A DAMPED AND DETUNED ACCELERATING STRUCTURE FOR THE MAIN LINACS OF THE COMPACT LINEAR COLLIDER

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Contents

1 Introduction 21
  1.1 Circular Colliders And Synchrotron Radiation . . . . . . . . . . . . 22
  1.2 Linear Collider . . . . . . . . . . . . . . . . . . . . . . . . . . 24

2 Compact Linear Collider An Overview 28
  2.1 Overview And Rationale . . . . . . . . . . . . . . . . . . . . . . 28
  2.2 CLIC Complete Layout . . . . . . . . . . . . . . . . . . . . . . 31
  2.3 Main Linac Structure . . . . . . . . . . . . . . . . . . . . . . . 33

3 RF Parameters 36
  3.1 Basic RF parameters . . . . . . . . . . . . . . . . . . . . . . . . 36
  3.2 Structure Type . . . . . . . . . . . . . . . . . . . . . . . . . . . 39
    3.2.1 Constant Impedance Structure . . . . . . . . . . . . . . . 40
    3.2.2 Constant Gradient Structure . . . . . . . . . . . . . . . 41

4 Wakefields excited by ultra relativistic beams 42
  4.1 Definition And Basic Concept . . . . . . . . . . . . . . . . . . . 42
    4.1.1 Definition . . . . . . . . . . . . . . . . . . . . . . . . . . 42
    4.1.2 Fundamental Theorem of Beam Loading . . . . . . . . . . 43
    4.1.3 Causality . . . . . . . . . . . . . . . . . . . . . . . . . . 43
  4.2 Normal Mode Expansions Of The Wake Potentials . . . . . . . . . 44
List of Tables

1.1 Circular colliders .................................................. 23
1.2 Power radiated in linear colliders [12] - [14], [16], [21] ........ 24
1.3 Frequency scaling of rf parameters ............................... 25
1.4 Key parameters of linear colliders [12] - [14], [16], [21] ........ 27

2.1 Overall parameters of the CLIC .................................. 30
2.2 Parameters of the optimised CLIC main linac structure ........ 35

7.1 Seven fiducial cell simulation results ........................... 98

8.1 Single cell results of the lowest dipole band of a 3.3 GHz detuned structure. .................................................. 113
8.2 CLIC_ZC1 structure parameters .................................. 121
8.3 CLIC_ZC2 structure parameters .................................. 122
8.4 CLIC_ZC structure parameters ................................... 124

9.1 Overall parameters of CLIC_DDS_C ........................... 146

10.1 RF parameters of nine fiducial cells of CLIC_DDS_E ........ 156

11.1 CLIC_DDS_A ......................................................... 174

E.1 Design parameters of seven fiducial cells of CLIC_DDS_C .... 219
E.2 RF properties of seven fiducial cells ............................. 220
F.1 H-field distribution in various undamped single cells . . . . . . . 222
F.2 Design parameters of nine fiducial cells of CLIC_DDS_E . . . . . 223
List of Figures

2.1 CLIC Layout ................................................. 31
2.2 CLIC RF power source complex .......................... 32
2.3 WDS cell .................................................. 34
4.1 WD cell .................................................... 49
4.2 SICA cell .................................................. 50
4.3 Choke mode cell ......................................... 51
4.4 NLC accelerating structure provided with detuning of dipole frequencies damping through attached manifolds. ............... 53
5.1 Single band circuit model ................................. 55
5.2 Single cell dispersion curves: Curves are predicted by circuit model and dots are simulation results. Light line is shown dashed. .... 58
5.3 Double band circuit model ............................... 59
5.4 Single cell dipole dispersion curves. Curves are predicted by circuit model and dots are simulation results. Light line is shown dashed. .................. 60
5.5 Double band circuit model with manifold geometry ........ 61
5.6 Dispersion curves for infinitely periodic manifold implemented single cell. The curves are predicted by a circuit model and dots are the results of the simulations performed with HFSS. The light line is shown dashed. Red dots are used to fit the curves. The dashed curves represent the uncoupled manifold and cell modes.

5.7 Spectral function for a 24 cell structure, black curve shows the designed uncoupled kick factor weighted density function.

5.8 Lowest dipole Q of the 24 cell structure.

5.9 Envelope of wake field for a 24 cell structure. To provide an adequate damping in CLIC, the wake-function must be within the dashed line.

6.1 Monopole mode E-field along the cross-section in a (complete 360°) cell.

6.2 Monopole mode E-field along the length in a structure consisting of three identical cells (10° slice), indicating a phase advance of $2\pi/3$ per cell and $\sim 60\%$ of field enhancement in the iris region with respect to the axial field.

6.3 Components of accelerating field for the structure indicated in Fig.6.2. The black dashed lines indicate periodicity of the cells.

6.4 Monopole mode group velocity versus phase advance.

6.5 RF parameters of monopole mode as a function of beam offset.

6.6 Variation of the group velocity, $Q$ and $R'/Q$ along the structure length.

6.7 Power in CLIC_DDS test structure.

6.8 Accelerating electric field in CLIC_DDS test structure.

6.9 Variation of the normalised electric field along the structure length.

6.10 Surface electric field in CLIC_DDS test structure.
6.11 Variation of normalised magnetic along the structure length . . . . 82
6.12 Surface magnetic field in CLIC_DDS test structure . . . . . . . . 82
6.13 RF pulse profile . . . . . . . . . . . . . . . . . . . . . . . . . . 84
6.14 Pulsed temperature rise in CLIC_DDS test structure . . . . . . . . 85
6.15 Variation of modified Poynting vector along the structure length . 86
6.16 Modified Poynting vector in DDS test structure . . . . . . . . . . 87

7.1 Dipole E-field in an accelerating cell . . . . . . . . . . . . . . . . . 91
7.2 Fiducial cell dipole mode properties . . . . . . . . . . . . . . . . . 92
7.3 Wakefield envelope of an identical cell structure . . . . . . . . . . 94
7.4 Linearly tapered structure . . . . . . . . . . . . . . . . . . . . . . . 95
7.5 Wakefield envelope of a linearly tapered structure . . . . . . . . . . 95
7.6 Fiducial cell parameters . . . . . . . . . . . . . . . . . . . . . . . . 99
7.7 Fiducial cell properties . . . . . . . . . . . . . . . . . . . . . . . . 99
7.8 Gaussian distribution of the kick factor weighted density function 100
7.9 Dipole mode results of a structure with Gaussian distribution . . . 101
7.10 Dipole mode results of a structure with Gaussian distribution . . . 102
7.11 Wakefield envelope for various dipole bandwidths . . . . . . . . . . 105
7.12 Wakefield envelope for various sigmas . . . . . . . . . . . . . . . . 107
7.13 Two fold interleaving . . . . . . . . . . . . . . . . . . . . . . . . . . 109
7.14 Wakefield envelope for various structure interleavings . . . . . . . 110

8.1 A comparison between the uncoupled and coupled mode frequencies 113
8.2 Kick factors . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 114
8.3 Mode density and kick factor weighted density function . . . . . .115
8.4 Mode coupling coefficients of the first two dipole bands . . . . . .116
8.5 Resonating frequencies of the first two dipole bands . . . . . . . . . 117
8.6 Synchronous phase of the lowest dipole band . . . . . . . . . . . . . 117
8.7 Envelope of a DS wakefield for a 25 cell non-interleaved structure. 118
8.8 Wakefield envelope of an interleaved structure for first 24 bunches. 119
8.9 Wakefield envelope of an interleaved structure for entire bunch train. 120
8.10 Envelope of wakefield for an 8-fold interleaved structure over entire bunch train with several damping $Q$s. 120
8.11 RF structure properties of ZC$2$ structure. 123
8.12 Amplitude of the wakefield over the first four trailing bunches, indicating the zero crossing of the bunches. 125
8.13 Envelope of wakefield over the entire bunch train. 126
9.1 Quarter symmetry CLIC$DDS_{C}$ cell geometry 129
9.2 CLIC$DDS_{C}$ cell geometry: Left - details of the side view and right - details of the manifold 129
9.3 Design parameters of seven fiducial cells of CLIC$DDS_{C}$. All parameters are in mm except $\epsilon$ which is dimensionless. 130
9.4 E-fields in CLIC$DDS_{C}$ cell 132
9.5 RF parameters of seven fiducial cells of CLIC$DDS_{C}$ versus synchronous frequency. 133
9.6 RF structure properties of the first out of 8 structures of CLIC$DDS_{C}$. 134
9.7 RF structure properties of the last structure of CLIC$DDS_{C}$. 135
9.8 Peak input power and rf-to-beam efficiency CLIC$DDS_{C}$. 136
9.9 Surface E-field in unloaded condition in CLIC$DDS_{C}$ structures. 138
9.10 Pulsed temperature rise in unloaded condition in 8 CLIC$DDS_{C}$ structures. 139
9.11 Synchronous kick factors of seven fiducial cells of CLIC$DDS_{C}$. 140
9.12 Synchronous frequencies of 8-fold interleaved CLIC$DDS_{C}$. 141
9.13 Circuit parameters of seven fiducial cells of CLIC$DDS_{C}$. 142
9.14 HOM properties of CLIC$DDS_{C}$. 143
9.15 Spectral function of the 8-fold interleaved CLIC_DDS.C. The black dashed curve indicates the designed $Kdn/df$. 144
9.16 Envelope of wakefield in 8-fold interleaved CLIC_DDS.C. 145
10.1 A quarter symmetry undamped cell wall comparing various shapes 149
10.2 H-field optimisation for various undamped single cells 150
10.3 H-field optimisation for various manifold damped single cells 151
10.4 RF parameters versus iris thickness (t) 152
10.5 $1/8$ th of CLIC_DDS_E cell 154
10.6 Comparison of iris thickness and $R_c$ taper in CLIC_DDS_C and CLIC_DDS_E 155
10.7 RF parameters of nine fiducial cells of CLIC_DDS_E versus synchronous frequency 157
10.8 Comparison of E-field in CLIC_DDS_C and CLIC_DDS_E 158
10.9 Peak input power and rf-to-beam efficiency of CLIC_DDS_E 159
10.10 Surface E-field in CLIC_DDS_E structures 160
10.11 Pulsed temperature rise in CLIC_DDS_E structures 160
10.12 RF structure properties of the first out of 8 structures of CLIC_DDS_E 161
10.13 RF structure properties of the last structure of CLIC_DDS_E 162
10.14 Spectral function of the 8-fold interleaved CLIC_DDS_E 164
10.15 Envelope of wakefield in 8-fold interleaved CLIC_DDS_E 164
10.16 Spectral function of the 8-fold interleaved CLIC_DDS_ER 165
10.17 Envelope of wakefield in 8-fold interleaved CLIC_DDS_ER 166
11.1 Design parameters for CLIC_DDS_A 168
11.2 CLIC_DDS_A: Maxima of the fields in eighth symmetry single cells 170
11.3 Single cell rf parameters of CLIC_DDS_A 171
11.4 RF properties of CLIC_DDS_A. The lower and upper solid black lines indicate the allowable pulsed temperature rise and surface E-field respectively. The middle solid black line indicates the average loaded accelerating gradient. .......................... 173

11.5 Spectral function of CLIC_DDS_A. ................................. 175

11.6 Lowest dipole $Q$ of CLIC_DDS_A. ................................. 176

11.7 Envelope of wakefield of CLIC_DDS_A ............................. 176

11.8 CLIC_DDS_A: Lower energy (upstream) end of the structure . . . 178

11.9 CLIC_DDS_A: Upper energy (downstream) end of the structure . 179

11.10 CLIC_DDS_A disc .................................................. 180

11.11 CLIC_DDS_A disc: Disc cross-section .............................. 181

11.12 CLIC_DDS_A: Full 24 cell structure with two matching cells . . . 182

12.1 A comparison of various damping methods ............................ 186

12.2 CLIC_DDS optimisation considering various parameters ................. 187

12.3 CLIC_DDS_A qualification disc ...................................... 189

D.1 Maximum tolerable bunch population for various $<a>/\lambda$ .......... 216

D.2 Maximum tolerable bunch population for various $\Delta a/<a>$ ......... 217

D.3 Luminosity per bunch crossing for various $<a>/\lambda$ ................. 217

D.4 Luminosity per bunch crossing for various $\Delta a/<a>$ ............... 218

F.1 Circuit parameters of seven fiducial cells of DDS_E ................. 224

G.1 CLIC_DDS_A: Regular disc ........................................ 226

G.2 CLIC_DDS_A: Matching disc 1 .................................... 227

G.3 CLIC_DDS_A: Matching disc 2 .................................... 228

G.4 CLIC_DDS_A: Disc stack .......................................... 229

G.5 CLIC_DDS_A: Complete assembly .................................. 230
Abstract

Linear colliders are an option for lepton collision at several TeV. The Compact Linear Collider (CLIC) aims at electron and positron collisions at a centre of mass energy of 3 TeV. In CLIC, the main accelerating structures are designed to operate at an X-band frequency of 12 GHz with an accelerating gradient of 100 MV/m. Two significant issues in linear accelerators that can prevent high gradient being achieved are electrical breakdown and wakefields. The baseline design for the CLIC main linacs relies on a small aperture size to reduce the breakdown probability and a strong damping scheme to suppress the wakefields. The strong damping scheme may have a higher possibility of electrical breakdown. In this thesis an alternative design for the main accelerating structures of CLIC is studied and various aspects of this design are discussed. This design is known as a Damped and Detuned Structure (DDS) which relies on moderate damping and strong detuning of the higher order modes (HOMs). The broad idea of DDS is based upon the Next Linear Collider (NLC) design. The advantages of this design are: well damped wakefields, minimised rf breakdown probability and reduced size of the structure compared to the strong damping design. Procedures necessary to minimise the rf monopole fields and enhance the wakefield suppression are discussed. The rf as well as mechanical designs of a test structure are presented. This unique design forms the basis of this research and allows both the electrical breakdown and beam dynamics constraints to be simultaneously satisfied.
Declaration

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I was born in Mumbai, India in 1982. I completed my Bachelor of Science (BSc.) degree from Birla College, Kalyan, Mumbai University with physics as my major subject in 2003. I opted for Nuclear Physics as my major for doing a Master of Science (MSc.) degree from the University Dept. of Physics in Mumbai University and graduated in 2005. I joined SAMEER, Microwave Research Centre, in Mumbai in July 2005 to work in Medical Linac Division as a Research Scientist and served there until Sept 2007. In Sept. 2007, I joined the High Energy Physics group in the University of Manchester to do a PhD in Accelerator Physics. The accelerator group at the University of Manchester is also affiliated to the Cockcroft Institute of Accelerator Science and Technology (Daresbury).
Dedicated to my family,
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Chapter 1

Introduction

The Standard Model in elementary particle physics states that leptons and quarks are the fundamental particles and matter is made up of these fundamental particles. Particle accelerators are used to help verify the predictions of the Standard Model. Today, the standard model has been validated up to the 100 GeV energy scale. The next energy frontier in high energy physics is the TeV energy range and will first be explored by the Large Hadron Collider (LHC) [1]. If a fixed target (laboratory frame) collision is used then a very small fraction of the energy will be utilised to creation of new particles and the remaining energy will be acquired by these particles in the form of kinetic energy (so as to conserve the momentum). However, if two beams are designed to collide head-on (centre of mass frame) then the total momentum is zero and all the energy will be utilised to create new particles. This is why modern colliders are designed to collide beams head-on. The colliders are classified in two categories, namely circular and linear. An overview of several colliders is presented in this chapter.
1.1 Circular Colliders And Synchrotron Radiation

In circular colliders, particle beams (particle-particle or particle-antiparticle) are accelerated many times through same accelerating gap(s) one in the clockwise direction and the other in anticlockwise direction and meet almost head-on at a common point in the ring containing the beams. The point at which the beams collide is known as interaction point. In this type, as the particle beams gain energy by passing through the same accelerating gap many times, the size (circumference) of the collider is comparatively compact. Some of the recent circular colliders are the Large Electron - Positron Collider (LEP) at CERN [2] - [4] and the Hadron Electron Ring Accelerator (HERA) at DESY [5], [6] designed to collide particles at a 200 GeV and 300 GeV centre of mass energy, respectively. The most recent circular collider is the proton-proton collider, the LHC at CERN.

The results from the LHC collision will help unravel the physics at TeV energy scale. An $e^+e^-$ collider is a complementary machine to the hadron machine. Although there does exist the potential to build a muon collider but this has many technical difficulties which need to be overcome [7].

Circular colliders are restricted to hadron collision due to synchrotron radiation. The power emitted due to synchrotron radiation when a charged particle traverses a circular path is proportional to the fourth power of the energy and can be defined as follows [8]

$$P_{\text{syn}} = \frac{\beta^4 c C_\gamma E^4}{2 \pi \rho^2} \quad (1.1)$$

where $\beta = v/c$, $\rho$ is the bending radius, $C_\gamma = \frac{4\pi r_0}{3mc^2}$ and $r_0$ is the classical radius of an electron. The energy loss per revolution due to radiation can be defined as [8]

$$U_0 = \frac{\beta^3 C_\gamma E^4}{\rho} \quad (1.2)$$

The rest mass of an electron is $\sim 1836$ times smaller than that of proton, hence, its $\gamma$ (relativistic factor, $\gamma \approx E/mc^2$) for a 7 TeV energy will be 1836 times larger. In
In this case, synchrotron radiation emitted by a 7 TeV electron will be \( \sim 10^{13} \) times larger than that of a proton. In LEP, electrons and positrons circulating at 100 GeV energy radiate a significant fraction of their energy in the form of synchrotron radiation. The energy radiated per turn due to synchrotron radiation emission in a select of circular colliders is compared in Table 1.1.

<table>
<thead>
<tr>
<th>Colliders</th>
<th>Particles</th>
<th>Beam energy</th>
<th>Circumference</th>
<th>( \Delta E/\text{rev.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHC</td>
<td>p p</td>
<td>7000</td>
<td>27</td>
<td>4.3</td>
</tr>
<tr>
<td>HERA</td>
<td>p e(^-)</td>
<td>820, 30</td>
<td>6.336</td>
<td>( 3.4 \times 10^{-3}, 71 \times 10^3 )</td>
</tr>
<tr>
<td>LEP</td>
<td>e(^+)e(^-)</td>
<td>100</td>
<td>26.7</td>
<td>( 2.1 \times 10^6 )</td>
</tr>
</tbody>
</table>

It is clear that the synchrotron radiation is negligible for hadrons but for electrons it is significant. Building a lepton circular collider beyond 200 GeV centre of mass energy is not practical. Thus, designing lepton colliders for achieving energies of the order of multiple TeV naturally leads to linear acceleration. In linear accelerators (linacs), particle motion is along a straight path. Though every accelerated particle radiates, this radiation is negligibly small compared to radiation in circular motion. The power radiated by a charged particle along a straight path may be calculated as \[ P_{\text{rad}} = \frac{1}{4\pi\varepsilon_0} \frac{2e^2}{3m^2c^3} \left( \frac{dE}{dx} \right)^2 \] (1.3)

where \( dE/dx \) is the rate of change of energy per unit distance or can be defined as an average accelerating gradient. It is interesting to look at the ratio of the power radiated to the power supplied by an external source and is defined as \[ \frac{P}{dE/dt} = \frac{1}{4\pi\varepsilon_0} \frac{2e^2}{3m^2c^3} \left( \frac{dE}{dx} \right) \approx \frac{2}{3} \frac{r_0}{mc^2} \left( \frac{dE}{dx} \right) \] (1.4)

23
The 2 mile long linear collider at the SLAC National Accelerator Laboratory (SLC) [12], S-Band Linear Collider (SBLC) [13], NLC [14], [15], International Linear Collider (ILC) [16] and CLIC [21] are among the recent proposals of the linear colliders\(^1\). It can be observed from Table 1.2 that the ratio of power loss to power supplied is almost negligible for \(e^+e^-\) linear colliders.

<table>
<thead>
<tr>
<th>Colliders</th>
<th>Beam energy</th>
<th>Accelerator length</th>
<th>(\frac{p}{dE/dx})</th>
<th>(\times 10^{-14})</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLC</td>
<td>46</td>
<td>3</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>SBLC</td>
<td>250</td>
<td>16</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>ILC</td>
<td>250</td>
<td>15</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>NLC</td>
<td>250</td>
<td>15</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>CLIC</td>
<td>1500</td>
<td>21</td>
<td></td>
<td>26</td>
</tr>
</tbody>
</table>

This emphasises why linear colliders are preferred over circular colliders for lepton-lepton collision at several TeV energies. If we consider a hypothetical case of accelerating electrons (positrons) up to 55 TeV along a 20 km length i.e. an accelerating gradient of \(\sim 2.75 \text{ GV/m}\) then the power loss will be \(\sim 1\%\) of the supplied power. At present, it will be beyond scope to achieve accelerating gradients of the order of few GV/m using conventional rf cavities; plasma accelerators may achieve gradients as high as GV/m.

### 1.2 Linear Collider

In linear colliders, charged particles are accelerated linearly through several thousand cavities. The linac structures accelerating charged particle bunches, upon

\(^1\)SLC is the only linear collider that has been built to date.
achieving the nominal energy, direct the particle bunches to collide almost head on (at an angle of the order of few milli radians). For a given accelerating gradient, higher collision energy demand can only be met by increasing the collider length. The length can be reduced by increasing the operating accelerating gradient.

A linear collider can be designed to operate either at room temperature, referred to as normal conducting (NC), or at 2 K, termed as superconducting (SC). In the former, the power loss on the cavity walls is significant; in the latter, the power loss is almost negligible but is limited by the maximum allowed accelerating gradient. In high gradient linac structures, the power dissipated on the cavity walls is large and this leads to heating of the walls. In high gradient machines if superconducting technology is employed then the upper limit of the gradient is decided by the maximum allowed surface magnetic field. Exceeding this limit will quench the superconductivity. The frequency dependence of various rf parameters\(^2\) in NC and SC structures (Appendix A) are presented in Table 1.3.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Normal conducting</th>
<th>Superconducting</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF surface resistance ((R_s))</td>
<td>(\sqrt{\omega})</td>
<td>(\omega^2)</td>
</tr>
<tr>
<td>Power dissipated in cavity walls ((P_{dis}))</td>
<td>(1/\sqrt{\omega})</td>
<td>(\omega)</td>
</tr>
<tr>
<td>Quality factor ((Q))</td>
<td>(1/\sqrt{\omega})</td>
<td>(1/\omega^2)</td>
</tr>
<tr>
<td>Shunt impedance per unit length ((R'))</td>
<td>(\sqrt{\omega})</td>
<td>(1/\omega)</td>
</tr>
<tr>
<td>(R'/Q)</td>
<td>(\omega)</td>
<td>(\omega)</td>
</tr>
</tbody>
</table>

Furthermore, a linac structure can either be operated in a standing wave (SW) mode or in a traveling wave (TW) mode [9]. In a SW structure the rf power flows in either direction forming a standing wave pattern of the fields inside the accelerating structure and hence it does not require an output load. A TW structure

\(^2\)To be discussed in Chapter 3
has an output load and provided the structure is well matched, the rf power travels only in one direction before extraction in the output loads. SW structures reflect significant amount of input rf power back to the source. Failure to divert this reflected power may damage the rf source; a high power isolator is often used to absorb this reflected power.

In order to build linear colliders which meet the luminosity demand of $\sim 10^{34}$ cm$^{-2}$s$^{-1}$, pulses with a large current and multiple bunches within a pulse are required. The field excited by intense multi-particle beam is referred to as wakefield. The transverse component of the wakefield can dilute the beam emittance or in the worst case lead to Beam Break Up (BBU) [10], [11]. The longitudinal wakefield can give rise to energy spread. The transverse and longitudinal wakefields are proportional to the third and second power of the accelerating frequency$^3$ respectively and hence this problem is much more severe in high frequency linacs. However, for high gradient NC structures, choosing a too low frequency will not be cost efficient due to the fact that it will require comparatively more input power to achieve the desired accelerating gradient$^4$. The high input power demand will subsequently have high surface fields. Linear colliders with beam energies of the order of a few 100 GeV are designed with low gradient (up to 20 MV/m) and low frequency (S-band). However, achieving energies of the order of multi TeV necessitate high gradient (above 60 MV/m) operation in order to retain feasible collider lengths. This leads to high frequency (X-band) operation. Key parameters of the SLC, SBLC, NLC, ILC and CLIC are presented in Table 1.4.

As the focus of this thesis is to study the main accelerating structure of CLIC, details of the innovative CLIC scheme are discussed in the next chapter.

---
$^3$Discussed in Chapter 4
$^4$Low frequency structures are bigger in dimension
Table 1.4: Key parameters of linear colliders [12] - [14], [16], [21]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units</th>
<th>SLC</th>
<th>SBLC</th>
<th>NLC</th>
<th>ILC</th>
<th>CLIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega/2\pi$</td>
<td>GHz</td>
<td>2.856</td>
<td>2.856</td>
<td>11.424</td>
<td>1.3</td>
<td>11.9942</td>
</tr>
<tr>
<td>Technology</td>
<td>–</td>
<td>NC</td>
<td>NC</td>
<td>NC</td>
<td>SC</td>
<td>NC</td>
</tr>
<tr>
<td>$E_{c.m.}$</td>
<td>GeV</td>
<td>100</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>3000</td>
</tr>
<tr>
<td>$&lt;E_{acc}&gt;$</td>
<td>MV/m</td>
<td>16.7</td>
<td>17</td>
<td>52</td>
<td>32</td>
<td>100</td>
</tr>
<tr>
<td>Luminosity</td>
<td>cm$^{-2}$s$^{-1}$</td>
<td>$6\times10^{30}$</td>
<td>$5\times10^{33}$</td>
<td>$2\times10^{34}$</td>
<td>$2\times10^{34}$</td>
<td>$5.4\times10^{34}$</td>
</tr>
<tr>
<td>Total site length</td>
<td>km</td>
<td>3</td>
<td>33</td>
<td>30</td>
<td>33</td>
<td>48</td>
</tr>
</tbody>
</table>
Chapter 2

Compact Linear Collider An Overview

2.1 Overview And Rationale

The CLIC design has been studied for more than two decades as part of an international collaboration led by CERN. As the proposed energy is in the multi TeV range, the accelerating gradient needs to be high enough to keep the overall site length feasible. The accelerating gradient for the present CLIC design is 100 MV/m and will operate at an X-band frequency of 11.9942 GHz [18], [19].

It is essential to have cost-effective technology for achieving high energies with linear colliders. In conventional linacs, klystrons are used as a source of rf power to accelerate the beam. Bearing in mind the feasibility of the overall site length and the cost to achieve a centre of mass energy of the order of few TeV, very high accelerating gradients are required. There are two issues which make high accelerating field gradients difficult to achieve in linear colliders, namely the susceptibility of the structures to electrical breakdown and the ability of being able to accelerate multiple bunches whilst maintaining beam stability. It was observed
that high frequency allows high gradients [22] due to the fact that the peak power required in the accelerating structure is smaller for a given gradient compared to that of low frequencies [23]. For this reason, the CLIC scheme was initially proposed to operate at 30 GHz with an ambitious gradient of 150 MV/m leading to an overall site length of \( \sim 33 \) km [24].

One disadvantage of 30 GHz operation is the lack of high power rf source availability at this frequency. Even if such sources (klystrons) are made available, the number of sources required to achieve energy of the order of a few TeV will be large (few thousands). A solution to avoid the use of a large number of klystrons is Two Beam Acceleration (TBA) [21], [23] scheme. The CLIC proposal is based on a novel scheme of TBA [21] in which a low energy high current beam referred to as drive beam is decelerated to provide energy to accelerate the main beam. The power extraction and transfer structures (PETS) [18] are specially designed to extract power from the drive beam and transfer it to the main beam.

In 2005-06, the experimental results obtained at CTF3 (CLIC Test Facility, CERN) revealed that the data collected during CTF2 (30 GHz at 150 MV/m and short pulse length) cannot be extrapolated for the nominal CLIC operation. The increasing trend of accelerating gradient with rf frequency below 3 GHz is not valid in the high frequency domain [18]. It was also observed that in the frequency range of 12 - 30 GHz the maximum achievable gradient is independent of the operating frequency for a given pulse length [18]. Using the experimental data from CTF3 and NLC, an optimisation procedure considering various parameters such as operating frequency, accelerating gradient, luminosity and the total cost resulted in a 100 MV/m optimum gradient and 14 GHz frequency. In order to test the structures at the CTF3 facility the revised frequency should be an integer multiple of the CTF3 frequency which is 2.99855 GHz. This constraint has resulted in the choices of 11.9942 GHz and 14.9928 GHz frequency [18]. To
take advantage of the available data and expertise from the X-band NLC (11.424 GHz) project, 12 GHz frequency was preferred. Thus the revised parameters for the present CLIC structure for frequency and gradient are 11.9942 GHz and 100 MV/m respectively, other parameters are presented in Table 2.1 [18].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Designed value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{cm} )</td>
<td>3</td>
<td>TeV</td>
</tr>
<tr>
<td>Main linac RF frequency</td>
<td>11.9942</td>
<td>GHz</td>
</tr>
<tr>
<td>Total luminosity</td>
<td>( 5.9 \times 10^{34} )</td>
<td>cm(^{-2})s(^{-1})</td>
</tr>
<tr>
<td>Luminosity above 99% of the nominal ( E_{cm} )</td>
<td>( 2 \times 10^{34} )</td>
<td>cm(^{-2})s(^{-1})</td>
</tr>
<tr>
<td>Linac repetition rate</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>Number of particles per bunch</td>
<td>( 3.72 \times 10^9 )</td>
<td>–</td>
</tr>
<tr>
<td>Number of bunches per pulse</td>
<td>312</td>
<td>–</td>
</tr>
<tr>
<td>Bunch separation</td>
<td>0.5 (6)</td>
<td>ns (rf cycles)</td>
</tr>
<tr>
<td>Bunch train length</td>
<td>156</td>
<td>ns</td>
</tr>
<tr>
<td>Beam power per beam</td>
<td>14</td>
<td>MW</td>
</tr>
<tr>
<td>Unloaded/loaded graient</td>
<td>120/100</td>
<td>MV/m</td>
</tr>
<tr>
<td>Overall two linac length</td>
<td>42.16</td>
<td>km</td>
</tr>
<tr>
<td>Total beam delivery length</td>
<td>( 2 \times 2.75 )</td>
<td>km</td>
</tr>
<tr>
<td>Proposed site length</td>
<td>48.4</td>
<td>km</td>
</tr>
<tr>
<td>Total site AC power</td>
<td>392</td>
<td>MW</td>
</tr>
<tr>
<td>Wall plug to main beam power efficiency</td>
<td>7.1</td>
<td>%</td>
</tr>
</tbody>
</table>

Table 2.1: Overall parameters of the CLIC
2.2 CLIC Complete Layout

A schematic layout of the CLIC scheme is presented in Fig. 2.1 [18]. The electrons and positrons after their production are accelerated up to 2.2 GeV and then pass through the pre-damping rings and then other damping ring in order to reduce the emittance. After the damping rings, electron and positron bunches pass through two bunch compressors where the bunch length is first reduced by a factor of 9 and then 4, thus at the entrance of the main linac the bunch length is 44 \( \mu \text{m} \). The production of the drive beam, which is a feature of the CLIC scheme, starts with the acceleration of a 138 \( \mu \text{s} \) pulse to 2.38 GeV at 1 GHz frequency using conventional klystrons. A complete schematic of CLIC power generation is illustrated in Fig. 2.2 [18]. This pulse consists of 24 \( \times \) 24 sub-pulses where
each sub-pulse is 240 ns long and bunches are separated by 60 cm (i.e. 2 ns). The beam current at this stage is 4.2 A. This pulse passes through a delay line and two combiner rings where the bunches interleave by a factor of two, three and four respectively. In the delay loop even and odd buckets of the bunches are separated using an rf deflector. Each even bunch is delayed with respect to the following odd bunch by 240 ns. Using second rf deflector, bunches are recombined by a factor of two. Similarly, bunches are combined in the combiner rings and the frequency is multiplied in a similar manner. The idea here is to interleave the bunches and make one pulse of 24 sub-pulses at 12 GHz frequency. Thus at the linac entrance a low energy high current beam is available to power the main accelerating structures. The $24 \times 24$ sub-pulses, after interleaving become $1 \times 24$ pulses, and the beam
current after the interleaving is 101 A \((24 \times 4.2 \, \text{A})\) [18]. The 24 drive beams (sub-pulses) feed the 24 decelerator sectors, which consist of 1491 PETS each [18]. The drive beam is decelerated in the PETS to produce 136 MW power. Each PETS feeds two main linacs with 63.9 MW peak power [18].

### 2.3 Main Linac Structure

It is important to ensure the main linacs are designed such that they are able to suppress the intense wakefields imposed by X-band operation. The structures use a combination of weak detuning and strong damping of the higher order modes to minimise the beam instabilities and emittance dilution due to transverse wakefields. Several designs for the accelerating structures have been studied such as Tapered Damped Structures (TDS) [21], [22], [25], Hybrid Damped Structures (HDS) [24], [26], [27] and the present baseline design, Waveguide Damped Structures (WDS) [18], [19]. The development of the WDS was necessary due to the high surface fields and unacceptable breakdown rates in TDS and HDS designs.

WDS is designed to operate at the revised frequency of 11.9942 GHz at the optimum loaded gradient of 100 MV/m. Compared to TDS and HDS the operating frequency of the WDS structure is reduced by a factor of \(\sim 2.5\). Since the transverse wakefield is proportional to the third power of frequency (will be discussed in Chapter 4) the wakefield effects in the present structure are less severe compared to that of a 30 GHz structure. The damping scheme in each cell incorporates four waveguides, the cell geometry of WDS is illustrated in Fig. 2.3 [19]. This design also benefits from a reduction in the pulse temperature rise.
The optimised WDS structure, known as CLIC\_G structure [19] is designed by considering several parameters including frequency, accelerating gradient, rf phase advance per cell, short-range wake, transverse long-range wake suppression, rf-to-beam efficiency, pulse temperature rise, surface electric field (E-field) and luminosity. The optimised parameters of the present CLIC\_G structure are presented in Table 2.2$^1$ [18], [19], [20].

Figure 2.3: WDS cell

$^1L_{bx}$ and $\mathcal{F}$ are discussed in Chapter 6
Table 2.2: Parameters of the optimised CLIC main linac structure

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Designed value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(&lt; E_{acc} &gt;)</td>
<td>100</td>
<td>MV/m</td>
</tr>
<tr>
<td>(f_{acc})</td>
<td>11.9942</td>
<td>GHz</td>
</tr>
<tr>
<td>RF phase advance per cell</td>
<td>120</td>
<td>deg</td>
</tr>
<tr>
<td>Cell length</td>
<td>8.333</td>
<td>mm</td>
</tr>
<tr>
<td>First, last iris radius</td>
<td>3.15, 2.35</td>
<td>mm</td>
</tr>
<tr>
<td>First, last iris thickness</td>
<td>1.67, 1.0</td>
<td>mm</td>
</tr>
<tr>
<td>First, last cell shunt impedance</td>
<td>89, 112</td>
<td>MΩ/m</td>
</tr>
<tr>
<td>First, last cell group velocity</td>
<td>1.66, 0.83</td>
<td>%c</td>
</tr>
<tr>
<td>Number of particles per bunch</td>
<td>3.72</td>
<td>(10^9)</td>
</tr>
<tr>
<td>Average iris to wavelength ratio</td>
<td>0.11</td>
<td>–</td>
</tr>
<tr>
<td>Number of cells per structure</td>
<td>24</td>
<td>–</td>
</tr>
<tr>
<td>Structure length (active)</td>
<td>229</td>
<td>mm</td>
</tr>
<tr>
<td>Bunch separation</td>
<td>0.5 (6)</td>
<td>ns (rf cycles)</td>
</tr>
<tr>
<td>Number of bunches in the train</td>
<td>312</td>
<td>–</td>
</tr>
<tr>
<td>Pulse length</td>
<td>240.8</td>
<td>ns</td>
</tr>
<tr>
<td>Input Power</td>
<td>63.8</td>
<td>MW</td>
</tr>
<tr>
<td>RF-to-beam efficiency</td>
<td>27.7</td>
<td>%</td>
</tr>
<tr>
<td>Luminosity per bunch crossing ((L_{px}))</td>
<td>(1.22 \times 10^{34})</td>
<td>(m^{-2})</td>
</tr>
<tr>
<td>Figure of merit ((\mathcal{F}))</td>
<td>9.1</td>
<td>arb. units</td>
</tr>
</tbody>
</table>
Chapter 3

RF Parameters

3.1 Basic RF parameters

In any accelerator it is very important to accelerate the beams efficiently so as to minimise the input power requirement. The characteristics of accelerating cavities are determined in terms of their figures of merit, some of which are discussed here. The figure of merit of an accelerating cavity is most commonly defined in terms of the quality factor $Q$ which is based on the amount of energy stored ($U$) in the cavity and the power dissipated in the cavity walls ($P$) as follows [28]

$$Q = \frac{\omega U}{P}$$

(3.1)

where $\omega/2\pi$ is the cavity frequency. The shunt impedance is another commonly defined figure of merit which measures how efficiently particles accelerate. It is defined in terms of an axial peak accelerating voltage ($V_0$) for a given power dissipation ($P$) as follows [28]

$$R = \frac{V_0^2}{P}.$$  

(3.2)

For a cavity of length $L$, the average axial electric field $E_0$ is defined as

$$E_0 = \frac{V_0}{L}.$$  

(3.3)
For a given field $E_0$, $V_0$ and $P$ both increase linearly with the cavity length [28]. Hence shunt impedance per unit length is more often used to define cavity figure of merit.

$$R' = \frac{R}{L} = \frac{E_0^2}{P/L}. \quad (3.4)$$

When a cavity is excited with rf power, some of the power is utilised to accelerate the beam ($P_b$) and some is dissipated in the cavity walls ($P$), hence the total power ($P_t$) is given as the sum of the two terms $P_t = P + P_b$. The beam power ($P_b$) is calculated in terms of the energy gain ($\Delta W$) and beam current ($I$) as follows [28]

$$P_b = \frac{\Delta W I}{q} \quad (3.5)$$

$$\Delta W = q V_0 T \cos \phi L \quad (3.6)$$

where $q$ is the particle charge, $\phi$ phase of the field relative to the crest, $T$ transit time factor which is taken into account due to the sinusoidal nature of the field. It is defined as follows [28]

$$T = \frac{\int_{-L/2}^{L/2} E(0, z) \cos(\omega z/c)dz}{\int_{-L/2}^{L/2} E(0, z)} \quad (3.7)$$

When a beam enters the accelerating cavity, it interacts with the electromagnetic fields excited in the cavity. The longitudinal voltage $V_\parallel$ across the cavity length $L$, at an offset $r$ is defined as [29]

$$V_\parallel(r) = \int_0^L E_z(r, z) e^{i \omega z/c} dz \quad (3.8)$$

where $E_z$ is the axial electric field. The electromagnetic energy stored in the cavity is defined as [29]

$$U = \frac{\varepsilon_0}{2} \int |E|^2 d^3r \quad (3.9)$$

The loss factor $k_\parallel$ is defined as the energy lost by the particle per unit charge squared. Using computer codes such as HFSS [30], CST microwave studio [31],
GdFidL [32], etc. the longitudinal loss factor can be evaluated from numerically calculated fields in terms of the longitudinal voltage and stored energy as follows [29]

\[ k_{\parallel}(r) = \frac{|V|^2}{4U}. \]  

(3.10)

The interaction of the beam may also be defined in terms of the quantity \( R/Q \) as follows [29]

\[ \frac{R}{Q}(r) = \frac{|V|^2}{\omega U r^{2m}} = \frac{4k_{\parallel}(r)}{\omega r^{2m}}. \]  

(3.11)

where \( m = 0, 1, 2, \ldots \). For the monopole mode, \( m = 0 \) and hence the loss factor and \( R/Q \) are independent of the offset. For the dipole and quadrupole modes \( m \) is 1 and 2 respectively. The transverse loss factor is calculated from the longitudinal loss factor using Panofsky-Wenzel theorem [33], [34] as follows

\[ k_{\perp} = \frac{k_{\parallel}c}{\omega}. \]  

(3.12)

As the loss factor varies with offset, it is generally defined as a unique quantity for a given cell or cavity geometry by normalising with respect to \( r^{2m} \), where \( r \) is the transverse offset from the electrical centre of the cavity and \( m \) is the order of multipole. For example, for a dipole mode, the loss factor varies as \( r^2 \) hence a kick factor (normalised loss factor) is defined, independent of the offset. The transverse kick factor per unit length of a dipole mode is defined as follows

\[ K_{\perp} = \frac{k_{\parallel}c}{r^{2m}\omega L}. \]  

(3.13)

Accelerating structures are essentially loaded waveguides and the frequency components of the modes launched in these structures travel at different velocities due to dispersion. The group velocity of the modes is defined as [28]

\[ v_g = \frac{d\omega}{dk} \]  

(3.14)

where \( \omega \) and \( k \) are the frequency and wave vector respectively. In other words, group velocity is defined as the rate of flow of rf energy in the structure. Individual
waves in the wave envelope travel with different velocities, the velocity of the individual waves is known as phase velocity which can be defined as [28]

\[ v_p = \frac{\omega}{k}. \]  

(3.15)

In the given structure \( v_p v_g = c^2 \) where \( v_p > c \) and \( v_g < c \). Detailed calculation of the group velocity in accelerating structures is discussed in Chapter 6.

### 3.2 Structure Type

In this section, the discussion is focused on the disc loaded TW structure as the CLIC main linacs are designed to operate in this mode\(^1\). The next step is to select the frequency of operation, phase advance per cell and the attenuation of the wave along the structure. The choice of the frequency is mainly governed by the gradient required for acceleration and is discussed in Chapter 2. It is observed that the higher the phase advance per cell (closer to \( \pi \)), the smaller the group velocity is. Hence, the rf breakdown probability is reduced [35]. The attenuation of the wave along the structure can either be kept constant or increased. The former type is known as constant impedance structure and the latter, constant gradient structure [36]. CLIC main linacs are constant gradient structures. In the following sub-sections both types are discussed briefly. In order to study the characteristics of both structures some of the fundamental rf parameters are redefined as follows [36], starting from the shunt impedance per unit length:

\[ R' = \frac{E^2}{-dP/dz} \]  

(3.16)

\[ Q = \frac{\omega(U/L)}{-dP/dz} \]  

(3.17)

\[ R'/Q(\Omega/m) = \frac{E^2}{\omega(U/L)} \]  

(3.18)

\[ P = v_g(U/L). \]  

(3.19)

---

\(^1\) The shunt impedance in TW mode is twice as large as SW mode.
Rearranging the above eqs. 3.17 to 3.19 it can be shown that

$$P = \frac{E^2 Q v_g}{\omega R}.$$  \hspace{1cm} (3.20)

The field attenuation per unit length $\alpha$ is defined as [36]

$$\alpha = \frac{\omega}{2v_g Q} = \frac{-dP/dz}{2P}.$$ \hspace{1cm} (3.21)

### 3.2.1 Constant Impedance Structure

In the constant impedance structure, the attenuation is kept constant by using identical iris radii of the accelerating cells along the length. The power decays exponentially along the structure due to losses in the cavity and beam loading (power delivered to the beam). The rf parameters such as $R$, $Q$, $\alpha$ and $v_g$ are constant (independent of $z$) and the power as a function of distance in this structure can be evaluated using eq. 3.21 as follows [36]

$$\frac{dP}{dz} = -2\alpha P.$$

Integrating eq. 3.22 we obtain power as a function of distance.

$$P(z) = P_0 e^{-2\alpha z}.$$ \hspace{1cm} (3.23)

From eq. 3.23 it can be seen that the power decays exponentially and the same exponential decay in the electric field can be obtained by substituting eq. 3.20 in eq. 3.22 and integrating over the structure length.

$$\frac{d}{dz} \left[ \frac{E^2 Q v_g}{\omega R} \right] = -2\alpha \frac{E^2 Q v_g}{\omega R}$$

$$\frac{dE}{dz} = -\alpha E$$

$$E(z) = E_0 e^{-\alpha z}.$$ \hspace{1cm} (3.24)
3.2.2 Constant Gradient Structure

In constant gradient structure, the attenuation along the structure is increased by systematically reducing the iris radii of the cells along the length of the structure. The power dissipation due to ohmic losses and beam loading is compensated by increasing the shunt impedance along the structure (tapered irises). The power decays linearly along the structure and the accelerating field is kept constant. The attenuation and group velocity are sensitive to a change in the iris radii. However, the rf parameters, such as $R$ and $Q$, are comparatively less sensitive. Hence, in the following derivation $R$ and $Q$ are kept constant. Eq. 3.22 now becomes

$$\frac{dP}{dz} = -2\alpha(z)P$$

Integrating Eq. 3.25 we obtain

$$P_L = P_0 e^{-2\tau}$$

where $\tau$ is defined as

$$\tau = \int_0^L \alpha(z) \, dz.$$  

Considering the assumption that $R$ is constant, $E$ will be constant and hence $\frac{dP}{dz}$. It can be concluded that power will decay linearly and can be defined as follows [36].

$$P(z) = P_0 + \frac{P_L - P_0}{L} z.$$  

By substituting 3.26 in 3.28 we obtain the following relation for power in constant gradient structure

$$P(z) = P_0 [1 - \frac{z}{L} (1 - e^{-2\tau})].$$  

(3.29)
Chapter 4

Wakefields excited by ultra relativistic beams

4.1 Definition And Basic Concept

4.1.1 Definition

Consider a particle of charge $q$ (driving charge) moving in a free space with velocity $v$, the radial electric field of this charge lies nearly in a transverse direction with the angle of opening of $1/\gamma$, where $\gamma = (1 - (\frac{v}{c})^2)^{-1/2}$ and $c$ is the speed of light. If the particle is travelling at a velocity close to speed of light ($v \approx c$) then the electric field associated with the charge is perpendicular to the direction of motion. In this case, a test charge following the driving charge will not experience any force. If the charges are moving in a metallic conductor then the fields associated with the driving charge get scattered from the geometrical discontinuities of the conductor (vacuum bellows, irises of the accelerating cells, etc.) and act back on the test charge. The fields of the driving charge acting back on the trailing (test charge in this case) are defined as wakefields. The wakefield experienced due to the geo-
metrical discontinuities is defined as geometric wake which has both longitudinal and transverse components [34]. If the charges are moving in a perfect conductor with no geometric discontinuities, then the fields terminate on the surface (image) charges and the surface charges move in synchronism with the driving charge. In case of a conductor with some resistivity, the surface charges lag behind the driving charge, in addition, magnetic field and surface currents diffuse in the walls of the conductor giving rise to fields which affect the motion of the test particle both transversely and longitudinally. This is a resistive wakefield [37].

4.1.2 Fundamental Theorem of Beam Loading

Consider an accelerator cavity, if a beam enters the cavity at position \( z = 0 \) and leaves it at \( z = L \), it induces a voltage \( V_b \). The fraction of the voltage experienced by the beam itself \( V_e \) is averaged to the half of the induced voltage. Thus the potential experienced by the charged particle itself is half of that experienced by the trailing particles behind the leading particle, this is called as fundamental theorem of beam loading [38].

\[
V_e = \frac{1}{2} V_b. \tag{4.1}
\]

\( V_e \) is independent of whether there is any stored energy or not when the beam enters the cavity.

4.1.3 Causality

When a charge or charge distribution moves with the speed of light \( v \approx c \) the electromagnetic (e.m.) fields associated with the charge lie perpendicular to the direction of motion. Hence, there is no wakefield ahead of the driving charge, this is called as causality [38]. For \( v < c \), e.m. fields exist ahead of the driving bunch i.e. non-causal behaviour [37].
4.2 Normal Mode Expansions Of The Wake Potentials

The wake potential (Appendix B) can be calculated by solving the Maxwell’s equations in both the time or frequency domain using various computer codes. In the frequency domain, the wakefield is obtained by performing a summation of all the modes excited at the synchronous phase (strongly excited mode in the cavity) of each mode. In modal sum method, the longitudinal and transverse wakefields are calculated for an empty cavity using the eigen modes excited in the cavity. This method is called as the Condon method. It was introduced in [39] and used in [37].

4.2.1 The Longitudinal Wake Potential

Consider a charge $Q$ (driving charge) entering a perfectly conducting closed cavity at $z = 0$ and leaving at $z = L$ with the speed of light. The total voltage lost by a test particle in traversing a distance $s$ along the same path divided by the charge $Q$ is defined as the longitudinal delta function wake potential $W_{z}(s)$ and can be written as follows [37]

$$W_{z}(s) = -\frac{1}{Q} \int_{0}^{L} E_{z}(z, \frac{z + s}{c}) dz \quad (4.2)$$

where $E_{z}(z, t)$ is the electric field along the $z$ axis due to the driving charge. $W_{z}(s) = 0$ for $s < 0$ from causality. After extensive analytical calculations [37] the expression for the longitudinal wakefield simplifies to the following form [37]

$$W_{z}(s) = \sum_{n} k_{n} \cos\left(\frac{\omega_{n}s}{c}\right) \begin{cases} 0 & s < 0 \\ 1 & s = 0 \\ 2 & s > 0. \end{cases} \quad (4.3)$$
4.2.2 The Transverse Wake Potential

The transverse wake potential is the total transverse momentum kick experienced by a test particle following the same path at a distance \( s \) behind the driving charge divided by the charge \( Q \).

\[
W_{\perp}(s) = -\frac{1}{Q} \int_0^L dz \left[ E_{\perp} + (v \times B)_{\perp} \right]_{r(z+s)/c}.
\]  

(4.4)

The transverse wakefield in terms of the modal sum is then given by [37]

\[
W_{\perp}(s) = \sum_n k_n \frac{\sin(\omega_n s/c)}{\omega_n c} \begin{cases} 
0 & s < 0 \\
1 & s = 0 \\
2 & s > 0.
\end{cases}
\]  

(4.5)

4.2.3 Short And Long Range Wakefields

Both the longitudinal and transverse wake potentials can be classified according to the region of calculation i.e. within or outside the bunch length, namely short-range and long-range wakefields respectively. The main impact of the longitudinal component of the wakefield is that it increases the energy spread of the bunches. The transverse component dilutes the emittance of the bunch. In order to achieve a luminosity of the order of \( 10^{34} \) cm\(^{-2}\) s\(^{-1}\) in linear colliders, it is essential to keep the wakefields within acceptable limits so as to achieve the desired luminosity.

Short-range Wakefield

The wake potential induced by the head of the bunch is experienced by the remainder of the bunch and is referred to as short-range or intra bunch wakefield [35]. The typical length of the bunches accelerated in linear colliders can be as small as few tens of a micrometers (for CLIC \( \sigma_z \sim 44 \mu \text{m} \) [18]. The longitudinal and transverse components of the short-range wakefields are inversely proportional to the
second and fourth power of the average aperture radius of the accelerating cells respectively [40], [41]. In principle, once the iris dimension is specified, the short-range wakefield cannot be damped or suppressed. The only way to keep it within the limit is to choose a suitable aperture size and populate the bunches accordingly. For an accelerating cell of aperture radius \( a \), cavity radius \( b \), iris thickness \( t \) accelerating gap \( g \) and cell period of length \( L \), the short-range geometric wakefield can be approximated as [40]

\[
W_{sr}^{\perp}(s) = \frac{4Z_0 c s_{00}}{\pi a^4} \phi(s) \exp \left[ 1 - \left( 1 + \sqrt{\frac{s}{s_{00}}} \right) \exp \left( - \sqrt{\frac{s}{s_{00}}} \right) \right]
\]  

(4.7)

\[
\phi(s) = \begin{cases} 
1 & \text{for } s > 0 \\
0 & \text{for } s < 0 
\end{cases}
\]

(4.8)

Here \( Z_0 = 120 \) is the impedance of free space and \( \alpha_1 = 0.4648 \). However, the eqns. 4.6 and 4.7 are valid for small \( s \). The beam also experiences a resistive wall wakefield [42], [43]. This will not be considered in this work as the geometric wakefield dominates beam dynamics in the accelerating cavities of the main linacs.

Long-range Wakefield

In linear colliders, multiple bunches are accelerated so as to meet the desired luminosity. The wake potential experienced by the trailing bunches due to a driving
bunch moving parallel but at an offset to the designed path is defined as long-range wakefields. The longitudinal and transverse components of the long-range wakefields are inversely proportional to the second and third power of the accelerating aperture radius respectively [44]. The long-range wakefield can be suppressed so as to keep the emittance within the acceptable limits. Various methods to suppress the long-range wakefield are briefly discussed in the following section.

The equations for calculating the longitudinal and transverse long-range wakefields are described in section 4.2.1 and 4.2.2 i.e. the case when $s > 0$. Along with the accelerating mode, other modes (HOMs) are also excited in the cavity. In principle, an infinite number of modes are excited by the particle beam. If $Q_n$ is the quality factor of a particular HOM, then it will decay in time $2Q_n/\omega_n$, where $\omega_n$ is the synchronous frequency of the mode $n$. The transverse long-range wakefield may be calculated as [45]

$$W_\perp(s) = 2\sum_n K_\perp n \sin(\frac{\omega_n s}{c}) \exp\left[-\frac{\omega_n s}{2Q_n c}\right] \theta(s)$$

(4.10)

where $\theta(s)$ is the Heaviside step function. The wakefield will decay with the damping term $\exp\left[-\frac{\omega_n s}{2Q_n c}\right]$. Though the wakefield will oscillate with the sinusoidal term in eq. 4.10, in practice, the envelope of these oscillations is calculated. It gives the maximum excursion of the wakefield even though the beam might experience a wake lower than that of the envelope amplitude. The envelope of the wakefield is calculated using the following equation [46].

$$W_\perp(s) = 2\left|\sum_n K_\perp n \exp\left[i\frac{\omega_n s}{c}(1 + \frac{i}{2Q_n})\right]\right| \theta(s).$$

(4.11)
4.3 Methods of wakefield suppression

As discussed in this chapter, the charged particle beam induces wakefields in the accelerating structure, the transverse component of the wakefields kicks the beam from the desired path causing emittance dilution. When any given structure is subjected to rf excitation, in principle, an infinite number of modes get excited. The lowest mode in accelerating structures is the $TM_{010}$ mode. All the modes above this mode are referred to as HOMs and the transverse component of these modes kicks the beam off axis. It is necessary to take energy out of these modes by means of some damping mechanism. At the same time the fundamental mode should not be perturbed. These modes can be grouped into a characteristic band structure. Extensive simulations have indicated that the dominant kick factors in X-band linacs operating at phase advance of $2\pi/3$, reside in the first band. Hence, the focus is on suppressing the modes in the first band. There are several methods which can be used to suppress the wakefields [47] and these are discussed in the following sections.

4.3.1 Waveguide Damping

In the waveguide damping (WD) method, HOMs are coupled out of the main accelerating structure by means of four waveguides\(^1\) which are connected to each main accelerating cell as shown in Fig. 2.3 and 4.1 [18], [19]. The dimensions of the waveguides are decided by the cut-off frequency which is above the fundamental mode. The advantage of this method is that a lowest dipole $Q$ of the order of 10, or even less can be achieved [19]. This is the baseline design for the main accelerating structures for CLIC (described in Chapter 2). In order to achieve this heavy damping of the HOMs it is necessary to couple the damping waveguides

\(^1\)Four waveguides to damp both the polarisations of the dipole wakefields.
strongly to the accelerating structure. The strong coupling causes a large perturbation in the cavity walls giving rise to high surface magnetic field (H-field). The excessive dissipation of power in the cavity walls leads to an unacceptably large pulse temperature rise\(^2\). In the baseline design of CLIC main linacs, the H-field is optimised by modifying the cavity wall. Another difficulty in designing these structures is the length of the damping waveguides. In order to keep the dielectric damping material away from the vicinity of the accelerating beam\(^3\), it is necessary to keep the length of the damping waveguides long enough.

Figure 4.1: WD cell

\(^2\)Pulsed temperature rise is discussed in detail in Chapter 6
\(^3\)To minimise the breakdown probability
4.3.2 Slotted Iris Constant Aperture Method

The drive beam accelerator (DBA) at CTF3 relies on a different technique for damping the HOMs. The HOMs are damped in this case using the slotted iris constant aperture structures (SICA) which is designed to operate at 3 GHz (S-band) [18], [49]. The SICA cell geometry is presented in Fig. 4.2 [18]. In these structures, as the name suggests, the aperture of the cells is constant. In addition to four damping waveguides, four radial slots are provided to each iris to enhance the dipole coupling. The tip of the iris is designed like a nose cone [18], [49]. The length of the nose cone varies from cell to cell providing the detuning (spread) of the HOMs. The advantage of this structure is, the lowest dipole $Q$ of $\sim20$ can be achieved. Another advantage is, the short-range wakefield is less severe as the aperture is large (aperture radius = 17 mm [18]) compared to the X-band tapered structures. The disadvantage is, there is a relatively high surface E-field. The ratio of E-field to accelerating gradient is $\sim 3.4$ [18] at the end of the radial slots, which are not a problem at a low gradient operation ($\sim 6.5$ MV/m loaded gradient for
DBA operation [18]) however, are very severe at high gradients such as the CLIC main linacs.

### 4.3.3 Choke Mode Damping

The choke mode cavity [50] is another promising approach to damp the HOMs. In this method, a radial slot is provided to damp the HOMs, the other end of the slot is terminated using a load. However, this slot will practically damp all the modes excited in the cavity. In order to preserve the fundamental mode for an efficient acceleration, a slot parallel to the beam axis is introduced at a quarter wavelength from the cavity wall as shown in Fig. 4.3 [50]. As the radial slot is at a quarter wavelength from the cavity wall, the fundamental mode is reflected by the choke, i.e. it is trapped. Any HOM at an odd multiple of the accelerating mode will also be trapped in the cavity. The advantages of this method are: it is easy to fabricate especially for mass production, significant damping of the HOMs can be achieved.

![Figure 4.3: Choke mode cell](image-url)
The disadvantage is, the trapped HOMs could be a problem if the geometry is not changed to shift them away from the odd multiples of the accelerating mode. The width of the radial slot ($h_2$) is an important parameter; $R'/Q$ can be optimised by varying this gap [51]. Minimising $h_2$ will improve the $R'/Q$, however, it may increase the concentration of the surface fields in the vicinity where the slot cuts into the cavity. However, due to a limited study, the performance of this structure at high gradients is not clear yet [47].

4.3.4 Detuning And Moderate Damping

The dominant component of the wakefield is a sum over a limited number of modes excited in the cavity with large kick factors. If we design a structure in such a way that the dipole modes of each cell are different than each other then the wakefield will not add coherently. The NLC studies have shown that the Gaussian spread of the frequencies results in a rapid decay of the wakefield compared to the linear spread [45]. This method was employed for the NLC design [35]. Further discussion on this method is the main objective of Chapter 7. Detuning the structure will not be sufficient alone to damp the wakefields as a finite number of modes eventually result in recoherence of the wake. This recoherence in the wakefield is damped down by coupling it out through slots attached to four manifolds. The manifold is a waveguide-like structure, running parallel to the main beam along the cavity walls. It takes the energy from the HOMs and dissipates it in loads which are located remotely as shown in Fig. 4.4 [35].

There are several potential advantages of this design: It results in a relatively compact structure as there is no need of long waveguides, hence it is mechanically easy to fabricate. Less perturbation of the fundamental mode as the coupling slots are smaller compared to WD method. Remote location of the damping material minimises the breakdown probability. For a given iris, pulsed temperature rise is
low compared to WD method. In addition to wakefield suppression, manifolds can be utilised for beam position monitoring. It is important to note that the NLC DDS structures were high power tested up to 65 - 70 MV/m (unloaded gradient) [15], [35], [52] - [59]. Hence, the structure performance and the comprehensive knowledge is readily available. The disadvantages are: In order to achieve sufficient bandwidth necessary for a rapid decay in the wakefield over the first few trailing bunches, a large spread in the dipole frequencies is required and is achieved by increasing the iris radii (tapering). This depletes the rf-to-beam efficiency, although it is a small effect. This increase in the iris radii will also increase the pulsed temperature rise. In order to efficiently design the structure and analyse the dipole modes excited, a circuit model and a spectral function method are utilised. This circuit model was developed for this express purpose. The main elements of these analyses are described in the next chapter.
Chapter 5

Circuit model

An accelerator cavity may be represented as a series of LC circuits [60], [61]. In this method, each LC circuit loop represents one accelerating cell [62] and the interaction of the neighbouring cells is represented by the mutual inductance of the LC loops. In linear colliders, each accelerating structure consist of several cells; each cell has its own resonating frequency. The e.m. fields in each cell leak through the iris to the neighbouring cells. This is known as coupling of the cells and it causes a shift in the the resonating frequencies of the coupled modes compared to single cells (uncoupled modes). It is important to calculate the coupled mode frequencies accurately. The circuit model method is a very powerful, accurate and time efficient method when it is required to calculate the coupled frequencies of an N-cell structure (where N is the number of accelerating cells) which otherwise takes a significant computational time using commercially available computer codes based on finite element and finite difference algorithms. In general, by adding a resistor to each loop, realistic losses can be introduced. Without this tool, accelerator designing will be significantly more time consuming. In this chapter, several circuit models are discussed.
5.1 Single Chain Model For Accelerating Mode

Accelerator cavities operate in a TM mode. The coupling of neighbouring cells is magnetic in nature. In single chain circuit model these accelerating cells are represented by a chain of LC circuits connected in series. The neighbouring cell interaction in this case is represented by mutual inductance of the LC loops as shown in the Fig 5.1.

![Figure 5.1: Single band circuit model](image)

The calculation of the circuit model begins with Kirchhoff’s law, the voltage in the $m^{th}$ loop is given by

$$\left(2j\omega L_m - \frac{j}{\omega C_m}\right)i_m + j i_{m-1}M_- + j i_{m+1}M_+ = 0 \quad (5.1)$$

where $M_\pm$ is the mutual inductance between the $m^{th}$ and $m\pm1^{th}$ loop respectively. By dividing the above eq. 5.1 by $2j\omega L_m$ it can be seen that

$$\left(1 - \frac{1}{2\omega^2 L_mC_m}\right)i_m + \frac{i_{m-1}M_-}{2\omega L_m} + \frac{i_{m+1}M_+}{2\omega L_m} = 0. \quad (5.2)$$

Defining a resonating frequency, $\omega_m$,

$$\omega_m = \frac{1}{\sqrt{2L_mC_m}} \quad (5.3)$$

and substituting eq.5.3 in to eq. 5.2 we obtain

$$\left(1 - \frac{\omega_m^2}{\omega^2}\right)i_m + \frac{i_{m-1}M_-}{2\omega L_m} + \frac{i_{m+1}M_+}{2\omega L_m} = 0. \quad (5.4)$$
The current in the loops can be normalised in terms of a variable $a_m$ as follows

$$i_m = \frac{a_m}{\omega_m \sqrt{L_m}} = \frac{a_m \sqrt{C_m}}{\omega_m} \quad (5.5)$$

this gives,

$$\left(1 - \frac{\omega_m^2}{\omega^2}\right) a_m + \frac{a_m-1}{\omega_m \sqrt{L_m}} \frac{M_-}{\sqrt{L_{m-1}}} + \frac{a_{m+1}}{\omega_{m+1} \sqrt{L_{m+1}}} \frac{M_+}{2 \omega L_m} = 0 \quad (5.6)$$

multiplying eq. 5.6 by $\sqrt{L_m}/\omega_m$, it can be simplified as

$$\left(1 - \frac{\omega_m^2}{\omega^2}\right) a_m + \frac{a_m-1}{\omega_m \sqrt{L_m}} \frac{M_-}{\sqrt{L_{m-1}}} = 0 \quad (5.7)$$

$$\left(1 - \frac{\omega_m^2}{\omega^2}\right) a_m + \frac{\kappa_m^{-\frac{1}{2}}}{2} a_{m-1} + \frac{\kappa_m^{+\frac{1}{2}}}{2} a_{m+1} = 0 \quad (5.8)$$

where the coupling between the neighbouring cells (loops) $\kappa$ is defined as

$$\kappa_m^{\pm\frac{1}{2}} = \frac{M_\pm}{\omega_m \omega_{m \pm 1} \sqrt{L_m L_{m \pm 1}}} \quad (5.9)$$

The coupling coefficient can also be expressed as

$$\eta_m^{\pm\frac{1}{2}} = \kappa_m^{\pm\frac{1}{2}} \omega_m \omega_{m \pm 1} \quad (5.10)$$

For $\kappa_m^{\pm\frac{1}{2}} = \kappa$, in a periodic structure the current will advance by a factor of $\phi$ per cell and we can write

$$a_m = a_0 e^{im\phi} \quad (5.11)$$

substituting eq.5.11 in to eq. 5.8

$$\left(1 - \frac{\omega_m^2}{\omega^2}\right) a_0 e^{im\phi} + \frac{\kappa}{2} \left(e^{i\phi} + e^{-i\phi}\right) a_0 e^{im\phi} = 0 \quad (5.12)$$

$$\left(1 - \frac{\omega_m^2}{\omega^2}\right) + \kappa \cos \phi = 0 \quad (5.13)$$
or in terms of $\eta$ it can be simplified as

$$
\left(1 - \frac{\omega_m^2}{\omega_r^2}\right) + \eta \cos \phi = 0.
$$

(5.14)

For $\phi = 90^\circ$, $\omega_m = \omega_r = \omega_{\pi/2}$ is the resonating frequency, hence, the frequency of the cell or mode can be expressed in terms of $\omega_r$ and $\eta$ as

$$
\omega = \frac{\omega_r}{\sqrt{1 + \eta \cos \phi}}
$$

(5.15)

Now, by substituting for $\phi = 0^\circ$, $90^\circ$ and $180^\circ$ in eq. 5.15, $\omega_{\pi/2}$ and $\eta$ can be expressed in terms of $\omega_0$ and $\omega_\pi$ as

$$
\omega_{\pi/2} = \sqrt{\frac{2\omega_0^2 \omega_\pi^2}{\omega_0^2 + \omega_\pi^2}}
$$

(5.16a)

$$
\eta = \frac{\omega_\pi^2 - \omega_0^2}{\omega_0^2 + \omega_\pi^2}.
$$

(5.16b)

Hence, using eq. 5.16 $\omega_{\pi/2}$ and $\eta$ can be calculated in terms of $\omega_0$ and $\omega_\pi$. These parameters can be calculated using computational tools such as HFSS. This allows dispersion curves to be obtained with the aid of eq. 5.15. In Fig. 5.2 single cell simulation results (obtained using HFSS) are compared to the circuit model prediction. When the cell is excited with rf power, a number of modes get excited. However, the beam will strongly excite only those modes whose phase velocity is in synchronous with the speed of light, these modes are defined as synchronous modes. In accelerating cells, the lowest synchronous mode is the accelerating mode. A dashed line indicating the speed of light is plotted along with the dispersion curves in Fig. 5.2. The intersection of the light line with dispersion curve gives the synchronous mode which will be seen by the beam. The frequency corresponding to the light line is defined as [29]

$$
f = \frac{\phi c}{2\pi L}
$$

(5.17)
where $\phi$ is the phase advance per cell, $c$ is the speed of light and $L$ is the cell length or period. In case of accelerating cells with a small iris, the prediction of the circuit model is exceptionally good. However, as the iris radius of the cell increases, there is more interaction with the neighbouring cells. In this case, the maximum difference in the prediction compared to the simulation results is no more than 5.0 MHz. However, this can be improved by using additional coupling terms [63] from next neighbouring cells in eq. 5.15. Further explanation on circuit model is available in [45], [62]-[66].

### 5.2 Double Chain Model

For an N-cell structure cell-to-cell coupling leads to frequency shifts which necessitates to incorporate for these shifts in the calculations of the eigen frequencies and wakefield. The fundamental mode in accelerating structures is TM in nature.
The single chain circuit model is sufficient for predicting the mode frequencies in this case. Out of the other multipoles, such as dipole and quadrupole, the dipole modes can severely kick the beam from the desired path for realistic off-sets of the order of a few \( \mu \text{m} \). The quadrupole kicks are not as severe for these realistic offsets. For this reason, it is necessary to account for the shifts in the dipole frequencies accurately. When we consider the dipole HOMs excited in cavities, the single chain of circuit model prediction is not sufficiently accurate [45]. This is due to the fact that the dipole HOMs\(^1\) are neither TE nor TM but hybrid. One has to consider the interaction between the TE-TE, TM-TM and TE-TM modes explicitly to get a reasonable prediction. Hence, a double chain of circuit model

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\(^1\)In particular, the lowest dipole mode has significantly large kick factors compared to other modes; hence is the subject of this discussion.
was introduced [45].

The detuned structure is modelled by using a double chain of circuits as shown in Fig. 5.3. It models lower two dipole bands, where each loop $m$ of the chain represents cell $m$ of the cavity, $\omega_m$ and $\tilde{\omega}_m$ are the TE and TM mode resonant frequencies respectively of the $m^{th}$ cell.

The network equations involved in describing double chain circuit model are briefly summarised in Appendix C. In this case, the characteristic dispersion relation for a single cell subjected to an infinite periodic condition is obtained by solving the determinant $\text{Det}(H - \lambda) = 0$ of eq. C.8 [65].

$$\left(\frac{1 + \eta \cos \phi}{\omega_m^2} - \frac{1}{\omega^2}\right)\left(\frac{1 - \tilde{\eta} \cos \phi}{\tilde{\omega}_m^2} - \frac{1}{\omega^2}\right) - \frac{\eta \tilde{\eta}}{\omega_m^2 \cdot \tilde{\omega}_m^2} \sin^2 \phi = 0. \quad (5.18)$$

The resultant dispersion curves of the first two dipole bands along with simulation results are illustrated in Fig. 5.4. It can be observed that the prediction and simulation results are in good agreement.

Figure 5.4: Single cell dipole dispersion curves. Curves are predicted by circuit model and dots are simulation results. Light line is shown dashed.
5.3 Double Chain Model With Manifold Geometry

The coupled mode frequencies of the dipole modes are calculated using the double band circuit model. However, this model is not sufficient to predict the coupled mode frequencies of a DDS cell\(^2\). In order to incorporate a manifold geometry running parallel to the accelerating beam\(^3\), a transmission line is capacitively coupled to the double band circuit model [35]. Fig. 5.5 represents a double band circuit model with manifold geometry. Here, each component of the cell is mag-

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\(^2\)A DDS cell geometry is introduced in Chapter 4.
\(^3\)on the walls of the cavity
netically coupled to the either neighbours. The interaction between TE modes is represented by $M$, TM interaction is represented by $\hat{M}$ and the cross coupling between the TE-TM is represented by $M_x$. The manifold geometry is represented by a transmission line. The manifolds are designed to propagate $TE_{10}$ modes\(^4\), hence the coupling of the manifold to the accelerating cell is capacitive. This coupling is represented by the coupling of the manifold capacitance to the capacitance of the TE mode of the cell. Details of manifold implemented circuit model along with various parameters necessary to obtain the degree of manifold coupling are presented in Appendix C. Previous studies have indicated that the prediction of the wakefield in the X-band DDS, using manifold implemented circuit model is in very good agreement with the measured wakefield [35]. This method is applied to calculate the eigenmodes in CLIC structures. Prior to understanding these calculations the parameters of the model must be determined and the procedure to do this is outlined here.

Using definitions from eq. C.24 in eq. C.23 we get the dispersion relation for an infinitely long periodic structure and is given by [35]

\[
\left\{ \left( \frac{1 + \eta \cos \phi}{\omega_0^2 - \omega^2} \right) \left( \frac{1 - \hat{\eta} \cos \phi}{\hat{\omega}_0^2 - \omega^2} \right) - \frac{\eta_x^2}{\omega_0 \hat{\omega}_0} \sin \phi \right\} (\cos \phi - \cos \psi) \\
= \Gamma^2 \frac{\pi P}{c} [(1 - \hat{\eta} \cos \phi)\hat{\omega}_0^2 - \omega^2] \frac{\sin \psi_{0n}}{\psi_{0n}}
\]

(5.19)

where $\eta_x = \sqrt{\eta \hat{\eta}}$ is a cross-coupling coefficient of the TE-TM modes. We obtain eigen mode frequencies for the lowest three HOMs\(^5\) for a given phase advance per cell for a manifold implemented accelerating cell of infinite periodicity. We choose eight particular phase frequency pairs from the simulations and then solve the above eight non-linearly coupled eq. 5.19 so as to get the best possible fit for

\(^4\)which is decided by investigating the fields in the manifold mode of a single cell simulation using HFSS code

\(^5\)lowest two dipole modes and the manifold mode
the three HOM curves. The parameters $\omega_0$, $\hat{\omega}_0$, $\eta$, $\hat{\eta}$, $\eta_x$, $F_n$, $\alpha$ and $\Gamma$ are accurately determined once the three curves pass through the eight phase frequency pairs.

Fig. 5.6 illustrates the dispersion curves of a typical manifold implemented CLIC periodic cell. The solid curves are the circuit model prediction, dashed curves are the uncoupled mode frequencies, and dashed line is the light line. The red dots are the simulation results we have used to predict the curves and the black dots are the indicators of how good the prediction is for the intermediate phases.

The region at $\sim 71^\circ$ is the avoided crossing region, where due to the manifold coupling, curves avoid crossing. If there is no coupling (i.e. $\Gamma = 0$) then the curves cross each other at this point as shown by the dashed curves. In case of strong coupling, the spacing between the lowest two curves in the vicinity of avoided crossing region is large. In the lowest curve in Fig. 5.6 (red curve), the modes from 0° to $\sim 70^\circ$ phase advance are dominantly manifold-like modes. At avoided crossing they are coupled out to the manifold and then the latter of the same curve is dominantly (lowest) dipole mode. On the other hand, the magenta curve is dominantly (lowest) dipole mode from 0° to $\sim 70^\circ$ phase advance and onward it is manifold-like mode. The region of avoided crossing depends on the cell geometry, i.e. iris radius, manifold radius, manifold penetration and coupling slot width. The optimisation of the manifold and cell parameters is discussed in Chapter 6.

By fitting eq. 5.19 to the dispersion curves (Fig. 5.6) we obtain circuit parameters necessary to calculate the wakefunction, these calculations are discussed in Appendix C.
Figure 5.6: Dispersion curves for infinitely periodic manifold implemented single cell. The curves are predicted by a circuit model and dots are the results of the simulations performed with HFSS. The light line is shown dashed. Red dots are used to fit the curves. The dashed curves represent the uncoupled manifold and cell modes.

5.4 Methods Of Wake Function Calculation For A Damped And Detuned Structure

5.4.1 Modal Analysis Method

Provided the synchronous frequencies and kick factors [45] for all N cells\(^6\) in a structure have been calculated, then the uncoupled transverse wakefield can be calculated using eq. 4.11. It is referred to as uncoupled model because cell-to-cell coupling is not taken into account. The prediction of this wakefield is good for a

\(^6\)In practise, we calculate \(\omega_{\text{syn}}\) and \(K_{\text{syn}}\) for 7 cells out of 24 and then use cubic spline interpolation to obtain the intermediate ones.
very short time scales up to a few nanoseconds (for the first few trailing bunches in the CLIC design). However, beyond this, it is necessary to take the cell-to-cell coupling into account.

For an accurate calculation of the transverse wakefield, the coupled mode frequencies and coupled kick factors are calculated using the double band circuit model coupled to the attached manifolds using C.23 and C.18 respectively. The dipole $Q_s$ for mode frequencies $(\omega/2\pi)$ are calculated from the real and imaginary part of the mode frequencies $Q = \Re e[\omega] / 2 \Im m[\omega]$. The minimum separation of the lowest two curves in the avoided crossing region (Fig. 5.6) accounts for the dipole $Q$. In this method, as we have discussed, the dipole $Q$ and wakefield are calculated from the eigen modes. For CLIC, DDS, we retain 24 cells per structure\(^7\), hence, we need to solve a matrix of $48 \times 48$ elements to obtain the eigen modes. However, in order to get adequate suppression of the wake, as demanded by the beam dynamics constraints, we interleave\(^8\) eight 24 cell structures (effectively representing 192 cells). Each structure has slightly different frequencies. In this case we need to solve a $384 \times 384$ matrix to obtain the eigen modes and corresponding kick factors. If the modes are strongly coupled to the manifolds then it results in a considerable shift in the mode frequencies. In this case, the wakefield prediction using this method is not as accurate. Due to this reason, a different method is used for the wake function calculation. This method also incorporates for the dipole $Q_s$ and is discussed in the next section.

5.4.2 Spectral Function Method

The modal sum method enables the wake-function to be calculated by summing over the damped modes. On the other hand, a more accurate method which in-

\(^7\)As specified in the present CLIC, G baseline design.
\(^8\)To be discussed in Chapter 7
cludes the regime of large frequency shifts and strong coupling of the manifold
modes, known as Spectral function method is necessary for the NLC and CLIC
structures [35], [69] - [72]. In this method, the impedance of the cavity is cal-
culated and the wakefield is obtained by taking the Fourier transform. The essential
parameters of the general method applied to an N-cell structure are discussed in
Appendix C. The Spectral function is defined as [35]

\[ S(\omega) = 4 \text{Im}(Z(\omega + j\epsilon)) \]  

(5.20)

where \( Z \) is the impedance of the structure and is defined in eq. C.31. The spectral
function (eq. 5.20) of the 24 cell CLIC DDS (1 GHz dipole bandwidth) structure
is illustrated in Fig. 5.7 [73]. This structure has end cells that are closely matched
to the CLIC_G baseline design. The intermediate cells have been varied in order
to achieve a Gaussian fall-off of the initial suppression in the wakefield. As there

Figure 5.7: Spectral function for a 24 cell structure, black curve shows the de-
signed uncoupled kick factor weighted density function.
are 24 cells, we expect to see the same number of resonating modes i.e. peaks in Fig. 5.7. However, some of the peaks merge and hence we do not see all the modes in the spectrum. The width of the peaks indicates the quality factor of the modes. In other words, it indicates how much the mode is damped due to the manifold coupling. Using Lorentzian fits [74] to the spectral function, the dipole $Q$s of the modes are calculated and the result is illustrated in Fig. 5.8 [73]. In this case, the average $Q$ is $\sim$500. However, there is a significant deviation in the $Q$s from 1000 to 100. The wake function eq. C.35 is calculated by taking the inverse Fourier transform of the spectral function and is shown in Fig. 5.9 [73]. This wake-function is not acceptable from a beam dynamics perspective\textsuperscript{9} and means of damping it will be discussed in Chapter 7.

\textsuperscript{9}Beam dynamics perspective is discussed in Chapter 6.
Figure 5.9: Envelope of wake field for a 24 cell structure. To provide an adequate damping in CLIC, the wake-function must be within the dashed line.
Chapter 6

Structure Design : Fundamental Mode Calculations And Strategy

The focus of this work is on high gradient linear accelerators. Various design issues for these accelerators will be discussed in this chapter. In designing high gradient and high frequency accelerating structures, it is important to minimise the surface fields on the cavity walls because, excessive surface fields degrade structure performance and life time. The accelerating structure should be optimised for input power, surface fields, and rf-to-beam efficiency. This chapter includes discussions on simulations and calculations of the rf parameters using computational tools along with various design constraints.

An accelerator structure or cavity consists of N cells, where these cells are either identical or have slightly different parameters however, the length of each cell is usually constant\(^1\) as the phase advance per cell is generally invariant for a given section of accelerators. The phase advance is designed according to \(2\pi L / \lambda = \phi\), and for a given accelerating frequency (= \(c / \lambda\)), this prescribes the length \(L\) of each cell.

\(^1\)This is true for a relativistic beam i.e. \(v \approx c\). However, in case of \(v < c\) the cell length varies along the structure.
The main parameters of each cell are the iris radius $a$, iris thickness $t$ and cavity radius $b$. The design of the structure begins with a single cell simulation using commercially available codes such as HFSS [30], CST Microwave Studio [31], GdfidL [32], etc. These codes solve Maxwell’s equations inside the structure numerically to obtain the eigen modes which constitute the e.m. fields and the frequency of the structure. Driven mode calculations are also used to ensure the structure is properly matched to the attached couplers. Once the single cell dimensions have been optimised, then the entire structure is suitably parameterised. In the next section two important rf parameters, namely the group velocity and shunt impedance which characterise the cells in each structure are described.

### 6.1 Monopole Mode Simulations

The accelerating mode interacts with the beam at the synchronous phase. Each cell in the multi-cell CLIC cavities corresponds to that in a periodic structure. The cell is designed by applying the Floquet [29] periodicity condition. The eigenmode module of HFSS is used and a $10^\circ$ slice\(^2\) with magnetic boundary conditions is imposed to ensure only monopole and quadrupole modes are excited in the simulation. This thin slice enables a rapid and accurate characterisation of the cells in the structure. The monopole field in a full cell is represented in Fig. 6.1. The accelerating field for a structure with a $2\pi/3$ phase advance is illustrated in Fig. 6.2 and Fig. 6.3.

\(^2\)utilising the symmetry of the monopole fields to minimise the computational time
Figure 6.1: Monopole mode E-field along the cross-section in a (complete 360°) cell.

Figure 6.2: Monopole mode E-field along the length in a structure consisting of three identical cells (10° slice), indicating a phase advance of $2\pi/3$ per cell and $\sim 60\%$ of field enhancement in the iris region with respect to the axial field.
Figure 6.3: Components of accelerating field for the structure indicated in Fig. 6.2. The black dashed lines indicate periodicity of the cells.

Dispersion curves of these cells are determined from these simulations and are discussed in Chapter 5. The derivative of the dispersion curves are related to the group velocity and this is an important quantity to keep within the prescribed limit. The group velocity and shunt impedance are described in the following sub-sections.

### 6.1.1 Group Velocity

The group velocity of the e.m. fields is an important quantity as it should be kept within certain limits to prevent electrical breakdown. It is a strong function of the iris radius $a$ and iris thickness $t$, which affects the cell-to-cell coupling. The inverse of the group velocity is related to filling time, which defines the time required to fill the accelerating structure with rf fields. The group velocity can be obtained in terms of the cell frequencies [64] using the standard definition of the
group velocity

\[ v_g = \frac{d\omega}{dk} \]  \hspace{1cm} (6.1)

where \( \omega = 2\pi f \) and \( kL = \phi \):

\[ \frac{v_g}{c} = \frac{\phi_{acc}}{f_{acc}} \frac{df}{d\phi} \]  \hspace{1cm} (6.2)

here \( \phi_{acc} \) and \( f_{acc} \) are the phase and frequency of the accelerating mode respectively. The circuit model described by eq. 5.15 allows eq. 6.2 to be written as:

\[ \frac{v_g}{c} = \frac{\pi f_r}{3f_{acc}} \frac{\eta \sin \phi}{[1 + \eta \cos \phi]^{3/2}}. \]  \hspace{1cm} (6.3)

Here, the present CLIC phase advance per cell has been applied \( \phi_{acc} = 2\pi/3 \). The group velocity for a typical cell, with an iris radius of 3 mm (corresponding to a bandwidth of 1.55\%) is illustrated in Fig. 6.4.

**6.1.2 \( R'/Q \)**

Once the geometry has been designed using this process, the quality factor and shunt impedance can be calculated. The quality factor is calculated using the
finite element code HFSS, with a finite conductivity boundary condition applied to the surface of the cavity. In order to calculate the $R'/Q$ (the ratio of shunt impedance per unit length to quality factor) of the fundamental mode, we extract the on-axis fields using the postprocessor of the simulation tools. The $R'/Q$ is calculated using the following relation

$$R'/Q = \frac{|V|^2}{\omega U L}$$  \hspace{1cm} (6.4)$$

where the peak potential $V = \int_0^L E_z e^{-i\omega z/c} dz$, $\omega = 2\pi f_{acc}$ and $U$ is the electromagnetic energy stored in the fundamental mode$^3$. The rf properties of a monopole mode, namely the loss factor, $R'/Q$ and $R'$ are illustrated in Fig. 6.5 as a function of offset for an accelerating cell with iris radius and thickness of 4.0 mm. Each of the cells is characterised in this manner. The overall accelerating electric field and power flow are determined from these single cell simulations and are discussed in the next section.

$^3$The rms potential is $V_{rms} = V/\sqrt{2}$

![RF properties of single cell](image)

Figure 6.5: RF parameters of monopole mode as a function of beam offset.
6.2 RF Properties Of A Structure: Fundamental Mode

Here fundamental energy conservation conditions are applied in order to derive power flow relations within the accelerating structures. These evaluations are then applied to constant gradient CLIC structures. When a structure is excited with rf power, some part of the power \( P \) is utilised to accelerate the beam (i.e. beam loading) and some fraction dissipates in the cavity walls due to Ohmic losses. The power dissipated in the structure can be given as \([78]\)

\[
\frac{dP}{dz} = -\omega \frac{P}{Q} v_g \tag{6.5}
\]

where \( z \) is the distance along the structure length. The power loss due to the beam loading depends on the local field, which is calculated in terms of the loss factor per unit length \( (k') \) and beam current \( (I) \). The field produced by an element of charge \( dq \) may be defined as \([78]\)

\[
dE = -2k'dq. \tag{6.6}
\]

Using eq. 3.11, it can be expressed in terms of shunt impedance per unit length \( (R') \) and \( Q \) as

\[
dE = -2\left(\frac{\omega R'}{4Q}\right)Idt \tag{6.7}
\]

The relation between power flow along the structure and axial electric field is given by eq. 3.20

\[
\frac{dP}{dz} = v_g \frac{E^2}{\omega R'} \tag{6.8}
\]

Though all the rf parameters vary along the structure (due to slow tapering), for instance, assume only power and field vary along the structure, and differentiate eq. 6.8

\[
\frac{dP}{dz} = v_g \frac{2}{\omega R'} E \frac{dE}{dz} \tag{6.9}
\]
substituting for $dE$ from eq. 6.7 and $E$ from eq. 6.8 in the above eq. 6.9 it can be simplified to

$$\frac{dP}{dt} = -\sqrt{\frac{\omega v_g}{Q}} I \sqrt{P}.$$  

(6.10)

Eq. 6.10 gives the rate of change of power flow in the structure. Now changing the time variable to a space variable using

$$\frac{dP}{dz} = \frac{dP}{dt} \frac{1}{v_g}$$

(6.11)

the power flow per unit length along the structure can be expressed as

$$\frac{dP}{dz} = -\frac{\omega P}{Q v_g} - \sqrt{\frac{\omega R'}{v_g Q} I \sqrt{P}}.$$  

(6.12)

where the first term on the right hand side of eq. 6.12 indicates the power dissipated on the cavity walls due to Ohmic losses and the second, beam loading. In deriving eq. 6.12 only power and field are assumed to vary along the structure. However, in tapered structures $Q, v_g, R'$ vary as well. Hence to calculate the exact power flow in the structure, these parameters should also be considered as variables. In [78] detailed calculations of the power flow for variable rf parameters are reported both numerically and analytically. An analytical solution of eq. 6.12 for $P(z)$ in terms of $Q(z), v_g(z)$ and $R'(z)$ will be mathematically complex. In general, eq. 6.12 can be solved numerically to incorporate the effect of the tapering on the rf parameters. The input power ($P_{in}$) required to set up the desired accelerating gradient in the structure is calculated using single cell rf parameters. The single cell parameters are obtained using HFSS simulations. A typical set of tapered group velocity, $Q$ and $R'/Q$ for a CLIC_DDS test structure\(^4\) is illustrated in Fig. 6.6 [79], [80]. The calculations involving beam loaded and unloaded rf properties in this structure are discussed in the following sub-sections.

\(^4\)CLIC_DDS test structure is discussed in detail in Chapter 11.
Figure 6.6: Variation of the group velocity, $Q$ and $R'/Q$ along the structure length

6.2.1 Loaded And Unloaded Accelerating Fields

In order to calculate $P_{in}$ to set-up the desired accelerating gradient, it is necessary to know the beam current (i.e. bunch population and bunch spacing). The bunches are populated by considering the limitations imposed by short-range wakefield. The bunch spacing is decided by considering the amplitude of long-range wakefield at the location of each bunch. Means to achieve the necessary long-range wakefield damping are discussed in Chapter 4. Once the beam current is known, then the input power $P_{in}$ is calculated using Runge-Kutta method [81], [82] followed by a calculation of the accelerating gradient. After calculating the accelerating field in the structure, the surface fields are calculated. In CLIC-DDS test structure, the objective is to calculate $P_{in}$ which is sufficient to maintain an average 100 MV/m loaded accelerating gradient for a bunch population of $4.2 \times 10^9$
particles in a train of 312 bunches, which are spaced by 0.67 ns (8 rf cycles). To calculate the power along the structure numerically, the power per unit length, $f_N(z)$ can be defined as:

$$f_N(z) = \frac{dP_N}{ dz} = -\frac{\omega}{Q_N v_{gN}} P_N - \sqrt{\frac{\omega}{v_{gN} Q_N}} I \sqrt{P_N}$$

(6.13)

The entire structure length is divided in 9 intervals such that each step is $h = NL/9$. The power at every interval is calculated using a second order Runge-Kutta method.

$$P_{N+1} = P_N + h f_{N+1} \left[ P_N + \frac{h}{2} f_{N+1}(P_N) \right].$$

(6.14)

In this way for $N = 1$ to 9 the power at an interval of $h$ is obtained. Eq. 6.8 allows the accelerating field to be calculated for the power within that interval. Integrating the field from $z = 0$ to $z = NL$ gives the average accelerating gradient. By varying $P_{in}$, an accelerating gradient of 100 MV/m is obtained. In this way the loaded power and accelerating gradient are calculated. The unloaded power is then obtained by letting the beam current $I = 0$ in eq. 6.13 and the unloaded accelerating gradient is calculated subsequently (using eq. 6.8). The power flow for both the loaded and unloaded conditions in the test structure are illustrated in Fig. 6.7 and the corresponding accelerating field in Fig. 6.8.

---

5Bunch population and bunch spacing are discussed in Chapter 9 and 10 respectively.
Figure 6.7: Power in CLIC-DDS test structure

Figure 6.8: Accelerating electric field in CLIC-DDS test structure
6.2.2 Surface Electromagnetic Fields

It is important to minimise the surface electric and magnetic fields, in order to reduce the potential for electrical breakdown and to minimise the surface pulse temperature rise. Both must be kept to within experimentally determined limits. The normalised (with respect to the accelerating electric field) surface E-field in the test structure is evaluated using simulation tools and is illustrated in Fig. 6.9. The actual surface E-field both in loaded and unloaded conditions is displayed in Fig. 6.10. In general, a structure whose iris tapers down, experiences a large surface E-field towards the smaller iris end. However, changing the ellipticity of the iris, allows the surface E-field in the iris region to be reduced. This method is used to reduce the surface E-field in the CLIC-DDS test structure. This accounts for the distribution in Fig. 6.9 and Fig. 6.10. Also, within a given cell, the maximum of the surface E-field occurs in the iris region. It is very important to

Figure 6.9: Variation of the normalised electric field along the structure length.
design a structure with minimum surface E-field as excessive field may lead to electron emission in the structure.

Unlike the E-field, the normalised surface H-field reduces down the structure. This is due to the fact that the maximum of the surface H-field within a cell occurs in the vicinity of the cavity walls ($H \propto J_1(kr)$ [28] where $J_1$ is a first order Bessel function). Hence, in a given structure it reaches a maximum value towards the lower energy energy end of the structure i.e. the largest iris\textsuperscript{6}. The maximum of the normalised surface H-field in CLIC,DDS test structure is illustrated in Fig. 6.11. The corresponding variation of the maximum of surface H-field in loaded and unloaded conditions in the test structure is presented in Fig. 6.12.

\textsuperscript{6}In detuned structures the iris and cavity radii tapers down along the structure length hence the first cell in the structure has the largest iris and cavity radii.
Figure 6.11: Variation of normalised magnetic along the structure length

Figure 6.12: Surface magnetic field in CLIC_DDS test structure
As the H-field causes a current flux in the cavity walls which is dissipated due to ohmic losses, it leads to temperature rise in this region. The CLIC linacs are designed to operate for short pulses (∼250 ns). Every time the structure is excited with high power, the H-field dissipates in the cavity walls and increases the temperature of the cavity for this short period of time. This momentary rise in the cavity temperature is known as pulsed surface temperature rise and may be calculated as [83], [84]

\[
\Delta T(°K) = \sqrt{\frac{\mu_0 \omega f_p r_p^e}{2\pi \sigma \lambda \rho C_H (H_{sur}^{max})^2}} = 3.9 \times 10^{-12} \sqrt{f_{acc} r_p^e (H_{sur}^{max})^2} \tag{6.15}
\]

where \(\mu_0 = 4\pi \times 10^{-7} \text{(H/m)}\) is the permeability of free space, \(\omega = 2\pi f_{acc} \text{(GHz)}\) accelerating frequency, \(\sigma = 6 \times 10^7 \text{(Ω)}\) electric conductivity of pure copper, \(\lambda = 401 \text{ (W m}^{-1}\text{K}^{-1})\) heat conductivity, \(\rho = 8.96 \times 10^3 \text{(kg/m}^3\)) mass density, \(C_H = 385 \text{ (J kg}^{-1}\text{K}^{-1})\) specific heat, \(r_p^e\) corrected pulse length (ns). The pulse length for a rectangular shape is given by

\[
t_p = T_b + t_r + t_f \tag{6.16}
\]

\[
T_b = N_b \times t_b \tag{6.17}
\]

\[
t_f = \frac{L}{<v_g>} \tag{6.18}
\]

where \(t_r\) is the rise time and \(t_f\) is the filling time of the pulse, \(t_b\) is the inter bunch spacing and \(N_b\) is the total number of bunches in a train, \(L\) is the total structure length and \(<v_g>\) is the average group velocity of the individual cells in the structure. A small correction was introduced in the pulse length to account for the finite rise time and filling time [20].

\[
r_p^e = t_p - [t_f (1 - p_x) + t_r (1 - p_x)] \tag{6.19}
\]

\[
p_x = \frac{P_{out}^{\text{loaded}}}{P_{out}^{\text{unloaded}}} \tag{6.20}
\]
The profile of the corrected pulse length [20] for the CLIC DDS test structure is displayed in Fig. 6.13. The pulsed temperature rise in the CLIC DDS test structure is presented in Fig. 6.14. The pulsed temperature rise is an important criterion in high gradient linacs as it varies as the square of the product of the surface H-field and gradient. For CLIC, the maximum unloaded accelerating gradient is of the order of 130 MV/m [18], [19], [79]. Hence, it is essential to design the cavities with minimum surface H-field. The momentary rise in the cavity temperature may not be high but it rises abruptly with the arrival of the pulse then falls back\(^7\) and it repeats for every pulse. This continuous change in the cavity temperature affects the structure lifetime.

\(^7\)In practise, this corresponds to a temperature interval of 0\(^\circ\) to \(\sim\) 60\(^\circ\).

Figure 6.13: RF pulse profile
6.2.3 Modified Poynting Vector

When structures operate at high frequency and high gradient, it is observed that the rf breakdowns in these structures are severe. The rf breakdown is a complex phenomenon and has not clearly been understood due to the fact that several factors such as high surface E-fields, high surface H-fields (leading to pulsed temperature rise) and high accelerating gradient can cause breakdown. However, the other factors, that may cause breakdown are: surface impurity, multipacting of the iris, structure materials such as copper (Cu) and molybdenum (Mo). It indicates that several factors play a role. In [85] a new local field quantity termed as modified Poynting vector ($S_c$) is defined. This factor represents the complex power flowing out of the structure. It gives the limit of the high gradient structure performance.

Figure 6.14: Pulsed temperature rise in CLIC DDS test structure.
in presence of rf breakdowns, $S_c$ is defined as [85]

$$S_c = \Re\{S\} + g_c \Im\{S\} \quad (6.21)$$

where $S = E \times H$ is the Poynting vector and $g_c$ is the geometrical factor. In practice, for CLIC accelerating cells, this factor can be approximated to $1/6$ [85]. The normalised $S_c$ (with respect to the accelerating field square) for single cells of the CLIC DDS test structure is illustrated in Fig. 6.15. The variation of $S_c$ along the structure in the beam loaded and unloaded conditions is displayed in Fig. 6.16.

![Figure 6.15: Variation of modified Poynting vector along the structure length](image-url)
6.2.4 RF To Beam Efficiency And Overall Figure Of Merit

The rf to beam efficiency is also an important figure of merit, as it defines how efficiently the pulse energy is transferred into beam energy. The pulse energy is calculated in terms of the pulse power and pulse width as

$$E_{\text{pulse}} = P_{\text{in}} t_p^*.$$  \hspace{1cm} (6.22)

The beam energy is calculated in terms of the beam current $I_b$, beam voltage $V_b$ and pulse width $T_b$ as

$$E_{\text{beam}} = I_b V_b T_b$$
$$= \frac{Q}{I_b} \langle E_{\text{acc}} \rangle L N_b t_b$$
$$= n_b q N_b \langle E_{\text{acc}} \rangle L.$$  \hspace{1cm} (6.23)
where \( Q = n_b q \) is the total charge per bunch, \( n_b \) is the number of particles per bunch and the electronic charge \( q = 1.6 \times 10^{-19} \) C. The rf-to-beam efficiency is then calculated from eq. 6.22 and eq. 6.23 as

\[
\eta_{\text{rf-beam}} = \frac{E_{\text{beam}}}{E_{\text{pulse}}}
\]  

(6.24)

In the optimisation procedure it is important to define a figure of merit for the structure which will include all the rf properties of the structure. Several structures can then be compared just by comparing this single property i.e. the overall figure of merit. In optimising the CLIC structure, the overall figure of merit is defined [20] in terms of its rf-to-beam efficiency \( (\eta_{\text{rf-beam}}) \), number of particles per bunch \( n_b \) and luminosity per bunch crossing \( (L_{\text{box}}) \) and is as follows:

\[
\mathcal{F} = \eta_{\text{rf-beam}} \frac{L_{\text{box}}}{n_b}
\]

(6.25)

The calculation of \( L_{\text{box}} \) and \( n_b \) is explained in Appendix D.

### 6.3 Overall Design Constraints

In order to achieve high gradients for a feasible overall linac length to achieve 3 TeV centre of mass collision energy, a reasonably high input power is required. If the accelerating frequency is high then the structure dimensions are small and it takes relatively less input power to achieve the desired accelerating gradient. High frequency linear accelerators must be carefully designed to ensure that the high e.m. fields do not produce excessive wakefields or cause the structure to breakdown [18], [19]. For CLIC main linacs, the rf breakdown constraints are based on X-band experimental data, mainly coming from NLC/GLC X-band programme [18],[19] and CTF experiments. These constraints are based upon the semi empirical formulae and are as follows
1) Surface electric field:
\[ E_{\text{sur}}^{\text{max}} < 260 \text{ MV/m} \quad (6.26a) \]

2) Pulsed surface heating:
\[ \Delta T^{\text{max}} < 56 \text{ K} \quad (6.26b) \]

3) Power:
\[ P_{\text{in}} t_p^{1/3}/C_{\text{in}} < 18 \text{ MW ns}^{1/3}/\text{mm} \quad (6.26c) \]

where \( P_{\text{in}} \) is the input power, \( C_{\text{in}} \) is the circumference of the first regular iris and \( t_p^{\text{p}} \) is the time for which \( P_{\text{in}}/P_{\text{load}} > 0.9 \) \[8\] \[20\].

On the other hand the wakefields excited by the beam are divided into the short range and long range wakefields. The short-range wakefield is constrained according to the number of particles per bunch and the aperture size. Structures need to be designed in such a way that the transverse long-range wakefields, which kick the beam off axis should decay adequately before the arrival of the next trailing bunch. The allowable limit on the transverse long-range wakefield can be given as \[18\]
\[ W_T < \frac{6.67 \times 4 \times 10^9 <E_{\text{acc}}>}{n_b} \text{ V/pC/mm/m.} \quad (6.27a) \]

For CLIC_DDS we take \( n_b = 4.2 \times 10^9 \) and thus the maximum allowable transverse long-range wakefield is \( W_T < 6.3 \text{ V/pC/mm/m.} \) Now, if the long-range wakefield is plotted against the time scale, then the time at which it is below 6.3 V/pC/mm/m is the minimum possible inter-bunch spacing. If a structure is designed in such a way that the wakefield decay is rapid (for a given \( Q \)), it will allow a compact inter-bunch spacing which will improve the overall efficiency of the collider.

\[8t_p^{\text{p}} \approx T_b + \frac{\gamma}{3} \quad [84]\]
Chapter 7

Structure Design : Wakefield

Optimisation

In this chapter, simulations, calculations and optimisation of the wakefield from the first dipole band\(^1\) are discussed. This is the dominant band for X-band travelling wave structures operating with a \(2\pi/3\) phase advance per cell [46].

7.1 Single Cell Dipole Mode

The dipole E-field in a typical X-band single periodic accelerating cell is presented in Fig. 7.1. By taking advantage of the symmetry in the field pattern of this mode, simulations can be performed with only one quarter of the cell. This improves the efficiency of the calculations in terms of the required simulation time. In this case, boundary conditions are defined on the symmetry planes as E-symmetry and H-symmetry, to minimise the other multipoles excited. However, sextupole modes also satisfy this boundary condition and can get excited. Visual inspection of the

\(^1\)A band can be defined as a series of modes of same or different multipoles of a single cell at different phase advance per cell, typically a dispersion curve is defined as a band.
fields readily allows dipole and sextupole modes to be distinguished. We take only dipole frequencies into account for further calculations. In case of the accelerating mode, the cell is designed for a specific phase advance (of the fields) per cell \( \phi = 2\pi/3 \) in this case. The synchronous mode is excited at this particular phase. However, in a dipole mode the synchronous phase is not known. It is necessary to do simulations for various phase advance from \( \phi = 0 \) to \( \pi \) and plot the dispersion curve of frequency against phase. The phase at which the light line [29] intersects the dispersion curve is the synchronous phase. Then the synchronous frequency is obtained by performing a simulation at the synchronous phase. The dipole dispersion curves for detuned structure (DS or undamped) and DDS single periodic cell are represented in Fig. 5.4 and Fig. 5.6 respectively. A similar procedure is repeated to obtain synchronous frequencies of several other cells, typically seven to nine single cell synchronous frequencies are obtained by simulations and the rest
are interpolated. Now, using the postprocessor simulation tools, the kicks factors are obtained for these cells.

The maximum of the wakefield (at a position \( z = 0 \) i.e. immediately after the leading bunch) is equal to twice the average kick factor (eq. 4.11) and hence it is important to calculate the kick factors accurately. A description of the kick factor calculation is presented in Chapter 3. The loss factor of the dipole mode in an X-band single periodic cell for several offsets is illustrated in Fig. 7.2(a). The corresponding kick factor in this cell is displayed in Fig. 7.2(b). Once the single cell synchronous frequencies and kick factors are known for all cells in the structure, then the uncoupled mode wakefield is calculated using eq. 4.11. The coupled mode wakefield relies on the circuit model, details of which are discussed in Chapter 5. The wakefield in the structure must satisfy the beam dynamics criteria described in Chapter 6. Optimisation of the wakefield involves several steps and is discussed in the following sections.

Figure 7.2: Fiducial cell dipole mode properties

\[
\omega_{\text{syn}}/2\pi = 15.88 \text{ GHz} \\
\beta_{\text{mod}}(x) = 0.1285 x^2 + 8.87 \times 10^{-6} x^6 \\
K = 46.7 \text{ V [pC mm m]^{-1}}
\]
7.2 Distribution Function Of Cell Parameters

Maintaining a constant iris radius, in an accelerating structure consisting of N cells, will generate HOMs at the same frequency in each cell. In this case the transverse wakefield which is the modal sum of these modes will add coherently and the overall kick experienced by the beam will be maximised. In high frequency accelerators (X-band), often it is preferred to taper the iris radius of the cells along the structure. The first advantage is, it provides a constant accelerating gradient along the path [36] as described in chapter 3. The second advantage is, the HOMs are generated with a slight shift in their frequencies [15], [35], [45]. In the following sub-sections the wakefield decay in a non-tapered, linearly tapered and error function (erf) tapered structure is discussed. The erf tapering results in a Gaussian distribution of frequencies, which is the base of CLIC-DDS, is described in detail.

7.2.1 Identical Cell Structure : No tapering

Consider a single undamped cell of iris radius 3.95 mm. The lowest synchronous dipole mode excited in this cell is at 16.35 GHz having a kick factor of 53 V/pC/mm/m. Now we construct a 200 cell structure from this cell i.e. structure with identical cells. As there is no tapering, the synchronous frequencies and kick factors for all the cells are identical. The resultant wakefield in this structure is severe as there is no room for it to decay. The wakefield on the first few bunches in this structure is displayed in Fig. 7.3(a). It of course is practically constant. The wakefield over the entire bunch train is displayed in Fig. 7.3(b). In this case, wakefield decays to some extent due to the losses experienced by the finite conductivity of the accelerating cells which corresponds to a $Q \sim 6000$.

Slowly detuning the frequencies of successive cells in a precisely defined man-
ner ensures the modes do not add coherently. The wakefield decay in such a tapered structure is described in the next sub-section.

![Wakefield envelope for the first four bunches](image)

![Wakefield envelope over the entire bunch train](image)

Figure 7.3: Wakefield envelope of an identical cell structure

### 7.2.2 Linear Tapering

In order to study the effect of tapering the structure on the wakefields, a taper in the iris from 4.95 mm (first cell) to 2.15 mm (last cell) was introduced in an undamped structure. This tapering excites the lowest dipole synchronous frequencies in the range 15 GHz to 18.36 GHz with kick factors of 22 and 91.6 V/pC/mm/m in the first and the last cell, respectively. The linear tapering in the synchronous frequencies is displayed in Fig. 7.4(a) along with the kick factors in Fig. 7.4(b). The tapering has resulted in a decay of the wakefields and is illustrated in Fig. 7.5(a). The wakefield on the first trailing bunch is now \( \sim 20 \) V/pC/mm/m which is almost 5 times smaller than that of the identical cell structure. The minimum separation in the frequencies is \( \Delta f_{\text{min}} \sim 17 \) MHz. The finite number of modes eventually lead to recoherence of the wake at a location \( \Delta t_{\text{max}} = 1/\Delta f_{\text{min}} \). In this structure, we can see two recoherences at locations 60 ns and 120 ns within the total bunch train.
Figure 7.4: Linearly tapered structure

length and is illustrated in Fig. 7.5(b). Even though the tapering has resulted in

decaying the wakefield, the beam dynamics constraint requires no more than 6.3
V/pC/mm/m wake$^2$ on the first trailing bunch. The recoherence in the wakes can
be damped by either strong or moderate damping schemes. The strong damping
(Chapter 4) may be achieved at the cost of enhanced breakdown probability. Alter-
natively, a moderate damping scheme (Chapter 4) may be used. In the latter

$^2$for a bunch population of $4.2 \times 10^9$
case, the wake on the first few trailing bunches can be minimised by increasing
the dipole bandwidth, increasing the bunch spacing or by choosing a different ta-
pering scheme. Increasing the bandwidth will exacerbate the surface fields, while
increasing the bunch spacing will reduce the overall efficiency of the collider. In
[45] a detailed study of linear tapering and erf tapering (Gaussian distribution)
is presented. We chose a Gaussian distribution for the present structure and this
approach is discussed in the next sub-section, although other distributions may
allow a more rapid decay in the wakefield.

7.2.3 Error Function Tapering: Gaussian Distribution

The wakefield is proportional to the Fourier transform of the term Kick factor
weighted density function, $Kdn/df$, where $K$ is the kick factor, $n$ is the cell num-
ber and $f$ is the synchronous frequency. If we enforce a Gaussian distribution to
$Kdn/df$ then the wakefield will fall off in a nearly Gaussian fashion for a short
time scale [45]. As the prescribed distribution truncates at a finite bandwidth$^3$, the
wakefield start to recohere in a sinc-like ($\sin x/x$) manner. The detailed procedure
to enforce a Gaussian distribution to the cell parameters is as follows.

If the iris radii of the cells are varied as an erf with respect to the cell number
($n$ vs $a$) then the corresponding synchronous frequencies ($f$) will also follow the
erf pattern with respect to the cell numbers. The derivative of an erf is of course a
Gaussian function. In this case the derivative is the distribution of the frequencies
with respect to the cell number i.e. the density function ($dn/df$) will also be a
Gaussian. This procedure begins by designing the end cells and calculating their
lowest dipole synchronous frequencies using commercially available computer
codes, such as HFSS. The fiducial mid-cell is designed by taking the mean of the
parameters of the end cells namely, the iris radius, iris thickness and ellipticity of

$^3$If $dn/df$ distribution truncates at infinity, then the wakefield will fall like a Gaussian
the iris. It is subsequently tuned to get the correct accelerating frequency, followed
by the synchronous frequency calculation. Taking 3 cell iris radii and synchronous
frequencies, an erf distribution (eq. 7.1) is enforced on these parameters.

\[ D(x) = A + B \text{erf} \left[ \frac{x - x_c}{\sqrt{2}\sigma} \right] \quad (7.1) \]

where \( x_c \) is the central frequency (or iris radius) of the Gaussian distribution\(^4\), \( \sigma \)
is the standard deviation of the Gaussian distribution and \( A \) and \( B \) are constants.

It is not sufficient to rely on only 3 cells to represent a structure of \( N \) cells.
The iris radius is not the only parameter which is variable as there will be a signif-
ificant change in the synchronous frequencies with iris thickness and cavity radius
variation. Hence, it is important to know the synchronous frequency dependence
of these parametric changes. This motivates us to design four more fiducial cells,
two in each half of the structure, to get the correct frequency dependence of the
cell parameters. Once we have these seven fiducial cells scaled to synchronous
frequencies then we can calculate any intermediate cell parameters for our de-
sired synchronous frequency. The parameters of seven fiducial cells are presented
in Table 7.1 and the variation of these parameters as a function of synchronous
frequency is illustrated in Fig. 7.6 and Fig. 7.7. Using these fiducial cells we can
evaluate the dependence of the kick factor on synchronous frequency.

If we take derivative of eq. 7.1 and multiply by the kick factor variation, then
we get an asymmetric Gaussian distribution of the \( Kdn/df \). The reason for the
asymmetry is, variation of the kicks with frequency. This asymmetry may lead
to a non-optimum wakefield. To avoid this asymmetry, a Gaussian distribution to
\( Kdn/df \) is enforced as follows,

\[ K(f) \frac{dn}{df} = e^{-(f-f_c)^2/2\sigma^2}. \quad (7.2) \]

\[ \frac{dn}{df} = \frac{e^{-(f-f_c)^2/2\sigma^2}}{K(f)}. \quad (7.3) \]

\(^4\)The \( x_c \) is not necessarily the mid cell synchronous frequency

97
The synchronous frequencies are obtained iteratively. Now, integrating from cell to cell we get

\[ 1 = C \int_{f_1}^{f_N} \frac{e^{-(f-f_c)^2/2\sigma^2}}{K(f)} df \]  

(7.4)

where \( C \) is the constant and for an \( N \) cell structure it is given by

\[ C = \frac{N - 1}{\int_{f_1}^{f_N} \frac{e^{-(f-f_c)^2/2\sigma^2}}{K(f)}}. \]  

(7.5)

By fitting a polynomial\(^5\) to the kicks and synchronous frequencies from Table 7.1 we obtain the kick factor as a function of frequency \( K(f) \) as follows:

\[ K(f) = 7970.9 - 1505.81 f + 93.7104 f^2 - 1.91 f^3 \]  

(7.6a)

The end cell frequencies are known from our fiducial cell simulations, i.e. the geometry correspond to label A and G from Table 7.1: \( f_1 = 15.0 \) GHz, \( f_N = 18.37 \) GHz, \( f_c = \frac{f_1 + f_N}{2} = 16.68 \) GHz. For \( \Delta f = f_N - f_1 = 3.6 \sigma \)\(^6\) we solve eq. 7.4 for an

\(^5\)In this case a polynomial fit is used, in general a cubic spline fit can be used.

\(^6\)\( \sigma \) is varied in order to optimise the wake suppression on the first trailing bunch and is discussed later in the chapter.
initially specified $f_1$ and iteratively calculate the remaining frequencies. For the above specified values, the kick factor weighted density function is given by:

$$Kdn/df = 26.96 e^{-(0.69)(16.68+f)^2}$$

(7.7)

The resultant Gaussian distribution for a 200 cell structure (interleaved structure\(^7\)) is displayed in Fig. 7.8. It can be observed that the density function is no longer a Gaussian. However, the product of $K$ and $dn/df$ is Gaussian, and is symmetric around $f_c$. The dipole mode properties, namely the variation of the synchronous

\(^7\)Interleaving of structures is discussed later in this chapter.
Figure 7.8: Gaussian distribution of the kick factor weighted density function

frequencies and kick factors along the structure are displayed in Fig. 7.9(a) and
Fig. 7.9(b) respectively. The Gaussian distribution presented in Fig. 7.8 is illus-
trated in Fig. 7.10(a) with more details. Here the $\Delta f = f_N - f_i$ corresponds to the
lowest dipole bandwidth. In a given structure there will always be a finite number
of cells to represent the Gaussian distribution, thus the width of the Gaussian will
be truncated at a given bandwidth, and in this case it is 3.3 GHz. The wakefield
is calculated by taking the inverse Fourier transform of $K_{dn}/df$. If the Gaussian
distribution of $K_{dn}/df$ is truncated at infinity then the wakefield, as a result, will
fall off like a Gaussian. The wakefield of a true Gaussian distribution is calculated
using eq. 7.8 and is shown by the blue dashed curve in Fig. 7.10(b).

$$W_G(t) = 2\bar{K}\exp[-2(\pi\sigma t)^2]$$  \hspace{1cm} (7.8)

where $\bar{K}$ is the mean kick factor. However, in reality, due to the truncation of
the $K_{dn}/df$ distribution at a finite value, the wakefield of this distribution is no
longer Gaussian. However, it is given by the convolution of the inverse Fourier
transform of the product of Gaussian and rectangular function (red dashed line in Fig. 7.10(a)). This convolution results in a sinc-like (sinx/x) properties in the wakefield. The wakefield falls off like a Gaussian for a short time scale (~0.5 ns) and then observes ripples. The wakefield for a truncated Gaussian distribution is calculated as [87]

\[ W_{TG}(t, \Delta f) = 2K|\chi[t, \Delta f]| \]  

(7.9)

\[ \chi[t, \Delta f] = \frac{\Re\{erf[(\Delta f - 4\pi\sigma^2 t)/(2\sqrt{2}\sigma)]\}}{erf[\Delta f/(2\sqrt{2}\sigma)]} \]  

(7.10)

Eq. 7.9 is represented by the red curve in Fig. 7.10(b). In general, the wakefield is calculated using the modal sum method (eq. 4.11) described in Chapter 4 and is illustrated by the black curve in Fig. 7.10(b). The wakefield over a complete bunch train is illustrated in Fig. 7.10(c). However, eq. 7.9 provides a rapid means of computing the wake, as it requires only a frequency spread (unlike modal sum method which requires individual frequencies of the cells in a given structure).

The minimum separation of the frequencies of the modes in this 200 cell structure is \( \Delta f_{\text{min}} = 10.68 \) MHz. The finite number of modes eventually leads to recoherence of the wake at a position \( \Delta t_{\text{max}} \sim 93 \) ns, this recoherence position is clearly visible in 7.10(c). Optimisation of the wakefield using a Gaussian distribution is dis-

Figure 7.9: Dipole mode results of a structure with Gaussian distribution
cussed in the next section and the suppression of the recoherence of the wakefield is discussed in Chapter 9 using a moderate damping scheme.

Figure 7.10: Dipole mode results of a structure with Gaussian distribution
7.3 Optimisation Of A Gaussian Distribution

In the CLIC baseline design, the main accelerating bunch train contains 312 bunches, each separated by 6 rf cycles i.e. 0.5 ns [18]. The wakefield generated by the first bunch should decay within the arrival of the next trailing bunch (0.5 ns) to a minimum value specified by the beam dynamics constraint. In a moderately damped structure, forcing a Gaussian distribution to the cell parameters primarily aims at minimising the wake on the first few trailing bunches. The wake decay is governed by the bandwidth of the distribution and standard deviation of the Gaussian distribution.

7.3.1 Wakefield Optimisation By Varying Bandwidth

Here the bandwidth under consideration is that of the lowest dipole mode. In order to minimise the wakefield on the trailing bunches, we need a bandwidth as large as possible. Increasing the iris radius of the cell reduces its dipole frequency. Hence, ideally, we need a large input iris (cell 1) and a small output iris (cell N) to fulfil the beam dynamics criterion. Increasing the input iris will also increase the group velocity of the fundamental mode and it will require a higher input power to maintain the desired accelerating gradient. Several analyses have shown that the power absorbed in the structures is proportional to the square of the group velocity [75] - [77]. More power absorption will lead to unacceptable breakdown rates. Also, a high input power will cause high unloaded gradients in the structure and hence there will be very high surface fields on the cavity walls, which will also contribute to the breakdown rates. For these reasons, breakdown constraints in particular, put an upper limit to increasing the input iris radius. One possible means to increase the input iris while keeping the group velocity low, is to increase the iris thickness of the cell. However, this will reduce the shunt impedance of the
structure, and hence the structure will need more input power and will be less efficient. On the other hand, decreasing the output iris radius is limited by the fact that too small iris opening will lead to a concentration of fields in a smaller area and hence will enhance the surface E-field unacceptably. This may lead to electron emission towards the higher energy end of the structure. Furthermore, reducing the iris radius will make the short-range wakefields larger. With these limitations in mind, a range of bandwidths have been studied to meet the beam dynamics as well as rf breakdown criteria, and selected results are displayed in Fig. 7.11.

In Fig. 7.11(a) the wakefield is illustrated for the first four bunches. As seen from Fig. 7.11(a), the choice of 3.3 GHz is well justified as it experiences only 0.6 V/pC/mm/m of the wakefield on the first trailing bunch, though the overshoot on the second trailing bunch is higher (1.8 V/pC/mm/m) than the first trailing bunch. The other structure with bandwidth 1.7 GHz experiences much higher, and unacceptable wake on the first trailing bunch which is 74 V/pC/mm/m. It should be noted that, the maximum wakefield (wake at $t = 0$) for both the bandwidths is different. This is due to the fact that for these bandwidths, the average iris radii are different and hence the average kick factor varies (the maximum of the wakefield is twice the average kick factor). The wakefield over the complete bunch train is illustrated in Fig. 7.11(b). As the structure bandwidth reduces, the minimum frequency separation of the modes also reduces$^8$. Hence, the recoherence position moves further away.

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$^8$The mode spacing is not uniform, it is minimum towards the centre of the Gaussian distribution and maximum towards the end of the distribution.
Figure 7.11: Wakefield envelope for various dipole bandwidths
7.3.2 Sigma Optimisation

The standard deviation of the Gaussian distribution $\sigma$ plays an important role in deciding how rapidly the wakefield will decay once it is generated by the leading bunch or particle. The number of $\sigma$ of the bandwidth decides the wake experienced by the first trailing bunch ($W_1$) and how fast the wake is falling. Increasing the number of $\sigma$ results in reducing $W_1$ till it reaches a minimum value. At the same time, it reduces the amplitude of the ripples. Reducing the number of $\sigma$ makes the distribution more Gaussian-like and hence it results in a fast roll-off of the wake. However, it also results in large ripples on the trailing bunches. The best result for the total minimum wake will result in a large possible number of sigma for the given bandwidth. However, it will not have an optimum wake on the first trailing bunch and this is illustrated in Fig. 7.12(a). The first few bunches are sensitive to $\sigma$. Thereafter the wakefield is nearly insensitive to $\sigma$. This behaviour is illustrated in Fig. 7.12(b).
Figure 7.12: Wakefield envelope for various sigmas

(a) Wakefield envelope for the first four bunches

(b) Wakefield envelope over the entire bunch train


7.4 Interleaving Structures

For nominal CLIC operation more than 140,000 structures will be needed to achieve a final 1.5 TeV beam energy [47]. It is important to optimise the number of cells per structure from the perspective of the input power requirement. The rf breakdown constraints motivate a design with less cells per structure, in order to minimise the input power requirement for a given bunch population. However, a longer structure will have a better shunt impedance and better efficiency. This is because, as it is a normal conducting structure, the Ohmic losses will reduce the power available along the structure. However, demanding larger input power will cause enhanced surface fields on the cavity walls. On the other hand, the Gaussian distribution is better sampled with more cells per structure and hence the wakefield will decay more effectively.

Throughout the optimisation procedure for the Gaussian distribution, we have discovered that about 200 cells are needed to optimise the wakefield. In order to reconcile the sampling of Gaussian (more cells per structure) and input power (less cells per structure) requirement, interleaving of the neighbouring structure frequencies is employed in order to maintain 25 accelerating cells per structure. Hence, the input power requirement is kept reasonably low and eight structures are interleaved to keep the beam dynamics criterion satisfied. Eight structures have slightly different frequencies (non-identical structures). In general, the betatron wavelength is of the order of few meters. For example, in CLIC main linacs, the betatron wavelength towards the beginning of the lattice is 3.5 m and towards the end is 16.6 m [88]. As long as the total length of the interleaved structures is less than the betatron wavelength, the beam will not see different structures. It will effectively see only one structure, hence, the total number of modes or cells will be given by the summation of all the interleaved structures. As an example of the procedure, two-fold interleaving of the neighbouring structure frequencies is
illustrated in Fig. 7.13. Structure 1 in this case will be built with odd cell numbers (1 through 25) and Structure 2 will be built with even cell numbers (2 through 50). The beam will effectively experience a single structure consisting of 50 cells. Eight-fold interleaving was necessary in the present design and is illustrated in Fig. 7.14.
Figure 7.14: Wakefield envelope for various structure interleavings

(a) Wakefield envelope for the first four bunches

(b) Wakefield envelope over the entire bunch train
The wakefield optimisation procedure discussed in this chapter relies on the uncoupled wakefield calculations. Cell-to-cell coupling causes a frequency shift and is considered after finalising the end cell dimensions (based on the uncoupled wakefield calculations). This can be justified by the fact that in the procedure of wakefield optimisation, the first bunch is taken into account in order to minimising the wake experienced. Both the uncoupled and coupled predictions of the wakefields are in good agreement up to a few ns (the first trailing bunch arrives at 0.5 ns). Coupled mode calculations are discussed Chapter 5. After finalising the structure bandwidth i.e the end cell iris dimensions for the wakefield optimisation, the rf properties are calculated. Various structures, designed in the process of optimisation of the CLIC_DDS, are discussed in the next chapters.
Chapter 8

Detuned Structure: DS

This chapter focuses on the rf properties of purely detuned accelerating structures (DS). The influence of various parameters on the wakefield is analysed using both a coupled and uncoupled model.

8.1 Large Bandwidth Structure

Detailed analysis indicate that a structure with a dipole bandwidth of 3.3 GHz provides a sufficiently rapid wakefield decay which meets the beam dynamics constraint. A dipole bandwidth of ~3.3 GHz is obtained by tapering the iris radii from 4.95 mm to 2.15 mm. The parameters of seven cells in a structure of 25 cells are presented in Table 8.1. The mean deviation of the Gaussian distribution in terms of the bandwidth is $\Delta f = 3.6 \sigma$ and the detuning spread is $\sim 20\%$ [87] of the central frequency. The average aperture to accelerating wavelength ratio for this structure is $<a>/\lambda = 0.142$. This ratio is important as far as the rf breakdown rates are concerned. Reduced value of $<a>/\lambda$ results in a reduction of rf breakdown [35]. A damping $Q$ (ranging from 500 to 2000) is artificially imposed in these calculations.
Table 8.1: Single cell results of the lowest dipole band of a 3.3 GHz detuned structure.

<table>
<thead>
<tr>
<th>Cell no.</th>
<th>a (mm)</th>
<th>b (mm)</th>
<th>t (mm)</th>
<th>$v_g/c$ (%)</th>
<th>$f_{syn}$ (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.95</td>
<td>11.23</td>
<td>5.72</td>
<td>1.93</td>
<td>15.0006</td>
</tr>
<tr>
<td>5</td>
<td>4.53</td>
<td>10.79</td>
<td>4.83</td>
<td>1.86</td>
<td>15.5621</td>
</tr>
<tr>
<td>9</td>
<td>4.23</td>
<td>10.53</td>
<td>4.19</td>
<td>1.73</td>
<td>15.9745</td>
</tr>
<tr>
<td>13</td>
<td>3.95</td>
<td>10.34</td>
<td>3.65</td>
<td>1.62</td>
<td>16.3542</td>
</tr>
<tr>
<td>17</td>
<td>3.65</td>
<td>10.16</td>
<td>3.24</td>
<td>1.47</td>
<td>16.7502</td>
</tr>
<tr>
<td>21</td>
<td>3.26</td>
<td>9.99</td>
<td>2.4</td>
<td>1.3</td>
<td>17.2450</td>
</tr>
<tr>
<td>25</td>
<td>2.15</td>
<td>9.69</td>
<td>0.5</td>
<td>1.03</td>
<td>18.3657</td>
</tr>
</tbody>
</table>

Figure 8.1: A comparison between the uncoupled and coupled mode frequencies.
The synchronous and resonating frequencies, synchronous kick factors and coupling coefficients are the required parameters to calculate the coupled mode frequencies and kick factors. These circuit parameters are calculated using single cell simulation results as described in Chapter 5. A comparison between the uncoupled mode synchronous frequencies and coupled mode frequencies is presented in Fig. 8.1. The coupled mode frequencies are shifted by \(~200\) MHz with respect to the uncoupled mode frequencies due to cell-to-cell interactions. This resulted in \(~3.5\) GHz bandwidth of the coupled mode frequencies. The uncoupled

![Figure 8.2: Kick factors](image)

mode and coupled mode kick factors are illustrated in Fig. 8.2. The variation in the uncoupled kick factors is governed by the degree of taper in the iris. The coupled model allows cell-to-cell interactions to be taken into account and it is an accurate representation of the physics in the dipole mode interactions in the structure. The prescribed density function \((dn/df)\) and kick factor weighted density

114
function ($K_{dn/df}$) are illustrated in Fig. 8.3 [87]. The non-smooth behaviour of the coupled $K_{dn/df}$ is due to the variation in the coupled kick factors.

Figure 8.3: Mode density and kick factor weighted density function
The circuit parameters required for the coupled mode frequency calculations are the coupling coefficients (TM mode - \( \hat{\eta} \), TE mode - \( \eta \)), and resonating frequencies (TM mode - \( \hat{f}_0 \), TE mode - \( f_0 \)). These parameters are displayed in Fig. 8.4 and Fig. 8.5 respectively. The other parameters required for the calculation of the coupled mode kick factors are the fitting parameter \( \epsilon \), and synchronous phase \( \psi \).

The ratio of TE to TM mode content is indicated by \( \epsilon \) and this is calculated using eq. C.20. In this case, the lowest dipole mode is TM-like throughout the structure, and \( \epsilon \) is equal to unity. The synchronous phase follows the nature of the tapered distribution and is displayed in Fig. 8.6.

![Figure 8.4: Mode coupling coefficients of the first two dipole bands](image)
Figure 8.5: Resonating frequencies of the first two dipole bands

Figure 8.6: Synchronous phase of the lowest dipole band.
A comparison of the envelope of the wakefield using both the uncoupled and coupled calculations for a 3.3 GHz bandwidth is presented in Fig. 8.7 [87]. As expected, the uncoupled model prediction is in reasonably good agreement with the coupled calculations up to \( \sim 1 \) ns. However, 25 cells do not provide an ade-

\[
\begin{align*}
W_t/V[pC \text{ mm m}^{-1}] & - \text{ Uncoupled mode} \\
W_t/V[pC \text{ mm m}^{-1}] & - \text{ Coupled mode}
\end{align*}
\]

Figure 8.7: Envelope of a DS wakefield for a 25 cell non-interleaved structure.

equate sampling of the distribution. Interleaving successive structures will aid the sampling and hence will improve the wakefield suppression. As discussed in Chapter 7, eight-fold interleaving satisfies the beam dynamics criterion. In Fig. 8.8 and Fig. 8.9 the wakefield suppression in an eight-fold interleaved structure is presented [87]. In Fig. 8.10 the wakefield suppression with various damping \( Q_s \) is presented. It can be observed that a damping \( Q \) of \( \sim 1000 \) will be needed to satisfy the beam dynamics criterion.

In this structure, the normalised maximum surface E-field in the first iris is \( \sim 5 \). However, for this structure to satisfy the surface E-field constraint (eq. 6.26a) the maximum tolerable (unloaded) accelerating gradient cannot be more than 52
Figure 8.8: Wakefield envelope of an interleaved structure for first 24 bunches.

MV/m (= 260/5). This is approximately half of the specified nominal CLIC gradient. This unacceptably large surface E-field arises as a consequence of the large aperture required to maintain the 3.3 GHz bandwidth. The average iris radius in this design is 3.55 mm. It was necessary to increase the iris thickness to reduce the average group velocity to ~1.5%. In this structure the ellipticity of the iris was also changed to minimise the group velocity. Both these changes adversely affected the surface E-field. Though the beam dynamics criterion is well satisfied, this structure fails to satisfy the rf breakdown criterion. In order to meet this criterion, the dipole bandwidth is modified. A structure with a smaller bandwidth is discussed in the next section.
Figure 8.9: Wakefield envelope of an interleaved structure for entire bunch train.

Figure 8.10: Envelope of wakefield for an 8-fold interleaved structure over entire bunch train with several damping $Q$s.
8.2 Reduced Bandwidth Structure

The surface fields are reduced by modifying the minimum and maximum dimensions of the end cells. This impacts the dipole bandwidth severely and results in a bandwidth of ~0.84 GHz (CLIC_ZC1). The bandwidth of this structure is similar to the CLIC_G structure. Also, to facilitate a direct comparison with CLIC_G baseline design, the number of cells for CLIC_DS was reduced to 24. The bandwidth was reduced by reducing the iris radius of the first cell from 4.95 mm to 2.99 mm. This results in a Gaussian distribution whose standard deviation in terms of the bandwidth is \( \Delta f = 3.0 \sigma \) with a detuning spread of ~ 4.7% of the central frequency. The average aperture to accelerating wavelength ratio for this structure is \(<a>/\lambda = 0.1\). The parameters of the ZC1 structure [89] are presented in Table 8.2. This structure still has a surface E-field which is too large to meet the breakdown constraint imposed. However, modifying the ellipticity of the iris thickness, reduces the surface E-field significantly [90] (CLIC_ZC2). Parameters of the ZC2 structure are presented in Table 8.3 and the rf properties along the structure are

<table>
<thead>
<tr>
<th>Cell no.</th>
<th>a (mm)</th>
<th>b (mm)</th>
<th>t (mm)</th>
<th>( v_g/c ) (%)</th>
<th>( f_{syn} ) (GHz)</th>
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<td>9.68</td>
<td>0.83</td>
<td>0.83</td>
<td>18.40</td>
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presented in Fig. 8.11 [90]. The average aperture size of this structure is 2.5 mm. As the average aperture is small, the bunches can be populated only up to $2.9 \times 10^9$ particles per bunch. The input power requirement of the structure is only about 47 MW due to a reduced beam loading. Once the manifold geometry is employed, it perturbs the cavity walls and the pulsed temperature rise is expected to increase by a factor of $\sim 2$ [91]. Detailed rf parameters of both the CLIC_ZC structures are presented in Table 8.4. A comparison of CLIC_G structure with CLIC_ZC structures is presented in [90].

Table 8.3: CLIC_ZC2 structure parameters

<table>
<thead>
<tr>
<th>Cell no.</th>
<th>$a$</th>
<th>$b$</th>
<th>$t$</th>
<th>$v_g/c$</th>
<th>$f_{syn}$</th>
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Figure 8.11: RF structure properties of ZC₂ structure.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>CLIC ZC1</th>
<th>CLIC ZC2</th>
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<tbody>
<tr>
<td>$&lt;a&gt;/\lambda$</td>
<td>–</td>
<td>0.102</td>
<td>0.1</td>
</tr>
<tr>
<td>First, last iris radius ($a$)</td>
<td>mm</td>
<td>2.99, 2.13</td>
<td>2.87, 2.13</td>
</tr>
<tr>
<td>First, last iris thickness ($t$)</td>
<td>mm</td>
<td>1.6, 0.7</td>
<td>1.6, 0.7</td>
</tr>
<tr>
<td>First, last cell group velocity ($v_g/c$)</td>
<td>%</td>
<td>1.49, 0.83</td>
<td>1.45, 0.83</td>
</tr>
<tr>
<td>First, last cell shunt impedance ($R'$)</td>
<td>MΩ/m</td>
<td>107, 138</td>
<td>108, 138</td>
</tr>
<tr>
<td>Filling ($t_f$)</td>
<td>ns</td>
<td>56.8</td>
<td>58.6</td>
</tr>
<tr>
<td>Peak input power ($P_{in}$)</td>
<td>MW</td>
<td>48</td>
<td>47</td>
</tr>
<tr>
<td>Number of bunches in the train ($N_b$)</td>
<td>–</td>
<td>312</td>
<td>312</td>
</tr>
<tr>
<td>Number of particles per bunch ($n_b$)</td>
<td>$10^9$</td>
<td>3.0</td>
<td>2.9</td>
</tr>
<tr>
<td>$E_{sur}^{max}$</td>
<td>MV/m</td>
<td>285</td>
<td>231</td>
</tr>
<tr>
<td>$\Delta T_{max}$</td>
<td>K</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>RF-beam efficiency ($\eta_{rf-beam}$)</td>
<td>%</td>
<td>27.09</td>
<td>26.11</td>
</tr>
<tr>
<td>$P_{in} t_p^{1/3} / C_{in}$</td>
<td>MW ns$^{1/3}$/mm</td>
<td>14.07</td>
<td>14.36</td>
</tr>
</tbody>
</table>

After satisfying the rf breakdown criteria in the CLIC ZC2 structure, we investigated the wakefields excited in the structure. As the lowest dipole bandwidth is reduced from 3.3 GHz to 0.84 GHz we expect poorly damped wakefields. As described in the previous section, the circuit parameters required to obtain the coupled mode wakefields are calculated first. In these calculations it is observed that the wakefield experienced by the first two trailing bunches is unacceptable large. In this case, if we increase the bunch spacing by a factor of 3 (i.e. to 1.5 ns) compared to the CLIC_G bunch spacing, then both the constraints will be satisfied. However, this also impacts the rf-to-beam efficiency of the collider, which will be 8 to 10 %. Provided the cells are carefully designed and manufactured such that the bunches are located at the zero crossing in the wakefield, the beam dynamics
Figure 8.12: Amplitude of the wakefield over the first four trailing bunches, indicating the zero crossing of the bunches.

constraint is satisfied. The amplitude of the wakefield under these conditions is illustrated in Fig. 8.12. It can be observed that the amplitude of the wakefield at the location of the bunches is almost zero. The phase of the wakefield experienced by the bunches is governed by the average synchronous frequency. The zero crossing of the bunches is maintained by tuning the structure irises. The accelerating frequency is retained by tuning the cavity radius. The zero crossing of the wakefield is important for the first few bunches only. Because, for a reduced bandwidth a damping $Q \sim 500$ will be sufficient to damp the wakefield on the remaining bunches. In Fig. 8.13 the wakefield amplitude for an 8-fold interleaved CLIC.ZC2 structure is presented with several artificially imposed damping $Q$s.
Figure 8.13: Envelope of wakefield over the entire bunch train.
8.3 Summary

In this chapter a trade-off between the rf breakdown and beam dynamics constraints has been discussed in order to design a structure capable of withstanding high accelerating gradient. The structure with a large bandwidth provides an excellent suppression of the wakefield, however, it is limited by severe surface fields. The zero crossing scheme offers a solution to satisfy both the rf breakdown and beam dynamics criteria. However, the fact that more than 140,000 structures are required for CLIC [47], will result in very stringent fabrication tolerances if the zero crossing scheme is adopted. A possible solution, which will satisfy both the constraints at the cost of a few percent reduction in the rf-to-beam efficiency is discussed in the next chapter. The geometry discussed in the next chapter incorporates damping manifolds. The pulsed temperature rise due to magnetic field enhancement in the vicinity of the manifold coupling slots is an important effect and is also discussed.
Chapter 9

A Damped and Detuned Structure With Circular Geometry: CLIC_DDS_C

A trade-off between the rf breakdown and beam dynamics constraints leads to the bandwidth considered in this chapter. Herein circular cells are used and HOM damping manifolds discussed. This structure is known as circular cell damped and detuned structure, CLIC_DDS_C.

9.1 Geometry of CLIC_DDS_C

In CLIC_DDS_C, the iris radii tapers down from 4.0 mm to 2.3 mm. This taper results in a lowest dipole bandwidth of ~2.3 GHz, which is considered a moderate bandwidth compared to the 0.83 GHz (ZC) and 3.3 GHz structures. Details of the cell geometry are presented in Fig. 9.1 and Fig. 9.2.
Figure 9.1: Quarter symmetry CLIC\_DDS\_C cell geometry

Figure 9.2: CLIC\_DDS\_C cell geometry: Left - details of the side view and right - details of the manifold

129
The structure is designed by relying on seven fiducial cells. All the design parameters, such as iris and cavity radii and iris thickness, are mapped to synchronous frequency. The intermediate cell parameters are obtained from interpolation fits. Details of seven cell parameters are illustrated in Fig. 9.3 (Table E.1).

![Graph showing design parameters of seven fiducial cells of CLIC_DDS_C. All parameters are in mm except \( \epsilon \) which is dimensionless.](image-url)

Figure 9.3: Design parameters of seven fiducial cells of CLIC_DDS_C. All parameters are in mm except \( \epsilon \) which is dimensionless.
As discussed in the earlier chapters, a taper in the iris radius is necessary to produce the constant gradient structure and to provide a spread in the dipole frequencies. The iris thickness also tapers down and maximises the shunt impedance. The parameter $\epsilon$ defines the ellipticity of the iris. For the first cell, $\epsilon = 1$ (circular). However, it increases along the structure to minimise the surface E-field in the vicinity of the iris. In contrast, the manifold radius $R_m$ reduces along the structure, in order to ensure the cut-off frequency of the manifold is above that of the accelerating frequency. This ensures accelerating mode will not propagate through the manifolds. If $R_m$ is invariant, the other parameters must be changed to ensure the manifold cut-off is above the accelerating mode. The manifolds sample the dipole field through coupling slots. The manifold slots in the close vicinity of the dipole fields will enhance coupling. The parameter $R_c$ defines the radial distance of the coupling slots from the electrical centre of the cavity. This parameter can extend up to the cavity iris. However, in this extreme, the slot will cut into the iris and will experience high E-field (as in the case of SICA structures [18]). In CLIC_DDS_C we have chosen $R_c$ for the first cell as 6.2 mm. Reducing it further will increase the dipole coupling. However, mechanical fabrication will be difficult in this case. Increasing $R_c$ will reduce the dipole coupling. The TE-to-TM mode ratio of the dipole fields gets stronger along the structure, hence $R_c$ needs to be increased along the structure to maintain the critical coupling throughout the structure. In order to minimise the surface fields at the sharp corners of the tip of the manifold coupling slots, a rounding radius $r_1 = 0.85$ mm is provided. The width of the manifold slot is $h_1$. Increasing the width will improve the coupling, however, at the same time it will perturb the cavity walls and will lead to an increased pulsed temperature rise.

The E-fields excited in CLIC_DDS_C corresponding to monopole, manifold and dipole modes are illustrated in Fig. 9.4(a), Fig. 9.4(b) and Fig. 9.4(c) respec-
After finalising the geometry of the fiducial cells, the fundamental rf properties of the individual cells are studied. These properties are illustrated in Fig. 9.5 as a function of the synchronous frequency. It should be noticed that the structure parameters are designed to vary in an erf fashion in order to maintain the erf distribution of the synchronous frequencies. Hence, the variation of these parameters with synchronous frequency is nearly linear (as can be observed from Fig. 9.5). However, the surface E-field reduces along the structure almost linearly up to 17.5 GHz, before rising sharply. This can be understood in terms of the variation between the iris radii and thicknesses of the last fiducial cell with respect to the penultimate fiducial cell. As both the iris radius and thickness reduce significantly, it leaves a very small surface area near the iris region and thus experiences a concentration of the fields. It was necessary to maintain this sharp change in order to preserve the erf distribution.

9.2 Fundamental Mode RF Properties Of CLIC_DDS_C

(a) Fundamental mode \( \omega/2\pi = 11.994 \text{ GHz} \) (b) Manifold mode \( \omega/2\pi = 14.4 \text{ GHz} \) (c) Lowest dipole mode \( \omega/2\pi = 15.0 \text{ GHz} \)

Figure 9.4: E-fields in CLIC_DDS_C cell
Figure 9.5: RF parameters of seven fiducial cells of CLIC_DDS_C versus synchronous frequency.

An 8-fold interleaved structure is necessary to enhance the wakefield suppression to the acceptable level defined by the beam dynamics criterion. The rf parameters of the first and the last structure are illustrated in Fig. 9.6 and Fig. 9.7 respectively. The number of particles per bunch used for these calculations are $4.2 \times 10^9$ for 312 bunches (discussed later in this section). The average filling time of the 8 structures is 41.6 ns with a rise time of 23 ns\(^1\). The pulsed temperature rise

---

\(^1\)The rise time is approximately taken as 23 ns which is similar to the baseline CLIC design.
is calculated for the corrected pulse length of $r_p = 248.8 \, \text{ns}^2$.

Figure 9.6: RF structure properties of the first out of 8 structures of CLIC_DDS_C.

$^2r_p$, for 8 structures is nearly similar as it is dominated by the total bunch train length ($T_b$).
Figure 9.7: RF structure properties of the last structure of CLIC_DDS_C.
The input power requirement of the 8 structures for the above mentioned rf parameters to maintain the loaded accelerating gradient of 100 MV/m is illustrated in Fig. 9.8. As the iris radii taper down the structure, the $Q$ and $R'/Q$ improve and $v_g$ decreases. All these changes help reducing the input power requirement for a given gradient (eq.6.8). Hence, the required input power reduces for the structures, as the average iris of these structures reduces. The average power required for

\[
\langle P_{in}\rangle = 72.85 \text{ MW}
\]

\[
\langle \eta_{rf-beam} \rangle = 23.14 \%
\]

Figure 9.8: Peak input power and rf-to-beam efficiency CLIC\_DDS\_C.

the 8 structures is $\sim 73$ MW which is about 16% higher than that of the CLIC\_G structure. This is due to the fact that the average aperture of the CLIC\_DDS\_C (3.15 mm) is $\sim 15\%$ larger than that of the CLIC\_G (2.75 mm). In addition to this, the bunch charge of the CLIC\_DDS\_C ($4.2 \times 10^9$) is $\sim 13\%$ higher than that of the CLIC\_G ($3.72 \times 10^9$) which requires more input power. The average aperture of the CLIC\_DDS\_C allows us to populate the bunches up to $5 \times 10^9$. However, in this case, the structure will require an input power of about 76 MW to achieve the
desired accelerating gradient. The higher input power will cause severe surface fields on the cavity walls. The number of particles per bunch is reduced in order to alleviate this condition to some extent.

The bunch spacing in CLICDDS_C is 8 rf cycles compared to 6 in CLIC_G. The increase in the bunch spacing allows the wakefield to be damped to the acceptable level before the arrival of the first trailing bunch. On the other hand, the increase in the bunch spacing reduces the efficiency of acceleration. The rf-to-beam efficiencies of the 8 structures are illustrated in Fig. 9.8. The average $\eta_{rf-beam}$ of CLICDDS_C is $\sim 23\%$. It is inversely proportional to the input power and hence it linearly increases from structure to structure for a linearly decreasing power requirement.
The surface E-fields in 8 structures of CLIC_DDS_C are illustrated in Fig. 9.9. The maximum surface E-field, as expected, occurs in the vicinity of the last iris of the last structure and is 320 MV/m. This is an unacceptable E-field as it is 23 % higher than the allowed limit. The cavity geometry is further optimised to bring down the surface E-field.

Figure 9.9: Surface E-field in unloaded condition in CLIC_DDS_C structures.
The pulsed temperature rise in 8 structures of CLIC_DDS_C is illustrated in Fig. 9.10. The maximum pulsed temperature rise occurs in the first cell of the first structure and is 72° K which is ∼29% above the acceptable limit. The variation in temperature rise along the structure is due to the variation in H-field (Fig. 9.5) and unloaded accelerating gradient (Fig. 9.6). Means to reduce this will be discussed in the next chapter.

Figure 9.10: Pulsed temperature rise in unloaded condition in 8 CLIC_DDS_C structures.
9.3 Wakefield Suppression in CLIC_DDS_C

The variation of the uncoupled kicks as a function of synchronous frequency is illustrated in Fig. 9.11. The synchronous frequencies for the 8-fold interleaved structures are presented in Fig. 9.12. The dipole bandwidth of the structure is $\Delta f = 2.33$ GHz. The mean deviation of the Gaussian distribution in terms of the bandwidth is $\Delta f = 3.6 \sigma$. The detuning spread is $\Delta f/f_c = 13.7 \%$. The average iris radius to wavelength ratio of the structure is $<a>/\lambda = 0.126$. As

![Figure 9.11: Synchronous kick factors of seven fiducial cells of CLIC_DDS_C.](image)

CLIC_DDS_C geometry incorporates manifolds, we utilised a double chain circuit model equipped with manifold representation (by a transmission line) to predict the coupled mode wakefields excited in the structure [35]. Details of the circuit model are discussed in Appendix C. The calculation of the coupled mode wakefield begins with calculating various circuit parameters using eq. 5.19. The cut-off
frequency \( (f_c) \), TE-TM mode resonating frequencies \((f_0 - \hat{f}_0)\) and the cross coupling term \( A \) are illustrated in Fig. 9.13(a). The terms involving coupling of the dipole mode to the manifold, namely the shunt capacitance of the manifold \( \alpha \), the coupling coefficient of the manifold \( \Gamma \) and the width of the avoided crossing \( \Gamma_x \) are presented in Fig. 9.13(b). By changing \( R_c \) along the structure, \( \Gamma \) is maintained nearly constant. As \( \Gamma_x \) reduces down the structure a weak coupling occurs in that region. The nature of the dipole modes and their coupling with the manifold can be understood by studying the dispersion curves of the dipole modes and this is illustrated in Fig. 5.6. The modes excited at the phase advance per cell of 0, \( \pi \) and synchronous phase are important modes. The fourth important mode is the one excited at the avoided crossing \((f_x)\), where the lowest dipole mode and the manifold mode avoid crossing (Fig. 5.6). These four modes give an indication of how the fields are coupling out of the structure. The frequencies of these modes
Figure 9.13: Circuit parameters of seven fiducial cells of CLIC_DDS_C are illustrated in Fig. 9.14. The lowest HOM excited at the zero phase advance per cell is the manifold mode. At the avoided crossing, due to manifold coupling, this mode is coupled out of the accelerating cells. The synchronous modes in the structure are excited very close to the \( \pi \) mode.
The calculation of the circuit parameters is followed by the spectral function calculations (Appendix C). In calculating the spectral function realistic losses in the copper cavities are considered. We have used the lowest dipole $Q_{cu} = 5800$\footnote{Calculated for single infinitely periodic cells using HFSS} for 191 cells. The last cell is a special cell as it is provided with a HOM coupler and vacuum ports. This last cell has a low $Q$ [92]. In calculating the spectral function of CLIC\_DDS\_C we have used $Q_N = 36$\footnote{Based on NLC $Q_N$}. The spectral function of the 8-fold interleaved CLIC\_DDS\_C is illustrated in Fig. 9.15. Each spike in the spectral function corresponds to one coupled mode in the structure. The width of the spike corresponds to the coupling of the dipole mode to the manifold. The broader the width, the stronger the coupling will be. The dipole $Q$ of the modes are calculated by fitting a Lorentzian to each mode of the spectral function [74].
Figure 9.15: Spectral function of the 8-fold interleaved CLIC_DDS_C. The black dashed curve indicates the designed $K_{dn}/df$.

The spectral function indicates that modes towards the lower frequency end are not well damped. The wakefield excited in CLIC_DDS_C is illustrated in Fig. 9.16. Here a comparison between the detuned structure (DS) and the manifold damped structure (CLIC_DDS_C) wakefield is presented. It can be seen from Fig. 9.16 that the structure satisfies the beam dynamics criterion as the wakefield on all the bunches is well within the acceptable level.
9.4 Conclusion

The structure considered in this chapter had a lowest dipole bandwidth of 2.3 GHz. The wakefield meets the beam dynamics constraint. However, the surface fields are still unacceptable from the perspective of breakdown and structure lifetime. The overall parameters of CLICDDS.C are summarised in Table 9.1. Means to reduce the surface fields are discussed in the next chapter.
Table 9.1: Overall parameters of CLIC-DDS_C

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Designed value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(&lt;a&gt;/\lambda)</td>
<td>0.126</td>
<td>–</td>
</tr>
<tr>
<td>First, last iris radius ((a))</td>
<td>4.0, 2.3</td>
<td>mm</td>
</tr>
<tr>
<td>First, last iris thickness ((t))</td>
<td>4.0, 0.7</td>
<td>mm</td>
</tr>
<tr>
<td>First, last (Q)</td>
<td>4708, 6602</td>
<td>mm</td>
</tr>
<tr>
<td>First, last cell shunt impedance ((R'))</td>
<td>47.89, 131.77</td>
<td>MΩ/m</td>
</tr>
<tr>
<td>First, last cell group velocity ((v_g/c))</td>
<td>2.06, 1.07</td>
<td>%c</td>
</tr>
<tr>
<td>Number of particles per bunch ((n_b))</td>
<td>4.2</td>
<td>(10^9)</td>
</tr>
<tr>
<td>Average iris to wavelength ratio (&lt;a&gt;/\lambda)</td>
<td>0.126</td>
<td>–</td>
</tr>
<tr>
<td>Number of cells per structure</td>
<td>24</td>
<td>–</td>
</tr>
<tr>
<td>Number of structures per interleaving</td>
<td>8</td>
<td>–</td>
</tr>
<tr>
<td>Bunch separation ((t_b))</td>
<td>0.67 (8)</td>
<td>ns (rf cycles)</td>
</tr>
<tr>
<td>Number of bunches in the train ((N_b))</td>
<td>312</td>
<td>–</td>
</tr>
<tr>
<td>Filling ((t_f)), rise ((t_r)) time</td>
<td>41.5, 23</td>
<td>ns</td>
</tr>
<tr>
<td>Pulse length ((t_p, t_p^r, t_p^p))</td>
<td>272.6, 248.8, 216.4</td>
<td>ns</td>
</tr>
<tr>
<td>Peak input power ((P_{in}))</td>
<td>73</td>
<td>MW</td>
</tr>
<tr>
<td>(E_{sur}^{max})</td>
<td>320</td>
<td>MV/m</td>
</tr>
<tr>
<td>(\Delta T^{max})</td>
<td>72</td>
<td>°K</td>
</tr>
<tr>
<td>(P_{in} t_p^{1/3}/C_{in})</td>
<td>17.4</td>
<td>MW ns(^{1/3})/mm</td>
</tr>
<tr>
<td>Average rf-beam efficiency ((\eta_{rf-beam}))</td>
<td>23</td>
<td>%</td>
</tr>
<tr>
<td>Dipole bandwidth (\Delta f)</td>
<td>2.33</td>
<td>GHz</td>
</tr>
<tr>
<td>Standard deviation of Gaussian distribution</td>
<td>(\Delta f = 3.6 \sigma)</td>
<td>GHz</td>
</tr>
<tr>
<td>Detuning spread (\Delta f/ f_c)</td>
<td>13.7</td>
<td>%</td>
</tr>
</tbody>
</table>
Chapter 10

A CLIC DDS With Elliptical Cell Geometry

Here the surface field in the vicinity of the manifold slot is investigated by modifying the geometry of the outer cell wall. Various ellipticities are explored. The manifolds discussed in the previous chapter are coupled to accelerating cells by slots which cut into the cells to sample the HOM fields. Increasing the width of the slots and increasing the penetration into the cells are two means available to increase the coupling and hence enhance the wakefield suppression. However, a large slot width causes an enhanced current in the cavity wall and hence can dramatically increase the H-field. This may lead to unacceptably large surface pulsed temperature rise. In the CLIC-DDS-C study it has been observed that, the manifold slots increase the H-field by $\sim 50\%$. Furthermore, the irises have a relatively small radius and thickness. This enhances the E-field in the vicinity of the iris. Means to reduce the excessive fields are explored in this chapter. Various geometries of accelerating cells are discussed with a view to optimise the surface e.m. fields. Details of the optimised geometry, rf and wakefield properties are discussed in the following sections. Mechanical problems involved in building the
structure and possible solutions are discussed in the penultimate section. Considering a range of issues from a physics and mechanical engineering point of view, the last section concludes our study on CLIC_DDS_E.

10.1 Geometry Optimisation To reduce Surface Fields

The H-field in a circular cell is uniformly distributed along the surface of the cavity wall. When the manifold is coupled, it enhances the field in the vicinity of the slot. If a cavity geometry is such that the H-field along the cavity wall is non-uniform before introducing the manifold (un-damped cell), then the manifold can be introduced in the wall where the H-field is minimum. A detailed study of different geometries to minimise the H-field enhancement is presented in the next sub-section. The surface E-field is optimised by changing the average iris thickness of the structure. The change in the iris thickness changes the rf parameters such as the shunt impedance, dipole bandwidth and efficiency of acceleration in particular. An optimisation scan of the iris thickness is discussed in the last sub-section.

10.1.1 H-field Optimisation

In order to re-distribute the surface H-field, various geometries such as rectangular, convex and concave elliptical geometries were proposed [93], [94]. A detailed sketch comparing the cavity wall of several geometries is presented in Fig. 10.1. In this case, the ellipticity of the cavity wall is defined in terms of the parameters $A$ and $B$ in such a way that a circular geometry gives an ellipticity of 1.

$$\epsilon = (\sqrt{2} - 1) \frac{A}{B} \quad (10.1)$$
Also, for a rectangular cell $\epsilon = \infty$. For $\epsilon > 0$ the wall is convex elliptical and for $\epsilon < 0$ it is concave. The maximum H-field occurs in the first cell. Hence, the

Figure 10.1: A quarter symmetry undamped cell wall comparing various shapes

simulations are focussed on reducing the H-field in this cell. In these simulations $a$ and $t$ are both set to 4 mm. Details of various (quarter symmetry un-damped) geometries and their H-field distributions along the cell wall are presented in Table F.1. As the maximum in the H-field observed in concave elliptical shapes is higher than the convex, the latter is focussed on in reducing the H-field. The surface H-fields on the cavity walls for various convex ellipticities along with the reference field\(^1\) are presented in Fig. 10.2 [80]. The field on the centre of the quarter wall is minimum for $\epsilon = 0.41$. However, away from the centre of the quarter wall, the

\(^1\)The field on the circular wall is defined as a reference.
field rises sharply. At a location where the manifold coupling slot is intended to be inserted, the field is considerably higher than that of the circular shape. This undamped geometry gives a coarse indication of the distribution of the H-field. A detailed understanding of the enhancement needs a complete geometry equipped with manifold slots and this is illustrated in Fig. 10.3 [80]. It is clear that though the field at the centre of the quarter cavity wall (contour length = 0) is minimum for a circular cell, the enhancement in the field due to manifold slots is maximum. For a geometry with $\epsilon = 0.82$ the field in the centre is 5% more than the circular shape, however, the overall field is reduced by 17.6%. For $\epsilon = 1.38$ the field in the centre is 15.8% higher however, the overall field is reduced by 21.8%. This is because, in this case, the maximum of the H-field occurs at the centre of the quarter cavity wall and in other geometries it is in the vicinity of the manifold.
10.1.2 E-field Optimisation

In optimising the H-field, only the first cell geometry is optimised. The rest of the cells are designed with the same ellipticity. In order to minimise the surface E-field on the iris, the iris ellipticity\(^2\) and thickness are varied. The ellipticity of the first iris is \(\epsilon_1 = 1\) and for the last cell is \(\epsilon_N = 2\). The variation of \(\epsilon\) along the structure is linear. Reducing the ellipticity will increase the surface E-field, whereas increasing it will also increase the group velocity. The end cell ellipticities are chosen in order to optimise the group velocity (first cell) and surface E-field (last cell). In principle, the ellipticity of the last cell can be increased. However, as

\[^2\]This is the ellipticity of the iris thickness and is different than the ellipticity of the cavity wall discussed in the previous sub-section.
the iris is already thin, a large ellipticity may not be mechanically feasible. The E-field is minimised by varying the iris thickness. In order to study the dependence of the rf properties on iris thickness, a coarse sampling of the simulation is performed. In each case, simulations are performed for only the end cells and the middle cell to represent a complete 24 cell structure. The iris thickness of the first cell is kept fixed at 4.0 mm. The last iris thickness is allowed to vary in the range 0.7 mm to 1.9 mm. The middle cell iris thickness is ascribed as the mean of the end cell iris thicknesses. Five different structures, each represented by 3 cells, with an average iris thickness ranging from 2.35 mm to 2.95 mm are studied. The results are illustrated in Fig. 10.4 [80]. All the mentioned rf parameters are com-

![Figure 10.4: RF parameters versus iris thickness (t)](image)

Figure 10.4: RF parameters versus iris thickness (t)
pared with respect to a reference structure whose cavity wall is elliptical and has the design parameters of CLIC_DDS_C e.g., $t_1 = 4.0$ mm and $t_{24} = 0.7$ mm.

As expected, it is clear that reducing the iris thickness results in an enhanced E-field. This is due to the smaller surface area left for power dissipation leading to a concentration of E-field in this region. After an average iris thickness of $\sim 2.65$ mm, there is no significant reduction in the surface E-field. Another consequence of changing the iris thickness is $v_g$ and $R'/Q$ change. The group velocity is a strong function of $t$. However, $R'/Q$ is relatively less sensitive in the range chosen.

The input power needed to achieve acceleration is directly proportional to group velocity and inversely proportional to $R'/Q$. From the perspective of wakefields, increasing $t$ reduces the lowest dipole bandwidth. From the beam dynamics point of view a bandwidth of $\sim 2.3$ GHz is necessary. Reducing the bandwidth causes a less effective suppression of the wakefield for the first few bunches.

Bearing in mind these constraints, an average iris thickness of 2.65 mm ($t_1 = 4.0$ mm and $t_{24} = 1.3$ mm) has been chosen. This reduces the surface E-fields by $\sim 30\%$ and bandwidth by $\sim 3\%$. The input power requirement reduces by $\sim 3\%$ hence the efficiency increases by the same amount with respect to the reference structure.

### 10.2 Geometry of CLIC_DDS_E

The CLIC_DDS_E geometry is optimised with the cavity wall ellipticity of 1.38 and iris thickness taper from 4.0 mm to 1.3 mm. As the DDS geometry consists of various ellipticities and manifolds, the structure is complex to define mesh using computational tools. To optimise the computational time and memory needed in the simulations, we make use of the symmetry of the geometry and use an 1/8 th
of a cell (45 deg. slice) for the fundamental mode\(^3\). The geometry of the 1/8 th of the CLIC\_DDS\_E cell is presented in Fig. 10.5.

![Figure 10.5: 1/8 th of CLIC\_DDS\_E cell](image)

To improve the prediction of the intermediate cell parameters, nine fiducial cells are modelled (instead of seven cells such as in CLIC\_DDS\_C) to represent the full structure. A comparison between the iris thickness taper in CLIC\_DDS\_C and CLIC\_DDS\_E is presented in Fig.10.6. The average iris thickness in this case is increased by \(\sim 12.8\)% to optimise various rf parameters discussed in the last section. The geometry of CLIC\_DDS\_E has changed significantly compared to that of CLIC\_DDS\_C which has resulted in reducing the monopole surface fields by \(\sim 25\)%.

Simulations of the dipole modes on single cells have revealed that the change in the geometry has re-distributed the dipole fields as well. The coupling of these fields to the manifold is reduced. The average manifold penetration \(<R_c>\) need to be reduced from 6.85 mm to 6.5 mm to maintain sufficient dipole coupling. Increasing \(<R_c>\) will reduce the coupling further and hence, the wakefield

\(^3\)For dipole mode simulations a quarter symmetry cell is used.
suppression will be inadequate. A comparison of $R_c$ in both CLIC-DDS structures is presented in Fig. 10.6. Detailed design parameters of CLIC-DDS_E structure are presented in Table F.2.

10.3 Fundamental Mode RF Properties Of CLIC_DDSE

After completing the design-optimisation of CLIC-DDS_E, the fundamental mode rf properties of the structure are studied. The fundamental mode rf properties of nine cells of CLIC-DDS_E are presented in Table 10.1. The change in the iris thickness has reduced the average group velocity by $\sim 10\%$. Other rf parameters such as $Q$ and $R'/Q$ are almost constant, as these parameters are more sensitive to the iris radius. The change in the nature of the rf parameters with the synchronous frequency (along the structure) is illustrated in Fig. 10.7.
<table>
<thead>
<tr>
<th>Cell Label</th>
<th>$v_g$</th>
<th>$Q$</th>
<th>$R'/Q$</th>
<th>$E_{\text{sur}}/E_{\text{acc}}$</th>
<th>$H_{\text{sur}}/E_{\text{acc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.07</td>
<td>4921</td>
<td>10.208</td>
<td>2.26</td>
<td>4.9</td>
</tr>
<tr>
<td>B</td>
<td>1.65</td>
<td>5177</td>
<td>11.476</td>
<td>2.08</td>
<td>4.5</td>
</tr>
<tr>
<td>C</td>
<td>1.56</td>
<td>5356</td>
<td>12.208</td>
<td>2.0</td>
<td>4.1</td>
</tr>
<tr>
<td>D</td>
<td>1.46</td>
<td>5597</td>
<td>13.288</td>
<td>1.85</td>
<td>3.8</td>
</tr>
<tr>
<td>E</td>
<td>1.39</td>
<td>5819</td>
<td>14.274</td>
<td>1.78</td>
<td>3.32</td>
</tr>
<tr>
<td>F</td>
<td>1.31</td>
<td>6028</td>
<td>15.291</td>
<td>1.71</td>
<td>3.13</td>
</tr>
<tr>
<td>G</td>
<td>1.19</td>
<td>6251</td>
<td>16.642</td>
<td>1.65</td>
<td>2.91</td>
</tr>
<tr>
<td>H</td>
<td>1.03</td>
<td>6409</td>
<td>17.797</td>
<td>1.68</td>
<td>2.75</td>
</tr>
<tr>
<td>I</td>
<td>0.77</td>
<td>6621</td>
<td>19.082</td>
<td>1.7</td>
<td>2.61</td>
</tr>
</tbody>
</table>
Figure 10.7: RF parameters of nine fiducial cells of CLIC_DDS_E versus synchronous frequency.
A comparison between the normalised surface E-field of both the CLIC-DDS structures is presented in Fig. 10.8. It can be observed that the surface E-field in both structures is nearly similar up to 17.5 GHz, and from thereon CLIC-DDS-C structure experiences a sharp rise due to its small iris thickness. A major difference between the normalised H-field of CLIC-DDS-C and CLIC-DDS-E is observed throughout the structure. A comparison of the H-field in both the structures is presented in Fig. 10.8. As discussed in this section, the group velocity in CLIC-DDS-E structure is reduced by ~10% however, the $Q$ and $R'/Q$ are similar to CLIC-DDS-C. Hence, the average input power requirement of the 8-fold interleaved structure is reduced by ~4.7% (Fig. 10.4). The input power requirement of CLIC-DDS-E for the beam current of CLIC-DDS-C is presented in Fig. 10.9. The decrease in the input power requirement naturally increases the efficiency of acceleration. In this case the rf-to-beam efficiency has increased by ~3.6% (Fig. 10.4).
Figure 10.9: Peak input power and rf-to-beam efficiency of CLIC_DDS_E.

The rf-to-beam efficiency of CLIC_DDS_E is presented in Fig. 10.9. The maximum unloaded gradient and normalised E-field in CLIC_DDS_C are 137.3 MV/m and 2.33 respectively. Hence, the surface E-field is 320 MV/m. In CLIC_DDS_E, the maximum unloaded gradient is 147.5 MV/m and the maximum normalised E-field is 1.7. Thus the surface E-field is 251 MV/m. The unloaded E-fields in the 8 interleaved structures are illustrated in Fig. 10.10. The modification in the cavity walls has reduced the pulsed temperature rise to 52°K (from 72°K). Though the maximum H-field occurs in the first cell, the maximum pulsed temperature rise is in the second cell. This is due to the fact that the unloaded accelerating gradient in the second cell is higher than that of the first cell due to the nature of the erf distribution. The unloaded condition pulsed temperature rise in 8 interleaved structures is illustrated in Fig. 10.11. The rf properties of the first and last structure are presented in Fig. 10.12 and Fig. 10.13 respectively.
Figure 10.10: Surface E-field in CLIC_DDS_E structures.

\[ E_{\text{sur}} = 251 \text{ MV/m} \]

Figure 10.11: Pulsed temperature rise in CLIC_DDS_E structures.

\[ \Delta T_{\text{max}} = 52 \, ^\circ\text{K} \]
Figure 10.12: RF structure properties of the first out of 8 structures of CLIC_DDS_E.
Figure 10.13: RF structure properties of the last structure of CLIC_DDS_E.
10.4 Wakefield Suppression in CLIC_DDS_E

Changing the cavity shape has reduced the surface fields. Equally important is the effect on the dipole mode properties. Details of the circuit parameters are presented in Fig. F.1(a) and Fig. F.1(b). The dipole mode coupling $\Gamma_x$ has been reduced by 25%. Increasing the coupling slot width will recover the lost coupling, however, will also increase the pulse temperature rise. On the other hand, increasing the slot penetration into the cavity is limited by the mechanical design considerations. In this manner the design is constrained.

The spectral function of the 8-fold interleaved CLIC_DDS_E structure is presented in Fig. 10.14. As it is expected, due to a reduced coupling, the width of the spikes in the spectral function (represents the quality factor of the modes) is reduced. The structure is poorly coupled to the manifolds, compared to CLIC_DDS_C. The wakefield excited in the structure is illustrated in Fig. 10.15 [80]. It can be observed that the wakefield amplitude in this case is larger compared to the CLIC_DDS_C structure. However, it is still within the acceptable limits defined by beam dynamics [18].
Figure 10.14: Spectral function of the 8-fold interleaved CLIC_DDS_E.

Figure 10.15: Envelope of wakefield in 8-fold interleaved CLIC_DDS_E.
10.5 Mechanical constraints On Building CLIC_DDS_E

The edges and corners of CLIC_DDS_E are required to be rounded\(^4\) to prevent the field enhancements. In this case, the radius of curvature is 0.5 mm which is limited by the radius of the tool used to fabricate the cavity [95]. This forces \(R_c\) to be increased, and hence the coupling of the dipole modes is reduced. This modified structure is referred to as CLIC_DDS_ER (\(R = \text{Rounded}\)). The spectral function of CLIC_DDS_ER is illustrated in Fig. 10.16. The decrease in the coupling is visible in the lower frequency end of the spectral function. The envelope wakefield of CLIC_DDS_ER is presented in Fig. 10.17. The wakefield suppression in this case has been degraded compared to CLIC_DDS_E. This structure is subjected to slightly tighter machining tolerances.

![Figure 10.16: Spectral function of the 8-fold interleaved CLIC_DDS_ER.](image)

\(^4\)The structures discussed, namely CLIC_DDS_C and CLIC_DDS_E do not have realistic roundings on the manifold edges.
10.6 Conclusion

Modifying the geometry of CLIC_DDS_C has two main implications on its rf properties. Firstly, the surface e.m. fields has been substantially reduced. Secondly, the wakefield suppression has slightly diminished. In order to validate high power performance of this design a single structure will be high power tested. This is described in the next chapter.
Chapter 11

Test Structure: CLIC_DDS_A

The CLIC_DDS_A [80] corresponds to the first structure out of the 8-interleaved structures discussed in Chapter 10. A single structure will be tested at high power of 71 MW in order to ascertain the ability of the structure to sustain high surface e.m. fields. This accelerator does not have optimised wakefield coupling. In the interests of mechanical simplicity, the manifold parameters are kept fixed along the length of the structure. This has resulted in radial distance of manifold tip $R_c$ of 6.8 mm, manifold radius $R_m$ of 2.1 mm and manifold centre $m_c$ at 14.45 mm from the electrical centre of the cavity. A manifold designed with these dimensions ensures the cut-off frequency of the manifold ~0.5 GHz above the accelerating mode in each cell. The following two sections describe rf properties and wakefield suppression in CLIC_DDS_A structure. The penultimate section describes mechanical details of the structure. In the last section, the present status of CLIC_DDS_A and upcoming tests are discussed.


11.1 Design Parameters: CLIC_DDS_A

The first structure is focussed on as it requires the largest input power and experiences the largest pulsed temperature rise, compared to the other interleaved structures. It allows the worst case scenario to be studied. It is a 24 cell structure with a taper in iris radius from 4.0 mm to 2.5 mm. The iris thickness varies from 4.0 mm to 1.47 mm. Detailed design parameters of each of the 24 cells are illustrated in Fig. 11.1(a). The maximum differences in the iris radii, cavity radii and iris thicknesses are 150 \( \mu \text{m} \), 80 \( \mu \text{m} \) and 177 \( \mu \text{m} \) respectively. A comparison of various design parameter differences is illustrated in Fig. 11.1(b).

![Design Parameters for CLIC_DDS_A](image)

(a) Cell parameters
(b) Cell-to-cell parameter difference

Figure 11.1: Design parameters for CLIC_DDS_A
11.2 Fundamental Mode RF Parameters: CLIC_DDS_A

It is interesting to note the properties of this structure with respect to the surface e.m. fields. For example, the maximum surface H-field is located in the first cell and this is a representation of the fact that this is the largest aperture in the structure. The opposite is true for the surface E-field which is maximum in the last cell. The net cross product of these fields, corresponding to the modified Poynting vector, is maximum in the first cell. The maximum surface e.m. fields in single cells are illustrated in Fig. 11.2 [80]. The H-field is minimised by varying the cavity parameters and this results in a reduced pulsed temperature rise of $40^\circ$ K. However, a further $\sim20\%$ enhancement in pulsed temperature rise is observed in the vicinity of the coupling slots. Thus, there is a hot spot in the cavity in the vicinity of these slots and it is $\sim51^\circ$ K. The slot width is optimised to keep the pulsed temperature rise within the acceptable limits. The pulsed temperature rise in this location can further be reduced by $\sim5\%$ by reducing the slot width at the cost of dipole coupling.
Figure 11.2: CLIC_DDS_A: Maxima of the fields in eighth symmetry single cells
The rf properties of each cell are illustrated in Fig. 11.3. The erf function tapering of the iris radii is clearly visible in the rf properties such as group velocity and $R'/Q$. Also, the variation of $Q$ and normalised (E and H) fields is almost linear along the structure. $S_c$ reduces up to cell 21 and flattens out thereafter. In order to achieve an average accelerating gradient of 100 MV/m, for a bunch populated with $4.2 \times 10^9$ charged particles, 71 MW of rf power must be supplied to the
structure. The inter-bunch spacing in a train of 312 bunches is 8 rf cycles (0.67 ns). The rf properties of the structure are illustrated in Fig. 11.4. In the absence of beam, the maximum accelerating gradient peaks up to 132 MV/m in the last cell. In the presence of beam, the maximum accelerating voltage is 105 MV/m. This maxima occurs in cell 6 (0.05 m along the structure length), this functional behaviour is governed by the erf distribution. The maximum surface E-field in the structure is 220 MV/m and the maximum pulsed temperature rise is 51° K. The modified Poynting vector peaks at 6.75 W/µm² in cell 2. Complete details of the rf parameters are presented in Table 11.1.
Figure 11.4: RF properties of CLIC_DDS_A. The lower and upper solid black lines indicate the allowable pulsed temperature rise and surface E-field respectively. The middle solid black line indicates the average loaded accelerating gradient.
Table 11.1: CLICDDS_A

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units</th>
<th>CLICDDS_A</th>
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<tbody>
<tr>
<td>$&lt;a&gt;/\lambda$</td>
<td>–</td>
<td>0.13</td>
</tr>
<tr>
<td>First, last iris radius ($a$)</td>
<td>mm</td>
<td>4.0, 2.5</td>
</tr>
<tr>
<td>First, last iris thickness ($t$)</td>
<td>mm</td>
<td>4.0, 1.47</td>
</tr>
<tr>
<td>First, last $Q$</td>
<td>–</td>
<td>5020, 6534</td>
</tr>
<tr>
<td>First, last cell group velocity ($v_g/c$)</td>
<td>%</td>
<td>2.07, 1.0</td>
</tr>
<tr>
<td>First, last cell shunt impedance ($R'$)</td>
<td>MΩ/m</td>
<td>51.1, 118.46</td>
</tr>
<tr>
<td>Filling ($t_f$), rise ($t_r$) time</td>
<td>ns</td>
<td>45.4, 23</td>
</tr>
<tr>
<td>Pulse length ($t_p$, $t'_p$, $t''_p$)</td>
<td>ns</td>
<td>276.5, 251, 217.2</td>
</tr>
<tr>
<td>Peak input power ($P_{in}$)</td>
<td>MW</td>
<td>70.8</td>
</tr>
<tr>
<td>No. of bunches ($N_b$)</td>
<td>–</td>
<td>312</td>
</tr>
<tr>
<td>Bunch population ($n_b$)</td>
<td>$10^9$</td>
<td>4.2</td>
</tr>
<tr>
<td>$E_{\text{max, acc}}$: Loaded/Unloaded</td>
<td>MV/m</td>
<td>105, 132</td>
</tr>
<tr>
<td>$E_{\text{sur}}^\text{max}$</td>
<td>MV/m</td>
<td>220</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>K</td>
<td>51</td>
</tr>
<tr>
<td>$S_c$</td>
<td>W/μm²</td>
<td>6.75</td>
</tr>
<tr>
<td>$P_{int}^{1/3}/C_{in}$</td>
<td>MW ns$^{1/3}$/mm</td>
<td>16.93</td>
</tr>
<tr>
<td>$\mathcal{L}_{bx}$</td>
<td>m$^{-2}$</td>
<td>$1.36 \times 10^{34}$</td>
</tr>
<tr>
<td>RF-to-beam efficiency ($\eta_{r-f-beam}$)</td>
<td>%</td>
<td>23.5</td>
</tr>
<tr>
<td>$F$</td>
<td>arb. units</td>
<td>7.6</td>
</tr>
<tr>
<td>Dipole bandwidth $\Delta f$</td>
<td>GHz</td>
<td>2.0</td>
</tr>
<tr>
<td>Standard deviation of Gaussian distribution $\sigma$</td>
<td>–</td>
<td>$\Delta f / 3.48$</td>
</tr>
<tr>
<td>Detuning spread $\Delta f / f_c$</td>
<td>%</td>
<td>11.8</td>
</tr>
</tbody>
</table>
11.3 Wakefield Suppression: CLIC_DDS_A

The lowest dipole mode bandwidth is $\sim 2$ GHz and the corresponding average dipole mode kick factor is $70 \, \text{V/pC/mm/m}$. The spectral function for CLIC_DDS_A is illustrated in Fig. 11.5 [80]. The dipole $Q$s are calculated using Lorentzian fits

![Figure 11.5: Spectral function of CLIC_DDS_A.](image)

(Chapter 5) and are presented in Fig. 11.6 [80]. The mean $Q$ of the modes is $\sim 1650$. The minimum separation of the modes in this structure is 62.3 MHz, leading to a rapid wake recoherence after 16 ns. The wakefield is displayed in Fig. 11.7 [80]. It is poorly suppressed as a direct consequence of the insufficient sampling of the dipole distribution. A fully interleaved version of CLIC_DDS_A will properly suppress the wakefields and this corresponds to CLIC_DDS_ER. Furthermore, there is no provision in CLIC_DDS_A to couple out HOM fields from the manifolds to dielectric damping material. However, a structure, equipped with HOM couplers, CLIC_DDS_B, is presently being designed.

175
Figure 11.6: Lowest dipole $Q$ of CLIC_DDS_A.

Figure 11.7: Envelope of wakefield of CLIC_DDS_A.
11.4 Mechanical Details: CLIC_DDS_A

Power is supplied to CLIC_DDS_A by means of a mode launcher (ML) [96]. In order to achieve a minimum reflection of power, matching cells are designed to each end of the structure [97] - [99]. By suitably modifying the matching cell parameters, namely iris radii, cavity radii and cavity length allows a good match to be achieved. A detailed sketch of the complete structure is presented in Fig. 11.8 and Fig. 11.9. In the design process, we focus on single cells (with half irises at the either sides of the accelerating gap). However, in fabricating the structure, discs having a full iris at one end and full gap at the other are designed from the perspective of mechanical convenience. A CAD drawing\(^1\) of the first regular disc of CLIC_DDS_A is presented in Fig. 11.10, together with a cross-section view in Fig. 11.11. The full 24 cell structure with ML and matching cells is presented in Fig. 11.12.

\(^1\)Courtesy of V. Soldatov, CERN, Geneva.
Figure 11.8: CLIC_DDS_A: Lower energy (upstream) end of the structure
Figure 11.9: CLIC_DDS_A: Upper energy (downstream) end of the structure
Figure 11.10: CLIC_DDS_A disc
Figure 11.11: CLIC_DDS_A disc: Disc cross-section
Figure 11.12: CLIC_DDS_A: Full 24 cell structure with two matching cells
11.5 Discussion

The rf and mechanical designs of the structure have been completed [99]. Single cells will be tested with a view of ascertaining them monopole and dipole properties and compare to their designed values. A complete bonded structure is expected to be available by the end of 2012. High gradient studies will be made in this structure and it will also be available for studying the dipole wakefield suppression properties. Enhancing the wakefield suppression can be achieved by interleaving and by allowing the manifold to be in closer proximity to the cells.
Chapter 12

Conclusion

12.1 Summary

In order to achieve a centre of mass energy of 3 TeV for CLIC, more than 140,000 main linac structures are required [47]. Hence, it is very important to operate these structures at the nominal parameters within tolerable breakdown limits. It is also important to minimise the production cost for fabricating these structures.

The present baseline design for CLIC, i.e. CLIC_G, suppresses the long-range wakefields induced by the lowest dipole mode using strong damping ($Q \sim 10$). The relatively small irises optimise the shunt impedance in this structure and hence reduce the input power requirement to achieve the desired beam loaded accelerating gradient. This leads to an rf-to-beam efficiency of the structure to $\sim 28\%$. The luminosity per bunch crossing in this case is $1.22 \times 10^{34}$ m$^{-2}$ and the overall FOM (discussed in Chapter 6.) is 9.1 (arb units) [18], [20]. However, the strong damping scheme, in which the dielectric damping material is in relatively close proximity of the accelerating beam, has the potential to give rise to rf breakdown. Experimental results performed on this structure will reveal whether the breakdown rates are within the tolerable limits. The dielectric damping material
can be placed away from the cavity. However, in this case, the length of the damping waveguide will be excessively long. This will add to the fabrication cost and it will also affect the compactness of the structure.

The other damping scheme that is under consideration for CLIC is choke mode damping. Choke mode cavities are easy to fabricate especially for mass production and significant dipole damping can be achieved in this method. However, achieving a comparable $R'/Q$ in these cavities is limited by the width of the choke slot as discussed in Chapter 4 [92]. A low $R'/Q$ will require more input power to achieve the desired accelerating gradient. This will also increase the surface fields and will decrease the rf-to-beam efficiency of the structure. The $R'/Q$ in these cavities can be improved by decreasing the slot width. However, this will incur a concentration of surface field in the vicinity of the slot which may lead to enhanced electrical breakdown [92].

The third damping scheme that has been studied for CLIC, which is the main focus of this thesis, is the DDS method. The DDS cavities have been studied and tested at high gradients operating at X-band frequency for more than a decade for the NLC. The technical knowledge gained is documented and readily available. The research performed on CLIC-DDS builds upon the previous work and meets the even more challenging demands presented by CLIC. As the DDS method relies on a moderate damping scheme and remote location of the dielectric damping material, it minimises the breakdown probability. The manifolds, running parallel to the accelerating beam on the cavity wall, can be utilised for mounting beam position monitors (BPMs). The compactness of the structure, is a mechanical as well as cost-effective advantage.

A comparison of the above mentioned damping methods, namely a waveguide damping structure (WDS), choke mode damping structure (CDS) and DDS
by considering their breakdown rates\textsuperscript{1} at high gradient, structure size (disc diameter), and tolerances to build these structures is presented in Fig. 12.1. It can be observed that the disc size in DDS is relatively smaller and the structures of this nature have been operated with accelerating gradient (unloaded) up to 65 - 70 MV/m [15], [35], [52] - [59] for the NLC. Though the tolerances in DDS are a little tighter, achieving tolerances up to a few micrometers is achievable in practice. Mass production techniques will also meet these tolerance requirements. Experimental results gained from CLIC\_DDS\_A will provide further evidence on the

\textsuperscript{1}Expected and based on previous results
predicted performance at high gradients (up to 130 MV/m\(^2\)). Similarly, experimental results of WDS and CDS will also provide further data of their performance at high gradient.

The present CLIC_DDS_A design results from a relaxed bunch spacing (8 rather 6 rf cycles) and has reduced surface e.m. fields. Furthermore, a fully interleaved structure suppresses the wakefield beyond the beam dynamics requirements. This design is the result of two initial designs, subsequently rejected as a result of unacceptably high surface fields and potentially severe tolerances. A

![Diagram of CLIC_DDS optimisation considering various parameters](image)

**Figure 12.2:** CLIC_DDS optimisation considering various parameters

\(\Delta f_{\text{syn}}(\text{GHz})\) vs. \(\langle a \rangle/\lambda\)

2The average accelerating gradient is 100 MV/m; however, the peak unloaded gradient is \sim 130 MV/m.
strong contender to meet the stringent surface field requirements is a moderate bandwidth structure equipped with modified cavity wall and a relaxed inter bunch spacing of 8 rf cycles (0.67 ns). A structure incorporating these changes, satisfies both the constraints, and has similar tolerances to the CLIC_G baseline design. The rf-to-beam efficiency in this case is $\sim 23\%$. This is approximately 5\% lower than the CLIC_G structure. On the other hand, the luminosity per bunch crossing is $1.36 \times 10^{34}$ m$^{-2}$. This is $\sim 10\%$ higher than the CLIC_G. The overall FOM is 7.6, which is $\sim 16.5\%$ smaller compared to CLIC_G. Increasing the average aperture radius (by $\sim 18\%$ compared to CLIC_G$^3$) has benefited the increased bunch population and hence luminosity per bunch crossing. This also decreases the shunt impedance and hence the efficiency of acceleration. Means to improve the efficiency are being investigated and include modifying the monopole phase advance to $5\pi/6$ per cell from the existing $2\pi/3$. A schematic summary of the choices available is illustrated in Fig. 12.2. In this figure the blue dashed curve represents failure of the circular cells to meet the rf breakdown criteria due to excessive pulsed temperature rise as discussed in Chapter 9.

At present CLIC_DDS_A is under fabrication. The mechanical design of the structure has been completed. Detailed drawings of the cells and the complete assembly of the structure are presented in Appendix G$^4$. The qualification cells of CLIC_DDS_A have been fabricated and are displayed in Fig. 12.3. The structure will be tested for high gradients at an input power of 71 MW at CERN. Further prospect for enhancing the wakefield suppression and means to minimise the surface fields are discussed in the following section.

$^3$In Chapter 9, average aperture of CLIC_DDS_C is compared with CLIC_G

$^4$Courtesy of V. Soldatov and G. Riddone, CERN, Geneva.
12.2 Future of CLIC-DDS

Several approaches are being considered to further optimise the structure. These approaches have the potential to enhance the beam luminosity and efficiency. Three potential aspects of this are: a high phase advance structure (HPA), insertion of additional manifolds and insertion of dielectric damping rods into the manifolds.

The phase advance per cell for CLIC is $2\pi/3$. An HPA structure such as that designed for the NLC [35] operating at a $5\pi/6$ phase advance is being studied [100] to analyse the rf properties and wakefield suppression in HPA structure. An initial study of the CLIC_HPA [100] has shown that the wakefield damping in HPA structure can be improved compared to the standard ($2\pi/3$) structure i.e. CLIC/DDSA. Preliminary study has already been conducted on the first six dipole modes of the standard and HPA structures. Due to the enhanced coupling and less severe kick factors, the HPA structure has the potential to offer an improved wakefield sup-
pression. Wakefield suppression can further be improved in both the standard and HPA structure by introducing addition manifolds (up to 8 manifolds in total). A detailed study is needed to design additional manifolds to minimise the fundamental mode perturbation. Another approach to further enhance the wakefield suppression is to introduce a dielectric damping material in the manifolds (SiC). In this case a SiC rod will run through the manifolds along the structure [101]. It will reduce the dipole $Q$. The effects of introducing such damping material in the manifold on the fundamental mode properties needs a detail investigation.
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Appendix A

Frequency scaling of cavity parameters

In a circularly cylindrical cavity, the accelerating mode is the TM$_{010}$ mode. The cutoff frequency for this mode is given by [28]

$$\omega_c = \frac{2.405c}{b}. \quad (A.1)$$

Here, $b$ is the cavity radius. The accelerating frequency is also inversely proportional to the cavity radius. The total electromagnetic energy stored in the cavity is calculated from the peak electric field $E_0$ and can be given as [28]

$$U = \frac{\pi \epsilon_0 L b^2}{2} E_0^2 J_1^2(2.405). \quad (A.2)$$

Here $L$ is the length (period) of the cavity, $J_1$ is the Bessel’s function.

The average power dissipated on the walls of the cavity of surface area $ds$ is given by [28]

$$P_{dis} = \frac{R_s}{2} \int_s H^2 ds = \frac{\pi b R_s E_0^2 \epsilon_0}{\mu_0} J_1^2(2.405)L + b. \quad (A.3)$$

Here $R_s$ is the surface resistance.

201
The quality factor of the cavity is expressed in terms of the frequency, energy stored in the cavity and the power dissipated in the cavity walls [28].

\[ Q = \frac{\omega U}{P_{\text{dis}}} \tag{A.4} \]

The rf surface resistance in a normal conducting cavity is given by [28]

\[ R_s = \sqrt{\frac{\mu_0 \omega}{2\sigma}} \tag{A.5} \]
where \( \sigma \) is the conductivity of the cavity walls (copper in this case).

The rf surface resistance of superconducting niobium is approximately given by [28]

\[ R_s = 9 \times 10^{-5} \frac{\omega^2 (\text{GHz})}{T(K)} \exp\left(-\frac{T_c}{T}\right) + R_{\text{res}} \tag{A.6} \]
where \( \alpha = 1.92 \) and \( T_c = 9.2 \degree K \) is the critical temperature. Imperfections in the surface are calculated in terms of the residual resistance, \( R_{\text{res}} \) and for superconducting cavities is typically of the order of \( 10^{-14} \Omega \) to \( 10^{-13} \Omega \).

The shunt impedance per unit length is a measure of excellence of a structure as an accelerator and is defined as follows [28].

\[ R' = \frac{E_0^2}{P_{\text{dis}} L}. \tag{A.7} \]

From the above eq. A.2 to A.7 the frequency dependence of the rf parameters in normal and superconducting cavities can be conclude as

**RF Surface resistance**

\[ R_s \propto \begin{cases} \sqrt{\omega} & \text{NC} \\ \omega^2 & \text{SC.} \end{cases} \tag{A.8} \]

**Power dissipated in the cavity walls**

\[ P_{\text{dis}} \propto \begin{cases} 1/\sqrt{\omega} & \text{NC} \\ \omega & \text{SC.} \end{cases} \tag{A.9} \]
Quality factor of the cavity

\[ Q = \frac{\omega U}{P_{dis}} \propto \begin{cases} 
1/\sqrt{\omega} & \text{NC} \\
1/\omega^2 & \text{SC.}
\end{cases} \quad (A.10) \]

Shunt impedance per unit length

\[ R' \propto 1/P_{dis} \propto \begin{cases} 
\sqrt{\omega} & \text{NC} \\
1/\omega & \text{SC.}
\end{cases} \quad (A.11) \]

Ratio of shunt impedance per unit length to quality factor

\[ R'/Q \propto \begin{cases} 
\omega & \text{NC} \\
\omega & \text{SC.}
\end{cases} \quad (A.12) \]
Appendix B

Wake Potential

In the frequency domain, the Fourier components of the voltage $V(\omega)$ and current $I(\omega)$ allow a beam impedance to be defined as [38].

$$Z(\omega) = \frac{V(\omega)}{I(\omega)}.$$  \hspace{1cm} (B.1)

If a unit point-like charge (at $v \approx c$) passes through a cavity represented by this impedance followed by a test point-like charge, then the potential experienced by the test charge at a distance $ct$ behind the leading particle is defined as wake potential $W_t$. It is also known as delta function wake potential or Green’s function. The wake potential, for a current rather than a point charge, induced $dV(t)$ at a position $ct$ can be evaluated by considering an element of charge (driving charge) $dq$ ahead in the bunch by a distance $c(t - t')$ and is given by [38]

$$dV(t) = W(t - t') dq' = W(t - t') I(t') dq'.$$  \hspace{1cm} (B.2a)

This is the convolution of the wake function with the current distribution. For $v < c$ there is some wakefield present ahead of the driving bunch i.e. the case of the non-causal behaviour where charge to charge interaction (space charge forces)
cannot be neglected [37]. In other case, the total potential due to all the charge elements in the bunch ahead of the distance $ct$ is given by integrating the contribution of the potential at point $t$ due to the charge $dq$ [38].

$$V(t) = \int_{-\infty}^{t} W(t-t')I(t')dt' = \int_{0}^{\infty} W(\tau)I(t-\tau)d\tau. \quad (B.2b)$$

The wake potential can be expressed in the frequency domain using Fourier transform as follows [38]

$$V(\omega) = \tilde{V}(t) = \int_{-\infty}^{\infty} e^{i\omega t}V(t)dt = \int_{-\infty}^{\infty} e^{i\omega t}dt \int_{0}^{\infty} W(\tau)I(t-\tau)d\tau. \quad (B.3)$$

For $t = t' + \tau$, changing the order of integration we get

$$V(\omega) = I(\omega) \tilde{W}(\tau) \quad (B.4)$$

using B.1 and $I(\omega) = \tilde{I}(t)$

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} = \tilde{W}(\tau). \quad (B.5a)$$

The wake function is defined in terms of the impedance using an inverse Fourier transform as follows

$$W(\tau) = \tilde{Z}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\omega \tau} Z(\omega) d\omega. \quad (B.6)$$
Appendix C

Circuit Model

Here, the calculations involving monopole (single band) coupled mode frequencies for a structure consists of N cells are discussed. Eq. 5.8 is an eigen system with $a_m$ eigenstates and $\omega^{-2}$ eigenvalues. In order to solve eq. 5.8 for, say, $m = 1, \ldots, N$, where for example, $N = 3$ we need to consider physical boundary conditions because the electromagnetic fields will terminate at both the ends of the structure. Assuming the structure has full cells at either ends the boundary conditions are imposed as [45],

\begin{align*}
a_0 &= a_1 \\
a_{N+1} &= a_N \quad \text{(C.1a)} \\
\kappa_\frac{1}{2} &= \kappa_1 \\
\kappa_{N+\frac{1}{2}} &= \kappa_N \quad \text{(C.1b)}
\end{align*}

Let the eigenvalue $\omega^{-2} = \lambda$ for $m = 1$ to 3, eq. 5.8 can be written as for $m = 1$

\begin{equation}
\frac{a_1}{\omega_1^2} + a_0 \frac{\kappa_\frac{1}{2}}{2} + a_2 \frac{\kappa_\frac{1}{2}}{2} = \lambda a_1 \quad \text{(C.2a)}
\end{equation}

for $m = 2$

\begin{equation}
\frac{a_2}{\omega_2^2} + a_1 \frac{\kappa_\frac{1}{2}}{2} + a_3 \frac{\kappa_\frac{1}{2}}{2} = \lambda a_2 \quad \text{(C.2b)}
\end{equation}
for m = 3

\[ \frac{a_3}{\omega_3^2} + a_2 \frac{\kappa_2^2}{2} + a_4 \frac{\kappa_4^2}{2} = \lambda a_3. \]  

(C.2c)

Using eq. C.1 in to eq. C.2 the condensed matrix form can be written as

\[
\begin{pmatrix}
\frac{1}{\omega_1^2} + \frac{\kappa_1^2}{2} & 0 & \frac{\kappa_3^2}{2} \\
\frac{\kappa_3^2}{2} & \frac{1}{\omega_2^2} + \frac{\kappa_5^2}{2} & 0 \\
0 & \frac{\kappa_5^2}{2} & \frac{1}{\omega_3^2} + \frac{\kappa_5^2}{2}
\end{pmatrix}
\begin{pmatrix}
a_1 \\
a_2 \\
a_3
\end{pmatrix}
= \lambda
\begin{pmatrix}
a_1 \\
a_2 \\
a_3
\end{pmatrix}
\]  

(C.3)

which can also be expressed as

\[ \Lambda a = \lambda a \]  

(C.4)

where

\[ \Lambda = \begin{pmatrix}
\frac{1}{\omega_1^2} + \frac{\kappa_1^2}{2} & 0 & \frac{\kappa_3^2}{2} \\
\frac{\kappa_3^2}{2} & \frac{1}{\omega_2^2} + \frac{\kappa_5^2}{2} & 0 \\
0 & \frac{\kappa_5^2}{2} & \frac{1}{\omega_3^2} + \frac{\kappa_5^2}{2}
\end{pmatrix} \]  

(C.5)

\[ a = \begin{pmatrix}
a_1 \\
a_2 \\
a_3
\end{pmatrix}. \]  

(C.6)

The resonating frequencies and the coupling coefficients are obtained from single cell simulations using commercially available computational tools. Using eq. C.3 the eigenvalues i.e. coupled mode frequencies for the entire N-cell structure can be calculated.
C.1 Double Chain Model

Following a similar procedure as described in 5.1, for TE modes the coupled modes are represented as [45]:

\[
\left(1 - \frac{1}{\omega^2}\right) a_m + \frac{\kappa_m + 1/2}{2} a_{m+1} + \frac{\kappa_m - 1/2}{2} a_{m-1} = \frac{\sqrt{\kappa_m + 1/2}}{2} \hat{a}_m + \frac{\sqrt{\kappa_m - 1/2}}{2} \hat{a}_{m-1}, \quad \text{(C.7a)}
\]

whereas for TM modes:

\[
\left(1 - \frac{1}{\hat{\omega}^2}\right) \hat{a}_m - \frac{\hat{\kappa}_m + 1/2}{2} \hat{a}_{m+1} - \frac{\hat{\kappa}_m - 1/2}{2} \hat{a}_{m-1} = -\frac{\sqrt{\hat{\kappa}_m + 1/2}}{2} \hat{a}_m + \frac{\sqrt{\hat{\kappa}_m - 1/2}}{2} \hat{a}_{m-1}, \quad \text{(C.7b)}
\]

Considering a special case of a structure with uniform cells (no tapering) subjected to an infinite periodic condition and applying Floquet condition [28], [29] eq. C.7 can be written in matrix form as

\[
\begin{pmatrix}
1 + \eta \cos \varphi \\ \eta \sin \varphi
\end{pmatrix} \begin{pmatrix}
\omega_m^2 \\ \eta \omega_m \omega_0
\end{pmatrix} \begin{pmatrix} a_m \\ \hat{a}_m \end{pmatrix} = \begin{pmatrix}
\omega^2 \\ \eta \omega_0 \omega_0^T
\end{pmatrix} \begin{pmatrix} a \\ \hat{a} \end{pmatrix}
\]

where \( \varphi \) is the phase advance of the fields per cell.

Now returning to the case of a non-uniform tapered structure, to solve for the eigenvalue, the boundary conditions need to be specified. If we assume that the cavity consists of N full cells (\( m = 1 \) to N), with metallic end walls, then the boundary conditions correspond to [45]

\[
a_0 = a_1, \quad \hat{a}_0 = -\hat{a}_1, \quad \kappa_{1/2} = \kappa_1, \quad \hat{\kappa}_{1/2} = \hat{\kappa}_1
\]

\[
a_{N+1} = a_N, \quad \hat{a}_{N+1} = -\hat{a}_N, \quad \kappa_{N+1/2} = \kappa_N, \quad \hat{\kappa}_{N+1/2} = \hat{\kappa}_N
\]

\[
\begin{pmatrix}
H & H_x \\ H_x^T & \hat{H}
\end{pmatrix} \begin{pmatrix} a \\ \hat{a} \end{pmatrix} = \lambda \begin{pmatrix} a \\ \hat{a} \end{pmatrix}
\]

208
where $H(=\Lambda)$ and $a$ are defined in eq. C.5 and C.6 respectively; $H_x^T$ is the transpose of $H_x$ matrix. Now for example, if $N = 3$ then,

$$\hat{H} = \begin{pmatrix}
\frac{1}{\omega_1} - \frac{k_1}{2} & -\frac{k_3}{2} & 0 \\
-\frac{k_2}{2} & \frac{1}{\omega_2} & -\frac{k_5}{2} \\
0 & -\frac{k_4}{2} & \frac{1}{\omega_3} - \frac{k_3}{2}
\end{pmatrix}$$  \hspace{1cm} (C.11)

and

$$\hat{a} = \begin{pmatrix}
\hat{a}_1 \\
\hat{a}_2 \\
\hat{a}_3
\end{pmatrix}.$$  \hspace{1cm} (C.12)

The coupling coefficients in eq. C.11 are negative because the lowest dipole band TM mode is a backward wave i.e. its group velocity$^{1}$ is negative ($\omega_0 > \omega_\pi$).

Defining a cross coupling term $\kappa$ as

$$\kappa_{m\pm 1} = \sqrt{\kappa_{m\pm 1}^2} \frac{\hat{\kappa}_{m\pm 1}}{2}$$  \hspace{1cm} (C.13)

the matrix of the cross coupled terms becomes

$$H_x = \begin{pmatrix}
\kappa_{1} & -\kappa_{\frac{3}{2}} & 0 \\
\kappa_{\frac{3}{2}} & 0 & -\kappa_{\frac{5}{2}} \\
0 & \kappa_{\frac{5}{2}} & -\kappa_{3}
\end{pmatrix}.$$  \hspace{1cm} (C.14)

Eq. C.10 can be written in further compact manner as

$$\tilde{H} \tilde{a} = \lambda \tilde{a}$$  \hspace{1cm} (C.15)

where

$$\tilde{H} = \begin{pmatrix}
H & H_x \\
H_x^T & \hat{H}
\end{pmatrix}$$  \hspace{1cm} (C.16)

$^{1}$The calculation of group velocity using $\omega_0, \omega_\pi$ and $\kappa$ will be discussed in Chapter 6
\[
\tilde{a} = \begin{pmatrix} a \\ \hat{a} \end{pmatrix}.
\]  
(C.17)

The matrix defined in eq. C.15 is a $2N \times 2N$ matrix and is solved for calculating $2N$ coupled modes. The kick factors of the lowest dipole modes are large compared to other HOMs\(^2\). Hence we focus on optimising the wakefield generated due to the lowest dipole mode. The calculation of the wake involves the coupled mode frequencies which are calculated using eq. C.15 and coupled mode kick factors. The coupled mode kick factor for the double-band model is calculated \([45]\) as

\[
K_p = \frac{\sum_n \hat{a}_n^{(p)} \sqrt{\epsilon(n) K_i^{(n)} f_i^{(n)} e^{i\psi_p}}^2}{N f_p} 
\]  
(C.18)

where, $\psi_p$ is phase shift per cell \([45]\) and $f_i = \frac{\omega_i}{2\pi}$ is the mode frequency, and

\[
\psi_p = 2\pi f_p L / c
\]  
(C.19)

where $L$ is the cell length and $\epsilon$ is a fitting parameter and $p$ is the mode nearest the synchronous point \([45]\). This fitting parameters is important because the ratio of TM to TE dipole modes vary along the structure hence it is necessary to weight the ratio accordingly.

\[
\epsilon = 1 + \frac{|a_p|^2}{|\hat{a}_p|^2}. 
\]  
(C.20)

Only $\hat{a}_n$ appears in the numerator of eq. C.18 since only the $TM_{110}$ and not $TE_{111}$ will kick the beam.

**C.2 Double Chain Model With manifold Geometry**

Referring to Fig. 5.5, if $V_n$, $C_n$ are the voltage and capacitance of the manifold representation respectively and $v_n$, $c_n$ are the voltage and capacitance of the TE

\(^2\)This is true for a TW X-band structure with accelerating phase advance of $2\pi/3$ per cell \([46]\).
component of the cell mode respectively then [35], [68]

\[ V_n = -\frac{j}{\omega} \left( \frac{I_n}{C_n} + i_n \frac{\kappa_n}{\sqrt{C_n c_n}} \right) \]  
(C.21)

\[ v_n = -\frac{j}{\omega} \left( \frac{i_n}{c_n} + I_n \frac{\kappa_n}{\sqrt{C_n c_n}} \right) \]  
(C.22)

where \( I_n \) and \( i_n \) are the currents through \( C_n \) and \( c_n \) respectively. The network equations of the circuit model can be expressed in terms of a compact matrix form as [35], [68]

\[
\begin{pmatrix}
\dot{H} & H_x^T \\
H_x & H - GR^{-1}G
\end{pmatrix}
\begin{pmatrix}
\dot{a} \\
\dot{a}
\end{pmatrix}
- f^2
\begin{pmatrix}
\dot{a} \\
0
\end{pmatrix}
= B.
\]  
(C.23)

Various terms involved in the above eq. C.23 are defined as follows [35], [68]

\[ R_{nn} = -2 \cos \psi_n \]  
(C.24a)

\[ R_{nn+1} = 1 \]  
(C.24b)

\[ \cos \psi_n = \cos \psi_{0n} - \alpha \left( \frac{\pi L f}{c} \right)^2 \sin \psi_{0n} \]  
(C.24c)

\[ \alpha = \frac{2C}{LC_T} \]  
(C.24d)

\[ \psi_{0n} = \left( \frac{2\pi L}{c} \right) \sqrt{f^2 - F_{cn}^2} \]  
(C.24e)

\[ H_m = \frac{1}{f_n^2} \]  
(C.24f)

\[ \dot{H}_m = \frac{1}{f_n^2} \]  
(C.24g)

\[ H_{mn+1} = \frac{\eta_{mn+1/2}}{2f_n f_{n+1}} \]  
(C.24h)

\[ \dot{H}_{mn+1} = -\frac{\dot{\eta}_{mn+1/2}}{2f_n f_{n+1}} \]  
(C.24i)

\[ H_{x,nn+1} = \frac{\eta_{x,nn+1/2}}{2f_n f_{n+1}} \]  
(C.24j)
Here, $C_T$ is the capacitance per unit length of the transmission line, $\psi_{0n}$ is the local phase advance per waveguide section, $B$ is a representation of a driving current ($B$ is defined in the next section), the $a$’s and $H$ matrices have already been discussed in the previous section. The manifold is represented by $R$ matrix and the coupling of the manifold to TE cell mode is represented by $G$ matrix in which $\Gamma$ is the manifold to cell coupling coefficient. $F_n$ is the cut-off frequency of the manifold and $L$ is the cell length (periodicity). If we set $\Gamma = 0$, corresponding to no coupling of the manifold to the cell modes then we retain eq. C.10.

### C.3 Spectral Function Method

In order to define a Spectral function over the propagating modes in the manifolds a network equation in terms of a $3N \times 3N$ matrix is defined as [35], [69],[72]

\[
(H - \lambda U) \bar{a} = \lambda \bar{B}
\]

where

\[
\bar{H} = \begin{pmatrix}
\hat{H} & H_\times^T & 0 \\
H_\times & H & -G \\
0 & G & -R
\end{pmatrix}.
\]

Here $H, \hat{H}, H_\times, G$ and $R$ matrices are all $N \times N$ matrices and retains the same form as discussed in eq. C.24 and

\[
\bar{a} = \begin{pmatrix}
\hat{a} \\
a \\
A
\end{pmatrix}.
\]
where $a$ and $\hat{a}$ are the eigenvectors of the cell modes and have been defined in eq. 5.5. $A$ is the eigen vector of the TE mode in the manifold and is defined as [35]

$$A_n = \frac{j2\pi V_n}{\omega} \sqrt{\frac{e}{P}} \left( e^{-j\sin \psi_{0n}} \right).$$

(C.28)

The matrix involved in defining the beam current $B$ is

$$\overline{B} = \begin{pmatrix} B \\ 0 \\ -A \end{pmatrix}$$

(C.29)

where $B$ is normalised to unit charge per unit displacement.

$$B = \sqrt{\left( \frac{2\omega^2}{c} K_s L \right)} \exp \left( -\frac{j\omega}{c} n \right)$$

(C.30)

where $f_s$ and $K_s$ are the single infinitely periodic cell synchronous frequencies and kick factors respectively. The impedance is calculated from the following relation

$$Z(\omega) = \frac{1}{2\pi} \sum_{n,m}^N \sqrt{K_n K_m} \omega_n \omega_m \exp \left[ -\frac{j\omega P}{c} (n - m) \right] \tilde{H}_{nm}$$

(C.31)

where $\tilde{H}$ is a $3N \times 3N$ matrix

$$\tilde{H} = \overline{H}(U - \lambda^{-1}\overline{H})^{-1}.$$

(C.32)

The transverse wakefield experienced by a trailing bunch per unit charge, a distance $s$ behind the driving bunch\(^3\) moving with $v \approx c$ can be written as [35], [69], [72]

$$W_\perp(s) = \frac{1}{2\pi} \int Z(\omega - je) \exp \left[ (js/c)(\omega - je) \right] d\omega$$

(C.33)

where $e$ is an infinitesimal displacement introduced to broaden the resonant peaks, which helps to sample the impedance properly. It can be ignored when we consider sufficient Ohmic losses and manifold damping. It should be noted that the

\(^3\)drive bunch charge is normalised to unit displacement
circuit model calculation introduces a small non-physical quantity, a precursor, which contributes to a small wakefield ahead of the driving bunch i.e. $W_{\perp} \neq 0$ for $s < 0$ which contradicts with the causality. In order to remove this contradiction a causal wake is often defined as follows [35]

$$W_c(s) = 0(s)[W_{\perp}(s) - W_{\perp}(-s)]$$  \hspace{1cm} (C.34)

where $0(s)$ is the unit step function. The envelope of the causal wake is given as [35]

$$W_e(s) = \frac{0(s)}{2\pi} \int_0^{\infty} S(\omega) \exp(j\omega s/c) d\omega.$$  \hspace{1cm} (C.35)
Appendix D

FOM calculation

For a structure with end cell iris radii of $a_1$ and $a_N$, the average and the difference in the aperture size of the structure are defined as [84], [86]

\[
\langle a \rangle = \frac{a_1 + a_N}{2} \quad \text{(D.1a)}
\]

\[
\Delta a = \frac{a_1 - a_N}{2}. \quad \text{(D.1b)}
\]

The maximum number of particles that can be allowed per bunch is decided by the limitations imposed by the short-range wakefield. For a given structure, $\langle a \rangle$, $\Delta a$ and accelerating wavelength ($\lambda$) are the known parameters. For three different $\Delta a/\langle a \rangle$, $n_b$ is calculated such that the short-range wakefield is within the allowable limit by using Fig. D.1 [84], [86]. The first special case (black curve) of $\Delta a/\langle a \rangle = 0$ represents a structure with no tapering i.e. $\Delta a = 0$ (identical irises), and hence it allows maximum number of particles per bunch compared to the other two curves. The second curve (blue curve) represents the case of moderate tapering. The third curve (red curve) with $\Delta a/\langle a \rangle = 0.3$ represents a heavily tapered structure, hence the average iris radius of the structure is relatively smaller, and this restricts the bunch population. From Fig. D.1, the intersection of three curves with the dashed line, representing the designed $\langle a \rangle/\lambda$ for the CLIC_DDS
Figure D.1: Maximum tolerable bunch population for various $\langle a \rangle/\lambda$

The test structure gives the required data for plotting the curve in Fig. D.2. Now, for the designed $\langle a \rangle/\lambda$ and $\Delta a/\langle a \rangle$, $n_b$ is calculated using Fig. D.2. Similarly, the luminosity per bunch crossing is calculated using Fig. D.3 and Fig. D.4 [84], [86].
Figure D.2: Maximum tolerable bunch population for various $\Delta a/\langle a \rangle$

Figure D.3: Luminosity per bunch crossing for various $\langle a \rangle/\lambda$
Figure D.4: Luminosity per bunch crossing for various \( \Delta a/\langle a \rangle \)
## Appendix E

**CLIC_DDS_C parameters**

Table E.1: Design parameters of seven fiducial cells of CLIC_DDS_C

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<th>Cell</th>
<th>Label</th>
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<th>$b$</th>
<th>$t$</th>
<th>$\epsilon$</th>
<th>$R_m$</th>
<th>$R_c$</th>
<th>$f_{\text{syn}}$</th>
<th>$K_{\text{syn}}$</th>
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<td>3.05</td>
<td>1.16</td>
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Table E.2: RF properties of seven fiducial cells

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<th>$R'/Q$</th>
<th>$E_{\text{sur}}^{\text{max}}/E_{\text{acc}}$</th>
<th>$H_{\text{sur}}^{\text{max}}/E_{\text{acc}}$</th>
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<td>–</td>
<td>kΩ/m</td>
<td>–</td>
<td>mA/m</td>
</tr>
<tr>
<td>A</td>
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<td>4708</td>
<td>10.173</td>
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</tr>
<tr>
<td>B</td>
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<td>5390</td>
<td>12.754</td>
<td>1.96</td>
<td>4.86</td>
</tr>
<tr>
<td>C</td>
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<td>5628</td>
<td>13.844</td>
<td>1.86</td>
<td>4.56</td>
</tr>
<tr>
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<td>5809</td>
<td>14.732</td>
<td>1.77</td>
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</tr>
<tr>
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<td>5989</td>
<td>15.725</td>
<td>1.71</td>
<td>4.02</td>
</tr>
<tr>
<td>F</td>
<td>1.33</td>
<td>6214</td>
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</tr>
<tr>
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<td>6602</td>
<td>19.959</td>
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<td>3.37</td>
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</table>
Appendix F

CLIC_DDS_E parameters
Table F.1: H-field distribution in various undamped single cells

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Field distribution</th>
<th>$A/B$</th>
<th>$\epsilon$</th>
<th>$H_{\text{max}}^{\text{sur}}$ (mA/V)</th>
<th>Geometry</th>
<th>Field distribution</th>
<th>$A/B$</th>
<th>$\epsilon$</th>
<th>$H_{\text{max}}^{\text{sur}}$ (mA/V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>![Circle Image]</td>
<td>$1/(\sqrt{2} - 1)$</td>
<td>1</td>
<td>3.64</td>
<td>Concave ellipse</td>
<td>![Concave ellipse Image]</td>
<td>10</td>
<td>4.14</td>
<td>4.17</td>
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<tr>
<td>Rectangle</td>
<td>![Rectangle Image]</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>4.86</td>
<td>Convex Ellipse</td>
<td>![Convex Ellipse Image]</td>
<td>5</td>
<td>2.07</td>
<td>4.54</td>
</tr>
<tr>
<td>Concave Ellipse</td>
<td>![Concave Ellipse Image]</td>
<td>10</td>
<td>-4.14</td>
<td>4.99</td>
<td>Convex Ellipse</td>
<td>![Convex Ellipse Image]</td>
<td>2</td>
<td>0.82</td>
<td>3.75</td>
</tr>
<tr>
<td>Concave Ellipse</td>
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<td>5</td>
<td>-2.07</td>
<td>5.11</td>
<td>Convex Ellipse</td>
<td>![Convex Ellipse Image]</td>
<td>1</td>
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Table F.2: Design parameters of nine fiducial cells of CLIC-DDS_E

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<th>Cell</th>
<th>$a$</th>
<th>$b$</th>
<th>$t$</th>
<th>$\epsilon$</th>
<th>$R_m$</th>
<th>$R_c$</th>
<th>$f_{syn}$</th>
<th>$K_{syn}$</th>
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</thead>
<tbody>
<tr>
<td>Label</td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
<td>–</td>
<td>mm</td>
<td>mm</td>
<td>GHz</td>
<td>V/pC/mm/m</td>
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<td>2.99</td>
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<td>2.89</td>
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<td>16.7517</td>
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<td>2.3</td>
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<td>6.67</td>
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</tr>
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<td>6.8</td>
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Figure F.1: Circuit parameters of seven fiducial cells of DDS_E
Appendix G

Detailed Mechanical Drawings Of CLIC_DDS_A
Figure G.1: CLIC_DDS_A: Regular disc
Figure G.4: CLIC.dds.A: Disc stack
Figure G.5: CLIC_DDS_A: Complete assembly