 approach can be overly conservative and that the $g$ approach is less so. The results also show that both the R6 and $g$ approaches demonstrate very little sign of non-conservatism for the finite element results here. This would,
however, be expected as the finite element results were used to define $g$. The issue of conservatism within the $g$ and R6 results is discussed further within Section 7, in conjunction with experimentally based comparisons in Section 6.

It is also possible to define a $V_g$ term that can be implemented within the R6 framework. It is this implementation that has been considered for the experimental implementation, as it is common to plot failure points upon the FAD and not assess experiments as a $J$-estimation scheme.

The work presented within this section is also summarised within a paper to the International Journal of Pressure Vessels and Piping [101], a copy of which can be found in Appendix 3.
5. Experimental Programme to Investigate Failure under Combined Loading at Different Levels of Plasticity

5.1. Preface

This section provides an overview of an experimental programme designed to assess the effect of combined primary and secondary stresses on crack extension under different levels of plasticity (i.e. over a range of \( L_r \) values). These experiments were designed such that the crack growth occurred under both brittle fracture and ductile tearing. The experimental programme was performed to extend the range of experimental measures of the interaction of secondary stresses with primary stresses with which the \( V_g \) approach presented in Section 4 could be validated. The experiments were also performed in conjunction with similar experiments conducted by Inspectra AB, Stockholm, Sweden [76]. Involvement at the design stage of these experiments and provision of material ensured exchange of results from both sets of experiments on similar experimental programmes. The main difference between the experiments detailed in this section and those in Sweden was the method used to change the fracture load. Here, one material was cooled to different temperatures to change the fracture toughness, and hence level of plasticity when the crack extended, whereas the tests in Sweden chose different materials and tested all at room temperature. It is also worth noting that the aim of the Swedish experiments was to consider plastic conditions only, and not necessarily test under elastic conditions. Details of the Swedish experiments are not provided within this section, but are detailed in Appendix A1 (also summarised in Section 2).

The review of experimental approaches to study failure under combined primary and secondary stress presented in Section 3 only included a limited number of experiments [77], [62], [78], [79], [28], [80], [81] [82–86]. Further, of these there even less experiments that cover a large range of \( L_r \) values from elastic conditions (low \( L_r \)) to fully plastic conditions (i.e. \( L_r > 1 \)). The aim of the experiments performed in this research was therefore to provide additional experimental data points over intermediate values of \( L_r \) so.
that the redistribution of secondary stress with primary stress induced plasticity can be considered.

Because of the load capacity of testing facilities and availability of suitable material it was not possible to use large specimens as used in [77], [62], [82–86]. Subsequently, the type of sample was limited to typical laboratory specimens for use of conventional testing facilities. Broadly speaking two approaches for inducing a residual stress field in a laboratory specimens are used: (a) the tests in References [77], [62] induce the residual stress by welding, and (b) the tests in References [78], [79], [28], [80], [81] induce the residual stress by mechanical pre-compression.

The test method adopted here uses the pre-compression method and specimen design similar to that developed by Mirzaee-Sissan [78]. This approach was chosen since a well-controlled residual stress field can be created in a standard fracture toughness specimen. As shown below the stress field can easily be measured and predicted with good accuracy.

5.2. Overview of Experimental Programme

5.2.1. Specimen geometry

The specimen geometry used in this work was modified from that used by Mirzaee-Sissan [78]: a scallop notched-bend specimen with a crack at the root of the notch as in Figure 54. The specimens were refined as follows:

- The pre-compression notches on the ends of the specimen were made into round notches to prevent shearing of the corner from the root of the sharp V-notch.
- The specimen thickness was increased from 10 mm to 25 mm to prevent the possibility of buckling under compression.
- The radius of the scalloped notch was larger to increase the volume of the tensile residual stress field.
Based on the results of a series of design analyses described in the next section, a crack length of 3.5 mm below the notch root was employed. This ensured that the crack tip was located a sufficient distance from the notch root to minimise any interaction between the crack tip and the free surface and shallow enough to minimise the stress redistribution due to first inserting the notch and then the crack. The specimens contained an Electric
Discharge Machined (EDM) notch length of 2 mm inserted to the specimen before the crack was sharpened and grown by 1.5 mm to the desired length by fatigue loading. The drawing detail for the EDM notch can be seen in Figure 55.

5.2.2. Material

The material used in this study was low alloy ferritic steel, A533B-1. The steel, prior to sectioning, was in plate form as manufactured by Japan Steel Works in 1991 [102]. The plate was 70 mm thick and originally 2.5 m² square. As sections of this plate were adopted for other tests in the intervening period the remaining section of plate was 1280 by 650 mm². Cross-rolling to manufacture the plate followed the initial forging process. The plate then went through a heat treatment including heating to approximately 900 °C for 4.25 hours before being water quenched, tempered at approximately 650 °C for 6.7 hours and then allowed to cool in air. The chemical composition specified during manufacturing for the plate can be seen in Table 5.

Table 5 – Chemical composition of A533B plate [102]

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>Ni</th>
<th>Cr</th>
<th>Cu</th>
<th>Mo</th>
<th>V</th>
<th>Al</th>
<th>Co</th>
<th>As</th>
<th>Sn</th>
<th>Sb</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.21</td>
<td>0.26</td>
<td>1.40</td>
<td>0.006</td>
<td>0.66</td>
<td>0.10</td>
<td>0.04</td>
<td>0.50</td>
<td>&lt;0.003</td>
<td>0.02</td>
<td>0.012</td>
<td>0.003</td>
<td>0.004</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The specimens were all manufactured from the same plate of A533B. A sectioning drawing for the plate is shown in Figure 56. All specimens were provided with a unique identifier number such that the location of all the specimens within the original plate could be traced. Sections of material were also used for providing tensile specimens.

Two types of tensile specimens were employed, one standard sized (M12) specimen as shown in Figure 57 and one half the size specimen (M6). The full sized specimen was orientated along the cut sections, L-direction, whilst the other was in the through thickness direction of the plate which meant a full sized specimen could not be used. Specimens in the short transverse, T-direction, were not considered. This was because the plate manufacturing process included cross-rolling, meaning the materials data in these directions should be identical.
5.2.3. Tensile Tests

Tensile properties are reported in Reference [103], although additional tensile tests are described here. These were performed according to BS EN 10002-5:1992 standard [104] to provide full stress-strain curves at four temperatures. These temperatures reflected the temperatures at which the fracture mechanics tests were performed, as based on the finite element analyses detailed below. A minimum of three tensile tests was performed at each temperature and orientation.
Tensile tests were performed on round bar specimens extracted in the crack opening direction, Figure 57, at a strain rate of 0.0083% per second and were interrupted once the ultimate tensile strength (UTS) had been achieved. The material properties in the through thickness direction (of the original plate) were also performed at a strain rate of 0.0083% per second and were interrupted once the ultimate tensile strength (UTS) for the half sized specimens was achieved. Figure 58 through to Figure 61 show the measured true stress versus true strain data for all temperatures considered.

In addition to the tensile tests, compression-tension tests were performed in the crack opening direction at room temperature to compress the material to strains of 1.0 and 2.0% followed by a tensile test to the UTS at a rate of 0.004% per second. Finally, one set of low cycle fatigue samples underwent a 30 compression-tension cycles to ± 1.5% strain at room temperature to form the material’s hysteresis loop, at a rate of 0.002% per second. These compression tests were performed to consider the material’s hardening characteristics and are compared with finite element predictions below. These tests were only performed in the crack opening direction.

Figure 58 through to Figure 61 show that the material exhibited Lüders strain following yield at all temperatures. The presence of Lüders strain is characteristic of normalised ferritic material in tensile tests. An interpretation of the stress-strain curves is required which does not include the Lüders strains for implementation in the finite element analyses. A number of approaches can be considered for this;

1) the French fracture code RSE-M [60] suggests undercutting the stress-strain curve at 85% of the yield stress and joining this, by a straight line to the end of the Lüders strain;

2) the Lüders band can simply be removed and the stress-strain curve “shifted” to the left; or

3) a curve fit can be produced which covers the initial yield and re-joins the stress strain curve at a later stress.

The first of these is considered to be inappropriate for trying to predict the most realistic materials response, as this will not accurately capture yield. It is also likely that a unique curve cannot be defined, as the magnitude of the Lüders strain will also alter from test to
test. The second case might initially seem the most appropriate case but it is difficult to properly interpret the start and finish of the Lüders strain effect and how this might alter the onset of yielding. It is therefore the third option that has been considered here, where a curve fit is applied to fit the initial yielding region and to re-join or form a close tangent to higher strains. It is noted that this solution has somewhat been confirmed by comparison to the cyclic tensile tests discussed in Section 5.3.2 below and by comparison of the finite element analyses to the test measurements. The curve fit made, by use of a Ramberg Osgood curve fit (as defined in Equation 66), to the tensile measurements at each temperature has also been included in Figure 58 through to Figure 61. The fit properties used are shown below in Table 6.

**Table 6 – Ramberg Osgood curve fit to tensile curves**

<table>
<thead>
<tr>
<th>Temperature, $T$ (°C)</th>
<th>Young's modulus, $E$ (MPa)</th>
<th>Yield Stress, $\sigma_y$ (MPa)</th>
<th>Strain Hardening Index, $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>203000</td>
<td>500</td>
<td>10.0</td>
</tr>
<tr>
<td>-50</td>
<td>216000</td>
<td>540</td>
<td>9.2</td>
</tr>
<tr>
<td>-90</td>
<td>219000</td>
<td>560</td>
<td>8.5</td>
</tr>
<tr>
<td>-150</td>
<td>228000</td>
<td>700</td>
<td>15.0</td>
</tr>
</tbody>
</table>
Figure 58 – Tensile test results at -150 °C for M12 specimens in the crack opening direction (black), the smaller M6 specimens parallel to the crack and a Ramberg Osgood curve fit

Figure 59 – Tensile test results at -90 °C for M12 specimens in the crack opening direction (black), the smaller M6 specimens parallel to the crack and a Ramberg Osgood curve fit
Figure 60 – Tensile test results at -50 °C for M12 specimens in the crack opening direction (black), the smaller M6 specimens parallel to the crack and a Ramberg Osgood curve fit.

Figure 61 – Tensile test results at 20 °C (room temperature) for M12 specimens in the crack opening direction (black), the smaller M6 specimens parallel to the crack and a Ramberg Osgood curve fit.
5.2.4. Compressive Pre-Load

Two sets of fracture tests were performed. In the first, a compressive pre-load was applied at the ends of the specimens such that a tensile residual stress was introduced into the material ahead of the notch after unloading. The pre-load was applied to specimens at room temperature at a rate of 0.5 mm/min until a displacement of 4 mm was measured at the notch mouth using a calibrated clip gauge. The level of compression was considered through sensitivity studies within finite element analyses of the test process outlined further in Section 5.3. A photograph of the experimental set-up is shown in Figure 62 prior to compression and in Figure 63 after compression. The loading was applied with a Schenck-Trebel 250 kN uniaxial servo-electric test machine controlled via a desktop computer interface.

![Figure 62 – Photograph of pre-compression specimen set-up prior to compression.](image-url)
The EDM notch was introduced to 2 mm after unloading and a sharp crack grown by fatigue using a $\Delta K$ of 21 MPa$\sqrt{m}$ and an R-ratio of 0.1 to get the total desired total crack size of 3.5 mm. The size of the crack considered was also determined by the finite element analyses described in Section 5.3. The magnitude maximum value of $K$ used in the fatigue crack growth was sufficiently high to ensure the crack grew quickly and controllably but was less than half the secondary stress intensity factor predicted by finite element analyses from the residual stress field. The fatigue crack growth was undertaken using an Amsler electro-resonant Vibrophore.

In the second set of specimens, no pre-load was applied. The specimens contained no residual stress and are referred to as “as-received” specimens. For consistency the crack
was also grown to 3.5 mm by the same process of a 2 mm EDM notch followed by fatigue crack growth using a $\Delta K$ of 21 MPa$\sqrt{m}$ and an $R$-ratio of 0.1 for the subsequent 1.5 mm.

### 5.2.5. Measurement of Residual Stress

The residual stress field was quantified in three ways. First, Digital Image Correlation (DIC) was used to measure the displacement of material in the vicinity of the notch root during preloading. Two cameras were positioned approximately 560 mm from the surface of the specimen and focussed on the material in the vicinity of the notch root with a field of view of 70 by 80 mm. A series of images were captured throughout the pre-loading and unloading with a camera resolution of 2,000 by 2,400 pixels. The Vic3D System post-processor was used to derive displacement vectors in both in-plane and out-of-plane directions with a spatial resolution of approximately 5 $\mu$m. Normalising the displacement by the cell size then led to derived principal strains. The resultant strain field was then compared with the results from three-dimensional finite element analysis described below. It was this analysis that was used as the means to relate the measured surface strain field to a calculated residual stress field.

Secondly, neutron diffraction was performed on a pre-loaded, but uncracked, specimen at the ENGIN-X instrument on the ISIS beam-line at the Rutherford Appleton Laboratory. Neutron diffraction was used as it allows the residual stress to be quantified below the surface of a metal so that, for this specimen, measurements could be taken through the entire thickness. The residual stress is measured by determining the intensity and angular distribution of a number of detection banks positioned such that the central detector is normal to the incident beam, thus allowing the measurement to be focused to a specific location. The angular distribution of scattered neutrons is then examined as a diffraction pattern following Bragg’s law to calculate the lattice spacing, and hence localised strains, of the metallic structure when compared to an equivalent, unstrained, section of metal referred to as the “$d_0$” measurement. This localised strain can then be converted to a stress by assuming an elastic response for the “gauge volume” over which the measurements are made [105]. For the tests performed at ENGIN-X a gauge volume of 2 mm over the initial 10 mm from the root of the notch and 4 mm over the remaining ligament and a “$d_0$” measurement was taken from an unstrained section of material near the materials edge.
Such a relatively large gauge volume was used for efficiency as time on the beamline was considerably restricted. Ideally a volume size of less than 1 mm would be used for added refinement but this would have significantly increased the time taken as both the required number of locations and time at each location would have increased. The samples were positioned by use of optical theodolites focused on the incident location of the beamline and focus of the bank detectors.

Finally, a measurement of the strain 2 mm ahead of the notch on the specimen centre-line was performed using the incremental surface hole drilling method on two similarly compressed specimens. The basic premise of the method is to relieve the residual stress, by way of drilling into the material, so that surface strains can be measured by strain gauges on the materials surface as material is incrementally removed. These deformed stains can then be used to calculate the original strain field that can be related to stress by assuming standard, elastic relations [105].

5.2.6. Test Temperatures

In order to change the level of plasticity in the specimens when the crack extended the material was cooled to different temperatures. Fracture toughness tests were performed on as-received and preloaded specimens at temperatures of -150, -90 and -50 °C. Note that these temperatures were defined from a series of design analyses described below in the finite element analyses. The temperature of each specimen was measured via a thermocouple spot-welded to the surface a few millimetres from the crack tip. The test temperature was controlled to within ±2 °C. Test temperatures were chosen to explore residual stress effects on failure at three differing levels of plasticity as defined by the \( L_r \) parameter: (a) \( L_r < 0.3 \), (b) \( 0.4 < L_r < 0.8 \) and (c) \( 0.9 < L_r < 1.2 \). The temperature range spans the lower to upper ductile-to-brittle transition temperature range. Specimens tested at -150 °C were expected to fail by cleavage with no prior ductile tearing. Specimens tested at -90 °C were expected to exhibit some plasticity prior to cleavage and specimens tested at -50 °C were expected to fail by ductile tearing. Failure in all tests was defined by the initiation of crack growth, with an estimate of this provided by the material \( J - R \) curve where required (note that as the assessment was performed in terms of \( K, K_f - R \) curves are used for easier comparison).
5.2.7. Fracture Test Method

All specimens were tested under three-point bending at a loading rate of 0.5 mm/min and were in accordance with ASTM E 1820-08 [106]. The fracture toughness tests were performed on the same 250 kN Schenck-Trebel test machine as used to perform the in-plane compression. A picture of the fracture tests, after cooling and just prior to testing, can be seen in Figure 64.

![Figure 64](image)

**Figure 64 – Pre-compressed specimen under three-point bend within an environmental chamber cooled to the test temperature**

The applied load and displacement were continuously measured during each test; displacement being measured via a clip gauge attached to knife-edges positions at the notch mouth. Tests were performed within an environmental chamber cooled to the required test temperature. The specimens were placed into the chamber and the surface temperature of the material monitored. When the sample cooled to the specified temperature a three-point bend load was applied to the specimen and increased to failure. For the cases where cleavage fracture was not expected, i.e. at -50 °C, the specimens were allowed to tear by differing amounts so that a multi-specimen $K_f - R$ curve could be constructed.
In all cases, the value of $K_I$ at failure was derived from the results of 3D elastic-plastic finite element analyses as described in the following section.

After testing, each specimen was placed in methylated spirits and warmed to prevent environmental corrosion of the fracture surface. Upon completion of the tests, the samples were fully cracked open and the initial crack length and subsequent ductile tearing for each specimen measured using a computer numerically controlled (CNC) travelling microscope. The distance from the front face of the specimen to the end of the fatigue pre-crack was measured at nine equally spaced points across the thickness of the specimen. The outer points (i.e. the points nearest to the specimen surface) were taken a distance 1% of the specimen thickness from the specimen surface. Any ductile crack growth at each of the nine points was also measured using the same 9-point averaging approach. This allowed an average measure of crack size, and any associated crack growth, to be obtained for each sample.

5.3. Finite Element Analyses

5.3.1. Finite element model

Two finite element models were used to simulate the behaviour of the test specimen illustrated in Figure 54. Both models were developed using Version 6.9 of the ABAQUS finite element code [107]. The two models were 2D and 3D representations of the specimen such that the 2D model could be run relatively quickly when considering scoping calculations and the 3D model for an improved measure of the specimen response and for comparison to the measurement of strain at the outer surface.

The 2D model comprised 8-noded plane strain reduced integration elements, ABAQUS type CPE8R. The model simulated one half of the specimen with appropriate symmetry boundary conditions applied to nodes located on the uncracked ligament. An illustration of the model can be seen in Figure 65 along with the rigid surface loading pins. The crack tip was modelled with a focused mesh containing elements with degenerate nodes at the
crack tip to correctly model both blunting and the crack tip stress field, following guidance in [98], [107], which is further illustrated in Figure 66. The minimum dimension of elements directly ahead of the crack tip was 100 µm by 5 µm.

The 3D brick model utilised 20-noded reduced integration elements, ABAQUS type C3D20R. The model simulated one quarter of the specimen with appropriate symmetry boundary conditions applied to nodes located on the uncracked ligament and at the specimen centre. This model, illustrated in Figure 67, highlights the increased mesh refinement in the vicinity of the notch root, the crack tip and at the loading points. The crack tip was modelled with a focused mesh containing elements with degenerate nodes at the crack tip to correctly model both blunting and the crack tip stress field, following guidance in [98], [107]. The minimum dimension of elements directly ahead of the crack tip was 250 µm by 10 µm. In both models the mesh density at the contact locations of the loading pins was increased to ensure correct transference of load from the pins.
Figure 66 – Figure of the 2D finite element mesh crack tip in undeformed and deformed states with degenerate crack tip nodes allowing displacements representative of crack tip blunting under elastic-plastic conditions.

Figure 67 – Figure of the 3D finite element mesh showing boundary conditions and the focused mesh around the crack tip.
5.3.2. Material properties

The tensile material properties adopted for the tests was taken from the Ramberg Osgood curve fit as defined by Table 6 and Poisson’s ratio, $\nu$, was set to 0.3 as defined in Reference [103]. Previous experimental and numerical studies on an A533B-1 material by Lee et al [79] has revealed that A533B-1 material exhibits a Bauschinger effect when it is pre-loaded to cause yield in compression and then re-loaded in tension, thus requiring a kinematic or mixed isotropic-kinematic hardening approach. The results from the compression-tension material tests and the low cycle tests were used to assess the behaviour of the kinematic hardening model. A simple single-element model was used to assess the materials behaviour under both isotropic and kinematic hardening; isotropic hardening was included to assess if it was necessary to consider a mixed hardening model. The results of this comparison can be seen in Figure 68 at 1% and 2% compression before loading to failure (for the kinematic hardening case only). When the single element was cycled the results compared to the cyclic data can be seen in Figure 69 for the kinematic hardening model. Clearly the kinematic hardening model describes the hysteresis loop and tensile loading portions well, but not necessarily the initial compression, which demonstrates the Bauschinger effect. Nonetheless, this would be difficult to capture within standard hardening models. Therefore, only the kinematic hardening model has been used to describe the tests.
Figure 68 – Comparison of kinematic hardening model comparison to cyclic test data for the compression-load cycle

Figure 69 – Comparison of kinematic hardening model comparison to cyclic test data for the hysteresis loop (bottom)
Fracture toughness data were available from Reference [103] over the temperature range of -150 °C to +50 °C. The variation in lower-bound fracture toughness, $K_{mat}$, as a function of temperature is defined by Equation 79. Toughness estimates were also obtained from the as-received specimens for this material.

$$K_{mat} = 473.6e^{0.015T}$$

Equation 79

Figure 70 includes the estimate of fracture toughness by way of Equation 79 from the literature as well as the estimates from the tests outlined in the results section below (where the mean and lower bound estimates for the -90 and -50 °C cases are defined from $J - \Delta a$ curves in Section 5.4.3 and are hence below the results that allow for tearing).

Figure 70 – Change in fracture toughness with temperature from literature (red curve), individual test results (blue circles), mean estimate of fracture toughness from the tests (light blue triangles) and lower bound estimate of fracture toughness from the tests (red stars)
5.3.3. **Boundary conditions**

In addition to the symmetry conditions described above, boundary conditions were applied to simulate the compressive pre-loading, the unloading, and the three-point bend loading applied to the specimens. All loading was simulated via the circular rigid surfaces illustrated in Figure 65 or Figure 67 (depending on the model) that represented the pins used to apply loads in the tests. During the compressive pre-loading a displacement of 2 mm was applied to the end-loading rigid surface in the direction of the notch; note the investigation to determine the compressive load is outlined below. The rigid surface was positioned at grooves on the end of the model allowing the pre-compression to be applied by rigid surface contact without inducing significant shearing at the corners. The compressive pre-load was subsequently removed by defining a displacement of 10 mm to the rigid surface in the other direction.

Three-point bending was simulated by a second set of circular rigid surfaces located centrally on the symmetry line directly under the notch, and on the upper surface of the model towards the right-hand edge. Negligible load was applied to these rigid surfaces to ensure contact throughout and that no spurious restraining boundary condition was applied to the model during pre-loading. Fixing the position of the central rigid surface, and applying a vertical displacement to the rigid surface located on the upper surface simulated three-point bending of the specimen.

All contact was considered using “frictionless” contact. A short sensitivity study was performed with “hard” contact\(^{10}\) that did not show significant differences in the results.

5.3.4. **Design analyses: crack length**

A short investigation was performed to consider the length of the crack to be inserted to the notch root. The length of the pre-crack was chosen to ensure that: (a) the shielding influence of the notch was minimal, but (b) the crack did not significantly relieve the residual stress. To determine the appropriate crack length to use, the 2D plane strain model

\(^{10}\) where hard contact is represents an infinite coefficient of friction
was used to calculate the elastic stress intensity factor for cracks of increasing length inserted into the model with primary tensile loading applied. The resultant values of elastic $K$ were compared with the handbook (R6) solution for a standard edge cracked plate, i.e. without the scalloped notch, when loaded in tension [3].

Figure 71 shows a comparison between the finite element analyses and handbook solutions. The solid circles give the R6 solution, and the open circles are the finite element results. The shielding influence of the notch on $K$ can be seen by the large difference in derived $K$ for effective crack lengths between 17.5 and 20 mm. The effective crack length used was 21 mm, i.e. 3.5 mm below the notch root. This gives a stress intensity factor that is within 10% of the R6 solution [3], shown by the dashed curve in Figure 71.

![Figure 71 – Comparison of the variation in stress intensity factor as a function of crack length derived from the R6 handbook solution [3] and 2D finite element analyses with a nominal applied tensile stress of 10 MPa.](image)

These analyses also provided elastic validation of the 2D model, showing that the derived stress intensity factor was within one percent of an appropriate handbook solution. When the nominal crack depth is adopted within the 3D model outlined above the agreement with
handbook stress intensity factor solutions was unchanged and thus helps validate the 3D model.

5.3.5. Design analyses: compressive pre-load

With the crack size determined as above, it was then required to determine the level of pre-compression applied to the specimen. This was achieved through use of plane strain 2D finite element analyses with differing levels of pre-compression applied through displacement control of the loading pin. In the analyses, as only half the geometry was modelled, the level of displacement of the loading pin was doubled to represent the same compression in the tests where only one pin displaced, not two. The resultant residual stress fields after pre-compression can be seen in Figure 72 for pre-compression levels of 0.5, 1, 2, 4, 6 and 8 mm (as would be measured in the tests). The plot shows that for lower levels of compression, i.e. the 0.5 mm compression, the residual stress field is much lower than for other levels of compression with a maximum value of 400 MPa \(0.8 \sigma_y(T = 20^\circ C)\) at the root of the specimen notch and not increasing above 200 MPa \(0.4 \sigma_y\) for the remainder of the ligament. As the level of compression is increased a maximum stress at the notch root is seen in the 1 mm compression case of approximately 450 MPa \(0.9 \sigma_y\) which decreases as the level of compression is increased. However, as the level of compression is increased the size of the resultant tensile region at the notch root increases to a maximum tensile length of \(0.25 \frac{u}{(t - a)}\), where \(u\) is the distance along the ligament.

The lowest level of compression seen to have this size of tensile zone is the 4 mm compression case, which also has similar profiles to the 6 and 8 mm compression cases over the remaining ligament.

The reason for the observed initial increase and subsequent decrease in maximum stress with applied compression can be explained by the degree of yielding under compression. The 0.5 mm compression does not provide significant yielding on compression and, hence, is closest to an elastic case where a residual stress field will not exist. However, the larger levels of displacement induce such high levels of compressive yielding that, on unloading, the reverse yielding of the kinematic hardening model prevents the stress from increasing to the levels seen for some of the lower levels of compression. This therefore means that
the 2 mm or 4 mm compression should provide the highest elastic-plastic stress intensity factor.

![Graph showing stress tangential to the crack for different compression levels](image)

**Figure 72 – Resultant residual stress field after compression to different total loading pin displacements**

The elastic-plastic stress intensity factors resulting from the residual stress fields, $K_f^S$, can be seen in Figure 73, where the JEDI postprocessor was required to account for the initial stress field providing non-proportional loading effects. Note that information regarding the derivation of $K_f^S$ from the finite element analyses is considered further in Section 5.3.7.

As predicted from the residual stress profiles the two highest estimates of $K_f^S$ are the 2 mm and 4 mm compression cases. It is also clear that there is a large initial increase in $K_f^S$ up to 2 mm compression but then only a minor decrease in $K_f^S$ with more applied compression. To ensure a large value of $K_f^S$, required so that the contribution from the residual stress is significant, a level of compression of 4 mm was taken in the tests. This is because the estimate of $K_f^S$ is only slightly less than the maximum estimate but is not likely to vary significantly with potential material variations as the compression at 1 mm or 2 mm would.
To provide an improved measure of the effects of pre-compression, the 4 mm compression was also applied to the 3D model. The residual stress field resulting from the 3D model can be seen in Figure 74 which shows that the 3D model does not achieve plane strain conditions, even at the centre of the specimen, such that the residual stress results in a lower magnitude at the notch root.

When inserting a crack into this residual stress field, for the chosen crack length and pre-compression within the more representative 3D model, the value of the elastic-plastic crack driving force for secondary loads alone, $K^S_f$, was found to be 35 MPa.m$^{0.5}$ at the centre of the specimen. Again, the JEDI [25] post-processor to ABAQUS [107] was used to account for the presence of plastic strains prior to the cracks insertion. The difference between this value and the estimate of approximately 42 MPa.m$^{0.5}$ from the 2D model can be explained by the difference in the residual stress field under the more realistic conditions within the 3D model compared to the 2D plane strain model (shown in Figure 74) which allows the stresses and strains in the thickness direction to be accounted for, thus modifying the stress at which the material yields and hence the resultant stress field.

Figure 73 – Resultant elastic-plastic secondary stress intensity factor after compression to different total loading pin displacements.
5.3.6. Design analyses: test temperature

Test temperatures were chosen to ensure that failure occurred at different levels of $L_f$. The change in failure load, and hence $L_f$, can be linked to the change in material fracture toughness given by Equation 79, which is further shown in Figure 70 by the red curve. Three temperatures were chosen to test material properties that covered a range of fracture toughness values. The temperatures chosen were -50 °C, -90 °C and -150 °C which corresponded to lower bound fracture toughness values of approximately 50, 125 and 225 MPa m$^{0.5}$ which should provide a large difference in the load at fracture, and hence level of plasticity and mode of fracture. These temperatures therefore provided the values at which to test the material properties, which should also allow the effects of plasticity to be assessed.

Nonetheless, to fully assess the likely values of $L_f$ at fracture at these temperatures and show a difference in the failure mode, a series of R6 [3] handbook calculations was performed to simulate the tests at temperatures in the range -150 to +50 °C. The material
properties were interpolated between the measured values over the range of temperatures and the Option 2 failure assessment curve adopted as the plasticity enhancement factor. The value of $L_r$ at failure was derived from the load at which the elastic-plastic crack driving force was equal to the material fracture toughness defined from [103]; i.e. implementing a $J$ estimation scheme. The estimate of $K_f^S$ was taken as 35 MPa m$^{0.5}$ from the 3D finite element model (note that if $K_f^S$ is provided $K_f^P$ is not required), $K_f^P$ and the elastic-perfectly plastic limit load, $P_L$, were taken from conventional three-point bend specimen solutions. The plasticity correction factor was taken as the complex $\rho$ of R6 [3]. It is noted that these simple calculations are approximate for a number of reasons; $K_f^S$ was not assumed to change with cooling; the notch was not assumed to change $K_f^P$ or $P_L$ and the plasticity correction of the Complex $\rho$ is conservative. An illustration of this simple assessment can be seen in Figure 75.

Figure 75 – R6 prediction of failure of pre-compressed specimens at a range of different temperatures, T (T in °C)

This assessment shows that at temperatures of -150, -90 and -50 °C a conservative estimate of failure is approximately $L_r = 0.1, 0.6$ and 1.0, respectively. Although it is considered likely that the tests may failure at a slightly higher estimate of $L_r$ these temperatures should provide failure conditions within the elastic, contained plasticity conditions (small
scale yielding) and gross yielding (uncontained plasticity) conditions. This should help provide a suitable assessment of the $V_g$ plasticity interaction term.

5.3.7. Derivation of J

The estimates of crack driving force $J$ have been obtained from the JEDI [25] post-processor to ABAQUS [107] such that the initial residual stress field and any non-proportional loading effects can be correctly accounted for. The path dependence in the ABAQUS calculation can be seen in Figure 76 compared to the path independence of the JEDI estimate for the estimate of $K_f^P$ under differing levels of compression. It can clearly be seen that the non-proportional loading resulting from the mechanical pre-compression has a significant effect on the ABAQUS estimates that is also related to the severity of the pre-compression. The ABAQUS estimates are also seen to become negative, which is clearly incorrect. The JEDI values, however, are relatively constant over the initial 10 contours and allow a good estimate of $K_f^P$. It is noted that the ABAQUS estimates are, actually, relatively close to the JEDI estimates within the 1st contour. The reason for this is not clear but it is possible that the energy at this point is translated directly the localised change in strains (i.e. follow proportional loading) which helps minimise any contribution from the initial stress field.

The adopted values of $K_f$ used in the analyses were obtained from Equation 80 below from the estimates of $J$ of the finite element analyses. The estimate of $K_f^P$ is obtained after the pre-compression and cooling but prior to the three-point bend load. The values for $K_f^P$ are found with application of the three-point bend load alone, such as for the as-received samples. Values of $K_f$ are found when the three-point bend load was applied following the pre-compression.

$$K_f = \sqrt{E'J} \quad \text{Equation 80}$$

It is further noted that for cases where there was no pre-compression of the sample, i.e. the estimates of $K_f^P$, the values of $J$ defined from ABAQUS and JEDI were the same.
Figure 76 – Contour dependence of $J$ from ABAQUS and JEDI estimates with the crack inserted to a residual stress field generated under different levels of mechanical pre-compression.

5.4. Results

5.4.1. Characterisation of residual stress and strain fields

The measured displacement fields were converted to the measured strain fields as measured from the DIC on the surface of the specimens. These can be seen in Figure 77 and Figure 78 at the maximum compression and for the residual strain field after the compression is removed. The results are qualitatively similar and show how the resultant strain field is negative, with a strain of more than 7%, at the notch root region. This compressive strain field extends to just over half of the remaining ligament. The strain at the back face of the specimen has a tensile strain of over 4%. This comparison shows that the predicted strain field and the measured strain field follow very similar spatial
distributions where the neutral axis is positioned at the same location and the contours of strain extend in the crack opening (x) axis to a close approximation.

A quantitative comparison of the DIC and finite element results can be seen in Figure 79 for the strains in the crack opening direction (x) and crack growth direction (y) when taken along a path defined by the uncracked ligament. The DIC results are shown as curves and the finite element results as the data points. There is good agreement between the predicted strain fields with no points more than 5% from the measured values. This observed correlation therefore provides good validation to the finite element model and the kinematic hardening model for the A533B-1 material used.

The predicted distribution of opening mode residual stress, $\sigma_{xx}$, generated ahead of the notch as a result of pre-loading is illustrated in Figure 80. This includes results taken along the surface and at the centre of the specimen, at a depth of 12.5 mm below the surface. Comparisons with measurements made by neutron diffraction and hole drilling are also shown in Figure 80. The results show that there is good agreement between the neutron diffraction measurements and the results from the 3D finite element model, for normalised depths greater than 5 mm with, perhaps, a slight over prediction of the maximum compressive stress. The difference in this maximum compressive stress region may be a result of sampling a large volume in the neutron diffraction experiment. For normalised depths closer to the notch, i.e. within 5 mm, the neutron diffraction data are greater than the predicted values. It is unclear what the effect of significant plastic strain in this initial region would be which could modify the $d_0$ measurement. Nonetheless, taken altogether, the results provide strong evidence for the validity of the finite element model and hardening material model.
Figure 77 – Digital image correlation measured strain field in the vertical axis, $\varepsilon_{xx}$, (top) compared to the predicted strain field from the 3D finite element analyses, $\varepsilon_{xx}$, (bottom) at maximum compressive load
Figure 78 – Digital image correlation measured residual strain field in the vertical axis, $\varepsilon_{xx}$, (top) compared to the predicted residual strain field from the 3D finite element analyses, $\varepsilon_{xx}$, (bottom) after compressive load
Figure 79 – Predicted residual strain field ahead of scalloped notch compared to DIC results

Figure 80 – Neutron diffraction and surface hole drilling measurements of residual stress compared to finite element predictions moving away from the scalloped notch.
Figure 80 also includes the results from shallow hole drilling, illustrating the average stress measured on two specimens, with the associated error ranges. This measurement of the residual stress near the surface shows good agreement with the stress field predictions at both the surface and centre of the specimen.

5.4.2. Load verses clip gauge displacement comparisons

The load versus displacement curve for the as-received specimen can be seen in Figure 81 for the specimen tested at -150 °C. As expected the test shows very little plastic deformation. The plot compares the finite element prediction as red points and the experimental result as a grey line. Very little variation was seen within the experiments and hence only one line is shown. The comparison shows a reasonable agreement between the test and finite element prediction, with the worst agreement remaining within 5% of the test.

Figure 82 shows the load versus displacement curve for the as-received specimen tested at -90 °C. This plot clearly shows significant levels of plastic deformation, with the onset of plastic deformation at 60 kN and gross plastic deformation between 80 and 90 kN. The figure shows the upper and lower bound test results as the solid grey and black lines, and the finite element predictions at the centre and outside of the 3D model as the green and red points (note that these are coincidental). The match between the finite element and test results is very good until an applied load of 80 kN, just as gross yielding occurs. The reason for this difference is likely to be both the characterisation of the material ultimate tensile stress and crack growth. It is likely that, as some level of crack growth was seen in all as-received cases before the maximum load, the crack length changed over the gross yielding region, thus changing the compliance of the specimen and causing the results to differ still further. In the finite element analyses, as this is also used for J estimation purposes, the stress strain curve was extrapolated beyond what would be the failure strain. The use of non-linear geometry should also be considered if trying to provide an exact match. However, when performing the analyses under non-linear geometry conditions the analyses would not solve under the gross levels of displacement seen here. There are approaches that can be used to provide such an estimate and to account for crack growth,
such as re-meshing the model partway through the analyses, but this was considered unnecessary for these cases, as the agreement to the onset of gross yielding was good.

The comparison of clip gauge displacement against load for the experimental and finite element results for the -50 °C case can be seen in Figure 83. The comparison shows good agreement under elastic response but the finite element results are seen to yield slightly earlier than the tests. It is further noted that the results do converge at higher loads once the material becomes fully plastic. This earlier onset of yielding could be explained by the over prediction of the tensile curve from the measured tensile behaviour when trying to remove the Lüders strain effect; it is however noted that the load versus displacement curve for the pre-compressed specimen detailed below does provide a good fit to the experiments when using this fit to the stress-strain data. This would further suggest some Lüders strain effect on initial loading within this larger specimen.

Given the good match of material behaviour and the clip gauge displacement with load, it can be concluded that these results show that the finite element analyses provides a suitable means to estimate the primary crack driving force, \( K_f \), on loading. This can then be used to estimate the fracture toughness from the as-received specimens.

The pre-compression of the samples was governed by the clip gauge displacement and, hence, allows this to be to a further compare experimental data with the finite element analyses. The clip gauge displacement under specimen compression from the tests can be seen in Figure 84 as the solid line compared to the finite element prediction shown as the open data points. Note that all test results matched this curve to within 2.5%. The results can be seen to follow the same features as the compress-load tensile samples in Figure 68; they provide a good match of the hysteresis effect on unloading, and follow the loading curve well after plasticity, but do not match the finite element analysis so well at the initial onset of plasticity. This is likely to be a result of the stress-strain curve being slightly different under compression, for the initial cycle, or a further effect of Lüders strain on initial loading. The results, despite this, are considered to provide a suitable match to the analyses.
It was also possible to provide comparison of the clip gauge displacement under three-point bend loading of the pre-compressed samples. These results can be seen in Figure 85 and Figure 86 for the specimens tested at -90 and -50 °C respectively (note that the results from tests performed at -150 °C are not shown as these remain elastic). The data from tests performed at -90 °C show good, although not exact, agreement with the finite element prediction. The match is always within 5% over all applied loads. The most significant differences are seen at higher loads, under most significant plasticity. This is the region where the match is expected to be least good as the combined effects of crack growth and stress-strain curve extrapolation are most significant here. The comparison of the data generated at -50 °C with the finite element analyses results shows very good agreement, within 2.5% over all applied loads. It is therefore considered that the finite element results show a consistently good correlation with the test results. This therefore means that it is possible to adopt the finite element estimates of crack driving force at a given load to correspond to those from the tests.

Figure 81 – Load versus clip gauge displacement for as-received specimen at -150 °C compared to finite element prediction
Figure 82 – Load versus clip gauge displacement for as-received specimen at -90 °C compared to finite element prediction

Figure 83 – Load versus clip gauge displacement for as-received specimen compared to finite element prediction at -50 °C
Figure 84 – Load versus clip gauge displacement under pre-compression at room temperature compared to finite element prediction

Figure 85 – Load versus clip gauge displacement for pre-compressed specimen at -90 °C compared to finite element prediction
5.4.3. Fracture Tests

The results for all tests are illustrated in Figure 87 and Figure 88. These show the applied load at failure and the measured values of $K_I^p$ as a function of temperature, where the open and solid symbols are the as-received and pre-compressed specimens, respectively. The results are also summarised in Table 7 and Table 8. The $K_I^p$ values shown here were derived using the results from 3D finite element analysis of the as-received specimen for the applied primary load. The fracture toughness data presented only represents the crack driving force due to primary (applied) load, $K_I^p$. It is noted that the conventional experimental measure of $K_I^p$ for a three point bending specimen was not appropriate to this non-conventional geometry, i.e. one containing a large blunt notch. It was therefore necessary to deduce a value from finite element analyses for the given load or clip-gauge value. Note that this is equivalent to defining a finite element based correction factor to the conventional approach.

The results show that the presence of a residual stress field has a significant effect on both the failure load and the value of $K_I^p$ at failure when compared with data from as-received
specimens. This is most pronounced at -150 °C where the average failure load with the residual stress (24 kN) is just under 50% of that without the residual stress (49 kN). As the temperature is increased the influence of the residual stress is seen to decrease so that the average failure load with the residual stress at -90 °C is 77% (20 kN difference in load) of that without the residual stress and there if no difference in load for the -50 °C cases. This clearly illustrates that the effect of secondary stress is still significant for the -90 °C cases but not for -50 °C cases once significant plasticity occurs. The effect of plasticity can be seen in the estimates of $K_f^p$ which again exhibit a difference of the residual stress cases to the as-received cases of 35 MPa.m$^{0.5}$ for the -150 °C cases, 78 MPa.m$^{0.5}$ for the -90 °C cases and a negligible difference for -50 °C cases. This helps to show that the relative contribution of secondary stress is enhanced under moderate levels of plasticity but removed under gross plasticity. Here it is noted that the -50 °C cases were interrupted at different stages of the tests but the load is not seen to change significantly between any of the specimens regardless of when the test was stopped.

The tests showed that there is a larger amount of scatter in the failure load at lower temperatures. A greater level of scatter in load would be expected for cleavage fracture as the cleavage event can be considered as probabilistically determined by the likelihood of finding a suitable initiating particle at a given load [108]. At higher loads the effect of plasticity is to remove this statistical dependence as such initiating particles will start to form voids. It is also noted that under plastic conditions there will be more strain for a small increase in load. These two effects of plasticity may cause the reduction in scatter for the pre-loaded specimens. Conversely the added strains at higher temperatures are captured in the estimates of $K_f^p$ which is why the higher temperature tests show a greater scatter in $K_f^p$.

The derived $K_f^p - R$ curves for the pre-loaded and as-received specimens are shown in Figure 89 at -50 °C and for the as-received specimens in Figure 90 at -90 °C. The -50 °C cases showed no significant difference in load or $K_f^p$ between the specimens with residual stress and the as-received cases and could be considered under the same statistical variation. The $K_f^p - R$ curves help confirm this with similar numbers of specimens from each group above and below the average fit line. Also included is a lower bound fit to the data generated by shifting the fitted curve to cover all failure points. The plot of the -90 °C
results only includes the as-received specimens as the residual stress specimens were seen to fail in cleavage whereas the as-received specimens showed some signs of ductile tearing. The degree of tearing was negligible but does allow a crude $K^F - R$ curve to be plotted which can also be shifted to provide a lower bound fit to the data.

<table>
<thead>
<tr>
<th></th>
<th>-150°C Residual Stress</th>
<th>-150°C As-Received</th>
<th>-90°C Residual Stress</th>
<th>-90°C As-Received</th>
<th>-50°C Residual Stress</th>
<th>-50°C As-Received</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1</td>
<td>25.82</td>
<td>29.58</td>
<td>55.19</td>
<td>89.42</td>
<td>88.43*</td>
<td>86.33*</td>
</tr>
<tr>
<td>Sample 2</td>
<td>20.29</td>
<td>45.79</td>
<td>71.69</td>
<td>89.29</td>
<td>83.88*</td>
<td>86.47*</td>
</tr>
<tr>
<td>Sample 3</td>
<td>25.85</td>
<td>65.86</td>
<td>71.61</td>
<td>84.91</td>
<td>85.50*</td>
<td>89.25*</td>
</tr>
<tr>
<td>Sample 4</td>
<td>27.56</td>
<td>62.94</td>
<td>79.69</td>
<td>88.23</td>
<td>87.43*</td>
<td>87.95*</td>
</tr>
<tr>
<td>Sample 5</td>
<td>19.84</td>
<td>39.81</td>
<td>63.44</td>
<td>91.94</td>
<td>88.10*</td>
<td>86.91*</td>
</tr>
<tr>
<td>Average</td>
<td>23.87</td>
<td>48.79</td>
<td>68.32</td>
<td>88.76</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The loads shown are where the test was stopped

**Figure 87 – Failure load for all specimens tested: open symbols are as-received samples; closed symbols are pre-compressed specimens, i.e. containing a residual stress**
Table 8 – $K_J^p$ at failure for all specimens

<table>
<thead>
<tr>
<th>Sample</th>
<th>-150 °C Residual Stress</th>
<th>-150 °C As-Received</th>
<th>-90 °C Residual Stress</th>
<th>-90 °C As-Received</th>
<th>-50 °C Residual Stress</th>
<th>-50 °C As-Received</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1</td>
<td>37.08</td>
<td>42.48</td>
<td>86.44</td>
<td>198.68</td>
<td>379.80</td>
<td>325.5141</td>
</tr>
<tr>
<td>Sample 2</td>
<td>29.14</td>
<td>65.75</td>
<td>123.95</td>
<td>197.89</td>
<td>267.22</td>
<td>329.9034</td>
</tr>
<tr>
<td>Sample 3</td>
<td>37.13</td>
<td>94.61</td>
<td>123.72</td>
<td>171.58</td>
<td>303.32</td>
<td>404.4957</td>
</tr>
<tr>
<td>Sample 4</td>
<td>39.58</td>
<td>90.35</td>
<td>148.53</td>
<td>191.49</td>
<td>354.90</td>
<td>366.506</td>
</tr>
<tr>
<td>Sample 5</td>
<td>28.49</td>
<td>57.17</td>
<td>103.44</td>
<td>216.58</td>
<td>370.66</td>
<td>344.1225</td>
</tr>
<tr>
<td>Average</td>
<td>34.28</td>
<td>70.07</td>
<td>117.21</td>
<td>195.24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 88 – $K_J^p$ at failure for all specimens tested: open symbols are as-received samples; closed symbols are pre-compressed, i.e. containing a residual stress
Figure 89 – $K_J - R$ curve for specimens tested at -50 °C

Figure 90 – $K_J - R$ curve for specimens tested at -90 °C
The resultant fracture toughness values from the as-received samples are shown in Table 9 and have previously been shown in Figure 70. The comparison of the lower bound predictions from the tests to the lower bound values from the literature is within 3% at -90 °C but under-predicts the curve at -50 °C and over-predicts the curve at -150 °C. The under prediction at -50 °C might be explained by the low number of tests. The lower estimate at -150 °C may be an erroneous result at the next lowest result is almost 35% higher.

<table>
<thead>
<tr>
<th>Table 9 – $K_{mat}$ for as-received specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{mat}$ (MPa.m$^{0.5}$)</td>
</tr>
<tr>
<td>-150 °C</td>
</tr>
<tr>
<td>Mean Test</td>
</tr>
<tr>
<td>Lower Bound Test</td>
</tr>
<tr>
<td>Lower Bound Literature</td>
</tr>
</tbody>
</table>

5.5. Discussion

5.5.1. Bauschinger Effect

Experimental and numerical studies by Lee et al [79] and Bordet et al [109] have demonstrated that A533B plate exhibits a Bauschinger effect when it is pre-loaded. This phenomenon tends to reduce the residual stress compared to an isotropic hardening model but is more closely aligned to a kinematic hardening model for the compressive pre-load and release used here. Measured residual stress from DIC, neutron diffraction and surface hole drilling are closely aligned to the kinematic hardening model predictions. These results show that the multi linear kinematic hardening model adopted in the analyses clearly provides good predictions for this material. Some of the predictions of load versus clip gauge displacement from pre-cracked tests and those for cyclic tensile tests do show some discrepancies in the region near the onset of yielding under compression but are observed to provide close agreement on subsequent loading. This may be a result of the
Bauschinger effect but as most tests are compressed beyond the onset of yielding this is of little significance overall.

5.5.2. Warm Pre-Stress (WPS) Considerations

That the tests are pre-loaded and cooled means that WPS is a potential concern within this work. The WPS effect is a feature of specimens that have been pre-loaded at a higher temperature and subsequently cooled before being tested to failure. The phenomenon is described as “the effective enhancement of the cleavage fracture toughness at low temperature following the application, at a higher temperature, of a stress intensity factor which exceeds the fracture toughness of the virgin material at low temperature” [109].

There are several methods available to predict WPS effects, such as those discussed by Bordet et al [109], Pickles and Cowan [110] and Section III.10.4 of R6 [3].

It is considered that the experiments performed in this research can neglect the effect of WPS because the secondary stress intensity factors, both before and after cooling, are lower than the material toughness for all cases. This means that there should be negligible WPS effects at failure. This also reflects the guidance in R6 Section III.10.4 [3]. Nonetheless, for added confidence the approach to estimate the WPS enhancement (Section III.10.4 [3]) within R6 was considered as shown below.

\[
K_f = K_2 + \sqrt{K_{mat}\Delta K_u} + 0.15K_{mat}
\]  

\textit{Equation 81}

where \(K_f\) is the failure stress intensity factor after WPS effects, \(K_2\) is the stress intensity factor before cooling (or unloading), \(K_{mat}\) is the fracture toughness at the temperature of interest and \(\Delta K_u\) is the change in stress intensity factor on unloading. In these experiments the finite element analyses has indicated no change in stress intensity factor on cooling, which means \(\Delta K_u = 0\), and the square root term is zero. If \(K_f > K_{mat}\), WPS needs to be taken into account. When Equation 81 is applied to the experiments here it leads to a potential WPS enhanced mean fracture toughness of 46 MPa.m\(^{0.5}\) and a lower bound toughness value of 41.5 MPa.m\(^{0.5}\) at -150 °C. As both of these estimates are less than the actual material toughness being considered WPS need not be considered at this temperature. From inspection of Equation 81, it can be seen that the greatest WPS effect is
predicted for the lowest fracture toughness and, hence, WPS effects can be neglected for all other temperatures considered also. It is also noted in the following section that excellent agreement to the failure load at -150 °C is found when using the value of \( K_{mat} \) when not considering WPS effects.

5.6. **Interim Conclusions**

An experimental program has been presented to quantify the fracture toughness of a laboratory specimen containing secondary residual stress from mechanical pre-compression. The aim of these experiments was to provide additional experimental data points at intermediate values of \( L_r \) so that the changing nature of the secondary stress contribution with primary stress induced plasticity could be considered.

The experiments have provided failure loads for the pre-compressed specimens that cover a range of \( L_r \) and provide significantly different values of primary stress intensity factor, \( K^p \), at failure. The influence of plasticity is reflected in the degree of plasticity observed in the load verses clip gauge displacement results and the amount of crack growth observed in the specimens. It is therefore considered that the experiments provide failure over different regions of plasticity interaction for the contribution of the secondary stress and meet the original aims of the experiments. Good qualitative correlation has been observed between the finite element predictions and direct measurements of the stress and strain field from DIC, neutron diffraction and incremental surface hole drilling when using a kinematic hardening material model in the finite element analyses. It is therefore considered that the material fit to the tensile data is valid, as are the range of finite element analyses considered to describe these experiments. It is therefore concluded that the values of crack driving force defined from these finite element analyses are valid.

Consideration of potential WPS and Bauschinger effect issues has also been discussed. WPS has been shown not to be of concern as at the most susceptible testing condition, which is that of the pre-compressed specimen at -150 °C, WPS effects as estimated by R6 [3] have been shown to be negligible. The potential for Bauschinger effects has not been excluded but comparison with the finite element analyses has shown good comparison indicating that any effect is adequately included in the kinematic hardening model. The
conclusion that can be drawn from this discussion is that the experiments can be considered a valid test for the $R_6 V$ Factor [3] and $V_g$ Function methods applied in the following section.

The work presented in this section, in combination with the application of the $R_6 V$ Factor [3] and $V_g$ Function methods applied in the following section, is also contained in a paper submitted to the International Journal of Pressure Vessels and Piping [111], which is also presented within Appendix 3.
6. Application of the \( V_g \) approach to the Available Experiments

6.1. Introduction

This section provides details of comparisons of the \( V_g \) approach developed within Section 4 to a range of experiments subject to the combined influence of primary and secondary stresses, including those detailed in Section 5. To provide comparison with existing approaches the \( V \) approach of R6 [3] is also included. Also shown, where possible, is the case of neglecting the secondary loading influence entirely.

The range of experiments available for useful comparisons are somewhat limited beyond those detailed in Section 5 as, of the experiments detailed in Section 2 and Appendix A1, only the aluminium plate tests [77], the CT and SENT tests by Lee et al [79], the 16MnR welded plate tests by Hong-Liang [112] and the Inspectra AB tests [76] provide results above \( L_r = 0.5 \). This, however, should provide sufficient data by which to consider the effect of plastic redistribution of the secondary stress on fracture toughness. For completeness, however, all the tests outlined in Section 2 and Appendix A1 are included to provide further data within the comparison so that any potential non-conservatism can be examined further. In this comparison the tests are assessed by means of plotting against the FAD so that an illustrative approach to failure can be provided. It has already been noted that this is equivalent to the use of the approaches as a \( K_f \) estimation scheme as presented in Section 2.

In many of the experiments the full material stress-strain curve was not available with which to define \( V_g \) and it was necessary to interpret the data between the available yield and ultimate tensile stresses. A number of approaches are available within R6 to provide this information in Section II.1, all of which adopt a Ramberg Osgood strain hardening relationship and provide means to estimate the strain-hardening coefficient. The approach adopted here is considered to provide the best description of the hardening curve and is suggested in R6 to define Option 2 FADs. For consistency this approach has been used for all cases where a stress-strain curve is not available. To provide an estimate of the strain...
hardening index within the Ramberg Osgood curve Equation II.1.3 of R6 was used (which originates from Considère’s construction):

\[
\frac{\sigma_{UTS}}{\sigma_y} = \left( \frac{1}{n} \right)^{\frac{1}{n}} \frac{1}{0.002} e^{-\frac{1}{n}}
\]

Equation 82

Where this has been adopted the estimates of \( n \) are defined. It is also noted that the use of this approximation is validated by the results presented.

This section is separated into sub-sections as follows:

- Subsection 6.2 compares the results of the tests identified in the previous section,
- Subsections 6.3 to 6.6 provide comparison to identified experiments that include a range of results over different values of plasticity and include the aluminium plate tests [77], the A533B bend tests [77], welded plate tests [112] and the spinning cylinder tests [82–86], respectively.
- Subsections 6.7 to 6.9 consider experiments performed in the elastic regime including the pre-compression tests of Mirzaee-Sisan [78], C-ring tests of Kamel [28] and the side-punched Tests of Hurlston [81], respectively.
- Subsections 6.10 and 6.11 detail the comparison against experiments under medium to significant levels of plasticity and include the pre-compressed tests of Lee [79] and the pre-compressed Inspactra AB tests [76], respectively.
- Subsection 6.12 provides a summary of all results.

6.2. Application to the Pre-Compression tests outlined in Section 5

6.2.1. Summary of Tests

This subsection provides details of the application of the \( V_g \) and R6 Complex \( V \) approaches to the tests outlined in Section 5. The secondary stress within these tests is a residual stress field resulting from a mechanical pre-compression of a three-point bend specimen with a scalloped notch. After the end load pre-compression, and establishment of residual stress
field on unloading, a short crack was introduced at the base of the scalloped notch. The specimen was then cooled to one of three temperatures before a three point bending load applied. A number of samples were also tested that did not undergo the pre-compression and hence did not include a residual stress so that an estimate of $K_{mat}$ could be obtained.

A summary of the required inputs to an assessment of the tests can be found in Table 10 below.

**Table 10 – Assessment inputs for Pre-Compressed tests of Section 5**

<table>
<thead>
<tr>
<th>Variable</th>
<th>-150 °C</th>
<th>-90 °C</th>
<th>-50 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As Received</td>
<td>Residual</td>
<td>As Received</td>
</tr>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Three-point bend specimen with 17.5 mm radius scalloped notch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W$ (mm)</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$ (mm)</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$ (mm)</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Crack size, $a$ (mm)</td>
<td>3.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Material Properties</strong></td>
<td></td>
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</tr>
<tr>
<td>$E$ (MPa)</td>
<td>228000</td>
<td>219000</td>
<td>216000</td>
</tr>
<tr>
<td>$v$</td>
<td>0.3</td>
<td></td>
<td></td>
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<tr>
<td>Tensile curve</td>
<td>Defined by Ramberg Osgood Curve</td>
<td></td>
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</tr>
<tr>
<td>Strain hardening index, $n$</td>
<td>15</td>
<td>8.5</td>
<td>9.2</td>
</tr>
<tr>
<td>Yield Stress (MPa)</td>
<td>700</td>
<td>560</td>
<td>540</td>
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<tr>
<td>Mean Fracture Toughness $(MPa.m^{1/2})$</td>
<td>70</td>
<td>145</td>
<td>295</td>
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<td>Lower Bound Fracture Toughness</td>
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### Loading

<table>
<thead>
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<th>Primary load</th>
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<tr>
<td>$P(kN)$</td>
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</tr>
<tr>
<td>29.6, 45.8,</td>
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<tr>
<td>65.9, 62.9,</td>
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<td>89.3, 88.0,</td>
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<tr>
<td>86.9</td>
<td>88.4, 83.9,</td>
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<td>85.5, 87.4,</td>
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</tr>
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<td>88.1</td>
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$K_f^p$ (from FE),

$K_f^S$ (from FE),

$K_f$ (MPam$\frac{1}{2}$)

<table>
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<tr>
<th>Secondary stress</th>
<th>From a mechanical pre-compression</th>
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</thead>
<tbody>
<tr>
<td>$K_f^p$ (from FE),</td>
<td></td>
</tr>
<tr>
<td>$K_f^S$ (from FE),</td>
<td></td>
</tr>
</tbody>
</table>

6.2.2. **Assessment Method**

6.2.2.1. **$K_r$ Position**

To assess the tests, when plotted against the FAD, the proximity to failure by fracture needs to be considered by the $K_r$ parameter. This term captures the proximity to fracture by taking the summation of the elastic primary and secondary contributions (when allowing for plasticity interaction) and normalising by the material fracture toughness.

When the level of plasticity is increased the critical value of $K_r$ is decreased, as governed by the FAD with the R6 Complex $V$ and $V_g$ approaches $K_r$ is given by:

$$K_r = \frac{K_f^p + VK_f^S}{K_{mat}}$$  \hspace{1cm} \text{Equation 83}

$$K_r = \frac{K_f^p + V_g K_f^S}{K_{mat}}$$  \hspace{1cm} \text{Equation 84}
In these equations the terms that require defining in an assessment are the primary and secondary stress intensity factors, the material fracture toughness and $V$ or $V_g$.

For the tests summarised in Section 5, the as-received samples were used to provide the material fracture toughness at each temperature. This is beneficial as it incorporates any geometric constraint effects, due to the possible influence of the notch. The results were considered twice, as the value of fracture toughness used at each test temperature was taken as both the average of the measured results and the lower bound estimate as detailed in Table 10. The average value adopted should provide a prediction of the mean failure condition in the pre-loaded specimens. As such, when failure assessment positions are plotted on a failure assessment diagram, there should be an equal number of failure positions inside and outside the curve. The lower bound toughness estimate should provide an estimate with all assessment points outside the failure assessment curve. Here it was considered beneficial to adopt both measures as the average value provides the best measure of accuracy of the approaches considered and the lower bound value provides confidence in the conservatism of the approaches.

It is noted that as the tests at -90 °C showed small amounts of crack growth or tearing before cleavage failure and the estimate of fracture toughness at initiation of tearing was defined from the $K_f - R$ curves. For the -90 °C cases, as nominally pure cleavage was also observed for the specimens with residual stress and some of the as-received specimens, an estimate of the toughness at the blunting line was used. However, as the -50 °C cases showed no sign of cleavage the more conventional 0.2 mm offset to the blunting line was used in its $K_f - R$ curve.

The values for $K_f^P$ and $K_f^S$ were both available from the finite element analyses of the tests. It is noted that $K_f^S$ is not required in either of the R6 Complex $V$ or $V_g$ approaches as this is cancelled from the equations if $K_f^S$ is known. The value of $K_f^S$ used has been adopted from the 3D finite element analyses in Section 5.2 where it was found to be 35 MPam$^{0.5}$. It is noted that, when compared, the finite element solutions matched the R6 solution for $K_f^P$ in an un-notched standard three-point bend specimen to within approximately 4%, which is better than the 10% criterion set within the design analyses of the crack depth. However,
the values of $K_I^p$ from the finite element analyses have been used as this allows the effect of the notch to be included.

### 6.2.2.2. $L_r$ Position

To determine the abscissa on the FAD, $L_r$ has been determined. This was obtained for these tests from the finite element analyses, by way of elastic-perfectly plastic analyses, to obtain the limit load at each temperature and checked with handbook solutions from R6 [3]. The R6 handbook limit load, $N_L$, solution used for comparison, is given by.

$$N_L = \frac{1.22(1 - a/W)^2\gamma W^2B\sigma_y}{2S}$$  \hspace{1cm} \text{Equation 85}$$

where $\gamma$ is 1 or $2/\sqrt{3}$ for Tresca or von Mises yield criteria, respectively. The difference in the handbook and finite element predictions of limit load were approximately 2% assuming von Mises stress conditions. It is also noted that the finite element solutions were between the Tresca and von Mises estimates. This comparison can be seen in Table 11.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>-150 °C</th>
<th>-90 °C</th>
<th>-50 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_L$, FE (kN)</td>
<td>101.4</td>
<td>81.1</td>
<td>78.3</td>
</tr>
<tr>
<td>$N_L$, Tresca (kN)</td>
<td>89.8</td>
<td>71.8</td>
<td>69.3</td>
</tr>
<tr>
<td>$N_L$, von Mises (kN)</td>
<td>103.7</td>
<td>82.9</td>
<td>80.0</td>
</tr>
</tbody>
</table>

### 6.2.3. Results and Comment

The results from applying the R6 Complex $V$ or $V_g$ approaches to the test data in Section 5 can be seen in Figure 91, when adopting the average fracture toughness estimate, plotted against the R6 Option 1 FAD. Figure 92 shows the same results when adopting the lower bound fracture toughness. The cases considered in these figures are the results from the as-received specimens, the results of the pre-compressed specimens when neglecting the secondary stress contribution and the results of the pre-compressed specimens with the R6
Complex $V$ or $V_g$ approaches. As with all figures presented within this section the as-received specimens are shown by the open stars, the estimates when neglecting the contribution from secondary stress are shown as open circles, the R6 Complex $V$ approach by the closed squares and the $V_g$ approach shown by the open diamonds. The blue symbols represent the results at -150 °C, the green results are at -90 °C and the red results are those at -50 °C.

**Figure 91 – Pre-compressed tests of Section 5 assessed by R6 Complex $V$ and $V_g$ approach of Section 4 when adopting mean fracture toughness estimates compared to an Option 1 FAD**

The results for the case that referenced the average materials fracture toughness in Figure 91 show a number of features. When considering the position of the pre-compressed specimens, with the different approaches, all approaches have the same applied primary load, and hence $L_r$, so the only difference between the approaches is the value of $K_r$.

When considering this difference in the approach for defining $K_r$, it can be seen that (1) the results for the R6 Complex $V$ and $V_g$ approaches are closely positioned to each-other and evenly spread about the failure assessment curve, indicating that both approaches are accurate and that the $V_g$ approach only provides moderate benefit over $V$ for these cases, and (2) that the consequence of not including the influence of secondary stress causes the
assessment points to be biased inside the FAD, indicating that this approach leads to inaccurate and non-conservative predictions of failure. When progressing from the -150 °C to the -50 °C pre-compressed specimens it can be seen that the average assessment location diverges from the FAD to further outside (i.e. more conservative) for the R6 Complex $V$ and $V_g$ approaches, and that the non-conservative consequence of not including the secondary stress contribution diminishes. This second point is exemplified at -50 °C where neglecting secondary stresses is conservative, which agrees with the results observed by Ainsworth [77] when assessing aluminium plate tests, when the influence of secondary stress is effectively removed for $L_r > 1$.

When considering the average fracture toughness estimate of failure at -150 °C it is clear that the prediction of $K_f^S$ from the kinematic hardening material, taken when adopting the JEDI postprocessor, provides a very good estimate of failure. The accuracy of the JEDI estimate is easier to demonstrate under cleavage conditions where the estimate of failure is simply provided as $K_f^P + K_f^S$. This helps add evidence to support the use of the modified J-integral as implemented in the JEDI postprocessor for pre-strained, non-proportional loading, as well as adding further support to the finite element results of Section 5.

The results when assessed against the mean fracture toughness also show that the as-received specimens are evenly spaced across the FAD for the specimens tested at -150 °C only. The as-received specimens tested at -50 °C are next closest to the FAD, but are seen to be outside the FAD, which is a result of the specimens experiencing differing levels of crack growth and, as such, do not represent initiation. However, the as-received samples tested at -90 °C are significantly outside the FAD which would not be explained by allowance for the small levels of crack growth. Note that this feature may be explained by comparison of the results to the Option 1 FAD, and is considered further by comparison to the Option 2 FAD below.
Figure 92 – Pre-compressed tests of Section 5 assessed by R6 Complex $V$ and $V_g$ approach of Section 4 when adopting lower bound fracture toughness estimates compared to an Option 1 FAD

The results for the case referencing the lower bound material’s fracture toughness in Figure 92 also show a number of features. Some of these, such as the similarity of the R6 complex $V$ and $V_g$ approaches and the consequence of not including secondary stresses, remain the same as discussed in context of Figure 91. However, when progressing from the pre-compressed specimens tested at -150 °C to the -50 °C it can be seen that the minimum assessment point at -150 °C is significantly outside the FAD but that for the tests at -90 °C and -50 °C is either on or very close to the FAD. This indicates that the approaches are adequately conservative and that the lower bound fracture toughness at -150 °C may be too low. This can be considered by comparison to fracture toughness properties described in literature [103], where the minimum fracture toughness is approximately 20% higher (50 MPam$^{0.5}$). When adopting this value the assessment points were still outside the FAD but the level of conservatism was reduced by approximately 50% and the potentially “incorrect” as-received specimen then falling within the FAD.

The as-received specimens tested at -150 °C and -50 °C are positioned just outside the FAD, as would be expected. However, the as-received samples at -90 °C are again
significantly outside the FAD, which would not be expected. It is noted that it is generally in this region that the divergence between different Options of FAD is greatest. To consider this point further, both the mean fracture toughness and lower bound fracture toughness results are re-considered against Option 2 FADs in Figure 93 and Figure 94. It was necessary to consider two different material tensile curves in this comparison; that from the tensile tests and that from the curve fit adopted within the finite element analyses. In the figures the shaded FAD described by the black curve is the Option 2 FAD from the tensile curves at -150 °C and the red curves are the Option 2 FAD from the Ramberg-Osgood curve fit. These Option 2 curves provide an improved fit at larger values of $L_r$. However, the Option 2 curve from the tensile properties better describes the as-received results and the Ramberg-Osgood fit used in the finite element results best represents the pre-compressed specimens. The main difference in the two tensile curves is the inclusion of the Lüders band behaviour in the tensile tests but not in the Ramberg-Osgood fit. This is considered more in Section 7.

Figure 93 – Pre-compressed tests of Section 5 assessed by R6 Complex $V$ and $V_g$ approach of Section 4 when adopting mean fracture toughness estimates compared to Option 2 FADs from tensile data (black) and Ramberg Osgood fit (red)
6.3. Application to Aluminium Plate Tests

6.3.1. Summary of Tests

This subsection provides details of the application of the \( V_g \) and R6 Complex \( V \) approaches to the aluminium plate tests detailed in Reference [77]. The aluminium plate tests were designed to consider a range of loading conditions by including two different geometries, one larger and one \( \frac{1}{4} \) sized, and two different grades of aluminium alloy. The full size plates were 25 mm thick, 1 m wide and 2.3 m tall. To introduce the residual stress two parallel electron beam welds were performed, thus creating a region of high tensile stress at the centre of the plate contained within a region of compressive stress in the outer ligaments. Within the \( \frac{1}{4} \) sized plate the residual stress was introduced by inserting two cuts in the ends of the specimen to the same location as the electron beam welds in the full size plate. A tensile stress was applied to the central tab of material and compressive stresses applied to the outside regions, reflecting the tensile and compressive regions of the
larger plate specimen. These stresses were then “locked in” by re-welding the slots together.

Following the introduction of secondary residual stress a through-wall crack was inserted and the plates subject to uniform tensile loading. Combining the two geometries and different aluminium grades allowed a full range of test conditions to be considered. During the tests, crack growth was measured by potential difference (ACPD) techniques. More detail can be found in Appendix 1 and [77].

A summary of the required inputs to an assessment of the tests can be found in Table 12 below.

Table 12 – Assessment inputs for aluminium plate tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>1P</th>
<th>1S</th>
<th>2P</th>
<th>2S</th>
<th>3P</th>
<th>3S</th>
<th>P1</th>
<th>S1</th>
<th>K1</th>
<th>K3</th>
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<tr>
<td>Geometry</td>
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<td>0.575</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Initial Crack size, a (mm)</td>
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<td>78.5</td>
<td>12.5</td>
<td>16.5</td>
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<td>0.2</td>
<td>0.2</td>
<td>0.43</td>
<td>0.2</td>
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<td>7.1</td>
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<td>24.5</td>
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<td>5.6</td>
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<td>3.9</td>
<td>1.65</td>
<td>1.45</td>
<td>0.6</td>
<td>1.3</td>
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<td>Crack growth 4, (∆a)4, (mm)</td>
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<td>36.9</td>
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<td>11.5</td>
<td>10</td>
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<td>1.85</td>
<td>1.7</td>
<td>1.3</td>
<td>2.5</td>
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<td>Crack growth 5, (∆a)5, (mm)</td>
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<td>42.3</td>
<td>15.9</td>
<td>20</td>
<td>-</td>
<td>16.1</td>
<td>2.55</td>
<td>2.35</td>
<td>2.2</td>
<td>4.1</td>
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<td>Crack growth 6, (∆a)6, (mm)</td>
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<td>-</td>
<td>-</td>
<td>46</td>
<td>-</td>
<td>-</td>
<td>3.95</td>
<td>3.2</td>
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<td>Crack growth 7, (∆a)7, (mm)</td>
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<td>4.35</td>
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<td>-</td>
<td>-</td>
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<td>7</td>
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Material Properties

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<th>AL2024</th>
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Page 183 of 349
<table>
<thead>
<tr>
<th>$E$ (MPa)</th>
<th>73700</th>
<th>69900</th>
<th>74050</th>
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<td>$\nu$</td>
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<td>Yield Stress, $\sigma_y$ (MPa)</td>
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<td>437</td>
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<td>Tensile Strength, $\sigma_{UTS}$ (MPa)</td>
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<td>301</td>
<td>528</td>
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<td>Fracture Toughness Defined from $J - \Delta a$ curves</td>
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<td>$K_{mat,\Delta a_1}$ (MPa.m$^{1/2}$)</td>
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<td>146.3</td>
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<td>$K_{mat,\Delta a_7}$ (MPa.m$^{1/2}$)</td>
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<td>$K_{mat,\Delta a_{12}}$ (MPa.m$^{1/2}$)</td>
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</table>

**Loading**

<table>
<thead>
<tr>
<th>Primary load</th>
<th>Tensile load applied to ends of plate, remote from crack tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\Delta a_1}$ (MN)</td>
<td>2.2</td>
</tr>
<tr>
<td>$P_{\Delta a_2}$ (MN)</td>
<td>4.03</td>
</tr>
<tr>
<td>$P_{\Delta a_3}$ (MN)</td>
<td>5.15</td>
</tr>
<tr>
<td>$P_{\Delta a_4}$ (MN)</td>
<td>5.25</td>
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<td>$P_{\Delta a_5}$ (MN)</td>
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<td>$P_{\Delta a_6}$ (MN)</td>
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<td>$P_{\Delta a_7}$ (MN)</td>
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<tr>
<td>$P_{\Delta a_{11}}$ (MN)</td>
<td>-</td>
</tr>
<tr>
<td>$P_{\Delta a_{12}}$ (MN)</td>
<td>-</td>
</tr>
</tbody>
</table>

**Secondary stress**

| $\sigma_{res,}$ (MPa) | 0 | 91.3 | 0 | 68.9 | 0 | 71.9 | 0 | 249.1 | 0 | 69.9 |

*Defined from $J - \Delta a$ curves.*

*Self-balancing stress distribution over plate width.*
6.3.2. Assessment Method

6.3.2.1. \( K_r \) Position

In the aluminium plate tests \( J - R \) curves are provided in [77] and Chapter V of R6 [77] and the resultant toughness values are provided in Table 12 for measured levels of crack growth. As lower bound \( J - R \) are used the estimates of failure should be positioned outside the failure assessment curve.

Values for \( K_r^P \) and \( K_r^S \) need to be defined from handbook estimates. The stress intensity factor solution adopted is that for a through-wall crack in a finite sized plate within R6. This can be seen in Equation 86 below.

\[
K_l = \sigma_m \sqrt{\pi a} \left(1 - 0.025(a/W)^2 + 0.06(a/W)^4\right) \frac{\sqrt{\cos(\pi a/W)}}{\pi a/W}
\]  

\text{Equation 86}

where \( \sigma_m \) is the remotely applied tensile membrane load. When using Equation 86 for the estimate of \( K_l^P \) the applied load at failure was simply converted to the applied stress to find \( \sigma_m \) at failure.

As the residual stress has been expressed as a tensile load in [77] it is also possible to use Equation 86 to define \( K_l^S \). When applying this estimate of \( K_l^S \) in [77] the width of the plate was reduced to the width of the region of the tensile stress, so that the compressive regions at the edge of the plate were not considered. This was not applied here as the crack size was small enough for this to have negligible difference and, if this was applied, the result was to marginally enhance the estimate of \( K_l^S \), thus making it easier to demonstrate a conservative result.

As the secondary stress is applied as a tensile stress field originating remote from the crack tip it is also necessary to consider an enhancement of \( K_l^S \) under elastic-plastic conditions so that \( K_l^P > K_l^S \). The \( a_{eff} \) approach within R6 (Section II.6 [3]) was considered to enhance \( K_l^S \) by a measure of an enhanced crack length by accounting for the plastic zone size. Therefore \( K_l^S \) was provided by:
\[ K_j^S = K_j^S \frac{a_{eff}}{a} \]  \hspace{5cm} \text{Equation 87}

\[ a_{eff} = a + \frac{1}{2\pi \beta} \left( \frac{K_j^S}{\sigma_y} \right)^2 \]

where \( \beta \) accounts for plane stress and plane strain conditions and has been set to unity here.

### 6.3.2.2. \( L_r \) Position

The \( L_r \) position of the assessment points was determined from the R6 limit load solution for a centre cracked plate as shown below:

\[ N_L = 2\sigma_y t(W - a) \]  \hspace{5cm} \text{Equation 88}

### 6.3.2.1. Estimation of Material Tensile Curve

In order to provide an estimate of the materials stress-strain curve for the different grades of aluminium used in the tests it was necessary to interpret the data between the materials yield and ultimate tensile stress as detailed in Section 6.1 based upon a Ramberg-Osgood material. When solving this equation for \( n \) for the three different aluminium types used the following values of strain hardening index were found.

<table>
<thead>
<tr>
<th>Material</th>
<th>( \sigma_y ) (MPa)</th>
<th>( \sigma_{UTS} ) (MPa)</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL2024</td>
<td>397</td>
<td>508</td>
<td>11.3</td>
</tr>
<tr>
<td>AL2024 (¼ sized)</td>
<td>437</td>
<td>528</td>
<td>13.7</td>
</tr>
<tr>
<td>AL5083</td>
<td>153</td>
<td>301</td>
<td>5.3</td>
</tr>
</tbody>
</table>
6.3.3. Results and Comment

Figure 95 shows the results of the application of the assessment of the aluminium plate tests when including all test results, including the as-received samples that did not include the residual stress and consideration of the residual stress specimens when not including the secondary stress contribution. Figure 96 repeats these results but only includes the assessment results of the cases that include the residual stress when assessed by the R6 Complex $V$ or $V_g$ approaches. Where shown the as-received specimens are shown by the open stars, the estimates when neglecting the contribution from secondary stress are shown as open circles, the R6 Complex $V$ approach by the closed squares and the $V_g$ approach shown by the open diamonds.

![Graph showing aluminium plate test results assessed by R6 Complex V and Vg approaches compared to Option 1 FAD](image)

**Figure 95 – Aluminium Plate tests assessed by R6 Complex $V$ and $V_g$ approach of Section 4 compared to an Option 1 FAD**

With respect to Figure 96, when considering the as received samples that include primary stresses only most points are seen to be located outside the FAD at low values of $L_r$ and marginally inside the FAD at high values of $L_r$. This was also observed for the tests reported in [77], although the points were marginally closer to the FAD than seen here. The difference in the assessment point is solely in the position of $K_r$ and, therefore, must
be a result of the stress intensity factor solution considered. It is emphasised that the
difference is not significant, less than 0.02 $K_r$, and these results are considered to
effectively reproduce those of [77] under primary loads alone. It can also be seen that one
assessment point, at approximately $L_r = 0.45$, lies within the FAD. This was also reported
in reference [77] and attributed to uncertainty in the initiation load, causing initiation to be
conservatively interpreted, potentially pulling an assessment points within the FAD. Note
that, because of uncertainty in the tensile properties an Option 2 FAD could not be
considered, although this might allow the potentially non-conservative cases at $L_r > 1$ to
fall within the FAD.

Once again it is clear that not accounting for the influence of secondary stress leads to non-
conservative results as these assessment points fall within the FAD. It is also seen that
once $L_r > 1$ the secondary stresses contribution can be neglected.

![Figure 96](image)

**Figure 96** – Aluminium plate tests assessed by R6 Complex $V$ and $V_k$ approach of
Section 4 compared to an Option 1 FAD when only considering the specimens
including residual stresses

When considering the specimens with residual stress it can be seen that all but two of the
assessment points using the Complex $V$ and $V_k$ approaches lie outside the FAD (or within
very close proximity). The two points within the FAD were also reported in [77] and also explained by the uncertainty in the initiation load. Therefore the Complex $V$ and $V_g$ approaches can be considered to provide conservative estimates of failure for these tests.

The differences in the results of the Complex $V$ and $V_g$ approaches are seen to be minimal when adopting one grade of aluminium but further apart for the other. This is shown by the comparison of the results at approximately $L_r = 0.8$ where the lower strength alloy AL5083 shows a bigger reduction (12\% reduction in $K_r$) when adopting the $V_g$ approach that the AL2024 alloy (3.5\% reduction in $K_r$). It is also noted that the reduction of conservatism, if an estimate can be provided by the distance between the FAD and the assessment point, will provide well over 50\% reduction in conservatism for the lower yield stress alloy AL5083. This, however, may be misleading as the assessment points are close to the FAD so that even small benefits can result in a large reduction in conservatism. Nonetheless, the results indicate that the differences are closely related to the material properties. It is therefore worth noting the potential for the results altering with different estimates of the material properties for the aluminium alloys used here. This is not thought likely to provide significantly different results though. This is because below $L_r = 1$ the materials responses are characterised predominately by the well characterised elastic properties and post yielding the influence of secondary stress is removed. For the AL2024 alloy to show significantly different results the strain hardening index would need to decrease by approximately half (assuming a Ramberg-Osgood style relation) or show more plastic strain before yield. Even then, the results would likely only change between $L_r = 0.5$ and 1.0 under small-scale yielding.

### 6.4. Application to A533B Bend Tests

#### 6.4.1. Summary of Tests

Also considered in reference [77] are tests on an A533B steel. Four-point bend specimens measuring 600 x 600 x 70 mm$^3$ that contained a residual stress field were used in the tests. The specimens were made by welding the two halves of a cut plate back together using a double V preparation, thus creating a self balancing through thickness residual stress. The
plate was subsequently cut in half and one of these subjected to a post weld heat treatment (PWHT). This reduced the residual stress magnitude in the plate to between 10 and 30% of that in the welded condition. Test specimens, a block for residual stress measurement, and material characterisation specimens were then manufactured from each of the plates. The test specimens then had a shallow surface crack with an $a/t$ ratio of 0.25 to maximise the stress intensity factors predicted. The samples were then placed under 4-point bending and monitored for crack growth at two test temperatures of -120 and -30 °C so that initiation occurred at low and high values of $L_r$, respectively. The different samples were defined depending on their heat treatment state and test temperature as: Low $L_r$ As-Welded (LLAW), Low $L_r$ Heat Treated (LLHT), High $L_r$ As-Welded (HLAW) and High $L_r$ Heat Treated (HLHT). A summary of the required inputs to an assessment of the tests can be found in Table 14 below as obtained from reference [77] and R6.

<table>
<thead>
<tr>
<th>Variable</th>
<th>LLAW</th>
<th>LLHT</th>
<th>HLAW</th>
<th>HLHT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Four point bend plate specimens</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2W$ (mm)</td>
<td>600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$ (mm)</td>
<td>70</td>
<td>71</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Crack depth, $a$ (mm)</td>
<td>19.4</td>
<td>18.6</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Crack length, $2c$ (mm)</td>
<td>175</td>
<td>174.2</td>
<td>175</td>
<td></td>
</tr>
<tr>
<td><strong>Material Properties</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td>200000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Stress, $\sigma_y$ (MPa)</td>
<td>618</td>
<td>551*</td>
<td>596</td>
<td>561*</td>
</tr>
<tr>
<td>Tensile Strength, $\sigma_{UTS}$ (MPa)</td>
<td>791</td>
<td>762*</td>
<td>772</td>
<td>694*</td>
</tr>
<tr>
<td>Fracture Toughness $(MPa.m^{1/2})$</td>
<td>37</td>
<td>46</td>
<td>62</td>
<td>321</td>
</tr>
<tr>
<td><strong>Loading</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary load</td>
<td>Four point bend loading</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P$ (MN)</td>
<td>1.27</td>
<td>2.19</td>
<td>5.1</td>
<td>4.83</td>
</tr>
<tr>
<td>Secondary stress</td>
<td>Residual stress provided in paper</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* - These material properties correspond to the weld material. Corresponding un-starred values are for the base material.
6.4.2. Assessment Method

6.4.2.1. $K_r$ Position

$K_r^P$ and $K_r^S$ were defined from solutions for a finite surface crack in an infinite width plate in R6 [3]. As the crack length is less than 50% of the plate width, and the assessment has been based on the deepest point of the crack, the use of an infinite plate rather than a finite plate is not expected to provide significant differences in the elastic result. The solution used was taken from R6, Equation IV.3.1, can be seen in Equation 89 below.

\[
K_i = \sqrt{\pi a} \sum_{i=0}^{i=3} \sigma_i F_i
\]  

\text{Equation 89}

where $\sigma_i$ are the coefficients of a polynomial fit to the stress field over the crack length and $F_i$ are the influence coefficients determined from look-up tables depending on $a/c$ and $a/t$. When using Equation 89 for the estimate of $K_r^P$, the applied moment at failure was simply converted to the applied bending stress at failure and only $\sigma_1$ and $F_1$ used. To determine $K_r^S$ the same equation was used but the stress distribution was fitted over the crack length; note that a function describing the stress distribution over half the wall thickness in R6 [3], Section V, is provided as in Equation 90 for the as-welded and Equation 91 for the heat treated cases:

\[
\sigma_{res} = -105.5 + 3408.2 \left(1 - \frac{x}{t}\right) - 8640.8 \left(1 - \frac{x}{t}\right)^2
\]  

\text{Equation 90}

\[
\sigma_{res} = -28 + 609.1 \left(1 - \frac{x}{t}\right) - 1590.8 \left(1 - \frac{x}{t}\right)^2
\]  

\text{Equation 91}

As the secondary residual stress is very localised and bending in nature (as it has to self-balance) no account of elastic follow-up was considered necessary so that $K_r^S$ was set to be equal to $K_r^S$. 
6.4.2.2. \( L_r \) Position

The \( L_r \) position of the assessment points was determined from the R6 [3] limit load solution for a semi-elliptical centre-cracked plate under pure bending as:

\[
L_r = \frac{2 \sigma_b}{3 \sigma_y} \left( 1 - 20 \left( \frac{a}{2c} \right)^2 \zeta^3 \right) \frac{1}{(1 - \zeta)^2}
\]

Equation 92

where \( \sigma_b \) is the applied bending stress and \( \zeta \) is defined by:

\[
\zeta = \frac{2ac}{t(2c + 2t)}
\]

Equation 93

6.4.2.3. Estimation of Materials Tensile Curve

As with the aluminium plate tests it was again necessary to provide an estimate of the strain-hardening index from Equation II.1.3 of R6, as detailed in Section 6.1. When for \( n \) for the different temperatures and heat treatment conditions for the weld and plate material the following values of strain hardening index were found.

Table 15 – Strain hardening exponents for A533B bending plate tests

<table>
<thead>
<tr>
<th></th>
<th>( \sigma_y ) (MPa)</th>
<th>( \sigma_{UTS} ) (MPa)</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parent, -120 °C, As Welded</td>
<td>618</td>
<td>791</td>
<td>10.0</td>
</tr>
<tr>
<td>Weld, -120 °C, As Welded</td>
<td>551</td>
<td>762</td>
<td>8.5</td>
</tr>
<tr>
<td>Parent, -120 °C, Heat Treated</td>
<td>596</td>
<td>772</td>
<td>9.8</td>
</tr>
<tr>
<td>Weld, -120 °C, Heat Treated</td>
<td>561</td>
<td>694</td>
<td>11.5</td>
</tr>
<tr>
<td>Parent, -30 °C, As Welded</td>
<td>520</td>
<td>677</td>
<td>10.0</td>
</tr>
<tr>
<td>Weld, -30 °C, As Welded</td>
<td>431</td>
<td>612</td>
<td>8.5</td>
</tr>
<tr>
<td>Parent, -30 °C, Heat Treated</td>
<td>520</td>
<td>677</td>
<td>10.0</td>
</tr>
<tr>
<td>Weld, -30 °C, Heat Treated</td>
<td>425</td>
<td>561</td>
<td>10.2</td>
</tr>
</tbody>
</table>
6.4.3. Results and Comment

Figure 97 shows the results of the application of the assessment approaches to the A533B plate bend tests. The estimates when neglecting the contribution from secondary stress are shown as open circles, the R6 Complex $V$ approach by the closed squares and the $V_g$ approach shown by the open diamonds.

Figure 97 – A533B bend plate tests assessed by R6 Complex $V$ and $V_g$ approach of Section 4 compared to an Option 1 FAD

All assessment points are significantly outside the bounds of the FAD shown in Figure 97. This was also observed within the validation of R6 when considering these tests and is explained in [3], [77] by not taking account of crack-tip constraint and poor estimates of the material fracture toughness. It was also considered that the limit load solution used was conservative; here however a different, more recent solution has been adopted which still shows similar values of $L_r$ at failure. It is therefore considered more likely that the fracture toughness values adopted may be too low. Indeed, the values used here, and presented in Table 14, are much lower than those for the A533B plate used in the
experiments in Section 5 for the parent material that had not experienced a heat treatment. It would also appear that there may be a mistake in the fracture toughness values provided in reference [77] for the High $L_r$ As Welded (HLAW) case as this is also far too low. It can also be considered that as the tests only considered one test of each case that the material’s fracture toughness might be better represented by the mean toughness value. Adopting the average toughness values in Section 5 for the Low $L_r$ As Welded (LLAW) and HLAW cases reduces the assessment points to $K_r = 1.28$ and $0.48$, respectively, which is significantly more aligned to the heat treated results.

The difference between the Complex $V$ and $V_{a}$ approaches is only observable for the HLAW case, where the Complex $V$ approach is out of the bounds provided by the chart at $K_r = 3.24$. If the potentially more realistic values of fracture toughness are adopted the benefit is approximately a 15% reduction in $K_r$. This is, however, of little consequence as the maximum value of $L_r, L_r^{\text{max}}$, is only 1.2, which means that the HLAW and High $L_r$ Heat Treated (HLHT) cases would both be considered to have failed by plastic collapse. Nonetheless both approaches provide equivalent, conservative results.

### 6.5. Application to Welded Plate Tests

#### 6.5.1. Summary of Tests

Fracture experiments were performed by Hong-Liang [112] where weld residual stress and primary loads were combined. These experiments incorporated three different specimen designs where the horizontal crack was located (1) between two vertical welds, (2) just above a horizontal weld and (3) across a vertical weld. The material used was a 16MnR pressure vessel steel. The plate tests were then cooled to a range of temperatures from -65 °C to room temperature and tested under a tensile primary load. A summary of the required inputs to an assessment of the tests can be found in Table 16 below.
Table 16 – Assessment inputs for Welded Plate Tests [112]

<table>
<thead>
<tr>
<th>Specimen</th>
<th>T-1</th>
<th>T-2</th>
<th>T-3</th>
<th>T-4</th>
<th>T-5</th>
<th>T-6</th>
<th>T-7</th>
<th>T-8</th>
<th>T-9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Type</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2w$ (mm)</td>
<td>178</td>
<td>179</td>
<td>177</td>
<td>180</td>
<td>175</td>
<td>180</td>
<td>180</td>
<td>180</td>
<td>180</td>
</tr>
<tr>
<td>$t$ (mm)</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$l$ (mm)</td>
<td>400</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Crack size, $a$ (mm)</td>
<td>15</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>12.5</td>
<td>10</td>
<td>7.5</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Material Properties</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Stress, $\sigma_y$ (MPa)</td>
<td>355</td>
<td>382</td>
<td>413</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile Strength, $\sigma_{UTS}$ (MPa)</td>
<td>545</td>
<td>579</td>
<td>616</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fracture Toughness $(\text{MPa.m}^{1/2})$</td>
<td>228</td>
<td>154</td>
<td>123</td>
<td>80</td>
<td>123</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loading</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary load</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$	ext{Tensile load applied to ends of plate, remote from crack tip}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P$ (MN)</td>
<td>0.596</td>
<td>0.452</td>
<td>0.565</td>
<td>0.377</td>
<td>0.335</td>
<td>0.396</td>
<td>0.214</td>
<td>0.366</td>
<td>0.465</td>
</tr>
<tr>
<td>Secondary stress</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{res}$ (MPa)</td>
<td>291</td>
<td>252</td>
<td>323</td>
<td>252</td>
<td>360</td>
<td>360</td>
<td>360</td>
<td>360</td>
<td>360</td>
</tr>
</tbody>
</table>

6.5.2. Assessment Method

6.5.2.1. $K_r$ Position

Values for $K_r^P$ and $K_r^\xi$ were defined by Equation 86 as for the aluminium plate tests and normalised by the material’s fracture toughness to provide values of $K_r$.

Some interpretation of the residual stress fields within [112], under the different weld and crack combinations, was required to provide estimates of $K_r^\xi$. In doing so, the residual
stress was assumed to act as a membrane stress equal to the average value over the crack length. For the case where the defect is located across the weld the stress distribution is given in [112] by:

\[
\sigma(x) = 353 \left(1 - \left(\frac{x}{25}\right)^2\right) \exp\left(-0.5 \left(\frac{x}{25}\right)^2\right)
\]

Equation 94

In the paper two further plots are shown for the stress distribution in a plate with the duplicate welds and in the case where the defect is parallel to the weld. These two figures are repeated in Figure 98 below. It is not clear in the paper which of these distributions pertains to which defect case but the paper does, however, mention that the stress in the duplicate weld reaches the material’s yield stress (~360 MPa at room temperature) which would indicate Figure 98 (a) is for the parallel weld and (b) is for the duplicate weld. In the duplicate weld, as the defects are small (to fit between the welds), the membrane tensile stress is taken at the peak value shown in Figure 98 (b) of 360 MPa for use in Equation 86. For the case of the parallel weld the defect was assumed to be within the peak region of stress and hence has a tensile membrane stress of 200 MPa.

Figure 98 – Residual stress distributions for crack opening stress when the defect is (a) parallel to the weld and (b) between two duplicate welds [112]
As the secondary stress is not a membrane stress field and the location of the defects was close to the weld it was not considered necessary to provide an enhancement of $K_f^s$ under elastic-plastic conditions so that $K_f^s > K_f^s$. Therefore $K_f^s$ was set to be equal to $K_f^s$.

6.5.2.2. L_r Position

The $L_r$ position of the assessment points was determined from the R6 handbook limit load solution shown by Equation 88 for a centre cracked plate, as used for the aluminium plate tests.

6.5.2.3. Estimation of Material Tensile Curve

It was again necessary to provide an estimate of the strain-hardening index from Equation II.1.3 of R6, , as detailed in Section 6.1. When solving for $n$ for the different temperatures and heat treatment conditions the following values of strain hardening index were found.

Table 17 – Strain hardening exponents for welded plate tests

<table>
<thead>
<tr>
<th>Temperature</th>
<th>$\sigma_y$ (MPa)</th>
<th>$\sigma_{UTS}$ (MPa)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 °C</td>
<td>355</td>
<td>545</td>
<td>9.5</td>
</tr>
<tr>
<td>-30 °C</td>
<td>382</td>
<td>579</td>
<td>9.8</td>
</tr>
<tr>
<td>-60 °C</td>
<td>413</td>
<td>616</td>
<td>10.1</td>
</tr>
</tbody>
</table>

6.5.3. Results and Comment

Figure 99 shows the results of the application of the assessment of the welded plate tests to both the R6 Complex $V$ or $V_g$ approaches as well as results when not including the secondary stress contribution. The estimates when neglecting the contribution from secondary stress are shown as open circles, the R6 Complex $V$ approach by the closed squares and the $V_g$ approach shown by the open diamonds.
Figure 99 shows that all assessment points when accounting for the secondary stress contribution closely bound the FAD and that ignoring the secondary stress contribution is non-conservative. It can also be seen that there is a general benefit to adopting the $V_g$ approach over the Complex $V$ approach contained within R6. In terms of a reduction in estimates of $K_r$, the results indicate an increase in the percentage reduction from 5% at low values of $L_r$ to 25% at high values of $L_r$. As detailed before it is difficult to provide a measure of the reduction in conservatism because of the assumptions made on material properties, as well as the use of the Option 1 FAD.

Figure 99 shows a divergence from the FAD at $L_r$ values of 0.72 and 0.87. It is not clear why these positions would indicate higher fracture toughness values. It is possible that the secondary stress contribution is not accounted for correctly. However, this is considered unlikely as one of the cases is where the defect is across the weld and is, therefore, the same as other cases shown that do predict failure correctly. Further consideration of the difference between the Option 1 and Option 2 FADs also shows little difference for this material. One potential explanation would be a loss of constraint when approaching the yield stress, creating a perceived increase in the materials fracture toughness, but this is difficult to validate here because of the lack of relevant data, although the distribution of assessment points in Figure 99 is similar to that of a constraint modified FAD (i.e. as in [34]).
Figure 99 – Welded plate tests [112] assessed by R6 Complex V and $V_g$ approach of Section 4 compared to an Option 1 FAD

The results presented in Figure 99 do, however, provide good evidence of the conservative use of the R6 Complex V or $V_g$ approaches and also provides evidence of the potential benefits in adopting the $V_g$ approach.

6.6. Application to Spinning Cylinder Tests

6.6.1. Summary of Tests

A series of large-scale experiments are reported in references [82–86] that consider the effects of primary hoop stress and thermal shock on ductile crack initiation and growth. To cover a range of combinations of primary and secondary stress, crack size and shape, a total of six tests were performed. These tests are referred to as Spinning Cylinder Tests 1 to 6 (SC1 to SC6 [82–86]).
All the tests were performed on forged cylinders that underwent subsequent heat treatment, water quenching and tempering to match an ASTM A508 Class 3 forging and had a nominal internal diameter of one metre. These large cylinders were rapidly rotated to induce a primary hoop stress within a specially commissioned test house and rig. During testing the cylinder was also heated, to approximately 300 °C, before the internal surface was sprayed with cold water to induce a severe thermal shock in some cases. A brief overview of the different tests is detailed below:

- **SC1** – Primary load only [82], [83]. The spinning cylinder 1 (SC1) test was of a full-length axial internal crack in the cylinder that was tested under rapid rotation with no secondary load with the crack growth measured.

- **SC2** – Secondary load only [82], [83]. The spinning cylinder 2 (SC2) test was of a full-length axial internal crack in a cylinder that was heated to approximately 320 °C before being rapidly cooled on the internal surface by water spray. To ensure an even temperature distribution a negligible rotational speed, and hence primary load, was also applied.

- **SC3** – Primary and secondary loads [82], [83]. The spinning cylinder 3 (SC3) test was a combination of SC1 and SC2. For this test the cylinder was heated to a steady state temperature before cold water was sprayed internally coupled with an increase in rotational speed to maintain an equal ratio of primary to secondary loading of 1:5.

- **SC4** – Secondary loads only [84]. The spinning cylinder 4 (SC4) test was of two surface breaking semi-elliptical cracks positioned 180° apart on the internal surface of the cylinder. The cylinder was heated to approximately 320 °C before being rapidly cooled on the internal surface by water spray. To ensure an even temperature distribution a negligible rotational speed was also applied.

- **SC5** – Primary and secondary stresses in a weld after PWHT (post weld heat treatment) [85]. The spinning cylinder 5 (SC5) was of an extended axial surface breaking defect within the heat-treated area of a weld. The cylinder was heated to a steady state before being rapidly cooled on the internal surface under a constant rotational speed. This test was specifically designed to reflect the weld and
component behaviour that would be expected for a defect in the Sizewell B reactor pressure vessel.

- SC6 – Primary and secondary loads [86]. The spinning cylinder 6 (SC6) test was designed to be a repeat of SC4 but with one of the semi-elliptical cracks elongated and a constant primary load applied.

The data required for an R6 based assessment of these tests can be seen in Table 18 below.

### Table 18 – Assessment inputs for Spinning Cylinder Tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>SC1</th>
<th>SC2</th>
<th>SC3</th>
<th>SC4</th>
<th>SC5</th>
<th>SC6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Cylinder with internal extended axial crack (1-3,5), and semi-elliptical crack (4,6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_i$ (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>$t$ (mm)</td>
<td></td>
<td></td>
<td></td>
<td>200</td>
<td>150</td>
<td>200</td>
</tr>
<tr>
<td>$l$ (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1300</td>
<td></td>
</tr>
<tr>
<td>Initial Crack depth, $a$ (mm)</td>
<td>116</td>
<td>104</td>
<td>40 and 60</td>
<td>85</td>
<td>40 and 35</td>
<td></td>
</tr>
<tr>
<td>Initial crack length, $c$ (mm)</td>
<td>-</td>
<td>-</td>
<td>80 and 120</td>
<td>-</td>
<td>80 and 210</td>
<td></td>
</tr>
<tr>
<td>Crack growth 1, $(\Delta a)_1$, (mm)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_2$, (mm)</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.12</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_3$, (mm)</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.32</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_4$, (mm)</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.68</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_5$, (mm)</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.93</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_6$, (mm)</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.36</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_7$, (mm)</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.87</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_8$, (mm)</td>
<td>7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.46</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_9$, (mm)</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_{10}$, (mm)</td>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$(\Delta a)_{11}$, (mm)</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Material Properties</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td>193120</td>
<td>212350</td>
<td>189100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.275</td>
<td>0.28</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Stress, $\sigma_y$ (MPa)</td>
<td>539</td>
<td>565</td>
<td>521</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density, $\rho$, ($kg/mm^3$)</td>
<td>$7.787 \times 10^9$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile Strength, $\sigma_{UTS}$ (MPa)</td>
<td>707</td>
<td>660</td>
<td>660</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fracture Toughness</td>
<td>$J_{mat} = 0.208\Delta a^{0.329}$ ($\Delta a$ in mm)</td>
<td>$K_{mat} = 54.2e^{0.00977}$</td>
<td>$J_{mat} = 0.288\Delta a^{0.368}$</td>
<td>As SC4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Loading**

<table>
<thead>
<tr>
<th>Primary load</th>
<th>Cylinder rotated at high speed to generate hoop stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{au1}$ (rpm)</td>
<td>2600</td>
</tr>
<tr>
<td>$P_{au2}$ (rpm)</td>
<td>2600</td>
</tr>
<tr>
<td>$P_{au3}$ (rpm)</td>
<td>2600</td>
</tr>
<tr>
<td>$P_{au4}$ (rpm)</td>
<td>2600</td>
</tr>
<tr>
<td>$P_{au5}$ (rpm)</td>
<td>2600</td>
</tr>
<tr>
<td>$P_{au6}$ (rpm)</td>
<td>2600</td>
</tr>
<tr>
<td>$P_{au7}$ (rpm)</td>
<td>2600</td>
</tr>
<tr>
<td>$P_{au8}$ (rpm)</td>
<td>2600</td>
</tr>
<tr>
<td>$P_{au9}$ (rpm)</td>
<td>2600</td>
</tr>
<tr>
<td>Secondary stress</td>
<td>Cylinder heated then sprayed, internally, with cold water</td>
</tr>
<tr>
<td>$\sigma^S$</td>
<td>-</td>
</tr>
</tbody>
</table>

**6.6.2. Assessment Method**

**6.6.2.1. $K_r$ Position**

Values for $K_I^P$ and $K_I^S$ were defined by Equation 95 below [3] for the extended axial cracks and by Equation 89 for the semi-elliptical cracks (note that although the solution is the
same the influence coefficients are different for the different geometries and at both the surface breaking and deepest points of the crack).

\[
K_i = \frac{1}{\sqrt{2\pi a}} \int_0^a \sigma(u) \sum_{i=1}^{i=3} F_i \left(1 - \frac{u}{a}\right)^{i-\frac{3}{2}} du
\]

Equation 95

where \(\sigma(u)\) is the value of stress at position \(u\) through thickness and \(F_i\) are influence coefficients found from look-up tables dependent on the cracked geometry. The solution of this equation requires numerical integration.

To solve for \(K_p\) for the spinning cylinder it was necessary to estimate the stress distribution through the thickness of a thick cylinder under rapid rotation. This was obtained as shown below:

\[
\sigma_h(u) = \frac{3 + (v/(1 - v))}{8} \rho \omega^2 \left(R_o^2 + R_i^2 + \frac{R_i^2 R_o^2}{(u + R_i)^2} - \frac{1 + 3(v/(1 - v))}{3 + (v/(1 - v))} (u + R_i)^2\right)
\]

Equation 96

where \(u\) is the through thickness position, \(R_o\) is the outer radius, \(R_i\) is the inner radius, \(\rho\) is the materials density and \(\omega\) is the angular velocity.

Likewise, for the secondary stress intensity factor the stress distribution through thickness resulting from the thermal transient applied to the cylinders was required. These transients were obtained from finite element predictions of the thermal distribution through thickness with time, calibrated against thermocouple data, available from the respective test reports. At each position through thickness and time increment these thermal distributions were converted to an elastic thermal stress as:

\[
\sigma_{th}(u, t) = \frac{E \alpha \Delta T(u, t)}{1 - \nu}
\]

Equation 97

where \(\alpha\) is the coefficient of thermal expansion and \(\Delta T(u, t)\) is the temperature difference between that at position \(u\) and the average value at time \(t\). An example of these thermal
transient data can be seen in Figure 100 for the case of SC5, where time 0 is the instant where the cold water spray starts and the transient begins and the dimensions to the right of figure are the distance from the inner radius.

Figure 100 – Example thermal transient data from spinning cylinder test 5

6.6.2.2. \( L_r \) Position

The \( L_r \) position of the assessment points was determined from the solution shown below, contained in Section V of R6.

\[
L_r = \left( \frac{\omega}{\omega_i} \right)^2 \quad \text{Equation 98}
\]

\[
\omega_l = \sqrt{\frac{2\sigma_j ln\left(\frac{R_i + a}{R_o}\right)}{\sqrt{3\rho \left(\frac{R_i^3 - (R_i + a)^3}{3(R_i + a)} + \frac{1}{2}\left((R_i + a)^2 - R_o^2\right)\right)}}} \quad \text{Equation 99}
\]
6.6.2.3. Estimation of Material Tensile Curve

It was again necessary to provide an estimate of the strain-hardening index as detailed in Section 6.1. When solving for $n$ for the different forgings, the values of strain hardening index shown in Table 19 were found. It is also noted that tensile material properties were available in the reports but these contained inconsistencies between different tests and could be well characterised by the Ramberg-Osgood fit. This therefore allows the approaches outlined in previous sections to be validated. This comparison is shown in Figure 101, where the data points are the measured points and the solid line is the fit, for the material used in SC1 to SC4, which is nominally the same as SC6. This therefore helps validate the approach taken to establish the material properties in other tests. It is noted that the difference in results for SC5 in Table 19 was due to the post weld heat treatment applied to the cylinder.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_y$ (MPa)</th>
<th>$\sigma_{UTS}$ (MPa)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC1-4</td>
<td>539</td>
<td>707</td>
<td>9.7</td>
</tr>
<tr>
<td>SC5</td>
<td>565</td>
<td>660</td>
<td>14.5</td>
</tr>
<tr>
<td>SC6</td>
<td>521</td>
<td>660</td>
<td>10.7</td>
</tr>
</tbody>
</table>

Figure 101 – Comparison of tensile data for spinning cylinder tests (open circles) to Ramberg Osgood fit when adopting R6 estimate of the strain hardening index, $n$
6.6.3. Results and Comments

Figure 102 shows the results of the assessment of the spinning cylinder tests when adopting the R6 Complex $V$ or $V_g$ approaches; where the R6 Complex $V$ approach is shown by the closed squares and the $V_g$ approach shown by the open diamonds for spinning cylinder tests SP2 to SP6. SP1 is also included, which did not include secondary stresses, as the open diamonds. The cracked type is also indicated in the figure legend as either an extended defect (Ex D) or a semi-elliptical defect (S-E D).

In Figure 102 the results presented only include the cases that showed crack growth. For example, only the surface breaking locations of SC4 and SC6 showed any crack growth (which was confirmed by post test analyses) and, as such, only the assessment point for the surface breaking location is presented. The assessment points for the deepest points were, nonetheless, seen to remain within the FAD at all but the highest loads. Along a similar line the initiation time for SC5 was not certain but estimated. Therefore, the times before the estimated initiation of crack growth was observed are not included.

When considering the results from Figure 102 collectively it is clear that all experiments are conservatively predicted by both the Complex $V$ and $V_g$ approaches. It can also be seen that the use of the $V_g$ approach provides a reduction in conservatism for all cases, even those that do not include significant levels of primary load. This reduction under negligible primary loads is a result of the magnitude of the thermal loads creating a secondary reference stress above yield ($K_f^S / (K_f^P / L_r) > 1$), which causes the lookup tables in R6 to increase $K_r$ even without the inclusion of primary stresses. The $V_g$ approach does not include this feature but is still seen to provide an accurate prediction of failure. The reduction in conservatism seen for each case is up to 20% in terms of $K_r$ for SC2, 3, 4 and 6, but is less for SC5, which is the one case that showed a different hardening curve (as this material underwent a post welding heat treatment). It is also noted that the effect of the residual stress was not considered in the assessment of SC5 as [85] stated that the PWHT removed the majority of the residual stress.
The results presented in Figure 102 also show some divergence between the experiments performed on extended defects and those performed on semi-elliptical defects. All the cases considered for extended defects are seen to fail at a value of $K_r$ close to the FAD in most cases and less than $K_r = 1.5$ for all. However, the two cases (SC4 and SC6) with testing semi-elliptical defects failed at assessment points close to $K_r = 2$. It has already been noted that the failure location for these defects was closer to the point where the defect intercepts the internal surface of the cylinder than the deepest location. One potential explanation for the observed increase in $K_r$ might be the loss of constraint of the material close to the internal surface increasing the effective fracture toughness.
6.7. Pre-Compression Tests of Mirzaee-Sisan

6.7.1. Summary of Tests

The tests performed by Mirzaee-Sisan [78] include the residual stress through a mechanical pre-load in the form of end-load compression. As already noted, these tests formed the basis for the experimental programme in Section 5. The specimens were fabricated from A533B ferritic pressure vessel steel. Following a pre-compression of 73 kN the specimen was unloaded and cooled to -150 °C before incrementing the applied load under 3-point bending until a brittle failure event occurred. As a complex loading history was present, the British Energy Code “J-Mod” [23] was used to accurately calculate the crack driving force parameter \( J \).

The data required for an R6 based assessment of these tests can be seen in Table 20 below.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>As Received (1-8)</th>
<th>Residual Stress (1-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Type</strong></td>
<td>Three-point bend specimen with scalloped notch</td>
<td></td>
</tr>
<tr>
<td>( W ) (mm)</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>( S ) (mm)</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>( B ) (mm)</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Initial Crack size, ( a ) (mm)</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td><strong>Material Properties</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E ) (MPa)</td>
<td>220000</td>
<td></td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Yield Stress, ( \sigma_y ) (MPa)</td>
<td>636</td>
<td></td>
</tr>
<tr>
<td>Tensile Strength, ( \sigma_{UTS} ) (MPa)</td>
<td>889</td>
<td></td>
</tr>
<tr>
<td>Fracture Toughness (MPa.m(^{1/2}))</td>
<td>70.3</td>
<td></td>
</tr>
<tr>
<td><strong>Loading</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary load</td>
<td>Three point bend load</td>
<td></td>
</tr>
<tr>
<td>( P ) (kN)</td>
<td>45.7, 35.2, 30.2, 24.1, 30.6, 16.6, 13.8, 12.7, 25.1, 12.4, 17.8</td>
<td></td>
</tr>
</tbody>
</table>
6.7.2. Assessment Method

6.7.2.1. $K_r$ Position

Values for $K_r^I$ and $K_r^S$ are defined in the paper from finite element analyses, which have been validated against analytical solutions. These finite element values have also been adopted here to provide $K_r$.

6.7.2.2. $L_r$ Position

The $L_r$ position of the assessment points was determined from Equation 85.

6.7.2.3. Estimation of Material Tensile Curve

The tensile curve was defined by a Ramberg-Osgood curve with the strain-hardening index estimated as described in Section 6.1.

6.7.3. Results and Comment

The results from applying the assessment approaches to the results of the pre-compressed tests of Mirzaee-Sisan can be seen in Figure 103. In the figure, the R6 Complex $V$ approach is again shown by the closed squares and the $V_g$ approach shown by the open diamonds, open circles as the cases that do not account for the secondary stress in the pre-compression.
compressed cases and the open stars are the as-received specimens that did not undergo a pre-compression.

![Graph showing fracture toughness](image)

**Figure 103 – Pre-compressed tests by Mirzaee-Sisan assessed by R6 Complex V and \( V_g \) approach of Section 4 compared to an Option 1 FAD**

Under the elastic conditions of these experiments there is little difference between the Complex \( V \) and \( V_g \) approaches, with the \( V_g \) approach marginally less conservative. Treating the pre-compressed cases as primary only is again non-conservative, as would be expected, especially under brittle conditions, as all assessment points are within the FAD. The assessment of the as-received samples shows the assessment points to be evenly spread over the FAD, which would be expected as the mean fracture toughness has been considered. This, however, then indicates that the pre-compressed samples are not being correctly assessed as all the assessment points are outside the FAD.

That the pre-compressed samples are exterior to the FAD may be explained by the estimate of \( K_f^S \), where the value adopted has been defined from the finite element analyses. It is stated in [78] that isotropic hardening has been assumed for the hardening model in the finite element analyses. Therefore, within the pre-compression, release step of the
analyses it is possible that the magnitude of the tensile stress field has been over-predicted, leading to an enhanced value of $K^S_f$, which would then increase the estimate of $K_r$. By comparison to the results of Section 5 where, also for A533B material, kinematic hardening is best suited the estimate of $K_r$ is 35 MPam$^{0.5}$, which is 10 MPam$^{0.5}$ lower (but on a thicker specimen under a different levels of compression). It is noted that by comparison of the values of $K^S_f$ and $K^P_f$ in Table 20, a value of 25 MPam$^{0.5}$ would provide the best agreement. By simply scaling the applied loads of these tests compared to those in Section 5, whilst accounting for the difference in the area of the uncracked ligament of the specimen, the finite element analyses of Section 5 would suggest end load compression between 0.5 to 1 mm. This would then suggest a value of $K^S_f$ between 21 and 33 MPam$^{0.5}$. It is however noted that the specimens are not identical, i.e. different thickness and scalloped notch size, and this potential range of $K^S_f$ is only suggestive but would provide an improved prediction of failure.

### 6.8. Pre-Compression C-Ring Tests

#### 6.8.1. Summary of Tests

The pre-compressed C-Ring tests performed by Kamel [28] inserted a tensile or compressive residual stress field by mechanical compression or tension of the specimen before inserting the defect. Finite element analyses of the tests provided stress intensity factors from a residual stress field validated by neutron diffraction.

The material used within the tests was a high strength, low alloy tubing steel, AISI 4333 M4. Monotonic stress-strain curves, orientated to match the hoop direction in the experiments, showed almost elastic-perfectly plastic behaviour with a yield stress of 1050 MPa, a UTS of 1100 MPa and a significant Bauschinger effect under compression. On loading all specimens were observed to fail under brittle conditions without any crack growth.

The data required for an R6 based assessment of these tests can be seen in Table 21 below.
Table 21 – Assessment inputs for C-Ring tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>As Received</th>
<th>Pre-Tensioned</th>
<th>Pre-Compressed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>C-Ring</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_i ) (mm)</td>
<td>26.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_o ) (mm)</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t ) (mm)</td>
<td>27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Crack size, ( a ) (mm)</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Material Properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E ) (MPa)</td>
<td>180000 (derived from tensile curve provided)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.3 (estimated)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile curve</td>
<td>Tensile curve provided</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fracture Toughness ( (MPam^{1/2}) )</td>
<td>105</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Loading</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary load</td>
<td>Tensile load applied to pins inducing a bending stress field at crack location</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P ) (kN)</td>
<td>76</td>
<td>91</td>
<td>60</td>
</tr>
<tr>
<td>( K_i^P ) (from FE), ( (MPam^{1/2}) )</td>
<td>105</td>
<td>125.7</td>
<td>82.9</td>
</tr>
<tr>
<td><strong>Secondary stress</strong></td>
<td></td>
<td>C-Ring pulled in tension to induce a compressive stress field at crack on release</td>
<td>C-Ring compressed to induce a tensile stress field at crack on release</td>
</tr>
<tr>
<td>( K_i^S ) (from FE), ( (MPam^{1/2}) )</td>
<td>-</td>
<td>-34</td>
<td>434</td>
</tr>
</tbody>
</table>
6.8.2. Assessment Method

6.8.2.1. \( K_r \) Position

An estimate of \( K_f^S \) has been defined in [28], which is stated as being equivalent under tensile and compressive stress fields. These estimates are defined from finite element analyses. Values of \( K_f^P \) are also provided in [28]. These finite element values have also been adopted here to provide \( K_r \).

For the case that undergoes a pre-tension it is recommended in R6 that no benefit be taken from the compressive residual stress field. Therefore, when assessing this case it is only the primary stress contribution to \( K_r \) that has been plotted. This aspect is, however, discussed further below.

6.8.2.2. \( L_r \) Position

For this geometry no definition of the limit load could be identified. However, as the results are brittle, this is not of significant concern. As a simple approximation the average tensile stress in the uncracked ligament was used (which has to balance the applied load) as a fraction of the material yield stress. This is akin to the net section collapse solutions for plates. Therefore the solution used was:

\[
L_r = \frac{P}{\sigma_y (W - a)t}
\]

Equation 100

It is appreciated that this does not include the bending component of the stress field and how this would affect the plastic collapse limit load. However, the results from this approach predict value of \( L_r \) less than 0.2 so even if the contribution of the bending component reduces the limit load by a factor of 2, which is considered unlikely, the value of \( L_r \) will remain low, corresponding to essentially an elastic response, and not affect the interpretation of these tests.
6.8.3. Results and Comment

The results from applying the assessment approaches to the results of the pre-compressed C-Ring tests can be seen in Figure 104 below. In the figure, the R6 Complex V approach is again shown by the closed squares and the \( V_g \) approach shown by the open diamonds.

Note that the results for the R6 V and the \( V_g \) analyses lie on top of each other.

**Figure 104 – C-Ring tests assessed by R6 Complex V and \( V_g \) approach of Section 4 compared to an Option 1 FAD**

It can be seen that, as the tests are essentially elastic, there is no difference between the Complex V and \( V_g \) approaches and all results are conservative.

It can be seen by comparison of the data in Table 21 that the approach of simply adding the primary and secondary contributions provides estimates of combined crack driving force of 91 and 117 MPam\(^{1/2}\) for the pre-tensioned and pre-compressed cases respectively. It is clear that these results are not near the fracture toughness value of 105 MPam\(^{1/2}\) and that including the compressive residual stress is potentially non-conservative. This is,
however, considered unusual for the case under elastic conditions as the solutions should be additive and inherently contain little conservatism. Note that when including the effects of stress redistribution with plasticity the conservatism in the Complex $V$ and $V_g$ approaches will act to be non-conservative and the guidance of R6 is valid. Therefore it is considered that under elastic conditions it should be possible to provide an improved estimate of failure for the pre-tensioned and pre-compressed cases.

By simply modifying $K_f$ it is possible to make all assessment points fall upon the FAD if $K_f = \pm 22$ for the pre-compressed and pre-tensioned cases. This would simply suggest that the estimate of $K_f$ is incorrect. In [28] $K_f$ has been determined from the finite element analyses that adopted both isotropic and kinematic hardening models and introduced the residual stress field in an unloaded model by means of the SIGINI command in ABAQUS. It is noted in [28] that neither hardening model accurately captured the hardening behaviour of the material used as it exhibited a Bauschinger effect on unloading, which would impact on the predicted stress field. Some evidence of this can be seen in the comparison to neutron diffraction measurements within reference [28].

It is also considered that the use of the SIGINI command is not correct. In ABAQUS, SIGINI is used to define an initial stress field in an initially unstressed model and is normally used to model excessive deformation, such as may be seen under buckling analyses, where the mesh deforms to the point where a new mesh is necessary. This approach, however assumes proportional loading, i.e. the geometry is monotonically loaded with no unloading so that stress-strain points are always on the stress-strain curve. This, however, is unlikely here and the use of SIGINI might have introduced the correct stress, but not necessarily the correct strain, so that the evaluation of $f$ is no longer valid.

---

11 The SIGINI command takes a pre-defined stress field and applied this to an unstressed finite element model; i.e. it allows the definition of an initial stress field. This is commonly used where a specialist model is used to calculate the stress field before this is applied to a second finite element model for structural analyses; such as weld modeling followed by crack insertion.
6.9. Application to Side Punched Tests

6.9.1. Summary of Tests

Hurlston [81] constructed experiments that include both primary and secondary stresses. The secondary stress introduced was by way of mechanical pre-compression out-of-plane, parallel to the crack front inducing a tensile residual stress acting to open the crack on unloading. Relatively shallow (\(a/W = 0.22\)) and deep cracks (\(a/W = 0.42\)) were considered, both with and without the residual stress. The specimens were cooled to brittle conditions and tested under three-point bending. The specimens were made from A533B steel. It can be considered that these tests improved upon similar tests by Mahmoudi [113] (and, as such the tests in [113] are not considered here).

The data required for an R6 based assessment of these tests can be seen in Table 22 below.

Table 22 – Assessment inputs for side-punched tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Shallow Crack As-Received</th>
<th>Shallow Crack Residual</th>
<th>Deep Crack As-Received</th>
<th>Deep Cracked Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Three point bend specimen with two crack depths</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(W (mm))</td>
<td></td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S (mm))</td>
<td></td>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B (mm))</td>
<td></td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Crack size, (a (mm))</td>
<td>11</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Material Properties</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E (MPa))</td>
<td>220000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\nu)</td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Stress, (\sigma_y (MPa))</td>
<td>695</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile curve</td>
<td>Provided by Ramberg Osgood relationship</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ramberg Osgood Strain</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardening Index, (n)</td>
<td>7.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fracture Toughness (lower)</td>
<td>76</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bound value), $(MPam^{1/2})$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Loading

<table>
<thead>
<tr>
<th>Primary load</th>
<th>Three point bend loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ (kN)</td>
<td>86, 96, 110, 116, 119, 124, 132</td>
</tr>
<tr>
<td></td>
<td>38, 43, 59, 61, 68, 69, 73</td>
</tr>
<tr>
<td></td>
<td>47, 50, 51, 56, 61, 63, 64</td>
</tr>
<tr>
<td></td>
<td>30, 31, 33, 37, 39, 42, 48</td>
</tr>
<tr>
<td>Secondary stress</td>
<td>Residual stress from side punch</td>
</tr>
<tr>
<td>$K_r^S$ Solution (from FE), $(MPam^{1/2})$</td>
<td>-</td>
</tr>
</tbody>
</table>

#### 6.9.2. Assessment Method

##### 6.9.2.1. $K_r$ Position

An estimate of $K_r^S$ has been defined in [81] from finite element analyses. Values of $K_r^p$ were defined as below [78]. These values are then used to estimate $K_r$.

$$K_r = \frac{FS}{BW^{3/2}} f\left(\frac{a}{W}\right)$$  \hspace{1cm} \text{Equation 101}

where $f\left(\frac{a}{W}\right)$ is given by:

$$f\left(\frac{a}{W}\right) = \frac{3}{2(1 + 2\frac{a}{W})(1 - \frac{a}{W})^{3/2}}\left(1.99 - \frac{a}{W}\left(2.15 - 3.93\frac{a}{W} + 2.7\left(\frac{a}{W}\right)^2\right)\right)$$  \hspace{1cm} \text{Equation 102}

##### 6.9.2.2. $L_r$ Position

The $L_r$ position of the assessment points was determined from Equation 85.
6.9.3. Results and Comment

Figure 105 shows the results of an assessment of the side-punched tests where the R6 Complex $V$ approach is shown by the closed squares, the $V_g$ approach shown by the open diamonds, the consequence of not including the secondary stress is shown by the open circles and the samples without a residual stress are shown as the open stars. The black points are for the shallow cracked specimens and the red points are for the deeply cracked specimens.

![Figure 105 – Side-punched tests assessed by R6 Complex $V$ and $V_g$ approach of Section 4 compared to an Option 1 FAD when adopting the lowest fracture toughness from the as received samples observed in the tests as $K_{mat}$ in the calculations](image)

It can be seen that, as the tests are approaching intermediate levels of $L_r$, there is a slight difference between the Complex $V$ and $V_g$ approaches, but this is only minor. The results show that the deep crack is conservative, whereas the results of the shallow crack indicate some level of non-conservatism for both the Complex $V$ and $V_g$ approaches. In Figure 105 the fracture toughness value has been taken as the minimum value of the deeply cracked,
high constraint, specimens, 75 MPam$^{1/2}$. This value used however, is significantly higher than the fracture toughness of this material reported in the literature [103], which would provide an estimate of fracture toughness of 50 MPam$^{1/2}$ at -140 °C. The results of the assessment when adopting this value of fracture toughness can be seen in Figure 106 which clearly shows that all results are now conservative. This difference in fracture toughness is simply believed to be a result of the relatively low number of specimens tested and the large potential variability in fracture toughness under brittle conditions.

![Figure 106 – Side-punched tests assessed by R6 Complex V and Vg approach of Section 4 compared to an Option 1 FAD when adopting the lower bound fracture toughness from the literature [103] as $K_{mat}$ in the calculations](image-url)
6.10. Pre-Compression Compact Tension and Single Edge Notched Tensions Tests

6.10.1. Summary of Tests

A series of studies were performed by Lee et al [79] on both high constraint compact tension, CT, specimens and low constraint single edge notched, SENT, specimens both with and without a residual stress. The specimens that contained residual stress underwent a compressive preload to establish the stress field prior to pre-cracking. The samples were made from A533B pressure vessel steel and tested in the cleavage fracture regime.

The data required for an R6 based assessment of these tests can be seen in Table 23 below.

Table 23 – Assessment inputs for pre-compressed CT and SENT tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>CT</th>
<th>SENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As Received</td>
<td>Residual</td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>CT Specimen</td>
<td>CT Specimen electron beam welded to a tensile rig</td>
</tr>
<tr>
<td>W (mm)</td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>B (mm)</td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>Initial Crack size, a (mm)</td>
<td></td>
<td>30.4</td>
</tr>
<tr>
<td>Material Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td></td>
<td>210000</td>
</tr>
<tr>
<td>$\nu$</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>Tensile curve</td>
<td>Tensile curve provided</td>
<td></td>
</tr>
<tr>
<td>Yield Stress (MPa)</td>
<td></td>
<td>400</td>
</tr>
<tr>
<td>Lower Bound Fracture Toughness ($MPa.m^{1/2}$)</td>
<td>89</td>
<td>112</td>
</tr>
</tbody>
</table>

Loading
### 6.10.2. Assessment Method

#### 6.10.2.1. $K_r$ Position

An estimate of $K_r^S$ has been defined in [79] from finite element analyses and adopted the JEDI postprocessor to account for the pre-stressed and non-proportional effects.

Values of $K_r^P$ are defined from R6 [3] as shown in Equation 103 for the CT specimen and in Equation 104 for the SENT specimen under fixed grip conditions.

\[
K_r = \frac{F}{BW^2} \left( \frac{2 + a}{W} \right)^2 f \left( \frac{a}{W} \right)
\]

**Equation 103**

\[
f \left( \frac{a}{W} \right) = 0.886 + 4.64 \left( \frac{a}{W} \right) - 13.32 \left( \frac{a}{W} \right)^2
\]

\[
+ 14.72 \left( \frac{a}{W} \right)^3 - 5.6 \left( \frac{a}{W} \right)^4
\]

where, for the CT geometry, $F$ is the applied load, $B$ is the specimen thickness, $W$ is the distance from the loading pin to the back face and $a$ is the distance from the loading pin to the end of the crack (i.e. crack depth).
\[ K_I = \sigma \sqrt{\pi a f \left( \frac{a}{W} \right)} \]

where for the SENT geometry \( \sigma \) is the applied stress, \( W \) is the plate thickness and \( a \) is the crack depth.

These values are then used to estimate \( K_r \) when adopting the fracture toughness defined as the minimum value from the cases that did not undergo a pre-compression. This allows constraint effects from the two loading conditions to be included.

### 6.10.2.2. \( L_r \) Position

The \( L_r \) position of the assessment points was determined from the geometric limit loads defined in Equation 105 and Equation 106 for the CT and SENT (fixed grip) geometries from R6 Equations IV.1.5.1 and IV.1.5.5, respectively.

\[ N_L = W B \sigma_y \left( \sqrt{\left( 1 + \frac{2}{\sqrt{3}} \right) \left( 1 + \frac{2}{\sqrt{3}} \frac{a}{W} \right)^2} - \left( 1 + \frac{2}{\sqrt{3}} \frac{a}{W} \right) \right) \]  
\[ \text{Equation 105} \]

\[ N_L = W B \sigma_y \left( 1 - \frac{a}{W} \right) \]  
\[ \text{Equation 106} \]

### 6.10.3. Results and Comment

Figure 107 shows the results of an assessment of the pre-compressed CT and SENT tests where the R6 Complex \( V \) approach is shown by the closed squares, the \( V_g \) approach shown by the open diamonds, the consequence of not including the secondary stress is shown by
the open circles and the samples without a residual stress are shown as the open stars (where the lines between the stars represent the assessment points between the two extreme cases). The black points are for the CT specimens and the red points are for the SENT specimens.

Figure 107 – Pre-compressed CT and SENT tests assessed by R6 Complex $V$ and $V_g$ approach of Section 4 compared to an Option 1 FAD

These tests represent results under intermediate levels of $L_r$. In the figure it can be seen that there is a relatively large difference between the Complex $V$ and $V_g$ approaches, with both approaches remaining conservative. The maximum potential benefit of adopting the $V_g$ approach over the Complex $V$ approach is a reduction of $K_r$ of approximately 25% and over 50% reduction in the contribution of the secondary stress allowing more than 75% reduction in the levels of conservatism inherent to the Complex $V$ approach (as demonstrated at $L_r = 0.5$ and 0.75). Also, as with other cases considered the consequence of neglecting the secondary stress is non-conservative results.
This case shows that there is clearly a large potential benefit to be found when adopting the $V_g$ approach when increasing the levels of plasticity considered.

### 6.11. Application to Inspectra AB pre-compressed tests

#### 6.11.1. Summary of Tests

Bolinder, of Inspectra Technology AB, [76] performed a range of tests to consider combined loading at high values of $L_r > 1$. The tests were based upon the Mirzaee-Sisan [76] experiments but are performed on materials at room temperature so that the intermediate and high values of $L_r$ are realised. As such the tests introduced the residual stress by a pre-compression on the end of the specimen.

To test under plasticity two materials were considered; Weldox 700 and A533B. Two different geometries were considered for the Weldox 700 material so that two values of $L_r$ resulted. Therefore three cases are considered to have the failure condition at:

- Case 1; $L_r = 0.9$, Weldex 700,
- Case 2; $L_r = 1.0$, Weldex 700, and
- Case 3; $L_r = 1.1$, A533B.

The data required for an R6 based assessment of these tests can be seen in Table 24 below.

#### Table 24 – Assessment inputs for Inspectra AB pre-compressed tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weldex 700 – $L_r = 0.9$</td>
<td>2 x As Received</td>
<td>3 x Residual</td>
<td>2 x As Received</td>
</tr>
<tr>
<td>Weldex 700 – $L_r = 1.0$</td>
<td>2 x As Received</td>
<td>3 x Residual</td>
<td>3 x As Received</td>
</tr>
<tr>
<td>A533B – $L_r = 1.1$</td>
<td>3 x As Received</td>
<td>5 x Residual</td>
<td>3 x As Received</td>
</tr>
</tbody>
</table>

**Geometry**

<table>
<thead>
<tr>
<th>Type</th>
<th>Three point bend specimens with notch of radius 0.25W</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$ (mm)</td>
<td>100</td>
</tr>
<tr>
<td>$B$ (mm)</td>
<td>50</td>
</tr>
<tr>
<td>----------</td>
<td>----</td>
</tr>
<tr>
<td>$S$ (mm)</td>
<td>400</td>
</tr>
<tr>
<td>$a$ (mm)</td>
<td>35</td>
</tr>
<tr>
<td>Crack growth 1, $(\Delta a)_1$, (mm)</td>
<td>0.41, 0.27</td>
</tr>
<tr>
<td>$(\Delta a)_2$, (mm)</td>
<td>0.72, 0.63</td>
</tr>
</tbody>
</table>

**Material Properties**

| $E$ (MPa) | 207500 | 205300 |
| $\nu$ | 0.3 | |
| Yield Stress, $\sigma_y$ (MPa) | 655 | 471 |
| Tensile curve | Tensile properties provided |
| Fracture Toughness | $J - R$ data provided |

**Loading**

| $P_{\Delta a_1}$ (kN) | 469(x2) | 432, 465(x2) | 234, 241 | 236(x3) | 26, 27, 26 | 27, 26(x2), 27(x2) |
| $P_{\Delta a_2}$ (kN) | 488(x2) | 483(x3) | 243, 250 | 245(x3) | 27(x3) | 26, 27(x2), 28(x2) |

Secondary stress: End compression and tensile residual stress field on release

| $K_f^S$ from FE (MPam$^{1/2}$) | - | 96.5 | - | 115.8 | 73.8 |
| Limit Load (FE) (kN) | 460 | 227 | 24 |

### 6.11.2. Assessment Method

#### 6.11.2.1. $K_r$ Position

An estimate of $K_f^S$ has been defined in [76] from finite element analyses. Values of $K_f^P$ can be defined from a three-point bend test as shown in Equation 101.
6.11.2.2. \( L_r \) Position

The \( L_r \) positions of the assessment points have been determined from finite element analyses.

6.11.3. Results and Comment

Figure 108 shows the results of an assessment of the pre-compressed bend tests performed by Inspectra AB where the R6 Complex \( V \) approach is shown by the closed squares, the \( V_g \) approach shown by the open diamonds, the consequence of not including the secondary stress is shown by the open circles and the samples without a residual stress (as-received) are shown as the open stars. The black points are for Case 1, the red points are for Case 2 and the blue points are for Case 3. Also included in the figure are Option 2 FADs for the Weldex 700 material in red and the A533B material in black in addition to the Option 1 FAD in black. Note that in order to show the data effectively the plot starts from an \( L_r \) value of 0.8.

As the results for the as-received cases make the plot complex to consider Figure 109 repeats Figure 108 without the Option 1 FAD, the results when not including the secondary stress contribution, or the as-received cases.
Figure 108 – Inspectra AB pre-compressed tests assessed by R6 Complex $V$ and $V_g$ approach of Section 4 compared to an Option 1 FAD including all data points

Figure 109 – Inspectra AB pre-compressed tests assessed by R6 Complex $V$ and $V_g$ approach of Section 4 compared to an Option 1 FAD only considering the pre-compressed cases
In Figure 108 it can be seen that the use of the Option 1 FAD is not correct for the A533B material at these levels of applied primary load as even the cases that include only primary load fall within the FAD. The Option 2 FADs provide an improved estimate of failure compared to the data as no points fall far from the FAD, even if very slightly inside. For these results, with this level of plastic redistribution of the residual stress, the consequence of not including the secondary stress contribution is close to the as-received samples but still more than 0.2 in terms of \( K_r \). This helps demonstrate that even at such high levels of \( L_r \) the secondary stress contribution cannot necessarily be neglected, but its respective contribution is significantly diminished.

In Figure 109 it can be seen that there is a large difference between the Complex \( V \) and \( V_g \) approaches for the Weldex 700 material but a much smaller difference for the A533B material, but with both approaches remaining conservative in all cases. As with the pre-compressed CT and SENT cases the maximum potential benefit of adopting the \( V_g \) approach over the Complex \( V \) approach is a reduction of \( K_r \) of approximately 25%. This again converts to more than a 50% reduction in the contribution of the secondary stress and more than 75% reduction in the levels of conservatism (as demonstrated at \( L_r = 0.95 \)).

This case shows that there can be a large benefit when adopting the \( V_g \) approach at high levels of plasticity and that neglecting the secondary stress contribution for ductile materials around \( L_r = 1 \) can, potentially, lead to non-conservative results.

### 6.12. Summary of results

The application of the R6 Complex \( V \) and \( V_g \) approaches to the numerous experiments here has shown a number of consistent features. These are summarised below:

- Both the R6 Complex \( V \) and \( V_g \) approaches have been applied to the experiments considered in the validation of R6, Revision 4, as well as a number of additional experiments, including those outlined in Section 5, so that a large number of geometries, materials and types of secondary stress have been considered. It is
clear from the results that both the R6 Complex $V$ and $V_g$ approaches are conservative.

- The results for the cases that allow for different levels of plasticity, i.e. those that are not purely brittle in nature, have been summarised by the approach considered in Figure 110 and Figure 111 for the R6 Complex $V$ and $V_g$ approaches, respectively.

- The use of the $V_g$ approach when compared with the Complex $V$ approach of R6 can provide significant benefit, of up to 25% in terms of $K_r$, which can relate to a reduction in the perceived conservatism in the approach of over 50%. This reduction can be seen in Figure 99, Figure 107 and Figure 108 to be a result of the relative contribution of the secondary stress where the $V_g$ approach provides up to a 50% reduction in the multiplication term to $K_i^s$.

  - Figure 112 summarises the levels of reduction in conservatism between the R6 Complex $V$ and the $V_g$ approaches for each assessment point, for all tests compared to an Option 1 FAD (note that the cut-off has arbitrarily been made at $L_r = 1.25$ for illustration and does not affect the results). Therefore, the larger the reduction in conservatism, the longer the line (if no reduction was seen, no line is present).

- The reduction in the level of conservatism is material dependent. This might be a result of the assumptions made in defining the material stress-strain curve.

- Some cases with the R6 Complex $V$ approach, such as for the spinning cylinder tests, increase the value of $V$ as a result of the level of secondary stress alone; effectively increasing $K_i^S$ above $K_i^S$. The tests considered here show that this is not necessary as with the $V_g$ approach no enhancement is made but the method still remains conservative.

- It is non-conservative to neglect the contribution of the secondary stress completely. Figure 108 shows this to even be the case for $L_r > 1$, although the level of non-conservatism is small. It can therefore be considered that the contribution is never completely removed; it just becomes less influential to failure.
Figure 110 – Summary plot of all tests assessed by R6 Complex V compared to an Option 1 FAD when only considering cases with primary induced plasticity

Figure 111 – Summary plot of all tests assessed by $V_e$ compared to an Option 1 FAD when only considering cases with primary induced plasticity
Figure 112 – Illustration of the reduction in conservatism for all tests assessed by adopting the $V_g$ approach
7. Discussion

7.1. Preface

This section provides further discussion surrounding the work presented though Section 4 to Section 6. It is separated into a number of sections, which are then broken to further subsections, to consider different aspects of the work including the:

7.2) “Development and features of the \( V_g \) approach”,
7.3) “Experimental studies”
7.4) “Validation of the \( V_g \) approach by identified experiments”.

Following these initial three subsections further consideration is then given to;

7.5) “Potential knowledge gaps and analytical consideration”; where a potential gap in the validation is examined numerically for cylinders with a weld residual stress.

7.2. Development and Features of the \( V_g \) Approach

This section considers the redistribution of secondary stresses on combination with primary stresses as described by the \( V_g \) term. To allow discussion of a number of features observed the section is split to the following subsections:

7.2.1) Qualitative description of secondary stress redistribution,
7.2.2) Dependence of \( V_g \) on the magnitude of secondary stress,
7.2.3) Material’s dependence on \( V_g \) and the modification to the Option 2 FAD,
7.2.4) Conservatism of \( V_g \) and the R6 complex \( V \) approaches,
7.2.5) Effect of elastic follow-up,
7.2.6) Summary.
7.2.1. Qualitative Description of Secondary Stress Redistribution

The work presented in Section 4 details the development of a new plasticity interaction parameter, $\lambda_f$, to describe how the secondary stress contribution to fracture is enhanced under moderate levels of plasticity and then subsequently reduced under larger levels of plasticity. To consider the definition of $\lambda_f$ it is first important to understand the behaviour and interactions between primary and secondary stresses. This section therefore provides a qualitative description of secondary stress redistribution.

Table 1 provides an overview comparison of the main differences between primary and secondary stresses. The most significant difference is that secondary stresses result from deformations within the material whereas primary stresses originate from externally applied loads. This means that increasing plastic deformation within a structure will influence the secondary stresses distribution and magnitude; however the enhancement in the levels of strain will also contribute to the crack driving force. As primary stresses are applied remotely any level of deformation can be obtained, leading to plastic collapse.

From the work reviewed and presented it is clear that the interaction of primary and secondary stresses can be complex as secondary stress will reduce with increasing plasticity whilst the primary stress will continue to increase with increasing plasticity, providing an interdependent relation. Figure 113 provides a physical interpretation of this interaction based on the increase in crack tip plasticity as a function of increasing primary stress. The results shown in Figure 113 are from the finite element analyses performed in Section 4 where the graphs above each contour plot show the total stress applied in red (in these cases a thermal bending stress of $\sigma_p = 300$ MPa (yield) and an increasing tensile primary load). From simply subtracting the tensile primary stress the resultant secondary stress is shown in blue. The contour plots in the figure show the degree of plastic strain in the structure, where the red areas are above 0.2% plastic strain and the grey areas are above 0.4% plastic strain.

This relatively simple illustration of the development of the plastic zone with applied primary stress demonstrates that the main feature of combined primary and secondary loading is that the secondary stress will redistribute with increasing plasticity. It has been shown in [58], [61], [63], [75], [77] that the predominant reason for redistribution
processes is plasticity, where plastic flow allows the strain mismatch causing the secondary stress to be reduced to different extents depending on the degree of plasticity. The plasticity can either be from the secondary stress itself or from primary stresses that, by definition, do not reduce with plastic strains, or a combination of both.

Figure 113 – Illustration of secondary stress redistribution on the cracked plane under an increasing primary stress and hence plastic zone size for a fully circumferential cracked cylinder subject to a through thickness thermally induced bending stress and a primary end load as in Section 4.

Figure 113 shows that under higher primary loads, \( 1.1 < L_r < 1.5 \), the proportion of the total stress above yield increases, subsequently redistributing the secondary stress so that it decreases in magnitude, where eventually, at \( L_r = 1.5 \), the total stress is almost membrane through thickness and is dominated by primary loads; the secondary bending contribution is negligible. However, for the stress to redistribute the plastic strain must increase both in magnitude and extent. This can also be seen in Figure 113 where the point where the plastic zone crosses the ligament is at approximately \( L_r = 1 \). It is after this point that the secondary stress redistribution accelerates as the net section yielding results in significant
stress relief. The results of Ainsworth et al [77] and Bolinder [76] show this behaviour experimentally, as well as the experimental results described in Section 5 and assessed in Section 6.

There are cases for the $V_g$ approach in Section 4 that appear to not follow this reduction as the relative contribution of the secondary stress to the total crack driving force was seen to increase, i.e. $V_g > 1$, with intermediate to high levels of plasticity, $0.5 < L_r < 1.1$. These are nominally where the region of interest, i.e. the crack tip vicinity, is elastic-plastic but the material surrounding the crack is elastic and preventing a more complete relaxation. Therefore the stress level is highly coupled with an increase in crack tip strain. This enhancement from the surrounding material is essentially a manifestation of elastic follow-up (leading to a plastic enhancement of $J$); i.e. the stiffness of the surrounding material away from the region of interest is reducing the relaxation levels within that region as described by [2], [54].

When considering the behaviour of this interaction the $V_g$ method follows the basic features noted above. The development of $V_g$ was effectively based on an empirical fit to a range of finite element data relating to a fully external circumferentially cracked cylinder. However, the range of loading applied to the cylinder allowed many different combinations of primary axial and hoop stress, and with different magnitudes of a thermally induced secondary biaxial bending stress. From this approach an interim function, $g$, has been defined to describe the redistribution of secondary stresses when combined with primary stress, as the level of plasticity increases. This has been shown to be compatible with the existing R6 $V$ factor approach by conversion to $V_g$.

7.2.2. Dependence of $V_g$ on the Magnitude of Secondary stress

In the development of the $V_g$ approach it is noted that there was a negligible difference in the finite element prediction of $g$ with an increase in the level of secondary loading. It is upon this relative insensitivity to the level of secondary stress that that the $V_g$ approach is based. This insensitivity can also be seen in the $V$ approach of R6, in the $V/V_0 = \xi$ term, as shown in Figure 114 for levels of secondary residual stress up to $L_r = 0.8$, and for all
values of secondary reference stress when $L_r > 1$. Note that in the construction of Figure 114 the $\xi$ table of R6 [3] is plotted with the secondary reference stress calculated from $K^S_f/(K^P_f/L_r) = \left(\sigma_{ref}^S/\sigma_y\right)/f\left(\sigma_{ref}^S/\sigma_y\right)$. In this derivation it is assumed that the $\bar{a}$ terms for the primary and secondary stress are equivalent, which has been considered potentially incorrect by James et al [71] under some loading combinations. However, here it is still used as it is inherent to the assumptions in R6 and this comparison is general. It is also noted that this independence would not be seen in the $\rho$ implementation of R6.

This helps show that this relative insensitivity to the level of secondary reference stress to $g$ is correct for smaller values of secondary reference stress and when the primary reference stress is greater than yield. It is noted in Section 4 that the demonstrated independence might be an artefact of the secondary stress type adopted or the geometry used in the analyses. This is not considered to be likely as features such as the stress type and geometry are encompassed by the use of the reference stress concept. The reference stress concept has been shown [39–42], [46] to effectively treat a large range of geometries, including the cracked cylinder used here, as well as a number of stress types, also including bending stresses. Moreover, the use of the well established Option 2 FAD, such as shown in Figure 13 [47], contains the greatest difference between materials under small scale yielding conditions. Therefore any potential variation in the use of reference stress concept should only exist between $L_r = 0.6$ and 1 and/or $\sigma_{ref}^S/\sigma_y = 0.6$ and 1. Therefore, it is considered that the independence of $g$ as found in Section 4 is correct, as further supported by Song [74] in Figure 27 where the level of stress redistribution is clearly the same for a number of different levels of secondary stress and crack orientations. It is also noted that the development of the simplified interpretation of the $V$ approach [63] is also possible because of an observed insensitivity of $V$ to the level of secondary stress applied.
Figure 114 – Plot of $V/V_0$ as a function of $L_r$ for a range of secondary reference stress values obtained from the look-up tables in R6 [3]

It is therefore necessary to understand the difference in values of $V$ in R6 at lower values of $L_r$ for larger values of secondary reference stress shown in Figure 114. A potential explanation of this is provided by considering the original deformation-bounding theorem, as used to define the original $\rho$ term by Ainsworth [58]. It is noted by Ainsworth that the approach is a conservative interpretation of the bounding theorem. In this conservative approach, it is postulated that when the level of secondary stress approaches yield the deformation stress applied to represent the secondary load, $R$, will not account for the potential self-relaxation of the secondary stress. This in turn will enhance the contribution of the secondary reference stresses close to and above yield. Therefore the deformation load includes both primary and secondary stress, as might be considered with moderate levels of elastic follow-up. This is considered further below when examining the conservative nature of R6. **It is therefore considered that the level of secondary stress should have a negligible effect on the value of $V_g$, as demonstrated by the finite element analyses.**
7.2.3. Material’s Dependence on $V_g$ and the Modification to the Option 2 FAD

The results used to define $V_g$ show a very well defined relation to the materials hardening coefficient, sharing features seen for an Option 2 FAD. Such results are not currently considered in the approaches of R6 where only an upper bound approach irrespective of material properties is presented. The material’s hardening dependence is correct, as the crack driving force, or equivalently the crack tip opening, is governed by both stress and strain at the crack tip [13], [14]. Therefore the contribution of secondary stresses to the crack driving force can alter depending on the relative contribution of the stress and strain in this region. For a material that shows high strain hardening (low $n$) the crack tip stresses will be lower than for a low strain hardening (high $n$) material at the same load, but will have higher stresses in the remaining ligament (as the applied force must balance). This means that, on application of primary loads, the low strain hardening material will induce more crack tip plasticity but will have a smaller plastic zone. Therefore, on contained yielding, the low strain hardening material will have a bigger increase in crack driving force with plastic strain but, on gross yielding that allows the secondary stress to redistribute, the low strain hardening will have a larger strain level for a given stress. This explains why the estimates of $V_g$ show the greatest increase and subsequent decrease for cases with larger values of $n$, i.e. lower strain hardening.

That the redistribution of the secondary stresses follows the form described by one of the R6 FAD options provides some further justification of the methodology. The development of the FAD as outlined in [43] is to describe the evolution of plasticity in a cracked component. Under primary loads this enhances the primary stress intensity factor. However, under secondary loads it is therefore not surprising that an equation based upon the FAD reduces the secondary stress contribution. The benefit of an Option 2 FAD above an Option 1 FAD is that this simply allows the material’s dependence to be included in this assessment. Therefore the use of $V_g$, as adopting the Option 2 FAD, provides an additional complexity but also allows a more accurate estimate of secondary stress redistribution. It is therefore considered that the $V_g$ approach follows the hierarchical philosophy of R6.

The $V_g$ approach is materials dependent and this dependence is straightforward to implement and could provide an additional hierarchy to the existing approaches in R6.
In the $V_g$ approach the estimate of the $g$ term adopts a modified form of the Option 2 FAD. In Section 4 it was found that the modification to the Option 2 FAD is determined by two fitting parameters, the A and B terms detailed below:

$$g = \left[ \frac{E \varepsilon_{ref}^{mod}}{\sigma_{ref}^{mod}} + A \left( \frac{\sigma_{ref}^{mod}}{\sigma_y} \right) \right]^{1/2} \quad \text{Equation 107}$$

$$\sigma_{ref}^{mod} = \sigma_{ref}^p / B = \sigma_{ref}^p / 1.25 \quad \text{Equation 108}$$

$$A = \frac{\sigma_{in \ plane}}{1.25 \sigma_{mises}} \quad \text{Equation 109}$$

The change in the modified reference stress, $\sigma_{ref}^{mod}$, from the primary reference stress was found by the location of the skeletal point of the finite element predictions of $g$. As demonstrated by [40], [41] the point at which the skeletal point occurs for a range of different materials with different strain hardening index provides a stress that can describe the reference stress for the cracked body. This could indicate that this is the equivalent primary stress to where the combined reference stress is at yield. However, the modified reference stress is less than the primary stress meaning that it cannot represent the combined reference stress (which has to be equal to or greater than the primary stress alone). Therefore it is likely that this stress represents the stress at which the primary stress becomes dominant and the secondary stress is effectively removed, which will be further affected by elastic follow-up.

The values for $B$ seen in Section 4 were between 1 and 1.2, but a value of 1.25 was chosen for conservatism. This range of values for $B$ is also compatible with the experimental results seen by Ainsworth [77] and Bolinder [76] where the effective contribution of secondary stress is negligible for $\sigma_{ref}^p / \sigma_y$ values between 1 and 1.2. It is therefore considered that the value of 1.25 used here should be conservative for most conditions encountered.
The value for $A$ has been found, empirically, from fitting finite element results. The use of the $A$ term is to increase $g$ above unity and provide the increase in the secondary stresses contribution to failure under plasticity. It has been outlined above that the mechanism whereby the secondary stress can enhance the crack driving force with plasticity means that the plastic zone will be contained within the structure. This, therefore, means that the enhancement must be linked to the small-scale yielding correction to the Option 2 FAD, and that the need for an enhancement dictates that the value of $A$ should be negative.

In determining $A$ the value taken is from the ratio of the applied remote stress and the von Mises equivalent stress, when considered as a remotely applied tensile stress field. This ratio is similar to the definition of triaxiality, which can be defined by the ratio of the hydrostatic stress to the von Mises equivalent stress. Therefore it is considered that the definition provided above for $A$ may be linked to the triaxiality of the structure, and as described by Thaulow [37], can then be linked to the level of constraint in the structure. This is recommended as an area for future investigation.

### 7.2.4. Conservatism of $V_g$ and the R6 Complex $V$ Approaches

Comparison of the developed $V_g$ approach to the finite element results and the R6 $V$ factor approach showed that the existing R6 approach can be overly conservative, by over 50%, and that the $V_g$ approach is less so. The results presented in Section 4 indicate, as would be expected as the finite element results were used to define $g$, that the $V_g$ approach removes a significant level of the conservatism. This level of conservatism within the existing R6 approach when secondary stresses are present was also noted by TAGSI [7], Goldthorpe [17], [66], [67], Song [74] and Banahan [4] by comparison to experimental results, finite element analyses and plant experience. It is also clear that such levels of conservatism can lead to unnecessary replacement and downtime costs in major components [4], [7]. It is therefore necessary to suggest a reason for the conservatism in the existing approaches.

All existing approaches within R6 are essentially derived from the $\rho$ factor approach originally developed by Ainsworth by the approximate deformation-bounding theorem, as identified in [58], with a further modification by Hooton and Budden [61]. It has already been suggested that such an approach may correspond to some level of elastic follow-up.
when the secondary reference stress is greater than yield. It is further commented that the Spinning Cylinder test results [82–86], available finite element results [17], [66], [67], [74] and the RSE-M code [60], suggest that such an enhancement under secondary stresses alone is very conservative. However, this will cause the secondary stress contribution to be enhanced without any degree of primary stress induced plasticity and can be considered separate to the condition when the primary and secondary stresses are combined, as it is assumed (also assumed here) that this is captured in the value of $K_f^S$.

Under conditions where the primary and secondary stresses are combined, the deformation bounding theorem used in R6 suggests a significantly higher increase in the secondary stress contribution to the crack driving force with higher levels of plasticity. It is suggested here that the conservative implementation of the bounding theorem causes this conservatism. In the appendix of [58] is can be seen that Equation A2 shows the displacements applied, $R\Delta u$ (where the starred case is with the dummy load), compared to the change in both elastic and plastic components of strain, given by the $A$ terms, at time $t = 0$ and $t = T$:

$$R\Delta u \leq R\Delta u^* + A(0) - A(T) \leq R\Delta u^* + A(0)$$  \hspace{1cm} \text{Equation 110}$$

The implementation of this relationship in the definition of $\rho$ is that given by the final inequality. This therefore indicates that including the “$-A(T)$” term, which is the strain at time $t = T$, would provide a reduction in the level of conservatism of the approach, and that the inclusion of the conservatism is intentional. A definition for $A(T)$ can be provided, by following the logic outlined in the Appendix of [58] and is shown below:

$$A(T) = \frac{1}{2}(\sigma^P_{\text{ref}} + \sigma^R_{\text{ref}} - \sigma_{\text{ref}}) \left(\varepsilon(\sigma^P_{\text{ref}} + \sigma^R_{\text{ref}}) - \varepsilon(\sigma_{\text{ref}})\right)$$  \hspace{1cm} \text{Equation 111}$$

where $\sigma^P_{\text{ref}}, \sigma^R_{\text{ref}}$ and $\sigma_{\text{ref}}$ are the primary, equivalent secondary (as an additional primary load) and total combined reference stresses respectively and the $\varepsilon(\sigma)$ terms are the strain corresponding to the reference stress in the bracket. Under elastic conditions where $\sigma^P_{\text{ref}} + \sigma^R_{\text{ref}} = \sigma_{\text{ref}}$ will result in $A(T) = 0$ and have no effect. However, under
conditions with plastic redistribution and $\sigma_{ref}^p + \sigma_{ref}^r \neq \sigma_{ref}$ the magnitude of $A(T)$ will be greater than 1 for all potential variations in $\sigma_{ref}^r$. It is noted that it is not easy to include this equation within the original derivation subsequently used to define $\rho$ as $A(T)$ introduces many additional co-dependent $\sigma_{ref}$ terms that prevent the minimisation of $\sigma_{ref}^r$ required. Neglecting the $A(T)$ term does, however, help demonstrate that the conservatism is fundamental to the approach, that this added conservatism is directly related to the magnitude of the stresses applied, but the conservatism of the approach is very difficult to quantify.

Where the $V_g$ approach has been defined from finite element analyses means that it is not subject to these conservatisms. However, that the approach can be related to the reference stress approach helps demonstrate a more universal applicability. The cases considered within Section 4 and the experiments considered in Section 6 all show $V_g$ to remain conservative, but less so than the R6 approaches. The $V_g$ approach is less conservative than the existing $V$ factor approach.

7.2.5. Effect of Elastic Follow-Up

One potential detrimental point regarding the reduction in conservatism when adopting $V_g$ compared to the R6 complex $V$ approach is that potential difficulties surrounding elastic follow-up are emphasised. Under the current R6 approach the implementation of the deformation-bounding theorem used leads to the inclusion of the reference load that might be over estimated, especially for cases that have a high secondary stress. The conservatism added here can be considered to relate to moderate levels of elastic follow-up inherently being considered in the applied secondary stress. However, as this is not included in the $V_g$ approach, where elastic follow-up does occur it is likely that the $V_g$ approach will become non-conservative. It is also noted that significant levels of elastic follow-up are unlikely to be observed within the finite element analyses considered as a self-balancing bending stress field is adopted. It is also unlikely that elastic follow-up would be expected within the experiments considered, as these have to remain relatively small so they can be tested in laboratories and the secondary stress cannot act over a large enough distance to induce elastic follow-up on its own. Song has demonstrated the effect of elastic follow-up [74] by
altering the thermal stress field to create a tensile regions in order to induce elastic follow-up effects. From this work an approach to detail an interaction has been defined under differing values of elastic follow-up [114]. However, a numerical estimate is still required for this to be used, which is difficult to achieve, and is dependent on many unknowns. It has also been noted that the existing R6 approaches can potentially become non-conservative, as shown in Figure 28 when elastic follow-up is significant. It would therefore be expected that the $V_g$ approach would also be non-conservative, and by a more significant degree that the R6 methods, in the presence of significant elastic follow-up. It is possible that an indication of elastic follow-up would be provided from comparison of $K_f^g$ to $K_f^6$, from finite element analyses, to see any increase under elastic-plastic conditions. This, however, requires finite element analyses and full knowledge of the thermal loading applied both at the cracked region and further away from the defect. It is also noted that the degree of elastic follow-up might also be influenced by the primary load applied as this will dictate the evolution of the plastic zone. It is therefore recommended that more work be considered into the evaluation and effects of elastic follow-up and how to predict it numerically. In the meantime it is simply suggested, as in R6, that if elastic follow-up is considered likely that the secondary stress be treated as a primary load. An investigation to the effect of elastic follow-up is suggested for future work.

7.2.6. Summary

To summarise the development of the $V_g$ approach: A new approach to describe the relative contribution of secondary stresses to the failure of components with a crack-like defect has been developed from the results of finite element analyses of a fully circumferentially cracked cylinder. The new approach has been shown to be compatible with the R6 $V$ factor approach, where an alternate definition of the $V$ term has been defined, called $V_g$.

A number of features of this approach are detailed below:

- The $V_g$ approach is less conservative than the existing $V$ factor approach.
• *The level of secondary stress was shown to have negligible effects on the value of $V_g$ derived from finite element analyses.*

• *The $V_g$ approach is materials dependent and this dependence is straightforward to implement, and could provide an additional hierarchy to the existing approaches in R6. This materials dependence therefore provides a distinct advantage over existing approaches.*

7.3. **Experimental Studies**

This section considers the experimental study detailed in Section 5 to provide further experimental results when combining primary and secondary stresses. To allow discussion of a number of features observed the section is split to the following subsections:

7.3.1) Basis of the experimental studies,

7.3.2) Comparison of residual stress measurements and finite element predictions,

7.3.3) Material’s modelling,

7.3.4) Summary.

7.3.1. **Basis of Experimental Studies**

Within Section 2, as further detailed within Appendix 1, a large database of experiments has been constructed where the loading applied to the tests comprises both primary and secondary stresses. This database provides all necessary input data for an assessment of the tests considered and covers a broad range of loading conditions, geometry and material. The cases considered include numerous geometries including CT, SENT, three-point bend, four point bend, centre cracked plates and cylinders. Different types of secondary stress considered include weld residual stresses, thermal stresses and mechanically induced residual stresses. The materials considered also range from structural steels such as A533B, the Chinese PV steel 16MnR and an A508 Class 3 forging, to an offshore low alloy tubing steel AISI 4333 M4 and the Swedish material Weldex 700 that demonstrate very low strain hardening, through to two different grades of aluminium.
The number of experiments included in this database (i.e. Appendix 1) can be separated to those that consider elastic conditions alone (i.e. $L_r < 0.3$) [28], [77], [81], [91], those that consider limited levels of plasticity (i.e. $L_r < 0.6$) [82–86], [115], those that consider more extensive plasticity (i.e. $L_r > 0.6$) or that include a range of values of $L_r$ [76], [77], [112]. Those of more interest in validating the $V_g$ approach are these final sets of experiments where the same tests are used over a large range of $L_r$. However, these tests are limited and only include the aluminium plate tests [77] and the Chinese pressure vessel steel plate tests [112]. It was therefore considered that a knowledge gap existed in the number of useful experiments by which the $V_g$ could be validated. This is because:

1) neither a mechanical pre-compressive residual stress field nor a thermal stress field had been considered,
2) the number of tests is currently restricted to plates and
3) the materials used are not necessarily applicable to structural materials used in the UK.

It was therefore considered necessary to extend the number of results by considering a further experimental programme. It was chosen to extend the suggestive results of Mirzaee-Sisan [91] and Lee [115] to consider a number of different failure modes on pre-compressed laboratory specimens made from A533B steel. These tests are outlined in Section 5 and complement the tests performed by Inspectra AB [76], Sweden, which were performed in parallel with some initial collaboration but designed and tested independently. It is, however, noted that the aim of the Swedish tests was to consider failure of specimens under primary and secondary stresses under gross plasticity and, therefore, only provide additional data close to and above $L_r = 1$. Where a gap in the experimental database was identified further tests have been performed to provide additional data.

7.3.2. Comparison of Residual Stress Measurements and Finite Element Predictions

Of the tests outlined in Section 4 a number of features were noted. These include a good match between the finite element prediction and measured values of the residual stress
field when adopting the kinematic hardening model for the A533B material with a Ramberg Osgood fit and demonstrating that warm pre-stressing (WPS) effects can be neglected. When considering the match between the measured residual stress field and the finite element predictions the good match is seen predominantly for the digital image correlation and surface hole drilling approaches, but less so for the neutron diffraction measurements. The neutron diffraction results show good agreement with the 3D finite element model, for normalised depths greater than 5 mm with, perhaps, a slight over prediction of the maximum compressive stress. It was stated in Section 5 that the difference in this maximum compressive stress region might be a result of sampling a large volume. This is considered a potential explanation as the stress field incorporates a large change in stress over a short distance. That the neutron diffraction results show a lower than predicted magnitude of stress indicates that, if the finite element prediction is correct, the larger sampling volume is incorporating more lower stressed values leading to a lower average stress for a given location.

7.3.3. Material’s Modelling

One feature of the tests that has been shown to have some effects, and potentially on the comparison of the test results with the $V_g$ approach, is the treatment of Lüders strain in the material. The approach adopted in Section 5, which was selected from three options, was to provide a curve fit which covers the initial yield and re-joins the stress strain curve at a later stress. Hurlston [81] also used this curve fit approach when using the same material. This approach was chosen over the other suggested methods as; it is difficult to interpret where the Lüders strain starts to affect the stress strain curve; and the approach considered in RSE-M [60] is for handbook assessments and is therefore conservative. It has also been noted in Section 5 that the curve fit approach provides good predictions to the material’s characterisation tests that have gone through a load cycle, such as the compression-tension tests and the low cycle fatigue tests. This agreement is provided as the Lüders strains are removed after the initial loading condition. The hysteresis loops provided by Bolinder within Reference [76] also show that the Lüders strains are removed in cyclic conditions for the same material. However, the cyclic tests of Bolidiner can be seen to show Lüders strains on the initial load. It is noted that for tests reported in the literature on similar material, Bolinder chose not to remove the Lüders strains, Lee [115] only tested...
compression-tension tests and did not see Lüders strains which resulted in similar results as determined here, and Mirzaee-Sisan [91] did not explicitly state how Lüders strains were treated.

When comparing the load versus displacement plots, the results are generally close to the finite element prediction, but some cases, for the same temperature, are seen to provide different levels of accuracy. It can be seen in Section 6 that the effect of Lüders strains should be included for the as-received specimens when comparing the Option 2 FAD to the failure locus, which is best seen for the -90 and -50 °C as-received specimens (see Figure 93). However, the pre-compressed specimens show excellent agreement when the Lüders strain is removed. This demonstrates that, for this material, the loading history has a significant effect not only on the residual stress field but the tensile curve that should be used to assess the tests.

This can also be seen for compression-tension tests, where the effect is different to that when loaded in tension initially. This therefore helps demonstrate the Bauschinger effect observed by Lee [115] for this material. It was also noted that the predictions of load versus clip gauge displacement in the tests of Section 5 and cyclic tensile tests do show some differences in the region near the onset of yielding under compression but are seen to provide close agreement on subsequent tensile loading. This helps show that the Bauschinger effect does not adversely affect the finite element predictions, which is also confirmed by the comparison of the predicted and measured residual stress fields.

It is considered that the experiments performed provide useful additional data to the initial database available in the published literature by which to consider combined primary and secondary loading with different cases predicting initiation over a range of values of $L_p$. These experiments have been well-characterised and potential variations explored so that the finite element predictions of $K_f$ can be adopted in an assessment of these tests. The accuracy of the finite element analyses has been confirmed by comparison to measures of the residual stress field. **These tests have been reviewed and demonstrate valid data by which to extend the database.**
7.3.4. Summary

To summarise, the experiments considered and performed: A single database of experiments containing the combined influence of primary and secondary stresses has been collated alongside an extraction of all necessary input data to perform an assessment for each case. This database can be found in Appendix 1. Where a gap in the knowledge was identified, further tests have been performed to provide additional data. These tests have been reviewed and demonstrate valid data by which to extend the database. Some features of this database are detailed as:

- The database has been formed to provide fracture toughness data under combined primary and secondary loading for numerous geometries including CT, SENT, three point bend, four point bend, centre cracked plates and cylinders.
- The different types of secondary stress considered include weld residual stresses, thermal stresses and mechanically induced residual stresses.
- The materials considered also range from structural steels such as A533B, the Chinese PV steel 16MnR and an A508 Class 3 forging, to an offshore low alloy tubing steel AISI 4333 M4 and the Swedish material Weldex 700 that demonstrate tensile with very low strain hardening, through to two different grades of aluminium.

7.4. Validation of the $V_g$ Approach to Identified Experiments

The application of the R6 Complex $V$ and $V_g$ approaches, performed in Section 6, to the experiments, identified in Appendix 1, has identified a number of features which are discussed further within this section:

7.4.1) Application of $V_g$ and R6 complex $V$,
7.4.2) Material effects,
7.4.3) Secondary stresses in isolation,
7.4.4) Neglecting contribution from secondary stresses,
7.4.5) Use of the modified $J$-integral,
7.4.6) Summary.
7.4.1. Application of $V_g$ and R6 Complex $V$

The main feature of this is that both the R6 Complex $V$ and $V_g$ approaches conservatively predict the failures seen within the tests. A number of cases are seen to be marginally inside the failure assessment curve, such as those seen for the aluminium plate tests, but this was also seen in [3], [77] when assessing these results as part of the validation of R6 and explained by difficulty in identifying the precise load at initiation such that a conservative estimate of initiation load was used. In the vast majority of cases the results are always outside the FAD, showing a conservative prediction of initiation or additional crack growth within the tests. This demonstrates the universality of both the R6 complex $V$ and $V_g$ approaches as the range of cracked geometries and loading types considered is more significant that those used to validate the current R6 approach [3]. It can also be noted that many of the conservative assumptions adopted, such as the use of Tresca limit load solutions or material properties, may make it more difficult to show non-conservative results. The $V_g$ approach has shown conservative results irrespective of the material, geometry or secondary stress type assessed.

It was noted in Section 6 that the use of $V_g$ approach compared to the Complex $V$ approach of R6 can provide significant benefit, of up to 25% in terms of $K_T$, which can relate to a reduction in the perceived conservatism in the approach of over 50%. This has been shown to be a result of the relative contribution of the secondary stress where the $V_g$ approach provides up to approximately a 50% reduction in the multiplication term to $K_T^g$ (see the Lee results where the value of the R6 $V$ is 1.9 compared to $V_g$ value of 1.05). This level of reduction is also allowed within the French RSE-M [60] approach by the $k_{th}$ term but not within the R6 approach until approaching plastic collapse. This level of reduction is also seen in the results of Song [74] for the two thermal stress fields, as well as the finite element results within Section 4. A potential explanation of the origin of the conservatism within the R6 approach has already been discussed above, with relation to the deformation-bounding theorem. The R6 Complex $V$ factor approach also showed conservative results. However, the $V_g$ showed less conservatism in all cases corresponding to potential benefits of more than a 25% reduction in $K_T$, which could correspond to over 50% reduction in the value of $V$. 
7.4.2. Material Effects

The accuracy of the results and the reduction in the level of conservatism between the R6 Complex $V$ and $V_g$ approaches can be seen to be geometry, load and material dependent. It has been suggested that this might be a result of the assumptions made in defining the material’s stress-strain curve. In many cases the assessments only included the material’s yield and ultimate tensile stresses. The approach considered in Section II.1 of R6 [3] that defines the strain-hardening index for a Ramberg-Osgood curve describing the stress strain curve has been used. However, the data available for the Spinning Cylinder tests indicated that the Ramberg-Osgood approach should be used but also allowed limited data for comparison. This comparison was made in Figure 101 that showed excellent agreement, somewhat validating this approach. Nonetheless, where available, the measured stress strain curve has been used. Natural variability in materials, differences in the method used to obtain the tensile and fracture data, and the assumptions inherent to the data in the literature (such as the treatment of Lüders strains), are thought to contribute to the variability when comparing similar materials. However, as the $V_g$ is material dependent it is also likely that some materials will be closer to the R6 $V$ approach than others, showing differing levels of conservatism, as demonstrated by the two grades of aluminium in the aluminium plate tests. It is therefore confirmed in the tests that the level of the reduction in conservatism is dependent on the material considered.

7.4.3. Secondary Stresses in Isolation

Some cases assessed with the R6 Complex $V$ approach, such as the spinning cylinder tests, increase the value of $V$ despite the secondary stress acting in isolation; effectively increasing $K_I^S$ above $K_I^S$. The results from spinning cylinder test 4 in particular show this clearest, as the applied primary stress is insignificant compared to the level of secondary stress, leading to values of $L_r$ less than 0.1. Therefore, the effect of the secondary stress can be considered as the major contributor to initiation and subsequent crack growth.

When applying the R6 approach the value of $K_r$ is over 2 at initiation and throughout the subsequent crack growth. When adopting the $V_g$ approach, so the enhancement of the elastic value from the look-up tables is not included (as discussed above), the elastic estimate of secondary stress intensity factor leads to a value of $K_r$ over 1.7. This indicates
that, for the significant thermal transient within this test, the estimate of \( K_f^S \) should be up to 40% reduced from \( K_f^S \). It is noted that the approach within R6 of taking finite element analysis results for the un-cracked body stress field might allow some reduction but finite element analyses of these tests were not considered here. It is noted that the French RSE-M code would allow a potential reduction of up to 50% under such conditions. When considering both the R6 \( V \) and the \( V_g \) approaches it is noted that the value of \( K_f^S \) is used as an input to these calculations, meaning that the \( V_g \) approach would also be very conservative when the guidance of R6 is used to estimate \( K_f^S \). Although not considered explicitly in either the R6 \( V \) and the \( V_g \) approaches, the effects of the secondary stress acting in isolation is shown in the tests to allow a reduction in the elastic-plastic crack driving force compared to the elastic value.

7.4.4. Neglecting the Contribution from Secondary Stresses

Within all the assessment results of the experiments, where possible, additional points have been included that demonstrate the effect of neglecting the secondary stress contribution on fracture. In each case where the magnitude of primary load prevents significant plasticity \( (L_r < 1) \) it is clearly non-conservative to neglect the contribution of the secondary stress completely. However, the results from the Swedish Inspectra AB tests [76] do show this to also be true for some of the cases with \( L_r > 1 \), although the level of non-conservatism is small. At first this might be considered somewhat contradictory to the results by Ainsworth et al [77] where the contribution of the secondary stress is seen to be removed for ductile materials. However, it is noted that a number of results in [77] show a contribution from secondary stress to failure at values of \( L_r \) greater than 1 and suggest it can only really be neglected by \( L_r \geq 1.2 \). It is therefore possible that the contribution is never completely removed, even under widespread plasticity; it just becomes less influential on failure. This is also predicted by the finite element results in Section 4 where the estimate of \( g \) was seen to reduce the secondary stress contribution to less than 10% under gross plasticity, depending on the strain hardening coefficient, but never completely removed. This was even the case for values of \( L_r \) greater than the estimate of \( L_r^{\text{max}} \) as calculated in Table 4. Clearly this is in direct contradiction to the treatment of ductile materials in assessment codes such as ASME [1] where the secondary stress is commonly
neglected. *It was shown that neglecting the contribution from the secondary stress is non-conservative. This was even seen, admittedly at lower levels, for values of $L_r$ above yield where it might be considered that the contribution of the secondary stress can be neglected for ductile materials.*

7.4.5. **Use of the Modified $J$-Integral**

When examining the results of the experiments performed in Section 5 it is clear that the estimate of $K_J^S$ provides a very good estimate of failure for the tests, especially at $-150 \, ^\circ C$. This can be seen as the failure locations are equally spaced about the FAD when adopting the mean fracture toughness from the as-received tests. The estimate of $K_J^S$ was taken from the finite element analyses when adopting the JEDI postprocessor [25]. This therefore shows that the finite element analyses is correct and that $J$-contour integral in JEDI accurately predicts the crack driving force for the non-proportional, pre-strained, stress field generated in the tests. The calculation within JEDI [25] (and J-MOD [23]) to evaluate the $J$-contour integral includes an additional term to account for the prior work used to generate the residual stress field, thus better accounting for the energy release when inserting or growing a crack. As the entire loading history was known in the specimens, it is also possible to have an accurate measure of each element’s location on, or within, the hysteresis loop so that an accurate measure of $J$ is possible, therefore correctly accounting for non-proportional effects. The results presented by Lee [115] also demonstrated the accuracy of the results when implementing JEDI [25] compared to experimental results. The pre-compressed tests considered by Inspectra AB [76] implemented an in-house development of the modified $J$-contour integral that also showed good comparison to experimental results. *The use of the modified $J$-integral, such as defined by JEDI, has been shown to provide excellent agreement with experimental results in the elastic regime when considering the experiments outlined in Section 5. This helps provide evidence for the use of the modified $J$-integral to accurately define the crack driving force for cases with non-proportional loading due to the presence of a residual stress field.*
7.4.6. Summary

To summarise the assessment of the experiments outlined with the R6 $V$ and $V_g$ approaches: All experimental results presented have been assessed using the $V_g$ approach. Where there were insufficient experiments that fail between moderate and high loads, a further set of experiments has been performed. The application of the $V_g$ approach to all published experiments has shown the approach to be conservative. A number of additional features are detailed below:

- The $V_g$ approach has shown conservative results irrespective of the material, geometry or secondary stress type assessed.
- The R6 Complex $V$ factor approach also showed conservative results. However, the $V_g$ method showed less conservatism in all cases corresponding to potential benefits of more than a 25% reduction in $K_r$ which could correspond to over 50% reduction in the actual level of conservatism.
- The level of the reduction in conservatism is dependent on the material considered.
- The effects of the secondary stress acting in isolation is shown in the tests to allow a reduction in the elastic-plastic crack driving force compared to the elastic value.
- It was shown that neglecting the contribution from the secondary stress is non-conservative. This was even seen, admittedly at lower levels, for values of $L_r$ above yield where it might be considered that the contribution of the secondary stress can be neglected for ductile materials.
- The use of the modified $J$-integral, such as defined by JEDI, has been shown to provide excellent agreement with experimental results in the elastic regime when considering the experiments outlined in Section 5. This helps provide evidence to the use of the modified $J$-integral to accurately define the crack driving force for cases with a non-proportional residual stress field.
7.5. Potential Knowledge Gaps and Further Analytical Consideration

The R6 Complex $V$ and $V_g$ approaches have been applied to an extensive range of different geometries and loading conditions. There are, however, some cases where the approaches have not been applied. This section provides a review of the cases considered and, with reference to the work in Appendix 2, adds further analytical validation. This section is separated as below:

7.5.1) Identification of knowledge gap,
7.5.2) Reproduction of results from Appendix 2,
7.5.3) Discussion of weld residual stress results,
7.5.4) Summary.

7.5.1. Identification of Knowledge Gap

As detailed above, the finite element analyses and experimental considerations include a large number of sample geometries and loading conditions. Table 25 demonstrates the cases that have been considered by either finite element analyses or experiments.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Primary Loading</th>
<th>Secondary Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tension</td>
<td>Bending</td>
</tr>
<tr>
<td>Centre cracked plate</td>
<td>Tension</td>
<td>NA</td>
</tr>
<tr>
<td>Circumferentially cracked</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>cylinder</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Axially cracked cylinder</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>CT</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>C-Ring</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

* Points marked with a star are considered in the following section

Of the potential cases that have not been considered some are not realistic, such as thermal loads in a CT or C-Ring specimen. Therefore of the cases not considered the most significant were for the axially and circumferentially cracked cylinders subject to a residual stress. It is detailed in [4] that weld residual stresses in piping is one of the most significant forms of residual stress commonly assessed in a nuclear power station. The
most severe form of residual stress is from repair welds, as these do not always undergo a post weld heat treatment to reduce the residual stress level. Therefore, additional cases of a repair weld adjacent to an extended crack in both axially cracked and circumferentially cracked cylinders have been considered as part of a short finite element investigation. Details of this investigation can be found in Appendix 2.

7.5.2. Reproduction of Results from Appendix 2

The resulting residual stress field modelled in Appendix 2 can be seen in Figure 115. This was combined with different combinations of primary axial and hoop stress so that the crack opening stress was in ratio with the out of plane stress as 1:0, 1:1 and 1:2. The stress intensity factor results can be seen in Figure 116 comparing $K_I$ and $K_I^P$ for the axially (top) and circumferentially (bottom) cracked cylinders. The results comparing the R6 Complex $V$ Approach and the $V_2$ Approach can be seen in Figure 117 and Figure 118 for the axially and circumferentially cracked cylinders, respectively.

Figure 115 – Finite element residual stress field predictions for a single pass weld axially and circumferentially at the weld location offset from the weld centre line
Figure 116 – Finite element prediction of $K_J^P$ and $K_J$ for the axially (top) and circumferentially (bottom) cracked cylinders with a weld residual stress and different combinations of hoop and axial stress loads.
Figure 117 – R6 Complex V (top) and $V_g$ (bottom) estimates of $K_J$ normalised by the finite element prediction for the axially cracked cylinder with a weld residual stress and different combinations of hoop and axial stress loads as a function of $L_r$. 
Figure 118 – R6 Complex $V$ (top) and $V_k$ (bottom) estimates of $K_J$ normalised by the finite element prediction for the circumferentially cracked cylinder with a weld residual stress and different combinations of hoop and axial stress loads as a function of $L_r$. 
7.5.3. Discussion of Weld Residual Stress Results

When comparing the results for $K_f$ the results respond as would be expected when including secondary stresses so that the value for $K_f$ starts above $K_f^p$ (at $K_f^p$) but becomes more aligned at higher values of $L_r$. One feature, however, seen for these weld residual stress cases in the circumferentially cracked cylinder is that between $L_r = 0.8$ and $1.4$ the estimate of $K_f^p$ increases beyond $K_f$. This suggests that the weld residual stress at this point is having a negative contribution to the total stress intensity factor. This is considered possible as the redistribution of secondary stresses can only occur for cases that become plastic. However, a weld residual stress will include both tensile and compressive regions so that, when an additional primary stress is applied, the compressive regions will not redistribute until beyond $L_r = 1$. This therefore means that the remaining component of the weld residual stress may have negative effect on $K_f$ under redistribution, depending on the proximity of the compressive region to the crack tip plastic zone. The proximity of the crack-tip to the compressive region may explain why only the circumferentially cracked cylinder shows this effect as the axially cracked cylinder has a much deeper tensile region and the crack tip for the circumferentially cracked defect is actually just inside the compressive region of residual stress.

The application of the complex $V$ Factor and the $V_g$ approach outlined in Section 4, both when adopting an Option 3 FAD for $f(L_r)$, can be seen in Figure 117 and Figure 118 for the axially and circumferentially cracked cylinders. The results show that both approaches can be considered to remain conservative; the $V_g$ Approach does appear to become marginally non-conservative at low values of $L_r$ for the circumferentially cracked cylinder but this is still within 3% and not considered significant as this is less than the 5% accuracy of the reference stress approach of R6 as detailed by Miller [46].

The R6 $V$ Factor approach shown incorporates the Option 3 FAD defined from the finite element analyses and the more accurate complex $V$ approach. This therefore can be considered the most accurate estimate capable by the R6 approach. However, the $V_g$ approach still demonstrates a reduction in conservatism. The maximum conservatism seen in the R6 approach in Figure 117 and Figure 118 is approximately 40%, whereas the
maximum seen for the $V_g$ Approach is approximately 20%. This level of conservatism for R6 is similar to that observed by Goldthorpe [17], [66], [67].

It is noted that in the application of the $V_g$ approach here, the inputs to the approach and the comparative value of $K_f$ all come from finite element analyses. Thus uncertainties associated with large-scale tests cannot influence these results here.

The short analyses contained in Appendix 2 have shown that the $V_g$ Approach can be successfully applied to the non-proportional stress field resulting from the welding simulation. The predictions of $K_f$ from the $V_g$ Approach compared to the R6 Complex V Factor Approach allows for a reduction in the level of maximum conservatism seen by up to 20%.

It was also noted that the non-linear and self-balancing effect of the weld residual stress could lead to cases where the contribution of the residual stress becomes negative as the tensile stress is redistributed first.

7.5.1. Summary

It is therefore considered that where evidence is not available from experiments further finite element analyses have been performed to consider weld residual stresses in a cylinder. These results show the $V_g$ approach to remain conservative although additional experimental validation is recommended. All analyses undertaken in this research have, therefore, shown the application of the $V_g$ approach to be suitable.
8. Conclusions

This section provides a summary of the main developments and conclusions from the research performed within this project to better understand the contribution of secondary stresses to the failure of crack like defects under the combined influence of primary and secondary stresses. Initially the developments made within this work are presented, followed by the main conclusions drawn from this work.

The main developments made within this research are:

1. A new approach, $V_g$, to characterise the relative contribution to fracture from secondary stresses has been defined. This approach has been shown to provide a less conservative approach when compared with the $R6V$ factor approach.
2. A large database of experiments considering both primary and secondary stresses has been collated.
3. Further experiments have been performed to increase the number of experiments that allow validation of the $V_g$ approach.
4. An extensive validation exercise for the $V_g$ approach has been undertaken with respect to the experiments performed and those from the literature.

The main conclusions from the work are as follows:

1. A new approach to describe the relative contribution of secondary stresses to the failure of components with a crack like defect has been developed from the results of finite element analyses of a fully circumferentially cracked cylinder. The new approach has been shown to be compatible with the $R6V$ factor approach, where an alternate definition of the $V$ term has been defined, called $V_g$. A number of features of this approach are detailed below:
   a. The $V_g$ approach is less conservative that the existing $V$ factor approach.
   b. The level of secondary stress was shown to have negligible effects on the value of $V_g$ derived from finite element analyses.
c. The $V_g$ approach is materials dependent and this dependence is straightforward to implement and could provide an additional hierarchy to the existing approaches in R6. This materials dependence therefore provides a distinct advantage over existing approaches.

2. A single database of experiments addressing the combined influence of primary and secondary stresses has been collated alongside an extraction of all necessary input data to perform an assessment for each case. This database can be found in Appendix 1. Where a gap in the knowledge was identified, further tests have been performed to provide additional data. These tests have been reviewed and are considered to provide valid data by which to extend the database. Some features of this database are detailed as:

   a. The database has been formed to provide fracture toughness data under combined primary and secondary loading for numerous geometries including CT, SENT, three point bend, four point bend, centre cracked plates and cylinders.
   b. The different types of secondary stress considered include weld residual stresses, thermal stresses and mechanically induced residual stresses.
   c. The materials considered also range from structural steels such as A533B, the Chinese PV steel 16MnR and an A508 Class 3 forging, to an offshore low alloy tubing steel AISI 4333 M4 and the Swedish material Weldex 700 with very low strain hardening, through to two different grades of aluminium.

3. All experimental results presented have been assessed using the $V_g$ approach. The application of the $V_g$ approach to all published experimental data and the new data, has shown the approach to be conservative. A number of additional features are detailed below:

   a. The $V_g$ approach has shown conservative results irrespective of the material, geometry or secondary stress type assessed.
b. The R6 Complex $V$ factor approach also showed conservative results. However, the $V_g$ method showed less conservatism in all cases corresponding to potential benefits of more than a 25% reduction in $K_r$ which could correspond to over 50% reduction in the actual level of conservatism.

c. The level of the reduction in conservatism is dependent on the material considered.

d. The effects of the secondary stress acting in isolation is shown in the tests to allow a reduction in the elastic-plastic driving force compared to the elastic value.

e. It was shown that neglecting the contribution from the secondary stress is non-conservative. This was even seen, admittedly at lower levels, for values of $L_r$ above yield where it might be considered that the contribution of the secondary stress could be neglected for ductile materials.

f. The use of the modified $J$-integral, such as defined by JEDI, has been shown to provide excellent agreement with experimental results in the elastic regime when considering the experiments outlined in Section 5. This helps provide evidence for the use of the modified $J$-integral to accurately define the crack driving force for cases with a non-proportional residual stress field.

4. Where evidence is not available from experiments further finite element analyses have been performed. These include the consideration of weld residual stresses in cylinders. These results show the $V_g$ approach to remain conservative although additional experimental validation is recommended. All analyses undertaken in this research have, therefore, shown the application of the $V_g$ approach to be suitable.
9. Further Work

The work presented here aimed to address the interaction of primary and secondary stresses on the crack driving force in cracked components. A new approach has been developed and assessed against a large number of experiments. Through the course of the work, and the discussions made in Section 7, the following areas are considered important areas for further consideration:

1. Further validation of $V_g$ Approach
   Although a large number of experiments have been considered within Section 6 it is still important to consider validating the approach further where possible. If the approach is to be included in R6 it is vital that as many cases are considered so that any potential non-conservatism can be examined.

2. Elastic follow-up
   By reducing the level of conservatism as part of the $V_g$ Approach, when compared to the Complex R6 $V$ Approach, it is more likely that elastic follow-up is likely to lead to non-conservative results. Currently R6 suggests that for cases where elastic follow-up is considered likely, the secondary stress is considered as primary. This, however, may be very conservative. It is also recognised that elastic follow-up is an area that is not very well described, especially within fracture assessments. It is therefore recommended that further work be considered to address this issue specifically.

3. Constraint
   The level of constraint for a given geometry, load and material will influence the evolution of the plastic zone and, hence, the redistribution of secondary stresses. It is therefore recommended that ranges of constraint conditions be considered when combining primary and secondary stresses whilst evaluating $V_g$ and $Q$ to see if any trends can be inferred.
10. References


Appendix 1 - Experiments Considering Combined Primary and Secondary Stresses

Appendix 1.1 Introduction

This appendix provides details of available experiments that include the effect of combined primary and secondary stresses on fracture. Initially a summary table is provided, that is also repeated in the main body of the thesis, which summarises the experiments detailed in the remainder of the appendix. Within each subsection, detailing the experiments individually, a summary table is included that provides the required information to perform the comparisons made in Section 6 of the thesis.
Table A1.1 – Overview of Experiments Considering Combined Primary and Secondary Stresses

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reference</th>
<th>Geometries</th>
<th>Details of Secondary Stress</th>
<th>Primary Load</th>
<th>Material</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium Plate Tests</td>
<td>Ainsworth et al [A1.1]</td>
<td>Centre cracked plate. Two sizes.</td>
<td>(1) Two parallel weld beads with crack in middle region of high tensile stress. (2) End tabs cut into plate with centre tab pulled and outer tabs compressed prior to welding to provide fit-up residual stresses.</td>
<td>Tensile membrane stress with crack growth measured.</td>
<td>Two grades of aluminium alloy were used to provide different levels of plasticity at failure.</td>
<td>Used to validate R6. Failure observed over a range of values of $L_r$.</td>
</tr>
<tr>
<td>A533B Steel Plate Tests</td>
<td>Ainsworth et al [A1.1]</td>
<td>Centre cracked plate.</td>
<td>Weld residual stress from welding two halves of a plate back together. Some samples underwent PWHT.</td>
<td>Four Point Bending with crack growth measured.</td>
<td>A533B steel (to provide structurally relevant material)</td>
<td>Used to validate R6. Experiments showed that R6 method is overly conservative. Some potential explanations given.</td>
</tr>
<tr>
<td>16MnR Plate Tests</td>
<td>Hong-Liang [A1.2]</td>
<td>Centre cracked plate.</td>
<td>Weld residual stress with three different weld / crack combinations to create different residual stress profiles</td>
<td>Tensile membrane stress.</td>
<td>16MnR steel (Chinese specification)</td>
<td>Show that R6 is conservative apart from case with a high tensile secondary stress. Failure observed over a range of values of $L_r$.</td>
</tr>
<tr>
<td>A533B bend tests</td>
<td>Mirzae-Sisan [A1.3]</td>
<td>Bend specimen with central semi-circular scallop with crack at base of scalloped notch.</td>
<td>The specimens were pre-compressed at room temperature in the longitudinal direction to establish a plastic residual stress field. The tests were then unloaded, cooled and tested to fracture.</td>
<td>Three point bend test at cooled temperature (to ensure brittle fracture)</td>
<td>A533B steel</td>
<td>Showed that R6 methods are conservative. Direct finite element estimate is still conservative.</td>
</tr>
<tr>
<td>A533B steel constraint tests</td>
<td>Lee et al. [A1.4]</td>
<td>Compact-tension and Single-edge notched plate</td>
<td>Residual stress induced by in-plane compressive preload prior to precracking</td>
<td>Bending stress for compact-tension specimen, and tension for single-edge cracked plate.</td>
<td>Heat treated A533B steel</td>
<td>Showed that residual stresses can increase crack-tip constraint.</td>
</tr>
<tr>
<td>Test Type</td>
<td>Reference</td>
<td>Description</td>
<td>Stress Field and Behavior</td>
<td>Comments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------------------------------------</td>
<td>--------------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A433 M4 C-ring with mechanically induced residual stress</td>
<td>Kamel [A1.5]</td>
<td>C-Ring with shallow crack. One level of pre-compression to induce a tensile residual stress field on unloading. An opposite level of pre-tension to result in a compressive residual stress field.</td>
<td>Tensile load at loading pins (note that this induces a bend at the crack). AISI 433 M4. High yield (1050 MPa) and very low hardening (UTS = 1100 MPa). Also showed strong Bauschinger effect.</td>
<td>Argued that tests are brittle but elastic summation of K appears to be incorrect. Neutron diffraction used to measure stress field. Used to investigate constraint effects.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A533B Side-punched residual stress</td>
<td>Hurlston [A1.6]</td>
<td>Bend specimen with side punches. A controlled secondary stress field is inserted by compressing the bend specimen in the thickness direction by use of side punches.</td>
<td>3 point bend test at cooled temperature (to ensure brittle fracture)</td>
<td>A533B steel.</td>
<td>Showed that crack-tip constraint can be included in the assessment of defects under secondary loads.</td>
<td></td>
</tr>
<tr>
<td>A533B and Weldex 900 Swedish Tests</td>
<td>Bolinder [A1.7]</td>
<td>Bend specimen with central semi-circular scallop with crack at base of scalloped notch. The specimens were pre-compressed at room temperature in the longitudinal direction to establish a plastic residual stress field. The tests were then unloaded and tested to failure without cooling. A number of different cases (material and crack length) were considered.</td>
<td>3 point bend test at room temperature</td>
<td>Weldex 900 and A533B steel.</td>
<td>Showed that R6 methods are conservative and the levels of stress redistribution at higher levels of plasticity.</td>
<td></td>
</tr>
<tr>
<td>A508 Spinning Cylinder Tests 1-6</td>
<td>AEAT Reports [A1.8–A1.12]</td>
<td>Large scale rotating cylinder. The cylinders were heated (note exact temperatures vary between experiments) and cold water sprayed internally to produce a thermal shock profile through thickness.</td>
<td>High rotational velocity to simulate pressure hoop stress in a cylinder</td>
<td>Forged cylinders heat treated to represent A508 steel.</td>
<td>Provided validation of R6. One of the only experiments able to include a thermal transient.</td>
<td></td>
</tr>
</tbody>
</table>
Appendix 1.2  Ainsworth et al - Aluminium Plate Tests

Tests performed on two aluminium alloys (AL2024 and AL5083) when including residual stress were reported by Ainsworth, Sharples and Smith [A1.1]. The aluminium tests were designed to consider a range of loading conditions including two different geometries and two different grades of aluminium alloy. The two geometries shown in Figure A1.1 introduced the residual stress by different means. The full size plates were 25 mm thick, 1 m wide and 2.3 m tall. To introduce the residual stress two parallel electron beam welds were performed, thus creating a region of high tensile stress contained within a region of compressive stress in the outer ligaments. The second plate geometry was made to be exactly ¼ the size of the large plate. For this geometry the residual stress was introduced by inserting two cuts in the ends of the specimen to the same location as the electron beam welds in the full size plate. A tensile stress was applied to the central region of the material and compressive stresses to the outside regions, reflecting the tensile and compressive regions of the larger plate specimen. These stresses were then “locked in” by re-welding the slots together.

The two materials used were Aluminium alloy AL2024 and alloy AL5083. The AL2024 alloy represents a material with a high ratio of yield stress to fracture toughness and the AL5083 alloy has a low ratio of yield stress to fracture toughness. Full ranges of test conditions were therefore possible with the two different geometries and two materials. These are summarised in Table A1.2 along with the test specification reference used.
Figure A1.1 – Specimen dimensions used in [A1.1]

Table A1.2 – Aluminium test samples used in [A1.1] with alloy type and test reference

<table>
<thead>
<tr>
<th>Test</th>
<th>Material</th>
<th>Specimen width 2w (m)</th>
<th>Specimen thickness (mm)</th>
<th>Crack length 2a (m)</th>
<th>Residual stress (MPa)</th>
<th>Residual stress/yield stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1P</td>
<td>Al alloy 2024</td>
<td>1.0</td>
<td>25</td>
<td>0.133</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1S</td>
<td>Al alloy 2024</td>
<td>1.0</td>
<td>25</td>
<td>0.133</td>
<td>91</td>
<td>0.23</td>
</tr>
<tr>
<td>2P</td>
<td>Al alloy 5083</td>
<td>1.0</td>
<td>25</td>
<td>0.157</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2S</td>
<td>Al alloy 5083</td>
<td>1.0</td>
<td>25</td>
<td>0.157</td>
<td>69</td>
<td>0.45</td>
</tr>
<tr>
<td>3P</td>
<td>Al alloy 5083</td>
<td>1.0</td>
<td>25</td>
<td>0.025</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3S</td>
<td>Al alloy 5083</td>
<td>1.0</td>
<td>25</td>
<td>0.025</td>
<td>73</td>
<td>0.47</td>
</tr>
<tr>
<td>P1</td>
<td>Al alloy 2024</td>
<td>0.25</td>
<td>6.25</td>
<td>0.033</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S1</td>
<td>Al alloy 2024</td>
<td>0.25</td>
<td>6.25</td>
<td>0.033</td>
<td>252</td>
<td>0.57</td>
</tr>
<tr>
<td>SK1</td>
<td>Al alloy 2024</td>
<td>0.25</td>
<td>6.25</td>
<td>0.041</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SK3</td>
<td>Al alloy 2024</td>
<td>0.25</td>
<td>6.25</td>
<td>0.041</td>
<td>70</td>
<td>0.16</td>
</tr>
</tbody>
</table>
The specimens were tested by the application of an increasing tensile primary stress and stable ductile crack growth was measured. Significant ductile crack growth was observed in all tests (measured by a.c. potential drop and inspection).

Under testing the load carrying capacities of the plates containing a residual stress were found to become aligned to that of a plate with only primary loads applied near primary yield (i.e. values of $L_r$ between 0.9 and 1.2). This can be seen in Figure A1.2 which shows the ratio of the load carrying capacity of the test containing secondary stresses compared to its equivalent with primary loads alone. The solid curves in Figure A1.2 are simply bounding curves to the data available.

Figure A1.3 shows the failure loci from the complex R6 V factor method for the interaction of primary and secondary stresses for the cases considered in the experiments plotted against an Option 1 failure assessment diagram. The inputs to the calculations were taken from the stress state of the plate at initiation and at increasing amounts of stable tearing. The results shown are, in most cases, conservative, which are demonstrated by the failure loci being outside the curve. Where some apparent non-conservatism is observed it was thought to be a result of using mean material values and uncertainty in the initiation load within the tests. The aluminium alloy experiments demonstrate the reduction in the contribution of secondary stresses on fracture under increasing applied primary loading and the levels of conservatism inherent to R6, particularly at low values of $L_r$. 
Figure A1.2 – Observed reduction in apparent load capacity [A1.1]

Figure A1.3 – Reproduction of failure loci from the R6 V Factor Method plotted against an Option 1 FAD [A1.1]

The required information to perform an R6 based analyses can be seen in Table A1.3 below, where the information has been taken both from the information within R6 [A1.13] and in [A1.1].
Table A1.3 – Variables required for R6 assessment of aluminium plate tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>1P</th>
<th>1S</th>
<th>2P</th>
<th>2S</th>
<th>3P</th>
<th>3S</th>
<th>P1</th>
<th>S1</th>
<th>5K1</th>
<th>5K3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Type</td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2w (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>t (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td>6.5</td>
<td></td>
</tr>
<tr>
<td>l (m)</td>
<td></td>
<td></td>
<td></td>
<td>2.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.575</td>
<td></td>
</tr>
<tr>
<td>Initial Crack size, a (mm)</td>
<td>66.5</td>
<td>78.5</td>
<td>12.5</td>
<td>16.5</td>
<td>20.5</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Crack growth 1, (Δa)₁, (mm)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.43</td>
<td>0.2</td>
<td>0.35</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>(Δa)₂, (mm)</td>
<td>7.1</td>
<td>7.1</td>
<td>0.95</td>
<td>1.4</td>
<td>1.7</td>
<td>1.6</td>
<td>0.65</td>
<td>0.9</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>(Δa)₃, (mm)</td>
<td>24.3</td>
<td>24.5</td>
<td>4.4</td>
<td>5.6</td>
<td>4.8</td>
<td>3.9</td>
<td>1.65</td>
<td>1.45</td>
<td>0.6</td>
<td>1.3</td>
</tr>
<tr>
<td>(Δa)₄, (mm)</td>
<td>36.0</td>
<td>36.9</td>
<td>8.3</td>
<td>11.5</td>
<td>10</td>
<td>10.1</td>
<td>1.85</td>
<td>1.7</td>
<td>1.3</td>
<td>2.5</td>
</tr>
<tr>
<td>(Δa)₅, (mm)</td>
<td>-</td>
<td>42.3</td>
<td>15.9</td>
<td>20</td>
<td>-</td>
<td>16.1</td>
<td>2.55</td>
<td>2.35</td>
<td>2.2</td>
<td>4.1</td>
</tr>
<tr>
<td>(Δa)₆, (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>46</td>
<td>-</td>
<td>-</td>
<td>3.95</td>
<td>3.2</td>
<td>3.4</td>
<td>-</td>
</tr>
<tr>
<td>(Δa)₇, (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4.35</td>
<td>3.45</td>
<td>7.8</td>
<td>-</td>
</tr>
<tr>
<td>(Δa)₈, (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>8.25</td>
<td>4.15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(Δa)₉, (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>9.95</td>
<td>5.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(Δa)₁₀, (mm)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(Δa)₁₁, (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(Δa)₁₂, (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>7.45</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

| **Material Properties**         |    |    |    |    |    |    |    |    |     |     |
| E (MPa)                         | 73700| 69900| 74050|     |     |     |     |     |     |     |
| ν                               | 0.3 |    |    |    |    |    |    |    |     |     |
| Yield Stress, σᵧ (MPa)          | 397 | 153 | 437 |     |     |     |     |     |     |     |
| Tensile Strength, σₜₚₛ (MPa)    | 508 | 301 | 528 |     |     |     |     |     |     |     |
| Fracture Toughness              |     |     |     |     |     |     |     |     |     |     |
| Defined from J – Δa curves      |     |     |     |     |     |     |     |     |     |     |
| Kₘₐᵗₜₐ₁ (MPa.m²/³)              | 37.1| 37.1| 71.6| 71.6| 77.9| 71.6| 61.5| 55.6| 49.0| 55.6|
| Kₘₐₜₜ₂ (MPa.m²/³)               | 76.0| 76.0| 90.3| 99.7| 105.4|103.6| 68.9| 73.1| 55.6| 63.1|
| Kₘₐₜₜ₃ (MPa.m²/³)               | 97.3| 97.5|146.3|160.9|151.4|139.8| 81.7| 79.8| 67.9| 78.2|
| Kₘₐₜₜ₄ (MPa.m²/³)               |105.4|105.9|189.1|217.4|204.7|205.5| 83.4| 82.1| 78.2| 88.1|
| Kₘₐₜₜ₅ (MPa.m²/³)               | -   |108.8|250.5|277.5| -   |251.9| 88.4| 87.1| 86.1| 96.4|
| Kₘₐₜₜ₆ (MPa.m²/³)               | -   | -   | -   |405.3| -   | -   | 95.8| 92.1| 93.2| -   |
| Kₘₐₜₜ₇ (MPa.m²/³)               | -   | -   | -   | -   | -   | -   | 97.5| 93.4|108.5| -   |
| Kₘₐₜₜ₈ (MPa.m²/³)               | -   | -   | -   | -   | -   | -   |109.6| 96.7| -   | -   |
\[
\begin{array}{cccccccccc}
K_{\text{mat.}\Delta a_0} (\text{MPa.m}^{1/2}) & - & - & - & - & - & - & 113.4 & 100.7 & - & - \\
K_{\text{mat.}\Delta a_{10}} (\text{MPa.m}^{1/2}) & - & - & - & - & - & - & 103.4 & - & - & - \\
K_{\text{mat.}\Delta a_{11}} (\text{MPa.m}^{1/2}) & - & - & - & - & - & - & 106.3 & - & - & - \\
K_{\text{mat.}\Delta a_{12}} (\text{MPa.m}^{1/2}) & - & - & - & - & - & - & 107.5 & - & - & - \\
\end{array}
\]

**Loading**

<table>
<thead>
<tr>
<th>Primary load</th>
<th>Tensile load applied to ends of plate, remote from crack tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{\Delta a_1} (\text{MN}))</td>
<td>2.2 1.1 2.8 1.45 4.0 3.3 0.45 0.23 0.25 0.22</td>
</tr>
<tr>
<td>(P_{\Delta a_2} (\text{MN}))</td>
<td>4.03 1.1 3.25 2.5 4.1 4.25 0.45 0.23 0.34 0.29</td>
</tr>
<tr>
<td>(P_{\Delta a_3} (\text{MN}))</td>
<td>5.15 2.4 3.36 3.3 4.4 4.39 0.46 0.25 0.4 0.38</td>
</tr>
<tr>
<td>(P_{\Delta a_4} (\text{MN}))</td>
<td>5.25 3.0 3.47 3.5 4.63 4.59 0.47 0.25 0.44 0.43</td>
</tr>
<tr>
<td>(P_{\Delta a_5} (\text{MN}))</td>
<td>- 3.2 3.56 3.6 - 4.65 0.47 0.27 0.45 0.46</td>
</tr>
<tr>
<td>(P_{\Delta a_6} (\text{MN}))</td>
<td>- - - 3.75 - - 0.48 0.27 0.46 -</td>
</tr>
<tr>
<td>(P_{\Delta a_7} (\text{MN}))</td>
<td>- - - - - - 0.48 0.29 0.47 -</td>
</tr>
<tr>
<td>(P_{\Delta a_8} (\text{MN}))</td>
<td>- - - - - - 0.49 0.29 - -</td>
</tr>
<tr>
<td>(P_{\Delta a_9} (\text{MN}))</td>
<td>- - - - - - 0.49 0.31 - -</td>
</tr>
<tr>
<td>(P_{\Delta a_{10}} (\text{MN}))</td>
<td>- - - - - - 0.31 - -</td>
</tr>
<tr>
<td>(P_{\Delta a_{11}} (\text{MN}))</td>
<td>- - - - - - 0.33 - -</td>
</tr>
<tr>
<td>(P_{\Delta a_{12}} (\text{MN}))</td>
<td>- - - - - - 0.33 - -</td>
</tr>
<tr>
<td>Secondary stress</td>
<td>Self-balancing stress distribution over plate width. Width of tensile region taken as plate width in SIF solution so that tensile solutions can be used.</td>
</tr>
<tr>
<td>(\sigma_{\text{res.}} (\text{MPa}))</td>
<td>0 91.3 0 68.9 0 71.9 0 249.1 0 69.9</td>
</tr>
</tbody>
</table>

**Appendix 1.3**  
Ainsworth *et al* - A533B Plate Tests

Also considered in [A1.1] are tests on an A533B steel, which were performed to repeat the aluminium alloy results for a structurally relevant material. Four-point bend specimens measuring 600 x 600 x 70 mm³ that contained a residual stress field were used in the tests. The specimens were made by welding the two halves of a cut plate back together using a double V preparation, thus creating a self balancing through thickness residual stress. The plate was subsequently cut in half and one of these subjected to a post weld heat treatment (PWHT). This reduced the stress magnitude in the plate by between 10 and 30%. Test specimens, a block for residual stress measurement and material characterisation
specimens were then manufactured from each of the plates. The test specimens then had a shallow surface crack with an $a/t$ ratio of 0.25 to maximise the stress intensity factors predicted. The samples were then placed under 4-point bending and monitored for crack growth at two test temperatures of -120 and -30 °C. The failure results demonstrated a similar failure load with and without residual stress at -30 °C and a large amount of associated plasticity. At -120 °C an increase in load of 1.7 times the welded sample was required to cause complete fracture in the sample that underwent the PWHT process.

Table A1.4 shows the comparison when estimating the failure load from the R6 V factor method, in a similar way to the aluminium alloy results, at both the surface and deepest points for the crack and the actual failure loads.

**Table A1.4 – Comparison of predicted and actual failure loads for the A533B material bend tests [A1.1]**

<table>
<thead>
<tr>
<th>Test</th>
<th>Temperature (°C)</th>
<th>$K_{\text{ref}}$ (MPa m$^{1/2}$)</th>
<th>$\sigma_{0.2%}$ (MPa)</th>
<th>Ultimate tensile strength (MPa)</th>
<th>Residual stress included</th>
<th>Predicted failure load (MN)</th>
<th>Experimental failure load (MN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Deepest point</td>
<td>Surface point</td>
</tr>
<tr>
<td>As welded</td>
<td>-120</td>
<td>32.2</td>
<td>534</td>
<td>757</td>
<td>No</td>
<td>0.86</td>
<td>1.16</td>
</tr>
<tr>
<td>PWHT</td>
<td>-120</td>
<td>41.0</td>
<td>546</td>
<td>686</td>
<td>No</td>
<td>1.09</td>
<td>1.51</td>
</tr>
<tr>
<td>As welded</td>
<td>-30</td>
<td>59.0</td>
<td>412</td>
<td>586</td>
<td>Yes</td>
<td>1.47</td>
<td>1.83</td>
</tr>
<tr>
<td>PWHT</td>
<td>-30</td>
<td>321.2</td>
<td>413</td>
<td>546</td>
<td>Yes</td>
<td>2.73*</td>
<td>2.73*</td>
</tr>
</tbody>
</table>

*Plastic collapse failure at $L_r = L_{r,\text{max}}$.*

These results show significant conservatism when compared to the experimental failure load, for all cases assessed. The level of conservatism in the assessment was thought to be a result of both crack-tip constraint, inaccurate estimate of fracture toughness and of an overly conservative limit load solution. The tests, however, do show that the effect of the residual stress is significant under brittle conditions but is less so when the material behaves in a ductile manner.

Table A1.5 below shows the main data from the tests required to perform an assessment of the tests.
Table A1.5 – Variables required for R6 assessment of A533B plate tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Low $L_r$ as-welded (LLAW)</th>
<th>Low $L_r$ heat treated (LLHT)</th>
<th>High $L_r$ as-welded (HLAW)</th>
<th>High $L_r$ heat treated (HLHT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Four point bend plate specimens</td>
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<td></td>
</tr>
<tr>
<td>$2W$ ($m$)</td>
<td>600</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$ ($mm$)</td>
<td>70</td>
<td>71</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Crack depth, $a$ ($mm$)</td>
<td>19.4</td>
<td>18.6</td>
<td>18</td>
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</tr>
<tr>
<td>Crack length, $2c$ ($mm$)</td>
<td>175</td>
<td>174.2</td>
<td>175</td>
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<tr>
<td>Material Properties</td>
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<tr>
<td>$E$ ($MPa$)</td>
<td>200000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Stress, $\sigma_y$ ($MPa$)</td>
<td>618</td>
<td>551</td>
<td>596</td>
<td>561</td>
</tr>
<tr>
<td>Tensile Strength, $\sigma_{UTS}$, ($MPa$)</td>
<td>791</td>
<td>762</td>
<td>772</td>
<td>694</td>
</tr>
<tr>
<td>Fracture Toughness ($MPa.m^{1/2}$)</td>
<td>37</td>
<td>46</td>
<td>62</td>
<td>321</td>
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<tr>
<td>Loading</td>
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</tr>
<tr>
<td>Primary load</td>
<td>Four point bend loading</td>
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</tr>
<tr>
<td>$P_{2a1}$ ($MN$)</td>
<td>1.27</td>
<td>2.19</td>
<td>5.1</td>
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</tr>
<tr>
<td>Secondary stress</td>
<td>Residual stress provided in paper</td>
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</tr>
</tbody>
</table>

Appendix 1.4    Hong-Liang et al

Fracture experiments were performed by Hong-Liang [A1.2] where weld residual stress and primary loads were combined. These experiments incorporated three different specimen designs as shown in see Figure A1.4. These specimens were initially welded together from two plates and the crack introduced by a wire saw cutting device. The material used by Hong-Liang was a 16MnR steel, to reflect its use in Chinese pressure vessels. The tests were then cooled to a range of temperatures from $-65$ °C to room temperature and tested under a tensile primary stress field.
The experiments were designed to follow previous work by the same authors [A1.14] within which another definition of \( \rho \) was provided to treat a potentially non conservative case of a small crack in a high tensile secondary stress field. However, it may be best the estimate of \( K_f^S \) needed improving under these conditions, rather than the \( \rho \) factor. The results, illustrated in Figure A1.5, show that the R6 \( \rho \) factor method is generally conservative for all other cases, and is within the materials variability, even for the case where non-conservatism was expected.

Table A1.6 below shows the main data from the tests required to perform an assessment of the tests.

![Figure A1.4 – Schematic of welded plate experiment samples used by Hong-Liang [A1.2]](image-url)
Figure A1.5 – Experimental results presented by Hong-Liang [A1.2]

Table A1.6 – Variables required for R6 assessment of Hong-Liang plate tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>T-1</th>
<th>T-2</th>
<th>T-3</th>
<th>T-4</th>
<th>T-5</th>
<th>T-6</th>
<th>T-7</th>
<th>T-8</th>
<th>T-9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Plate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2w$ (mm)</td>
<td>178</td>
<td>179</td>
<td>177</td>
<td>180</td>
<td>175</td>
<td>180</td>
<td>180</td>
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<tr>
<td>$t$ (mm)</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$l$ (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>400</td>
</tr>
<tr>
<td>Initial Crack size, $a$ (mm)</td>
<td>15</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>12.5</td>
<td>10</td>
<td>7.5</td>
<td>5</td>
<td>25</td>
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<td><strong>Material Properties</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>200,000</td>
</tr>
<tr>
<td>$v$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>Yield Stress, $\sigma_y$ (MPa)</td>
<td>355</td>
<td>382</td>
<td></td>
<td></td>
<td>413</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile Strength, $\sigma_{UTS}$ (MPa)</td>
<td>545</td>
<td>579</td>
<td></td>
<td></td>
<td>616</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fracture Toughness ($MPa.m^{1/2}$)</td>
<td>228</td>
<td>154</td>
<td>123</td>
<td></td>
<td>80</td>
<td></td>
<td>123</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Loading</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary load</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P$ (MN)</td>
<td>0.596</td>
<td>0.452</td>
<td>0.565</td>
<td>0.377</td>
<td>0.335</td>
<td>0.396</td>
<td>0.214</td>
<td>0.366</td>
<td>0.465</td>
</tr>
</tbody>
</table>
Secondary stress & Average tensile load from weld residual stresses over crack length
\( \sigma_{\text{res}}, \text{(MPa)} \) & 291 & 252 & 323 & 252 & 360 & 360 & 360 & 360 \\

### Appendix 1.5 Mirzaee-Sisan et al

More recently Mirzaee-Sisan [A1.3] has performed tests that included a residual stress in significantly smaller samples. The residual stress was inserted into the specimens through a mechanical pre-load in the form of end load compression. The tests were performed on a single edge notched bend specimen shown in Figure A1.6 that was fabricated from A533B ferritic pressure vessel steel. The tests were performed to investigate the interaction between a residual stress and an applied mechanical load.

![Figure A1.6 – Schematic of test specimen used by Mirzaee-Sisan [A1.3]](image)

The specimens were subjected to an end load pre-compression at the positions indicated by the arrows in Figure A1.6. This was subsequently unloaded and the specimens cooled to -150 °C before incrementing the applied load under 3-point bending until a brittle failure event occurred. This process has been termed compression-unload-cool-fracture (CUCF). The failure tests were performed at -150 °C to ensure brittle conditions.

To provide an estimate of the effect of the residual stress some as-received samples, which did not undergo the pre-compression, were also tested under 3-point bending. The mean estimate of \( K^P \) at failure for these as-received specimens was subsequently used as an estimate of the materials fracture toughness \( K_{\text{mat}} \). It is noted that the determination of all stress intensity factors used in the work was from finite element analyses. As a complex
loading history was present, the British Energy Code “J-Mod” [A1.15] was used to accurately calculate the crack driving force parameter $J$.

The failure points were then plotted for both the as-received and CUCF cases against an Option 1 failure assessment curve using different estimates of the crack driving force at failure (see Figure A1.7). Within the plot, three different means of estimating the combined crack driving force were used for the CUCF specimens where the simple R6 estimate was used, taking $K_j$ from the finite element results and using it in the detailed R6 method and by taking an estimate of $K_j$ straight from the finite element results.

![Figure A1.7 – Plot of experimental results on an R6 Option 1 FAD for the CUCF and As-Received specimens from [A1.3]](image)

The figure shows that the results from as-received samples lie over the failure assessment curve, which would be expected as this has been used to define the fracture toughness. Of more interest is the added conservatism seen in the pre-stressed samples. This conservatism has already been noted in the main body of the thesis, but the conservatism that remains even when the finite element estimate of the combined crack driving force is used. This would indicate that providing methods that predict the interaction of primary and secondary stresses based upon finite element results should be conservative compared to experiments. One factor that could provide improved estimates for the failure prediction
may be an altered \( K_{mat} \), through the influence on constraint from the residual stress field [A1.16, A1.17]. Under a constraint modified failure assessment curve [A1.16–A1.18] with the presence of secondary stress, the value of \( K_r \) at \( L_r = 0 \) is increased, which could provide improved estimates of failure for the experimental results. The data relevant to assessing the tests within an assessment can be seen in Table A1.7 below.

**Table A1.7 – Variables required for R6 assessment of Mirzaee-Sissan pre-compressed three-point bend tests**

<table>
<thead>
<tr>
<th>Variable</th>
<th>As Received (1-8)</th>
<th>Residual Stress (1-5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Type</strong></td>
<td>Three-point bend specimen with scalloped notch</td>
<td></td>
</tr>
<tr>
<td>( W \ (mm) )</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>( S \ (mm) )</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>( B \ (mm) )</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Initial Crack size, ( a \ (mm) )</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td><strong>Material Properties</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E \ (MPa) )</td>
<td>220000</td>
<td></td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Yield Stress, ( \sigma_y \ (MPa) )</td>
<td>636</td>
<td></td>
</tr>
<tr>
<td>Tensile Strength, ( \sigma_{UTS} \ (MPa) )</td>
<td>889</td>
<td></td>
</tr>
<tr>
<td>Fracture Toughness ( (MPa.m^{1/2}) )</td>
<td>70.3</td>
<td></td>
</tr>
<tr>
<td><strong>Loading</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary load</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P (kN) )</td>
<td>45.7, 35.2, 30.2, 24.1, 30.6, 16.6, 30.9, 28.1</td>
<td>13.8, 12.7, 25.1, 12.4, 17.8</td>
</tr>
<tr>
<td>Secondary stress</td>
<td>-</td>
<td>Residual stress profile from mechanical pre-compression</td>
</tr>
<tr>
<td>( R^P_f ) Solution (from FE), ( (MPa.m^{1/2}) )</td>
<td>106.4, 82.1, 70.3, 56.1, 71.3, 38.6, 71.8, 65.5</td>
<td>32.2, 29.5, 58.4, 28.9, 41.5</td>
</tr>
<tr>
<td>( R^S_f ) Solution (from FE), ( (MPa.m^{1/2}) )</td>
<td>-</td>
<td>45.9</td>
</tr>
</tbody>
</table>
Appendix 1.6  Lee et al

A series of studies were performed by Lee et al [A1.4] to consider how a residual stress field may affect constraint and hence fracture toughness. The experimental portion of this investigation was performed on both high constraint compact tension specimens and low constraint single edge notched specimen. A schematic illustration of the two specimens can be seen in Figure A1.8. The specimens that contained residual stress underwent a compressive preload to establish the stress field prior to pre-cracking. The samples were made from A533B pressure vessel steel and tested in the cleavage fracture regime.

![Figure A1.8 – Illustration of CT and SENT specimens used in [A1.4]](image)

The results from these tests showed similar distributions of the crack driving force (derived using the Serco code JEDI [A1.19]) at failure for the high constraint CT specimens with and without the residual stress field. However, the pre-loaded SENT specimens showed a lower crack driving force than the non-preloaded SENT specimens. It is argued in [A1.4] that this shows a reduction in constraint benefit arising from cracks in a highly bending residual stress field.

The experiments were coupled with detailed finite element analyses that considered both differences in the materials hardening characteristics and in estimations of the constraint parameter, $Q$. The coupled experimental and finite element work showed that the behaviour of specimens containing residual stress need to be quantified by two parameter fracture mechanics, i.e. including constraint. It also showed that the effect of constraint in
a residual stress field is generally increasing the constraint parameter, reducing any perceived benefit.

The experiment showed that the use of the modified form of the \( J \) contour integral provided a good estimate of \( J \) compared to the experimental difference between the pre-loaded and normal specimens. The stress intensity factor from residual stress was found to be approximately 25 MPa.m\(^{0.5}\).

All values required for an assessment of these tests can be seen in Table A1.8 below.

**Table A1.8 – Variables required for an R6 assessment of the Lee pre-compressed CT and SENT tests**

<table>
<thead>
<tr>
<th>Variable</th>
<th>CT</th>
<th>SENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As Received</td>
<td>Residual</td>
</tr>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>CT Specimen</td>
<td>CT Specimen electron beam welded to a tensile rig</td>
</tr>
<tr>
<td>( W ) (mm)</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>( B ) (mm)</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Initial Crack size, ( a ) (mm)</td>
<td>30.4</td>
<td></td>
</tr>
<tr>
<td><strong>Material Properties</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E ) (MPa)</td>
<td>210000</td>
<td></td>
</tr>
<tr>
<td>( v )</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Tensile curve</td>
<td>Tensile curve provided</td>
<td></td>
</tr>
<tr>
<td>Yield Stress (MPa)</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>Lower Bound Fracture Toughness (MPa.m(^{0.5}))</td>
<td>89</td>
<td>112</td>
</tr>
<tr>
<td><strong>Loading</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary load</td>
<td>Pin loading to CT specimens to apply high constraint bending load</td>
<td>End load applied via rig to provide low constraint tensile load</td>
</tr>
<tr>
<td>( P ) (kN)</td>
<td>19 specimens with load range between 35.3 to 25.7, 29.4, 31.6, 29.7, 28.7, 45.0,</td>
<td>10 specimens with load range between 214.4 to 159.0, 168.1, 166.1, 186.4, 223.6, 160.2,</td>
</tr>
</tbody>
</table>
Appendix 1.7  Kamel – Compressed C-Ring Tests

A mechanical pre-load was applied to compress a C-Ring specimens [A1.5], thus creating a tensile residual stress on unload. The residual stress was measured through neutron diffraction and compared to finite element analyses before fracture toughness tests were performed on the specimens.

An illustration of the specimen design chosen can be seen in Figure A1.9. The specimen was chosen to allow a residual stress field to be applied through bending whilst ensuring that the specimen allows a relatively low level of constraint. The specimen had an outer diameter of 100 mm, a width of 23.5 mm and a thickness of 27 mm. 10 % thickness side grooves were also applied to ensure surface effects were reduced, thus enhancing levels of crack tip triaxiality. Eight specimens were tested, five with a mechanical pre-load. All specimens had a 2 mm crack inserted after pre-loading, where applied, by a 0.1 mm diameter wire on an electro-discharge machine.

<table>
<thead>
<tr>
<th></th>
<th>51.8</th>
<th>27.4, 30.1, 36.5, 29.0 to 273.8</th>
<th>188.6, 206.7, 194.8, 154.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secondary stress</td>
<td>-</td>
<td>Residual stress field</td>
<td>-</td>
</tr>
<tr>
<td>Residual stress field</td>
<td>-</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

$K^c$ Solution (from FE), $(MPam^{1/2})$
The material used within the tests was of a high strength, low alloy tubing steel, AISI 4333 M4. Monotonic stress-strain curves, orientated to match the hoop direction in the experiments, showed almost elastic-perfectly plastic behaviour with a yield stress of 1050 MPa, a UTS of 1100 MPa and a significant Bauschinger effect under compression.

Three specimens were pre-compressed under displacement-controlled conditions between 2.2 and 2.5 mm. Two specimens were pre-tensioned under displacement control between 1.9 and 2.2 mm; the pre-tensioned specimens resulted in a compressive residual stress field over the crack length. One specimen from both the pre-compressed and pre-tensioned specimens was then subjected to neutron diffraction measurements.

All specimens were tested to determine the experimentally defined fracture toughness by application of a monotonic tensile load. All specimens were observed to fail under brittle conditions without any crack growth, but the fracture surface demonstrated a ductile, void growth mechanism under scanning electron microscopy. The effect of residual stress was observed for all failure loads for the different conditions tested: pre-compressed was less than the as-received case, which was less than the pre-tensioned case.
Both finite element analyses and elastic interpretation of the results were used to provide estimates of the fracture toughness, $K_m$, at the failure load, $P_c$, from primary loads alone. Also provided were estimates of the secondary stress intensity factor, $K_{res}$, from finite element analyses using the ABAQUS SIGINI command to import the measured residual stress field. The primary and secondary stress intensity factors have then been summed to provide an estimate of the materials fracture toughness, $K_c = K_m + K_{res}$. An overview of these results can be seen in Table A1.9.

### Table A1.9 – Summary of LEFM based results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$P_c$ (kN)</th>
<th>$K_m$ (MPa√m)</th>
<th>$K_{res}$ (MPa√m)</th>
<th>$K_c$ (MPa√m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>As-received</td>
<td>76</td>
<td>105</td>
<td>0</td>
<td>105</td>
</tr>
<tr>
<td>Pre-tensioned</td>
<td>91</td>
<td>125</td>
<td>-34</td>
<td>91</td>
</tr>
<tr>
<td>Pre-compressed</td>
<td>60</td>
<td>82</td>
<td>+34</td>
<td>115</td>
</tr>
</tbody>
</table>

The variables required to perform an assessment of these tests can be seen in Table A1.10 below.

### Table A1.10 – Data required for an assessment of Kamel C-ring tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>As Received</th>
<th>Pre-Tensioned</th>
<th>Pre-Compressed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>C-Ring</td>
<td></td>
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</tr>
<tr>
<td>$R_i$ (mm)</td>
<td>26.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_o$ (mm)</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$ (mm)</td>
<td>27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Crack size, a (mm)</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Material Properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td>180000 (derived from tensile curve provided)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>0.3 (estimated)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile curve</td>
<td>Fracture Toughness $(MPam^{1/2})$</td>
<td>105</td>
<td></td>
</tr>
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<td>-------------------------------</td>
<td>-----------------------------------</td>
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<td></td>
</tr>
</tbody>
</table>

### Loading

<table>
<thead>
<tr>
<th>Primary load</th>
<th>Tensile load applied to pins inducing a bending stress field at crack location</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(kN)$</td>
<td></td>
</tr>
<tr>
<td>Secondary stress</td>
<td></td>
</tr>
<tr>
<td>$K_f$ Solution (from FE / tests), $(MPam^{1/2})$</td>
<td></td>
</tr>
</tbody>
</table>

## Appendix 1.8  Hurlston – Side Punched Tests

Hurlston [A1.6] constructed experiments to consider constraint effects on the treatment of specimens that include both primary and secondary stresses. The secondary stress introduced was by way of mechanical pre-compression out of plane tangential to the crack front inducing a tensile residual stress acting to open the crack on unloading. The effect of constraint was investigated by considering a shallow ($a/W = 0.22$) and a deep crack ($a/W = 0.42$). A number of specimens were also tested that did not include a residual stress so that the effect of residual stress could also be considered.

Within the tests performed by Hurlston a conventional three-point bend specimen was used with the dimensions outlined in Figure A1.10, made from the same A533B material used in Section 5 of the main body of the thesis. The reported materials characterisation for the A533B plate can be seen in Table A1.11.
The secondary residual stress was introduced to the specimen by “punching” the side of the specimen 15 mm ahead of the crack tip with a punch of radius 5 mm. A schematic of the side-punching set-up can be seen in Figure A1.12. The side-punching approach was adopted as it can create a controlled stress field that has minimal plastic strain and, hence, little effect on the material ahead of the crack but can alter the constraint level.

To apply the mechanical pre-compression the top punch was displaced by 0.5 mm and held for 30 seconds to allow the stress field to stabilise. The resultant stress field predicted from associated finite element analyses can be seen in Figure A1.12 compared to neutron diffraction measurements for two crack depths. In performing the finite element analyses for the A533B material a multi-linear kinematic hardening model was used to predict the residual stress field. Clearly the comparison of the predicted and measured residual stress fields is good, as were comparisons of load displacement plots from the tests. This helped
validate the finite element analyses performed and confirmed the use of the kinematic hardening model for this material.

Figure A1.11 – Illustration of side-punching of the three-point bend specimen [A1.6]

Figure A1.12 – Predicted residual stress field after side-punching as measured versus predicted moving away from the notch tip (prior to crack insertion) [A1.6]

After the side-punching the specimens were cooled to -140 °C to ensure an elastic material response. The measured load at failure can be seen in Figure A1.13 and the corresponding combined primary and secondary stress intensity factor in Figure A1.14.
The measured load at failure clearly shows that the short cracked standard specimen attains a higher load to fail than the deeply cracked specimen, which is clear indication of low constraint conditions in the shallow cracked specimen. It can also, therefore, be seen that the effect of the residual stress field is to increase the constraint of the specimen. The
fracture toughness values were also seen to change with the level of constraint seen and can be fit to the expected failure curve in $J - Q$ space.

From the experiment and finite element analyses the values required for an R6 based assessment of these tests are shown in Table A1.12. It can be considered that these tests improved upon similar tests by Mahmoudi [A1.20] (and, as such [A1.20] is not considered here).

<table>
<thead>
<tr>
<th>Table A1.12 – Variables required for R6 assessment of Hurlston tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>Geometry</td>
</tr>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Width ($mm$)</td>
</tr>
<tr>
<td>Depth ($mm$)</td>
</tr>
<tr>
<td>Thickness ($mm$)</td>
</tr>
<tr>
<td>Initial Crack size, $a$ ($mm$)</td>
</tr>
<tr>
<td>Material Properties</td>
</tr>
<tr>
<td>Young's Modulus, $E$ (MPa)</td>
</tr>
<tr>
<td>Poisson's Ratio, $\nu$</td>
</tr>
<tr>
<td>Yield Stress, $\sigma_y$ (MPa)</td>
</tr>
<tr>
<td>Tensile curve</td>
</tr>
<tr>
<td>Ramberg Osgood Strain</td>
</tr>
<tr>
<td>Hardening Index, $n$</td>
</tr>
<tr>
<td>Fracture Toughness (lower bound value), $(MPa^{1/2})$</td>
</tr>
<tr>
<td>Loading</td>
</tr>
<tr>
<td>Primary load</td>
</tr>
<tr>
<td>Load ($kN$)</td>
</tr>
<tr>
<td>Secondary stress</td>
</tr>
<tr>
<td>$K_f$ Solution (from FE), $(MPa^{1/2})$</td>
</tr>
</tbody>
</table>
Appendix 1.9 Bolinder - Inspectra AB Tests

Tests have been performed in Sweden by Bolinder under Inspectra Technology AB [A1.7] (funded by the Swedish radiation safety authority, Strål-Säkerhets-Myndigheten) to consider combined loading at both intermediate and high values of \( L_r \). The tests were based upon the Mirzaee-Sisan [A1.3] experiments but are performed on materials at room temperature so that the intermediate and high values of \( L_r \) are realised. Specifically, the aim of the experimental programme was to consider the effect of the secondary stress at values of \( L_r > 1 \). This was to provide evidence towards reducing levels of conservatism when assessing ductile materials within the Swedish fracture handbook, ProSACC [A1.21], itself based upon R6.

A range of previous experiments had been considered for the test design, including the aluminium tests [A1.1], and the 3 point bend notched test specimen scheme recently undertaken by Mirzaee-Sisan [A1.3] was finally chosen. To induce the secondary stress field, the specimen underwent a compressive tensile force that creates a residual stress field upon release. The test geometry was a three-point bend specimen with a notch and pre-crack in the longest dimension (see Figure A1.15).

To test under plasticity, the Swedish materials Weldox 700 and Weldox 900 were initially chosen for their high yield strength (700 and 900 MPa respectively) and high fracture toughness. However, it is also worth noting that both materials have a similar post yield strain hardening. The effect of material hardening and (compressive) loading magnitude on the residual stress profile was examined which showed little variation in the range of the tensile region and (generally) only minor changes in the stress fields. Because of the similarity in the results of the Weldex 700 and 900 materials a further material (A533B) was provided and the Weldox 900 material was not used.
Considered factors affecting the experimental programme such as geometry, magnitude of pre-loading, material and specimen size had also been considered by their effect on the stress field in finite element analyses. An idealised geometry, which acts to maximise the tensile region over the crack, was found to have the dimensional parameters shown in Figure A1.15. It was also shown that, for high tensile strength, a small width ($W$) is needed to obtain a high $L_r$. There is also a need to ensure the residual stress zone is smaller than the plastic zone at failure, so a high width ($W$) is needed. The final geometry proposed had an $a/W = 0.35$, an initial scalloped notch radius of $0.25W$ and a thickness of $0.5W$. The value of $W$ used depended on the material and failure load desired. Three tests were considered with and without residual stress to fail at values of $L_r = 0.9$, 1.0 and 1.1. Details of the $W$ used for these conditions, with the material considered, can be seen in Table A1.13.

Figure A1.15 – Test specimen within Swedish experimental programme for the treatment of secondary stresses in ductile materials [A1.7]
It is also worth noting that under these conditions the Q stress crack-tip constraint factor for the geometry (under primary loads) is shown to be negative for smaller crack depths. This therefore means that the measure of fracture toughness from the specimens without a residual stress should be considered for each specimen.

A comparison between the experimental results and finite element analyses has been conducted. These comparisons show very good agreement for all materials and loads applied. As such, both the experiments and finite element analyses show that the secondary stress contribution is negligible once yielding occurs. The analyses of the results by comparison to the R6 assessment procedure has also been performed which additionally show the R6 predictions are very conservative compared to the experiments.

The details required to perform an assessment of these tests can be seen in Table A1.13.

Table A1.13 – Variables required for R6 assessment of Inspectra AB pre-compressed three-point bend tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Weldex 700 – ( L_r = 0.9 )</th>
<th>Weldex 700 – ( L_r = 1.0 )</th>
<th>A533B – ( L_r = 1.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 x As Received</td>
<td>3 x Residual</td>
<td>2 x As Received</td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Three point bend specimens with notch of radius 0.25W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( W (mm) )</td>
<td>100</td>
<td>70</td>
<td>27</td>
</tr>
<tr>
<td>( B (mm) )</td>
<td>50</td>
<td>35</td>
<td>13.5</td>
</tr>
<tr>
<td>( S (mm) )</td>
<td>400</td>
<td>280</td>
<td>108</td>
</tr>
<tr>
<td>( a (mm) )</td>
<td>35</td>
<td>24.5</td>
<td>9.45</td>
</tr>
<tr>
<td>Crack growth 1, ( (\Delta a)_1, (mm) )</td>
<td>0.41, 0.27</td>
<td>0.24, 0.32, 0.31</td>
<td>0.15, 0.46</td>
</tr>
<tr>
<td>( (\Delta a)_2, (mm) )</td>
<td>0.72, 0.63</td>
<td>0.58, 0.73, 0.8</td>
<td>0.59, 0.91</td>
</tr>
<tr>
<td>Material Properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E (MPa) )</td>
<td>207500</td>
<td>205300</td>
<td></td>
</tr>
</tbody>
</table>
\[ \nu \]
\[ \sigma_y (MPa) \]
\[ 0.3 \]
\[ 655 \]
\[ 471 \]

<table>
<thead>
<tr>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary load</td>
</tr>
<tr>
<td>( P_{\Delta a_1} (kN) )</td>
</tr>
<tr>
<td>( P_{\Delta a_2} (kN) )</td>
</tr>
</tbody>
</table>

| Secondary stress | End compression and tensile residual stress field on release |
| \( K_f^S \) from FE (MPam\(^{1/2}\)) | - | 96.5 | - | 115.8 | 73.8 |

| Limit Load (FE) (kN) | 460 | 227 | 24 |

**Appendix 1.10  AEAT Reports – Spinning Cylinder Tests**

A series of large-scale experiments [A1.8–A1.12] were performed by the UK Atomic Energy Authority Technology (AKAEA) to consider the effects of primary hoop stress and thermal shock. To cover a range of combinations of primary and secondary stress, crack size and shape, a total of six tests were performed. These tests are referred to as Spinning Cylinder 1 to 6 (SC1 to SC6).

All the tests were performed on forged cylinders which went subsequent heat treatment, water quenching and tempering to match an ASTM A508 Class 3 forging. All cylinders had a nominal diameter of one metre. This large cylinder was rapidly rotated to induce a primary hoop stress within a specially commissioned test house and rig. During testing the cylinder could also be heated, to the desired steady state temperature, before the internal surface was sprayed with cold water to induce a severe thermal shock. A figure of the experimental set-up of the tests can be seen in Figure A1.16. A brief overview of the tests is detailed below:
• SC1 – Primary load only [A1.8,A1.9]. A full-length axial internal crack was tested under rapid rotation and the crack growth measured.

• SC2 – Secondary load only [A1.8,A1.9]. A full-length axial internal crack was heated to approximately 300 °C before being rapidly cooled on the internal surface by water spray. To ensure an even temperature distribution a negligible rotational speed was also applied.

• SC3 – Primary and secondary loads [A1.8,A1.9]. A combination of SC1 and SC2 where the cylinder was heated to a steady state temperature when cold water was sprayed internally coupled with an increase in rotational speed to maintain an equal ratio of primary to secondary loading of 1:5.

• SC4 – Secondary loads only [A1.10]. Two surface breaking semi-elliptical cracks were positioned 180° apart on the internal surface of the cylinder that was then heated to approximately 300 °C before being rapidly cooled on the internal surface by water spray. To ensure an even temperature distribution a negligible rotational speed was also applied.

• SC5 – Primary and secondary stresses in a weld after PWHT (post weld heat treatment) [A1.11]. Extended axial surface breaking defect within a heat-treated weld. The cylinder was heated to a steady state before rapidly cooling on the internal surface under a constant rotational speed. Crack growth was recorded within these tests. This test was specifically designed to reflect the weld and component behaviour that would be expected for a defect in a Sizewell B power station.

• SC6 – Primary and secondary loads [A1.12]. Similar to SC4 but with one of the semi-elliptical cracks elongated and a constant primary load applied.

These tests have been very well characterised, both experimentally and by finite element analyses. Prior to the tests each cylinder forging underwent material tests to obtain tensile properties, fracture toughness, thermal characterisation and density measurements. Likewise during the tests the thermal shock was measured by a series of thermocouples, which was compared to finite element predictions, and crack growth measured by potential difference approaches. Post-test analyses included fractographic examination of the crack locations to correlate with measured data.
The data required for an R6 based assessment of these tests can be seen in Table A1.14 below. A number of the values within the table required interpretation of the plots in the respective reports or judgement. Where this judgement was required, as with the $J - R$ curve of SC5, the lower bound value has been taken. It is also noted that tabulated tensile properties were also included but these were not consistent between equivalent tests and could be represented by idealised curves as adopted here.

![Diagram of test rig](image)

**Figure A1.16 – Test specimen at centre of test rig for spinning cylinder tests [A1.11]**
Table A1.14 – Variables required for R6 assessment of SC1 to SC6 cylinder tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>SC1</th>
<th>SC2</th>
<th>SC3</th>
<th>SC4</th>
<th>SC5</th>
<th>SC6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Cylinder with internal extended axial crack (1-3,5), and semi-elliptical crack (4,6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_i ) (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>500</td>
</tr>
<tr>
<td>( t ) (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>200</td>
</tr>
<tr>
<td>( l ) (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1300</td>
</tr>
<tr>
<td><strong>Initial Crack size, ( a )</strong> (mm)</td>
<td>116</td>
<td>104</td>
<td>40 and 60</td>
<td>85</td>
<td>40 and 35</td>
<td></td>
</tr>
<tr>
<td><strong>Initial crack length</strong> (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>80 and 120</td>
<td>-</td>
<td>80 and 210</td>
</tr>
<tr>
<td>Crack growth 1, ( (\Delta a)_1 ) (mm)</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 2, ( (\Delta a)_2 ) (mm)</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>0.12</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 3, ( (\Delta a)_3 ) (mm)</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>0.32</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 4, ( (\Delta a)_4 ) (mm)</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>0.68</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 5, ( (\Delta a)_5 ) (mm)</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>0.93</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 6, ( (\Delta a)_6 ) (mm)</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>1.36</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 7, ( (\Delta a)_7 ) (mm)</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>1.87</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 8, ( (\Delta a)_8 ) (mm)</td>
<td>7</td>
<td>-</td>
<td>-</td>
<td>2.46</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 9, ( (\Delta a)_9 ) (mm)</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 10, ( (\Delta a)_{10} ) (mm)</td>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Crack growth 11, ( (\Delta a)_{11} ) (mm)</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Material Properties</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E ) (MPa)</td>
<td>193120</td>
<td>212350</td>
<td>189100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( v )</td>
<td>0.275</td>
<td></td>
<td></td>
<td>0.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Stress, ( \sigma_y ) (MPa)</td>
<td>539</td>
<td></td>
<td></td>
<td>565</td>
<td>521</td>
<td></td>
</tr>
<tr>
<td>Density, ( \rho ) (Kgmm(^{-3}))</td>
<td></td>
<td></td>
<td></td>
<td>7.787 x 10(^{9})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile Strength, ( \sigma_{UTS} ) (MPa)</td>
<td>707</td>
<td></td>
<td></td>
<td>660</td>
<td>660</td>
<td></td>
</tr>
<tr>
<td>Fracture Toughness</td>
<td>( J_{mat} = 0.208\Delta a^{0.329} ) (( \Delta a ) in mm)</td>
<td>( K_{mat} = 54.2e^{0.00977} )</td>
<td>( J_{mat} = 0.288\Delta a^{0.368} ) As SC4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Loading</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Page 305 of 349
<table>
<thead>
<tr>
<th>Primary load</th>
<th>Cylinder Rotated at high speed to generate hoop stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{\Delta a_1} ) (rpm)</td>
<td>2600 485 Ramped from 1566 to 2267 500 1733 2000</td>
</tr>
<tr>
<td>( P_{\Delta a_2} ) (rpm)</td>
<td>2600 - - - 1825 -</td>
</tr>
<tr>
<td>( P_{\Delta a_3} ) (rpm)</td>
<td>2600 - - - 1875 -</td>
</tr>
<tr>
<td>( P_{\Delta a_4} ) (rpm)</td>
<td>2600 - - - 1900 -</td>
</tr>
<tr>
<td>( P_{\Delta a_5} ) (rpm)</td>
<td>2600 - - - 1900 -</td>
</tr>
<tr>
<td>( P_{\Delta a_6} ) (rpm)</td>
<td>2600 - - - 1900 -</td>
</tr>
<tr>
<td>( P_{\Delta a_7} ) (rpm)</td>
<td>2600 - - - 1900 -</td>
</tr>
<tr>
<td>( P_{\Delta a_8} ) (rpm)</td>
<td>2600 - - - 1900 -</td>
</tr>
<tr>
<td>( P_{\Delta a_9} ) (rpm)</td>
<td>2600 - - - 1900 -</td>
</tr>
<tr>
<td>( P_{\Delta a_{10}} ) (rpm)</td>
<td>2600 - - - - -</td>
</tr>
<tr>
<td>( P_{\Delta a_{11}} ) (rpm)</td>
<td>2600 - - - - -</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Secondary stress</th>
<th>Cylinder heated then sprayed, internally, with cold water</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma^S )</td>
<td>-</td>
</tr>
</tbody>
</table>

**Appendix 1.11 References**


Appendix 2. Finite Element Investigation into Weld Residual Stresses and the Plasticity Interaction Term $V_g$

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Appendix 2.1 Preface

This appendix provides an overview of an investigation to consider how typical welds within a cylinder can be treated with the $V_g$ term defined in Section 4. The main reason for this comparison is the lack of experimental or analytical cases that include a weld residual stress in a cylinder. As cylindrical geometries are the most predominant pressure vessel in nuclear power plant it was considered important for validation to be available for cases that experience a repair weld. The case of a repair weld was chosen as as-welded conditions nominally represents the worst case scenario for a structural assessment [A2.1] as repair welds do not normally experience a post weld heat treatment to reduce the level of residual stress. It was also considered important to consider a welded component to provide further confidence in non-proportional, pre-loaded secondary stresses in cylinders. As such, it is not important that the residual stress matches a measured residual stress distribution, only that the profile shares its characteristics.
Two cracked geometries are considered here, one of a fully circumferential defect and one of a fully axial defect, both cases with a deep crack of $a/t = 0.4$. Extended defects were chosen as these represent a severe cracked case. The crack depth was chosen as this represented the most significant enhancement in $g_{FE}$ seen in Section 4.

This appendix is structured as follows. Initially the finite element analyses are considered in Appendix 2.2. The results from the finite element analyses are presented in Appendix 2.3. A short discussion is made in Appendix 2.4 before conclusions are made in Appendix 2.5.

Appendix 2.2  Finite Element Analyses

This section provides a review of the finite element analyses performed to define $g_{FE}$ for cases that include a single pass weld, similar to a repair weld. The geometry adopted is a cylinder with either an external axial or external circumferential extended defect. The approach taken to introduce the residual stress field was of a coupled thermal and mechanical simulation. This approach requires the thermal response to a weld arc to be modelled and the thermal response transferred to a mechanical model to simulate the stress and strain response.

Appendix 2.2.1  Geometry

Appendix 2.2.2  Axial Defect

The cylinder with the full-length external axial crack has a mean radius, $R_m$, of 500 mm, wall thickness, $t$, of 25 mm and a total length of 500 mm. These dimensions ensured that thin cylinder approximations hold for this geometry. To reduce the complexity of the model, symmetry boundary conditions have been utilised where possible. A symmetry condition in the ‘y’ direction has been applied to the base of the model so that only 250 mm of the cylinder length need be modelled. 180° of the cylinder (measured from the crack plane) was modelled and a subsequent symmetry boundary condition in the ‘z’ direction was applied. Modifying the symmetry condition in the ‘z’ direction allowed the
crack faces to open under loading, introducing a constant depth crack. An illustration of this can be seen in Figure A2.1.

![Figure A2.1 – Illustration of axially cracked cylinder with boundary conditions shown](image)

Note that when the model was applied to the weld analyses, the asymmetry of the model required that the complete geometry was modelled and the boundary conditions were applied as fixed boundary conditions, not symmetry boundary conditions. Under these conditions the crack was inserted as a seam into the model as the crack was no longer positioned on a boundary condition. The weld representative heat flux was also positioned offset from the crack so that the crack was positioned in the region of highest tensile stress. This offset was also replicated for the circumferential defect.

The mesh for the cylinder model with an external axial crack has been generated using ABAQUS [A2.2]. The mesh comprises three dimensional quadratic, reduced integration elements (ABAQUS type C3D20R). The main body of the model contains standard, brick-shaped elements with a focused region close to the crack tip, see Figure A2.2. When performing the sequentially coupled thermal-mechanical welding simulation the element type must be altered. During the thermal stage of the analysis diffusive heat transfer
elements (AB AQUS type DC3D8) are used, allowing only the temperature degree of freedom to be active.

Figure A2.2 – Illustration of mesh adopted in axially cracked cylinder

Appendix 2.2.3  **Circumferential Defect**

The model used for the fully circumferential constant depth crack similar to the axially cracked cylinder. The models used took advantage of the axi-symmetry of a circumferentially cracked cylinder. In doing so it took a 2D perspective of the cylinder to include the out of plane hoop stress, instead of a full 3D model.

Where possible, a y-symmetrical boundary condition was positioned at the ligament to take advantage of the geometrical symmetry. In the case where a welding residual stress was included, the whole length of the cylinder was modelled as the weld would cause an asymmetric stress distribution. The boundary condition used fixed the bottom surface of the component in the y-direction. No boundary condition in the x-direction was necessary.
as a result of the axis-symmetric nature of the model. An illustration of the mesh and boundary conditions can be seen in Figure A2.3.

![Figure A2.3 – Illustration of circumferentially cracked cylinder with boundary conditions shown](image)

The meshes for the circumferentially cracked cylinder were generated using ABAQUS [A2.2] and consisted of 8-node bi-quadratic, reduced integration elements (ABAQUS type CAX8R), when mechanical loading was applied and 8-node bi-quadratic diffusive heat transfer elements (ABAQUS type DCAX8) for the inclusion of the welding heat flux.

**Appendix 2.2.4 Material**

The elastic material used was to be representative of a type 304 stainless steel, had a Young’s modulus, $E$, of 196,000 MPa, a Poisson’s ratio, $\nu$, of 0.296 and a coefficient of thermal expansion, $\alpha$, of $1.46 \times 10^{-5} \degree C^{-1}$. The material used had plastic properties defined from a Ramberg Osgood fit to a Type 304 stainless steel shown in Equation A2.1, where the material’s yield stress, $\sigma_y$, and strain hardening component, $n$, were temperature dependent.
\[ \varepsilon = \frac{\sigma}{E} + 0.002 \left( \frac{\sigma}{\sigma_y} \right)^n \]  
Equation A2.1

To be representative of a welding analyses the strain hardening index was fitted to the available yield and ultimate tensile stress (assumed to be at a true strain of 0.1) for the 304 steel, as shown in Figure A2.4 for different temperatures. The curve fit applied therefore extended the strain that the finite element analyses includes beyond the measured ultimate tensile stress and far beyond what would be required within the assessment. This has the added benefit of maintaining the asymptotic stress / strain behaviour as the crack tip position is approached, therefore bringing the model closer to a HRR stress field and, hence, closer to a J controlled stress field. The material was assumed to have an annealing temperature of 1400 °C.

![Figure A2.4 – Example of true stress-strain data used in the assessment](image)

**Appendix 2.2.5 Secondary Loading**

Applying the sequentially coupled thermal-mechanical welding simulation is complex. The method adopted simulates a detailed single sided single pass welding process, similar to a repair weld, and as a result a highly representative residual stress profile is created.
The method used to obtain the weld profile is adapted from many sources, the most similar being [A2.3] which also used 304 steel.

This method contains two stages; thermal and mechanical. Within the thermal stage of the analysis a heat flux, defined by the Goldak double ellipsoid heat source model [A2.4], is passed along the desired weld path. This causes a thermal response and an output of temperatures over time is created. After the heat flux is applied, the model is left to run to allow the material to cool to the surrounding temperature of 20°C. The mechanical stage of the analysis then applies the temperatures obtained from the thermal stage, but includes the mechanical response to the change in temperature. This stage therefore uses the mechanical stress elements to represent the welding pass in terms of stress and strains.

At the end of the mechanical analysis a complex residual stress and strain distribution is present, which strongly represents one that would be created during the welding process. Although the weld centre-line created the highest magnitude residual stress the external surface was compressive. Therefore, to allow the crack to open, the welding position was offset from the crack. The crack opening stress at the crack position can be seen in Figure A2.5.

Figure A2.6 compares the finite element residual stress field for the single pass circumferential weld in a cylinder with the most similar case (4 pass weld) defined with [A2.5], in a guidance manual for welding finite element analyses. It is noted that they would not be expected to be identical, but the similarity of the two provides some additional verification to the method used in the welding simulation.
Figure A2.5 – Predicted residual stress fields at crack location

Figure A2.6 – Comparison of weld profiles from a single pass axial weld in a cylinder (bottom) from the FE compared to a four-pass weld in a cylinder (top) from [A2.5]. In the figure the red regions are tensile and blue compressive.
The apparent difference in the residual stress fields shown in Figure A2.5 can be explained by the geometric constraint offered by the transverse weld direction. As the circumferentially cracked cylinder offers less constraint in the axial direction, a typical “S” shaped residual distribution is found. As the desire of this work was to include residual stresses in fracture, not create an exact representation of the residual stress itself, these effects are not considered further.

**Appendix 2.2.6 Primary Loading**

Both models had different in- and out-of-plane primary loads, applied as uniform tensile pressure, in addition to the residual stresses. For the circumferentially cracked cylinder the in-plane primary load was, of course, the axial stress and the out-of-plane primary stress was the hoop stress. The axially cracked cylinder was the opposite way round to the circumferentially cracked cylinder. These primary loads were applied with different out-of-plane load providing (elastically defined, remotely applied) ratios of 1-0, 1-1 and 1-2 (in plane – out of plane) for all geometries. A range of magnitudes of primary load were applied to each model corresponding to \( L_r = 0.1, 0.2, \ldots \ldots 1.6 \). To obtain the required applied pressures and/or stresses, the elastic-perfectly plastic limit load was from a finite element RIKS analyses, thus including any effects from biaxial loading.

In all cases under combined loading, the primary load was applied after the weld residual stress.

**Appendix 2.2.7 Finite Element Runs**

**Appendix 2.2.8 Calculating Elastic Crack Driving Forces, \( K_1 \)**

Values of the \( J \)-contour integral for an elastic material, \( J_e \), were recorded for each magnitude of primary loading (from \( L_r = 0.1 \) to \( L_r = 1.6 \)) by use of the *Contour Integral term in the ABAQUS input deck when only an elastic material was modelled. Calculations to obtain \( K_1^P \) were then undertaken using Equation A2.2, where \( E' \) for plane
strain is defined in Equation A2.3. It is worth noting that a secondary stress intensity factor, $K_f^S$, could also be obtained but is not necessarily required for this investigation.

$$K_1 = \sqrt{E'J_e} \quad \text{Equation A2.2}$$

$$E' = \frac{E}{1 - \nu^2} \quad \text{Equation A2.3}$$

**Appendix 2.2.9 Calculating Elastic-Plastic Crack Driving Forces, $K_f$**

Values of $J$ were recorded for each magnitude of primary loading (as described for $J_e$ above) by use of the JEDI postprocessor. Calculations to obtain $K_f^P$ were then undertaken using Equation A2.4, where $E'$ for plane strain is defined in Equation A2.3.

$$K_f = \sqrt{E'J} \quad \text{Equation A2.4}$$

This was again used to evaluate $K_f^S$, when only the residual stress was present, and $K_f$, when both primary and secondary loads were present.

In the analyses the crack was inserted “instantaneously” into the residual stress field just aside from the centre of the weld path.

**Appendix 2.3 Finite Element Results**

The results from the analyses can be separated into a number of sections. First the results from the elastic and elastic-plastic stress intensity factors are shown. These are then used to provide comparisons of the R6 and $V_g$ estimates of $K_f$.

**Appendix 2.3.1 Elastic and Elastic-Plastic Stress Intensity Factors**

A comparison of the elastic primary stress intensity factors, $K_f^P$, and the elastic-plastic value, $K_f^P$, can be seen in Figure A2.7 for the axially (top) and circumferentially (bottom)
cracked cylinders when plotted against \( L_r \). The elastic cases are shown as a dashed lines and the elastic-plastic results as the solid lines. The different loading ratios (in-plane : out-of-plane) are shown as blue = 1:0, red = 1:1 and orange = 1:2. It is noted that this colour theme is continued throughout the remainder of this Appendix.

The combined elastic-plastic stress intensity factor, \( K_f \), can be seen in Figure A2.8 compared to \( K_f^p \) for the axially (top) and circumferentially (bottom) cracked cylinders when plotted against \( L_r \). It is noted that the value of \( K_f \) extrapolated to zero primary load of course corresponds to the elastic-plastic secondary stress intensity factor, \( K_f^s \). The values for \( K_f^s \) were 16.0 and 7.4 MPam\(^{0.5}\) for the axial and circumferential cracks, respectively.

**Appendix 2.3.2  Estimates of \( K_f \) from R6 \( V \) and \( V_g \) Approaches**

The results of applying the R6 \( V \) Factor (complex) and the \( V_g \) Approach outlined in Section 4, both when adopting an Option 3 FAD for \( f(L_r) \), can be seen in Figure A2.9 and Figure A2.10 for the axially and circumferentially cracked cylinders, respectively, when plotted against \( L_r \). In both figures, the R6 \( V \) Approach [A2.6] is shown as the top figure and the \( V_g \) Approach as the bottom figure. The results are presented as normalised to the finite element results, meaning that a non-conservative estimate will show below the 1:1 line (hashed black line in figures).
Figure A2.7 – Comparison of $K^p_I$ and $K^p_f$ for the axially (top) and circumferentially (bottom) cracked cylinders with different ratios of applied hoop and axial stress
Figure A2.8 – Comparison of $K_f^p$ and $K_f$ for the axially (top) and circumferentially (bottom) cracked cylinders with different ratios of applied hoop and axial stress
Figure A2.9 – Comparison of the estimate of $K_f$ from the R6 Complex V Approach (top) and the $V_\theta$ Approach (bottom) when normalised to the finite element value for the axially cracked cylinder
Figure A2.10 – Comparison of the estimate of $K_f$ from the R6 Complex $V$ Approach (top) and the $V_g$ Approach (bottom) when normalised to the finite element value for the circumferentially cracked cylinder
Appendix 2.4 Discussion

Appendix 2.4.1 Stress Intensity Factors

The results presented in Figure A2.7 show the comparison of $K_i^p$ and $K_f^p$ for the axial and circumferentially cracked cylinders. It can be seen that the axially cracked cylinder is relatively independent of the combination of the hoop and axial hoop stress, where the cases with hoop to axial load in ratios of 1:0 and 1:1 are exactly the same. The effect of increasing the axial stress to double the hoop (i.e. the 1:2 case) can be seen to reduce the elastic and elastic plastic stress intensity factors. This is a result of the portion of the applied load in plane reducing for an equivalent value of $L_r$. The reason that the cases where the loads are in proportions 1:0 and 1:1 are the same is that the effect of the out of plane stress on the materials limit load is negligible under the von Mises stress criterion used in the limit load analyses, meaning the applied in plane loads are the same at each value of $L_r$.

For the circumferentially cracked cylinder the results are seen to approximately follow this trend for the elastic results. However, under elastic-plastic analyses when the out of plane load is increased the estimate of $K_f^p$ increases. This can be considered an effect of the way that the loads are applied to the geometry. For the axially cracked cylinder the hoop stress is generated from an internal pressure and the out of plane loading from an axial load; this means that the out of plane loading only has the effect of inducing a tensile stress. Conversely, the out of plane hoop stress for the circumferential crack will induce an additional bulging in the cracked section from the induced radial stress that will enhance the crack driving force, especially under more significant levels of plasticity. This is why this effect is not seen for the elastic results.

When comparing the results for $K_f$ the same trends can be seen as for the primary loads described above. The results respond as would be expected when including secondary stresses and the value for $K_f$ starts above $K_f^p$ (at $K_f^p$) but becomes more aligned at higher values of $L_r$. One feature, however, seen for these weld residual stress cases in the
circumferentially cracked cylinder is that between $L_r = 0.8$ and $1.4$ the estimate of $K_f$ increases beyond $K_f$. This suggests that the effect of the weld residual stress at this point is having a negative contribution to the stress intensity factor. This is considered possible as the redistribution of secondary stresses can only occur for cases that become plastic. However, a weld residual stress will include both tensile and compressive regions so that, when an additional primary stress is applied, the compressive regions will not redistribute until well beyond $L_r = 1$. This therefore means that the remaining component of the weld residual stress may have negative effect on $K_f$ under redistribution, depending on the proximity of the compressive region to the crack. The proximity of the crack-tip to the compressive region may explain why only the circumferentially cracked cylinder shows this effect as the axially cracked cylinder has a much deeper tensile region and the crack tip for the circumferentially cracked defect is actually just inside the compressive region of residual stress.

**Appendix 2.4.2 Estimates of $K_f$ from R6 V and $V_g$ Approaches**

The application of the R6 V Factor (complex) and the $V_g$ Approach outlined in Section 4, both when adopting an Option 3 FAD for $f(L_r)$, can be seen in Figure A2.9 and Figure A2.10 for the axially and circumferentially cracked cylinders. The results show that both approaches can be considered to remain conservative; the $V_g$ Approach does appear to become marginally non-conservative at low values of $L_r$ for the circumferentially cracked cylinder but this is still within 3% and not considered significant.

The R6 V Factor approach shown incorporates the Option 3 FAD defined from the finite element analyses and the more accurate complex $V$ approach. This therefore can be considered the most accurate estimate capable by the R6 approach. However, the $V_g$ approach still demonstrates a reduction in conservatism. The maximum conservatism seen in the R6 approach in Figure A2.9 and Figure A2.10 is approximately 40%, whereas the maximum seen for the $V_g$ Approach is approximately 20%.

It is noted that in the application of the $V_g$ approach here, the inputs to the approach and the comparative value of $K_f$ all come from finite element analyses. This is considered a more
influential validation than experiments with a weld residual stress as the uncertainties associated with large-scale tests can not influence the results here.

Appendix 2.5 Conclusions

The short analyses presented within this Appendix have considered a representative welding profile for a cylinder subjected to a single pass repair weld for defects both axially and circumferentially. The work has shown that the $V_g$ Approach can be successfully applied to the non-proportional stress field resulting from the welding simulation for cylinders. The predictions of $K_f$ from the $V_g$ Approach compared to the R6 Complex $V$ Factor Approach allow for a reduction in the level of maximum conservatism seen by up to 20%.

It was also noted that the non-linear and self-balancing effect of the weld residual stress can lead to cases where the contribution of the residual stress to fracture become negative as the tensile stress is redistributed first.

Appendix 2.6 References


Appendix 3. Copies of Published Journal Papers

This appendix provides copies of two published journal papers that have been written from the work presented within Sections 4, 5 and some of the results from Section 6 of the main body of the thesis. These two papers have both been submitted to the International Journal of Pressure Vessels and Piping as consecutive papers, published in January 2013.

The first paper, titled “Predictions of elastic-plastic crack driving force and redistribution under combined primary and secondary stresses – Part 1: Analytical investigation”, details the development of, and analytical comparisons using, the $V_g$ approach. The second paper, titled “Predictions of elastic-plastic crack driving force and redistribution under combined primary and secondary stresses – Part 2: Experimental application”, records the experimental work detailed in Section 5 of the thesis and the assessment of the test results when adopting the $V_g$ approach.

The following pages provide copies of the Part 1 paper followed by the Part 2 paper. Note that the pages are numbered as published in the journal and, as such, the remaining page numbers corresponding to this thesis are not shown.
Predictions of elastic–plastic crack driving force and redistribution under combined primary and secondary stresses – Part 1: Analytical investigation

P.M. James a,*, D.G. Hooton a, C.J. Madew a, A.H. Sherry b

a AMEC Technical Services, Walton House, Birchwood Park, Risley, Warrington, Cheshire WA3 6GA, UK
b Dalton Nuclear Institute, The University of Manchester, Sackville Street, Manchester M13 9PL, UK

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ABSTRACT

Many engineering components, particularly those containing weldments, may enter service with or develop small crack-like defects. In non-stress-relieved welds, or under thermal stresses, such defects will experience both primary and secondary stresses combined. This paper describes a new function, $g$, that has been introduced to detail the plasticity interaction under combined loading within structural integrity assessment methods. This function has been based on the Option 2 failure assessment curve in the R6 approach to structural integrity assessment and has been determined from finite element analyses of an externally cracked cylinder under combined primary and secondary loadings. The $g$ function method has been used in the assessment of a series of other cracked geometries subject to loadings which include secondary stresses arising from both thermal gradients and weld residual stresses. The results presented show that the $g$ approach can successfully be extended to the assessment of these other geometries and loading combinations and provides reasonably accurate and less conservative assessments than existing methods presented in R6. A second paper, “Part 2: Experimental Application”, compares the application of the $g$ approach with data from experiments performed over a range of plasticity development.

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1. Introduction

Many engineering components, particularly those containing weldments, may enter service with small crack-like defects and experience both primary and secondary stresses combined. It is therefore integral to the safe operation of these components that a measure of the integrity of these components is provided. In this paper, as in the R6 defect assessment procedure [1], secondary stresses are defined as those which result from an internal strain mismatch within the material and generally arise from thermal expansion, weld residual stress or a component fit-up stress. Tensile secondary stresses will enhance the localised stress field but do not contribute to plastic collapse. Conversely, primary stresses arise from an applied load such as an end load, self weight or internal pressure, and do contribute to plastic collapse.

Within R6 [1] a measure of the criticality of a crack-like defect is provided by comparing an estimate of the crack driving force (CDF), or $K$, due to both primary and secondary stresses to the materials resistance to fracture, $K_{\text{mat}}$. To account for uncertainty and variation in both the loads and the properties of the material assessed it is common to use conservative (upper bound) estimates of CDF. However, further conservatism is also provided within the approach to combine primary and secondary stresses in the estimate of the total CDF. This additional conservatism can lead to the repair or replacement of components with crack-like defects which may in fact be safe to continue operation; causing additional downtime and financial implications to the operator. Within the assessment of defects under combined primary and secondary stresses it is generally acknowledged that existing procedures, such as those in R6 [1], for detailing the combined CDF are in some cases overly conservative, particularly for cases of high secondary stress and relatively low primary stress. The current R6 methods also rely on the use of ‘look-up’ tables that provide the parameters which describe the redistribution of secondary stresses, making the approach complex to implement. These points have led to an extensive research programme within the UK over recent years to try and reduce this conservatism and improve the usability, whilst maintaining a well-founded structural integrity assessment capability.

A range of procedures exist for the treatment of secondary stresses such as BS7910 [2], RSE-M [3] and API579 [4] which follow a similar underlying approaches to R6 but with some variations in
how the interaction of primary and secondary stresses is defined. However, here focus is maintained on the approaches in R6 [1]. Within R6 [1], two options for predicting the CDF exist within the presence of combined primary and secondary stresses. These are the \( \rho \) and \( V \) methods, both of which have further options for simplified and complex approaches. The complex approaches of each method are equivalent. Both \( \rho \) and \( V \) are functions which allow for an enhancement and subsequent reduction of the secondary stress contribution to the CDF under increased primary load.

Recently Hooton et al. [5] proposed an alternative method to that outlined within R6, which does not require \( \rho \) or \( V \) to be calculated and hence does not require the use of ‘look-up’ tables. The alternative method is based on the treatment of combined loading in the Time-Dependent Failure Assessment Diagram (TDFAD) approach of the R5 [6] assessment procedure for the high temperature response of structures.

This paper details the further development of this method and the estimation of a new interaction parameter, \( g \), which reduces the conservatism of the current \( \rho \) and \( V \) methods. This development is considered over two papers. In this paper, Part 1, a definition of \( g \) is derived from, and compared to, the results from detailed non-linear finite element analyses. Part 2 [7] compares \( g \) with data obtained from a new set of experiments performed, which are also reported in the paper, to validate the new approach over a range of plasticity levels.

This paper is structured as follows. Section 2 provides a review of existing methods and observations relating to estimates of the combined CDF. The finite element analyses performed to estimate \( g \) are provided in Section 3. Results from these analyses are then assessed in Section 4 to derive a descriptive form for \( g \). This is subsequently used in Section 5 to assess other cracked geometries under a thermally induced through-wall equi-biaxial bending stress that are also analysed using further finite element analyses. Section 6 then considers the inclusion of a weld residual stress profile to the same cracked geometries considered in Section 5. The main features of the results are discussed in Section 7 and conclusions summarised in Section 8.

The second paper, Part 2 [7], details a new experimental programme to provide estimates of \( g \) over a range of primary stresses by altering the test temperature, thus altering the amount of plasticity developed in the ligament at failure.

2. Review of existing methodology

This section provides a review of previous analytical approximations made to quantify the interaction of primary and secondary stresses in the context of fracture assessment. In all cases, the aim of the approach is to provide a combined primary and secondary CDF for elastic–plastic conditions, \( K_f \), which can be compared to the material toughness properties to assess proximity to failure. However, it is noted that under single parameter fracture mechanics, i.e. where crack-tip constraint effects are not considered, the value of \( K_f \) (or equivalently the energy release per unit length crack growth, \( J \)) is assumed to fully describe the stress field in the vicinity of a crack [8,9]. As such, any change in \( K_f \) should also describe the redistribution of the stress field ahead of the crack.

The current R6 procedure [1] for treating combined primary and secondary stress was derived from the reference stress approach developed by Ainsworth [10] and extended by Hooton and Budden [11]. The generalised procedure originally outlined by Ainsworth aimed to treat the total combined stress in the same manner as that used for primary stress acting alone. This was achieved by defining a reference stress, \( \sigma_{ref} \), that generates the same total crack driving force, when used as a primary stress, as the actual combination of primary and secondary stresses. Thus:

\[
K_f = \frac{K_f}{f(\sigma_{ref}/\sigma_y)} = \frac{\sigma_{ref} \sqrt{\pi a}}{f(\sigma_{ref}/\sigma_y)}
\]

where \( K_f \) is the linear elastic stress intensity factor (SIF) under combined loading, \( f(\sigma_{ref}/\sigma_y) \) is the R6 failure assessment curve, \( \sigma \) is a reference length that depends on both the stress field and the geometry and \( \sigma_y \) is the material yield stress. In these analyses \( \sigma_y \) is taken to correspond to the 0.2% yield stress. Under primary loads alone the reference stress is defined by the primary reference stress, \( \sigma_{ref}^{P} \), itself defined by \( P \), the applied load, and \( \sigma_y \), the perfectly plastic limit load for the geometry and material, which is captured in R6 by the \( L_r \) term:

\[
\sigma_{ref}^{P} = \frac{P}{P_L} = L_r
\]

The contribution from the secondary stress to the CDF is quantified with reference to the linear elastic SIF which is related to the secondary reference stress, \( \sigma_{ref}^{s} \), as follows:

\[
K_f^{s} = \sigma_{ref}^{s} \sqrt{\pi a}
\]

where \( K_f^{s} \) is the linear elastic SIF for the secondary stress acting alone. It is widely assumed that \( \pi \) is the same for both primary and secondary stresses [16,10,11], so that \( (K_f^{s}/\sigma_{ref}^{s})^2 = \pi a \) can be determined from handbook solutions; where \( K_f^{s} \) is the linear elastic SIF under primary stresses alone. The secondary reference stress \( \sigma_{ref}^{s} \) is defined as the reference stress when only secondary stresses are present and, likewise, the primary reference stress, \( \sigma_{ref}^{P} \), is the reference stress when only primary loads are present. The derived relationship between the combined reference stress and the primary reference stress normalised by the yield stress can be seen in Fig. 1 [10] for different levels of secondary reference stress.

Ainsworth [10] then showed that incorporating a shifting factor, \( \rho \), into the definition of \( K_f \) results in the following equation:

\[
K_f = \left( \frac{K_f^{P} + K_f^{s}}{f(\sigma_{ref}^{P}/\sigma_y)} \right) - \rho
\]

A simplified approach to estimating \( \rho \) was provided by Ainsworth [10], which was found to be overly conservative for small values of primary stress [11]. A further methodology was therefore
developed to reduce this conservatism at low primary stresses and to allow for any redistribution of the secondary stress, by Hooton and Budden [11]. The redistribution of secondary stresses results from the internal strain mis-match being reduced by plastic flow within the material as it attempts to reach its lowest energy state.

When adopting this revised methodology the resulting value for \( \rho \) was found to be:

\[
\rho = \psi + \varphi \left( 1 - \frac{K_1^S}{K_j^S} \right)
\]  

(5)

where \( \psi \) is provided by Ainsworth [10] and shown in Equation (6), \( \varphi \) is defined in Ref. [11] and shown in Equation (7), and \( K_j^S \) is the elastic–plastic SIF under secondary loads alone.

\[
\psi = f\left(\frac{\sigma_{\text{ref}}^p/\sigma_y}{\sigma_{\text{ref}}}\right) - \frac{f\left(\frac{\sigma_{\text{ref}}^p/\sigma_y}{\sigma_{\text{ref}}}\right)}{f\left(\frac{\sigma_{\text{ref}}^S/\sigma_y}{\sigma_{\text{ref}}^S}\right)}\left[\frac{\sigma_{\text{ref}}^p}{\sigma_{\text{ref}}} + \frac{\sigma_{\text{ref}}^S}{f\left(\frac{\sigma_{\text{ref}}^S/\sigma_y}{\sigma_{\text{ref}}^S}\right)}\right]
\]  

(6)

\[
\varphi = \frac{f\left(\frac{\sigma_{\text{ref}}^S/\sigma_y}{\sigma_{\text{ref}}^S}\right)}{f\left(\frac{\sigma_{\text{ref}}^S/\sigma_y}{\sigma_{\text{ref}}^S}\right)}
\]  

(7)

The variation of \( \varphi \) and \( \psi \) with normalised primary stress, \( \sigma_{\text{ref}}^p/\sigma_y = L_r \), is illustrated for increasing \( \sigma_{\text{ref}}^p/\sigma_y \) in Fig. 2 calculated from Equations (6) and (7). The influence of \( \varphi \) is to allow for the influence of \( \rho \) for higher values of \( L_r \) when crack-tip plasticity reduces the influence of \( \sigma_{\text{ref}}^p \) on the CDF. This formulation of \( \rho \) is currently provided within R6 with “look-up” tables for \( \varphi \) and \( \psi \), dependent on values of \( K_j^S, K_1^S \) and \( L_r \).

The \( V \) factor approach [12] was derived to simplify the assessment methodology, but provides equivalent values of \( K_j \) to those derived from Equation (5). The terms \( \varphi \) and \( \psi \) can be related to \( V \), as part of an \( \xi \) term, which is also provided as “look-up” tables in R6 [11]. This relationship is shown in Equation (8) below.

\[
K_j = \frac{K_j^p + VK_j^S}{f\left(\frac{\sigma_{\text{ref}}^p/\sigma_y}{\sigma_{\text{ref}}}\right)} = \frac{K_j^p + \xi K_j^S}{f\left(\frac{\sigma_{\text{ref}}^p/\sigma_y}{\sigma_{\text{ref}}^p}\right)} = \frac{K_j^p + (1 + \varphi/\psi)K_j^S}{f\left(\frac{\sigma_{\text{ref}}^p/\sigma_y}{\sigma_{\text{ref}}^p}\right)}
\]  

(8)

Hooton et al. [5] detail the derivation of an alternative method for estimating the combined reference stress. This alternative method is less conservative than the R6 methods described above, and less complex to use, since look up tables for \( \varphi \) and \( \psi \) are not required. This alternate definition of \( K_j \) is given by:

\[
K_j = \left\{ \left( \frac{K_j^p}{f\left(\frac{\sigma_{\text{ref}}^p/\sigma_y}{\sigma_{\text{ref}}}\right)} \right)^2 + (K_j^S)^2 + 2K_j^pK_j^S \right\}^{1/2}
\]  

(9)

It was recently shown by James et al. [13] that this expression can be further reduced to provide a conservative estimate as:

\[
K_j = \frac{K_j^p}{f\left(\frac{\sigma_{\text{ref}}^p/\sigma_y}{\sigma_{\text{ref}}}\right)} + K_j^S
\]  

(10)

By assessing the validity of the assumption that the \( \pi \) term in Equations (1) and (3) can be taken from primary loads alone, James et al. [13] also showed that a weighted average of the primary and secondary contributions to \( \pi \) can result in the similar description of \( K_j \) in Equation (11) below. In this form the \( g \) allows for redistribution effects under combined primary and secondary loading.

\[
K_j = \frac{K_j^p}{f\left(\frac{\sigma_{\text{ref}}^p/\sigma_y}{\sigma_{\text{ref}}}\right)} + K_j^S g
\]  

(11)

Song et al. [14] have explored the variation of the \( V \) term for a range of primary and secondary stress combinations using detailed finite element analyses. These analyses covered a range of secondary loading types for a fully circumferential external crack in a cylinder. The results showed that the existing approaches within R6 are conservative and that the \( V \) factor can be better approximated by an R6 Option 2 FAD than by the existing look-up tables. The results of Song et al. [14] can be related to \( g \) by comparison of Equations (8) and (11), i.e. \( g = V/(fL) \).

3. Finite element analyses

This section describes a series of finite element analyses of a fully circumferential, externally cracked cylinder, used to provide an estimate of \( g \).

3.1. Geometry, mesh and material properties

One geometry with two cracked depths was modelled using ABAQUS, version 6.8 [12], for use in estimating \( g \). The geometry was a cylinder with a mean radius, \( r_m \), of 5 m, an overall model height of...
10 m and a wall thickness, \( t \), of 0.5 m. This provides a cylinder with an \( r_m/t \) of 10, which allows the use of thin shell approximations when validating the mesh and in defining the applied loads. Two crack depths of \( a/t = 0.1 \) and 0.4 were used coupled with two combinations of pressure loading to provide four different cases (see Table 1). The geometry was chosen solely to represent the normalised \( r_m/t \) ratio and does not represent a specific component, although it should be possible to apply the results to any cylinder which follows the thin shell approximation.

Reduced integration, eight nodded, axi-symmetric elements (ABAQUS type CAX8R) were used to create the finite element mesh. A focused crack-tip mesh was adopted with positions of the mid side nodes moved to a quarter element distance under elastic conditions. The degenerate behaviour of the crack-tip nodes was also altered to remain fixed, to ensure a sharp crack, under elastic conditions and free to move, to allow crack-tip blunting, under elastic–plastic conditions, to reflect the guidance in references [15,16]. The only geometric boundary condition required in the axi-symmetric model was a symmetry condition applied to the uncracked ligament.

The material had elastic properties based on those for type 316L stainless steel with plastic properties represented by an idealised Ramberg–Osgood form shown below with Young’s Modulus, \( E \), taken as 196,000 MPa, Poisson’s ratio, \( \nu \), as 0.296, \( \sigma_y \) as 290 MPa and \( n \) is the strain hardening exponent. Four different values of 5, 7.5, 10 and 12.5 for \( n \) were used.

\[
e = \frac{\sigma}{E} + 0.002 \left( \frac{\sigma}{\sigma_y} \right)^n
\]

where \( e \) is the von Mises equivalent strain and \( \sigma \) is the von Mises equivalent stress. As loading was proportional with no unloading steps included, a simple isotropic hardening law was used with small strain approximations.

### 3.2. Loading

The primary load was simulated via a pressure to the element faces located on the end surface of the cylinder and, where a hoop stress was to be induced, to the elements faces on the internal surface. To ensure the pressure on the end surface correctly transferred as an axial stress, nodes located at the top surface were constrained to maintain equal axial displacements with no rotation. This is analogous to a remote load applied to a long cylinder.

To validate the model the resultant axial, \( \sigma_a \), and hoop, \( \sigma_h \), stresses were compared to standard cylinder theory. The derived stresses were within 0.1% of the theoretical values for all cases when measured away from the influence of the crack.

The magnitudes of the applied primary loads were defined by the R6 \( L_r \) parameter. The loads were applied in 0.1 increments of \( L_r \) up to a value of 1.6. In the analyses the pressures required for each values of \( L_r \) were obtained from \( L_r = P/P_i \), where \( P \) is the applied load and \( P_i \) is the elastic–perfectly plastic limit load for the geometry and material.

For each loading condition a value of \( P_i \) was obtained by elastic–perfectly plastic finite element analyses. A potential range of values of \( P_i \) were found depending on the local limit load, which is where the plastic zone extends across the uncracked ligament, and the global limit load, which is defined by the load at which the structure can not take further stress the finite element analyses fails. The value for \( P_i \) used provided Option 3 FADs, from different Ramberg–Osgood strain hardening coefficients. The value of limit load chosen was to force agreement of the different curves at \( L_r = 1 \), as also applied by Song [14]. This was to ensure the primary reference stress was equal to the yield stress, i.e. \( \sigma_{ref} = \sigma_y \), at \( L_r = 1 \), which is a feature of reference stress approaches [17]. The value of \( P_i \) used was always between the local and global solution, which were also seen to match Handbook limit load solutions [1] when assuming Mises stress criterion.

A secondary stress field was applied to the model that corresponded to an elastic bending stress with \( \sigma_{ref}/\sigma_y \) values ranging from \( \sigma_{ref}/\sigma_y = 0, 0.1, 0.2, \ldots, 1.6 \). As secondary loads do not contribute to plastic collapse it is not possible to determine the secondary reference stress from finite element analyses. Instead, Handbook solutions defined for primary loads were used to determine the secondary reference stress. In each case the secondary stress was applied via thermal gradient ranging from \( +T \) on the inner surface of the cylinder model to \( -T \) on the outer surface. These values of \( T \) were defined within a user-subroutine to equate to the through thickness values of \( \sigma_{ref}/\sigma_y \) required. In all cases the crack was present from the start of the analyses with the secondary stress acting to open the crack.

### 3.3. Data interpretation

Within the estimates of \( g \) it was assumed that the value of \( J \), estimated by the \( J \)-contour integral in ABAQUS, and \( K_J \) are related by Equation (13).

\[
K_J = \sqrt{\frac{EJ}{1 - \nu^2}}
\]

To estimate values of \( K_J^p \) and \( K_J^p \) finite element analyses were performed with only the primary load applied under elastic and elastic–plastic conditions, respectively. Likewise estimates of \( K_J^p \) and \( K_J^p \) were obtained from elastic and elastic–plastic finite element analyses, respectively, with only the secondary stress present. Estimates of \( K_J^p \) were obtained with all loads applied in elastic–plastic finite element analyses.

To verify the mesh and modelling approach, derived values of \( K_J^p \) were shown to be within 1.5% of handbook solutions [1] for a given load and the material and geometry specific Option 3 FADs, constructed from \( K_J^p \) and \( K_J^p \), were qualitatively compared to the generalised Option 1 and material specific Option 2 FADs of R6 [1].

### 4. Calculation of \( g \)

To calculate \( g \), Equation (11) was rearranged as Equation (14):

\[
\frac{K_J - K_J^p}{K_J^p} = g_{FE}
\]

where \( g_{FE} \) is the value of \( g \) derived from the finite element analyses. When calculating the value of \( g \) for a given load all inputs \( (K_J^p, K_J^p \) and \( K_J^p) \) were obtained from the elastic and elastic–plastic finite element analyses. It was found that there was a maximum deviation of either 15%, or 0.1 in absolute terms, on \( g_{FE} \) when comparing different magnitudes of the secondary stress. Therefore, to reduce the number of solutions whilst maintaining conservatism in the approach the upper bound values were used.
to define $g_{FE}$ for each strain hardening and loading condition considered. It was also noted that $K_f^2$ remained within 3% of $K_f^2$ for the cases considered.

A short sensitivity analyses of the effect of different yield stresses was also considered by doubling the yield stress for select cases. This change was shown to have almost no effect on the estimate of $g$ as a function of $L_e$. This helps to demonstrate that the reference stress approach, such as those of R6 [1], is suitable for developing $g$, since reference stress approaches should be independent of the yield stress.

The results for $g_{FE}$ are illustrated in Fig. 3 as solid lines. The strain hardening exponent for each curve is also shown. The open circles, and the $A$ and $B$ parameters are explained below. Within these plots it is noted that, in general, the following trends are maintained:

1) All curves tend to unity at $L_e = 0$.
2) All curves have a skeletal point (i.e. a point where all strain hardenings have approximately the same value) at a specific $L_e$.
3) All curves appear to reduce quickly at large values of $L_e$.
4) The lower the strain hardening exponent (larger $n$), the higher the value of $g$ prior to the skeletal point; and the lower the value of $g$ beyond the skeletal point, i.e. at high $L_e$.

Some of these observations are also characteristics of the R6 failure assessment curve, $f(\sigma_{ref}^P/\sigma_y)$. Features of the plots which are not shared with the failure assessment curve are: (a) the value of $g$ rises above unity, whereas the value of $f(\sigma_{ref}^P/\sigma_y)$ reduces below unity with increasing $L_e$ and (b) the skeletal point for $g$ is at $L_e$ of approximately 1.2, whilst that for $f(\sigma_{ref}^P/\sigma_y)$ is at $L_e = 1$ [17]. However, the qualitative similarity of the form of the $g$ with $L_e$ curves to a failure assessment curve suggest that an equation of the form used to define the Option 2 FAD may be used to represent $g$ as a function of $L_e$. The R6 Option 2 FAD is given by:

$$f\left(\frac{\sigma_{ref}^P}{\sigma_y}\right) = \frac{E_{\text{mod}}}{E_{\text{ref}}} + \frac{A\left(\frac{\sigma_{mod}^P}{\sigma_y}\right)^2}{E_{\text{mod}}/\sigma_y} - 1/2$$

(15)

After fitting an equation of this form to the results, the following definition based upon the Option 2 FAD was obtained.

$$g = \frac{E_{\text{mod}}}{\sigma_y} + \frac{A\left(\frac{\sigma_{mod}^P}{\sigma_y}\right)^2}{E_{\text{mod}}/\sigma_y} - 1/2$$

(16)

where $\sigma_{mod}^P$ is a shifted, or modified, reference stress and is defined by $\sigma_{ref}^P = B\sigma_{mod}^P$. The $A$ and $B$ terms are additional constants. The value for $B$ is linked to the position of the skeletal point and the $A$ term provides a measure of the enhancement above unity seen under small scale yielding. Results of this curve-fit can be seen as the open circles within Fig. 3, with the values of $A$ and $B$ included. From this fit, over 90% of the points were within 10% of the finite element results, with approximately 40% of the remaining 10% showing a potential non-conservatism. The values of $A$ and $B$ can be conservatively defined by Equations (17) and (18).

$$B = 1.25$$

(17)

$$A = \frac{\sigma_{\text{in-plane}}}{1.25 \times \sigma_{\text{mises}}}$$

(18)

where $\sigma_{\text{in-plane}}$ is the remotely applied primary stress acting to open the crack, i.e. axial stress, and $\sigma_{\text{mises}}$ is the von Mises equivalent stress for all remotely applied primary stresses, i.e. the equivalent stress defined from the hoop and axial stress.

This added conservatism removes all the points showing a potential non-conservatism of more than 10% and means that less than 1% of all points shows a potential non-conservatism of more

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**Fig. 3.** Plots showing results for $g_{FE}$ (solid lines) compared to the Equation (16) (open circles) for (a) Case 1, (b) Case 2, (c) Case 3 and (d) Case 4.
than 5%. However, it does mean that approximately 30% of cases show a conservatism of more than 10%, with a quarter of these showing a conservatism of more than 100% in terms of $g$, i.e. the value of $g_{\text{ref}}$ is half that predicted using Equations (16)–(18). It is also noted that the rapid variation of $J$ with load means that it is difficult to obtain a good fit but these differences in $g$ may correspond to relatively small differences in load at fracture.

It is this conservative interpretation provided by Equations (16)–(18) which is adopted as $g$ in this paper.

The remainder of the paper provides further results from finite element analyses which have been used to explore the use of $g$ for different geometries and types of secondary stress.

5. Application of $g()$ to thermally induced biaxial through-wall bending stresses

5.1. Finite element analyses

5.1.1. Geometry, material properties and mesh

Three geometries were investigated: (a) a cylinder with a full length external axial crack, (b) a cylinder with an external fully circumferential crack and (c) a centre cracked plate with a finite depth semi-elliptical crack. In each case two crack depths, $a$, corresponding to values of $a/t = 0.2$ and 0.4 were used. Both of the cylinder geometries had a mean radius, $r_{\text{m}}$, of 500 mm, a total length of 500 mm and a wall thickness, $t$, of 25 mm. The cylinder geometry was chosen to provide a thin shell cylinder which simplifies the analyses and allows comparison to large range of components. The centre cracked plate was 1 m square and 25 mm thick. A semi-elliptical crack in the plate was modelled, i.e. $a/c = 1$, where $c$ is the semi crack length. The large plate was chosen as: (a) it provides another simplified geometry which many components with defects can be approximated to and (b) the reference stress approach has historically shown the worst agreement for geometries with a small crack in a uniform, highly tensile stress field [18].

Reduced integration, twenty noded, three-dimensional elements (ABAQUS type C3D20R) were used to create the mesh for the axially cracked cylinder and plate geometries. Reduced integration, eight noded, axi-symmetric elements (ABAQUS type CAX8R) were used for the circumferentially cracked cylinder as the geometry allowed for further symmetry. Again, as described in Section 3, a focused crack-tip mesh was adopted with the positions of the mid side node and the degenerate behaviour of the crack-tip nodes modified under elastic and elastic–plastic conditions. An illustration of the mesh adopted for the axially cracked cylinder is shown in Fig. 4. Symmetry conditions were applied for each model: (a) the axially cracked cylinder was modelled as a quarter model, defined by a half length of 250 mm and a 180° section from the axial crack, (b) the circumferential crack was modelled as a half model, defined by a half length of 250 mm and, (c) the centre cracked plate was modelled as a quarter model of 500 mm square. An additional nodal constraint was applied to the centre of the plate geometry to prevent whole-body motion in the vertical axis. Further constraints were applied to all models on their top surface to ensure equal vertical displacement and prevent rotation when a through-wall thermal gradient was applied. For all cases the crack was present from the start and created by only applying the symmetry condition to the uncracked ligament. Model verification was performed by comparing the elastic stress field resulting from applied loads and elastic SIFs with handbook solutions.

To remain consistent with the finite element analyses outlined in Section 3 the same elastic material properties, relating to a 316L stainless steel, were used. It is however noted that the plastic strains were not estimated by a Ramberg–Osgood relation but more representative 316L data [19] were used. These data were used to provide a more meaningful comparison to real components. The 316L plastic strains show strong similarity to a Ramberg–Osgood curve with a strain hardening of 10 at lower levels of yield but show a higher hardening beyond 1% plastic strain.

The values for $K_f$ were again taken from $J$ when adopting Equation (13) to provide the conversion. For the semi-circular crack in the extended plate only the results relating to the deepest point were used.

5.1.2. Loading

As with the calculation of $g$, a range of primary loads were applied corresponding to a range of $L_r = 0, 0.1, 0.2, \ldots, 1.6$. For the three geometries, the loading was applied so that a crack opening stress, $\sigma_y$, and stress acting normal to the crack opening stress, $\sigma_n$, (i.e. hoop stress in a circumferentially cracked cylinder) had the ratio of 1:0, 1:1 or 1:2. This was to consider the effect of stresses acting to influence the evolution of plastic strain, and hence the redistribution of secondary stresses, which do not act to open the crack.

The thermally induced secondary bending stress was applied in the same manner as that described in Section 3 to calculate $g$. The temperature gradient from $-T$ to $+T$, from the outer to inner surfaces, was applied to provide an elastic through-wall bending stress ranging from $+\sigma_y$ to $-\sigma_y$.

5.2. Application of $g$ and R6 method to finite element results

The finite element analyses were used to obtain the estimates of the primary and secondary SIFs under both elastic and elastic–plastic conditions. These values were then used as the input variables to the simplified methods being assessed and compared to the elastic–plastic finite element estimate of the combined SIF. An example of the results from applying the thermally induced, equibiaxial, bending stress field can be seen in Fig. 5 for the centre cracked plate and in Fig. 6 for the externally circumferentially cracked cylinder. Fig. 5 compares the estimate of $K_f$ defined using the R6 approach (or $V$) factor when adopting the Option 1 failure assessment curve as a $J$-estimation scheme (Equation (4)), $K^{R6\_01}$, the Option 3 failure assessment curve, $K^{R6\_03}$, alongside that defined from Equation (11) when assuming $g = 1$, $K^{g=1}$, and when defined using $g$ from Equation (16), $K^{g}$. All estimates of $K_f$ have been normalised by the value of $K_f$ obtained by the finite element analyses, $K^F$. As the trends shown in Fig. 5 are indicative of all results, Fig. 6 only compares $K^{R6\_03}$ and $K^F$. 

![Fig. 4. Illustration of mesh design used for the axially cracked cylinder with the deeper crack.](image-url)
All results are shown normalised by $K_{J}^{FE}$; therefore a non-conservative result (i.e. when $K_J$ is underestimated) is less than unity. This is indicated by the shaded regions in all figures. It is also considered that a potential error margin on the finite element analyses exists, given accumulation of error when manipulating the results, and is estimated at 10%. This estimate of potential error in the results is based on a potential error of 5% in reference stress estimates and 5–15% in finite element analyses as outlined by Miller [18]. It is also argued in Ref. [18] that these levels of error would be insignificant when considering those from the SIF solution used or material properties, for example. It is also argued that these differences would also be observed with less than 5% variation on load or

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**Fig. 5.** Estimates of $K_{J}/K_{J}^{FE}$ for different ratios of in-plane and normal stresses for a centre cracked plate containing a semi-circular crack of $a/t = 0.2$ and 0.4 with a through-wall thermally induced biaxial secondary bending stress when assessed using (a) the R6 method with an Option 1 FAD, (b) the R6 method with an Option 3 FAD, (c) the $g$ function approach with $g = 1$ and (d) the $g$ function approach with $g$ from Equation (16).

**Fig. 6.** Estimates of $K_{J}/K_{J}^{FE}$ for different ratios of in-plane and normal stresses for the circumferentially cracked cylinder containing a semi-circular crack of $a/t = 0.2$ and 0.4 with a through-wall thermally induced biaxial secondary bending stress when assessed using (a) the R6 method with an Option 3 FAD and (b) the $g$ function approach with $g$ from Equation (16).
limit load. A strongly shaded region is also included in the figures to indicate where a result is non-conservative by more than 10%. All results are presented within a region of ±25% deviation about $K_{FE}^0$. A result outside this ±25% region is either very conservative or non-conservative and the approach used to derive $K_f$ may not be suitably accurate for industrial application.

The results illustrated in Figs. 5 and 6 show similar trends. For all results the R6 $K_{FE}^{R6}$ (1) results are the most conservative. For example, in Fig. 5 the shallow cracked $K_{a1}/K_{a2} = 1.2$ results for $K_{FE}^{R6}$ (1) start at just over 5% conservative and increase to become more than 25% conservative between $L_r = 0.6$ and 1.5; the level of conservatism in this case was actually just over 50% at about $L_r = 1$. For the same case the $K_{b1}/K_{b2} = 1:0$ results are more accurate, especially for the deeper crack, where the results are almost at low values of $L_r$ and increase to just under 20% conservatism at $L_r = 1.1$. The level of conservatism using $K_{FE}^{R6}$ (3), $K_{FE}^{R6}$ (1) and $K_f$ methods is reduced, with $K_f$ yielding the most accurate estimate. This can be seen by comparing the shallow cracked $K_{a1}/K_{a2} = 1:2$ results for the $K_{FE}^{R6}$ (3) case which is always more than 15% conservative and, between $L_r = 0.9$ and 1.4, is more than 25% conservative, with the $K_f$ result which is generally less than 5% conservative and rises to approximately 20% at $L_r = 1.1$. It is perhaps not surprising that the use of $K_f$ yields the most accurate results since a through-wall secondary bending stress was used to derive $g$.

Based upon the centre cracked plate and the externally, fully circumferentially cracked cylinder results, the use of $K_f$ is observed to be more accurate than either the $K_{FE}^{R6}$ (1) or the $K_{FE}^{R6}$ (3) methods.

The results for the axially cracked cylinder are illustrated in Fig. 7 and indicate a similar trend in $K_f$ accuracy as observed for the plate and circumferentially cracked cylinder in Figs. 5 and 6, respectively. For this geometry, the g results show up to 30% non-conservatism for the deepest crack. However, the results exhibit this non-conservatism for all methods considered, including those from R6.

In reviewing the cause of the non-conservatism, it is noted that as the out-of-plane stress, $\sigma_2$ (axial stress), is increased, and as the crack length is reduced, the results become more conservative. It is also noted that this non-conservatism does not occur for the centre cracked plate and circumferentially cracked cylinder. Given these trends it is considered that the most likely reason for the non-conservatism is a constraint effect, reducing the redistribution of the secondary stress, especially given that the extended crack has the lowest natural constraint of the geometries used in this analysis. The additional constraint provided by an increased out-of-plane stress and a shallower crack depth reduces the level of non-conservatism.

6. Application to components containing weld residual stresses

6.1. Finite element analyses

The same three geometries as those described in Section 5 were used as the basis for considering the $g$ approach applied to a crack within a weld residual stress field. In this comparison only the deeper crack depth has been considered, which was chosen as it exhibited the least conservative results in Section 5.

The residual stress field was applied within the finite element model using a welding simulation. Applying a sequentially coupled thermal–mechanical welding simulation is a complex, but also reasonably realistic, means of establishing a residual stress within a finite element model. The method adopted here simulated a single pass welding process, similar to a repair weld.

The welding simulation contained a thermal and mechanical stage, and closely resembled the procedure defined in Ref. [20]. Within the thermal stage of the analysis a heat flux, defined by the Goldak double ellipsoid heat source model [21], was passed along the desired weld path. This caused a thermal response from the underlying elements and an output of temperature distribution as a function of time created. After the heat flux was applied, cooling of the model to the initial temperature of 20 °C was simulated. The mechanical stage of the analysis then applied the temperature distribution versus time obtained from the thermal stage to the model which included the mechanical response of the material to the change in temperature.

Within the thermal response to the welding simulation, diffuse heat transfer elements of ABAQUS type DC3D8 for the axially cracked cylinder and plate and DCAX8 for the circumferentially cracked cylinder were applied to the same mesh configuration as that described in Section 5. Within the mechanical response to the welding simulation, and within the thermally induced bending stress cases, the models described in Section 5 were used. However, as the weld was offset from the crack, the model was no longer symmetrical about the crack and it was necessary to reflect the finite element model about the uncracked ligament, remove the now redundant symmetry conditions and reapply single node constraints to prevent whole-body motion in that axis.

When modelling the sequential thermal–mechanical welding simulation a more extensive set of materials properties were required over different temperatures. These properties were constructed by fitting a Ramberg–Osgood function through limited data for a Type 304 austenitic stainless steel [16]. The annealing temperature and Poisson’s ratio were assumed to remain constant.

![Fig. 7. Estimates of $K_f/K_{FE}$ for different ratios of in-plane and normal stresses for the axially cracked cylinder containing a semi-circular crack of $a/t = 0.2$ and 0.4 with a through-wall thermally induced biaxial secondary bending stress when assessed using (a) the R6 method with an Option 3 FAD and (b) the $g$ function approach with $g$ from Equation (16).](image-url)
at 1400 °C and 0.3, respectively. An illustration of these material elastic–plastic properties can be seen in Fig. 8.

The weld simulation was also designed so that the crack was located at the region where the stress remained tensile at the largest distance through thickness. The residual stress fields across the crack-plane can be seen in Fig. 9 for the different geometries. It is noted that the process of including a weld residual stress was to consider the effect of a residual stress field when using g and not in modelling the residual stress field in the most accurate means or in reflecting a specific weld profile. The benefit to modelling the welding process here, rather than applying a stress field, is to capture the pre-strained loading history that will be seen in a residual stress field.

As the residual stress field was present prior to the crack being introduced, the JEDI postprocessor [22], was used to derive values of J following the mechanical analyses. JEDI was developed to account for the influence of pre-strains and non-proportional loading effects when calculating J in a finite element analysis. In all cases the crack was inserted to the residual stress field by removing a tied surface contact placed on the crack flanks after the welding process but prior to the application of any primary loads and a value for $K_I^E$ obtained.

6.2. Application of g and R6 method to finite element results

Normalised estimates of $K^{R6 \, O1}$, $K^{R6 \, O3}$, $K^{g \, O1}$ and $K^{g \, O3}$ are illustrated as a function of $L_o$ Fig. 10 for the plate geometry, and equivalent results for $K^{R6 \, O3}$ and $K^{g \, O3}$ are shown in Figs. 11 and 12 for the axially and circumferentially cracked cylinders, respectively, for the relevant residual stress fields shown in Fig. 8.

Collectively the plots show that the estimates of J is most conservative when the primary loading acts entirely to open the crack, i.e. $\sigma_2 = 0$. When the loading parallel to the crack is increased, the conservatism is reduced and the non-conservatism, particularly at low values of $L_o$, is increased. Nevertheless, the results provide good estimates of $K_I$ with the use of g through Equation (16) providing improved estimates of $K_I$ over the existing R6 method.

It is also noted that slightly non-conservative estimates are found for some cases at low values of $L_o$, i.e. before widespread plasticity occurs. The reason for this is the presence of the residual stress field providing regions of tensile and compressive strain. As the crack is inserted into a tensile region of a self balancing stress field the stress field over the uncracked ligament is mostly compressive. This means that the strain stress and strain field will not redistribute readily, even though the material is plastic from the welding process, and the stress and strain field at the crack-tip will be enhanced. This then means that the value of $K_I^{FE}$ used will be slightly enhanced over this stress range.

7. Discussions

A new function for $g$ has been defined to describe the redistribution of secondary stresses through the application of a primary stress induced plasticity. Many important features of $g$, as defined through finite element analyses, are shared with an Option 2 failure assessment curve of the R6 reference stress approach, upon which a function for $g$ has been based. However, one important difference to an Option 2 FAD is that the curve increases above unity under moderate to high levels of plasticity. The use of an Option 2 FAD to detail the plasticity redistribution effects was also noted by Song [14], as was the increase above unity. However no means was provided to account for this increase. The increase itself shows that the contribution to the combined crack driving force from the secondary stress is actually enhanced under applied primary loads before redistribution reduces the secondary stress. This is somewhat contradictory to the view that secondary loads can be neglected in a ductile material where it is often assumed that residual stresses are reduced by increasing plasticity prior to crack initiation. These results demonstrate the need to account for secondary stresses if it is possible that the crack initiates prior to plastic collapse.

The results found when comparing different estimates of $K_I$ for the through-wall bending stress and the weld residual stress are generally conservative for all methods assessed. Of all the geometries considered, with the two types of applied secondary stresses, the best estimate of $K_I$ is for the circumferentially cracked cylinder with the thermally induced secondary bending stress field when adopting $g$. This is not surprising as this was the geometry used to define $g$. However, the results for the other geometries do show that the same estimate of $g$ can be successfully extended to other geometries and secondary stress types and still be less conservative than existing methods.

One potential reason for the observed conservative nature of the R6 methods may be in an attempt to cope all potential eventualities, including cases with moderate levels of elastic follow-up. Under secondary stresses this is easily demonstrated in a finite element analyses by a difference in the elastic and elastic–plastic

![Fig. 8. Ramberg–Osgood data relevant to 304 stainless steel materials properties used in the welding analyses.](image-url)
SIFs. This is because components with little elastic follow-up will remove the internal mis-match and residual stress is relieved through crack-tip plasticity. Consequently there is little difference in the elastic and elastic–plastic SIFs. However, a large elastic follow-up will act as a remote stress and not be relieved through crack-tip plasticity, although the crack-tip plasticity will enhance the elastic–plastic SIF. In such cases it might be necessary to treat the secondary stresses as a primary stress. The secondary stresses considered here showed little evidence of elastic follow-up, whereas one of the cases presented by Song [14] does include elastic follow-up and is closer to, and sometimes above, existing R6 estimates. It would therefore be recommended that cases with significant elastic follow-up be considered as additional primary stresses.

In some limited cases the g approach has been observed to be excessively non-conservative and, when it does occur, it is normally

![Fig. 10. Estimates of $K_R/K_{FE}$ for different ratios of in-plane and normal stresses for a centre cracked plate containing a semi-circular crack of $a/t = 0.4$ with a weld residual secondary stress when assessed using (a) the R6 method with an Option 1 FAD, (b) the R6 method with an Option 3 FAD, (c) the g function approach with $g = 1$ and (d) the g function approach with $g$ from Equation (16).](image)

![Fig. 11. Estimates of $K_J/K_{FE}$ for different ratios of in-plane and normal stresses for an external circumferentially cracked cylinder containing an extended crack of $a/t = 0.4$ with a weld residual secondary stress when assessed using (a) the R6 method with an Option 3 FAD and (b) the g function approach with $g$ from Equation (16).](image)
observed in all of the approaches studied. However, the R6 method has been extensively used and verified over many years and no problems have required methods to be modified. One potential explanation is that approach is strongly dependent on the estimate of primary reference stress, both in g and f(Lr), which can be estimated by a number of approaches. As a small change in the primary reference stress could have large implications on Kt under plasticity it is possible that a different estimate would provide more conservative results. It is also noted that the majority of cases where the larger non-conservatism exists is under extreme levels of plasticity, making the validity of J, and hence Kt, controlled fracture debatable due to likely constraint loss.

Of all cases, the thermally induced bending stress fields applied to the axially cracked cylinder shows the highest degree of non-conservatism, which was also seen for the existing R6 methods. It is considered that the reason for the non-conservatism is a result of the geometries tendency to a lower level of constraint.

It is possible that the ideal values for A and B used to define g will change for different geometries and loading conditions. However, the conservative estimates presented here appear adequate for an engineering approach, at least for the geometries considered. It is also considered possible that any ideal values of A and B will also be linked to a measure of elastic follow-up or geometric constraint; this is also indicated by the crude measure of crack-tip triaxiality adopted in Equation (18). As the values of A and B predicted by Equations (17) and (18) are adequate for the centre cracked plate with a shallow crack, which one of the lowest constrained geometries normally considered, it is likely that they can successfully be extended to other geometries without issue. Further work may be needed to confirm whether this is indeed the case.

8. Conclusions

A new function, g, has been developed to describe the plasticity interaction under combined primary and secondary loading for the estimation of crack driving force. This function has been determined from finite element analyses of an externally cracked cylinder under a through-wall thermally induced bending stress. The function has been used to assess a series of cracked geometries which include both a through-wall bending stress and a weld residual stress. The main conclusions from these analyses are discussed below.

- Many important features of g are shared with an Option 2 FAD, upon which the derived function for g has been based. However, one important difference to an Option 2 FAD is that the curve increases above unity under moderate to high levels of plasticity.
- Although the calculated value for g is based on a fully external, externally cracked cylinder, the results for the other geometries do show that the same estimate of g can successfully be used to provide a reasonable assessment of Kt.
- The calculated form for g can be successfully applied to both proportional, i.e. thermally induced stresses, and non-proportional secondary stress fields, i.e. weld residual stresses.
- The derived form of g is less conservative than existing methods, such as the p approach of R6.
- Only in some limited cases are excessively non-conservative results observed. When it does occur, it is normally seen irrelevant of the approach used, and may be explained by other processes.

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Predictions of elastic-plastic crack driving force and redistribution under combined primary and secondary stresses – Part 2: Experimental application

P.M. James, P. Hutchinson, C.J. Madew, A.H. Sherry

AMEC Technical Services, Walton House, Birchwood Park, Risley, Warrington, Cheshire WA3 6GA, UK
Dalton Nuclear Institute, The University of Manchester, Manchester M13 9PL, UK

ABSTRACT

Engineering components may contain small crack-like defects that experience combinations of primary and secondary stresses during service. A new function, \( g \), was introduced in the accompanying Part 1 paper to quantify the influence of plasticity interaction under combined primary and secondary loading. This paper reports new experiments to examine the interaction of primary and secondary loads with plasticity. These experiments were performed on three point bend specimens that had experienced a pre-compression to induce a residual stress field before being tested to failure at \(-150\), \(-90\) and \(-50\) \(\degree\)C to correspond to varying levels of plasticity, and hence \(l_N\), at fracture. Both 2D and 3D finite element analyses have been performed that show excellent agreement with experimental measures of the residual stress field from digital image correlation, neutron diffraction and surface hole drilling when adopting an incremental kinematic hardening model. On applying the existing R6 and the \( g \) plasticity interaction parameters to these experiments it was found that the accuracy of the methods diverged with increasing plasticity, with the \( g \) approach providing the more accurate result. However, since both the existing R6 and the \( g \) plasticity interaction parameters were shown to provide conservative results, the experiments provide useful validation for both methods.

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1. Introduction

Engineering components may contain small crack-like defects. Engineering components such as piping or pressurised containment vessels also experience combinations of both primary and secondary stresses during operation. It is therefore integral to the safe operation of these components that an understanding of the structural integrity is established. Secondary stresses are displacement controlled and generally arise from thermal gradients, weld residual stress or component fit-up stresses. Primary stresses arise from an applied load, such as an end load or internal pressure, and do contribute to failure by plastic collapse.

The effect of the combined influence of primary and secondary stresses on defective structures is currently considered within the R6 defect assessment procedure [1]. However, the previous Part 1 paper [2] detailed the development of a new approach for quantifying the total crack driving force arising from the interaction of primary and secondary stresses acting on a component containing a defect. The developed approach uses a new interaction parameter, \( g \), which reduces the conservatism of the \( \rho \) and V-factor methods currently found within R6 [1]. The definition of \( g \) was derived from, and compared to, a series of elastic–plastic finite element analyses performed with respect to a number of cracked geometries under two types of secondary stress; a thermal gradient and a weld residual stress [2].

This Part 2 paper describes a series of laboratory experiments performed on a ferritic pressure vessel steel to validate the new \( g \) approach across a range of plasticity levels; from small- to large-scale yielding. A review of experiments performed to study failure under combined primary and secondary stress was performed. These include the pre-compressed bend specimen tests by Mirzaei-Sissan et al [3], the aluminium plate tests by Ainsworth et al [4] used to validate R6, welded A533B plate tests by Pan and Li [5], the pre-compressed compact tension tests of Lee et al [6] and the series of spinning cylinder tests [7–11] also noted in the R6 validation. This showed that there is only a limited experimental data set that covers a large range of plasticity levels from predominantly elastic conditions to fully plastic conditions. The aim of these experiments therefore is to provide additional experimental data points over intermediate values of plasticity so that the changing nature of the secondary stress contribution with primary stress induced plasticity can be considered.

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The specimen design chosen builds upon the work of Mirzaee-Sissan [3] in which a local residual stress field is induced in a fracture mechanics specimen via a compressive pre-load. This provides a review of the 2D and 3D finite element analyses used to initially scope and subsequently compare to the experiments, coupled with the necessary material modelling. To provide confidence in the results of the finite element analyses experimental measures of the residual stress field from digital image correlation (DIC), neutron diffraction and surface hole drilling have also been obtained and compared.

The paper is structured as follows. Section 2 summarises the simplified methods currently used to estimate the interaction of primary and secondary stresses. The tests performed are then described in Section 3, followed by details of the complementary finite element analyses in Section 4. Experimental results, including the measurements of the residual stress field, and the fracture loads and obtained toughness values are presented in Section 5. The application of $g$ to assess the failure condition from these tests is then presented in Section 6. A discussion is provided in Section 7 before the main conclusions are summarised in Section 8.

2. Overview of simplified methods for the interaction of primary and secondary stresses

2.1. Generalised approach

For the R6 $V$-factor approach [1], the estimate of the $K_r$ fracture parameter is provided by:

$$K_r = \frac{K_f^p + VK_s^s}{K_{f,\text{mat}}}$$  \hspace{1cm} (1)

where $K_f^p$ and $K_f^s$ are the elastic stress intensity factors associated with primary and secondary stresses, respectively, and $K_{f,\text{mat}}$ is the material fracture toughness at the temperature of interest. The $L_r$ plastic collapse parameter is defined by the magnitude of applied primary load relative to the plastic limit load, or equivalently by the ratio of the primary reference stress to the yield stress:

$$L_r = \frac{P}{P_l} = \frac{\sigma_{\text{ref}}^p}{\sigma_y}$$  \hspace{1cm} (2)

where $P$ is the applied load, $P_l$ is the plastic limit load, $\sigma_{\text{ref}}^p$ is the primary reference stress and $\sigma_y$ is the 0.2% proof stress. The use of the reference stress allows a simplified way to account for load and geometrical effects within one term.

The assessment of the integrity of a structure is performed by comparing the location of the failure assessment point, defined by $K_r$ and $L_r$, with an appropriate failure assessment curve. The curve is defined by one of three options. The Option 1 curve $f_1(L_r)$ is a general lower-bound curve that is independent of both the material and geometry of the component and is defined by Equation (3). The Option 2 curve $f_2(L_r)$ is less general, being material dependent but geometry independent. This is derived from the stress–strain properties of the material and is provided by Equation (4). The Option 3 curve $f_3(L_r)$ is specific to the material and the component geometry of interest and is derived from the results of an elastic–plastic finite element analysis as defined by Equation (5).

$$f_1(L_r) = \left[0.3 + 0.7e^{-0.6L_r} \right] \left[1 + 0.5L_r^2 \right]^{-0.5}$$  \hspace{1cm} (3)

$$f_2(L_r) = \left[ \frac{E_{\text{mod}}^p}{\sigma_{\text{ref}}^p} + \frac{0.5(\sigma_{\text{ref}}^p/\sigma_y)^2}{E_{\text{mod}}^p/\sigma_{\text{ref}}^p} \right]^{-1/2}$$  \hspace{1cm} (4)

$$f_3(L_r) = \frac{K_f^p}{K_{f,\text{mat}}}$$  \hspace{1cm} (5)

where $E$ is Young’s Modulus, $\epsilon_{\text{ref}}^p$ is the strain corresponding to the primary reference stress, $\sigma_{\text{ref}}^p$, as defined by the material stress–strain curve, and $K_f^p$ is the equivalent stress intensity factor derived from the J-integral where $K_f^p = \sqrt{E/((1-\nu^2)f)}$ under plane strain conditions, $\nu$ being Poisson’s ratio.

As inferred above, the failure assessment curves become increasingly more accurate when progressing from Option 1 to Option 3, but are more complex to determine. The curves themselves quantify the enhancement to the crack driving force from plasticity when considering primary stresses acting alone. However, they do not account for the interaction of crack-tip plasticity with the crack driving force due to secondary stress. This interaction is captured by the $V$-factor. In this paper two methods for estimating this plasticity interaction are adopted. First, the R6 $V$-factor [1] is utilised and secondly the $g$ term derived in the Part 1 paper [2] is applied.

2.2. The R6 plasticity interaction factor $V$

In R6 [1] the influence of plasticity on the combined crack driving force due to primary and secondary stresses is defined by the $V$ plasticity interaction factor. There are two approaches to define this parameter. Under the complex $V$-factor methodology the $V$ is estimated by:

$$V = \frac{K_f^s}{K_{f,\text{mat}}}$$  \hspace{1cm} (6)

where $K_f^s$ is the elastic–plastic secondary stress intensity factor and $\xi$ is an interaction parameter provided in look-up tables [1]. These tables are defined in terms of $L_r$ and the parameter $K_f^s / (K_f^p / L_r)$. It is the complex $V$-factor approach which is adopted in this paper and is subsequently simply referred to as the ‘$V$-factor method’.

2.3. The $g$ plasticity interaction factor

The $g$ plasticity interaction parameter, described in the associated Part 1 paper [2] can be incorporated into Equation (1) as an alternative approach to estimate $V$. This new method of estimating $V$ from $g$, $V_g$, is summarised below. This approach is subsequently referred to as the ‘$g$-Function method’.

$$V_g = \frac{g(L_r)K_f^s}{K_{f,\text{mat}}}$$  \hspace{1cm} (7)

where $g$ is provided by an equation of similar form to the Option 2 failure assessment curve, Equation (4):

$$g = \left[ \frac{E_{\text{mod}}^p}{\sigma_{\text{ref}}^p} + \frac{A(\sigma_{\text{ref}}^p/\sigma_y)^2}{E_{\text{mod}}^p/\sigma_{\text{ref}}^p} \right]^{-1/2}$$  \hspace{1cm} (8)

$$A = \frac{-\sigma_{\text{in,plane}}}{1.25 \times \sigma_{\text{mises}}}$$  \hspace{1cm} (9)

$$\sigma_{\text{ref}}^p = L_r \sigma_y / 1.25$$  \hspace{1cm} (10)

where $\sigma_{\text{mod}}$ is the modified reference strain defined from $\sigma_{\text{mod}}$ in Equation (10), via the material stress–strain properties, $\sigma_{\text{in,plane}}$ is
the remotely applied primary stress in the crack opening direction and $\sigma_{\text{mises}}$ is the Von-Mises equivalent stress defined for the primary loads acting alone.

3. Experimental test programme

3.1. Scope

The test programme was designed to assess the effect of combined primary and secondary stresses on failure over a range of $L_r$ values with fracture occurring at three levels of plasticity. A review of experimental approaches to study failure under combined primary and secondary stress was undertaken \[3-11\]. This showed that there are limited experimental data that incorporate residual stresses and cover a large range of $L_r$ values. Broadly speaking two approaches for inducing a residual stress field in a laboratory specimen are normally used: (a) the tests in References 4 and 5 induced the residual stress by welding, and (b) the tests in References 3 and 6 induced the residual stress by mechanical pre-compression. The test approach adopted here uses the pre-compression method with a similar specimen design to that developed by Mirzaee-Sissan \[3\]. This approach was chosen since a well-controlled residual stress field can be created in a standard fracture toughness specimen.

3.2. Specimen geometry

The specimen geometry used in this work was a scallop notched-bend specimen with a crack at the root of the notch as in Fig. 1. The specimens were modified slightly from that in Reference [3], as follows:

- The pre-compression notches on the ends of the specimen were made into round notches to prevent shearing of the corner from the root of the sharp V-notch.
- The specimen thickness was increased to 25 mm to prevent buckling under compression.
- The radius of the scalloped notch was larger to increase the volume of the tensile residual stress field.

Based on the results of a series of design finite element analyses described in the next section, a crack length of 3.5 mm below the notch root was employed. This ensured that the crack tip was located a sufficient distance from the notch root to minimise any interaction between the crack tip and the free surface whilst remaining short enough to minimise the stress redistribution due to first inserting the notch and then the crack. The specimens contained an Electric Discharge Machined (EDM) notch of 2 mm inserted to the specimen before the crack was sharpened and grown by 1.5 mm under fatigue loading.

3.3. Material

The material used in this study was a low alloy ferritic steel, A533B-1 \[12\]. Tensile tests were performed according to BS EN 10002-5:1992 standard \[13\] to provide full stress–strain curves at four temperatures, reflecting the temperatures used for the fracture mechanics tests. The tensile tests were performed on round bar specimens of diameter 7.5 mm and gauge length 20 mm at a strain rate of 0.1 mm per minute and were interrupted once the ultimate tensile strength (UTS) had been achieved. Fig. 2 shows the resultant true stress versus true strain data, indicating that the material exhibited a Lüders strain plateaux following yield at all temperatures.

In addition to the tensile tests compression-tension tests were performed at room temperature to compress the material to strains of 1.0 and 2.0% followed by a tensile test to the UTS at a rate of 0.05 mm per minute. Finally, one set of low cycle fatigue samples underwent a 30 compress–load cycles to $\pm$1.5% strain at room temperature to form the materials hysteresis loop, at a rate of 0.05 mm per minute. These compression tests were performed to consider the materials hardening characteristics following a pre-load and are shown by comparison to finite element predictions below.

3.4. Compressive pre-load

Two sets of fracture tests were performed. In the first, a compressive pre-load was applied at the ends of the specimens such that a tensile residual stress was introduced into the material ahead of the notch after unloading. The pre-load was applied to specimens at room temperature at a rate of 0.5 mm/min until a displacement of 4 mm was measured at the notch mouth using a clip gauge. The EDM notch was introduced after unloading and a sharp crack grown by fatigue using a $\Delta K$ of 21 MPa$\sqrt{m}$ and an $R$-ratio of 0.1.

Fig. 1. Specimen design (all dimensions in mm). Note that Detail-A show EDM starter notch at the position of crack.
3.5. Measurement of residual stress

The induced residual stress field was quantified in three ways. First, Digital Image Correlation (DIC) was used to measure the displacement of the specimen surface in the vicinity of the notch root during pre-loading. Two cameras were positioned 560 mm from the surface of the specimen and focussed on the vicinity of the notch root. A series of images were captured throughout the pre-loading and unloading. A post-processor was used to derive displacement vectors in both in-plane and out-of-plane directions using a cell size of 35 μm. Principal strains were then derived by normalising the displacement by the cell size. The resultant strain field was then compared with the results from three-dimensional finite element analysis (described in Section 4). It was these data that were used as the means to relate the measured surface strain field to a calculated residual stress field, i.e. by finite element analyses.

Secondly, neutron diffraction was performed on a pre-loaded, but uncracked, specimen at the ENGIN-X instrument on the ISIS beam-line at the Rutherford Appleton Laboratory. This enabled the residual stresses to be measured through the entire thickness of the 25 mm thick specimen. The residual stress is measured by determining the intensity and angular distribution of a number of detection banks positioned such that the central detector is normal to the incident beam, thus allowing the measurement to be focused to a specific location. The angular distribution of the diffracted neutron beam using scattered neutrons is then examined as a diffraction pattern following Bragg’s law to calculate the lattice spacing, and hence localised strains, of the metallic structure when compared to an equivalent, unstrained, section of metal referred to as the “00” measurement. This localised strain can then be converted to a stress by assuming an elastic response for the “gauge volume” over which the measurements are made [14]. For the tests performed at ENGIN-X a gauge volume of 2 mm was used with measurements every 2 mm over the initial 10 mm from the root of the notch and 4 mm over the remaining ligament. The “00” measurement was taken from an unstrained section of material near the materials edge.

Finally, a measurement of the strain 2 mm ahead of the notch on the specimen centre-line was performed using the incremental surface hole drilling method on two similarly compressed specimens. The basic premise of the method is to relieve the residual stress, by way of drilling into the material, so that surface strains can be measured by strain gauges on the materials surface. These deformed strains can then be used to calculate the original strain field which can be related to stress by assuming standard, elastic relations [14].

3.6. Test temperatures

Fracture toughness tests were performed on As-Received and pre-loaded specimens at temperatures of −150, −90 and −50 °C. These temperatures were chosen to explore residual stress effects on failure at three differing levels of plasticity as defined by the Lc parameter: (a) Lc < 0.3, (b) 0.4 < Lc < 0.8 and (c) 0.9 < Lc < 1.2. The temperature range spans the lower to upper ductile-to-brittle transition temperature range. Specimens tested at −150 were expected to fail by cleavage with no prior ductile tearing. Specimens tested at −90 °C were expected to exhibit some plasticity prior to cleavage and specimens tested at −50 °C were expected to fail by ductile tearing. Failure in all tests was defined by initiation, with an estimate of this provided by the materials J−R curve where required (note that as the assessment was performed in terms of K, K−R curves are used for easier comparison).

3.7. Fracture test method

All specimens were tested in three-point bending at a loading rate of 0.5 mm/min in accordance with ASTM E 1820-08 [15]. The applied load and displacement were continuously measured during each test; displacement being measured via a clip gauge attached to knife-edges positions at the notch mouth. Tests were performed within an environmental chamber cooled to the required test temperature. The specimens were placed into the chamber and the surface temperature of the material monitored. When the sample had cooled to the specified temperature a three-point bend load was applied to the specimen and increased to failure. For the cases where cleavage fracture was not expected, i.e. at −50 °C, the specimens were allowed to tear by differing amounts so that a multi-specimen Kf−R curve could be constructed and an estimate of a 0.2 mm tearing fracture toughness for the material provided by a standard blunting line. Following each test the amount of pre-cleavage crack growth was measured using a computer controlled travelling microscope. The mean ductile tearing was measured and defined using a nine-point average. For specimens tested at −90 °C and −50 °C, in which pre-cleavage tearing was evident, multi-specimen Kf−Δa (or Kf−R) curves were constructed such that failure could be derived from the value of Kf at 0.2 mm of tearing. This value was used as the material fracture toughness, Kmatri.

In all cases, the value of J at failure was derived from the results of 3D elastic–plastic finite element analyses as described in the following section and converted to Kf as detailed below.

4. Finite element analyses

4.1. Finite element model

A finite element model was used to simulate the behaviour of the test specimen illustrated in Fig. 1. The model was developed using Version 6.9 of the ABAQUS finite element code [16].
The model was a 3D brick model utilising 20-noded reduced integration elements, ABAQUS C3D20R. The model simulated one quarter of the specimen with appropriate symmetry boundary conditions applied to nodes located on the uncracked ligament and specimen centre. This model illustrated in Fig. 3 highlights the increased mesh refinement in the vicinity of the notch root, the crack tip and at the loading points. The crack tip was modelled with a focused mesh containing elements with degenerate nodes at the crack tip to correctly model both blunting and the crack tip stress field, following guidance in [16,17]. The minimum in-plane dimension of elements directly ahead of the crack tip was 250 μm by 10 μm.

4.2. Material properties

The room temperature Young’s Modulus, \( E \), was equal to 208,920 MPa and Poisson’s ratio, \( \nu \), was set to 0.3 [12]. The plastic properties followed the data illustrated in Fig. 2. The Lüders strain visible in Fig. 2 was included in the material data used in the analyses.

Previous experimental and numerical studies on an A533B-1 material by Lee et al [6] has revealed that A533B-1 material exhibits a Bauschinger effect when it is pre-loaded to cause yield in compression and then re-loaded in tension, thus requiring a kinematic or mixed isotropic-kinematic hardening approach. The results from the compression-tension material tests and the low cycle test were used to assess the behaviour of the kinematic hardening model. A single element model was used to assess the materials behaviour under both isotropic and kinematic hardening; isotropic hardening was included to assess if it was necessary to consider a mixed hardening model. The results of this comparison can be seen in Fig. 4 (top) at 1% and 2% compression before loading to failure. When the single element was cycled the results compared to the cyclic data can be seen in Fig. 4 (bottom) for the kinematic hardening model. The kinematic hardening model describes the hysteresis loop and tensile loading portions well, but not necessarily the initial compression, which demonstrates the Bauschinger effect. Nonetheless, this would be difficult to model within standard hardening models. Therefore, only the kinematic hardening model has been used to describe the tests.

For design analyses fracture toughness data were available from Reference 7 over the temperature range \(-150 \degree C \to +50 \degree C\). The variation in lower bound fracture toughness, \( K_{\text{mat}} \), as a function of temperature is defined by Eq. (11) where \( K_{\text{mat}} \) is given by MPa m\(^{0.5} \) and \( T \) in \( \degree C \). Estimates of the fracture toughness were also obtained from the As-Received specimen tests discussed previously for this material as detailed below.

\[
K_{\text{mat}} = 473.6e^{0.015T} \tag{11}
\]

4.3. Boundary conditions

In addition to the symmetry conditions described above, boundary conditions were applied to the model simulate the compressive pre-loading, the unloading, and the three-point bend loading applied to the specimens. All loading was simulated via the circular rigid surfaces illustrated in Fig. 3 that represented the pins used to apply loads during the tests. During the compressive pre-loading a compressive displacement of 2 mm was applied to the end-load rigid surface in the direction of the notch. The rigid surface was positioned at grooves on the end of the model allowing the pre-compression to be applied by rigid surface contact without inducing significant shearing at the corners. The compressive pre-load was subsequently removed by defining a displacement of \(-10 \text{ mm} \) to the rigid surface to move it away from the specimen.
Three-point bending was simulated by a second set of circular rigid surfaces located centrally on the symmetry line directly under the notch, and on the upper surface of the model towards the right-hand edge. Negligible load was applied to these rigid surfaces to ensure contact throughout the analysis and that no spurious restraining boundary condition was applied to the model during pre-loading. Three-point bending of the specimen was simulated by fixing the position of the central rigid surface, and applying a vertical displacement to the rigid surface located on the upper surface.

4.4. Design analyses: crack length

The length of the pre-crack was chosen to ensure that: (a) the shielding influence of the notch was minimal, and (b) the crack did not significantly relieve the residual stress. To determine the appropriate crack length, a 2D plane strain model was used to calculate the elastic stress intensity factor for cracks of increasing length inserted into the model following the pre-loading simulation. The resultant values of $K$ were compared with the handbook (R6) solution for a standard edge cracked plate, i.e. without the scalloped notch, when loaded in tension [1].

The shielding influence of the notch on $K$ was found to act over effective crack lengths between 17.5 and 20 mm (when including the notch) but diminished over deeper crack depths so that the handbook and finite element solutions matched. The effective crack length used was 21 mm, i.e. 3.5 mm below the notch root. This gave a stress intensity factor that is within 10% of an appropriate handbook solution. When the effective crack length is adopted within the 3D model outlined, above the agreement with the R6 handbook solution was the same as the 2D model and thus validated the 3D model for elastic analyses.

4.5. Design analyses: test temperature

The test temperatures were chosen to ensure that failure in each tests series occurred at different levels of $L_r$: from failure under predominantly elastic, small-scale yielding to large-scale yielding. To define appropriate test temperatures, a series of design analyses were performed on the 2D plane strain finite element model to simulate the entire loading process using material properties appropriate to temperatures in the range $-150$ to $+50$ °C. The analyses steps were: (a) pre-compression at room temperature, (b) instantaneous crack insertion at room temperature, (c) cooling (or heating) to test temperature and (d) loaded under three-point bending. The predicted value of $L_r$ at failure was from the load at which the crack driving force was equal to the material fracture toughness. The test temperatures chosen were $-150$, $-90$ and $-50$ °C, which corresponded to failure at $L_r < 0.3$, $0.4 < L_r < 0.8$ and $0.9 < L_r < 1.2$, respectively.

To determine the value of $L_r$ the limit load was determined from an elastic–perfectly plastic RIKs analysis relevant to each temperature and validated against R6 handbook solutions [1]. The difference in the R6 handbook and finite element predictions of limit load were within 2%.

4.6. Derivation of $J$

The elastic–plastic crack driving force, $J$, was obtained as a function of load using the JEDI [18] post-processor to ensure that the initial residual stress field and any non-proportional loading effects were correctly accounted for. The effective stress intensity factors, $K_f$, were obtained from Equation 12 below. The estimate of $K_f$ due to the residual stress field alone, $K_f^r$, was obtained after the pre-compression and cooling steps but prior to the simulation of three-point bending. The values for $K_f^r$ were found by simulating the three-point bend loading alone, i.e. step (d) above.

$$K_f = \sqrt{E'J}$$  \hspace{1cm} (12)

where $E' = 1/(1-\nu^2)$ under plane strain conditions.

When inserting a crack into the residual stress field, for the chosen crack length and pre-compression, the values of the elastic–plastic crack driving force for secondary loads alone, $K_f^p$, was found to be 35 MPa m$^{0.5}$. These results were taken from the 3D finite element analyses using the JEDI [18] post-processor to ABAQUS to account for the presence of plastic strains prior to the cracks insertion.

5. Results

5.1. Characterisation of residual stress and strain fields

A comparison of the residual opening mode strain field, $e_{xx}$, on the surface following pre-loading measured using DIC and predicted by 3D elastic–plastic finite element analysis is illustrated in Fig. 5 following the release of the pre-load. The results show that the resultant strain field is negative, with a strain of more than $-7\%$, at the notch root region, which will result in a tensile residual stress in this region due to the bulk elastic response of the surrounding
material. This compressive strain field extends to just over half of the remaining ligament. The strain at the back face of the specimen is tensile and over +4%, which will result in a compressive residual stress field. This comparison shows that the predicted strain field and the measured strain field follow very similar spatial distributions where the neutral axis is positioned at the same location and the contours of strain extend in the crack opening axis (x) to a close approximation.

A quantitative comparison of the DIC and finite element results is shown in Fig. 6 for the strains in the crack opening direction (x) and crack growth direction (y) when taken along a path defined by the uncracked ligament. The DIC results are shown by the lines and the finite element results as the data points. There is excellent agreement between the measured and predicted strain field with predicted values within 5% from the measured values at all positions. This agreement provides good validation to the finite element model and hardening model for the A533B-1 material used.

The predicted distribution of opening mode residual stress, $\sigma_{xx}$, generated as a function of distance from the notch root as a result of pre-loading is illustrated in Fig. 7. This includes results taken relevant to along the surface and the centre of the specimen. The finite element results are compared with measurements made by neutron diffraction and surface hole drilling in Fig. 7. The results show that there is good agreement between the neutron diffraction measurements the 3D finite element model predictions, for normalised depths greater than 0.2 with, perhaps, a slight over prediction of the maximum compressive stress. As the sampling volume taken was relatively large compared to the potential changes in the stress field the results might not accurately reflect the stress at that position. Nonetheless, the results provide further evidence for the validity of the finite element model and hardening material model.

Fig. 7 also includes the results from shallow hole drilling, illustrating the average stress measured on two specimens, with the associated error range. This measurement of the residual stress near the surface shows excellent agreement with the finite element predictions at the surface with the values being within 6% and 20% of each other for the two shallow hole drilling measurements and both within the potential error margins.

5.2. Fracture tests

The results for all tests are illustrated in Figs. 8 and 9, which show the applied load at failure and the measured $K_{ij}$ as a function of temperature. The results are also summarised in Tables 1 and 2. The $K_{ij}$ values were derived using the results from 3D finite element analysis of the As-Received specimen for the applied primary load because the conventional experimental measure of $K_{ij}$ for a three point bending specimen was not appropriate to this non-conventional geometry. Note that additional comparison to the load versus clip-gauge displacement was also made between the finite element analyses and the tests. This comparison was made under both the compression and loading of all cases. The fracture toughness presented only represents the crack driving force to due primary (applied) load, $K_{ij}$.

The results show that the presence of a residual stress field has a significant effect on both the failure load and the value of $K_{ij}$ at failure when compared with data from As-Received specimens. This is most pronounced at $-150 \, ^\circ C$ and $-90 \, ^\circ C$ cases where the failure load is reduced by approximately 50% and 25% respectively. At $-50 \, ^\circ C$ the data are equivalent as the tests were interrupted at a maximum load which was not influenced by the presence of the residual stress. The effect of plasticity can be seen in the estimates of $K_{ij}$ which again show a difference between the residual stress
cases to the As-Received cases of 35 MPa m⁰.₅ for the −150 °C cases, 78 MPa m⁰.₅ for the −90 °C cases and a negligible difference for −50 °C cases. This helps to show that the relative contribution of secondary stress is enhanced under moderate levels of plasticity but removed under gross plasticity.

The derived Kₚ − R curves for the pre-loaded and As-Received specimens are shown in Fig. 10 at −50 °C. At −50 °C there is no significant difference in load or Kₚ between the specimens with residual stress and the As-Received cases and could be considered under the same statistical variation. The Kₚ − R plot helps confirm this with similar numbers of specimens from each group above and below the average fit line. Also included is a lower bound fit to the data by shifting the fitting curve to cover all failure points so that a lower bound estimate of toughness can be considered. It is noted that the validity limits of the Kₚ − R plot are not included as, when adopting the approach of ASTM E 1820-08 [15] for this geometry, are outside the scales of the plot. It is also noted that an exclusion line for values above 1.5 mm crack growth also exists within ASTM E 1820-08 but as only one point is marginally outside this region all points have been used.

6. Application of the plasticity interaction term, g, to the experimental results

6.1. As-Received specimens

The As-Received samples were used to define the material fracture toughness at each temperature. This is beneficial as it incorporates any geometric constraint effects due to the notch, although it is noted that the residual stress may also change the level of constraint in the specimens that underwent the pre-compression. The value of fracture toughness used at each test temperature was taken as the average of the measured results with a second assessment with the lower bound value. The average value adopted should provide a prediction of the mean failure condition in the pre-loaded specimens. As such, when failure assessment positions are plotted on a failure assessment diagram, there should be an equal number of failure positions inside and outside the curve. The lower bound toughness value should provide an assessment with all points falling outside the failure assessment curve. It is considered important to consider both approaches as the average fracture toughness will best assess the accuracy of the methods used and the lower bound is akin to a safety case assessment which reflects the worst case scenario.

As both the tests at −90 °C and −50 °C showed small amounts of crack growth or tearing before cleavage failure the estimate of fracture toughness was defined from Kᵢ−R curves. However, the residual stress specimens at −90 °C did not show significant tearing and could be considered to fail by cleavage. Therefore, at the −90 °C cases the measure of fracture toughness was taken from the As-Received specimens where the Kᵢ−R curve crossed the blunting line. Conversely, as the −50 °C cases showed no sign of cleavage the more conventional 0.2 mm blunting line was used. All of the specimens at −150 °C were seen to fail by cleavage. Therefore the average and minimum results for the As-Received specimens define the mean and lower bound fracture toughness at this temperature. The values of fracture toughness defined from these tests can be seen in Table 3 compared to values reported in literature [12]. The similarity of the lower bound values and those from literature help validate the measure of fracture toughness.

6.2. Assessment of pre-loaded-specimens

The assessment results from applying the g-Function and V-Factor methods outlined in Section 2 to the test data are shown in Fig. 11, with respect to the average fracture toughness using the R6 Option 1 FAD. The data points within these plots are the As-Received data points as the open stars, the primary contribution to the samples with the residuals stress as the open circles, the R6 V-Factor results as the solid squares and the g-Function results as the solid diamonds.

7. Discussion

7.1. Application of g to experimental results

The results presented show that the difference between the V-Factor and g-Function predictions of failure is only minor at low values of Lₛ. However, there is a moderate benefit shown when using the g-Function approach at higher values of Lₛ where the reduction is seen to be approximately −8% in terms of Kᵢ at both −90 °C and −50 °C in Fig. 11. The results presented therefore show

### Table 1

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Residual Stress</th>
<th>As-Received</th>
</tr>
</thead>
<tbody>
<tr>
<td>−150</td>
<td>25.8</td>
<td>29.6</td>
</tr>
<tr>
<td>−90</td>
<td>55.2</td>
<td>89.4</td>
</tr>
<tr>
<td>−50</td>
<td>88.4*</td>
<td>86.4*</td>
</tr>
<tr>
<td>Average</td>
<td>23.9</td>
<td>48.8</td>
</tr>
</tbody>
</table>

*The loads shown are where the test was stopped.*
that the results improve the estimate of fracture load. When applying the lower bound fracture toughness all data points were found to fall outside the FAD, which helps show that the approach can be used in a lower bound assessment. It is noted that the level of reduction in $K_I$ when considering the g-Function approach is less than may be inferred from the Part 1 paper [2]. It is possible that this is a result of the change in material and geometry, causing the $V$-Factor and g-Function estimates to become more similar. Nonetheless Fig. 11 provides considerable confidence in the method whilst demonstrating a reduction in conservatism.

The prediction of $K_f^*$ from the kinematic hardening material, taken when adopting the JEDI [18] post-processor, provides a very good estimate of failure, especially under cleavage conditions. This helps add evidence to support the use of the modified J-Integral as implemented in the JEDI post-processor as there is no additional plasticity enhancement, i.e. both $V$ and $g = 1$, so the secondary contribution is solely defined by $K_f^*.$

In nearly all cases, both the R6 $V$-Factor and g-Function methods provide very good estimates of failure in the tests, with the g-Function providing a potential benefit of up to 8% at approximately $L_r = 0.9.$ An accurate prediction of failure would be expected for the $-150 \, ^\circ C$ case, as long as the estimate of $K_f^*$ is correct, as the response is predominantly elastic, and there is little difference between the $V$-Factor and g-Function methods. The observed agreement between the $V$-Factor and g-Function methods is perhaps of more note in $-90 \, ^\circ C$ and $-50 \, ^\circ C$ cases as the secondary stress contribution is either enhanced or reduced respectively. This agreement between the methods and comparison to experimental data can be considered as very useful validation to both methods.

### 7.2. Warm Pre-stress (WPS) considerations

Warm Pre-Stress (WPS) is a potential concern within this work as it can provide an enhanced fracture toughness when the material has experienced changes in load and temperature. Commonly the WPS effect is a feature of specimens that have been pre-loaded at a higher temperature and subsequently cooled before being tested to failure, such as those used here. The phenomenon is described as “the effective enhancement of the cleavage fracture toughness at low temperature following the application, at a higher temperature, of a stress intensity factor which exceeds the fracture toughness of the virgin material at low temperature” [19]. There are several methods available to predict WPS effects, such as those discussed by Bordes et al [19], Pickles and Cowan [20] and Section III.10.4 of R6 [1].

It is considered that the experiments performed in this report can neglect the effect of WPS as the secondary stress intensity factors, both before and after cooling, are lower than the material toughness for all cases. This means that there should be negligible WPS effects at failure. This also reflects the guidance in R6 Section III.10.4 [1].

Nonetheless, for added confidence the approach to estimate the WPS enhancement (Section III.10.4 [1]) within R6 was considered as shown below.

$$K_f = K_2 + \sqrt{K_{mat}\Delta K_u + 0.15K_{mat}}$$  \hspace{1cm} (13)

where $K_f$ is the failure stress intensity factor after WPS effects, $K_2$ is the stress intensity factor after cooling (or unloading), $K_{mat}$ is the fracture toughness at the new (cooled) condition and $\Delta K_u$ is the change of stress intensity factor on cooling (or unloading). In these experiments finite element analyses indicated that there was no significant change in $K_f^*$ on unloading which means the square root term is zero and the new failure load is simply $K_f = K_2 + 0.15K_{mat}.$ R6 states that if $K_f$ is less than $K_{mat}$ at the temperature in question, then WPS can be neglected [1].

When considering the applied preload of 35.4 MPa m$^{-0.5}$ and the $-150 \, ^\circ C$ fracture toughness estimates of 42.5 MPa m$^{-0.5}$ and 70 MPa m$^{-0.5}$ the potential WPS enhanced toughness would be 41.5 MPa m$^{-0.5}$ and 46 MPa m$^{-0.5}$ for the lower bound and average toughness respectively. Both of these estimates are less than the actual material toughness being considered which means that the measured fracture toughness should be used and the WPS effect can be neglected at $-150 \, ^\circ C.$ From inspection of Equation (13), it can be seen that greatest WPS effect is predicted for the lowest fracture toughness and, hence, WPS effects can be neglected at all other temperatures also. For added confidence the Chell model as outlined in Reference [21] was also considered for these cases. The predictions of a WPS modified toughness from the Chell model was

### Table 2

<table>
<thead>
<tr>
<th>Sample</th>
<th>$-150 , ^\circ C$ residual stress</th>
<th>$-150 , ^\circ C$ As-Received</th>
<th>$-90 , ^\circ C$ residual stress</th>
<th>$-90 , ^\circ C$ As-Received</th>
<th>$-50 , ^\circ C$ residual stress</th>
<th>$-50 , ^\circ C$ As-Received</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1</td>
<td>37</td>
<td>42</td>
<td>86</td>
<td>199</td>
<td>380</td>
<td>326</td>
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<tr>
<td>Sample 2</td>
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<td>66</td>
<td>124</td>
<td>198</td>
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<td>330</td>
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<tr>
<td>Sample 3</td>
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<td>95</td>
<td>124</td>
<td>172</td>
<td>303</td>
<td>404</td>
</tr>
<tr>
<td>Sample 4</td>
<td>40</td>
<td>90</td>
<td>149</td>
<td>191</td>
<td>355</td>
<td>367</td>
</tr>
<tr>
<td>Sample 5</td>
<td>28</td>
<td>57</td>
<td>103</td>
<td>217</td>
<td>371</td>
<td>344</td>
</tr>
<tr>
<td>Average</td>
<td>34</td>
<td>70</td>
<td>117</td>
<td>195</td>
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</table>

Table 3

<table>
<thead>
<tr>
<th>Temperature ($^\circ C$)</th>
<th>Mean estimate</th>
<th>Lower bound estimate</th>
<th>From literature [7]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-150$</td>
<td>70</td>
<td>42</td>
<td>50</td>
</tr>
<tr>
<td>$-90$</td>
<td>158</td>
<td>137</td>
<td>123</td>
</tr>
<tr>
<td>$-50$</td>
<td>226</td>
<td>248</td>
<td>224</td>
</tr>
</tbody>
</table>

![Fig. 10. $K_J$–R curve for specimens tested at $-50 \, ^\circ C.$](image-url)
8. Conclusions

An experimental programme has been designed and successfully performed to quantify the influence of residual stresses on fracture at different levels of plasticity, i.e. at different positions on an R6 FAD.

Excellent correlation between the finite element predictions and direct measurements of the stress and strain field from DIC, neutron diffraction and incremental surface hole drilling has also been seen when using a kinematic hardening material model in the finite element analyses.

Applying the g-Function method to the experimental results provides a reduction compared to the R6 V-Factor method. This reduction in conservatism is seen to increase with increased levels of plasticity with a maximum observed reduction of approximately 8% of $K_c$.

In all cases both the R6 V-Factor and g-Function methods remain conservative. This can be considered as useful validation to both methods.

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References