Left-handed metamaterials realized by complementary split-ring resonators for RF and microwave circuit applications

A Thesis submitted to The University of Manchester for the degree of Doctor of Philosophy in the Faculty of Engineering and Physical Sciences

2012

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ACKNOWLEDGEMENT

There are numerous people who I wish to thank for their assistance during this PhD study. Firstly, I want to take this chance to give a special thanks to my supervisor, Dr. Zhirun Hu, who has inspired this work. I also want to thank him for his knowledge encouragement and support in both my studies and in my life. Also my utmost gratitude to my sponsor, the Royal Thai Government, who’s funding has supported my living and study costs.

I would also like to thank Dr. Cahyo Mustiko for his assistance discussions and advice over my PhD both with respect to lab work and theory. Thanks are also due to Dr. Abid Ali and Dr. Mahmoud Abdalla for their previous works without which the inspiration for my thesis would not have been realized.

Finally, I would like to thank sincerely to my husband, Mr. Ross Allen, and the rest of my family for their support and sacrifices for me to achieve what I have.

This thesis is an opportunity to express my thanks for all their support and encouragement.
ABSTRACT

A new equivalent circuit of left-handed (LH) microstrip transmission line loaded with Complementary split-ring resonators (CSRRs) is presented. By adding the magnetic coupling into the equivalent circuit, the new equivalent circuit presents a more accurate cutoff frequency than the old one. The group delay of CSRRs applied with microstrip transmission line (TL) is also studied and analyzed into two cases which are passive CSRRs delay line and active CSRRs delay line. In the first case, the CSRRs TL is analyzed. The group delay can be varied and controlled via signal frequency which does not happen in a normal TL. In the active CSRRs delay line, the CSRRs loaded with TL is fixed. The diodes are added to the model between the strip and CSRRs. By observing a specific frequency at 2.03GHz after bias DC voltages from -10V to -20V, the group delay can be moved from 0.6ns to 5.6ns.

A novel microstrip filter is presented by embedding CSRRs on the ground plane of microstrip filter. The filter characteristic is changed from a 300MHz narrowband to a 1GHz wideband as well as suppression the occurrence of previous higher spurious frequency at 3.9GHz. Moreover, a high rejection in the lower band and a low insertion loss of <1dB are achieved.

Finally, it is shown that CSRRs applied with planar antenna can reduce the antenna size. The structure is formed by etching CSRRs on the ground side of the patch antenna. The meander line part is also added on the antenna patch to tune the operation frequency from 1.8GHz downward to 1.73GHz which can reduce the antenna size to 74% of conventional patch antennas. By using the previous antenna structure without meander line, this proposed antenna can be tuned for selecting the operation frequency, by embedding a diode connected the position between patch and ground. The results provide 350MHz tuning range with 35MHz bandwidth.
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<td>ADS</td>
<td>Advanced design system</td>
</tr>
<tr>
<td>BW</td>
<td>Bandwidth</td>
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<tr>
<td>CPW</td>
<td>Co-planar waveguide</td>
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<tr>
<td>CRLH</td>
<td>Composite right/left-handed</td>
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<td>CSRR</td>
<td>Complementary split ring resonator</td>
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<tr>
<td>DNG</td>
<td>Double negative material</td>
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<tr>
<td>EM</td>
<td>Electromagnetic</td>
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<tr>
<td>FBW</td>
<td>Fractional bandwidth</td>
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<tr>
<td>HFSS</td>
<td>High frequency structure simulator</td>
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<tr>
<td>LH</td>
<td>Left-handed</td>
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<tr>
<td>LHM</td>
<td>Left-handed metamaterial</td>
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<tr>
<td>MTM</td>
<td>Metamaterial</td>
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<tr>
<td>NRI</td>
<td>Negative refraction index</td>
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<tr>
<td>PCB</td>
<td>Printed circuit board</td>
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<tr>
<td>PLH</td>
<td>Purely left-handed</td>
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<tr>
<td>PRH</td>
<td>Purely right-handed</td>
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<tr>
<td>RF</td>
<td>Radio frequency</td>
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<tr>
<td>RH</td>
<td>Right-handed</td>
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<tr>
<td>SNG</td>
<td>Single negative material</td>
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<tr>
<td>SRR</td>
<td>Split-ring resonator</td>
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<td>TEM</td>
<td>Transverse electromagnetic modes</td>
</tr>
<tr>
<td>TL</td>
<td>Transmission line</td>
</tr>
<tr>
<td>TW</td>
<td>Thin wire</td>
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CHAPTER 1

INTRODUCTION

1.1 Overview

Electromagnetic metamaterials (MTMs) are manmade and can be treated as homogeneous electromagnetic structures with unusual-unnatural properties [1.1]. Left-Handed (LH) MTMs have negative electric permittivity (\(\varepsilon\)) and permeability (\(\mu\)) properties simultaneously. Because LH MTMs exhibits these double negative parameters, LH MTMs are said to have anti-parallel phase and group velocities, therefore gives a negative refractive index (\(n\)) [1.2]. Left-handedness was theorized first by Veselago [1.3] and was proved experimentally by Pendry [1.4] and Smith [1.5]. Building on this initial work, LH structures can be categorized into main configurations: thin wires (TW) and composite materials made from an array of splitting ring resonators (SRRs) [1.6]; and periodically loaded transmission lines (TLs) usually using shunt shorted inductance (stubs, meandered or spiral lines), and series capacitances (gap or inter-digital capacitors) [1.7]. As a result of what is previously mentioned, in the last decade, there are many new frontiers of microwave circuits and components in the form of LH applications [1.8-1.11], for performance enhancement, and size reduction.

Planar circuit technology is a compatible technique to fulfill a main aim in RF-Microwave industry for making mass produced components of high frequency, and a wide frequency operating range. However, in microstrip technology, SRRs particles exhibit weak H-field excitation by the incident field, as in a co-planar waveguide.
(CPW) structure. Its specific effect is not noticeable by maintaining size [1.12-1.14]. In order to overcome these limitations, a new configuration of the radial E-field, excites the particle, and thus has been introduced [1.13-1.14]. Complementary split ring resonators (CSRRs) are the dual form of SRRs. Since this structure is etched on the ground plane, under the conductor strip position, and is excited by the electric field, induced by the conductor line. This coupling can be modeled by a series, connecting the line capacitance to the CSRRs, and therefore, due to these benefits, CSRRs will be used for this thesis.

1.2 Aim and Objectives

The aim of this research is to use the unique properties of MTMs via CSRRs and transmission lines to achieve both performance and size reduction in planar microwave applications, such as transmission line, filter, and antenna.

As mentioned, there are three main objectives of this study one of the objectives is to present a distinctive characteristic of group delay on the left handed passband of a novel CSRRs transmission line, which can minimize the length as well as enhance performance of the signal delay. In communication systems, signal delay can degrade the system quality. Delay suppressions are the recommended part of communication systems. However, this thesis presents a new method concerning the group delay of LH passband of CSRRs transmission line. The group delay can be managed, as a result more signals can be sent simultaneously without any concern over interval time of each input signal.
Filter is a component used to select the wanted signal and suppress the unwanted noise. The filter performance is a crucial part resulting in system quality, therefore, it is essential to design a high performance filter which is covered all criterion of the selectivity, high and wide rejection out-of-band, spurious suppression, as well as low insertion loss. In planar microstrip filter, the structure and length of transmission line are related to LC parameters and resonant frequencies. CSRRs, acting as a LC resonator, compatible in planar microstrip technology, presented a narrow rejection band with high selectivity, therefore, with these attractive specific properties, it is the selected particle to fulfill the planar microstrip filter criteria, as it exhibits good functionality and size.

Antenna is the front end in transmitter and receiver part. In antenna technology, the patch antenna size is larger when there is a lower operating frequency. Therefore, miniaturization of microstrip antenna is another objective in this thesis. Because of the specific permittivity characteristic of CSRRs at resonant frequency, it is again the proper particle to apply in minimization the patch antenna.

1.3 Thesis Outline

The thesis is organized into seven chapters. Chapter 2 presents the general theory of metamaterials by explaining the Maxwell’s equations and how they are applied. The MTMs’ specific properties, such as negative refraction and reversal Doppler Effect, are described. The general MTMs formed in planar technology are presented; for example, thin wires, split ring resonators (SRRs) and its dual part called “complementary split ring resonators (CSRRs)”. In order to make a clear point with
regard to MTMs, the chapter will end up with some applications of MTMs in today’s work.

In chapter 3 the general equivalent circuit models of metamaterial on microstrip structures are presented and analyzed, which includes the purely left-handed (LH) structure and the composite left-right handed (CRLH) structure. The left-handed particle structure is formed by complementary split ring resonators (CSRRs), are analyzed. In addition, the new equivalent circuit of the left-handed particle structure with the magnetic coupling effect between the two close rings included is also represented, which is not appeared in the previous works. In addition, the LH passband region of CSRRs loaded TL structure is also indicated and analyzed by its phases and dispersion relations.

Since there is very limited work to date in the delay property on planar microstrip circuitry using MTMs, the study of delay lines using left-handed Materials (LHMs) dispersive properties, will be explored in chapter 4. There are two separated cases in this chapter. The passive CSRRs delay line is first analyzed. The group delays are studied via the displayed dispersive delayed signals, as both continuous waves (CWs) and pulse signals. In active CSRRs delay line, the presence of varactor diodes for a tunable structure is proposed. The group delay on a specific frequency is studied.

In chapter 5, a new configuration of microstrip wideband passband filter is demonstrated. The filter theory is initially presented. Then, the prototype microstrip filter is demonstrated and analyzed by introducing the rectangular CSRRs on the
ground plane of this prototype structure, which results in the novel microstrip filter characteristics being changed. The filters performance is changed from a narrow bandpass filter to a wide bandpass filter. Other filter criteria are also displayed and investigated. In addition, the higher spurious suppression is also presented.

In chapter 6, a criterion for ‘electrically small’ size, using the negative phase property has been developed via antenna design. The etching of CSRRs on the group plane part of a microstrip patch antenna is presented for specific propagation and minimization. The meander line is another option for fulfilling further size reduction. Furthermore, the tunable patch antenna is proposed by embedding a diode connected between the ground and the patch side. The frequency selective antenna is proposed.

Chapter 7 describes the conclusions of this thesis and future works of the CSRRs applications which are resultant of the conclusions that were able to come about from the study in this thesis.
1.4 References


CHAPTER 2

FUNDAMENTALS OF METAMATERIALS - A REVIEW

2.1 History

In 1968, the theoretical verification of negative index material and left handed (LH) phenomenon in a medium with negative permittivity ($\varepsilon$) and negative permeability ($\mu$) was postulated and pointed out by Veselago [2.1]. His research showed the exhibition of the phase velocity direction opposite to the Poynting vector of the wave propagation in such a medium; the occurrence of backward wave propagation. Therefore, his study is to support the theory of negative index of refraction or the ‘left handed’. Nevertheless, the substances which have negative magnetic permeability ($\mu<0$) are still not done until three decades later.

![Diagram](image.png)

Figure 2.1 (a) The first artificial dielectrics lenses: Lattice of conducting disks arranged to form lens by Kock and Cohn in 1948 [2.2], and (b) J.B. Pendry designed periodic structure of negative $\varepsilon$ and $\mu$ [2.3].
The first structure that showed negative permittivity in certain frequency bands was proposed by Pendry et al [2.3, 2.4]. Owing to the ideas of Pendry, Smith [2.5] designed a composite medium which have both negative permittivity and permeability. Then more studies, such as Shelby et al [2.6] designed the periodical structures by using a split-ring resonators and wire strips, have been investigated of. Metamaterials (MTMs), their properties, and their applications have been appeared. The Left-Handed Materials (LHMs), Double Negative (DNG) Materials, and Single Negative (SNG) Materials, are the example names of the above mention. However, MTMs can be referred to many materials that provide the permittivity or permeability less than 1.

Before study, it should be firstly clarified about the definition of MTM and its concept. MTMs are artificial materials, consisting on sub-wavelength periodic (or quasi-periodic) inclusions (‘atoms’) of metals and/or dielectrics, whose electromagnetic, or optical, properties can be controlled through structuring, rather than through composition [2.7]. Since the period is smaller than a wavelength, effective (continuous) media properties are achieved, and it is possible to obtain properties beyond those available in nature. For these reasons, it is necessary to study and control these unnatural material characteristics for the specific purpose in wireless communication systems.
2.2 Maxwell’s Equations and Left-Handed Metamaterial Properties

Most electromagnetic phenomena can be described by Maxwell’s equations, first formulated by James Clerk Maxwell in 1860’s. The relations of electric and magnetic field quantities are presented which describe the electromagnetic wave are [2.8]:

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M}, \]  
\(2.1\)

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \]  
\(2.2\)

\[ \nabla \cdot \vec{D} = \rho_e \]  
\(2.3\)

\[ \nabla \cdot \vec{B} = \rho_m \]  
\(2.4\)

All quantities are real values in the functions of space and time. \(\nabla\) and \(\frac{\partial}{\partial t}\) are the spatial and temporal differentiations, respectively.

\(E\) = the electric field vector (V/m),

\(H\) = the magnetic field vector (A/m),

\(D\) = the displacement of electric flux density vector (C/m\(^2\)),

\(B\) = the displacement of the magnetic flux density vector (W/m\(^2\)),

\(J\) = the electric current density vector (A/m\(^2\)),

\(M\) = the magnetic current density (V/m\(^2\)).

\(\rho_e\) is the scalar electric charge density (C/m\(^2\)),

\(\rho_m\) is the scalar magnetic charge density (W/m\(^2\)).
In order to fully depict the interaction of the electromagnetic fields, Maxwell’s equations have to be extended by the constitutive relations as

\[ \vec{D} = \varepsilon \vec{E} \]  
\[ \vec{B} = \mu \vec{H} \]

(2.5)  
(2.6)

where \( \varepsilon_0 \) is the free space permittivity \((8.854 \times 10^{-12} \text{ C/Vm}^2)\) and \( \mu_0 \) is the free space permeability \((4\pi \times 10^{-7} \text{ W/Am}^2)\).

Then, the time dependence is assumed as \( e^{+j\omega t} \). Therefore, the time derivatives in equation (2.1) and (2.2) can be replaced as \( j\omega \). Maxwell’s equations can be then presented as [8,12]:

\[ \nabla \times \vec{E} = -j\omega \mu \vec{H} - \vec{M}_s \]  
\[ \nabla \times \vec{H} = j\omega \varepsilon \vec{E} + \vec{J}_s \]

(2.7)  
(2.8)

In the case of absence of sources, the equation (2.7) and (2.8) can be expressed without electric and magnetic current densities:

\[ \vec{k} \times \vec{E} = \omega \mu \vec{H} \]  
\[ \vec{k} \times \vec{H} = -\omega \varepsilon \vec{E} \]

(2.9)  
(2.10)

where the \( k \) is the wave vector along the direction of the phase velocity \( k = \omega \sqrt{\mu \varepsilon} \).
Before the study of LHMs, the right hand rule in electromagnetism will be reviewed. When the direction of both normal $E$ and $H$ fields are represented by the thumb and the index finger of the right hand respectively, placed at right angles to each other then the middle finger placed at right angle to both the fingers gives the direction of propagation of the wave. In nature, all electromagnetic waves such as light are followed this rule which can be stated in mathematical form as equation \((2.1)\):

$$\vec{E} \times \vec{H} = \vec{S}$$ \hspace{1cm} \text{(2.1)}

where $\vec{E}$ is the electric field, $\vec{H}$ is the magnetic field and $\vec{S}$ represents the Poynting vector and the direction of energy and wave propagation.

![Diagram showing electric field, magnetic field, wave vector (E, H, k) and Poynting vector (S) for electromagnetic wave propagation in right-handed and left-handed media.](image)

**Figure 2.2** The diagrams of electric field, magnetic field, wave vector $(E, H, k)$ and Poynting vector $(S)$ for electromagnetic wave propagation in right-handed and left-handed medium \([2.1, 2.8]\)
As from the previous Maxwell equations, it can be assumed that in a medium where the permittivity and permeability is negative (LHMs), the phase velocity will be anti-parallel to the direction of wave propagation or energy flow. In another word, the wave has a ‘negative phase velocity’ in that medium. Even though, the direction of energy flow is always sent from the transmitter to the receiver, the phase moves in the opposite direction, illustrated of the $S$ vector and the anti-parallel of $k$ vector by Figure 2.2.

### 2.2.1 LHMs and its Entropy condition

Assuming that the external supply of electromagnetic energy to the system is shut off, the internal energy per unit volume of medium ($W$) is positive as illustrated in equation (2.12-2.24).

In general medium, the internal energy per unit volume of the medium ($W$) by time varying is expressed as:

$$ W = - \int \nabla \cdot \vec{S} dt $$  \hspace{1cm} (2.12)

where $S$ is Poynting vector in time domain with the observed distance ($r$) as

$$ S(r,t) = \vec{E}(r,t) \times \vec{H}(r,t) $$  \hspace{1cm} (2.13)

Thus, the divergence of the Poynting vector is

$$ \nabla \cdot \vec{S} = \left( \nabla \times \vec{E} \right) \vec{H} - \left( \nabla \times \vec{H} \right) \vec{E} = \left( \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} \right) $$ \hspace{1cm} (2.14)
In general medium, the relations of electric flux density \( \mathbf{D} \) and electric field intensity \( \mathbf{E} \) as well as magnetic flux density \( \mathbf{B} \) and magnetic field intensity \( \mathbf{H} \) are defined as

\[
\mathbf{D} = \varepsilon(t) \otimes \mathbf{E}(r,t) = \int_{-\infty}^{t} \varepsilon(t-t') \mathbf{E}(r,t') dt' \tag{2.15}
\]

\[
\mathbf{B} = \mu(t) \otimes \mathbf{H}(r,t) = \int_{-\infty}^{t} \mu(t-t') \mathbf{H}(r,t') dt' \tag{2.16}
\]

Then, the time varying electric field intensity at a quasi-harmonic field with median frequency \( \omega_0 \) can be presented as

\[
\mathbf{E}(r,t) = \text{Re}(\mathbf{E} e^{j\omega t}) = \frac{1}{2} \left( \mathbf{E} + \mathbf{E}^* \right) \tag{2.17}
\]

Since the following terms are zero at averaged over time

\[
\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} = \mathbf{E}^* \cdot \frac{\partial \mathbf{D}^*}{\partial t} = \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} = \mathbf{H}^* \cdot \frac{\partial \mathbf{B}^*}{\partial t} = 0 \tag{2.18}
\]

Therefore the equation (2.14) can be written as

\[
\nabla \mathbf{S} = -\frac{1}{4} \left( \mathbf{E} \cdot \frac{\partial \mathbf{D}^*}{\partial t} + \mathbf{E}^* \cdot \frac{\partial \mathbf{D}^*}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}^*}{\partial t} + \mathbf{H}^* \cdot \frac{\partial \mathbf{B}^*}{\partial t} \right) \tag{2.19}
\]

where the field components can be analyzed via Fourier series expansion to

\[
\frac{\partial \mathbf{D}}{\partial t} = j \omega \varepsilon(\omega) \mathbf{E} + \frac{d(\omega \varepsilon)}{d\omega} \frac{\partial \mathbf{E}}{\partial t} e^{j\omega t} \tag{2.20}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = j \omega \mu(\omega) \mathbf{H} + \frac{d(\omega \mu)}{d\omega} \frac{\partial \mathbf{H}}{\partial t} e^{j\omega t} \tag{2.21}
\]

Finally, the Poynting vector and the internal energy per unit volume in medium can be reformed as
\[
\n\n
\n
(2.22)

\[
W = \frac{1}{4} \left( \frac{d(\omega \varepsilon)}{d\omega} E^2 + \frac{d(\omega \mu)}{d\omega} H^2 \right)
\]

(2.23)

As a result, the entropy condition can be written as \( W > 0 \) or it can be rewritten in the terms of medium parameters as

\[
\frac{d(\omega \varepsilon)}{d\omega} > 0 \quad \text{and} \quad \frac{d(\omega \mu)}{d\omega} > 0
\]

(2.24)

From equation (2.24), it is impossible that the entropy condition can be happened while the constitutive parameters are both negative. However, this can be occurred in frequency dependent medium which the constitutive parameters would be positive in some frequency regions in order to compensate for their negative values. This condition causes LHM as a causal and dispersive medium with frequency dependent constitutive parameters.

### 2.2.2 LHMs Phenomena

#### 2.2.2.1 Wave radiation in LHMs

As the wave number (\( k \)) is anti-parallel with the power flow in LHMs as well as frequency is always a positive quantity. Therefore, the phase velocities in RH medium are reverse compared to the LH medium. The phase velocity is defined as:

\[
\n\n
\n
(2.25)
The power flows of LH medium are similar to the RH medium and can be presented as

\[ S = E \times H^* \]  \hspace{1cm} (2.26)

\[ P_0 = \frac{1}{2} \int_s E \times H^* \, ds \]  \hspace{1cm} (2.27)

where \( S \) = Poynting vector

\( P_0 \) = Power flow

In summary, the LH mediums provide the backward wave phenomenon due to the reverse general direction of the phase velocity. The phase velocity is opposite of the oriental direction while the power and group velocity are not affected [2.8].

In a homogeneous, a transverse electromagnetic (TEM) wave has a propagation constant (\( \beta \)) equally to the wave number (\( k_n \)) which is defined as

\[ \beta = k_n = \eta k_0 = \eta \omega / c \]  \hspace{1cm} (2.28)

Equation (2.9) and (2.10) can be deduced to

\[ \vec{\beta} \times \vec{E} = s \omega |\mu| \vec{H} \]  \hspace{1cm} (2.29)

\[ \vec{\beta} \times \vec{H} = -s \omega |\epsilon| \vec{E} \]  \hspace{1cm} (2.30)
while $s$ is the sign that $s=+1$ in case of RH medium and $s=-1$ in case of LH medium, respectively.

To understand a LH system, the equation of refraction index has been defined by the square root of the constitutive parameters which are:

$$\eta = \pm \sqrt{\varepsilon_r \mu_r}$$ \hspace{1cm} (2.31)

where $\eta$ is refractive index and provides negative value in LH medium, $\varepsilon_r$ is electric permittivity of material, while $\mu_r$ is magnetic permeability of material

The possible real number of the signs of $\varepsilon$ and $\mu$ are (+, +), (+, -), (-, +), and (-, -) which lead to a double positive (DPS), single negative (SNG) or double negative (DNG) medium, respectively. However, these negative permeability and permittivity will not generally show the negative refractive index. However, Ziolkowski [2.9] used the mathematics to prove the square root choice that leads to a negative index of refraction (NRI)

It can be noted that ‘$\eta$’ can still be positive while the values of $\mu$ and $\varepsilon$ are negative. However, in mathematics, ‘$\eta$’ for DNG materials is negative; for example $\eta=-1$ can be derived from phase $\pi$ of both $\varepsilon$ and $\mu$. In negative medium, the expression of permittivity and permeability in terms of magnitude and phase are considered as:
\[ \varepsilon_r = |\varepsilon_r| e^{i\phi_\varepsilon}, \quad \mu_r = |\mu_r| e^{i\phi_\mu}, \quad \text{where} \quad \phi_\varepsilon \in \left( \frac{\pi}{2}, \pi \right), \quad \phi_\mu \in \left( \frac{\pi}{2}, \pi \right) \] (2.32)

From equation (1.32), the refractive index and the wave impedance of the medium can be written as:

\[ \eta = \sqrt{|\mu_r \varepsilon_r|} e^{i\phi_\eta} \] (2.33)

\( \phi_\eta \) is the total phase of \( \mu_r \) and \( \varepsilon_r \) which leads to negative \( \eta \)

Figure 2.3 Energy flow and wave vector diagram between RH and LH interface

The energy flow vectors are in the same direction, while the wave vectors are in opposite direction (antiparallel) in \( n = -1 \) medium, shown in Figure 2.3.

Figure 2.4 represents the electromagnetic applications based on the signs of the material permittivity, permeability, and refraction index at the interface between air and each medium. There are four regions in the diagram. Plasma belongs to the region with negative permittivity and positive permeability. Split rings belong to the
region of negative permeability and positive permittivity. It can be obviously seen that when the two signs opposite, there is no wave transmission in medium. This is because the wave vector becomes imaginary. When both parameters are positive, refraction occurs positively and vice versa.

Figure 2.4 Material classifications according to $\varepsilon$, $\mu$ pairs and $\eta$, type of waves in the medium, and example of structures.

2.2.2.2 Negative Refraction Phenomenon

In the negative materials, the traveling wave passes from the air through the medium and bends to the same side of the normal as the incident ray. This phenomenon is called negative refraction index, given by the equation (2.31) and illustrated in Figure 2.5.
The Snell’s law supports the wave propagation through LHMs that bend in the wrong way [2.10]. In Figure 2.5, the refractive index of $n_2' = -n_2$ and the wave is refracted to the opposite side compared to the ray propagating in Right-Handed (RH) medium. Although the wave bends the opposite direction, the Snell’s law is still been satisfied when a negative value of $n$ is substituted and $\theta_2 < 0$ into the equation,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$  \hspace{1cm} (2.34)

Due to its negative refractive index, the wave propagating travelling through a LHM slab would be internally focused inside the slab and then create an image point after leaving the slab. Pendry et al [2.11] stated the useful of this material property in realization in ‘perfect lenses’, shown in Figure 2.6(a).
2.2.2.3 Reversal Doppler Effect in LHMs

Doppler Effect is the phenomenon that change in frequency of a wave when the observer moving away the wave source.
Consider the source $P$ moving along $z$ direction, while the angular frequency of the radiated electromagnetic wave $\omega$, illustrated in Figure 2.7. In far-field, the radiated field can be given as [2.14]

$$
\bar{E}(z,t), \bar{H}(z,t) \alpha \frac{e^{i\phi(\omega t)}}{r}, \text{where} \quad \phi(\omega, t) = \omega t - kr
$$

whereas $k$ refers to the wave number in medium and $r$ is the standard radial variable of the spherical coordinates.

In both cases, when source moves towards the positive $z$ direction with velocity $v_z$, the wave position function of time is $z = v_z t$, which $r=z$ in $\theta=0$. Therefore, the seen phase by observer from $P$ to $z$ axis is

$$
\phi = \omega t - knz = \omega \left(1 - \frac{k}{\omega} v_z\right) t = \omega \left(1 - \frac{v_z}{|v_p|}\right) t
$$
Replaced $v_p$ with $\omega/k$

Thus, the coefficient of $t$ is the Doppler frequency ($\omega_{\text{Doppler}}$) defined as

$$\omega_{\text{Doppler}} = \omega - \Delta \omega, \text{ with } \Delta \omega = s \frac{v_s}{v_p}$$ (2.37)

In RH medium, the sign $s=+1$ and $\Delta \omega > 0$, the Doppler frequency is shifted downward when the observer is at point $P$ and shifted upwards when the observer position is at the right hand of moving source, displayed in Figure 2.7(a). In LH medium, sign $s=-1$ and $\Delta \omega < 0$, the phenomenon is opposite, shown in Figure 2.7(b), respectively.

**Summary of the LHMs properties**

There are fundamental phenomena of DNG media by Veselago in 1968 [2.1] such as

- DNG medium presents the propagation of EM waves with $E$, $H$, and $k$ in a left-handed triad ($E \times H$ antiparallel to $k$).
- The phase in DNG medium propagates backward to the source (backward wave) with the phase velocity antiparallel to the group velocity.
- Because of the negative permittivity and permeability, the refractive index is negative.
- The constitutive parameters of DNG medium have to be under frequency dependent as a dispersive medium. In composite material, the permittivity and permeability are presented as $\varepsilon_{\text{eff}}$ and $\mu_{\text{eff}}$, respectively.
2.3 Realization of Left-Handed Materials

In 1999, Pendry et al [2.3] proved that the negative effective permeability ($\mu_{\text{eff}}(\omega)$) can modify the permeability of the host substrate from an array of conducting non-magnetic rings. The cause of bulk $\mu_{\text{eff}}(\omega)$ variation for a very large positive value of $\mu_{\text{eff}}(\omega)$ at the lower resonance frequency and a significantly large negative $\mu_{\text{eff}}(\omega)$ at the higher resonance frequency is from the considerable enhancement of magnitude of $\mu_{\text{eff}}(\omega)$ when the constituent unit cells are resonantly made. Schultz et al [2.5] are the scientists who first realized the LH materials by creating a periodical array of interspaced conducting non-magnetic split-ring resonators and continuous wires. Before their success of realizing the LHMs, the attempts to produce the negative permittivity materials were made earlier by Pendry [2.16]. In order to create the negative permittivity, a three dimensional mesh of conducting wires was used as a structure to alter the permittivity with supporting substrate. The exhibition at the frequency region in his experiment can show the simultaneously negative values of both effective permeability $\mu_{\text{eff}}(\omega)$ and effective permittivity $\varepsilon_{\text{eff}}(\omega)$. Then Schultz et al [2.6] used these two concepts to create a LH structure. The wire strips and a mesh of interspaced split-ring resonators are introduced for this achievement. The wire strips generate $\varepsilon$ while the split-ring resonators (SRRs) alter $\mu$. Therefore, the frequency dependent negative material with both negative parameters was realized. It should be noticed that this would only happen under the condition that the size of unit cell is considerably smaller than the smallest operation wavelength. As a result, these periodic structures can give a uniform isotropic alteration of the base material properties. In order to consider the actually effective parameters of a homogeneous medium, the constraint of the wave on a unit cell dimension is analyzed.
For a typical electromagnetic wave of frequency ($\omega$), the characteristic dimension of the structure ($a$) should be in the condition as follow [2.3]:

$$a \ll \lambda = \frac{2\pi c}{\omega}$$  \hspace{1cm} (2.38)

Figure 2.8 shows a genetic view of the periodical structure that gives an effective bulk permittivity and permeability for MTMs.

Figure 2.8 (a) Photograph of the Metamaterial cube [Physic today, June 2004], and (b) Generic view of a host medium with periodically placed structures constituting a MTM.

### 2.3.1 Metal Wire Geometry

Pendry 1998 [2.16, 2.17] applied the individual properties of thin metallic wires which can alter the effective permittivity of the host medium when excited appropriately. His evaluation used a long metallic cylinder embedded in a
homogeneous medium. The geometry of the composite medium with periodically placed wire inclusions is shown in Figure 2.9.

![Figure 2.9 Metamaterial structures: (a) a medium composed with metallic wire and (b) thin wire lattice exhibiting negative $\varepsilon_{\text{eff}}$ if $E$ applied along the wires.](image)

The array of thin metal wire, illustrated in Figure 2.9(a), presents the effective negative $\varepsilon$. If the electric field $E$ is applied along the wires, the induced current along the wires will generate equivalent to electric dipole moment. There are two factors affecting the electron movement in a wire radius $r$; the average electron density $n_{\text{eff}}$ and the effective mass of electrons by magnetic effects.

$$n_{\text{eff}} = \frac{n \pi r^2}{a^2} \quad (2.39)$$

where $n$ is the density of electrons in the wire
From Ampere’s law, a current flowing through a wire produces a magnetic field, where the direction of the field is depended on the direction of the current. In addition, the presence of the magnetic field alters the momentum of electrons. The magnetic field $H(R)$ is defined in equation (2.40) which is equivalent to the momentum per unit length of wire as

$$H(R) = \frac{I}{2\pi R} \equiv \frac{r^2 n e v}{2R}$$  \hspace{1cm} (2.40)

where $I$ = the current flow through the wire

$R$ = the distance from the wire

Then, the effective mass of an electron $m_{\text{eff}}$ is expressed as

$$m_{\text{eff}} = \frac{\mu_0 e^2 r^2 n}{2} \ln\left(\frac{a}{r}\right)$$  \hspace{1cm} (2.41)

where $e$ = the electron charge

$v$ = the average electron velocity

It is noticed from equation (2.40) and (2.41) that the bigger radius of the wire the more effective mass of the electrons. This observation is later investigated in the plasma frequency ($\omega_p$). Plasma frequency is the fundamental oscillation frequency of the electrons while returning to the equilibrium position, termed as

$$\omega_p^2 = \frac{n_{\text{eff}} e^2}{\varepsilon_0 m_{\text{eff}}} = \frac{2\pi c_0^2}{a^2 \ln\left(\frac{a}{r}\right)}$$  \hspace{1cm} (2.42)

where $c_0$ is the speed of light in a vacuum.
In equation (2.42), it is noticed that decreasing the effective mass provides a large shift in the plasma frequency. Moreover, in order to maintain the wire array as a homogenous material, the wire radius must keep small as compared to the lattice dimension \((a)\). Therefore, the effective \(\varepsilon\) of the composite medium can be evaluated from an effective homogeneous medium. In lossless metal, the plasmonic-type permittivity is derived as

\[
\varepsilon_{\text{eff}}(\omega) = 1 - \frac{\omega_p^2}{\omega^2}
\]  

(2.43)

From equation (2.43), it has been appeared that the permittivity is negative when \(\omega < \omega_p\). Since there is no magnetic material employed and no magnetic dipole moment is created, the permeability is simply \(\mu = \mu_0\) for all frequencies.

### 2.3.2 Split-Ring Resonator Geometry

Because the specific property of SRRs embedded in a host medium can give the bulk composite permeability and become negative in a certain frequency band, the study of the region above the SRR resonant frequency has been widely observed.

![Figure 2.10 Metamaterial structures: Split ring resonators lattice exhibiting negative \(\mu_{\text{eff}}\) if magnetic field \(H\) is perpendicular to the plane of the ring.](image)

Figure 2.10 Metamaterial structures: Split ring resonators lattice exhibiting negative \(\mu_{\text{eff}}\) if magnetic field \(H\) is perpendicular to the plane of the ring.
While applying the magnetic field $H$ perpendicular to the plane of the ring, the induced currents then are generated around the ring equivalent to the appearance of the magnetic dipole moments [2.3]. The permeability frequency function is formed as

$$\mu_{\text{eff}}(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_{0m}^2 + j\omega \varsigma}$$

(2.44)

where $\omega_{0m}$ is the resonant frequency in GHz range given by

$$\omega_{0m} = c \left[ \frac{3a}{\pi \ln \left( \frac{2wr^3}{d} \right)} \right]$$

(2.45)

$F$ is the filling fraction of the SRR, while $\varsigma$ is the damping factor due to metal loss, these parameters are expressed as

$$F = \pi \left( \frac{r}{a} \right)^2 \quad \text{and} \quad \varsigma = \frac{2aR'}{r\mu_0}$$

(2.46)

whereas $r =$ inner radius of the smaller ring

$w =$ the width of the ring

$d =$ the radial spacing between the inner and outer rings

$R' =$ the metal resistance per unit length.
Figure 2.11 some geometries of SRR used to realise artificial magnetic materials [2.7]

The SRR structure has a magnetic response due to the presence of artificial magnetic dipole moments by the ring resonator. At a resonant frequency range, these artificial magnetic dipole moments are larger than the applied field which leads to the presence of the real part of negative effective permeability, $\text{Re}(\mu_{\text{eff}})$. In lossless case ($\zeta=0$), the range around the resonant frequency providing negative permeability is under the condition as $\omega_{\text{om}} < \omega < \omega_p = \frac{\omega_{\text{om}}}{\sqrt{1-F}}$, where $\omega_p$ is the plasma frequency of the SRR particle. In other words, in the medium the discontinuity of the dispersion relation of the permeability is occurred between $\omega_{\text{om}}$ and $\omega_p$ because of the negative $\mu_{\text{eff}}$ at that frequency range.

The introduction of capacitive elements that enhances the magnetic effect is produced by the splits of rings. The strong capacitance between the two concentric rings helps the flow of current along the SRR configuration. Since in the SRR the capacitive and inductive effects nullify, the $\mu_{\text{eff}}$ has a resonant form. At resonant frequency, owing to the capacitive effects due to the gap interacts with the inherent
inductance of the structure; the electromagnetic energy is shared between the external magnetic field and the electrostatic fields within the capacitive structure. Therefore, normally the experiments are focused on a certain frequency band which is around and above the resonance frequency in order to get the negative effective permeability [2.18].

Depicted in Figure 2.12, is the first LHM prototype designed by Smith et. al. [2.5]. In this work, the combined particles of the thin wire structure and SRR structure appeared an overlapping frequency range that have both negative permittivity and permeability. After applying an electromagnetic wave through this composite structure, the pass band is presented at the frequency range of interest that the constitutive parameters are simultaneously negative.

![Image of LHM prototype](image)

(a) (b)

Figure 2.12 the first DNG metamaterial structures [2.5] Smith et al., 2000-2001: (a) Mono-dimensionally DNG structure and (b) Bi-dimensionally DNG structure [2.6] (the rings and wires are on opposite sides of the boards)
2.3.3 Complementary Split-Ring Resonator

![Figure 2.13 Topology of CSRRs and the stack CSRRs, E is parallel to the CSRRs plane][2.19]

In 2004, CSRRs are firstly introduced by Falcone et al.[2.20]. The CSRRs, a dual counterpart of SRRs or sometimes called ‘slotted split-ring resonator’, are comprised of slots which is the same dimensions as the corresponding SRR. By the principle of duality, the CSRRs properties are in dual relation of the SRRs properties. The SRRs behave as a magnetic point dipole, whereas the CSRRs present an electric point dipole with negative polarization. In CSRRs, the $E$ field is applied parallel to the CSRRs plane in order to generate a strong electric dipole which affects the CSRRs resonant frequency [2.15]. The CSRRs, as shown in Figure 2.13, $r$ can be used to obtain the effective permittivity of a bulk medium. Both SRRs and CSRRs present approximately the same resonant frequency due to their shared dimensions.

The CSRRs can be formed in planar transmission media by etching these resonators in the ground plane of the microstrip. An example of this structure is demonstrated in Figure 2.14(a). The HFSS simulation is used for design. The 50Ω line is chosen to
match the port impedance. This CSRRs structure provides the inhibition of signal propagation at a resonance frequency as a narrow band.

![Figure 2.14 Geometry of the CSRRs (a) with and (b) without capacitive gap](image)

Figure 2.14 Geometry of the CSRRs (a) with and (b) without capacitive gap ($r_{in}=3\text{mm}$, $c=0.3\text{mm}$, $d=0.4\text{mm}$ on Roger/RO6006 duroid with $\varepsilon_r=6.15$ and $h=1.39\text{mm}$)

Considering on Figure 2.15(a) and (b), at the resonant frequency, the dip in transmission coefficient is displayed at 3.52GHz under the sudden change of phase to zero degree. At the frequency band above its resonant frequency, the CSRRs proposes a negative effective permittivity in real part (unlike SRRs), whereas exhibits positive effective permittivity at frequency band below the resonant frequency, demonstrated in Figure 2.15(c).
Figure 2.15 The simulated results of (a) scattering parameters, (b) the output phase, and (c) the effective permittivity of a combined CSRRs structure and microstrip line.
Since the $E$ field excitation in the CSRRs configuration is better matched in microstrip transmission line, CSRRs is widely used for applying to the artificial LH transmission lines. This structure is consisted of a microstrip line section with a series capacitive gap etched in the conductor strip and loaded with CSRRs which etched in the ground plane. The unit cell CSRRs transmission line is shown in Figure 2.14(b). In a composite CSRRs transmission line, these capacitive gaps need to be etched periodically on the microstrip line to nullify the tank inductance and obtain LH wave propagation.

Figure 2.16 The simulated results of (a) insertion, return loss, and (b) phase of $S_{21}$ for Figure 2.14(b) and a series gap of 0.3mm.
The LH passband range is then appeared, shown in Figure 2.16(a). At the resonant frequency of CSRRs, the advance phase 90 degrees is obtained which is one of its specific properties of the LHMs, illustrated in Figure 2.16(b) [2.13], [2.20] The CSRRs analysis in terms of equivalent LC circuits will be explained in the next chapter.

2.4 CSRRs applications in recently works

2.4.1 CSRRs and its stop band characteristic
Since CSRRs presents stop band characteristic when applied with microstrip, it has been used to develop the filter functions. The recent work [2.21] presents a wide stop band filter by using the co-operation of the different CSRRs dimensions. It is found that the size of CSRRs relates to its resonant frequency in opposition. In the other words, the small size of CSRRs is presented; the higher resonant frequency is obtained.

Figure 2.17 the two different sizes of CSRRs [2.21]

Figure 2.17 demonstrates the two CSRRs lengths after tuning to merge their resonant frequencies. The two resonant frequencies are now presented as one stopband, shown in Figure 2.18.
Figure 2.18 the S-parameters of the two CSRRs sizes [2.21]

This application can be further applied to bandpass filter to widen the stop band as well as increase rejection level.

### 2.4.2 CSRRs and antenna applications

#### A. Compact Patch antenna

Figure 2.19 (a) Photograph of the patch antenna loaded with CSRRs and (b) simulated reflection coefficient by varying $l_1$ ($l_1$ is the CSRRs length) [2.22]

The compact antenna loaded with CSRRs and reactive impedance surface (RIS) is shown in Figure 2.19(a). The CSRRs are modeled as a shunt LC resonator which creates resonant frequency. The photograph shows the patch size around
0.099\lambda_0 \times 0.153\lambda_0 which is very compact. By HFSS simulation, the antenna resonant frequency is varied inversely to the CSRRs size, illustrated in Figure 2.19(b).

### B. Frequency selective antenna

![Figure 2.20](image)

Figure 2.20 (a) the ultra-wideband antenna with quadruple-band rejection and (b) The measured transmission loss [2.23]

The four rejection bands at 2.6, 3.5, 5.5, and 8.2GHz are selected by LC resonator of the depicted SRRs and CSRRs on patch and ground. The resonant frequency of each particle can support the wideband antenna performance by suppressing signal interferences.
2.5 References


*ISBN: 007123201X.*


CHAPTER 3

METAMATERIAL TRANSMISSION LINES

INTRODUCTION

Metamaterial transmission lines (MTM TLs) are artificial lines where Right-Left Handed composite parts embedded on a host such as microstrip transmission lines or coplanar waveguides loaded with reactive elements. Their relevant characteristics in either impedance or phase of these propagating structures can be controlled beyond what can achieve in conventional transmission lines. These artificial lines, formerly proposed in the early 2000’s [3.1-3.5], are inspired on MTMs which exhibits similar properties and, in some cases, fabricated using identical constituent particles [3.6, 3.7].

MTM-TLs are artificial lines with controllable characteristics. Furthermore, these artificial lines can be designed to exhibit LH wave propagation in certain frequency bands. Such lines are normally implemented by means of lumped or semi-lumped reactive elements. Since these elements are electrically small, the conditions to achieve homogeneity can also be achieved, that is a small period compared to signal wavelength. Only under these conditions, the terms of effective constitutive parameters $\mu_{\text{eff}}$ and $\varepsilon_{\text{eff}}$ are studied. However, homogeneity is not a fundamental requirement in transmission lines. Indeed homogeneity can only be achieved in a certain region of the allowed band [3.1].
From the point of view of microwave circuit design, the advantages of MTM-TLs rely on miniaturization and on the possibility to control the dispersion diagram and characteristic impedance, rather than on homogeneity. Thus, MTM-TLs are defined as artificial lines, consisting on a host line loaded with reactive elements, with controllable characteristics. Homogeneity is not considered to be a requirement for such lines [3.8, 3.9]. Notice that according to this, there is no need for a minimum number of unit cells to implement these artificial lines. Indeed, in most of the cases, a single cell is considered since this reduces line dimensions, as will be shown later.

With regard to the implementation of MTM-TLs, there are two main approaches: LC-loaded lines or dual transmission line [3.1-3.5, 3.10], and resonant type approaches [3.6, 3.7]. In LC-loaded lines, proposed by Eleftheriades [3.2] and Caloz [3.1], the key element to achieve left-handedness consists of a host line loaded with series capacitances and shunt inductances. These lines can be implemented by using lumped loading elements, or, alternatively, by means of semi-lumped planar components such as series gaps, interdigital capacitors, grounded stubs or vias. The Resonant type MTM-TLs can be implemented by loading a host line with SRRs and shunt inductive elements [3.11, 3.12], or, alternatively, by loading a host line with CSRRs and series capacitances [3.13-3.15]. Both LC-loaded lines and resonant type MTM-TLs exhibit similar characteristics and dispersion. The simultaneously negative values of the effective permeability and permittivity of the medium for both types of the LH approaches occurred in an interval frequency that the reactive elements dominate over the per-section capacitance and inductance of the line [3.1, 3.13, 3.14, 3.16]. Therefore, both line types are useful for the implementation of the state-of-the-art microwave and millimeter wave circuits.
In this chapter, the CSRRs, a dual counterpart of SRRs, are formed to show a left handedness in planar transmission media by etching these resonators in the ground plane of the microstrip transmission line. Due to the presence of CSRRs, the inhibition of signal propagation is acted at a resonance frequency as a narrow band. This phenomenon interprets the negative effective permittivity of the medium. The series capacitive gap is added to the structure in order to obtain the negative permeability as well as a band pass with left handed wave propagation [3.13-3.15].

Therefore, SRR and CSRRs based transmission lines act as the frequency selective structures with electrically small unit-cell dimensions. The concentration of this chapter is focused on the analyzing of one-dimensional CSRRs on microstrip transmission line and its equivalent circuits as a specific performance.

Figure 3.1 Topology of (a) SRR and (b) CSRRs with relevant dimensions.
3.1 Dual Transmission Line Approach: Equivalent Circuit Model and Limitations

Based on general transmission line concept, the fundamental electromagnetic properties of LHMs and the physical realization of these materials are reviewed. Because of the unavoidable RH properties on transmission line that occur naturally in practical LHMs, the CRLH TL structure is also analyzed. The study of LH transmission line is explained in the next topic as well as its characterization. Then the design and implementation of the CSRRs TL and their applications are presented.

➢ Analysis for Periodic LH Transmission Line Model

Periodic analysis of the LH TL model was first introduced by Eleftheriades [3.2]. It assumes that the periodic loaded elements are infinity. In this case, the dispersion characteristics of the LH TL model can be calculated by applying the standard periodic structure analysis for microwave periodic networks. $ABCD$ matrix is used to describe the transmission characteristics of a two port network through its input and output currents [3.1, 3.17].

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}$$  \hspace{1cm} (3.1)

For a wave propagating through a line, the voltage and current at the cells $(n+1)$ and $(n)$ are also related as

$$V_{n+1} = V_n e^{-\gamma d}$$ \hspace{1cm} (3.2)

$$I_{n+1} = I_n e^{-\gamma d}$$ \hspace{1cm} (3.3)

where $\gamma$ is the propagation constant and it evaluated as
\[ \gamma = \alpha + j\beta \]  

(3.4)

where \( \alpha \) is the attenuation constant and \( \beta \) is the phase propagation constant. From the previous equations, it can concluded that

\[
\begin{bmatrix}
A - e^{\alpha d} & B \\
C & D - e^{\alpha d}
\end{bmatrix}
\begin{bmatrix}
V_{n+1} \\
I_{n+1}
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]  

(3.5)

So, for a non-trivial solution, the characteristic equation for (3.5) is

\[
(A - e^{\alpha d})(D - e^{\alpha d}) - BC = 0
\]  

(3.6)

Making use of the relation of reciprocal for any two-port passive network,

\[ AD - BC = 1 \]  

(3.7)

Substituting the previous equations, the propagation condition can be written as

\[
\cosh(\alpha d)\cos(\beta d) + j\sinh(\alpha d)\sin(\beta d) = 0.5(A + D)
\]  

(3.8)

Then to analyze the propagation of a very short LH TL model, the planar equivalent circuit of a unit cell is introduced. The LH transmission line model has a section of a transmission line of length \( d \) and loaded with a series impedance \( Z \) and a shunt admittance \( Y \). For a lossless cell the equivalent circuit is shown in Figure 3.2.
In the following analysis, $Z$, $Z_0$ and $Y$ are the load series impedance, characteristic impedance and shunt admittance respectively. $\theta$ is the propagation angle of the hosting transmission line. The periodic length of the cells is assumed very small compared to the guided wavelength; otherwise the effect of the hosting elements should be compensated \[3.2\]. The $ABCD$ matrix of the periodic structure is expressed as cascade of three two port sections as \[3.1, 3.17\]

$$
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} =
\begin{bmatrix}
1 & \frac{Z}{2} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos\left(\frac{\theta}{2}\right) & jZ_0 \sin\left(\frac{\theta}{2}\right) \\
\frac{1}{jY_0} \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right)
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
Y & 1
\end{bmatrix}
$$

\[(3.9)\]

It can be concluded that
\[ A = \cos(\theta) + \frac{1}{2} ZY \cos^2\left(\frac{\theta}{2}\right) + j \frac{1}{2} (Z_0 Y + ZY_0) \sin\left(\frac{\theta}{2}\right) \]  \hspace{1cm} (3.10) \]

\[ B = Z \cos\left(\frac{\theta}{2}\right) + Y \left(\frac{Z}{2}\right)^2 \cos\left(\frac{\theta}{2}\right) - Z_0 \sin^2\left(\frac{\theta}{2}\right) + j \frac{1}{2} \left(Z Y Z_0 + 2Z_0 + 2\left(\frac{Z}{2}\right)^2 Y_0\right) \sin(\theta) \]  \hspace{1cm} (3.11) \]

\[ C = Y \cos^2\left(\frac{\theta}{2}\right) + jY_0 \sin\left(\frac{\theta}{2}\right) \]  \hspace{1cm} (3.12) \]

\[ D = A \]  \hspace{1cm} (3.13) \]

where \( Z_0 \) and \( Y_0 \) are the characteristic impedance and admittance of host transmission respectively, and \( \theta \) is the propagation angle of the hosting transmission line given as

\[ \theta = \beta d \]  \hspace{1cm} (3.14) \]

where \( \beta \) is the wave constant along the hosting transmission line.

From (3.8) and (3.10) to (3.13), the propagation condition can be written as

\[ \cosh(\alpha d) \cos(\beta d) + j \sinh(\alpha d) \sin(\beta d) \]

\[ = \cos(\theta) + \frac{1}{2} Z_L Y_L \cos^2\left(\frac{\theta}{2}\right) + \frac{1}{2} \left(Z_0 Z_L + Y_0 Y_L\right) \sin\left(\frac{\theta}{2}\right) \]  \hspace{1cm} (3.15) \]

For propagation criterion given when \( \alpha = 0, \beta \neq 0 \), the dispersion equation is
\[
\cos(\beta d) = \cos(\theta) + \frac{1}{2} Z_L Y_L \cos^2\left(\frac{\theta}{2}\right) + i \frac{1}{2} (Z_0 Z_L + Y_0 Y_L) \sin\left(\frac{\theta}{2}\right)
\] (3.16)

3.1.1 Left-Handed Transmission Line

3.1.1.1 Principle of Left-Handed Transmission Line

The dual of conventional transmission line is used to analyze the transmission line approach of LHMs, which the equivalent circuit is shown in Figure 3.3. Comparing with the RH- transmission line, The LH TL is contained with inductance/capacitance by inverting the series/parallel arrangements. The RH TL obviously exhibits low-pass band of frequencies in nature, whereas the LH transmission line exhibits high-pass band [3.1].

![Circuit model of a LH TL per unit length](image)

Figure 3.3 The circuit model of a LH TL per unit length [3.1]

The wave number \(\gamma(\omega)\) of such a line is derived from per unit length impedance

\[
Z' = Z / dz = (G' + j\omega C')^{-1}
\]

and admittance \(Y' = Y / dz = (R' + j\omega L')^{-1}\) as

\[
\gamma(\omega) = \alpha(\omega) + j\beta(\omega) = \sqrt{Z'Y'}
\]

\[
= \sqrt{(G' + j\omega C')^{-1}(R' + j\omega L')^{-1}}
\] (3.17)
\[
\alpha(\omega) = \frac{1}{2}\sqrt{\left[R'G' - \omega^2L'C'\right]^2 + \omega^2(L'G' + R'C')^2 + \left[R'G' - \omega^2L'C'\right]} \]  \hspace{1cm} (3.18a)
\]
\[
\beta(\omega) = -\frac{1}{2}\sqrt{\left[R'G' - \omega^2L'C'\right]^2 + \omega^2(L'G' + R'C')^2 + \left[R'G' - \omega^2L'C'\right]} \]  \hspace{1cm} (3.18b)

where \(G', C', R', L'\) are the values per unit length quantities. The equations (3.18a) and (3.18b) show the attenuation factor \(\alpha(\omega)\) and propagation factor \(\beta(\omega)\) where the negative sign in \(\beta(\omega)\) indicates a negative phase velocity and its nonlinearity indicates frequency dispersion. In addition, the characteristic impedance of the line is given by
\[
Z_c(\omega) = \frac{R' + j\omega L'}{\sqrt{G' + j\omega C'}} \]  \hspace{1cm} (3.19)

In the lossless case \((G' = R' = 0)\), equations (3.1a) and (3.18b) reduce to
\[
\alpha = 0 \]  \hspace{1cm} (3.20a)
\[
\beta(\omega) = -\frac{1}{\omega \sqrt{L'C'}} \]  \hspace{1cm} (3.20b)

\(\beta(\omega)\) exhibits the dispersion diagram shown in Figures 3.4 with the anti-parallel hyperbolic phase and group velocities which indicate the left-handedness [3.1].
\[
v_p = -\omega^2 \sqrt{L'C'} \quad \text{and} \quad v_g = +\omega^2 \sqrt{L'C'} \]  \hspace{1cm} (3.21)
The characteristic impedance simply becomes \( Z_c = \sqrt{L' C'} \), while the group delay will then be

\[
\tau_p = -\frac{d\varphi}{d\omega} + \frac{d(\beta p)}{d\omega} = \frac{p}{\sqrt{L'C'}} \cdot \frac{1}{\omega^2}
\]  

(3.22)

where \( p \) represents the physical length of the line. The group delay is in the relation form of \( 1/\omega^2 \) dependence, indicating the dispersion becomes increasingly larger when the frequency decreases. Considering equation (3.21), the multiplication of

\[
v_p \cdot v_g = -\omega^2 (L'C')
\]

is in a LH TL.

Figure 3.4 shows the dispersion diagram of a purely LH TL and purely RH TL. The LH area is illustrated on the negative area of dispersion (red), while the RH area is presented on the positive dispersion (blue).
In natural, because of unavoidable parasitic series inductance and shunt capacitance, the dispersion from the PRH contribution is increased with frequency. From dispersion graph, the backward wave propagation is existed along the LH line as presenting in negative values (red line). In this range, the line becomes a backward leaky-wave opposite of the case of conventional RH TL that the dispersion can be predicted in positive dispersion band (blue line).

### 3.1.1.2 Equivalent Material Parameters

Both $\mu(\omega)$ and $\varepsilon(\omega)$ values in LH material can be derived from the analogy between the plane wave solution in homogeneous material (Maxwell equations) as well as the wave along the LH TL (TL equations). In RH TL, the impedance and admittance of the material are supposed to $Z' = j\omega\mu$ and $Y' = j\omega\varepsilon$, respectively, which they have been changed to $Z' = 1/(j\omega C')$ and $Y' = 1/(j\omega L')$ in the LH TL as shown in equations (3.23a) and (3.23b)

\[
-\frac{1}{\omega C'} = \omega\mu \Rightarrow \mu(\omega) = -\frac{1}{\omega^2 C'}(0) \tag{3.23a}
\]

\[
-\frac{1}{\omega L'} = \omega\varepsilon \Rightarrow \varepsilon(\omega) = -\frac{1}{\omega^2 L'}(0) \tag{3.23b}
\]

These above equations both demonstrate the negative of $\mu$ and $\varepsilon$ in a LHM together with the relative dispersion of the frequency dependence by $1/\omega^2$. Although such a material does not seem to exist in nature, it might be realized artificially as MTMs using appropriate implants, as in recent attempts [3.1-3.5]. Only at the occurrence of frequency dispersion, the simultaneous negative values of $\varepsilon$ and $\mu$ can be realized. It
can be concluded that the LHMs with the dispersive constitutive parameters given by equations (3.23a) and (3.23b) satisfy the generalized entropy conditions for dispersive media [3.1], shown in below

\[
\frac{\varepsilon[\omega\mu(\omega)]}{\partial \omega} = \frac{1}{C'\omega^2} \right) \quad \text{and} \quad \frac{\varepsilon[\omega\varepsilon(\omega)]}{\partial \omega} = \frac{1}{L'\omega^2} \right) \quad (3.24)
\]

Then, the refractive index is derived by taking the square root of the product in (3.23a) and (3.23b), \( k = \sqrt{Z'Y'} = \sqrt{(j\omega\mu(\omega)j\omega\varepsilon(\omega))} = \omega\sqrt{\mu, \varepsilon_r} / c_0 \) in plane wave. For \( \beta = -1/(\omega\sqrt{L'C'}) \), it yields as:

\[
\frac{\omega}{c_0} \sqrt{\mu, \varepsilon_r} = -\frac{1}{\omega\sqrt{L'C'}} \Rightarrow \eta(\omega) = -\frac{c_0}{\omega^2 \sqrt{L'C'}} \left(0 \right) \quad (3.25)
\]

This last equation confirms the negative value of \( \eta(\omega) \) associated with the reversal of Snell’s law at the interface.

### 3.1.2 CRLH Theory

In this section, the transmission line approach to CRLH MTMs will be discussed. The equivalent circuit model of equivalent homogeneous CRLH TL can be shown in Figure 3.5. For simplicity, only the lossless case of transmission line will be analyzed. [3.1, 3.17]
Figure 3.5 Equivalent circuit model of homogeneous CRLH TL [3.1].

**Homogeneous Case**

The homogeneous model of a CRLH lossless transmission line shown in Figure 3.5 consists of an inductance $L’_R$ in series with a capacitance $C’_L$ and a shunt capacitance $C’_R$ in parallel with an inductance $L’_L$. The propagation constant of TL is given by $\gamma = \alpha + j\beta = \sqrt{Z’Y’}$, where $Z’$ and $Y’$ are the per-unit length impedance and per-unit length admittance, respectively. $Z’$ and $Y’$ of CRLH TL are defined as

$$Z’(\omega) = j\left(\frac{\omega L’_R - 1}{\omega C’_L}\right)$$

$$Y’(\omega) = j\left(\frac{\omega C’_R - 1}{\omega L’_L}\right)$$

Thus, the dispersion relation for a homogenous CRLH TL is

$$\beta(\omega) = s(\omega) \sqrt{\frac{\omega^2 L’_R C’’_R + 1}{\omega^2 L’_L C’_L} - \left(\frac{L’_R}{L’_L} + \frac{C’_R}{C’_L}\right)}$$

(3.26)
where

\[
s(\omega) = \begin{cases} 
-1 & \text{if } \omega \omega_1 = \min \left( \frac{1}{\sqrt{L_R' C_L''}}, \frac{1}{\sqrt{L_L' C_R''}} \right) \\
+1 & \text{if } \omega \omega_2 = \max \left( \frac{1}{\sqrt{L_R' C_L''}}, \frac{1}{\sqrt{L_L' C_R''}} \right)
\end{cases}
\tag{3.28}
\]

The equation (3.27) of phase constant $\beta$ can be made in the form of purely real or purely imaginary which is upon the sign of radicand. In the case of frequency range where $\beta$ is purely real, a pass band is present since $\gamma = j\beta$. While a stop band occurs in the frequency range where $\beta$ is purely imaginary since $\gamma = \alpha$.

This stop band is a unique characteristic of the CRLH transmission line, which is not clearly presented from the Purely Right-Handed (PRH) and the Purely Left-Handed (PLH) cases. In CRLH TL, the LH characteristics presents at lower frequencies and the RH properties at higher frequencies. From the dispersion diagram, the group velocity $\left( v_g = \partial \omega / \partial \beta \right)$ and phase velocity $\left( v_p = \omega / \beta \right)$ of these transmission lines can be extracted.
Considering this diagram, it is found that in a PRH transmission line, \( v_g \) and \( v_p \) are parallel \((v_g v_p, 0)\), whereas in PLH transmission line, \( v_g \) and \( v_p \) are antiparallel \((v_g v_p, 0)\). In conclusion, the CRLH transmission line has LH \((v_g v_p < 0)\) and RH \((v_g v_p > 0)\) regions. Moreover, Figure 3.6 also illustrates the stop-band that occurs when \( \gamma \) is purely real for a CRLH transmission line. In the case of the series and shunt resonances of the CRLH transmission line are equal which is called the balanced case, which is shown in equation (3.29)

\[
L'_R C'_L = L'_L C'_R
\]  

(3.29)

This means the LH and RH contribution are exactly balanced, shown in Figure 3.7

The impedance matching is achieved at the frequency where the Bloch impedance coincides with the reference impedance of the ports; while the phase matching occurs at those frequencies where the phase shift of the structure is a multiple of \( \pi \).
The ripple occurring in Figure 3.7 is consequence of periodicity and it may limit filter performance, which means the ripples increase at the edges of the band since the Bloch impedance takes extreme values. The electrical simulation has been obtained through ADS. From the example, the 30-cell CRLH TL has the resonate frequency at $f_0=3.18\,\text{GHz}$. The left handed cut off ($f_L$) is 1.31GHz and the right handed cut off frequency ($f_H$) is 7.71GHz, respectively.
The dispersion graph of the LH and RH region are connected in one line in the case of balanced CRLH TL. The LH band is presented on the $\beta d<0$, while $\beta d>0$ is demonstrated RH region. In addition, there is no dispersion at the resonate frequency of CRLH TL, shown in Figure 3.8.

Under the condition in equation (3.29), the propagation constant is reduced to the simpler form:

$$\beta = \beta_R + \beta_L = \omega \sqrt{L_R' C_R'} - \frac{1}{\omega \sqrt{L_L' C_L'}}$$  \hspace{1cm} (3.30)$$

where the phase constant of CRLH distinctly splits up into the RH phase constant $\beta_R$ and the LH phase constant $\beta_L$. 

---

Figure 3.8 Dispersion diagram of the 30-stage CRLH TL balance case in Figure 3.7
From the dispersion diagram, it can be noted that the phase velocity \( v_p = \omega / \beta \) of CRLH TL becomes increasingly while the frequency is higher. Moreover, the diagram illustrates the dual characteristic of the CRLH TL which at low frequencies the CRLH TL is dominantly LH, while at high frequencies the CRLH TL is dominantly RH. In balanced case of CRLH TL the dispersion diagram indicates that an LH to RH transition occurs at [3.1-3.5, 3.16]:

\[
\omega_0^{\text{unbalanced}} = \frac{1}{\sqrt{L' C_R' L' C_L'}} \quad \text{balanced} \quad \frac{1}{\sqrt{L' C'}}
\] (3.31)

The \( \omega_0 \) is referred to the transition frequency. Because of the purely imaginary of \( \gamma \) in the balanced case, the connecting of dispersion graph of the LH and RH line appeared. This means the curve of balanced CRLH TL dispersion does not have a stop band. Although \( \beta \) is zero at \( \omega_0 \), which corresponds to an infinite guided wavelength \( (\lambda_g = 2\pi / |\beta|) \), wave propagation still occurs since \( v_g \) is nonzero at \( \omega_0 \). In addition, at \( \omega_0 \) the phase shift for a TL of length \( d \) is zero \( (\phi = -\beta d = 0) \). Phase advance \( (\phi(0)) \) occurs in the LH frequency range \( (\omega(\omega_0)) \), and phase delay \( (\phi(0)) \) occurs in the RH frequency range \( (\omega(\omega_0)) \). Generally, the characteristic impedance of a normal transmission line is given by \( Z_0 = \sqrt{Z'/Y'} \). While in the CRLH TL, the characteristic impedance is

\[
Z_0^{\text{unbalanced}} = Z_L \sqrt{\frac{L_R' C_L' \omega^2 - 1}{L'_C C'_R \omega^2 - 1}} \quad \text{balanced} \quad Z_L = Z_R,
\] (3.32a)

\[
Z_L = \frac{L'_L}{\sqrt{C'_L}}
\] (3.32b)
\[ Z_R = \sqrt[2]{\frac{L'_R}{C'_R}} \]  
(3.32c)

\( Z_L \) and \( Z_R \) represent the PLH and PRH impedances, respectively. Consider the equation (3.32a), the characteristic impedance for the unbalanced case is frequency dependent; therefore, the balanced case is frequency independent. As stated previously, the propagation constant of a transmission line is \( \gamma = j\beta = \sqrt{Z'Y''} \). Due to the propagation constant of a material is \( \beta = \omega \sqrt{\mu \varepsilon} \), the following relation can be set up:

\[ \omega^2 \mu \varepsilon = Z'Y'' \]  
(3.33)

Similarly, the characteristic impedance of TL \( Z_0 = \sqrt{Z'Y''} \) can be related to the material’s intrinsic impedance \( \eta = \sqrt{\mu \varepsilon} \) by

\[ Z_0 = \eta \quad \text{and} \quad \frac{Z'}{Y''} = \frac{\mu}{\varepsilon} \]  
(3.34)

As equation (3.33), the permeability and permittivity of a material relate to the impedance and admittance of its equivalent transmission line model

\[ \mu = \frac{Z'}{j\omega} = L'_R - \frac{1}{\omega^2 C'_L} \]  
(3.35a)

\[ \varepsilon = \frac{Y''}{j\omega} = C'_R - \frac{1}{\omega^2 L'_L} \]  
(3.36b)
The refractive index \( \eta = c \beta / \omega \) for the balanced and unbalanced CRLH TL is displayed in Figure 3.9.

![Graphs showing refractive index for balanced and unbalanced CRLH TL](image)

**Figure 3.9** (a) Typical index of refraction for the balanced (green) and unbalanced CRLH TL (red-orange), (b) the refraction index of 30-stage CRLH TL in Figure 3.7

As illustrated in Figure 3.9, a refractive index of the CRLH TL in the LH range is negative, while it shows the positive values in the RH range (a) [3.1]. In balanced
case CRLH TL, the refractive index is equal to zero on the resonate frequency, displayed in Figure 3.9(b).

### 3.2 CSRRs Resonant Type of MTMs TL: Topology, its Equivalent Circuit and Synthesis

Implementation of resonant type approach can either use CPW or microstrip transmission line technologies. The CSRRs particles are etched in the ground plane, underneath the conductor strip so that the CSRRs can be excited by the time varying electric field of the quasi-TEM signal propagating in the line, while the time varying magnetic field in CSRRs is applied parallel to the plane of the particle [3.15, 3.18]. The topology of one cell CSRRs can be analyzed by lumped element equivalent T-circuit model depicted in Figure 3.10 which ignores the losses and inter-resonators coupling. The model is valid under the condition of small electrical size of CSRRs.

![Figure 3.10](image)

Figure 3.10 Basic cell of CSRRs-based transmission line (a) and equivalent circuit model, (b) The upper metallization is depicted in black; the slot regions of the ground plane and depicted in grey [3.19].
These resonators are extracted to parallel resonant tanks of inductance $L_c$ and capacitance $C_c$ [3.19-3.24], whereas their coupling to the host line is modeled by the capacitance $C$. The series gaps etching above the CSRRs to enhance line-to-CSRRs coupling are replaced by the capacitance $C_g$. The series impedance must be dominated by $C_g$ to achieve the left-handedness condition. In order to simplify the analysis of the equivalent circuit, the line inductance $L$ is neglected. Normally, $L$ may not be omitted to describe the accurate structure. These parameters will be used to model the layout of the CSRRs unit cell to obtain some specific targets.

The study of phase shift of the elemental cell and Bloch impedance; as shown in equations (3.37) and (3.38), confirms the highly dispersive of the structure:

$$\cos \phi = 1 + \frac{Z_s(j\omega)}{Z_p(j\omega)}$$

(3.37)

$$Z_B = \sqrt{Z_s(j\omega)[Z_s(j\omega) + 2Z_p(j\omega)]}$$

(3.38)

where $Z_s$ and $Z_p$ are the series and shunt impedance of the equivalent T-circuit model of Figure 3.10, respectively. The $Z_B$ and $\phi$ are the key electrical characteristics of artificial LH lines by CSRRs method, where $\phi$ has to set as the condition of $\phi = \beta l$, $\beta$ is the phase constant for the Bloch waves, at the operating frequency, while $l$ is the period of the structure. Therefore, the structure exhibits a band pass behavior with backward wave propagation in the allowed band from the analysis of equations (3.37) and (3.38). The flexibility for the artificial lines is dependent on these four parameters ($Z_s$, $Z_p$, $Z_B$ and $\phi$).
The analysis of the limitation of these circuit parameters are described as follows. In order to obtain the limit of this LH transmission band, the $Z_B$ or $\phi$ have to be forced to real numbers or positive values and possible for physical implementation according to:

\[
f_L = \frac{1}{2\pi} \left[ \frac{1}{L_c + \left( C_c + \frac{4}{\frac{4}{C_g} + \frac{4}{C}} \right)} \right] \quad (3.39)
\]

\[
f_H = \frac{1}{2\pi \sqrt{L_c C_c}} \quad (3.40)
\]

$f_L$ and $f_H$ represent the lower and higher frequency of the interval, respectively. At these frequencies, both the phase and Bloch impedance take extreme values, for instance; $Z_B \to \infty$ and $\phi = 0$ at $f_H$, whereas $Z_B = 0\ \Omega$ and $\phi = \pi$ at $f_L$. In order to wider the bandwidth, these $f_L$ and $f_H$ have to be set apart from the cut off frequency ($f_c$) as much as possible. Analysis of the intrinsic limits of the operative bandwidth is described as below;

From equations (3.39) and (3.40), the capacitance value of the series gap can be inferred by

\[
C_g = \frac{1}{2\omega_c Z_c} \sqrt{\frac{1 + \cos \phi_c}{1 - \cos \phi_c}} \quad (3.41)
\]
where $\omega_c = 2\pi f_c$. Then, the other parameters as follows can be extracted by using equations (3.37) to (3.40):

$$L_c = \frac{Z_c}{2} \sqrt{\frac{1 + \cos \phi_c}{1 - \cos \phi_c}} \frac{\omega_c}{\omega_h} \frac{\left(\omega_h^2 - \omega_c^2\right)}{\left(\omega_c^2 - \omega_L^2\right)}$$

(3.42)

$$C_c = \frac{1}{L_c \omega_h^2}$$

(3.43)

$$C = \frac{2 \omega_h^2 \left(\omega_c^2 - \omega_h^2\right)}{Z_c \omega_c \left[ (1 + \cos \phi_c) \left(\omega_h^2 - \omega_c^2\right) - 2 \omega_h^2 \left(\omega_c^2 - \omega_L^2\right) \right]}$$

(3.44)

where $\omega_L = 2\pi f_L$ and $\omega_H = 2\pi f_H$. As from equations (3.42) to (3.44), the values of $C_s$, $L_c$, and $C_c$ should be positive to provide the condition of $\omega_L \omega_c \omega_H$. Since the angular frequencies which $C$ may be negative, the operative bandwidth is limited by $C$ which has to force in the real positive number as shown in the condition:

$$1 + \cos \phi_c \frac{2 \omega_h^2 \left(\omega_c^2 - \omega_h^2\right)}{\omega_c^2 \left(\omega_h^2 - \omega_L^2\right)}$$

(3.45)

The value of the operating angular frequency, $\omega_c$, and phase, $\phi_c$, only limit the range of $\omega_L$ and $\omega_H$. At the phase $\phi_c = \pi/2$, following this rule, by neglecting the $L$ parameter, the intrinsic limits for $\omega_L$ and $\omega_H$ are $1/\sqrt{2} \omega_c \omega_L$ and $\omega_H \omega_c$ which might not possible for physical implementation. Thus at the operating frequency, the
possibility to control phase and line impedance with a single cell structure will be analyzed later.

Another interesting point is the transmission zero frequency, \( f_Z \), which is extracted from the simulation or experiment results. At the transmission zero frequency, the value of phase will be under the condition of \( \phi = \pi / 2 \), or \( f_{\pi/2} = \omega_{\pi/2} / 2\pi \), then after consider the mentioned condition, the latter condition can be demonstrated as:

\[
Z_{s}(j\omega_{\pi/2}) = -2Z_{p}(j\omega_{\pi/2})
\]  (3.46)

Another frequency point, where the second allowed (right handed) band starts up, is the operating frequency \( f_c \). At the operating frequency, the phase variation can be represented by \( \theta_c = \theta(f_c) \) and the image impedance is \( Z_c = Z_B(f_c) \). These frequencies are given by the following expressions:

\[
f_Z = \frac{1}{2\pi \sqrt{L_c (C + C_c)}}
\]  (3.47)

\[
f_c = \frac{1}{2\pi \sqrt{LC_g}}
\]  (3.48)

In order to extract the parameters at the resonant frequency of the CSRRs, the null condition of the shunt admittance has been forced, while the impedance from the input port is set by adding the output impedance (50\(\Omega\)) and the reactive impedance of the \( L \) and \( C_g \). Using equations (3.40), (3.47), and (3.48), the three elements of the shunt reactance can be extracted. Because the line inductance, \( L \), cannot be ignored,
the $C_g$ must be considered as an effective capacitance by using the simulation of the equivalent circuit model.

It is noted from all the previous equations that the bandwidth is limited by the achievable values of $C$ and $C_g$ and it is impossible to simultaneously obtain abrupt transition bands at the lower and upper edges of the LH band, in addition, the in-band ripple is expected as consequence of periodicity. While the transmission zero frequency is forced to be nearer the lower edge of the band, the gap capacitance $C_g$ have to make highly as comparing to the coupling capacitance $C$. As a result, the appearance of a highly selective filter is provided at the lower band edge. As refer to equations (3.39) and (3.40), it appears that the bandwidth is dependent on the values of $C_g$ and $C$, and it is difficult to design wide (or even moderate) band structures [3.22].

3.3 Electric, Magnetic Coupling and the new Equivalent Circuit Model

All previously published works of CSRRs have assumed that the coupling between the rings in adjacent cells is dominated by the electric coupling and magnetic coupling can be neglected [3.19-3.24]. However, it is revealed that there are strong magnetic couplings between the adjacent rings, resulting in significant effects on circuit performance. This section presents a new equivalent circuit model. The model has been developed and verified both numerically and experimentally to take into account of the magnetic coupling between the rings in adjacent cells, which has been neglected so far.
To observe this effect, a simple 2-cell CSRRs microstrip TL, shown in Figure 3.11 was designed and analyzed, aiming to understand the coupling effects between the CSRRs.

![Diagram](image)

Figure 3.11 (a) The topology of CSRRs unit cell and (b) 2-cells CSRRs microstrip TL and its magnetic field distribution at 2.4GHz.

The example in Figure 3.11 demonstrates the qualification of the 2-cell CSRRs when added the series gaps on the strip line. As shown in this figure, the narrow pass band is occurred and this also generates the left hand wave. The Rogers RO3010 substrate has been chosen (thickness $h=1.27\text{mm}$, $\varepsilon_r=10.2$) for this electromagnetic simulation.
In the design the transmission line strip width is of 3.2mm, the thickness of substrate 1.27mm, the external radius of the ring ($R_{ext}$) 3.5mm, the metal ring width ($c$) 0.4mm, and the gap between the inner and outer rings ($d$) 0.3mm. The total length of the 2-cell CSRRs line is 15mm. Separation between the two cells is initially set to 0.2mm and finally reaches to 1mm by increasing 0.2mm at each step.

![Electric_field](image1)

(a)

![Magnetic-field](image2)

(b)

Figure 3.12 Field variations with ring separation of the two CSRRs (a) Electric field and (b) Magnetic field at 2.4GHz
Electric and magnetic fields at the centre point between the two rings are shown in Figure 3.12(a) and (b), respectively, illustrating that there exists strong magnetic coupling between the adjacent cells apart from electric coupling especially when the cell separation is reducing. This magnetic coupling, which has so far been neglected in all previously published works, must be included in the equivalent circuit model of a CSRRs TL for more accurate circuit modeling and design.

A new equivalent circuit model for a 2-cell CSRRs TL has been developed as shown in Figure 3.13. A transformer, \( N \), is introduced to take into account the mutual magnetic coupling effects between the cells. The results in Figure 3.14 prove that magnetic coupling has an influence in changing S-parameters by giving the cut off frequency more correctly as compared to the full wave electromagnetic simulation (HFSS), therefore it should be recommended in the equivalent circuit in order to offer more accurate results (0.2mm is set as a gap between the two CSRRs).

![Figure 3.13 Equivalent circuit model of a two adjacent-cell CSRRs TL](image)

It can be seen that the two are agreed reasonably well (loss mechanism was not included in the circuit model). Extracted element values with three different
separations between the two cells are given in Table 3.1. It can be seen, as expected, from the table that $C_M$ and $N$ change their values whereas all other element values do not vary while the distance between the two CSRRs cells changes.

Figure 3.14 (a) The S-parameters of the 2-cells CSRRs TL with cell separation of 0.2mm and (b) Smith Chart, equivalent circuit model and full wave simulation.
It has been found that the magnetic coupling which takes into account in the case of
the two adjacent CSRR cell can provide the $S_{11}$ graph obviously closer to the $S_{11}$
from HFSS full wave simulation at 2.27GHz. If considering the circuit simulation
without taking into account of the magnetic coupling, the $S_{11}$ graph (dash line)
displays the centre frequency at 2.3GHz which is not exposed the precise centre
frequency. On Smith Chart, the $S_{11}$ of the new equivalent circuit provides a better
match of the graph especially on the passband than the equivalent circuit without
magnetic coupling. The rational of transformer proves that there is strong magnetic
coupling between the rings when they are closely placed.

Table 3.1 Extracted Element Parameters for the 2-cell CSRRs TL

<table>
<thead>
<tr>
<th>Distance between Cell (mm)</th>
<th>$C_1$(pF)</th>
<th>$C_2$(pF)</th>
<th>$L_1$(nH)</th>
<th>$L_2$(nH)</th>
<th>$C_M$(pF)</th>
<th>N(turns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.3573</td>
<td>2.5075</td>
<td>0.8092</td>
<td>1.57</td>
<td>5.4</td>
<td>1.36</td>
</tr>
<tr>
<td>0.5</td>
<td>1.3573</td>
<td>2.5075</td>
<td>0.8092</td>
<td>1.57</td>
<td>4</td>
<td>1.12</td>
</tr>
<tr>
<td>1</td>
<td>1.3573</td>
<td>2.5075</td>
<td>0.8092</td>
<td>1.57</td>
<td>0.6</td>
<td>1.09</td>
</tr>
</tbody>
</table>

3.4 Analyzing the LH operating area of CSRRs TL

As mention at the beginning of this chapter, a CSRRs applied with series gap on host
line exhibits LH properties. In order to verify the Left handed characteristics, the LH
region of CSRRs TL has to be designated. In this section, a 4-unit cell CSRRs
transmission line, shown in Figure 3.15, was represented for consideration. The 4-
cell CSRRs TL has been fabricated on the Rogers RO3010 substrate, with a thickness $h$ of 1.27mm, dielectric constant $\varepsilon_r=10.2$, $r_{ext}=3.4$mm, $c=0.4$mm, $d=0.3$mm. The width of the TL was designed to achieve 50$\Omega$ characteristic impedance at the operating frequency.

Based on the analysis in magnetic coupling, the geometry of a 4-cells CSRRs TL placed by cells 1-2 is 1mm, cells 2-3 2mm, cells 3-4 1mm, as shown in Figure 3.15 (a).

![Figure 3.15](image)

(a) The prototype of 4-cell CSRRs TL, (a) backside view and (b) top view.
CSRRs TLs has a total physical length of 35 mm and were fabricated. The S-parameters were measured using Agilent E8364A Network Analyzer, shown in Figure 3.16. The lower passband and higher passband of the CSRRs TL are measured to be 2.2 and 2.5 GHz respectively, displayed as narrow passband on Figure 3.16(a), the centre of operating area gives highest transmitted signal with lower loss; however there are some substrate and metal losses. The insertion loss at this point is approximately 2.49 dB. In addition, the phase compression has appeared within the LH passband (2-3 GHz) of the CSRRs TL, shown on Figure 3.16(b).

Figure 3.16 The measured results of (a) S-parameters of 4 CSRRs cells and (b) Phase of $S_{21}$. 
The circuit model as shown in Figure 3.13 is used here again to extract the loading parameters ($C_1$, $C_{sub}$, $C_2$, $L_2$, and $L_1$). These parameters were used to model a cell of CSRRs before cascading them as 4 cells to fit with the measurement. By fitting the curve with the measurement results, the extracted parameters can be found as ($C_1=1.1787\,\text{pF}$, $C_{sub}=1.4476\,\text{pF}$, $C_2=3.007\,\text{pF}$; $L_2=1.276\,\text{nH}$, $L_1=1.009\,\text{nH}$, $N=1.08$ and $C_M=0.59\,\text{pF}$), seen in Figure 3.17. The S parameters ($S_{21}$ and $S_{11}$) of both measurement and simulation agree well. The passband has been correctly predicted.

![Figure 3.17 The S-parameters ($S_{21}$ and $S_{11}$) by circuit simulation and measurement](image)

The discrepancies between the simulated and measured data in the stop band are probably due to the fabrication tolerance in our PCB laboratory, especially the alignment between the metal strip on the top of the substrate and the CSRRs on ground plane. However, the new equivalent circuit model that takes into account of
magnetic coupling effect between the CSRRs, which has been neglected so far in published works, is proposed and verified both numerically and experimentally.

In order to indicate the LH area, the 4-cell CSRRs TL is analyzed by dispersion criterion, shown in Figure 3.18. By taking the simulated S parameters, the dispersion of the 4-cell CSRRs TL can now be plotted and be shown in Figure 3.18. It can be seen that the $\omega$-$\beta$ dispersion diagram can be used to confirm LH and RH characteristics of the device - the desired passband (~2.2-2.55GHz) is in the LH area (below lower cutoff frequency $f_{c1}$) whereas the undesired passband (4.7-7GHz) is in the RH area (above higher cutoff frequency $f_{c2}$).

![Figure 3.18 Dispersion diagram of 4-cell CSRRs TL.](image)
The dispersion graph exhibits a very different behavior of the CSRRs TL especially in the LH passband that provides higher dispersion slope comparing to the RH area. This means when the input wave gets close to the left hand side of the lower cutoff ($f_{c1}$), the observed wave is obviously transformed the shape of the original wave since the high dispersion which cannot be predicted as in the ideal TL case. This behavior is explained by the fact that the backward wave effect of the periodic (artificial) MTM transmission line which becomes dominant over the ideal TL. At the dispersion effect higher than $f_{c1}$; the dispersion $\omega-\beta$ curve exhibits a zero slope there, corresponding to $v_g=0$, until reaching the RH area.
3.5 Conclusion

In this chapter, we have derived the three-type TL; the purely LH TL, the CRLH TL, and the CSRRs TL. The dispersion properties versus frequency of them have been analyzed, including their phase and group velocities. In LH area, the sign of phase and group velocities in LH TL are opposite, unlikely in RH area owing to the backward wave propagation. Moreover, it is found that both CRLH and CSRRs TL provide LH and RH area. In balanced CRLH TL case, the LH and RH area can be merged to exhibit broader passband. Although the equivalent circuit of CSRRs TL has been analyzed in many years, the magnetic coupling is still not taken into account to the equivalent circuit. This chapter also recommends the new equivalent circuit model of the CSRRs TL under the two more unit cell adjacent. The simulated scattering parameters are presented in the case of with and without the magnetic coupling. The results show that the magnetic coupling should be added in the equivalent circuit model as a good agreement of both simulation and measurement results as well as the indicating of LH and RH area.
3.6 References


CHAPTER 4

METAMATERIAL DELAY LINE USING COMPLEMENTARY SPLIT RING RESONATORS

INTRODUCTION

Digital communications have become more and more important in our life; as a result highly complex data processing is required. Group delay seems to be a main problem that degrades system performance for frequency selective components. There have been many works in analyzing and suppressing group delay on signal distortion [4.1-4.8] such as a loaded transmission line with a varactor diode [4.5], a surface acoustic wave (SAW) [4.6], and a magneto-static wave (MSW) [4.7, 4.8]. In communication systems, there are many sources, especially co-channel interference, leading to interference between communication channels. Recently, a controlled signal delay has been used in signal processing instead of simply suppressing the group delay [4.9-4.14].

However, the characteristic of metamaterial CSRRs lines as dispersive transmission lines in group delay has not been well developed. Due to its compactness and unique electrical properties, the prototype of the 4-cell CSRRs TL has been investigated as a passive delay line. The simulated and measured group delay and signal delay results are well established with the dispersion relation of CSRRs TL. Also, a significantly larger group delay of the CSRRs TL than the conventional TL is presented. Furthermore, the active tunable delay line has been studied by embedding the 4-cell
CSRRs TL with varactor diodes. By observing a fixed frequency, the group delay can be obtained.

4.1 Group Delay and Dispersion on CSRRs Transmission Lines

4.1.1 Group Delay and Systems

Group delay is a time used for transmitting in a system. In other words, it is a measurement of how long the signal, in time, takes to transit a system. Normally, it is mainly a function of system properties. [4.15, 4.16]

![Figure 4.1 Transfer function block diagram $H(j\omega)$][1]

In linear system, the transfer function of signal in frequency domain is represented as

$$H(j\omega) = A(j\omega)e^{ip(j\omega)}$$

The group delay (G.D.); represented as \( \tau \), is a time dimension which relates to the changing rate of a total phase shift with angular frequency as follows;

---

[1]: image_url
\[ G.D. = \tau(\omega) = -\frac{\partial \phi(\omega)}{\partial \omega} \]  \hspace{1cm} (4.2)

where \( \phi \) and \( \omega \) are the total phase shift and the angular frequency, respectively.

Group delay; sometimes called “envelope delay”, also can be expressed as the form of a differential equation of phase variation at the operating frequency. Group delay can be described as the time delay of the amplitude envelope of a narrow group at the frequencies around specific frequency.

As shown in equation (4.2), the group delay is negatively proportional to frequency. The delay signal \( \tau \) represents as a function of

\[ H(j\omega) \approx e^{-j\omega\tau} \]  \hspace{1cm} (4.3)

Which shows that the envelope of the signal delay is a form of group delay.

4.1.2 The dispersion properties and group delay of a CSRRs TL

Dispersion is the phenomenon where the original wave change of waveform after propagating through a medium which is dependent upon that wave’s frequency. This phenomenon also provides a difference in phase velocities due to different frequencies. However, in some specific materials, the dispersion can be controlled and functionalized with various phase manipulation effects [4.17-4.19].
In order to compare the group delay of CSRRs TL, the fundamental mode of dispersion with relation to MTMs TL can be analyzed as [4.20]:

\[
\beta^{\text{CSRR}}(\omega) = \beta^{\text{RH}}(\omega) + \beta^{\text{LH}}(\omega) = \frac{\omega}{\omega_R} - \frac{\omega}{\omega_L}
\]

(4.4)

, with \( \omega_R = 1/\sqrt{L_RC_R} \) and \( \omega_L = 1/\sqrt{L_CL_L} \)

where \( \omega_R \) and \( \omega_L \) are the cut off frequency of right-handed and left-handed passbands.

The first term is represented as the conventional part of the line, while the last term is the effect by left-handed wave propagation. Therefore, in a LH passband, the phase velocity \( (v_p) \) and the corresponding group velocity \( (v_g) \) are

\[
v_p = \frac{\omega}{\beta} = \frac{-\omega^2}{\omega_L}
\]

(4.5)

\[
v_g(\omega_c) = \left( \frac{d\beta(\omega)}{d\omega} \right)^{-1} \bigg|_{\omega_c} = \frac{\omega_c^2}{\omega_L}
\]

Then, the group delay in a CSRRs TL on a LH area can be given as [4.20, 4.21]:

\[
\tau_g(\omega_c) = -\frac{d\phi}{d\omega} = +\frac{d(\beta_n)}{d\omega} = N \left[ \frac{\omega_L}{\omega_c^2} \right]
\]

(4.6)

Where \( \phi \) is the transmission phase, \( \omega_c \) is the observed frequency on the LH passband and \( N \) is the number of CSRRs cells.
In the case of a conventional TL, the group delay is given only by the RH effect which can be presented as:

\[ \tau_g = l \left[ \frac{1}{\omega_R} \right] \]  \hspace{1cm} (4.7)

where \( l \) is the length of transmission line.

As shown in equation (4.6) and (4.7), the group delay of any signals travelled through a CSRRs TL can be varied in the LH area which is dependent on the input signal frequency (\( \omega_c \)) and a number of CSRRs cells (\( N \)). Whereas, the delay time of different signal frequencies through a conventional TL is constant. Hence at a LH certain frequency band, a CSRRs TL can provide different time delay by frequency function. Therefore, by fixing the parameter \( N \) with 4-cell CSRRs in this chapter, a signal delay at the another end of the CSRRs TL are observed.

### 4.2 Passive CSRRs Model and Delay Line Design Procedures

Due to unique dispersion characteristics of MTM TLs, as compared to conventional TLs, the group delay and their signal delays of the 4-cell CSRRs microstrip TLs are analyzed by both simulation and measurement. Since the CSRRs loaded transmission line is fixed, both applied models are called “Passive model delay line”.
Figure 4.2 (a) The topology of a unit cell CSRRs TL and Photographs of the designed 4-cell CSRRs TL;(b) ground view, (c) microstrip top view

Figure 4.2 illustrates the dimensions of designed 4-cell CSRRs delay line which is implemented on the Roger (RO3010) substrate with $\varepsilon_r = 10.2$ and 1.27mm thickness. In the design the transmission line strip width is adjusted to obtain 50Ω matching network, the external radius of the ring ($R_{ext}$) 3.4mm, the metal ring width ($c$) 0.4mm, and the gap between the inner and outer rings ($d$) 0.3mm. The total length of the 4-cells CSRRs TL is 35mm. The equivalent circuit model of this optimized 4-cell CSRRs delay line is shown in the previous chapter. The S-parameter measurement is carried out using Network Analyzer (Agilent E8364A).
The simulated and measured frequency responses of the designed 4-cell CSRRs TL are shown in Figure 4.3. The appearance of the LH passband, indicated in the previous chapter, has the bandwidth of 300MHz from 2.2GHz to 2.5GHz, which provides the limitation in the system of that frequencies can travel through this dispersive CSRRs delay line.

![Figure 4.3 The S-Parameters of the 4-cell CSRRs TL by both simulations and measurement.](image)

It should be noticed that the substrate and metal losses have been taken in account in the HFSS simulation, which is not included in the equivalent circuit model of ADS simulation; however, it can be seen that the passband has been correctly predicted. The discrepancies between the simulated and measured data in the higher frequency band are probably due to the fabrication tolerance in our PCB laboratory, especially the alignment between the metal strip on the top of the substrate and the CSRRs.
etched on the back of it. The measured transmission coefficient seems having narrower bandwidth.

Figure 4.4 the group delay on passband of the 4-cells CSRRs TL by both simulations (dot and dash blue) and measurement (solid blue) as well as a conventional TL at length=35mm (red), respectively.

Figure 4.4 shows the equivalent circuit model, full wave simulation and measured delay time as a function of frequency within the passband of the designed 4-cell CSRRs TL(blue). With consideration to the operating frequency band, there are some differences on the level of delay time with respect to the measured result. This provides a higher group delay than both simulations which is due to the losses by fabrication, environment, and measurement, etc which take in to account the measured results. However, the relationship of these two simulated group delay graphs is parallel which supports the simulations and their measured results.
Especially, the HFSS full wave simulation results (dash blue), the group delay graph seems to get closer to the measured group delay result (solid blue).

In both simulated and measured results, the slope of group delay obviously rises at the frequency near the lower cutoff frequency which is 2.2GHz at 4.6ns, (by measurement) 2.3GHz at 3ns and 2.4GHz at 2ns, respectively. At 2.4 and 2.5GHz, the delay can be assumed to be stable at 2ns which supports the analyzed group delay relation happening in LH area of the delay line that it is inversely proportional with respect to frequency.

For comparison purpose, the full wave simulated group delay of the conventional TL with the same length (35mm) is represented as RH TL in Figure 4.4 (solid red). Its group delay tends to be stable at 0.3ns all frequencies which presents as 10 times of the CSRRs TL when observing at the mid passband 2.3GHz. These significantly group delay results are very useful for size and cost reduction in communication system especially in radar applications.

Delay lines can be very useful in signal processing such as for some radar or telecommunication applications. It is necessary to multiplex the multiple signals at the transmitter in the form of a single, complex signal and de-multiplex each individual signal at the receiver end. The diagram of a simple RF system is shown in Figure 4.5.
Figure 4.5 The dispersive delay line in a simple RF system [4.13].

The analyzing delay signal was designed on Advanced Design System (ADS) which was simulated by loading the measured S-parameters of 4-cell CSRRs TL to the data block. Then, the three sinusoidal continuous waves (CWs) with magnitude 1V at frequencies of 2.2, 2.3 and 2.4GHz were fed into the proposed CSRRs TL.

Figure 4.6 The three continuous wave (CW) output signals in Time domain of 4-cell. CSRRs delay lines by measurement: $f_{cw1}=2.2$GHz (blue), $f_{cw2}=2.3$GHz (pink), and $f_{cw3}=2.4$GHz (red), respectively.
At the output end, the different signals in phase and times are extracted and shown in Figure 4.6. Because the CSRRs TL acts as a narrow bandpass filter, the delay of a continuous wave cannot obviously be indicated. However, the signal properties and delays can be analyzed by observing the steady state of the signals.

Due to the relation between dispersion and group delay, the three sinusoidal waves can clearly be seen both time differences of their amplitudes and phases at the output port. The CW frequencies that are nearer the higher cutoff frequency (2.3 and 2.4GHz), presents less suffering from dispersion and consume less delay than the CW frequency (2.2GHz) which is near the lower cutoff frequency.

In order to present the clear view of group delay in CSRRs TL, the pulse signals, which are represented as digital data in communication systems, are fed in the system in Figure 4.5. It is shown that the modulated pulse with carrier $f_{c2}=2.5$GHz (solid blue) is delayed less as it transmits with the peak propagated time at 5ns, whereas the modulated pulse with the lower carrier $f_{c1}=2.25$GHz (dot red) has a time delay at 6ns which displays the peak voltage, illustrated in Figure 4.7.
Figure 4.7 the modulated output signals with different carriers ($f_{c1}=2.25\text{GHz}$ in dot red and $f_{c2}=2.5\text{GHz}$ in solid blue).

In order to make a clear view of the signal delay and group delay, the envelope detection of the two input pulses have been considered in the magnitude voltage and time relation, illustrated in Figure 4.8.

It is obvious that the pulse signal of carrier frequency at 2.25GHz (dash pink) consumes the time via the 4-cell CSRRs TL more than the pulse signal of the carrier frequency at 2.5GHz (red) by 1ns delay difference and 3.5ns from the midpoint of the original pulse. These time differences between the input and output pulses are matched with the group delay graph in Figure 4.4.
Figure 4.8 The Envelopes of RF Pulse $f_{c1}$ and $f_{c2}$ in time domain compared to input after travelling through the 4-cell CSRRs by the measured S-parameters

Therefore, the signals at the output of the delay line can be functioned and extracted in both CW and Pulse signals. It can be seen clearly that the signals sent through the CSRRs TL provide different delays for different frequency signals and a much longer delay with much compact size in comparison with a conventional TL, the more complex data can be sent at the same time without increasing the system complexity.
4.3 Active CSRRs Model and Delay Line

There are many reviews that present a varactor diode for a tuning part in transmission lines. Especially in the recent years, the applied diodes for a large group delay have been reviewed [4.23, 4.24]. Besides the passive tunable CSRRs delay line, that is presented previously, the active tunable CSRRs delay line, by embedding varactor diodes into the 4-cell CSRRs TL is now performed. Because the varactor diode can act as a small inherence capacitor after changing the applied voltage, the four varactor diodes (BB833 Infineon Technologies) are placed in the 4-cell CSRRs TL as demonstrated in Figure 4.9(a). \( L_{\text{choke}} \), representing a thin wire linked between the varactor diode and the strip, \( R_{\text{var}} \) and \( C_{\text{var}} \) representing the varactor loss and junction capacitor, respectively.

The fabricated active tunable CSRRs TL for 4-cascaded cells is depicted in Figure 4.9(b) and (c) by following the dimensions of the 4-cell CSRRs TL. The measurement is carried out using Agilent E8364A Vector Network Analyzer. The DC bias voltages vary from 0V to -20V.
Figure 4.9 (a) The equivalent circuit of two adjacent CSRR cells with varactor diodes, (b) and (c) the photographs of the fabricated 4-cells CSRRs active delay line on both sides.

This new configuration can be tuned to provide varying delay time and passband. When more negative DC voltages are applied, the diode capacitance reduces and the group delay is prolonged. Figure 4.10 depicts the insertion loss for the different DC bias.
The insertion response (dB) after DC bias voltage of 4-cell CSRRs TL is illustrated in Figure 4.11. By observing a specific carrier frequency of 2.03GHz, it can be seen that the diodes begin to have significant effects as the DC bias varied from -10V to -20V. The DC changes the delay from 0.6ns to 5.6ns. However, owing to the diode addition, which leads to the changing of the characteristic impedance, it is also observed that the passband of CSRRs TL has moved toward higher frequencies and suffered from addition loss due to the inherit diode properties, as illustrated in Figure 4.10.

Figure 4.10 The insertion response (dB) after DC bias voltages of 4-cell CSRRs TL
Figure 4.11 Measured delay time of the 4-cell active CSRRs delay line at the frequency of 2.03GHz with different applied voltages

It is worth pointing out that the group delay of the 4-cell CSRRs delay line can be tuned without increasing the overall physical dimensions. This tunable CSRRs delay line, with embedded varactor diodes, provides the tuning rate of 0.5ns/V from -10 to -20V (bias).
4.4 Conclusion

In this chapter, the 2 types of CSRRs delay line have been presented; the passive CSRRs delay lines and the active CSRRs delay line.

In passive CSRRs delay lines, the compact model, consisting of 4-cell CSRRs particle, has been demonstrated. It is found that the group delay of 4-cell CSRRs TL, are dependent on the frequencies over the left-handed passband. Then, the two waveforms, represented as the continuous wave and digital signal, are investigated, in order, to verify the signal delays in CSRRs TL and group delays. The measured results confirmed the group delays of LH passband are related as a function of frequency. The 35mm length of 4-cell CSRRs delay line displays the delay of signal 2.25GHz modulated pulse, at 6ns and 2.5GHz modulated pulse, at 5ns, relating to the group delay graph, respectively. In addition, the 35mm long 4-cell CSRRs TL is presented 3ns delay at 2.3GHz which is approximately 10 times higher than those can be provided by a conventional TL with the same length, which means the shorter TL can be used in the system to obtain the same delay as in conventional case.

The later remarkable point in this work is the experimental demonstration of the tunable delay line by embedding varactor diodes into the model of 4-cell CSRRs TL. When DC bias varies from -10V to -20V, the average tuning rate 0.5ns/V is presented. Even though, this method leads to the mismatching of characteristic impedance of transmission line, the loading of varactor diode can make a large change of group delay with compact size, which can be proved very useful especially in for various radar and communications.
4.5 References


CHAPTER 5

FILTER THEORY AND ITS APPLICATION WITH CSRRS

INTRODUCTION

Wireless communication has been an important role in human’s life for the last few decades. Owing to the rapid growth of communication technologies, there have been many researches to develop devices and circuits for its applications in multiple frequency bands [5.1-5.5]. In addition, the requirement of reducing interference between channels must be met for all communication systems. Microwave filter is an essential part to get rid of unwanted signal from the system and allow the signal in specific frequency to pass through. Planar filters are widely used because of the convenience in manufacturing, low profile and ease to apply with microwave integrated circuits.

It is essential to design compact filters with high frequency selectivity, high rejection at the band edge as well as wide rejection band. To achieve these demands, many design techniques have been reported [5.6-5.11, 5.17, 5.18]. Since resonators are the basic components of planar filters and from the view point of this filter design section, filters with CSRRs have been introduced to accomplish above requirements [5.12, 5.13, 5.19-5.22]. Generally, CSRRs with microstrip line presents the narrow band properties [5.14-5.16]. In addition, CSRRs on microstrip line with a series gap presents a passband frequency [5.27]. For this reason, a key contribution of this
Chapter is to provide microstrip wideband filters in L-Band frequency spectrum from 0.9-1.9GHz with good frequency selectivity by the aid of rectangular CSRRs and their coupling properties.

5.1 Definitions and Fundamentals of Filters

As mention above, filters are presented as a very essential component in radio transmitter/receiver systems for recovery signals. On other words, it can be defined as “a transducer for separating waves on the basis of their frequencies”. There are 3 main types of filter; Active filter (need external source for operating), passive filter (no requiring external source for operating), and hybrid filter [5.23]. The two-port network and lumped elements have been used to distribute filters and their frequency responses [5.1, 5.23, 5.24].

5.1.1 2-port network analysis

Most RF/microwave components can be represented by a two-port network as shown in Figure 5.1. The various parameters have been used to analyze the two-port network, for instance; network variables, scattering parameters(S-parameters), and short-circuit admittance parameters (Y-parameters), open-circuit impedance parameters (Z-parameters), and ABCD parameters. These parameters can be converted to each other. The S-parameters are used to present in this section which are comprised of:

- The Input Reflection Coefficient with the output port terminated by a matched load or $S_{11}$ (Return loss)
- The Forward Transmission (insertion) gain with the output port terminated in a matched load or $S_{21}$ (Insertion loss)
The Reverse Transmission (insertion) gain with the input port terminated in a matched load or $S_{12}$

The Output Reflection Coefficient with the input terminated by a matched load or $S_{22}$

Figure 5.1 Two-port network with the input reflection coefficient and the output reflection coefficient [5.1].

The 2 port network, shown in Figure 5.1, has the parameters $V_1$, $V_2$ and $I_1$, $I_2$ which are voltage and current variables in complex amplitudes at port 1 and port 2, respectively. $Z_{01}$ and $Z_{02}$ are the terminal impedances and $V_s$ is the voltage generator. At port 1, the sinusoidal voltage is given by [5.1]

\[ V_1(t) = |V_1| \cos(\omega t + \phi) = |V_1| e^{j(\omega t + \phi)} \]  \hspace{1cm} (5.1)

Therefore, the complex amplitude can be given by

\[ V_1 = |V_1| e^{j\phi} \]  \hspace{1cm} (5.2)
The wave variables $a_1$, $b_1$ and $a_2$, $b_2$ in Figure 5.1 are introduced for simplying the voltage and current of the 2 port network which $a$ is signified as the incident waves and $b$ is represented as the reflected waves. Thus, the voltage and current are

$$V_n = \sqrt{Z_{0n}} (a_n + b_n) \quad \text{and} \quad I_n = \frac{1}{\sqrt{Z_{0n}}} (a_n - b_n), \quad n=1 \text{ and } 2 \quad (5.3)$$

or

$$a_n = \frac{1}{2} \left( \frac{V_n}{\sqrt{Z_{0n}}} + \sqrt{Z_{0n}} I_n \right) \quad \text{and} \quad b_n = \frac{1}{2} \left( \frac{V_n}{\sqrt{Z_{0n}}} - \sqrt{Z_{0n}} I_n \right), \quad n=1 \text{ and } 2 \quad (5.4)$$

The scattering parameters of 2 port network are represented in the terms of wave variables as

$$S_{11} = \frac{b_1}{a_1} \bigg|_{a_2=0} \quad \text{and} \quad S_{12} = \frac{b_1}{a_2} \bigg|_{a_1=0}$$

$$S_{21} = \frac{b_2}{a_1} \bigg|_{a_2=0} \quad \text{and} \quad S_{22} = \frac{b_2}{a_2} \bigg|_{a_1=0} \quad (5.5)$$

While $a_n = 0$ in the case of balanced impedance and no wave reflection at port $n$, the analysed matrix form is:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (5.6)$$

Both $S_{11}$ and $S_{22}$ are the reflection coefficient, whereas $S_{12}$ and $S_{21}$ are the reverse and forward insertion coefficient, respectively. These specific parameters are used to analyse the components in microwave frequency.

The amplitude and phase is represented by
\[ S_{mn} = |S_{mn}|e^{j\phi_{mn}} \] while \( m \) and \( n \) are the integer 1, 2, … (5.7)

The amplitude is normally considered in dB from \( 20\log|S_{mn}| \)

While the insertion loss between port \( n \) and \( m (L_A) \) and return loss at the port \( n (L_R) \) are

\[ L_A = -20\log|S_{mn}|...m \neq n \] (5.8)
\[ L_R = 20\log|S_{mn}| \] (5.9)

Which the return loss is related to the proportion of the Voltage Standing Wave Ratio (VSWR) by

\[ VSWR = \frac{1 + |S_{mn}|}{1 - |S_{mn}|} \] (5.10)

Beside the scattering parameters are used for microwave filter design, there are other two parameters that use for analyzing the design. First, the Phase Delay \( (\tau_p) \) represents the difference phase of the wave between the input and output port in 2-port network:

\[ \tau_p = \frac{\phi_{21}}{\omega} \] (5.11)

Another one is the Group Delay \( (\tau_d) \) which is considered the difference phase of the baseband wave on the input port and output port:
In 2-port network, the reflection parameter \((S_{11})\) can be analyzed in the term of terminal impedance \((Z_{01})\) by replacing with \(Z_{in1}=V_1/I_1\). \(Z_{in1}\) is called the input impedance by looking into port1, then

\[
S_{11} = \left. \frac{b_1}{a_1} \right|_{a_{in2}=0} = \frac{V_1/\sqrt{Z_{01}} - I_1\sqrt{Z_{01}}}{V_1/\sqrt{Z_{01}} + I_1\sqrt{Z_{01}}}
\]  
\tag{5.13}

\(V_1\) is replaced by \(Z_{in1}I_1\), therefore

\[
S_{11} = \frac{Z_{in1} - Z_{01}}{Z_{in1} + Z_{01}}
\]  
\tag{5.14}

In the same means, if consider the port 2 as the input and \(Z_{in2}=V_2/I_2\), \(Z_{in2}\) is the input impedance looking into port2.

\[
S_{22} = \frac{Z_{in2} - Z_{02}}{Z_{in2} + Z_{02}}
\]  
\tag{5.15}

In balanced network, the relation of S-parameters are

\[
S_{12}=S_{21} \text{ and } S_{11}=S_{22}
\]  
\tag{5.16}
In lossless and passive network, the total the input power is from the transmission power and the reflection power:

\[ S_{21}^* S_{21} + S_{11}^* S_{11} = 1 \text{ or } |S_{21}|^2 + |S_{11}|^2 = 1 \]  
\[(5.17)\]

\[ S_{12}^* S_{12} + S_{22}^* S_{22} = 1 \text{ or } |S_{12}|^2 + |S_{22}|^2 = 1 \]

Table 5.1 presents the summarization of the main four type of two-port network and their parameters

**Table 5.1** The overview of the relationship between two-port network parameters and incident and reflected wave variables [5.1, 5.23, 5.24].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
<th>Dependent relations and properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scattering parameters</strong> (S-parameters)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( S_{11} = \frac{b_1}{a_1} )</td>
<td>( S_{12} = \frac{b_1}{a_2} )</td>
<td>• ( S_{SWR} =</td>
</tr>
<tr>
<td>( S_{21} = \frac{b_2}{a_1} )</td>
<td>( S_{22} = \frac{b_2}{a_2} )</td>
<td>• Amplitude : ( 20 \log</td>
</tr>
<tr>
<td>( [b_1] = \begin{bmatrix} S_{11} &amp; S_{12} \end{bmatrix} )</td>
<td>( [a_1] = \begin{bmatrix} a_1 \end{bmatrix} )</td>
<td>• at port ( m &amp; n )</td>
</tr>
<tr>
<td>( [b_2] = \begin{bmatrix} S_{21} &amp; S_{22} \end{bmatrix} )</td>
<td>( [a_2] = \begin{bmatrix} a_2 \end{bmatrix} )</td>
<td>• Insertion Loss : ( L_i = -20 \log</td>
</tr>
<tr>
<td>( S_{11} = \frac{Z_{in1} - Z_1}{Z_{in1} + Z_1} )</td>
<td>( S_{22} = \frac{Z_{in2} - Z_2}{Z_{in2} + Z_2} )</td>
<td>• at port ( n )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Return Loss : ( L_r = 20 \log</td>
</tr>
<tr>
<td>• Voltage Standing Wave Ratio : ( VSWR = 1 +</td>
<td>S_{sw}</td>
<td>) ( 1 -</td>
</tr>
<tr>
<td>• Group Delay : ( \tau_g = \frac{d\phi_{11}}{d\omega} ) \text{ seconds}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( k_{in} \) is the incident wave, \( k_{out} \) is the reflected wave, and \( Z_1, Z_2 \) are the characteristic wave impedances of the associated network.
<table>
<thead>
<tr>
<th>Y-parameters (Admittance Parameters)</th>
<th>Z-parameters (Impedance Parameters)</th>
<th>ABCD Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-circuit admittance parameters</td>
<td>Open-circuit impedance parameters</td>
<td>Also referred to as transfer or chain matrix.</td>
</tr>
<tr>
<td>$Y_{11} = \frac{I_1}{V_{11}}$</td>
<td>$Z_{11} = \frac{V_1}{I_{11}}$</td>
<td>$A = \frac{V_1}{I_1}$, $C = \frac{I_1}{V_1}$</td>
</tr>
<tr>
<td>$Y_{12} = \frac{I_2}{V_{12}}$</td>
<td>$Z_{12} = \frac{V_2}{I_{12}}$</td>
<td>$B = \frac{V_1}{-I_2}$, $D = \frac{I_1}{-V_1}$</td>
</tr>
<tr>
<td>$Y_{21} = \frac{I_1}{V_{21}}$</td>
<td>$Z_{21} = \frac{V_2}{I_{21}}$</td>
<td>$\text{For a lossless network, } A \text{ and } D \text{ are purely real, } B \text{ and } C \text{ are purely imaginary.}$</td>
</tr>
<tr>
<td>$Y_{22} = \frac{I_2}{V_{22}}$</td>
<td>$Z_{22} = \frac{V_1}{I_{22}}$</td>
<td>$AD-BC=1$, for a reciprocal network</td>
</tr>
</tbody>
</table>

- **Reciprocal network**: $Y_{12} = Y_{21}$ and $Z_{12} = Z_{21}$
- **Symmetrical network**: $Y_{12} = Y_{21}$ and $Y_{11} = Y_{22}$
- **For lossless network**: $Y$ parameters are purely imaginary.
- **For lossless network**: $Z$ parameters are purely imaginary.
- $[Z] = [Y]^{-1}$
### 5.1.2 The terminated two-port network in Z-parameters

Table 5.2 Summarization of the terminated two port circuits [5.1, 5.25, 5.26]:

<table>
<thead>
<tr>
<th>Connection Types</th>
<th>Z parameter extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>T Network</td>
<td>$Z = \begin{bmatrix} \frac{Z_1 + Z_2}{Z_2} &amp; \frac{Z_2}{Z_2 + Z_3} \ \frac{Z_2}{Z_2 + Z_3} &amp; \frac{Z_2}{Z_2 + Z_3} \end{bmatrix}$</td>
</tr>
<tr>
<td>Pi Network</td>
<td>$Z = \begin{bmatrix} \frac{Z_1(Z_2 + Z_3)}{Z_1 + Z_2 + Z_3} &amp; \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3} \ \frac{Z_2}{Z_1 + Z_2 + Z_3} &amp; \frac{Z_2}{Z_1 + Z_2 + Z_3} \end{bmatrix}$</td>
</tr>
<tr>
<td>Symmetric T-Bridge Network</td>
<td>$Z = \begin{bmatrix} \frac{Z_2^2 + Z_1 Z_3}{2Z_1 + Z_1} + Z_2 &amp; \frac{Z_2^3}{2Z_1 + Z_3} + Z_2 \ \frac{Z_2^2}{2Z_1 + Z_3} + \frac{Z_2^3}{2Z_1 + Z_3} &amp; \frac{Z_2^2}{2Z_1 + Z_3} + Z_2 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
### 5.1.3 The interconnection of two-port circuits

The 2 port networks can be connected as in Figure 5.2 [5.26]:

(a) Series connection of two 2-port networks: \( Z = Z_1 + Z_2 \)

(b) Parallel connection of two 2-port networks: \( Y = Y_1 + Y_2 \)
Figure 5.2 Interconnection of two-port network (a) Series, (b) Parallel, and (c) Cascade [5.26]

If the 2 port networks with Z-parameters are connected in series, the equivalent port is given by

\[ [Z]_{eq} = [Z]_1 + [Z]_2 + [Z]_3 \ldots + [Z]_n , \quad n \text{ is the number of 2 port network with Z-parameters} \]

If the 2 port networks with Y-parameters are connected in parallel, the equivalent port is

\[ [Y]_{eq} = [Y]_1 + [Y]_2 + [Y]_3 \ldots + [Y]_n , \quad n \text{ is the number of 2 port network with Y-parameters} \]

If the 2 port networks are connected in cascade which each individual networks have transmission parameters \([A]_1, [A]_2, [A]_3, \ldots, [A]_n\), the total equivalent 2 port parameter will have a transmission parameter by

\[ [A]_{eq} = [A]_1 \cdot [A]_2 \cdot [A]_3 \ldots \cdot [A]_n , \quad n \text{ is the number of each individual network} \]
5.2 RF & Microwave Filter Characteristics

In passive RF/Microwave filter, lumped components, such as inductors and capacitors, are commonly used in RF filter design [5.1]. However, distributed components such as transmission lines (interdigital structure, comb line or coupled line) are another alternative way for RF filter design. The characteristics of filter (lowpass, highpass, bandpass, and bandstop) and some specific functions (Butterworth, Chebyshev, Bessel and Elliptic) are described by their output response and some parameters as follow in Figure 5.3:

Figure 5.3 The transfer function of Bandpass filter

- Bandwidth \((BW)\) or the half-power bandwidth: the difference between the upper and lower frequency \((f_H, f_L)\) of the circuit at which the amplitude is 3dB below the passband response.

The Fractional bandwidth of a filter is defined as the bandwidth divided by its centre frequency, \(BW/f_c\).
Ripple ($R_P$): in frequency domain is the periodic variation of insertion loss, normally known as the difference between the maximum and minimum of the insertion loss in passband.

Insertion loss ($IL$): the loss of signal in power after transmitted to device, normally presented in dB as $10\log (P_T/P_R)$, where as $P_T$ is the beginning power transmitted in to the load and $P_R$ is the power received by load. In scattering parameter, the insertion loss ($IL$) is defined as:

$$IL = -20\log |S_{21}| dB$$

Return loss ($RL$): the difference in dB between the forward and reflected power measured at the output point of filter.

Selectivity: the desired attenuation of the unwanted frequencies. In filter design, filter’s selectivity defines how much the filter will reject unwanted frequencies.

$Q$ factor: The inverse fractional bandwidth defined as $Q = \frac{f_c}{BW}$; a high $Q$ filter gives narrow passband, while a low $Q$ filter will have a wide passband.

Group delay: is the amount of time for signal to pass through the filter, most types of filter the group delay is defined as $d\phi/df$ varied with frequency.

### 5.3 Overview of the Design Filter

There are many methods to design RF/microwave filters. Because of the high efficiency, easy manufacture, the coupling techniques are recommended, especially in planar filters [5.4-5.11]. In this chapter, a high rejection in the lower band and improved wideband pass filters are designed using CSRRs and coupling stub arrangement. The two main parts of this design are
1. The coupling part on microstrip line width and length of the balanced-capacitive stub and the inductive coupling line are provided to control its frequency selectivity and upper transition band.

2. The CSRRs which is etched on the ground plane and a coupling plate are used for enhancing the properties in lower transition band by giving a transmission zero. A lower band good rejection and the low loss in band operating frequency are represented by the help of CSRRs coupling properties.

### 5.3.1 The main Coupling structure

The coupling structure with loaded capacitance is presented in Figure 5.4. The initial structure of the design bandpass filter is comprised of the two coupling parts; firstly, the coupling part between both stubs and their inductive lines while another coupling is occurred in coupling plate. With these strong coupling parts, the structure provides a sharp narrow passband as shown in Figure 5.5

![Diagram](image)

Figure 5.4 The layout of a balanced load capacitance without CSRRs
The simulation results by Ansoft HFSS are run on RO3010 substrate with thickness \( h = 1.27 \text{mm} \) and dielectric constant \( (\varepsilon_r) = 10.2 \) and \( \tan \delta = 0.0035 \). The capacitive stubs \( (L_{stub}) \) are 16mm. The lengths of both inductive coupling lines \( (L_{inductance}) \) are 24mm.

![Graph of S11 and S21](image)

Figure 5.5 The insertion and return loss of balanced load capacitance structure without CSRRs by HFSS simulation

This coupling structure generates a narrow passband filter from 1.7-2GHz which has the scattering parameters shown in Figure 5.5. The simulated \( S_{11} \) and \( S_{21} \) present that the balanced load capacitance structure has the insertion loss in passband at -1dB and give a sharp cut-off at the higher band edge which provide transition band approximately 50MHz. In addition, the structure can present a high resolution down to -30dB. The stopband area is clearly from 2.1GHz to 3.6GHz. These properties will be used in design the higher passband of the proposed filter.
5.3.2 Electromagnetic Properties of CSRRs

The rectangular CSRRs loaded transmission line and its equivalent circuit model have been present [5.14-5.16], shown on Figure 5.6(a) and (b).

Figure 5.6 (b) The unit cell of CSRRs loaded with transmission line topology and (b) its equivalent circuit [5.14-5.16]

Figure 5.6(a) represents the topology of rectangular CSRRs which is composed of the three factor as following air slot on ground plane \(d\), its conductance \(c\), and the dimension length \(a\), respectively. Owing to the behavior of CSRRs which acts as \(LC\) resonator, the equivalent circuit can be modeled by these following parameters: CSRRs is formed by the parallel combination of \(L_r\) and \(C_r\), and its coupling to the host line is represented by the capacitance \(C_c\). The series gap on transmission line is represented by \(C_g\).
As a result of input port at the resonant frequency, the parameter \( L \) and \( C_g \) can be ignored. Therefore, the intrinsic resonant frequency of CSRRs is given by [14-16]

\[
f_r = \frac{1}{2\pi \sqrt{L_r C_r}}
\]  

(5.18)

The transmission zero is given by

\[
f_z = \frac{1}{2\pi \sqrt{L_r (C_r + C_c)}}
\]  

(5.19)

From equation (5.18) and (5.19), CSRRs has been applied in this design model for selecting the passband of filter and controlling the transmission zero which is related to a high rejection in the lower band.

### 5.3.3 Combination model of the microstrip coupling structure and CSRRs

As previous mentioned, this research presents a new configuration of wideband filter under the perspective of metamaterials in microstrip configuration by using the rectangular complementary split-ring resonators (CSRRs). The design technique of this filter is based on combining of rectangular CSRRs on ground plane and the coupling structure formed by microstrip line on Figure 5.4. The layout of proposed filter is shown in Figure 5.7.
The width of strip ($W$) and stubs ($W_{stub}$) are 2mm while the lengths of balanced inductive stubs ($L_{stub}$) are 14mm which is the same length as coupling plate, respectively. The width of coupling plate is 4mm. The length of both inductance lines ($L_{inductance}$) is 29.5mm, whereas their width ($W_{inductance}$) is 0.3mm and all gaps are maintained at 0.3mm, respectively. The rectangular CSRRs size ($a$) is 10mm.
Figure 5.8 The current distributions of the proposed bandpass filter at (a) 0.72GHz (no transmission) (b) 1.4GHz (centre frequency), respectively

As shown in Figure 5.8(a), the unwanted signal is reflected, hence no current is transferred to port2 on the transmission zero frequency at 0.72GHz. At 1.4GHz which is in the passband of the proposed filter, the wanted signal has been propagated to port2, illustrated in Figure 5.8(b).

Figure 5.9 The magnetic field distribution on ground plane at (a) 0.72GHz and (b) 1.4GHz, respectively
The magnitude distribution of the magnetic field on ground plane is demonstrated in Figure 5.9. This shows that there is the existence of the H field on the ground plane which is strongly concentrated on the CSRRs. It again illustrated that little magnetic energy has been transferred to port2 at 0.72GHz, whereas the energy propagates to port2 at 1.4GHz.

![Figure 5.10](image1)

(a)

![Figure 5.10](image2)

(b)

Figure 5.10 The photograph of the fabricated filter (a) Top view, (b) Bottom view.

This proposed filter is fabricated on RO3010 substrate with thickness $h=1.27\text{mm}$, total dimension of the filter is $40\times41.5\text{mm}^2$ and dielectric constant ($\varepsilon_r$) =10.2.
The photograph of fabricated filter is shown in Figure 5.10. The frequency characteristics are measured using Agilent Technologies ENA series E5071B Network Analyzer.

### 5.3.4 Experimental Results and Discussion

![Figure 5.11 The equivalent Circuit of proposed filter](image)

The equivalent circuit of the proposed filter, presented in Figure 5.11, is explained by the combination model between the π model and T model. The extracted parameters of this designed filter is as followed: \(C_M=2.08\text{pF},\) \(C_r=7.2028\text{pF},\) \(L_r=2.8072\text{nH},\) \(C_c=4.3114\text{pF},\) \(C_l=0.8084\text{pF},\) \(C_s=0.2096\text{pF},\) \(L_1=5.6719\text{nH},\) and \(L_2=1.189\text{nH},\) respectively.
The ADS simulated S-parameters (a), and (b) comparison of ADS and HFSS simulated frequency responses on the designed filter at 0.9-1.9GHz.

The equivalent circuit simulated results of the proposed filter is shown in Figure 5.12(a). In Figure 5.12(b), the comparison of the two simulated results by ADS equivalent circuit and HFSS are illustrated. Reasonably good agreements can be found in the two simulated results. However, there are some discrepancies. The
equivalent circuit analysis provides wider band and lower transmission zero. This is because the equivalent circuit analysis cannot provide the total effects from magnetic loss tangents, substrate loss, and measurement loss, etc.

Figure 5.13 shows the simulated and measured results of the return loss $S_{11}$ and insertion loss $S_{21}$ in wideband. The results of the frequency response measured on the fabricated band pass filter substrate show satisfactory agreement with the simulated frequency responses by the HFSS in the region of interest.

![Graph of S11 and S21](image)

Figure 5.13 Measured and HFSS simulated frequency response on the designed filter at 0.9-1.9GHz.

The measured insertion loss is about 0.9dB at the centre frequency and the passband return loss is less than -10dB. The filter presents wideband properties which covers the frequencies from 0.9 to 1.9GHz. The 3dB FBW is approximately 77%. These
values are adequate enough to be used in communication channel filtering. Moreover, it is noticed that the excellent characteristic of the out-of-band rejection can be achieved due to the induced between a parallel inductive line coupling and the rectangular CSRRs by obtaining up to 3.4GHz. This results show the effect of applied CSRR on conventional filter that can be modified the filter properties without changing a fundamental dimension. Owing to the CSRR properties, a sharp transition band with transmission zero deep down to -60dB by measurement at the lower cut-off edge is presented.

5.4 Conclusion

In this chapter, the design and fabrication of the wideband pass filter with the bandwidth 1GHz based on the complementary split-ring resonators (CSRRs) have been presented. It was found that the presence of CSRRs on conventional coupling filter can modify the filter properties from 300MHz narrowband filter to 1GHz wideband filter without changing any profile. In this work, the measured insertion loss of this design filter at operation band is very low which presents <1dB at the centre frequency as well as the passband return loss is less than -10dB. With CSRR properties, the transmission zero and a sharp transition band on the lower cut-off frequency has been demonstrated. At the out of band on higher frequency, this proposed filter exhibits a frequency suppression which is occurred on the conventional coupling filter at 3.9GHz. This design filter provides the excellent characteristic of both in band and out of band.
5.5 References


CHAPTER 6

ANTENNA THEORY AND APPLICATIONS WITH CSRRs

INTRODUCTION

The specific-electromagnetic properties of metamaterials, which give the negative permeability and negative permittivity, have led to the first Split Ring Resonators (SRRs) being reported by Pendry [6.1, 6.2]. There are many applications of metamaterials, which have been applied in the last decade, including antennas, in order to enhance their performance [6.3, 6.4]. Due to the properties of metamaterials that can manipulate the electromagnetic field, the higher impedance substrate can be generated. This can be used to reduce the size as well as maintain its efficiency.

This chapter mainly consists of three parts. The first part will briefly mention about the basic microstrip patch antennas and its properties. The second part will present a new technique of antenna size reduction, by etching CSRRs on the ground plane as well as the inductively meander line loaded patch. By this method, the size reduction of the proposed antenna can reach 74% of the conventional rectangular patch antenna. In last part, the technique of tunable CSRRs antenna has been introduced by reversely biasing the varactor diode between its patch and ground side. The wide tuning range has been achieved to 350 MHz without changing any dimension. The tunable CSRR microstrip antenna still remains compact.
6.1 Antenna Theory and its Definition

Antenna is one of the most crucial components in wireless communication systems. The IEEE defines antenna as “The part of a transmitting or receiving system that is designed to radiate or receive electromagnetic waves”. Webster's Dictionary defines antenna as “a usually metallic device (as a rod or wire) for radiating or receiving radio waves”. In other words, antenna is a structure used to transit the signal between free space and a guiding device e.g. transmission line. Then, transmission line will transport electromagnetic energy from the antenna to the receiver or from the transmitting source to the antenna [6.5].

Generally, there are two categories of antennas according to their application [6.5]:

- Omnidirectional antennas are called the group that radiate or receive in all directions. It is normally used when the relative position of the other station is arbitrary or unknown.

- Directional or beam antennas are used to specially receive or radiate in a certain direction or directional pattern.

If consider in terms of type, antennas can be divided into various types such as wire antennas, aperture antennas, lens antennas, reflector antennas, and microstrip patch antennas [6.5].
6.1.1 Antenna Radiation and its Characteristic

An antenna radiates its signal which is occurred basically due to the time varying (AC) current or acceleration of charge. It is simply said that without motion of charges in a wire (no current), no radiation happens. The antenna radiation can be explained in Figure 6.1 [6.5].

A voltage source, connected to a two conductor transmission line, applies a sinusoidal voltage wave across the transmission line. A sinusoidal electric field is generated which causes the appearance of electric lines of force which are tangential of the electric field.

The electric field’s magnitude is shown by the high concentration of the electric line of force. The movement of the free electrons on the conductors under the electric lines of force leads to the appearance of current flow which forms a magnetic field. The time varies of magnetic and electric fields will then create the electromagnetic waves traveled between the conductors. The free space waves are appeared at the open ends of the electric lines which is connected to the open space. Because the electromagnetic waves are continuously generated by the sinusoidal wave from the source, its electric disturbance is constant which leads to these waves can propagate through the transmission line and the antenna as well as the free space radiating.

Owing to the charges, inside the transmission line and antenna part, the electromagnetic waves are continuing upheld. While in the free space; they are radiated after creating closed loops [6.5].
6.1.2 Conventional Microstrip Patch Antenna

Microstrip patch antennas are very popular in airborne systems, mobile radio, pagers, radar systems and a variety of wireless and satellite communications because of its attractive features which have advantages over other antenna structures such as low profile, low fabrication cost, light weight, easy fabrication, simplicity and capability in integrating with microwave integrated circuits technology [6.6]. However, microstrip antennas still have a bulky size especially in lower frequencies, therefore, in order to respond for high demand of wireless communications with limited space in the present-day; many techniques of minimization have been proposed [6.7, 6.8].
Figure 6.2 Microstrip Patch Antenna structure.

Generally, a microstrip patch antenna comprises of the feed lines and a radiating patch, made of conducting material such as copper, on one side of a dielectric substrate and has a ground plane on the other side. The example of a rectangular microstrip patch antenna, shown in Figure 6.2, has the length $L$, width $W$ placing on a substrate of height $h$. The radiating patch can be of many shapes and size, for example, rectangular, circular, elliptical, triangular or square etc. For a rectangular patch, the length $L$ of the patch is usually $\frac{\lambda_0}{3} < L < \frac{\lambda_0}{2}$, where $\lambda_0$ is the free-space wavelength. The patch thickness ($t$) is chosen to be quite thin by $t \ll \lambda_0$. The height $h$ of the dielectric substrate is typically $0.003\lambda_0 \leq h \leq 0.05\lambda_0$. The dielectric constants ($\varepsilon_r$) of substrate of the design microstrip antennas can be ranging from 2.2 to 12 [6.5].

The feeding method is an important factor which can affect the performance of the design patch antenna. In the other words, the different feeding methods exhibit
different characteristics of the antennas. Figure 6.3 demonstrates the common feeding methods used in industries:

(a) Coaxial feed, (b) Inset-feed, (c) Proximity-coupled feed, and (d) Aperture-coupled feed. [6.5]
The most common and easiest feed is coaxial type, while inset feed is widely used for array antennas. The proximity-coupled feed is recommended to use for reducing spurious radiation from the feed line in multilayer fabrication. The aperture-coupled feed type, used in thick substrate, can eliminate feed line radiation [6.5, 6.6].

Patch antennas radiate mainly because of the fringing fields between the ground plane and the edge of the patch. The antenna that has a thick dielectric substrate with a low dielectric constant is desirable because it provides better efficiency, larger bandwidth and better radiation; however, it has a larger size [6.5]. In order to design a compact patch antenna, higher dielectric constants must be used which are less efficient and result in narrower bandwidth. Compensation must be made between antenna performance and dimensions while design an antenna.

### 6.1.3 The Advantages and Disadvantages of Microstrip Patch Antennas

Since 1950s, microstrip antennas have been widely used, as compared as the conventional microwave antennas. Table 6.1 illustrates the gain of microstrip antennas over the conventional microwave antennas [6.6];
Table 6.1 Comparison of microstrip antennas and conventional microwave antennas

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages and limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Light weight, low volume and thin profile configurations</td>
<td>• Narrow bandwidth and associated tolerance problems</td>
</tr>
<tr>
<td>• Low fabrication cost (readily amenable to mass production)</td>
<td>• Some, lower gain (-6dB)</td>
</tr>
<tr>
<td>• Compatible with printed-circuit technology (easy to manufacture as standalone elements or as arrays elements)</td>
<td>• Larger ohmic loss in the feed structure of arrays</td>
</tr>
<tr>
<td>• In case of very thin substrate, they may also be conformable such as bending that leads to unobtrusive antenna</td>
<td>• Most radiate into half-space</td>
</tr>
<tr>
<td>• Linear and circular polarizations can be applied with simple feed</td>
<td>• Complex feed structures required for high-performance arrays</td>
</tr>
<tr>
<td>• Dual frequency and dual polarization antennas can be easily made</td>
<td>• Polarization purity is difficult to achieve</td>
</tr>
<tr>
<td>• No cavity backing is required</td>
<td>• Poor end-fire radiator, except tapered slot antennas</td>
</tr>
<tr>
<td>• Easily integrated with microwave integrated circuits</td>
<td>• Extraneous radiation from feeds and junctions</td>
</tr>
<tr>
<td>• Feed lines and matching networks can be fabricated simultaneously with the antenna structure</td>
<td>• Lower power handing capability (approx. 100W)</td>
</tr>
<tr>
<td></td>
<td>• Reduced gain and efficiency as well as unacceptably high levels of cross-polarization and mutual coupling within an array environment at high frequencies</td>
</tr>
<tr>
<td></td>
<td>• Excitation of surface waves</td>
</tr>
<tr>
<td></td>
<td>• Most fabricated on a high dielectric constant of substrate, leads to poor efficiency and narrow bandwidth</td>
</tr>
<tr>
<td></td>
<td>• Lower radiation efficiency</td>
</tr>
</tbody>
</table>

Although microstrip antennas have these limitations, most can be minimized. Several techniques have been proposed to overcome these limitations such as array configuration techniques with power handling [6.9] and choke structure to gain the limitation of surface-waves [6.10, 6.11], etc.
6.1.4 Fundamental Antenna Parameters

An equivalent circuit of antenna is shown in the right side of Figure 6.4. $V_s$ represents the source comprised of the internal impedance ($Z_s = R_s + jX_s$), while the antenna input impedance, represented by $Z_{in} = R_{in} + jX_{in}$, is connected. The real part of antenna consists of the radiation resistance ($R_{rad}$) and the antenna losses ($R_{loss}$).

The input impedance can be used to analyze the reflection coefficient $\Gamma$ by [6.5]

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = S_{11} \quad (6.1)$$

where $Z_0$ is the characteristic impedance of the transmission line connecting the antenna to the generator or source. The reflection coefficient shows the amount of
power reflected to the source. In ideal case, $\Gamma$ should be zero. Therefore, the voltage standing wave ratio is

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (6.2)$$

The return loss is given by

$$RL = -20\log|\Gamma| \quad (6.3)$$

Moreover, the input impedance is used to determine the antenna’s resonant frequencies on Smith Chart with less reflection to the source by matching with the feed line. In order to receive the maximum power transfer to antenna, the input impedance, ideally, contains only a resistive real part which is the same value as the internal resistance of the signal source.

6.1.5 Analysis of patch antenna

The basic formulation and relationship between dimensions of rectangular patch antennas as well as the resonant frequency can be analyzed as follow [6.5]:

- The substrate used for this case is RO3010 with a dielectric constant=10.2 and a height of 1.27mm.

The initial given values of the width ($W$) of the patch is calculated by using the following formula. $c$ is the speed of light in vacuum =3x10$^8$m/s, $f_r$ is the design resonant frequency.

$$W = \frac{1}{2f_r\sqrt{\mu_0\varepsilon_0}} \sqrt{\frac{2}{\varepsilon_r + 1}} = \frac{v_0}{2f_r}\sqrt{\frac{2}{\varepsilon_r + 1}} \quad (6.4)$$
The effective dielectric constant of the conventional patch antenna is calculated by

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left[ 1 + 12 \frac{h}{W} \right]^{-1/2}$$  \hspace{1cm} (6.5)

Then, calculate the extended incremental length of the patch by

$$\frac{\Delta L}{h} = 0.412 \frac{(\varepsilon_{\text{eff}} + 0.3) \left( \frac{W}{h} + 0.264 \right)}{\left( \varepsilon_{\text{eff}} - 0.258 \right) \left( \frac{W}{h} + 0.8 \right)}$$  \hspace{1cm} (6.6)

The value of the effective length of the patch is calculated by

$$L_{\text{eff}} = \frac{c}{2f_0 \sqrt{\varepsilon_{\text{eff}}}}$$  \hspace{1cm} (6.7)

The actual length of the patch is then provided by

$$L = L_{\text{eff}} - 2\Delta L$$  \hspace{1cm} (6.8)

Finally, the resonant frequency is calculated by using the formula:

$$f_r = \frac{c}{2L \sqrt{\varepsilon_{\text{eff}}}}$$  \hspace{1cm} (6.9)

6.2 CSRRs Properties and Antenna Design

The CSRRs loaded transmission line, and its equivalent circuit model, have been proposed [6.12, 6.13], shown in Figure 6.5 (a) and (b). Figure 6.5 (a) represents the topology of CSRRs which is composed of three parameters: air slot on ground plane
(d), its conductance (c), and the average ring dimension (r₀), respectively. Owing to
the behavior of a CSRRs which acts as LC resonator, the equivalent circuit can be
modeled by Figure 6.5 (b). It can be seen that CSRRs are formed by the parallel
combination of Lᵣ and Cᵣ and its coupling to the host line, which are represented by
the capacitance Cₑ. The series gap on transmission line is represented by C₉

![Figure 6.5](a) The unit cell of CSRRs TL and (b) its T-equivalent circuit.

The intrinsic parallel resonant frequency (fₑ) of the CSRRs can be given by [6.12,
6.13].

To obtain fₑ, the shunt patch to the ground is opened. This gives directly the value of
L from the negative permittivity line. Then, the additional condition is,
\[ Z_s(j\omega_{\pi/2}) = -Z_p(j\omega_{\pi/2}) \]  \hspace{1cm} (6.10)

\( Z_s \) and \( Z_p \) are the series and shunt impedance of the T-equivalent circuit model of the structure, respectively. Thus, the first factor to design is the radius of inner and outer ring which mainly controls the resonant frequency of antenna. In this proposed antenna, there is another factor which has been used to control the resonant frequency. That is the meander line which will be explained in later.

### 6.2.1 Meander line antenna concept

Generally, a conventional microstrip patch antenna consists of a radiating patch printed on one side of a dielectric substrate which has a ground plane on the other side. The patch is normally made of conducting material such as copper. The radiating patch and feed lines are usually etched on the dielectric substrate. In this paper, the efforts on miniaturization were achieved mainly by focusing on etching CSRRs on the ground plane with inductively meander line and capacitive gap on the patch.

The idea of introducing a CSRRs particle on the ground plane of antenna structure is to generate the negative permittivity at the design frequency which this coupling on CSRRs is the first main key used to control the resonant frequency of antenna. Then, the meandered line patch, acts as inductance, is applied for further controlling the resonant frequency. The proposed rectangular microstrip patch antenna was designed
by using Ansoft HFSS simulation programme and validated experimentally using Vector Network Analyzer (HP8510).

### 6.2.2 Design procedures

The antenna in this research was designed and adjusted to obtain a proper compact dimension on Roger/RO3010 substrate and operates at 1.73GHz by Ansoft HFSS simulation programme.

![Antenna model by HFSS simulation programme](image)

Figure 6.6 Antenna model by HFSS simulation programme

The antennas in three different meander line turns, as shown in Table 6.2, have been studied and designed. The layout of the meander line and its geometrical parameters are given in Figure 6.6 and 6.7. The introduced meander line with the capacitive gap, acting as a series of inductors, by using the equation (6.10) to (6.12), helps to finely match and tune the resonant frequency lower with maintain the patch size as electrical small.
Figure 6.7 Layout of the meander line (4 turns), a=0.6mm and the capacitive gap=0.2mm.

Table 6.2 Antennas and Meander line turns

<table>
<thead>
<tr>
<th>antenna</th>
<th>Meander Line Turns (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
The simulated antenna return losses are illustrated in Figure 6.8. The meander line turns can move the resonant frequency lower and achieve better matching. However, the CSRRs on the ground plane are a sensitive model, a few limited meander line turn, as a fine tuning, can be accepted. The Bandwidth of antenna3 (dash blue line) is narrow approximately 35MHz, reaching about 2% fractional bandwidth.

After applied the voltage source to antenna, the electric field has been generated as well as fringing zone polarizing through the patch. Figure 6.9(a) and (b) show the electric field distribution on the patch surface of the proposed antenna1 and antenna3 (x-y plane), respectively. Moreover, there is another electric field occurred by the coupling between the inner and outer of the two rings of CSRRs which is opposite direction to the original one. As a result, the electric field distributions on the coupling area on patch have been created which provides a new resonant frequency.

The magnetic field distributions, illustrated by Figure 6.9(c) and (d), are perpendicular to the electric field distribution.
Figure 6.9 The simulated field distributions at resonant frequencies (a) E field of antenna1 at 1.8GHz and (b) E field of antenna3 at 1.73GHz, while (c) H field of antenna1 at 1.8GHz and (d) H field of antenna3 at 1.73GHz, respectively.

Figure 6.10(a) and (b) demonstrate the surface’s current distributions cover all on patch and ground of antenna3. At the resonate frequency of CSRRs, the current direction was dominated by rearranging the direction on patch as well as happening on ground. In addition, the inductive meander line on patch is also used to give better coupling with CSRRs as tuning to get an optimum matching the antenna resonant frequency.
Figure 6.10 The surface’s current distribution at 1.73GHz of antenna3 (a) on patch (b) on ground, respectively.

6.2.3 Experimental results

The photos of the fabricated antenna are shown in Figure 6.11. The antenna was fabricated on the RO3010 substrate, whose dielectric constant ($\varepsilon_r$) is 10.2, loss
tangent (\(\tan \delta\)) 0.02, and thickness \((h)\) 1.27mm, respectively. The size of outer
ground plane on the front side of the substrate is \(W_s \times L_s\), which is 22 \(\times\) 28mm\(^2\).

Figure 6.11 The photos of antenna3 (a) the meander line patch with gap (b) The
CSRRs on the ground plane.

The antenna was designed to have the centre frequency of 1.73GHz. The meander
line patch size \((W_p \times L_p)\) is controlled by the ring radius \(R_o\). In this work \(R_o=8.2\)mm.
The CSRRs was etched on the ground plane with the parameters of \(c\) and \(d\) which are
0.2 and 0.2mm, respectively. The inductive meander patch gap is designed to get
better power matching at 0.2mm.
Figure 6.12 shows the simulated and measured return losses for the proposed antenna which has a meander line of 4 turns. It can be seen that the measured results are well agreed with the simulated one. As mentioned in the previous section, the size of this proposed CSRRs resonant antenna with meander line can be reduced to reach the patch size of $0.095\lambda_0 \times 0.095\lambda_0$. This is much smaller compared to the conventional patch antenna which has the patch size of $0.21\lambda_0 \times 0.16\lambda_0$. The size of the proposed antenna can be reduced dramatically because the resonant frequency can be controlled by the reaction of inner and outer coupling of the CSRRs instead of the physical length of conventional patch. Moreover, with the help of meander line and the capacitive cap, acting as inductive and capacitive loadings, the antenna can reduce the size further.
6.3 A compact and tunable antenna

This section introduces a novel tunable compact antenna loaded with CSRRs. The varactor diode (Infineon BB535) is used to tune the operation frequency. The dimension of antenna is maintained the same as the previous section. The operation frequency of a CSRRs loaded antenna without meander line as in the previous section is 1.8GHz. The proposed varactor loaded CSRRs antenna can be tuned by changing the DC bias.

![Diagram of the proposed antenna with varactor diode.](image)

Figure 6.13 The simple structure of the proposed antenna with varactor diode.

Varactor diode can vary its inherit capacitance by reverse bias. The low reversed bias voltage provides high inherit capacitance, while higher the reversed bias voltage will decrease the diode capacitance, respectively. The equivalent circuit of the tunable antenna is illustrated in Figure 6.13.
Figure 6.14 The layout of tunable patch antenna (a) ground view and (b) patch view.

Figure 6.14(a) is the photograph of the varactor diode connecting on ground plane via the second patch, whereas Figure 6.14(b) shows the patch view and the connecting point of the varactor diode to the ground plane which is placed on the second half of the patch in order to allow the constant electric wave pass through the total patch size.

The bias can control the resonant frequency, shown in Figure 6.15. Without DC bias, the antenna resonates at 1.15GHz with return loss of 33dB. After applied reversed DC bias, the junction capacitance inside varactor diode decreases, resulting in moving the resonate frequency higher. In the other words, the tunable antenna provides the resonant frequency under the condition of the lower diode capacitance, the higher resonant frequency. Moreover, in the frequency ranges of the working antenna, the operated bandwidths are stable at 35MHz. After biasing DC to +12V, there is no radiation from antenna (antenna is off) that because the completed
connecting of the depletion region inside diode presents a completed circuit loop and is shorted to ground which is no diode capacitance at this frequency.

Figure 6.15 The measured return loss of the tunable CSRRs patch antenna with different DC bias voltages.

As a result of diode applied CSRRs patch antenna, a fine tuning range is covered 350MHz from 1.15 to 1.5GHz with maintaining bandwidth at 35MHz. This is the first shown the co-operation of CSRRs as mention before as well as the work of diode application to fulfill the aim to minimize and tuning antenna.
6.4 Conclusion

Novel compact CSRRs loaded patch antennas with tunability have been studied, designed, fabricated and characterized.

Microstrip patch antenna using CSRRs and capacitive loaded meander line can reduce size by 74% in comparison with conventional rectangular patch antenna. The size reduction was achieved by effectively manipulating the electromagnetic field distributions within and between the meander line patch and CSRRs. More turns of the meander line can move the operating frequency lower without changing any dimension of patch which is maintained at 16.4x16.4mm$^2$. In tunable antenna, the size of antenna is still compact by keeping the previous dimension of without diode case. The operation frequency of CSRRs antenna with diode is at 1.15GHz which is lower than CSRRs antenna without diode at 1.8GHz in same dimension. After applied DC voltage, the inherit capacitance of diode changes, leading to the change of the overall impedances of the antenna. Therefore, the tunable CSRRs antenna can be achieved. In this research, the operation frequency tunable range is 350MHz cover the frequencies from 1.15 to 1.5GHz with stable 35MHz narrow bandwidth.


6.5 References


CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

The aim of this research was to propose and examine a novel approach of CSRRs and microstrip in communication components. In microstrip planar structure, the left-handed band of the resonant-type metamaterial structure is formed by CSRRs particle etched onto the ground plane. This CSRRs exhibit negative effective permittivity, whereas the negative effective permeability can be obtained by etching series gaps on the conductor strip, shown in Chapter 2.

The equivalent circuit and the left-handed area of CSRRs TL were analyzed in Chapter 3. It was found that the magnetic coupling component should be added in the lumped-element equivalent circuit model in the case of more than two adjacent CSRRs cells. The analysis of the magnetic coupling effect was carried on by HFSS full-wave electromagnetic simulation. The 2-unit cell of CSRRs TL was modeled and placed. By varying the distance between these two CSRRs, the detected magnetic couplings were captured and analyzed. The graph result of magnetic coupling presented that there were some currents that appeared from 200A/m to 50A/m between 0.2mm to 1mm distances of these two rings, respectively. By using ADS extracted the parameters of the 0.2mm distance case, the $S_{11}$ graph of the proposed equivalent circuit can present the resonant frequency ($f_c$) at 2.27GHz matching the $S_{11}$ from full wave simulation, which was better than the old equivalent circuit, which provides $f_c$ at 2.3GHz. The $S_{11}$s on smith chart also agreed with the
new equivalent circuit. Another noticeable outcome in this chapter was the indicating LH operating area of CSRRs TL. The 4-cell CSRRs TL was fabricated. It was noticed that there were two passbands that appeared (≈2.2-2.5GHz and 4.7-6.5GHz). The dispersion graph clearly indicated the left-handed area (<2.55GHz) and right-hand area (>4.7GHz) of this fabricated 4-cell CSRRs TL. This left-handed area was then further analyzed in chapter 4.

Because metamaterials present a specific dispersion property, under these dispersion conditions, the first application in this thesis was to study the delay property in CSRRs applied to microstrip TLs (chapter 4). Initially, the group delay relation of both LH and RH areas were analyzed. It was found that in LH areas the group delay is inversely proportional to frequency. Then, the group delays of these two cases were investigated, which are passive delay line and active delay line. The group delay of the previous 4-cell CSRRs TL of 35mm length was measured and referred to passive delay line. The group delay graph results of both simulations and measurement agreed with the previous analyzed theory, whereas in 35mm long normal TL the group delay remained the same overall frequency at 0.3ns. In comparison, at 2.3GHz the group delay of this 4-cell CSRRs TL displayed 3ns which was 10 times that of a conventional TL. Then, the signal delays on a simple RF system were examined. The two waveforms, CWs and pulses, were studied. By loading the measured S-parameters to the data block in ADS, the three different sinusoidal waves of 2.2, 2.3 and 2.4GHz were fed in the 4-cell CSRRs TL simultaneously. The results showed that these three signals perform differently in times, phases, and amplitudes at the port end, relating to the group delay graph. Next, the two modulated different pulses with carrier 2.25 and 2.5GHz were fed in
simultaneously. At the output port, the delayed pulses consumed the time in CSRRs TL at 6 and 5ns of the carrier wave at 2.25 and 2.5GHz, respectively. In active delay line, the 4 diodes were embedded in the 4-cell CSRRs TL. Because of the change of characteristic impedance resulting from diode addition, the operating frequency was shifted. The group delays were detected by monitoring one frequency. For example, after applied the DC bias to diodes from -10 to -20V, the group delay displayed the tuning rate of 0.5ns/V at 2.03GHz.

Both simulation and measurement results in this chapter prove that the group delay can be varied, and controlled, without changing the transmission line length, which is not found in normal microstrip structure.

The later application was to develop a wideband passband filter, by using rectangular CSRRs (chapter 5). The new filter configuration was achieved by the modified structure of a narrow passband filter. A rectangular CSRRs was placed on ground plane in the prototype model. The coupling between CSRRs and the coupling plate of the host line generated a passband on the lower frequency side of the proposed filter; as a result a high rejection in the lower band and the transmission zero were presented. The novel filter exhibited a wide bandpass with a passband up to 77% of the bandwidth covering the range from 0.9 to 1.9GHz. The insertion loss is less than -1dB. Furthermore, the proposed filter presented the better performance in frequency suppression on spurious frequency at 3.9GHz which appeared in the prototype filter. The validated results were agreed through both equivalent circuit simulation and measurement.
The last application is to minimize the size of planar antenna, which was done by etching a CSRRs on the ground side of the patch antenna. A capacitive gap was also etched on patch side. The originated frequency of the initial antenna was controlled by the LC resonator of CSRRs, then it was applied the meander line part on the middle of the patch to 2 and 4 turns. The simulated results showed that the operating frequency was lower when applying more meander turns from 1.75GHz to 1.73GHz, respectively. The measured $S_{11}$ of the antenna with 4 turns of meander line agreed with the HFSS full wave simulation. This novel planar antenna has shown 35MHz bandwidth and significantly reduced the size to 74% of a conventional microstrip patch antenna. In addition, inserting a diode from patch to ground can change the proposed antenna to be active in selecting the operation frequency. After DC bias was applied from 0V to 8V, the antenna provided the tuning range to 350MHz covering the frequency 1.15GHz to 1.5GHz and maintaining its physical dimensions. This proposed antenna maintained 35MHz bandwidth over all tuning ranges.

In summary, this thesis presents three areas of work, one showing that the transmission line can control signal delay and reduce size, which means more data can be multiplexed simultaneously. This thesis also shows the development of enhanced bandwidth of a filter, from narrowband to wideband, by introducing only the CSRRs particle. Finally the development of an antenna of a smaller size is also demonstrated in this thesis. This thesis shows that introducing CSRRs in RF/microwave components that can serve both miniaturization and performance enhancement purposes.
7.2 Future works

This thesis provides the most fundamental part of communication systems. However, there are some details that should be applied for more quality results. The future works in this subject are provided in the list below;

- In CSRRs applied microstrip transmission lines, the requirement of system analysis is needed. This simple system is includes the signal and carrier generator, mixer, and filter. Each part has to be placed on the same board. Therefore, the more correct delay in the real system should be analyzed.

- Because of the low gain appearance in electrically small antenna, in this work, it is hard to get the radiation pattern. Placing the antenna in the form of array antennas can support the gain enhancement. The further implementation is very useful in analyzing the radiation.
LIST OF PUBLICATIONS


Appendix

Figure 5.11 can be simplified as the equivalent circuits as follow:

The proposed filter is composed of network N1 and N2. N1 is in T-model, while N2 is in Pi-model, respectively.

The Z parameters of network N1 (T-model) are [5.1]
\[
[Z]_T = \begin{bmatrix}
Z_1 + Z_2 & Z_2 \\
Z_2 & Z_2 + Z_3
\end{bmatrix}
\]

Then, convert the Z parameters to Y parameters by
\[
Y_{11T} = \frac{Z_2 + Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}, \quad Y_{12T} = \frac{-Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}
\]
\[
Y_{21T} = \frac{-Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}, \quad Y_{22T} = \frac{Z_1 + Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}
\]

The Z parameters of network N2 (Pi-model) are \([5.1, 5.25, 5.26]\)
\[
[Z]_T = \begin{bmatrix}
\frac{Z_4(Z_4 + Z_6)}{Z_4Z_5 + Z_6} & \frac{Z_4Z_6}{Z_4 + Z_5 + Z_6} \\
\frac{Z_4 + Z_5 + Z_6}{Z_4 + Z_5 + Z_6} & \frac{Z_6(Z_4 + Z_5)}{Z_4 + Z_5 + Z_6}
\end{bmatrix}
\]

Change the Z parameter to Y parameters by
\[
Y_{11T} = \frac{Z_6(Z_4 + Z_5)}{Z_4Z_5Z_6}, \quad Y_{12T} = -\frac{1}{Z_5}
\]
\[
Y_{21T} = -\frac{1}{Z_5}, \quad Y_{22T} = \frac{Z_4(Z_5 + Z_6)}{Z_4Z_5Z_6}
\]

N1 and N2 are in parallel connection. Therefore, the proposed filter can be derived in

Y parameters as
\[
Y_{11} = \frac{Z_2 + Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} + \frac{Z_6(Z_4 + Z_5)}{Z_4Z_5Z_6}
\]
\[
Y_{12} = Y_{21} = \frac{-Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} - \frac{1}{Z_5}
\]
\[
Y_{22} = \frac{Z_1 + Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} + \frac{Z_4(Z_5 + Z_6)}{Z_4Z_5Z_6}
\]