Enhancement of Liquids Mixing Using Active Pulsation in the Laminar Flow Regime

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Nomenclature

\( A \) Area \( m^2 \)

\( c \) Tracer concentration \( mg/L \)

\( d \) Dimension of jet orifice, \( m \)

\( D \) Diffusion coefficient, \( m^2/s \)

\( D \) Diameter of piston or orifice, \( m \)

\( f \) Frequency, \( Hz \)

\( h \) Width of the mixing channel, \( m \)

\( h \) Depth of cavity or orifice

\( I \) Fluorescence intensity as in tif iamge

\( l \) Length in the third dimension \( m \)

\( l \) of the mixing channel

\( p \) Pressure \( Pa \)

\( Q \) Flow rate \( L/min \)

\( pitch \) Distance between orifices \( m \)

\( t \) Time, \( s \)

\( T \) Period of Pulsation, \( s \)

\( U,u \) Velocity, \( m/s \)

\( V \) Characteristic velocity, for defining \( Re \) \( m/s \)

\( W \) Interrogation windows size \( pixel \)

\( x,y,z \) Coordinate, \( m \)

Greek Symbols

\( \Delta \) Piston displacement, \( m \)

\( \Delta x, \Delta y \) Pixel displacement \( m \)

\( \mu \) Dynamic viscosity, \( Pa\cdot s \)

\( \rho \) Density, \( kg/m^3 \)

\( \nu \) kinematical viscosity, \( m^2/s \)

\( \omega \) Angular velocity , \( rad/s \)
Dimensionless Number

$L$ Stroke length

$M$ Magnification

$N$ Number of synthetic jet along the channel

$R_{Q}$ Ratio of flow rate

$R_{h}$ Ratio of jet orifice to channel width

$R_{pitch}$ Ratio of jet pitch distance to channel width

$Pe$ Peclet number

$Re$ Reynolds number

$Re_{L}$ Re based on Stroke length

$S$ Stokes number

$Str$ Strouhal number

Subscript

$a$ Axial

$b$ Background

$c$ Synthetic jet cavity

$eff$ Effective

$j$ jet

$o$ Synthetic jet orifice

$r$ Radial

$n$ Net flow to be mixed

Abbreviation

$CoV$ Coefficient over variation

$DC$ Duty cycle

$LSJ$ Lateral synthetic jet

$M.D.$ Mixing degree

$PF$ Primary flow

$S.D.$ Standard deviation

$SF$ Secondary flow

$SV$ Solenoid valve
Abstract

Both the need for mixing highly viscous liquids more effectively and the advance of micro-scale applications urge the development of technologies for liquid mixing at low Reynolds numbers. However, currently engineering designs which offer effective jet mixing without structural and operational complexity are still lacking.

In this project, the method of enhancing liquid mixing using active pulsation in the laminar flow regime is explored experimentally. This work started by improving the inline pulsation mechanism in an existing confined jet configuration whereby the fluid from a primary planar jet and two surrounding secondary planar jets are pulsed by active fluid injection control via solenoid valves in the out-of-phase mode. The influence of Reynolds number, pulsation modes, frequency, duty cycle on mixing is then investigated using PLIF and PIV experimental techniques. A combination of different mixing mechanisms is found to be at play, including sequential segmentation, shearing and stretching, vortex entrainment and breakup. At a given net flow Reynolds number, an optimal frequency exists which scales approximately with a Strouhal number ($Str=flh/U_j$) about 1. This optimal frequency reflects the compromise of the vorticity strength and segmentation length. Furthermore, a lower duty cycle is found to produce a better mixing due to a resultant higher instantaneous Reynolds number in the jet flow. Overall, the improvement of the rig has resulted in an excellent mixing being achieved at a net flow Reynolds number of 166 which is at least order of magnitude lower than in the original rig.

In order to achieve fast laminar mixing at even lower Reynolds numbers, the active pulsation mechanism using lateral synthetic jet pairs is designed and tested at a net flow Reynolds number ranging from 2 to 166 at which a good mixing is achieved. The influence of actuation frequency and amplitude, and different jet configuration is evaluated using PLIF and PIV experimental techniques. At the mediate to high Reynolds numbers tested in this study, the interaction and subsequent breakup of vortices play a dominant role in provoking mixing. In contrast, at the lower end of Reynolds numbers the strength of vortex rollup is weakened significantly and as a result folding and shearing of sequential segments provide the main mechanism for
mixing. Therefore it is essential to use multiple lateral synthetic jet pairs to achieve good mixing in both mixing channel and synthetic jet cavity at this Reynolds number. It is found that an increase in both the actuation magnitude and frequency improves mixing, thereby the velocity ratio represents the relative strength of the pulsation velocity to the mean flow velocity is crucial for mixing enhancement. In order to identify actuation conditions for good mixing, a regression fit is conducted for the correlation between the dimensionless parameters, net flow Reynolds number $\text{Re}_n$, stroke length $L$ and Strouhal number $\text{Str}$. Over the tested range of the net flow Reynolds number from 2 to 83, the relationship of parameters is found as $\text{Re}_n^{0.14} L^{2.3} \text{Str} = 18.5$ and the velocity ratio at least above 2.0. Suggested by the comparatively small exponent, net flow Reynolds number is less influential than stroke length and Strouhal number.

The success in obtaining excellent mixing using lateral synthetic jet pairs at low Reynolds numbers in the present work has opened up a promising prospect of their applications in various scenarios, including mixing of highly viscous liquids at macro-scale and micro-mixing.
Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institution of learning.
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Journal Papers

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Qingfeng XIA, Shan ZHONG, Quantification of Y-shaped jet mixing by active injection control, Experimental Thermal and Fluid Science (under review)

Conference

Qingfeng XIA, Shan ZHONG, Presentation on 7th International Symposium on Mixing in Industrial Processes, Sep. 2011 (presented)
Software developed during PhD research

(1) clUtils v0.2 Basic Linear OpenCL C library (released by BSD licensed),

(2) clFoam v0.2 OpenCL solver plugin for OpenFoam1.7 (Open source computational fluid dynamics software, GPLv3),
First announced in 6th OpenFoam workshop, 2011

(3) Matlab Flow Visualization Toolbox v1.0, Support image post-processing for:
  PLIF (Planar Laser Induced Fluorescence),
  PSP (Pressure Sensitive Paint),
  TSP (Temperature Sensitive Paint),
  BOS (Background Oriented Schlieren),
  TSCL (Thermal Sensitive Liquid Crystal);
Chapter 1. Introduction

1.1. The need for mixing in laminar flow regime

Mixing is a complicated process which occurs in both industry and nature. Mixing is defined as the reduction of inhomogeneity to a satisfied condition; the inhomogeneity may refer to concentration, phase, or temperature difference (Paul et al., 2004). As shown in Figure 1.1, the mixing phenomena span a wide range of length scales and Reynolds numbers. From the turbulent mixing in the stars to the laminar mixing in the mantle of the Earth, the Reynolds number varies by forty orders of magnitude. For the mixing processes in chemical, biochemical and food engineering, the Reynolds numbers and length scales can span up to ten orders of magnitude.

Stirred tanks and inline pipe mixing are the two commonly used facilities in industrial mixing processes (Nasr et al., 2002). Stirred tanks are effective for batch mixing of a large volume. In contrast, in-line mixing has the advantages of compactness,
energy-effectiveness, and capability of changing the mixing ratio dynamically. Furthermore, the continuous processing will rely on the in-line mixing facilities. Although batch mixing using stirring vessels has been well-established in industries, a shift from batch mixing to inline (continuous) mixing is forecasted, especially in micro-mixers (Hessel et al., 2005). Some common applications of micro-mixing technologies in chemical and biological engineering are shown Figure 1.2 (Jeong et al., 2010).

Miscible liquid mixing is regarded as simple due to the absence of inter-phase interactions and chemical reactions. But it is difficult if the Reynolds number is below the critical value for the onset of turbulent flow, which is 1800 for a circular pipe and 1000 for planar flow (Klaassen and Peltier, 2006). Mixing at such low Reynolds numbers is several order of magnitude slower than turbulent mixing. Furthermore, molecular diffusion of highly viscous liquids deteriorates as a result of the low molecular diffusion coefficient, due to the high viscosity matrix (Paul et al., 2004).

Figure 1.2 Applications of micro-mixing in chemical and biological engineering (Jeong et al., 2010)

Therefore, passive or active mechanisms for increasing interfacial areas are
indispensable for mixing at low Reynolds numbers. However, there is no precisely
defined range for “low Reynolds numbers”. In a review of micro-mixers (Nguyen and
Wu, 2005), Re > 100 is considered as high, 10 < Re < 100 as intermediate and Re <
10 as low. Whereas, in the field of macro-mixing, the upper limit of “low Reynolds
number” may extend to 200 (Mackley and Stonestreet, 1995).

Both the need for mixing highly viscous liquids more effectively and the advance of
micro-scale application urge the development of technologies for liquid mixing at
low Reynolds numbers. Furthermore, an improved mixing efficiency can reduce
energy consumption dramatically, in favour of the low carbon economy trend.

1.2. Mixing techniques for low Reynolds numbers

Due to the absence of turbulent mixing at low Reynolds numbers, passive or active
means of provoking mixing is dispensable in order to achieve desired level of mixing.
Passive mixing is characterised by no exterior energy consumption except the
pumping action or pressure source. Whereas, active mixing relies on the extra energy
input to disturb the flow to be mixed, according to the active control logic.

Several passive mixing means are experimentally proved effective at Reynolds
numbers under 500 approximately (Nguyen and Wu, 2005), such as multi-lamination
(Branebjerg et al., 1996; Lee and Kwon, 2009), flow split-and-recombine (SAR)
(Miranda et al., ; Suzuki and Ho, 2002; Niu and Lee, 2003; Ryrie, 2006; Zumbrunnen
et al., 2006; Metcalfe and Lester, 2009), chaotic advection (Ryrie, 2006). However,
those technologies have disadvantages, such as difficulty in cleaning and structural
complexity. On the other hand, the active mixing mechanisms need extra energy
inputs and pulsation control, which complicate the operations. For example, a high
voltage electrical source is necessary to generate disturbance by the electromagnetic
forces (Oddy et al., 2001).

Jet mixing, which permits both rapid inline mixing and convenient maintenance, has
attracted a lot of research interest. Jet mixing, in principle, may involve the use of the
passive mixing techniques, such as collision of jets and eddy formation (Yang et al.,
2004; Ashar Sultan et al.), and active mixing techniques, such as periodic injection
switching (Niu and Lee, 2003; Hessel et al., 2005). In jet mixing, intensive flow
shearing initiated by an intermittent injection of fluid flow and sequential segmentation generated by out-of-phase periodic injection can enhance mixing effectively. For instance, controlled injection has been proven effective in droplet mixing in lab-on-a-chip systems (Paik et al., 2003). Although, jets collision has been reported effective for laminar mixing above Reynolds number of several hundreds (Wong et al., 2004), a simple passive jet mixer is still lacking at lower Reynolds numbers.

1.2.1. Introduction to synthetic jets

A synthetic jet is a zero-mass-flux jet, which is synthesized from the ambient fluid. A typical synthetic jet actuator consists of a small cavity with either an oscillating membrane or a reciprocal piston at one side and an orifice on the other, see Figure 1.3. As a result of the volume change of the cavity, a train of vortical structures is produced which propagate from the orifice at their own self-induced velocity (Glezer and Amitay, 2002; Holman, 2005; Zhou et al., 2009). These vortical structures are capable of entraining the ambient fluid into its cores thereby providing an effective mechanism in transporting mass and momentum across a flow field. Together with their unique feature of injecting vorticity and momentum into the flow field without the need of an external fluid source, synthetic jets bear many potential promises to control and management of fluid flows.

![Figure 1.3 Schematic of a typical circular synthetic jet actuator](image_url)

Synthetic jets have received intense research attention since the 1990s. Most of these studies has been carried out for the potential applications of delaying flow separation over an aerodynamic body in which synthetic jets are ejected through a slot or an
array of circular orifices upstream of a separated flow (Shuster and Smith, 2007; Tang et al., 2007; Jabbal and Zhong, 2008; Tesar, 2009; Zhang and Zhong, 2010). In more recent years, there has been an increased interest in surface cooling using impinging synthetic jets (Travaicek and Tesar, 2003; Pavlova and Amitay, 2006; Li., 2009; Chaudhari et al., 2010). Some preliminary studies of mixing by synthetic jets also have been carried out (Ritchie et al., 2000; Travaicek and Tesar, 2003).

Lateral synthetic jets with orifices oriented normally or at an angle to an incoming flow in a channel are capable of generating intensive local disturbances hence bear the potential promises to enhance mixing. In addition, the strength of synthetic jets can be varied independently from that of the main flow such that the mixing can be effective in low Reynolds number flows where turbulence is absent. However, the use of synthetic jets in mixing enhancement has been explored very little and the mechanisms by which synthetic jets promote mixing are not yet understood.

1.3. Aims and objectives

1.3.1. Background of this project

At the time this project started, a confined jet mixing rig became available due to the completion of a PhD project (Kamou 2007). Figure 1.4 illustrates the geometry of the jet confluence region and the details of the nozzle outlets and the cross-section of the initial part of the mixing channel are also shown. In this rig, water mixing was studied with an upstream velocity pulsation created by rotation valves at a Reynolds number ranging from 2000 to 10000. In the experiment, both the pulsation frequency of the valves and the duty cycle were varied in order to study their effect on the mixing. The velocity difference between the time-averaged velocity in the central confined jet flow and that in the surrounding jets flow, is found to be critical for fast mixing with a large difference resulting in a better mixing. It was also found that mixing deteriorated as the Reynolds number reduces and a sufficient mixing cannot be achieved at a Reynolds number below 2000 at which turbulence and flow instability are substantially reduced.

In view of the poor performance of the rotation valves used in creating jet pulsation,
the initial goal of this project is to improve the flow pulsation mechanism so as to push the value of the Reynolds number at which good mixing can be achieved to a Reynolds number as low as possible. Therefore, the rotation valves are replaced by solenoid valves to provide a more effective control of jet pulsation. Pulse width modulation (PWM) is employed to allow a periodic switching of fluid via a square wave signal with a chosen duty cycle.

![Figure 1.4 Illustration of the jet mixing part of water mixing rig (Kamau, 2007)](image)

Figure 1.5 shows the schematic of confined jets mixing by actively control of injections by PWM. Sequential segmentation and stretching can be achieved by the alternative injection from the confined jet and the surrounding jets, controlled by the solenoid valves according to the PWM signals. The advantage of the PWM mixing enhancement is the simplicity of the control logic which can be integrated into the flow rate control module.

With the modified experimental rig, the Reynolds number at which good mixing can be achievable is pushed down to 166. After the achievement of this initial goal, the rig is modified further to facilitate the use of lateral synthetic jet pairs so as to achieve good mixing at an even lower Reynolds numbers. Again, the control parameters and the configuration of lateral synthetic jet pairs are varied to allow the effect of these parameters to the effectiveness of mixing to be explored fully to the lowest Reynolds number which is achievable in this rig.
Figure 1.5 Schematic of confined jet mixing enhanced by active injection control

The schematic of a 2D mixer enhanced by lateral synthetic jet pairs is shown in Figure 1.6. Two different liquids flow parallel into the mixing channel from the bottom, which are referred as liquid A and liquid B and shown in red and blue colour respectively. One or more lateral synthetic jet pairs deployed along the straight mixing channel provide intensive local perturbation at the jet exit regions.

Figure 1.6 Schematic of liquid mixing enhanced by lateral synthetic jet pairs

The lateral synthetic jet (LSJ) pair refers to two synthetic jets with an opposite or
slightly staggered configuration on the mixing channel (see Figure 1.6). In addition, the LSJ pair operates in the out-of-phase mode, i.e. two lateral synthetic jets share the identical piston movement amplitude but in an opposite direction. Compared with the integral zero-mass in one pulsation period of the single synthetic jet, the LSJ pair maintains a constant volume flow rate in the mixing channel.

Mixing pulsated by lateral synthetic jet (LSJ) pairs has the following advantages.

(1) The mixing effect is not sensitive to the flow rate of the incoming liquids to be mixed, i.e. the mixing effect is stable for a moderate range of Reynolds numbers based on incoming fluids velocity.

(2) Local pulsation generated by the out-of-phase lateral synthetic jet pair will influence little on upstream or downstream flow fields; this is useful in the circumstances when maintaining the steady flow rate is crucial.

(3) Lateral synthetic jet pairs mixer is easy to manufacture due to its geometrical simplicity: the smooth straight mixing channel and the simple synthetic jet cavity geometry.

(4) Lateral synthetic jet pairs are easy to control. Although external energy input and control scheme is necessary, all the lateral synthetic jet pairs share the identical displacement actuator, therefore, the overall system complexity can be reduced.

1.3.2. Objectives

As mentioned above, the current research aims to achieve fast laminar mixing at a Reynolds number as low as achievable using the existing mixing rig. To achieve this aim, the objectives of this project are specified as follows:

1. To improve the jet pulsation mechanism in the existing mixing rig, examine the effects of changing pulsation frequency, duty cycle, pulsation mode on the level of fluid mixing using PLIF and PIV, quantify the effectiveness of laminar mixing and identify the mixing mechanisms via active injection controlled by solenoid valves in a confined jet configuration.

2. To design the actuator and control system for the lateral synthetic jets pairs
arrangement, examine the effects of changing actuation intensity, frequency and orifice configuration on the level of fluid mixing using PLIF and PIV, quantify the effectiveness of liquid mixing and identify the mixing mechanisms by single or multiple lateral synthetic jets pairs.

3. To establish the parametric conditions for achieving effective mixing with lateral synthetic jets pairs in the laminar flow regime.

1.4. Research methodology

In this work, all the results are obtained from experiments. Particle image velocimetry (PIV) and planer laser induced fluorescence (PLIF), are used as the primary experimental techniques to study the effect of mixing. In addition, the inertial dye is utilized in stereoscopic dye visualization to reveal the presence of 3D flow structures in the mixing channel. PLIF provides contours of fluorescence concentration allowing the flow structures present in the mixing channel to be visualised and offering a visual comparison of the level of mixing at different flow conditions. It also allows the quantitative evaluation of the mixing degree. On the other hand, PIV is indispensible to provide the velocity data required in the quantification of the mixing degree. It also reveals the vortex structure and its interaction in the mixing channel.

Throughout this project, water and sugar solution are used as the fluid media. In this setup, the lowest volume flow rate achievable is dictated by the stable flow rate permitted by the pumps. In order to reduce the Reynolds number, sugar solutions with different levels of sugar concentration with a viscosity up to 50 times of water, are used. Here sugar solutions are considered be more desirable compared to other viscous fluids, such as silicone oil or glycerol, because they are cheaper to reproduce and easier to be disposed in relatively large quantities.

1.5. Outline of this Thesis

This thesis consists of seven chapters. After this introduction chapter, Chapter 2 provides a review of the literature relevant to in-line mixing at low Reynolds numbers. It includes a review of the basic mechanisms of laminar mixing, a description of
passive and active mixing techniques as well as jet mixing, and an introduction to the mass and heat transfer enhancement by synthetic jets.

Chapter 3 describes the setups used in the experiments for studying the effect of fluid mixing enhanced by actively injection control using solenoid valves as well as fluid mixing augmented by lateral synthetic jet pairs. The experimental techniques, including dye visualisation, PIV and PLIF, and the method of quantifying the mixing degree are also described.

Chapter 4 discusses the experimental results obtained from the confined jet water mixing which is enhanced by active injection control via solenoid valves. The flow patterns in the mixing channel when both the confined jet and the surrounding jets are pulsed in the out-of-phase mode are analysed, and the influence of various control parameters on the mixing degree is illustrated. The effect of pulsation applied to either the confined jet or the surrounding jets only are also investigated and compared with the results obtained when both the confined jet and the surrounding jets are pulsed.

Chapter 5 presents the experimental results of the water mixing enhanced by single and three lateral synthetic jets pairs in the opposing or staggered configuration at a Reynolds number in the range of 50 to 166. The influence of jet pair configurations, the Reynolds number, pulsation amplitudes and frequencies on the mixing effect is then investigated.

In Chapter 6, the results obtained from the mixing experiments using single and three lateral synthetic jets pairs in sugar solution at a Reynolds numbers ranging from 2 to 20 are presented. The influence of jet pair configurations, the Reynolds number, pulsation amplitudes and frequencies on the mixing effect is then investigated.

Finally, the key findings from this work are summarised and the suggestions for future work are given in Chapter 7.
Chapter 2. Literature Review

This chapter aims to provide a literature review of the continuous liquids mixing enhanced at the laminar flow regime. First of all, the principles for the laminar mixing are briefly introduced. These are followed by introduction to several typical active and passive mixing techniques, in order to identify their strengths and weaknesses. Secondly, the application of synthetic jets in heat and mass transfer is reviewed, in order to highlight the need for further research on the laminar mixing by synthetic jets.

2.1. Principles of laminar mixing

Although mixing is associated with an extremely wide range of time and length scales (Figure 1.1), it can be categorized into laminar and turbulent flow regimes. As the name suggests, laminar flow is the layered flow state that layers of fluid with different velocities do not exchange fluid particles greatly in the perpendicular direction. On the other hand, the turbulence flow is characterized by the random velocity fluctuation and a large amount of inter-layer mixing (Schlichting et al., 2000). The flow regime can be judged by the dimensionless Reynolds number, which is defined in Equation 2.1.

\[
\text{Re} = \frac{L V}{\nu}
\]  

(2.1)

where \( L \) is characteristic length; \( V \) is characteristic velocity; \( \nu \) is the kinetic viscosity.

Above the critical Reynolds number, the flow can be turbulent flow; whereas the flow remains laminar under the critical Reynolds number. The critical value for pipe flow is 2300, observed by O. Reynolds in 1883. Recent studies reports that the transition from laminar flow to turbulent flow commonly occurs at a typical Reynolds number below 1800 in a circular pipe or 1000 in a planar flow (Klaassen and Peltier, 2006). Mixing in turbulent flow can be 100-1000 times faster than laminar mixing, due to interlayer convection. Therefore, laminar mixing is more challenging at low Reynolds numbers, in the circumstances of mixing of highly viscous fluids or mixing
within micro scale geometry.

Without the turbulent mixing phenomena, laminar mixing depends only on the laminar diffusion, characterized by the molecular diffusion coefficient \( D \). It is defined by Fick’s law, shown in \textbf{Equation 2.2}.

\[
\Phi = D \frac{dc}{dx}
\]  
\text{(2.2)}

where \( \Phi \) is the species flux, \( c \) is the concentration and \( x \) is the length coordination.

Peclet number is another dimensionless number relating to the rate of advection of a flow to its rate of diffusion.

\[
P_e = \frac{LV}{D}
\]  
\text{(2.3)}

The Peclet and Reynolds number diagram in Figure 2.1 shows the typical operation ranges of micro-mixers (Nguyen and Wu, 2005). There is a proportional relation between Reynolds number and Peclet number. Mixing at high Peclet numbers is dominated by flow advection (zone at the top right corner), whereas diffusion mixing dominated in the region of Peclet numbers lower than 1000 (zone at the bottom left corner). Therefore, the design or selection of mixer should refer to this diagram, since each mixer has its own optimum range of Reynolds numbers and Peclet numbers for the underlying mixing mechanisms. For example, passive mixing is a better choice for mixing at high Peclet numbers and Reynolds numbers, whereas multi-lamination is suitable for a situation when there are both low Peclet numbers and Reynolds numbers.
2.2. Mechanisms for laminar mixing

As it is suggested in Figure 2.1, laminar mixing can be categorized into diffusion dominated and advection dominated regimes. Mixing in both regimes is constrained by Fick’s Law of diffusion. According to Equation 2.2, the following measurements can accelerate the diffusion mixing:

1. An increase in the dwelling time of fluid in mixing vessel/channel by lower fluid velocity or longer mixing channel.

2. An enhancement in the diffusion coefficient of some polarized solvent by plate electrodes.

3. An increase in the solvent gradient by increasing the interfacial areas of fluids to be mixed, such as flow advection and multiple laminations.

It is common sense that mixing as fast as possible, despite the wide range of flow regimes, is the design target of the mixer. Thus, increasing the dwelling time in the mixer disobeys the design principles of timeliness and flexibility. Since it is rarely
convenient to change the diffusion coefficient of fluids, increasing the interfacial areas is the most practical way to enhance the laminar mixing. Common approaches for interfacial areas augmentation are interface stretching or chaotic advection.

2.2.1. Segmentation and interface stretching

In the case of the Poiseuille flow in a straight pipe driven by a constant pressure gradient, it has a parabolic velocity profile on the cross-section. The velocity magnitude is high in the centre, but the magnitude is low in the near-wall region. If two liquids flow in parallel through a straight channel without any pulsation, the cross-section velocity gradient does not improve the molecular diffusion. However, if the liquids are sequentially segmented, the parabolic velocity profile on the cross section can improve the mixing. This is called “shear augmented diffusion”. Similarly, flow between parallel plates, driven by a constant pressure gradient, is called planar Poiseuille flow.

The interface stretching of the sequential segmentations is illustrated in Figure 2.2. The cross-section velocity profile is set steady as the parabolic shape, according to the presumption of the fully development flow in a 2D channel. The length of the channel is dimensionless based on the channel width. At first, two interwoven rectangular fluid plugs in blue and red colour are placed at the bottom of the channel at the initial condition (Figure 2.2(a)). As the velocity of flow in the centre of the channel is much higher than the near wall region, the plugs deform into crescent shape as they flow downstream, shown in Figure 2.2 (b-d). Furthermore, the shearing and stretching will continue as the plugs flow down the channel. The increase in the total contacting interface of two liquids can accelerate the molecular diffusion greatly. Notably, the diffusion on the interface is not demonstrated, since the segment deformation is simulated in the kinetic manner.
For the shearing augment mixing system, Taylor (1953) found that the cross-section average concentration of solute eventually spreads as though there was a diffusion coefficient. The effect, also known as Taylor dispersion, is named after the British fluid dynamicist G. I. Taylor. Consequently, the diffusion augmentation can be described by an effective diffusion coefficient $D_{\text{eff}}$ (Nguyen et al. 2005). The effective diffusivity including both axial and transverse molecular diffusion effect (Dorfman and Brenner, 2001), is given by Equation 2.4.

$$D_{\text{eff}} = D + \frac{\overline{V}^2 W^2}{210D} \tag{2.4}$$

Where $\overline{V}$ is the mean velocity of the pipe or channel; $W$ is the width of the channel.

For Poiseuille flow in pipe, the efficient diffusivity is given by Equation 2.5

$$D_{\text{eff}} = D + \frac{\overline{V}^2 R^2}{48D} = D(1 + \frac{Pe^2}{48}) \tag{2.5}$$

where $R$ is the radius of pipe.
In practice, Taylor dispersion mixing is implemented via sequential segmentation, which can increase the interfaces area as the interwoven segments shearing and stretching. Sequential segmentation mechanism divides the different liquids to be mixed into interwoven segments, and the stretching of the segments due to a non-uniform cross-sectional velocity profile increases the area of interface for molecular diffusion. Sequential segmentation can be implemented either by the switching of valves (Nguyen and Huang, 2005), the utilization of the electrokinetic effect (Coleman et al., 2006), the control of the external pumps (Fujii et al., 2003) or by lateral periodic perturbation (Niu and Lee, 2003).

Multi-lamination is also effective in increasing the diffusion interface areas. Alternate feeds via bifurcation and inter-digital structure are popular implementation of multi-lamination. Tested models of multi-lamellae system include the cyclone mixer, SuperFocus, StarLaminar, etc (Hessel et al., 2005). Besides, flow split-and-recombine structure can generate multi-lamination flow patterns as well.

### 2.2.2. Chaotic advection

Chaos is the condition between “order” and “disorder”, and chaotic systems are systems that look random but are not (Ottino, 1990). Chaotic advection refers to the mass transfer in chaotic system, and it is best illustrated by the homogeneous dispersion of the particle clouds in kinematic system. The chaos is featured by initially close particles that separate from each other exponentially in time. Therefore, chaotic advection has a potential to enhance mixing efficiency.

Chaos will not appear in steady 2D laminar flow, but it is possible in the unsteady conditions (Niu and Lee, 2003). Figure 2.3 illustrates how the particles cloud disperses in the 2D channel, due to the temporal pulsation from side channels. Figure 2.3(a) show the schematic setup of the chaos system. Two liquid streams flow in parallel passing the horizontal channel along the x axis. Meanwhile, three pairs of side channels induced lateral sine flow movement crossing the horizontal channel. Due to the lateral pulsation, the initially gathered particles in the red colour stretched into the irregular particle scattering lines eventually, see Figure 2.3(b).
On the other hand, chaotic advection can be initiated by the elaborate 3D geometrical structure of the flow channel (Deval and Ho, 2002). The basic idea is to modify the channel shape for splitting, stretching, folding and breaking of the flow passing by. Various 3D structures, such as Kenics static mixer (Jaffer and Wood, 1998), the twisted pipe (Tuttle, 2006), patterned wall like herringbone grooves (Stroock et al., 2002), are designed to generate macro-scale chaotic advection. Some of these static chaotic mixers are mature industry products for processing engineering.

Moreover, chaotic advection is also popular in micromixing. The design concept of micromixers based on chaotic advection is similar to their macroscopic counterparts, which are well investigated and summarized (Ottino, 1990). In addition to the pressure pulsation, electromagnetic force (Gopalakrishnan and Thess, 2010),
dielectrophoretic force (Deval and Ho, 2002) and electrokinetical force (Lee, 2001) are employed to initialize the chaos.

### 2.2.3. Classification of passive and active mixing techniques

Figure 2.4 gives an overview of methods for micro-mixing, categorized into the active and passive approaches (Nguyen and Wu, 2005). Although this classification is based on the micro-scale mixing, it is applicable to the mixing of highly viscous liquid at macro-scale, due to the dimensionless definition of low Reynolds numbers.

![Figure 2.4 Classification of techniques for micromixers (Nguyen and Wu, 2005)](image)

Passive mixing, characterised by no exterior energy consumption except the pumping action or pressure source, is competitive in energy saving and system simplicity. Several passive mixing approaches have been experimentally proved to be effective at Reynolds numbers under 500 approximately (Nguyen and Wu, 2005), such as multi-lamination, flow split-and-recombine (SAR), chaotic advection and jet impinging mixing.

Active mixing relies on the extra energy input to disturb the flow to be mixed,
according to the active control logic. The geometry of the mixer can be as simple as a straight and smooth channel. Yet, active disturbance consumes the external energy to generate the pressure fluctuation, electromagnetic instability, and so on. Nevertheless, the control logic and the extra energy source complicate the overall mixing system. For example, a high voltage power source is required to generate electrokinetic forces.

2.3. Passive techniques

2.3.1. Serial and parallel lamination

Figure 2.5 shows some forms of the split-and-join serial laminar mixers (Nguyen and Wu, 2005): (a) join–split–join, (b) split–join (c) split–split–join, (d) multiple intersecting micro-channels. In order to increase the interfacial areas, a cascade structure is usually adopted. However, it is difficult to manufacture and clean the complex geometrical structure.

![Figure 2.5 Various split and join laminar mixers (Nguyen and Wu, 2005)](image)

To simplify the manufacturing of micro mixer with splitting and joining structures, some modification is necessary. Figure 2.6 shows a fast passive planar liquid micromixer (Melin et al., 2004) with simplified split-and-rejoin configuration in which the zigzag main channel segments are connected by lateral orifices. The micromixer was fabricated by patterning tiny orifices of a depth of 50 mm on a silicon wafer. Two sample streams flow into the first main channel segment in
parallel; the dyed sample 2 is underneath the non-dyed sample 1. Then, the dyed sample 2 flow on top of the sample 1 after the main stream reverses the flow direction in the second main channel segment. The dyed sample passing through the orifices between the first and second main channel rejoins the non-dyed sample in the next channel segment as a thin layer on top. Ideally, one more interweaved liquid layers forms as the main channel path change passes at each segment. However, it is hard to manufacture distinctly independent channels in a micro-mixer.

**Figure 2.6 Micro mixer with Split-and-recombine structures (Melin et al., 2004)**

Instances of parallel laminating mixing include layer compressing and cyclo-mixing (Hardt et al., 2002). The SuperFocus™ mixer is an example of layer compressing. The focusing structures compress the 138 interwoven liquid streams from the verge of the circular plate to the outlet at the centre (Figure 2.7 (a)). Since multiple tangential streams flow into a much narrower outlet at the centre, the layer width has been substantially reduced as the streams are squeezed toward the centre. As a result, the higher concentration gradient leads to faster molecular diffusion mixing.

A cyclo-mixer with a rotating shaft in the centre of the glass cylinder is capable of multiple laminating, shown in Figure 2.7 (b). Radial interwoven fluid streams are issued from the rotation shaft of the mixing vessel. Flow patterns in the cyclone micromixer are illustrated by CFD simulations (left) and a dilution-type dye experiment (right) in Figure 2.7 (b). Although the image obtained from the experiments result is not as clear as the numerical simulation, the matching of the
flow patterns is obvious. The flow rates of the two liquids are 30 mL/h and 20 mL/h respective for the top images, and moderate stream folding is observed. If the flow rates increased to 64 mL/h and 42 mL/h respectively, longer laminating and stronger curving is demonstrated in the circular cavity at the bottom of Figure 2.7 (b).

![Figure 2.7 Mixing by multiple laminating](image)

(a) SuperFocus™ mixer  
(b) cyclomixer by numerical (left) and experimental (right) investigation

Figure 2.7 Mixing by multiple laminating (a) SuperFocus™ mixer; (b) cyclomixer  
(Hardt et al., 2002)

Obviously, a disadvantage of multiple laminating mixers is the structural complexity, which not only troubles the manufacturing and maintaining, but also increases the pressure drop of the mixing process.

### 2.3.2. Chaotic advection mixers

Generally, faster chaotic advection occurs only at relatively higher Reynolds number (Re=25~70) (Nguyen and Wu, 2005). Comparatively simple structures such as zig-zag channel or serpentine pipe can initialize chaos at higher Reynolds numbers. Whereas, a series of elaborate 3D structures are indispensable at low Reynolds numbers. For Re<1, flow reorientation techniques may be involved to enhance the mixing at the diffusion dominated regime (Xia et al., 2005). Notably, each chaotic mixer has its own range of effective Reynolds numbers, for example, the Herringbone mixer has a wide effective Reynolds number ranging from 1 to 100.

Figure 2.8 shows some planar channel structures which are effective only at high
Reynolds numbers. Obstacles on the wall change the cross-section shape of the mixing channel, leading to vortices and 3D flow structure at high Reynolds numbers. Although the obstacle in a microchannel (Figure 2.8 (b)) can also enhance mixing at high Reynolds numbers (Wang et al., 2002), it cannot generate eddies or circulations at low Reynolds numbers. Specifically, there is a critical Reynolds number of 80 found in the experimental study for the zig-zag mixer (Figure 2.8 (c)).

More 3D structures generating chaotic advection are illustrated in Figure 2.9. The 2D Tesla structure illustrated in Figure 2.9 (a) can generate well mixing at a wide range of Reynolds number (Re>5). In contrast, the 3D serpentine channel in Figure 2.9(b), works only at higher Reynolds numbers of Re=25-70. Nevertheless, Figure 2.9 (d) shows the combination of flow split-and-join structure and the serpentine channel, which is more effective at low Reynolds numbers (Re=0.1~2). In summary, the more complicated the 3D geometric structure the more effective mixing it generates at low Reynolds numbers.
Figure 2.10 demonstrates the circulation flow pattern in the staggered herringbone mixer (SHM) (Stroock et al., 2002). The staggered herringbone refers to the two groups of asymmetric “<” shape grooves, which are “0 cycle” and “1/2 cycle” shown in the centre of Figure 2.10. Each group of grooves initiates two counter-rotating circulation on the cross-section of the mixing channel, illustrated by the streamlines on the left top of Figure 2.10 (a). The boundary between the two circulations is determined by the turning point of the “<” shape asymmetric grooves. After the half cycle, the herringbone asymmetry is swapped, so is the ratio of the two circulation areas, shown in top right corner of Figure 2.10. In addition to the theoretic analysis, the circulation pattern change is confirmed by the fluorescence dye visualization image captured by the confocal micrographs, see Figure 2.10 (b).

Moreover, Figure 2.11(a)-(c) shows micromixers with grooved or ribbed channel, which are reported effective at Reynolds numbers from 1 to 100. Besides, the chaotic advection can be introduced by the electrokinetic force on smooth channel with patterned surface charge. Micromixers (Biddiss et al., 2004) in Figure 2.11(d) and (e) have electrical anode/cathode patterns to act as the grooves and ribs in Figure 2.11(b) and (c). The heterogeneous surface charge can initiate the flow circulation like Figure 2.10, under the applied potentials ranging from 70 to 555 V/cm. Although a mixing degree up to 95% can be achieved at $Re<1$, the manufacturing process of charge surface is also complex as the etching of the micro grooves on the micro channel.
Generally speaking, chaotic mixers will not work effectively at very low Reynolds numbers ($\text{Re} < 1$), but a chaotic mixer combining with split-and-join structures can achieve fast mixing at $\text{Re} = 0.2$ (Xia et al., 2005). Numerical study suggests that the normal serpent pipe chaotic mixer generates poor mixing at such a low Reynolds number; see Figure 2.12 (a). In contrast, the structure in Figure 2.12 (b) shows excellent homogeneity after 10 periodic split-and-join structures.

**Figure 2.11 Illustration of ribs and grooves and electrokinetic initiated chaotic mixers**

*(Nguyen and Wu, 2005)*
2.3.3. Static mixers

In the early 1950s, many devices were designed to reduce the temperature inhomogeneity in polymer piping. One of the first commercial units was the Kenics device. They are also called static mixers because the mixer does not move, although
the liquids have both radial and axial movement (Meyer, 2004). In short, static mixer refers to the periodically placed baffles within the mixing pipeline. Figure 2.13 shows the baffle structures and flow division for a typical static mixer: Kenics\textsuperscript{TM} KM Series static mixers. The baffles which are patented helical surface elements (Figure 2.13(a)), directs the flow towards the pipe walls then back to the pipe centre. The radial velocity reversal and flow division are resulted from alternating right-hand and left-hand baffle elements, thus the mixing efficiency is improved. Given enough baffles elements along the pipeline, the streamwise and radial gradients in temperature, velocity and material composition is eliminating gradually (Kenics, 2008).

![Figure 2.13 Typical structure of Kenics\textsuperscript{TM} static mixer (Kenics, 2008)](image)

The flow in the Kenics\textsuperscript{TM} mixer with Reynolds numbers of 0.15, 1, 10, 100, and 1000, has been studied by FLUENT/UNS\textsuperscript{TM} software (Hobbs and Muzzio, 1998). It suggests that higher Reynolds numbers does not mean better mixing, in contrary to common sense. Surprisingly, the mixing is independent of Reynolds numbers, if Reynolds number is no greater than 10. At higher Reynolds numbers, for example Re = 100, the flow is not global chaotic, but some islands impede the mass transfer with
the rest fluid. Nevertheless, if Reynolds number is greater than 1000, the flow is mainly chaotic again, except for some small islands. Therefore, mixing at Reynolds numbers higher than 100 but lower than 1000 may be not as effective as lower Reynolds numbers.

In addition, more energy is consumed at higher Reynolds numbers, as the pressure drop in the Kenics\textsuperscript{TM} static mixer is proportional to Reynolds number (Joshi et al., 1995) for $\text{Re}<10$ while it is approximately proportional to $\text{Re}^{3/2}$ for $10 < \text{Re} < 1000$. Thus, $\text{Re} < 10$ is the most energy efficient Reynolds number range for the Kenics\textsuperscript{TM} static mixers.

Similarly, the KMX static mixer is excellent for mixing applications involving fluids with extreme viscosity or volume ratios. Figure 2.14 illustrates the KMX static mixer which seems more complex than the Kenics\textsuperscript{TM} KM Series. The KMX utilizes flow splitting to achieve very rapid mixing. Each KMX element is a cylinder consisting of multiple interweaving blades, which split fluid into layers as the mixture flows downstream. Each blade features concave construction offering better cross-stream flow movement than flat blades for superior mixing per unit length in the tough applications with high viscosity ratio. Therefore, Reynolds numbers as low as practical are preferred for KMX static mixer, considering both the mixing rates and energy requirements (Zalc et al., 2002).

There are many different static mixers available for industry usage; some of them are shown in Figure 2.15. Each of them is optimized for the specific application; the selection of various static mixers is tabulated in Cullen’s book on Food mixing (Cullen, 2009), according to the industrial applications along with Reynolds number regime. In addition, some guidelines for static mixer selection according to the flow regimes and application circumstances been summarized in \textit{Handbook of Industrial Mixing Science and Practice} (Meyer, 2004), shown in Figure 2.16.
Figure 2.14 Illustration of the Kinees KMX static mixer

Figure 2.15 Static mixer design options (Meyer, 2004) From left: vortex mixer (type KVM), corrugated plate (type SMV), wall-mounted vanes (type SMF), cross-bar (type SMX), helical twist (type KHT), cross-bar (type SMXL)
Figure 2.16 Rough guidelines for mixing applications in laminar and turbulent flow regimes (Meyer, 2004)

### 2.3.4. Baffled tube mixer

The concept of oscillatory flow in baffled tubes has been steadily studied since 1989; a brief research history and some dimensionless numbers about oscillatory flow in a baffled tube is discussed (Ni and Gough, 1997). The typical system of an oscillatory baffled tube consists of a tube with baffled structures, an oscillating unit providing the reciprocal pulsation, flow input and output facilities, and measurement and control accessories. A scheme of a meso-scale oscillatory baffled tube mixing system is illustrated in Figure 2.17. Particularly, the close-up view of the baffled structure, which is the key to the mixing enhancement, is shown at the right top of the Figure 2.17. Moreover, the baffled structure is diverse in geometry and layout. Figure 2.18 (a) shows the ring-shaped baffles in meso-scale tube, and Figure 2.18 (b) demonstrates the multiple holes plate in the scaled-up application.
The chaotic advection in the oscillatory flow in baffled tubes has been studied steadily since 1989 (Ni and Gough, 1997; Ni et al., 2003). The baffled tube schemed in Figure 2.19 (b) are mounted with equally-spaced baffled structure which traverse to periodically reversing flow in the tube. The flow oscillation in tube generates back-to-back vortex rollup between the baffles (Figure 2.19 (c)). On the contrary, no vortex structure is found in Figure 2.19 (a) for the oscillating flow in the smooth wall tube. For higher oscillating Reynolds numbers, the formation and development of the stronger vortex results in the chaos in the cavity between the baffles (Figure 2.19 (d)).
Besides, baffle tubes with and without oscillation is studied at net flow Reynolds number of 128 on an oscillatory baffled bioreactor (OBB). The bioreactor is designed for better control over the cultivation Pullulan (Gaidhani et al., 2003), in order to
improve the homogenization of mixing at a very low shear rate. Experiments indicate that fluid oscillation in the absence of baffles does not significantly affect the laminar flow profiles substantially. In contrast, chaotic advection flow occurs for oscillatory flow with baffles (Mackley and Ni, 1991).

### 2.3.5. Impinging jets

A passive mixer using impinging jets is advantageous due to the simple structure, but it is only effective at high Reynolds numbers. There is a commercial micromixer based on the impinging jets. Figure 2.20(a) shows the micromixer with a dimension of 10x35x10mm (microinnova.com, 2011(retrieved)). The mixer consists of two inlet tubes on the top driven by two separated pumps and two 350µm wide nozzles on the bottom (Figure 2.20(a)). Furthermore, the impinging-Jet micromixer works in different flow patterns depending on Reynolds numbers, shown in Figure 2.20(b). After all, the fanned-out jet pattern is preferred for fast mixing at higher Reynolds numbers.

Jet impinging in a micro T-mixer has been studied at high Reynolds numbers ranging from 20 to 1400 (Wong et al., 2004). The impinging jet T-mixer, illustrated in Figure 2.21, consists of two horizontal and inlet channels on the top and one vertical outlet channel. There is no baffled structure in the smooth channels, or any pulsation on the inlet flow streams. Therefore, mixing by jet impinging relies on natural flow instability and vortex structure, etc.
The dominant effect of high Reynolds numbers on the jet impinging is suggested by Figure 2.21, which shows the dye visualization of the mixing at the junction of the micro T-mixer at different applied pressures: (a) 1.12 bar, (b) 1.88 bar, (c) 2.11 bar, (d) 2.48 bar, (e) 2.77 bar and (f) 4.27 bar. The corresponding Reynolds number based on the outlet channel width is ranging from 200 to 600, since Reynolds number is approximately linear with the applied pressure. At first, there is no obvious mixing but distinct stream striations, if Reynolds number is below 200 (Figure 2.21(a)). Then, similar striations of dye can be observed in the mixing channel in the Reynolds number range from 150 to 400. Meanwhile, the liquid streams break up into striations progressively at higher Reynolds number. Finally, there is a sudden improvement of mixing performance shown in Figure 2.21(f). Therefore, it is suggested that fast mixing in the micro impinging jet T-mixer can be achieved at Reynolds numbers between 400 and 500 or above.
In addition to the experimental study, the three-dimensional numerical simulation by Fluent™ 6.1 demonstrates the flow regions and patterns for the jet impinging mixer (Wong et al., 2004). Figure 2.22 (a) shows the different regions at the T-shape junction: crossing flow with secondary flow, the region of vortices and the separation of boundary layer. The swirling flow and vortices initialized in the micro T-mixer explain why fast mixing can be achieved at Reynolds numbers ranging from 500 to 1400. On the other hand, Figure 2.22 (b) illustrates the secondary flow patterns on the cross section X-X. The secondary flow that occurs during the change of flow direction after the impinging sweeps the partially mixed liquid to the walls of the mixing channels. This leads to the blurring of the striations observed in the mixing experiments.
Furthermore, the flow structures in the mixing channel of the T-shaped mixer has been categorised into 3 regimes (Soleymani et al., 2008): stratified flow, vortex flow, and engulfment flow, see Figure 2.23. The engulfment flow refers to the breakup of the flow symmetry for the high velocities, which has been confirmed by the 3D flow structure obtained by stereo micro particle image Velocimetry (Hoffmann et al., 2006). Transition from vortex flow (Figure 2.23 (a) and (c)) to engulfment flow (Figure 2.23 (b) and (d)) is reported to improve the mixing dramatically (Soleymani et al., 2008). The importance of the flow transition in the jet mixing is consolidated by the studying of the impinging flow in a micro T-mixer with the conclusion that
fast mixing is caused by streams breaking up into striations at Reynolds numbers between 400 and 500 or above (Wong et al., 2004).

Figure 2.23 Path lines at the entrance of the mixing channel for both vortex and engulfment flow regimes in a T-mixer (Soleymani et al., 2008)

2.4. Active mixing techniques

2.4.1. Various active mixers

Figure 2.24 shows various active mixing system: (a) sequential segmentation, (b) pressure disturbance along the mixing channel, (c) integrated micro-stirrer in the mixing channel, (d) electrohydrodynamic disturbance, (e) dielectrophoretic disturbance, (f) electrokinetic disturbance in the mixing chamber and (g) electrokinetic disturbance in the mixing channel. Except for the pressure fluctuation or mechanic stirring, (b) and (c)), micro-mixing also is initialized by the electromagnetic forces. Experiments has confirmed that chaotic mixing can be generated by both the electrokinetic and pressure driving in micro mixer (Lee, 2001).
The flow electrokinetic instability (EKI) generated by the fluctuating electric field, can be adopted in micro-mixing. The electrokinetic instability refers to the flow instability initialized by the oscillating electro-osmotic movement, and an EKI micromixer is investigated experimentally at $Re=1$ approximately (Oddy et al., 2001). Fluid A and fluid B, introduced into the mixing chamber from the left channel 1 and 2, are directed to outlet channel 5, see Figure 2.25 (a). An opposite electric field strength above 100V/mm is applied to the channel 3 and channel 4, which are connected to the mixing chamber on the top and bottom respectively. Meanwhile, the EKI and mixing happen in the rectangular chamber with a width of 1mm, which is illustrated as black block in the centre of the mixer.

Figure 2.25 (b) shows the temporal sequence of the fluorescence dye visualization of the mixing chamber. Firstly, no observable mixing at $t=0$s, before the electric field fluctuation acts on the liquid. After the application of high amplitude voltage oscillating, the interface is disturbed by the electro-osmotic effect and substantial mixing in the chamber can be observed at $t=1.5$s. Finally, homogenous fluorescence
texture is found at the outlet at $t=3s$. In conclusion, the mixing time estimated as $t=2.5s$ has shown the high efficiency of the micro mixing by EKI.

![Schematic of EKI micromixer](image1.png)

![PLIF experiment result](image2.png)

**Figure 2.25 mixing by electrokinetic instability (EKI) micromixer**

(Oddy et al., 2001)

Notably, in order to take advantages of electrical field disturbance, the flow to be mixed needs to be sensitive to the electrical field applied on the mixing channel wall. Fortunately, the common solvent water is capable of reacting to the high voltage electrical field. Although there is no moving part but electrical anode or cathode plates within the mixers, a high electrical field (300V/cm in typical case) is necessary to generate significant electrophoresis or electro-osmotic effect in micro-mixing device (Fushinobu and Nakata, 2005).

### 2.4.2. Sequential segmentation by active injection control

The ideal way to produce sequential segmentation is to switch different liquids into microstructure devices alternatively and periodically. The jets confluence configuration can be either a T-shaped or a Y-shaped junction with a variable flow confluence angle. Notably, it has been reported that the variation of the angle can not substantially improve the mixing (Niu and Lee, 2003). Figure 2.26 shows the schematic illustration of sequential segmentation in a Y-shape mixer. Ideally, the short plugs in the mixing channel generate fast mixing due to the increasing interfacial areas. Therefore, the higher switching frequency is preferred if segmentation is successful. However, the upper limit of the switching frequency is...
limited by the capability of the flow control mechanism to generate segmentation in the micro-channel.

![Schematic illustration of alternative switching in Y-mixer](image)

Figure 2.26 Schematic illustration of alternative switching in Y-mixer

(Niu and Lee, 2003)

In practice, sequential segmentation can be implemented by valve switching or active controlled pumping. Solenoid actuators can switch the different fluids into the mixing channels alternatively, and the ratio of fluids to be mixed can be controlled by the switching ratio (Huiqian et al., 2006). Except for valve switching, a continuous micromixer equipped with pulsatile micro-pumps is capable of generating sequential segmentation (Deshmukh and Pisano, 2000).

An innovative micromixer, which utilizes an expansion chamber to amplify the interfacial area of two liquids (Coleman et al., 2006), demonstrates the potential of mixing enhancement by sequential segmentations in Figure 2.27. Sequential segmentation forms interweaved flow plugs in the straight channel before the chamber, driven by electrokinetic effect from a reservoir A and B. The flow pathway is illustrated as the black line and the reservoirs as black blocks on the top of Figure 2.27. These plugs deform into interweaved arcs in the suddenly expanded chamber, and eventually the diffusion finishes quickly in the chamber, shown in the close-up view of the expansion chamber at the bottom of Figure 2.27.
Furthermore, Figure 2.28 illustrates the segmentation in micro-channel via periodic perturbation from a pair of opposing lateral jets (Niu and Lee, 2003). The lateral jets are driven by alternatively switching between the high and low pressure sources via the solenoid valves. Due to low Reynolds numbers in micro-scale liquid mixing, only sequential segmentation without vortex rollup is indicated by the crescent shape in the dye visualization.

In addition, pulse width modulation (PWM) flow control can be used for segment manipulation, besides the flow rate control function via successively switching valve on and off (Rangel et al., 2010). Sequential segmentation can be generated by the
alternating injections. Moreover, the intensive flow shearing initialized by the fluctuation of jet flow rate can enhance mixing effectively. For instance, the precise control of injection has been experimentally proved effective in droplet mixing on a lab-on-a-Chip system (Paik et al., 2003). Using a square wave signal with various duty cycles, PWM is distinguished from the periodic flow switch (Glasgow and Aubry, 2003) between a high and low pressure source according to a sine wave signal. Nevertheless, the pressure sources switching is regarded too complex to control (Niu and Lee, 2003), while the PWM pulsation mechanism can be integrated into the flow rate control unit.

2.4.3. Lateral jet pulsation

Injection form a lateral channel, which initializes either vortices or large scale structures in shearing layers, can substantially enhance mixing in the main channel. Figure 2.29 shows two of various lateral jet confluence configurations. The top one is referred as Y-shaped jet configuration; the horizontal channel is regarded as the main channel and the lower channel as the side channel. The bottom one is called the confined jet configuration, where the centre channel is confined by the two surrounding jets. The pulsed jet from the side channel is capable of generating vortex rollups, segmentations and flumes in the primary channel. Further discussion on lateral jet configuration and mixing mechanism is presented by Kamau (2007).

![Figure 2.29 Schematic illustration of jet mixing configuration (Kamau, 2007)](image)

Particularly, T-mixer refers to the Y-shaped confluence configuration whose angle between the side channel and the main channel is 90°. The mixing of two aqueous
reagents in a T-shaped channel was studied both numerically and experimentally (Glasgow and Aubry, 2003), and the tracer concentration contours are shown in Figure 2.30. The out-of-phase temporal variations of velocity for left and top inlet channels are shown in Figure 2.30 (a); the fluctuation amplitude of the velocity is much higher than the time averaging magnitude. As a result, the inlet velocity is negative for a duration, which means the channel injects and sucks alternatively in one pulsation period. Figure 2.30 (b) shows the concentration contours at the different phases, as qualitative indicators for the mixing effect. The crescent shape segmentation is observed, but homogeneity is not achieved for presented channel length.

![Image](image_url)

**Figure 2.30 Inlet velocity profile and mixing effects (Glasgow and Aubry, 2003)**

### 2.4.4. Oscillating jets

Oscillating jets (Tesar, 2009) refer to two impinging jets periodically switching the
output channels under pulsation. Figure 2.31(a) shows an oscillating jet mixer which consists of two perpendicular inlets at the top and bottom and two horizontal outlets. Fluid A and fluid B are temporally pulsed at the entrance of the oscillator. The periodic pulsation mechanism makes liquid A injected into the one of the horizontal outlet channels in half pulsation period, while liquid B is injected into another outlet channel. Then, liquid A and liquid B swap the output channel in the second half of pulsation period. Thus, interwoven segments are generated in the outlets channels, and segments are stretched and mixed in two horizontal outlet channels (Figure 2.31(b)). Although no extra energy input and no moving parts are needed for the jet oscillation, there is a threshold for the dynamic effects, which is estimated as Reynolds numbers above 30. It means the oscillating jet may not work at lower Reynolds numbers.

![Figure 2.31](image)

(a) Schematic of oscillator micro-mixer and (b) annotated dye visualization segmentation by oscillation (Tesar, 2009)

2.4.5. Active chaotic mixer

Despite the complex 3D geometrical structures, the unsteady 2D flow is capable of initiating chaos. Chaotic advection in micro-channel is generated by pulsation from multiple side channels (Niu and Lee, 2003), as mentioned in Figure 2.3. Another example is the chaotic particle transport in time-dependent Rayleigh-Bénard convection (Solomon and Gollub, 1988), shown in Figure 2.32. In addition to the primary rotating flow movement produced by the rotating disk on the bottom, secondary counter-rotating secondary circulations is observed in the laser illuminated
plane. Thus, chaotic advection is initiated by the secondary flow in the rotation vessel. In contrast to the conventional stirring tank mixer, there are no stirring pads within the vessel.

Moreover, chaotic advection can be enabled by two rotating rods in a barrel (Figure 2.32 (a)), applied in the smart polymer mixing (Zumbrunnen et al., 2006). The height of the barrel cross-section is 1.41 cm; all other dimensions are relative to this characteristic length. Figure 2.32 (b) demonstrates the chaotic condition by the progressive multilayered structures from finite element simulation, corresponding to the different operational modes. The extent of mixing is controlled by the way two stirring rods are rotating periodically. For example, N=1 in Figure 2.32 (b) means the procedure that the right rod rotates 3 cycles while the left rod only completes one cycle; this is followed by the left rod finishing 3 cycles while the right rod rotates only one cycle. The difference in the rod rotating speed leads to substantial shearing deformation and layering, due to the strong wall shearing at very low Reynolds numbers.

As the flow is extruded to the outlet on the right, the layers continue interweaving
and thinning. Therefore, more layers form up on the cross-section near the outlet, shown at the top of Figure 2.32 (b). The longer the flow is dwelling in the barrel, the better is the mixing. This is indicated by dye visualization on the cross-section of the barrel as the procedure repeats at N=2. Finally, For N=8 (the rotating procedures repeat 8 times), the layers thickness can not be recognized in the images.

Figure 2.33 Smart mixer by double rotating rods: (a) system schematic; (b) dye visualization (Zumbrunnen et al., 2006)
2.5. Mixing by synthetic jets

2.5.1. Introduction to synthetic jet

Understanding of the formation and evolution of synthetic jets at low Reynolds numbers is indispensable for the application of synthetic jets in heat and mass transfer at a small scale. Synthetic jet formation is marked by the presence of the outward velocity along the jet axis (Holman, 2005). The formation criterion is concluded by Strouhal number $Str$ and the dimensionless stroke length $L$: $Str = \pi/L$, where $L$ is the plug length of synthetic jet blowing structure. The corresponding dimensionless stroke length is $L > 0.5$ for a circular synthetic jet and $L > 3.1$ for a two-dimensional synthetic jet (Holman, 2005). Another threshold value is given as $L > 0.25$ in cross flow (Milanovic and Zaman, 2005). This difference indicates the complexity to quantify the precise threshold for a synthetic jet formation which depends on various factors such as orifice shape, aspect ratio of orifice, the synthetic jet actuation program, etc. On the other hand, the criterion of a Stokes number higher than 10 for a strong vortex rollup is reached by theoretical analysis. Numerical studies have proposed that a circular synthetic jet vortex rollup would occur with a minimum Stokes number of about 8.5 at a dimensionless stroke length higher than 4 in water (Zhou et al., 2009).

![Diagram of synthetic jets actuation methods](image)

(a) Electromagnetic driving (Kercher et al., 2003)  (b) Piezoelectric driving (Lee et al., 2003)

Figure 2.34 actuation methods of micro synthetic jets
Electromagnetic force and piezoelectric force are common actuation methods for the synthetic jets. For example, Figure 2.34 (a) shows the micro synthetic jet cooling driving by electromagnetic force generated by a solenoid coil on the cylinder permanent magnet, which is attaching on the centre of the vibration membrane with a diameter of 25 mm and a thickness of 225 um. The actuator has a resonant frequency of about 70 Hz; it can be driven by a very low AC voltage of 450mV. Experiments suggest the 45% relative temperature reduction for a driving power of 0.55W, with a typical orifice diameter of 2.38 mm.

On the other hand, the piezoelectric actuator has a simpler structure than the electromagnetic counterpart. Active flow control in the wind tunnel by micro synthetic jet is driven by a piezoelectric membrane of a diameter of 40mm (Figure 2.34 (b)). Opposing to the membrane, a diameter of 0.5mm jet orifice in drilled on the inner wall of the wind tunnel. The synthetic jet actuator is driven at frequencies between 0 and 1.6 kHz, according to the sine wave with two different driving voltages of $\pm 7.5$ and $\pm 10V$. Experiments suggest that the forcing frequency is critical for the boundary layer control. Although the piezoelectric actuator is easy to be integrated into the control system, the working frequency range is narrow and fixed by the characteristic frequency of the piezoelectric actuator. Nevertheless, a high driving voltage (up to 90V) is needed, in contrast to the electromagnetic actuator need only 4V to driving an acoustic speaker of diameter 50 mm (Chaudhari et al., 2010).

In more recent years, there has been an increased interest in surface cooling using impinging synthetic jets (Travaicek and Tesar, 2003; Pavlova and Amitay, 2006; Li., 2009; Chaudhari et al., 2010). Employing synthetic jets to a cross flow parallel to the impinging surface, has been investigated experimentally (Milanovic and Zaman, 2005). They find the penetration depth is a function of the magnitudes of the jet velocity and the velocity of the flow parallel to the surface. After all, they obtained a correlation equation between the jet penetration and the jet-to-cross-stream momentum ratio.

Aiming at the electronic element cooling, heat transfer enhancement by synthetic jet impinging is quantitatively studied (Li, 2009). The experimental setup consists of
three parts: the synthetic jet actuator which generates the impinging jet from the circular orifice $D_o=5\text{mm}$, the heated plate on which the jet is impinging, and the imaging system which records the colour hue changes of the temperature sensitive liquid crystal layer coated on the test surface. Moreover, hotwire is utilized to measure the jet velocity near the orifice, and turbulent flow regime is identified for $Re>500$. The experimental results show improved heat transfer compared with the steady impinging jet.

![Experiment setup of surface cooling by synthetic jet impinging (Li, 2009)](image)

Figure 2.35 Experiment setup of surface cooling by synthetic jet impinging (Li, 2009)

### 2.5.2. Mixing by synthetic jets

Turbulent gas-air fuel mixing enhanced by synthetic jet has been experimentally studied on a coaxial pipe configuration (the outer diameter $D_o$ is 2.54cm, whereas the inner diameter $D_i$ is 1.41cm). The dyed surrounding gas flow is pulsated by the circular synthetic jet in the centre; Figure 2.36 shows a series of PLIF contours at different phase of the pulsation period. The mixing enhancement by the large-scale structures of synthetic jet is demonstrated by the break-up of the surrounding jet.
stream in red colour, due to the interaction of the surrounding jets with the central synthetic jet.

Figure 2.36 Contours of dye mass fraction for coaxial mixing enhanced by synthetic jet at different phases, pulsing frequency 60 Hz (Ritchie et al., 2000)

Another example of mixing by synthetic jet plumes is conducted on the circular synthetic jet (diameter $d=2\text{mm}$) in cross-flow (Gordon and Soria, 2001). The cross flow is along the X axis, and the synthetic jet injection is vertical to the cross flow. The PLIF experiment utilizes the Kiton Red 620 fluorescence dye, which is dispersed into the synthetic jet cavity, see Figure 2.37 (a). The synthetic jet Reynolds number is fixed at 5820, while the momentum ratio of injection to the cross-flow $R$ is 20 and 10 respectively for the presented cases. Both PLIF contours demonstrate the capability of synthetic jet to disperse stream into the cross flow. Furthermore, the experiments suggest the Strouhal number is critical for jet plume patterns. If the Strouhal number is greater than 0.02, multiple plumes are observed in the concentration contour (Figure 2.37 (b)). Otherwise, a single plume is generated, shown in Figure 2.37(c).
Figure 2.37 PLIF study of synthetic jet plume in cross-flow (a) experimental setup; 
(b) multiple trajectory jet $St = 0.058; Re = 2110; R = 10$; 
(c) single trajectory jet $St = 0.012; Re = 5280; R = 20$ (Gordon and Soria, 2001)

Notably, periodic lateral perturbation (Niu and Lee, 2003) has a similar effect of interface folding and stretching as the synthetic jet flow in the mixing channel (Figure 2.3), but there is key differences between the piston driven lateral synthetic jet and pressure driven lateral jets. First of all, the mass flow rate of synthetic jet blowing is guaranteed for liquids with different viscosities, if the synthetic jet pulsation is provided by rigid diaphragm moving reciprocally near the neutral position. Secondly, the steady flow rate condition in the mixing channel is guaranteed by synthetic jet, due to zero-mass characteristics of synthetic jet pulsation. This may be critical for
some application with a special requirement of steady flow rate.

In the mean time, several precursor investigations suggest inviting mixing performance by single synthetic jet, but further parameters optimizing is not yet conducted. Mixing augmented by single lateral synthetic jet is numerically investigated for feasibility of the air cooling enhancement in a 200µm 2D micro channel (Timchenko et al., 2004). Another numerical study (Mautner, 2004) using lattice Boltzmann method investigates the mixing enhanced by single or double in-phase synthetic jet attached the side wall in a biosensor at low Reynolds numbers of 1 or 10. Notably, the high geometric ratio of the synthetic jet orifice to the mixing channel width is not common for the micro channel manufacturing.

Figure 2.38(a) and (b) shows the concentration contour and velocity vectors and streamline of mixing by the in-phase double synthetic jets at Re=10 and Re=1 respectively (Mautner, 2004). Despite the distinct Reynolds numbers, the streamlines and the contours indicate the substantial flow disturbance by the synthetic jet on the bottoms of the horizontal channel. However, the vortex rollup at Re=10 is much stronger than that of Re=1, indicated by the streamlines.
Although there have been some preliminary studies on mixing enhanced by lateral synthetic jets at low Reynolds numbers (Mautner, 2004; Tesar, 2009), detailed experimental investigations aiming at understanding the flow physics involved are still lacking. Such an improved understanding will be beneficial for designing more effective devices for mixing in many applications, such as inline flow mixers for viscous fluids and bi-sensors where level of mixing is restricted by the inherent low Reynolds numbers.
2.6. Summary

In this chapter, the underlying mixing mechanisms for laminar mixing is introduced, followed by a brief review of typical passive and active inline mixers, as well as the introduction to the synthetic jet and its application.

In spite of a rich collection of passive and active mixers, the structural complexity of some passive mixers and operational inconvenience of some active mixers invites further research of jet mixing with or without active pulsation. Jet mixers usually have simple geometry, and do not require the complicate controlling system. Therefore, jet mixing, as a compromise of active and passive approaches, is worth further research.

Jet mixing involves various approaches, for examples, 3D flow instability for impinging jet, vortex rollup for jet mixing, interfacing stretching in sequential segment and chaotic advection by lateral disturbance. Passive jet mixer, such jet impingement, is promising due to its simple structure without extra operation, but it is ineffective at Reynolds number below 400 approximately. Therefore, another effective and conveniently-implemented jet pulsation is desirable for fast jet mixing at Reynolds numbers below 400.

On the other hand, synthetic jet, a zero-mass-flux flow control method, which has been experimentally confirmed to be effective for flow control in boundary layer, is promising in heat and mass transfer enhancement at low Reynolds numbers. Some preliminary numerical studies have demonstrated this potential. However, the experimental exploration of its potential in mass transfer enhancement at low Reynolds numbers is limited, to the best knowledge of the author. Hence, the concept of the mixing by lateral synthetic jets pair is worth experimental validation.

Therefore, the following research areas are proposed and investigated in this thesis: (1) The proposal of an active injection control mechanism effective for laminar jet mixing at Reynolds number below 400 with the minimized modification to the existent passive jet mixing configuration (Chapter 4). (2) The design of an inline laminar mixing enhanced by the lateral synthetic jet pairs and experimental investigation on the effective Reynolds numbers range (Chapter 5 and 6).
Chapter 3. Experimental Setup and Visualization Methods

In this chapter, the experimental setup and experimental methods employed to study the effectiveness of solenoid valves and lateral synthetic jet pairs in achieving mixing enhancement are described. This chapter starts with the description of the experimental rig which enhances water mixing using solenoid valves. It is followed by a description of the driving system and jet orifice configuration of the single and multiple lateral synthetic jet pairs. The operation principle and experimental setup of the Particle Image Velocimetry (PIV), which provides the information about the flow field in the mixing channel, and the Planar Laser Induced Fluorescence (PLIF) technique, which is used to evaluate the mixing degree, are then presented. Finally, the method of quantifying the mixing degree based on the downstream cross-section concentration profile is described and the uncertainties of the measurement are discussed.

3.1. Jet mixing by active injection control

3.1.1. Confined jets mixing rig

The experimental implementation of jet mixing by active injection control is based on the confined jets water mixing rig (Kamau, 2007). The Reynolds numbers range from 2000 to 10000 had been investigated via inert dye, PIV and PLIF with and without the flow rate fluctuation generated by two rotation valves on the confined jet flow and surrounding jet flow. Experimental result shows the velocity ratio of the confined jet to the surrounding jets are crucial for the mixing effect, and fast mixing can be obtained at Re=2917 at high velocity ratio. However, the rotation valves improve little on the water mixing.

In order to investigate the laminar mixing enhancement capability of the confined jets configuration at lower Reynolds numbers, small pumps designed for lower flow rates
substitute the original high flow rate pumps, and two water flow meters are replaced by two direct displacement volume meters which are capable to measure volume flow rates for a wide range of fluid viscosity. Furthermore, the rotation valves are replaced by solenoid valves to generate sufficient pulsation in the laminar flow regime. The solenoid valves actively control the fluid injection into the mixing channel according to pulse width modulation signal. As a result, flow segmentation and vortex rollup are expected to enhance mixing in the mixing channel.

The schematic of the experimental setup for mixing by solenoid valves is shown in Figure 3.1. The experimental setup of the planar confined jet configuration consists of a centred confined jet referred to as the primary flow (PF) jet and two surrounding jets referred to as the secondary flow (SF) jets. In the primary flow case, liquid illustrated in red colour pumped from the PF tank reservoir flows along a circular pipe through the PF solenoid valve and PF flow meter, eventually reaches the circular pipe to rectangular channel adaptor at the inlet to the mixing channel. In the secondary flow case, liquid illustrated in blue colour is also pumped along a circular pipe from the SF tank reservoir, flows through the SF solenoid valve and SF flow meter before the pipe reaches a T-junction and divides for the two secondary jets. Following the division of the SF, fluid in each branch reaches a circular pipe to rectangular channel adaptor at the inlet to its respective mixing channel.

Both the primary flow and the secondary flows are fed into the jet confluence component upstream of the mixing channel. The geometry of the jet confluence component is shown in Figure 3.2(a). The outlet of the central converging duct in the jet confluence component is 4mm in width. The secondary flow, which has been divided into two rectangular ducts, flows parallel on either side of the primary flow rectangular duct. Both the secondary flows join the primary flow through converging and bending ducts, and the confluence angle between the primary flow jet and each secondary flow jet is 30°. The straight Perspex channel after the jet confluence is referred as the mixing channel where the mixing between the primary and secondary flows takes place. The mixing channel has a cross-section of 8 mm x 40 mm and it is one meter in length.
Experimental Setup and Visualization Methods

Figure 3.1 Schematic illustration of liquid mixing enhanced by solenoid valves

The detailed geometry of the jet confluence configuration is shown in Figure 3.2(b). The width of primary flow jet exit is 4 mm. It is to be noted, the two secondary jets, which share the same flow meter and solenoid valve and, are divided into two streams before flowing into two circular pipe to rectangular channel adaptors. Hence the flow rate of the secondary flow is the sum of the flow through the two surrounding jets.

The straight Perspex channel after the jet confluence, which is referred as the mixing channel, is the place where the mixing occurs. The mixing channel is 8 mm wide and 1 metre long. The dimension of the cross-section of the mixing channel is 8 mm x 40 mm, with an aspect ratio of 5.
Two Berkurt™ 3-way Type 330 solenoid valves, each with a response time of 10-20 ms for both opening and closing, provide active control on the primary and secondary flows. The switching of the solenoid valve is controlled by an N-channel MOSFET power switch according to a pulse width modulation signal from the NI™ PCI 6211 data acquisition card.

Two Oval™ III Model 41 positive displacement volumetric flow meters, which are appropriate for a range of incompressible liquids and provide a measuring range from approximately 0.03 L/min to 1.5 L/min, measure the flow rates of the primary flow and secondary flow. The meters have a design measurement precision of 1.0% (OVAL, 2006), and have been calibrated to be effective for pulsated flow by comparing the volume calculated from the mean flow rate of the pulsated flow with the actual volume collected at the outlet of the mixing channel over a period of 5 minutes.

3.1.2. Control parameters

In the current experiment setup of liquids mixing enhanced by the solenoid valves, the solenoid valve switching frequencies and opening duration can be actively
controlled, in addition to the time-averaged flow rate for either PF or SF adjusted by the corresponding control valves before the solenoid valves.

The Reynolds number for pulsated flow in the mixing channel is based on the spatial and temporal mean velocity for unsteady flow pulsated by solenoid valves:

\[
\text{Re} = \frac{h\bar{V}}{\nu} = \frac{Q_s + Q_p}{l\bar{V}}
\]

(3.1)

where \( h \) is the width of the mixing channel after the jet exits, which is 8mm in the current setup; \( l \) is the other length dimension of the mixing channel cross-section; \( Q_p \) is the time-averaged flow rate of the primary flow; \( Q_s \) is the time-averaged flow rate of the secondary flow, \( \bar{V} \) is the time and spatial mean velocity in the mixing channel; \( \nu \) is the kinetic viscosity.

Flow rate ratio \( R_Q \) in Equation 3.2 may indicate the interaction between the confined jet flow and the surrounding jets flow in the mixing channel.

\[
R_Q = \frac{Q_p}{Q_s}
\]

(3.2)

Pulsation frequency \( f \) is an important parameter determining the flow structure in the mixing channel. Since there is an upper threshold for switching frequency limited by the response time of 10-20 ms for the opening and closing of the solenoid valve, the operation frequency is controlled less than 10Hz, to guarantee the alternative fluid injection of either PF or SF.

The duty cycle is the duration of the valve opening as a percentage of one pulsation cycle, is defined in Equation 3.3.

\[
\text{duty cycle} = 100 \frac{t_o}{T}
\]

(3.3)

Where \( t_o \) is the valve opening duration in one pulsation cycle \( T \).

The two solenoid valves can be configured to open simultaneously or open with a phase difference. If the phase difference is exactly half cycle, pulsation mode is referred as out-of-phase. On the other hand, in-phase refers to the mode if both
solenoid valves opening simultaneously without phase difference. The current study concentrates on the out-of-phase mode, where the phase difference is half the pulsation period, with the duty cycles of 50% and 25% respectively.

The ideal periodic variation of the flow rate of both the primary flow and the secondary flow, pulsated with the solenoid valves opening in the out-of-phase mode is shown in Figure 3.3(a) and (b). The instantaneous flow rate is represented by the ratio of the simultaneous flow rate to the time-averaged total flow rate in the mixing channel. In particular, if both solenoid valves operate out-of-phase with a duty cycle of 50% and $R_Q = 1$, then the ideal total flow rate in the mixing channel is time independent, illustrated as the horizontal green line in Figure 3.3(a). On the other hand, Figure 3.3(b) shows that if both solenoid valves operate out-of-phase with a duty cycle of 25%, then there is a period of time when the total flow rate is zero in the mixing channel.

![Figure 3.3 Ideal flow rate of both PF and SF pulsed by solenoid valves out-of-phase](image)

(a) both PF and SF pulsed, duty cycle 50%  
(b) both PF and SF pulsed, duty cycle 25%

The pulsation configuration of pulsating both PF and SF is the focus of the current experimental study, however, pulsating only either PF or SF is also investigated to identify the individual contribution on the confined jet mixing. Figure 3.4 shows the ideal periodic variation of the instantaneous flow rate of both the primary flow and the secondary flow, if only PF is pulsed by solenoid valves with duty cycles of 50% and 25% respectively. The on-and-off switching acts only on PF, while SF remains constant flow rate for the whole pulsation period. As a result, the total flow rate in the mixing channel rises during the PF injection and drops when only SF is pumped into
the mixing channel. Likewise, pulsating only the SF is similar with diagrams shown in Figure 3.4, except that PF flow rate is steady and SF is injected at $t=0.5T$ for each pulsation cycle.

(a) Only PF pulsatated, duty cycle 50%  
(b) Only PF pulsatated, duty cycle 25%

**Figure 3.4 Ideal flow rate of only PF pulsatated by solenoid valves**
3.2. Lateral synthetic jet pairs mixer

3.2.1. Concept of mixing by lateral synthetic jet pairs

The schematic of a 2D mixer enhanced by lateral synthetic jet pairs is shown in Figure 3.5. One or more lateral synthetic jet pairs deployed along the straight mixing channel provide intensive local perturbation near the synthetic jet orifices. The intensive perturbation is capable of achieving substantial mixing within the region near the synthetic jet pairs, which operates out-of-phase.

The lateral synthetic jet (LSJ) pair refers to two synthetic jets with an opposite or slightly staggered orifice configuration on both sides of the mixing channel. Particularly, the driving piston is connected by a rigid metal displacement transfer frame, so both pistons deliver identical piston movement amplitude but change the LSJ cavity volume in an opposite direction. Therefore, the flow rate in the mixing channel remains stable as the input stream if the pairing LSJ pulsating out-of-phase, in contrast to the temporal fluctuation of the single synthetic jet pulsation.

![Figure 3.5. (a) Schematic of the fluid mixing by LSJ pair (b) Temporal variations of spatial mean jet velocity](image)
Furthermore, the periodical variations of mean jet velocity across the jet orifices $u$ are demonstrated in Figure 1(b). As it can be seen from this figure, at $t=0$ both pistons begin to move from their left-most positions to the right. Later at $t=0.25T$, the left jet reaches its maximum injection velocity and ejects fluid from the left cavity into the mixing channel, while the right jet acquires its maximum suction velocity and inhales fluid from the mixing channel into the right cavity. This process is then reversed in the suction cycle of the left LSJ, from $t=0.5T$ to $t=1.0T$. As a result of the left and the right lateral synthetic jets’ out-of-phase pulsation with the identical amplitude, the total flow mass injected into the mixing channel is zero at any time.

3.2.2. Multiple LSJ pairs mixing rig

![Schematic for the liquids mixing by lateral synthetic jet pairs](image)

The experimental setup of the liquid mixing by LSJ pairs is based on the setup water mixing by solenoid valves. Both setups share the piping system, flow meters, flow confluence module and the mixing channel downstream, shown in Figure 3.6. Instead of the confined jet confluence configuration in previous section, two different
liquids flows are injected into the mixing channel via Y-junction at the bottom. The input liquids are referred as left flow (\textit{LF}) and right flow (\textit{RF}) which are shown in red and blue colour respectively. Notably, the solenoid valves do not operates, so there is no pulsation before the LF and RF confluence. Consequently, each liquid takes up half of the mixing channel width and flow parallel to the lateral synthetic jet orifices in the laminar flow regimes.

Overview of the \textbf{LSJ} mixing module assembly is shown in Figure \textit{3.7}. Lateral synthetic jet pairs pulsating module is added about 400mm downstream after the Y-shaped jet confluence. The \textbf{LSJ} pairs are driven by pistons connected to a shaker, located at the left side of the \textbf{LSJ} cavity. An extra supporting rack is not displayed in this schematic, but it is indispensable to prevent unnecessary vibration of the shaker and mixer system. The photograph of the experimental setup is illustrated in Appendix 1.

A Labview program is designed to control the shaker movement and image capturing trigger signals for \textbf{PIV} experiments. The sinusoid signal generated by DAQ card is sent to the shaker after amplification. The displacement of piston movement is measured by the eddy-current displacement sensor and digitalized by DAQ card.
3.2.3. Synthetic jet actuator

The lateral synthetic jet pair mixing module consists of two lateral synthetic jet cavities with piston connected to the shaker, and part of the mixing channel between the cavities. Pistons of both synthetic jet cavities, which are connected by a rigid frame and rods as a rigid body, are driven by only one shaker on the left side. On the top of mixing module is the laser sheet reflection module, whose function and structure will be described in later part of this chapter. Finally, the lateral synthetic jet pair mixing module and the laser sheet reflection module are connected with the mixing channel, since all the module’s mixing channel are designed with the identical inner and outer cross-section geometry.

A permanent magnetic shaker, provided by LING DYNAMIC SYSTEM LTD, is driven by a sine wave amplified by a model P30AE power amplifier with a maximum...
power of 30W. The pistons on both sides of the mixing channel are bonded together by the displacement transfer structure as a single rigid body, due to the out-of-phase pulsation design. The displacement transfer structure, which is fixed to the rod of the shaker at the left side, transfers the displacement output from the shaker to pistons on both sides. In such an arrangement, the two pistons oscillate with the same oscillating displacement but with a phase difference of 180°. Furthermore, the piston peak-to-peak displacement giving by the shaker is measured via an eddy-current displacement sensor which can measure a displacement ranging from 0 to 3 mm.

The explosion view of the LSJ mixer assembly in Figure 3.8 provides more details on the structures of the lateral synthetic jet pair mixing module. To simplify the view, fasteners such as screws and nuts are not shown in this assembly. In particular, the right LSJ cavity is symmetrical with the left cavity, so only the left cavity is exploded.

![Figure 3.8 Explosion view of the lateral synthetic jet mixer assembly](image)

The cavity of the lateral synthetic jet consists of the square Perspex cavity,
interchangeable jet plate and jet plate sealing frame, shown in Figure 3.9. The structural complexity is required by convenience for changing the LSJ configuration. For example, there is no need to make a brand new mixing module but replace the jet plate, if mixing by multiple LSJ pairs is investigated after a single LSJ pair.

Firstly, the square Perspex synthetic jet cavities, glued together with the mixing channel forms the primary structure of the mixing module, shown in Figure 3.9(a). Those two Perspex base plates, together with two 8mm square Perspex columns, form the mixing channel. The cavity of the synthetic jets has an inner dimension of 126 mm x 126 mm and a depth of 10 mm.

Secondly, the flexibility of synthetic jet pair configuration is achieved by interchangeable jet plate with rectangular slot orifices, shown at the top of Figure 3.9(b). The interchangeable jet plate is made of 4 mm Perspex sheet, and fixed into a concaved socket at the bottom of the LSJ cavity. Functionally, there are single or multiple 40mm wide rectangular slots on the jet plate which are identical width with the mixing channel.

Finally, the jet plates sealing frame, screwed onto the bottom of the SJA cavity, fixes the interchangeable jet plate to the concaved slot shown at the bottom of Figure 3.9(b). The assembly of jet plates sealing frame is shown in Figure 3.8. Furthermore, there is a rubber seal together with jet plates sealing frame provides a water-proof seal between the cavity and the mixing channel.
Experimental Setup and Visualization Methods

Figure 3.9  Perspex lateral SJA cavity and base plate

(a) Explosion view of the synthetic jet cavity  (b) Changeable jet plate and seal frame

c) Dimensions of the synthetic jet cavity

Synthetic jet blowing and sucking is implemented by the oscillating movement of the piston which increases or decreases the volume of the LSJ cavity. The piston is made of a thin rubber sheet sandwiched between the two aluminium disks. This sandwich structure is widely used in the design of piston driven synthetic jet actuator (Tang et al., 2007). The diameter of the aluminium disks is 120mm, to guarantee sufficient mass blowing and suction during the synthetic jet pulsation. Meanwhile, the aluminium disks should provide satisfied rigidness to prevent warping, which is
guaranteed by the single disk thickness of 1.5mm. Besides, the rubber sheet between the circular plates, which is clamped between two rubber seal frames screwed to the side walls of the SJA cavity, seals the SJA cavity but makes the piston movable.

The LSJ mixing module is a pluggable assembly, which can be easily inserted to the existent mixing channel. The LSJ mixing module’s incoming and outgoing ports have the identical cross-section inner dimension of $8 \times 40$ mm and outer dimension of $24 \times 56$ mm with existent mixing channel. The mixing channel connectors, comprised by 4 Perspex cubic blocks interconnected by screws, fixes the LSJ mixing module incoming or outgoing port with existent mixing channel.

3.2.4. Geometric consideration for the cavity of lateral synthetic jet

A shallow cavity depth is preferred in the mixing application circumstances. Direct numerical simulation suggests that the shallower synthetic jet cavity can generate stronger vortex rings inside and outside the synthetic jet activities (Frizzetta 1999). Furthermore, the initially unmixed liquid residing inside the cavity has a negative impact on the mixing, since the blowing stroke of the LSJ may push out the unmixed into the mixing channel. Hence, a shallow cavity depth 10mm is selected in current experiment setup, in order to alleviate the negative effect of the unmixed liquid.

On the other hand, orifice depth $h_o$ and cavity depth $h_c$ (Figure 1.3) is not critical to the orifice exit velocity profile (Tang, 2006). However, the ratio of orifice depth to orifice diameter should be unity for stable vortex ring (Crook 2002).

For the given width of the mixing channel, a proper orifice width $d$ should be carefully selected. $R_h$, the ratio of the width of the orifice $d$ to the width of the mixing channel $h$, is defined in Equation 3.4.

$$R_h = \frac{d}{h}$$  \hspace{1cm} (3.4)

There is no space for vortex rollup if $R_h$ is big, while LSJ can not penetrate through the mixing channel and form segmentation if $R_h$ is small. Preliminary numerical study has compare the effect of different ratios, $R_h=$0.5, 1.0 and 2.0, but $R_h=0.5$ is
preferred for the vortex rollup. Therefore, in current experimental setup, the LSJ orifice is tentatively selected as 4mm, i.e. half of the mixing channel width, leading to a fixed ratio $R_h$=0.5.

### 3.2.5. Orifice configuration for lateral synthetic jet pair

The configuration of the synthetic jet is illustrated in Figure 3.10. Single opposing lateral synthetic jet configuration refers to the layout of the left and the right LSJ in opposite position on the side walls; while the LSJ orifices are slightly staggered in the single staggered configuration. Multiple lateral synthetic jet pair configuration is the chain-up of single LSJ pair along the mixing channel. It is to be noted, “opposite” is the synonym for “opposing” in the current jet configuration; both words are used interchangeably in current result discussion.

The origin of the coordinate is selected as centre of the piston not the orifice LSJ centre in Figure 3.10, and the length unit is dimensionless based on the mixing channel width ($h$=8mm). The location of the left LSJ orifice is at $y= -0.5h$, 2mm deviating from the centre of the LSJ cavities. This is to ensure that the right jet orifice (located at $y=0.5h$) for the staggered LSJ pair configuration has the identical deviating distance form the centre of the LSJ cavities. Nevertheless, since the LSJ cavity has dimension of 126mm×126mm, a distance of 2mm deviating from the centre of cavity will not influence the blowing velocity profile at the orifice.

The dimension of the interchangeable plate for the current experiment is shown in Figure 3.11. There are three 4×40 mm rectangular slot jets on the Perspex plate. The rectangular slots are equally-spaced with a pitch of 16 mm. The interchangeable plates are manufactured in pair for both left and right cavities to form multiple opposite or staggered lateral synthetic jet pairs configuration.

For the multiple lateral synthetic jet pairs, the distance between the LSJ is a critical geometric parameter affecting the jet flow in the mixing channel. $R_{pitch}$, the ratio of is the distance between jets on the same side to the dimension of the orifice $d$, is defined in Equation 3.5
where $pitch$ is the distance between jet orifices on the same side of the channel.

$R_{pitch} = \frac{pitch}{d}$ \hfill (3.5)

Figure 3.10 Illustration of single and multiple LSJ pairs configuration

Figure 3.11 Dimensions of the jet plate for 3 staggered configuration (unit: mm)
3.2.6. Sugar solutions preparation

Cane sugar solutions are Newtonian fluid for concentration up to 78% (Quintas et al., 2006), and the viscosity of sugar solution is predictable by mass fraction and temperature (Mageean et al., 1991). Thereby, sugar solution is an economical choice of working liquid for mixing at low Reynolds numbers, whose viscosity can be flexibly adjusted by the mass fraction during preparation. For example, the sugar solution with a dynamic viscosity of $\mu=5.0 \times 10^{-2}$ Pa·s has a mass fraction of 58.2% and a density 1277kg/m³ at 18°C. Furthermore, the actual viscosity of the sugar solution is validated by a GILMONT falling ball viscosity meter before each experiment. Consequently, the difference between the theoretical calculation and measurement by the viscosity meter is within 3% for the identical temperature.

In the current mixing experiments, sugar solution with dynamic viscosity $\mu=1.0 \times 10^{-2}$ Pa·s and $\mu=5.0 \times 10^{-2}$ Pa·s are prepared to investigate the mixing capability of LSJ pair mixer at low Reynolds numbers. In order to investigate the influence of fluid viscosity on the mixing effect, the total net flow rate in the mixing channel is designedly fixed at 0.2 or 0.4 L/min for both water and sugar solutions. Therefore, the net flow Reynolds numbers are 2 and 4 respectively in the sugar solution with a dynamic viscosity $\mu=5.0 \times 10^{-2}$ Pa·s, corresponding with $Re_n=83$ and $Re_n=166$ in water mixing. Accordingly, the net flow Reynolds numbers are 10 and 20 respectively for the sugar solution with a dynamic viscosity $\mu=1.0 \times 10^{-2}$ Pa·s.

Notably, temperature measurement and monitor is indispensible for the experiments of current sugar solution mixing, since the viscosity of sugar solution is temperature dependent. Hence, the temperature in the mixing channel is monitored for each experiment, and the actual viscosity is corrected according to the theoretic prediction relationship (Sugartech, 2010), if the temperature deviates away from the designed temperature point more than 0.5°C.
3.3. Inert Dye visualization

Inert dye, which is convenient to obtain and disperse in the liquid to be mixed, is a basic qualitative flow visualization method for the preliminary study. The environment-friendly food dye “Super Cook” in 38ml bottle is purchased from the supermarket, and blue is selected for best contrast under the current lamp illumination. Moreover, this food dye is sold as water solution, so it is well dispersed in the water tank.

However, inert dye can not provide quantitative evidence of the fully mixing condition. Alternatively, pH value sensitive dye (chemical reaction indicator) is more reliable to identify the fully mixing condition by colouring or decolouring the dye. However, the layer overlapping due to the volumetric visualization, the degree of mixing estimated by dye visualization may not be accurate. Hence, quantitative technique like **PLIF** is preferred to evaluate the mixing degree.

Stereoscopic visualization is capable to show flow structure in two orthogonal directions simultaneously. Prism stereoscopic visualization technology has been used for the qualitative estimation of the 3D flow structure visualization of synthetic jet development from both front and lateral views (Jabbal and Zhong, 2008). In the current experiments, the 3D flow instability is evaluated via this stereoscopic visualization technology.

The configuration of the stereoscopic visualization is illustrated in Figure 3.12. The principle of stereoscopic visualization is that views reflected by two mirrors are merged onto a prism then captured by camera with a proper angle. The dimension of the cross-section of the quasi-2D mixing channel is 8x40mm. The front view which is 8mm width provides the primary view for vortex rollups and evaluation of mixing degree. While, the lateral view with a width of 40mm shows 3D flow structure near the jet exits region.
In the current experimental setup, the blue inert dye is dispersed only in the SF tank, while the PF is kept clean without dye, since preliminary test show that dye in the SF gives clearer flow structure than in the PF. After both SF and PF is injected into the mixing channel, A Fujifilm S6500 colour CCD camera is used to take photograph from the front view. A white paper is placed behind the mixing channel, to clean up the background and improve the contrast of the dyed flow structure in the mixing channel. Finally, the photos are cropped to the width of the mixing channel and basic contrast enhancement is applied to the cropped images.

3.4. Particle Image Velocimetry (PIV)

PIV can provide the quantitative details of the velocity fields in the mixing channel, and occurrence of the vortex rollups can be evidently located. Furthermore, an accurate description of the flow condition downstream is worth investigation. Besides, the downstream velocity information is necessary for the current quantification of mixing degree weighted by cross-section velocity profile. Also, the time-variation of the velocity fields reflect the influence of the solenoid valves and LSJ pulsation on the downstream channel flow.

3.4.1. Laser illumination for injection control mixer

The PIV image capturing equipments and the configuration of laser illumination for
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water mixing by solenoid valves are shown in Figure 3.13(a). The laser sheet, produced by New Wave™ Nd:YAG 532nm twin laser head and cooling system (Model: SOLO IV 120), illuminates the central plane of the mixing channel. The TSI “POWERVIEW™ Plus 4MP” model 630059 camera, which has 12-bit gray-scale intensity range and 15 frames per second image capture rate at a maximum resolution of $2048 \times 2048$ pixels and a pixel size of $7.4 \, \mu m \times 7.4 \, \mu m$, is used to capture PIV and PLIF images. In addition, the camera was focused onto the laser sheet plane with a viewing area of $8mm \times 40mm$ utilizing the high standard Micro-Nikkon AF 60mm f2.8 D lens. The camera was fixed on a 3D traverse; so the region of interest can be conveniently move upstream or downstream. Finally, a synchronizer from TSI Inc. (LASERPULSE™ model 610034) and Insight 3G™ software are used to control the firing of the laser and trigger the camera.

Hollow glass spheres (HGS) with a size distribution of 2-20 µm, a mean diameter of 10 µm and an equivalent density of 1.1 g/cm³, are seeded into the water for the PIV study. According to the Stokes drag law in Equation 3.7, the settling velocity of the HGS seeding is estimated as $5.5\mu m/s$ in water. Compared with the low operation frequency of the solenoid valves or lateral synthetic jets, the vibration period of particle is neglectable.
\[ U_p = \frac{d_p (\rho_p - \rho) g}{18 \mu} \]  \hspace{1cm} (3.7)

Where, \( d_p \) is the particle diameter, \( \rho_p \) and \( \rho \) are density of particle and working liquid respectively; \( g \) the gravity acceleration and \( \mu \) is the dynamic viscosity.

### 3.4.2. Laser illumination for lateral synthetic jet pair mixer

The calculation of the mixing degree is based on the concentration profiling across the mixing channel downstream 12.5 \( h \) from the orifice centre of the single opposing LSJ pair, which is consistent with water mixing quantification. The laser illumination setup for PLIF and PIV after the LSJ cavity is shown in Figure 3.14(a), similar with setup for the mixing enhanced by solenoid valve in Figure 3.13. The laser sheet passing through the Perspex wall of the mixing channel, reaches the region of interest on the top of the LSJ cavities, i.e. 9.5\( h \)-15\( h \) downstream from the centre of the LSJ cavity.

On the other hand, the illumination of the mixing channel between the LSJ cavities is not straightforward, since the laser sheet is blocked by the metal pistons of LSJ cavity. Because visualizing the flow structure between the LSJ cavities is primary objective of the experimental study, a slice of 4mm wide flat mirror is inserted downstream. Thereby, the mixing channel between the jet orifices can be illuminated from the laser reflected from the mirror slice. The schematic of laser sheet reflection by the mirror slice and capture deployment is shown in Figure 3.14(b). It is to be noted that the region of interest (ROI) is the 8mm wide mixing channel between the LSJ cavities, captured via the 8mm wide front window of the mixing channel.
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(a) Illumination of downstream region  
(b) Laser sheet reflected into mixing channel

Figure 3.14 Schematic of the PIV and PLIF laser illumination for LSJ mixer

Figure 3.15 Schematic of laser illumination of mixing channel between the jet orifices via mirror reflection

The details of the laser reflection module are presented in Figure 3.16. Essentially, the mirror reflection module consists of a slice of flat mirror glued onto a copper rod
which is inserted into a rectangular Perspex channel section. The Edmund™ flat mirror coated with Enhanced Aluminium is selected for laser reflection since it provides >90% reflection in the visible light range. In order to minimize the flow disturbance, the flat mirror with a thickness of 1mm is cut into a rectangular slice with a dimension of 4x13mm, and glued to a copper half-cylinder rod with a diameter of 6 mm as the only mechanical support. Besides, the geometry of the channel section is identical the existent mixing channel, which enable convenient integration into current experimental rig. Notably, the reflected laser sheet can only illustrated the central plane ($Z=0$) as the mirror fixing rod is fixed into the mixing channel with an angle of 45 degree on the central plane, however, the mirror fixing rod can be unplug for maintenance.

![Figure 3.16 Detail of the laser reflection mirror installation](image)

The presumption that the blocking effect of the mirror mixing rod to the upstream flow in the channel is ignorable is confirmed by commercial CFD software Fluent™ 6.3 in design stage and validated by the PIV experiment. The velocity profiles on the cross-section of the channel 15$h$ (120mm) downstream from the centre of the LSJ orifices is compared for the conditions with and without mirror rod, shown in Figure 3.17. Numerical studies indicate the blockage effect is not noticeable at $Re=166$, shown as the superposition of blue and red lines in Figure 3.17. Furthermore, the experimental result shows excellent agreement with the theoretical prediction of the parabolic velocity profile.
The design of laser sheet reflection module is experimentally validated by PLIF/PIV visualization in the 2D channel mixing. In spite of various restrictions, such as the inhomogeneous energy distribution, mirror overheating, corrosion, laser glaring on side walls, energy attenuation of the laser sheet, the feasibility of PLIF and PIV illumination by laser sheet reflection is proved by experimental result of concentration contours and velocity vectors. Moreover, the module design without extra optical equipment accelerates the fast manufacturing process and minimizes modification to the existent experimental setup. Besides, the application of laser reflection may be extended to the field of micro-mixing when flow visualization on specific plane in micro structure is necessary.

Although, continuous experiments prove the feasibility of the mirror reflection without burnout or overheating, the Aluminium reflection film of the mirror is exposed to the chemical corrosion of the working flow or moisture. Hence, the mirror fixing bar should be pulled out and conserved in a cool atmosphere for further utilization.

3.4.3. Uncertainty of PIV measurement

The principle of PIV measurement is to estimate the particles displacements on one double exposed image or two sequential images in selected pulse illumination time interval. Based on the known time interval and pre-calibrated ratio of particles
displacements in pixel and geometrical instance, the velocity of particles can be calculated indirectly.

\[ u = \frac{M \Delta x}{\Delta t} \]  

(3.8)

Where \( u \) is the particle velocity; \( \Delta t \) selected laser pulses time interval; \( \Delta x \) the distance travelled by the seeding particles in the given time interval; \( M \) is the map factor representing the ratio of the distance measured in pixels on the CCD and the physical distance measured in the area of interest. In the current experiments, \( M \) is equal to 0.024mm/pixel for the 4 million pixel CCD camera, based on a typical field of view of 48mm×48mm.

The quality of PIV measurement depends on the selection of key parameters, such as seedling density, interrogation windows size and laser pulses time interval. After the careful selection of window size and seedling, high PIV measurement accuracy relies on the estimation of the pulses time interval \( \Delta t \) for each run of particle image capturing. There is an empirical rule for \( \Delta t \) selection, shown in Equation 3.9, that the maximum particle pixel displacement should be less than one fourth of the pixel window size \( W \). Thus, the optimum \( \Delta t \) needs to be estimated by the maximum velocity in the region of interest in each PIV measurement run.

\[ \frac{\Delta x}{W} = \frac{MU_{\text{max}} \Delta t}{W} < 0.25 \]  

(3.9)

The main uncertainty in the velocity measurement arises from the determination of the particle displacement by the cross-correlation algorithm in the PIV post-processing software. Currently, the timing precision of the pulse synchroniser is on the magnitude of the tens of nanoseconds, thus the timing error is ignorable comparing with image correlation time interval \( \Delta t=500-5000\text{us} \) in current experiments. In contrast, the relative error of pixel displacement is dominant, since the minimum error of pixel displacement is only 0.05-0.1 pixel for the typical interrogation windows of 32×32 pixels (Westerweel, 1997). Therefore, the error of velocity calculation relative to the maximum flow velocity is no more than 1.25% (0.1pixel/8pixel), for an average particle displacement of the maximum pixel displacement (one fourth of the 32×32 pixel interrogation window size).
In the present experiment of water mixing by solenoid valves, PIV provides flow field near the jet exits and the velocity profile of the cross-section of the end of observation zone. Observation zone is 8×40mm ($1h \times 5h$) for both downstream region and near jets injection region, corresponding to a ROI of 360×1800 pixels. A spatial resolution of 2.25 vectors/mm is reached since the interrogation zone of 32×32 pixels is selected. Finally, 4 or 8 phase points are captured in a full cycle of the synthetic jet. 100 pairs of images are taken at each phase point to produce the phase-averaged vector field.

### 3.5. Planar laser induced fluorescence (PLIF)

Quantification of the mixing effect is always a challenge for mixing in compact geometry, yet, indispensable for evaluating the effectiveness of the mixing system and mixing mechanisms underlying. Fortunately, Planar laser induced fluorescence (PLIF) can rapidly quantify the tracer concentration and quantitatively evaluate the mixing effect.

PLIF as an experimental quantitative measurement of tracer concentration has been widely used in mixing evaluation on the macro-scale, and gains more popularity in micro-mixing. The quantitative concentration measurement is based on the fact that the laser induced fluorescence emission intensity is linear with fluorescence concentration in the calibrated range (Melton and Lipp, 2003). PLIF has been widely adopted for flow structure visualization and mixing evaluation in macro-scale applications (Hébrard et al., 2007). Furthermore, the micro laser induced fluorescence (µLIF) which utilizes a confocal laser scanning microscope (CLSM) is not only utilized to qualitatively illustrate flow structure (Hoffmann et al., 2006), but also quantify the passive mixing in micro-channels with geometric variations (Hsieh and Huang, 2008).

#### 3.5.1. PLIF calibration

Rhodamine B is selected as the passive fluorescence dye for PLIF for its excellent stability, emission performance and linearity. In addition, it is independent of the pH of the experimental fluid over a the range $3 < \text{pH} < 11$, Rhodamine B is an ideal...
fluorescence for the **PLIF** study for its high water solubility and low toxicity (Melton and Lipp, 2003). Furthermore, it has a central excitation (absorption) wavelength of 543 nm and a central emission fluorescence wavelength of 580 nm. Hence, the Nd: YAG 532 nm laser of the **PIV** system is a particularly suitable excitation source, since the wavelength of the fluorescence light is sufficiently different from the laser energy wavelength.

Although Kamau (2007) has validated linearity of the fluorescence intensity with the Rhodamine B concentration up to 0.6mg/L in water within the identical Perspex channel. Furthermore, the linearity has been confirmed by a 6 equally-spaced concentration points calibration from 0 mg/L to 0.5 mg/L for the specific laser energy after the modification and changing working flow.

Figure 3.18 shows the calibration diagram for water mixing by solenoid valves. The intensity value is the spatial averaged pixel intensity from 16bit gray scale monochrome image in the region of interest. This region is from confined jets confluence to 12.5h downstream, and it has been temporally averaged over 80 sequential images. This calibration reconfirmed the conclusion of Kamau(2007) that linearity is up to 0.6mg/L in water.

![Figure 3.18 Calibration of the linearity of Rhodamine B concentration with the emission fluorescence intensity](image)

Therefore, the Rhodamine B solution with a concentration of 0.2 mg/L, which locates in the excellent linearity region, is selected as the default concentration in current
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water mixing experiments. Moreover, the similar calibration has been conducted in the sugar solution to confirm the linear relationship.

3.5.2. Concentration measurement and uncertainty

Due to the excellent emission characteristics of Rhodamine B, the PLIF experiments share identical image capturing equipments and the configuration of laser illumination with PIV shown in Figure 3.13(a), except that one high pass optical filter is mounted on the lens to filter out the YAG 532nm laser scattering light. Also, it makes the simultaneous PIV and PLIF measurement possible.

During the PLIF experiment, eighty PLIF images in 16-bit grey scale are acquired on the central plane. Each image is then corrected for spatial non-homogeneity in the laser sheet by subtracting the background intensity distribution before being converted into a contour plot of dye concentration in Matlab™. More specifically, the instantaneous value of fluorescence concentration \( c \) at any point of interest is obtained by using the equation given as:

\[
    c = C_{cc} \times \left( \frac{I_{raw} - \overline{I}_b}{\overline{I}_{cc} - \overline{I}_b} \right) 
\]

where \( I_{raw} \) is the fluorescence intensity at that point, \( \overline{I}_b \) is the time-averaged background fluorescence intensity when no fluorescence dye is added, \( C_{cc} \) is a constant reference concentration value and \( \overline{I}_{cc} \) is the time-averaged fluorescence intensity measured from the solution with the reference concentration \( C_{cc} \).

The uncertainty of the tracer concentration measurement is less than 2% in the current PLIF experiments. The concentration measurement error mainly comes from the temporal and spatial energy fluctuation in the laser sheet (Melton and Lipp, 2003) which is common for the pulse lasers. The integrated laser energy on the whole ROI is analyzed for a sequence of 100 pulses, and the standard deviation of laser energy is found to be 1.67%. Other sources of errors, such as the linearity calibration error and background subtracting error, can be well controlled and here can be neglected.
3.6. Mixing quantification

Several quantification methods have been proposed to quantify the mixing effect efficiently, such as probability density functions (PDF), intensity of segregation (Unger and Muzzio, 1999; Kling and Mewes, 2004; Lehwald et al.) and Coefficient of Variation (CoV) (Kresta et al., 2009). Essentially, both intensity of segregation and CoV share the definition for two-species mixing with a flow rate ratio of 1:1. Applications of CoV include quantification based on the concentration contour of a rectangular region in the SMX static mixer (Pust et al., 2006) and the micro-LIF (Hsieh and Huang, 2008). Furthermore, standard deviated weighted by the velocity profile has been introduced to evaluate micro-mixing in simulation of the jet mixing via periodic injection switching (Glasgow and Aubry, 2003).

Notably, the value of CoV is not sufficient to identify the mixing degree, but just the first dimension of the three dimensions description of mixing extent (Kresta et al., 2009). The second dimension is the scale of segregation, i.e. the spatial resolution of the concentration measurement, and the last dimension is the potential to reduce segregation, which is temporal trend of mixing. In the current experimental setup, more than 250 pixels are captured for the 8mm wide mixing channel in the PLIF images. Therefore, the spatial resolution of 31µm/pixel is sufficiently high for the scale of segregation in macromixing. Besides, the temporal and spatial variation of the mixing degree sequence is evaluated in later section.

3.6.1. Mixing quantification by concentration profiling

Quantitative evaluation of the mixing in current experiments is based on the velocity weighted CoV (Glasgow and Aubry, 2003) of the Rhodamine B concentration profile. The concentration profile on the cross-section of a specific position is extracted from the concentration contour obtained in the PLIF experiments. In the case of water mixing enhanced by solenoid valves experiments, the maximum PLIF observation zone begin with the confluent of jets and ended at 12.5h (100 mm) downstream, where the concentration profile is selected to calculate the mixing degree. The mixing degree based on the velocity weighted CoV is defined in Equation 3.11:

\[ \text{Equation 3.11} \]
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\[
Mixing\ degree = 1 - CoV = 1 - \sqrt{\frac{\int_{-h/2}^{h/2} \left( \frac{c}{\bar{c}} - 1 \right)^2 \frac{u}{\bar{u}} \, dx}{h}}
\]  

(3.11)

where, \(c\) is Rhodamine B concentration on the cross section of the mixing channel; \(\bar{c}\) is the Rhodamine B concentration at the fully mixed condition, which depends on the flow rate ratio and concentration of the primary flow, can be calculated before the experiments; \(u\) is local streamwise velocity of the cross-section, and \(\bar{u}\) is mean streamwise velocity on the cross-section. If the concentration profile is homogeneous at the cross-section, the mixing degree will be 1, which implies a fully mixing condition. In practice, mixing degree >0.95 is regarded as homogeneous mixing, and the minimal length the liquids need to pass is the mixing length (InvotecSolution, 2004).

Notably, the PLIF image capturing frequency is selected as 3.7Hz, to obtain a series of images with the pseudo random phases. The recording frequency is chosen since it is not a multiple of the actuation frequency of the solenoid valves/LSJ pulsation. Therefore, the mixing degree calculation is independent of the concentration profile fluctuation locking to the periodic pulsation.

In addition, the image sequence is captured after the stable pulsation, to minimize the initial effect on the mixing. In the current mixing experiments, images are captured at least 1 minute after the action of solenoid valves or shaker.

### 3.6.2. Downstream velocity profile

The selection of the specific downstream position for mixing degree quantification is based on the stability of cross-section velocity profile. Velocity distribution on the cross-section should be temporally steady, i.e. independent of the upstream periodic pulsation. Thus, a fixed velocity distribution can be presumed to simplification of the mixing degree calculation. In the current experiment setup, a parabolic velocity profile is adopted according to the entrance length calculation.

According to the entrance length estimation (Equation 3.12) based on the strictest criterion that the pressure gradient reaches 99% of its asymptotic value (Sadri and Floryan, 2002), an entrance length relative to the hydraulic diameter \(L_e=12.1\) is
needed at $Re=83$. However, this is excessively strict for a simple velocity profile shape assumption. Sadri also reported the entrance length can be reduced to half of that value, if the criterion is selected as the centre line velocity reaching 99% of its asymptotic value. Hence, an entrance length of $13h$ is enough at $Re=83$. Therefore, the end of the observation region ($12.5h$) can be an ideal position to illustrate the influence of the active pulsation.

$$L_c = 0.166 Re^{-1.69} \pm 0.1\%, 50 \leq Re < 100,$$

$$L_c = -121.617 Re^{-1} + 1.63 Re^{-0.126} \pm 1.3\%, 100 \leq Re < 1000.$$  

(3.12)

In order to simplify the mixing degree evaluation, the concentration profiling position is fixed at $y=12.5h$, yet the cross-section velocity profile is investigated at higher mean Reynolds numbers by PIV experiments.

The downstream velocity profile approximates a parabola, if both PF and SF are pulsedated with a duty cycle of 50% at $Re=166$. Figure 3.19 (a) shows the actual velocity profiles $12.5h$ downstream from the jet exit appear to be close to parabolic shape, if both the primary flow and the secondary flow are pulsedated out-of-phase with the same flow rate ratio $RQ=1$ and a duty cycle of 50%. Ideally, the mean flow rate is kept constant in the mixing channel and the cross-section velocity profile should be a steady parabola of the fully developed laminar flow downstream, but there are small time variations during a pulsation period. Notably, the velocity measurement error near the wall of the mixing channel is higher than that in the centre of the channel.

On the other hand, the velocity profiles are approximately parabolas for only PF pulsedated with a duty cycle of 25%, presented in Figure 3.19 (b). Despite the intense fluctuation of the peak velocity, the velocity profile seems to be approximately parabolic. However, the profile is more flat in the centre of the channel when PF is injected at $t=1/8T$ and $t=2/8T$, and weak reversed flow occurs at $t=4/8T$. 

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For the case of mixing by the LSJ pairs, the parabolic downstream velocity profile is guaranteed by design, since the pulsation will not change the total mass flow rate in the mixing channel. According to the entrance length theory, the LSJ pair generates only intense local disturbance, but leave the far downstream flow field undisturbed. Therefore, the velocity profile remains stable as a parabola, after the necessary entrance length, which is \( y = 12.5h \) from the opposing LSJ orifices.

Furthermore, the time variation of the velocity profile of mixing channel is confirmed by Figure 3.20. The phase-locked time averaged velocity profiles at four different phases during an actuation cycle at \( f = 2 \text{Hz} \) and \( L = 6 \) along with the velocity profile without the actuation of synthetic jets (No pulsation) and theoretical parabola are

(a) \( f = 2 \text{Hz}, \ L = 6 \)

(b) \( f = 6 \text{Hz}, \ L = 2 \)

Figure 3.20 Temporal variation of the Y-axis velocity profile on the cross section \( y = 12.5h \) for water mixing by single LSJ pair, \( \text{Re}_n = 166 \)
shown in Figure 3.20 (a). It can be seen that the parabolic shape of the velocity profiles is evident and there is only a small variation between the velocity profiles obtained with and without synthetic jet actuation. This indicates the pulsation of the synthetic jet pair has minor impact on the flow rate fluctuation downstream. Similar velocity profile is found at higher pulsation frequency of 6Hz, shown in Figure 3.20 (b). Moreover, a distance of 12.5$h$ from the jet orifices guarantees the disturbance has diminished substantially (Equation 3.12). The above finding also applies in all the other cases of liquids mixing by LSJ pairs at lower Reynolds numbers. Therefore, the channel flow at this location can be regarded as fully developed laminar whose velocity profile has a steady parabolic shape.

It is time-consuming and unnecessary to measure instantaneous cross-section velocity profile for every experimental case. Therefore, in order to save time in calculating the mixing degree, a parabolic instantaneous velocity profile on the cross-section of $y=12.5h$ is assumed for all cases. This argument is based on the fully development channel flow approximation and entrance length theory (Equation 3.12).

The velocity profile of fully developed 2D channel flow can be normalized on the spatially averaged velocity $\overline{u(x)}$:

$$\frac{u(x)}{\overline{u(x)}} = \frac{3}{2} \left( 1.0 - \left( \frac{x}{h/2} \right)^2 \right)$$  \hspace{1cm} (3.13)

The velocity ratio of the local streamwise velocity $u(x)$ to the spatially averaged velocity $\overline{u(x)}$ is independent of the viscosity and the magnitude of the mean net flow velocity $\overline{u(x)}$. It depends only on the spanwise position $x$ across the mixing channel. Since the velocity ratio is constant for the parabolic streamwise velocity profile, the instantaneous velocity measure is not necessary for the mixing degree calculation of each case.

### 3.6.3. Uncertainty of mixing quantification

The accuracy of mixing degree for a single PLIF concentration contour on the central plane depends only on the concentration measurement, since steady velocity profile is selected. For cases of mixing by solenoid valves with a duty cycle of 50% and liquids
mixing by lateral synthetic jet pairs, the parabolic velocity profile on the cross section of the mixing channel is guaranteed by design. Furthermore, the parabola velocity profiles with moderate variation in pulsation period have been shown in Figure 3.19. Therefore, the error from velocity measurement is not discussed.

In addition to the concentration measurement error on the cross section, the temporal variation of mixing degrees for the sequential concentration contours and spatial deviation at different lateral positions contribute to the uncertainty of mixing quantification. For the independent sources of uncertainty, the overall standard deviation of mixing degree can be estimated by **Equation 3.14**.

\[
\text{Overall standard deviation of mixing degree} = \sqrt{E_t^2 + E_c^2 + E_z^2}
\]  

where \( E_t \) is the standard deviation for the sequence of mixing degrees for each case, \( E_z \) is the standard deviation of averaged mixing degree at different Z axis positions and \( E_c \) is the standard deviation of the tracer concentration measurement. It is to be noted, the laser sheet spatial and temporal fluctuations not only affect the concentration measurement, also contribute to the temporal variation of mixing degrees sequence. Therefore, the uncertainty can be overestimated.

First of all, the temporal fluctuation of the mixing degree based on the concentration profiling is shown in Figure 3.21. Due to the periodic pulsation at the jet exits and stretching of segments in the mixing channel, the mixing degree varies with the fluctuation of concentration profile on the specific cross-section. In order to minimize the impact of temporal fluctuation of the concentration profile, the mixing degree for each of 80 consecutively-captured images is calculated according to **Equation 3.11**, thus the final value of mixing degree takes the temporally-averaged value of the mixing degree sequence.

Figure 3.21 shows the mixing degrees calculated for a sequence of consecutive images. The fluctuation of the mixing degree is evident in Figure 3.21 (a) for the case, only PF pulsed at 1 Hz with a duty cycle of 50%. The standard deviation of the time series is 0.050. Moreover, Figure 3.21 (a) demonstrates periodic fluctuation locking to the image capturing frequency is 4 Hz, which should be avoided. Instead of the phase-locked image capturing during PIV, image sequence of non-repeated
phases are preferred to calculate the time-averaged mixing degree for cases with periodic pulsation. Therefore, a capturing frequency of 3.7Hz is selected to captures images sequence with pseudo random phases.

Figure 3.21 (b) suggests a sequence of 80 images is sufficient to estimate the temporal fluctuation of mixing degree. The standard deviation for the first 80 images is 0.046, while the standard deviation for the entire sequence of 160 images is 0.047. Only 2% difference in standard deviation indicates that 80 sequential images is a reasonable compromise between huge dataset volume and measurement accuracy.

Figure 3.21 Time series of mixing degree for the sequential images for PF only pulsated with a duty cycle of 50%

Moreover, the temporal standard deviation for mixing degree of well-mixed cases is less than the not well-mixed cases. Figure 3.22 shows the temporal standard deviations are 0.014 and 0.012, for both PF and SF pulsated with a duty cycle of 25% at \( f = 2 \) Hz and 6Hz respectively. The relative standard deviations are \( 0.014/0.905 = 1.6\% \) and \( 0.012/0.86 = 1.4\% \) respectively. Apparently, the standard deviation for well-mixed case (Figure 3.21(a)) is smaller than that of not well-mixed case (Figure 3.21(b)).
Secondly, the mixing degree based on the standard deviation of the concentration should use the $X-Z$ plane concentration contour on the cross-section (an 8×40mm rectangle illustrated at the bottom of Figure 3.23) of the mixing channel (Lee and Kwon, 2009), instead of the line concentration profile (from $x=-0.5h$ to $0.5h$) at $Z=0$. Yet, it is practically difficult to obtain the concentration contour on such cross-section for the current experimental setup. In order to accelerate the quantification of mixing degree, the line concentration profile is only extracted from the central plane ($Z=0$), shown as the central line in at the bottom of Figure 3.23.

Nevertheless, the spatial fluctuation of mixing degree is evaluated on the different lateral positions. Except for the central plane $Z=0h$, PLIF is also conducted on the other 4 planes of equally-spaced position along the $Z$ axis: $Z=-0.5, -1.0, -1.5, -2.0 h$, in order to evaluate the spatial standard deviation of the mixing degree. The schematic of laser sheet positions on the cross-section of the mixing channel is illustrated in Figure 3.23.
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The difference of the mixing degree calculated from concentration profiles of different lateral planes is smaller for well-mixed cases with a duty cycle of 25% than those with a duty cycle of 50%. The maximum standard deviation for cases with a duty cycle of 25% is 0.020 at $f=6$ Hz, corresponding to the spatially-averaged mixing degree of 0.90, whereas the mixing degree calculated on the central plane seems higher than spatial averaged mixing degree, shown in Figure 3.23 (a). On the other hand, the fluctuation of mixing degree is higher for the not well-mixed cases when both PF and SF are pulsated with a duty cycle of 50%, $Re=166$. The maximum standard deviation is 0.045 at $f=3$ Hz, corresponding to the spatially-averaged mixing degree of 0.74.

In summary, the time-averaged mixing degree calculated from PLIF result on the central plane is time-saving to identify good mixing, although there are substantial discretions in mixing degree for the individual cases. For example, the average mixing degree is 0.91 for the case: both PF and SF pulsed with a duty cycle of 25%, $f=2$ Hz and $Re=166$, the standard deviation of temporal and spatial variation is...
0.014/0.91 and 0.020/0.91 respectively. In addition to the standard deviation of laser sheet energy for consecutive 100 pulses: 1.67%, the overall standard deviation is 2.8%, according to Equation 3.14. Although the overall standard deviation for not well-mixed cases is higher, time-averaged mixing degree calculated based on the cross-section concentration profile on the central plane is sufficient to distinguish the well-mixed cases.

For the cases of mixing by LSJ pairs, the uncertainty of mixing degree quantification is found to be less than that of water mixing by solenoid valves, due to the steady streamwise velocity profile and weak flow instability at lower Reynolds numbers.
Chapter 4. Mixing by Active Control of Fluid Injection

In this chapter, jet mixing between the fluid from a primary planar jet and two surrounding secondary planar jets which are pulsed out-of-phase is studied experimentally by PIV and PLIF. Solenoid valves are used to actively control the fluid injection into the mixing channel with pulse width modulation. The experiments are conducted using water at a range of mean Reynolds number between 83 and 250, and the influence of duty cycles, pulsation frequency, and pulsation configuration on mixing degree is discussed. Moreover, other pulsation modes of only the primary flow or the secondary flow pulsed by solenoid valve are investigated.

4.1. Dimensionless parameters

For the mixer used in present experiment, the mixing degree in the channel can be described using the following functional expression:

\[ MixingDegree = f(\bar{U}_n, h, f, \nu, DC) \]  

(4.1)

where \( \bar{U}_n \) is the time-averaged mean velocity across the mixing channel, \( f \) is the operating frequency of the Solenoid valves, \( \nu \) is dynamic viscosity of the fluid respectively. \( h \) is the width of the rectangular mixing channel where the mixing happens; \( DC \) refers to the pulse duty cycle of the solenoid valves controlling the injection duration.

The Reynolds number for pulsed flow in the mixing channel is based on the spatial and temporal mean velocity for unsteady flow pulsed by solenoid valves:

\[ Re = \frac{h\bar{U}_n}{\nu} = \frac{Q_s + Q_p}{lv} \]  

(4.2)

where \( h \) is the width of the mixing channel after the jet exits; \( l \) is the other length
dimension of the mixing channel cross-section; \( Q_p \) is the time-averaged flow rate of the primary flow; \( Q_s \) is the time-averaged flow rate of the secondary flow, \( \bar{U}_n \) is the time and spatial mean velocity in the mixing channel; \( \nu \) is the kinetic viscosity.

Flow rate ratio \( R_Q \) in Equation 4.3 may indicate the interaction between the confined jet flow and the surrounding jets flow in the mixing channel.

\[
R_Q = \frac{Q_p}{Q_s} \tag{4.3}
\]

Where the velocity ratio of the primary jet flow to the secondary jet flow is not a meaningful parameter for the intermittent flow condition, as the primary flow and secondary flow are injected into the mixing channel alternately.

Giving flow rate ratio of unity, \( \bar{U}_j \) is time-averaged velocity during the fluid injection velocity in the mixing channel, and it depends on duty cycle \( DC \) (the ratio of the solenoid valve opening duration to the whole period).

\[
\bar{U}_j = 2\bar{U}_n / DC \quad \text{if} \ (R_Q=1) \tag{4.4}
\]

And the Reynolds number based on the averaged injection velocity represents the local disturbance near the confined jet exits.

\[
Re_j = \frac{h\bar{U}_j}{\nu} \tag{4.5}
\]

Finally, the Strouhal number as a dimensionless representation of the actuation frequency \( f \) provides a measure of the level of sequential segmentation occurring in the mixing channel,

\[
Str = \frac{f h}{\bar{U}_j} \tag{4.6}
\]

The dimensional analysis shows that the mixing degree is determined by four independent dimensionless parameters. They are the net flow Reynolds number \( Re_n \), the velocity ratio \( R_Q \), the Strouhal number \( Str \) and duty cycle \( DC \).

\[
MixingDegree = f(Re, Str, R_Q, DC) \tag{4.7}
\]
4.2. Testing conditions

Without pulsation, no significant mixing is achieved at $\text{Re} = 416$ and $R_Q = 1.0$ for the current confined jets configuration. Mixing can be enhanced via variation of the flow rate ratio of the primary flow to the secondary flow. For example, better mixing is achieved at $R_Q = 0.25$ than the case with $R_Q = 1.0$. However, fully mixing is not available at $\text{Re} = 416$, as the flow rate ratio $R_Q$ increases from 0.25 to 4.0. Therefore, it is suggested that an effective pulsation mechanism is indispensible for fast laminar mixing at lower Reynolds numbers.

![Confined jets water mixing without pulsation at the different flow rate ratio, Re= 416](image)

$R_Q = 0.25 \quad R_Q = 0.33 \quad R_Q = 0.5 \quad R_Q = 1.0 \quad R_Q = 2.0 \quad R_Q = 3.0 \quad R_Q = 4.0$

Figure 4.1 Confined jets water mixing without pulsation at the different flow rate ratio, $\text{Re} = 416$

The current confined jet mixing experiments focus on water mixing at a range of pulsation frequency (from 1Hz to 10Hz), two duty cycles (25% and 50%) and a mean Reynolds numbers between 100 and 250. The flow rate ratio between the primary and secondary flow is kept as unity ($R_Q = 1$). Both PLIF and PIV techniques are used to visualise the flow patterns and to quantify the mixing degree in the mixing channel. The effects of pulsation frequency, duty cycle and Reynolds number are examined for both PF and SF pulsed. Moreover, other pulsation modes of only PF or SF pulsed by solenoid valve are investigated, to evaluate the respective contribution on water mixing.
4.3. Both primary flow and secondary flow pulsated

In this section, the mixing extent in the mixing channel when both PF and SF pulsated by the solenoid valves is analyzed. First of all, the fluorescence concentration contour plots from the PLIF experiments serve as a qualitative approach to the analysis of the vortex rollup and sequential segmentation in the mixing channel immediately after the jet exit. The observation zone for the PLIF experiment begins with the confluence of jets and ends at 12.5h (100 mm) downstream, where the concentration profile is taken to calculate the mixing degree. Then, the time variations of the velocity distributions at the different phases of the injection are demonstrated. The vortex rollup can be confirmed by the velocity vectors at the jet exits and the cross-section velocity profiles are evaluated at the end of the observation zone where the mixing degree is quantified. Finally the effect of Reynolds number, pulsation frequency and duty cycle are analyzed quantitatively by the calculated mixing degree.

In the PLIF experiment, the primary flow is a Rhodamine B solution with a concentration of 0.2 mg/L whereas the secondary flow contains pure water. Since the flow rate ratio is 1, the concentration in the mixing channel is expected to vary from 0 to 0.2 mg/L with the concentration level equal to 0.1mg/L in the fully mixed condition. The colour map of small rainbow in TecPlot™ 10 is adopted to translate the concentration distribution into colour contour. The scale for the concentration of Rhodamine B is shown on the top right corner of Figure 4.2. A high concentration is represented by the colour red, whilst a low concentration is represented by the colour blue. The condition of fully mixing can be indicated by the homogeneous green colour on the cross-section downstream, as the calculated concentration of the ideal fully mixing condition is 0.1 mg/L for \( R_Q = 1 \).

PIV measurement of the velocity distribution in the mixing channel is indispensible to illustrate the importance of the vortex rollup. The development of vortex rollup can be visualized by the phase-locked averaged velocity fields of key phase sequences in one pulsation period (T). The phase sequences are selected with an even-distributed interval of \( \frac{1}{4} T \) or \( \frac{1}{8} T \). Moreover, velocity magnitude contours illustrate the influence of actively controlled injection on the velocity disturbance. However, the
velocity field of the region near the jets exit is not obtainable due to the insufficient laser illumination. Therefore, the observation zone is $0.5-5.5\ h$ downstream from the jet exit on the central plane.

### 4.3.1. Duty cycle of 50%

![Concentration Contours for Different Time Stages](image_url)

**Figure 4.2 Sequences of instantaneous concentration contours for the case: both PF and SF pulsated with a duty cycle of 50%, $f=2\ Hz, Str=0.8, Re =166$**

Figure 4.2 show the formation and development of PF segments for the case of both PF and SF pulsated with a duty cycle of 50%, $f=2\ Hz$ and $Re =166$. After PF is injected into the mixing channel, the front of the PF stream expands in both lateral directions immediately after the jet exit, while the SF stream in blue colour forms into a triangular shape above the PF stream, see Figure 4.2 (a). At the end of PF injection $t= 4/8\ T$, the PF stream fills up the whole width of the mixing channel, and weak rollup is observable at the right wing of PF expansion, shown at the bottom of Figure 4.2 (b). As the result of the SF injecting and PF rolling up, the wings of the PF segment fold back.

However, the SF injection formed a triangular shape segment at the jet exit, instead of the mushroom shape of the PF segment. The triangle is formed by the impinging
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of the SF streams from two surrounding jets with a 60° confluence angle. Moreover, the triangle wedges into the PF segment formed in the PF injection cycle. Thus, the PF segment on the top deforms into the crescent shape, shown in Figure 4.2 (d).

The presence of vortices can be indicated by the appearance of mushroom structures in concentration contours, shown in Figure 4.2(b). Furthermore, the counter-rotating vortex pairs which form the mushroom shape PF segments in the mixing channel can be clearly seen in the velocity fields obtained from PIV experiments (Figure 4.3). In particular, only one vortex pair is generated at the end of the PF injection (Figure 4.3 (b)) for each pulsation period, while the SF injection does not initialize observable vortex rollups.

![Figure 4.3 Sequences of velocity vector with vorticity contours: both PF and SF pulsated with a duty cycle of 50%, f=2Hz, Re=166](image)

Furthermore, the velocity fields suggest that the vortex structures occur only within 3 h after the jet exits and the velocity vectors return to parallel conditions at 5 h downstream from the jet exits. The PIV results suggest the regions of intensive
velocity fluctuations are limited within $5h$ from the jet exit. The vortex structure generated by the PF injection in previous cycle disappears after the PF is injected again. Thus, Figure 4.2 (a) shows only a region of higher velocity magnitude in the centre of the mixing channel at $y=1-2h$, where vortex kernel runs out the vorticity. Finally, the region of higher velocity magnitude diminishes when it approaches $y=4h$. Therefore, mixing at further downstream depends mainly on the flow stretching and shearing.

Mixing of both primary flow and secondary flow pulsated out-of-phase with a duty cycle of 50% relies mainly on sequential segmentation is suggested by the concentration contours in Figure 4.4. In Figure 4.4, it can be seen that segments with vortices are produced by the primary flow with a frequency up to 8Hz and as the frequency increases; both the size and strength of the vortices are reduced.

![Figure 4.4 Concentration contours of both PF and SF pulsated with duty cycle 50%, at different frequencies: Re= 166](image-url)
First of all, in the baseline case, the interface between the primary flow stream and the surrounding secondary flow streams remains almost in straight lines with a negligible level of mixing. This is indicated by the clear and straight interface between the primary flow stream in red and the surrounding secondary flow streams in blue (Figure 4.4 (g)).

In Figure 4.4(a), at \( f = 1 \text{Hz} \) a vortex-like structure associated with a high dye concentration (in red) is seen at the inlet of the mixing channel, which is produced by the pulsation of the primary flow. A further three ‘Λ’-shaped isolated regions of higher concentrations (also in red) with an almost equal spacing are observed downstream along the mixing channel. They must be the remainders of the vortex structures produced in the previous pulsation cycles. These ‘Λ’-shaped structures appear to become progressively elongated in the streamwise direction as a result of the shearing effect. These structures are separated by fluid of low dye concentration (in blue) from the secondary flow, a clear evidence of successful sequential segmentation as a result of alternative feeding of primary and secondary flows in the mixing channel. Overall, at \( f = 1 \text{Hz} \) a moderate level of mixing is evident as indicated by the changes in the contour colour of the secondary flow. Similar segmentation structure is observed at \( f = 2 \text{Hz} \), shown in Figure 4.4(b).

As the frequency increases towards \( f = 4 \text{Hz} \), the number of vortex structures increases whereas the streamwise extent of individual structures decreases as a result of the reduced time interval during which they are formed (Figure 4.4 (b-d)). At \( f = 1 \text{Hz} \) and 2Hz, the near-wall region appears to be filled with the residual of primary flow of high dye concentration due to the local low flow velocity there. However, the PF segments formed by alternative injection at the jet exit break up in the centre of the mixing channel at \( f = 3 \text{Hz} \) (Figure 4.4(c)). The breakup and stretching of PF segments increase the interface areas between the PF and surrounding SF stream. Therefore, the concentration level appears to decrease progressively as the pulsation frequency increases to 4Hz.

Beginning at \( f = 6 \text{Hz} \) the spanwise extent of the vortex structures becomes less than the width of the mixing channel. Although the mixing effect of sequential segmentation indicated by the interwoven of primary and secondary flow intensifies
due to a reduction in their size at higher frequencies, the segments become increasingly concentrated in the central part of the mixing channel, resulting in deteriorated mixing in the region near the walls (Figure 4.4(e) and (f)).

Similarly, PF segmentation is shown in Figure 4.5 at higher Reynolds number of 250, but better mixing is suggested by the larger green colour regions. The counter-rotating vortex rollup structures are universal in the rear of the PF segments, for the tested frequencies ranging from 1Hz to 8Hz. Likewise, as the pulsation frequency increases, the PF segment in red colour at the jet exit becomes shorter. However, the segmentation length is longer due to the increased total flow volume injected into the mixing channel, comparing with the case with the identical pulsation frequency at Re=166. Nevertheless, the increased injection volume and velocity lead to the higher momentum in a single injection, which is beneficial to the mixing for its promoted flow disturbance capability.
Besides, a tendency of segmentation breakup at higher Reynolds number of 250 is suggested by Figure 4.5(b), (c) and (d). The shapes of the PF segments become asymmetrical and irregular, as they are moving downstream. The motion in the third dimension may contribute to this asymmetrical segmentation structure, which will be evaluated later by the stereoscopic dye visualization.
4.3.2. Duty cycle of 25%

The critical role of vortex rollup on the fast mixing in near jet exit region, is indicated by the sequence of the concentration contours for the case: both PF and SF pulsed out-of-phase with a duty cycle of 25%, $Re=166$, $f=2$ Hz (Figure 4.6). Significant mixing is indicated by the large zone of green colour shown in Figure 4.6. The critical role of vortex rollup on the fast mixing in near jet exit region, is indicated by the sequence of the concentration contours for the case: both PF and SF pulsed out-of-phase with a duty cycle of 25%, $Re=166$, $f=2$ Hz (Figure 4.6). Significant mixing is indicated by the large zone of green colour shown in Figure 4.6.

First of all, the vortex rollup is suggested by the mushroom shape PF segment at the jet exit (Figure 4.6(b)) at the end of the PF injection at $t=2/8$ T, which are confirmed by the counter rotating vortices pairs in PIV results shown in Figure 4.7. After the PF injection, without further pushing effect of PF injection, the PF segment separates from jet exit as a result of entraining surrounding fluid by the vortex rollup. Meanwhile, the vortex rollup structure does not move downstream significantly, comparing Figure 4.6(c) with the Figure 4.6(b). Furthermore, the sudden dropping of flow rate in the mixing channel accelerates the break up of the vortex structure then enhances the mixing near the vortex pair, which is suggested by the fading of PF segment in Figure 4.6(d).

Similar to the cases with duty cycles of 50%, no strong vortex rollup is initialized during SF injection cycle. The injected SF in blue colour shows up in the mixing
channel at \( t = 5/8 \ T \). As the joint effort of SF injection and PF rollups, the PF segments transformed into the balloon shape, shown in Figure 4.6(e). At the end of the SF injection at \( t = 6/8 \ T \), the SF injection streams nearly join at the centre of the mixing channel. However, it is not until \( t = 7/8 \ T \), the SF streams are merged into a single segment. Although, strong vortex structure is not evident in the concentration contours, successful SF segment is formed for further mixing by interface stretching downstream.

Figure 4.7 Sequences of velocity vector with vorticity contours: both PF and SF pulsated with a duty cycle of 25%, \( f = 2 \text{Hz}, Str = 0.8, Re = 166 \)

Figure 4.7 confirms the presence of stronger vortex rollups, if the duty cycle is lowered from 50% to 25% for both PF and SF pulsation. Firstly, the enhanced vorticity is suggested by the appearance of second vortex pair in the region within 1.5-2.5\( h \) downstream from the jet exit, shown in Figure 4.7 (c). This vortices pair is formed in previous PF pulsation cycle, yet it is hidden by the overwhelming PF injection velocity when the flow rate is high in the mixing channel during the PF injection at \( t = 1/8 \ T \), see Figure 4.7 (a). Then, Figure 4.7 (b-d) illustrates clearly the
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diminishment of the previous vortex pair. Secondly, the new vortices pair generated in the current pulsation period is observed in Figure 4.7 (c) for the first time. The pair is generated by PF injection in the current pulsation period, but it is not until \( t = 5/8T \) the pair shown up at \( y=0.5-1.0h \). Finally, the counter-rotating vortices pair is fully demonstrated in Figure 4.7 (d).

Segmentation patterns similar to Figure 4.6 are observed in the instantaneous concentration contours in the mixing channel, when both PF and SF are pulsated out-of-phase at different pulsation frequencies with a duty cycle of 25% and a Reynolds number of 166 (Figure 4.8). It should be noted that a high instantaneous velocity of both the primary and secondary flow will be required, in order to obtain the same time-averaged Reynolds number in the mixing channel at a lower duty cycle. This, in fact, is reflected by the stronger initial vortex structures, their more rapid breakdown and the resultant higher mixing extent as seen in Figure 4.8.

Likewise, the group of cases shown in Figure 4.8 suggest that pulsation frequency is an important pulsation parameter to determine the segmentation pattern. Although there is apparent sequential segmentation and deformation of PF stream in Figure 4.8(a), the segment is too long to be rollup into an oval shape. The strong vortex rollup of the primary flow is evident at \( f=2 \) Hz, and the segment is shorter than that that of \( f=1 \) Hz. Besides, the unsymmetrical structures in Figure 4.8(a) and (b) hint that the flow movement in the third dimension is intensive in the mixing channel. On the other hand, vortex pair rollup is not evident in the contours for higher pulsation frequencies, but sequential segmentation play prominent role in mixing. The primary flow breaks up into two parts immediately after the jet exit for the high pulsation frequency up to 8 Hz, shown in Figure 4.8 (d), (e) and (f). In summary, fast mixing is achieved at wide range of pulsation frequencies via both vortex rollup and segmentation.
Figure 4.9 demonstrates that the pulsation frequency affects the vorticity strength if both PF and SF are pulsed with a duty cycle of 25%. The identical phase of $t=3/8T$ are selected since vortex rollup is generated at the end of PF injection. Figure 4.9(a) shows vortex rollup occurs only within the region $1h$ from jet exit, and no vortex is found further downstream at $f=1Hz$. It suggests the primary flow segment should not be longer than the width of the mixing channel, or it may prevent the rollup of the whole PF segment into a circular shape (Figure 4.8(a). In contrast, the PF segment is about $1h$ in length at $f=2Hz$, which is a proper length of the PF segment makes the vortex rollup travels farther with less constraint from the walls in Figure 4.9 (b). Figure 4.9(c) shows only one main vortex kernel with two much weaker kernels downstream at $t=3/8T$ for the case: $Re=166, f=4Hz$. However, the vorticity strength of the main vortex is not as strong as that case of $f=2Hz$, due to the reduced PF
segmentation momentum at higher pulsation frequencies. Therefore, the stronger vortex rollup occurs at the frequency with a compromise of segment length and injection momentum.

Figure 4.9 Velocity vector with vorticity contours for both PF and SF pulsated with a duty cycle of 25%, at different pulsation frequency, \( t=3/8T, \text{Re}=166 \)

4.3.3. Stereoscopic dye visualization of 3D flow structure

The 3D flow structure, which has been reported to be effective for laminar mixing (Soleymani et al., 2008), invites qualitative evaluation by stereoscopic dye visualization. The motion on the third dimension may be caused by intensive velocity fluctuations and the sudden geometrical change upon entry to the mixing channel with a limited aspect ratio of 5.

Favourable 3D flow structure is observed from the front and lateral views simultaneously by the stereoscopic visualization configuration illustrated by Figure 3.12 in Chapter 3. The dyed primary flow structures, visualized from the front and
lateral views simultaneously, are shown in Figure 4.10 and Figure 4.11. The observation region is 5 mm to 85 mm (0.6\textit{h} to 10.6 \textit{h}) downstream from the jet exit in the streamwise direction. In the front direction, the whole width (8 mm/1h) of the mixing channel is shown; in the lateral direction, only a central region of 25 mm(-1.6\textit{h}-1.6\textit{h}) out of the lateral window with a total length of 40 mm is presented, because the debris and screws impedes the observation of the flow condition in the lateral view.

Figure 4.10 suggests the intensive motion in the third dimension depends on the proper pulsation frequency to generate vortex rollup. PLIF and PIV experiments have confirmed that the formation and breakup of the strong vortex rollup relies on optimum pulsation frequency, see Figure 4.4. Obvious vortex rollup structures are observed in concentration contours only at low pulsation frequencies, in particular, \(f=2\text{Hz}\) and \(f=3\text{Hz}\). This is consistent with the stronger fluctuation observed from the lateral view in Figure 4.10 (b) and (c). On the contrary, the 3D flow structure is not evident with a pulsation frequency of 1 Hz in Figure 4.10 (a) or 5 Hz in Figure 4.10 (d).

Figure 4.10 Stereoscopic dye visualization of the 3D flow structure: Both PF and SF pulsated with a duty cycle of 50%, Re=166
Furthermore, apparent 3D flow structures, accompanied by strong vortex rollups, are more likely to be found at a higher injection Reynolds number with a duty cycle of 25%. The 3D flow structure develops immediately after the injection from the jet exit, indicated by the wavy boundary of the dyed PF from the lateral view in Figure 4.11(a), (b) and (c). The time-averaged Reynolds number $Re$ in one pulsation period is identical for cases in Figure 4.11 with a duty cycles of 25% and Figure 4.10 with a duty cycles of 50%. However, the averaged injection Reynolds number $Re_J$ is predicted as 333 during either the primary flow or the secondary fluid injection period in Figure 4.11. Whereas, the averaged injection Reynolds number $Re_J$ is 166 by theoretical prediction for the duty cycle of 50%, which shows only moderate 3D flow structures in Figure 4.10. Nevertheless, the sudden drop of flow rate in the mixing channel also contribute to the flow motion in the third dimension, when both PF and SF injection is shutdown.

![Stereoscopic dye visualization of the 3D flow structure: Both PF and SF pulsated with a duty cycle of 25%, Re=166](image)

**Figure 4.11** Stereoscopic dye visualization of the 3D flow structure: Both PF and SF pulsated with a duty cycle of 25%, $Re=166$

### 4.3.4. Quantification of mixing degree

The variations of time-averaged mixing degree with changing pulsation frequency for
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The cases with a duty cycle of 50% at three different Reynolds numbers is shown in Figure 4.12(a). From this figure, a Reynolds number dependence of the mixing degree can be clearly seen, with a higher Reynolds number resulting in a higher degree of mixing. This Reynolds number dependence is not surprising since a higher Reynolds number will not only lead to the formation of stronger vortices at the entrance of the mixing channel but also a quicker breakdown of these structures, both of which are favourable to mixing.

It can also be seen that, at a given Reynolds number, the mixing degree varies with the pulsation frequency and an optimal frequency exists, especially at the two lower Reynolds numbers. The optimal frequency scales approximately with the Strouhal number $Str = \frac{f h}{U}$, around 1. Above this optimal frequency the size of the vortex structures produced by the pulsating primary flow has reduced to such an extent that they are unable to span the entire width of the mixing channel. This results in a reduced level of mixing, especially in the near-wall region, despite a strong presence of sequential segmentation effect in the central region. Better mixing relies on the appropriate pulsation frequency, which reflects a balance between the primary flow plug length and successful sequential segmentation. The lower pulsation frequency guarantees the segmentation and generate vortex rollup but decreases the area of interface between the PF and SF. While the higher pulsation frequency can break the primary flow into smaller segments, but the no vortex rollup is observed.

The above observation appears to apply to the cases with a duty cycle of 25% as
shown in Figure 4.12 (b). However, at the same Reynolds number the mixing degree is apparently higher due to a higher instantaneous primary and secondary flow velocity required to maintain the same mean Reynolds number in the mixing channel. For example, the mixing degree for the case: \( \text{Re} = 166 \) with a duty cycle of 25\%, \( R_Q = 1 \) is better than the case: \( \text{Re} = 166 \) with a duty cycle of 50\%, \( R_Q = 1 \). Cases with the lower duty cycle of 25\% have a stronger vortex rollup than those cases with a duty cycle of 50\%; this has been proved by the stronger vortex in the velocity contour. For the same mean Reynolds number, lower duty cycles means higher instantaneous flow velocities in the mixing channel, which may lead to the stronger 3D structure flow. Besides, there is a dramatic drop of the total flow rate to near zero in the mixing channel if the duty cycle is 25\%, since there is a period of time when there is no additional flow rate injected into the mixing channel. The sudden drop of flow rate in the upright channel also contributes to the rollup and breakup of the vortex.
4.4. Pulsating either primary flow or secondary flow

Investigation on either PF or SF pulsated aims to compare the pulsating effect on the confined jet and surrounding jets. It is to be noted that the Reynolds number is based on the mean flow velocity in the mixing channel, consistent with the definition of both PF and SF pulsated. The ideal flow rate fluctuation during one pulsation period $T$ is illustrated in Figure 3.4.

4.4.1. Pulsating only primary flow

Duty cycle of 50%

Figure 4.13 Sequences of concentration contours: only PF pulsated with a duty cycle of 50%, $Re = 166$, $Re_f = 1$, $f = 2Hz$

Figure 4.13 demonstrates the transformation of the PF segmentation after the injection. The leading edge of the PF segmentation stretches laterally in both directions and touches the side walls of the mixing channel when the PF injection finishes at $t = 4/8T$. The following drop of total flow rate results in the contours of the stretching parts. Compared with the case in which both PF and SF pulsated with a duty cycle of 50%, there is a bulky tail formed by the trailing part of the PF injection.
The bulky tail decreases in length as it flows downstream. After the completion of the next PF injection at $t = 4/8 \, T$, the tail transforms into a crescent head for the faster velocity in the central part of the channel, while the curved parts flow slowly along the walls of the channel. Finally, the crescent head catches up the previous PF segment and encloses the SF in the centre.

Figure 4.14 Sequences of velocity vectors with vorticity contours: only PF pulsed with a duty cycle of 50%, $Re=166$, $R_Q=1$, $f=2 \, Hz$

Weak vortex rollups are found within $2h$ from the jet exit in Figure 4.14 (c) and (d), for the case of $Re=166$, $R_Q=1$, $f=2 \, Hz$, PF only pulsed with a duty cycle of 50%. Only one weak vortex rollup pair is generated for each pulsation period. The rollup of the PF injection appears firstly in Figure 4.14 (c), after the ending of the PF injection. The vortex rollup pair is constrained by the limited width of the mixing channel; this pair depletes its vorticity quickly as it move downstream.

The flow patterns of PF only pulsed with a duty cycle of 50% at different frequencies, $Re= 166$, $R_Q=1$, are presented in Figure 4.15. Similar umbrella-shaped segmentation can be found at low pulsation frequencies of 1-3 Hz; while segmentation is not achieved at pulsation frequencies higher than 6 Hz. For the low
pulsation frequency of 2 Hz shown in Figure 4.15 (b), the leading edge curved backwards, but the rollup is not evident until the appearance of the next umbrella-shaped PF segmentation. As the leading edge rolls up at both sides of the mixing channel, the stem of the umbrella-shaped segmentation transforms into a crescent shape. On the other hand, the pattern of arrow-shaped segmentations without rollup dominates at higher pulsation frequencies, illustrated in Figure 4.15 (d) and (e). Although sequential segmentation is narrowly achieved for $f=6$ Hz, PF segmentations can not stretch into the near wall regions of the mixing channel.

![Concentration contour of only PF pulsated with a duty cycle of 50% at different frequencies, Re= 166, $R_e=1$](image)

**Figure 4.15** Concentration contour of only PF pulsated with a duty cycle of 50% at different frequencies, $Re= 166, R_e=1$

Certainly, there is an upper limit of pulsation frequency above which the mixing deteriorates, due to the incomplete sequential segmentation. The threshold frequency
for successful segmentation constrained by the specific pipeline configuration and the dynamic responding characteristics of the solenoid valves. For current confined jets configuration, the PF can be segmented for frequencies less than 8 Hz. The flow pattern of \( f=8 \text{Hz} \) falls back into streamwise parallel flows. The parallel flow pattern can not improve the mixing significantly, due to the failure of increasing the interfacial areas as the PF and SF flow downstream. Thus, the central region and the near wall regions of the mixing channel are ill-mixed, since the primary flow concentrates in the centre of the mixing channel while the SF dominates the near wall region.

After all, pulsating PF only is disadvantageous to the completion of the PF segmentation. Without the stopping of SF injection, the PF injection struggles to expand to the whole width of the mixing channel. Thus, PF can not reach the side walls of the mixing channel at the high pulsation frequency.

**Duty cycle 25%**

![Figure 4.16 Sequences of concentration contours: only PF pulsated with a duty cycle of 25%, Re = 166, \( R_\theta=1 \), \( f=2 \text{Hz} \)](image)

Figure 4.16 gives a close up view of the process of vortex rollup and break up for the case, \( Q_P=Q_S=0.2 \text{ L/min} \), \( f=2 \text{Hz} \), when only PF is pulsated with a duty cycle of 25%. The pulsation frequency of 2Hz is selected, since segmentation length approximately equal to the mixing channel width is favourable for the vortex rollups. Furthermore, eight evenly-spaced sequential images are captured for one pulsation period, which give more details on the rollup structure.
The lateral expansion of the leading edge of the PF segmentation is observed at $t = 1/8\ T$. When the PF solenoid valve begins to close at $t = 2/8\ T$, the PF segmentation looks like a mushroom and the front part has touched the walls of the mixing channel on both sides. The vortex rollup is not evident until $t = 3/8\ T$, when the rollup structure appears at both the wings. Then, the PF segmentation moves slowly as only SF constantly flows into the mixing channel with a low flow rate of 0.2 L/min. Since the vortices continue to entrain fluid into the kernel regions, the tail of the PF segmentation disappears. Later at $t=6/8\ T$, the mushroom-shaped PF segmentation transforms into a circular shape with a back-to-back double vortex pair inside. Finally, the vortex pair swirls into a closed circle at $t=6/8\ T$.

Moreover, the vortex rollup structure formed in the current pulsation period can move further downstream before the breakup at $y=3\ h$. Thus, two vortex pairs can be observed simultaneously from $t=1/8\ T$ to $t=8/8\ T$. At first, the downstream vortex pairs are stable from $t=1/8\ T$ to $t=3/8\ T$. Then, an asymmetrical structure is found at $y=2\ h$. Finally, the vortex structure runs out its vorticity and breaks up at $y=3\ h$.

Compared with the case of both PF and SF pulsated shown in Figure 4.6, rollup structures are symmetrical and clear if only PF pulsated with the duty cycle of 25%. However, the double vortex pair becomes asymmetrical at $y=2-3\ h$; the left vortex dominates over the right one. Then, the right vortex disappears due to breakup and exhaustion of vorticity, at $t = 7/8$. 
During the PF injection from $t = 0/8T$ to $t = 2/8T$, the rise in the total flow rate in the mixing channel is indicated by the higher maximum velocity magnitude, indicated by bigger vector size. When the flow rate is high in the mixing channel, the vortex kernels are hidden by the overwhelming PF injection velocity. Then, a strong vortex pair reappears at $y=2h$, shown in Figure 4.17 (b) at $t = 4/8T$, due to the dropping flow rate in the mixing channel. This vortex pair, formed in the previous PF injection cycle, takes up the whole width of the mixing channel. Meanwhile, a near vortex kernel is forming at the PF jet exit, which shows up in Figure 4.17 (c) at $t = 6/8T$. The SF which is steadily injected into the mixing channel pushes the vortex pair slowly downstream. Finally, when the downstream vortex pair moves to $y=2.5h$ at $t = 8/8T$, the vortex pair formed in previous cycle are depleting their vorticity, shown in Figure 4.17 (d).

Furthermore, there is clear correspondence between the vortex pairs in Figure 4.17 and the rollup structure in Figure 4.16. For example, Figure 4.17 (c) shows two
vortex pair centres at \( y=2.5h \) and \( y=2.5h \) respectively, which correspond to the PF rollup structures at the identical positions. Likewise, the correspondence of plateaux of higher velocity magnitude in the PIV results and the PF segments in concentration contours is confirmed.

Compared with the case with both PF and SF pulsed with a duty cycle of 25% (Figure 4.7), the vortex rollups structure is similar to a great extent, except that the vortex pairs can survive a longer distance downstream. Hence, it suggests that the PF injection is a primary contributor for the vortex rollup.

![Figure 4.18 Concentration contour of only PF pulsed with a duty cycle of 25% at different frequencies, Re= 166, \( R_Q=1 \)](image)

Figure 4.18 shows that sequential segmentations and vortex rollups are achievable with the primary flow only pulsed by a solenoid valve with a duty cycle of 25%, \( \text{Re} = 166 \). Vortex rollups are apparent in a wide range of frequencies from 2Hz to 4Hz,
but not the case with a pulsation frequency of 1Hz. The vortex generated at the leading edge of the PF injection can not roll up the excessively long PF segment for \( f=1 \)Hz. Figure 4.18 (b) suggests that vortex generation and breakup lead to fast mixing within 5h from the jet exit at \( f=3 \) Hz, while the vortex breakup is not observed at \( f=3 \)Hz. Although strong vortex rollup is not evident if \( f > 4 \) Hz, fast diffusion mixing is obtained by the increasing interfacial areas. However, the failure of segmentation at \( f=8 \) Hz leads to deteriorated mixing, shown in Figure 4.18(f). Although fast mixing is obtained at the centre of the mixing channel, the near wall region requires a much longer time to reach a homogeneous concentration

4.4.2. Pulsating only secondary flow

Duty cycle of 50%

The partial PF segmentation generated by SF pulsation with a duty cycle of 50% is shown in Figure 4.19. The partial segmentation is achieved by the cutting effect of SF injection from two surrounding jets, but PF segments cannot expand to the side walls of the mixing channel before the next SF injection.

![Concentration contours](image)

Figure 4.19 Sequences of concentration contours: only SF pulsed with a duty cycle of 50%, Re = 166, \( R_d=1, f = 2 \)Hz

Notably, the SF injection begins at \( t=4/8T \) and stops at \( t=8/8T \), thus the rear part the PF expands into a vase shape from \( t=1/8T \) to \( t=4/8T \). After the SF injection, the
momentum of the SF comes from two inclined surrounding jets which punches the PF into segments, but there is a tail linking the PF stream and the PF segment, shown in at $t=6/8T$ Figure 4.19. Then, both PF and SF go parallel when they are injected into the mixing channel simultaneously. Eventually, the vase-shaped PF segments transforms into a delta shape at $y=2h$ and stretches further into a “Λ” shape downstream. Moreover, no evidence of vortex rollup is observed by only SF pulsation, which is concluded in previous PIV experiments.

Figure 4.20 shows no vortex rollup but partial PF segmentation for a wide range of pulsation frequencies, if only SF is pulsated with a duty cycle of 50%. Without the active control of PF injection, the segmentation is partially achieved since PF can not reach the near wall regions. As the pulsation frequency increases, the PF segment length is shortened. Although the PF segment cut by the SF injection has various shapes at the jet exit, it will be stretched into the “Λ” shape downstream, shown in Figure 4.20 from (a) to (e). Thus, only moderate mixing is induced by the shearing and stretching effect of the sequential segmentations of PF.
In particular, SF can not cut PF into complete segments at the jet exit at $f=6$ Hz, but the molecular diffusion separates the segments eventually. Nevertheless, partial segmentation fails at $f=8$ Hz (in Figure 4.20 (f)), since the PF stream is not segmented but sheathed by the SF injected from the surrounding jets.

**Duty cycle 25%**

Figure 4.21 shows that the steadily injected PF is fully segmented by the SF pulsated with the duty cycle of 25%, $Re = 166, f = 2$ Hz. The PF segments in the red colour can touch the side walls of the mixing channel due to the longer duration for PF expansion, instead of partial segmentation with the duty cycle of 50%. The steadily injected PF emerges as mushroom shape at $t=4/8T$. Then, the PF stream is segmented into a flatfish shape by the SF injection at $t=6/8T$. Later, the tail of the flatfish-shaped
PF segment disappears. Finally, the PF segment transforms into a crescent shape and stretches into a “Λ” shaped segment downstream. Therefore, better mixing is achieved than that of duty cycle of 50%, since injection from two SF jets can join at the centre at $t=2/8T$ shown in Figure 4.19.

![Figure 4.21 Sequences of concentration contours: only SF pulsated with a duty cycle of 25%, $Re = 166, f = 2Hz$](image)

No vortex rollup but segmentation is observed in Figure 4.22, if only SF is pulsated at low pulsation frequencies with a duty cycle of 25%, $Re = 166$. PF segments are well-formed by the scissor effect of the SF injection from two surrounding jets at $f=1Hz$ and 2Hz, shown in Figure 4.22 (a) and (b). For higher pulsation frequencies, such as 4Hz and 6Hz, bat shape PF segments are formed immediately at the jet exit, then broken up into the isolated islands in the red colour, illustrated in Figure 4.22(c) and (d). Finally, the SF remains in the near wall region and flows parallel with the PF, if only SF is pulsated with a frequency of 10 Hz. In conclusion, a certain amount of mixing can be achieved by molecular diffusion on stretching the interface, as the SF can be distributed into the central part the mixing channel shown in Figure 4.22(b-e).
Figure 4.22 Concentration contours of only SF pulsated with a duty cycle of 25% at different frequencies, $R_e=1$, $Re = 166$

4.4.3. Quantification of mixing degree

The extent of the mixing with only the PF pulsated by a solenoid valve is shown in Figure 4.23, which consolidates the conclusion that better mixing relies upon higher Reynolds numbers, lower duty cycles and the appropriate pulsation frequencies. The tendency of higher Reynolds numbers to generate better mixing is evident, and the mixing with PF only pulsated with a duty cycle of 25% is apparently better than that with a duty cycle of 50%. For example, the case $Q_p = Q_s = 0.3$ L/min, $Re = 250$ and a duty cycle of 25% in Figure 4.23(a), has a better mixing performance than the group of cases: $Q_p = Q_s = 0.3$ L/min, $Re=250$, duty cycle of 50% in Figure 4.23(a).

The mixing pulsated by only PF with a duty cycle of 50% is not sensitive to the
pulsation frequency for the tested Reynolds number, suggested by the less fluctuation of mixing degree with Strouhal number. The degree of mixing does not decrease for the case with a duty cycle of 50% at Reynolds number of 333, indicated in Figure 4.23(a). The molecular diffusion improves as the higher pulsation frequency increases the interface area for the molecular diffusion. However, there is a drop in the degree of mixing as the Strouhal number increases from 1.5 to 3.0 at Reynolds number of 166, due to the ill-formation of segmentations, which are illustrated by the concentration contours of $f=8\text{Hz}$ in Figure 4.15(f).

However, the degree of mixing is independent of pulsation frequency at Reynolds number of 250 with a duty cycle of 25%. Since flow instability or 3D structure flow begins to play an important role in laminar mixing at higher Reynolds numbers, there is a Strouhal number range of 0.3 to 1.5 (corresponding to the frequency range of 2 Hz to 6 Hz), in which the degree of mixing is equally good, indicated in Figure 4.23(b), except that the segments length is excessively long. On the other hand, at lower Reynolds numbers, optimum pulsation frequency exists when mixing relies only on sequential segmentation and vortex structure. In contrast, the mixing deteriorates with increasing pulsation frequency with a duty cycle of 25% at lower Reynolds numbers of 83 and 166, due to the ill-formed segmentation at high pulsation frequency. Comparatively best mixing can be achieved with a pulsation frequency of 2 Hz (corresponding to Strouhal number is near 0.5) at Reynolds number of 166, since a proper segmentation length and adequate vortex rollups is produced by PF pulsation.

In contrast, mixing degree higher than 0.9 is not achievable with only SF pulsed with a duty cycle of 50% or 25%, suggested by the mixing degree shown in Figure 4.24. Firstly, the mixing is worse than only PF pulsed for the same Reynolds numbers and pulsation frequencies, due to the failure of the vortex rollup at low pulsation frequencies or sequential segmentation at higher frequencies. For example, the mixing degree is 0.71 for the case: pulsating with only the SF with the duty cycle of 25%, $\text{Re}=166$, $R_\theta=1$, $f=2\text{Hz}$, while the degree of mixing for the case pulsating with only the PF with the duty cycle of 25% is 0.80, indicated in Figure 4.23(b).
Similarly, increasing Reynolds number is inclined to improve mixing for duty cycle of both 25% and 50%, but the trend is not universal for cases with the duty cycle of 50%. On the other hand, there is an optimal pulsation frequency for the cases with a duty cycle of 50% or 25% in Figure 4.24. The peak of the degree of mixing occurs between the Strouhal number 0.5 and 1.0, which is apparently indicated by the variation of mixing degree at $Re=166$ in Figure 4.24(b). The occurrence of the peak is due to the partial segmentation achieved at a low pulsation frequency, such as the case $f=3$ Hz at $Re=166$.

### 4.5. Summary

In this chapter, mixing between the fluid from a primary planar jet and two
surrounding secondary planar jets which are pulsed out-of-phase has been studied experimentally. Solenoid valves are used to actively control the fluid injection into the mixing channel with pulse width modulation. The experiments are conducted using water at a range of pulsation frequencies (1Hz-10Hz), two duty cycles (50% and 25%) and a mean Reynolds number between 100 and 250.

This mixing enhancement method is shown to be effective with a mixing degree as high as 0.9 achieved at a mean Reynolds number of about 166 with a duty cycle of 25%. The effective Reynolds number has been largely reduced from about \( \text{Re}=2000 \) (Kamau, 2008), due to the active injection control. A combination of different mixing mechanisms is found to be at play, sequential segmentation, shearing and stretching, vortex entrainment and breakup. For the highest Reynolds number tested here, the mixing degree is independent of the pulsation frequency. However, at a given Reynolds number, an optimal frequency exists which scales approximately with a Strouhal number (\( \text{Str} = f h / \bar{U}_j \)) of 1. This optimal frequency reflects the compromise of the vorticity strength and segmentation length. Furthermore, at a given mean Reynolds number a lower duty cycle is found to produce a better mixing due to a resultant higher instantaneous Reynolds number in the jet flow.

Finally, the effect of pulsating the primary flow (\( \text{PF} \)) or the secondary flow (\( \text{SF} \)) alone on water mixing is studied. The results show the pulsating \( \text{PF} \) is more effective than pulsating SF. Umbrella shape segmentations with weak vortex rollup are found if \( \text{PF} \) is pulsed alone with a duty cycle of 50%. Pulsating \( \text{PF} \) alone with a duty cycle of 25% generates similar segmentation and vortex rollup structure with the cases if both \( \text{PF} \) and \( \text{SF} \) pulsed with the same duty cycle, but the mixing degree is slightly deteriorated. On the other hand, pulsating \( \text{SF} \) only generates partial segmentation without observable vortex structure, despite the duty cycle.
Chapter 5. Mixing by Lateral Synthetic Jet Pairs (\(\text{Re}_n=83,166\))

In this chapter, mixing enhancement by means of by single or multiple LSJ pairs is experimentally investigated in water via PIV and PLIF. The mixing effect of opposite and staggered LSJ pair configuration is compared. Furthermore, the more effective mixing by three opposing LSJ pairs along the mixing channel is studied.

5.1. Dimensionless analysis

For the mixer used in the present experiment, the mixing degree in the channel can be described using the following functional expression

\[
\text{MixingDegree} = f(\bar{U}_n, \bar{U}_j, h, d, f, \rho, \mu)
\]

(5.1)

where \(\bar{U}_n\) is the time-averaged mean velocity across the mixing channel, \(\bar{U}_j\) is the jet blowing velocity over the whole synthetic jet actuation cycle, \(f\) is the actuation frequency of the synthetic jet, \(\rho\) and \(\mu\) is the density and dynamic viscosity of the fluid respectively. Since \(\bar{U}_n, h\) and \(\rho\) are the repeating parameters, the dimensional analysis shows that the mixing degree is determined by three independent dimensionless parameters. They are the net flow Reynolds number \(\text{Re}_n\), the dimensionless stroke length \(L\) and the Strouhal number \(\text{Str}\). The first parameter characterizes the flow conditions in the mixing channel whereas the latter parameters characterize the operating conditions of the synthetic jets.

The net flow Reynolds number \(\text{Re}_n\) is defined as:

\[
\text{Re}_n = \frac{\rho h \bar{U}_n}{\mu}
\]

(5.2)

The Reynolds number based on the mean jet blowing velocity \(\bar{U}_j\) is defined as:
\[
\text{Re}_j = \frac{\rho d \bar{U}_j}{\mu} \tag{5.3}
\]

where \( \bar{U}_j \) is the mean jet blowing velocity over the whole actuation cycle, which is given by

\[
\bar{U}_j = \frac{1}{T} \int_0^T u dt \tag{5.4}
\]

where \( T = 1/f \), and \( u \) is the spatial averaged jet velocity across the orifice. In the present setup, since the fluid is incompressible, \( \bar{U}_j \) is linearly proportional to the piston actuation frequency and displacement. For a piston driven \textbf{LSJ}, the mean jet blowing velocity is proportional to the piston vibration amplitude and frequency. Therefore, \textbf{Equation 5.4} can be rewritten as:

\[
\bar{U}_j = \frac{A \Delta f}{n}, \quad \text{where } A \text{ is the area ratio of the piston and the width of rectangular jet orifice, } n \text{ is the number of the LSJ slots with an identical orifice cross-section, } f \text{ is the piston vibration frequency and } \Delta \text{ is the peak-to-peak vibration amplitude.}
\]

The velocity ratio represents the relative strength of the synthetic jet blowing to the mean flow.

\[
R_v = \frac{\bar{U}_j}{U_n} \tag{5.5}
\]

Notably, \( \text{Re}_j = \frac{d}{h} R_v, \text{Re}_n = 0.5 R_v, \text{Re}_n, \) for the fixed geometry of the mixing channel and \textbf{LSJ} orifices in current experimental setup. Therefore, the combination of the net flow Reynolds number \( \text{Re}_n \) and velocity ratio \( R_v \) is frequently used to denote the pulsation strength, instead of synthetic jet Reynolds number \( \text{Re}_J \).

The stroke length, \( L_o \), in a physical term, can be taken as a measure of the height of the fluid column containing all the fluid which is ejected during a synthetic jet actuation cycle. A dimensionless stroke length \( L \), can be defined as the ratio of the stroke length to the width of the orifice, i.e.
Mixing by Lateral Synthetic Jet Pairs (Ren=83,166)

\[ L = \frac{L_0}{d} = \frac{U_j}{fd} \]  

(5.6)

Based on the definition of \( U_j \) and \( L \), the dimensionless stroke length is independent of the frequency but proportional to the displacement of the oscillating piston of the synthetic jet actuator. Thereby, Equation 5.3 can be rewritten as: \( \text{Re}_j = \frac{\rho d^2}{\mu} L f \).

Particularly, the stroke length for the multiple LSJ pairs configuration, is calculated from piston peak-to-peak displacement equally divided by the total LSJ number on the same side of the mixing channel. In other word, the piston displacement is shared by these lateral synthetic jets, and the dimensionless stroke length \( L \) is calculated by the average \( U_j \). Therefore, a bigger piston displacement is needed to achieve the identical stroke length of the single LSJ pair.

Stokes number is the dimensionless representation of the pulsation frequency, which is commonly used to the vortex rollup capability of the synthetic jet.

\[ S = \sqrt{\frac{2\pi fd^3}{\nu}} \]  

(5.7)

On the other hand, the Strouhal number as a dimensionless representation of the actuation frequency \( f \) provides a measure of the level of sequential segmentation occurring in the mixing channel.

\[ \text{Str} = \frac{fd}{U_u} \]  

(5.8)

5.2. Test conditions

In order to evaluate the mixing effect of LSJ pair, the tracer concentration contours and velocity fields are investigated in the mixing channel via PLIF and PIV. Particularly, the concentration profile at the cross-section 12.5\( h \) from the centre of the LSJ cavity is used for mixing degree quantification; similar to water mixing by solenoid valves.
In the present PLIF experiment, the fluid in the left stream is made of a Rhodamine B solution with a concentration of 0.2 mg/L whereas that in the right stream contains pure water. Since the ratio between the flow rates of the two streams is 1, the concentration in the mixing channel is expected to vary from 0 to 0.2 mg/L with the concentration level being equal to 0.1mg/L in the fully mixed condition. On the other hand, the PIV experimental results are phase-locked time-averaged velocity fields in the region of interest.

In this experiment, in order to evaluate the mixing enhancement effect of single and three opposing/ staggered LSJ pairs, the peak-to-peak displacement and frequency of the synthetic jet actuators are varied independently from 0.1 to 1mm and from 1 to 8 Hz respectively. Correspondingly, the synthetic jet Reynolds number varies from 16 to 256, the dimensionless Stroke length $L$ from 1 to 8, and the Strouhal number from 0.4 to 3.2. Notably, the mean velocity in the mixing channel is mainly tested at 0.01m/s, giving a net flow Reynolds number of 83. Nevertheless, the influence of net flow Reynolds numbers is briefly evaluated at $Re_n=166$.

Firstly, mixing by the single opposing LSJ pair is investigated in detail; the influence of pulsation frequency, stroke length and net flow Reynolds number is evaluated qualitatively and quantitatively. The results are compared to mixing by the single staggered jet configuration at the identical LSJ actuation parameters. Finally, chaining up three opposing LSJ pairs along the mixing channel is investigated and compared to the single opposing jet configuration. Although experimental study of mixing by three staggered LSJ pairs is also conducted, but it is not presented due to the insignificant improvement over the opposing jet configuration.

Before moving to the experimental result analysis, the definition of pulsation phases is demonstrated in Figure 5.1. The definition of pulsation phases in the current experiments of LSJ pairs is based on the spatial-average velocity $\overline{U}_j$ of left LSJ orifice. The LSJ actuator is driven by the amplified sine wave signal; the starting of the pulsation period referred to the minimum cavity volume for the left LSJ. Then the left LSJ begins to blowing fluid into the mixing channel, until the end the blowing stroke at $t=2/4T$. Meanwhile, the equal volume is sucked into the right LSJ cavity, which leads to the constant flow rate in the mixing channel. In the second half of the
pulsation period, the flow movement reverses during the suction stroke of the left LSJ, i.e., the left LSJ sucks the equivalent volume which is blown from the right LSJ.

![Figure 5.1 Definition of pulsation phases based on the left LSJ velocity](image)

**5.3. Single opposing LSJ pair**

First of all, the variation of stroke length at fixed actuation frequency $f=1\text{Hz}$ and $2\text{Hz}$ is investigated in detail at $Re_n=83$. Some typical cases are demonstrated by phase sequences of concentration contours and velocity fields. It is followed by the variation of pulsation frequencies at the fixed stroke length $L=2$. Then, the mixing effect of pulsation frequency and stroke length is evaluated quantitatively. Finally, the influence of net flow Reynolds number is studied at $Re_n=166$.

**5.3.1. Influence of Stroke length $L$**

In Figure 5.2, the time sequence of instantaneous fluorescence concentration contours in the mixing channel at $L=3$ and $f=1\text{Hz}$ in a synthetic jet actuation cycle.

At $t=0T$, the piston of the left synthetic jet starts to move to the right from its left most position indicating the start of the blowing cycle (Figure 5.2 (a)). At $t=T/8$, a small amount of fluid (in red) ejected from the left cavity can be seen at the exit of the left orifice ($y=-0.5h$). The ejected fluid presents an obstacle to the incoming flow from upstream, forcing the incoming fluid to flow around it and exhibit a crescent shape (Figure 5.2 (b)). At $t=T/4$, the piston of the left synthetic jet reaches its neutral position ensuring the maximum jet velocity at the orifice exit (Figure 5.2 (c)). At this
instant, a vortex pair appears which grows further in size as the blowing cycle continues till $t=T/2$. As the same time, under the impact of the net flow in the mixing channel, the vortex pair also becomes inclined downstream with the crescent shaped incoming fluid being stretched and wrapping around it (Figure 5.2(e)). Noticeably, as the vortex pair propagates downstream its downstream branch is strengthened whereas its upstream branch becomes increasingly weakened and squashed due the impact of the incoming flow. At $t=T/2$, the blowing cycle of the right synthetic jet starts and a vortex pair of low concentration (in blue) fully emerges at $t=7T/8$ from the right orifice. It can be seen that during the ejection process of the right jet, the vortex pair from the left orifice has propagated a sufficient distance downstream such that when the vortex pair from the right jet is fully formed the former has already moved out the way. As a result, the level of interaction between the two opposing jets is low.

The development of the two counter-rotating vortices produced in the current cycle can be inferred from the vortices produced in the previous cycles. It can be seen that, similar to what happens to the upstream branch of the vortex pair of high fluorescence concentration, the upstream branch of the vortex pair of low concentration is also weakened and eventually diminishes by $y=1h$. As such, further downstream the mixing channel is filled up with two rows of counter-rotating vortex rollers formed by the opposing pair of synthetic jets. These vortex rollers entrain the surrounding fluid into their cores as they propagate downstream. An interaction between the counter-rotating vortices can also be seen, which results in the outer layer of the anti-clockwise vortices being peeled off and pulled through the gap between the vortex rollers. The vortex rollers are seen to become increasingly flattened further downstream due to the continuous action of the local shear. In this case, mixing is clearly enhanced through the entrainment induced by these vortices. Sequential segmentation is also present due to alterative injection of fluid as a result of the out-phase actuation of the synthetic jet pair. Nevertheless, it is not the main mechanism of mixing due to the relative low jet actuation frequency in this case.
Figure 5.2 Sequence of concentration contours for water mixing by single opposing LSJ pair at $L=3$, $R_v=1.2$, $f=1\text{Hz}$, $\text{Str}=0.4$, $\text{Re}_n=83$

Figure 5.3 Velocity vector fields with vorticity contours for water mixing by single opposing LSJ pair at $L=3$, $R_v=1.2$, $f=1\text{Hz}$, $\text{Str}=0.4$ ($f=1\text{Hz}$), $\text{Re}_n=83$

The PIV result, shown in Figure 5.3, confirms the presence of vortices in the flow field. The phase-locked time averaged PIV velocity vector fields with superimposed
vorticity contours at four phases during the actuation cycle in the above case are shown in Figure 5.3. At $t=1/4T$, a region of high blowing velocity at the exit of the left orifice is observed and the accompanying vortex rollup is also evident with opposite vorticity occurring at its upstream and downstream side (Figure 5.3 (a)). This confirms that the structure having a mushroom shaped cross-section at the exit of the left orifice seen in Figure 5.2 (c)) is indeed a newly formed vortex pair. At $t=T/2$, a distinct vortex with anti-clockwise rotation is seen at $y=0$ and it appears to correspond to the downstream branch of the vortex pair seen in Figure 5.2 (e). A weak clockwise vorticity is also observed upstream which corresponds to the weakened upstream branch of the vortex pair. At $t=3T/4$ and $t=0$ T the aforementioned process is repeated for the right synthetic jet. Further downstream, the presence of vortices is less well defined since they are relatively weak and are embedded in a shear layer. Nevertheless, the locations of the isolated regions of negative and positive vorticity appear to correspond to those of the vortex rollers seen in Figure 5.2. Furthermore, the velocity vectors in the middle part of the mixing channel trace out wavy streamlines in response to the presence of these two rows of vortices.

Figure 5.4 shows the instantaneous fluorescence concentration contours in the mixing channel at the same frequency as in Figure 5.2 but at different $L$ including that at $L=3$. The concentration contour in the baseline case without synthetic jet actuation is also inserted in the figure for reference (Figure 5.4 (a)). In the baseline case, the interface between the left and the right flow streams remains almost in a straight line and the level of mixing is negligible. It can be seen that at $L=2$ the interface is disturbed. However, the strength of the vortices is insufficient to reach the opposite wall and the associated entrainment does not produce an appreciable level of mixing (Figure 5.4 (b)). As $L$ increases to 3, a substantial improvement in mixing is achieved due to the formation of vortices with sufficient strength and the mixing channel is essentially filled with counter-rotating vortex rollers (Figure 5.4 (c)). As $L$ increases further, the strength and the size of the vortices are increased correspondingly and the level of mixing improves progressively. At $L=4$ and 5, the mixing channel appears to be occupied by similar vortex structures as seen at $L=3$. At $L= 6$ and 8, although a strong initial vortex pair is clearly observed, the individual vortices revealed by the gradient in fluorescence concentration become less
discernable in the downstream locations, and the fluid appears to be well mixed at $y=7h$ (Figure 5.4(f) and (g)).

Figure 5.4 Velocity fields and vorticity contours for water mixing by single opposing LSJ pair with different $L$ at $f=1$Hz, $t=T/2$, $Str=0.4$, $Re_n=83$

In Figure 5.4, the phased-averaged PIV velocity vector fields obtained at the end of the blowing cycle of the left jet ($t=T/2$) of the above cases are shown. As it can be seen from this figure, the formation of the vortex pair from the left orifice is evident and the vortex pair becomes progressively bigger and stronger as $L$ increases. Nevertheless, despite the initial stronger vortex pair at a higher $L$, the strength of the
vortices appears to decay sharply after a short distance downstream. Thus, second vortex pair is not observed downstream, for \( L > 4 \).

In order to track the development of vortex pair, the phase-locked time averaged PIV velocity vector fields at four phases during an actuation cycle at \( L = 5 \) and \( f = 1 \text{Hz} \) are hence examined (Figure 5.5). It is found that due to the higher jet velocity and relatively larger vortex size at a large \( L \) of 6, during the time the new vortex pair from the right orifice is being formed the remaining downstream branch of the previous vortex pair from the left orifice has not managed to move out of its way (Figure 5.5 (c)). Consequently, the latter is significantly weakened by the former, resulting in a rapid decay of the initial vortex. This kind of interaction between vortex pairs from the opposite walls appears to result in a good mixing, most probably due to the breakdown of the initial 2D flow to 3D flow.

![Figure 5.5 Sequence of velocity fields and vorticity contours for water mixing by single opposing LSJ pair at \( L = 5, f = 1 \text{Hz} \) (Str=0.4), \( R_v = 2.0 \) and \( \text{Re}_n = 83 \)](image)

In Figure 5.6, the instantaneous fluorescence concentration contours in the mixing channel at \( L = 2 \) to 5 at \( f = 2 \text{Hz} \) are shown. PLIF images at \( L > 5 \) were not taken since the vortical structures are not discernable as the result of a good mixing. It should be noted that the velocity ratio is doubled when the frequency increases from 1 to 2Hz at the identical \( L \). Therefore a better mixing is expected at the identical \( L \) at \( f = 2 \text{Hz} \).

Vortices pair is observed at the left LSJ orifice, but it is buried in the LF stream due to the limited velocity ratio at \( L = 2 \) (Figure 5.6 (a)). In contrast, at \( L = 3 \), strong vortices are produced by the synthetic jets which become disintegrated after \( y = 1h \) (Figure 5.6)
(b)). At $L=4$ and 5, except the initial vortex near the orifice exit, the contour becomes almost uniformly green after $y=0h$, indicating a good mixing at two LSJ orifices (Figure 5.6 (c) and (d)). The phase-locked time averaged velocity vector fields shown in Figure 5.6, confirm the presence of an initial vortex near the orifice exit and the absence of vortices further downstream.

Figure 5.6 Instantaneous concentration contours and velocity fields for water mixing by single opposing LSJ pair, at $t=T/2$, $Str=0.8$ ($f=2\mathrm{Hz}$) and $Re_n=83$

To understand why the vortices breakdown rapidly after the initial formation, the
phase-locked time averaged velocity vector fields at $L=4$ and $f=2\text{Hz}$ are illustrated in Figure 5.7. As the blowing cycle of the left synthetic jet starts at $t=0$, the previously formed vortex pair propagates only a short distance downstream (about $0.3h$), since the period of the actuation cycles is reduced at a higher frequency. In particular, the counter-clock rotation vortex kernel is exactly on the orifice of LSJ, it is severely destroyed by the LSJ blowing (Figure 5.7 (c)). Meanwhile, the downstream vortex kernel survives until $t=3T/8$, when the new vortex pair formed at the both sides of the left LSJ orifices. After, the process of vortex pair rollup and breakup repeat in second half of the pulsation cycle.

Moreover, the vortex intensity is stronger than that case of $f=1\text{Hz}$ and $L=4$ observed in Figure 5.3, due to the increasing velocity ratio. Both upstream and downstream vortex kernels are evident at the end of LSJ blowing, and the upstream branches vortex is actively involved into the jet interactions. The breakup of the upstream vortex kernel by the LSJ blowing accelerates the mixing greatly. In addition, the diameter of the strong vortex rollup is comparable to the width of the mixing channel, thus the vortex effectively entrains the surrounding liquids. Therefore, fast mixing is suggested by the large green region after the LSJ orifice, shown in Figure 5.6 (d).

Figure 5.7 Sequence of velocity vector fields and vorticity contours for water mixing by single opposing LSJ pair at $L=4$, $Re_s=3.2$, $f=2\text{Hz}$, $Str=0.8$ and $Re_n=83$
5.3.2. Influence of pulsation frequency ($L=2$)

In Figure 5.8, the effect of increasing actuation frequency is illustrated further using the fluorescence concentration contours at $L=2$ at a range of frequency from 1 to 8Hz. It is evident from Figure 5.8 that at $L=2$ the strength of the vortices produced by the synthetic jets is quite low and they barely reach the opposite wall. Nevertheless, it can be clearly seen that at a given $L$ the level of mixing is progressively improved as the actuation frequency is increased and a good mixing is obtained at a frequency of 5Hz and above. It is because that, as the frequency increases, the level of interaction between the opposing jets (Figure 5.5) is increased, resulting in a breakdown of the initially 2D flow into 3D hence a good mixing at high frequencies.

![Instantaneous concentration contours for water mixing by single opposing LSJ pair at different actuation frequencies, at $L=2$ and $Re_n=83$](image)

(a) $f=1$ Hz, $R_v=0.8$  (b) $f=2$Hz, $R_v=1.6$  (c) $f=3$Hz, $R_v=2.4$  (d) $f=4$Hz, $R_v=3.2$  (e) $f=5$Hz, $R_v=4.0$  (f) $f=6$Hz, $R_v=4.8$  (g) $f=8$Hz, $R_v=6.4$

In summary, increasing either stroke length or pulsation frequency can improve the mixing. Therefore, the synthetic jet Reynolds number $Re_J$, which is proportional to the multiple of the stroke length and pulsation frequency (Equation 5.3 and Equation 5.6) for the given working liquids and jet orifices geometry, is found to be an important parameter determining the mixing effect. The effect of synthetic jet Reynolds number will be discussed in the later section.
5.3.3. Downstream flow patterns

In order to compare the extent of mixing at different synthetic jet operating conditions quantitatively, the mixing degree is calculated from the concentration profile on the cross-section at \( y = 12.5h \), which is extracted from the PLIF concentration measurement on the central plane of the mixing channel. It should be noted that the mixing degree is evaluated at a distance of 5.5\( h \) downstream of the end of the measurement window (\( y = 7h \)) of the PLIF images presented earlier. Therefore, the mixing degree evaluated based on the measurement at \( y = 12.5h \) will be slightly higher than that obtained from the upstream measurement window.

In order to further explain the mixing mechanism of the LSJ pair, the downstream flow patterns near the mixing degree qualification position \( y = 12.5h \) is illustrated in Figure 5.9. The downstream observation zone is 9.2-13.8\( h \) from the piston centre at \( y = 0h \); the concentration profile at \( y = 12.5h \) (100 mm) is taken to calculate the mixing degree.

![Figure 5.9 Downstream concentration contours of water mixing pulsated by single opposing LSJ pair, \( f = 1Hz, Str = 0.4, Re_n = 83 \)](image)

(a) \( L = 1, R_e = 0.4, Re_J = 16 \), (b) \( L = 2, R_e = 0.8, Re_J = 32 \), (c) \( L = 3, R_e = 1.2, Re_J = 48 \), (d) \( L = 4, R_e = 1.6, Re_J = 64 \), (e) \( L = 5, R_e = 2.0, Re_J = 80 \), (f) \( L = 6, R_e = 2.4, Re_J = 96 \), (g) \( L = 8, R_e = 3.2, Re_J = 128 \)

Figure 5.9 presented the downstream flow structure for water mixing pulsated by single opposing LSJ pair at \( f = 1Hz, Str = 0.4, Re_n = 83 \). It suggests the mixing degree is sensitive to the dimensionless stroke length. At first, periodic structure without significant mixing is observed in Figure 5.9 (a) and (b), since the LF stream in red colour never reaches
the right side of mixing channel. At $L=3$, the partial ‘Λ’-shaped structure can be distinguished in Figure 5.9 (c), but LF in red still concentrates in the left side of the mixing channel.

Later, the mixing effect is improved as the stroke length increases. There is no clear red and blue interface for segmentation but partial mixed strips, shown in Figure 5.9 (d). Meanwhile, the regular segmentation structure can not be recognized any longer, but interwoven red and blue strips. Although, vortex structure generated by the LSJ blowing is not observable downstream, since the vorticity strength exhausts immediately after the orifices. The vortex rollup has distorted the segments and increased the interface for molecular diffusing. Therefore, the periodic flow structure can not be distinguished in Figure 5.9. Finally, at $L=8$, most of the zone in the observation is homogenously green, which indicate a good mixing.

5.3.4. Quantification of mixing by single opposing LSJ pair

Figure 5.10 (a) Variations of mixing degree with $L$ and $f$ (b) curve fitting of $L$ and $f$ at a mixing degree of 0.9 for the single LSJ pair configuration, $Re_n=83$

Figure 5.10(a) shows the variations of time-averaged mixing degree with changing dimensionless stroke length for a range of actuation frequencies tested on the single LSJ pair configuration. This figure clearly indicates that at a fixed actuation frequency a higher $L$ will result in a better mixing and as the actuation frequency increases a lower $L$ is required to achieve a given mixing degree. It also shows that a mixing degree greater than 0.9 can be achieved when $L$ is sufficiently high for a given frequency. In order to obtain a correlation for the synthetic jet operating
condition at which a good mixing is guaranteed, a best fit line is found for the relationship between the mixing degree and the dimensionless stroke length at each frequency. The value of $L$ at the mixing degree of 0.9 is then extracted from the intersection point between the line of mixing degree=0.9 and the best fit line at each frequency (Figure 5.10 (a)). Finally, by best fitting the values of $L$ and its corresponding $f$ in the logarithmic scales with a straight line the functional relationship between $f$ and $L$ is obtained. This relationship is found to be $fL^{1.82}=27.5$ at $Re_n=83$, as shown in Figure 5.10 (b). When the frequency is replaced by the Strouhal number, this relationship becomes

$$Str L^{1.82} = 11 \quad (5.10)$$

The above relationship allows either the synthetic jet actuation frequency or piston displacement to be chosen for good mixing when one of these two parameters is fixed. When the characteristics of the driving mechanism of the synthetic jet actuators is available, the optimal synthetic jet operating condition in terms of $L$ and $f$, which produces the desired mixing at the minimum power consumption can be found. It should be noted, however, Equation 5.10 may only apply for $h/d = 2$ which is chosen in the present setup and the sensitivity of the above correlation to $Re_n$ is yet to be investigated.

According to Equation 5.10, a high velocity ratio is required for fast mixing by single opposing LSJ pair. For the given net flow Reynolds number and the fixed mixer geometry, the synthetic jet Reynolds number, which represents the relative strength of the pulsation velocity, is proportional to the velocity ratio and thus the product of actuation frequency and amplitude ($R_c \propto Re_j \propto fL$). However, Equation 5.10 suggests that increasing the pulsation amplitude is more effective than the frequency.

Particularly, mixing degree of 0.9 is selected for the correlation. Although higher threshold values such as 0.95 and 0.99, are widely adopted, they do not apply properly in the current experiments. Noticeable standard deviations exist in the current mixing quantification, due to the spatial and temporal fluctuation of the laser sheet, and the intrinsic variation of flow structure. For example, the mixing degree for the case of $L=6$, $f=1$Hz at $Re_n=83$ is 0.906, whereas no flow structure can be
distinguished in the homogenous green concentration contour (Figure 5.9 (f)). Additionally, increasing the stroke length to 8 does not improve the mixing significantly (Figure 5.10 (a)). Therefore, a mixing degree higher than 0.9 is regarded as good in the current experiments.

Figure 5.11 indicates most energy-effective pulsation frequency is about 5Hz and $L=3$ at $Re_n=83$. The time-averaged total power consumption of the synthetic jet actuator was obtained by measuring the temporal variations of current and voltage supplied to the actuator. The electronic power consumes on the friction of the rubber of the sandwich-structured piston, propulsion the movement of the liquid through the LSJ orifice, the inert effect of the LSJ system and mechanical and electrical losses of the shaker (Zhang and Zhong, 2010). However, the pressure forces, acting on the two pistons on each side of the mixing channel, compensate each other due to the rigid connection.

Besides, the Helmholtz resonance of the flow in the LSJ cavity has negligible influence on the power consumption, since the Helmholtz resonance frequency (Zhang and Zhong, 2010) is 3.5kHz for the current LSJ cavity and orifice geometry:

$$f_{H} = \frac{c}{2\pi} \sqrt{\frac{A}{Vh_o}}$$  \hspace{1cm} (5.11)
Where $A$ is the cross-section area of the synthetic jet orifice, $h_o$ is the orifice depth, $V$ is the cavity volume, and $c$ is the speed of sound in water, which is 1800 m/s at 20°C.

Figure 5.11(b) shows the power consumption drops with the increasing actuation frequencies; after the energy-effective pulsation frequency of 5Hz, energy consumption rises sharply for the identical stroke length. However, $L=2$ is not reachable for $f=12$Hz, it is limited by the voltage measurement range of the DAQ card. On the other hand, fast mixing can be achieved at either a small stroke length at the high actuation frequency or big low stroke length at the low actuation frequency. Therefore, the selection of optimum actuation parameters is based on the most energy consumption of the LSJ actuation. In addition, doubling the net flow rate ($Ren=166$) has no noticeable impact on the power consumption.

5.3.5. Influence of net flow Reynolds number

Since the velocity ratio $R_v$ is crucial for mixing by single opposing LSJ pair, the impact of the increasing net flow rate on the mixing effect is worth investigating. At higher net flow Reynolds numbers, the velocity ratio $R_v$ decreased at the identical LSJ pulsation frequency and stroke length. The spatial averaged net flow velocity in the mixing channel of 0.02m/s, which is twice as high as that of $Ren=83$. It means the flow segmentation can move about $2.5h(20mm)$ in the mixing channel in one pulsation period($f=1$Hz), thus the dwelling time for the incoming liquids at the LSJ orifice is reduced.

Figure 5.12 shows both the fluorescence concentration contours and velocity vectors for cases of $f=1$Hz and $Ren=166$, at the end of left LSJ blowing. Similar trend of improved mixing at bigger stroke length is confirmed at $Ren=166$. Firstly, segmentation is not successful at $L=2$ (Figure 5.12 (b)), although the interface of the red and the blue streams curves towards the right orifice. At $L=3$, segmentation and rollup is observable, but the red stream still concentrates on the left side of the mixing channel (Figure 5.12 (c)). Secondly, substantial mixing can be achieved via vortex rollup and segmentation for $L=4$, $R_v=0.8$, shown in the fluorescence concentration contour of Figure 5.12 (d). Figure 5.13 shows the whole process of vortex rollup by phase sequence of contours.
and velocity fields. Compared to the counter-rotating vortex pair for $L=4$ at $Re_n=83$ (Figure 5.4(d)), only one weaker vortex rollup is observed at the end of left LSJ blowing in the velocity field of Figure 5.12(d).

Later, stronger vortex rollup is observed at the downstream wing of the LSJ orifice as the stroke length increases; the rollup fills up the width of the mixing channel at $L=6$, seen in Figure 5.12 (f). Finally, fast mixing due to the strong vortex rollup at $L=8$, $R_v=1.6$, is suggested by homogenous green zone immediate after the jet orifice in the fluorescence concentration contour (Figure 5.12 (g)).

![Figure 5.12 Concentration contours and velocity fields with vorticity contours for water mixing by single opposing LSJ pair at $t=T/2$, $f=1Hz$, Str=0.2 and Re$_n=166$](image)
Figure 5.13 demonstrates the development of the vortex rollup generated by the lateral synthetic jet at $L=4$ and $Re_n=166$. The concentration contours and velocity fields show two potential vortical kernels at both sides of the left LSJ orifice exit (Figure 5.13(c)), when the blowing velocity of left LSJ reaches the peak blowing velocity at $t=2/8T$. However, the upstream one is crushed later by the upward net flow momentum. Therefore, only one anti-clockwise vortex fills up the mixing channel at the end of left LSJ blowing (Figure 5.13(e)), instead of the double counter-rotating vortical structure in velocity vector (Figure 5.5 $L=5$, $f=1$Hz, $Re_n=83$).

![Figure 5.13 Sequences of contours and velocity fields with vorticity contours for water mixing by single opposing LSJ pair at $L=4$, $f=1$Hz, $Str=0.2$, $R_v=0.4$, $Re_n=166$]

Besides, the vortex structure move to $y=1\ h$ due to the higher net flow velocity,
therefore, the blowing from the right \textbf{LSJ} has little impact on the vortical structure generated by the left jet blowing, illustrated in the fluorescence contour in Figure 5.13 (f-h).

The first row of Figure 5.14 shows the concentration contours for water mixing by single opposing \textbf{LSJ} pair at $f$=2Hz and $Re_n=166$. Only interface fluctuation is observed in Figure 5.14(a); it suggests the weak jet penetration capability at $L=2$, Figure 5.17 (b) show sudden improvement of mixing at $L=3$, via segmentation and vortex rollup structure similar to those at $Re_n=83$. Then, Figure 5.14 (c) show fast mixing at $L=4$ and $R_v=1.6$; most part of the mixing channel is filled with green colour. Finally, mixing is completed at the $L=5$, leaving the mixing channel with homogeneous green.

The second row of the Figure 5.14 shows phase-locked time averaged velocity vectors &vorticity contours at the ending of the left \textbf{LSJ} blowing. Vortex rollups firstly shows up at $L=3$, $R_v=1.2$ in Figure 5.14 (b), at the end of the left \textbf{LSJ} blowing. Only one vortex structure is observed at the downstream corner of the jet orifice, whereas the upstream vorticity is too weak to roll up surrounding liquid at exit of the \textbf{LSJ} orifice. Finally, the upstream vortex grows up at $L=5$, $R_v=2.0$, and the double vortices take up the mixing channel completely. Therefore, fast mixing is achieved immediately after the blowing of the left \textbf{LSJ} due to the strong vortex rollup, indicated by homogenous green colour in the fluorescence contour of Figure 5.14 (d). Instead of the counter-rotating double vortex pair with similar intensity for $Re_n=83$, the downstream branch of the vortex pair dominates the vortical pattern for $f$=2Hz, $Re_n=166$. 
Mixing by Lateral Synthetic Jet Pairs (Ren=83,166)

The breakup of the vortex rollup due to the blowing from the opposing LSJ is illustrated in Figure 5.15. There is an anti-clockwise vortical structure between the LSJ orifices at $y=0.5h$, seen in Figure 5.15 (a) at $t=0$, however, the vortex is crushed by the joint impact of the net flow stream and LSJ blowing at $t=T/8$. Due to the strong wall shearing at the left LSJ orifice, new vortices kernels are initialized at $t=2T/8$ (Figure 5.15 (c)), which grow up into counter-rotating vortex pair (Figure 5.15(d) and (e)). The pulsation frequency has doubled for the identical stroke length;
therefore, the influence of the increased net flow rate is not as significant as $f=1\text{Hz}$, $Re_n=166$.

![Velocity vector fields with vorticity contours for water mixing by single opposing LSJ pair, at $L=4$, $R_v=1.6$, $f=2\text{Hz}$, $Str=0.4$ and $Re_n=166$](image)

Figure 5.15 Velocity vector fields with vorticity contours for water mixing by single opposing LSJ pair, at $L=4$, $R_v=1.6$, $f=2\text{Hz}$, $Str=0.4$ and $Re_n=166$

Strouhal number is recognized as an important parameter to determine the vortex rollup patterns and thus the mixing enhancement capability. Strouhal number represents the relative dwelling time of flow structure at the exit of LSJ pair. Only one vortex rollup is formed at the downstream wing of the LSJ orifices at $Str=0.2$, seen in PIV results of $f=1\text{Hz}$ and $Re_n=166$ (Figure 5.12). Whereas, vortex pair is found at $Str=0.4$, for the cases of $f=1\text{Hz}$ and $Re_n=83$ (Figure 5.4), and $f=2\text{Hz}$ and $Re_n=166$ (Figure 5.14). If no vortex rollup is formed at the upstream wing of the jet exit, there will be no immediate breakup accompanied with the 3D flow structure. Moreover, the reduced dwelling time also impairs the fluid entraining effect of vortex rollup, since strong vortex rollup exists only within $2-3h$ downstream from the LSJ orifice. Therefore, the mixing effect is deteriorated at low Strouhal number of 0.2.

In summary, fast mixing is achievable at $Re_n=166$ for given sufficiently big stroke length at the given actuation frequency, since the mixing relies on the vortex rollup and breakup at high synthetic jet Reynolds number. Although the vortex intensity is comparatively low for the small stroke length ($L<4$), mixing is steadily improved as the stroke length increases.

Figure 5.16(a) confirms the net flow Reynolds number has a negative compact on the mixing by single opposing LSJ pair at $f=1\text{Hz}$ ($Str=0.2$), compared with identical
Mixing by Lateral Synthetic Jet Pairs (Ren=83,166)

actuation amplitude at $Str=0.4$ ($f=1\,\text{Hz}$ and $Re_n=83$). Since higher $Re_n$ leads to a shorter incoming liquids dwelling time and a lower $Str$. Concentration contours show rollup and segmentation, but the vortex structures move faster as the time-averaged net flow velocity has doubled (Figure 5.12 (g)). On the other hand, the influence of net flow Reynolds number is not significant at $f=2\,\text{Hz}$ ($Str=0.4$), due to the faster rollup and breakup of vortex structure at 2Hz ($Str=0.4$). Vortex pair can be initialized again, hence the influence of the Strouhal number is not as evident as $f=1\,\text{Hz}$. Furthermore, stronger 3D flow structure, accompanying with vortex rollup and breakup at higher instantaneous Reynolds number and $Str\geq 0.4$, compensates the negative impact of the reducing dwelling time around the synthetic jets. Thereby, fast mixing is confirmed by the quantification study for the case $f=2\,\text{Hz}$, $L=4$ for both $Re_n=83$ and $Re_n=166$ (Figure 5.16(a)), despite the difference in $R_v$ and $Str$.

![Figure 5.16](image)

(a) Influence of net flow Reynolds number  
(b) Influence of synthetic jet Reynolds number  
(c) Influence of velocity Ratio $R_v$

*Figure 5.16 Influential effect of net flow Reynolds number and synthetic jet Reynolds number on the mixing degree of water mixing on single opposing LSJ configuration*
Figure 5.16(b) indicates the influence of the synthetic jet Reynolds number is significant. The synthetic jet Reynolds number represents the strength of synthetic jet pulsation, independent of incoming net flow Reynolds number. Mixing degree near 0.9 is achieved at $Re_J=128$, despite the increasing net flow Reynolds number. It is to be noted, higher $Re_J$ will be needed for the high pulsation frequencies, according to Equation 5.10.

At last, fast mixing at $Re_n=166$ is achievable at high velocity ratios ($Re>1.6$), shown in Figure 5.16(c). For example, both Figure 5.12 (g) ($f=1$Hz, $L=8$) and Figure 5.14 (c) ($f=2$Hz, $L=4$) show fast mixing at $Re =1.6$, if synthetic jet Reynolds number is sufficiently high. At such a high velocity ratio, the peak LSJ blowing velocity dominates over the net flow velocity in the mixing channel and strong rollups are universal, since the instantaneous jet blowing velocity can be 3 times higher than time-averaged blowing velocity $U_j$ (Figure 5.1). It is to be noted, net flow Reynolds numbers higher than 166 are not tested, since mixing at such Reynolds numbers can be achieved by solenoid valves on a simplified system.

### 5.4. Single staggered LSJ pair

Although the opposing LSJ pair configuration has demonstrated the capability of faster mixing enhancement via vortex rollups and sequential segmentations, the flow patterns and mixing effect of the LSJ pair whose orifices are in slightly staggered is also interesting. In the current experimental study, a single staggered LSJ pair configuration is also tested. The distance between the left and the right jet orifice centre is $1h$ (8mm), equal to the width of the mixing channel. Moreover, a longer distance between the orifices centre (two fold of the width of the mixing channel) is tested, but the results are not presented due to the insignificant improvement on mixing.

#### 5.4.1. Influence of stroke length $L$

Figure 5.17 illustrates the instantaneous concentration contours at the end of the left LSJ blowing for water mixing pulsated by staggered jet configuration. Firstly, the staggered jet configuration generates stronger disturbance at $L=1$ and 2 (Figure 5.17)
Mixing by Lateral Synthetic Jet Pairs (Ren=83,166)

than the opposing jet configuration (Figure 5.2(a) and (b)), but successful segmentation is observed at $L=3$, together with the vortex rollup structures. At $L=4$, the left LSJ blowing structure fills up the space between the LSJ orifices, and the downstream counter-clock rollup is evident in the concentration contour of Figure 5.17 (d). It seems the downstream right LSJ can penetrates deeper than the left LSJ. However, there are still some unmixed striations in the concentration contour. Furthermore, fast mixing is achieved at $L=5$ in the region between the LSJ orifices, via the giant rollup structure shown in Figure 5.17(e).

![Concentration contours and velocity fields with vorticity contours for water mixing by single staggered LSJ pair Str=0.4(f=1Hz), t=4/8T, Re*83](image)

Finally, the downstream striation of red spots disappears at $L=7$ and 8, seen in Figure 152.
Mixing by Lateral Synthetic Jet Pairs (Ren=83,166)

5.17(f) and (g). Mixing is finished immediately at the LSJ orifices. Particularly, the mixing is completed in the left LSJ cavity when the incoming streams are sucked into, hence the blowing structure from the left LSJ orifice is in green colour.

The PIV results in Figure 5.17 confirm the vortex rollup initialized by the single staggered LSJ pair. Single vortex kernel is clearly demonstrated in vector and vorticity contours in Figure 5.17 (c-g). The vortex strength grows up and the position of vortex kernel appears farther downstream as the stroke length \( L \) increases.

Instead of the double counter-rotating vortex pair found at the exit of the single opposing LSJ orifices, only one vortex kernel is found between the lower left LSJ orifice and the upper right orifice at the end of left LSJ blowing, shown in Figure 5.17. However, the shape of vortex rollup spans longer in the streamwise direction, which compensates the negative impact of single vortex rollup.

The phase sequence of instantaneous fluorescence concentration contours at \( L=3 \) and \( f=1 \) Hz is illustrated in Figure 5.18. Similarly, at \( t=0T \), the piston of the left LSJ starts to move from its left most position, but the blowing velocity magnitude is lowest as zero at the beginning of the blowing stroke for the single staggered LSJ pair configuration. Complete segmentation of LF is suggested by the red colour segment tilting between the left jet orifice and the right jet orifice (Figure 5.18 (a) and (b)). After that, the red stream is pushed out by left LSJ blowing and forms a single vortex rollup at \( y=0h \) (Figure 5.18 (e), \( t=4/8T \)). During the right LSJ blowing cycle, the rollup is distorted by the blowing from the right LSJ on the top and suction at the bottom, and the interface has been greatly stretching. Finally, the vortex structure generated by the left LSJ is destroyed before passing the right LSJ office.

On the other hand, a mushroom-shaped vortex pair is formed at the exit of right LSJ (\( y=0.5h \), after the blowing from the right LSJ (Figure 5.18 (a)). Vortex structure formed by the right jet blowing in previous cycle moves slowly downstream during left LSJ blowing (Figure 5.18 (b-d)). Noticeably, the downstream branch of vortical structure propagates downstream, while the upstream one is immediately destroyed by the suction of right LSJ. Therefore, only the rollup structure initialized by right LSJ is found downstream, instead of two vortex rollups formed by each of the opposing LSJ pair. Nevertheless, the size of rollup is bigger than that of opposing
Mixing by Lateral Synthetic Jet Pairs (Ren=83,166)

**LSJ** pair configuration.

![Sequence of instantaneous concentration contours and time-averaged velocity fields with vorticity contours for the case, L=3, Str=0.4(f=1Hz), R_v=1.2 and Re_n=83, is illustrated in Figure 5.19. This figure confirms the vortex breakup in the region between the left and the right LSJ orifices. At the beginning the left LSJ blowing (t=0T), a mushroom-shaped blowing structure at the right LSJ orifice is shown in Figure 5.19(a). When the left LSJ reaches the maximum blowing at t=2/8T, the upstream wing of rollup is destroyed by the suction at y=0.5h (Figure 5.19(c)). After the left LSJ blowing, a new vortex structure fills up the mixing channel between y=0h and 1h, seen in Figure 5.19(e). Since the rollup centre is just at the exit of right LSJ, most of the segment in red colour is broken down into pieces during the right LSJ blowing (Figure 5.19(f-h)). Therefore, better mixing is achieved at L=4 than that of L=3 shown in Figure 5.18.

The velocity fields (shown as the second row of Figure 5.19) clearly illustrate the domination of downstream vortex. The vortex rollup initially appears at the peak of the left LSJ blowing at t=2/8T, and grows up to its maximum size t=3/8T (Figure 5.19(e)). After that, the vortex kernel is pushed by the net flow to the right LSJ orifice at y=0.5 h, and the vortex is destroyed by the right jet blowing at t=5/8T.

![Figure 5.18 Sequences of concentration contours for water mixing by single staggered LSJ pair, L=3, Str=0.4(f=1Hz), R_v=1.2 and Re_n=83,](image-url)
Mixing by Lateral Synthetic Jet Pairs (Ren=83,166)

(Figure 5.19(g)). Similarly, only the vortex on the downstream corner of the right LSJ orifice grows up and propagates downstream, show in Figure 5.19(g-h). Although the vortex generated by the upper right LSJ will not be destroyed by any other lateral synthetic jet, the vorticity magnitude decays quickly.

Unlike the counter-rotating vortex pair structure for the opposite LSJ pair configuration (Figure 5.4(c)), a single but much stronger vortex rollup is found for the staggered jet configuration. The vortex spans the whole width of the mixing channel and effectively entrains the flow on both side of the mixing channel. Additionally, the strong vortex is broken down by the upper right LSJ blowing. Therefore, better mixing is achievable via the vortex rollup and 3D flow structure
accompanying with the breakup in the region between the left and the right LSJ orifices.

Figure 5.20 demonstrates better mixing effect than the opposing LSJ pair configuration at $f=2\text{Hz}$. Figure 5.20(b) shows most the LF stream in red colour remains on the left side other mixing channel at $L=1.5$, but the flow disturbance performance is comparable with $L=2$ for the opposing configuration shown in Figure 5.6(a). Furthermore, significant mixing is achieved at $L=2$, despite red striations with high Rhodamine B concentration (Figure 5.20(c)). As the stroke length increase to at $L=3$, the red striations are hardly found in Figure 5.20(d). Finally, no red striation but homogenous green in the mixing channel is found in Figure 5.20(e), which indicates good mixing at $L=4$, $R_v= 4.0$, $Re_J=160$. Although segmentation is not as evident as the mixing by single opposing LSJ pair at $f=2\text{Hz}$, mixing is achieved by the intensive vortex rollup near the LSJ orifices.

Figure 5.20 Instantaneous concentration contours for water mixing by single staggered LSJ pair, $St=0.8(f=2\text{Hz})$, $Re_w=83$

Figure 5.21 shown the sequence of time-averaged velocity vector fields and vorticity contours for the staggered LSJ pair, $L=4$, $f=2\text{Hz}$. Compared with the
counter-rotating vortices pair structure generated by the opposite LSJ pair at the exit of the right LSJ (Figure 5.7, $L=4$, $f=2$Hz), staggered configuration initializes the asymmetrical vortices pair structure at lower left LSJ orifice. Although vortex rollup pair is observed at the end of the left LSJ blowing, but the vortex kernel between the orifices ($y=-0.5h$ and $y=0.5h$) is stronger. Moreover, the size of the vortex rollup between the orifices is comparable with the width of the mixing channel, which means intensive blending in this region. In addition, the vortex rollup between the two LSJ orifices is broken by the right LSJ, during its blowing cycle. Therefore, intensive mixing is achieved by the 3D flow companied with vortex breakdown.

On the other hand, the vortices pair structure on the upper right LSJ orifice seems symmetrical. The upstream counter-clockwise vortex kernel, coincident with the vortex rollup generated by the lower left LSJ, contributes to the formation of strong vortex in the region between the two jet orifices.

![Figure 5.21 Sequence of time-averaged velocity vector fields with vorticity contours for water mixing by single staggered LSJ pair, $L=4$, $Str=0.8(f=2$Hz), $R_v=3.2$, $Re_n=83$](image)

**5.4.2. Influence of actuation frequency ($L=2$)**

Figure 5.22 suggests fast mixing is achievable at high pulsation frequency for the fixed stroke length $L=2$, similar with the single opposing LSJ pair configuration. Firstly, periodic pattern but no significant mixing is clearly demonstrated in Figure 5.22(a) at $f=1$Hz. After that, there is a sudden change of the mixing degree from at
$f=2$ Hz. The left stream in red colour can reach the right side of the channel (Figure 5.22(b)). The mixing is further improved at $f=4$ Hz, but fully mixing is not reached at the end of the observation region at $y=7h$ (Figure 5.22(d)). Finally, fast mixing at $f=6$Hz is shown in Figure 5.22(e).

![Instantaneous Concentration contours for water mixing by single staggered LSJ pair at different pulsation frequency, $L=2$ and $Re_n=83$](image)

(a) $f=1$ Hz, (b) $f=2$ Hz, (c) $f=3$ Hz, (d) $f=4$ Hz, (e) $f=6$ Hz,

$Re_n=0.8 \quad R_e=1.6 \quad R_e=2.4 \quad R_e=3.2 \quad R_e=4.8$

**Figure 5.22 Instantaneous Concentration contours for water mixing by single staggered LSJ pair at different pulsation frequency, $L=2$ and $Re_n=83$**

Furthermore, the velocity fields for water mixing by single staggered LSJ pair at $L=2$, $f=4$Hz and $Re_n=83$(Figure 5.23) confirms that a counter-rotating vortex pair can be observed at the both exits of the orifices, despite the slightly staggered configuration. However, the size of the vortex rollups is smaller than the half width of the mixing channel, which means the entraining effect of these rollups is limited. Moreover, the left LSJ blowing does not form a complete segmentation. Therefore, moderate mixing in the centre of the mixing channel with the ill-mixed near wall region is shown in the concentration contour (Figure 5.22 (d)).
5.4.3. Quantitative comparison to the opposing jet configuration

Figure 5.24(a) suggests the staggered LSJ pair configuration is preferred at small stroke lengths $L<4$ for water mixing pulsed by single LSJ pair at $Re_n=83$. The staggered configuration is inclined to generate stronger vortex rollup between the staggered jet orifices. Similarly, Figure 5.24(a) shows the staggered jet pair is slightly better than the opposing jet configuration at $f=2$ Hz. For example, fast mixing can be achieved at $L=3$ on the staggered configuration, while the mixing degree is about 0.6 by the opposing jet configuration. However, as the stroke length increases at $f=1$Hz and 2Hz, the advantage of the staggered configuration is not significant any longer, seen in Figure 5.24(a). Similar stroke length is needed to achieve a mixing degree higher than 0.9. For instance, $L>4$ at $f=2$ Hz and $L>6$ at $f=1$Hz are necessary for both the opposing and staggered configuration to obtain good mixing. Furthermore, Figure 5.24(a) suggests synthetic jet Reynolds number is more influential. Given the sufficient synthetic jet Reynolds number, a mixing degree higher than 0.9 is universal for both the opposing and staggered configuration.
In summary, stronger vortex rollups and breakup between the left and the right orifices of the single staggered LSJ pair are found at $Re_n=83$, despite the different patterns of velocity fields. Although the staggered configuration outperforms the opposite configuration slightly at low actuation frequencies such that a lower stroke length is required to achieve the same mixing degree, the performance of the opposite configuration catches up at high actuation frequencies. Therefore the demand on the actuation intensity and frequencies becomes similar for both configurations as far as achieving a mixing degree near 0.9 is concerned.

5.5. Multiple opposing LSJ Pairs

Single staggered LSJ has demonstrated the mixing enhancement via the interaction between the upper right LSJ blowing and flow structure generated by the lower left LSJ. Faster mixing is expected by chaining up multiple LSJ pairs along the mixing channel. In the current experiments, three LSJ pairs been arranged along the mixing channel with either opposite or staggered configuration. The distance between the jet orifices on one side of the mixing channel is selected as $2h$ (16mm), which is two fold of the staggered distance of the staggered LSJ pair. Moreover, the results for mixing by three staggered jet pairs are available, but they are not presented here.

5.5.1. Influence of stroke length $L$

PIV results, shown in Figure 5.25, suggests similar vortex pair structures for each
LSJ at the end of left LSJ blowing and the vorticity strength increases with the stroke length. Similar with the single opposite LSJ pair, the downstream vortex kernel is stronger than the upstream one. Moreover, the vorticity generated by uppermost jet is no stronger than the other two LSJ pair, except for the case $L=8$ shown in Figure 5.25(f).

However, concentration contours in Figure 5.26 evidently demonstrate the mixing enhancement by chaining up three LSJ pairs over the single LSJ pair. Although the PIV results suggest similar flow pattern for each LSJ pairs along the mixing channel, better segmentation structure is generated by the stronger disturbance of the uppermost LSJ pair, shown in Figure 5.26(b). Comparing with cases with single LSJ pair, chaining up of the LSJ pair has magnified the segmentation capability pair-by-pair at $L=2$. The first LSJ pair just curves the left and the right stream interface, whereas the second LSJ pair augments the interface curvature to touch the wall on the right side of the mixing channel. At last, complete segmentation is formed at the uppermost jet orifice at the end of left jets blowing ($t=4/8T$).

![Figure 5.25 Velocity fields with vorticity contours for water mixing by three opposing LSJ pairs, $t=4/8T$, $f=1$Hz, $Str=0.4$ and $Re_n=83$](image)
Mixing by Lateral Synthetic Jet Pairs (Ren=83,166)

Figure 5.26 Concentration contours for three opposing LSJ pairs, t=4/8T, f=1Hz, Str=0.4, Ren=83

Although the successful segmentation is evident at \( L=3 \), shown in Figure 5.26 (d), mixing is not completed within the observation region, due to the non-optimum distance between the pairs and weak vortex rollups. The segment in red colour generated by the lowest \( \text{LSJ} \) pair in the previous blowing cycle just moves to the middle \( \text{LSJ} \) pair downstream, when the left \( \text{LSJ} \) enters the suction cycle. The middle left \( \text{LSJ} \) mainly sucks the red stream and blows it out later. Therefore, the middle \( \text{LSJ} \) pair contributes little to the mixing enhancement.

For \( L=4 \), the partial mixing is achieved by stronger vortex rollup entraining the surrounding stream. Furthermore, the rollup structure formed by the lowest \( \text{LSJ} \) pair, is sucked into and blown from the middle left \( \text{LSJ} \) orifice (Figure 5.26(d)). Thereby, the substantial mixing occurs in the mixing channel and LSJ cavity. Similar process repeats for the middle and uppermost \( \text{LSJ} \) pairs. In addition, further mixing is augmented by the shearing effect of the blowing and sucking by the \( \text{LSJ} \) pair, and it is more effective than the purely sequential segmentation shearing and stretching.

Finally, Figure 5.26 (e) shows fast mixing is achieved at \( L=5 \); whereas similar mixing effect is achievable only at \( L=8 \) for single opposite \( \text{LSJ} \) pair. Therefore, it can be
concluded that multiple LSJ pairs can reduce the requirement of stroke length for satisfying laminar mixing at $f=1\, \text{Hz}$.

The phase sequence of concentration contours in Figure 5.27 demonstrates the mixing process via the cooperation of three opposing LSJ pairs. Vortex rollups are found for both the right and the left LSJ blowing at $t=0/8\, \text{T}$ and $t=4/8\, \text{T}$ respectively, outside the jet orifices of the lowest LSJ pair. The downstream wing of the vortex pair is stronger than the upstream one, without suppressing from the net flow velocity. This asymmetric vortex structure, similar with the case of single LSJ pair (PLIF result of Figure 5.4(c)), is confirmed by the PIV results (Figure 5.29 (a) and (e)).

Furthermore, the vortex rollups are interwoven in the mixing channel and move downstream as a joint effect of net flow velocity and blowing from the lowest LSJ pair. When the vortex rollups move to $y=-1\, \text{h}$, they are deformed by the blowing of the middle LSJ pair. In particular, the vortex rollup in red colour formed by the lowest left LSJ blowing is broken down by the suction of the middle left LSJ. Part of the red stream is sucked into left LSJ cavity; hence the blowing structure from the middle LSJ pair is in green and yellow colour. As a result, the blowing structure is not as clear as that of lowest LSJ pair.

Finally, the rollup structure from the middle LSJ pair undergoes the similar process of deformation and breakup by the uppermost LSJ pair, thus mixing improves gradually. After $y=3\, \text{h}$, the mixing mainly relies on the segment stretching. Fortunately, most of part of the mixing channel downstream is filled with green colour, indicating the significant mixing by the three LSJ pair. Besides, a strip in red colour is blown out from the uppermost left LSJ, which is originated from LSJ cavity.

The velocity fields of water mixing by three opposing LSJ pairs indicate similar flow pattern for each LSJ, shown in Figure 5.28. The blowing velocity magnitude for each LSJ is almost equal, and the vortex structure at the end of left LSJ blowing is similar with single LSJ pair pulsation (PIV result of Figure 5.4(c)). Due to the symmetrical jet configuration, velocity fields for the right LSJ blowing cycle is not presented.
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Figure 5.27 Sequences of instantaneous concentration contours for water mixing by three opposing LSJ pairs, $L=4, f=1$Hz, $Str=0.4$, $Re_{e}=1.6$, $Re_{n}=83$

Figure 5.28 Sequences of time-averaged velocity vector with vorticity contours for water mixing by three opposing LSJ pairs, $L=4, f=1$Hz, $Str=0.4$, $Re_{e}=1.6$, $Re_{n}=83$

Figure 5.29 (a-c) suggests multiple LSJ pairs do not outperform the single LSJ pair, due to the small stroke length ($L \leq 2$). Whereas, Figure 5.29 (d) shows distinct flow
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structures at $L=2.5$. The interaction between the orifice and the wavy stream interface generated by the upstream LSJ pairs enables the red stream touching the right wall of the mixing channel at $y=-1h$. Furthermore, the uppermost LSJ pair stretches some red stream slices, which lead to partial mixing in the mixing channel. Therefore, substantial mixing is achieved through by the segmentation at $L=2.5$.

Figure 5.29(e) shows mixing is improved suddenly at $L=3$, via the stronger interaction of blowing structure generated by the lowermost LSJ with middle LSJ blowing. The blowing out from the uppermost jet at $y=1.5h$ is green, which suggests the fast mixing happens inside the cavity. It is to be noted, the red or green stream blowing out from uppermost right jet is already dwell inside of the cavity. For higher dimensionless stroke length of $L=4$, the mixing is completed by the single lowermost LSJ pair.

![Figure 5.29 Concentration contours for water mixing by three opposing LSJ pairs with different $L$ at $f=2$Hz, $Str=0.8$ and $Re_J=83$](image)

Similar with water mixing by single opposing LSJ pair, Figure 5.30 indicates each LSJ pair generates similar vortex pair structures at the end of the left LSJ blowing, but the uppermost jet is slightly stronger than the upstream pairs. Weak vortex rollups are found at the both corners of the jet orifice, seen in Figure 5.30 (b) $L=3$, $Re_J=96$. 

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Although the rollups do not fill up the whole width of the channel, the mixing has been completed at $L=3$ (Figure 5.29(e)). This indicates that the fast mixing is not only achieved through vortex rollups, but also the high-speeded reciprocal blowing and suction contribute to the mixing.

![Mixing by Lateral Synthetic Jet Pairs](image)

**Figure 5.30 Sequences of velocity vector and vorticity contours for water mixing by three opposing LSJ pairs, $f=2$ Hz, $Str=0.8$, $t=4/8T$, $Re_n=83$**

Figure 5.31 suggests enlarging stroke length is more effective than increasing pulsation frequency for the given velocity ratio. Although the velocity ratio of the synthetic jet mean velocity to the net flow spatial averaged velocity is fixed at $R_v=2.4$, the flow patterns and mixing effect are different. Fast mixing can be achieved by the lowermost LSJ pair at $f=1$ Hz, $L=6$, shown in Figure 5.31(a). Whereas, no substantial mixing is observable at $f=6$ Hz and $L=1.0$ in Figure 5.31(f). The stroke length is too small that the blowing from the LSJ orifice can not cross the central line of the mixing channel. Suckling and blowing is limited to the same stream on one side as the stroke length reduces; the mixing thus deteriorates. As a compromise, partial mixing is achieved by the reciprocal movement of the red and blue stream interface in the mixing channel (Figure 5.31 (c) $f=3$ Hz, $L=2$).
5.5.2. Influence of actuation frequency ($L=2$)

No significant mixing is found at low pulsation frequencies ($f<3\text{Hz}$). Figure 5.32(a) show the complete segmentation without vortex rollup at $f=1\text{Hz}$, whereas no segments are formed at $f=2\text{Hz}$, seen in Figure 5.32(b). The failure of segmentation is caused by reduced jet penetration in shorter blowing cycle at $f=2\text{Hz}$. Besides, the vortex kernels buried under the liquids interface are not strong enough to entrain liquid on the opposite side of the channel. Therefore, the mixing deteriorates at $f=2\text{Hz}$, compared with the case with $f=1\text{Hz}$ at fixed stroke length of 2.

As the pulsation frequency increases, the influence of LSJ pulsation on the velocity fields at the end of blowing is indicated by the growing size of vortex kernels in Figure 5.33. Then, at $f=3\text{Hz}$, substantial mixing is achieved by the partial segmentation, shown in Figure 5.32(c). Although the size of vortex kernels is too small to entrain flow on the other side of the mixing channel, shown in Figure 5.33(c), the PLIF result suggests good mixing at $f=4\text{Hz}$ in concentration contour (Figure 5.32(d)). The blowing structure can not be distinguished for the downstream LSJ pairs.
Mixing by Lateral Synthetic Jet Pairs (Ren=83,166)

Figure 5.32 Instantaneous concentration contours for water mixing by three opposing LSJ pairs with different pulsation frequencies, $L=2$ and $Re_n=83$

Finally, fast mixing is obtained by the lowest LSJ pair at $f=6$Hz (Figure 5.32(e)). The vortex rollup is obviously showed at the end of left LSJ blowing and the green and
yellow rollup structure suggests significant mixing inside the mixing channel. Further studies have shown that a dimensionless stroke length \( L > 2 \) is necessary for the successful segmentation by opposing LSJ pairs. This criterion is derived from the geometrical ratio of the width of the mixing channel and width of orifice, which is fixed at 2 in the current experimental setup. Due to the definition of the dimensionless stroke length of synaptic jet (see Equation 5.6), \( L \) is the ideal penetration depth of the synthetic jet blowing without vortex rollup. However, a bigger stroke length is necessary at the higher pulsation frequency, due to the tendency of stronger vortex rollup at higher Stokes numbers. Part of the blowing velocity energy transfers to vortex rollups (Crook, 2002), hence the jet penetration depth is reduced at the presence of vortex rollups.

For example, segmentation is achieved at \( L = 2.5 \) and \( f = 2 \) Hz. Nevertheless, fast mixing can be achieved at high velocity ratio (Figure 5.32(e)), at the fixed stroke length \( L = 2 \).

In addition to the near orifice region in the mixing channel, substantial mixing happens in the synthetic jet cavities. Numerical studies on the synthetic jet has suggested the existence of the counter-rotating vortex pairs inside the cavities (Zhou et al., 2009), with similar vorticity strength outside the synthetic jet cavity. The vortex rollup can effectively mix inside the cavity, if both liquids to be mixed are sucked into the cavity. Moreover, the shearing effect of high frequency blowing and suction can augment the mixing, if the contribution of vortex rollup is limited at the small stroke length of 2. Therefore, the existence of synthetic jet cavity is potentially helpful to mixing enhancement.

### 5.5.3. Quantification of water mixing by multiple LSJ pairs

Figure 5.34 shows the variations of time-averaged mixing degree with changing dimensionless stroke length for a range of actuation frequencies tested on the single LSJ pair configuration. The trend is similar with water mixing by single LSJ that at a fixed actuation frequency a higher \( L \) will result in a better mixing and as the actuation frequency increases a lower \( L \) is required to achieve a given mixing degree. It also shows that a mixing degree greater than 0.9 can be achieved when \( L \) is sufficiently high for the tested frequency range from 1 Hz to 8 Hz. Furthermore, the requirement of dimensionless stroke length \( L \) for mixing degree greater than 0.9 is reduced using
three opposing LSJ pair.

![Figure 5.34 Variations of mixing degree with $L$ and $f$ for three opposing LSJ pairs](image)

![Figure 5.35 Comparison of mixing effect of single and three opposite LSJ pairs](image)

Similarly, a correlation for the synthetic jet operating condition at which a good mixing is guaranteed is fit between the mixing degree and the dimensionless stroke length $L$ at each frequency. The procedure of the fitting is identical to that of water mixing by the single opposing LSJ pair. The relationship is found to be $fL^{3.42} = 64.2$, shown as black line in Figure 5.35. When the frequency is replaced by the Strouhal number, this relationship becomes:

$$Str \ L^{3.42} = 25.7 \quad (5.12)$$

Compare with water mixing by single opposing LSJ pair, $fL^{1.82} = 27.5$ (shown as red
line in Figure 5.35), the minimum stroke length needed for fast mixing is reduced slightly. However, more energy consumption is needed for bigger piston displacement at the identical pulsation frequency. Using the single LSJ pair is preferred for the watering mixing, which relies mainly on the vortex rollup.

The advantage of multiple LSJ pair is not observed at higher pulsation frequency, since a small stroke length is needed for mixing degree higher than 0.9. The mixing degree is similar for both single and three opposing LSJ pairs, if the pulsation frequency is higher than 5Hz at the low stroke length of 2. Previous study on single opposing LSJ pair shows the mixing is completed immediately outside the jet orifices without visible segmentation and vortex structures downstream. Therefore, two fitting lines in Figure 5.35 are converging at high pulsation frequencies, since mixing is completed mainly by the lowest LSJ pair.

Indeed, the multiple LSJ pair mixer resembles the chaotic mixer with lateral jet pulsation (Glasgow and Aubry, 2003), if there are more LSJ pairs along the mixing channel. However, the existence of the synthetic jet cavities distinguishes the LSJ pair mixer from this 2D unsteady chaotic system. Instead of initiating swirling in the mixing channel, lateral synthetic jet sucks the incoming liquids into the cavities and blows out later. Thereby, the cavities are not only part of the pulsation actuator, but also a place for intensive mixing. Moreover, mixing is completed just near the LSJ orifices via vortex rollup, as well as segmentation stretching and folding. For example, water mixing by three opposing LSJ pair is completed by the most upstream LSJ pair. On the contrary, a long distance is required for significant mixing by chaotic system, which violates the objective of a short mixing length. Therefore, the current experimental result is not interpreted in the terminology of chaotic advection, since the LSJ pair mixer is not designed as a pure chaotic mixer.

Moreover, this experiment suggests that in order to maximise the effect of vortex entrainment and ensure successful segmentation the ratio between the width of the mixing channel and the width of LSJ orifice should be selected less than, say 4. As such, the vortex kernel can occupy a substantial portion of the mixing channel and to ensure that the flow structures produced by the synthetic jets can either penetrate the width of the mixing channel to be sucked back into the cavity.
5.6. Summary

In this chapter, the effect of lateral synthetic jet pairs on the mixing between two water streams of the same flow rate in a planar mixing channel at the net flow Reynolds numbers of 83 and 166 is examined in detail using PLIF and PIV. The synthetic jet pair is operated 180° out-of-phase and at a range of piston actuation frequencies and displacements.

The extent of mixing is evaluated using PLIF data at a distance of 12.5h from the centre of the jet cavity. It is found that at a fixed actuation frequency a higher $L$ results in a better mixing and as the actuation frequency increases a lower $L$ is required to achieve a given mixing degree. At a sufficiently high frequency or dimensionless stroke length, a nearly homogenous mixing with a mixing degree of greater than 0.9 can be obtained. A functional relationship between $f$ and $L$ is also obtained by best fitting the experimental data, which can guide the selection of the synthetic jet operating conditions to ensure a good mixing. The relationship is found to be $Str \cdot L^{1.82}=11.0$ at $Re_n=83$.

According to this relationship, a high velocity ratio is required for good mixing by single opposing LSJ pair. The velocity ratio, which represents the relative strength of the synthetic jet pulsation velocity to the net flow velocity, is recognized as an influential parameter ($R_v \propto Re_j \propto fL$). Besides, this relationship suggests that increasing the actuation amplitude is more effective than improving the actuation frequency.

In addition, it is found the dimensionless stroke number $L>2$ is prerequisite for the successful segmentation by single opposing LSJ pair. This criterion derives from the geometrical ratio of the width of the mixing channel and the width of LSJ orifice, which is fixed at 2 in the current experimental setup. However, a stroke length bigger than 2 is necessary for segmentation, at the presence of stronger vortex rollup, since the jet penetration depth is reduced due to the part of the energy transfer from blowing velocity to vortex rollups (Crook, 2002).

Moreover, Both PLIF and PIV results show that each synthetic jet produces vortex pairs which become stronger as the actuation frequency or amplitude increases; these
vortices play an important role in prompting mixing between the two fluid streams. The good mixing obtained at a high frequency or a high dimensionless stroke length is caused by a strong interaction between the vortices produced by the lateral synthetic jet pair, thereby the vortex pair which is formed on one side of the mixing channel is destroyed during the formation process of the vortex pair from the opposite side, resulting in the breakdown of the initial 2D flow into a 3D flow hence a greatly enhanced mixing.

Furthermore, the influence of the increasing net flow Reynolds number is investigated at $\text{Re}_n=166$. Experiments suggest the higher net flow Reynolds number has negative impact on the mixing degree, due to the reduced dwelling time and Strouhal number at the identical LSJ pulsation parameters. However, fast mixing is achievable at high velocity ratio ($R_v>1.6$), since the peak LSJ blowing dominates over the net flow velocity in the mixing channel and strong rollup structure is universal at such high synthetic jet actuation parameters. Moreover, it is found Strouhal number determines the distance between the consecutive structures for the opposing LSJ pair and $\text{Str} \geq 0.4$ is necessary to ensure sufficient interaction between vortex structures produced by the opposite jet orifices and their subsequent breakup.

In addition, stronger vortex rollups and breakup in the region between the left and the right orifices of the single staggered LSJ pair are found at $\text{Re}_n=83$, due to the different patterns of velocity fields. Although the staggered configuration outperforms the opposite configuration slightly at low actuation frequencies such that a lower stroke length is required to achieve the same mixing degree, the performance of the opposite configuration catches up at high actuation frequencies. Therefore the demand on the actuation intensity and frequencies becomes similar for both configurations as far as achieving a mixing degree near 0.9 is concerned.

Finally, experiments shows chaining up three opposing LSJ pairs is more effective than the single LSJ pair for the identical actuation parameters, but it is not as energy-effective as the single LSJ pair configuration. Similarly, the relationship of actuation frequency and stroke length to achieve good mixing is found to be $\text{Str} \cdot L^{3.42} = 25.7$ at $\text{Re}_n=83$. However, chaining up three LSJ pairs does not significantly improved mixing at such Reynolds number, since the mixing effect is mainly depends on the vorticity strength generated by the first LSJ pair.
Chapter 6. Mixing by Lateral Synthetic Jet Pairs (Reₙ=2~10)

In this chapter, the liquids mixing mechanisms of lateral synthetic jet pairs at low net flow Reynolds numbers are experimentally investigated in sugar solutions via PIV and PLIF techniques. Mixing by single and multiple opposing LSJ pairs is studied in sugar solutions, and the mixing effect of both three opposite and staggered LSJ pairs is compared at low net flow Reynolds number of 2. Finally, the mixing of sugar solutions with a different viscosity are studied, to confirm the mixing enhancement capability of LSJ pairs at a wide range of net flow Reynolds numbers.

6.1. Test conditions

In the previous chapter, single and multiple LSJ pairs have been proved to be effective for fast water mixing at Reₙ=83. In order to evaluate the mixing enhancement capability of LSJ pairs at lower Reynolds numbers, more viscous liquids are required for the fixed geometry of mixing channel and LSJ cavity in the current experimental setup. Therefore, sugar solutions, whose dynamic viscosities can be adjusted via variation of mass fraction, are utilized as the working liquids. The LSJ actuation parameters for the mixing of sugar solutions are similar with those of water mixing by LSJ pair. The peak-to-peak displacement of the pistons varies from 0.1 to 1.5 mm, corresponding to the dimensionless Stroke length from 1 to 12 for a single LSJ pair. Noticeably, the tested maximum dimensionless Stroke length is higher than that of water mixing, in order to generate stronger pulsation in the more viscous sugar solution. On the other hand, the actuation frequency of LSJ varies independently from 1 to 8 Hz respectively, similar with water mixing. Moreover, the net flow rate in the mixing channel is kept identical with water mixing at Reₙ=83. Therefore, the net flow Reynolds number is 2 in the sugar solution with dynamic viscosity \( \mu=5.0\times10^{-2} \) Pa·s, and the corresponding synthetic jet Reynolds number varies from 1.0 to 23, and the Strouhal number from 0.2 to 1.6.
Similarly, the current experiments of mixing of sugar solutions by the \textbf{LSJ} pairs focus on the opposing \textbf{LSJ} pairs configuration at $\text{Re}_n=2$. The staggered configuration is tested with the similar \textbf{LSJ} actuation parameters, but only part of the result is presented and compared with the opposing jet configuration. Finally, mixing of the sugar solution with different dynamic viscosity $\mu=1.0\times10^{-2}$ Pa·s is studied, in order to confirm the mixing enhancement capability of the \textbf{LSJ} pairs at a wide range of net flow Reynolds numbers. Likewise, only the cases with opposing \textbf{LSJ} pairs configuration at $\text{Re}_n=10$ is reported here.

6.2. Single LSJ pair mixing ($\text{Re}_n=2$)

The blowing structure of single opposing LSP is briefly investigated at low Reynolds numbers, before moving to the study of three \textbf{LSJ} pairs. The segmentation capability and mixing effect of the single \textbf{LSJ} pair is evaluated by the concentration contours. Basic knowledge of the blowing structure of single LSJ pair helps to understand the interaction of blowing structures with multiple \textbf{LSJ} pairs.

6.2.1. Single opposing LSJ pair

Figure 6.1 shows the interwoven segments in the sequence of tracer concentration contours, for the case of sugar solution mixed by single opposing \textbf{LSJ} pair at $f=1$Hz and $L=6$. When the left \textbf{LSJ} blowing reaches its maximum blowing velocity at $t=1/4T$, the upstream \textbf{LF} in red colour has been directed into the right \textbf{LSJ} cavity, see structure B1. At the same time, a small red spot appears at the left \textbf{LSJ} orifice, shown as segment A1 in Figure 6.1(a), as a result of left \textbf{LSJ} blowing.

At the end of the left \textbf{LSJ} blowing, the structure A1 form left jet orifice grows into A2 and wedges into the blue segment, shown in Figure 6.1(b). There are green strips inside the segment A2 pushed out from the left cavity, which suggests partial mixed in the left \textbf{LSJ} cavity (Figure 6.1(b)). Moreover, the structure B2, which horizontally bridges the left and the right jet orifices, suggests that the lateral flow movement is much stronger than the flow movement along the mixing channel.

Figure 6.1(c) shows the structure B3 is segmented from the incoming \textbf{LF} in red colour, since the upstream \textbf{RF} in blue is sucked into the left cavity during the left
LSJ suction stroke. Furthermore, the segment B3 is twisted at the bottom, which is caused by the strong lateral pushing from the right LSJ. Meanwhile, the tiny green spot at the right LSJ orifice indicates the right LSJ is pushing out the previously sucked red stream from $t=2/4T$ to $t=3/4T$. Particularly, if the blue and red colour are swapped and mirrored by Y axis, the flow patterns on contours at $t=1/4T$ matches with that of $t=3/4T$, due to the out-of-phase pulsation on the opposing LSJ pair configuration.

Figure 6.1 Sequence of instantaneous concentration contours for sugar solution mixed by single opposing LSJ pair, $Re_n=2, f=1Hz, L=6, Re_J=2.0$

At the end of left LSJ suction at $t=4/4T$, the incoming RF in blue is sucked into the left LSJ cavity; while another oval blue segment is pushed out from right LSJ orifice. The oval blue segment wedges into the LF segment in red, but it fails to split the red segment, see Figure 6.1(d). Similarly, the contours at $t=4/4T$ and $t=2/4T$ have similar flow patterns, except that the structure A2 does not shows as oval shape. Meanwhile, segment A3 is stretched thinly but not twisted as it moves downstream slowly, which suggests no obvious X-axis velocity gradient but a Y-axis velocity gradient cross the mixing channel away from the LSJ orifice.

The sugar solution mixing by single opposing LSJ pair at $f=1Hz$ and $L=6$ relies on the segmentation shearing and thinning, due to the absence of strong vortex rollup
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

structures at low Reynolds number of $Re_n=2$. Only crescent shape interwoven segments are observed at downstream. Thus, moderate mixing is observed at the end of observation region in Figure 6.1. In contrast, water can be well-mixed immediately after the single LSJ jet exit (Figure 5.4), for the identical LSJ pulsation parameters of $L=6$ and $f=1$Hz. Therefore, strong vortex rollup is confirmed as the main contributor for mixing enhancement by single LSJ pair.

For the viscous sugar solution, pulsation by single opposing LSJ pair can not provide adequate disturbance for fast laminar mixing at small stroke length, suggested by Figure 6.2(a) and (b). Although successful segmentation is indicated by the crescent segment shown in Figure 6.2(a) $L=3$, $Re_f=1.2$ and $Re_J=1.0$, the crescent-shaped segment is distorted at the front and the bottom right part. Increasing the dimensionless stroke length $L$ to 6, the crescent segment has been further wedged into 2 parts (Figure 6.2(b)). This increases the interface between the red fluid stream and blue fluid stream. However, without the strong vortex rollup, the increasing interfaces due to downstream stretching is not sufficient to achieved fast laminar mixing.

Nevertheless, a dimensionless stroke length $L$ more than 9 can sufficiently disturbs the flow, shown in Figure 6.2(c). The upstream liquids are sucked into the LSJ cavity and pushed out due to the big stroke length, and the intense shearing and deforming contribute the mixing greatly. Both the reciprocal movements between the orifices and parabolic velocity profile downstream increase the interfacial areas. Therefore, substantial mixing is achieved by the blowing and suction of opposing LSJ pair near the jet orifices, indicated by the large areas of green colour in the mixing channel. However, completed mixing is not available at the end of the observation region $y=7h$. This suggests that pure segmentation stretching is not as effective as vortex rollups.

Mixing is improved but not completed at $L=12$, shown in Figure 6.2(d). Nevertheless, the shaker can not work at a dimensionless stroke length higher than $L>12$ with a stable sine wave piston movement. Therefore, full mixing is not achieved in sugar solution by the single lateral synthetic jet pair in the tested stroke length range at $f=1$Hz.
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

To be noted, the quality of concentration contours is not as high as that in water mixing experiments done with brand-new reflection mirror slot. The quality of reflected laser sheet depends on the status of flat mirror inserted into the channel, but the mirror may be eroded or contaminated by water or sugar solution in the mixing channel. There are some vertical strips on the contour in the upstream region (marked out in Figure 6.2 (a)), where it should be homogenous red colour. These strips are caused by the slight inhomogeneity of the laser beam or the surface condition of the reflection mirror. Furthermore, the temporal and spatial energy fluctuation of the laser beam can move the trips time by time, which makes it impossible for a satisfied calibration to get rid of the unsteady laser intensity variation. Nevertheless, the concentration contour obtained from PLIF experiment is satisfying for the qualitative evaluation of the segment structure in the mixing channel. Fortunately, the downstream PLIF experimental data used by the quantitative analysis are as good as
water mixing, since the laser sheet reaches the mixing channel without the assistance of the reflection mirror.

Figure 6.3 shows crescent-shaped segments is generated by single opposing LSJ pair at $f=2\text{Hz}$, $Str=0.8$, $L=6$, $Re_n=2$. Firstly, the green stream pushed out from the left LSJ cavity indicates partial mixing within the cavity, when the left LSJ reaches the peak blowing velocity at $t=1/4T$. At the end of the left LSJ blowing, the LF in red colour takes up the space between the orifices. Meanwhile, the blue segment transforms into triangle shape which is slightly leans to the left side of the mixing channel (Figure 6.3(b)). Then, the asymmetry of blue segment is obvious in Figure 6.3(c), and the triangular shape deformed into “Λ” shape after the blowing of right LSJ. Finally, at the end of right LSJ pulsation at $t=4/4T$, the RF in blue takes up the space between the opposing jet orifices with a red triangle segment on the top.
Figure 6.4 Instantaneous concentration contours for sugar solution mixed by single opposing LSJ pair at $t=T/2, f=2\text{Hz}, Str=0.8, Re_{a}=2$

Figure 6.4 shows improved mixing by the increasing stroke length at $f=2\text{Hz}$. First of all, a similar crescent-shaped segmentation is formed at $L=3$ and $L=6$ at $f=2\text{Hz}$, illustrated in Figure 6.4(a) and (b). The red stream concentrates on the left side of the mixing channel at $L=3$. As the stroke length increases, the LF in red colour concentrates on the right side, while the RF in blue is swapped to the left side of the mixing channel (Figure 6.4(b)), as result of asymmetric segmentation. On the other hand, no clear crescent-shaped segments are observed at bigger stroke length, since the blowing structure from left LSJ is in green colour, shown in Figure 6.4(c). The higher blowing velocity leads to partial mixing via interface shearing and stretching. Furthermore, Significant mixing is achieved at $L=12$ and $f=2\text{Hz}$, but there are still sequential isles of unmixed red stream downstream (Figure 6.4 (d)), which can not be mixed efficiently by the flow stretching downstream.
6.3. Three opposing LSJ pairs (Reₙ=2)

Experiments on the mixing by single opposing LSJ pair has suggested that the extent of the mixing is largely determined by the initial disturbance near the jet orifice at low Reynolds numbers. Hence, multiple LSJ pairs, which create disturbances at different downstream locations of the mixing channel, would be more effective for mixing without intense vortex rollup.

6.3.1. Influence of stroke length (f=1Hz)

Figure 6.5 shows the sequence of velocity fields for sugar solution mixed by three opposing LSJ pairs at f=1Hz, L=4 and Reₙ=2. Only half of the pulsation cycle is presented, since velocity patterns is Y-axis symmetrical due the geometric symmetry of the opposing LSJ. First of all, three LSJ pairs generate the similar vector pattern near jet orifices, except that the uppermost LSJ blowing velocity is stronger that the jets below. Secondly, the LSJ can only affect the velocity field within 1h near the orifice, while all the other regions remain as undisturbed. In other word, the LSJ blowing has little influence on the region away from the LSJ orifices, and this is indicated by the stable velocity vectors parallel with Y axis in Figure 6.5. Especially, the velocity pattern at the end of blowing t=4/8T is similar with the parabolic velocity profile in the fully developed 2D channel flow, which means the pulsation effect diminishes immediately after the LSJ blowing. In contrast, the vortex rollups reach the maximum diameter for water mixing at higher Reynolds numbers.
Mixing by Lateral Synthetic Jet Pairs ($Ren=2\text{~to~}10$)

Figure 6.5 Sequence of velocity fields with vorticity contours for sugar solution mixing by three opposing LSJ pairs, $f=1\text{Hz} (S=1.6, Str=0.4)$, $L=4$, $R_e=1.6$, $Re_e=2$.

Notably, multiple lateral synthetic jets pair configuration may bypass part of the incoming fluid via the LSJ cavities. Figure 6.6(a) demonstrates the upstream flow branches at the lowest LSJ into left and right lateral synthetic jet cavities, then rejoins at the uppermost jet orifices. The bypass phenomenon is not significant for water flow passing the LSJ region without pulsation, but PIV results suggest the bypass effect is obvious in sugar solution. Figure 6.6(b) shows the velocity fields of incoming sugar solution passing the mixing channel between three opposing LSJ pairs without pulsation. The velocity magnitude drops obviously at the lowest synthetic jet orifice ($y=-2.5h$), which indicates part of the sugar solution flows into the LSJ cavities via the lowest LSJ orifices. Whereas, there is no observable flow rate changing at the middle LSJ pair orifices at $y=-0.5h$. Finally, the velocity magnitude in the mixing channel rises after the uppermost LSJ pair at $y=1.5h$, marked by the vector colour transiting form green to red in Figure 6.6(b). This indicates the bypass stream rejoins the mainstream at the uppermost LSJ orifice exits, due to the mass
Moreover, the bypass effect of multiple LSJ cavities is evidently indicated by the curve of red stream interface at the lowest LSJ orifices, shown as the baseline case \((L=0)\) in Figure 6.6(c). LF and RF streams pass the LSJ pair in parallel without noticeable mixing, if no pulsation is generated by the LSJ pairs. Nevertheless, the bypass phenomenon has the negative impact on the mixing, since the un mixed liquids in the LSJ cavity is continuously pushed into the mixing channel. The red stream sticking to the right wall after the uppermost right LSJ orifice is an evidence of the unmixed liquids residual in the right LSJ cavity. To be noted, it is possible that the interface of LF and RF streams curves to the right side, depending on the initial flow rate ratio of LF and RF during pump boosting.

![Figure 6.6](image)

**Figure 6.6** (a) Schematic of flow bypass effect; (b) Time-averaged velocity fields; (c) concentration contour for sugar solution in the mixing channel without pulsation

Figure 6.7 demonstrates the process of partial segmentation at \(L=4\) by the phase sequence of concentration contours. First of all, partial mixing is achieved without apparent vortex rollup structure, if the stroke length is big enough to guarantee segmentation. The LF stream is bridging between the orifices of the lowest LSJ pair at the end of left LSJ blowing, shown in Figure 6.7(d). After that, LF is
segmented by the right LSJ blowing, but the segment does not span the whole width of the mixing channel. Instead of the “Λ” shaped segmentation by single opposing LSJ pair in sugar solution, the segment in red colour is destroyed by the next LSJ pair before stretching into a crescent shape.

Secondly, the LF concentrates on the left side of the mixing channel, since the red segmentation reaching the middle LSJ at \( t=4/8T \) is sucked into the left LSJ cavity. Due to partial segmentation, the suction and blowing liquids of the middle LSJ pair is partial mixed, and the partial mixed liquids from the middle left LSJ is sucked by the uppermost LSJ in the similar way. However, the blowing from the uppermost left jet is never sucked back by any LSJ again. Instead, it forms asymmetrical “Λ” shaped segmentation downstream without further lateral pulsation. Additionally, the domination of red stream on the left side inhibits the diffusion mixing through the segmentation interface stretching.
Finally, without strong vortex rollup, the mixing relies on the flow segmentation and stretching via the flow blowing and suction of the chained LSJ pairs. Therefore, the gap between the LSJ on the one side of the mixing channel is important on the mixing enhancement at low Reynolds numbers. Given the successful segmentation, a longer distance between the LSJ pair is preferred for the segmentation shearing and stretching. If the segment is sucked back by another LSJ before sufficient stretching, the mixing augment effect is reduced.

Figure 6.8 Instantaneous concentration contours for sugar solution mixing by three opposing LSJ pairs, at $f=1Hz$, $S=1.6$, $St=0.4$, $t=0.5T$, $Re=2$

Figure 6.8 confirms that the increasing pulsation intensity can steadily improve the mixing by three opposing LSJ pairs. Slightly better segmentation is found in sugar solution than less viscous liquids at low stroke lengths. For example, complete segregation is observed at $L=2$ in Figure 6.8(b), whereas segmentation is not achieved in water by the identical stroke length (Figure 5.26(b)). The LF stream in red colour touches the opposing wall of the mixing channel; while the RF stream in blue colour forms an “Λ” shaped segmentation on the top of the LF stream. Therefore, the criterion of successful segmentation by opposing LSJ pair at $L>2$ is reliable in both water and sugar solutions.
Figure 6.8 (e) and (f) show similar flow pattern with $L=4$ in Figure 6.7, but better mixing is achieved for bigger stroke lengths. For instance, two red stream segments are generated by the lowest synthetic jet injection; the smaller one on the top is blown out from the left LSJ cavity. The segments are destroyed by blowing of the middle LSJ pair (Figure 6.9 (a-c)). Furthermore, the red segments are sliced by the blowing of the left middle LSJ, shown in Figure 6.9(d). Later, the slices are sucked into the left LSJ cavity during the suction cycle, and mixing process continues in the LSJ cavity. Finally, the blowing structure from the right middle LSJ is nearly homogenous in green colour (Figure 6.9(g) and (h)). Therefore, mixing mainly happens in the region between the lowest and middle LSJ, which confirms the effectiveness of mixing by chaining up LSJ pairs.

![Sequence of instantaneous concentration contours for sugar solution mixing by three opposite three LSJ pairs, $f=1\text{Hz}(S=1.6, \text{Str}=0.4)$, $L=6$, $\text{Re}_n=2$](image)
Finally, fully mixing is achieved at $L=7$, shown in Figure 6.8(g). The substantial mixing is finished by the lowest and the middle LSJ pairs, except that the uppermost LSJ may push out some sparks of unmixed liquids from the left LSJ cavity due to the higher blowing velocity.

In summary, chaining up LSJ pairs is more effective than the single LSJ pair at low Reynolds numbers. Comparing with limited mixing by single opposing LSJ pair at $L=6$ (Figure 6.1), fast mixing is completed at the orifices of the middle LSJ pair at $L=6$ for three LSJ pairs in Figure 6.8(f).

$f=2\text{Hz}$

Increasing the pulsation frequency from 1Hz to 2Hz not only augments the LSJ blowing velocity, but also initializes weak vortex rollups at the jet orifices, see Figure 6.11. A vortex kernel is found at downstream wing of each left LSJ at $t=2/8T$, shown in Figure 6.11(c). The vortex kernels survive until the end of the left LSJ blowing at $t=4/8T$; three weak counter-rotating vortexes pairs are indicated in the vorticity contour of Figure 6.11(d). Although the vortex rollups are weak at such a low Stokes number of 2.3, they have evidently altered the velocity patterns in the mixing channel.
at the end of LSJ blowing. As a result, the velocity vectors are not homogenously parallel.

![Figure 6.11 Sequence of velocity vectors and vorticity contours for sugar solution mixing by three opposing LSJ pairs at f=2Hz(S=2.3, Str=0.8), L=6, Re_n=2](image)

Figure 6.12 shows better mixing in sugar solution at \( f=2 \)Hz, compared with the case with the identical stroke length at \( f=1 \)Hz (Figure 6.8). Although the red stream can touch the opposite wall of the mixing channel at the end of left LSJ blowing, the segmentation does not fully span downstream at \( L=2 \) (Figure 6.12 (b)). After that, complete segmentation is achieved at \( L=3 \), and the incoming stream has been split into strips by the lowest LSJ pair. Similarly complete segmentation is achieved at \( f=1 \)Hz, which confirms the stroke length as the primary parameter for segmentation.

Moreover, the segments are further sliced by the middle LSJ pair, leading to a substantial mixing downstream (Figure 6.12 (c) and (d)). The process of the middle LSJ pair cooperating with the lowest LSJ pair is demonstrated by the phase sequence of concentration contours in Figure 6.13. No evident vortex structure is found in the velocity fields, but the development of segmentation at \( L=4 \) contributes greatly to the mixing. Shorter segments form at the lowest LSJ exits with higher frequency,
compared with the segmentation for $L=4$ and $f=1$ Hz in Figure 6.5. Furthermore, the segments are stretched into crescent shapes immediately by the lowest LSJ blowing.

Finally, Figure 6.12(e) shows the minimum stroke length needed for fast mixing is reduced to $L=5$ at $f=2$ Hz, instead of $L=7$ at $f=1$ Hz. The mixing is completed between the lowest and middle LSJ pair at $L=5$, suggested by the homogenous green colour after the middle LSJ pair in Figure 6.12(e).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6.12.png}
\caption{Instantaneous concentration contours for three Opposite LSJ pairs, at $t=T/2, f=2$ Hz ($S=2.3, St=0.8$), $Re=2$}
\end{figure}

It is to be noted that the sequence is captured in two different runs, due to the limitation of maximum image capturing speed of the current PLIF system. The flow patterns near the middle LSJ pair for the first capture (Figure 6.13 (a), (c), (e) and (g)) are slightly inconsistent with the second capture (Figure 6.13 (a), (c), (e) and (g)). The inconsistency is caused by the variation of flow structure and laser energy fluctuation. For example, the unmixed liquids in the cavities form isles of red colour downstream, which are not observed in the second capturing sequence. Nevertheless, it does not affect the qualitative evaluation of the segmentation structures.
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

6.3.2. Influence of actuation frequency (L=2)

Experimental study of the circular synthetic jet at low Reynolds numbers indicates that a Stokes number higher than 7 is indispensable for strong vortex rollups (Zhou and Zhong, 2009). Although, this criterion does not directly apply to the 2D synthetic jets in the channel, the tendency of increased vorticity is expected at higher pulsation frequencies. Therefore, mixing by opposing LSJ pairs at a frequency range from 1Hz to 12Hz deserves investigation.

Velocity fields with vorticity contours for sugar solution mixing by three opposing LSJ pairs at $f=6\text{Hz}(S=3.9)$, $L=2$, Re$_n$=2 are demonstrated in Figure 6.14. Similar vortex pair is initialized by each of the LSJ pair in Figure 6.14(d), but the vortices strength is stronger at the uppermost LSJ orifices. This is caused by the higher blowing velocity of the uppermost LSJ, due to the bypass effect of multiple LSJ pairs. Compared with velocity fields of $f=2\text{Hz}$ and $L=6$ at Re$_n$=2, the vortex rollup is more influential on the whole width of the mixing channel, since higher stroke number is beneficial to the vortex rollup. However, these weak vortex rollups are not sufficient
for fast mixing; worse mixing is observed for the identical velocity ratio of 6.4.

Figure 6.14 Sequence of velocity fields with vorticity contours for sugar solution mixing by three opposing LSJ pairs, at \( f=6\text{Hz} \) \( (\text{Str}=2.4, \text{S}=3.9), L=2, R_v = 4.8, \text{Re}_n=2 \)

Figure 6.15 confirms fast mixing of sugar solution is achievable at the fixed Stroke length \( L=2 \), if the pulsation frequency is high enough. In the highly viscous sugar solution, segmentation is not achieved at \( L=2 \) at low frequencies of 1 Hz and 2 Hz, shown in Figure 6.15(a) and (b). At the higher pulsation frequency of 4Hz, red stream strips are generated by the LSJ pair blowing and suction at the lower and middle LSJ pairs, and the strips provide more interfacial areas than the segmentation. However, these strips can not touch the opposite wall of the mixing channel, except for the uppermost LSJ pair. Instead, crescent shapes segments crossing the mixing channel are observed after the uppermost LSJ pair (Figure 6.15 (c)).
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

Figure 6.15 Instantaneous concentration contours for sugar solution mixed by three opposing LSJ pairs at different actuation frequencies at $L=2$, $Re_n=2$

Similar flow pattern of strips in the concentration contours are demonstrated in Figure 6.15(d-e), but mixing is gradually improved with the increasing pulsation frequency. For the fixed net flow rate, the liquids to be mixed undergo more passes of reciprocal movements and stronger shearing effect at the higher pulsation frequencies. Also, the vortex rollups contribute to mixing, as the intensity and diameter of vortex rollup increase. Therefore, full mixing is achieved at $f=12$ Hz, shown in Figure 6.15(f).

In addition, increasing the stroke length is more effective than the increasing pulsation frequency. Although vortex pairs are observable in sugar solution, but fully mixing is not achieved, shown in concentration contour of Figure 6.15(d). By contrast, PIV result for $L=6$ and $f=2$Hz($S=2.3$) does not shows strong vortex rollup at the identical mean LSJ blowing velocity $U_j$ (Figure 6.11), but fast mixing is achieved at the LSJ exit (Figure 6.12 (e)). Therefore, it is confirmed that increasing stroke length is more effective at low Reynolds numbers, similar with water mixing by three opposing LSJ pair at higher Reynolds numbers.
6.3.3. Quantification of mixing in sugar solution

The position for concentration profiling is selected at 12.5h from the piston center, which is consistent with the water mixing pulsed by LSJ pair. Although there are vertical strips in the concentration contour when the laser sheet is reflected by mirror to the region between LSJ orifices; downstream concentration measurement has no such drawback. To confirm the reliable tracer concentration measurement in sugar solution without the mirror reflection, the PLIF experimental data for downstream region is illustrated in Figure 6.16.

![Figure 6.16 Downstream Instantaneous concentration contours for sugar solution mixing by three opposing LSJ pairs, f=1Hz, f=2Hz(S=1.6, Str=0.4), Reₙ=2](image)

The flow pattern of the stretched crescent segmentation is observed downstream in sugar solution at L>1 in Figure 6.16. The flow structure in the contour is as clear as water mixing by single opposing LSJ pair (Figure 5.9). Therefore, the quantification of mixing degree based on the high quality downstream concentration profiling is reliable, if the laser illumination reaching the mixing channel directly without mirror reflection.

As shown in Figure 6.16, the well-formed ‘Λ’-shaped structures confirm the capability of three opposing LSJ pairs to generate the complete segmentation at such low Reynolds number. The ‘Λ’-shaped structures confirm the shearing effect on the segments which are progressively elongated in the streamwise direction. Meanwhile,
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

substantial mixing is achieved by shearing and stretching of segments. On the other hand, Figure 6.17 show the distance between the partial ‘Λ’-shaped segments is reduces as the pulsation frequency increases.

In addition, the entrance length for fully development flow is further reduced at lower Reynolds numbers, hence the parabolic velocity profile is guaranteed. The entrance length theory suggests the upstream disturbance should diminish after a distance of $12.5h$ for such a low Reynolds number. In addition to the PIV results of downstream velocity profile for water mixing in Chapter 3, the parabolic velocity profile is suggested by the periodic flow structures. The segments distance of $2h$ (16mm) at $f=1$Hz, indicates the velocity at the centre of the mixing channel is 16mm/s. Meanwhile, the mean spatial velocity in the mixing channel is 10mm/s for the $Re_n=2$, the velocity at the central line of the mixing channel for the fully developed 2D channel flow is 1.5 times higher than the spatial mean net flow velocity.

Figure 6.17 Downstream Instantaneous concentration contours for sugar solution mixing by three opposing LSJ pairs, $L=2$, $Re_n=2$

Figure 6.18 shows the variations of time-averaged mixing degree with changing dimensionless stroke length for a range of actuation frequencies tested for sugar solution mixed by three opposing LSJ pairs configuration. Similar correlation for the synthetic jet operating condition at which mixing degree is higher than 0.9 is
conducted, and a line is found for the relationship between the mixing degree and the dimensionless stroke length at each frequency. This figure quantitatively confirms that at a fixed actuation frequency a higher $L$ will result in a better mixing, and as the actuation frequency increases a lower $L$ is required to achieve a given mixing degree. It also shows that a mixing degree greater than 0.9 can be achieved when $L$ is sufficiently high for a given frequency.

Figure 6.18 Mixing degree for sugar solution mixing by three opposing LSJ pairs

![Figure 6.18 Mixing degree for sugar solution mixing by three opposing LSJ pairs](image)

Figure 6.19 Fitting for frequency and stroke length requirement for mixing degree equals 0.9 in sugar solution

![Figure 6.19 Fitting for frequency and stroke length requirement for mixing degree equals 0.9 in sugar solution](image)

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The value of $L$ at the mixing degree of 0.9 is then extracted from the intersection point between the horizontal line of mixing degree=0.9 and the best fit line at each frequency (Figure 6.19). Finally, by best fitting the values of $L$ and its corresponding $f$ in the logarithmic scales with a straight line, the functional relationship between $f$ and $L$ is obtained. This relationship is shown in Figure 6.19. The correlation does not include result for $f>8$Hz, since the pulsation system can not provide stably precise stroke length at such high frequencies.

When the frequency is replaced by the Strouhal number, this relationship becomes:

$$\text{Str} \cdot L^{1.98} = 11.4 \tag{6.1}$$

The above relationship allows either the synthetic jet actuation frequency or piston displacement to be chosen when one of these two parameters is fixed. When the characteristics of the driving mechanism of the synthetic jet actuators is available, the optimal synthetic jet operating condition in terms of $L$ and $f$, which produces the desired mixing at the minimum power consumption, can be found.

Compared with the correlation of water mixing by three opposing LSJ pairs: $fL^{3.42}=64.2$ (shown as red line in Figure 6.19), bigger stroke length is needed due to the absence of strong vortex rollups. For example, in order to achieve mixing degree higher than 0.9, $L>4.9$ for sugar solution mixing at $f=1$Hz, while $L>3.5$ is enough for water mixing at $Re_n=83$. However, these two fitting lines are inclined to converge at higher pulsation frequencies. Water mixing results have suggested the vortex kernel size is too small to determine the mixing degree for the small stroke length. Therefore, the mixing is mainly achieved by shearing of reciprocal flow movement at the lowest LSJ pair. On the contrary, all the LSJ pairs contribute to the mixing at lower Reynolds numbers. Figure 6.15 shows the gradually improved mixing passing through the multiple LSJ pairs.

At the absence of strong vortex rollup at low Reynolds number, the variation of Strouhal number is relative to the size of the segmentation. A big Strouhal number indicates the incoming fluids can be segmented into finer slices via more passes of reciprocal lateral movements (see Figure 6.15 for region between the jet orifices and Figure 6.17 for downstream region). On the other hand, higher Stokes number can
predicate the occurrence of the vortex in the mixing channel.

The dimensionless stroke length determines the place where the mixing mainly happens. For a big stroke length ($L=6$, $f=1$Hz in Figure 6.9 and $f=2$Hz in Figure 6.12(f)), part of the incoming liquids are sucked into the synthetic cavities for mixing. Hence, the blowing structure of is partial mixed, indicated by the green-yellow colour in the concentration contours (Figure 6.9). For the fixed $L=2$ at high pulsation frequency of 12Hz (Figure 6.15(f)), the incoming liquids is not substantially sucked into the synthetic jet cavities. Instead, mixing is achieved by strong slicing and shearing effect of the high speech reciprocal movements.

The last but not the least, the number of the LSJ pair along the mixing channel needs to be selected carefully. The mixing enhanced by three opposing LSJ pairs at $L=4$, $R_v=1.6$, $Re_J=7.7$ (Figure 6.13) is no better than mixing enhanced by single lateral synthetic jet configuration at $L=12$, $R_v=4.8$, $Re_J=3.9$ (Figure 6.2(d)), but the piston amplitude is identical. The product of number of LSJ pairs and average stroke length is proportional to the piston displacement, which will determine the power consumption of shaker. Therefore, the optimal LSJ pair configuration should be selected via comparison of the mixing effect single LSJ pair with big stroke number and multiple LSJ pairs with lower stroke number.

### 6.4. Three staggered LSJ pairs ($Re_n=2$)

Water mixing experiments indicates that the staggered or opposing LSJ pair configuration has no significant difference on the mixing enhancement effect, since mixing depends on the strength of vortex rollups at LSJ jet orifice. However, if the LSJ blowing is strong enough to push the liquid into the orifices of downstream LSJ on the opposite side of the mixing channel, mixing may be beneficial from a shorter distance between LSJ pairs for stronger jet interaction. Therefore, staggered configuration is worth investigating for the mixing enhancement due to interaction between consecutive jet pairs.

Moreover, water mixing experiment has suggested the increasing net flow Reynolds number has the negative impact on the mixing, since it reduces the velocity ratio for identical LSJ pulsation amplitude. Thus, sugar solution mixed at $Re_n=4$ is briefly
6.4.1. Influence of stroke length ($f=2$Hz)

First of all, Figure 6.20 demonstrates velocity pattern initialized by three staggered configuration at $L=6$ and $f=2$Hz, which is distinct from that of three opposing LSJ pairs configuration shown in Figure 6.11. Weak vortex kernels can be distinguished in the vorticity contour at the end of left LSJ blowing (Figure 6.20(d)). However, these vortices are not observable at the end of right LSJ blowing at $t=8/8T$ (Figure 6.20(h)). Nevertheless, the lowest left LSJ continues sucking stream into the left LSJ cavity at $t=8/8T$, due to the inertial effect of right LSJ blowing.

Also, Figure 6.20 shows different flow patterns during the left LSJ blowing and right LSJ blowing by the sequence of velocity fields. During the left LSJ blowing (Figure 6.20 (a-c)), the “∫” shape velocity vector bundles connect the left and the right LSJ pair orifices. The pairing of the left and the right LSJ is obvious, since net flow rate from the bottom and jet blowing flow rate from left LSJ are confluent into the right LSJ orifice which is $y=0.5h$ downstream. As a result, several weak vortex kernels are found along the mixing channel, at the end of left LSJ blowing. On the other hand, the blowing from right LSJ show vector bundles in “}” shapes (Figure 6.20 (e-g)) except that the uppermost right LSJ blowing structure is not completely visualized. The stream from each right LSJ orifices is sucked into two LSJ on the opposite side of the mixing channel during the right LSJ blowing. Although the blowing from right is mainly sucked by the paring left LSJ upstream, part of the flow goes into the next left LSJ downstream. Finally, the “S” shape wavy vectors are found at the end of right LSJ blowing (Figure 6.20 (h)), instead of the weak vortex structures.
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

Figure 6.20 Sequence of velocity fields with vorticity contours for sugar solution mixed by three staggered LSJ pairs at $f=2\text{Hz}$, $L=6$, $Re_{st}=2$

Similar with sugar solution mixing by three opposing LSJ pairs, Figure 6.21 shows the mixing degree is steadily improved with the increasing stroke length. Firstly, the segmentation at low stroke length is deteriorated by the staggered jet configuration. LF segment is not generated by the lowest LSJ pair at $L=3$, shown in Figure 6.21 (c). Secondly, the stronger by-pass effect is suggested by the huge segments in red colour pushed out from the uppermost left LSJ. The LF is mainly sucked into the left cavity by the lowest LSJ then pushed out from the uppermost left LSJ. Thus, the mixing is worse than that of three opposing LSJ pairs.

Finally, although the segmentation capability of the staggered jet pair configuration is not as good as that of the opposing jet configuration, fast mixing is still possible if stroke length is big enough. Figure 6.21(e) has shown fast mixing at $L=5$; the mixing is completed by the lowest and the middle LSJ pairs, similar with the opposing jet configuration at the identical actuation parameters (Figure 6.12(e)).
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

Figure 6.21 Instantaneous concentration contours for sugar solution mixed by three staggered LSJ pairs with different stroke length at $f=2$Hz, $t=T/2$, Re$_n=2$

Figure 6.22 show the sequence of the concentration contours for the case: $f=2$Hz, $L=4$, Re$_n=2$, $R_e=4.0$, Re$_J=15.4$. The liquids are mixed via stretching and folding by the lowest LSJ pair. When the partially mixed liquids (indicted by green or yellow colour near $y=-1h$) reach the next LSJ pair. The middle LSJ pair can split the liquids into smaller slices, contributing to the mixing via the molecular diffusion.

However, chaining up the third LSJ pair does not enhance the mixing as much as the second LSJ pair, because the red stream initially resides in the left LSJ cavity has a negative impact on the mixing. Due to the by pass effect of multiple LSJ pairs at low Reynolds numbers, the uppermost LSJ has strongest blowing velocity. Thereby, the uppermost LSJ pair pushes out the unmixed stream inside cavities, impairing the mixing effort of the lowest and middle LSJ pairs. Although the segment in red colour pushed out by the uppermost LSJ can be effectively deformed into the crescent shape, the mixing improves slowly without further enhancement from LSJ pair (Figure 6.22). Hence, the design of the jet configuration and cavity geometry should minimize such
negative effect of the existence of the LSJ cavities, through investigating the flow fields inside the cavities.

Figure 6.22 Sequence of Instantaneous concentration contours for sugar solution mixing by three staggered LSJ pairs \( f=2\text{Hz}, L=4, \text{Re}_n=2 \)

### 6.4.2. Influence of actuation frequency (\( L=2 \))

The mixing effect is not satisfying at low stroke length, shown in Figure 6.23 (a-c). Segmentation is not available at \( f=4\text{Hz} \) for three staggered LSJ pairs configuration, but curvature of incoming liquids interface. The limited stroke length can not blow the red stream to the opposite side wall of the mixing channel. Thus, mixing can not take advantages of the segmentation shearing and stretching. However, Figure 6.23(d) shows the moderate mixing by three staggered LSJ pairs at \( f=6\text{Hz} \). As the pulsation frequency increases, the incoming liquids undergo reciprocal blowing and suction when passing the LSJ orifices. The lateral reciprocal movement is more effective than parabolic velocity profile downstream for flow shearing at high
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

pulsation frequency. As a result, partial mixing is found at the uppermost LSJ orifice. Finally, fast mixing is observed at \( L=2 \) and \( f=8 \) Hz shown in Figure 6.23(e), and it suggests staggered configuration is as good as opposite configuration for the smaller stroke length and higher pulsation frequencies.

Weak vortex kernels are found at the downstream branch of left LSJ orifices at the end of the left LSJ blowing, shown in Figure 6.24(b). There is vortex structure near the left LSJ orifices at the end of blowing, but the uppermost LSJ is strongest. In particular, the obvious X axis velocity is found in the mixing channel, as a result of existence of vortex structure. Nevertheless, the vortex structure does not show up in the concentration contour of Figure 6.23 (d).

**Figure 6.23** Instantaneous concentration contours for sugar solution mixed by three staggered LSJ pairs at different pulsation frequencies, \( L=2, t=T/2, \text{Re}_n=2 \)

(a) \( f=1 \) Hz, \( \text{Re}_n=0.8 \)  
(b) \( f=2 \) Hz, \( \text{Re}_n=1.6 \)  
(c) \( f=4 \) Hz, \( \text{Re}_n=3.2 \)  
(d) \( f=6 \) Hz, \( \text{Re}_n=4.8 \)  
(e) \( f=8 \) Hz, \( \text{Re}_n=6.4 \)
6.4.3. Quantitative comparison of jet configurations

Similar trend that increasing stroke length or actuation frequency leads to better mixing degree is demonstrated in Figure 6.25, and there is no obvious preference on the opposing or staggered jet configuration at $f=2\text{Hz}$ and $L=2$ to achieve a mixing degree higher than 0.9. Admittedly, three staggered LSJ pair is less effective than the opposing jet configuration at $f=2\text{Hz}$, due to the negative impact of initially unmixed liquids which are pushed out by the uppermost LSJ blowing. Whereas the trend is contrary in water mixing, since the single staggered LSJ pair generates stronger vortex rollup at the downstream corner of orifice. Mixing at low Reynolds numbers relies on the segmentation and its folding and stretching by the downstream LSJ pair, due to the absence of strong vortex rollup structure. On the other hand, there is no significant difference for the cases with the fixed stroke length $L=2$, since the initially unmixed liquids is not likely to be pushed out at such a low stroke length.
After all, for the actuation parameters tested in the current experiments, both the segmentation structure near the jet orifices in the concentration contours and mixing quantification downstream do not show evident preference of the staggered LSJ pair configuration over the opposing one, in order to achieve good mixing.

6.4.4. Influence of net flow rate ($Re_n=4, f=2$Hz)

The PLIF results (Figure 6.26) confirm that doubling the net flow Reynolds number to 4 has insignificant influence on mixing, due to the high velocity ratios. Through chaining up the three staggered LSJ pairs, fast mixing is only achievable at $L=5$, $f=2$Hz and $Re_n=2$, shown in Figure 6.21(e). Similar extent of mixing is achieved at $L=5$ and 6, suggested by the large area of homogenous green at the end of observation region in Figure 6.26(e) and (f). Notably, the spatial-averaged jet velocity at the LSJ orifice is $\pi$ times of the mean LSJ velocity $\overline{U_j}$ (Equation 5.4), when LSJ blowing reaches the maximum velocity at $t=1/4T$. The high velocity ratio $R_v=2.4(L=6, f=2$Hz) means the peak LSJ blowing velocity can be 6-7 times higher than the streamwise incoming velocity near the LSJ orifices. At such a big stroke length, doubling the net flow rate does not alter the velocity pattern during the LSJ blowing. Therefore, fast mixing can be obtained at $Re_n=4$, if the $R_v$ is higher than 2.

Figure 6.27 show the velocity fields of sugar solution mixing by three staggered LSJ pairs at $f=2$Hz, $L=6$, $Re_n=4$. The flow pattern is similar with the identical LSJ actuation frequency and amplitude at $Re_n=2$ (Figure 6.20). The “S” shape velocity
pattern is found in the mixing channel at $t = 4/4 T$, as a result of the “{” vector pattern for the right LSJ blowing. Likewise, no strong vortex rollup is observed at the end of either the left or the right LSJ blowing.

![Instantaneous concentration contours for sugar solution mixing](image)

Figure 6.26 Instantaneous concentration contours for sugar solution mixing by three staggered LSJ pairs, at $t=T/2, f=2Hz, Re_n=4$

Unlike the water mixing at high Reynolds numbers, the net flow Reynolds number at $Re_n=4$ generate similar or slightly better mixing than those case with $Re_n=2$, shown in Figure 6.28. When strong entraining effect of vortex rollup is absent, the mixing in highly viscous liquids by the three staggered LSJ pairs relies on flow folding and stretching of sequential segments, as well as the mixing of partial blended liquids in the cavities. It is to be noted, the worse mixing effect at $Re_n=2$ is largely caused by the initially unmixed liquids in the cavities, whereas this is not evident for the increased net flow rate at $Re_n=4$.

Indeed, there is a threshold that the influence of the net flow rate can be ignored, in order to achieve good mixing. Since good mixing is only obtained with a high velocity ratio necessary, slightly increasing the net flow rate will not impair
segmentation in the channel or mixing in the cavities. Therefore, for the tested net flow Reynolds number of 4, $R_v > 2$ is required to keep the domination of jet blowing over net flow velocity in the channel, in addition to the actuation parameters to achieve good mixing at $Re_n = 2$.

Figure 6.27 Sequence of velocity fields with vorticity contours for sugar solution mixing by three staggered LSJ pairs at $L = 6$, $f = 2$ Hz, $R_v = 2.4$, $Re_n = 4$

(b) Variation of stroke length

(b) Variation of velocity ratio

Figure 6.28 Comparison of mixing degree by three staggered LSJ pairs at different net flow Reynolds numbers: $Re_n = 2$ and 4

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6.5. Influence of net flow Reynolds numbers (Re\textsubscript{n}=10)

In order to confirm the mixing capability of LSJ pair in a wider Reynolds number range, mixing of less viscous sugar solution ($\mu=1.0\times10^{-2}$ Pa·s) is studied. For the selected net flow rate of 0.2L/min in the mixing channel, the net flow Reynolds number is $Re\textsubscript{n}=10$, which is in-between $Re\textsubscript{n}=83$ for water and $Re\textsubscript{n}=2$ for the sugar solution ($\mu=1.0\times10^{-5}$ Pa·s). Thereby the transitional condition between the vortex dominated mixing ($Re\textsubscript{n}=83$) and shearing-and-stretching dominated mixing is explored.

6.5.1. Mixing by three opposing LSJ pairs

Similar to the opposing LSJ pairs mixing at $Re\textsubscript{n}=2$, fast mixing is achievable by chaining up the LSJ pairs along the mixing channel. Figure 6.29(b) shows segmentation is not achieved at $L=2$ by the lowest LSJ pair, but the red stream can be segmented downstream by the uppermost LSJ pairs. Whereas, Figure 6.29(c) demonstrates a complete segmentation at the lowest LSJ orifice, the red stream bridges the lowest left and right LSJ orifices just as the case of $L=3$ and $f=1$Hz at $Re\textsubscript{n}=2$(Figure 6.8(c)). Furthermore, similar segmentation but better mixing is achieved at $L=4$; the phase sequence of contours (Figure 6.30) clearly demonstrates the process of mixing enhancement via the segment slicing and stretching by the multiple LSJ pairs. Finally, Figure 6.29(f) shows homogenous green colour after the uppermost jet pair at $y=1.5h$, which means good mixing is achieved at $Re\textsubscript{n}=10$, given big enough stroke length $L=6$.

Figure 6.30 illustrates fast laminar mixing at $Re\textsubscript{n}=10$, due to the successful sequential segmentation and interaction between the segment with the LSJ pair downstream. At $t=0$, the piston of the left synthetic jet is starting to move from its left most position and the jet blowing velocity minimum, so the left and the right streams go parallel to the lowest jet orifice at $y=-2.5h$. Meanwhile, the right stream blown out from the lowest right synthetic jet touches the left wall of the mixing channel, and the left stream in red colour forms a “Λ” shaped segmentation on the top of the right stream, as the result of right LSJ blowing(Figure 6.30 (a)).
At the maximum blowing velocity of the left synthetic jet, the blowing of the lowest left synthetic jet pushes the left stream towards the right synthetic jet orifice and segments the right stream in blue colour from the upstream stream (Figure 6.30 (c)). After that, LF in red colour bridges the jet orifices of the lowest LSJ pair at the end of left LSJ blowing. At the same time, the “Λ” shaped LF segment on the top is distorted toward the right side (Figure 6.30 (e)). As the mean velocity of the net flow in the mixing channel is 0.01m/s for Reₙ=10, the previous “Λ” shaped segment has not moved to the next lateral synthetic pair at y=−0.5h during blowing cycle of the left LSJ. Therefore, the “Λ” shaped segment is stretched into a strip by the middle synthetic jet during its blowing cycle, while a new “Λ” shaped LF segment forms by the lowest LSJ pair (Figure 6.30 (h)).

Figure 6.29 Instantaneous concentration contours for sugar solution mixed by three opposing LSJ pairs at f=1Hz, S=3.6, t=T/2, Reₙ=10

No distinguishable flow structure is shown after the uppermost LSJ pair at y=1.5h, which indicates that fast mixing happens between the lowermost and uppermost LSJ pairs. However, the region in green colour is not homogenous; there are some orange and light green spots within the mixing channel, see the sequence of contours in
Nevertheless, there is no evidence that the mixing is purely relies on the vortex rollup of single LSJ pair, $t=T/2$. No vortex rollup is found in the velocity fields (Figure 6.31). Only a slight diversion the flow direction in the mixing channel at the end of left or right LSJ blowing at $t=0/4T$ and $t=2/4T$. Instead, the chain-up of the LSJ pairs plays a more important role on the fast mixing.

![Instantaneous concentration contours for sugar solution mixed by three opposing LSJ pairs, $f=1\text{Hz}, L=4$, $Re_n=10$](image)

Figure 6.30 Instantaneous concentration contours for sugar solution mixed by three opposing LSJ pairs, $f=1\text{Hz}, L=4$, $Re_n=10$
Figure 6.31 Sequence of time-averaged velocity vectors and vorticity contours for three opposite LSJ pairs, $Re_n=10, L=4, f=1\text{Hz}, S=3.5$

Figure 6.32 confirm the requirement of $L>2$ for the segmentation-based mixing. According to the plug model of the synthetic jet, the dimensionless stroke length needs to be greater than the ratio of mixing channel width to the orifice width ($h/d=2$), so that the LSJ blowing can reach the opposite side of the mixing channel. At the absence of strong vortex rollup, the segmentation capability by single opposing LSJ is even better. For example, Figure 6.32 (c) and (d) show partial mixing via segmentation at the uppermost LSJ orifices, thereby a crescent shape segmentation is observable after $y=3h$. However, red stream concentrates in the left side of the mixing channel.

As the stroke length increases to $L=3$, segmentation is completed at lowermost LSJ orifices, shown in Figure 6.32(e). Large areas of green colour is demonstrated in the concentration contour after $y=2.5h$. Although the upper LSJ pairs are found to more effective for mixing enhancement, the improved mixing effect is based on the partial mixing by the lowest LSJ pair.
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

Figure 6.32 Instantaneous concentration contours for sugar solution mixed by the three opposing LSJ pairs at \( t=T/2, f=2\text{Hz}, S=5.0, \text{Re}_n=10 \)

At last, fast mixing can be fulfilled at \( L=4, R_v=3.2 \), shown in Figure 6.32(f). The mixing happens mainly between the first and second jet pair, but no strong vortex rollup is observed in velocity vectors at \( L=4, f=2\text{Hz} \) (Figure 6.33). Only one weak vortex can be found in the velocity fields at the downstream corner at each LSJ orifices for Stokes number \( S=5.0 \) for \( f=2\text{Hz} \) (Figure 6.33 (a-d)). The vortex is found at the peak of LSJ blowing velocity at \( t=1/4T \), and the vortex survives until the end of blowing at \( t=1/4T \). During the right LSJ blowing cycle, similar vortex rollup is found at the downstream corner of the right LSJ blowing (Figure 6.33 (c) and (d)). Compared with the flow patterns at \( L=4 \) and \( f=1\text{Hz} \) (Figure 6.31), the velocity fields in the mixing channel is apparently curved at the end of LSJ blowing \( t=2/4T \) and \( t=4/4T \), due to the vortex structure generated by the right LSJ blowing. In contrast, mixing is not completed for \( L=4 \) and \( f=2\text{Hz} \) at \( \text{Re}_n=2 \)(Figure 6.12 (d)).
Figure 6.33 Sequence of phase-locked velocity fields and vorticity contours for sugar solution mixing by three opposing LSJ pairs, \( L=4, f=2\text{Hz}, S=5.0, \text{Re}_n=10 \)

Figure 6.34 confirms good mixing is achievable at high actuation frequencies with the fixed stroke length \( L=2 \). Figure 6.34(a) illustrates partial mixing is achieved by the successful segmentation at \( f=1\text{Hz} \). The LF in red colour touches the right wall of the mixing channel at the end of left LSJ blowing. However, the segmentation is deteriorating as the pulsation frequency increases at first, due to the failure of segmentation (Figure 6.34 (b) and (c)). For the identical synthetic jet blowing velocity, fast mixing is fulfilled at \( f=2\text{Hz} \) and \( L=4 \) (Figure 6.32(f)), but fast mixing is not achieved at \( f=4\text{Hz} \) and \( L=2 \), illustrated in Figure 6.34 (d). It suggests that the increasing \( L \) is more effective than the increasing frequency.

Nevertheless, the mixing effect is steadily improved with the increasing pulsation frequency (\( f>6\text{Hz} \)), due to the higher velocity ratio. At \( f=8\text{Hz} \), the blowing structures near the LSJ orifices are in green colour, although fully mixing is not achieved in the mixing channel. Moreover, the mixing is improved by chaining up the LSJ pair; the green colour structure from the uppermost LSJ pair spans almost the whole width of the mixing channel (Figure 6.34 (e)). Finally, Figure 6.34 (f) show fast mixing is achieved at the lowermost LSJ pair at \( f=10\text{Hz} \).
Mixing by Lateral Synthetic Jet Pairs (Ren=2~10)

Figure 6.34 Instantaneous concentration contours for sugar solution mixed by three opposing LSJ pairs at different pulsation frequencies, $L=2$, $t=T/2$, Re$_{n}$=10

Compared with the cases with the fixed $L=2$ at Re$_{n}$=2 (Figure 6.15), the flow structures are similar but the mixing degree is noticeably better at the same pulsation frequency. Partial segmentation is found at low pulsation frequencies ($f<4$Hz), for both Re$_{n}$=2 and Re$_{n}$=10, whereas the periodic segment structure is not observable in Figure 6.34 (e), due to the improved mixing.

6.5.2. Quantitative comparison

In order to evaluate the mixing enhancement capability of LSJ pairs in liquids with different viscosity, mixing three liquids by three opposing LSJ pairs are quantitatively compared in Figure 6.35. The mean net flow velocity is identical as 0.01m/s; so the difference of net flow Reynolds number is due to the dynamic viscous controlled by the mass fraction of the sugar solution. The velocity ratio and Strouhal number is same for each liquid at the given actuation frequency and amplitude (stroke length $L$), while only the synthetic jet Reynolds numbers and Stokes number varies...
with the changing viscosities.

Figure 6.36(a) indicates the mixing pulsated by three opposing LSJ pairs is effective for a wide range of dynamic viscosity. Although similar mixing degree is reached at $f=1$Hz and $L=6$, but the mechanism is different. Vortex rollups play an important role on the water mixing enhanced by single LSJ pair at higher Reynolds number. Thereby, mixing by three LSJ pairs improves slightly outperform the single LSJ pair at the identical pulsation frequency and stroke length, since the mixing effect is largely determined by the first upstream LSJ pair. However, when the strong vortex rollup is not available in more viscous sugar solutions, fast mixing relies on the segmentation and its interaction with the downstream LSJ pairs. For instance, good mixing is not achieved at $L=12$ for $f=1$Hz for single opposing LSJ pair, while $L=6$ is enough for mixing by three opposing LSJ pairs at $f=1$Hz.

Figure 6.36 (b) shows similar trend as Figure 6.36 (a) at $f=2$Hz; the increasing stroke length generates better mixing. On the one hand, the minimum stroke length need for fast mixing is reduced for higher net flow Reynolds number ($Re_n=83$), because the vortex dominating water mixing is faster than folding and stretching at lower Reynolds numbers. On the other hand, LSJ pair in sugar solution can generate stronger disturbance than water mixing at small stroke length ($L<2.5$) at $f=2$Hz, since the LSJ pair penetrates deeper in the sugar solutions without energy transfer to vortex rollup. Previous PIV results reveal that the blowing energy transfers into vortex rollups in water near the LSJ orifices, but the size of vortex kernel is too small to entrain flow near the opposite side of channel at the small stroke length. Whereas, the penetration effect is stronger in more viscous liquids due to the absence of vortex at low Stokes numbers, thereby partial or complete segmentation is formed due to the stream interface folding. Nevertheless, there is a sudden improvement of water mixing at $L=2.5$, after the vortex is strong enough to influence the whole width of the mixing channel.
Figure 6.36 Quantitative comparison of mixing of different liquids by three opposing LSJ pairs at (a) $f=1$ Hz, Str=0.4 and (b) $f=2$ Hz, Str=0.8

Figure 6.37 shows the extent of different liquids mixed by three opposite pairs at fixed stroke length of 2. First of all, mixing degree $>0.9$ is universally achieved at $f=10$Hz and $L=2$. The dropping and rising curve of the mixing degree can be explained via concentration contours in Figure 6.34. Secondly, increasing the pulsation frequencies can gradually improve the mixing degree at the fixed dimensionless stroke length $L=2$, except for the fluctuation at the low pulsation frequencies ($f<4$Hz). Since better segmentation is shown in Figure 6.34(a) for $L=2$ and $f=1$Hz, partial mixing with a mixing degree near 0.5 is achieved at $Re_n=10$. Then, the mixing degree drops at $f=2$Hz, due to the incomplete segmentations. Eventually, the mixing degree rises again with the increasing pulsation frequency, as a result of higher blowing velocity.
Table 6.1 is the collection of dimensionless parameters that mixing degree of 0.9 is achieved by three opposing LSJ pairs at Reₐ=2, 10 and 83. Notably, the stroke length needed for mixing degree equal to 0.9 is interpolated from the nearest tested stroke lengths for the given pulsation frequency. The net flow velocity is kept constant as 0.01 m/s, thus the net flow Reynolds number represents the effect of viscosity of liquids to be mixed and Strouhal number is proportional to the pulsation frequency.

For the fixed synthetic jet geometry in the current experiment (d/h=0.5), the mixing degree is dependent on three dimensionless numbers: net flow Reynolds number, dimensionless stroke length and Strouhal number, according to the non-dimensionless analysis in Chapter 5 (Equation 5.1). The correlations for mixing degree equals to 0.9 are \( \text{Str} \cdot \text{L}^{3.42} = 25.2 \) for water mixing at Reₐ=83 (Equation 5.10) and \( \text{Str} \cdot \text{L}^{1.98} = 11.4 \) for sugar solutions mixing at Reₐ=2 (Equation 6.1). Based on the experimental result (Table 6.1) of water and sugar solutions mixing by three opposing LSJ pairs at the fixed averaged net flow velocity of 0.01m/s, an approximate correlation between the mixing degree and dimensionless numbers are tentatively proposed as Equation 6.2.

\[
\text{Re} \cdot \text{Str} \cdot \text{L} = A \quad \text{for MixingDegree} = 0.9
\]  

(6.2)

where \( A \) is a constant, \( i, j \) are exponents which indicate the relative influential effect
on the mixing degree. The influential effect of net flow Reynolds number and stroke length are compared to Strouhal number via the magnitude of the exponents.

Table 6.1 Parameter table for mixing degree =0.9 (L interpolated)

<table>
<thead>
<tr>
<th>Re_n</th>
<th>L</th>
<th>Str</th>
<th>Rv</th>
<th>Re_j</th>
<th>Mixing degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>83</td>
<td>3.44</td>
<td>0.4</td>
<td>1.376</td>
<td>57.104</td>
<td>0.90</td>
</tr>
<tr>
<td>83</td>
<td>2.61</td>
<td>0.8</td>
<td>2.088</td>
<td>86.652</td>
<td>0.90</td>
</tr>
<tr>
<td>83</td>
<td>2.38</td>
<td>1.2</td>
<td>2.856</td>
<td>118.524</td>
<td>0.90</td>
</tr>
<tr>
<td>83</td>
<td>2.28</td>
<td>1.6</td>
<td>3.648</td>
<td>151.392</td>
<td>0.90</td>
</tr>
<tr>
<td>83</td>
<td>2.18</td>
<td>2.4</td>
<td>5.232</td>
<td>217.128</td>
<td>0.90</td>
</tr>
<tr>
<td>83</td>
<td>1.79</td>
<td>3.2</td>
<td>5.728</td>
<td>237.712</td>
<td>0.90</td>
</tr>
<tr>
<td>10</td>
<td>4.75</td>
<td>0.4</td>
<td>1.9</td>
<td>9.5</td>
<td>0.90</td>
</tr>
<tr>
<td>10</td>
<td>3.65</td>
<td>0.8</td>
<td>2.92</td>
<td>14.6</td>
<td>0.90</td>
</tr>
<tr>
<td>10</td>
<td>2.02</td>
<td>3.2</td>
<td>6.464</td>
<td>32.32</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>5.51</td>
<td>0.4</td>
<td>2.204</td>
<td>2.204</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>3.92</td>
<td>0.8</td>
<td>3.136</td>
<td>3.136</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>2.94</td>
<td>1.2</td>
<td>3.528</td>
<td>3.528</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>2.64</td>
<td>1.6</td>
<td>4.224</td>
<td>4.224</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>2.31</td>
<td>2.4</td>
<td>5.544</td>
<td>5.544</td>
<td>0.90</td>
</tr>
<tr>
<td>2</td>
<td>1.91</td>
<td>3.2</td>
<td>6.112</td>
<td>6.112</td>
<td>0.90</td>
</tr>
</tbody>
</table>

The logarithmic reorientation of Equation 6.2 is a multiple linear expression of Strouhal number, net flow Reynolds number and stroke length.

\[ i \ln \text{Re}_n + j \ln L + \ln \text{Str} = \ln A \quad \text{for} \quad \text{Mixing Degree} = 0.9 \]  \hspace{1cm} (6.3)

The coefficients of Equation 6.3 are solved via the multiple variables regress function of Matlab, as shown in Equation 6.4.

\[ \text{Re}_n^{0.14} L^{2.3} \text{Str} = 18.5 \quad \text{for} \quad \text{Mixing Degree} = 0.9 \]  \hspace{1cm} (6.4)

Figure 6.38 shows the parameter scattering of the experimental data and the multiple variable regression result (Equation 6.4). The regressed plane is shown as meshed quadrilateral (Figure 6.38 (a)) or a line (Figure 6.38 (a)) in the 3D logarithmic coordination due to the dedicated selection of view point. Thus the deviation of the scattering points from the multiple-variable linear regression is demonstrated.
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Equation 6.4 suggests net flow Reynolds number is less influential than Strouhal number whose exponent is unity. Mixing is largely determined by the synthetic jet actuation strength, i.e., increasing either actuation amplitude or frequency leads to better mixing. In addition, increasing the stroke length $L$ is more effective than increasing Strouhal number, which is the non-dimensional reorientation of the actuation frequency.

6.6. Summary

In this chapter, the effect of lateral synthetic jet pairs on the mixing between two sugar solution streams of the same flow rate in a planar mixing channel at low net flow Reynolds numbers is examined using PLIF and PIV.

First of all, mixing of sugar solution by single opposing LSJ pair is investigated at $Re_n=2$. Moderate mixing is achieved by the sequential segmentation and subsequent stretching at the absence of the strong vortex rollup. After that, mixing by three opposing LSJ pairs is studied at such low Reynolds number, it is followed by the comparatively study of three staggered LSJ pairs. Finally, the influence of the net flow Reynolds number is investigated.

Experiments show mixing at low Reynolds number is achieved by segmentation and
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further folding and stretching of the partial mixed liquids by the downstream LSJ pairs. The prerequisite of \( L>2 \) for successful segmentation still holds for mixing by three opposing LSJ pairs at \( \text{Re}_n=2 \); mixing inside the synthetic jet cavity is eminent in the multiple LSJ pair configuration. Moreover, Strouhal number indicates the fineness how the segments are sliced by the reciprocal blowing and suction, at the absence of strong vortex rollups.

Likewise, a functional relationship between \( f \) and \( L \), to achieve a mixing degree higher than 0.9 at \( \text{Re}_n=2 \), is also obtained for three opposing LSJ pairs, and the relationship is found to be \( \text{Str} \cdot L^{1.98}=11.4 \). Compared with water mixing by three opposing LSJ pair, a bigger stroke length is needed for fast mixing at the identical pulsation frequency, but the difference is reduced at higher pulsation frequencies. This suggests a higher velocity

Furthermore, the difference between the opposing and the staggered configurations are comparatively investigated at \( \text{Re}_n=2 \). Similar to water mixing by single LSJ pair, experiments do not suggest the staggered configuration outperforms the opposing one, despite the different patterns of velocity fields. Admittedly, mixing by three staggered LSJ pairs may deteriorate for existence of initially unmixed liquids inside the cavities. Nevertheless, the demand on the actuation intensity and frequencies becomes similar for both configurations as far as achieving a mixing degree near 0.9 is concerned.

Moreover, the net flow rate is doubled and thus the net flow Reynolds number is increased from 2 to 4, while the velocity ratio \( R_v \) is halved for the identical actuation frequency and amplitude. However, mixing degree higher than 0.9 is still achievable if \( R_v>2 \) for the tested frequencies, in addition to the minimum requirement \( L>2 \). Admittedly, an even higher velocity ratio is needed at high pulsation frequency, since increasing pulsation frequency is less effective than increasing \( L \). At such a high velocity ratio, the peak jet blowing velocity is about 7 times higher than the net flow and the increasing net flow rate only slightly alter the velocity patterns in the mixing channel and the synthetic jet cavities, thereby the negative impact on mixing is ignorable.

In addition, mixing by three opposing LSJ pairs is studied in a less viscous sugar
solution at the identical net flow rate. The velocity ratio remains for the identical actuation frequency and amplitude, but the absolute value of synthetic jet Reynolds number increases by 5 times. The results show mixing effect at $Re_n=10$ is slightly better than that at $Re_n=2$, with the assistance of weak vortex rollups. By comparing the mixing of different liquids by three opposing LSJ pairs at Reynolds numbers of 2, 10 and 83, it is confirmed that the mixing effect is not sensitive to the viscosities of the incoming liquids tested, despite the dominant mixing mechanism.

Besides the minimum requirement of stroke length $L>2$, the velocity ratio, which represents the relative strength of synthetic jet pulsation energy, is recognized as crucial parameters for mixing enhancement. In order to achieve a mixing degree high than 0.9 for the increased net flow velocity, a high velocity ratio, at least 2.0, is required for either strong vortex rollup at high Reynolds numbers or intensive stretching and folding of segments at low Reynolds numbers.

Finally, a regression fit is conducted for the correlation between the dimensionless parameters in the tested ranges, net flow Reynolds number, stroke length and Strouhal number. The relationship of parameters is found as $Re_n^{0.14} L^{2.3} Str = 18.5$, in order to achieve a mixing degree equals to 0.9. Suggested by the comparatively small exponent, net flow Reynolds number is less influential than stroke length and Strouhal number.
Chapter 7. Conclusions and Future Work

In this thesis, a detailed experimental study has been carried out with the aim of achieving fast laminar jet mixing via active jet pulsations at a Reynolds number obtainable in the existing rig. The aims of objectives set out in the first chapter have been successfully achieved. In the chapter, the key finding from this work is summarised followed by some recommendations for further work which could be carried out on the basis of this work.

7.1. Summary of key findings

In this section, the key finding from this work are summarised in terms of the original objectives as follows:

1. To improve the jet pulsation mechanism in the existing mixing rig, examine the effects of changing pulsation frequency, duty cycle, pulsation mode on the level of fluid mixing using PLIF and PIV, quantify the effectiveness of laminar mixing and identify the mixing mechanisms via active injection controlled by solenoid valves in a confined jet configuration.

In order to improve the liquids mixing on the existing confined jet configuration, the fluid from a primary planar jet and two surrounding secondary planar jets are pulsed by active fluid injection control via solenoid valves in the out-of-phase mode. The experiments are then conducted using water at a range of pulsation frequencies (1Hz-10Hz), two duty cycles (50% and 25%) and a mean Reynolds number between 100 and 250. This mixing enhancement method is shown to be effective with a mixing degree as high as 0.9 achieved at a mean Reynolds number of about 166 with a duty cycle of 25%. The improvement of the rig has resulted in an excellent mixing being achieved at a net flow Reynolds number which is at least order of magnitude lower than in the original rig. A combination of different mixing mechanisms is found to be at play, including sequential segmentation, shearing and stretching, vortex entrainment and breakup.
For the highest Reynolds number tested here, the mixing degree appears to be independent of the pulsation frequency. However, at a given Reynolds number, an optimal frequency exists which scales approximately with a Strouhal number (\(\text{Str}=\frac{fL}{U}\)) of 1. This optimal frequency reflects the compromise of the vorticity strength and segmentation length. Furthermore, at a given mean Reynolds number a lower duty cycle is found to produce a better mixing due to a resultant higher instantaneous Reynolds number in the jet flow.

Finally, the mixing effect of pulsating either the primary flow (PF) or the secondary flow (SF) alone is studied. The results show the pulsating PF is more effective than pulsating SF. Umbrella shape segmentations with weak vortex rollup are found if PF is pulsated alone with a duty cycle of 50%. Pulsating PF alone with a duty cycle of 25% generates similar segmentation and vortex rollup structure as when both PF and SF pulsated, but the mixing degree is slightly worse. On the other hand, pulsating SF only generates partial segmentation without observable vortex structures, disregard of the duty cycle.

2. To design the actuator and control system for the lateral synthetic jets pairs arrangement, examine the effects of changing actuation intensity, frequency and orifice configuration on the level of fluid mixing using PLIF and PIV, quantify the effectiveness of liquid mixing and identify the mixing mechanisms by single or multiple lateral synthetic jets pairs.

After the success completion of the first objective, a lateral synthetic jet pair mixer is then designed so as to achieve good mixing at an even lower Reynolds numbers. Mixing between two liquid streams of the same flow rate in a planar mixing channel is pulsated by lateral synthetic jet pairs operated out-of-phase at a range of piston actuation frequencies and displacements. Both single opposite and stagger lateral synthetic jet pairs have been tested. The mixing effect of multiple opposite lateral synthetic jet pairs also has been studied. The mixing effect is evaluated qualitatively and quantitatively by PLIF and PIV. With this setup good mixing between two fluid streams can be achieved at a Reynolds number at 2 which is the lowest Reynolds number which can be obtained in the present mixing rig.

At the net flow Reynolds numbers of 83 and 166, both PLIF and PIV results show
that each synthetic jet of the single opposing LSJ pair produces vortex pairs which become stronger as the actuation frequency or amplitude increases; these vortices play an important role in prompting mixing between the two fluid streams. The good mixing obtained at a high frequency or a high dimensionless stroke length is caused by a strong interaction between the vortices produced by the lateral synthetic jet pair. The vortex pair, which is formed on one side of the mixing channel, is destroyed during the formation process of the vortex pair from the opposite side, resulting in the breakdown of the initial 2D flow into a 3D flow hence a greatly enhanced mixing.

Furthermore, single staggered LSJ pairs are also found effective, despite the different patterns of velocity fields. Although the staggered configuration outperforms the opposing configuration slightly at low actuation frequencies such that a lower stroke length is required to achieve the same extent of mixing, the performance of the opposite configuration catches up at high actuation frequencies. Therefore the demand on the actuation intensity and frequencies becomes similar for both configurations as far as achieving a mixing degree near 0.9 is concerned.

For mixing of sugar solutions at low Reynolds number of 2, moderate mixing is achieved by single opposing LSJ pair via the sequential segmentation and subsequent stretching at the absence of the strong vortex rollup. Nevertheless, substantial mixing is obtained by the use of three opposing LSJ pairs via folding and shearing of sequential segments at the absence of strong vortex rollups and breakups. Substantial amount of mixing also happens in the synthetic jet cavities. The vortex rollups in the cavities can contribute effectively to mixing, if both liquids to be mixed are sucked into the cavity. Therefore, the existence of synthetic jet cavity is helpful on mixing enhancement. The aforementioned mixing mechanisms are expected to work at even lower Reynolds numbers. Similarly, no preference over three staggered LSJ pairs is suggested at such low Reynolds number.

Overall, it is found stroke length (the dimensionless actuation amplitude) higher than 2 is prerequisite for successful segmentation by opposing LSJ pairs. This criterion is dictated by the ratio between the width of the mixing channel and the width of LSJ orifice, which is fixed at 2 in the current experimental setup.

Moreover, this experiment suggests that in order to maximise the effect of vortex
entrainment and ensure successful segmentation the ratio between the width of the mixing channel and the width of LSJ orifice should be selected less than, say 4. As such, the vortex kernel can occupy a substantial portion of the mixing channel and to ensure that the flow structures produced by the synthetic jets can either penetrate the width of the mixing channel to be sucked back into the cavity.

Finally, the Strouhal number determines the distance between the consecutive structures produced by the synthetic jets and thus the spacing between sequential segments. At the presence of strong vortex rollups, \( Str \geq 0.4 \) is required to ensure sufficient interaction between vortex structures produced by the opposite jet orifices and their subsequent breakup.

3. To establish the parametric conditions for achieving effective mixing with lateral synthetic jets pairs in the laminar flow regime.

It is found that at a fixed actuation frequency a higher \( L \) results in a better mixing and as the actuation frequency increases a lower \( L \) is required to achieve a given mixing degree. At a sufficiently high frequency or dimensionless stroke length, a nearly homogenous mixing with a mixing degree of greater than 0.9 can be obtained. Hence a functional relationship between \( f \) and \( L \) is obtained by best fitting the actuation parameters required for a mixing degree higher than 0.9. This relationship could provide some guidance for selecting the synthetic jet operating conditions to ensure a good mixing.

This relationship is found to be \( Str \cdot L^{1.8} = 11.0 \) for water mixing by single opposing LSJ pair at \( Re_n = 83 \). The experiments also show that three opposing LSJ pairs are more effective than the single LSJ at an identical dimensionless stroke length, and the relationship is found to be \( Str \cdot L^{3.4} = 25.7 \) at \( Re_n = 83 \). Similarly, at low net flow Reynolds number of 2, the relationship of \( Str \cdot L^{1.98} = 11.4 \) is found for mixing of sugar solution by three opposing LSJ pairs. Compared with water mixing by three opposing LSJ pairs at \( Re_n = 83 \), a higher velocity ratio is required at \( Re_n = 2 \), which means either a slightly bigger stroke length or higher frequency is needed to achieve the same extent of mixing. However, the difference is reduced at higher pulsation frequencies.
For the fixed mixer geometry, the velocity ratio represents the relative strength of the pulsation velocity to the mean flow velocity in the mixing channel. Therefore, a sufficient velocity ratio should be maintained to ensure the creation of an intensive local disturbance and a strong vortex rollup required for good mixing. This is supported by the fact that over the range of the net flow Reynolds number from 2 to 166, when a good mixing indicated by a mixing degree of 0.9 is obtained the velocity ratio, $R_v$, is at least above 1.6 to 2.0.

Finally, based on the experimental data at net flow Reynolds number of 2, 10 and 83, a regression fit is conducted for the correlation between the dimensionless parameters, net flow Reynolds number, stroke length and Strouhal number. The relationship of parameters is found as $Re_n^{0.14} L^{2.3} Str = 18.5$, in order to achieve a mixing degree equals to 0.9. Suggested by the comparatively small exponent, net flow Reynolds number is less influential than stroke length and Strouhal number.

Therefore, multiple LSJ pairs are capable of enhancing laminar mixing at a wide range of Reynolds numbers, despite the difference in the dominant mixing mechanisms. The vortex rollup dominates the mixing at moderate or higher synthetic jet Reynolds numbers whereas stretching and folding of sequential segments dominates the mixing at low synthetic jet Reynolds numbers. Due to the dimensionless definition of Reynolds number, LSJ pairs is applicable to the mixing of highly viscous liquids at macro-scale and liquids mixing within micro-scale geometry.


7.2. Recommendation for future work

The success in obtaining excellent mixing using lateral synthetic jet pairs at low Reynolds numbers in the present work has opened up a promising prospect of their applications in various scenarios. Here some areas where further research could be carried out are stated. However, they are not intended to be exclusive.

Mixing by LSJ pairs at Re<1

Lateral synthetic jet pairs have shown good mixing augment effect for the tested range of Reynolds numbers from 2 to 166. The highest dynamic viscosity tested in the current experimental setup (5.0×10^{-2} Pa·s) is about 50 times more viscous than that of water at 20 °C. However, more viscous liquid is not tested, since the existent centrifugal pump is incapable of handling highly viscous liquids.

In order to reduce the Reynolds number even further to that typically occurring in micro-mixers, more viscous fluid should be tested. It is noted that the synthetic jet formation fails at Re<1 and S<1 in quiescent conditions, according to author’s experiment work on circular synthetic jet’s behaviour at very low Reynolds numbers (Xia and Zhong, 2012). Therefore, mixing enhanced by LSJ pairs at very low Reynolds numbers (Re<1) relies only on flow segmentation, shearing and stretching. In this situation, a narrower mixing channel is more preferable, in order to suck into and push out the incoming flow from the actuator cavity more effectively.

Micro mixing by LSJ pair

Due to the excellent mixing enhancement characteristics of the lateral synthetic jet pairs at low Reynolds numbers, an application in the micro-scale mixing is promising due to their structural and operational simplicity. The straight and smooth mixing channel is convenient to be manufactured and cleaned. In addition, the control logic is convenient to be implemented by integrated circuit, and the actuation of the LSJ can be driven by shape-memory alloy (SMA) or low frequency piezo-electric membranes. Besides, the mixing by LSJ pair is insensitive to the income flow rate fluctuation, in contrast to the Reynolds number dependency of static chaotic mixer.
Further experiments can be undertaken to apply lateral synthetic jet pairs for mixing at micro-scales.

**Scale-up for industrial mixing**

Industry may be more interested with the scale-up application of this jet mixing technique. In practice, the scale-up application may refer to mixing highly viscous liquid within a bigger geometry. The design of LSJ pair mixer should be applicable to a mixing channel wider than the current tested channel width of 8mm. However, the energy efficiency should be taken into account for mixing large volume of flow continuously since the interfacial area per volume will drop and the energy consumption will increase for the scale-up mixing equipment. Therefore, energy consumption calculations should be made in order to define the maximum size that this setup can be scaled up to.
References


Appendix 1: Photographs of the Mixing Rig
Appendix 2: Introduction to Flow Visualization

Toolbox

A general flow visualization Matlab toolbox has been developed for post processing images from PLIF (Planar laser induce fluorescence), BOS (background oriented Schlieren), TSLC (Temperature sensitive liquid crystal) and PSP (Pressure sensitive paint). It incorporates several common planar/surface visualization techniques: PLIF for tracer concentration, BOS for gas density, PSP for surface pressure and TSLC for surface temperature measurement.

This toolbox aims to provide an intuitive graphic user interface (GUI) for calibration and image processing of flow visualization images. It provides the functions such as image previews, data processing and result presentation, also the result data can be saved as image file, Matlab data file(*.mat) and Tecplot text file for further visualization.

The screenshots of the main GUI and calibration GUI, are attached for the PLIF and BOS application. Notably, the toolbox is still under development, GUI may slightly change in later version. Besides, a Power Point Presentation manual is provided for quick start on this toolbox.
PLIF result processing
(1) select images and value pair for calibration.
either add one by one(button: Add image & value pair) or import multiple files with file name as value(button: Import images from a folder)

(2) set ROI

(3) calibrate

(4) check the quality of polynomial fitting curve.
if OK, close this GUI to return main GUI

Calibration for Liquid crystal temperature
Calibration for Background Oriented Schlieren
Data processing for Background Oriented Schlieren